# Children's Consumption and Population Aging: a Realistic OLG Model

by

István Levente Fazakas

Submitted to

Central European University

Department of Economics

In partial fulfulment of the requirements for the degree of Master of Arts

Supervisor: Professor András Simonovits

Budapest, Hungary

2008

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Budapest, 8 June 2008

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# Acknowledgments

I would like to thank professor András Simonovits for introducing me to the field of pension economics through his research and for all the guidance he gave me as a supervisor. Thanks go also to my academic writing instructor, Thomas Rooney, for the help he provided in structuring my thesis.

# Abstract

One of the uses of OLG models in the relatively recent literature is the study of the effects of population aging, more specifically its repercussions on the pension systems. One of the models dealing with this issue is in Oksanen and Simonovits (2008). Their model characterizes the behavior of the household in a detailed manner, including among other elements, children's consumption, often neglected in the literature. In this paper I present and do a sensitivity analysis of children's consumption for the steady state version of their model. I find that, when children's consumption is accounted for, the consumption path of the household changes from a smooth line to a humped-shape curve. The humped shape appears when children are born in the household and increase its consumption, who later become adults and leave, the consumption returning to its original path. From this I conclude that OLG models, by including children's consumption in their setup, give better results.

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# Introduction

Population aging is a demographic phenomenon characterized by an increase in the share of elderly in the population. It was first noticed in the developed countries, but then it appeared in other countries, forecasted to eventually span the whole globe and to stay important throughout the 21st century. The major drivers of this phenomenon are the increase in life expectancy and the decrease in fertility rates. In the developed countries, people born today expect to live longer than those born half a century ago, and as a result the number of old people increases. In addition, fewer children are born during our days than fifty years ago, due to lower fertility rates, hence the number of young decreases.

A person born in Western Europe in 1960 was expected to live 63.6 years; in 2005 a person could expect to live 82.5 years. The forecasts of the evolution of the life expectancy predict that a person born in 2050 would live for 86.5 years (U.S. Census Bureau, 2008). The decreases in the fertility rates are also important. In 1960 the total fertility rate of women aged 15-49 was 2.3 which decreased to 1.53 in 2005. It is expected that the total fertility rate will increase to 1.7 in 2050 (U.S. Census Bureau, 2008). A dramatic consequence of this low rate is that it is not enough to sustain the growth of the population which requires at least a rate of 2.1 (McDevitt, 1999). The consequence of these two phenomena is that the share of old people in the distribution of the population increases.

This phenomenon has numerous economic, social and political implications. On the economic side, an important effect of population aging is the change in savings and investments. When working people are mostly young, investments may be higher than savings, but when most of the workers are close to their final years of employment savings may overcome investments. As a consequence, in an aging society an increase in savings to the detriment of investments can be noticed. With less investments, the result is a decrease in total output and the rate of economic growth decreases. In addition, due to high savings, the rates of return would decrease. (U.N., 2001)

Due to the great importance of population aging, many economists have been preoccupied with finding solutions to the different problems it creates. One of their main concerns revolves around the problems of the pensions systems, which has led to a rich literature on this topic. The most widely used framework for modeling pension systems is the Overlapping Generations Model (OLG )which allows the simulation of the demography and the economy of a country, with the households being the consumers, the business sector the producer, and the government the manager of the pension system. The OLG model starts from the household's optimization problem, thus being one of the macro models with micro foundations. It allows several generations to live simultaneously and, conform the life-cycle theory, to have different consumption and saving patterns depending on their age. In addition, computer simulations of the OLG model yield concrete results.

The OLG models have evolved considerably since they were introduced by Samuelson (1958), becoming richer and richer. My master's thesis extends the work of Oksanen and Simonovits (2008), who studied the issue of population aging with its effects on a "pay-as-you-go" (PAYG) pension system. In tackling the problem of population aging and its effects on the pension system, the authors insure intergenerational equity, which means that a worker who has contributed to the system will receive a fair benefit as pensioner. Their model assumes that prior to 1970 the world was in steady state, with a stationary population which allowed the self-sustainability of the PAYG system. But in 1970 the fertility rate started to decrease until 2000, the same year when the life expectancy increased. These changes modify the demographic structure of the population and make the pension system go through a transition phase. By 2080, the effects of the demographic changes disappear, the PAYG pension system is again self-sustainable, in a new steady state.

I have helped the development of the computer program that Oksanen and Simonovits used in order to simulate the OLG model and obtain the results. In their paper, they use a life-cycle model with a complex utility function which takes into consideration the characteristics of the household, most importantly the presence of children. Considering that the previous literature has mainly neglected the composition of the household's consumption, based on the model of Oksanen and Simonovits, I analyze how results change when the existence of children in the household is omitted. My results are obtained for the two steady states, the one which took place prior to the 1970's and for the one after the changes, when the fertility rate and life expectancy stabilized again, around the year 2080.

As a result of the sensitivity analysis I find that including children's consumption in the model gives a humped-shape consumption path and different results from the smooth path of consumption obtained when children's presence is ignored. I obtain these results for the original parameters from the model of Oksanen and Simonovits (2008), which I call "the O.–S. scenario", and also for the cases when I vary the values of the parameters for the decade the household is established and the decade children are born. From this I conclude that ignoring children's consumption in an OLG model does not give realistic results.

The rest of the paper is organized as follows. The second chapter is an overview of the literature on OLG and life-cycle models, and the way they treat the characteristics of the household, more specifically the children's consumption. The third chapter describes the steady state version of the OLG model from Oksanen and Simonovits (2008). The fourth chapter presents the results of the sensitivity analysis. It begins with the O.–S. scenario used in the original paper, after which it checks some other scenarios in order to check the robustness of the model. Finally some conclusions are presented with further recommendations of parameters or states for which sensitivity analysis can be done.

# Chapter 1. Review of Literature

Following the path of Samuelson (1958) the OLG models used in the literature usually do not take into account the humped shape path of household's consumption. But, from the empirical literature it can be seen that because at some point in any household children are born, the total consumption of the household increases; when they grow up and leave the household, the consumption returns to its normal level. This is why it is said that household's consumption path has a humped shape.

Table 1 presents the multitude of characteristics that can be encountered in OLG/Life-cycle models. Panel A shows the papers that do not include in the modeling of the household children's consumption. The Samuelson model is the starting point, being the most simple one which introduced the idea of OLG. Starting from his idea many authors have improved these models.

