Analyzing Oil Refinery Investment Decisions: A Game Theoretic Approach

By
Filip Ravinger

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Supervisor: Professor Andrzej Baniak

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Abstract

The objective of this thesis is to build a model of strategic behavior in an oligopolistic market structure, which takes into account key features of the oil refinery industry, such as heterogeneity of the output, large-scale investment with sunk costs and a high degree of uncertainty over future payoffs. In view of recent demand shifts toward lighter refinery products, the question arises whether it is profitable to undertake the upgrade investment that enables to increase product yields of those products. Using game theory tools, multi-product theory and a simple real-options analysis, a Cournot oligopoly model is devised to attempt to answer that question and to assess the investment behavior of refineries. It is argued that as long as some refinery enjoys a sufficient technological advantage, the equilibrium will have only this refinery investing. Further, the model gives insight into how changes in demand anticipations affect the equilibrium outcome of the investment game. Finally, the applicability of the model is illustrated by a case study, focusing on the Hungarian and Romanian oil refining sectors.

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Introduction

Since the emergence of the industrial revolution in the 18th century, the use of fossil fuels has increasingly become a driving force of economic development. Since then, a lot of economic research has been devoted to studying the use and management of non-renewable natural resources. In particular, as petroleum is one of the most valuable resources today, its efficient exploitation and processing has been a major field of interest for a large number of researchers.

More specifically, an integral part of the petroleum value chain is refining, a process that enables to deliver marketable oil output to the end consumers. From an economist's point of view, the oil refinery might be just an ordinary corporate entity, whose objective is to maximize its value, subject to constraints on the supply as well as on the demand side. Then, to examine the refinery's behavior, she might apply some of the traditional theories of the firm developed in economics. However, when doing so, the economist must be aware of certain issues that are specific to the oil refining industry.

First and utmost, the output of a refinery is a composite product, and as such, it is subject to competition on a variety of product markets. Second, the minimum capacity of modern refineries is rather large relative to the demand in a regional market, thus, adjusting the production process to meet the market demand involves large-scale investment, which carries significant sunk costs. Third, entrepreneurship in oil refining is subject to a high degree of uncertainty over future market conditions that can substantially affect the value of refinery projects.

The objective of this thesis is to build a model of strategic behavior in an oligopolistic market structure, which takes into account some key features of the refinery industry, and to apply the model to an analysis of refinery investment decisions. In the short run, refineries operate within given capacities and maximize their profit, given the market conditions (e.g. the price of crude oil and of refined products). In the long run, the refineries may alter their capacities based on their belief of future profitability (e.g. crude price outlook, demand for and supply of refined products). Since there is a significant lag between the investment decisions and their materialization, the decisions can be viewed as strategic commitments that carry considerable risk.

In view of the recent global demand shift from the low-value refined products (e.g. fuel oil) to the high-value ones (e.g. gasoline, diesel), various ways of upgrading the refining process to adapt this shift are fiercely debated. Apart from investing in higher capacities, the refineries may have a possibility to invest in advanced technologies that enable them to extract higher yields of the high-value products. However, both options are costly and subject to uncertainty over potential future rewards. Hence, a proper analysis of the investment motives must be undertaken, which is the main task of this thesis.

Due to a relatively high market share of each firm, the margin behavior of refineries is probably best analyzed in the framework of an oligopoly model. In this thesis, a Cournot oligopoly approach with production quantities as strategic variables is developed. If, however, short-run decisions are made in strategic interaction among refineries, so are long-run decisions. Thus, the investment decision can also be seen as a game. Hence, this thesis aims to propose an oligopoly model, incorporating uncertainty and using game theory tools to assess the investment behavior of the refineries.

The specific features of the refinery industry, the heterogeneity of output and investment under uncertainty, are to be addressed. The former will be treated as a special application of the multi-product oligopoly theory, while the latter can be modeled as stochastic demand with different states of the world: high and low demand situation for gasoline and diesel, the probabilities of which are common knowledge. These particularities together may lead to interesting dynamics in investment behavior, e.g. symmetric and asymmetric equilibria may exist. The model is to give some insight (in this simplified framework), for example, into how the outcomes change if demand anticipations are changed.

Some relevant literature that focuses on modeling the oil refining sector include the following. Manne (1951), one of the first authors to extensively study the refining industry, uses econometric techniques to estimate the cross-elasticities of the supply of the refined products, based on which one can predict their relative prices. Later, he develops these concepts to devise a linear programming model (Manne, 1958) and claims to answer questions of the substitutability of different products. More recently, Pompermayer et al. (2002) propose a spatial oligopoly model, which is close to the one treated in this thesis. Using linear programming techniques, the authors study the behavior of refiners in the Brazilian refinery market. Finally, Adhitya et al. (2006) use simulation techniques to evaluate refinery supply chain policies and investment decisions.

While the literatures on multi-product theory and investment under uncertainty are rich, treatment that would combine the two approaches is less so. Hence, through an application to the refining industry, one of the objectives of this study is to attempt to fill this gap in the literature.

The remainder of the thesis is organized as follows. In Chapter 1, the basic deterministic investment model is presented. Chapter 2 provides an extension by introducing stochastics into the model. In Chapter 3, applicability of the model is illustrated by a real-world case study.

Chapter 1

Modeling Oil Refinery Investment Decisions

The aim of the first chapter is to provide a detailed analysis of the basic deterministic refinery investment model. We start with a brief introduction to multi-product theory and a general description of the refinery industry. The investment model is then constructed in steps, followed a qualitative discussion of the simple case of only two refineries.

1.1 Multi-Product Oligopoly Models

The study of oligopolistic market structures has been in the focus of the economic literature since the publication of the seminal work of Cournot (1838). Oligopoly is an industry form where a small number of firms dominate the market for a single homogeneous product. With only a few producers in the market, it is reasonable to expect that the actions of individual producers affect the overall state of the industry and, in turn, the other producers' performance. Game theory tools have been proved very useful in analyzing such situations. In a setting where each firm pursues its own benefit but must also consider the potential responses of its rivals, the central problem is to find conditions which ensure the equilibrium state of the game. An intuitive notion of an equilibrium, a state where none of the firms can gain by deviating from its current strategy, is captured by what game theorists call the *Nash equilibrium*.

Many models have been proposed to analyze the behavior of firms in an oligopolistic market structure. A distinguishing element of most of them is the way they view the firms' responses to each other and to the market. In the model of the oil refinery oligopoly treated in this paper, we consider the production strategies to induce interaction among refineries, which leads us to applying the model of the Cournot quantity competition. In this model the firms are assumed to compete in quantities, that is, their decision variables are the amounts of output to be produced. The equilibrium concept to be used here is that of a *Cournot* or *Cournot-Nash equilibrium*.

In reality, however, firms often produce more than one product, and thus competition among producers involves interaction on more than one market. This important generalization of the standard oligopoly models has been widely addressed in the literature and is also in the focus of this paper. In a *multiproduct oligopoly* game the firms seek to maximize their profits by choosing quantities of each product. If the supply of and the demand for every product is independent of one another, this problem can be restated in terms of single-product games, so that in every market a standard Cournot game is played. Yet, this is seldom the case. On the supply side, the technology can be such that it involves a joint production process, while on the demand side, the products can be complements or substitutes. Then, more sophisticated methods of analysis are to be called for. Still, an underlying objective of these methods is to find conditions that guarantee the existence of the Cournot equilibrium. Okuguchi and Szidarovszky (1990) provide an extensive treatise on multi-product oligopoly theory. Fortunately, as we will see, the specificities of the refinery sector will allow us to devise a relatively simple multi-product model, which will be under certain circumstances representable also in a single-product form.

1.2 The Main Characteristics of the Refinery Sector

Crude oil is extracted from the ground in a form that is not suitable for market delivery. It contains many impurities and contaminants, which need to be separated through a refining process. This is a complex procedure involving a sequence of chemical processes, such as *distillation, catalysis* and *hydrotreating.* It includes a flow of various intermediate product streams, which yields as a final outcome a range of marketable end products as gasoline, diesel and fuel oil. The complexity of the whole process varies with different chemical properties of the oil input and so does the associated cost. In particular, the core of the refining process lies in the separation of crude-oil ingredients with diverse weight - hydrocarbons and other compounds such as sulfur, nitrogen and oxygen. It is primarily the weight of these ingredients that determines the qualitative properties of the crude (and hence its price), which in turn determine the yields of the refined end products and their qualitative properties (and their price).[1]

Hence, the refinery is viewed as a multi-product firm that uses crude oil as input

[1] For more on the subject of the quality and pricing of crude oil, see Manes (1964).

and produces a set of final products. It is important to realize that given the available technology, the type of crude oil, the operating mode of the refinery and the complexity of the refinery, the yields of the refined products are a result of a particular refining process and so are technology-specific. This means that the production of each product depends on the production of the other products. This special characteristics of the production can be referred to as the *inverse Leontief technology*,[2] where a certain amount of one product cannot be produced without producing certain amounts of other products at the same time.

Moreover, the demands for the refined products need not be independent. Specifically, gasoline and diesel can be viewed as substitutes (although imperfect) by consumers. Also, horizontal or vertical product differentiation by different refiners is likely to be an issue. These points may well complicate the building of our refinery oligopoly model.

An obvious consequence of the described characteristics of the refinery sector is a problem of a potential imbalance between the industry demand and supply. In fact, over years it has been a major challenge for the refineries to match their product slate with what the consumer wants. The demand for some products has been growing in exchange for a decreasing demand for some other ones and, due to the rigid character of the technology, the refinery supply has increasingly got out of balance with demand. Particularly, the preference for lighter products, those more easily refined from lighter oils, has become prevalent. For refiners it is the main challenge to adjust their refining process to be able to meet the varying preferences for different products.

Although a complex task, the adjustment can be done by investing in facilities that enable to chemically reprocess some intermediate or residual products to finally yield lighter products. As a result, the refinery is able to extract more of the lighter, more valuable products and to reduce the yields of heavier, less valuable ones. For example, through the process of *hydrocracking* the refineries can increase their yields of diesel, while *fluid catalytic cracking* enables them to raise gasoline yields. These processes, however, involve costly investment in new cracking plants, so the refinery must carefully analyze the profitability of their investment decisions. Moreover, since the minimum efficiency scale in the refinery industry is rather large relative to the market, investment is of a rather discrete nature.[3] Consequently, this investment is seen as a strategic action and must be analyzed in the context of the oligopolistic structure of the industry.

[2]Leontief technology assumes fixed proportions of input that produce a single output, while here we have a single input that produces fixed proportions of output, hence the term "inverse".

[3]That is, it is hardly feasible for a refinery to invest in a relatively small hydrocracking unit and then to continuously adjust its capacity.

The CERA[4] Report by Kennaby (2003) provides an insight into recent developments in the refinery sector in Europe. According to the report, the demand for European heavy fuel oil has been steadily decreasing over the 1990s and is expected to continue to fall further. On the other hand, the demand for middle distillates (e.g. diesel) has been on the rise and is expected to continue increasing. This shift in the demand profile seems to favor investing in hydrocracking processes, but the cost is a major barrier. Thus, many of the simple refineries have even started to face a choice between investment and closure. For the more complex refineries, the overall trend is to undertake investment in hydrocrackers, but which refinery actually invests and which does not can be viewed as a game. We will analyze this game by building a simple refinery investment model.

1.3 The Basic Refinery Investment Model

Our aim is to devise a multi-product model that would take into account the specific features of the refinery production technology outlined above. The underlying interaction of the players is modeled as a static non-cooperative game. The firms in the refinery industry make production decisions in advance, so they can be viewed as competing in quantities. Hence, the Cournot model appears to be convenient to be applied here.

We consider an elementary oligopoly model of oil refineries as follows. Although we mentioned earlier that crude-oil types vary significantly in their physical properties, here we assume that each refinery uses crude oil as a single homogeneous input. However, the product line is not a single homogeneous good, rather, it consists of several dozen fuels and chemicals. In particular, through a process of *simple distillation* the refineries successively separate lighter products (e.g. liquid petroleum gas, naphtha, gasoline), middle distillates (e.g. jet fuel, diesel, kerosene) and heaviest products (residual fuel oil). These products are differentiated by their qualitative characteristics and desirability, which determine their final value. Moreover, the products are refined in fixed proportions, that is, out of one unit of crude oil input, the refinery can extract a specific share of each output, depending on the current technology and the operating mode. It may then be in the interest of refineries to invest in improved technologies, which increase the yield of the higher-valued products. This is done through *downstream processing*, whereby the heavy feedstock is reprocessed and changed into lighter, more valuable output. Consequently, the question arises whether and to what extent it is profitable for the refineries to undertake this upgrade investment, taking into consideration the oligopolistic structure of the refinery industry and hence the strategic interaction among refineries.

