Tomasz Aleksander Makarewicz

An Evolutionary Model of Human Capital and Technology Accumulation

In partial fulfilment of the requirements for the degree of Master of Arts

Submitted to

Central European University

Department of Economics

Supervisors: dr. hab Andrzej Cieślik,

Proffesor Laslo Matyas

Budapest, Hungary

2008

Abstract

In this thesis, I propose a growth model based on an evolutionary framework. I discuss the mainstream growth theory to conclude that typical macroeconomic models are impotent to account for the historic, preindustrial growth regime or to link the economic and demographic changes that coincided during the Industrial Revolution. Based on this discussion, I propose a model, in which technology growth and human capital form a feedback system,each causing a growth of the other, through a selection of individuals with different approach towards investements in offsprings. However, as the model assumes that consumption necessities increase with the human capital endowment, the ultimate equilibrium is stagnation and extiction of individuals orientated towards high human capital investments. I conclude with an empirical analysis of UK data from a period 1260-1994, which confirms that dynamics predicted by the model fit the path of the average wage and GDP per capita.

Contents

Introduction **1**

1 Theoretical Background **3**

2 Theoretical Model **13**

2.1 General Information 13

2.2 Structure of the Model 15

2.2.1 Production 15

2.2.2 Consumer Problem 15

2.2.3 Technology 20

2.2.4 Human Capital 25

2.2.5 Income and the Population Growth 26

2.2.6 Population structure 27

2.3 Dynamics of the System 28

2.3.1 The Neolithic Scenario 29

2.3.2 The Bronze Age Scenario 30

2.3.3 The Greek Scenario 31

2.3.4 The Roman Scenario 33

2.3.5 Protestant Scenarios 35

2.3.6 The Colonial Scenario 39

2.4 The historical path of the GDP 41

3 Empirical Investigation **43**

3.1 The setup of the Investigation 43

3.2 1260-1860 Sample Analysis 47

3.3 The 1830-1994 Sample Analysis 50

4 Conclusions **55**

5 Bibliography **57**

Appendix A: : Proof of Lemma 1 **60**

Appendix B: Empirical estimation tables **67**

Sample 1260-1860 67

Sample 1830-1994 70

Appendix C: Proof of the Proposition 1 **74**

List of Figures

1 Effort's reaction curve for the technology growth. 19

2 Phase diagram for the effort and technology growth if $q_t = 0$. . . 22

3 Phase diagram for the effort and the technology level, if $q_t = 0$. In every period, the technology's reaction curve shifts rightward. 23

4 Phase diagram for the effort and technology growth if $q_t > 0$ and is stable. In every period, the technology's reaction function shifts rightwards, so the equilibriums E_1 and E_3 will eventually dissapear. 24

5 Natural growth of the ith type individuals as a function of the past technology level. 27

6 Comparison of the functional impact of the past technology level on the natural growth of both types of individuals - Case (1) as in P(9). 28

7 Comparison of the functional impact of the past technology level on the natural growth of both types of individuals - Case (2) as in P(9). 29

8 The GDP per capita (GDP PC) in England, 1260-1860. 47

9 The average wage divided by the GDP per capita (HH) in England in 1260-1860. 48

10 Estimated cointegration relationship for the 1550-1860 period. . . 49

11 Accumulated response functions to the unit shocks of the system, based on Cholesky's decomposition. Sample 1550-1860. 51

12 Estimated cointegration relationship for the 1830-1994 period. . . 52

13 Accumulated response functions to the unit shocks of the system, based on Cholesky's decomposition. Sample 1830-1994. 53

14 Resources dedicated to the procreation by the ith type individuals as a function of the past effort level. 63
15 Resources dedicated to the procreation by the ith type individuals as a function of the past technology level. 65

Introduction

In the following paper, I will propose and empirically verify a growth model based on an evolutionary framework, in which the growth of the production and consumption is linked with the selection of human capital and accumulation of the technology. I will discuss the growth theory literature, hence construct a dynamic model and show several scenarios it predicts. I will then try to evaluate the historical validity of those scenarios by an empirical investigation, based on historic data for Engand in the period 1260-1994. The model will be based on (Galor, Moav, 2002), with altered technology function and consumotion constraint.

The intended contribution of this paper is to explore the links between the evolution of mankind and its economic activities. In other words, I want to propose an interpretation of growth, in which the population structure both causes and is selected by the economic development. Hence, the focus of the paper is to account for the evolutionary feedback of humanity and its economic activities, and to check how one can coherently describe the historical path of the macroeconomic variables by the evolution. The proposed model will be based on microeconomic foundations that ecompass both the preindustrial (sometimes called agrarian) and postindustrial (or modern) path of economic and demographic variables - the gradual population growth combined with a stable consumption per capita in the preindustrial epoch, followed by the modern growth regime and demographic transition.

Economic development is one the most well discussed and controversial topics in the macroeconomic literature. Its importance is obvious, as the production, that is the degree to which we can adapt the reality to our will, constraints both the individual and social effort. Thus, richer individuals or groups can achieve more and enforce themselves upon the others, just as the Western civilization dominated the last three centuries of the world's history. Despite the topic's importance and the effort of economists to evaluate it, macroeconomics could not propose a single theory encapsulating this phenomenon coherently. In fact, different models describe well modern economic growth, yet they are impotent to adequately describe both the preindustrial and industrial economies or to show one process responsible for all the dramatic changes in the structure of the production and consumption between those epochs. Another problem is that those explanations usually have a limited scope and thus cannot link the post-industrial growth with the dramatic social, institutional and demographic changes connected with the Industrial Revolution. As a result, growth theory describes and even predicts well the behavior of the modern economy, but looses all of its explanatory power as is is focused on the ancient or medieval reality.

The structure of the paper follows its aim. The first chapter will consist of a literature discussion, in which I will describe the state of the growth theory, show its weaknesses and possibilities of overcoming those. The second chapter will be devoted exclusively to the model. I will discuss its overall structure, then following its different parts, examine its dynamics. The last part of this chapter will consist of scenarios predicted by the model. The third section will contain the empirical investigation, its methodology, data sources, results and their interpretation. Finally, the last chapter will summarize the whole article.

1 Theoretical Background

In the following section, I will discuss the state of modern mainstream growth theory, along with showing its weaknesses. Based on this, I will be able to formalize and detail the goals of my model and the framework on which I should base it.

The question of production growth is one of the most important and the most often analyzed one in the macroeconomic literature. In fact, there are three major problems connected with economic development - its source, its dynamics and its sustainability. Economists have been able to indicate four components of the product, technology, human capital and two factors of production suggested by classical economy, *i.e.,* the physical capital and labor - and an accumulation of all of these can fuel the growth in the short-run. As it is well known, data analysis indicates that the physical capital is not responsible for the development differences across the countries and models which incorporate the notion of a capital deprecation lead to a conclusion that no infinite growth is possible due to the accumulation of that production factor alone. Also, neither the labor variety can explain the differences in the production per capita nor theoretical models suggest a causal relationship between those two variables. It seems therefore that it must be the accumulation of the technology or the human capital that fuels the long-run growth. However, it is not clear how it actually happens.

The existing literature developed three major approaches towards the problem of the long-run cause of the production growth. First one is to describe the equilibrium of an economy given some level of the technology (or its growth). The short-run dynamics of the product are linked with the accumulation of the capital and may cause some temporary shifts in the system, yet capital growth must die out after some time and the economy reaches its steady state. Hence, we can construct a function linking those steady states and appropriate technology levels. Two notable examples of such a model are the AS-AD model and Solow-Swan model (Solow, 1956). We can actually assume some structure of the technology dynamics (as in later versions of the Solow-Swan model, we can assume it to grow in a linear manner) and compute the dynamic steady state of the economy. This approach yields three obvious problems. If we want to explain the technological growth, a model based on an assumption that the technology grows generates an *ignotum per ignotum* theory. Second, if we believe that the majority of the economic development is caused by the technological advancement, we cannot seriously consider an assumption that the technology growth is an exogenous constant, because the long-run economic development is characterized by two major breaks - the first and the second industrial revolutions - what clearly indicates that the technology grotwh has not been constant. Third, such an explanation *a priori* disallows any impact that we expect the economic situation of a society - its population size, accumulation of the capital, present level of the technology, etc. - may have over the technological progress.

Second approach is to link either the human capital or the technology with other factors of production in such a fashion that the economy could no longer be explained by the constant returns to scale. One of the first ideas within this approach was to say that the human capital is a function of the physical capital, and therefore the production function can be reduced to a linear function (or in general case, to a convex function) of the physical capital, AK model being the most famous and controversial example of such a theory (Jones, 1995). With a linear capital deprecation, such a model predicts no static steady state, but a constant growth. Even though this model seems convenient, it does not fit the data (see (Jones, 1995) for a discussion of AK models and their fitness to data).

A natural argument against two described approaches is that they lack any 'microfoundations', *i.e.,* that they assume some behavior of the whole system and its parts, without any analysis of economic agents' problems. The third approach is set upon this observation. Here, the technology creation is modeled explicitly. Sometimes, the technology is simply reduced to the human capital which - as in the Lucas-Uzawa model - is treated as the physical capital, *i.e.,* it is assumed that there exist a human capital production sector with a deprecation constraint (see (Lucas, 1988) for an example of such a model, or (Mulligan, Sala-I-Martin, 1995) for a discussion of a transitory dynamics of such a model and their fitness to data). However, depending on the assumed production function and the structure of the capital and the human capital production, this model predicts either no positive long-run growth or a constant growth. Neither of these conclusions fit the historical GDP path. An alternative is the class of models with an imperfect market of innovations. Those models are based on a production which is a function of production inputs of a variate number or quality (for a latter example, see (Aghion, Howitt, 1992)). New or better intermediate products are created by innovators who participate with the final goods' producers in an imperfect competition game. This approach has its own advantages, e.g. it allows for an incorporation of the Newton effect or graduate student's effect and therefore can account for some internal, exogenous dynamic of the technology, it also explicitly models the reason for the technological advancement (as a profit incentive for innovators). On the other hand, it restricts the technological creation only to the innovation sector, disregarding the role of learning-by-doing effect within the final goods production or links between the production, human capital and the technology. Another problem is that this model does not fit data well (see (Li, 2000) for a discussion and an example of an empirical investigation).

All three approaches are usually designed so that they can be able to explain growth in its modern form. In other words, when economists construct these models, they seem to have in mind the fact that for as long as we can measure the GDP, it grows stochastically around a linear trend (which is sometimes referred to as the natural growth). Hence the typical model predicts that the economy will at first accumulate different types of production factors (capital, human capital or some intermediate production factors), losing momentum as the stock of these becomes satiated. In the long run (*i.e.,* in the equilibrium), growth may be somehow generated by the technological progress, however it will be much lower compared to that from the accumulation period. Such a scenario can be observed in some modern cases, e.g. in the Western Europe after the II World War (when the post-war growth was accelerated by the reconstruction of what the warfare activities have damaged), however, from the historical point of view it is not the actual time path of the economy. In fact (see (Galor, Moav 2002) or (Galor, 2005)), the historical scenario is completely the opposite to the one predicted - the economy for thousands of years was mired in a stagnation to explode unexpectedly in the middle of 19th century. The initial momentum of the long-run growth did not die out, but contrary - it magnified two times in the 19th century and then stayed constant in spite of two global conflicts and a threat of another, nuclear one, wrecking havoc on the entire planet. These theories cannot explain as well the fact that prior to the Industrial Revolution, any temporal output growth caused the consumption per capita to increase. It is of course interesting to explain the modern dynamics of the growth, however such an explanation cannot be perceived as anything else but a model adequate only locally. The general problem of the economic growth cannot be evaluated by such a local explanation (for a detailed discussion, see (Galor, 2005)).

I think that the major problem of those approaches is that they lack a proper theoretical evaluation of the technology. Usually, technology is interpreted as a stock of human knowledge, with the emphasis on natural sciences and their applications. Obviously, technology contains those, however, it cannot be reduced to those. Comparison of developed countries suggests that states with equal or similar initial output, population, technology (in this sense of natural sciences) level and accumulated physical capital can diverge and after a lapse of time differ significantly in terms of the production. Notable examples of this phenomenon are the fate of postsocialist countries (see (Dunford, Smith, 2000) for a discussion) and the divergence between Northern and Southern Corea. Therefore, the notion of the technology cannot be void of the institutional aspects of economic phenomena, such as an existence of some sectors vital to the economy (like e.g. financial sector), existence of a stable political system able to diminish some transaction costs (an obvious example here is a proper judiciary system) or mechanisms that allow for or ease the diffusion of scientific knowledge. If one thinks about modern and prehistorical people, he must conclude that the decisive difference between modern and past humans lies not in the state of the external world, but in their ability to adapt to the world and to use it according to their will. Inhabitants of Neolithic communities had no combustion engines not because there was no oil during the neolithic epoch, but because neolithic men had no skills to obtain oil in larger quantities or a knowledge of its characteristics (and probably of its sole existence).

The adaptation to the reality is both a social and an individual phenomenon. On one hand, we create institutions that enable and increase the efficiency of our mutual effort to alter the environment, that decrease the friction of a group action (such as the transaction costs) and that enable the exchange of individual knowledge and therefore its public (or semi-public) accumulation. This notion I will refer to as the technology (in fact, this idea pervades modern institutional economics, see for example (Hall, Jones, 1999)). On the other hand, individuals differ in terms of ability to pursue their individual goals given the technology. For example, individuals differ in terms of efficiency of learning from and later enhancing the public pool of knowledge. Also, individuals with the same efficiency of obtaining access to public goods and institutions may vary in their ability in using those against the natural environment. We can summarize the discussed skill into a notion of the human capital.

The obvious and immediate question one can raise here is what are the links between so understood technology and human capital and what type of notions do we need to describe their long-run relationships. In a sense, the solution is already proposed in their formulation. Since I defined those two as an ability to adapt to the reality, I have already entangled them in an evolutionary framework ((Galor, 2005), see also (Gintis)). Also, the usual dynamics of evolutionary processes maybe the key to explain the sudden explosion of the technology level in XVIII century and two upward shifts in its momentum.

The problem that immediately appears here is that enhancing a model with the evolutionary framework necessarily brings forward the problem of the selection. On one hand, we may perceive this as an advantage. We know that the industrial revolution - the explosion of the production - coincided with dramatic changes in the demographic and social structure of western societies (see (Maddison, 1995)), to which they were not truly prepared (as seen in many historical examples of ideologies and movements opposite to either indutrialization or modernization - marxism, Catholic Church reaction, conservatism, etc.). In the classical approach towards growth, this fact is explained as a switch from an agrarian to an industrial production pattern, which then either enabled, enforced - *via* technological and economical progress - social changes or came to be neutral towards them, simply happened at the same time without any other relationships (Galor, 2005).

The final dissolution of the feudal order, along with a dawn of new institutions and recreation of the human perception of himself and the rest of the reality, triggered reaction movements which advocated either an abolishment of the new uncomfortable velocity of social phenomena or a change of its direction. The fact that those movements exist, enjoy mass support but always fail, demonstrates that the changes which took off somewhere in the XVIII century are both well rooted in the present state of humanity and yet not accepted by at least part of it. On the other hand, question omitted in the classical approach, which lacked historical perspective - why did the structure of the western economy shift from an agrarian to an industrial pattern - leads to an answer that the major difference between those lays not in the initial technology or resources endowment, but in the initial social institutions, as seen in the growth pattern among different countries and regions . As a result, the classical approach towards growth, the idea that the production somehow switched from one pattern to another independently from any social conditions, cannot be treated seriously (Galor, 2005). In other words, the notion of the Industrial Revolution - as such a switch of the social order - is flawed on itself. We should rather think of both social and economic changes of 18th, 19th and 20th century as a single, evolutionary process, in which the existing prior to the industrial revolution social and economic conditions somehow amassed and outburst through demographic and economic phenomenons. Therefore, there is no essential difference in the state of the world between and after 19th century, and we must construct a single theory linking both epochs, instead of thoughtlessly dividing the history into pre- and post-industrial age. Since modern human capital and technology seem to yield obvious evolutionary advantages for societies (e.g. a painful lesson given by British to Zulu), we can safely assume that they yield similar advantages for individuals living in a preindustrial times, such as wealthy merchants or bankers. Therefore, we can think of the preindustrial epoch as a time when technology and human capital necessary for the modern explosion were selected until amassed so that they could fuel an economic (and demographic) explosion.