There is a large number of theoretical studies concerned with population aging, which use as a framework OLG models. The model from Auerbach and Kotlikoff (1987) is more complex than the original one because pensioners receive a pension benefit and it includes also the business and the government sectors. In addition they obtain results not only for the steady states of the economy but also for the states of demographic transition. This rich setup enables the authors to analyze the dynamic impact of different fiscal policies and social security reforms. Also, they carry out sensitivity analysis of the different specifications of their model, and see how the outcomes are affected by the choice of the parameter values. In a similar setup, Graf and Schattenberg (2006) predict how the German economy will be affected by population aging. Krueger (2004) uses also an OLG model to analyze the impact of population aging on aggregate savings. It is as basic as the Samuelson model but useful in helping the author derive some conclusions about population aging. He analyzes the impact of increased life expectancy and decreased fertility rates in

Table 1: Comparison of OLG/Life-cycle Models

| Model | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| **A. No children's consumption** | | | | | | | |
| Samuelson (1958) | − | − | − | − | − | − | − |
| Auerbach and Kotlikoff (1987) | − | − | − | + | + | − | + |
| Graf and Schattenberg (2006) | − | − | − | + | + | − | + |
| Krueger (2004) | − | − | − | − | − | − | − |
| Hairault and Langot (2008) | − | + | + | − | + | + | − |
| Borsch-Supan, Ludwig, and Winter (2004) | − | + | − | + | + | + | + |
| Hubbard, Skinner, and Zeldes (1995) | − | − | + | − | − | + | − |
| **B. With children's consumption** | | | | | | | |
| Blundel, Browning, and Meghir (1994) | + | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| Scholz, Sheshadri, and Khitatrakun (2006) | + | − | + | − | + | + | − |
| Casarosa and Sparato (2007) | + | − | − | − | − | − | − |
| Oksanen and Simonovits (2008) | + | + | + | + | + | − | − |
| The present master's thesis | + | + | + | − | + | − | − |

Notes: (1) Children; (2) Bequest; (3) Credit Constraint; (4) Demographic transition; (5) Pension System; (6) Stochastic disturbance; (7) Business and government sectors;

a steady state model, in which he assumes for simplicity constant consumption-age profile.

The starting idea of Hairault and Langot (2008) is that PAYG systems redistribute wealth across and within generations because the benefits one receives do not totally depend on the contributions he paid. In efficient economies a funded system would be better, but the problem, from a welfare point of view, of transition from a PAYG to a funded system cannot be solved. The paper analyzes the effects of different reforms on the PAYG pension systems. They use a stochastic life-cycle model and analyze different steady states, with and without reform. In their model people are able to leave and receive bequests and they cannot borrow when their assets are negative due to the credit constraint. Borsch-Supan, Ludwig, and Winter (2004) also analyze the shift from a PAYG pension system to a (partially) funded one but in an open economy framework. Their setup is more complex and allows results to be obtained also for the demographic transition phase. Besides the standard sensitivity analysis that usually is done to OLG models, they do one with respect to model specification, by switching off some of its features and analyzing the obtained effects.

In an empirical paper and a setup different from population aging, Hubbard, Skinner, and Zeldes (1995) defend the life-cycle model which seems not to work in the case of low income families and argue that it is useful for explaining wealth accumulation once precautionary savings and social insurance programs are taken into consideration. Their life-cycle model incorporates uncertainties and credit constraint but it does not deal with more complex issues as children consumption or bequests.

The empirical literature has established the humped shape path of the household's consumption. This is why many theorists have reconsidered their models to include the observed pattern in the household's consumption in order to make them more realistic, thus, we now see more and more theoretical papers accounting for such household characteristics as children's consumption. Panel B of Table  presents some papers that present and/or try to solve this issue.

Blundel, Browning, and Meghir (1994) test the validity of the life-cycle model with micro level data. They study how household characteristics influence consumption and find that controlling for household characteristics, especially the presence of children, consumption is smooth along the life-cycle and it does not have the humped-shape. Scholz, Sheshadri, and Khitatrakun (2006) use an augmented stochastic life-cycle model and study the household consumption decisions and the way these decisions have an impact on their wealth. Their rich life-cycle model with uncertainties, credit constraint and pension system, fits well the data used, being very good for predicting the way people save; a sensitivity analysis finds for some well chosen parameter values an even better fit. Considering the presence of children in the household, the model obtains the humped-shape path of household's consumption.

Casarosa and Sparato (2007) consider a basic life-cycle model with the household as the decision unit and a per-adult-equivalent consumption level. The per-adult-equivalent consumption concept enables them to consider children's consumption as a 0.4 factor increase in the consumption when a new child is born. They make a sensitivity analysis of their model, considering, besides consumption, the fertility rate, the household age when children are born, and the length of the period during which the children are in the household and reflect on the micro but also on the macro implications of the different parameter values. Children's consumption explicitly appears also in the model of Oksanen and Simonovits (2008). However, the problem addressed by their article is that of the population aging and the different reforms that can be made in order to save a PAYG pension system. In their complex OLG model households face a credit constraint, pensioners finance their consumption out of the pension benefit and when they die leave a bequest. They obtain results for the household's consumption and savings not just for the steady states of the world but also during the state of demographic transition. In this paper, I am doing a sensitivity analysis for the steady states of their model by analyzing how the results change when children's consumption is omitted.

# Chapter 2. The Model

In order to study how demographic changes can hit the economy of a country, overlapping generations models are very suitable. In the following I present the simplified, time-invariant model from Oksanen and Simonovits (2008) which simulates the demographic changes that take place in a developed country and the way households respond to these changes. The business sector and the government are ignored, some simplifications being used when needed.

The household is the decision unit and it is composed of one adult and $f$ children, where $f$ is a real number (non-integer birth numbers come from averaging integers). The adult lives until age $I$. He is born in his parent's household where he lives until age $L$. At this age he becomes an adult and starts to work, so he leaves his old family in order to start a new one. At the age of $H$ the adult simultaneously gives birth to $f$ children. These children when aged $L$ leave the family as their parent did. The adult works until he is aged $J$ when he retires and he dies at $I$. In this model each period represents a decade in the life of a person.

## 2.1 Household's behavior

The household is founded when the adult has age $L$ when he maximizes his discounted lifetime utility, which is given by

$$U = \sum_{i=L}^{I} \delta^{i-L} u_i(c_i), \tag{1}$$

where $0 < \delta \leq 1$ is the discount factor and $u_i(c_i)$ is the utility of a person of age $i$ given that he consumes $c_i$. The amount the household consumes depends on its characteristics, so the utility function of the household takes the form

$$u_i(c_i) = \beta_i \, m_i \, u(c_i),$$

which includes an age independent part that depends on the consumption $u(c_i)$, adjusted with an individual utility multiplier $\beta_i$ and a household size indicator $m_i$. The household consumes more when it has to feed the children and less after the children start to work and leave the household. This paper underlines the importance of $m_i$ so much neglected by the literature by varying the values of $\mu$, the multiplier which introduces the effect of children's consumption in the model. As a result the consumption of the household will have the required humped-shape. Considering this, the size of the household is given by

$$m_i = \begin{cases} 1 + \mu f & \text{if } H \leq i < H + L; \\ 1 & \text{if } L \leq i < H \text{ or } H + L \leq i \leq I, \end{cases}$$

where $f$ is the fertility rate, representing the number of children born in the household. While the adult is its only component, he is also the only one who consumes. During his lifetime the adult reproduces himself only once, at age $H$. As a result, at this age, the consumption power of the household increases with $\mu f$, adult consumption equivalent. At the age the children become grown ups and leave the household, $L + H$, the adult of the household becomes again the only consumer. In addition, an individual enjoys consumption more while he is a child or worker and less when retired:

$$\beta_i = \begin{cases} 1 & \text{if } L \leq i \leq J; \\ \beta & \text{if } J < i \leq I. \end{cases}$$

