CAPITAL FLIGHT FROM THE DEVELOPING COUNTRIES

An investment risk based model

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Abstract

In the last three decades numerous developing countries suffered from the adverse impacts of fast capital outflow. In my thesis I concentrate on the explanation why and how these sudden shifts in the international capital movements occur. I set up a model investigating the behavior and the interplay of three different sectors; financial investors, governments and households. Using its outcomes, I determine how the single exogenous factors affect the optimal level of the developing country's investment share. Afterwards, I define the concept of capital flight in my model and I set up two scenarios with different initial conditions, but with the same final result: because of the interplay of the different risk factors a *circus vitious* situation occurs and as a consequence the previously invested capital flee out of the country within a short period. The general message of the model is simple: a highly indebted and risky developing country can not save itself once the capital flight procedure launched. Secondly, the developing country does not have to be 'guilty' to suffer from capital flight. Thirdly, the developing country should avoid creating 'stop and go' cycles. Finally, 'too' fast capital inflow can cause serious problems as well.

TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION[1]

One of the most spectacular economic phenomenons of the last three decade was the rapid expansion of the international capital market. The extent of cross-border credit relationship has been growing significantly; turnover on the foreign exchange markets has been going up to previously unimaginable heights and numerous innovative derivative products appeared creating new, fast growing financial markets. The other important feature was that this development spread out geographically as well; even more developing countries became active participant on the above mentioned markets. However, this process was not a smooth one, the fast increasing openness and financial vulnerability caused serious difficulties in some countries. Numerous developing country Capital account and/or debt crises occurred and as a result a significant part of the previously invested capital has flown out of the affected countries within a very short period. More interestingly, it seemed that in some cases not only 'guilty' countries but 'innocent' economies were hit seriously.

In my thesis I focus on why and how capital flight evolves. My major aim is to understand what the key explanatory factors in this process are. I discuss the issue from a theoretical point of view setting up a model in which first I investigate the major determinates of capital flight first separately and afterwards I focus on the effect of the interplay of the different factors.

The structure of my thesis is quite straight: in Chapter 2. I shortly summarize the related literature focusing on the definition of capital flight and the on the key explanatory elements found by the empirical researches. In Chapter 3. I set up the basics of my model, describing the optimization problem and the solution of the three involved economic sectors. At the end of this section I get a formula for the optimal level of the developing country's

[1] I am grateful to Julius Horváth and Thomas Rooney for their useful comments and their helpful attitude and to Èvi for being patient with me in most of the times.

share from the global investment which serves as a basis for Chapter 4. In this part I investigate how the change of the pre-defined exogenous variables affects the optimal level of investment allocation separately. Using these results I set up two scenarios in Chapter 5. where I assume that the different exogenous variables – risk factors – are interrelated. This feature helps to describe how capital flight procedure can develop. Finally, the last chapter summarizes my thesis and concludes the general message of my model.

CHAPTER 2: LITERATURE ON CAPITAL FLIGHT

The economic literature uses various definitions for capital flight simultaneously. The reason behind it is that the authors see differently what is the major force the major determinants of the capital movements. As a consequence the measurement of capital flight varied largely because of the different underlying definition.

Already in the early literature of capital flight the authors distinguish among the different versions. Dornbusch (1990) makes difference between the two forms of capital flight. He argues that the first type is a sudden outflow of the capital stock which is motivated by the fear of the capital owners that an expected significant change in the country's economic background results in large losses; whereas the second type of capital flight is a much slower but continuous process inspired by tax consideration or by the underdevelopment of the local capital market. Brown (1992)'s approach is close to the first type of definition where he considers the flight as a capital movement which is composed of funds fleeing across national borders in search of sanctuary. Gunter (2003) referring to this approach emphasises the importance of the second type of capital flight in his paper investigating the capital flight from China and Hong Kong in the last two decades. Currency crises literature and some political economy papers (like Le and Zak (2006)) use first-type definition of capital flight, whereas the in paper of development economics investigating the capital flight from a given region (e.g. Boyce and Ndikumana (2000) in case of Africa, Loungani and Mauro (2001) for Russia or Patnaik and Vasudevan (2000) for India) the second version of the capital flight is dominant.

Since there does not exist a single definition of capital in the international literature, I have to make clear what I understand below the concept of capital flight. The definition I use in my paper belongs to the first line of approaches; I think on the capital flight as a sudden movement of capital out of a country which is motivated basically by financial market panics where the investors being afraid of a potential loss withdraw the large extent of capital from a risky country. After setting up my theoretical model I give a more detailed and exact definition below[2].

Since there is not a standard definition of capital flight the results of the different researches varies significantly. In a theoretical paper I do not want to go into the details about the extent and the orientation of the capital flight. However, it is more interesting how the authors explained their results; which factors were the major determinant of the capital flight. Authors investigating region reported similar causes: according to Collier, Hoeffler and Pattillo (2001) the three variables which explained the African capital flight were exchange rate over-valuation, adverse investor risk ratings, and high level of indebtedness. Loungani and Mauro (2001) show that in case of Russia the macroeconomic instability, the random tax system, the lack of confidence in the banking and property right system and the large extent of corruption encouraged the excessive capital outflow. Patnaik and Vasudevan (2000) refer to a further cause in case of India, basically the different treatment of the resident and the foreign nationals. The capital account became open for the later group; however residents could not take out capital freely. This restriction led finally to a slow capital flight in a form of trade misinvoicing. Dooley and Kletzer (1994) mentions also that the "capital flight represents an arbitrage of the different treatment of resident and non-resident investors by domestic authorities."[3] Kant (1996) also investigates the effect of the preferential treatment of foreign capital on a broader sample. He finds that this factor is a significant determinant of capital flight, but he points out that economic mismanagement and inefficiencies play more important role. Auguste, Dominguez, Kamil and Tesar (2002) emphasises the adverse effect of the bad

[2] See section 5. Capital Flight.

[3] See Dooley and Kletzer (11), page 28.

economic policies in case of Argentina too.

Examining the group of the developing countries until the 1980s Rojas-Suarez (1990) finds that the two major causes of capital flight were “the risk of expropriation of domestic assets and the risk of losses in the real value of domestic assets resulting from inflation or exchange rate devaluations.”[4] Authors investigating the trends of later decades lay the emphasis on the impact of macroeconomic instability. Schineller (1997) - investigating seventeen developing countries - claims that the “substantial fiscal and current account deficits, overvalued exchange rates, high and/or volatile inflation, and ambitious financial sector liberalization most commonly generate flight.”[5] Another group of authors focus on the effect of the political and policy instability. Hermes and Lensink (2001) show that “policy uncertainty, measured by uncertainty of budget deficits, tax payments, government consumption and real interest rates appears to have a statistically significant positive impact on capital flight”[6]. According to them this process is quite simple; they argue that “as long as government policies and their impact on the real value of wealth are unclear, residents decide to take their money and run, since real returns on foreign assets are clear and certain.”[7] Le and Zak (2006) decomposes the policy and political risk in ‘sub-factors’. They note that there are such political events – like unconstitutional government change or uprising - which boost capital flight; nevertheless, they found negative relationship between other type of political events – e.g. constitutional government change or collective protest – and the extent of capital flight. Finally the motive of tax evasion in Schineller (1997-2) or the simple case of fraud in Eaton (8) were also mentioned that important determinants of capital flight.

[4] See Rojas-Suarez (1990), Summary section, Page iii,

[5] See Schineller (1997) page 19.

[6] Hermes and Lensink (2001), page 10.

[7] Hermes and Lensink (2001), page 10.

CHAPTER 3: THEORETICAL MODEL

In this section I set up a theoretical model trying to integrate the most important approaches described in Chapter 2[8]. The major goal of the model is to consider the key determinants influencing the level and the direction of the international capital movements on the financial markets. Besides reviewing the effect of the most important factors, the model focuses on explanation why sudden shifts, i.e. capital flights from the developing countries can occur.

The simplified world of my model consists of two countries: a developing and a developed economy. This structure helps to focus on the interaction of the two types of countries and to investigate the effect of the risk differentials among the entities. I assume that the advanced economy has reached steady state level of economic growth while the developing economy makes an effort to catch up with his neighbour. Furthermore, I presume that both the total output and the income per capita level are significantly larger in the advanced economy. This enables the developed country to follow independent monetary and fiscal policy.

The model examines the financial, the government and the household sectors. Each of those three optimizes according to their utility function; investors collect the savings of the households and maximize the risk adjusted return of their investment. Governments set the optimal level of their transfers to households from their resources originated from taxes or borrowing. Finally, the household sector optimizes its utility of consumption by dividing their income among consumption and savings.

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[8] I got the basic impressions for setting up my model from Le and Zak (2006). In that paper the authors construct a model which examines a developing country and its relationship with the rest of the world. This is similar to my set up. However, that study investigates only the behavior of one sector and focuses only on the effect of policy risk.

I do not investigate the production sector in the model explicitly, but I assume that it is integrated into the government's decision problem[9]. Furthermore I ignore all the revenue from the labour activity and I presume that in this world only capital income exists. The reason behind this is that I focus on the financial determinants of the capital flights and therefore – for simplicity – I try to ignore the real sector as much as possible. A further important feature of my model is that the free movement of capital is assumed; no capital control is introduced which means the investors in both countries can invest any part of the domestic savings abroad. Finally, I introduce such a notation in the model where – if not mentioned explicitly – the subscripts after the variables always refer to the period and the superscripts describes the country of origin (where A notes the advanced economy, while D regards the developing one.)

3.1 Order of decisions

All the representatives of three sectors have to make a decision. The process happens in the following order. Firstly, the Investor presents its optimal choice about the allocation of the total investment portfolio among the developed and the developing economy. This is a conditional decision since the result depends on the interest (r^A and r^D) and tax rates (τ^A and τ^D) offered by the two governments and on the savings level of both representative households. Secondly, the government of the developing country sets the rate of return on investment and the tax rate. The model treats the advanced economy's interest and tax policy independent and therefore its outcome is exogenous. The governments can already integrate the optimal choice of the Investor in its decision-making; however it is still depending on the savings level of the households. Finally, the representative household of the developing

[9] About the details, see section 3.3.

country defines its optimal savings level. It can already take into account the decisions of the Investor and the Government: it knows already the potential level of government transfer and the capital income. Again, I assume here that the savings level of the advanced economy's household is exogenous.

In the following sections I investigate the representatives in a more detailed way: I describe the most important features of the sectors, set their optimization problem and report the results of the decision making process.

3.2 Investors

The investor's major task is to collect all the savings from both economies and invest the raised amount into the available opportunities. Since I assume that at one decision period all the investors have the same level of risk aversion, the sector will be represented by one investor (the "Investor"). We can think of him as a representative who manages one big mutual fund[10]. The Investor's aim is to maximise the risk adjusted revenue of this fund.

The investment process is the following: firstly the Investor collects the new sums of savings from both countries, from the households. Afterwards he takes the investment decision. The available opportunities are quite limited in the model: the Investor can buy only government papers issues by either of the two governments. In his choice the Investor defines the optimal level of allocation of the resources among the two government bonds in order to ensure the highest possible risk adjusted revenue. Since the Investor is the first decision maker his choice is conditional on the interest rates, the tax rates and savings levels determined by the governments and the households later in the process. Technically this means that the Investor sets a financial investment function where the only depending variable

[10] Here I assume that the competition int he investment sector is full therefore they do not generate any profits. This results that the ownership structure is indifferent int he model since the owners do not get any extra revenue after their property.

is the ratio of the investments allocated to the developing economy compared to the total investment portfolio (α)[11] which is determined by three endogenous and by numerous exogenous variables. Once the optimal decisions of the governments and the households are known the Investor carries out the investments. At the end of the each period he gets back the full invested capital increased by the interests. Finally, the Investor divides the earned interest income among the households of both countries based on the ratio of their original savings and pays back the full amount of capital to its owner too. At the beginning of the next period the whole procedure starts again.

3.2.1 The Investment Function

The Investor's goal is to maximise the long run utility of the expected revenue from its investment. The utility function is quite simple:

(1)
$$\underset{\alpha_t}{Max} U = \sum_{t=0}^{\infty} \delta^t R_t$$

where t is the time variable, δ is discount factor measuring the weight of the future capital income in the utility function and R is the total revenue of the investment fund in each period. I assume that δ is positive, but smaller than 1, it takes the same value in both country and it is equal to the household discount factor[12]. However, it can differ from the government's discount factor (β).

The Investor maximises the total revenue by dividing the total investment budget into two parts: α part will be invested in the developing country and (1-α) part in the developed one. The investment budget is always the total savings of the new period originated from both the

[11] I presume that all the remaining fraction of the portfolio, 1-α part will be invested in the advanced country.

[12] This assumption reflects that the Investor does not have own time preference, but in his decision making procedure the Investor considers the Household's discount rate used at the consumption optimization.

advanced and the developing country:

$$I_t = S_t^{\ D} + S_t^{\ A} \tag{2}$$

where S is the saving of the households and I is the total investment of the mutual fund in each period. As mentioned above, the government should pay back all of their debt plus the offered interests in each period and they can borrow the necessary amount in the next period again. The investor pays the earned income and the original capital back to the households immediately and parallely collects the new amounts of savings and invests them into the government bonds.