Belief that one may perceive social changes as an embodiment of an evolutionary selection is a somehow distracting thought. Historically, idea that the biological objects have no essence and therefore there exist neither eternal species nor eternal biological strategies, is astonishingly fresh and can be dated as late as to the works of Carl Darwin from the half of the 19th century. And even if somehow that idea is allowed to enter the science, humans still value ourselves to much to recognize its implications as valid for themselves as well (see (Barkow, 1978) and (van den Berghe, 1990) for a description of the controversies connected with this issue). Therefore, people do not treat seriously the fact that the structure of our body is of no eternal significance, and that both our ancestors and possible decedents do not have to be humans. And even if they do, then they still like to point at culture, ethics and all the spirituality unknown to other animals, as something that makes humanity transcend from the biological evolution and something more than just a set of biological objects ((Fracchia, Lewontin, 1999) contains a good example of such a claim). A common argument against treating our social background as a mere adaptation mechanism is that we feel it to be something more than a plain biological phenomenon. Actually the present state of economics suggests that we should treat it as such, if it supposed to fulfil its role. As we know, the most interesting problems highlighted by almost any model based on the Game Theory is that usual equilibriums in economic games are not Pareto optimal, therefore if people are to play Pareto optimal strategies, there must exist a mechanism forcing them to abandon standard Nash equilibriums. Since playing former can yield higher payoffs compared to latter, societies consisting of individuals biased towards former have an evolutionary advantage and are being selected with a higher probability. Obviously that bias cannot come in a form of a conscious calculation, thus it must force itself upon individuals as a clear, unquestionable dogma (see (Gintis) for a discussion and interesting examples). Hence, the discussed argument is actually supporting the thesis that ethics and society are highly evolved adaptation mechanisms.

More sophisticated argument is that there are important modules of our culture that do not seem to yield an evolutionary advantage, so they should not be treated as biological objects (for example, an argument is formluated that those objects are not inherited in biological way but are rather spread among social groups, genders, national groups, generations, etc., *via* purely social forces, see (Bell et al., 2001)). A good example of such a phenomenon is arts, e.g. music. There exist theories that try to explain individuals' participation in arts as a sort of signaling game towards members of the opposite sex (being cultural is presumed to signal being intelligent). This does not however explain the emphasis we put on the religion or culture, nor the existence of individuals who devote themselves totally to those. A common idea of a counter-argument is that we simply observe an inflation process within this signalling game (culture being our peacock's tail, see (Barkow, 1978) for some examples). The far more intriguing approach is to notice that a self-awareness and intelligence are astonishingly useful for an adaptation to the environment, however they are not simple wings or claws, as they are all the time 'on' and cannot be put to a rest without the whole organism to sleep or lose self-control. Obviously, they are both complex and presumable therefore require complex maintenance, such as a constant stimuli and recreation. They also are not simple sets of algorithms like instincts, but rather devices that enable us to generate algorithms and norms according to necessities and thus necessarily have to resort to themselves. A side effect of this fact is that the self-awareness likes to question its purpose in general, its rationality and its well-being, something one can call a hunger for methaphysics. This hunger must be satiated by arts and hermeneutics as e.g. religion, *i.e.,* by complex social interaction consisting of symbolic, normative and esthetic stimuli. They are selected somehow in the evolutionary process (as discussed, *via* natural and sexual selection), however their current form is partially accidental - it enables us to cope with the side-effects of being sentient and apart from this it does not interfere in the adaptation process, so it was not eliminated. Obviously, this part must not be perceived by individuals as what it in fact is, as it can be used to strengthen the degree with which the culture can enforce our behavior to shift towards the evolutionary stable strategies.

2 Theoretical Model

2.1 General Information

The following model will describe the dynamic relationships between human capital, production, consumption and the technology. It is set on the discussion from the previous chapter, therefore it describes the path of the economy in evolutionary terms. I will base it on the model proposed by (Galor, Moav, 2002), modifying two of its assumptions - stability of the consumption constraint and the independence of the technological growth to its level. Both changes will be backed by a theoretical discussion as introduced. Even though the basic setup is similar to that in (Galor, Moav, 2002), and hence static solutions of my model are equivalent to those in the original paper, those two assumption changes will actually come to modify greatly the behavior of the model. The changes imposed are aimed on accounting for the natural growth decline in the post-industrial epoch, which the original paper could not explain.

(Galor, Moav, 2002) aims to be potent to describe the whole historical economic development, not just the post-industrial. Its basic structure is set on a discrete, infinite time, where each generation t is born and raised at the period $t-1$ and then replaces in the next period the previous generation in economic activities and the procreation. This model does not belong to the OLG class, however, as within the childhood, individuals do not participate in any economic behavior. Instead, individuals are assumed to be in their youth a subject of their parents will, *i.e.*, are raised and educated (are being invested in) in a fashion chosen by the parents and participate in an optimization problems only in the period of their adulthood, after which they die out and are replaced by a new generation, their children.

Individuals are assumed to have coherent preferences over the consumption and the number and the quality of children, where those preferences can be heterogeneously distributed over the society. For simplicity, authors assumed that all individuals have the same evaluation of the consumption and differ only in terms of the weight attached to the quality of children relatively to their number. It means that within the society, there can exist individuals (called 'quality type') who prefer less children of a higher human capital endowment than some other (called 'quantity type'). Adult individuals participate in economic enterprises and thus receive some income which they have to divide between their own consumption and the investment into the children, with a trade-off between the (costly) quality and quantity of the offsprings. As a result, both types of individuals participate in an evolutionary game, as depending on the level of the economic or technological development and their own human capital endowment, they will differ in terms of received income and thus in terms of the quality and the number of children. I must also emphasize that the term 'quality' refers only to the efficiency of adapting the technology to one's will. It does not imply any other advantage, especially in normative terms.

The model proposed in following paragraphs will be based on the same framework as (Galor, Moav, 2002). The goal of my thesis is to describe the relationship between the technology, production and the development of the human capital, therefore for simplicity I will impose two restrictions over the evolutionary process itself. First, I will restrict it to the social terms only. In other words, individuals are assumed here not to differ significantly in their physiology and not to evolve in it (neither by mutations nor by conscious allocation of resources). Second, I will not endogenize the mutation process, but instead treat the mutations as exogenous shocks to the population. Those will be the only exogenous elements of the model.

As it will become apparent, structure of the model imposes following path of evaluating its assumptions. First, I will state the production function, thus solve the static consumer's problem, obtaining a link between the current technology growth and the human capital level. This will allow me to determine the behavior of the technology (in fact, it will come to be a variable highly independent of other variables within the model). Then, I will show how the income and thus resources devoted to the procreation react to changes in the technology path. Combining those two elements, I will propose scenarios of evolution of the production, technology and human capital for different set of initial conditions and the type of the mutation shocks.

2.2 Structure of the Model

2.2.1 Production

Let $Y_t = BA_t^{\alpha}H_t^{1-\alpha}$, where B is a constant, A_t is the technology level and H_t is the human capital level and $\alpha \in (0,1)$. Since human capital is the only production function (knowledge being a public good), wage is given by

$$w_t = BA_t^{\alpha}H_t^{-\alpha}. \tag{1}$$

It will become clear later that the constant B sets the niche's capacity, *i.e.,* determines the maximum amount of individuals given the lowest possible level of both technology and human capital. It may be thus interpreted as a natural constraint of the environment over the humanity, *i.e.*, an initial endowment of land and other natural resources.

2.2.2 Consumer Problem

the following setup, apart from the differences in the technology and minimum consumption constraint, follow (Galor, Moav, 2002). Let the individual of ith type in the period t be characterized by the following utility function, in which $\gamma \in (0,1)$, c_t^i stands for the consumption, n_t^i for the number of his children (if $n_t^i < 1$, we can interpret it as a community of individuals of the ith type sharing costs of raising children of few chosen members of that community) and h_{t+1}^i for their quality:

$$u_t^i = (1-\gamma)\ln c_t^i + \gamma(\ln n_t^i + \beta^i \ln h_{t+1}^i) \tag{2}$$

Rasing a single child costs $\tau + e_{t+1}^i$, where τ is a fixed cost necessary for the basic child's needs and e_{t+1}^i represents the additional resources parents invest in the quality of their offspring. I will later refer to this variable as an effort. Let the h_{t+1}^i be a function of e_{t+1}^i, *i.e.*,

$$h_{t+1}^i = h(e_{t+1}^i, g_{t+1}), \tag{3}$$

where $g_{t+1} = \frac{A_{t+1}-A_t}{A_t}$ is the technological progress and $h(0,0) = 1$ (normalization, which will also ensure that the population actually receives an income even if it represents a hunter-gather scheme of survival). $\frac{dh}{dg} = h_g < 0$ (the erosion effect - rapid technological growth causes a deprecation of the present human capital efficiency), $\frac{dh}{de} = h_e > 0$, $\frac{d^2h}{de^2} = h_{ee} < 0$ (effort has a concave, positive effect over the human capital). Also, $\frac{d^2h}{dedg} = h_{eg} > 0$, $\lim\limits_{e_{t+1}^i \to \infty} h(e_{t+1}^i, g_{t+1})$ is infinite and $\lim\limits_{g_{t+1} \to \infty} h(e_{t+1}^i, g_{t+1})$ is equal to negative infinity.

Population can be divided into several groups according to the perceived importance of the quality of offsprings (visible in the β^i parameter - obviously, the larger it is, the more ith parents are focused on the quality of their children contrary to their quantity). The aggregate human capital endowment of the economy is

$$H_t = \sum_i L_t^i h_t^i, \tag{4}$$

where L_t^i denotes the number of ith type individuals and hence

$$L^i_{t+1} = n^i_t L^i_t. \tag{5}$$

The individuals are subject to a constraint

$$c^i_t \geq \overline{c}(e^i_t), \tag{6}$$

where $\frac{\partial \overline{c}}{\partial e} > 0$ and $\frac{\partial^2 \overline{c}}{\partial e^2} > 0$ - individuals have a minimum consumption necessities, which explode due to the parental effort investments. However, I also assume that at first, $\overline{c}$ grows slowly compared to other variables (*i.e.,* that the explosion accelerates after a certain threshold of e^i_t, before which is of small significance). The formulation of this assumption will be presented later, when the exact shape of $\overline{c}(e^i_t)$ will be necessary in order to determine the time path of variables.

The immediate question one can ask here is why the minimum consumption constraint should grow with the level of the effort. Two answers may be proposed, both based on the observation that $\overline{c}$ depends on e^i_t, which in itself determines the human capital level, hence we can interpret this constraint as a indirect function of the individual's endowment of the human capital. Obviously, human capital is a delicate resource that requires diligent maintenance, however it also imposes some degree of specialization upon the individual and requires investments in order to counter deprecation effects. Highly educated individuals are usually much more efficient in their work, yet they require a richer set of tools, may have higher expectations towards their consumption (Jenkins et al., 2003) (especially in terms of quality of consumed goods; another effect here may be connected with some signaling game of one's status, which may cause people to inflate their consumption in order to attract opposite sex with a promise of a high income or human capital level). They also may simply need higher quality goods for mere survival. For example, one needs a proper and stable diet (unavailable for the majority of pre-industrial societies) if he wants to participate in economic activities or in the education regardless of the natural conditions, especially the weather. Another factor here is that specialization makes individuals dependent on social supply of certain goods (e.g. in the modern societies, households are not self-sufficient as they used to be few centuries ago). Finally, a highly educated individual is not only a graduate of a good university, but also a person that invests in his human capital throughout all his professional life (Sargant et al, 1997). Those factors will obviously blow up the consumption constraint of those who represent a high level of the human capital.

Second argument comes from the fact that investments in the human capital are time consuming, just as the investment in children. For example, in modern societies one has to postpone child bearing until he finishes a university and then gain some professional experience, otherwise he may have no time both for the children and the accumulation of the human capital (this may be true especially for women). This obviously implies that individuals wishing to obtain a certain level of the human capital (and thus the income) will procreate relatively late compared to the age of their sexual maturity. On the other hand, we have evolved from organisms that procreate soon after they become adult and have no need for an ability to bear easily children in their old age, hence we are not adapted for a late child-bearing. This is visible e.g. in the fact that women attain a lower probability of a healthy offspring as they enter their middle thirties, hence by deciding on an university degree they may constrain heavily their expected quantity of children. From the point of view of the model's structure, both arguments imply that the minimum constraint rises (either because individuals must consume more or because they cannot spend their resources on children in some part of their youth).

Despite the change in the assumed form of the consumption constraint, it remains a constant for an individual, given his parents effort level (which

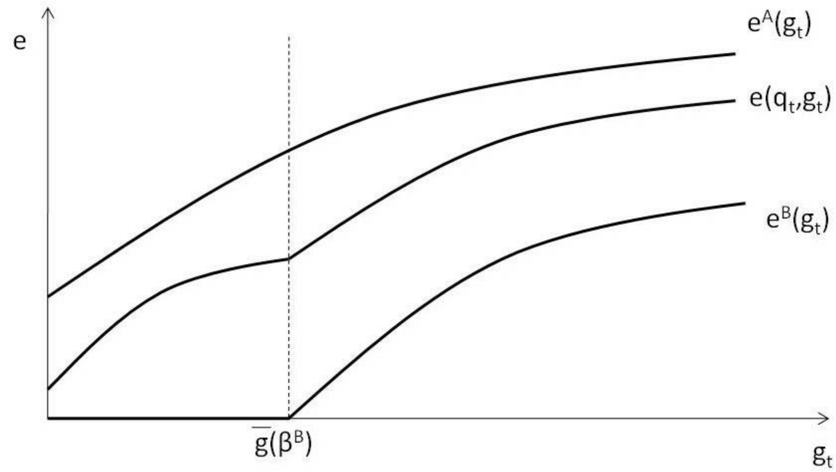

Figure 1: Effort's reaction curve for the technology growth.

obviously he cannot alter in any way). As a result, the static consumer problem in this model has exactly the same structure as the one in (Galor, Moav 2002). The solution of this problem can be summarized into:

Proposition 1 (P1) (represented by the Figure 1)

(1) The income and consumption constraint determine jointly the number of children.

(2) The effort put into the quality of children is always greater for type A individuals, for $g_t = 0$ is equal to zero for type B individuals and is greater than zero for type A individuals.

(3) There exists a threshold value $\overline{g}(\beta^B)$ such that for any technology growth rate above it type B individuals will expose a nonzero effort put into the quality of their children.

(4) For type A individuals and for type B individuals if $g_t > \overline{g}(\beta^i)$, the effort level is an increasing, concave function of the technology growth, $e^i_{t+1}(g_{t+1})$.

The proof for this proposition can be found in (Galor, Moav, 2002), however, I have also included it into the Appendix C.

2.2.3 Technology

Technology will have an explosive structure. Let

$$q_t = \frac{L_t^A}{L_t} \tag{9}$$

(share of type A individuals in the population) and let

$$e_t = q_t e_t^A + (1 - q_t) e_t^B \tag{10}$$

I assume

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} \equiv \psi(h(e_t), A_t) \tag{11}$$

to have following properties: $\psi(h(0), A_t) = \psi(1, A_t) = 0$, for any level of technology; $\psi(h(e_t), A_t)$ is strictly concave in respect to e_t, hence we can reduce it into $\psi(h(e_t), A_t) = \psi(e_t, A_t)$ and have $\frac{d\psi}{de_t} = \psi_e > 0$, $\frac{d^2\psi}{de_t^2} = \psi_{ee} < 0$; finally, $\psi(h(e_t), A_t)$ explodes with the technology level, *i.e.*, $\frac{d\psi}{dA_t} = \psi_A > 0$, $\frac{d^2\psi}{dA_t^2} = \psi_{AA} > 0$, however the explosion is somehow controlled by $\frac{d^3\psi}{dA_t^3} = \psi_{AAA} < 0$. Also, the explosion is assumed to accelerate significantly for high level of A_t (*i.e.*, if A_t is small, $\psi_A(A_t) \approx 0$). Obviously, $A_{t+1} = A_t(1+g_{t+1})$ is a convex function in respect to A_t, as $\frac{dA_{t+1}}{dA_t} = 1+g_{t+1}+A_t\psi_A > 0$ and $\frac{d^2 A_{t+1}}{dA_t^2} = 2\psi_A + A_t\psi_{AA} > 0$.

The reader's first reaction towards the proposed structure of (11) should be that it is counterfactual, because the technology (as seen in the product path) is not exploding. Even though we have at least two shifts in the economic growth speed, apart from those shifts the output growth rate is stable, what seems to indicate that g_{t+1} is a constant. The specified conditions do not imply that the explosion should be immediately significant. Second, the model is set in a discrete time, with a period meaning a generation (so a century consists of circa three to four periods), so the growth is expected to explode on a generation, not on a yearly basis. In fact, the first and second industrial revolutions can be interpreted as such 'generation jumps'. One must remember that those two events were followed by two global conflicts which presumably burdened the growth. In fact, it is interesting to notice that after the almost complete destruction of Europe, China and Japan, militarization of almost all economies (what definitely must have had a negative impact over their growth), death of millions (often with a high human capital endowment), for other millions youth spent in the army instead of the human capital accumulation - after all this, western economies managed not only to regenerate, but also to surpass both the prewar momentum and the prewar level of the development. Another aspect of the fact that the period in my model should be interpreted as a generation replacement time, not a fixed amount of time like a year, is that the technology growth may have a positive effect on one's life-span, therefore the output growth caused by the technological progress does not have to mean an increase in the income in a fixed period - the increased product may be so small that it will cover only the increase of income's length. However, we know empirically that both the income's spell and the income level per a fixed amount of time grow. Therefore, if the former grows as it did in the last two centuries and the latter grows linearly, we still observe an explosion of the life-time income[1].