The use of $0 < \beta < 1$ introduces more realism into the adults consumption behavior, making the consumption even more hump-shaped. A young person usually likes to consume and will consume as much as his income allows. But, after retiring, more leisure will compensate for the diminished consumption, so the $\beta$ adjusts household's consumption to the requirements of pensioners. The last component of the household utility function, which does not depend on age, transforms consumption into utility. It has a standard CRRA form that is easy to calculate analytically:

$$u(x) = \begin{cases} \dfrac{x^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1; \\ \log x & \text{if } \gamma = 1. \end{cases}$$

## 2.2 Household's budget constraint

In preparing the ground for the households budget constraint I have to introduce some concepts. The income of an adult worker is the wage he receives out of which his contribution to the pension system is subtracted. The contribution rate $\tau$ is calculated guaranteeing *intergenerational equity* (for for details, see Oksanen and Simonovits (2008)):

$$\tau = \frac{\theta \sum_{j=L}^{J} w_i \sum_{k=J+1}^{I} \nu^{-i}}{\sum_{j=L}^{J} w_i \nu^{-i} + \theta \sum_{j=L}^{J} w_i \sum_{k=J+1}^{I} \nu^{-i}}.$$

When he retires his income will be the pension benefit, which depends on his lifetime contributions

$$y_i = \begin{cases} (1-\tau) w_i & \text{if } L \le i \le J; \\ b_i & \text{if } J \le i \le I, \end{cases}$$

where the wage is given by the mincerian quadratic function multiplied with the growth factor of labor productivity, $g$. The use of the mincerian function is a natural way of modeling that at some point the returns to working are no longer increasing with time, but decreasing, as the person grows older:

$$w_i = (\omega_0 + \omega_1 i - \omega_2 i^2) g^{i-L}, \qquad i = L, \ldots, J.$$

The pension benefit is defined as the *initial benefit* received in the first decade of being a pensioner, $J+1$

$$b_{J+1} = \sum_{j=L}^{J} \theta_j (1-\tau) w_j,$$

followed by the *continued benefit* for the decades after $J+1$

$$b_{i+1,} = b_i g, \qquad i = J+1, \ldots, I-1.$$

The initial benefit is the sum of the net wages multiplied with the *accrual rates* of the working period and the continued benefits are the wage indexed initial benefits. The accrual rates represent the speed at which pension benefits build up with each decade of service.

Another element needed in the budget constraint is the interest factor, which is the interest rate + 1. In the steady state the growth rate of the population is constant, which simplifies the interest factor to

$$R = \alpha \nu g,$$

where $\nu$ is the growth factor of the population equal to $f^{\frac{1}{H}}$.

Finally, the lifetime budget constraint of the household is given by:

$$\sum_{i=L}^{I} R^{-i+L} (y_i - m_i \, c_i) = 0. \tag{2}$$

This is the *present value* of its lifetime earnings and consumptions. In each decade the household has an income of $y_i$ out of which it consumes $m_i \, c_i$, which represents the consumption of the adult, increased by the presence of children when they are also in the household. The savings from the income, given by the difference between income and consumption, are discounted with the interest factor. During its lifetime, the household some decades will save and other decades will consume more than its income, in order not to remain with unconsumed assets at its death.

## 2.3 Household's consumption

The household wants to smoothen its consumption by solving the maximization problem with the objective function (1) and the constraints (2). The optimized consumption looks like this:

$$c_L = \frac{\sum_{i=L}^{I} R^{-i+L}\, y_i}{\sum_{i=L}^{I} \delta^{(i-L)/\gamma}\, R^{(i-L)/(\gamma-1)}\, \beta_i^{1/\gamma}\, m_i}, \tag{3}$$

and

$$c_i = \delta^{(i-L)/\gamma}(R^{i-L}\,\beta_i)^{1/\gamma} c_L, \qquad i = L+1, L+2, \ldots, I. \tag{4}$$

where $c_L$ is the initial consumption and $c_i, i = L+1, L+2, \ldots, I$ is the continued consumption calculated based on $c_L$. Besides consuming, the household saves what remains, which adds up as to form the household's assets. The assets are given by the per period savings and the capitalized assets from previous periods:

$$a_i = R_i a_{i-1} + y_i - m_i\, c_i.$$

The savings are given by:

$$s_i = a_i - a_{i-1}.$$

## 2.4 Extensions

At this point the household model is a basic life-cycle model. In this section some enrichments of it will be presented, such as: habit formation, inheritance and credit constraint.

## Habit formation

The concept of *habit formation* means that a person gets used to consuming an amount which increases with time as labor productivity grows. The habit formation can be incorporated in the individual utility function as a discount of the consumption level to the initial level of consumption at age $L$:

$$u_i(c_i) = \beta_i\, m_i\, u(c_i/g^i).$$

As a result the consumption function has to be modified to incorporate the change in the utility function. Thus the initial consumption becomes

$$c_L = \frac{\sum_{i=L}^{I} R^{-i+L}\, y_i}{\sum_{i=L}^{I} \delta^{(i-L)/\gamma}\, R^{(i-L)/(\gamma-1)}\, \beta_i^{1/\gamma}\, m_i\, g^{i-L}} \tag{5}$$

and the continued consumption

$$c_i = \delta^{(i-L)/\gamma} (R^{i-L}\, \beta_i)^{1/\gamma} c_L\, g^{i-L}, \qquad i = L+1, L+2, \ldots, I. \tag{6}$$

## Inheritance

Until now a person (household) who died did not leave the assets that remained to his children. To incorporate more realism into the model, the problem of inheritance is solved in this section. A person leaves at his death, at age $i$, a share $\kappa$ of the capital value of his lifetime earnings. The amount left as *bequest* is denoted with $q_i$:

$$q_i = \kappa \sum_{j=L}^{I} R^{I-j}\, w_j.$$

The introduction of inheritance modifies the income of the household:

$$\hat{y}_i = y_i + \begin{cases} q_i/f & \text{if } i = F; \\ -q_i & \text{if } i = I; \\ 0 & \text{otherwise.} \end{cases}$$

A person receives a bequest at the age $F = I - H$ when his father dies and will leave a bequest to his children when he dies at the age of $I$. The bequest received is divided among all children that live in the household. As a result, the income of the household increases in decade $F$ with the bequest received and decreases in the last decade of life $I$ with the bequest left. Under these conditions the new initial consumption is given by

$$\hat{c}_L = \frac{\sum_{i=L}^{I} R^{-i+L}\, \hat{y}_i}{\sum_{i=L}^{I} \delta^{(i-L)/\gamma}\, R^{(i-L)/(\gamma-1)}\, \beta_i^{1/\gamma}\, m_i\, g^{i-L}}, \tag{7}$$

while the continued consumption considers $\hat{c}_L$ to start from. The assets and savings are calculated with the modified consumption also.