The total revenue of the fund consists of two parts: the normal interest revenue and capital redemption originated from the advanced economy and the risk adjusted interest income plus the repaid investment from the developing country. Formally the total revenue equation can be described as follows:

$$R_t = \left\{1 + r_t^{\ A}\left(1 - \tau_t^{\ A}\right)\right\}\left(1 - \alpha_t\right) I_t + \left\{\left[1 + r_t^{\ D}\left(1 - \tau_t^{\ D}\right)\right] + \Theta\left(\alpha, S^D\right)\right\}\alpha_t I_t \ , \tag{3}$$

where r is the rate of return offered by the governments for the next period; τ is the tax rate levied by governments on the next period's capital income; Θ_t is a comprehensive factor reflecting how the mutual fund evaluates the extra investment risk of the developing country. In the following sections I refer to Θ_t as the risk adjustment function.

3.2.2 Risk adjustment function

Θ_t is a comprehensive function which summarises the extra risk of the investments in the developing country and reflects how the investor evaluates these risks. The function is made up of four components: it covers the financial, the policy and the exchange rate risk of the developing country and incorporates the level of the global risk aversion as well. The function takes the following form:

(4) $$\Theta(\alpha, S^D) = \xi_t \left(b\gamma_t + c\eta_t + \varepsilon_t \right)$$

where ξ denotes the level of the global risk aversion, γ refers to the financial, η to the policy and ε to the exchange rate risk. Parameters b and c are the relative weights of the financial and the policy risk, respectively.

3.2.2.1 Financial risk

The financial risk parameter of γ tries to capture the risk of non-payment. It can be thought of as a typical credit risk: if the customer's (here the government's) financial conditions deteriorate then the chance of non-payment becomes higher and the investors are willing to finance the debtor only for higher expected income. In the model I measure the financial risk by the debt-to-savings level of the developing country compared to the debt-to-savings level of the advanced economy:

(5) $$\gamma_t = \frac{\dfrac{(1-\alpha_t)I_t}{S_t^{\,A}}}{\dfrac{\alpha_t I_t}{S_t^{\,D}}} - 1 = \frac{(1-\alpha_t)}{\alpha_t}\frac{S_t^{\,D}}{S_t^{\,A}} - 1$$

The idea behind this is that the higher is the total debt compared to the total domestic savings in a country the harder is to pay back the full amount of debt when the government can involve only local financing in the future. For instance, in such a case when sudden market turbulence happens and foreign investors are not willing to invest any money in the country[13]. If γ takes a positive value, which means that the developing economy is less indebted then the advanced country then the financial risk of investment in the less developed economy is low which helps the capital inflow. However, a negative parameter value of γ

[13] I presume here that the government can always impose such administrative constraint which ensures that the domestic capital can not leave the country. Without this assumption the level of the local savings would be much less important.

reflects that the developing country is more indebted, the financial risk is higher, and therefore the investment in the developing country's bonds becomes less attractive.

3.2.2.2 Policy risk

Policy risk (η) measures the stability of the political system and so the predictability of government policies. The underlying assumption is that the more instable the government the less the chance that it can follow its long term economic program. A more fragile political system issues in a weaker and less effective institutional background and in more unpredictable fiscal and/or monetary policy. This results in higher fluctuation in the budged deficit and/or in the inflation which deteriorates the investment opportunities and decreases the capital inflow.

Regarding the value of the parameter I presume that if the predictability of the developing country's government policies is at a lower level than in the advanced country then η is negative; but if it is more forecastable than η is positive. In the model η is an exogenous variable and I do not plug in any formula instead of it. However, to get an idea we can think of η as the ratio of the standard deviation of on important economic policy indicator (like the inflation or the budget deficit) in the two countries[14].

3.2.2.3 Exchange rate risk

The parameter ε is responsible for adjusting the potential return on investment based on the expected exchange rate fluctuations. My assumption is quite simple: the more foreign capital has flown into the developing country (at the beginning of the period) the higher the chance of a future net capital outflow from the country is (by the beginning of the next

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[14] To be more precise think on η as it suggested by the following formula: $\eta_t = \frac{ST\_DEV(X_t^A)}{ST\_DEV(X_t^D)} - 1$, where ST_DEV(X) refers to the standard deviation of the indicator X (budget deficit, inflation, etc.).

period), basically because the chance that the developing economy can attract even larger amount in the next period decreases. If a net capital outflow occurs - which means that the capital inflow at the beginning of the next period is smaller then the capital and interest outflow originated from the current period's investments - then the exchange rate will depreciate. This results in a lower rate of return on investment carried out in the developing country if it is measured in the currency of the advanced economy. The role of the exchange rate risk parameter is to incorporate these depreciations (appreciations) in order to make the two countries return level comparable.

Technically this means that the larger α (the current investment ratio) is the larger the chance for a future capital flight and indeed a depreciation of the exchange rate. I use a quite simple linear function to capture this phenomenon:

(6) $$\varepsilon = d - e\alpha \ ,$$

where d, e are exogenous parameters. The formula suggests that the exchange rate change is defined exclusively by the capital market, so we ignore the effect of the real sector, basically the effect of the international trade. Parameters d and e are set in such way that if the expected change of the exchange rate is zero then the parameter takes the value of zero as well, i.e. ε(α)=0. If there is no investment in the developing country in a given period, then the model expects d*100 percent appreciation by the end of this period basically because of expecting future capital inflow. Each percent increase of α result an e percent decrease of the expected appreciation. If the rise of α diminishes the value of ε below zero then the model expects depreciation of the developing country's currency which worsens the attractiveness of the investment.

3.2.2.3 Level of Investor's risk aversion

The interpretation of the risk aversion variable, ξ is somewhat different from the other three types of risk indicators: this exogenous factor measures how investors evaluate the above described class of risks in their investment decision. Think of ξ as a kind of risk premium; it mirrors how that extra capital gain varies over time which is expected by the Investor in charge of taking certain level of investment risk. Since I assume that the Investor is risk averse, the value of ξ should be always positive, i.e. the investment risk should be always incorporated into the Investors decision, with the proper sign. Furthermore, I presume that if the risk appetite is large; i.e. investors are willing to take reasonably high risks in exchange of low expected risk premium, then the value of ξ is close to zero; however if the investor is strongly risk averse and the expected premium is high then ξ is a positive number far from zero.

Based on the four factors described above I set the final form of the risk adjustment function:

(7) $$\Theta\left(\alpha, S^D\right) = \xi_t \left[b\left(\frac{(1-\alpha_t)S_t^{\ D}}{\alpha_t\, S_t^{\ A}} - 1 \right) + c\eta_t + d - e\alpha_t \right]$$

3.2.3 Optimal choice of the Investor

Equations (1), (2), (3) and (7) describe all the equations related to the Investor's problem. Plugging in (1) the other three functions we can set up the detailed utility function of the investor:

(8) $$\underset{\alpha_t}{Max} U = \sum_{t=1}^{\infty} \delta_t^{\ t} \left[\left\{1 + r_t^{\ A}\left(1-\tau_t^{\ A}\right)\right\}(1-\alpha_t) I_t + \left\{ \left[1 + r_t^{\ D}\left(1-\tau_t^{\ D}\right)\right] + \xi_t \left(b\left(\frac{(1-\alpha_t)S_t^{\ D}}{\alpha_t\, S_t^{\ A}} - 1 \right) + c\eta_t + d - e\alpha_t \right) \right\} \alpha_t I_t \right]$$

The utility function describes that the investor maximizes the long run adjusted rate of return by setting the allocation of the investments (α) depending the endogenous tax and interest and savings rates and the remaining exogenous variables. It is easy to see that the interest rate and the tax rate can complement each other: a lower interest rate can be 'compensated' by lower level of capital taxation and vica versa, the deterring effect of the higher taxes can be lessened by higher offered interest income.

Since the investors are interested in the net rate of return the model can be simplified by using a combined variable of ρ_t reflecting the total net income on capital. This indicator can be described as follows:

(9) $$\rho_t = 1 + r_t(1-\tau_t)$$

Plugging ρ_t in (8) we get the final form of the investor's problem:

(10) $$\underset{\alpha_t}{Max}\, U = \sum_{t=1}^{\infty} \delta_t^{\,t}\left[\rho_t^{\,A}(1-\alpha_t)I_t + \left\{\rho_t^{\,D} + \xi_t\left(b\left(\frac{(1-\alpha_t)S_t^{\,D}}{\alpha_t S_t^{\,A}} - 1\right) + c\eta_t + d - e\alpha_t\right)\right\}\alpha_t I_t\right]$$

Here I assume that the parameter value of b, c, d, e are all non negative, the discount factor δ_t is between zero and one, both ρ_t^A and ρ_t^D are larger than (or at minimum equals to) one, the savings level of S_t^D and S_t^A and so the global investment amount I_t and the ratio of the developing country, α are always nonnegative. Variable ξ_t is always positive; however η_t can take any kind of value.

I do not go into the mathematical details here[15]; I just describe the solution of the Investor's maximization problem which determines the optimal level of α:

(11) $$\alpha_t{*} = \frac{\rho_t^{\,D} - \rho_t^{\,A} + a\xi_t - b\left(\frac{S_t^{\,D}}{S_t^{\,A}} + 1\right) + c\eta_t + d}{2e}$$

The result is in most cases intuitive: the share of the developing country is increasing if the net interest rate offered by the developing country (ρ_t^D) increases; if the financial and policy risk decreases (i.e. ξ and η increases) or if the net interest rate in the developing country decreases. The parameters of the exchange rate adjustment functions behave as expected: the

[15] For the full mathematical derivation please find Appendix I.

increase of the constant (d) increases the value of α, while the hike of the multiplicator e decreases the optimal level of α. The only finding which seems to be counterintuitive is that an increase in the savings ratio in the two countries (S_t^D / S_t^A) decreases α in equilibrium.

3.3 Governments

In this model I focus only on the government of the developing country (the "Government"). The government of the advanced country is assumed to maintain a sustainable level of budget deficit and debt level and its policies are highly predictable[16]. We can think of the advanced country's government as an administration which is bound by legal constraint, for instance it has to follow rule based fiscal policies. As a result, the government is unable to take measures which contradict the long term interests of the whole economy but put the short run political interest forward.

About the legal environment of the developing countries I do not have such a hypothesis. The government does not have to follow any rule-based policy; it can run – technically – any level of budget deficit and it can change the internal discount rate (β^D) altering the scope of the policies according to the actual political interest of the ruling powers. This follows that the government policies are much less predictable.

Moreover, since in the model I focus on the behaviour of the financial sector, for simplicity I do not consider the production activity separately, but it is incorporated into the government activity. Regarding this model set up I assume that the governments allocate the

[16] This assumption suggests that the value of the relative policy risk indicator, η is negative in most cases, since the outcome of the government policies are more volatile in the developing country.

financial resources among the real investment opportunities optimally in order to maximise the real output and thus the government transfer.

3.3.1 Maximization Problem of the Government

The aim of the developing country's government is to maximise the long term utility of the government transfers to the households. I assume that the Government has two types of revenues: tax on the capital income and the debt borrowed from the Investor. The tax is levied on all the capital income earners in the own country, independently whether they are residents or foreigner households[17].

The Government possesses with the following policy tools: the interest rate which enables it to set directly the rate of return paid to the investors and the tax rate which is levied on the capital income. As I have mentioned above, the two policy tools can complement each other perfectly, therefore I do not investigate them separately. However, later on the distinction of the interest and the tax rate will be important.

The optimization process happens as follows: at the end for each period – simultaneously with the payoff of the pre-defined interests - the Government collects the capital income tax. Directly after this, at the beginning of the next period it observes the behaviour of the Investor (i.e. it notices the conditional optimal decision about α) and the economic and political environment (basically considers the values of the exogenous parameters). Afterwards, the government – knowing its need for external (debt) and internal (tax) financing – defines the conditional optimal value of the policy tools which maximizes the long run utility of the transfers. This is conditional, since the Government does not know the savings level yet. The tax rate is set always in advance and the defined rate will generate revenue only in the next

[17] Technically this means the tax is paid directly by the Investor. Simply, at the end of each period the Investor receives the net capital gain only which is the offered interests reduced by the taxes.

period therefore the interest rate is responsible for attracting the necessary amount of capital inflow in the given period.

Once the households take their optimal decision the Government receives the necessary amount of investments. This defines the income side of its budget and the government knows the present value of that amount[18] which has to be paid back to the Investor at the end of the period. The difference between these two determines the possible extent of the government transfers which is disbursed to the households immediately. Finally, at the end of the period the Government pays back the due amount and the proposed return after the previously borrowed debt, collects the capital income tax and the whole process starts again.

To reach the optimal long run utility of the transfers, the Government uses a quite simple utility function:

(12) $$\underset{\rho_t^D}{Max}\, U_G = \sum_{t=0}^{\infty} \left(\beta_t^{\ D}\right)^t G_t$$

Here β^D denotes the internal discount factor of the government in period 't' defining the weights of the future government transfers. Its value is between zero and one and the superscript 't' refers to the 't'-th power in the expression. β^D is usually different from the Investor's (and Household's) discount rate of δ^D. G_t^D is the value of the government transfer in the developing country in the 't'-th period.