We can consider a phase space for the variables (e_t, g_t), conditional on q_t and A_t (see the Figure 2, see also (Galor, Moav, 2001) - notice that their phase diagram shifts only due to changes in q_t, which, as discussed in this paragraph, is not true for my model). For $q_t = 0$, there are three equilibriums, one at the point $(0,0)$ (which will be denoted as ϕ) and two other at intersections of $e_t(g_t)$ and $g_t(e_t)$, one (denoted as E_1) is unstable and the second (E_2) is stable. However,

[1]As will become clear in the proposition 2 (P2), there is a direct link between the technological growth (hence technological level) and the human capital, therefor the same argument can be constructed in favor of the assumption, that the minimum consumption constraint should grow with the human capital (Galor, Moav, 2002).

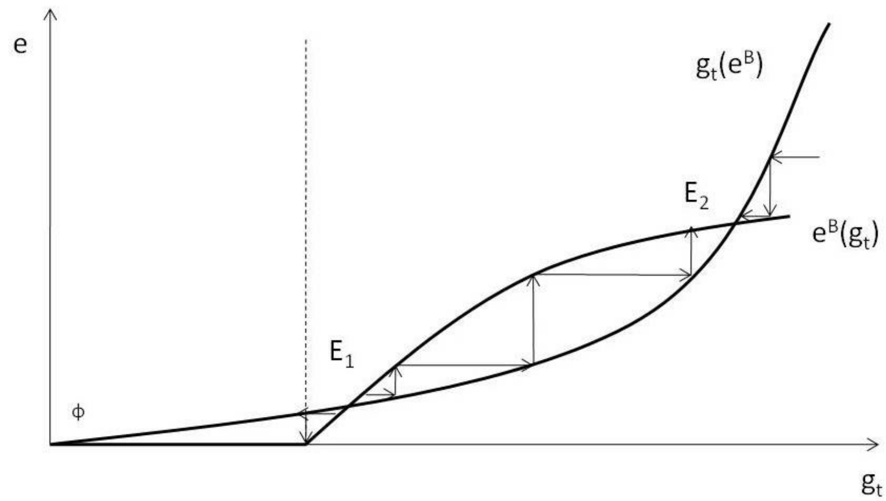

Figure 2: Phase diagram for the effort and technology growth if $q_t = 0$.

it follows from the properties of (11) that the line $g_t(e_t)$ is not stable in time, as for any $g_t > 0$, $A_{t+1} > A_t$ and $\psi_A > 0$. As a result, if the technology growth and the effort levels adjust to each other so that the technology growth will be positive (and hence the technology grows), g_t will be higher in the next period for any level of e_t (see the Figure 3). Notice that unless e_t falls dramatically, this effect will actually accelerate, so if e_t increases, technology will have a convex path time. In other words, in the phase diagram, after each adjustment, line $g_t(e_t)$ will shift rightward (I will call it a rabbit effect). Notice that this makes E_1 not only unstable, but also unsustainable - even if the pair (e_t, g_t) somehow phases to this point, $g_t > 0$ and hence the rabbit effect will render this point unstable and attracted by the E_2 equilibrium. Notice also that the latter has no fixed coordinates as well and will 'rabbit away' into the infinity.

Since ϕ and E_2 are sole attractors, there must exists a threshold value $g^*(A_t)$, which is a function of the initial level of A_t, such that if a random shock increases g_t over $g^*(A_t)$ then both effort and the technology will explode.

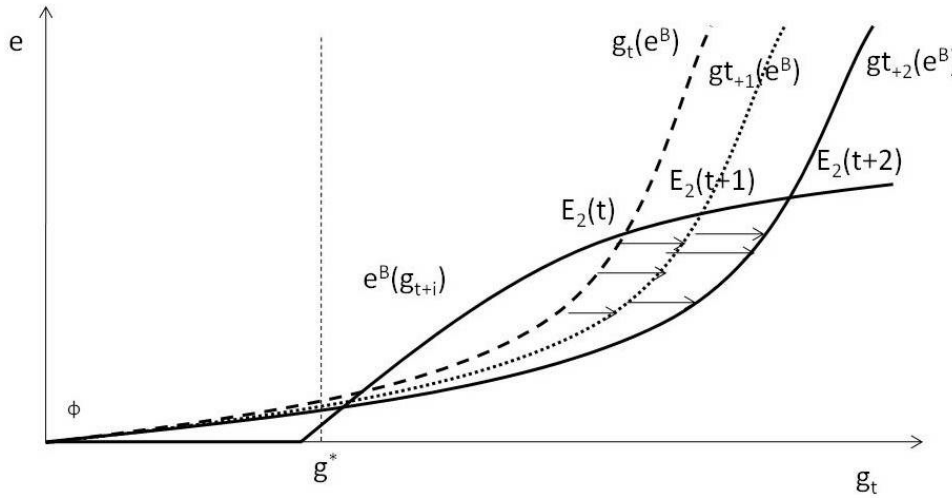

Figure 3: Phase diagram for the effort and the technology level, if $q_t = 0$. In every period, the technology's reaction curve shifts rightward.

However, if the shock render g_t positive but still smaller than $g^*(A_t)$, system will move towards ϕ.

If $q_t > 0$, ϕ is no longer a fixed point in the system (it follows from the properties of (8) that if $q_t > 0$ then $e_t(g_t)$ actually does not pass through the ϕ). Because of the kink in the $e_t(g_t)$ line, two additional equilibriums exist (E_1 and E_2). In fact, only E_2 will remain an attracting fixed point. As a result, any mutation of the population - holding all other factors constant - will cause a permanent explosion of the technology, as points near ϕ will be phased both upwards and rightward. The line $e_t(g_t)$ is fixed, whereas $g_t(e_t)$ will 'rabbit away', thus inevitably equilibriums E_1 and E_3 (if those points will actually ever occur) have to disappear in time (see Figure 4). As a result, mutation within the population will cause, holding all other factors constant, an explosion of both g_t and e_t.

If the system is growing but the q_t falls, we will observe two opposing forces for the dynamics of the system. First we have the rabbit effect, second,

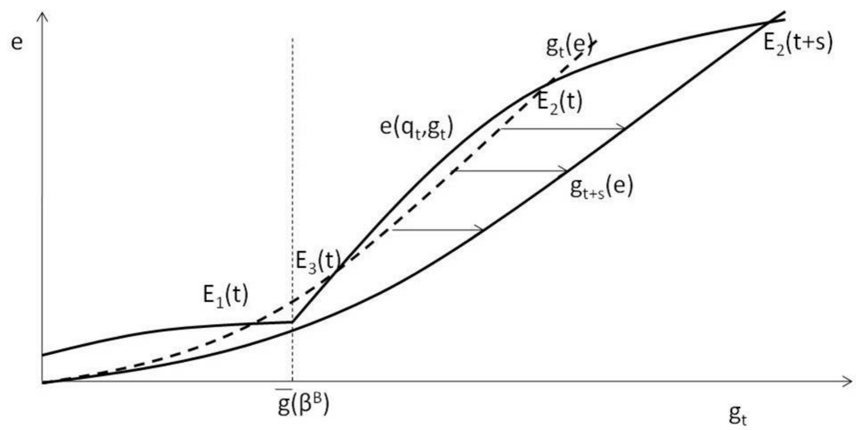

Figure 4: Phase diagram for the effort and technology growth if $q_t > 0$ and is stable. In every period, the technology's reaction function shifts rightwards, so the equilibriums E_1 and E_3 will eventually dissapear.

as follows from (P1) and properties of (10), the function $e_t(g_t)$ decreases for any value of g_t. In order to evaluate the effect per saldo of those two forces we would need to compare the appropriate derivatives and evaluate their ratio. However, we can notice that g_t is a concave function of q_t (as follows from (P1) and the properties of (10)) and a convex function of the A_t, so for a sufficiently high technology level (for sufficiently long period with a positive g_t) the effect of a decreased q_t will be smaller than the rabbit effect. Therefore, if the q_t falls once but remains positive, A_t will still increase and eventually rise to a level high enough for the technology growth function to increase above its level before the fall of q_t - in fact, for a sufficiently high A_t this will happen immediately. This implies that in the short-run the fall of q_t may cause the equilibrium E_2 coordinates to decrease, however this effect is negligible and can be omitted in the long-run analysis of the system.

Proposition 2 (P2)

If $q_t > 0$ or for a given technology level A_t, $g_t > g^*(A_t)$ then the system of (e_t, g_t) will explode to infinity. Otherwise, it will collapse to zero. If q_t falls during that explosion, as long as it remains positive or $g_t > g^*(A_t)$, it can be neglected for a long-run analysis.

2.2.4 Human Capital

An interesting problem arising here is that the effort level was shown in P1 to be a concave function of the technological growth, however technology may grow in a convex manner, as shown in P2. Thus, one can wander what will be the actual time path of the effort. We can easily see that based on the assumed structure of (11) and (8) $\frac{\partial e^i_{t+1}}{\partial A_t} = \frac{de^i_{t+1}}{dg_{t+1}} \frac{dg_{t+1}}{dA_t} = e_g \psi_A \equiv e_A > 0$. However, $\frac{\partial^2 e^i_{t+1}}{\partial A_t^2} = \frac{d^2 e^i_{t+1}}{dg^2_{t+1}} \frac{dg_{t+1}}{dA_t} + \frac{de^i_{t+1}}{dg_{t+1}} \frac{d^2 g_{t+1}}{dA_t^2} = e_{gg}\psi_A + e_g \psi_{AA} = e_{AA}$, which has an ambiguous sign. I assume that

Assumption 1 (A1)

$e_{gg}\psi_A + e_g\psi_{AA} = -\delta$, where δ is a positive constant.

Hence the effort level will increase in a concave manner if technology happens to grow (with the second derivative constant and negative). The assumption of the linearity of the second derivative is not essential and does not change any qualitative results (in fact, even stronger assumptions will be imposed on the behavior of $e^i_{t+1}(A_t)$ in the following proposition 4), however it will simplify greatly later derivations.

Assumed functional behavior of the human capital, (3), comes to be ambiguous, as $h_e > 0$ and $h_g < 0$, however the effort is an increasing function of the technology growth, hence an increase of the latter variable will directly decrease the human capital but also increase the effort and through it the human capital (P1). I will assume that per saldo effect is concave, thus

Assumption 2 (A2)

$\frac{\partial h}{\partial g} > 0$, $\frac{\partial^2 h}{\partial g^2} < 0$.[2]

Proposition 3 (P3)

Human capital level for both types of individuals is a concave, increasing function of the technology growth. Effort is an increasing, concave function of the technology level, with the second derivative constant.

In order to assure the immediate response of the natural growth to technological growth to be positive, we need to assume

Assumption 3 (A3)

For any A_t, it is true for any e^i_{t+1} that $e^i_{t+1} > \alpha A_t e_A$.

It will become clear later that the exact interpretation of the (A3) is that when technology grows, (1) grows fast enough to outweighed resources necessary to cover (7).

2.2.5 Income and the Population Growth

Lemma 1 (L1) (represented by the Figure 5)

There exists a value $\widetilde{A_i} \leq \overrightarrow{A_i}$ such that for any $A_t < \widetilde{A_i}$, $\frac{dn^i_{t+1}}{dA_t}(A_t) > 0$ and for any $A_t > \widetilde{A_i}$, $\frac{dn^i_{t+1}}{dA_t}(A_t) < 0$. For any $A_t > AN_i$, $n^i_{t+1} < 0$, i.e., for sufficiently high technological level, the ith type population will become extinct.

The proof of this lemma is technical and interesting on its own. It can be found in the Appendix A. Notice that if (L1) was not the case, then it is still true that $A_t > \overrightarrow{A_i}$, $\frac{dn^i_{t+1}}{dA_t} < 0$, so as the technology would grow, ith type population would behave in some undetermined way up to $\overrightarrow{A_i}$, when it would

[2] In their paper, Galor and Moav claimed the opposite ($\frac{\partial h}{\partial g} < 0$) seems natural. Unfortunately, since they showed that in the long run the technological growth is positive (contrary to the stagnation characteristic to the hunter-gather societies), by chance they obtained a conclusion that even though the equilibrium level of the effort is positive, post-industrial individuals represent lower value of the human capital than their tribal ancestors!

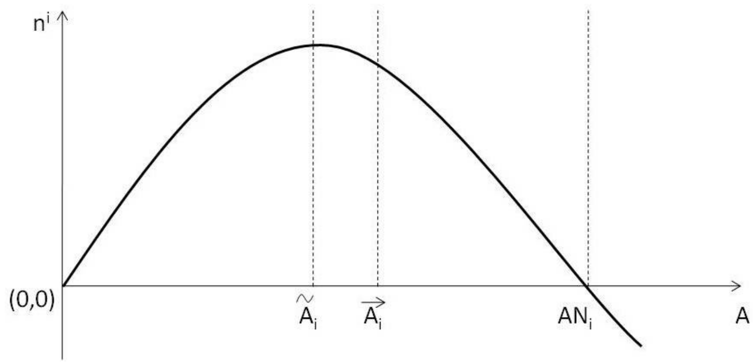

Figure 5: Natural growth of the ith type individuals as a function of the past technology level.

necessarily start to decrease and in time, as the technology would reach the AN_t level, ith type individuals would become extinct.

2.2.6 Population structure

As follows from (P1), for any positive technology growth or level, $e^A_{t+1} > e^B_{t+1}$. This implies that proper technology levels for $0 < eb < ec < ed < ef$ are different among the two groups of the population. Formally, combining (P1) and (P5), if $q_t > 0$, and defining $ek^{-1}(A_t)$ as a function such that $ek^{-1}(i, A_t) = A_{ki}$ iff $ek(A_{ki}) = ek$, where $k \in \{b, c, d, f\}$, we have that for any $k \in \{b, c, d, f\}$, $ek^{-1}(A, A_t) < ek^{-1}(B, A_t)$. This has an obvious influence over the structure of the population growth's reaction of both types of population to the technology:

Proposition 8 (P8)

It follows from P(3), P(7) and the properties of (15) that $\widetilde{A_A} < \widetilde{A_B}$. *It also follows from the properties of (15) that for* $A_t < \widetilde{A_A}$, $n^A_t > n^B_t$, $A_t > \widetilde{A_B}$, $n^A_t < n^B_t$ *and that therefore, as the intermediate theorem implies, there exists* $AQ_t \in [\widetilde{A_A}, \widetilde{A_B}]$ *such that* $n^A_t(A_t) = n^B_t(A_t)$. *It also implies that* $AN_B > AN_A$.

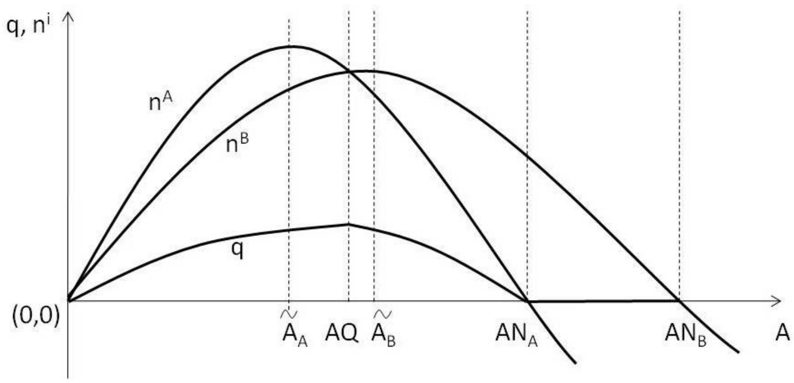

Figure 6: Comparison of the functional impact of the past technology level on the natural growth of both types of individuals - Case (1) as in P(9).