## Credit constraint

The realism of the model is enhanced by the introduction of the credit constraint. Young persons usually have problems obtaining credits due to having insufficient assets for collateral. A solution to the problem of the credit constraint is to change the way a household optimizes. Instead of optimizing for a length equal to the life expectancy the adult, it should optimize until a given age situated between the arrival of the bequest and the decade the children leave the family. So, the optimization should be split at age $V$:

$$F \leq V \leq L + H \quad \text{or} \quad L + H \leq V \leq F.$$

The household will optimize firstly until age $V - 1$ and then starting from $V$ until age $I$. The following notation makes the formulae simpler:

$$M = \begin{cases} V - 1 & \text{if } L \leq i \leq V; \\ I & \text{if } V < i \leq I. \end{cases}$$

The more complete initial consumption equation is given by

$$\tilde{c}_L = \frac{\sum_{i=L}^{M} R^{-i+L}\,\hat{y}_i}{\sum_{i=L}^{M} \delta^{(i-L)/\gamma}\,R^{(i-L)/(\gamma-1)}\,\beta_i^{1/\gamma}\,m_i\,g^{i-L}}, \tag{8}$$

and the continued consumption (6) is calculated based on $\tilde{c}_L$.

# Chapter 3. Results

The presence of children's consumption makes the model more realistic. In this chapter I present how the results change when children's consumption is omitted from the model, an approach similar to the one from Borsch-Supan, Ludwig, and Winter (2004). I do the sensitivity analysis for the two steady states of the model, the initial state which corresponds to the world not disturbed yet by demographic changes, and the final state after the wave of changes cooled down. I start with the O.–S. scenario, when $L = 2$ and $H = 3$, after which I check the results when $L$ is increased to 3, and finally a case in which $H$ is increased to 4.

The simulation for the household starts when a person is aged $L$, the decade he starts to work. At age $H$ he gives birth to $f$ children, retires at age $J = 5$ and dies at age $I$. The model is calibrated to the initial wage $w_L = 1$ and all the results presented in this section are normalized to it. The per-decade productivity increase of labor is $g = 1.0175^{10}$ and the relative interest factor $\alpha = 1.015^{10}$. The two parameters $\alpha$ and $g$ determine the discount factor $\delta = 1/(\alpha g)$. The utility multiplier $\beta = 0.7$, while the coefficient of intertemporal substitution on the CRRA utility function is $\gamma = 4$. The consumption of a household is multiplied by $\mu = 0.5$ when the household has to finance the consumption of its children. A person leaves a share of $\kappa = 0.05$ out of his lifetime earnings as bequest.

## 3.1 Importance of Children's Consumption

Many OLG models consider the consumption of the household being the amount the adult consumes. From the empirical literature it can be noticed that the lifetime consumption path is not smooth, but it has a hump, created by the presence of the children. This means that when children are born, the consumption of the household increases, after

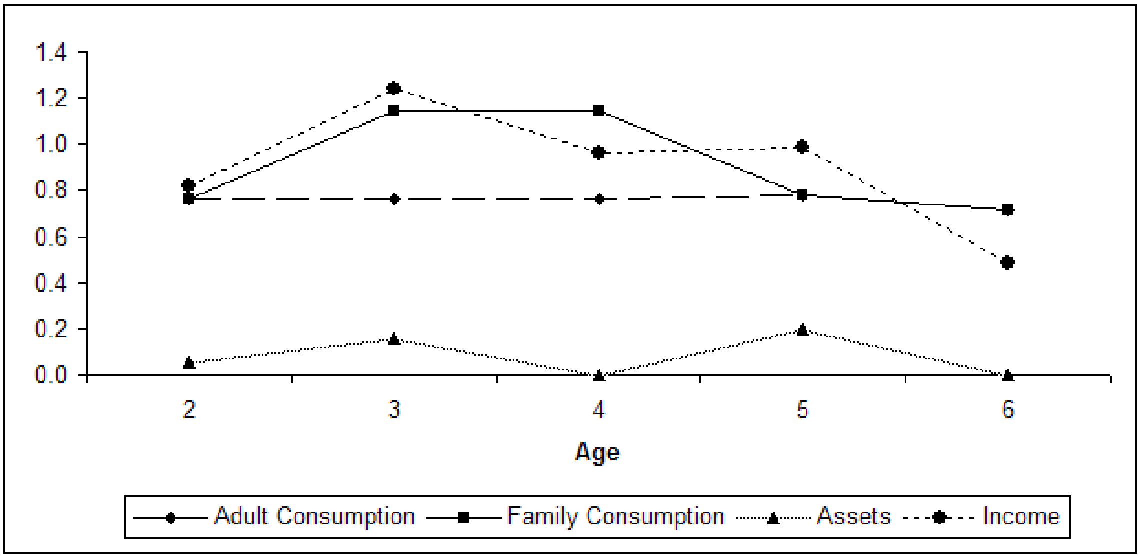


Figure 1: Initial Steady State – with Children's Consumption

which when the children leave the family, it decreases to a level close to the initial one. In this section I present the evolution of income, consumption and assets for the O.–S. scenario ($L = 2, H = 3$), and analyze the cases when children's consumption is accounted for and when it is not. I take separately both steady states and present the results of the simulations.

## Initial Steady State – Before Demographic Changes

Before 1970 the populations of the developed countries were growing at a constant rate, so the problems of self-sustainability of the PAYG pension systems were not present. In the simulation of this steady state the fertility rate is $f = 1$ and people are living until the age of $I = 6$. As can be seen in Figure 1 a person starts his own household at age $L = 2$ when he starts working and earning money. He solves his maximization problem which gives him a smooth consumption throughout his entire lifetime. Due to credit constraint he cannot smooth the consumption at will, he has to take into account that his assets cannot be negative. So he uses a heuristic algorithm and determines that the best age for the first period over which he should optimize is $M = 4$. It is a good age because in the next decade his children will leave the household so, having less burden, his income will again be enough for him. His consumption during the first three decades as the head of the household is 0.764, which decreases to 0.716 after he retires at age $J = 5$ (see Appendix). His consumption has its peak of 0.783 after his children leave the household in decade $L + H = 5$. But the consumption of the household is larger than the adult's consumption due to the presence of children. In decade $L = 2$ when there are no children yet, the consumption of the household is equal the adult's consumption. One decade later, in $H = 3$, when the children are born, the household's consumption increases to 1.146 due to children's consumption and maintains this level the next decade while the children are still in the household. At household age $L + H = 5$ children leave the household, so the children's component of consumption is again 0.

At age $L = 2$ the income of the household is just 0.82, the net wage the person gains. In the next decade, in $H = 3$ the person receives the bequest after his parent's death. This boosts the income of the household to the level of 1.237. During the next decade, the income returns to the net wage level of 0.965, which grew due to seniority and labor productivity growth. While retired, during decade 6, the person receives a pension of 0.483. The household cannot gather a large amount of assets due to the high level of expenditures. Its assets rise to 0.156 during the third decade, to fall to 0 again due to the consumption increased by the presence of the children. After the children leave, in decade 5 the household can gather a larger amount of assets 0.201, but it does not have much time to consume it. What it does not consume, it leaves as a bequest to its children.



