A Cheap Talk Game with Multiple Audiences

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Abstract

In this thesis I extend and develop a cheap talk game based on Farrell and Gibbons (1989) with one informed agent (a Sender) and two audiences (Receivers), with whom the Sender may communicate together in public or separately in private. I develop a model where the Sender has three possible types, describe the pure strategy perfect Bayesian equilibria in private and public communication, and find that, as opposed to the model of Farrell and Gibbons (1989), there exist cases where information can flow between the Sender and the Receivers in private, but no communication is credible in public. This scenario is named Mutual Subversion. I conduct a welfare analysis of the possible equilibria and conclude that while in private more informative equilibria are preferred by all players, this is no longer true in public. I apply an equilibrium refinement criterion, neologism-proofness (Farrell 1993) to the multiple available equilibria, and it turns out that sometimes no equilibrium satisfies the criterion. I then add a forum choice stage, made by the Sender and possibly contingent on his type, to the game and describe cases where the Sender can achieve superior payoffs by being able to choose the forum of communication himself. I examine forum choice equilibria from the point of view of neologism-proofness, and present a new type of equilibrium refinement which takes into account the forum in which a neologism is used. Finally, I present two further extensions to the model which provide further insights, one where the Sender has $k > 3$ possible types and one where there are three audiences.

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Chapter 1

Introduction

People never lie so much as after a hunt, during a war or before an election.

— Otto von Bismarck

A major party's political candidate travels to a small town during his campaign and visits a home for the elderly. He meets the residents, takes tea and promises to increase the amount of pension benefits upon his election. Later, he gives an interview on national television, and when asked to take a stand, stresses the efficiency gains resulting from a "smaller state", and the tax reductions that could take place if excess government spending were curtailed. A resident at the care home, while watching the televised interview, may experience a degree of uneasiness: although the candidate's statements do not directly contradict each other, they are certainly not identical. What should she believe? All of a political candidate's statements are *cheap talk*: his claims are costless, unverifiable, non-binding—he can say whatever he wants. Even so, our care home resident may be able to draw some conclusions about the politician's real plans by noticing the *difference* between what he says in a small community and what he says on national television; what he claims in a small, private environment as opposed to in front of the general public.

Different claims are made, different claims are believed, different claims are credible in private and in public. In this thesis, I extend and develop a cheap talk model in which one informed agent (a "Sender") tries to communicate with two or more audiences

("Receivers") who then each make a choice (take an action) that defines the payoffs of all players. I rely on the work of Farrell and Gibbons (1989), which provides the basic structure of the model in the thesis. Farrell and Gibbons (1989) were the first to analyse the question of cheap talk with two audiences. I extend their model first by broadening the scope of the Sender's private information (allowing the Sender to have more than two *types*, see Harsányi (1967)), then by adding a further stage to the game, an action stage for the Sender, enabling him to choose the forum of communication for the second stage.

Communication in a public forum does not necessarily lead to more credible claims than private communication, nor *vice versa*. The degree of credibility in each forum depends greatly on the extent to which the players' interests are aligned. The presence a second person may *discipline* communication with the first: the statements made may become credible (one reason for public engagement ceremonies: to convince worried parents that the commitment is a true one). But it might also *subvert* communication: statements previously credible may now not be so (the writer of a letter of recommendation may find it hard to express his true opinion if the subject is present). Some things can be said equally well in private and in public (friends can easily discuss where to go for dinner), while it is impossible to credibly communicate others via cheap talk (imagine a job applicant talking of her abilities: she will always describe them as high is her statements are unverifiable).

I categorise these situations using the parameters of the model and provide examples. I describe a possibility that only arises in my extended model: when the Sender has at least three types is when some information can flow between the Sender and the Receivers separately in private, but no communication is credible in public. In this case, the presence of one person subverts communication with the other, and *vice versa*: there is Mutual Subversion. Such a situation can arise, for example, when the Sender is explaining his plan of action to two potential sponsors, who both agree with the general aim of the Sender, but have diametrically opposed views as to the mode of execution.[1]

[1]Throughout the thesis, I will follow Farrell and Gibbons (1989) in taking the Sender to be a man and the Receivers to be women in the general case.

A prerequisite for Mutual Subversion to arise is the existence of a new type of equilibrium, in which the Sender partially reveals his private information. This *partially revealing* equilibrium naturally only appears in my extended model, where the Sender has at least three types.

I provide a welfare analysis of the possible pure strategy perfect Bayesian equilibria: the separating equilibrium and the partially revealing equilibrium (if they exist), and the pooling equilibrium. It turns out that while in private communication with one Sender and one Receiver, the players all prefer the more informative equilibria to the less informative, this is no longer true in public: it is possible that the Sender does better by remaining inscrutable. For example, if a boss has yet to decide to which of his employees to promote, while the candidates would rather leave than stay in their current position, the boss can induce them both to stay (if that is what he wishes) by not giving away his plans.

I then address the question of equilibrium refinement. Since in many cases there is more than one pure strategy perfect Bayesian equilibrium, in order to predict how events would unfold in real life, it is necessary to introduce refinement criteria. I survey the literature and check whether the equilibria described in the chapter survive a refinement criterion developed by Farrell (1993): *neologism-proofness*. I find that although sometimes neologism-proofness rules out implausible, uninformative equilibria, it does not always do so. Also, there are cases when neither available equilibrium is neologism-proof. When no equilibrium is neologism-proof, it becomes hard to predict how the game will unravel. Farrell (1993) briefly addresses the issue and suggests that an evolutionary interpretation of the game may help in making reasonable predictions.

An open question that Farrell and Gibbons (1989) mention as worthy of further work is the issue of how the forum in which communication takes place is selected: how and by whom is it decided whether the Sender talks to the Receivers in public or in private? In many cases, it is the Sender himself who, once in possession of his private information, decides who is present when he makes his statements. In that case, the Sender faces something akin to a mechanism design problem: he is constructing a game (in this case, choosing among games) to best meet his ends. However, the choice of forum in itself may provide information on the Sender's type to the Receivers. Information is carried in the public announcement of the Sender to make his claims in a private or a public environment: the Receivers can make inferences from the Sender's forum choice. Questions arise as to how much the Sender gives away by his choice, what the Receivers can infer, and what the welfare consequences are of introducing this further stage into the game.

I find that when the Sender has at least three possible types, he can sometimes improve his payoffs in comparison to the situation when the choice of forum cannot be made contingent on his type. The Sender's situation in the forum choice stage resembles that of the informed principal in Myerson (1983), who faces the same dilemma as the Sender in this thesis: he would not always like to give away all his information, but his optimal choice of mechanism (game) may force him to do so. In some cases, the Sender (in this thesis) or the principal (in Myerson (1983)) finds it best to remain inscrutable. I present cases (using the parameters of the model but also a real life example) when forum choice has favourable payoff-consequences for the Sender, and discuss the similarities among them. I rule out certain scenarios (for example, cases where at least one of the Sender's types can credibly identify itself in both fora) from among those where forum choice can result in superior payoffs to the Sender.

I then examine the case of the Sender's superior payoffs from the point of view of neologism-proofness on the example where forum choice enables the Sender to achieve such new, higher payoffs. I find that while only two equilibria are neologism-proof, only the equilibrium which gives the Sender the highest payoffs survives a new, generalised version of neologism-proofness that I develop, which takes into account that the Sender may deviate from equilibrium in the forum choice stage once the possibility of making unexpected speeches occurs to him, in order to be able to make such speeches in his preferred forum.

In the next chapter, I present two further extensions of the model which lead to some insights. The first is a model where the Sender has $k > 3$ possible types. I identify the pure strategy perfect Bayesian equilibria and point to welfare-equivalences between them in private communication. The second extension portrays the Sender in talk with three audiences. A scenario named Two-Sided Discipline emerges, in which the presence of all three Receivers is needed in order to discipline communication with one of the audiences. For example, an incumbent politician's incentives to always tell his voters what they would like to hear may be disciplined only by his need to tell the truth to *both* prospective foreign investors and the European Union.

1.1 Related Literature

The cheap talk game developed in the thesis belongs to the broad class of *signalling games*: games of incomplete information in which the informed player(s) send messages based on their private information to the uninformed player(s), who then take actions based upon the messages received. In more general terms, Sobel (2007) describes a signalling game as "any strategic setting in which players can use the actions of their opponents to make inferences about hidden information". One of the earliest works on signalling is Spence (1974)'s model about the job market. When signalling is costless (does not directly affect payoffs), non-binding and unverifiable, the game in question is referred to as a cheap talk game. The basic setting involves one informed Sender, and one Receiver who takes a single action after having heard the Sender's message and updated her beliefs accordingly. Crawford and Sobel (1982) analyse this baseline case and show that when the players have some but not entirely common interests, the Sender can only make imprecise statements credibly.

The theory of cheap talk games also shares some characteristics with the *mechanism design* literature, initiated among others by Hurwicz (1973). In mechanism design, players choose how to structure their communication in order to solve a given problem. Messages (reports on players' types) are often costless and unverifiable, and in fact, every cheap talk equilibrium is an incentive-compatible mechanism (Farrell and Rabin 1996). However, in a mechanism, the Receiver(s) usually commit in advance to a reaction rule: which action they will choose upon hearing a given message; and there is often a mediator (Farrell 1995).

A starting point for analysing a cheap talk game as opposed to a game with costly signalling or a mechanism design problem is the observation that most of the communication that takes place in real life is simple, unmediated, cheap talk. Many studies have applied the concept of a cheap talk game in various disciplines. Austen-Smith (1993) and Matthews (1989) analyse the legislative process, the former writing of expert referrals to the House during legislation, the latter of veto-threats. Stein (1989) applies Crawford and Sobel (1982)'s framework to the analysis of the imprecise statements of the Federal Reserve Bank concerning its desired level of the exchange rate. Forges (1990) investigates a job market example, while Fischer and Stocken (2001) develop a cheap talk model which applies to the case of an equity research analyst giving information about a firm's value to investors. Cheap talk games are also used to describe the behaviour of animals, for example by Enquist, Ghirlanda and Hurd (1998) or Bergstrom and Lachmann (1998) who use a so-called Sir Philip Sidney Game to describe the signalling that takes place among nesting birds.

Many extensions of the basic one Sender–one Receiver cheap talk game have been proposed and described. Enrichments of the structure of communication have been analysed among others by Aumann and Hart (2003) and Forges (1990), who describe games with many stages of communication. Battaglini (2002) analyses a game with two experts (Senders) reporting on a piece of information which has multiple dimensions. He finds that Crawford and Sobel (1982)'s results do not extend without change to a multidimensional setting. Many papers (apart from Battaglini (2002), for example Austen-Smith (1993), Park (2005) or Krishna and Morgan (2001)) use settings with multiple Senders.