Based on the expenditure and income sides described above the Government's budget can be formalized as follows:

(13) $$\tau_{t-1}^{\ D} r_{t-1}^{\ D} \alpha_{t-1} I_{t-1} + \alpha_t I_t = \alpha_{t-1} I_{t-1} + r_{t-1}^{\ D} \alpha_{t-1} I_{t-1} + G_t$$

[18] It is important to take into account that the government should not have the full amount which has to be given to the Investor at the end of the period. As I mentioned before the real production sector is not considered explicitly in the model, but it is integrated into the government's activity. This means that the Government 'invests' such level of capital in the domestic production sector that ensures that at the end of the period it gets back as much capital which is just enough to fulfill the Investor's claim.

The Government has two source of income: tax from capital income levied on the households and the borrowing from the investors. At the expenditure side there are three items: the government should pay back fully the debt borrowed in the previous period; the interests after the debt and the government transfer to the households. After expressing G_t and simplifying the equation[19] I get:

(14) $$G_t = \alpha_t * I_t - \rho_{t-1}{}^D \alpha_{t-1} * I_{t-1}$$

3.3.2 Optimal Choice of the Government

At the time of its decision-making the Government already knows the optimal level of α. Plugging the expression of (11) into (14) and the result into (12) I can set the decision problem of developing country's government:

(15) $$\underset{\rho_t{}^D}{Max}\, U_G = \sum_{t=1}^{\infty} \left(\beta_t{}^D\right)^t \left[\frac{\rho_t{}^D - \rho_t{}^A + a\xi_t - b\left(\frac{S_t{}^D}{S_t{}^A} + 1\right) + c\eta_t + d}{2e} I_t - \rho_{t-1}{}^D \frac{\rho_{t-1}{}^D - \rho_{t-1}{}^A + a\xi_{t-1} - b\left(\frac{S_{t-1}{}^D}{S_{t-1}{}^A} + 1\right) + c\eta_{t-1} + d}{2e} I_{t-1} \right]$$

This is already an intertemporal maximization problem where one should take into account the effect of the internal discount rate of the government. Appendix II describes the related mathematical derivations. As a result for ρtD I get:

(16) $$\rho_t{}^{D*} = \frac{1}{2}\left(\rho_t{}^A - a\xi_t + b\left(\frac{S_t{}^D}{S_t{}^A} + 1\right) - c\eta_t - d + \frac{1}{\beta_t{}^D} \right)$$

Again, the results are intuitive in most cases: decreasing financial and policy risk (i.e. increasing ξ and η) reduces the optimal net interest rate variable of $\rho_t{}^D$. The improving expectations related to the future exchange rate movement allow a cut in the optimal net

[19] For the mathematical derivation please see Appendix II.

interest rate. The positive sign of the fraction $\frac{1}{\beta_t{}^D}$ seems rational as well: a decreasing internal discount rate ($\beta_t{}^D$) means that the government focuses much more on the forthcoming periods and less on the far distant future. Therefore it tries to attract more capital at the cost of the future government expenditures. To achieve this it should set $\rho_t{}^D$ at higher value. Nevertheless, the positive sign before the savings ratio does not seem to be intuitive again.

3.4 Households

Similarly to the government sector, I investigate only the behaviour of the developing country's households in the model. My related assumptions are that all the households have the same size and the internal discount factor used for evaluation of the future consumption, $\delta_t{}^D$ are the same for each household. These conditions allow that a representative household (the "Household") can be used in the model. The household sector of the advanced economy does not play an explicit role in the model; I take his saving decision exogenous.

3.4.1 Maximization Problem of the Household

The aim of the developing country's representative household is to maximize its long run utility originated from the consumption. The Household is the last in order, hence it can incorporate already all the optimal choices of the other two sectors in its decision making process[20]. Knowing all the necessary information – most importantly the capital income of the previous period and the potential government transfer for the current one – the Household sets the optimal savings level in each period. This amount will be ordered to the Investor and

[20] Technically this means that the Household gets familiar with the Investment and the Government Transfer function.

parallely the Household receives the government transfer. From its disposable income the Household satisfies its consumption need during the period and finally, at the end of the time interval he gets back its former savings enhanced by the net capital income.

Again, I use a simple utility function:

(17) $$\underset{S_t^D}{Max}\, U_H = \sum_{t=1}^{\infty} \left(\delta_t^D\right)^t C_t^D ,$$

where δt^D is the households discount factor used for evaluation of consumptions of different periods and C_t^D is the consumption level of the representative household in the ‘t’-th period. In its decision the Household is facing further constraints: at the income side of its budget constraint are the capital income originated from the foreign country, plus the revenue from the developing country (both are granted by the Investor based on the previous period’s investment) plus the Government’s transfer of the current period. The expenditure side consists of two elements, the consumption and the saving of the current period:

(18) $$G_t + \rho_{t-1}{}^A (1-\alpha_{t-1}) S_{t-1}{}^D + \rho_{t-1}{}^D \alpha_{t-1} S_{t-1}{}^D = C_t + S_t{}^D$$

From (18) we can easily express the current consumption level:

(19) $$C_t = G_t + \rho_{t-1}{}^A (1-\alpha_{t-1}) S_{t-1}{}^D + \rho_{t-1}{}^D \alpha_{t-1} S_{t-1}{}^D - S_t{}^D$$

In its decision about the optimal level of S_t^D the Household can consider already the optimal level of α_t and ρ_t^D. Plugging (16), the expression of ρ_t^D into (11), we get such version of α which depends only on the Households decision about S_t^D.

(20) $$\alpha_t * = \frac{1}{4e}\left(a\xi_t - b\left(\frac{S_t{}^D}{S_t{}^A} + 1 \right) + c\eta_t + d + \frac{1}{\beta_t{}^D} - \rho_t{}^A \right)^{21}$$

Furthermore, the household knows the optimal level of the government transfer as well:

(21) $$G_t = \alpha_t * I_t - \rho_{t-1}{}^D * \alpha_{t-1} * I_{t-1}$$

[21] Here: the optimal level of ρ_t^D was already plugged into α – for the mathematical derivation, see Appendix II. after the calculation of the optimal level of ρ_t^D

where the total investment is equal to the savings originated from the two countries:

(22) $$I_t = S_t^{\ D} + S_t^{\ A}$$

Plugging (x+1), (x+2) into the utility function of () and simplifying the equation we get a more detailed version of the Household's maximization problem:

(23) $$\underset{S_t^D}{Max}\,U_H = \sum_{t=1}^{\infty}\left(\delta_t^{\ D}\right)^t C_t = \sum_{t=1}^{\infty}\left(\delta_t^{\ D}\right)^t\left(\alpha_t S_t^{\ D} + \alpha_t S_t^{\ A} - \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ A} + \rho_{t-1}^{\ \ A}S_{t-1}^{\ \ D} - \rho_{t-1}^{\ \ A}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D}\right)$$

3.4.2 Optimal Choice of the Household

Appendix III. describes the derivation of the above depicted intertemporal consumption optimization problem. After a long derivation we get the following result:

(24) $$S_t^{\ D}* = \frac{S_t^{\ A}}{2b}\left[\frac{1}{\xi_t \beta_t^{\ D}} - \frac{\rho_t^{\ A}}{\xi_t} + c\eta_1 + d - b - 4e - \frac{b\left(1 + \dfrac{\delta_t^{\ D}}{\beta_t^{\ D}}\right)}{1 - \delta_t^{\ D}\rho_t^{\ A}}\right]$$

Again, I get mostly consistent results. The optimal savings in the developing economy increases if the savings level in the advanced economy, S_t^A increases; if the policy risk or risk appetite diminished[22] (i.e. η grows and ξ drops); expectations about the future exchange rate movement improve (i.e. d increases, e decreases); the governments discount rate decreases or the net interest rate in the advanced country decreases[23]. Getting the solution for the optimal savings level α_t and ρ_t can be expressed with the help of the exogenous variables[24]:

[22] Assuming that the developing country offers more risky investment opportunities than the advanced economy.

[23] For a more detailed description about the effect of the change of each variable, please see the next chapter.

[24] For the mathematical background, see Appendix III.

(25) $$\alpha_t^* = \frac{1}{8e}\left(\frac{1}{\xi_t \beta_t^{\ D}} - \frac{1}{\xi_t}\rho_t^{\ A} + c\eta_1 + d - b + 4e + \frac{b\left(1+\dfrac{\delta_t^{\ D}}{\beta_t^{\ D}}\right)}{1-\delta_t^{\ D}\rho_t^{\ A}}\right)$$

(26) $$\rho_t^{\ D^*} = \frac{1}{4}\left(\frac{1}{\beta_t^{\ D}} + \rho_t^{\ A} - \xi_t\left(c\eta_1 + d - b - 4e + \frac{b\left(1+\dfrac{\delta_t^{\ D}}{\beta_t^{\ D}}\right)}{1-\delta_t^{\ D}\rho_t^{\ A}}\right)\right)$$

After getting familiar with the choice of each sectors and the obtaining the optimal values for α, ρD and SD, in the next chapter I turn to the problem that how does these dependent variables alter when the any among the exogenous variables changes.

CHAPTER 4: NET CAPITAL MOVEMENT

As a result of the three sectors maximization problem I get the optimal values for α_t, ρ_t^D, S_t^D. Going one step further in investigation of capital flights I calculate the optimal level of the capital inflow into the developing country in each year. This is simply the product of the current periods share (α_t) and the total investment (I_t). Assuming that all of the borrowed capital has to be paid back to its owner by the end of each period, with subtracting the previous period's capital inflow from the current period's value I get the net capital in- or outflow into or from the developing country:

(27) $$NCM = \alpha_t I_t - \alpha_{t-1} I_{t-1} = \alpha_t (1 + g_t) I_{t-1} - \alpha_{t-1} I_{t-1}$$

where g_t is the growth rate of the total global investment. Since I am focusing on the effect of the changes in the investment allocation process, I set g_t to zero:

(28) $$NCM = \alpha_t (1 + g_t) I_{t-1} - \alpha_{t-1} I_{t-1} = \alpha_t I - \alpha_{t-1} I = (\alpha_t - \alpha_{t-1}) I_{t-1} = \Delta\alpha_t I$$

Putting aside the effect of growth of the global investment it is clear that the sign of change of α forecasts whether there happens a net capital inflow into the less developed country or. Therefore it is important to define the partial effect of the exogenous parameters on α. Furthermore, in order to understand the underlying procedures the effect of these variables on ρ_t^D and S_t^D should be taken into consideration as well. Therefore at first I investigate the effects of different economic phenomena separately, using the partial derivatives of α_t, ρ_t^D, S_t^D with respect to the exogenous parameters[25].

4.1 Effect of the change of the advanced countries interest rate or tax level

Equation (29) below describe the partial effect of the change in ρ_t^A, the advanced country's interest rate on α:

(29) $$\frac{\partial \alpha_t^*}{\partial \rho_t^{A}} = \frac{1}{8e}\left(b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)\left(\frac{\delta_t^{D}}{\left(1-\delta_t^{D}\rho_t^{A}\right)^2}\right) - \frac{1}{\xi_t}\right) \gtrless 0$$

The sign of the derivative can be both positive and negative. If ξ_t, b, δ_t^D and ρ_t^A; the risk aversion, the relative weight of the financial risk, the Household's discount factor and the advanced economy's interest rates, respectively are all very large in the same time then a decrease in ρ_t^A can even diminish the investment share of the developing country. But I think these are seldom and extreme circumstances. In case of less severe conditions, basically in case of higher level of global risk appetite (lower ξ) this effect becomes negative and we get an intuitive outcome: shrinking level of the alternative investment's return results a reallocation of the investments toward the developing country.

The derivatives of ρ_t^D and S_t^D reflect similar picture, if all ξ, b, δ_t^D and ρ_t^A are all high then the change in ρ_t^A is followed by an movement in the value of ρ_t^D and S_t^D which seems to be irrational at first sight. However, in less extreme situation – in most of the cases – both variables behaves as expected, the offered interest rate grows and the domestic savings level declines when the alternative cost raises:

(30) $$\frac{\partial \rho_t^{D^*}}{\partial \rho_t^{A}} = \frac{1}{4}\left(1-\xi_t b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)\left(\frac{\delta_t^{D}}{\left(1-\delta_t^{D}\rho_t^{A}\right)^2}\right)\right) \gtrless 0$$

(31) $$\frac{S_t^{D}*}{\partial \rho_t^{A}} = -\frac{S_t^{A}}{2}\left(\frac{1}{b\xi_t} - \left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)\left(\frac{\delta_t^{D}}{\left(1-\delta_t^{D}\rho_t^{A}\right)^2}\right)\right) \gtrless 0$$

4.2 Effect of the change in the level of Investor's risk aversion

The picture is clearer in case of ξ. If the developing country is more risky than the advanced one – which happens when the value of the risk adjustment function is negative and therefore

[25] For the derivation see Apendix IV.

the derivative of (33) is positive – and the risk aversion is hikes then the developing country's government should offer higher net interest rates in optimum. This is intuitive. The impact on α and S_t^D depends on different determinants:

(32) $$\frac{\partial \alpha_t^*}{\partial \xi_t} = \frac{1}{8\xi_t^2 e}\left(\rho_t^A - \frac{1}{\beta_t^D}\right) \gtrless 0$$

(33) $$\frac{\partial \rho_t^{D^*}}{\partial \xi_t} = \frac{1}{4}\left(-c\eta_1 - d + b + 4e - \frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D \rho_t^A}\right) \gtrless 0$$

(34) $$\frac{S_t^D *}{\partial \xi_t} = \frac{S_t^A}{2b\xi_t^2}\left(\rho_t^A - \frac{1}{\beta_t^D}\right) \gtrless 0$$

Since basically both the offered interest rate, ρ_t^D and the risk aversion ξ_t increases, the effect is not straightforward: Equations (32) and (34) suggest that the sign of the determinants depends on the relation of ρ_t^A and $\frac{1}{\beta_t^D}$ since the quotient before the expression in parenthesis are assumed to be positive in both cases. If the developing country's government follows a very responsible economic policy (meaning that in its decision the future transfers have quite large weights or the advanced economy bids very high rate of return) then the difference of ρ_t^A and $\frac{1}{\beta_t^D}$ will be positive. In this case a possible decrease in ξ can lessen the level of α and S_t^D. However, typically the situation is different: the advanced country can afford to offer low level of interest and/or the developing country's government concentrates much more on the near future and therefore its internal discount rate is much lower. This suggests that normally an increasing level of risk aversion will diminish both, the ratio of the developing economy from the global investment resources and the savings level in the country.