[4]Cambridge Energy Research Associates.

1.3.1 Setup

We start with a general setting of the model. We denote by R the number of refineries operating on a particular market and by Z the number of refined products. Each refinery $r = 1, \ldots, R$ acquires crude oil from the crude market, the amount of which is denoted by x_r. This amount is processed and changed into final products via fixed conversion rates $\rho_{r,z}$, so the quantity of the refined output $z = 1, \ldots, Z$ is

$$q_{r,z} = \rho_{r,z} x_r.$$

The above equality captures the distinguishing feature of the refinery sector, the inverse Leontief technology described earlier. The total crude intake cannot exceed the refining capacity, denoted by K_r: $x_r \leq K_r$. The prices of the refined products p_z are determined by the inverse demand function

$$p_z = p_z\left(\sum_r q_{r,z}\right) = p_z(Q_z), \tag{1.1}$$

so the prices depend on the total amount of output delivered to the market, Q_z. We mentioned earlier that the demands for different products could be mutually dependent, i.e. they could be complements or, more likely in case of refinery products, substitutes. However, this issue is not in the focus of this paper, so in the above formulation we assume that the demands are independent. Rather, we wish to capture the fact that the products differ in the value they deliver to the consumers. This can be done by assuming different price elasticities of the demands.

Now, we can write the total revenue of the refinery as $TR_r = \sum_z q_{r,z} p_z(Q_z)$. Finally, denoting the total costs, being a function of the input and the capacity, by $C_r = C_r(x_r, K_r)$, we obtain the profit function[5]

$$\pi_r = TR_r - C_r = \sum_z \rho_{r,z} x_r p_z(Q_z) - C_r(x_r, K_r), \quad r = 1, \ldots, R. \tag{1.2}$$

The objective of each refinery is to choose the optimal crude oil intake x_r to maximize (1.2) subject to the capacity constraint $x_r \leq K_r$. This maximization problem yields the optimal input as a function of the remaining $R - 1$ firms' inputs. Taken together, these reaction functions then determine the Cournot equilibrium. Our ultimate goal will be to find and describe the equilibrium. Next, we proceed with the formulation of the

[5]Observe that our earlier assumption about crude as a homogeneous input can be slightly relaxed. In fact, having refinery-specific costs, we at least allow for every refinery to use a different type of crude.

investment model in steps.

1.3.2 Fixed Demand

Let us first consider a simple case, where the demand for each product is assumed to be fixed at D_z, so the equilibrium price is treated as given, and we may write $p_z = p_z^*$. This can be interpreted as a case of a perfectly competitive market, where all the firms are price takers. The firms' optimization problem then boils down to

$$\max_{x_r \leq K_r} \quad \pi_r = \sum_z \rho_{r,z} x_r p_z^* - C_r(x_r, K_r)$$
$$\text{s.t. } Q_z \leq D_z.$$

We can see that the solution of this problem gives the optimal crude input for every refinery, which is independent of other refineries' input unless the demand constraint is binding.

1.3.3 The General Case

We now turn to the general case, where the price relates to the quantity produced through the inverse demand function (1.1). We then formulate the firms' problem as

$$\max_{x_r \leq K_r} \quad \pi_r = \sum_z \rho_{r,z} x_r p_z \left(\sum_r \rho_{r,z} x_r \right) - C_r(x_r, K_r). \tag{1.3}$$

Solving this problem gives us the input of refinery r as a function of the remaining $R-1$ refineries' input, the reaction function. So we have a system of R equations with R unknowns, the solution of which yields the Cournot equilibrium.

1.3.4 Upgrade Investment

As discussed in the introductory paragraphs of this section, we would like to find out if it makes sense for the refineries to invest in more advanced technologies that enable to extract higher yields of more valuable products from a given amount of crude oil. We can examine this question by introducing a two-stage game. In the first stage the refineries (simultaneously) choose the level of investment, which affects their conversion rates but also their cost functions. Given these levels, an input-choice game is played in the second stage, as in Section 1.3.2 or as in the general case of Section 1.3.3.

Let us generalize the notation as follows. The conversion rates now depend on investment I_r, so that $\rho_{r,z} = \rho_{r,z}(I_r)$. In particular, after investing I_r, the conversion rate increases for the higher-valued products, while it decreases for the lower-valued products. Additionally, the investment affects the costs, so we write $C_r = C_r(x_r, K_r, I_r)$. Now, the 2^{nd}-stage profit-maximization problem (of Section 1.3.3) has the form

$$\max_{x_r \leq K_r} \quad \pi_r = \sum_z \rho_{r,z}(I_r) x_r p_z \left(\sum_r \rho_{r,z}(I_r) x_r \right) - C_r(x_r, K_r, I_r).$$

We can obtain the reaction function of optimal input, which is dependent not only on other firms' input, but also on the amount of investment by all the firms. We will then have $x_r = x_r^*(\boldsymbol{X_{\neg r}}, \boldsymbol{I})$, where $\boldsymbol{X_{\neg r}} = \{x_i | i \neq r\}$ and $\boldsymbol{I} = \{I_1, \ldots, I_R\}$. But since this holds for every $r = 1, \ldots, R$, we can solve for x_r as a function of $\boldsymbol{I}$ alone: $x_r = x_r^*(\boldsymbol{I})$.

Knowing the optimal choices of input in the 2^{nd} stage, we proceed backwards to find the optimal investment strategies. Specifically, we want to find such I_r that solves

$$\max_{I_r} \quad \pi_r = \sum_z \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}) p_z \left(\sum_r \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}) \right) - C_r \left(x_r^*(\boldsymbol{I}), K_r, I_r \right).$$

The solution of the above problem produces the optimal investment strategy of firm r as a function of the investment of the remaining $R-1$ firms. Solving this system of R equations with R unknowns, we obtain the Cournot equilibrium investment strategies.

In the above formulation, the conversion rates and costs are seen as continuously dependent on investment. However, in the solution of the model we define them for every particular level of investment, so that finding the Cournot equilibrium involves numerical comparison of profits for different investment strategies. This is a realistic restriction, since upgrade investment in the refinery sector is of a discrete nature.

1.4 Solution of the Model

In order to be able to solve the profit-maximization problems formulated in the previous section, we need to impose further assumptions on the function specifications. A standard form of the demand function used in the literature is the linear form, which we also use in this model except for the case of fixed demand. For each product $z = 1, \ldots Z$ we have

$$p_z(Q_z) = a_z - b_z Q_z = a_z - b_z \sum_i \rho_{i,z} x_i, \quad a_z > 0, b_z > 0.$$

As for the cost function, we will use a special logarithmic form:

$$C_r(x_r, K_r) = \alpha_r - \beta_r \log(K_r - x_r), \quad \alpha_r > 0, \beta_r > 0.$$

This function is convenient both intuitively and analytically. It captures the nature of the production process, where it becomes more difficult to produce as the input approaches the capacity constraint. Formally, it is a result of the marginal cost going to infinity with input approaching the capacity:

$$\lim_{x_r \to K_r} \frac{\partial C_r(x_r, K_r)}{\partial x_r} = \lim_{x_r \to K_r} \frac{\beta_r}{K_r - x_r} = \infty.$$

Moreover, this property prevents us from obtaining a corner solution of the maximization problem. Also, the cost function is convex, so the profit is concave, which is a necessary condition for the existence of maximum. An example of this form of a cost and marginal cost function is depicted in Figure 1.1. We may now proceed to providing guidance to

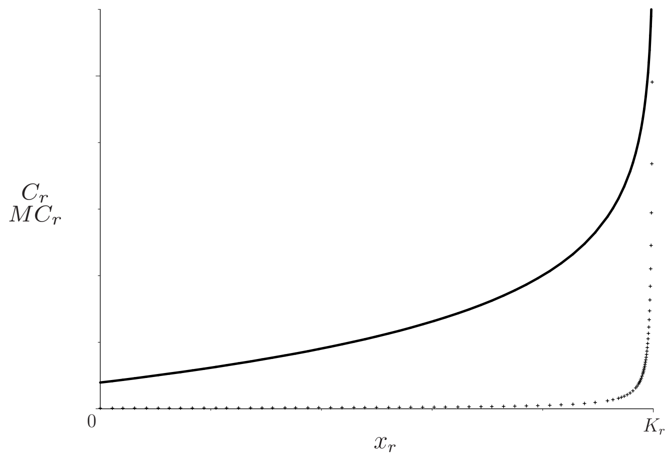

Figure 1.1: The cost (full line) and marginal cost (dotted line) functions

solving the model.

1.4.1 Fixed Demand

Each refinery $r = 1, \ldots R$ chooses $x_r \leq K_r$ to maximize the objective function

$$\pi_r(x_r) = \sum_z \rho_{r,z} x_r p_z^* - C_r(x_r, K_r) = \sum_z \rho_{r,z} x_r p_z^* - \alpha_r + \beta_r \log(K_r - x_r)$$

$$\text{s.t.} \quad \begin{cases} Q_1 \leq D_1, \\ \quad \vdots \\ Q_Z \leq D_Z. \end{cases}$$

Solving for x_r gives us[6]

$$x_r = K_r - \frac{\beta_r}{\sum_z \rho_{r,z} p_z^*} \tag{1.4}$$

Hence, we obtain an interior solution, so that $x_r < K_r$. However, we must assume that the demand for each product is large enough, so the demand constraints are not binding.[7] Thus, we obtain the Cournot equilibrium of optimal crude inputs that are independent among refineries.

1.4.2 The General Case

Every refinery faces the objective function

$$\pi_r(x_r) = \sum_z \rho_{r,z} x_r p_z(Q_z) - \alpha_r + \beta_r \log(K_r - x_r).$$

Solving the standard profit-maximization problem, we obtain the following expression for x_r :

$$x_r = \frac{\left(\sum\limits_z \rho_{r,z} a_z - \sum\limits_z \rho_{r,z} b_z \sum\limits_{i \neq r} \rho_{i,z} x_i + 2K_r \sum\limits_z \rho_{r,z}^2 b_z\right)}{4 \sum\limits_z \rho_{r,z}^2 b_z}$$
$$- \frac{\sqrt{\left(-\sum\limits_z \rho_{r,z} a_z + \sum\limits_z \rho_{r,z} b_z \sum\limits_{i \neq r} \rho_{i,z} x_i - 2K_r \sum\limits_z \rho_{r,z}^2 b_z\right)^2 - 8 \sum\limits_z \rho_{r,z}^2 b_z \left(K_r \sum\limits_z \rho_{r,z} a_z - K_r \sum\limits_z \rho_{r,z} b_z \sum\limits_{i \neq r} \rho_{i,z} x_i - \beta_r\right)}}{4 \sum\limits_z \rho_{r,z}^2 b_z} \tag{1.5}$$

So we have the optimal input of refinery r as a function of the choices of the remaining refineries.

[6] See Appendix for all the derivations.

[7] In fact, accounting for the demand constraints might become a rather complicated issue. Suppose that the total supply determined from (1.4) exceeds the demand for some product. Yet, that does not necessarily mean that the equilibrium involves firms reducing their production. It can still be the case that a loss from the particular product is compensated by a gain from some other product.

Let us consider an example with $R = 2$ refineries. Then we have the best response for refinery 1, given the choice of refinery 2:

$$x_1 = \frac{\left(\sum\limits_{z=1}^{Z} \rho_{1,z} a_z - x_2 \sum\limits_{z=1}^{Z} \rho_{1,z} b_z \rho_{2,z} + 2K_1 \sum\limits_{z=1}^{Z} \rho_{1,z}^2 b_z\right)}{4 \sum\limits_{z=1}^{Z} \rho_{1,z}^2 b_z} - \frac{\sqrt{\left(-\sum\limits_{z=1}^{Z} \rho_{1,z} a_z + x_2 \sum\limits_{z=1}^{Z} \rho_{1,z} b_z \rho_{2,z} - 2K_1 \sum\limits_{z=1}^{Z} \rho_{1,z}^2 b_z\right)^2 - 8 \sum\limits_{z=1}^{Z} \rho_{1,z}^2 b_z \left(K_1 \sum\limits_{z=1}^{Z} \rho_{1,z} a_z - x_2 K_1 \sum\limits_{z=1}^{Z} \rho_{1,z} b_z \rho_{2,z} - \beta_1\right)}}{4 \sum\limits_{z=1}^{Z} \rho_{1,z}^2 b_z}, \tag{1.6}$$

and we obtain an analogous expression for the best choice of x_2 given x_1. The Cournot equilibrium strategies x_1 and x_2 can then be found by numerical solution.