My thesis develops a model with two or more audiences (Receivers). Many of the works cited above implicitly assume that there is more than one audience (for example, in Fischer and Stocken (2001) there are many interested investors, and in Austen-Smith (1993) the experts talk to a House made up of many people), but do not incorporate this fact explicitly into their models. There are some exceptions, however. Farrell and Gibbons (1989) make a first contribution to this topic. Chakraborty and Harbaugh (2007), in their article on comparative cheap talk (in which an expert with private information about multiple issues can credibly rank these for a decision maker even if there is no credible communication about the issues separately—for example, a professor who would support each of his students in applying for a given job, but can rank them if asked to) mention that their results are robust to allowing different Receivers for each issue, as long as the expert's ranking is public. Sobel (2007) writes about cheap talk in advertising, when many consumer-Receivers are communicating amongst each other about the best available product. In the mechanism design literature, Myerson (1983) writes of a principal with private information wishing to coordinate his many subordinates.

The issue of the significance of forum choice is touched upon by Austen-Smith (1993) when comparing an open rule to a closed rule in legislative processes. The choice of forum defines how committees make their referrals, that is, whether other committees are present or not. Chakraborty and Harbaugh (2007) also underline the importance of a public message in ensuring credibility when there are many audiences. Their article concerns comparative cheap talk, where the Sender's private information has many dimensions. As long as all Receivers (each interested in a different dimension) are present, credible rankings may be achieved.

Equilibrium refinement in cheap talk games is first addressed by Farrell (1993) and Myerson (1986). Farrell's concept of neologism-proofness is extended in Matthews, Okuno-Fujiwara and Postlewaite (1991) and a different criterion is proposed by Rabin (1990). Austen-Smith (1993) analyses the problem of equilibrium refinement for pooling equilibria.

The further structure of the thesis is as follows. In Chapter 2 I derive a model with two audiences where the Sender has three possible types. I identify pure strategy perfect Bayesian equilibria, conduct a welfare analysis and discuss equilibrium refinement. In Chapter 3 I deal with the issue of forum choice: whether communication takes place in public or in private. Chapter 4 contains some further extensions of the model, notably to three audiences, and to the Sender having a possible $k > 3$ types. Chapter 5 concludes. Proofs can be found in the Appendix.

Chapter 2

A Cheap Talk Game with Two Audiences

Talk is cheap. Words are plentiful. Deeds are precious.

— H. Ross Perot

In this chapter, I present an extension of Farrell and Gibbons (1989)'s model to the case when the Sender has three possible types. I identify the pure strategy perfect Bayesian equilibria: a separating, a partially revealing and a pooling equilibrium, in private talk with each Receiver separately and in public with both of them present. I describe the scenarios that can take place depending on the degree of information transmission that may arise in private or in public, and find a case which does not occur in the original, two-state model: that of Mutual Subversion, where some information can flow between the Sender and each Receiver in private, but no communication is credible in public. In order to be able to make predictions as to which of the often multiple possible pure strategy equilibria will be played, I first conduct a welfare analysis to determine which equilibrium each player would like to play, and then address the issue of equilibrium refinement, presenting a problematic case when none of the available equilibria "survive" the application of an equilibrium refinement criterion for cheap talk games, Farrell (1993)'s *neologism-proofness*.

2.1 The Three-State Model

2.1.1 The Model

In the model developed in this section, there is one Sender, who is informed about a certain piece of information, which will be referred to as the “state of the world”, or the Sender’s *type*. This piece of information is the Sender’s private information: at the outset, no other player knows it. The Sender communicates with *two* uninformed (but interested) Receivers using cheap talk: costless (from the point of view of the players’ payoffs), non-binding claims that it is not possible to verify. Even though the Sender’s talk is costless, it can, in certain cases, be informative. It can therefore affect the beliefs of the Receivers regarding the state of the world. An important observation, which will be demonstrated below, is that the incentives for truthful revelation on the part of the Sender towards one of the Receivers can be conditional on the presence of the other, that is, whether an announcement is made in public or in private.

The timing of the game is as follows. First, the state of the world becomes clear to the Sender. Using Harsányi (1967)’s terminology developed for games of incomplete information, this means that the Sender is informed of his *type*. Then, the Sender sends some signal to the Receivers about his type, from a set of possible messages M. After having received the signal, the Receivers each decide which of their actions to take. Finally, payoffs are realised. Payoffs are contingent upon the state of the world, but not on the signal the Sender gave. Furthermore, the payoffs of the two Receivers are independent of each other. Notice that the Sender has no action to take, therefore no moral hazard problem surfaces, only that of adverse selection.

In Farrell and Gibbons (1989)’s model, there are two possible states of the world, and each Receiver takes one of two possible actions. The authors search for pure strategy perfect Bayesian equilibria, and characterise the conditions (the payoff structure) under which various equilibria exist. They find a pooling equilibrium in every case and a separating equilibrium in some cases, analysing both private and public communication. They continue by comparing the welfare consequences of these two types of equilibria, and briefly touch upon the issue of equilibrium selection, Pareto-ranking the possible equilibria and later applying Farrell (1993)'s concept of neologism-proofness, an equilibrium refinement criterion for cheap talk games.

I extend this model to the case with three states of the world, three possible types of the Sender. There are several reasons why investigating this extension is of interest. Firstly, a third type of pure strategy equilibrium emerges: the so-called partially revealing equilibrium. Secondly, with three types, it is possible that private communication with each Receiver separately is more effective (more revealing) than public communication (this case will be called Mutual Subversion and does not occur with only two types). And thirdly, the issue of *forum choice*, when the *Sender* makes the decision whether to talk to the Receivers in private or in public, becomes interesting and consequential. The question of forum choice is analysed in Chapter 3.

In this extended model, each Receiver chooses one of *three* possible actions. This enables the retainment of the symmetry of Farrell and Gibbons (1989)'s model. The payoff structure is shown in Table 2.1. The Receivers have prior beliefs regarding the state of the world: the prior probability that $s = s_i$ is π_i, where $\sum_{i=1}^{3} \pi_i = 1$ and $\pi_i \neq 0$. I assume that x_i and y_i, $i = 1, 2, 3$ are positive. This means that if the state of the world s_i were known to the Receivers, $\mathbf{Q}$ and $\mathbf{R}$, they would choose actions q_i and r_i respectively. This ensures that the Receivers are anxious to find out the Sender's type, whatever that type is—this is the case of interest. The Sender's payoff is the *sum* of the payoffs received from $\mathbf{Q}$ and $\mathbf{R}$, for example, if the Sender's type is s_1 and the Receivers choose actions q_1 and r_2, the Sender's payoff will be $v_1 + 0 = v_1$. The v_i and the w_i, $i = 1, 2, 3$, can be either positive or negative (or zero).

The claims the Sender makes to the Receivers are cheap talk and so do not appear in any player's payoff function. Thus the form of the actual messages is irrelevant—what matters is the meanings they convey in equilibrium. These meanings are constituted by the Receivers' reactions, which depend on their beliefs about the state of the world—which

Table 2.1: The payoff structure

Q	s_1	s_2	s_3	**S** from **Q**	s_1	s_2	s_3
q_1	x_1	0	0		v_1	0	0
q_2	0	x_2	0		0	v_2	0
q_3	0	0	x_3		0	0	v_3
R	s_1	s_2	s_3	**S** from **R**	s_1	s_2	s_3
r_1	y_1	0	0		w_1	0	0
r_2	0	y_2	0		0	w_2	0
r_3	0	0	y_3		0	0	w_3

in turn depend on the Sender's communication strategy. In a pure-strategy equilibrium these meanings can be "$s = s_1$", "$s = s_2$", "$s = s_3$", "$s = s_i$ or s_j" $(i, j = 1, 2, 3;\ i \neq j)$ or "no information". When considering the question of equilibrium selection (using neologism-proofness, to be defined later), however, I will assume that the players can in fact talk to each other in a more complex language.

2.1.2 Equilibria

When searching for the pure strategy perfect Bayesian equilibria, there are three distinct cheap talk games to consider. Two involve private communication: the Sender with Receiver **Q** and the Sender with Receiver **R**. The third is the case of public communication: the Sender talks simultaneously to both **Q** and **R**.

The Pooling Equilibrium

In all three games, irrespective of the values of the v_i and the w_i, a *pooling equilibrium* exists, in which the Sender always sends the same message irrespective of his type. This means that the message sent by the Sender in the cheap talk phase is uncorrelated with his private information on the state of the world, its meaning is thus "no information". The Receivers' posterior beliefs are the same as their prior beliefs, described by the π_i, therefore they take their pooling actions, which can be described as follows:

$$q_{pool} = \begin{cases} q_1 & \text{if } \pi_1 x_1 \geq \pi_2 x_2 \text{ and } \pi_1 x_1 \geq \pi_3 x_3 \\ q_2 & \text{if } \pi_2 x_2 \geq \pi_1 x_1 \text{ and } \pi_2 x_2 \geq \pi_3 x_3 \\ q_3 & \text{if } \pi_3 x_3 \geq \pi_1 x_1 \text{ and } \pi_3 x_3 \geq \pi_2 x_2, \end{cases} \tag{2.1}$$

and similarly:

$$r_{pool} = \begin{cases} r_1 & \text{if } \pi_1 y_1 \geq \pi_2 y_2 \text{ and } \pi_1 y_1 \geq \pi_3 y_3 \\ r_2 & \text{if } \pi_2 y_2 \geq \pi_1 y_1 \text{ and } \pi_2 y_2 \geq \pi_3 y_3 \\ r_3 & \text{if } \pi_3 y_3 \geq \pi_1 y_1 \text{ and } \pi_3 y_3 \geq \pi_2 y_2 \end{cases} \tag{2.2}$$

Out-of-equilibrium beliefs are assumed to be the same, prior beliefs: that is, the Sender's speech is simply ignored, whatever the message, and beliefs remain unchanged. The pooling equilibrium is a pure strategy form of the "babbling equilibrium" as it is known in the cheap talk literature (see, for example, Crawford and Sobel (1982)), since the Sender's message is interpreted as meaningless babble.