It therefore follows from the definition of q_t (9) that with an increasing series $\{A_t\}_{t=T_1}^{T_2}$, if $A_{T_1} < \widetilde{A_A} < AQ_t < \widetilde{A_B} < AN_A < AN_B < A_{T_2}$ (Case (1)) (see the Figure 6) or *$A_{T_1} < \widetilde{A_A} < AQ_t < AN_A < \widetilde{A_B} < AN_B < A_{T_2}$ (Case (2)) (see the Figure 7), for $A_t < AQ_t$, q_t will increase and for $A_t > AQ_t$, q_t will decrease, to reach zero when $A_t > AN_A$. However, when $A_t > AN_B$, both types of population extinct and therefore q_t becomes undefined.*

2.3 Dynamics of the System

Two most important propositions of the whole model are (P1), (P2) and (P8). (P2) shows that whenever the technology growth is initiated and $q_t > 0$, then the technology behaves almost independently from all other variables in the model, fueling itself in an infinite explosion. Even though the exact equilibrium between e_t and g_t depends in a given period on q_t, as long as $q_t > 0$, technology rabbits away and thus the equilibrium is shifted in every period so that the system will be attracted by a higher vector of (g_t, e_t) (P2). Even if the quality type population dies out, technology can still rise (the sufficient condition is that in the period after the quality type individuals become extinct, $g_t(A_t) > g^*$)

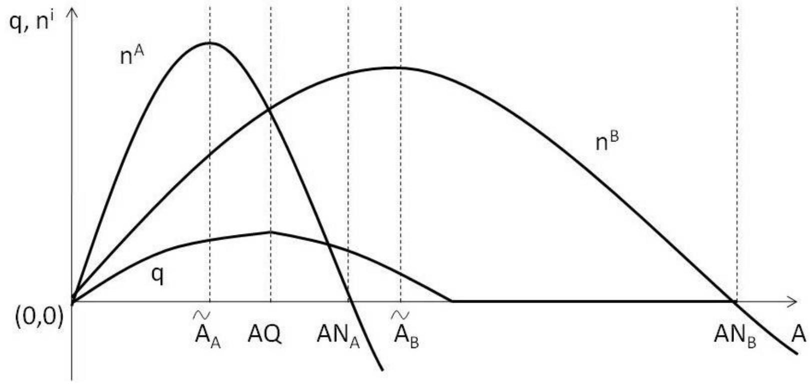

Figure 7: Comparison of the functional impact of the past technology level on the natural growth of both types of individuals - Case (2) as in P(9).

and eventually, the vector (g_t, e_t) will diverge to infinity.

On the other hand, the income of the population is a function of the technology level (as follows from (P8)). As once the technology is initiated, it will increase on its own, in the feedback between the technology and other variables, latter can somehow influence the exact path of the former, however it is the technology that determines the dynamics of the whole system. As follows from (P8), this implies that at first the technological advance has a positive, then a negative impact over the population growth. However, the exact long-run equilibrium depends on the set of initial conditions. (P1) combined with the (P3) enables us to evaluate the dynamics of the human capital.

In the following paragraphs, I will evaluate seven scenarios, each with a different set of initial conditions.

2.3.1 The Neolithic Scenario

Let the initial population consist only of quantity type individuals ($L_0 = L_0^B = L$), there be no mutation of the population and the technology level be smallest possible (*i.e.,* that of the simple hunter-gather communities, which for simplicity can be normalized to the unity). It follows from (P2) that $q_t = 0$, $g_t = 0$, $e_t = 0$ (*i.e.,* the system for the whole duration of the scenario remains in ϕ), as the initial set of conditions, combined with no emergence of the quality type individuals implies that the technology has no stimuli to expand. As a result, for any period, $Y_t = BL_t^{1-\alpha}$, where $L_t = L_t^B$, $w_t = BL_t^{-\alpha}$ and $n_t^B(\tau + e_{t+1}^B) = n_t^B\tau = d_t^B = BL_t^{-\alpha} - \overline{c}(0) \Leftrightarrow L_t = (\frac{B}{n_t^B\tau+\overline{c}(0)})^{1/\alpha}$, as follows from (1), (7) and the properties of (3). The population will be stable if $n_t^B = 1$, *i.e.,* when $L^* = (\frac{B}{\tau+\overline{c}(0)})^{1/\alpha}$. As the $(\frac{B}{n_t^B\tau+\overline{c}(0)})^{1/\alpha}$ is a decreasing function of n_t^B, it means that L^* is a stable equilibrium. If the initial population is low, *i.e.,* $L_0 > L^*$, it will grow until it reaches the equilibrium level L_t^*. We can interpret $\overline{c}(0)$ as the physical subsitence level consumption, therefore the population of humans in a most primitive society is strictly determined by our basic biological needs and the wealth of the environment.

This scenario obviously describes the initial fortune of the mankind, consisting of small groups following a hunter-gather scheme survival. Due to the abundance of the natural resources, initially small group of humans, located in the Africa, could spread across the globe, inhabiting new niches without any significant changes in the consumption per capita. In the equilibrium, only a constant number of individuals may survive, receiving the lowest possible income, which is high enough to cover only the physiological, subsitence necessities of the most uneducated individual.

2.3.2 The Bronze Age Scenario

This scenario has the same set of initial conditions, however the initial level of technology A_0 is greater than unity ($A_0 > 1$). Clearly, the structure of the equilibrium is the same as in the previous scenario, just that the equilibrium population size is $LB^* = (\frac{BA_0{}^\alpha}{\tau+\overline{c}(0)})^{1/\alpha} = A_0 \times L^*$. In other words, any technological growth over the most primitive technology causes a proportional increase of the population, however, as in the previous scenario, each individual receives income sufficient only for the pure biological survival (even though the whole society is more developed). Therefore, in this scenario, the technological progress means only a richer niche for mankind, but not an increase of the consumption per capita. In the next two scenarios, I will present two possibilities for a single shift in the technology level

2.3.3 The Greek Scenario

Let the initial population consist only of the quantity type of the population ($L_0 = L_0^B = L$). The initial technology level is set to A_0 (it does not have to be true that $A_0 = 1$) and does not growth, $g_0 = 0$. Also, for simplicity, let the population be in the Bronze Age equilibrium (or in the neolithic equilibrium, if $A_0 = 1$), so the system is stable. Now, let the population mutate twice. First, let in a period t_1 some individuals suddenly mutate into the quality type. As follows from (P1), those individuals will choose a positive amount of the effort in spite of the technology's stagnation and therefore, as is implied by (P2), the technology will start to grow, as the vector (e_t,g_t) will converge to E_1 (later, as the rabbit effect becomes significant, it may start to converge to the rabbiting away equilibrium E_2). As follows from (P8), the quality type individuals will (at least initially) gain an evolutionary advantage and spread among the society. Also, if $g_t > \overline{g}(\beta^i)$, the quantity type individuals will start to invest in their children. However, in a period t_2, another mutation happens so that all the quality type individuals die out. As a result, in the next period, $q_{t_2+1} = 0$. Let us assume that either $g_{t_2+1} < \overline{g}(\beta^B)$ or $g_{t_2+1} < g^*(A_{t_2+1})$. As follows from (P2), the first case means that the technology growth initialized by the quality type individuals is not high enough to attract the quantity type individuals to invest in their offspring's quality, whereas in the second case, those individuals have actually started to expose some effort, however as neither this effort nor the rabbit effect are high enough, the system is ultimately attracted by ϕ.

In both cases, the steady state is ϕ, therefore in a period t_3 (in the first case, $t_3 = t_2$, in the second, $t_3 > t_2$) the individual's effort drops to zero, hence the technological progress halts ($(e_{t_3}, g_{t_3}) = \phi$). Thus the system converges back to a bronze age equilibrium. Notice however that there is a positive amount of periods in which actually the technology was growing (for any $t \in [t_1, t_3)$, $g_t > 0$), thus $A_{t_3} > A_0$. It means that the equilibrium population size is $A_{t_3} \times L^* > A_0 \times L^*$. Therefore, a temporal existence of the quality type individuals could not alter the structure of the long-run equilibrium, still it managed to push the technology level and hence the population size up.

This scenario resembles the fate of the Hellenistic civilization. Greeks managed to develop science, mathematics, logic and philosophy to a level unsurpassed up to the end of the middle-ages. Still, the accumulation of the technology in the ancient Greece suddenly lost momentum in the second half of the fourth century BC, at the same time as Helens expanded politically and demographically beyond their homeland Greece, Macedonia and Ionia. The peak of Hellenic civilization - the Alexander's empire of the last decades of the fourth century BC - was followed by a stagnation in the third century and gradual collapse in following centuries. Indeed, we see here exactly the dynamics predicted by the scenario - a mutation causes the technological and demographic expansion, however the second mutation extinguishes the source of that development. Technology may grow due to some inertia of the system, however it quickly ceases. Population will still grow for some time until it reaches the new equilibrium and despite the higher technology, the consumption per capita will not increase.

That the first mutation must have had historically happened is clear, as otherwise humanity could not have develop over the neolithic age (this is clearly seen in the first scenario). Why then the second mutation? The natural answer is that the quality type individuals posses a relatively high level of the human capital, therefore we can expect them to become economic and scientific, but also political and military leaders. On the other hand, if the mutation is limited, *i.e.,* if $q_{t_1} \gtrapprox 0$ (what seems rather natural), it should be expected to occur locally (mutants should be concentrated in some specific area). In the pre-Alexander epoch, in which most European civilizations were decentralized or broken into a myriad independent political organisms, this meant that the whole civilization could start to develop even though only a minor fraction of its political entities enjoyed the mutation and new quality-oriented leadership. If, however, those entities were - apart from the technological and economic bounds - involved in political conflicts, in which the fortune shifted swiftly from one to the other local hegemon and victory was followed by a destruction of the enemy's elite (either directly by decimation or indirectly by the social status deprivation), the mutants - instead of spreading themselves among the population - were one by one exogenously removed from the system. In other words, the exogenous political distractions to the economic dynamics cleansed the economy from the quality mutation. Possibly the most notable example of this phenomenon is the fate of Athens, the mother of philosophy, who fell into intellectual stagnation after a series of conflicts with Sparta, other Greek cities and finally with Macedonia (Peloponnesian War, Corinthian War and the macedonian expansion being exactly those political distractions).

2.3.4 The Roman Scenario

Let the initial population, consisting only of the quantity type individuals, be in its Bronze Age equilibrium. Let the initial level of technology be A_0 and the technological growth be equal to zero (so this society is in ϕ equilibrium). However, let in a period t_1 happen an exogenous shock to the technology level, so that $A_{t_1} > A_0$, however $g_{t_1+1} = \frac{A_{t_1}-A_0}{A_0} < g^*(A_{t_1})$. As comes from (P2), the economy will eventually come back to the ϕ equilibrium. If $\overline{g}(\beta^B) > g_{t_1+1}$ then actually neither the effort nor the technology will grow in the following periods. Otherwise, individuals will choose a positive effort commitment and the technology will thus grow, however, both variables will decrease in time and the system will be attracted to the ϕ equilibrium. As a result, the economy will remain in the equilibrium described in the Bronze Age scenario, even though the terminal technology level is greater than the initial, hence after the technology starts to stagnate we still would observe some demographic expansion of the population, until it would satiate the newly enriched niche.

This scenario is actually one of the most commonly recurring processes in the history. There is a myriad of examples when a state manages by chance to seize its better developed rival and capture (at least partially) its technology, thus experiencing a single shift in the productivity and sometimes even a temporal development on its own, which however collapses into a stagnation. One may believe that the most notable example here is the case of the Roman empire, which when swallowing all of its better developed neighbors - especially Carthage and Hellenic civilization (*i.e.,* Greece, *Asia Minor* and Egypt) - enjoyed a major cultural and technological progress that suddenly started to loose the momentum when the Empire became short of civilized enemies to conquer. When discussing the reasons of the fall of the Western Roman Empire, it is often argued that the Romans, by destroying all of its advanced neighbors, simply became safe from a major invasion and thus lost stimuli to develop. From the perspective of the proposed scenario, it seems rather that the collapse of the Roman Empire started much earlier, when the expanding republic could not mutate on its own (*i.e.,* generate proper institutions) and thus used the captured Hellenistic technology only to expand its niche but not to change the structure of the equilibrium.

2.3.5 Protestant Scenarios

Let the initial population consists only of the quantity type individuals and the economy be in the bronze age equilibrium, with an initial level of technology A_0. Obviously, the society is in the stagnate steady-state ϕ. Let in the period t_1 a part of the society mutate so that some individuals become the quantity type. The initial behavior of the system is exactly as the one in the Greek scenario. It follows from (P1) that quality type individuals will start to invest in the quality of their children, what will cause the technological growth starting in the next period $t_1 + 1$. As follows from (P2), the long run relationship between the effort and the technological growth is the rabbiting away equilibrium E_2, so as long as the ration of the quality type individuals to the whole population does not fall, technology A_t, effort (both of the quality type individuals and the average) e_t^A, e_t and g_t will grow. As follows from (P8), this progress will initially increase the amount of resources quality type individuals devote for the procreation, who as a result will gain an evolutionary advantage and therefore their share in the population will grow.

Initially, quality type individuals will devote all of the growing surplus of the income over the consumption constraint to the procreation. However, when the technology reaches appropriate level at the period t_1, it follows from (P5) that the accumulated human capital will cause those individuals enjoy a sufficiently high level of income for an increased consumption. From this moment, this population will divide its income in a fixed proportion between the procreation and the consumption. The increase of the production will also allow the quantity type individuals to increase their natural growth, however at first they will enjoy a lower rate of growth than the other group. Thus, q_t will still be growing and this, combined with the technology's level impact on its growth, will ensure a constant, accelerating accumulation of the technology and hence of its velocity, as follows from (P2). Thus, the long-run equilibrium, E_2, will rabbit away and the quality type individuals will increase constantly their effort commitment, as follows from (P1) and (P2). The growth of the effort causes the human capital and thus the income to increase, it rises the minimum consumption constraint, though, and at a certain level of the technological progress at time t_2, the former effect will become impotent to outweigh the later, hence the natural growth of that part of the population will start to decrease, as is implied by (L1). Following (P5) and (P8), two things must happen - first, the consumption constraint will become so large that the quality type individuals will once again use all of the income's surplus over the constraint for the procreation; second, at t_3 quantity type individuals will gain an evolutionary advantage and thus enjoy a higher natural growth (and q_t will start to decrease).

As discussed in (P2), inevitably the rabbiting effect will outweigh the effect of the fall of q_t on the population, so the technology will not only remain growing, but its long-run growth will still accelerate. As a result, the effort commitment of both types of the population will remain increasing (or remain equal to zero) and finally at t_4 the quality type of the population will exhibit so high level of the effort, that the income will actually fall below the minimum consumption constraint. In this moment, the quality type population becomes extinct, even though the quantity group will still exist (P8). However, at this moment, $A_{t_4} > 0$ and $e^A_{t_4} > 0$, so in the final moment of the quality type group of the population, the technology is still growing ($g_t > 0$). Therefore, the evolution of the system depends on the approach of the quantity type of the population towards g_{t_4}. I will hence describe two possible endings to the Protestant scenario - Dutch and English.

The Dutch scenario Let us assume that either $g_{t_4} < g^*(A_{t_4})$ or even $g_{t_4} < \overline{g}(\beta^B)$. As follows from (P2) and was already discussed in the Greek and Roman scenarios, even if the first case was true and the population would put some effort into its offspring, the system will be ultimately attracted to ϕ. In this scenario, the economy therefore falls back into a bronze age equilibrium and will converge to the appropriate steady state (stable production and technology, no effort, lowest possible consumption per capita and $n = n^B = 1$), even though we can presume that it will exhibit a relatively high level of development.

To some extend this scenario resembles the Greek scenario, only that here the population is not exogenously purged of the quality type population, but rather the quality type individuals fall into a trap in which the technological growth makes them increase investments in children that are not outweighed by the income growth. This scenario greatly resembles the Dutch phenomenon prior to the industrial revolution. After the Netherlands became independent from the Habsburg Empire, they enjoyed a century long major economic development (the extend of which is represented by the fact that this country managed to replace Portugal in the Pacific area), which suddenly stopped and turned into a period of stagnation. One may believe that this is exactly what happened to the ancient Greece, however the Dutch case seems much more close to the described scenario, as contrary to the Hellenic civilization, Dutch were not exposed to such severe political shocks in the peak of their development.

Obviously, for this scenario to happen, the appropriate parameters, β^A and β^B, must be respectively large (close to 1) and small (close to 0), as otherwise it could not be true that $g_{t_4} < g^*(A_{t_4})$ or $g_{t_4} < \overline{g}(\beta^B)$. This means that the mutation must happen in a society highly disregarding the quality investments and cause some individuals to fall into the other extremum. In fact, Dutch freed themselves from a highly conservative empire (that remained in a relative stagnation up to the half of the 20th century!) in such a manner that the new Dutch state was founded as a merchant republic. One can easily interpret this fact as such a strong mutation.