1.4.3 Upgrade Investment

The 2$^{\text{nd}}$-stage objective (in the general case) is to choose $x_r \leq K_r$ to maximize

$$\pi_r(x_r, I_r) = \sum_z \rho_{r,z}(I_r) x_r p_z \left(\sum_r \rho_{r,z}(I_r) x_r\right) - C_r(x_r, K_r, I_r),$$

where I_r indicates the level of investment from stage 1. The conversion rates and the costs are now a function of investment. In particular, we may define

$$C_r(x_r, K_r, I_r) = \alpha_r(I_r) - \beta_r(I_r) \log(K_r - x_r),$$

so both the parameter of the fixed cost, α_r, and the parameter of the marginal cost, β_r, is affected by investment. Moreover, the implicit cost of investment is incorporated in the change in α_r.

Setting $R = 2$ and solving for the best response of refinery 1, given the choice of refinery 2 and stage-1 levels of investment by both refineries, we arrive at

$$x_1 = \frac{\left(\sum\limits_{z=1}^{Z} \rho_{1,z}(I_1) a_z - x_2 \sum\limits_{z=1}^{Z} \rho_{1,z}(I_1) b_z \rho_{2,z}(I_2) + 2K_1 \sum\limits_{z=1}^{Z} \rho_{1,z}^2(I_1) b_z\right)}{4 \sum\limits_{z=1}^{Z} \rho_{1,z}^2(I_1) b_z} - \frac{\sqrt{\begin{array}{l}\left(-\sum\limits_{z=1}^{Z} \rho_{1,z}(I_1) a_z + x_2 \sum\limits_{z=1}^{Z} \rho_{1,z}(I_1) b_z \rho_{2,z}(I_2) - 2K_1 \sum\limits_{z=1}^{Z} \rho_{1,z}^2(I_1) b_z\right)^2 \\ -8 \sum\limits_{z=1}^{Z} \rho_{1,z}^2(I_1) b_z \left(K_1 \sum\limits_{z=1}^{Z} \rho_{1,z}(I_1) a_z - x_2 K_1 \sum\limits_{z=1}^{Z} \rho_{1,z}(I_1) b_z \rho_{2,z}(I_2) - \beta_1(I_1)\right)\end{array}}}{4 \sum\limits_{z=1}^{Z} \rho_{1,z}^2(I_1) b_z}, \tag{1.7}$$

and we obtain an analogous expression for x_2. We can then numerically solve for the optimal input choices as functions of investment levels, so that $x_1 = x_1^*(\boldsymbol{I})$ and $x_2 = x_2^*(\boldsymbol{I})$, where $\boldsymbol{I} = \{I_1, I_2\}$.

Hence, the 1st-stage objective of refinery $r = 1, 2$ is to choose I_r to maximize

$$\pi_r(\boldsymbol{I}) = \sum_z \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}) p_z\Big(\rho_{1,z}(I_1) x_1^*(\boldsymbol{I}) + \rho_{2,z}(I_2) x_2^*(\boldsymbol{I})\Big) - C_r\Big(x_r^*(\boldsymbol{I}), K_r, I_r\Big).$$

Consistent with the discrete nature of the investment, suppose that the variable I_r can attain two discrete values as follows:

$$I_r = \begin{cases} 1 & \text{if refinery } r \text{ does not invest,} \\ 2 & \text{if refinery } r \text{ invests} \end{cases} \tag{1.8}$$

and the conversion rates are defined accordingly, so that $\rho_{r,z}(2) > \rho_{r,z}(1)$ for some set of products, while $\rho_{r,z}(2) \leq \rho_{r,z}(1)$ for the remainder. We can then find the Cournot equilibrium by forming a 2×2 payoff matrix:

I_1 \ I_2	$I_2 = 1$	$I_2 = 2$
$I_1 = 1$	$\pi_1(1,1), \pi_2(1,1)$	$\pi_1(1,2), \pi_2(1,2)$
$I_1 = 2$	$\pi_1(2,1), \pi_2(2,1)$	$\pi_1(2,2), \pi_2(2,2)$

(1.9)

1.5 Extensions of the Basic Model

In this section we analyze two important extensions of the basic model. While the first - the problem of the capacity choice - is straightforward and does not involve significant mathematical complications, the second - the problem of multiple markets - constitutes a serious analytical difficulty, and we will therefore exclude it from further consideration.

1.5.1 Capacity Games

A Simple Capacity Game

In the first stage the refineries simultaneously choose capacities. An input-choice game is then played in the second stage, similar to the one examined in preceding sections. Solving the maximization problem in (1.3) again yields the reaction functions of inputs, and - since we now explicitly treat capacities as endogenous variables - of capacities, so we write $x_r = x_r^*(\boldsymbol{X}_{\neg \boldsymbol{r}}, K_r)$, where $\boldsymbol{X}_{\neg \boldsymbol{r}} = \{x_i | i \neq r\}$. Hence, we have $x_r = x_r^*(\boldsymbol{K})$, where

$\boldsymbol{K} = \{K_1, \ldots, K_R\}$. Returning to the first stage, the objective is to find the capacity to solve

$$\max_{K_r} \quad \pi_r = \sum_z \rho_{r,z} x_r^*(\boldsymbol{K}) p_z \left(\sum_r \rho_{r,z} x_r^*(\boldsymbol{K}) \right) - C_r\Big(x_r^*(\boldsymbol{K}), K_r\Big). \tag{1.10}$$

For the case of two refineries and three products and with the function specifications as before, we obtain the best response as in (1.6). However, now the 1$^{\text{st}}$-stage problem consists of maximizing profits over capacities. It can be seen that after substituting the best response into (1.10), the objective function becomes a trivial function of K_1 and K_2. Thus, the Cournot equilibrium of capacity-choice strategies can be easily found.

Upgrade Investment with Capacity Choice

A straightforward generalization of the upgrade investment game (see Section 1.3.4) is to include the problem of the capacity choice. This can be done by introducing investment in capacity. We assume, as before, that investment I_r affects the conversion rates $\rho_{r,z}$ and the costs. In addition, now investment also affects the capacity, so that $K_r = K_r(I_r)$. Thus, the problem of capacity choice in the previous section is now translated into the problem of investment choice. Combining this with the upgrade investment model of Section 1.3.4, we arrive at the 2$^{\text{nd}}$-stage profit-maximization problem of the form

$$\max_{x_r \le K_r(I_r)} \quad \pi_r = \sum_z \rho_{r,z}(I_r) x_r p_z \left(\sum_r \rho_{r,z}(I_r) x_r \right) - C_r\Big(x_r, K_r(I_r), I_r\Big).$$

and consequently, the 1$^{\text{st}}$-stage problem of the form

$$\max_{I_r} \quad \pi_r = \sum_z \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}) p_z \left(\sum_r \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}) \right) - C_r\Big(x_r^*(\boldsymbol{I}), K_r(I_r), I_r\Big).$$

Suppose that demand is linear and the cost is given by

$$C_r(x_r, K_r, I_r) = \alpha_r(I_r) - \beta_r(I_r) \log\big(K_r(I_r) - x_r\big).$$

For the case of two refineries, we obtain a similar expression to (1.7) for the best response:

$$x_1 = \frac{\left(\sum_{z=1}^{Z}\rho_{1,z}(I_1)a_z - x_2\sum_{z=1}^{Z}\rho_{1,z}(I_1)b_z\rho_{2,z}(I_2) + 2K_1(I_1)\sum_{z=1}^{Z}\rho_{1,z}^2(I_1)b_z\right)}{4\sum_{z=1}^{Z}\rho_{1,z}^2(I_1)b_z} - \frac{\sqrt{\begin{array}{c}\left(-\sum_{z=1}^{Z}\rho_{1,z}(I_1)a_z + x_2\sum_{z=1}^{Z}\rho_{1,z}(I_1)b_z\rho_{2,z}(I_2) - 2K_1(I_1)\sum_{z=1}^{Z}\rho_{1,z}^2(I_1)b_z\right)^2 \\ -8\sum_{z=1}^{Z}\rho_{1,z}^2(I_1)b_z\left(K_1(I_1)\sum_{z=1}^{Z}\rho_{1,z}(I_1)a_z - x_2K_1(I_1)\sum_{z=1}^{Z}\rho_{1,z}(I_1)b_z\rho_{2,z}(I_2) - \beta_1(I_1)\right)\end{array}}}{4\sum_{z=1}^{Z}\rho_{1,z}^2(I_1)b_z}. \tag{1.11}$$

Then, in the simplest case of discrete investment defined by (1.8), the Cournot equilibrium can be found by forming a 2×2 matrix as in (1.9).

1.5.2 Multiple Markets

Up to now, we have implicitly assumed that all the refineries operate on a single geographic market, so that all the produced amount of output is delivered and sold within this market.[8] However, this is a slightly distorted depiction of reality, since most of the refineries operate and compete on more than one geographic market. In particular, interregional and international trade is an important aspect of the competition in the refinery sector. To properly analyze the interaction of the refineries on multiple markets, we would need to introduce a spatial oligopoly model and account for transportation costs. The profit-maximization problem of every refinery would then look similar to the following:

$$\max_{\substack{x_r,\\ q_{r,1}^m,\dots q_{r,Z}^m,\\ m=1,\dots,M}} \pi_r = \sum_{m=1}^{M}\left(\sum_{z=1}^{Z}\left(q_{r,z}^m p_z^m(Q_z^m) - TC_{r,z}^m(q_{r,z}^m)\right)\right) - C_r(x_r,K_r), \quad r = 1,\dots,R,$$

$$\text{s.t.} \sum_{m=1}^{M} q_{r,z}^m = q_{r,z} = \rho_{r,z}x_r, \quad z = 1,\dots,Z,$$

where the upper index m denotes the particular market and $TC_{r,z}^m(q_z^m)$ is the cost of transporting of $q_{r,z}$ amount of product z to market m. Hence, the decision problem of the refinery consists of choosing the total amount of crude oil input, x_r, and (once $q_{r,z} = \rho_{r,z}x_r$ of every product is refined) of choosing the amounts of every product to be delivered to each of the M markets. Clearly, since the spatial oligopoly game involves a choice of $M \times Z + 1$ variables and strategic interaction on M markets, the analysis of the model

[8] This is not to be confused with the markets for different products.

would go well beyond the framework of this paper.[9] Thus, in our basic model, we are forced to stick to the assumption of one geographic market and no external trade. In the application of the model in Chapter 3, we do return to this issue, yet with a few strict assumptions.

1.6 Comparison with a Single-Product Case

The legitimate question arises how the multi-product oligopoly model presented earlier differs from the standard single-product model. We saw that due to the specific technology that is characterized by a fixed relation between input and output, the firm's decision problem reduces to the choice of a single variable, the level of investment in the upgrade investment game, or the amount of crude oil intake in the capacity utilization game. It is then natural to think of an analogy with the single-product oligopoly model. In particular, let the technology be specified by a linear production function

$$q_r = \rho_r x_r.$$

Hence, the firm's output is a single homogeneous good, the price of which is again determined by the inverse demand function

$$p = p\left(\sum_r q_r\right) = p(Q).$$

The objective of firm $r = 1, \ldots, R$ is to maximize

$$\pi_r = \rho_r x_r p(Q) - C_r(x_r, K_r)$$

over input x_r subject to the capacity constraint $x_r \leq K_r$. In the case of fixed demand, the above problem can be written as

$$\max_{x_r \leq K_r} \quad \pi_r(x_r) = \rho_r x_r p^* - C_r(x_r, K_r)$$
$$\text{s.t. } Q \leq D.$$

[9]Pompermayer et al. (2002) analyze a refinery oligopoly model that accounts for transportation costs and uses sophisticated linear programming techniques.

Solving for the familiar linear demand and logarithmic cost specifications gives the optimal input

$$x_r = K_r - \frac{\beta_r}{\rho_r p^*}.$$

In the general case we solve the profit-maximization problem of the form

$$\pi_r(x_r) = \rho_r x_r p(Q) - C_r(x_r, K_r)$$

With two refineries the reaction curve is given by

$$x_1 = \frac{\left(\rho_1 a - x_2\rho_1 b\rho_2 + 2K_1\rho_1^2 b\right)}{4\rho_1^2 b} - \frac{\sqrt{\left(-\rho_1 a + x_2\rho_1 b\rho_2 - 2K_1\rho_1^2 b\right)^2 - 8\rho_1^2 b(K_1\rho_1 a - x_2 K_1\rho_1 b\rho_2 - \beta_1)}}{4\rho_1^2 b}.$$

We can similarly proceed with the upgrade investment game. Summing up, it is evident that the multi-product oligopoly model treated in the previous sections is a direct and straightforward generalization of the simple single-product model, the latter being its special case. Indeed, by combining the parameters of the multi-product model in a proper way, we may immediately arrive at the single-product model formulation.