4.3 Effect of the altering policy risk

One of the simplest cases occurs when I consider the effect of the change in the domestic policy risk. All the derivatives have a time invariant sign, the derivatives of α and S_t^D are nonnegative and the derivative of ρ_t^D is nonpositive, independently from the value of the exogenous parameters:

(35) $$\frac{\partial \alpha_t^*}{\partial \eta_t} = \frac{c}{8e} \geq 0$$

(36) $$\frac{\partial \rho_t^{D^*}}{\partial \eta_t} = -\frac{1}{4}\xi_t c \leq 0$$

(37) $$\frac{S_t^{D}*}{\partial \eta_t} = \frac{S_t^A}{2b} c \geq 0$$

This suggests that a decreasing policy risk helps the government to attract more capital (i.e. α grows) since the reallocation of investments because of the increasing risk adjusted capital gain favours the developing country. Simultaneously it enables the government to cut the interest rates (i.e. ρ_t^D will be larger) and inspires the households to save more.

4.4 Effect of the change in the parameters of the exchange rate risk

Even if I assumed that the equation describing the exchange rate expectations are reflecting the long run relationship, trend changes can occur. Typically, when there is a sharp change in the equilibrium future capital inflow. Therefore the examination of these parameters can be interesting. The impact of the change in the constant term, d is straight since the sign of the derivatives is unvarying over time:

(38) $$\frac{\partial \alpha_t^*}{\partial d} = \frac{1}{8e} > 0$$

(39) $$\frac{\partial \rho_t^{D^*}}{\partial d} = -\frac{1}{4}\xi_t \leq 0$$

(40) $$\frac{S_t^D *}{\partial d} = \frac{S_t^A}{2b} \geq 0$$

If d *ceteris paribus* increases then the exchange rate risk decreases since the change of a later depreciation of the currency shrinks. This allows the Government to cut the offered interest rates. Parallely, the decreasing exchange rate risk forces the Investor to reallocate its portfolio raising the ratio of the investments into the developing county. As a result, the government can enlarge the level of the transfers financed by the incoming capital surplus. The households – realising that their income level increases in short term – can afford to increase both part of the expenditures, consumption and savings.

The situation is similar in case of the determinant defining the slope in the equation (e):

(41) $$\frac{\partial \alpha_t *}{\partial e} = -\frac{1}{8e^2}\left(\frac{1}{\xi_t \beta_t^D} - \frac{\rho_t^A}{\xi_t} + c\eta_1 + d - b + \frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D \rho_t^A}\right) \gtreqless 0$$

(42) $$\frac{\partial \rho_t^{D^*}}{\partial e} = \xi_t > 0$$

(43) $$\frac{\partial S_t^D *}{\partial e} = -\frac{2S_t^A}{b} < 0$$

The derivative of ρ_t^D with respect to e is always positive; it shows that an increase in the slope term enforces the government to raise the equilibrium interest rate the higher the risk aversion is the larger this hike should be. Similarly, the effect of e on S_t^D has always the same sign: an increase of the slope term results diminishing savings level. This is partly counter-intuitive: households facing rising domestic interest rates and increasing capital income decrease the level of savings. However, it may be explained such way that the households having higher future income need fewer saving today to reach the optimal level of the long term consumption. Regarding α, the picture is less clear. Since the risk of the developing country simultaneously increased with the offered optimal interest rate, the effect of e on α depends on the other variables; it can be both positive and negative.

4.5 Effect of the change in the discount factors of the Government

Looking at the derivatives below it becomes clear that the sign of the expressions depends on the relation of δ_t^D and ρ_t^A. As I described Section 4.2, in practice the reciprocal of the households discount factor is in almost every case larger then the interest rate in the advanced economy. Therefore I can assume without leaving significant scenarios out of consideration that the denominator of 1- $\delta_t^D\rho_t^A$ is always positive. This helps in interpretation of (44)-(46) below:

(44) $$\frac{\partial\alpha_t*}{\partial\beta_t} = -\frac{1}{8e\left(\beta_t^{\ D}\right)^2}\left(\frac{1}{\xi_t} + \frac{b\delta_t^{\ D}}{1-\delta_t^{\ D}\rho_t^{\ A}}\right) \gtrless 0$$

(45) $$\frac{\partial\rho_t^{\ D^*}}{\partial\beta_t^{\ D}} = -\frac{1}{4}\frac{1}{\left(\beta_t^{\ D}\right)^2}\left(1-\frac{\xi_t b\delta_t^{\ D}}{1-\delta_t^{\ D}\rho_t^{\ A}}\right) \gtrless 0$$

(46) $$\frac{S_t^{\ D}*}{\partial\beta_t^{\ D}} = -\frac{S_t^{\ A}}{2b\left(\beta_t^{\ D}\right)^2}\left(\frac{1}{\xi_t} + \frac{b\delta_t^{\ D}}{1-\delta_t^{\ D}\rho_t^{\ A}}\right) \gtrless 0$$

In practice, a decline in β_t^D means that the Government shows increasing interest in the extent of the current periods transfer and focuses less on the future periods pay offs. This indicates that the Government - in order to attract larger amount of capital - raises its interest rates. (i.e. the derivative of ρ_t^D with respect to β_t^D should be negative. Equation (45) gives this result if I consider the assumption for 1- $\delta_t^D\rho_t^A$ above.) Reflecting the promise of the increasing government transfer, the Household increases its savings level from the extra income in order to get more future capital revenue compensating the expected lower level of future government transfers. (The negative sign of the derivative (46) captures correctly this phenomenon.) The simultaneously hiking savings level and interest rates attracts the capital inflow into the developing country. This is consequent with the negative sign of the derivative (44) which suggests the declining level of β lifts the level of α

I would like to emphasise an important feature of the model here: since ρ_t^D is endogenous, the Government can not set the net interest rate directly. Once the values of the exogenous variables become clear the optimal level of ρ_t^D is already given. However, the Government can have direct impact on the interest rate, basically with the setting of his internal discount rate[26]. As described above, in most cases the Government can reach an increase of ρ_t^D if he reduces β; and similarly, a hike of β will diminish the level of optimal influence

[26] The other opportunity of the Government is that he can influence the optimal value of η by setting the predictability of his policies.

CHAPTER 5: CAPITAL FLIGHT

After investigating the partial effects of the different factors, its time to turn back to the original topic and examine how capital flight can be described with the help of my model. Before going into the details I have to give a complex definition what I exactly mean of capital flight here. The key question of the previous chapter was what happens with the optimal level of α (and ρ_t^D and S_t^D) if the value of an exogenous variables *ceteris paribus* changes. In each of these 'stories' I assumed that all other external variables remains unchanged and there happens only a one-time adjustment in the level of the dependent variables. This approach can describe correctly an economy among 'normal' conditions where after a short adjustment period the dependent variables reach their new optimal value and the steady state economic growth continues until the appearance of the next external shock.

However, if I would like to model the real world then I can not presume that the effect of a shock dies away immediately when the different sectors set their new optimal decision reacting to the changing circumstances. In certain cases a shock in one economic subsegment generates turbulence in another field. The model should take into account that the exogenous variables are often interrelated with each other. At this point I reach the most interesting part of my thesis: the interdependency between the different exogenous variables means that a change in one parameter will cause the change of another in the next period which forces the different sectors into continuous reoptimization. The effect of the initial shock spreads out to many fields of the economy and the chain of these events can cause large shift in the capital market tendencies within a short period. This concept brings me closer to how I determine capital flight in my model: if after an external shock (i.e. change in one exogenous variable) the value of the other variables will be unchanged then I speak about a 'normal market adjustment' of capital movements. This case the effect of the shock dies away and the new optimum arises. However, if after the appearance of the shock alters the value of the other exogenous variable(s) significantly and - as a result - the new optimal level of α reaches a much lower level within a few periods then I speak about ‘capital flight’.

Going into the details of the model, equations (29) to (46) define how the equilibrium level of α, ρ^D and S^D changes in case of altering of one exogenous variables. However, they do not answer the question why and how sudden drop in capital inflow happens. As mentioned above, the key issue here is to consider that the different types of shocks are interrelating with each other therefore a small change in one parameter can cause large shift in the capital movement trends after a series of steps. Using the notation and results of my model I illustrate two scenarios with the purpose of showing how capital flight can occur. In the first scenario the initial shock appears in the developing country, in the second one the turbulence starts from the advanced economy. The outcome of both scenarios is that the majority of the capital leaves the developing economy within a short period.

5.1 Shock in the developing country

The first scenario starts from the following presumed initial conditions: I assume that the developing country’s risk level is higher then the advances country’s rate; the risk sensibility of the financial sector is low. Furthermore, I assume that the developing country’s government does not focus on the short term economic and political advantages of the government transfers, therefore it offers similarly high net interest rates than the advanced economy. In technical term this means that the value of $\boldsymbol{\Theta_t}$ is negative, where the weighted sum of the variables γ, η and ε are negative and the value of ξ is positive by definition, but small, i.e. close to zero. Since the Government does not follow short sighted political aims, the value of β is high, i.e. close to one. Furthermore, ρ_t^D is larger then ρ_t^A, but the difference is negligible. As a consequence, the optimal level of α is low; only a small portion of the investments is realised in the developing country and hence the level of government transfers are small as well[27].

However, I assume that there happens a change in the developing country's the political targets[28]: the Government – attempting to utilize the favourable international situation – would like to attract more capital with the intention of increasing the level of government transfers. This change is the initial shock in the developing economy. As described in section 4.5 this happens in such a way that the government starts to lessen the level of β[29] resulting in an increase in the optimal ρ_t^D. At first this can be done cheaply and easily: since α is still low, the financial and the exchange risk remains small as well[30]. As a consequence α increases, the developing country can enhance his transfers and the household sector increases his savings level[31]. Later one helps to decelerate the rise of the financial risk since not only α but S_t^D grows as well. Comparing (44) and (46) it becomes clear that the relative size of the parameters e and b determines whether α or S_t^D grows faster when β declines. If the later one does than the financial risk decreases continuously when the Government sets lower and lower level of β. Capital flight would hardly occur among such circumstances. But I think it is not a real scenario. Rather I assume that α increases faster then S_t^D, so the financial risk starts to grow slowly. But besides there is the appreciation pressure on the developing countries currency: at low level of α the exchange rate shock is positive

[27] The low level of α means that the financial risk of the developing country (measured by the variable γ) is low as well.

[28] This can be caused by various different factors. Nevertheless, in the democracies such change happens most often before elections.

[29] The other opportunity is to influence the value of η; However, I assume, the government chooses the easier way by altering β.

[30] See equations (5) and (6) in the financial and exchange rate risk sections in Chapter 3.

[31] See equation (46) and the related comment in section 3.5.

since the Investor expect revaluation because of the growing level of net capital inflow.

In the next period(s) the developing country's investment risk is still small and I assume that the Investor's risk sentiment level does not change, i.e. it is still low. The Government can go further on his way by further push down of β. It is still quite cheap to increase the government transfers. That is, relative low hike of the ρ_t^D results quite large change in α. Investment – basically financial and exchange rate – risk grows slowly compared to the previous period, however, the new optimal level of α represents still a sustainable investment allocation. Until this point the changes can be described as a 'normal market adjustment' process.

When will capital flight occur? If the Government stops at this sustainable situation, it will hardly happen. But if it keeps on lowering the value of β in order to reach a continuously growing transfer level, the flight will come surely. To see this, assume that the Government does not stop being even more short sighted. A further decrease of β causes that ρ_t^D should jump to an extremely high level[32], but the additional increase of the net capital inflow is negligible. The process reaches a turning point. The investment risk is pushed up to such a high level which disables the further increase of α. This has serious consequences: the drop of the next period's capital inflow is built into the general expectations. On the one hand this results a growing exchange rate risk: the expected capital outflow weakens the developing countries currency and – more importantly – alters the long term expectations related to the future exchange rate movements too. Technically, this means a shift in the parameters of the underlying exchange rate risk function; basically d decreases and e increases[33]. On the other hand, the drop of α cuts back the level of the government transfers simultaneously. Households decrease their savings in order to maintain the optimal consumption level. The

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[32] See equation (45) having a β being close to zero.

[33] This results in a decline in the risk function, i.e. a growth in the developing country's investment risk. See equation (7).

declining savings level gives a further lift to the financial risk level. Moreover, until this point the increase of the government transfer was forecastable and policies were predictable. But the effect of the shift in trend of capital movements appears in the policy risk as well: because of the decreasing predictability the value of η decreases.