The English Scenario Let, contrary to the Dutch scenario, $g_{t_4} > g^*(A_{t_4})$. This means that in t_4, when the quality type individuals disappeared, the quantity type group of the population was already exhibiting high enough level of the effort (and hence the human capital level) to render the technological growth sustainable even after the extinction of those who triggered it. Depending on A_0 and the vector of β^i, it follows from the (P8) that at t_4, $n^B_{t_4}$ was either decreasing (Case (2)) or increasing (Case (1)). Also, as follows from (P5), if the Case (1) is true, then quantity type individuals could have already accumulated so much of the human capital that were forced to again follow a corner solution in their problem of the division of the income between the consumption and the offspring (*i.e.,* they already fell into the same trap as the quality type population); if the Case (2) is true, then they may have managed to accumulate enough human capital in order to increase their consumption over the minimum level. However, in all cases, as already discussed, the future technology will still explode, therefore as follows from (P11), the quantity type will go along the same path of the resources available to the procreation and ultimately become extinct. Naturally, this means that the whole population dies out and no economic activity is possible anymore. Only the high technology level achieved by the dying society, embodied in tall buildings and beautiful gardens, may be witnessed by anything that is left after humanity fades away.

Contrary to the Dutch Scenario, here the whole population is caught in the trap of the accelerating technology and the increasing consumption demand, hence the whole society must eventually die. Ironically, as both the technology and the effort (and thus the average human capital) are constantly growing even in the terminal stages of the scenario, we still observe a positive (and presumably very high compared to the initial) production growth per capita and a very high consumption rate, closing to the whole (enormous) income. This is ironic indeed, as individuals in this scenario, being endowed with such a high level of both the human capital and the technology, are ultimately able to use less resources for the procreation than their primitive ancestors (*i.e.,* individuals in any previous scenario). However one may find strange a prediction that the abundance of resources (or income) should in time diminish the natural growth of a population (despite some initial push of the population size), this course of action is not a nonexisting phenomenon. In fact, we observe something very similar in modern western societies. When the first Industrial Revolution took off in England, at first the economic development coincided with the natural growth level unobserved in any period prior to the XVIII century. However, as the 19th century lapsed, three phenomenons happened at the same time - the production growth accelerated, the income and consumption per capita started to grow significantly for the first time in the history (and as discussed, the life-time consumption per capita exploded with the growth of the average life-span), but the population growth started to loose momentum to actually become negative in end of the following century.

2.3.6 The Colonial Scenario

Let the initial state of the world be as described in the Roman scenario and again, let the society be an object of an exogenous technological shock, which however is high enough to ensure that $g_{t_1+1} = \frac{A_{t_1}-A_0}{A_0} > g^*(A_{t_1})$. Here, as follows from (P2), contrary to the Roman scenario the system is attracted to the E_2, not the ϕ equilibrium. In other words, the technology start to grow and the will increase their effort in a level sufficient to enable the technological explosion. The development of this scenario resembles the English Protestant pattern of growth, only that there are no quality type individuals (obviously, as follows from (P1) and (P2), this implies that if two independent and identical economies are involved at the same time in those two different scenarios, the English Protestant one will exhibit a higher initial level of the growth and thus will explode faster, *i.e.,* will always exhibit a higher level of the technology and consumption per capita). However, the ultimate fate of this economy is exactly the same as in the previous scenario and here the whole population (*i.e.,* the quantity type individuals) will eventually die out for the same reason as in the English Protestant case.

The described scenario seems to have been happening in the peripheries of the Industrial Revolution. Regions such as Southern America, partly Africa, China and to some extend Russia were exposed to the European expansion of the preindustrial and industrial epoch that brought the industrial technology but not always caused the quality mutation of the population. In other words, due to some political reasons, those economies could at least partially imitate modern technology, however did not developed or were not allowed to develop their own quality orientated elite. As a result, those purely quantity type populations were exposed to a sudden change of the technological level, which was high enough to throw them out of the Bronze Age stagnation, and therefore entered a growth path, however this growth was inevitably slower than in the countries following the English Protestant Scenario.

Notice that the countries which were conquered and therefore exposed to a sudden technological shock that still was much to small for the $g_{t_1+1} = \frac{A_{t_1} - A_0}{A_0} > g^*(A_{t_1})$ condition to be true (as it seems in the case of some African countries), are actually following the Roman scenario.

2.4 The historical path of the GDP

Obviously, the real GDP path was marked by a mixture of all the described scenarios. In fact, it is possible that in some cases more than one scenario was happening at the same society - for instance, the Carolinian renaissance may have been an example of both the Roman and the Greek scenarios. Further more, it seems possible that the mutations were not always happening one by one, so that more than two groups with different evaluation of the human capital (*i.e.,* with different β^i) emerged at a same time, or close to each other in time. Also, at the same period different societies may follow different scenarios, as it seems to have been the case in the divergence of the Asia and Europe at the dawn of the Industrial Revolution. What is perfectly clear is that the initial stage of human development, the Neolithic scenario, was followed by a long series of a mixture of all scenarios apart from the English Protestant and the Colonial ones. Those two are completely unique and happened historically once, the former proceeding the latter by a small margin of time and therefore presumably causing it. Obviously, the mutation that finally caused the economy to escape the Bronze Age stagnation, causing the English Protestant pattern of development, happened in Europe, presumably in England, in the second half of the previous millennium. Even though before the industrial epoch there existed some societies that managed to achieve astonishing level of development, they all ultimately fell back to the Bronze Age equilibrium, where the technology raises the population size, but not the standards of living.

It was for the mutation that triggered the Industrial Revolution to completely alter that ancient scheme of the development. That mutation caused the technology to explode and hence spread in an astonishing velocity among other societies, triggering them either to mutate on their own or to acquire technological level that pushed them into an endogenous production growth per capita.

As a result, in the last 300 years, humanity suddenly changed the traditional order of the nature, where the niche growth causes only the population growth, not an improvement of conditions of the individual existence.

This prediction of my model does not only suit well the observed facts about the historical path of discussed variables, but also was generated in a consistent manner, without any qualitative differences of the analysis of the preindustrial and the postindustrial epoch. Therefore, one may find my model most convenient. However, it generates one prediction more, which on its own is rather distracting - the extinction of the humanity. Obviously, if not the whole mankind is following one of the Protestant or the Colonial scenarios, some societies will eventually return to the Bronze Age equilibrium (even if it will be a Bronze Age enhanced by a futuristic technology). Still, is there any hope for the societies leading the modern growth? First, one can notice that if a due to an another mutation some individuals with $\beta^i = 0$ will emerge within those societies, they will survive the extinction of their more quality orientated kinsmen (although they will converge to the Bronze Age equilibrium as well). Second, the model on itself is flawed by one restriction - the whole evolutionary game is played only in the field of culture, whereas our biology remains intact. However, it is possible that either the purely biological selection will somehow diminish the growing pressure of the consumption necessary to maintain the human capital (e.g. individuals with more efficient brains, or women able to give a birth at a very old age may enjoy an obvious evolutionary advantage over their kinsmen), or that we will use our technology to achieve the same effect (e.g. we will use modern medicine to enhance our brains, fasten the maturity process, extend the procreation period or simply find a substitute for the traditional procreation). In fact, one should extend my model by linking the cultural and the biological evolution of the mankind.

3 Empirical Investigation

3.1 The setup of the Investigation

In the following section, I will conduct an empirical investigation based on my theoretical model. In this investigation, I will use the historical data for English (and then British) GDP per capita (taken from (Clarke, 2001) and the average wage (taken from (Makridakis, 1998). As discussed in the scenario section, England and than United Kingdom is the only probable candidate for a country in which the English Protestant Scenario was initiated. This scenario predicts the historical path of English natural growth, consumption and production per capita, it has one more implication, though, which cannot be observed directly. The model implies that the economic growth in that country should be caused by the technological progress, to which the endowment of the human capital adapts in time. In other words, human capital may cause technological progress, but not independently - it is rather technology that fuels itself directly and indirectly through the human capital. The whole empirical analysis will focus on testing this prediction.

Even though the model generates interesting qualitative results that seem to fit the historical data, it is mostly constructed on functions with a defined behavior, yet without a proposed functional form. Therefore, if one wanted to use the model in order to estimate any kind of a structural econometric model, he would have to specify all the proposed functional relationships between the variables. This yields a major problem - if one used functions that may fit their assumed structure but apart from this are chosen *ad hoc*, without any foundations in the basic models of economic agents' behavior, he would always face a risk of misspecification, which would make the empirical investigation reject the model not because the model itself is wrong, but because it is not supported with a proper microeconomic discussion. To avoid this risk, one thus would have to conduct a proper investigation for any assumed relationship in the model. On the other hand, each of those relationships is presumably complex and demanding in terms of cognition, hence a proper investigation for a functional form of any part of the model - functional form that would be consistent both with the present and the historical data and across the different countries - is a major task on its own.

Another problem is that the major advantage of the model - *i.e.,* that it is constructed in order to fit the behavior of the economy also outside its modern shape - suggests an investigation that would not be based on the modern data alone. What would be a reason for constructing such a model, if one could not confront it with historical data as well? However, the data sets that can be said to be 'immediately observed' are modern, as the available and reliable macroeconomic statistics cover usually the period past the Second World War. Any data accounting for the size of economic variables prior to 1950ties are usually estimates on their own, have necessarily higher measurement error, lower frequency and finally they usually describe political entities that no longer exist, what yields obvious econometric problems.

A choice of a reduced form estimation technique is not constrained by all those problems, on the other hand. Also, it can work very well for the available data - far from the desired precision and frequency (and thus far from the desired length). The dynamic structure of the model immediately implies a dynamic reduced form model - VAR or VECM.

One can easily retrieve a proxy for the human capital from the wage and the GDP per capita. Obviously, the observed wage is not the same object as the wage (or income) of the model, w_t. The empirical wage is a remuneration for the labor, which is just one of many possible forms of one's contribution to the production. Other sources of income include rents from land, capital market, etc. Thus, w_t in practice will consist of many different elements, whose structure depend on the state of the economy and which were summed for the cause of simplicity. However, one can easily retrieve the labor wage in the model. We can assume an efficient labor market (especially as the model is based on a period which obviously has a 'long-run' duration for simple microeconomic processes), so that the remuneration for a single unit of labor contribution is $w_t^{labor} = \frac{dY_t}{dL_t} = (1-\alpha)BA_t^{\alpha}H_t^{-\alpha}\overline{h_t}$, where $\overline{h_t} = H_t/L_t$. A single person will receive on average this remuneration multiplied by his human capital endowment (*i.e.,* the number of his labor efficiency units), so the observed average wage is $w_t^{empirical} = \overline{h_t}w_t^{labor}$ (and model's w_t consists of w_t^{labor}). The ratio of the observed average wage to the product per capita is given by

$$\frac{w_t^{empirical}}{Y_t/L_t} = \overline{h_t}(1-\alpha)\frac{BA_t^{\alpha}H_t^{-\alpha}\overline{h_t}L_t}{BA_t^{\alpha}H_t^{1-\alpha}} = (1-\alpha)\overline{h_t}, \qquad (16)$$

i.e., it is equal to the average human capital endowment in the economy.

On the other hand, as $Y_t/L_t = BA_t^{\alpha}H_t^{1-\alpha}L_t^{-1}$, $\ln Y_t = \ln B + \alpha \ln A_t + (1-\alpha)\ln H_t - \ln L_t$. Therefore,

$$\frac{w_t^{empirical}}{Y_t/L_t} + \xi \ln Y_t = \overline{h_t} + \xi(1-\alpha)\ln \overline{h_t} - \xi\alpha \ln L_t + \xi\alpha \ln A_t + \xi \ln B. \qquad (17)$$

(17) has a straightforward econometric interpretation, based on the model. As established in (P3), human capital *via* effort is strictly linked with the technology level, that is it is its concave function. Also, as established in (L1) and (P8), the population size is a strict function of the technology level, so we can treat it as an additional embodiment of the technology. Therefore, there exists a long-run causal relationship between those two variables. One can use a logarithm to approximate this bond with a linear relationship. We know that in England, the consumption per capita in the last two centuries exhibited a growing behavior, so the model indicates that both variables must have been

growing as well (either in the English Protestant or the Colonial scenario), so they are not stationary. However, if the relationship predicted by the model is true, those two variables should be cointegrated in the sense that (17) must be stationary, and also that the technology level is exogenous in this relation, contrary to the human capital.

The natural testing procedure for this hypothesis is therefore to follow the Johansen procedure and then to construct a VECM. The hypothesis of the analysis is that GDP per capita is the exogenous variable of the system, while the ratio of the average wage to the GDP per capita follows the former variable path. The system consists of a mechanism correcting any divergence of those two variables from their long-run relationship, but which is somehow influenced by some short-run fluctuations and inertias of the dynamics of the economy. However, I must emphasize the fact that this cointegration relationship works only for the English Protestant Scenario - as in other scenarios, the human capital changes are negligible even if there are some shocks to the technological development. It is not clear when the mutation that pushed England form the Bronze Age Scenario to the English Protestant Scenario happened, however it is natural to assume that the accumulation of the human capital and technology caused by that mutation started at least in the late Middle-Ages, so the data should overlap this scenario.

The data set consisted of two different series for the production - one from 1260 to 1860 with a decade frequency, the second from 1830 to 1994, with a yearly frequency. Due to the difference in the frequency and the sources of the data, I divide the analysis between those periods and hence will propose two separate estimation results. All the tables I reffer to are in the Appendix B.

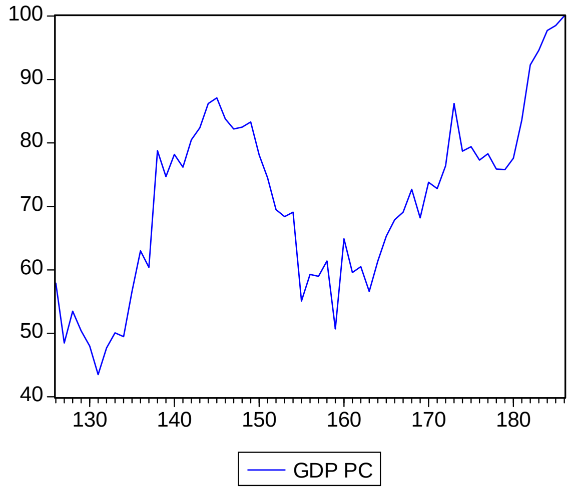

Figure 8: The GDP per capita (GDP PC) in England, 1260-1860.

3.2 1260-1860 Sample Analysis

The main surprise is that the claim that the sample should overlap with the period, in which the English Protestant scenario was triggered, came to be false. In fact, up to the beginning of the XVI century the proxy for the human capital fluctuated slightly around one constant value, while the product per capita enjoyed some major shifts and both Johansen procedure and Engel-Granger procedure rejected the null of a cointegration relationship between those two variables (Table 1.1) . This implies that England at that period was a subject to some dynamics of the Bronze Age scenario. However, one can notice a sharp decline of the former variable in the XVI century, that is in the period of Reformation, and afterwards variables seem to share a common trend (see Figure 8 and 9). This implies that the actual mutation coincided with the Reformation and that it is the XVI century in which England switched its development path from the Bronze Age scenario to the English Protestant scenario.

Immediately a question of an interpretation of this fact emerges. One may

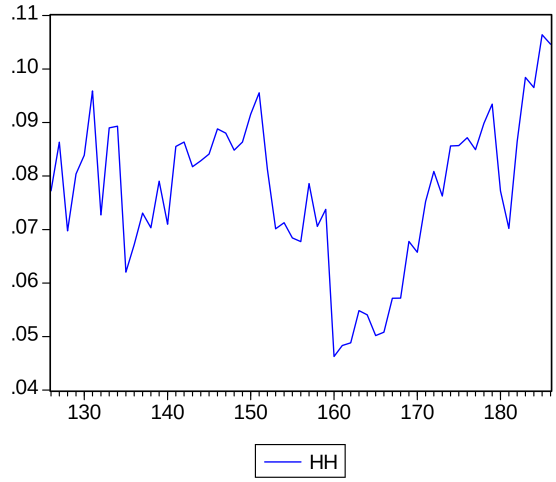

Figure 9: The average wage divided by the GDP per capita (HH) in England in 1260-1860.

be easily convinced by the Old Weber's thesis that it was the protestant ethics that made people change their approach towards the welfare and hence behave in a more productive way. This hypothesis however is not sufficient, as it is not clear why other protestant countries did not participate in the Industrial Revolution as early as England did. I think that a proper explanation can be based upon examining the impact of the Reformation upon the society, connected however with one process marking the early Tudor reign. Tudor dynasty captured the English throne due to the War of the Roses, which inflicted heavy casualties on English nobility and also caused a decline of their military and feudal power. As a result, old aristocracy was almost totally recycled and came to be less focused on military and more on the economic activities.