1.7 Qualitative Analysis

Let us now turn our attention back to the basic investment model formulated in Section 1.3. In Section 1.4 we showed that under the assumption of specific functional forms we are able to find the Cournot equilibrium of the underlying input-choice game, using expression (1.4) for the case of fixed demand or the reaction functions (1.5) in the general case. Consequently, defining a discrete relationship between the levels of investment and the final payoffs enables us to construct a payoff matrix and to find the Cournot equilibrium of the upgrade investment game. Apparently, the actual solution of the game involves cumbersome mathematical expressions. Instead of presenting them here, we rather attempt to provide a simple qualitative analysis of the equilibrium. In particular, we focus on the case of two refineries that face a decision whether to invest or not in an upgrading facility. The starting point is the reaction curve derived in the upgrade investment game, with capacity choice given by (1.11). We simplify the notation by omitting the dependence on investment and by denoting sums of products of multiple parameters by a single letter, as follows:

$$A_r = \sum_{z=1}^{Z} \rho_{r,z} a_z, \quad B_r = \sum_{z=1}^{Z} \rho_{r,z}^2 b_z, \quad \Gamma = \sum_{z=1}^{Z} \rho_{1,z}\rho_{2,z} b_z$$

Then the reaction function in (1.11) can be rewritten as

$$x_1 = \frac{(A_1 - \Gamma x_2 + 2K_1B_1) - \sqrt{(-A_1 + \Gamma x_2 - 2K_1B_1)^2 - 8B_1(K_1A_1 - K_1\Gamma x_2 - \beta_1)}}{4B_1}. \tag{1.12}$$

1.7.1 The Input Choice Game

First, let us examine the optimal input choices and the corresponding profits, given a particular combination of the investment strategies of the two firms. What we can see from (1.11) or (1.12) is that the reaction function is downward-sloping[10] (as opposed to the constant inputs in (1.4)), which is a manifesting feature of Cournot oligopoly models. Figure 1.2 depicts a reaction curve for a particular choice of parameters.

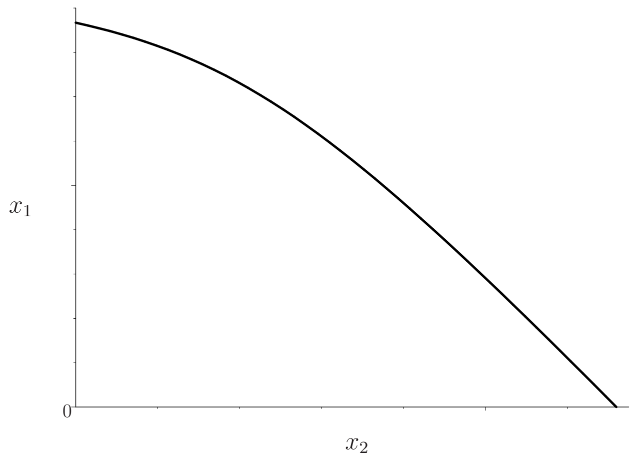

Figure 1.2: Reaction curve

Suppose next that the products vary in the value perceived by the consumers in terms of the price elasticity of demand. In particular, the higher-valued products are less price-elastic than the lower-valued ones, or, their demand parameters a_z and b_z are greater.[11] Then, we may investigate what happens if - all other parameters holding fixed - refinery 1's yield of some higher-valued product (denote by h) increases in exchange for a decreased yield of a lower-valued one (denote by l). We can see that this unambiguously increases parameter A_1 and, if the yield of product h is already higher at both refineries, also parameters B_1 and Γ.

A brief qualitative examination of (1.12) reveals that parameter A_1 shifts the reaction function upwards, while parameters B_1 and Γ shift and rotate it downwards. It remains

[10] See Appendix for a formal derivation of this claim.

[11] In case of linear demand $p_z = a_z - b_zQ_z$, the price elasticity for a current price-quantity pair is given by $e_z = -p_z/(p_z - a_z)$ or $e_z = p_z/(b_zq_z)$.

to determine which of these effects prevails. It turns out that for a reasonable choice of parameters the effect of A_1 outweighs the other two parameters only up to some point, presumably in a region where refinery 1's yield of product h is lower than that of product l yet, that is, where parameter B_1 can act reversely. After this point, the effect of B_1 and Γ prevails. So, initially the reaction curve gradually shifts up and then returns back down. Figure 1.3 shows an illustration of a reaction function varying with the yield of the valued product (and of the lower-valued one). As a consequence, transferring some of the refining yield from a heavy product to a lighter one affects the optimal crude oil input such that it increases in the beginning and then bends backward.

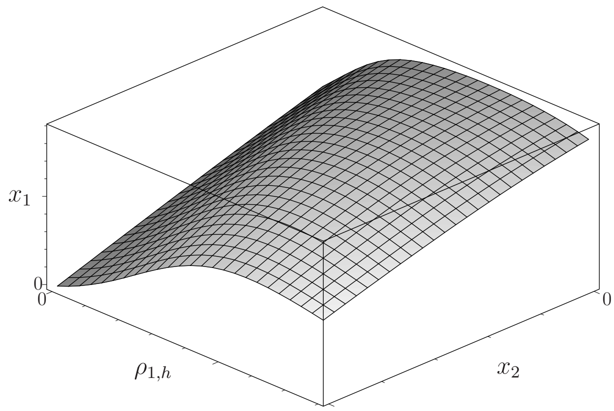

Figure 1.3: Reaction curve varying with $\rho_{1,h}$

The above result is of little surprise. With a higher yield of the valued product it is optimal for the refinery to attain more crude input to gain the additional profit from this product. But, at some point, the yield is so high that it can suffice with less input, so with lower costs. On the other hand, a too high yield of the valued product need not be beneficial for the refinery. It can happen that even with less input a significant amount of the product is delivered to the market, which in turn pushes its price down and hence, the refinery's profit. Further, we may examine how refinery 2's optimal choice and corresponding profit changes with increasing refinery 1's yield of the valued product. Since the reaction curve is downward-sloping, the optimal input of refinery 2 will follow the exact opposite pattern, and so will its profit. Figure 1.4 illustrates the discussed behavior of optimal input choices and the corresponding profits.

Finally, let us compare the above outcome with the case of fixed demand (perfect competition). According to (1.4), the optimal input choice increases with a higher yield of product h, assuming that $p_h^* > p_l^*$ and that the demand constraints are not binding. This, however, makes the refinery approach its capacity constraint faster, which in turn

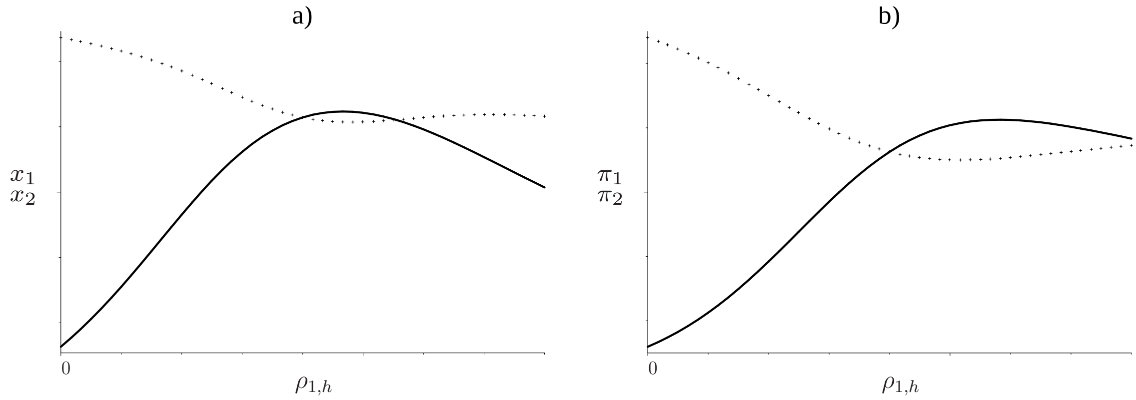

Figure 1.4: (a) Optimal input and (b) profit of refinery 1 (full line) and refinery 2 (dotted line) varying with $\rho_{1,h}$

significantly affects the costs. In the end, if the competitive prices are sufficiently high, there might be overproduction, and high costs close to the capacity constraint may actually make the firms worse off than in the oligopoly case, consistent with what the theory of perfect competition would suggest.

All these findings confirm that the possibility of investment in technologies which enable the transfer of yields is an interesting and relevant issue to study. Even more so if one needs to account for strategic interaction among refiners and for the discrete character of investment.

1.7.2 The Upgrade Investment Game

We may now proceed with an analysis of the upgrade investment game. In the simplest case where the two refineries decide whether to invest or not, we have four possible combinations of investment strategies, of which we can construct a payoff matrix as in (1.9). Our aim is to determine which of the four combinations can constitute the Cournot equilibrium. We have just seen that - all else holding fixed - increasing the yield of the valued product may be profitable up to some point. However, we omitted two important factors. First, the other refinery also has an opportunity to invest to increase its yields of higher-valued products. As noted previously, the reaction curves are downward-sloping, which means that any action of one refinery induces an opposite response of the competitor.

Second, the upgrade investment is costly. It is natural to think that the investment is reflected in the change of the cost parameters as follows. The new cost function will be flatter, so that the marginal cost approaches infinity slower. This is implicitly taken care of in case of capacity increase. Then, parameter β_r is used to adjust the speed of marginal-cost convergence to infinity. However, the fixed cost of increased capacity is higher and the refinery must also build the new plant, so it incurs the actual cost of investment. These two elements are transmitted into a change in parameter α_r. An illustration of how investment can affect the cost and marginal cost curves is shown in Figure 1.5.

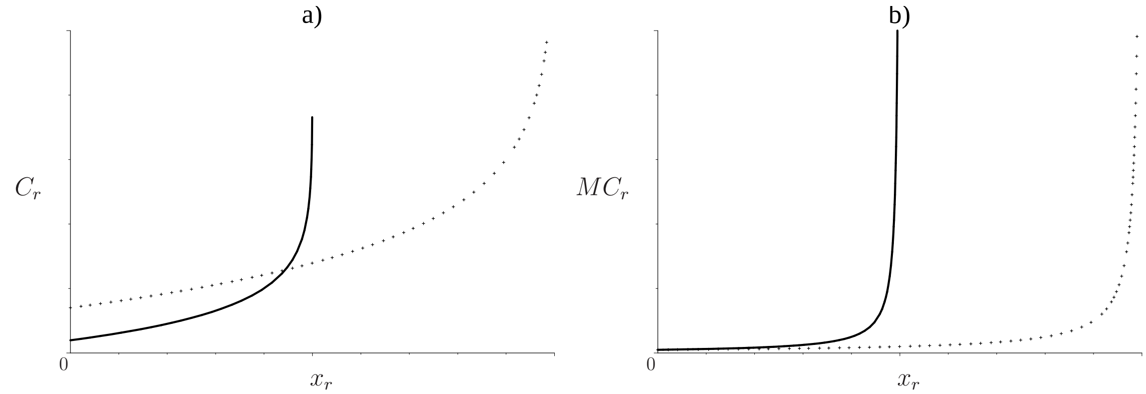

Figure 1.5: (a) Total cost and (b) marginal cost before (full line) and after (dotted line) investment

Hence, the resulting equilibrium is an outcome of a few diversely acting forces. Summing up, to evaluate the profitability of the investment strategy, one must take the following factors into consideration. First, increased yields of valued products raise the profit only within a certain range. Second, an action by one refinery aimed at increasing its profits induces a reaction of the competitor, which pushes the profits down. Third, investment incurs cost.