As it can be seen all the risk variables deteriorate at the same time and so α starts to decline in a continuously growing pace. But it is important to emphasize that the economy does not fall back to similar previous situation were the value of α was lower. A new phenomenon appears: investors become more cautious toward the growing level of investment risk; i.e. the risk appetite declines or in technical terms, the level of ξ increases. This has a further downward pushing effect on the level of α; the financial sector does not want to undertake the same investment risk as before. The Government can try to push up the net interest rates, but it can only prolong the fallback of α; the flight of the capital is unavoidable anymore.

Why will the capital flight surely happen? The combination of high investment risk and the growing level of risk aversion induce a fast reallocation of the capital favoring the less risky opportunities of the advanced economy. The sudden decrease of α have again serious feedback to the different risk factors: further exchange rate depreciation is expected[34]; financial risk also grows further[35]; the policy risk increases[36]. Consequently, the level of risk aversion, ξ gets bigger again. This process happens much faster then the upward going trend did since the fast growing global risk aversion accelerates it and put larger emphasise on the

[34] Again, this means a change in the form of the exchange rate risk function: d decreases while e grows.

[35] In this situation the Government had to pay the expensive cost of borrowing (since ρ^D was pushed up in a very high level) but it can reach only a continuously decreasing amount from borrowing. This two factors together results that the government transfers drop significantly, therefore the households savings level falls back largely as well. Therefore, at the early stage of capital flight the decreasing level of α can not compensate effect of the savings level decline and as a consequence, financial risk increases further. See equations (5)

[36] Because of the declining level of predictability of the public policies.

other type of risks as well. Where is the end of the capital flight procedure? Once α drops back to such a low level where in the next period no further decrease can be expected than the investment risk conditions can start to improve. This can make investors reconsider their capital allocation. However, it takes some time until the sentiment toward risk falls back to its initial level and capital inflow can grow significantly.

5.2 Shock in the advanced country

The second scenario begins from a somewhat different starting point: the equilibrium level of α reached the steady state at high level; i.e. there is a constant significant net capital inflow into developing economy. The later is still more risky then the advanced economy, but this drawback is compensated by a large positive interest rate spread. Furthermore, I assume that this situation is sustainable; the developing country's government sets a limit for himself in order to avoid the emergence such a capital flight activity as described in Section 5.1.[37]. So in this scenario the Government behaves much more responsible. Moreover, I presume that the global risk appetite is high, similarly to the other setting above.

Nevertheless, in this scenario the adverse changes begin in the advanced economy: assume that such a serious exogenous shock appears which causes non-financial market turbulence[38] in the advanced economy. At the first stage this shock does not need to have any direct impact on the financial markets. This suggests that neither of the risk parameters should change its value, except ξ, the level of the risk aversion. The reason is simple: because of the interrelation of the different markets any negative phenomenon can cause a potential loss

[37] Technically this means that the Government sets a minimal value of β and it can not set any lower value for these parameters.

[38] For instance, we can think on the collapse of housing markets as happened in US recently. Other recent example for such a shock can be the fast growth of the oil prices and the following inflation pressure.

which makes the investors toward the investment risks more sensible. At the new level of ξ the previous level of α is not an optimum anymore; the investors does not want to undertake the same risk for the same risk premium as before. If the Government can compensate the Investor with an increased the capital income than only a normal market adjustment happens. However, if there is no room for the Government to raise its interest rates[39] the flight of capital will begin. This process takes place similarly as described in the first scenario. Since the initial exogenous shock is a quite serious one it is easy to assume that the level of the risk aversion increases continuously. This results that the expected future level of α will diminish which pushes up the developing country's investment risk level: as above, financial risk grows because of the falling savings level; exchange rate risk increase because of the growing expectations for currency depreciation; and the growing volatility of the governments policies results a hike in the policy risk as well. By that time the shock reaches the financial markets which makes the investors more sensible toward any risk; ξ grows again, α shrinks further and investment risk reaches new heights. The situation becomes a vicious circle, resulting in capital flight from the developing country in a very short period. Again, the process stops only if the expectations related to the future level of α do not deteriorate further. Then the investment risk can decrease significantly and ξ can fall back and the interest rate differential is enough to attract more capital then in the previous period.

[39] This can happen if for example the Government has reached the minimum level of β, or it does not want an increase in the interest rates for any economic or political reason.

CHAPTER 6: SUMMARY

The central aim of my thesis was to create a theoretical model for explaining why and how sudden shifts in the international capital movements occurred resulting in a capital flight from the developing countries. In the 'world' of my model there existed two countries, a developing and an advanced economy. Here I investigated the behavior and the interplay of three different sectors. Each sector had an own utility function to optimize. The financial investors maximized the risk adjusted return of its investments. Here I constructed a risk adjustment function based on the experience of the empirical literature which consisted of the most important risk factors affecting the optimal decision of the investors. Regarding the second sector, the developing country's government maximized the long run utility of his transfers to households. Finally, household sector of the less developed economy set the optimal level of the long run consumption level. The government and the household sector of the advanced economy had an exogenous role in the model. After each sector set his decision, I got the optimal level of the α, the developing country's share from the total 'global' investments; the optimal level of ρ^D, the net interest rate offered by the developing country's government; and the optimal value of S^D, the savings level in the developing country. All these optimums were defined as a function of the different risk factors and other exogenous parameters.

Using these outcomes, I could define how the single exogenous factors affect the optimal level of the developing country's investment share (α). The results were in most cases intuitive: the growing exchange rate and policy risk diminished the optimal level of α but a decreased in the relative weight of the financial risk in the optimization process pushed up α. If the alternative cost of investment (i.e. the advanced country's net interest rate) rose then the model suggested a decline in the developing country's savings rate. Finally, if the internal discount factor of the government increased that resulted in a drop of α; while if the households discount rate rose, than the optimal level of α increased as well. The related analysis described the underlying explanation in a more detailed way.

Afterwards, I determined the concept of capital flight in my model and I set up two scenarios with different initial conditions, but with the same final result: because of the interplay of the different risk factors a *circus vitious* situation occurred and as a consequence, the previously invested capital has flown out of the country within a short period.

The model can be augmented by the estimation of the value of the different parameters to define the exact effect of the different risk factors. Furthermore, the potential interrelations between the different type of risks may be captured by measuring (or assuming) the stochastical relationships among the different factors. These two enhancements allows to analyze statistically the outcome of the model (e.g. by running Monte Carlo simulation).

Finally, I summarize some general messages of the model. Firstly and most importantly, the outcome of the second scenario clearly describes that the developing country does not have to be 'guilty' to suffer from capital flight. The problems originated from the advanced country can have serious adverse effect on him. Therefore the second consequence is that the developing country should let room for himself to be able to act in the bad times. Since if it does not do so, i.e. if the investment risk level in the developing country is significantly higher and the investment ratio (α) of the less developed economy is large, then even a very high interest rate spread can not stop a break out of the capital flight once the market turbulence takes place. Thirdly, the developing country should avoid creating 'stop and go' cycles. This variation of public policies decreases the predictability of the government activity resulting lower level of the equilibrium investment share or even a potential capital flight. The final message is that a „too" fast capital inflow can cause serious problems inducing capital flight procedures. Therefore – even in good times – the developing country should practice temperance and attract that kevel of capital which can be absorbed without causing future problems.

APPENDIX[40]

APPENDIX I. - INVESTORS MAXIMIZATION PROBLEM

The Investor's utility function is:

$$\underset{\alpha_t}{Max U} = \sum_{t=1}^{\infty} \delta_t^{\ t} R_t \text{ , where}$$

$$R_t = \left\{1 + r_t^{\ A}\left(1 - \tau_t^{\ A}\right)\right\}\left(1 - \alpha_t\right) I_t + \left\{\left[1 + r_t^{\ D}\left(1 - \tau_t^{\ D}\right)\right] + \Theta\left(\alpha, S^D\right)\right\} \alpha_t I_t$$

Since all information which is considered in the decision making process is related to the same period the Investor maximisation problem should not take into account intertemporal decisions. This results that it is enough to solve the problem only for one period and the general solution we get is valid for each further period. Furthermore, as I described in section 3.2.3 I introduce ρ_t instead of r_t and τ_t:

$$R_t = \rho_t^{\ A}\left(1 - \alpha_t\right) I_t + \left[\rho_t^{\ D} + \xi_t \left(b\left(\frac{\left(1 - \alpha_t\right) S_t^{\ D}}{\alpha_t S_t^{\ A}} - 1 \right) + c\eta_t + d - e\alpha_t \right)\right] \alpha_t I_t$$

$$R_t = \rho_t^{\ A}\left(1 - \alpha_t\right) I_t + \rho_t^{\ D} \alpha_t I_t + \xi_t b \frac{\left(1 - \alpha_t\right) S_t^{\ D}}{\alpha_t S_t^{\ A}} \alpha_t I_t - \xi_t b \alpha_t I_t + \xi_t c \eta_t \alpha_t I_t + \xi_t d \alpha_t I_t - \xi_t e \alpha_t^{\ 2} I_t$$

Finalizing the simplifications we get the Lagrangien:

$$L = \rho_t^{\ A} I_t - \rho_t^{\ A} \alpha_t I_t + \rho_t^{\ D} \alpha_t I_t + \xi_t b \frac{S_t^{\ D}}{S_t^{\ A}} I_t - \xi_t b \frac{S_t^{\ D}}{S_t^{\ A}} \alpha_t I_t - \xi_t b \alpha_t I_t + \xi_t c \eta_t \alpha_t I_t + \xi_t d \alpha_t I_t - \xi_t e \alpha_t^{\ 2} I_t$$

The First Order Condition of the problem is:

$$\frac{\partial R}{\partial \alpha} = -\rho_t^{\ A} I_t + \rho_t^{\ D} I_t - \xi_t b \frac{S_t^{\ D}}{S_t^{\ A}} I_t - \xi_t b I_t + \xi_t c \eta_t I_t + \xi_t d I_t - 2\xi_t e \alpha_t I_t = 0$$

We can divide the expression with I_t. This suggests that the result will be independent of the

[40] In the Appendix I do not show the meaning of each variable again. For understanding the notation or the economic background please find the related section of the model description above.

size of the total investments.

$$2\xi_t e\alpha_t = -\rho_t{}^A + \rho_t{}^D - \xi_t b\frac{S_t{}^D}{S_t{}^A} - \xi_t b + \xi_t c\eta_t + \xi_t d$$

$$2\xi_t e\alpha_t = -\rho_t{}^A + \rho_t{}^D - \xi_t b\frac{S_t{}^D}{S_t{}^A} - \xi_t b + \xi_t c\eta_t + \xi_t d$$

$$\alpha_t = \frac{-\rho_t{}^A + \rho_t{}^D - \xi_t b\frac{S_t{}^D}{S_t{}^A} - \xi_t b + \xi_t c\eta_t + \xi_t d}{2\xi_t e}$$

The optimal value of α:

$$\alpha_t = \frac{1}{2\xi_t e}\rho_t{}^D - \frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_t + \frac{d-b}{2e}$$

APPENDIX II - THE GOVERNMENT MAXIMIZATION PROBLEM

The Utility Function of the Government is:

$$\underset{\rho_t^D}{Max}\, U_G = \sum_{t=1}^{\infty} \left(\beta_t^D\right)^t G_t$$

S.T.

$$G_t = \alpha_t{}^* I_t - \rho_{t-1}{}^D \alpha_{t-1}{}^* I_{t-1}$$

$$\alpha_t{}^* = \frac{1}{2\xi_t e}\rho_t{}^D - \frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_t + \frac{d-b}{2e}$$

After plugging the two equations into the utility function we can set the Lagrangien:

$$L = \sum_{t=1}^{\infty}\left(\beta_t^D\right)^t \left[\left(\frac{1}{2\xi_t e}\rho_t{}^D - \frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_t + \frac{d-b}{2e}\right)I_t - \rho_{t-1}{}^D\left(\frac{1}{2\xi_{t-1} e}\rho_{t-1}{}^D - \frac{1}{2\xi_{t-1} e}\rho_{t-1}{}^A - \frac{b}{2e}\frac{S_{t-1}{}^D}{S_{t-1}{}^A} + \frac{c}{2e}\eta_{t-1} + \frac{d-b}{2e}\right)I_{t-1}\right] =$$

$$= \sum_{t=1}^{\infty}\left(\beta_t^D\right)^t \left[\frac{1}{2\xi_t e}\rho_t{}^D I_t + \left(-\frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_t + \frac{d-b}{2e}\right)I_t - \frac{1}{2\xi_{t-1} e}\left(\rho_{t-1}{}^D\right)^2 I_{t-1} - \rho_{t-1}{}^D\left(-\frac{1}{2\xi_{t-1} e}\rho_{t-1}{}^A - \frac{b}{2e}\frac{S_{t-1}{}^D}{S_{t-1}{}^A} + \frac{c}{2e}\eta_{t-1} + \frac{d-b}{2e}\right)I_{t-1}\right]$$

F.O.C. is:

$$\frac{\partial L}{\partial \rho_t{}^D} = \left(\beta_t^D\right)^t \frac{1}{2\xi_t e} I_t - \left(\beta_t^D\right)^{t+1}\frac{1}{2\xi_t e}2\rho_t{}^D I_t - \left(\beta_t^D\right)^{t+1}\left(-\frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_1 + \frac{d-b}{2e}\right)I_t = 0$$