Around one generation later, the Reformation started. It must be emphasized that the Catholic Church, apart from all the religious services, played an important role in the economy, as was its cultural, educational and scientific center. Also, catholic faith enabled to some extend institutions necessary for

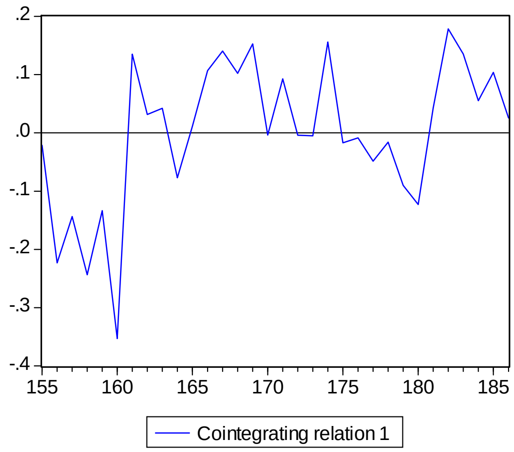

Figure 10: Estimated cointegration relationship for the 1550-1860 period.

economic agents for credible commitment and contract enforcement, e.g. in the case of guild (Richardson, 2005). When the Protestantism spread in England, all this had to disappear suddenly and an institutional vacuum emerged (*Reformation to revolution...*, 1995). It seems probable that the mutation is connected with part of the gentry filling this vacuum and thus becoming the basis for the future capitalism.

It seems that by 1550ties England was a protestant country, at least in terms of its elite (*Reformation to revolution...*, 1995), so the mutation must have happened within the two decades of the Reformation - 1530-50, and therefore a sample restricted to 1550-1860 should be cointegrated. Unfortunately, this sample consists only of 31 observations, so the Johansen test results are not reliable. Instead, I used the Engel-Granger procedure, *i.e.,* estimated a linear regression between the two variables in question and checked if thus obtained residuals are stationary (Table 1.2). In fact, they were.

Based on this, a VECM was estimated (only for those two variables, with one lag of their differences). The result came to be successful and stable - all but one restricted roots were greater in modulus than one (Table 1.5), and there was no autocorrelation left in the model (Table 1.6). Also, the cointegration relationship seems to be stationary (see the Figure 10), however one may think that it enjoyed a shift at the beginning of th XVII century. Still, this shift is not significant and accounting for it could happen only at the cost of additional sample shortening and thus loosing even more degrees of freedom. Surprisingly, the residuals were not rejected to be normal (Table 1.7), even though the sample was short. As follows from the Granger causality test, both variables influence each others short-run dynamics. Variance decomposition (based on Cholesky's decomposition, Tables 1.8 and 1.9) confirms my expectations - the main source of model's variability is the logarithm of GDP per capita, *i.e.,* the technology. As can be seen in the Figure 11, a single shock to the $\log Y_t/L_t$ persists in the system, causing both itself and the human capital proxy to grow linearly. On the other hand, a unit shock to the human capital causes a single shift of the level of that variable and has almost no effect on the output. Interestingly, the variability of the model is largely explained by its explanatory variables, as the SE accounted for not more than 1.5% of the behavior of the variables. As a result, one can safely claim the data shows there exists a long-run relationship between the technology and the human capital, in which the human capital follows, is attracted by the unaided technological progress.

3.3 The 1830-1994 Sample Analysis

Contrary to the previous data set, this sample had no structural breaks and the existence of a single cointegration relationship (see the Figure 12) between the logarithm of the GDP per capita and the proxy for the human capital was confirmed by one of the two tests of the Johansen procedure (the trace test,

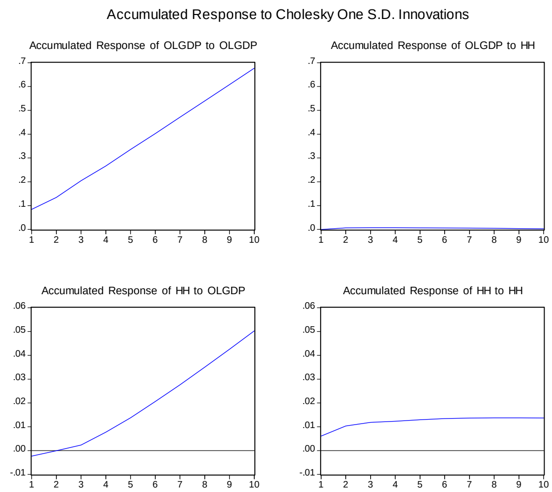

Figure 11: Accumulated response functions to the unit shocks of the system, based on Cholesky's decomposition. Sample 1550-1860.

Table 2.1). As the second test rejected this relationship only slightly, with a p-value equal to 0.061 (Table 2.1), VECM with two lags of the differences of those variables was estimated. The results were successful and stable - again, all but the one restricted roots were greater than zero in modulus (Table 2.4) and there was no autocorrelation left within the model (Table 2.5). Unfortunately, the kurtosis of the residuals was rejected to be normal (and hence the Jarque-Bera rejected the null of normality) (Table 2.6), so one may find himself reluctant to the Granger-causality tests' results. However, this does not burden estimation's major result, *i.e.,* the variance decomposition (Tables 2.7 and 2.8).

Here, the human capital has much greater impact over the system, as it explains most of its own variability over time and also has a strong impact on the product. The response graph (see the Figure 13) indicates that a shock in the logarithm of the GDP per capita again persists in that variable, but also causes the decline of the human capital. One may be astonished by this

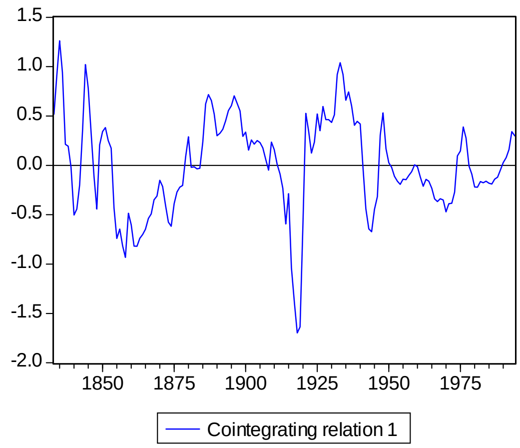

Figure 12: Estimated cointegration relationship for the 1830-1994 period.

remark, however, it is possible to interpret it using the model. The natural growth in modern societies is low or negative, whereas the model suggests that in the English Protestant scenario, this first should happen to the quality type individuals. As a result, when the technology grows, both the quality and the quantity groups should increase their investments in the human capital, however this is followed by a decreasing share of the former group in the population. If the latter effect is stronger, so if the q_t declines rapidly, whereas the quantity type population values relatively little the quality of their offsprings (so β^B is small), then the average human capital may decline as the quality population dies out. This also explains why the human capital has such a strong impact on itself and the logarithm of the GDP per capita - contrary to the previous case, when the technology fueled both variables, here, as discussed in (P2), technology causes the rabbit away effect, which is however somehow contained by the decreasing q_t, so in a sense, technology triggers on its growth two opposing forces at the same time (a positive stronger, though), whose friction is visible in the impact

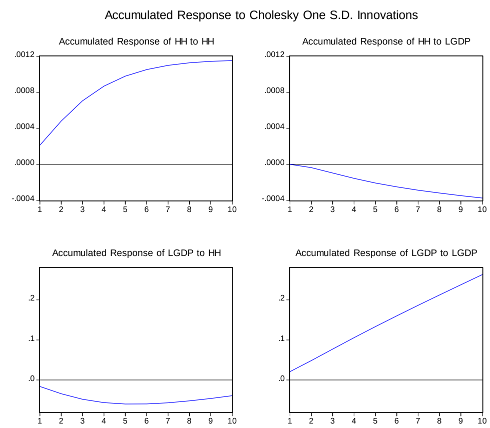

Figure 13: Accumulated response functions to the unit shocks of the system, based on Cholesky's decomposition. Sample 1830-1994.

of a unit shock in the human capital over the whole system.

The comparison of the two estimates shows how different the two investigated periods are. In the outbreak of the Industrial Revolution (or as the empirical analysis indactes, from the middle of 16th century), technology growth pushed the accumulation of human capital, however, in the modern era, thus accumulated capital inflated the consumption necessities and hence burdened the quality type individuals' natural growth - as a result, those individuals systematically vanish from the population, having no evolutionary advantage over the other group. Ironically, the velocity of technological progress is high enough to be self-sufficient and even though the human capital decreases, the production still grows. This proves the expectation that the English Protestant scenario did happen in England (and hence deserves its name) is correct, but also shows that the model does fit the reality and we can explain the historical path of the consumption, product, human capital, technology, natural growth and their relationships with the evolutionary framework. Also, the empirical investigation revealed the period, in which the actual mutation triggering the English Protestant scenario happened.

4 Conclusions

In this thesis, I proposed an evolutionary framework, in which individuals with a high evaluation of the investments in the quality of offsprings trigger a human capital accumulation and thus ignite technological advancement. Here, the quality oriented group of the society enjoys initially a growing income, thus gains an evolutionary advantage and spreads within the society, enlarging their human capital endowment with the technological growth. However, as human capital enforces individuals to increase their consumption, in time the income benefits of a high technology are outweighed by the increasing consumption constraint and either the quality type group or the whole population becomes extinct. A VECM estimation for the historical data for England confirms the expectations of the model towards the relationship between the technology and the human capital - it is the former variable that gives the whole system its momentum, to which the latter simply adapts.

The historical development of mankind's economic activities is difficult to asses, as it comes in a complex feedback with other demographic and social variables. Proposed model may be believed to show some insight into this phenomenon. In fact, the model generates scenarios that resemble historical events and processes, such as a sudden expansion of the Hellenic and Roman civilizations, an explosion followed by a stagnation of Netherlands or a seemingly endless explosion of the Industrial Revolution. The advantage of the model is that it links mutations and evolution of the individual preferences towards the procreation with technology and the accumulation of human capital. Thus, it explains the relationships between economy, especially the output and the consumption per capita, and the population size and structure. As a result, it yields a unified and coherent vision of the overall economic and demographic history that is supported by historical evidence. Also, an empirical investigation based on the model confirms its predictions and also indicates the historical location of the mutation responsible for the current economic growth. Hence, the evolutionary framework can be perceived as potent to explain coherently different economic phenomena of mankind's history.

Model predictions seem to be distracting, though. It seems that the reality allows only for a temporal solace from the poverty trap, in which individuals enjoy a consumption satisfying mere biological necessities and devote all the remaining income to procreation, with an inevitable extinction as a reward for those who try to defile this first law of nature. Also, one may dislike a thesis that his culture is only a matter of a biological evolution. In fact, we like to think of ourselves as different from animals because we can enjoy a life full of spirituality and culture and free from the unpleasantness of the existence dedicated solely to a struggle for biological survival. Both of these come to be an illusion, if the model predictions are correct.

It must be emphasized, however, that the model itself should be extended, as in its current form is still incomplete. First, it lacks biological evolution and hence mechanisms with which we could use our technology to alter our biology and thus to render false the grim conclusion of the previous paragraph. It seems therefore that the model should be enhanced by a mechanism, in which the technological level could either ease the consumption constraint or increase the efficiency of the investments in both the quality and the quantity of the offsprings. Another problem is that the model does not explicitly account for mutations, treating them as exogenous shocks. Instead of this approach, a stochastic process, that presumably could somehow depend on the state of the economy, should be added into the discussed structure.

5 Bibliography

Margo Todd (ed), (1995): *Reformation to revolution : politics and religion in early modern England*, Routledge, London

Aghion, Philippe Howitt, Peter, (1992): *A* Model of Growth Through Creative Destruction, *Econometrica*, Vol. 60, No. 2

Barkow, Jerome H., (1978): Culture and Sociobiology, American Anthropologist, *New Series*, Vol. 80, No. 1

Barro, Robert J., Sala-i-Martin, Xavier, (1995): *Economic Growth*, McGraw-Hill, Inc.

Bell, Duncan S.A., MacDonald, Paul K., Thayer, Bradley A., (2001): Start the Evolution without Us, I*nternational Security*, Vol. 26, No. 1

van den Berghe, Pierre L., (1990): *W*hy Most Sociologists Don't (And Won't) Think Evolutionarily, *Sociological Forum*, Vol. 5, No. 2

de Boer, Paul, van Dijk, Herman, Franses, Philip Hans, Heij, Christiaan, Kloek, Teun, (2004): *Econometric Methods with Applications in Business and Economics*, Oxford University Press

Clark, Gregory, (2001): The secret history of the Industrial Revolution, available at http://www.econ.ucdavis.edu/faculty/gclark/papers/secret2001.pdf

Dunford, Mick, Smith, Adrian, (2000): Catching up or Falling behind? Economic Performance and Regional Trajectories in the "New Europe", *Economic Geography*, Vol. 76, No. 2

Enders, Walter, (1995): *Applied Econometric Time Series*, John Wiley & Sons, Inc.

Fracchia, Joseph, Lewontin, R. C., (1999): Does Culture Evolve?, *History and Theory*, Vol. 38, No. 4, (Theme Issue 38: The Return of Science: Evolutionary Ideas and History)

Galor, Oded, (2005): From Stagnation to Growth: Unified Growth Theory, *Minerva Center for Economic Growth Working Paper* No. 2004-1

Galor, Oded, Moav, Omer, (2002) From Physical to Human Capital Accumulation: Inequality and the Process of Development

Gintis, Herbert, (2008): Towards a Unity of the Human Behavioral Sciences, available at http://www.umass.edu/preferen/gintis/NewUnity.pdf

Hall, Robert, Jones, Charles, (1999): Why Do Some Countries Produce So Much More Output Per Worker Than Others?, *Quarterly Journal of Economics*

Jenkins, Andrew, Vignoles, Anna, Wolf, Alison, Galindo-Rueda, Fernando, (2003): *T*he Determinants and Effects of Lifelong Learning, *Applied Economics* 35

Jones, Charles I., (1995): Time Series Tests of Endogenous Growth Models, *The Quarterly Journal of Economics*, Vol. 110, No. 2

Li, Chol-Won, (2000): Endogenous vs. Semi-Endogenous Growth in a Two-R&D-Sector Model, *The Economic Journal*, Vol. 110, No. 462, Conference Papers

Lucas, Jr, Robert E., (1998): On the mechanics of economic development, *Journal of Monetary Economics*, Volume 22, Issue 1

Maddison, Angus, (1995): *Monitoring the World Economy 1820-1992*, Organization for Economic Co-operation and Development, Paris

Makridakis, Wheelwright, Hyndman, (1998): *Forecasting: Methods and Applications*, 3rd edition, available at: (personal.buseco.monash.edu.au/~hyndman/TSDL/data/wagesuk.dat)

Mulligan, Casey B., Sala-I-Martin, Xavier, (1993): Transitional Dynamics in Two-Sector Models of Endogenous Growth, *The Quarterly Journal of Economics*, Vol. 108, No. 3

Richardson, Gary, (2005): Craft Guilds and Christianity in Late Medieval England: A Rational Choice Analysis, *Rationality and Society* 17

Sargant, N., J. Field, H. Francis, T. Schuller, and A. Tuckett, (1997): The Learning Divide: A Study of Participation in Adult Learning in the United Kingdom, National Institute of Adult Continuing Education (NIACE)

Solow, Robert M., (1956): A Contribution to the Theory of Economic Growth, *The Quarterly Journal of Economics*, Vol. 70, No. 1

Appendix A: Proof of Lemma 1

As was stated in (P2), if $q_t > 0$ or $g_t > g^*(A_t)$, the effort commitment and the technology will explode. However, it is not clear whether this will cause an everlasting population growth, as the technology (directly and through the human capital) increases income (due to properties of (1)), however - through the effort commitment it also causes the minimum consumption constraint to explode (due to properties of (6) and under P2), what under P1 may decrease the population growth (of both groups) through the income. Thus, the overall effect of the technology over d_t^i must be evaluated.

As follows from the properties of (2), three contemporary to the income variables are human capital, technology level and the population growth. Let

$$x_t^i = w_t h_t^i \qquad (14)$$

denote the total income of ith individual.