The equilibrium of the upgrade investment game can be found by a standard method for finding the Cournot-Nash equilibrium, that is, by indicating the best response of one player given the strategy of the second player. This means comparing particular cells of the matrix in (1.9). For instance, given that refinery 2 decides to invest, refinery 1 compares its payoffs $\pi_1(1,2)$ and $\pi_1(2,2)$. The profit from investing, $\pi_1(2,2)$, will be greater than $\pi_1(1,2)$ if, first, refinery 1 is not on its backward bending part of the profit function, that is, $\rho_{1,h}(2)$ is sufficiently low and, second, the associated cost of investment and the production cost after investment is not too high, that is, $\alpha_1(2)$ is sufficiently low. If the same holds for refinery 2, the Cournot equilibrium will have both refineries investing.[12]

Other types of equilibria may arise. If one of the refineries has some technological advantage, for example, in terms of costs or in terms of capacity, it can be optimal only

[12]This result is consistent with a rather general finding in the literature that if the firms compete à la Cournot in both investment and production stages, the equilibrium exhibits tendencies toward over-investment by the oligopolists. For reference, see Brander and Spencer (1983) or Reynolds (1986).

for this refinery to invest. The disadvantaged refinery would either find it too costly to invest, or it would be discouraged by the fact that already a substantial fraction of the market is served by the other refinery, so the prices are too low. Another kind of equilibrium can occur if the cost of investment is too high for both refineries. Then, obviously, none of them will invest. Finally, a special type of equilibrium arises if the refineries are technologically symmetric, but the demands are insufficient to accommodate the increased yields of both refineries. In such a case, only one of the refineries invests in equilibrium, but which of them actually does cannot be determined by this static analysis. Rather, it might be the subject of a commitment analysis, where one of the refineries has a 1^{st}-mover advantage and can commit to investment.

Lastly, let us briefly examine how the upgrade investment game can evolve in the case of the fixed demand problem in the 2^{nd}-stage. In this case the firms have lower incentive to invest, since their profit functions tend to bend backward faster, as we mentioned previously. In fact, a type of the equilibrium where only one refinery invests is then more likely.

This concludes the first chapter. We are now ready to introduce a major extension of the model that brings it closer to real-world phenomena, uncertainty. Modeling refineries' decisions under uncertainty is the focus of the second chapter.

Chapter 2

Modeling Refinery Investment under Uncertainty

In the present chapter we introduce the second key feature of the refining industry that this thesis aims to model, uncertainty. First, a preliminary guide to modeling uncertainty in the literature is provided. Then, a simple two-period application to our refinery investment model is presented. A qualitative analysis concludes the chapter.

2.1 Investment under Uncertainty

Investment is defined as an act of incurring expenses now with the prospect of profits generated at some point in the future. Associated with this act is some degree of uncertainty over the potential payoff. Specifically, the future reward from the investment follows a certain probability distribution, the realization of which is not known at the time when the investment decision is taken. Economists have struggled to develop a general rule that would be able to evaluate the attractiveness of a particular investment project and hence to help form optimal investment decisions. The traditional neoclassical theory presents the net present value (NPV) rule as an appropriate criterion for valuing investment projects. It assumes calculating the present value of the future cash inflow generated by the project, from which the present value of the cost necessary to launch the project is deducted. A positive NPV implies that the project should be undertaken.

However, as pointed out by Dixit and Pindyck (1994), the NPV criterion is based on one of two crucial assumptions, which are often overlooked. First, the investment project is considered as fully reversible, or, second, the investor is facing a now-or-never decision. Yet, these two conditions are rarely met in practice. The cost incurred to initiate the project is at least partially sunk, so it cannot be fully recovered, if the firm decides later to retract the project. This is because the project often involves transaction specificity, so the purchased assets cannot be sold, or if yes, then only at a discount. Also, unless strategic considerations such as entry deterrence force a firm to decide quickly, it usually has some flexibility about the timing of the investment. That is, the firm can postpone its decision until it acquires more information.

The violation of the two conditions has fostered the development of a new view of investment, the *real options approach*. This approach recognizes the opportunity cost of investment, which stems from the two features - the irreversibility and the ability to delay investment. This is where the notion of a real option emerges. Similar to financial options, the firm can choose to invest now (to exercise the option) or to wait until the uncertainty is at least partly resolved, and thus to take the risk that the value of the project changes. The actual investment decision is then based on comparing the present value of investing now with the present value of investing at possible future dates.

Research has shown that the opportunity cost of investment can be large and ignoring it might lead to erroneous investment decisions. In fact, first attempts to capture uncertainty together with irreversibility and timing flexibility in investment valuation models date back to 1970s. Most of the valuation techniques originate in the papers of Merton (1973) and Black and Scholes (1973), the pioneering works on financial options pricing. Myers (1977) argues that the optimal exercise of real options can create a significant corporate value. Since then, the literature has seen many attempts at applying the real options framework to investment models, among which, some of the prominent ones being Pindyck's (1988) analysis of the optimal capacity of a project and Trigeorgis' (1990) treatment of investment in natural resources. More recently, Imai and Watanabe (2004) examine investment under uncertainty in a market with the presence of a first-mover advantage and devise a model which could also be applied to the oil refinery sector. Cruz and Pommeret (2005) analyze investment with embodied technological progress and energy price uncertainty.

The standard real options model[1] utilizes the tools of stochastic calculus. In particular, the value of the project is assumed to follow a geometric Brownian motion. Then, the solution of the model calls for an application of dynamic programming techniques through the use of the Bellman equation and Ito's lemma. However, in this paper, being complicated enough by the multi-product part, we consider only a simple discrete-time application of the real options framework.[2] We will assume that the value of the investment can change at some discrete time points. This makes the calculation of the

[1] See Dixit and Pindyck (1994).

[2] Perotti and Kulatilaka (1998) present a discrete-time strategic real option model.

present values of investing at particular dates considerably simpler.

2.2 Uncertainty in Refinery Investment Models

Since there is a significant lag between investment decisions and their materialization, the decisions can be viewed as strategic commitments that carry considerable risk. The future value of the investment project is uncertain due to changing economic conditions. In our refinery model uncertainty is imposed on the demand side. In particular, we will assume that at the time when investment decisions are made, the refineries do not know the future state of the demand when the investment comes into practice.

Demand uncertainty will be modeled as stochastic demand with different states of the world with commonly known probabilities. Consequently, the refineries base their decisions on their rational expectations about demand. Consistent with the vast literature on uncertainty, we will apply the real options approach in our model. To this end, investment is viewed as, first, irreversible sunk cost and, second, as possible to be delayed. As opposed to the traditional NPV theory, this approach allows to explicitly account for the ability to wait and to value the option of delaying investment.

It is important to note that besides demand uncertainty, the refineries can face other kinds of uncertainty. For instance, at the time when the investment decision is taken, the refinery has some expectation about the associated cost, but does not know it precisely. At the time of the materialization of the investment, the cost can turn out to be larger than previously calculated. Hence, cost uncertainty may emerge as a remarkable issue. Technically, however, assuming no asymmetry in information,[3] modeling cost uncertainty would not differ much from the treatment of demand uncertainty presented below. Consequently, only demand uncertainty is considered here.

2.3 The Basic Model with Uncertainty

2.3.1 Setup

In the first chapter we examined the investment behavior of refineries in a static and fully deterministic setting. We showed that if the refineries have perfect information about the state of demand for each product, and if they simultaneously choose their investment plans and crude oil intakes, we can determine the Cournot equilibrium of their investment

[3] That is, the probability distribution of the costs of every refinery is of common knowledge.

strategies. In what follows, we develop this model by introducing demand stochasticity and a multi-period decision algorithm.

Hence, the prices of the refined products are now determined by the inverse demand function

$$p_z = p_z(Q_z, \varepsilon_z), \quad z = 1, \dots Z, \tag{2.1}$$

where ε_z's are parameters of random shocks, the (joint) distribution of which is known. The investment decision is made prior to the realization of this distribution, and therefore, the refineries' 2^{nd}- and 1^{st}-stage objective is (in the general case of Section 1.5.1) to maximize the expected profits:

$$\max_{x_r \leq K_r(I_r)} \quad E[\pi_r] = \sum_z \rho_{r,z}(I_r) x_r E\left[p_z\left(\sum_r \rho_{r,z}(I_r) x_r, \varepsilon_z\right)\right] - C_r\Big(x_r, K_r(I_r), I_r\Big),$$

and

$$\max_{I_r} \quad E[\pi_r] = \sum_z \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}) E\left[p_z\left(\sum_r \rho_{r,z}(I_r) x_r^*(\boldsymbol{I}), \varepsilon_z\right)\right] - C_r\Big(x_r^*(\boldsymbol{I}), K_r(I_r), I_r\Big),$$

respectively.

Now, suppose that the refineries have the opportunity to delay the investment decision and wait until the realization of the demand shock becomes known. Then, waiting one period will enable them to resolve demand uncertainty, and they will thus face a deterministic problem as in Section 1.5.1.

Following the above description, we can construct a simple stepwise decision tree. At the beginning of the first period, the firm forms its expectation about demand and can choose either to invest or not to invest. At the end of the period, the random shock is realized and the firm's payoff is delivered, based on its decision. Then, at the beginning of the second period, if the firm decided to invest in the first period, no choices are left now, but if it decided not to invest (that is, to postpone its decision), the firm - already knowing the state of demand - can again choose to invest or not to invest. The payoff is then again realized at the end of the period.

Evidently, a trade-off between deciding to invest now and postponing the decision until the next period may arise. If the firm decides to invest now, it may gain additional profit from this investment, provided that the realized demand turns out to be favorable. On the other hand, if it decides to wait, it may gain from resolving demand uncertainty and thus making a decision based on actual demand conditions. In particular, should the demand conditions turn unfavorable, the firm might refrain from investing.

2.3.2 The Case of One Refinery

Let us examine the above decision process in more detail. We start with the simplest case, where we assume that only one of the refineries has the investment opportunity.[4] Suppose that demand stochasticity is represented by

$$p_z = \varepsilon_z p_z(Q_z), \quad z = 1, \dots Z, \tag{2.2}$$

where ε_z is a binomial random variable defined as

$$\varepsilon_z = \begin{cases} u_z & \text{with probability } \lambda_z, \\ d_z & \text{with probability } 1 - \lambda_z, \end{cases}$$

where $u_z > 1$ and $d_z < 1$. Thus, in addition to what is determined by inverse demand, the price of each product z can randomly increase by factor u_z or decrease by factor d_z. The realization of this distribution comes at the and of the first period and the firm takes its period-1 decision based on the expectation of (2.2). The refinery's choices, together with the payoffs, are depicted in the tree in Figure 2.1.

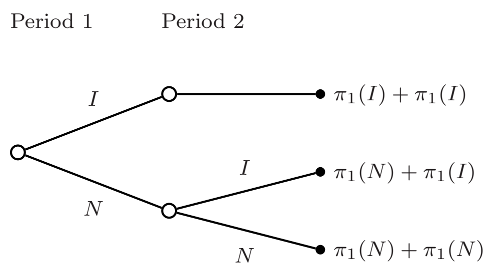

Figure 2.1: The tree of the game with one refinery

The refinery evaluates its options based on the expected payoffs. Denote by ϕ_i the probabilities of all 2^Z states of the world

$$\boldsymbol{\phi} = \{\phi_i | i = 1, \dots, 2^Z\} = \left\{ \prod_{k=1}^{Z} \lambda_k^{\theta_k} (1 - \lambda_k)^{(1-\theta_k)} | \{\theta_1, \dots, \theta_Z\} \in \{0,1\} \times \cdots \times \{0,1\} \right\}$$

and by $\pi_1^i(I_1)$ the corresponding realized profits in state i[5]

$$\pi_1^i(I_1) = \sum_z \rho_{1,z}(I_1) x_1^*(I_1) \Big(\theta_z u_z + (1 - \theta_z) d_z \Big) p_z(\cdot) - C_1 \Big(x_1^*(I_1), K_1(I_1), I_1 \Big).$$

[4]That is, for now, we disregard the competitors from strategic consideration.

[5]Note that since only refinery 1 can invest, the optimal crude input depends now only on the investment of refinery 1, so that $x_1^* = x_1^*(I_1)$.

Now, for some states of the world investing can yield higher profit, while for other ones not investing can be more profitable. We denote these subsets of states by G and B, respectively:

$$G = \{i | \pi_1^i(I) > \pi_1^i(N)\},$$
$$B = \{i | \pi_1^i(I) \leq \pi_1^i(N)\}.$$

Then, the total expected profit of investing in the first period is

$$2 \sum_{i=1}^{2^Z} \phi_i \pi_1^i(I) \tag{2.3}$$

and the total expected profit of delaying investment is

$$\sum_{i=1}^{2^Z} \phi_i \pi_1^i(N) + \sum_{i \in G} \phi_i \pi_1^i(I) + \sum_{i \in B} \phi_i \pi_1^i(N) \tag{2.4}$$

Expression (2.3) says that if the refinery chooses to invest in the first period, it will gain the profit of investing in both periods, while (2.4) says that if the refinery does not invest in the first period, it knows that it will invest in the second period if the state of the world turns out to be good ($i \in G$) and refrain from investing otherwise ($i \in B$). Consequently, our objective will be to determine which of the two expected profits is greater and hence to find the optimal investment rule.