The Separating Equilibrium

In the pooling case, communication was completely ineffective, and was ignored by the Receivers. There are equilibria, however, when cheap talk is taken into account. In a *separating equilibrium*, the Sender truthfully reveals the state of the world to the Receivers, that is, the meaning of the messages sent are $s = s_i$ if the state of the world is s_i. The Receivers believe them and their posterior beliefs become $\pi_i = 1$ if the meaning of message received is $s = s_i$, and $\pi_{-i} = (0, 0)$, where π_{-i} stands for "all π excluding π_i". The Receivers, now in possession of the information they are interested in, take action i if the meaning of the message heard is $s = s_i$, $i = 1, 2, 3$. Upon hearing any out-of-equilibrium, unexpected message, the Receivers always choose their pooling actions.

The condition for a separating equilibrium to exist is that the Sender have no incentive to lie. Once the Sender is truthful, the Receivers' reactions and beliefs are optimal and the Receivers will not deviate. The Sender has no incentive to lie if his utility when stating the truth is larger (or equal to) his utility when lying. Let $U(s_i, \text{“}s = s_j\text{”})$ signify the

Sender's utility when the state of the world is s_i, and the Receivers believe the Sender's message that the state is s_j.

$$U(s_i, \text{“}s = s_i\text{”}) \geq U(s_i, \text{“}s = s_j\text{”}); i, j = 1, 2, 3, i \neq j. \tag{2.3}$$

Using the parameters of the model, a separating equilibrium exists in private with Receiver **Q** if the Sender has no incentive to lie in *any* of the states, which requires that $v_i \geq 0 \ \forall i \in \{1, 2, 3\}$. Similarly, in the case of private talk with Receiver **R**, the conditions for a separating equilibrium to exist are $w_i \geq 0 \ \forall i \in \{1, 2, 3\}$. In the game where the Sender talks to both Receivers at once (in public), a separating equilibrium exists if, again, the Sender has no incentive to lie in *any* state. For this to be the case, $v_i + w_i \geq 0$ is needed $\forall i \in \{1, 2, 3\}$. From the above it is obvious that the following Proposition, which is stated also in Farrell and Gibbons (1989), holds.

Proposition 2.1.1 *While incentives for complete honesty in both relationships in private imply incentives for complete honesty in public, the converse is not true, that is: if there exists a separating equilibrium with both Receiver **Q** and Receiver **R**, there exists a separating equilibrium in public, too, however, public separation does* not *necessarily imply private separation.*

The Partially Revealing Equilibrium

It is also possible that the Sender has incentives to truthfully reveal one of the states, but would rather pool the other two together. This will be referred to as a *partially revealing equilibrium*. Let us assume that the revealed state is state i, and the two non-revealed states are j and k. In this case, the meaning of the Sender's cheap talk is $s = s_i$ if the state of the world is s_i, but “no information” otherwise. Accordingly, the posterior beliefs of the Receivers are $\pi_i = 1$, $\pi_{-i} = 0$ if the state of the world is i, but for states s_j and s_k, $j, k \neq i$, they need to be calculated using Bayesian updating:

$$\begin{aligned} & Pr(s = s_j | s \neq s_i) = \\ & = \frac{Pr(s \neq s_i | s = s_j) Pr(s = s_j)}{Pr(s \neq s_i | s = s_j) Pr(s = s_j) + Pr(s \neq s_i | s = s_k) Pr(s = s_k)} = \\ & = \frac{\pi_j}{\pi_j + \pi_k}, \end{aligned} \tag{2.4}$$

and similarly:

$$Pr(s = s_k | s \neq s_i) = \frac{\pi_k}{\pi_j + \pi_k}, \tag{2.5}$$

while of course

$$Pr(s = s_i | s \neq s_i) = 0. \tag{2.6}$$

If the state of the world is s_i, the Receivers take actions q_i and r_i, while if the state of the world is not s_i, they take pooling actions over states s_j and s_k. I shall refer to such actions as "partially revealing actions", not because they themselves reveal anything, but because they are the actions taken in a partially revealing equilibrium. For example, the partially revealing actions of Receiver **Q** are:

$$q_{pr} = \begin{cases} q_j & \text{if } \frac{\pi_j x_j}{\pi_j + \pi_k} \geq \frac{\pi_k x_k}{\pi_j + \pi_k} \\ q_k & \text{otherwise,} \end{cases} \tag{2.7}$$

which simplifies to

$$q_{pr} = \begin{cases} q_j & \text{if } \pi_j x_j \geq \pi_k x_k \\ q_k & \text{otherwise,} \end{cases} \tag{2.8}$$

and similarly for Receiver **R**:

$$r_{pr} = \begin{cases} r_j & \text{if } \pi_j y_j \geq \pi_k y_k \\ r_k & \text{otherwise.} \end{cases} \tag{2.9}$$

The condition for a partially revealing equilibrium to exist differs slightly in private and public communication. Recall that in private communication with, for example, Receiver **Q**, both the Sender and the Receiver acquired zero payoffs in any state s_i, if the Receiver chose any action other than q_i. If however, the decision of the Receiver *corresponded* to the actual state of the world, the Receiver got a positive payoff x_i in state s_i, and the Sender received a payoff v_i, which had an unspecified sign. A partially revealing equilibrium exists as long as there are at least two states in which the Sender's payoff, given that the Receiver's decision corresponds to the actual state of the world, is nonnegative; and also, the Receiver's partially revealing action calls for one of these two cases. For example, if $v_1 > 0$, $v_2 > 0$ and $v_3 < 0$, a partially revealing equilibrium exists with state one as the revealed case if $\pi_2 x_2 \geq \pi_3 x_3$, that is, Receiver **Q**'s partially revealing action is $q_{pr} = q_2$. A partially revealing equilibrium exists with state two as the revealed case if $\pi_1 x_1 \geq \pi_3 x_3$, that is, Receiver **Q**'s partially revealing action is $q_{pr} = q_1$. So, one of the states with the nonnegative v parameter will be the *revealed state*, the state in which the Sender truthfully reveals his type. He only has incentives to do so if his payoff is higher than if he would lie, that is:

$$\text{U}(s_i, \text{``}s = s_i\text{''}) \geq U(s_i, \text{``}s = s_j\text{''}); j \neq i. \tag{2.10}$$

The other nonnegative case is needed to cover the Receiver's partially revealing action. The Receiver takes her partially revealing action if she hears the signal "no information". Thus, in the two unrevealed states, the Receiver takes the same action. The Sender, knowing the parameters of the model, knows which action this will be. His incentive to pool two states together follows from this exact desire: to make the Receiver play the same action in two states. In one of the states the Sender will get a nonnegative payoff, in the other, zero, which is an improvement on the negative payoff he would get if he revealed the true state.[1]

[1]Notice that in a *pooling* equilibrium, a similar condition regarding the correspondence of the Receiver's action to a nonnegative payoff of the Sender is not needed, since in the pooling equilibrium, the Sender has no way to deviate from the equilibrium: whatever he says, it will be taken by the Receiver to be meaningless. In the partially revealing case, the Sender can deviate: by claiming that the state is the

The case of partially revealing in public is similar, but not entirely identical. The payoff the Sender receives in the revealed state s_i is $v_i + w_i$, which must naturally be nonnegative. The sign of the other two separating payoffs depends on the partially revealing actions of the Receivers. They could both be negative, which is not true in the private case. For example, if the Receivers' partially revealing actions are (q_1, r_2) (when the revealed state is s_3), then a partially revealing equilibrium exists as long as $v_3 + w_3 \geq 0$, $v_1 \geq 0$ and $w_2 \geq 0$. Such partially revealing actions with a common revealed state stem from the differences in the Receivers' x and y parameters which reflect how much they are desirous of finding out the Sender's private information. In the above case, the two Receivers place different weights on the two unrevealed states: they are differently affected by the various states of the world. They could, for example, represent firms of radically different sizes, who are both interested in finding out about the state of market, but put different weight on various specific types of information. Conversely, if their weighting is more similar, their partially revealing actions may be (q_1, r_1), in which case a partially revealing equilibrium exists if $v_3 + w_3 \geq 0$ and $v_1 + w_1 \geq 0$.

The question arises whether a statement similar to Proposition 2.1.1 holds for the relationship between partially revealing in public and in private in this, extended model. It follows from the above construction of the partially revealing equilibrium that such a statement is true only under a certain condition:

Proposition 2.1.2 *If there exists a partially revealing equilibrium with both Receivers in private revealing the* same state s_i, *then there exists a partially revealing equilibrium in public which reveals state* s_i.

If there are partially revealing equilibria with both Receivers in private, but neither two reveal the same state, then it is possible that there will be no partially revealing equilibrium in public: this is a case of *Mutual Subversion* which I will describe presently.

It can also be seen from the above that, firstly, if a separating equilibrium exists, than so does a partially revealing equilibrium, secondly, if a partially revealing equilibrium

revealed state. This move yields the Sender a payoff of zero if it is untrue. Adhering to the equilibrium by stating "no information" in the two unrevealed states must thus yield at least a payoff of zero.

exists, then so does a pooling equilibrium, and thirdly, a pooling equilibrium always exists. These three types of equilibria can be ranked according to their degree of informativeness, with the separating, fully revealing equilibrium being the most informative and the pooling equilibrium the least informative.

Scenarios

Using the statements in the previous paragraph and Propositions 2.1.1 and 2.1.2, it is possible to isolate various cases that may occur in private vs public communication, according to which equilibria exist. The results can be seen in Table 2.2. One purpose of such a categorisation of scenarios is to answer questions like: is public communication always more credible than private communication? As can be seen from Table 2.2, the answer is that it is not. In the table, "s" stands for the case when a separating (and thus also a partially revealing and a pooling) equilibrium exists in the given forum (there are three possible fora, corresponding to the three games the model concerns: talking in private with Receiver **Q**, talking in private with Receiver **R**, and talking in public to both Receivers); "pr" signifies the case when no separating equilibrium exists, but a partially revealing (and a pooling) equilibrium does; and "p" means that only a pooling equilibrium is attainable.