I can divide both side with I_t and $(\beta_t^D)^t$:

$$\frac{1}{2\xi_t e} - \frac{2\beta_t^D}{2\xi_t e}\rho_t{}^D - \beta_t^D\left(-\frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_1 + \frac{d-b}{2e}\right) = 0$$

$$\frac{2\beta_t^D}{2\xi_t e}\rho_t{}^D = \frac{1}{2\xi_t e} - \beta_t^D\left(-\frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_1 + \frac{d-b}{2e}\right)$$

$$\rho_t{}^D = \frac{1}{2\xi_t e}\frac{2\xi_t e}{2\beta_t^D} - \beta_t^D\left(-\frac{1}{2\xi_t e}\rho_t{}^A - \frac{b}{2e}\frac{S_t{}^D}{S_t{}^A} + \frac{c}{2e}\eta_1 + \frac{d-b}{2e}\right)\frac{2\xi_t e}{2\beta_t^D}$$

$$\rho_t{}^D = \frac{1}{2\beta_t^D} - \left(-\frac{1}{\xi_t}\rho_t{}^A - b\frac{S_t{}^D}{S_t{}^A} + c\eta_1 + d - b\right)\frac{\xi_t}{2}$$

$$\rho_t{}^D = \frac{1}{2\beta_t^D} + \frac{1}{\xi_t}\frac{\xi_t}{2}\rho_t{}^A + b\frac{\xi_t}{2}\frac{S_t{}^D}{S_t{}^A} - \frac{\xi_t}{2}c\eta_1 - \frac{\xi_t}{2}d + \frac{\xi_t}{2}b$$

The optimal level of ρ_t^D is:

$$\rho_t^{D*} = \frac{1}{2}\left(\frac{1}{\beta_t^D} + \rho_t^A + \frac{\xi_t b}{S_t^A} S_t^D - \xi_t c \eta_1 - \xi_t d + \xi_t b \right)$$

We can write back ρ_t^{D*} into α:

$$\alpha_t * = \frac{1}{2\xi_t e}\rho_t^D - \frac{1}{2\xi_t e}\rho_t^A - \frac{b}{2e}\frac{S_t^D}{S_t^A} + \frac{c}{2e}\eta_t + \frac{d-b}{2e}$$

$$\alpha_t * = \frac{1}{2\xi_t e}\frac{1}{2}\left(\frac{1}{\beta_t^D} + \rho_t^A + \frac{\xi_t b}{S_t^A} S_t^D - \xi_t c \eta_1 - \xi_t d + \xi_t b \right) - \frac{1}{2\xi_t e}\rho_t^A - \frac{\xi_t b}{2\xi_t e}\frac{S_t^D}{S_t^A} + \frac{\xi_t c}{2\xi_t e}\eta_t + \frac{\xi_t d - \xi_t b}{2\xi_t e}$$

$$\alpha_t * = \frac{1}{2\xi_t e}\left[\frac{1}{2}\frac{1}{\beta_t^D} + \frac{1}{2}\rho_t^A + \frac{1}{2}\frac{\xi_t b}{S_t^A} S_t^D - \frac{1}{2}\xi_t c \eta_1 - \frac{1}{2}\xi_t d + \frac{1}{2}\xi_t b - \rho_t^A - \frac{\xi_t b}{S_t^A} S_t^D + \xi_t c \eta_1 + \xi_t d - \frac{1}{2}\xi_t b \right]$$

$$\alpha_t * = \frac{1}{2\xi_t e}\left(\frac{1}{2}\frac{1}{\beta_t^D} - \frac{1}{2}\rho_t^A - \frac{1}{2}\frac{\xi_t b}{S_t^A} S_t^D + \frac{1}{2}\xi_t c \eta_1 + \frac{1}{2}\xi_t d - \frac{1}{2}\xi_t b \right)$$

$$\alpha_t * = \frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^D} - \rho_t^A - \frac{\xi_t b}{S_t^A} S_t^D + \xi_t c \eta_1 + \xi_t d - \xi_t b \right)$$

APPENDIX III. - THE HOUSEHOLD'S MAXIMISATION PROBLEM

$$\underset{S_t^D}{Max}\, U_H = \sum_{t=1}^{\infty} \left(\delta_t^{\ D}\right)^t C_t^{\ D}$$

S.T.

$$C_t = G_t + \rho_{t-1}^{\ \ A}(1-\alpha_{t-1})S_{t-1}^{\ \ D} + \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D}$$

$$G_t = \alpha_t I_t - \rho_{t-1}^{\ \ D}\alpha_{t-1}I_{t-1}$$

$$I_t = S_t^{\ D} + S_t^{\ A}$$

$$\rho_t^{\ D^*} = \frac{1}{2}\left(\frac{1}{\beta_t^{\ D}} + \rho_t^{\ A} + \frac{\xi_t b}{S_t^{\ A}} S_t^{\ D} + \xi_t b - \xi_t c\eta_1 - \xi_t d\right)$$

$$\alpha_t{*} = \frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{\ D}} - \rho_t^{\ A} - \frac{\xi_t b}{S_t^{\ A}} S_t^{\ D} + \xi_t c\eta_1 + \xi_t d - \xi_t b\right)$$

Plugging in G_t and I_t into C_t we can simplify the equation:

$$\begin{aligned}
C_t &= \alpha_t I_t - \rho_{t-1}^{\ \ D}\alpha_{t-1}I_{t-1} + \rho_{t-1}^{\ \ A}(1-\alpha_{t-1})S_{t-1}^{\ \ D} + \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D} = \\
&= \alpha_t\left(S_t^{\ D} + S_t^{\ A}\right) - \rho_{t-1}^{\ \ D}\alpha_{t-1}\left(S_{t-1}^{\ \ D} + S_{t-1}^{\ \ A}\right) + \rho_{t-1}^{\ \ A}(1-\alpha_{t-1})S_{t-1}^{\ \ D} + \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D} = \\
&= \alpha_t S_t^{\ D} + \alpha_t S_t^{\ A} - \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ D} - \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ A} + \rho_{t-1}^{\ \ A}S_{t-1}^{\ \ D} - \rho_{t-1}^{\ \ A}\alpha_{t-1}S_{t-1}^{\ \ D} + \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D} = \\
&= \alpha_t S_t^{\ D} + \alpha_t S_t^{\ A} - \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ A} + \rho_{t-1}^{\ \ A}S_{t-1}^{\ \ D} - \rho_{t-1}^{\ \ A}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D}
\end{aligned}$$

The Lagrangien is:

$$L = \sum_{t=1}^{\infty}\left(\delta_t^{\ D}\right)^t C_t = \sum_{t=1}^{\infty}\left(\delta_t^{\ D}\right)^t\left(\alpha_t S_t^{\ D} + \alpha_t S_t^{\ A} - \rho_{t-1}^{\ \ D}\alpha_{t-1}S_{t-1}^{\ \ A} + \rho_{t-1}^{\ \ A}S_{t-1}^{\ \ D} - \rho_{t-1}^{\ \ A}\alpha_{t-1}S_{t-1}^{\ \ D} - S_t^{\ D}\right)$$

F.O.C.:

$$\begin{aligned}
\frac{\partial L}{\partial S_t^{\ D}} &= \left(\delta_t^{\ D}\right)^t \frac{\partial \alpha_t}{\partial S_t^{\ D}} S_t^{\ D} + \left(\delta_t^{\ D}\right)^t \alpha_t \frac{\partial S_t^{\ D}}{\partial S_t^{\ D}} + \left(\delta_t^{\ D}\right)^t \frac{\partial \alpha_t}{\partial S_t^{\ D}} S_t^{\ A} - \left(\delta_t^{\ D}\right)^{t+1} \frac{\partial \rho_t^{\ D}}{\partial S_t^{\ D}} \alpha_t S_t^{\ A} - \left(\delta_t^{\ D}\right)^{t+1} \rho_t^{\ D} \frac{\partial \alpha_t}{\partial S_t^{\ D}} S_t^{\ A} \\
&+ \left(\delta_t^{\ D}\right)^{t+1} \rho_t^{\ A} \frac{\partial S_t^{\ D}}{\partial S_t^{\ D}} - \left(\delta_t^{\ D}\right)^{t+1} \rho_t^{\ A} \frac{\partial \alpha_t}{\partial S_t^{\ D}} S_t^{\ D} - \left(\delta_t^{\ D}\right)^{t+1} \rho_t^{\ A} \alpha_t \frac{\partial S_t^{\ D}}{\partial S_t^{\ D}} - \left(\delta_t^{\ D}\right)^t \frac{\partial S_t^{\ D}}{\partial S_t^{\ D}} = 0
\end{aligned}$$

We can divide with the expression with $(\delta_t^D)^t$ and simplify where possible:

$$\frac{\partial\alpha_t}{\partial S_t^{D}} S_t^{D}+\alpha_t+\frac{\partial\alpha_t}{\partial S_t^{D}} S_t^{A}-\delta_t^{D}\frac{\partial\rho_t^{D}}{\partial S_t^{D}}\alpha_t S_t^{A}-\delta_t^{D}\rho_t^{D}\frac{\partial\alpha_t}{\partial S_t^{D}} S_t^{A}+\delta_t^{D}\rho_t^{A}-\delta_t^{D}\rho_t^{A}\frac{\partial\alpha_t}{\partial S_t^{D}} S_t^{D}-\delta_t^{D}\rho_t^{A}\alpha_t-1=0$$

After plugging in α and ρ I calculate the derivatives separately:

$$\frac{\partial\alpha_t}{\partial S_t^{D}}=\frac{\partial\left[\frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)\right]}{\partial S_t^{D}}=-\frac{\xi_t b}{4\xi_t e S_t^{A}}=-\frac{b}{4e S_t^{A}}$$

$$\frac{\partial\rho_t^{D}}{\partial S_t^{D}}=\frac{\partial\left[\frac{1}{2}\left(\frac{1}{\beta_t^{D}}+\rho_t^{A}+\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t b-\xi_t c\eta_1-\xi_t d\right)\right]}{\partial S_t^{D}}=\frac{\xi_t b}{2 S_t^{A}}$$

$$\frac{\partial\rho_t^{D}}{\partial S_t^{D}}\alpha_t=\frac{\xi_t b}{2 S_t^{A}}\frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)=\frac{b\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)}{8 e S_t^{A}}$$

$$\rho_t^{D}\frac{\partial\alpha_t}{\partial S_t^{D}}=\frac{1}{2}\left(\frac{1}{\beta_t^{D}}+\rho_t^{A}+\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t b-\xi_t c\eta_1-\xi_t d\right)\left(-\frac{b}{4e S_t^{A}}\right)=\frac{-b\left(\frac{1}{\beta_t^{D}}+\rho_t^{A}+\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t b-\xi_t c\eta_1-\xi_t d\right)}{8 e S_t^{A}}$$

Plugging the results into the original equation:

$$\begin{aligned}&\left(-\frac{b}{4e S_t^{A}}\right)S_t^{D}+\frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)+\left(-\frac{b}{4e S_t^{A}}\right)S_t^{A}\\&-\delta_t^{D}\frac{b\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)}{8 e S_t^{A}}S_t^{A}+\delta_t^{D}\frac{-b\left(\frac{1}{\beta_t^{D}}+\rho_t^{A}+\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t b-\xi_t c\eta_1-\xi_t d\right)}{8 e S_t^{A}}S_t^{A}\\&+\delta_t^{D}\rho_t^{A}-\delta_t^{D}\rho_t^{A}\left(-\frac{b}{4e S_t^{A}}\right)S_t^{D}-\delta_t^{D}\rho_t^{A}\frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)-1=0\end{aligned}$$