**Sensitivity Analysis**

Neglecting children's consumption in the household by having $\mu = 0$, changes the results. The consumption of the household is equal to the consumption of the adult in the house-

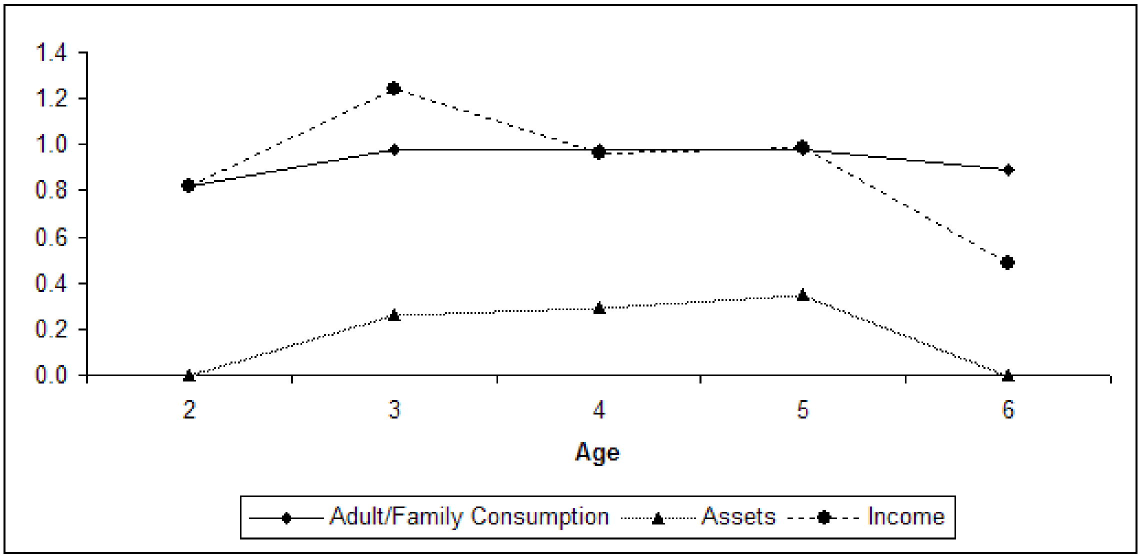


Figure 2: Initial Steady State – no Children's Consumption

hold. Children's consumption is zero in this case. As Figure 2 shows, the income of the household remains the same, but the consumption and the assets change. Disregarding children's consumption, the adult can consume more. His initial consumption is 0.82 in this case, increases to 0.975 and remains at this level for the next three periods and decreases to 0.891 after retirement. Not dealing with the consumption of children, the consumption of the household did not increase for the household ages of $H = 3$ and 4 while the children were present. As a result of solving the optimization, using the heuristic method the household determined that $M = 2$, so it consumed all its income during the first decade of work and then reoptimized for the rest of its life. The assets that it could gather is much higher, constantly increasing until the age of 5 where it reaches a peak level of 0.352. During his last decade of life, the adult consumes a part of the remaining assets and leaves as bequest the rest.

A more direct comparison of the two consumptions cannot be done for the O.–S. scenario. The reason is that when children's consumption is taken into account the heuristic algorithm cuts the optimization in decade $M = 4$, while in the other case in$M = 2$. In this section it could be seen that the value chosen for $\mu$ matters, changing the shape of the paths of consumption and assets. The next two sections present scenarios which permit a direct comparison of the consumptions and assets, the optimization process being the same.

## Final Steady State – After Demographic Changes

It is forecasted in Oksanen and Simonovits (2008) that the effects of the demographic changes will have ended by 2080. The new steady state is characterized by a fertility rate of $f = 0.79$ and a life expectancy of $I = 7$. Again the PAYG pension system is sustainable because the number of young is enough to guarantee a decent pension for the elderly. As presented in Figure 3, the adult's initial income, his net wage, at age $L = 2$ is 0.644. It is lower than in the initial steady state because the pension contributions deducted from the wage are higher, and because the adults live longer with one decade so there is the need to guarantee a pension for a longer period of time. During the next decades the income increases due to the increase in seniority and labor productivity. The adult's father dies when he is $F = 4$ decades old and leaves him a bequest which boost his period 4 income to 1.121. This is the largest level of income the household experiences throughout its lifetime. From age 4 on, the income decreases, especially because the adult retires when he is $J = 5$ decades old and lives on his pension until he dies in $I = 7$. Due to the increase of the life expectancy of people, they are pensioners for two decades instead of one, while receiving a pension, which is a relatively low income.

In his last decade of life $I = 7$, the adult's income drops to 0.35, way below his level of consumption. The adult has to maintain the household's consumption out of his income. So he solves his maximization problem and smoothens his consumption throughout his lifetime. He thinks appropriate to optimize until age $H = 3$, the decade before the arrival of the bequest and reoptimize from the decade of receiving the bequest until he dies.

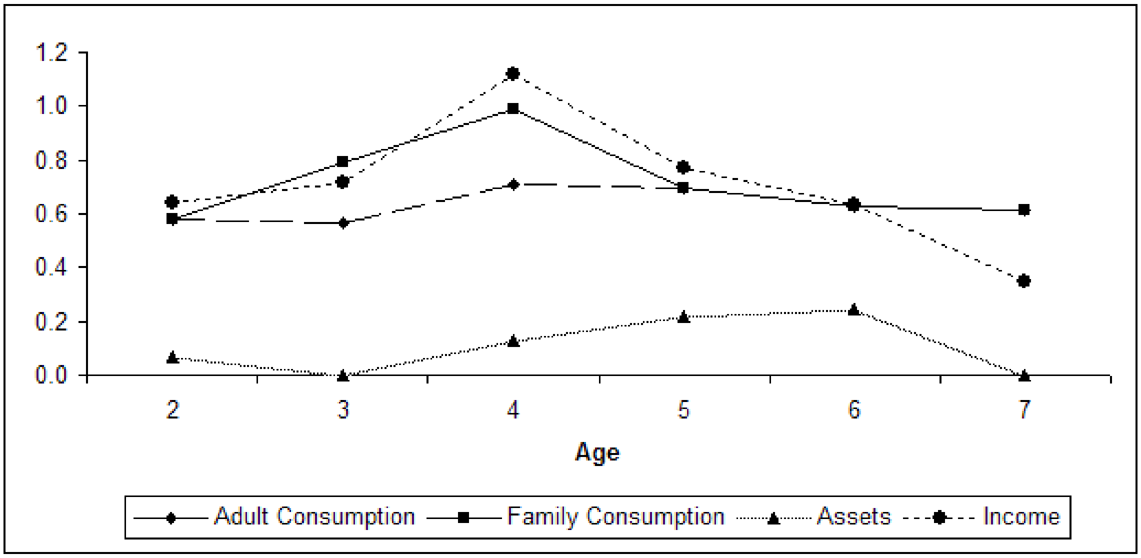


Figure 3: Final Steady State – with Children's Consumption

As a result he starts off with an initial consumption of 0.577 which decreases a little to 0.565 the decade his children are born. His level of consumption reaches its peak the next decade, when he receives the bequest from his father, which allows for an increase in the consumption. Starting with decade 5 his consumption starts to decrease slowly, reaching 0.612 in his last decade of life. The consumption of the household is increased to 0.789 by the arrival of the children during decade $H = 3$, being larger than the adult's consumption, also the next decade when the children are still in the household. The household succeeds in gathering assets mainly after the arrival of the children and the receipt of the bequest, peaking at 0.245 during decade 6.