2.3.3 The Case of Two Refineries

Let us proceed with a more general case, where two refineries operate on the market and both have the investment opportunity. Then, in forming the investment rules, strategic interaction between the two refineries must be taken into consideration. Starting from the first period, four possible scenarios may arise. First, if both refineries decide to invest, then no choices are left in period 2. Second and third, if only one refinery invests and the other one delays the decision, the latter's choice between investing and not investing constitutes the problem of period 2. Fourth, if both refineries delay the decision, then in period 2 a simple investment game between the two refineries is played. The tree of the whole game with possible strategies and payoffs is depicted in Figure 2.2.

Let again ϕ_i be the probabilities of the states of the world, with the corresponding

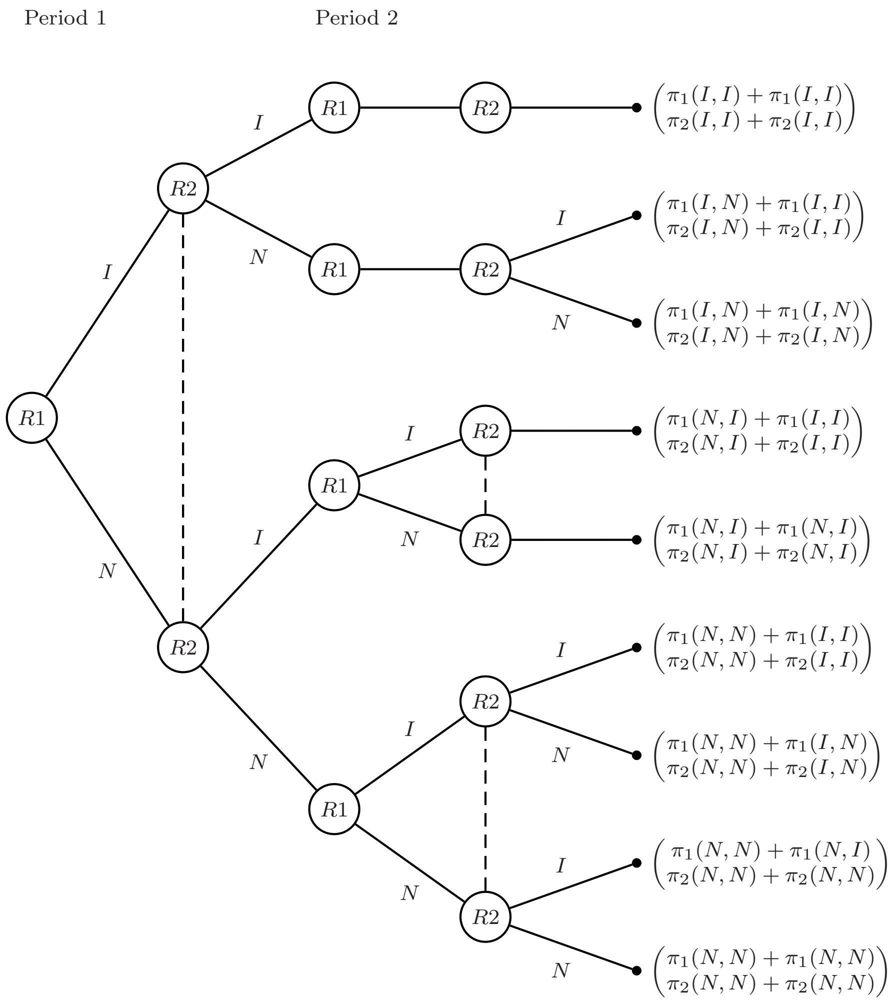

Figure 2.2: The tree of the game with two refineries

realized profits of refinery $r = 1, 2$ in state i given investment $\boldsymbol{I} = \{I_1, I_2\}$ denoted by

$$\pi_r^i(\boldsymbol{I}) = \sum_z \rho_{r,z}(I_r)x_1^*(\boldsymbol{I})\Big(\theta_z u_z + (1-\theta_z)d_z\Big)p_z(\cdot) - C_r\Big(x_r^*(\boldsymbol{I}), K_r(I_r), I_r\Big).$$

Now, similarly to the one-refinery case, we need to distinguish among the states of the world that determine optimal strategies of the refineries in the second period. In particular, we denote by G_1^I (B_1^I) the subset of states in which investing (not investing) is more profitable for refinery 1, given that refinery 2 invests, and similarly by G_1^N (B_1^N) the states in which investing (not investing) is more profitable for refinery 1, given that refinery 2 does not invest:

$$G_1^I = \{i|\pi_1^i(I,I) > \pi_1^i(N,I)\}, \qquad G_1^N = \{i|\pi_1^i(I,N) > \pi_1^i(N,N)\},$$
$$B_1^I = \{i|\pi_1^i(I,I) \leq \pi_1^i(N,I)\}, \qquad B_1^N = \{i|\pi_1^i(I,N) \leq \pi_1^i(N,N)\}.$$

For refinery 2, the subsets G_2^I (B_2^I) and G_2^N (B_2^N) are defined analogously.

Then, the expected profits of both refineries for the four possible period-1 scenarios can be summarized in Table 2.1. Again, our objective is to compare the expected payoffs and to find the equilibrium of optimal investment rules.

$R1\backslash R2$	Invest	Delay
Invest	$2\sum_{i=1}^{2^Z}\phi_i\pi_1^i(I,I)$	$\sum_{i=1}^{2^Z}\phi_i\pi_1^i(I,N)+\sum_{i\in G_2^I}\phi_i\pi_1^i(I,I)+\sum_{i\in B_2^I}\phi_i\pi_1^i(I,N)$
	$2\sum_{i=1}^{2^Z}\phi_i\pi_2^i(I,I)$	$\sum_{i=1}^{2^Z}\phi_i\pi_2^i(I,N)+\sum_{i\in G_2^I}\phi_i\pi_2^i(I,I)+\sum_{i\in B_2^I}\phi_i\pi_2^i(I,N)$
Delay	$\sum_{i=1}^{2^Z}\phi_i\pi_1^i(N,I)+\sum_{i\in G_1^I}\phi_i\pi_1^i(I,I)+\sum_{i\in B_1^I}\phi_i\pi_1^i(N,I)$	$\sum_{i=1}^{2^Z}\phi_i\pi_1^i(N,N)+\sum_{i\in G_1^I\cap G_2^I}\phi_i\pi_1^i(I,I)+\sum_{i\in G_1^N\cap B_2^I}\phi_i\pi_1^i(I,N)+\sum_{i\in B_1^I\cap G_2^N}\phi_i\pi_1^i(N,I)+\sum_{i\in B_1^N\cap B_2^N}\phi_i\pi_1^i(N,N)$
	$\sum_{i=1}^{2^Z}\phi_i\pi_2^i(I,N)+\sum_{i\in G_1^I}\phi_i\pi_2^i(I,I)+\sum_{i\in B_1^I}\phi_i\pi_2^i(N,I)$	$\sum_{i=1}^{2^Z}\phi_i\pi_2^i(N,N)+\sum_{i\in G_1^I\cap G_2^I}\phi_i\pi_2^i(I,I)+\sum_{i\in G_1^N\cap B_2^I}\phi_i\pi_2^i(I,N)+\sum_{i\in B_1^I\cap G_2^N}\phi_i\pi_2^i(N,I)+\sum_{i\in B_1^N\cap B_2^N}\phi_i\pi_2^i(N,N)$

Table 2.1: The expected payoffs in the two-refinery game

2.4 Qualitative Analysis

Without knowing the exact values of the parameters, it appears rather laborious to analytically find the solution of the game described above. Instead, we provide a brief qualitative analysis of the equilibrium and examine how the outcome is affected by varying different parameters.

2.4.1 One Refinery

In the case of one refinery, the decision problem involves comparing the two expressions for investing and delaying, (2.3) and (2.4). A trivial case arises if the expected profit of investing is lower than that of not investing. It is easy to see that the optimal decision (in the first period) is the same as without the option to delay - the firm will choose not to invest. Therefore, we consider only the case where the expected profit of investing is higher, that is, where the conventional NPV rule would suggest to invest. Clearly, in order for the value of the option to delay to exist, the refinery must expect that such a period-2 state exists in which it is profitable to refrain from investing, or, the set B is nonempty. Then, the decision rule reduces to comparing the expected relative gain in "bad" states in period 2 to the expected relative loss from not investing in period 1, that is, to determining the inequality

$$\sum_{i\in B}\phi_i\pi_1^i(N)-\sum_{i\in B}\phi_i\pi_1^i(I)\gtrless\sum_{i=1}^{2^Z}\phi_i\pi_1^i(I)-\sum_{i=1}^{2^Z}\phi_i\pi_1^i(N),$$

or

$$\sum_{i\in B}\phi_i\left(\pi_1^i(N)-\pi_1^i(I)\right)\gtrless\sum_{i=1}^{2^Z}\phi_i\left(\pi_1^i(I)-\pi_1^i(N)\right).$$

The analysis of how investment affects the realized profit in the current state was conducted in Section 1.7. It remains to examine the effect of stochasticity. Thus, if the probabilities of the "bad" states are sufficiently high and avoiding investment in these states promises a substantial gain (as a result of a substantial price fall, for instance), it will be optimal for the refinery to delay its decision and wait until period 2 to see how the state of the economy evolves. Otherwise the refinery will invest immediately. In particular, in case of only two states,[6] a "good" (g) and a "bad" (b) state, the condition for delaying to be optimal is given by

$$2\frac{\phi_b}{\phi_g}>\frac{\pi_1^g(I)-\pi_1^g(N)}{\pi_1^b(N)-\pi_1^b(I)}.$$

2.4.2 Two Refineries

To analyze the two-refinery game we start from Table 2.1. Again, a trivial case arises if for both refineries not investing is (in expectation) more profitable than investing, no matter what the other refinery does. Then, the equilibrium will have both firms delaying their decision. Hence, we suppose that this is not the case.

Next, consider the case when refinery 2 decides to invest in the 1st period. Given this, refinery 1 compares its expected payoff from investing and delaying. But since in the 2nd period no game between the two refineries will be played, this problem is analogous to the one treated above. Specifically, refinery 1 would decide to invest immediately if the probabilities of the "bad" states are not too high or the relative gain in these states is low. Conducting the same comparison for refinery 2, we can determine the conditions for

[6]Or equivalently, two subsets of states that yield the same payoff.

immediate investment by both refineries to be an equilibrium outcome of the game. In particular, in case of only two states for each refinery, a "good" (g_1^I and g_2^I, respectively) and a "bad" (b_1^I and b_2^I, respectively) state,[7] both refineries will choose to invest if the following two conditions are met:

$$2\frac{\phi_{b_1^I}}{\phi_{g_1^I}} < \frac{\pi_1^{g_1^I}(I,I) - \pi_1^{g_1^I}(N,I)}{\pi_1^{b_1^I}(N,I) - \pi_1^{b_1^I}(I,I)} \quad \text{and} \quad 2\frac{\phi_{b_2^I}}{\phi_{g_2^I}} < \frac{\pi_2^{g_2^I}(I,I) - \pi_2^{g_2^I}(I,N)}{\pi_2^{b_2^I}(I,N) - \pi_2^{b_2^I}(I,I)}.$$

If one of the above conditions is violated, it can be seen that the equilibrium will have the respective refinery delaying investment and the other one investing.[8] If none of the conditions holds, it is possible that both refineries will delay their decision. However, deriving an exact condition in such case would be more complex, as one needs to account for the possibility of a game played in the 2$^{\text{nd}}$ period. Intuitively, this type of equilibrium can arise if both the probabilities of "bad" states and the corresponding relative gains are high for both refineries.

This closes our discussion of uncertainty in the refinery investment models. In the last chapter we apply concepts presented so far to a real-world case study.

[7] Again, these are to be interpreted rather as subsets of states that yield the same profit for the particular refinery, given that the other refinery invests.

[8] This is simply because if, say, refinery 1 preferred investing to delaying given that refinery 2 invests, it would exhibit the same preference given that refinery 2 delays, since the prospect of refinery 2 refraining from investment even in period 2 cannot "hurt" its opponent.