The case of Full Communication occurs when there is a separating equilibrium with both Receivers in private, and thus, by Proposition 2.1.1, also in public. This is the case when the interests of the Sender and the Receivers are aligned, and there is no cause to lie, for example, when one person (the Sender) issues invitations to his two friends (the Receivers) inviting them to some event (which may be taking place in any of three locations). Conversely, in the case of No Communication, the Sender's interests are opposed to the interests of both Receivers, causing meaningless communication to occur in the cheap talk stage, resulting in a pooling equilibrium in all three fora. For example, a job applicant's claims about his reliability are not inherently credible when his claims are cheap talk—regardless of whether he makes them in his Curriculum Vitae (a public

Table 2.2: Scenarios

No.	Name	**Q**	**R**	**Public**
1	Full Communication	s	s	s
2	No Communication	p	p	p
3	One-Sided Discipline	p	s	s
		pr	s	s
4	Subversion	p	s	p
		p	s	pr
		pr	s	pr
5	Mutual Discipline	p	p	s
		p	pr	s
		pr	pr	s
6	PR—Full Communication	pr	pr	pr
7	PR—One-Sided Discipline	p	pr	pr
8	PR—Subversion	p	pr	p
9	PR—Mutual Discipline	p	p	pr
10	**Mutual Subversion**	pr	pr	p

The roles of **Q** and **R** can be reversed.

document) or in the various interviews separately.

The third case listed is that of One-Sided Discipline. In this scenario, there is a separating equilibrium with one of the Receivers in private, but not with the other, however, there is a separating equilibrium in public. This is the case when the presence of one of the Receivers *disciplines* communication with the other. The interests of the Sender are aligned with those of one of the Receivers, but not with the other. However, when both are present, the Sender cares more about giving the correct information to one Receiver than about misleading the other. A possible real-life example for this case arises when an informed firm sends a signal about its own position (profitability, or some other measure of "strength") to a rival firm contemplating entry, and the capital market. The firm's objective is to dissuade the rival firm from entering, thus the interests of the firm-Sender and the rival-Receiver are at odds. However, we assume the firm prefers to give credible signals to the capital market. If the rival firm can find out the firm's message to the capital market (that is, communication is public), then, as long as the firm is more concerned about giving the right information to the capital market than dissuading the rival firm from entering, communication will be credible. The presence of the capital market thus disciplines the communication with the rival firm.

It is also possible that the firm's primary objective is to stop the rival firm from entering, and it cares less about the information given to the capital market. In this case, if communication is public, the presence of the rival firm *subverts* communication with the capital market, and honesty is compromised. This is the case of Subversion, when there is a separating equilibrium with one of the Receivers, but not with the other, and there is also no separating equilibrium in public. The presence of a third possible state of the world lends richness to the example. The firm's inability to credibly communicate with the rival firm and its willingness to give the correct information to the capital market may in public result in an "intermediate" degree of communication: a partially revealing equilibrium, in which the firm's strongest type can credibly reveal itself, however the two weaker types can do no better than pool. The original example is mentioned in the finance literature, for example in Bhattacharya and Ritter (1983), where it is examined with the assumption of costly signalling.

Another example of subversion is the case of a letter of recommendation. The Sender is the writer of the letter, while the Receivers are the subject of the letter and the addressee of the letter. The Sender can either be utterly convinced that the subject is in possession of the required qualities (state one), he may be utterly unconvinced (state three), or he may have some favourable, but not overly favourable opinion (state two). If the letter is private, the interests of the Sender and the addressee are assumed to be aligned: the Sender recommends the subject only if he believes her to be adequate (since if he lied, his reputation may be compromised). However, if the subject also reads the letter, the Sender may be more worried about possibly offending the subject than being strictly honest concerning her abilities. Letters of recommendation are usually meant to be private, perhaps for this very reason.

A fifth possibility is Mutual Discipline. In this case, there is no separating equilibrium with either Receiver in private, but there is in public. The presence of one Receiver disciplines communication with the other and vice versa. This is the case when the interests of the Sender and the Receivers are not aligned, but not aligned in opposite directions. For example, a politician talking to various constituents in private has an incentive to tell all of them what they want to hear—thus, his communication is not credible. However, when forced to talk in public, he has incentives to “lie in opposite directions”, and his claims may become credible as a result. For example, he may be talking of the optimal trade-off between various tax rates and the degree of state-provided social services. He would always emphasise low taxes to one group of constituents and the high degree of social services to the other, regardless of whether he plans on cutting taxes (state one), leaving the current compromise unchallenged (state two) or increasing taxes to finance growing social expenditures (state three). A similar situation arises when a firm has to deal with bond-raters and a workers' union (as in Farrell and Gibbons (1989)). It would like to get high bond ratings, but also avoid having to raise wages. The former implies that the firm would like to report high profitability to bond-raters and low profitability to the union, regardless of the actual state of the world. But when talking in public, these two incentives to lie may effectively cancel each other out.

The cases of Full Communication, One-Sided Discipline, Subversion and Mutual Discipline can also be defined in an environment where there is no separating equilibrium, but some information may be revealed via partially revealing. For example, in the case of PR—One-Sided Discipline, the presence of one Receiver ensures that at least *some* information is revealed in public that would not have been in private.

Mutual Subversion

The tenth and final case is Mutual Subversion. A consequence of Proposition 2.1.1 is that there is no Mutual Subversion in the two-state model of Farrell and Gibbons (1989). Also, in this extended model, there is no Mutual Subversion in connection to a separating equilibrium. It is not possible that there is a separating equilibrium with both Receivers in private, but not in public: this would be the exact opposite of what Proposition 2.1.1 claims. However, in this three-state model with partial revealing, it *is* possible that even though some information is revealed in private to both Receivers, none would be revealed in public.

Expressed using the parameters of the model, Mutual Subversion occurs, for example, if for one, the conditions in Table 2.3 hold; for another, $q_{pr} = q_2$, $r_{pr} = r_2$; and $\pi_3 x_3 \geq \pi_1 x_1$ by Receiver **Q**, and/or $\pi_1 y_1 \geq \pi_3 y_3$ by Receiver **R** *ex ante*.

Table 2.3: Mutual Subversion

Q	**R**	**Public**
pr	pr	p
$\mathbf{v_1 > 0}$	$w_1 < 0$	$v_1 + w_1 < 0$
$v_2 > 0$	$w_2 > 0$	$v_2 + w_2 > 0$
$v_3 < 0$	$\mathbf{w_3 > 0}$	$v_3 + w_3 < 0$

The conditions in Table 2.3 and $q_{pr} = q_2$, $r_{pr} = r_2$ ensure that there is a partially revealing equilibrium in private with both Receivers **Q** and **R**, the revealed state being state s_1 with Receiver **Q** and state s_3 with Receiver **R**. The condition $\pi_3 x_3 \geq \pi_1 x_1$ by Receiver **Q**, and/or $\pi_1 y_1 \geq \pi_3 y_3$ by Receiver **R** ensures that there is no partially revealing equilibrium in public: since from among the $v_i + w_i$, only $v_2 + w_2$ is positive, a partially revealing equilibrium could only exist (with the revealed state being state two) if $(q_{pr}, r_{pr}) = (q_1, r_3)$. The conditions given above preclude this.

In a less formal manner, Mutual Subversion signifies a case when the Sender would like to tell only one piece of information, be it true, to each Receiver—for example, the information the Receivers would be happy on hearing. Unfortunately, the Receivers would like to hear different things, and in public, the Sender ends up only babbling. Imagine, for example, the case when the Sender is assigned to seek sponsors for an upcoming project in his organisation. The project may be Project One (in this case, the state of the world, known to the Sender but not the Receivers, is s_1), Project Two (the state of the world is s_2) or Project Three (corresponding to s_3). The two possible sponsors, Receivers **R** and **Q**, are prepared to donate various items to help create one of the three possible projects. Their donations are represented by the v and w parameters, and are positive (donations) or negative (harmful actions) according to Table 2.3.[2] The sponsors, however, have harshly different, let us say, politically based views about the desirability of

[2]The donations can only be used in the specific project they are linked to, otherwise they would be transferable from one Project to another.

the various projects. Sponsor **Q** is greatly in favour of Project One, accepts Project Two, but would strive to harm the Organiser if the project turned out to be Project Three (this is represented by $v_1 > 0$, $v_2 > 0$ and $v_3 < 0$. Sponsor **R** is highly appreciative of Project Three, is agreeable to Project Two, but would attempt to thwart the realisation of Project One (using the parameters, this means that $w_1 <$, $w_2 > 0$ and $w_3 > 0$, as seen in Table 2.3). In this case, the Sender would, in private, reveal that the project he seeks sponsors for is Project One, if it were indeed so, to Sponsor **Q**, but would rather not elaborate on the subject if it were Project Two or Three. Similarly, the Sender would reveal state three to Sponsor **R**, but would give general (uninformative) information in the other two states. Thus, there is a partially revealing equilibrium with each Receiver in private. In public however, even revealing Project Two proves to do more harm than good (as shown by the facts that although $v_2 + w_3 > 0$, $(q_{pr}, r_{pr}) \neq (q_1, r_3)$, thus no partially revealing equilibrium exists in public), and the Sender will ask for donations after having given only cursory, and ultimately uninformative, information on the nature of the project.

Such cases of sponsors with harshly opposed interests can often be found in policy-related issues. If the goal of the Sender's organisation is to assure that most babies are born to married couples rather than unmarried couples or single mothers, then one project could be to campaign for laxer abortion laws (Project One), another to distribute information on family planning (Project Two), and a third to run a campaign strongly discouraging sexual intercourse before marriage (Project Three). Obviously these options appeal to different organisations which all, however, agree with the ultimate goal of the Sender. Or alternatively, consider a current issue in Hungarian social policy regarding begging and the homeless (see, for example, Schartzenberger (2007)). All parties agree that the problem of begging and homelessness should be handled, but whether these people should be banned from the inner city (Project One: as in Budapest's fifth district), redirected to shelters (Project Two) or asked to take part in reintegration programs and job training (Project Three) is a matter of debate between various organisations.

The next section tackles the question of which equilibrium the players prefer in the various fora. It seems plausible, for example, that if a separating equilibrium exists, all players prefer to use it, instead of resorting to partially revealing or pooling that are also possible. This intuition turns out to be true only under certain conditions.

2.1.3 Welfare

In this section, I will describe equilibrium preferences for all players, both in the private communication games and the public communication game, both *ex post* and *ex ante*, comparing first separation to pooling, then to partially revealing, and finally partially revealing to pooling. I will also match these results with the two-state model's results when applicable.

First, let us consider the case of the Receivers. Receivers always prefer separation to partially revealing, and partially revealing to pooling (whenever the former two exist), often strictly, but at least weakly. The reason for this is the fact noted earlier, that the three types of equilibria can be ranked according to informativeness. Since Receivers only obtain a positive payoff if they can correctly infer the state of the world, they prefer more informative equilibria to less informative equilibria. Consequently, from now on, the focus will be on the preferences of the Sender.