Then, $\frac{dx_{t+1}^i}{dh_{t+1}^i} = w_{t+1} + h_{t+1}^i B {A_{t+1}}^{\alpha}(-\alpha){H_{t+1}}^{-\alpha-1}L_{t+1}^i =$ $= w_{t+1}^i(1-\alpha\frac{h_{t+1}^i L_{t+1}^i}{H_{t+1}}) > 0$, since from the (4) $\frac{h_{t+1}^i L_{t+1}^i}{H_{t+1}} \in [0,1]$ and $\alpha \in (0,1)$. $\frac{d^2 x_{t+1}^i}{{dh_{t+1}^i}^2} = -\alpha B {A_{t+1}}^{\alpha}{H_{t+1}}^{-\alpha-1}L_{t+1}^i - w_{t+1}\alpha\frac{L_{t+1}^i(H_{t+1}-h_{t+1}L_{t+1}^i)}{{H_{t+1}}^2} < 0$, again from the definition of the variables. Thus, the immediate response of the income to the changes of the human capital is concave. Notice that this implies that the response of the income of an individual to his parents effort is concave as well (as follows from the properties of (3)). Trivially, $\frac{dx_{t+1}^i}{dh_{t+1}^j} = h_{t+1}^i B {A_{t+1}}^{\alpha}(-\alpha){H_{t+1}}^{-\alpha-1}L_{t+1}^j < 0$, $\frac{d^2 x_{t+1}^i}{{dh_{t+1}^j}^2} = h_{t+1}^i B {A_{t+1}}^{\alpha}(-\alpha)(-\alpha-1){H_{t+1}}^{-\alpha-2}{L_{t+1}^j}^2 > 0$, $\frac{dx_{t+1}^i}{dA_{t+1}} > 0$ and $\frac{d^2 x_{t+1}^i}{dA_{t+1}^2} < 0$ (${A_{t+1}}^{\alpha}$ is obviously a concave function). Finally, using (5), we can derive $\frac{dx_{t+1}^i}{dn_t^s} =$ $= h_{t+1} B {A_{t+1}}^{\alpha}(-\alpha){H_{t+1}}^{-\alpha-1}h_{t+1}^s L_t^s < 0$, $\frac{d^2 x_{t+1}^i}{{dn_t^s}^2} = h_{t+1}^i B {A_{t+1}}^{\alpha}(-\alpha)(-\alpha-1){H_{t+1}}^{-\alpha-2}{h_{t+1}^s}^2{L_{t+1}^j}^2 > 0$, for $s \in \{A, B\}$. Obviously, at all those fact are true for d_t^i as well, as that variable is a linear combination of the income and the consumption constraint which on its own does not depend on any contemporaneous variables (as follows the properties of (7)).

It will be necessary to link the income with the past effort. Using facts shown in the above paragraph and properties of (3), $\frac{dx_{t+1}^i}{de^j} = \frac{dx_{t+1}^i}{dh_{t+1}^j}h_e > 0$ and $\frac{d^2x_{t+1}^i}{{de_{t+1}^s}^2} = \frac{d^2x_{t+1}^i}{{dh_{t+1}^j}^2}h_eh_e + \frac{dx_{t+1}^i}{dh_{t+1}^j}hee < 0$.

Finally, in order to avoid a situation when the income would never be greater than the consumption constraint, no matter the technology level, I need to impose two additional assumptions over the income:

Assumption 4 (A4)

First, for any technology level there exists $\overrightarrow{e_t^i}$ *for which* $(1-\gamma)x_t^i(\overrightarrow{e_t^i}) > \overline{c}(\overrightarrow{e_t^i})$ *(notice that for this* e_t^i*,* $d_t^i = \gamma x_t^i$*). Second, for the lowest possible technology level* A_0*,* $x_t^i(0) > \overline{c}(0) > (1-\gamma)x_t^i(0)$*.*

The implication of this assumptions is that in the hunter-gather society, $d_t^i = h_t^i w_t - \overline{c}(e_t^i)$, however, as the effort is accelerated by the technological growth, individuals will be able to switch their behavior so that $d_t^i = \gamma h_t^i w_t$.

Proposition 4 (P4)

The ith type individual income and resources allocated to the procreation are:

(1) concave functions of the individual's human capital and his parents' effort;

(2) decreasing with a positive second derivative functions of the jth type individuals human capital and both types of individuals population growth;

(3) concave function of the contemporaneous technology level.

Under (7), $n_{t+1}^i = \frac{d_{t+1}^i}{(\tau + e_{t+2}^i)}$, is a linear function of the resources allocated to the procreation. Under (6), (A4), (P4) and holding all other variables constant, $d^i_{t+1} = h^i_{t+1}w_{t+1} - \bar{c}(e^i_{t+1})$ for relatively small e^i_{t+1}, it will grow in a concave manner with e^i_{t+1} up to a threshold $e^i_{t+1} = eb$ such that $h^i_{t+1}(eb)w_{t+1}(eb) = \frac{\bar{c}(eb)}{1-\gamma}$. As long as $e^i_{t+1} < eb$, any of its increase will increase the surplus of the income over the constraint, which will be used exclusively in order to raise more children. When e^i_{t+1} crosses the threshold value eb, individuals will start to increase their consumption with the income increase, thus they will devote only γ fraction of their income to the children, *i.e.*, for $e^i_{t+1} > eb^i$, $d^i_{t+1} = \gamma h^i_{t+1}w_{t+1}$. However, under (P4) and from the properties of (5), in respect to the effort, the income is a concave, whereas the constraint is a convex function, *i.e.*, $\frac{\partial^2 \bar{c}}{\partial {e^i_{t+1}}^2} > 0 > \frac{d^2 x^i_{t+1}}{d{e^s_{t+1}}^2}$, hence there exists second threshold value for the effort, ec, such that for any $e^i_{t+1} > ec^i$, $\frac{\partial \bar{c}}{\partial e^i_{t+1}}(e^i_{t+1}) > \frac{dx^i_{t+1}}{de^i_{t+1}}(e^i_{t+1})$. This implies that after e^i_{t+1} reaches ec level, the constraint will start to grow more rapidly than the wage, d^i_{t+1} is a decreasing function of the effort for $e^i_{t+1} > ec$. Hence, there exist a third threshold, ef such that for $e^i_{t+1} > ef$, $h^i_{t+1}w_{t+1} - \bar{c}(e^i_{t+1}) < 0$ (which implies that the ith type of population will disappear in the next generation) and a fourth threshold ed such that for any $ef > e^i_{t+1} > ed$, recourses devoted to the procreation will be again $d^i_{t+1} = h^i_{t+1}w_{t+1} - \bar{c}(e^i_{t+1})$ (although this time as a decreasing function).

Interestingly, $\frac{d^2 d^i_{t+1}}{d{e^s_{t+1}}^2} = \frac{d}{de^i_{t+1}}[\frac{dx^i_{t+1}}{dh^j_{t+1}} - \frac{\partial \bar{c}}{\partial e^i_{t+1}}] = \frac{d^2 x^i_{t+1}}{d{h^j_{t+1}}^2} - \frac{\partial^2 \bar{c}}{\partial {e^i_{t+1}}^2} < 0$. In other words, the d^i_{t+1} is always convex in respect to the effort (see the Figure 14). This can be summarized into

Proposition 5 (P5)

Holding all other variables constant, for the ith type of the population, the resources allocated to the procreation are a convex function of the effort level from the last period, with a kink at eb and ed, an increasing function for any $0 < e^i_{t+1} < ec$, *a decreasing function for any* $e^i_{t+1} > ec$, *a function with positive*

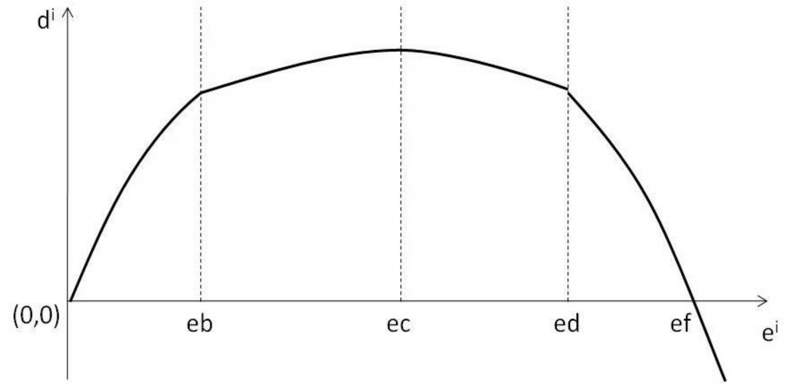

Figure 14: Resources dedicated to the procreation by the ith type individuals as a function of the past effort level.

values for any $0 < e^i_{t+1} < ef$ *and negative values for any* $e^i_{t+1} > ef$*, where* $0 < eb < ec < ed < ef$*.*

We can use (P4) with (A1) and (P1) to link the resources allocated to the procreation with the technology from the previous period. As it follows from the properties of (7), (A1), (P4) and (P5), there are four mechanisms of transmission, with which A_t can influence d^i_{t+1}: e^i_{t+1}, A_{t+1}, n^i_t and n^j_t.

$$\frac{dd^i_{t+1}}{dA_t} = \frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{de^i_{t+1}}{dA_t} + \frac{\partial d^i_{t+1}}{\partial A_{t+1}}\frac{dA_{t+1}}{dA_t} + \frac{\partial d^i_{t+1}}{\partial n^i_t}\frac{dn^i_t}{dA_t} + \frac{\partial d^i_{t+1}}{\partial n^j_t}\frac{dn^j_t}{dA_t} \tag{15}$$

The problem with (15) is that we have to evaluate $\frac{dn^j_t}{dA_t}$, the immediate response of the natural growth to the level of the technology. On one hand, technology increases d^i_t, however, through the effort level it also increases e^i_{t+1} and thus decreases d^i_t. Formally, $\frac{dn^i_t}{dA_t} = \frac{\partial n^i_t}{\partial d}\frac{dd^i_t}{dA_t} + \frac{\partial n^i_t}{\partial e^i_{t+1}}\frac{de^i_{t+1}}{dA_t} = \frac{1}{\tau+e^i_{t+1}}\frac{dd^i_t}{dA_t} - \frac{d^i_t}{(\tau+e^i_{t+1})^2}\frac{de^i_{t+1}}{dA_t} = \frac{1}{(\tau+e^i_{t+1})^2}(\frac{dd^i_t}{dA_t}(\tau+e^i_{t+1}) - e_A d^i_t)$. We can notice that if $d^i_t = \gamma h^i_t w_t$, then it follows from properties of (1) that $d^i_t/\frac{dd^i_t}{dA_t} = \alpha A_t$, hence $\frac{dn^i_t}{dA_t} = \frac{1}{(\tau+e^i_{t+1})^2}\frac{dd^i_t}{dA_t}(\tau + e^i_{t+1} - e_A\alpha A_t) > 0$ as follows from (A3) and (P4). On the other hand, if $d^i_t = h^i_t w_t - \overline{c}(e^i_t)$ then $d^i_t/\frac{dd^i_t}{dA_t} = \alpha A_t - \frac{\overline{c}(e^i_t)}{\alpha h^i_t B A^{\alpha-1}_t H^{-\alpha}_t}$ and $\frac{dn^i_t}{dA_t} = \frac{1}{(\tau+e^i_{t+1})^2}\frac{dd^i_t}{dA_t}(\tau + e^i_{t+1} - e_A\alpha A_t + e_A\frac{\overline{c}(e^i_t)}{\alpha h^i_t B A^{\alpha-1}_t H^{-\alpha}_t}) > 0$, which is again true due to (A3) and (P4).

Proposition 6 (P6)

The population growth of any type of individuals is an increasing function of the contemporaneous technology level.

We can now evaluate (15). As follows from the properties of (11) and (P4), $\frac{\partial d^i_{t+1}}{\partial A_{t+1}}\frac{dA_{t+1}}{dA_t} > 0$. As follows from (P4) and (P6), $\frac{\partial d^i_{t+1}}{\partial n^j_t}\frac{dn^j_t}{dA_t} < 0$ and $\frac{\partial d^i_{t+1}}{\partial n^i_t}\frac{dn^i_t}{dA_t} < 0$. The most interesting is the behavior of $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{de^i_{t+1}}{dA_t}$. From (P3), $\frac{de^i_{t+1}}{dA_t} > 0$, however it follows from (P5) that $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}} > 0$ for $0 < e^i_{t+1} < ec^i$, to become negative if e^i_{t+1} crosses the threshold of ec^i. It means that at first $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{de^i_{t+1}}{dA_t} > 0$ to switch to negative values as e^i_{t+1} increases. A question arises of how the system will behave jointly. As $\frac{d^2 n^i_t}{dA_t{}^2}$ is a complicated function, we can disregard it in this moment and concentrate on the $\Delta \equiv \frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{de^i_{t+1}}{dA_t} + \frac{\partial d^i_{t+1}}{\partial A_{t+1}}\frac{dA_{t+1}}{dA_t}$. $\frac{d\Delta}{dA_t} = \frac{d^2 d^i_{t+1}}{de^s_{t+1}{}^2}(\frac{de^i_{t+1}}{dA_t})^2 + \frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2} + \frac{d^2 d^i_{t+1}}{dA_{t+1}{}^2}(\frac{dA_{t+1}}{dA_t})^2 + \frac{dd^i_{t+1}}{dA_{t+1}}\frac{d^2 A_{t+1}}{dA_t^2}$. From (P3) and (P4), $\frac{d^2 d^i_{t+1}}{de^s_{t+1}{}^2}(\frac{de^i_{t+1}}{dA_t})^2 < 0$ and $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2} < 0$. From the properties of (11) and (P4) $\frac{d^2 d^i_{t+1}}{dA_{t+1}{}^2}(\frac{dA_{t+1}}{dA_t})^2 < 0$ and $\frac{dd^i_{t+1}}{dA_{t+1}}\frac{d^2 A_{t+1}}{dA_t^2} > 0$. Notice however that again from the properties of (11) and (P4), both $\frac{dd^i_{t+1}}{dA_{t+1}}$ and $\frac{d^2 A_{t+1}}{dA_t^2}$ have negative derivatives in respect to A_t, so this term is concave in respect to A_t. On the other hand, as follows from (P3) and (P5), $\frac{d}{dA_t}[\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2}] = e_{AA}\frac{d^2 d^i_{t+1}}{de^s_{t+1}{}^2}\frac{\partial e^i_{t+1}}{\partial A_t} > 0$. As a result, $\frac{dd^i_{t+1}}{dA_{t+1}}\frac{d^2 A_{t+1}}{dA_t^2}$ grows slower than $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2}$ decreases, so there must exist a positive A^*_i, such that for for any $A_t > A^*_i$, $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2}(A_t) + \frac{dd^i_{t+1}}{dA_{t+1}}\frac{d^2 A_{t+1}}{dA_t^2}(A_t) < 0$. Therefore, $\Delta < 0$ and hence (15) is negative for at least any $A_t > A^*$.

As I assumed that the technology explodes, it is natural to assume that at first (15) is positive, *i.e.,* that when the effort level (via the human capital and technology growth) still has a greater influence over the product growth than on the consumption constraint, $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{de^i_{t+1}}{dA_t} + \frac{\partial d^i_{t+1}}{\partial A_{t+1}}\frac{dA_{t+1}}{dA_t} > \frac{\partial d^i_{t+1}}{\partial n^i_t}\frac{dn^i_t}{dA_t} + \frac{\partial d^i_{t+1}}{\partial n^j_t}\frac{dn^j_t}{dA_t}$. Here, the technological pressure on the wage (*i.e.,* fact that today's

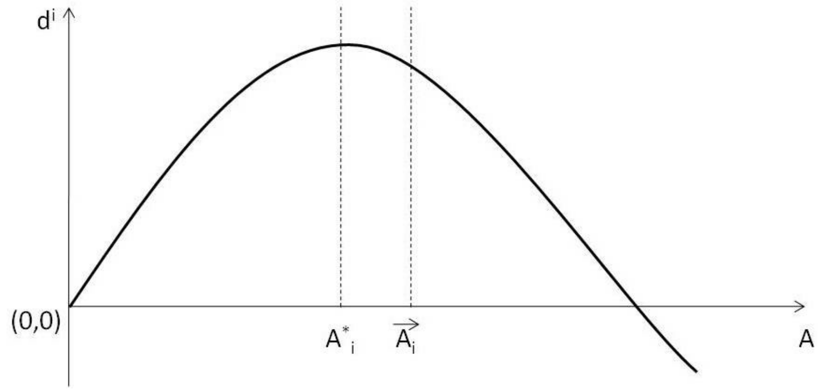

Figure 15: Resources dedicated to the procreation by the ith type individuals as a function of the past technology level.

technology level causes future human capital to be more abundant and thus relatively less expensive) is outweighed by the increase of the product caused by the accumulation of the technology and the human capital. As a result, technology growth at first causes a major population growth. However, as the technology causes levels of effort and of human capital to increase, in time the explosion of the necessary consumption eats all the production gains and population starts to decrease rapidly.