Product by 4ξe and simplify:

$$\begin{aligned}&-\frac{b\xi_t}{S_t^{A}}S_t^{D}+\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b-b\xi_t-\delta_t^{D}\frac{\xi_t b}{2}\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)\\&-\delta_t^{D}\frac{\xi_t b}{2}\left(\frac{1}{\beta_t^{D}}+\rho_t^{A}+\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t b-\xi_t c\eta_1-\xi_t d\right)+4\xi_t e\delta_t^{D}\rho_t^{A}+\frac{\delta_t^{D}\rho_t^{A}b\xi_t}{S_t^{A}}S_t^{D}\\&-\delta_t^{D}\rho_t^{A}\left(\frac{1}{\beta_t^{D}}-\rho_t^{A}-\frac{\xi_t b}{S_t^{A}}S_t^{D}+\xi_t c\eta_1+\xi_t d-\xi_t b\right)-4\xi_t e=0\end{aligned}$$

$$-\frac{b\xi_t}{S_t^A}S_t^D+\frac{1}{\beta_t^D}-\rho_t^A-\frac{\xi_t b}{S_t^A}S_t^D+\xi_t c\eta_1+\xi_t d-\xi_t b-b\xi_t-\delta_t^D\frac{\xi_t b}{2}\frac{1}{\beta_t^D}+\delta_t^D\frac{\xi_t b}{2}\rho_t^A$$
$$+\delta_t^D\frac{\xi_t b}{2}\frac{\xi_t b}{S_t^A}S_t^D-\delta_t^D\frac{\xi_t b}{2}\xi_t c\eta_1-\delta_t^D\frac{\xi_t b}{2}\xi_t d+\delta_t^D\frac{\xi_t b}{2}\xi_t b-\delta_t^D\frac{\xi_t b}{2}\frac{1}{\beta_t^D}-\delta_t^D\frac{\xi_t b}{2}\rho_t^A$$
$$-\delta_t^D\frac{\xi_t b}{2}\frac{\xi_t b}{S_t^A}S_t^D-\delta_t^D\frac{\xi_t b}{2}\xi_t b+\delta_t^D\frac{\xi_t b}{2}\xi_t c\eta_1+\delta_t^D\frac{\xi_t b}{2}\xi_t d+4\xi_t e\delta_t^D\rho_t^A+\frac{\delta_t^D\rho_t^A b\xi_t}{S_t^A}S_t^D$$
$$-\delta_t^D\rho_t^A\frac{1}{\beta_t^D}+\delta_t^D\rho_t^A\rho_t^A+\delta_t^D\rho_t^A\frac{\xi_t b}{S_t^A}S_t^D-\delta_t^D\rho_t^A\xi_t c\eta_1-\delta_t^D\rho_t^A\xi_t d+\delta_t^D\rho_t^A\xi_t b-4\xi_t e=0$$

$$-\frac{b\xi_t}{S_t^A}S_t^D-\frac{\xi_t b}{S_t^A}S_t^D+\delta_t^D\frac{\xi_t b}{2}\frac{\xi_t b}{S_t^A}S_t^D-\delta_t^D\frac{\xi_t b}{2}\frac{\xi_t b}{S_t^A}S_t^D+\frac{\delta_t^D\rho_t^A b\xi_t}{S_t^A}S_t^D+\delta_t^D\rho_t^A\frac{\xi_t b}{S_t^A}S_t^D$$
$$+\frac{1}{\beta_t^D}-\delta_t^D\frac{\xi_t b}{2}\frac{1}{\beta_t^D}-\delta_t^D\frac{\xi_t b}{2}\frac{1}{\beta_t^D}-\delta_t^D\rho_t^A\frac{1}{\beta_t^D}-\rho_t^A+\delta_t^D\frac{\xi_t b}{2}\rho_t^A-\delta_t^D\frac{\xi_t b}{2}\rho_t^A+\delta_t^D\rho_t^A\rho_t^A$$
$$+\xi_t c\eta_1-\delta_t^D\frac{\xi_t b}{2}\xi_t c\eta_1+\delta_t^D\frac{\xi_t b}{2}\xi_t c\eta_1-\delta_t^D\rho_t^A\xi_t c\eta_1+\xi_t d-\delta_t^D\frac{\xi_t b}{2}\xi_t d+\delta_t^D\frac{\xi_t b}{2}\xi_t d-\delta_t^D\rho_t^A\xi_t d$$
$$-\xi_t b+\delta_t^D\frac{\xi_t b}{2}\xi_t b-\delta_t^D\frac{\xi_t b}{2}\xi_t b+\delta_t^D\rho_t^A\xi_t b-4\xi_t e+4\xi_t e\delta_t^D\rho_t^A-b\xi_t=0$$

$$S_t^D\left(-2\frac{\xi_t b}{S_t^A}+2\frac{\delta_t^D\rho_t^A b\xi_t}{S_t^A}\right)+\frac{1}{\beta_t^D}\left(1-2\delta_t^D\frac{\xi_t b}{2}-\delta_t^D\rho_t^A\right)-\rho_t^A\left(1-\delta_t^D\rho_t^A\right)+\xi_t c\eta_1\left(1-\delta_t^D\rho_t^A\right)$$
$$+\xi_t d\left(1-\delta_t^D\rho_t^A\right)-\xi_t b\left(1-\delta_t^D\rho_t^A\right)-4\xi_t e\left(1-\delta_t^D\rho_t^A\right)-b\xi_t=0$$

$$S_t^D\frac{2\xi_t b}{S_t^A}\left(1-\delta_t^D\rho_t^A\right)=\frac{1}{\beta_t^D}\left(1-\delta_t^D\rho_t^A\right)-2\delta_t^D\frac{\xi_t b}{2}\frac{1}{\beta_t^D}-\xi_t b+\left(1-\delta_t^D\rho_t^A\right)\left(-\rho_t^A+\xi_t c\eta_1+\xi_t d-\xi_t b-4\xi_t e\right)$$

$$S_t^D\frac{2\xi_t b}{S_t^A}\left(1-\delta_t^D\rho_t^A\right)=\left(1-\delta_t^D\rho_t^A\right)\left(\frac{1}{\beta_t^D}-\rho_t^A+\xi_t c\eta_1+\xi_t d-\xi_t b-4\xi_t e\right)-\xi_t b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)$$

It is clear that if $\frac{1}{\delta_t^D}=\rho_t^A$ then S_t^D can take any value and the maximization problem does not have any solution. Therefore I assume that $\frac{1}{\delta_t^D}<>\rho_t^A$ and I divide both sides with $(1-\delta_t^D\rho_t^A)$:

$$S_t^{D}* = \frac{S_t^{A}}{2\xi_t b}\left[\frac{1}{\beta_t^{D}} - \rho_t^{A} + \xi_t c\eta_1 + \xi_t d - \xi_t b - 4\xi_t e - \frac{\xi_t b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right]$$

Finally, we get the optimal value of S_t^D

$$S_t^{D}* = \frac{S_t^{A}}{2b}\left[\frac{1}{\xi_t\beta_t^{D}} - \frac{\rho_t^{A}}{\xi_t} + c\eta_1 + d - b - 4e - \frac{b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right]$$

Using this result above I can get the final optimal value of α_t:

$$\alpha_t* = \frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}} - \rho_t^{A} - \frac{\xi_t b}{S_t^{A}} S_t^{D} + \xi_t c\eta_1 + \xi_t d - \xi_t b\right) =$$

$$= \frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}} - \rho_t^{A} - \frac{\xi_t b}{S_t^{A}}\frac{S_t^{A}}{2\xi_t b}\left[\left(\frac{1}{\beta_t^{D}} - \rho_t^{A} + \xi_t c\eta_1 + \xi_t d - \xi_t b - 4\xi_t e\right) - \frac{\xi_t b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right] + \xi_t c\eta_1 + \xi_t d - \xi_t b\right) =$$

$$= \frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}} - \rho_t^{A} - \left[\left(\frac{1}{2}\frac{1}{\beta_t^{D}} - \frac{1}{2}\rho_t^{A} + \frac{1}{2}\xi_t c\eta_1 + \frac{1}{2}\xi_t d - \frac{1}{2}\xi_t b - \frac{1}{2}4\xi_t e\right) - \frac{1}{2}\frac{\xi_t b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right] + \xi_t c\eta_1 + \xi_t d - \xi_t b\right) =$$

$$= \frac{1}{4\xi_t e}\left(\frac{1}{\beta_t^{D}} - \frac{1}{2}\frac{1}{\beta_t^{D}} - \rho_t^{A} + \frac{1}{2}\rho_t^{A} + \xi_t c\eta_1 - \frac{1}{2}\xi_t c\eta_1 + \xi_t d - \frac{1}{2}\xi_t d - \xi_t b + \frac{1}{2}\xi_t b + \frac{1}{2}4\xi_t e + \frac{1}{2}\frac{\xi_t b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right) =$$

$$= \frac{1}{4\xi_t e}\left(\frac{1}{2}\frac{1}{\beta_t^{D}} - \frac{1}{2}\rho_t^{A} + \frac{1}{2}\xi_t c\eta_1 + \frac{1}{2}\xi_t d - \frac{1}{2}\xi_t b + \frac{1}{2}4\xi_t e + \frac{1}{2}\frac{\xi_t b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right)$$

$$\alpha_t* = \frac{1}{8e}\left(\frac{1}{\xi_t\beta_t^{D}} - \frac{1}{\xi_t}\rho_t^{A} + c\eta_1 + d - b + 4e + \frac{b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right)$$

… and ρ_tD:

$$\rho_t^{D^*} = \frac{1}{2}\left(\frac{1}{\beta_t^D} + \rho_t^A + \frac{\xi_t b}{S_t^A} S_t^D + \xi_t b - \xi_t c\eta_1 - \xi_t d\right) =$$

$$= \frac{1}{2}\left(\frac{1}{\beta_t^D} + \rho_t^A + \frac{\xi_t b}{S_t^A}\frac{S_t^A}{2b}\left[\frac{1}{\xi_t \beta_t^D} - \frac{\rho_t^A}{\xi_t} + c\eta_1 + d - b - 4e - \frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right] + \xi_t b - \xi_t c\eta_1 - \xi_t d\right) =$$

$$= \frac{1}{2}\left(\frac{1}{\beta_t^D} + \rho_t^A + \frac{\xi_t}{2}\frac{1}{\xi_t\beta_t^D} - \frac{\xi_t}{2}\frac{\rho_t^A}{\xi_t} + \frac{\xi_t}{2}c\eta_1 + \frac{\xi_t}{2}d - \frac{\xi_t}{2}b - \frac{\xi_t}{2}4e - \frac{\xi_t}{2}\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A} + \xi_t b - \xi_t c\eta_1 - \xi_t d\right)$$

$$\rho_t^{D^*} = \frac{1}{2}\left(\frac{1}{2}\frac{1}{\beta_t^D} + \frac{1}{2}\rho_t^A - \frac{1}{2}\xi_t c\eta_1 - \frac{1}{2}\xi_t d + \frac{1}{2}\xi_t b + \frac{1}{2}4\xi_t e - \frac{1}{2}\frac{\xi_t b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right)$$

Dividing with ξ_t we get the final form of ρ_t^D:

$$\rho_t^{D^*} = \frac{1}{4}\left(\frac{1}{\beta_t^D} + \rho_t^A - \xi_t\left(c\eta_1 + d - b - 4e + \frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right)\right)$$

APPENDIX IV. - PARTIAL DERIVATIVES

The derivatives of α_t, ρ_t, S_t^D with respect to ξ_t. In order to

$$\frac{\partial \alpha_t *}{\partial \xi_t} = \frac{\partial \left[\frac{1}{8e} \left(\frac{1}{\xi_t \beta_t^D} - \frac{1}{\xi_t} \rho_t^A + c\eta_1 + d - b + 4e + \frac{b\left(1 + \frac{\delta_t^D}{\beta_t^D}\right)}{1 - \delta_t^D \rho_t^A} \right) \right]}{\partial \xi_t} = \frac{1}{8e}\left(-\frac{1}{\beta_t^D \xi_t^2} + \frac{\rho_t^A}{\xi_t^2} \right) = \frac{1}{8\xi_t^2 e}\left(\rho_t^A - \frac{1}{\beta_t^D} \right)$$

$$\frac{\partial \rho_t^{D^*}}{\partial \xi_t} = \frac{\partial \left[\frac{1}{4} \left(\frac{1}{\beta_t^D} + \rho_t^A - \xi_t \left(c\eta_1 + d - b - 4e + \frac{b\left(1 + \frac{\delta_t^D}{\beta_t^D}\right)}{1 - \delta_t^D \rho_t^A} \right) \right) \right]}{\partial \xi_t} = \frac{1}{4}\left(-c\eta_1 - d + b + 4e - \frac{b\left(1 + \frac{\delta_t^D}{\beta_t^D}\right)}{1 - \delta_t^D \rho_t^A} \right)$$

$$\frac{S_t^D *}{\partial \xi_t} = \frac{\partial \left[\frac{S_t^A}{2b} \left[\frac{1}{\xi_t \beta_t^D} - \frac{\rho_t^A}{\xi_t} + c\eta_1 + d - b - 4e - \frac{b\left(1 + \frac{\delta_t^D}{\beta_t^D}\right)}{1 - \delta_t^D \rho_t^A} \right] \right]}{\partial \xi_t} = \frac{S_t^A}{2b}\left(-\frac{1}{\beta_t^D \xi_t^2} + \frac{\rho_t^A}{\xi_t^2} \right)$$

$$= \frac{S_t^A}{2b\xi_t^2}\left(\rho_t^A - \frac{1}{\beta_t^D} \right)$$

The derivatives of α_t, ρ_t, S_t^D with respect to η_t:

$$\frac{\partial \alpha_t *}{\partial \eta_t} = \frac{\partial \left[\frac{1}{8e} \left(\frac{1}{\xi_t \beta_t^D} - \frac{1}{\xi_t} \rho_t^A + c\eta_1 + d - b + 4e + \frac{b\left(1 + \frac{\delta_t^D}{\beta_t^D}\right)}{1 - \delta_t^D \rho_t^A} \right) \right]}{\partial \eta_t} = \frac{c}{8e} \geq 0$$

$$\frac{\partial \rho_t{}^{D^*}}{\partial \eta_t} = \frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t{}^D} + \rho_t{}^A - \xi_t\left(c\eta_1 + d - b - 4e + \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D\rho_t{}^A}\right)\right)\right]}{\partial \eta_t} = -\frac{1}{4}\xi_t c \leq 0$$

$$\frac{S_t{}^D *}{\partial \eta_t} = \frac{\partial\left[\frac{S_t{}^A}{2b}\left[\frac{1}{\xi_t\beta_t{}^D} - \frac{\rho_t{}^A}{\xi_t} + c\eta_1 + d - b - 4e - \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D\rho_t{}^A}\right]\right]}{\partial \eta_t} = \frac{S_t{}^A}{2b}c \geq 0$$