Private Talk

Separating vs Pooling

In the two-state model, the Sender always prefers separating to pooling, if a separating equilibrium exists. This result extends to this three-state model. The explanation is that the pooling action of the Receiver (for example, Receiver **Q**) can be q_1, q_2 or q_3, depending on the relative sizes of $\pi_1 x_1$, $\pi_2 x_2$ and $\pi_3 x_3$ (see above). In a separating equilibrium, the Sender can induce either of these actions, thus also the action the Receiver would choose in the pooling equilibrium. In this way, the Sender has a greater choice set in the separating equilibrium than in the pooling equilibrium, and the larger choice set contains the smaller. Consequently, the Sender prefers separation to pooling. It is worth noting that this result hinges on the fact that the number of states of the world and the number of possible actions of the Receiver are equal. Were this not so, for example, if there were three states of the world but four possible actions, the Sender would not necessarily be able to induce every pooling action of the Receiver. This is one of the reasons that the fact that I have retained the symmetry of the two-state model is significant.

Separating vs Partially Revealing

The above argument based on the Sender's expanded choice set in the separating equilibrium can be extended without modification to the case of comparing separating to partially revealing. The actions of the Receiver under a partially revealing equilibrium can all be induced by the Sender in a separating equilibrium, thus, he prefers to separate, if such an equilibrium exists.

Partially Revealing vs Pooling

The case of comparing partially revealing to pooling, assuming the former exists, may appear more complex, but in fact, in private the Sender's preferences are again in line with the Receiver's:

Proposition 2.1.3 *In private talk, the Sender never strictly prefers pooling to partially revealing, if the latter exists.*

The proof of the proposition can be found in Appendix A.1. Altogether we can state that in private talk, the Sender's and the Receiver's equilibrium preferences are aligned: both players prefer the more informative equilibrium, if it exists.

Public Talk

Separating vs Pooling

In the case of public talk, it is no longer always true that the Sender prefers separating to pooling. To prefer pooling means that the Sender decides to stay inscrutable, and not give away any information that he has—even though an equilibrium exists where he does.

To see when this happens, it is useful to differentiate between two cases depending on whether the pooling actions of the two Receivers coincide. To be exact, if the pooling actions of the two Receivers are (q_i, r_i), then the game is *coherent* (see Farrell and Gibbons (1989)). For example, if $\pi_1 x_1 \geq \pi_2 x_2, \pi_1 x_1 \geq \pi_3 x_3$ and $\pi_1 y_1 \geq \pi_2 y_2, \pi_1 y_1 \geq \pi_3 y_3$ then $(q_{pool}, r_{pool}) = (q_1, r_1)$ and the game is coherent. Conversely, if the Receivers pooling actions do not coincide, the game is *incoherent*. We can state the following:

Proposition 2.1.4 *In a coherent game, the Sender prefers the separating equilibrium to the pooling equilibrium, both* ex post *and* ex ante.

The proof of Proposition 2.1.4 can be found in Appendix A.1. The incoherent case is when the Sender may prefer pooling to separating. We are of course investigating a case when public separation exists. This means that from among the cases in Table 2.2, the three possible ones are Full Communication, One-Sided Discipline and Mutual Discipline. We can state the following:

Proposition 2.1.5 *If the game is incoherent, then it is possible for the Sender to prefer pooling to separating. Specifically, in the case of Full Communication, the Sender prefers separating both* ex post *and* ex ante*. In the case of One-Sided Discipline, the Sender may prefer either separating or pooling* ex ante*, but never prefers pooling* ex post*. Finally, in the case of Mutual Discipline, the Sender may prefer either separating or pooling* ex ante*, and may even* weakly *prefer pooling* ex post*.*

The proof of Proposition 2.1.5 can be found in Appendix A.1. Proposition 2.1.5 closely resembles Proposition 3 in Farrell and Gibbons (1989), the only difference being that under Mutual Discipline in the two-state model pooling can be strictly preferred to separating *ex post*, but only weakly preferred in this extended, three-state model. In either case, the Sender can gain from remaining inscrutable. A very similar Proposition holds for the case of separating vs partially revealing. I will now present this case, and subsequently a real life example for a case when the Sender prefers to stay inscrutable.

Separating vs Partially Revealing

The comparison between separating and partially revealing closely resembles that of comparing separating and pooling. Proposition 2.1.4 becomes:

Proposition 2.1.6 *In a coherent game, the Sender prefers the separating equilibrium to the partially revealing equilibrium, both* ex post *and* ex ante*.*

The proof of Proposition 2.1.6 can be found in Appendix A.1. In the incoherent case, assume that the revealed state is state three, resulting in (q_3, r_3). In the other two states, however, partially revealing can result in either (q_1, r_2) or (q_2, r_1), hence the incoherence. Proposition 2.1.5 becomes:

Proposition 2.1.7 *If the game is incoherent, then it is possible for the Sender to prefer partially revealing to separating. Specifically, in the case of Full Communication, the Sender prefers separating both* ex post *and* ex ante*. In the case of One-Sided Discipline, the Sender may prefer either separating or partially revealing* ex ante*, but never prefers partially revealing* ex post*. Finally, in the case of Mutual Discipline, the Sender may prefer either separating or partially revealing* ex ante*, and may even* weakly *prefer partially revealing* ex post*.*

The proof of Proposition 2.1.7 can be found in Appendix A.1. An example for a case when the Sender can gain from remaining inscrutable is the following (extended from a similar example in the two-state case). The Sender is the boss of a company seeking a new manager. The two Receivers are vying for the promotion, but there is also a possibility to advertise the job opening and seek the new employee in the wider job market. The internal candidates, Receivers **Q** and **R** must each decide whether to stay or to leave the firm. Leaving implies switching to a different job, better than serving under a previous colleague, but worse than remaining in the same position with a new boss, or staying at the current firm following a promotion. Therefore, each candidate would stay if she knew she was going to get promoted, and leave if the other internal candidate was chosen. If an outsider came to the firm, both the candidates would stay. Assume that state one is that Receiver **Q** gets promoted, state two that Receiver **R** gets promoted and state three that a job advertisement is placed. Thus, the Receivers' third actions are to stay; Receiver **Q**'s first action q_1 is to stay, and her second action q_2 is to leave; and conversely, Receiver **R**'s first action r_1 is to leave, while her second action r_2 is to stay. The boss's payoffs can be seen in Table 2.4. The case described is one of Mutual Discipline. The boss can credibly communicate in any forum that a job advertisement is being placed, since all the players' interests are aligned in that case. The Receivers' partially revealing actions are (q_1, r_2) (the game is incoherent), which means that they will both stay if the boss gives them no information about which of them he plans to promote.

Table 2.4: Promotion Example—Mutual Discipline

Q	**R**	**Public**
pr	pr	s
$v_1 > 0$	$w_1 < 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 > 0$
$v_3 > 0$	$w_3 > 0$	$v_3 + w_3 > 0$

Now let us examine the boss's options when he must announce his plans to the firm in public. He may either separate, partially reveal or pool. Since both v_2 and w_1 are negative, he will in states one and two rather partially reveal than separate: although the fact that an internal candidates will be chosen will be announced, the boss would rather not name his choice between the two ahead of time. This way, by remaining inscrutable, he manages to retain both his workers, and get superior payoffs of v_1 in state one and w_2 in state two (as opposed to $v_i + w_i$ by separation), and the unchanged payoffs $v_3 + w_3$ in state three.[3]

Partially Revealing vs Pooling

It is worth first introducing/clarifying some concepts. Recall that the Receivers' pooling actions may be *coherent* or *incoherent*. They are coherent if their pooling actions coincide,

[3]Pooling is also payoff-inferior to partially revealing in this case, since in state three, the boss would receive a payoff of zero instead of the positive payoff of $v_3 + w_3$.

that is, they are (q_i, r_i). For example, if $\pi_1 x_1 \geq \pi_2 x_2$, $\pi_1 x_1 \geq \pi_3 x_3$, $\pi_1 y_1 \geq \pi_2 y_2$ and $\pi_1 y_1 \geq \pi_3 y_3$, then $(q_{pool}, r_{pool}) = (q_1, r_1)$ and the Receiver's pooling actions are *coherent*. Conversely, the Receivers' pooling actions may not coincide, for example if $\pi_2 x_2 \geq \pi_1 x_1$, $\pi_2 x_2 \geq \pi_3 x_3$, $\pi_1 y_1 \geq \pi_2 y_2$ and $\pi_1 y_1 \geq \pi_3 y_3$, leading to $(q_{pool}, r_{pool}) = (q_2, r_1)$, in which case the Receivers' pooling actions are *incoherent*.

Similarly, one can categorise the Receivers' *partially revealing* actions as either coherent or incoherent. For example, if the revealed state is state s_1, and $\pi_2 x_2 \geq \pi_3 x_3$, then if $\pi_2 y_2 \geq \pi_3 y_3$ then the partially revealing actions are (q_2, r_2), coherent; while if $\pi_3 y_3 \geq \pi_2 y_2$ then the partially revealing actions are (q_2, r_3), incoherent.

Notice that within the same constellation of parameters v_i and w_i, it is possible for the Receivers' partially revealing actions to be coherent while their pooling actions are incoherent, and *vice versa*. For example, if $\pi_1 x_1 \geq \pi_2 x_2 \geq \pi_3 x_3$ and $\pi_1 y_1 \geq \pi_3 y_3 \geq \pi_2 y_2$, then the Receivers' pooling actions are coherent: $(q_{pool}, r_{pool}) = (q_1, r_1)$. However, if there is a partially revealing equilibrium where the revealed state is state s_1, then the Receivers' partially revealing actions will be the incoherent pair (q_2, r_3). Conversely, if $\pi_1 x_1 \geq \pi_2 x_2 \geq \pi_3 x_3$ while $\pi_2 y_2 \geq \pi_3 y_3 \geq \pi_1 y_1$, then the Receivers' pooling actions are incoherent: $(q_{pool}, r_{pool}) = (q_1, r_2)$; while if there is a partially revealing equilibrium where the revealed state is state s_1, then the Receivers' partially revealing actions are coherent: $(q_{pr}, r_{pr}) = (q_2, r_2)$.

Bearing these definitions in mind, consider the following Proposition:

Proposition 2.1.8 *Assume a partially revealing equilibrium exists in public, without loss of generality with revealed state s_3. Then:*

1. *If $\pi_3 x_3 \leq \pi_1 x_1$, $\pi_3 x_3 \leq \pi_2 x_2$, $\pi_3 y_3 \leq \pi_1 y_1$ and $\pi_3 y_3 \leq \pi_2 y_2$, then the Sender weakly prefers partially revealing to pooling, both* ex post *and* ex ante.