Since $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2}(A_i^*) + \frac{dd^i_{t+1}}{dA_{t+1}}\frac{d^2 A_{t+1}}{dA_t^2}(A_i^*) = 0$, and $\frac{\partial d^i_{t+1}}{\partial n^j_t}\frac{dn^j_t}{dA_t}(A_i^*) + \frac{\partial d^i_{t+1}}{\partial n^i_t}\frac{dn^i_t}{dA_t}(A_i^*) < 0$, $\frac{\partial d^i_{t+1}}{\partial e^i_{t+1}}\frac{\partial^2 e^i_{t+1}}{\partial A_t^2}(A_i^*) + \frac{dd^i_{t+1}}{dA_{t+1}}\frac{d^2 A_{t+1}}{dA_t^2}(A_i^*) + \frac{\partial d^i_{t+1}}{\partial n^j_t}\frac{dn^j_t}{dA_t}(A_i^*) + \frac{\partial d^i_{t+1}}{\partial n^i_t}\frac{dn^i_t}{dA_t}(A_i^*) < 0$. Therefore, the threshold value $\overrightarrow{A_i}$ such that $\frac{dd^i_{t+1}}{dA_t}(\overrightarrow{A_i}) = 0$ (and for all greater A_t, that derivative is negative) is smaller than A_i^* (see the Figure 15).

Proposition 7 (P7)

There exists a positive value of technology $\overrightarrow{A_i}$ *such that for any* $A_t > \overrightarrow{A_i}$, $\frac{dd^i_{t+1}}{dA_t}(A_t) > 0$ *and for any* $A_t > \overrightarrow{A_i}$, $\frac{dd^i_{t+1}}{dA_t}(A_t) < 0$.

Population growth from the period t+1 n^i_{t+1} is influenced by the technology level from period t A_t by two mechanisms - by the resources allocated to the procreation d^i_{t+1} and by the optimal choice of the effort e^i_{t+2}. Thus, as comes from (7), $\frac{dn^i_{t+1}}{dA_t} = \frac{\partial n^i_{t+1}}{\partial d^i_{t+1}}\frac{dd^i_{t+1}}{dA_t} + \frac{\partial n^i_{t+1}}{\partial e^i_{t+2}}\frac{de^i_{t+2}}{dA_{t+1}}\frac{dA_{t+1}}{dA_t} = \frac{1}{(\tau+e^i_{t+2})^2}[\frac{dd^i_{t+1}}{dA_t} - d^i_{t+1}\frac{de^i_{t+2}}{dA_{t+1}}\frac{dA_{t+1}}{dA_t}]$. For $A_t > \overrightarrow{A_i}$, this derivative is negative, so for sufficiently high technology level, the ith type population growth is negative. It means that - as A_t follows the 'rabbit effect' - eventually this population may become extinguished. In fact, this follows from (P3) and (P5) (even if it is to happen only in the limit). $\frac{d^2n^i_{t+1}}{dA_t{}^2} = \frac{1}{(\tau+e^i_{t+2})^2}[\frac{d^2d^i_{t+1}}{dA_t{}^2} - \frac{dd^i_{t+1}}{dA_t}\frac{de^i_{t+2}}{dA_{t+1}}\frac{dA_{t+1}}{dA_t} - d^i_{t+1}e_{AA}(\frac{dA_{t+1}}{dA_t})^2 - d^i_{t+1}\frac{de^i_{t+2}}{dA_{t+1}}\frac{d^2A_{t+1}}{dA_t{}^2}] - 2\frac{1}{(\tau+e^i_{t+2})^3}[\frac{dd^i_{t+1}}{dA_t} - d^i_{t+1}\frac{de^i_{t+2}}{dA_{t+1}}\frac{dA_{t+1}}{dA_t}] < 0$ for $A_t > \overrightarrow{A_i}$, as follows from (11), (P3) and (P7). It means not only that after the threshold value $\overrightarrow{A_i}$ the growth of a ith type population is decreasing with the technological development, but also that for an infinite values of the technology level, it will diverge to the negative infinity. This implies that there exists another threshold value, AN_i such that for any $A_t > AN_t$, $n^i_{t+1} < 0$.[3] Both the first and the second derivatives have an ambiguous sign for $A_t < \overrightarrow{A_i}$. It is natural to assume that the structure of the consumption constraint ((6) and (P7)) and the technology impact over the growth rate (L1) are so that initially, $\frac{dd^i_{t+1}}{dA_t} > d^i_{t+1}\frac{de^i_{t+2}}{dA_{t+1}}\frac{dA_{t+1}}{dA_t}$ (with an explanation similar to that in P6). Thus the (L1) is true.

[3] $n^i_{t+1} < 0$ cannot actually happen, as the quantity of children cannot be negative - if that variables falls below zero, ith type individuals will become extinct.

Appendix B: Empirical Estimation Tables

In all tables below, (*), (**) and (***) denote test statistic with p-values below 10%, 5% and 1% respectively. No mark indicates p-value above 10% threshold. All tests are done for the logarithm of GDP per capita and the average wage, unless stated otherwise. In all cases, the variance decomposition is made according to the Cholesky's decomposition procedure.

Sample 1260-1860

Note: because the wage data has a yearly frequency, while GDP per capita - decade frequency, wage observation for a given decade was obtained by taking an average of wages from the five procedding and four successive years of the reported date (e.g. observation for the year 1360 is an average of 1355-1364 observations).

1.1. Cointegration tests - Johansen procedure and Engel-Granger procedure

Test	$\mathbf{H_0}$	**test statistic**
Trace test	no cointegration	15.49471
Eigenvalue test	no cointegration	14.26460
ADF	residuals of a linear regression have a unit root	-1.881342

1.2. Cointegration tests - Johansen procedure and Engel-Granger procedure - for sample restricted to 1550-1860 period

Test	$\mathbf{H_0}$	**test statistic**
Trace test	no cointegration	15.49471
Eigenvalue test	no cointegration	14.26460
ADF	residuals of a linear regression have a unit root	-3.509910***

VECM estimation results

1.3. Cointegration vector

Variable	constant	$human\_capital_{t-1}$	$\log GDP\_per\_capita_{t-1}$
Coefficient	-3.588127	-9.371367	1.000000

1.4. VECM

error correction between variables	$\Delta \log GDP\_per\_capita_t$	$\Delta human\_capital_t$
Cointegration relationship	0.019750	0.040008
$\Delta \log GDP\_per\_capita_{t-1}$	-0.407575	0.011972
$\Delta human\_capital_{t-1}$	1.232176	0.072007
constant	0.014800	0.000824

1.5. VECM - inverse roots graph

Inverse root	Modulus of the inverse root
1.000000	1.000000
0.630530	0.630530
-0.470065	0.470065
0.148789	0.148789

1.6. Autocorrelation test (LM) for residuals (H_0 - no autocorrelation)

Residuals' lag	LM test statistic
1	8.872554*
2	6.470871
3	0.392888
4	3.299246
5	5.146379
6	4.571027
7	0.937097
8	2.883424
9	4.677384
10	0.578323
11	1.362796
12	0.854138

1.7. Test for residuals' normality (H_0 - normality)

Component	Test statistic
Skewness	3.031323
Kurtosis	1.324041
Jarque-Bera	4.355364

1.8. Variance decomposition of $\log GDP\_per\_capita_t$ in percents

Period	SE	$\log GDP\_per\_capita_t$	$human\_capital_t$
1	0.081472	100.0000	0.000000
2	0.094540	99.53052	0.469478
3	0.116377	99.68474	0.315262
4	0.131540	99.74870	0.251302
5	0.147315	99.79571	0.204288
6	0.161097	99.82337	0.176629
7	0.174289	99.83825	0.161749
8	0.186507	99.84725	0.152751
9	0.198097	99.85222	0.147778
10	0.209054	99.85529	0.144711

1.9. Variance decomposition of $human\_capital_t$ in percents

Period	SE	$\log GDP\_per\_capita_t$	$human\_capital_t$
1	0.006582	11.66162	88.33838
2	0.008309	17.62825	82.37175
3	0.009476	27.32287	72.67713
4	0.010947	43.19267	56.80733
5	0.012409	55.14663	44.85337
6	0.013949	64.37032	35.62968
7	0.015437	70.88271	29.11729
8	0.016878	75.64214	24.35786
9	0.018247	79.15923	20.84077
10	0.019550	81.83977	18.16023

Sample 1830-1994

2.1. Cointegration tests - Johansen procedure

Test	$\mathbf{H}_0$	**test statistic**
Trace test	no cointegration	15.49471**
Trace test	one cointegration relationship	3.841466*
Eigenvalue test	no cointegration	14.26460**
Eigenvalue test	one cointegration relationship	3.841466**

Notice that (**) significance (in the interval [5%,10%]) is interpreted as a lack of significance.

VECM estimation results

2.2. Cointegration vector

Variable	constant	$human\_capital_{t-1}$	$\log GDP\_per\_capita_{t-1}$
Coefficient	-0.011467	1.000000	0.000899

2.3. VECM

error correction between variables	$\Delta \log GDP\_per\_capita_t$	$\Delta human\_capital_t$
Cointegration relationship	19.64520	-0.178033
$\Delta \log GDP\_per\_capita_{t-1}$	0.309756	-0.001610
$\Delta \log GDP\_per\_capita_{t-2}$	-0.035136	-0.000138
$\Delta human\_capital_{t-1}$	-2.536960	0.303645
$\Delta human\_capital_{t-2}$	-3.147115	-0.093137
constant	0.010188	1.16E-05

2.4. VECM - inverse roots

Inverse root	Modulus of the inverse root
1.000000	1.000000
0.653523	0.653523
0.427930	0.427930
0.137462 - 0.293450i	0.324050
0.137462 + 0.293450i	0.324050
0.096653	0.096653

2.5. Autocorrelation test (LM) for residuals (H_0 - no autocorrelation)

Residuals' lag	LM test statistic
1	2.041432
2	5.220259
3	1.943437
4	6.556147
5	6.476580
6	5.670714
7	9.928775**
8	1.457345
9	1.993744
10	1.167956
11	1.771718
12	7.387175

2.6. Test for residuals' normality (H_0 - normality)

Component	Test statistic
Skewness	0.972353
Kurtosis	34.17758***
Jarque-Bera	35.14994***

2.7. Variance decomposition of $\log GDP\_per\_capita_t$ in percents

Period	SE	$\log GDP\_per\_capita_t$	$human\_capital_t$
1	0.026426	61.73062	38.26938
2	0.042242	66.72991	33.27009
3	0.052975	71.97356	28.02644
4	0.060718	76.74313	23.25687
5	0.066830	80.52354	19.47646
6	0.072053	83.24372	16.75628
7	0.076760	85.09047	14.90953
8	0.081136	86.30748	13.69252
9	0.085269	87.10158	12.89842
10	0.089209	87.62208	12.37792

2.8. Variance decomposition of $human\_capital_t$ in percents

Period	SE	$\log GDP\_per\_capita_t$	$human\_capital_t$
1	0.000212	0.000000	100.0000
2	0.000344	1.141866	98.85813
3	0.000415	2.839707	97.16029
4	0.000450	4.195415	95.80459
5	0.000467	5.127028	94.87297
6	0.000474	5.781720	94.21828
7	0.000478	6.273545	93.72646
8	0.000480	6.664623	93.33538
9	0.000481	6.989788	93.01021
10	0.000482	7.270931	92.72907

Appendix C: Proof of the Proposition 1

Since each parent is endowed with h_t^i of human capital (which can be interpreted as efficiency units of labour), his optimal division of resources between the consumption and children is constrained by $h_t^i w_t = n_t^i(\tau + e_{t+1}^i) + c_t^i$ (in the original paper, parents had to divide their time, not the overall resources) and again, $c_t^i \geq \overline{c}(e_t^i)$. Thus, we can substitute $c_t^i = h_t^i w_t - n_t^i(\tau + e_{t+1}^i)$ and set a Lagrangian based on (2) and constrained by $h_t^i w_t - n_t^i(\tau + e_{t+1}^i) \geq \overline{c}(e_t^i)$. FOC are:

$\frac{\partial L}{\partial n_t^i} = \lambda_0[(1-\gamma)(h_t^i w_t - n_t^i(\tau+e_{t+1}^i))^{-1}(-(\tau+e_{t+1}^i))+\gamma(n_t^i)^{-1}] - \lambda_1(\tau+e_{t+1}^i) = 0,$

$\frac{\partial L}{\partial e_{t+1}^i} = \lambda_0[(1-\gamma)(h_t^i w_t - n_t^i(\tau + e_{t+1}^i))^{-1}(-n_t^i) + \gamma\beta^i(h(e_{t+1}^i, g_{t+1}))^{-1}he] - \lambda_1(n_t^i) = 0,$

$\frac{\partial L}{\partial \lambda_1} = h_t^i w_t - n_t^i(\tau + e_{t+1}^i) - \overline{c}(e_t^i) = 0.$

Obviously, $\lambda_0 = 0$ yields a contradiction ($e_{t+1}^i = -\tau$ or $\lambda_1 = 0$), so $\lambda_0 > 0$. If $\lambda_1 = 0$ (*i.e.,* if the constraint is not binding), the solution is given by $c_t^i = (1-\gamma)h_t^i w_t$ and $n_t^i(\tau + e_{t+1}^i) = \gamma h_t^i w_t$. Contrary, if $\lambda_1 > 0$ (when the constraint is binding), individual will choose will choose the corner solution $c_t^i = \overline{c}(e_t^i)$ and $n_t^i(\tau + e_{t+1}^i) = h_t^i w_t - \overline{c}(e_t^i)$. Finally, if $h_t^i w_t < \overline{c}(e_t^i)$, individual receives to little income to survive and will become extinct. Therefore,

$$n_t^i(\tau + e_{t+1}^i) = d_t^i = \begin{cases} 0 & \text{iff } h_t^i w_t \leq \overline{c}(e_t^i) \\ h_t^i w_t - \overline{c}(e_t^i) & \text{iff } h_t^i w_t \in (\overline{c}(e_t^i), \frac{\overline{c}(e_t^i)}{1-\gamma}) \\ \gamma h_t^i w_t & \text{otherwise} \end{cases} . \tag{7}$$

Thus, we can substitute d^i into the Lagrangian and recalculate in respect to e_{t+1}^i, to get a condition:

$$G(\beta^i, e_{t+1}^i, g_{t+1}) \equiv \beta^i h_e - \frac{h(e_{t+1}^i, g_{t+1})}{\tau + e_{t+1}^i} = 0 \tag{8}$$

As follows from the properties of (3) , $\frac{dG(\beta^i,e^i_{t+1},g_{t+1})}{d\beta^i} > 0$, $\frac{dG(\beta^i,e^i_{t+1},g_{t+1})}{dg_{t+1}} = \beta^i h_{eg} - \frac{h_g}{\tau+e^i_{t+1}} > 0$. On the other hand, $\frac{dG(\beta^i,e^i_{t+1},g_{t+1})}{de^i_{t+1}} = \beta^i h_{ee} - \frac{h_e(\tau+e^i_{t+1})-h(e^i_{t+1},g_{t+1})}{(\tau+e^i_{t+1})^2} < 0$. As a result, for given individual i and for given technology growth rate g_{t+1}, $G(\beta^i, e^i_{t+1}, g_{t+1})$ is a strictly decreasing function. If we assume $h_e > \frac{1}{\tau}$, then $G(1, 0, 0) > 0$, hence even in the absence of the technological growth, some individuals with high enough β (close to 1) will prefer to set $e^i_{t+1} > 0$. On the other hand, $G(0, 0, 0) < 0$, hence individuals with low enough β (close to 0) will choose $e^i_{t+1} = 0$ as a corner solution. Using the intermediate value theorem we can show that the threshold value $\overline{\beta}$, which distinguishes latter and former groups is given by a solution to $G(\overline{\beta}, e^i_{t+1}, g_{t+1}) = 0$. Since $\frac{dG(\beta^i,e^i_{t+1},g_{t+1})}{dg_{t+1}} > 0$, for any $0 < \beta^i < \overline{\beta}$ there exists a threshold value $\overline{g}(\beta^i)$, such that for any $g_{t+1} > \overline{g}(\beta^i)$ there exists a e^i_{t+1} such that $G(\beta^i, e^i_{t+1}, g_{t+1}) = 0$ (*i.e.,* for any higher technology growth rate, ith individual will prefer to increase his effort over zero). Notice that if $\beta^i = 0$, then i type individuals are simple not interested in the quality of their children at all and will exhibit a zero level effort regardless of the state of the world. Since $G(\beta^i, e^i_{t+1}, g_{t+1})$ is a strictly decreasing function in respect to e^i_{t+1}, for any β^i and g_{t+1} there exists a unique solution for (8). Notice that it means that the level of income cannot influence the amount of the effort parents will put into raising their children. Instead, it is only the g_{t+1} that for any ith member of the population determines e^i_{t+1}. We can further assume that this effect is concave.

Let the population be divided into two groups denoted by A and B, such that $0 < \beta^B < \overline{\beta} < \beta^A$. Obviously, type A individuals have a much higher marginal propensity to invest in the quality of their offsprings and will do so no matter the level of the technology growth. Type B individuals, on the other hand, will at first care only for the number of their offsprings and start to invest into their quality after the technology growth will increase to the appropriate level. Thus, the (P1) is true.