Chapter 3

Application: Refinery Investment in Hungary and Romania

The aim of the present chapter is to apply the model exposed in the first two chapters to a specific case study. The players of the upgrade investment game are refineries in two CEE countries - Hungary and Romania, and their strategic behavior on their common market is studied. The main task is to adopt the available data on these two countries, so that the parameters of the model can be calibrated. However, since the model is rather stylized, to be able to fit the data accurately a number of restrictions will be imposed. We begin with a brief description of the refining sectors and the demand profiles in the two countries. Then, the calibration is carried out and the results are discussed.

3.1 The Refining Industry

3.1.1 Hungary

All three of Hungary's refineries - Duna, Tisza and Zala - are owned by MOL Hungarian Oil and Gas Plc. (MOL), from 2006 an almost 100%-ly privatized company. However, the Duna refinery is the only active crude processing refinery, with a distillation capacity of 164 tb/d.[1] The reported capacity utilization rate was rather low until 2001. Then, MOL closed the distillation capacities at the other two refineries , so the utilization figures approached the EU average, around 90%. The other two refineries are still used for the desulfurization of fuels, gasoline blending and bitumen and petrochemicals production. However, MOL has the option of reactivating the distillation capacity at Tisza in case demand suddenly increases.

[1]This and subsequent data come from PFC Energy (2006) report.

In 2005 the crude intake rose by 10%, bringing volumes back to levels of 2001. Also in 2001, a delayed coker was installed, which enabled the refinery to produce equal product yields, with 12.5% less crude intake since then.

The Duna refinery is the second largest in the region. Moreover, a majority stake in a small Slovak site Slovnaft allows MOL to coordinate its commercial strategies. In its domestic market MOL yields surplus production and exports significant product volumes to the former Yugoslavia, to Germany and to Austria. The refinery is linked to both the Druzhba and the Adria crude pipeline systems, but currently makes no use of the latter. In 2005 it processed 13% domestic crude, the remaining 87% crude was of Russian origin, Russia's Lukoil being the company's main crude supplier. The inland location of Duna efficiently protects it from product imports, except from Austria's Schwechat, which enjoys no technological advantage, though.

Having launched the coker in 2001, Duna is a relatively complex refinery. The site has reduced the share of heavy fuel oil in its product yield below 3% and increased gasoline and gasoil yields. Also, MOL invests in upgrading the Duna refinery in order to further raise gasoline production capacity, but mainly to raise the desulfurization capacity.

The completion of MOL's privatization gives it the freedom to develop its own long-term growth strategy, with the prospects of expanding in both upstream and downstream, in order to remain an influential player in the region. However, increasing dependence on Russian crude supplies can be risky, since Russian operators may become direct competitors for regional dominance. Another challenge may come from Austria's OMV, which is planning to import additional fuels from its refineries in Romania.

3.1.2 Romania

Romania's refining sector is one of the longest-established in Europe, and among the largest and most complex ones in the region. In 2003, the overall refining capacity stood at 495 tb/d.[2] Ten crude-processing refineries operate in Romania, dominated by two integrated operators, which together control over half of total capacity. The 70 tb/d Arpechim and the 69 tb/d Petrobrazi refineries are owned by Petrom, while Rompetrol operates the 100 tb/d Petromidia and the 10 tb/d Vega sites. The rest of the sector consists of three sites, with a capacity between 56 and 70 tb/d, and very small refineries, with capacities of less than 10 tb/d.

The capacity utilization rates were very low for a long time, with only about 50% in 2002. Nevertheless, the production remains sufficient to export a substantial fraction

[2]This and subsequent data come from PFC Energy (2005) report.

of it. In 2005, production increased significantly after Lukoil reopened its Petrotel site. However, further capacity reductions can be expected in the coming years, due to EU accession-driven deregulation. The overall conversion capacity is high relative to the region, but further investment will be required mainly in the middle distillates desulfurization capacities.

The refining margins have been among the lowest in Europe, mainly due to an unofficial price capping system facilitated by the government prior to OMV acquiring its stake in Petrom. Also, the absence of a connection to the Druzhba pipeline system bars the sector from acquiring cheap Russian crude. Nearly half of the crude currently processed is of domestic origin. Nevertheless, the margins are expected to increase after the entrance of foreign strategic investors - Lukoil and OMV.

Romania's largest refiner, Petrom, operates the refineries Petrobrazi and Arpechim. Both source their crude intake partly from imports. In 2003, the Petrobrazi site began producing EU-compliant gasoline, but was unable to produce the equivalent diesel. Rather, it has become Romania's largest LPG producer. In 2002, the fluid catalytic cracking units were upgraded in both refineries. The Arpechim site is more complex and is one of Romania's most advanced refineries in terms of product quality. In 2003, most of its diesel and gasoline export was EU-compliant.

The privatization of part of Romania's refining sector promises further improvements. Importantly, OMV's acquisition in Romania fits its regional strategies, among which penetrating the Hungarian market is particularly challenging.

3.2 Demand Profiles

3.2.1 Hungary

The increased use of gas and the decline of the industrial and agricultural sectors caused Hungarian oil demand to fall through the 1980's and 1990's. Accordingly, gasoline demand declined dramatically in the early 1990's and continued to fall slowly in the late 1990's, only to recover in 2000 to reach an average growth of 1% per annum between 2000 and 2005, driven by increasing car ownership.[3] On the other hand, increased road freight transport and the share of diesel cars triggered a diesel demand growth reaching 9% per annum between 1995 and 2005. Further, demand for LPG has been growing since 2000, as was demand for naphta. Finally, demand for fuel oil in power generation rose in the early 1990's, but began dropping in the late 1990's and even more sharply in the early

[3] Taken from Wood Mackenzie (2006a) report.

2000s, as a result of switching to gas.

Recently the overall oil demand stabilized and it is anticipated that it grows from 6.8 Mt in 2005 to 8.8 Mt in 2020. Over 75% of that growth will come from the transport sector, the rest from industry. Thus, gasoline demand is forecast to continue growing moderately until 2010, when a small drop is predicted due to lower car park growth and increasing vehicle efficiency. A steady rise in road freight is forecast to further induce diesel demand to reach 3.4 Mt in 2020, at an annual growth rate of 3%. A modest growth of LPG demand is forecast to continue, mainly due to a lower level of excise duty on LPG. Naphta demand is also expected to continue growing to reach 1.4 Mt by 2020. Finally, fuel oil demand is projected to slowly decline until 2020, driven by the decline of the heavy industry and by further gasification.

The trends in the supply-demand balance of the refinery products have changed over the years. Gasoline has been steadily in deficit recently, but surplus is expected to emerge in the long term, due to presumed increased refinery production in the next years and a slight decline in demand between 2015 and 2020. Diesel has been in surplus but is predicted to fall into large deficits until 2010, due to an expected demand growth from the transport sector. Finally, fuel oil is expected to be balanced slightly in surplus, as a consequence of declining demand.

3.2.2 Romania

Following the deep restructuring of the economy in the early 1990's, Romanian oil demand decreased at an annual rate of 5%.[4] However, economic growth in the late 1990's and the prospects of the EU accession promising further economic restructuring have boosted the demand to attain a 2.5% annual growth. This development has also affected the demand for refined products. Gasoline demand has been increasing since 1995 at a 6% annual rate. The economic recession in the 1990's had caused freight transport to decline, and consequently, to decrease diesel demand by 8% per annum between 1996 and 2000. The subsequent economic recovery triggered a high growth of freight transport resulting in 5% annual growth in diesel demand. Fuel oil demand declined sharply between 1990 and 2000, as a result of switching to nuclear, coal and gas capacities for power generation.

Total oil demand in Romania is forecast to grow at an average yearly rate of 2% from 11.2 Mt in 2005 to reach 15.2 Mt by 2020. The majority of the growth is predictably attributed to the transport sector. Gasoline demand is expected to grow strongly at a 5% rate until 2010 due to increased car ownership and economic growth, which is also

[4] Taken from Wood Mackenzie (2006b) report.

predicted to drive 3% annual growth of diesel demand until 2015. Fuel demand trends are also anticipated to continue with more replacement by alternative sources.

The trends in the supply-demand balance are rather stabilized. Increased utilization of the refineries is predicted to outweigh gasoline demand growth, so the gasoline surplus is expected to rise. Investment in desulphurisation by main refineries will enable them to boost exports. The diesel surplus is expected to slightly decrease, due to growing transport demand. Fuel oil has been in a deficit but is expected to shift to a balanced position by 2015.

3.3 Application of the Investment Model

3.3.1 Assumptions

Our goal is to apply the upgrade investment model to the framework outlined above and to study the investment behavior of the Hungarian and Romanian refineries. However, it is clear that we face a few challenges regarding the applicability of our model. Most importantly, although MOL is a single Hungarian refiner, and thus can be considered player 1 in the investment game, defining the Romanian player 2 is a little obscure. Ten refineries operate in Romania, so the supply is rather segmented. We will therefore focus our attention on the largest Romanian refiner, Petrom, which was recently privatized by MOL's major regional competitor, Austria's OMV. Petrom operates two refineries, Petrobrazi and Arpechim, accounting for more than half of the total Romanian refinery production. Hence, a significant restriction to the model must be imposed, the supply of Romanian fringe is taken as given and thus disregarded from strategic consideration both by MOL and by Petrom.

Related to this issue is the problem of the model's single-market requirement.[5] Obviously, Hungary and Romania are two markets with MOL and Petrom delivering products mostly to their domestic markets. Moreover, both refiners export a fraction of their output and at the same time refined products are imported by some foreign operators. However, for the purposes of our model it is not a dramatic diversion to treat Hungary and Romania as a common market with two major competitors, due to their geographic proximity. Also, similar to the problem of fringe supply, we take the exports to and imports from out of the region as fixed and thus exclude from refiners' decision factors.

A consequence of these limitations is the problematic way the demand is viewed. The

[5] Recall from Section 1.5.2 what are the difficulties associated with multiple markets.

demand the refiners are facing is a residual demand, that is, adjusted after accounting for fringe supply, imports and exports (since we take these as fixed). Accordingly, the parameters of the demand functions must be adjusted to reflect this issue.

Hence, taking into account the above issues, we construct a refinery duopoly investment model as follows. We consider the two refineries, MOL and Petrom that operate on the Hungarian-Romanian market. Their product slate is approximated by three groups of products, light distillates, medium distillates and residual oil, the representative products of which being gasoline, diesel and heavy fuel oil, respectively. Based on the demand profiles, an attractive investment opportunity appears to arise. In particular, the projected demand shift from fuel oil toward diesel seemingly favors investing in a hydrocracking unit, which enables to increase diesel yields. This is, however, a costly investment. Which of the refineries will invest is the core of the game we wish to model. The final but implicit assumption, thus, is that the real-world problem can be approximated by our stylized model, mainly in terms of the specifications of the technological, as well as the demand side.

3.3.2 Calibration

We may now proceed to calibrating the parameters of the basic investment model based on the available data.[6] We use the 2005 data on prices, demands and supplies of refined products in both countries. Together with the data on price elasticities of demand, which are adjusted for residual demand, we are able to construct the demand functions. The way we treat the problematic issues discussed previously is simply by calculating the current imbalance between supply and demand and attribute this to fringe supply and international trade. This imbalance is then held fixed. The calibrated demand parameters are summarized in Table 3.1.[7]

Product	Supply	Demand	a_z	b_z	$p_z = a_z - bQ_z$
Light	6012	5907	1601	0.17	594
Middle	4310	6558	1575	0.23	596
Heavy	1929	2093	574	0.17	248

Table 3.1: Calibration of the demand parameters (quantities in kt/year, prices in USD/t)

Further, knowing the refineries' current conversion rates, capacities and crude inputs,

[6] The data sources include the PFC Energy (2005, 2006) and the Wood Mackenzie (2006a, 2006b) reports, Petrom 2005 Annual Report, as well as MOL's private resources.

[7] The average prices and the aggregate demands and supplies for the two countries were used. Then, using the price elasticities of 0.6, 0.4 and 0.7 (these values are realistic, taking into account the fringe supply and foreign trade) for light, middle and heavy products, respectively, the demand parameters were calculated. Further, a's were adjusted to capture the fixed supply-demand imbalance.

we may calibrate the status-quo part of the model, that is, where none of the refiners invests in upgrading capacity. We do this by equating the crude intake to the expression in (1.6) and assuming that the players are rational profit maximizers. From (1.6) we then obtain the only unknown variables that remain to be specified, the parameters of marginal costs, β_1 and β_2. Apparently, the parameters of fixed costs (α_1, α_2) can be taken rather arbitrarily, since it is only their difference after investment that matters in determining the equilibrium, as it turns out.