2. *If $\pi_3 x_3 \geq \pi_1 x_1$, $\pi_3 x_3 \geq \pi_2 x_2$, $\pi_3 y_3 \geq \pi_1 y_1$ and $\pi_3 y_3 \geq \pi_2 y_2$, then the Sender again weakly prefers partially revealing to pooling, both* ex post *and* ex ante.

3. *When the Receivers' pooling actions are incoherent, but their partially revealing*

actions are coherent, pooling may be strictly preferred to partially revealing by the Sender, both ex post *and* ex ante.

4. *When both the Receivers' pooling and partially revealing actions are incoherent, pooling may be strictly preferred to partially revealing by the Sender* ex ante, *but not* ex post.

The proof of Proposition 2.1.8 can be found in Appendix A.1. Consider the following example for a case when the Sender, in public, prefers to remain completely inscrutable rather than play a partially revealing equilibrium:

Example 2.1 The Sender is a young entrepreneur with a small business. He is awaiting the results from a deal he did a couple of weeks ago. A lot depends upon it: his business may turn out to be doing very well (State One), getting along (State Two) or on the brink of bankruptcy (State Three). The entrepreneur may talk to his prospective business associates (Audience **Q**) about the outcome of this deal. The associates are contemplating an investment into the entrepreneur's business. They may either negotiate a large deal (Action One), a small one (Action Two) or avoid cooperation (Action Three). The entrepreneur is of course in favour of a large deal and thus cannot credibly reveal his company's situation. This is shown in Table 2.5, where v_2 and v_3 are negative and there is thus only a pooling equilibrium with Receiver **Q** in private. Luckily for the entrepreneur, if no information is transmitted, the business associates will assume that the company is doing well (based on their prior beliefs and possible gains from a deal) and will invest. This means that the associates' pooling action is Action One (q_1).

Table 2.5: The Entrepreneur's Obfuscation

Q	**R**	**Public**
p	p	pr
$v_1 > 0$	$w_1 < 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 < 0$	$v_2 + w_2 < 0$
$v_3 < 0$	$w_3 > 0$	$v_3 + w_3 > 0$

The entrepreneur is also lucky enough to have a concerned family member, an aunt, who is willing to bail out the company if matters go too pear-shaped. A large donation would be in order (Action Three) if the aunt believed that the company was doing badly. She would also make a smaller, congratulatory investment (Action One) if the firm was a success, and would wait it out and stay away (Action Two) if the firm was a modest success. The entrepreneur is unfortunately blinded by the prospect of a large donation from his aunt and will always tell her that the firm is doing badly. The aunt knows this and thus cannot trust his statements. Again, there is only a pooling equilibrium in private, this time with the aunt, Audience **R**. Again, luckily for the Sender, when no information is transmitted, the aunt decides to send a large donation (her pooling action is Action Three). It is important that the large deal can only be implemented if the firm *is* in fact doing well, and the small deal if it is getting along all right (in state two). Also, the aunt's donation is useless if the firm is not on the brink of bankruptcy (it can be imagined to be a specific investment), and her small investment only useful if the firm is doing well. These conditions ensure that the entrepreneur receives a payoff of zero if a Receiver chooses Action i while the state of the world is s_j, $i \neq j$, as specified in the payoff structure in Table 2.1.

Suppose, however, that the firm has the opportunity to make a public statement about its finances, for example, in a financial daily. Would it choose to do so? It can be seen from Table 2.5 that a partially revealing equilibrium is possible in public communication, specifically, the Sender can credibly reveal the fact that it doing badly, be it so. In this equilibrium, if no information is transmitted (the entrepreneur decides not to make the public statement), the two audiences both play their first action: $(q_{pr}, r_{pr}) = (q_1, r_1)$, that is, the Aunt invests a small amount of money in the firm and the associates negotiate a large deal. However, if the firm does badly and the entrepreneur reveals this, he will get a large donation from his aunt but must forgo the new deal with the associates. The large donation compensates him for this loss, but can he do better? Indeed he can. By remaining entirely silent, he can play a pooling equilibrium in public, and get the best of both worlds—regardless of how his company is in fact doing. Using the parameters of the model, the entrepreneur will receive a payoff of v_1 in state one, zero in state two and w_3 in state three instead of the inferior $v_1 + w_1$ in state one, the same zero in state two and the inferior $v_3 + w_3$ in state three. Therefore, he prefers to remain inscrutable.

The entrepreneur may of course be reprimanded by his audiences for not conveying useful information to them when he could do so. To avoid such a situation, the entrepreneur would do well to ignore the possibility of a public statement entirely: effectively, he could choose a private *forum*, and "hide" behind this choice of forum. In private, all the players know that no meaningful information can be credibly conveyed and pooling equilibria are played. The issue of *forum choice* is treated more thoroughly in the next chapter. ∎

2.1.4 Equilibrium Selection

Pareto-ranking

One conclusion that can be drawn from the above welfare discussion is that in private, since all players prefer separation to pooling, separation to partially revealing and partially revealing to pooling, if the more informative equilibrium exists, it will most probably be played. This is a conclusion drawn from *Pareto-ranking* the possible equilibria: in private, separation (when it is possible) weakly Pareto dominates partially revealing, and partially revealing (when it exists) weakly Pareto dominates pooling.[4] Such ranking is an example of an equilibrium selection criterion: when cheap talk leads to many possible equilibria, it is important to be able to make plausible predictions as to what will actually happen in real life in the situation described—which equilibrium will be used? As the above welfare discussion showed, there is no simple Pareto-ranking of equilibria in public communication: sometimes, the Sender can get the best of both worlds by remaining inscrutable, and playing a pooling equilibrium even though a partially revealing equilibrium is available.

Equilibrium Refinement—Neologism-Proofness

Another possible way to make predictions about which equilibrium will be used is by using an equilibrium refinement criterion developed by Farrell (1993), *neologism-proofness*. The

[4] Pareto dominance means in this case that switching from one equilibrium to another harms none of the players, and strictly benefits at least one of them.

multiplicity of equilibria is a problem common to all signalling games, of which cheap talk games can be thought of as forming a subset. Several refinement criteria for signalling games have been developed,[5] but many have no effect when studying *costless* signalling. Farrell (1993)'s neologism-proofness, however, concerns cheap talk games. His starting point is the issue of meaning and language. He asserts that in most cheap talk situations, the players share a rich, common language, and therefore understand each other, not only through the meanings of the messages conveyed in a given equilibrium, but irrespective of the equilibrium being played. And if people can make out-of-equilibrium speeches, unexpected messages, which benefit all parties, then they can reasonably be expected to do so—and if the speech is compelling, the listeners may reasonably be expected to believe them. An unexpected speech is called a *neologism*, and one that will be believed is a *credible neologism*. But when is a neologism credible? The purpose of any neologism is to induce actions differing from the equilibrium actions. It is useful to first define a *self-signalling set*: A subset K of the set of the Sender's types T is self-signalling if "precisely the types in K gain by making a statement that induces the action that is a best response to the information that the type of the Sender is in K." (Sobel 2007). Farrell (1993) states that an equilibrium is *neologism-proof* if there are no self-signalling sets relative to the equilibrium[6]. A more formal definiton of neologism-proofness can be found in Matthews,

[5] Sobel (2007) lists some of them: Condition D1 and the weaker Intuitive Criterion by Cho and Kreps (1987), and Divinity by Banks and Sobel (1987):

"Condition D1: An equilibrium refinement that requires out-of-equilibrium beliefs to be supported on types that have the most to gain from deviating from a fixed equilibrium.

Intuitive Criterion: An equilibrium refinement that requires out-of-equilibrium beliefs to place zero weight on types that can never gain from deviating from a fixed equilibrium outcome.

Divinity: An equilibrium refinement that requires out-of-equilibrium beliefs to place relatively more weight on types that gain more from deviating from a fixed equilibrium." (Sobel 2007, 2)

[6] Farrell (1993)'s neologism-proof equilibrium concept was critised and further developed by Rabin (1990), Matthews, Okuno-Fujiwara and Postlewaite (1991), and Austen-Smith (1993). Rabin builds on the observation that in reality, people do not seem to lie as much as game theory would predict. This observation is later stated explicitly in Farrell and Rabin (1996) who reference Valley, Thompson, Gibbons and Bazerman (1995)'s experimental evidence. He proposes a new criterion, *Credible Message Rationalisability*, without reference to a putative equilibrium. Matthews, Okuno-Fujiwara and Postlewaite (1991) bring examples of cases when neologism-proofness is not restrictive enough, and suggest a new criterion, *announcement-proofness*, where an announcement is the generalisation of a neologism: one that takes into account other possible announcements. Austen-Smith (1993) analyses the problem of equilibrium refinement for pure strategy babbling equilibria, that is, pooling equilibria.

Okuno-Fujiwara and Postlewaite (1991). The gist of their definiton is as follows:

Definition Neologism, credible neologism, neologism-proofness.

1. For every nonempty set $K \subseteq T$ a *neologism* is an object represented as "K". *(The meaning of the neologism is "My type is in K".)*
2. A neologism is *believed* if it causes the Receivers to update their beliefs using the Bayes rule according to the information that the Sender's type is in K.
3. A neologism is *credible* relative to a putative equilibrium if
   - All of the Sender's types in K can increase their payoffs by convincing the Receivers that their type is in K.
   - None of the Sender's types not in K can do so.
4. A putative equilibrium is *neologism-proof* if no neologism is credible relative to it.
5. An outcome is neologism-proof if the equilibria giving rise to it are neologism-proof.

In Farrell and Gibbons (1989), which serves as the two-state, two-audience baseline model for the extensions in the previous sections, the authors examine the issue of the neologism-proofness of the possible equilibria: the separating and the pooling equilibrium. One of their propositions is that in private communication, whenever the more informative equilibrium exists, it is neologism-proof, while the less informative is not. However, when there is only one equilibrium (the pooling equilibrium, since that always exists), then it is neologism-proof. The similar statements hold here, too:

Proposition 2.1.9 *1. When a separating equilibrium exists in private communication, then the separating equilibrium is neologism-proof, while the partially revealing and the pooling equilibrium are not.*

2. When no separating equilibrium exists in private communication, but a partially revealing equilibrium does, then the partially revealing equilibrium is neologism-proof, but the pooling equilibrium is not.