**Sensitivity Analysis**

Having a $\mu = 0$, in other words overlooking the presence of children in the household, reveals different results. Surprisingly, the heuristic algorithm, that searches for an intermediary decade in the lifespan, $M$, where the optimization should be split does not give meaningful results. In other worlds, the credit constraint is violated for all $M$. Searching for a solution

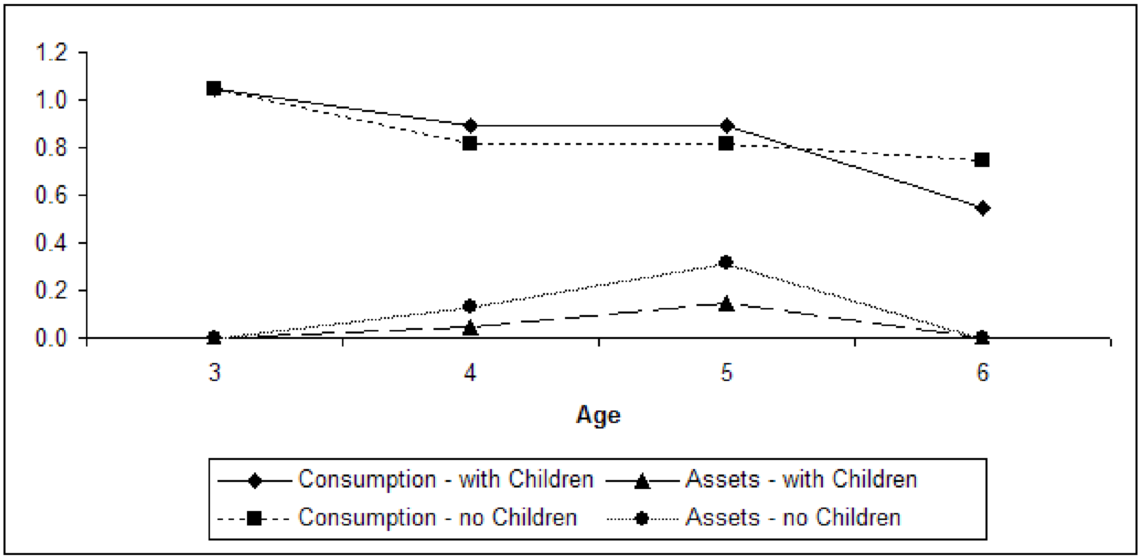


Figure 4: Consumption and Assets in the Initial Steady State, $L = 3, H = 3$

to this problem is not the focus of the current paper. As a result the sensitivity analysis cannot be done for this final steady state, for the O.–S. scenario.

## 3.3 Late Start of Work

In this section I investigate what happens if a person starts to work later, in year $H = 3$, and gives birth to children in the same year, $H = 3$. This situation has become popular in recent years; people study more so they start working later, and usually they give birth to children the moment they have a steady job.

## Initial Steady State – Before Demographic Changes

The difference between having children's consumption in the model or not, can be seen from Figure 4 which plots consumption and assets in both cases. As it can be seen the value of $\mu$ is important. During the second decade both types of household consume all their income. The income of this decade is high because the household received a bequest. The consumption of both types of household decreases during the fourth decade. When

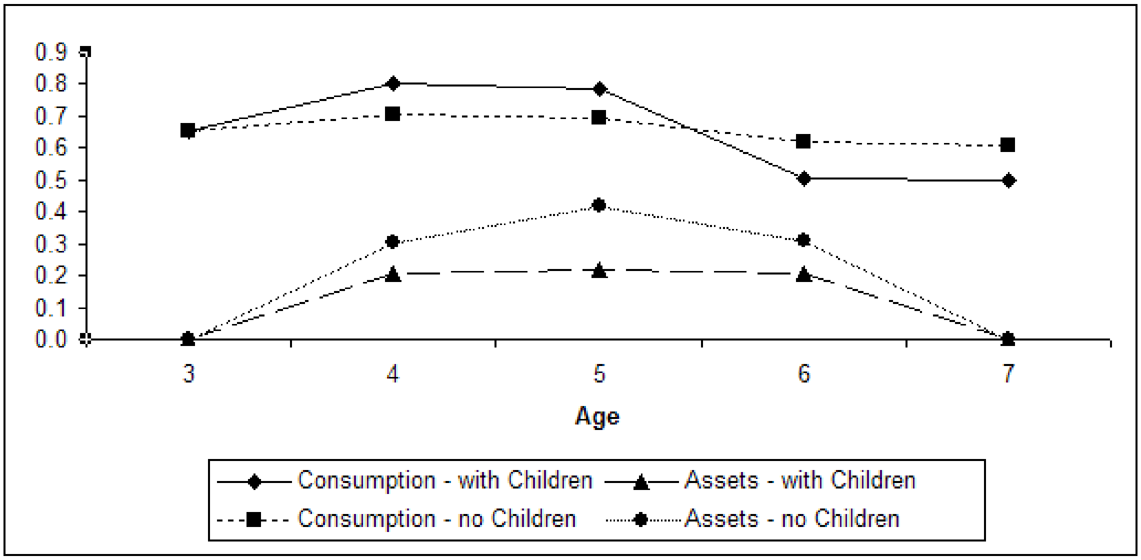


Figure 5: Consumption and Assets in the Final Steady State, $L = 3, H = 3$

children's consumption is taken into account, the new consumption level is 0.895. In the other case, the decrease is larger, being 0.815. This level of consumption is maintained also during the next decade. The consumption of the last decade, when the adult is pensioner, decreases further, reaching 0.546 when children's consumption is accounted for, and 0.746 in the other case. The difference in the two types of consumption being significantly larger during this last decade.

Changing the focus to the assets, it can be noticed that the optimizations of the consumption paths are different in the two cases. The level of the assets is determined by the choice of consumption. Because during the first decade the household consumes all its income, the assets are zero. When children's consumption is not taken into account, the level of the assets is larger than in the other case for each decade, and during decade $J = 5$ is double. Having the highest level of assets during the last decade of work agrees with the life-cycle hypothesis.

### Final Steady State – After Demographic Changes

When children's consumption is considered, the consumption path gets the humped-shape noticed by the empirical literature as can be seen in Figure 5. The consumption in both types of household starts from the same level, 0.653; during the first three decades, when children increase the consumption level of the household, it is larger. On the other hand, after they leave the family, the other type of consumption becomes the larger one. This is intuitive since ignoring children's consumption, its path is smoother than disturbed by the birth of children. During this steady state the difference between the two types of consumption is larger than before the demographic changes.

Having a longer life span, the household can gather more assets, which reach the peak during the last decade of work. The levels of assets when children's consumption is neglected is larger. During decade $J = 5$, the household which does not account for children's consumption has twice as many assets as the other type of household.

## 3.4 Very Late Children

In this last section I present the case when the household is formed at the age of $L = 2$ and children are born at age $H = 4$. There is a long time until the consumption increases due to the children, so the household can gather more assets in the meanwhile and face easier the higher consumption.