The derivatives of α_t, ρ_t, S_t^D with respect to ρ_t^A:

$$\frac{\partial \alpha_t *}{\partial \rho_t{}^A} = \frac{\partial\left[\frac{1}{8e}\left(\frac{1}{\xi_t\beta_t{}^D} - \frac{\rho_t{}^A}{\xi_t} + c\eta_1 + d - b + 4e + \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D\rho_t{}^A}\right)\right]}{\partial \rho_t{}^A} =$$

$$= -\frac{1}{8\xi_t e} + \frac{1}{8e}b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(-\frac{1}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2}\right)\left(-\delta_t{}^D\right) = \frac{1}{8e}\left(b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(\frac{\delta_t{}^D}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2}\right) - \frac{1}{\xi_t}\right) \gtrless 0$$

$$\frac{\partial \rho_t{}^{D^*}}{\partial \rho_t{}^A} = \frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t{}^D} + \rho_t{}^A - \xi_t\left(c\eta_1 + d - b - 4e + \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D\rho_t{}^A}\right)\right)\right]}{\partial \rho_t{}^A} =$$

$$= \frac{1}{4} - \frac{1}{4}\xi_t b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(-\frac{1}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2}\right)\left(-\delta_t{}^D\right) = \frac{1}{4}\left(1 - \xi_t b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(\frac{\delta_t{}^D}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2}\right)\right) \gtrless 0$$

$$\frac{S_t^{D}*}{\partial\rho_t^{A}}=\frac{\partial\left[\frac{S_t^{A}}{2b}\left[\frac{1}{\xi_t\beta_t^{D}}-\frac{\rho_t^{A}}{\xi_t}+c\eta_1+d-b-4e-\frac{b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right]\right]}{\partial\rho_t^{A}}=$$

$$=-\frac{S_t^{A}}{2\xi_t b}-\frac{S_t^{A}}{2b}b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)\left(-\frac{1}{\left(1-\delta_t^{D}\rho_t^{A}\right)^2}\right)\left(-\delta_t^{D}\right)=-\frac{S_t^{A}}{2}\left(\frac{1}{b\xi_t}-\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)\left(\frac{\delta_t^{D}}{\left(1-\delta_t^{D}\rho_t^{A}\right)^2}\right)\right)\gtrless 0$$

The derivatives of α_t, ρ_t, S_t^D with respect to β_t:

$$\frac{\partial\alpha_t*}{\partial\beta_t}=\frac{\partial\left[\frac{1}{8e}\left(\frac{1}{\xi_t\beta_t^{D}}-\frac{\rho_t^{A}}{\xi_t}+c\eta_1+d-b+4e+\frac{b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right)\right]}{\partial\beta_t}=-\frac{1}{8\xi_t e\left(\beta_t^{D}\right)^2}+\frac{1}{8e}\frac{b\delta_t^{D}}{\left(1-\delta_t^{D}\rho_t^{A}\right)}\left(-\frac{1}{\left(\beta_t^{D}\right)^2}\right)$$

$$=-\frac{1}{8e\left(\beta_t^{D}\right)^2}\left(\frac{1}{\xi_t}+\frac{b\delta_t^{D}}{1-\delta_t^{D}\rho_t^{A}}\right)$$

$$\frac{\partial\rho_t^{D^*}}{\partial\beta_t^{D}}=\frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t^{D}}+\rho_t^{A}-\xi_t\left(c\eta_1+d-b-4e+\frac{b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right)\right)\right]}{\partial\beta_t^{D}}=-\frac{1}{4}\frac{1}{\left(\beta_t^{D}\right)^2}-\frac{1}{4}\frac{\xi_t b\delta_t^{D}}{\left(1-\delta_t^{D}\rho_t^{A}\right)}\left[-\frac{1}{\left(\beta_t^{D}\right)^2}\right]=$$

$$=-\frac{1}{4}\frac{1}{\left(\beta_t^{D}\right)^2}\left(1-\frac{\xi_t b\delta_t^{D}}{1-\delta_t^{D}\rho_t^{A}}\right)$$

$$\frac{S_t^{D}*}{\partial\beta_t^{D}}=\frac{\partial\left[\frac{S_t^{A}}{2b}\left[\frac{1}{\xi_t\beta_t^{D}}-\frac{\rho_t^{A}}{\xi_t}+c\eta_1+d-b-4e-\frac{b\left(1+\frac{\delta_t^{D}}{\beta_t^{D}}\right)}{1-\delta_t^{D}\rho_t^{A}}\right]\right]}{\partial\beta_t^{D}}=\frac{S_t^{A}}{2b}\left(-\frac{1}{\xi_t\left(\beta_t^{D}\right)^2}\right)-\frac{S_t^{A}}{2b}\left(-\frac{b\delta_t^{D}}{1-\delta_t^{D}\rho_t^{A}}\right)\left(-\frac{1}{\left(\beta_t^{D}\right)^2}\right)=$$

$$=-\frac{S_t^{A}}{2b\left(\beta_t^{D}\right)^2}\left(\frac{1}{\xi_t}+\frac{b\delta_t^{D}}{1-\delta_t^{D}\rho_t^{A}}\right)$$

The derivatives of α_t, ρ_t, S_t^D with respect to δ_t:

$$\frac{\partial \alpha_t{}^*}{\partial \delta_t} = \frac{\partial\left[\frac{1}{8e}\left(\frac{1}{\xi_t \beta_t{}^D} - \frac{\rho_t{}^A}{\xi_t} + c\eta_1 + d - b + 4e + \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D \rho_t{}^A}\right)\right]}{\partial \delta_t} = \frac{1}{8e}\frac{\frac{b}{\beta_t{}^D}\left(1-\delta_t{}^D\rho_t{}^A\right) - b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(-\rho_t{}^A\right)}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2} =$$

$$= \frac{1}{8e}\frac{\frac{b}{\beta_t{}^D} - \frac{\delta_t{}^D\rho_t{}^A b}{\beta_t{}^D} + b\rho_t{}^A + \frac{\delta_t{}^D\rho_t{}^A b}{\beta_t{}^D}}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2} = \frac{b\left(\frac{1}{\beta_t{}^D} + \rho_t{}^A\right)}{8e\left(1-\delta_t{}^D\rho_t{}^A\right)^2}$$

$$\frac{\partial \rho_t{}^{D*}}{\partial \delta_t} = \frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t{}^D} + \rho_t{}^A - \xi_t\left(c\eta_1 + d - b - 4e + \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D \rho_t{}^A}\right)\right)\right]}{\partial \delta_t} = -\frac{1}{4}\xi_t\frac{\frac{b}{\beta_t{}^D}\left(1-\delta_t{}^D\rho_t{}^A\right) - b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(-\rho_t{}^A\right)}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2} =$$

$$= -\frac{1}{4}\xi_t\frac{\frac{b}{\beta_t{}^D} - \frac{\delta_t{}^D\rho_t{}^A b}{\beta_t{}^D} + b\rho_t{}^A + \frac{\delta_t{}^D\rho_t{}^A b}{\beta_t{}^D}}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2} = -\frac{\xi_t b\left(\frac{1}{\beta_t{}^D} + \rho_t{}^A\right)}{4\left(1-\delta_t{}^D\rho_t{}^A\right)^2}$$

$$\frac{\partial S_t{}^{D*}}{\partial \delta_t} = \frac{\partial\left[\frac{S_t{}^A}{2b}\left[\frac{1}{\xi_t \beta_t{}^D} - \frac{\rho_t{}^A}{\xi_t} + c\eta_1 + d - b - 4e - \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D \rho_t{}^A}\right]\right]}{\partial \delta_t} = \frac{S_t{}^A}{2b}\frac{-\frac{b}{\beta_t{}^D}\left(1-\delta_t{}^D\rho_t{}^A\right) - \left[-b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)\left(-\rho_t{}^A\right)\right]}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2} =$$

$$= \frac{S_t{}^A}{2}\frac{-\frac{1}{\beta_t{}^D} + \frac{\delta_t{}^D\rho_t{}^A}{\beta_t{}^D} - \rho_t{}^A - \frac{\delta_t{}^D\rho_t{}^A}{\beta_t{}^D}}{\left(1-\delta_t{}^D\rho_t{}^A\right)^2} = -\frac{S_t{}^A\left(\frac{1}{\beta_t{}^D} + \rho_t{}^A\right)}{2\left(1-\delta_t{}^D\rho_t{}^A\right)^2}$$

Change in the exchange rate relationship - The derivatives of α_t, ρ_t, S_t^D with respect to d:

$$\frac{\partial \alpha_t{}^*}{\partial d} = \frac{\partial\left[\frac{1}{8e}\left(\frac{1}{\xi_t \beta_t{}^D} - \frac{\rho_t{}^A}{\xi_t} + c\eta_1 + d - b + 4e + \frac{b\left(1+\frac{\delta_t{}^D}{\beta_t{}^D}\right)}{1-\delta_t{}^D \rho_t{}^A}\right)\right]}{\partial d} = \frac{1}{8e} > 0$$

$$\frac{\partial \rho_t^{D^*}}{\partial d}=\frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t^D}+\rho_t^A-\xi_t\left(c\eta_1+d-b-4e+\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right)\right)\right]}{\partial d}=-\frac{1}{4}\xi_t\leq 0$$

$$\frac{S_t^D*}{\partial d}=\frac{\partial\left[\frac{S_t^A}{2b}\left[\frac{1}{\xi_t\beta_t^D}-\frac{\rho_t^A}{\xi_t}+c\eta_1+d-b-4e-\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right]\right]}{\partial d}=\frac{S_t^A}{2b}\geq 0$$

…and with respect to e:

$$\frac{\partial \alpha_t*}{\partial e}=\frac{\partial\left[\frac{1}{8e}\left(\frac{1}{\xi_t\beta_t^D}-\frac{\rho_t^A}{\xi_t}+c\eta_1+d-b+4e+\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right)\right]}{\partial e}=-\frac{1}{8e^2}\left(\frac{1}{\xi_t\beta_t^D}-\frac{\rho_t^A}{\xi_t}+c\eta_1+d-b+\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right)\gtrless 0$$

$$\frac{\partial \rho_t^{D^*}}{\partial e}=\frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t^D}+\rho_t^A-\xi_t\left(c\eta_1+d-b-4e+\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right)\right)\right]}{\partial e}=\frac{1}{4}(-\xi_t)(-4)=\xi_t>0$$

$$\frac{\partial S_t^D*}{\partial e}=\frac{\partial\left[\frac{S_t^A}{2b}\left[\frac{1}{\xi_t\beta_t^D}-\frac{\rho_t^A}{\xi_t}+c\eta_1+d-b-4e-\frac{b\left(1+\frac{\delta_t^D}{\beta_t^D}\right)}{1-\delta_t^D\rho_t^A}\right]\right]}{\partial e}=\frac{S_t^A}{2b}(-4)=-\frac{2S_t^A}{b}\geq 0$$

The derivatives of α_t, ρ_t, S_t^D with respect to b:

$$\frac{\partial \alpha_t^*}{\partial b}=\frac{\partial\left[\frac{1}{8e}\left(\frac{1}{\xi_t\beta_t^{\,D}}-\frac{\rho_t^{\,A}}{\xi_t}+c\eta_1+d-b+4e+\frac{b\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}\right)\right]}{\partial b}=-\frac{1}{8e}+\frac{1}{8e}\frac{\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}=\frac{1}{8e}\left(\frac{\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}-1\right)=$$

$$=\frac{1}{8e}\left(\frac{\frac{\beta_t^{\,D}+\delta_t^{\,D}-\beta_t^{\,D}+\beta_t^{\,D}\delta_t^{\,D}\rho_t^{\,A}}{\beta_t^{\,D}}}{1-\delta_t^{\,D}\rho_t^{\,A}}\right)=\frac{1}{8e}\left(\frac{\delta_t^{\,D}\left(1+\beta_t^{\,D}\rho_t^{\,A}\right)}{\beta_t^{\,D}\left(1-\delta_t^{\,D}\rho_t^{\,A}\right)}\right)$$

$$\frac{\partial \rho_t^{\,D^*}}{\partial b}=\frac{\partial\left[\frac{1}{4}\left(\frac{1}{\beta_t^{\,D}}+\rho_t^{\,A}-\xi_t\left(c\eta_1+d-b-4e+\frac{b\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}\right)\right)\right]}{\partial b}=-\frac{1}{4}\xi_t\left(\frac{\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}-1\right)=$$

$$=-\frac{1}{4}\xi_t\left(\frac{\delta_t^{\,D}\left(1+\beta_t^{\,D}\rho_t^{\,A}\right)}{\beta_t^{\,D}\left(1-\delta_t^{\,D}\rho_t^{\,A}\right)}\right)$$

$$\frac{\partial S_t^{\,D^*}}{\partial b}=\frac{\partial\left[\frac{S_t^{\,A}}{2b}\left[\frac{1}{\xi_t\beta_t^{\,D}}-\frac{\rho_t^{\,A}}{\xi_t}+c\eta_1+d-b-4e-\frac{b\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}\right]\right]}{\partial b}=\frac{S_t^{\,A}}{2b}\left(-\frac{\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}-1\right)=$$

$$=-\frac{S_t^{\,A}}{2b}\left(\frac{\left(1+\frac{\delta_t^{\,D}}{\beta_t^{\,D}}\right)}{1-\delta_t^{\,D}\rho_t^{\,A}}+1\right)$$

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