3. When only a pooling equilibrium exists in private communication, then it is neologism-proof.

The proof of Proposition 2.1.9 can be found in Appendix A.1. Farrell and Gibbons (1989) also find that their statement holds in public in the *coherent* case as well. This result can also be extended to the model in this chapter:

Proposition 2.1.10 *Assume that public communication is taking place, and the players' partially revealing and pooling actions are coherent.*

1. *When a separating equilibrium exists in public communication, then the separating equilibrium is neologism-proof, while the partially revealing and the pooling equilibrium are not.*
2. *When no separating equilibrium exists in public communication, but a partially revealing equilibrium does, then the partially revealing equilibrium is neologism-proof, but the pooling equilibrium is not.*
3. *When only a pooling equilibrium exists in public communication, then it is neologism-proof.*

The proof of Proposition 2.1.10 can be found in Appendix A.1. The coherent cases can thus be treated in the same way as the private communication cases, and neologism-proofness eliminates all but the most informative equilibrium possible: a plausible result. However, the incoherent cases are somewhat different. Farrell and Gibbons (1989) briefly reflect on the incoherent case and state that in some instances both the separating and the pooling equilibrium may be neologism-proof. In the three-state case described in this thesis, there are cases when there are two equilibria available, but *neither* of them is neologism-proof. Observe, for example, the following case:

Example 2.2 The situation is that of Subversion (4 (c)), in which the Sender can credibly communicate with Receiver **R**, can partially reveal with Receiver **Q** and also in public. The revealed state in both cases is state three. Assume that the Receivers' partially revealing actions are (q_1, r_2) (and that their pooling actions are also (q_1, r_2)). The parameters of the model can be seen in Table 2.6.

Table 2.6: No Neologism-Proof Equilibrium

Q	**R**	**Public**
pr	s	pr
$v_1 > 0$	$w_1 > 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 < 0$
$v_3 > 0$	$w_3 > 0$	$v_3 + w_3 > 0$

In public, the Sender can thus either play a partially revealing equilibrium or a pooling equilibrium. However, neither of these is neologism-proof. In the partially revealing equilibrium, state s_1 is self-signalling. According to the equilibrium, in states one and two, the Sender should convey no information. Thus, in state one he will receive a payoff of v_1, and in state two, a payoff of w_2. The latter is the best he can achieve, in the former however, he would be better off with a payoff of $v_1 + w_1$, which he could achieve if both Receivers were under the impression that the state of the world was state one. The Sender can convince them of this, using a speech along the following lines:

> "My dear **Q** and **R**, although you were expecting no meaningful message from me, take a moment to listen. The state of the world is in fact state one. You should believe me, since I would only lose from convincing you of this in any other state (I would get a payoff of zero). However, in state one, we all gain from my sharing this information: I get a higher payoff than I would otherwise, and so do you. Please each choose your first action."

Similarly, in the pooling equilibrium, both states s_1 and s_3 are self-signalling. In each of these states, the Sender has an incentive to reveal his type, resulting in payoffs of $v_1 + w_1$ in state one and $v_3 + w_3$ in state three if believed, both superior to his pooling payoffs; and he has no incentive to convince the Receivers of the state being either s_1 or s_3 in any of the other states. Thus, pooling is also not neologism-proof. What will happen in this situation? According to the above analysis, it is hard to say. If one of the not neologism-poof equilibria are played, it will most likely be the partially revealing equilibrium, which Pareto-dominates the pooling equilibrium. ∎

Farrell (1993) mentions that situations can arise where there is no neologism-proof equilibrium. For such cases, he tentatively suggests an evolutionary approach which might lead to dynamic equilibria, since no neologism-proof static equilibrium exists. The evolutionary approach points to the interpretation that the lack of a static, neologism-proof equilibrium simply means that things may not "settle down", and not that there is no prediction to be made.

Chapter 3

Forum Choice

No man should advocate a course in private that he's ashamed to admit in public.

— George McGovern, American politician, b. 1922

A question left open by Farrell and Gibbons (1989) and the extension of their model detailed above, but recommended as worthy of further investigation, is the issue of how the forum in which communication takes place is selected: how and by whom is it decided whether the Sender talks to the Receivers in public or in private? Since it is the Sender who actually makes his cheap talk claim to the Receivers regarding his type, in whichever forum, it is often plausible to assume that the forum choice decision is made by the Sender himself, either before or after the state of the world becomes known to him. When the forum choice decision is made by the Sender, it may in itself provide information to the Receivers regarding the state of the world. Something may be inferred from the fact that a Sender gives a Receiver some information in private that he could have given in public, or vice versa. How much does the Sender give away by his choice of forum? What can the Receivers infer from the Sender wishing to speak to them together or separately? Can the Sender improve his payoffs by being able to select the forum of communication? Are people, as suggested in the quotation at the head of the chapter, prone to advocate a course in private that they would not in public? These and related issues are discussed in this chapter and illustrated by an example concerning a firm trying to dissuade another firm from entering the market.

The previous chapter described the games with private and public communication separately, with no indication as to how and by whom the choice of forum is made. The unspoken assumption was that this decision was made prior to the state of the world becoming known to anyone, that is, *ex ante*, by some unspecified person or entity. The choice of forum could not thus be contingent on the state of the world: it could be either private or public, regardless of the Sender's type. The above mentioned unspecified person may be the Sender himself (or some agent of the Sender whose interests are perfectly aligned with the Sender's). This case will be our point of departure. We shall compare the Sender's *ex ante* choice of forum with his choice *ex post*, when the decision *can* depend on the Sender's type, to find out whether the Sender can improve his payoffs by requesting different fora in different states of the world.[1] Regarding terminology, I will call a strategy where the Sender selects the same forum in every state of the world an *undifferentiated* strategy, since the Sender does not differentiate his choice of forum according to his type, and one where he does a *differentiated* strategy: a strategy only available if forum choice is made by the Sender *ex post*.

The model with *ex post* forum choice describes a new game with three stages, with the following steps:

1. Nature selects the Sender's type with prior probabilities π_i, $i = 1, 2, 3$, $\sum_{i=1}^{3} \pi_i = 1$. The Sender learns his type.
2. The Sender's action: a choice of forum (either "public communication", or "private communication"). The Sender's action is a binding announcement heard by both Receivers.
3. The Receivers update their prior beliefs about the state of the world.

[1] Notice that an *ex ante* choice of, for example, private communication is not entirely equivalent to an *ex post* choice of private communication in each state: one may be an equilibrium strategy while the other is not. Specifically, *ex ante* "private" and "public" can only both be equilibria if they result in the exact same payoffs and the decision maker is indifferent between them. This is not true *ex post*. However, since this chapter focuses on possible *new* equilibrium strategies which lead to higher payoffs than any *ex ante* strategy, this issue is of no real relevance and does not change any of the results presented in the chapter.

4. The Sender's cheap talk signal about the state of the world is made in his chosen forum.

5. The Receivers update their beliefs again about the state of the world.

6. The Receivers' actions. The Receivers each take an action and payoffs are realised.

The Sender's strategy in the forum choice stage is a choice of either "public" or "private" communication in each of his possible types, for example, (private, private, public) is a strategy which requires the Sender to request private communication if he is of type one or two and public communication is he is of type three. The Sender's strategy in the cheap talk phase is to announce either s_i or "no information" in each of his types. For example, (s_1, no information, no information) is a strategy in which the Sender reveals the first state but not the others. Receivers have prior beliefs π_i which they update twice, once upon hearing the Sender' choice of forum and once upon hearing his cheap talk. They then each take an action out of their respective three-element action sets.

Farrell and Gibbons (1989) claim that the issue of forum choice is of little interest in their two-state, two-audience model, since a separating equilibrium in the forum choice stage leaves nothing to be said in the cheap talk phase. However, the two-state case can serve as a benchmark for analysing forum choice in the extended models. The detailed description of forum choice in the two-state model is in Appendix A.2. I show that in the two-state model, the Sender cannot gain from using a differentiated strategy, and I consider the possibility of using mixed strategies in the forum choice phase, concluding that they add no new insight to the analysis. I now present the case when forum choice *is* of interest: in the three-state model presented in the previous chapter.

3.1 Forum Choice in the Three-State Model

In the three-state, two-audience model, partially revealing equilibria are also possible, leading to the multitude of possible scenarios detailed in Table 2.2, and repeated for convenience here, in Table 3.1. This section is organised as follows: first, an example is presented of a case when the possibility of making the choice of forum contingent on the state of the world offers the Sender an equilibrium strategy which is payoff-superior to any undifferentiated strategy—that is, adding a forum choice stage to the game provides new payoffs. Three further examples are presented in Appendix A.3. Second, some limitations on the number of scenarios in which the forum choice stage has such consequences are identified. Finally, the results and equilibrium refinement possibilities are discussed.

Table 3.1: Scenarios

No.	Name	**Q**	**R**	**Public**
1	Full Communication	s	s	s
2	No Communication	p	p	p
3 (a)	One-Sided Discipline	p	s	s
3 (b)		pr	s	s
4 (a)	Subversion	p	s	p
4 (b)		**p**	**s**	**pr**
4 (c)		**pr**	**s**	**pr**
5 (a)	Mutual Discipline	p	p	s
5 (b)		p	pr	s
5 (c)		pr	pr	s
6	PR—Full Communication	pr	pr	pr
7	PR—One-Sided Discipline	p	pr	pr
8	PR—Subversion	p	pr	p
9	PR—Mutual Discipline	p	p	pr
10	**Mutual Subversion**	pr	pr	p

The roles of **Q** and **R** can be reversed.

Forum Choice with Consequences—an Example

First of all, notice that any pure differentiated forum choice strategy will inevitably reveal one of the states of the world in the forum choice stage. For example, the strategy (public, private, private) (which means that the Sender requests public communication if he is of type one, and private communication if he is of type two or type three) will reveal state one. I will show an example of such a case, when forum choice enables the Sender to achieve superior payoffs, under Subversion (case 4 (b)).

Example 3.1 The signs of the parameters are presented in Table 3.2. The revealed state in public is state one, and assume that $\pi_2 x_2 \geq \pi_3 x_3 \geq \pi_1 x_1$ and $\pi_2 y_2 \geq \pi_3 y_3 \geq \pi_1 y_1$, thus $(q_{pr}, r_{pr}) = (q_{pool}, r_{pool}) = (q_2, r_2)$.

Table 3.2: Subversion

Q	**R**	**Public**
p	s	pr
$v_1 > 0$	$w_1 > 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 > 0$
$v_3 < 0$	$w_3 > 0$	$v_3 + w_3 < 0$

With these parameters, the Sender prefers public communication to private communication in state one ($v_1 + w_1 > w_1$), is indifferent in state two (payoffs are $v_2 + w_2$ in both cases) and prefers private communication in state three ($0 < w_3$). When forum choice is made *ex post*, it may be possible to implement these preferences. Indeed, the strategy (public, private, private) turns out to be an equilibrium strategy of the forum choice stage. It also provides better payoffs for the Sender than either (private, private, private) or (public, public, public).