### Initial Steady State – Before Demographic Changes

When children's consumption is ignored the household's consumption path is rather smooth, being a little larger during the first decade, when the bequest is received. From Figure 6 it can be noticed that, when children's consumption is accounted for, the household's consumption is higher during decades 4 and $J = 5$. But because the income in both cases

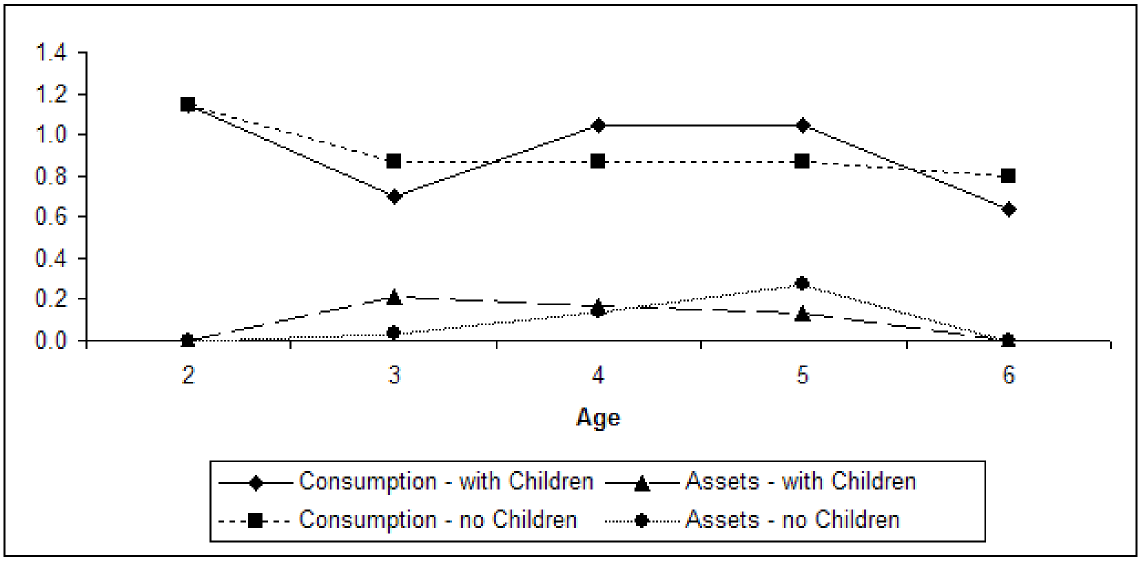


Figure 6: Consumption and Assets in the Initial Steady State, $L = 2, H = 4$

is the same, during the other decades it has to be the same or smaller.

The increase in consumption has a negative effect on assets. In the case when children's consumption is accounted for, the household's consumption increases from decade 3 until decade $J = 5$, but the corresponding assets decrease in this period. When children's contribution to the consumption is ignored, its decreasing path goes along with the increasing path of the assets. During retirement the assets in both cases decrease, because the household consumes them. The difference in the assets for the two types of consumption can be noticed during decades 3 and $J = 5$.

## Final Steady State – After Demographic Changes

Because life expectancy has increased to $I = 7$ and the households live longer, the ancestors receive the bequest during decade 3, which is not the household's first decade of existence. This can be seen in Figure 7 when children's contribution to the consumption is ignored. The level of consumption during the second decade is 0.657, which increases due to the bequest to 0.808 a decade later. The rest of its path is smooth and slowly

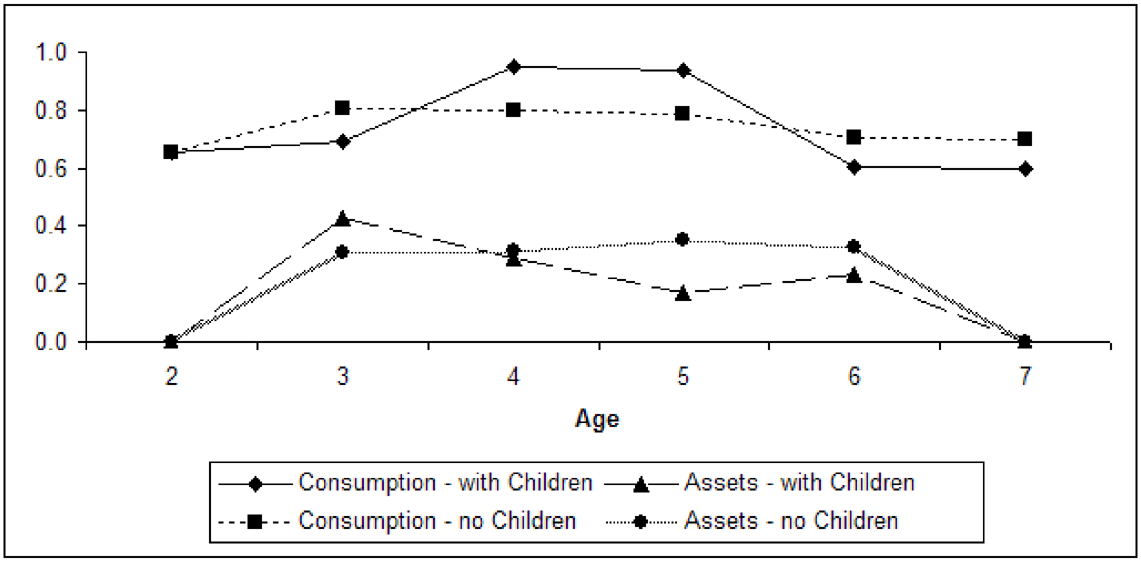


Figure 7: Consumption and Assets in the Final Steady State, $L = 2, H = 4$

decreasing, reaching 0.697 in the last decade. Taking into account children's consumption, the consumption level rises to 0.950 and 0.936 during decades 4, respectively $J = 5$. These values are well above those when children's consumption is ignored. On the other hand, during the other decades, 3, 6, $I = 7$ it is lower because the income constraints it to lower values.

The assets, when children's consumption is not accounted for, follow the life-cycle theory, by increasing until the last decade of work and decreasing afterwards. In the other case, children's consumption disturb its path by making a hole in the assets during decades 4 and $J = 5$, when the size of the household is larger. The differences between the two setups of the model being again clear.

# Conclusion

In this thesis I did a sensitivity analysis of a simplified version of the OLG model from Oksanen and Simonovits (2008), which did not include the demographic transition phase. More precisely, I analyzed the importance of modeling children's consumption and the robustness of the results as the values of some parameters change, in the steady states of their model.

A feature of the model is that household's consumption increases when children are born. I found that the path of consumption has the well known hump shape when children's consumption is accounted for and that it is smooth if children's consumption is ignored. For the decades when children were present in the household and their consumption considered, the level of consumption was always higher than the other case when children's consumption was ignored. But this was reversed for the decades when children were not born yet or left already the household. This behavior of the consumption in the two cases is because the income remains the same: if in some decade one type of consumption is higher, during some other decade the other type has to be higher. As a result, the total consumptions stayed approximately the same. The assets are determined by the consumption; in the case when children's consumption was accounted for, the household gathered less assets than in the other case.