The next step is to specify how the parameters change when the refineries decide to invest in upgrading capacity. In this we rely on MOL's expert opinions. In fact, MOL contemplates investing in a hydrocracking unit, which would increase its diesel yield, as well as the total refining capacity. In case of Petrom, an increase in the diesel yield is also possible, but without affecting the total capacity. The cost parameters are then adjusted as follows. First, before the investment, the fixed costs of both refineries are approximately at the same level. After the investment, the cost functions become flatter. For MOL, this is incorporated in the capacity increase, while for Petrom, parameter β_2 is adjusted. Then, the change in parameters α_1 and α_2 captures the proportional capacity increase (for MOL) and the cost of investment. In Table 3.2, the parameters for both the status-quo and the investment part are summarized.

	MOL		Petrom	
Yields	Before	After	Before	After
Light	0.43	0.40	0.46	0.44
Middle	0.38	0.44	0.25	0.38
Heavy	0.11	0.11	0.18	0.11
Capacity	8,100	9,400	8,000	8,000
Crude intake	**7,100**	?	**6,400**	?
Production				
Light	3,074		2,938	
Middle	2,691		1,619	
Heavy	802		1,126	
α	2,000,000	2,050,000	3,270,000	3,290,000
β	40,883	40,883	183,008	178,000
Costs	1,717,587	?	1,919,811	?
Revenue	3,627,285	?	2,987,659	?
Profit	**1,909,698**	?	**1,067,848**	?

Table 3.2: Parameter values before and after the investment (quantities in kt/year)

3.3.3 Results

We now have sufficient data to solve the basic upgrade investment game as in Chapter 1. To fill the question marks in Table 3.2, we construct the matrix of optimal inputs and the payoff matrix, as in (1.9). These matrices are shown in Table 3.3 and Table 3.4, respectively. We can immediately find the Cournot equilibrium of investment strategies. It turns out that the equilibrium has none of the refineries investing. Thus, the prediction of the static model is that it is not profitable for the refineries to undertake the upgrade investment, either due to high costs associated with it, or due to relatively low demands.

MOL/Petrom	Not invest		Invest	
Not invest	7,100	6,400	6,926	6,041
Invest	6,852	6,446	6,594	6,062

Table 3.3: The matrix of optimal inputs

MOL/Petrom	Not invest		Invest	
Not invest	1,909,698	1,067,848	1,705,238	1,063,634
Invest	1,799,626	1,122,322	1,537,241	1,086,746

Table 3.4: The payoff matrix

Let us proceed with the more realistic setting, the two-period investment game under uncertainty analyzed in Chapter 2. Based on the demand profiles, we project the future demand shocks as follows. A moderate growth in gasoline demand and a fair growth in diesel demand are forecast, while a decline in fuel oil demand is anticipated. Applying to our framework, we approximate the price shocks by the demand shifts of these products. In particular, suppose that next year the price of the light and middle distillates may increase by 5% and 10%, respectively, with a 50% probability, independently of each other, while the price of the residual fuel may decrease by 10% with a 50% probability. Assuming that these estimates are common, we wish to find the optimal investment rules of the two refineries, that is, whether it is profitable to invest now or to postpone the decision until next year, when the uncertainty is resolved. What we need is to construct the payoff matrix of the four combinations of the investment strategies, as in Table 2.1. After some calculations we obtain the payoff matrix shown in Table 3.5.

MOL/Petrom	Delay		Invest	
Delay	3,866,434	2,305,107	3,597,967	2,310,037
Invest	3,805,237	2,415,661	3,250,397	2,363,942

Table 3.5: The payoff matrix of the two-period game

Hence, we can see that for Petrom the equilibrium investment rule is to invest immediately, while MOL should delay its decision. It turns out that this result is robust against various other demand scenarios, provided that the projected diesel demand increase is sufficiently high. The explanation can be the following. Petrom utilizes the benefits of investment in all states and investing is a dominant strategy. The potential gain from the increased diesel yield outweighs the particularly high cost of investment. On the other hand, MOL, despite being slightly advantaged in terms of costs and capacity, the advantage in yields is so high that it is already located on the backward bending part of its profit function (see Section 1.7.2) and thus, rather paradoxically, investing seems unprofitable for MOL.

We conclude that in the static setting, high investment costs prevents both MOL and Petrom from investing in upgrading capacity. A prospect of a future demand growth of diesel and gasoline, though, apparently benefits Petrom and encourages it to invest. However, this result should be regarded with caution, due to some notable restrictions of the model's application.

Conclusion

The aim of this thesis was to build an oligopoly model which took into account key characteristics of the oil refining industry. Using game theory tools and elementary stochastic modeling techniques, the purpose was to capture the following particularities. First, the output of a refinery is a heterogeneous composite product, so the refinery is considered as a multi-product firm simultaneously competing in multiple product markets. Second, due to a relatively high share of each refinery in the regional market, adjusting the production to meet changing demand involves large-scale investment that carries significant sunk costs. Third, a high degree of uncertainty over future payoffs is associated with operating in the refinery market, due to fluctuating market conditions.

In view of recent changes in the demand profiles from heavy products to lighter products the question arises whether and to what extent it is profitable for the refineries to undertake the upgrade investment whereby they can increase the refinery yields of lighter, higher-valued products. In an attempt to answer this question, strategic interaction among refineries was taken into account, and a two-stage Cournot investment game was designed. In the first stage, the refineries choose their investment strategies by which they can build an upgrading capacity, enabling them to reprocess heavy residual output to obtain higher yields of lighter output. Then, given the firms' stage-1 choices, a capacity-utilization game is played, where the refineries choose the optimal crude oil intake, which, due to the special character of the refining technology, is a single decision variable of the second stage.

In this static and deterministic setting, the goal was to study the equilibrium behavior of the refineries and to determine the conditions under which a particular set of investment strategies is optimal. Due to considerable mathematical complications, the investment game between only two refineries was focused on, presumably, without loss of economic insights. Also, consistent with the discrete nature of investment, the game was reduced to two choices - to invest or not in the upgrading capacity. A simple qualitative analysis revealed that, as long as one of the refineries enjoyed a sufficient technological advantage, the equilibrium would have only this refinery investing, otherwise, if the demands were high enough and the costs of investment not too high, both refineries would invest in the equilibrium, consistently with the findings in the literature.

Furthermore, to model uncertainty over future payoffs, stochasticity was imposed on the demand side. Particularly, in addition to what is determined by the inverse demand, the price can increase or decrease by some random factor, the probabilities and magnitudes of these shocks being of common knowledge. Again, to keep the analysis simple, a two-period game with two refineries was designed. In each period an upgrade investment game is played, with the difference that the refineries resolve uncertainty only in the second period, while in the first period they base their decisions on their expectations. Also, consistent with the real options approach, the investment is irreversible, so once the refinery decides to invest in the first period, it cannot undo its decision later, should the demand turn out to be unfavorable.

The purpose of this model was to examine whether it was profitable for the refineries to invest immediately or to use the option to wait and see the realization of the demand shock, and, possibly, even to refrain from investing at all, if it turned out to be unprofitable. Again, a short qualitative analysis revealed that if the probabilities of the unfavorable states were too high as was the relative gain in these states compared to preemptive investment, then the refinery would delay its investment decision.

Finally, the applicability of the refinery investment model was illustrated by a case study. The investment behavior of Hungarian and Romanian refineries was studied. The projected demand shift from fuel oil toward diesel seemingly favors investing in a hydrocracking unit, which makes it possible to increase diesel yields. The application of the investment model attempted to answer the question which of the refineries would actually invest. Consistently with previous theoretical findings, the results suggest that although the Hungarian refinery enjoys a slight technological advantage, the cost of the investment is rather high, resulting in both refineries refraining from investment. However, when the model with uncertainty was applied and a sufficiently high probability for the upward shift of the diesel demand was assumed, it was found that it would be profitable at least for the Romanian refinery to invest immediately, while the Hungarian refinery would, rather paradoxically, delay the investment decision.

Lastly, it must be noted that throughout the construction of the model, a few simplifications and diversions from reality were necessary. Most importantly, refined product markets are regionally segmented by transportation costs, which was, due to mathematical complication, omitted in the model. Thus, the model deserves further elaboration and development. However, the major contribution of this thesis is the combination of two traditional theories in the economic literature - multi-product oligopoly and investment under uncertainty - and applying them to analyze strategic decision making of firms in a particular industry - in the refinery industry. The author believes that this goal has been fulfilled.

Appendix

A.1 Derivation of Optimal Input in the Basic Model

For the case of fixed demand the profit-maximization problem yields the first-order condition (FOC)

$$0 = \frac{\partial \pi_r(x_r)}{\partial x_r} = \sum_z \rho_{r,z} p_z^* - \frac{\partial C_r(x_r, K_r)}{\partial x_r} = \sum_z \rho_{r,z} p_z^* - \frac{\beta_r}{K_r - x_r}.$$

Solving for x_r gives

$$x_r = K_r - \frac{\beta_r}{\sum_z \rho_{r,z} p_z^*}.$$

In the general case we have the FOC:

$$\begin{aligned} 0 = \frac{\partial \pi_r(x_r)}{\partial x_r} &= \sum_z \rho_{r,z} p_z(Q_z) + \sum_z \rho_{r,z} x_r \frac{\partial p_z(Q_z)}{\partial x_r} - \frac{\partial C_r(x_r, K_r)}{\partial x_r} \\ &= \sum_z \rho_{r,z} \left(a_z - b_z \sum_i \rho_{i,z} x_i \right) - x_r \sum_z \rho_{r,z}^2 b_z - \frac{\beta_r}{K_r - x_r} \\ &= \sum_z \rho_{r,z} a_z - \sum_z \rho_{r,z} b_z \sum_{i \neq r} \rho_{i,z} x_i - 2x_r \sum_z \rho_{r,z}^2 b_z - \frac{\beta_r}{K_r - x_r} \end{aligned}$$

Rearranging gives

$$\begin{aligned} & \left(2 \sum_z \rho_{r,z}^2 b_z \right) x_r^2 \\ + & \left(-\sum_z \rho_{r,z} a_z + \sum_z \rho_{r,z} b_z \sum_{i \neq r} \rho_{i,z} x_i - 2K_r \sum_z \rho_{r,z}^2 b_z \right) x_r \\ + & \left(K_r \sum_z \rho_{r,z} a_z - K_r \sum_z \rho_{r,z} b_z \sum_{i \neq r} \rho_{i,z} x_i - \beta_r \right) \\ = & \; 0 \end{aligned}$$

Solving for x_r and applying the capacity constraint $x_r < K_r$ we have

$$x_r = \frac{\left(\sum_z \rho_{r,z}a_z - \sum_z \rho_{r,z}b_z \sum_{i\neq r} \rho_{i,z}x_i + 2K_r \sum_z \rho_{r,z}^2 b_z\right)}{4\sum_z \rho_{r,z}^2 b_z}$$
$$- \frac{\sqrt{\left(-\sum_z \rho_{r,z}a_z + \sum_z \rho_{r,z}b_z \sum_{i\neq r} \rho_{i,z}x_i - 2K_r \sum_z \rho_{r,z}^2 b_z\right)^2 - 8\sum_z \rho_{r,z}^2 b_z \left(K_r \sum_z \rho_{r,z}a_z - K_r \sum_z \rho_{r,z}b_z \sum_{i\neq r} \rho_{i,z}x_i - \beta_r\right)}}{4\sum_z \rho_{r,z}^2 b_z}$$

A.2 Derivation of the Slope of the Reaction Function

To show that the reaction curve is downward-sloping, let us differentiate (1.12) with respect to x_2. We obtain

$$\frac{-\Gamma + \frac{A_1\Gamma - x_2\Gamma^2 - 2\Gamma K_1 B_1}{\sqrt{A_1^2 - 2A_1x_2\Gamma - 4A_1K_1B_1 + x_2^2\Gamma^2 + 4x_2\Gamma K_1B_1 + 4K_1^2B_1^2 + 8B_1\beta_1}}}{4B_1}.$$

Denoting the expression in the numerator by N and solving it for x_2 we obtain

$$x_2 = \frac{2A_1N\Gamma + A_1N^2 - 4K_1B_1N\Gamma - 2K_1B_1N^2 \pm 2\sqrt{2}(\Gamma + N)\sqrt{-N(2\Gamma + N)B_1\beta_1}}{\Gamma N(2\Gamma + N)}.$$

We can see that for $N > 0$ the expression under the root sign is negative, since all the other parameters are positive. For $N = 0$, the expression does not make sense. It follows that there exists no real x_2 for which the reaction is upward-sloping or constant. Hence, it must be downward-sloping.

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