In addition, I checked how the model works when the decades of starting to work and of giving birth changed. I found that the results were similar to the O.–S. scenario, both when the adult started to work a decade later, in $L = 3$ and gave birth in $H = 3$ and when he started to work in $L = 2$ and gave birth in $H = 4$. The results for the O.–S. scenario were obtained for the initial steady state, and the results for the modified $L$ and $H$ scenarios were obtained for both steady states.

In conclusion, this steady state model based on Oksanen and Simonovits (2008) does not give robust results for the values of consumption and assets because, by excluding children's consumption from the model, the results change. This means that in order to correctly model the life-cyle of the household one has to take into consideration the presence of children. By ignoring the fact that children increase the consumption of the household, one does not obtain correct results. Further sensitivity analysis can be done to children's consumption in the transition phase of the original Oksanen and Simonovits (2008) model, but also for other specifications like the presence of bequest, the moment of death or habit formation. Including all these characteristics that a household has, makes the OLG models closer to reality and more suitable for analyzing population aging and its impact on the economy of a country. This paper has shown that children's consumption is such a characteristic.

# Appendix

Table 2: Initial Steady State: The O.–S. Scenario

| Age | Income | Adult Consumption | Family Consumption | Assets |
|---|---|---|---|---|
| **A. With Children's Consumption** $(I = 6, f = 1, \mu = 0.5)$ | | | | |
| 2 | 0.820 | 0.764 | 0.764 | 0.056 |
| 3 | 1.237 | 0.764 | 1.146 | 0.156 |
| 4 | 0.965 | 0.764 | 1.146 | 0 |
| 5 | 0.984 | 0.783 | 0.783 | 0.201 |
| 6 | 0.483 | 0.716 | 0.716 | 0 |
| **B. No Children's Consumption** $(I = 6, f = 1, \mu = 0)$ | | | | |
| 2 | 0.820 | 0.820 | 0.820 | 0 |
| 3 | 1.237 | 0.975 | 0.975 | 0.263 |
| 4 | 0.965 | 0.975 | 0.975 | 0.296 |
| 5 | 0.984 | 0.975 | 0.975 | 0.352 |
| 6 | 0.483 | 0.891 | 0.891 | 0 |



Table 3: Final Steady State: The O.–S. Scenario

| Age | Income | Adult Consumption | Family Consumption | Assets |
|---|---|---|---|---|
| **With Children's Consumption** $(I = 7, f = 0.79, \mu = 0.5)$ | | | | |
| 2 | 0.644 | 0.577 | 0.577 | 0.068 |
| 3 | 0.716 | 0.565 | 0.789 | 0 |
| 4 | 1.121 | 0.71 | 0.991 | 0.131 |
| 5 | 0.773 | 0.696 | 0.696 | 0.217 |
| 6 | 0.636 | 0.625 | 0.625 | 0.245 |
| 7 | 0.350 | 0.612 | 0.612 | 0 |

Table 4: Initial Steady State: $L = 3, H = 3$

| Age | Income | Adult Consumption | Family Consumption | Assets |
|---|---|---|---|---|
| **A. With Children's Consumption** ($I = 6, f = 1, \mu = 0.5$) | | | | |
| 3 | 1.045 | 0.697 | 1.045 | 0 |
| 4 | 0.943 | 0.597 | 0.895 | 0.048 |
| 5 | 0.984 | 0.597 | 0.895 | 0.144 |
| 6 | 0.379 | 0.546 | 0.546 | 0 |
| **B. No Children's Consumption** ($I = 6, f = 1, \mu = 0$) | | | | |
| 3 | 1.045 | 1.045 | 1.045 | 0 |
| 4 | 0.943 | 0.815 | 0.815 | 0.127 |
| 5 | 0.984 | 0.815 | 0.815 | 0.316 |
| 6 | 0.379 | 0.746 | 0.746 | 0 |

Table 5: Final Steady State: $L = 3, H = 3$

| Age | Income | Adult Consumption | Family Consumption | Assets |
|---|---|---|---|---|
| **A. With Children's Consumption** ($I = 7, f = 0.79, \mu = 0.5$) | | | | |
| 3 | 0.653 | 0.468 | 0.653 | 0 |
| 4 | 1.012 | 0.576 | 0.804 | 0.208 |
| 5 | 0.783 | 0.565 | 0.788 | 0.217 |
| 6 | 0.481 | 0.507 | 0.507 | 0.207 |
| 7 | 0.275 | 0.497 | 0.497 | 0 |
| **B. No Children's Consumption** ($I = 7, f = 0.79, \mu = 0$) | | | | |
| 3 | 0.653 | 0.653 | 0.653 | 0 |
| 4 | 1.012 | 0.705 | 0.705 | 0.306 |
| 5 | 0.783 | 0.692 | 0.692 | 0.420 |
| 6 | 0.481 | 0.620 | 0.620 | 0.311 |
| 7 | 0.275 | 0.608 | 0.608 | 0 |

Table 6: Initial Steady State: $L = 2, H = 4$

| Age | Income | Adult Consumption | Family Consumption | Assets |
|---|---|---|---|---|
| **A. With Children's Consumption** $(I = 6, f = 1, \mu = 0.5)$ | | | | |
| 2 | 1.146 | 1.146 | 1.146 | 0 |
| 3 | 0.911 | 0.697 | 0.697 | 0.214 |
| 4 | 0.965 | 0.697 | 1.045 | 0.168 |
| 5 | 0.984 | 0.697 | 1.045 | 0.133 |
| 6 | 0.483 | 0.638 | 0.638 | 0 |
| **B. No Children's Consumption** $(I = 6, f = 1, \mu = 0)$ | | | | |
| 2 | 1.146 | 1.146 | 1.146 | 0 |
| 3 | 0.911 | 0.872 | 0.872 | 0.038 |
| 4 | 0.965 | 0.872 | 0.872 | 0.138 |
| 5 | 0.984 | 0.872 | 0.872 | 0.271 |
| 6 | 0.483 | 0.798 | 0.798 | 0 |

Table 7: Final Steady State: $L = 2, H = 4$

| Age | Income | Adult Consumption | Family Consumption | Assets |
|---|---|---|---|---|
| **A. With Children's Consumption** $(I = 7, f = 0.79, \mu = 0.5)$ | | | | |
| 2 | 0.657 | 0.657 | 0.657 | 0 |
| 3 | 1.119 | 0.691 | 0.691 | 0.428 |
| 4 | 0.774 | 0.681 | 0.950 | 0.292 |
| 5 | 0.788 | 0.671 | 0.936 | 0.171 |
| 6 | 0.649 | 0.605 | 0.605 | 0.232 |
| 7 | 0.342 | 0.596 | 0.596 | 0 |
| **B. No Children's Consumption** $(I = 7, f = 0.79, \mu = 0)$ | | | | |
| 2 | 0.657 | 0.657 | 0.657 | 0 |
| 3 | 1.119 | 0.808 | 0.808 | 0.310 |
| 4 | 0.774 | 0.797 | 0.797 | 0.317 |
| 5 | 0.788 | 0.785 | 0.785 | 0.350 |
| 6 | 0.649 | 0.707 | 0.707 | 0.325 |
| 7 | 0.342 | 0.697 | 0.697 | 0 |

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