Assume the Sender uses the (public, private, private) strategy. I will now deduce the Receivers' best responses. In state one, the Receivers hear the announcement that communication in the second stage should be public. This immediately tells them that the state of the world is state one. Regardless of the message in stage two, they thus take actions (q_1, r_1) and the Sender realises the best possible payoff, $v_1 + w_1$. In state two, the Receivers hear a request for private communication, and adjust their beliefs accordingly: they now know that the state is either state two or state three. Their posterior beliefs are the following:

$$\mu_{e1} = \text{Prob}(s = s_1 | \text{“private”}) = 0 \tag{3.1}$$

$$\mu_{e2} = \text{Prob}(s = s_2 | \text{“private”}) = \frac{\pi_2}{\pi_2 + \pi_3} \tag{3.2}$$

$$\mu_{e3} = \text{Prob}(s = s_3 | \text{“private”}) = \frac{\pi_3}{\pi_2 + \pi_3}. \tag{3.3}$$

Based on these beliefs, in the second stage Receiver **Q** will play q_2, while the Sender will play a separating equilibrium with Receiver **R**, who will consequently choose r_2, affording the Sender a cumulative payoff of $v_2 + w_2$, just as in both undifferentiated forum choice cases. In state three, the Receivers' beliefs are the same as in Equations 3.1—3.3. Receiver **Q** will choose q_2 after having received no additional information in the second stage, while Receiver **R** will choose r_3 following second stage separation, giving the Sender a payoff of w_3, the best possible payoff in the state in question. If the Receivers behave in this way, the Sender has no incentive to deviate. He is receiving a positive payoff in every state of the world, while any deviation would result in a payoff of zero. In this example, the Sender manages to get the "best of both worlds".

To sum up this equilibrium:

1. The Sender's strategy in the forum choice stage is (public, private, private).
2. The Receivers update their beliefs, which become $\mu_{c1} = 1$, $\mu_{c2} = \mu_{c3} = 0$ if the Sender's action was "public" and are described in Equations 3.1–3.3 if it was "private".
3. The Sender's strategy in the signalling stage is the following:
    - State One: "$s = s_1$"
    - State Two: to Receiver **Q**: "no information"; to Receiver **R**: "$s = s_2$"
    - State Three: to Receiver **Q**: "no information"; to Receiver **R**: "$s = s_3$"
4. The Receivers update their beliefs. Receiver **Q**'s beliefs are unchanged. Receiver **R**'s beliefs are also unchanged in communication took place in public. If it took place in private, her beliefs become $\mu_{e2}(\text{“}s = s_2\text{”}) = 1$, $\mu_{e1}(\text{“}s = s_2\text{”}) = \mu_{e3}(\text{“}s = s_2\text{”}) = 0$ if the message received was "$s = s_2$", and $\mu_{e3}(\text{“}s = s_3\text{”}) = 1$, $\mu_{e1}(\text{“}s = s_3\text{”}) = \mu_{e2}(\text{“}s = s_3\text{”}) = 0$ if it was "$s = s_3$".
5. The Receivers out-of-equilibrium beliefs. The Receivers ignore out-of-equilibrium messages by taking them to mean "no information".
6. The Receivers' actions. Both Receivers choose their first action if communication took place in public. Receiver **Q** takes her second action if communication took place in private. Receiver **R**, under private communication, takes her second action if she hears the message "$s = s_2$" in the cheap talk stage, and her third action if she heard "$s = s_3$". She takes her second action if she hears any other message.
7. Payoffs are realised: in state one, $v_1 + w_1$ for the Sender, x_1 for Receiver **Q** and y_1 for Receiver **R**; in state two, $v_2 + w_2$ for the Sender, x_2 for Receiver **Q** and y_2 for Receiver **R**; and in state three, w_3 for the Sender, 0 for Receiver **Q** and y_3 for Receiver **R**.

This concludes the example. ∎

To put the above example in context, recall the real-life example cited in Chapter 2 regarding the firm who is trying to dissuade another firm from entering its market. The firm would like to give credible signals to the credit market. However, when signalling its strength to the entrant, the firm would always prefer to pretend to be very strong. Thus its private message is not credible. In public, the firm loses its ability to credibly communicate with the credit market in some states of the world (let us say, state two and three), and still has no credibility in communication with the entrant. In these states, the firm prefers private communication. But when the firm is indeed very strong (state one), the firm and the entrant would both like this fact to be credibly revealable—this can be done only in public. The forum choice stage solves this problem and the firm can get "the best of both worlds": by choosing the forum to be public in state one and private in states two and three, the firm manages to credibly reveal state one, yet enjoy the benefits of private communication in the other two states. The problem of credibility in communication with the entrant has been resolved to the Sender's advantage with the help of an additional stage of play.

Three further examples of differentiated forum choice having a positive effect on the Sender's achievable payoffs are presented in Appendix A.3. They occur in cases 3 (a) One-Sided Discipline, 5 (b) Mutual Discipline and 7 PR—One-Sided Discipline. There is more than one constellation of the signs of the parameters of the model and the Receivers' pooling- and partially revealing actions which enables the Sender to benefit from *ex post* forum choice within these cases.

Other Scenarios

Consider the following lemma:

Lemma 3.1.1 *If there is a partially revealing or a separating equilibrium with both Receivers in private, then the undifferentiated forum choice strategy (private, private, private) is a (weakly) dominant equilibrium strategy for the Sender in the forum choice stage.*

The proof of Lemma 3.1.1 can be found in Appendix A.1. A few remarks regarding Lemma 3.1.1 are in order. First, the fact that the undifferentiated (private, private, private) strategy provides the best possible payoffs to the Sender does not mean that it is the only strategy that does so (hence the assertion of only weak dominance). For example, under Full Communication, private and public talk always result in the same payoffs, thus (public, public, public) and (private, private, private) both lead to the best possible payoffs (as do many other strategies). Second, the cases that the Lemma refers to are the following ones: case 1 (Full Communication), case 3 (b) (One-Sided Discipline), case 4 (c) (Subversion), case 5 (c) (Mutual Discipline), case 6 (PR—Full Communication) and finally case 10 (Mutual Subversion). Third, even in these cases, many differentiated equilibria exist—however, they result in inferior payoffs. An example which, for the sake of comparison with the example in the previous subsection also analyses a case of Subversion, is presented below.

Example 3.2 Observe the case of Subversion (case 4 (c)) in Table 3.3, and assume without loss of generality that the revealed state is state one both in private with Receiver **Q** and in public; and that $(q_{pr}, r_{pr}) = (q_2, r_2)$. More specifically, assume that $\pi_2 x_2 > \pi_1 x_1 > \pi_3 x_3$ and $\pi_2 y_2 > \pi_3 y_3 > \pi_1 y_1$.

Table 3.3: Subversion

Q	**R**	**Public**
pr	s	pr
$v_1 > 0$	$w_1 > 0$	$v_1 + w_1 > 0$
$v_2 > 0$	$w_2 > 0$	$v_2 + w_2 > 0$
$v_3 < 0$	$w_3 > 0$	$v_3 + w_3 < 0$

In this case, always communicating in private leads to the following payoffs for the Sender:

$$\text{U}(s = s_1,\ \text{private}) = v_1 + w_1 \tag{3.4}$$

$$\text{U}(s = s_2,\ \text{private}) = v_2 + w_2 \tag{3.5}$$

$$\text{U}(s = s_3,\ \text{private}) = w_3, \tag{3.6}$$

while always communicating in public leads to:

$$\text{U}(s = s_1,\ \text{public}) = v_1 + w_1 \tag{3.7}$$

$$\text{U}(s = s_2,\ \text{public}) = v_2 + w_2 \tag{3.8}$$

$$\text{U}(s = s_3,\ \text{public}) = 0. \tag{3.9}$$

Thus, *ex ante* the Sender weakly prefers to always communicate in private. However, the *ex post* strategy of (public, private, public) is an equilibrium strategy also. Assume that the Sender uses this strategy. I shall derive the Receivers' best responses. When the Receivers hear the announcement "private", they infer that the state of the world is state two, and accordingly choose (q_2, r_2) in the action stage (regardless of the signals in the second stage). If the Receivers hear the announcement "public", they update their beliefs using the fact that the state of the world is not state two. Their updated beliefs are the following:

$$\mu_{c1} = \text{Prob}(s = s_1 |\ \text{“public”}) = \frac{\frac{1}{2}\pi_1}{\frac{1}{2}\pi_1 + \frac{1}{2}\pi_3} = \frac{\pi_1}{\pi_1 + \pi_3} \tag{3.10}$$

$$\mu_{c2} = \text{Prob}(s = s_2 |\ \text{“public”}) = 0 \tag{3.11}$$

$$\mu_{c3} = \text{Prob}(s = s_3 |\ \text{“public”}) = \frac{\pi_3}{\pi_1 + \pi_3}. \tag{3.12}$$

Based on these beliefs and using the assumptions made above on the relationship between the π_i, x_i and y_i, Receiver **Q** takes action q_1, while Receiver **R** takes action r_3. The Sender's payoff will thus be:

$$\text{U}(s = s_1,\ \text{public}) = v_1 \tag{3.13}$$

$$\text{U}(s = s_2,\ \text{private}) = v_2 + w_2 \tag{3.14}$$

$$\text{U}(s = s_3,\ \text{public}) = w_3. \tag{3.15}$$

These payoffs are all nonnegative, which implies that if the Sender believes that the Receivers will act in the way described above, he has no incentive to deviate from this strategy, which is thus an equilibrium strategy. However, the payoffs earned are inferior to the payoffs earned under (private, private, private). Again, assume that the Receivers' ignore out-of-equilibrium messages in the second stage by taking them to mean "no information". $\blacksquare$

It is also possible to establish that several other scenarios provide no scope for payoff-improvement for the Sender through the choice of forum. These deductions are more tedious, and while some are relegated to Appendix A.1, others remain in need of further work. Now, using the examples above and in Appendix A.3, and Lemma 3.1.1, I will now further analyse the cases when forum choice enables the Sender to achieve superior payoffs.

Discussion

Let us briefly investigate the conditions for a differentiated equilibrium to exist at all. In a pure strategy differentiated equilibrium, a deviation from equilibrium involves requesting a different forum from the one assigned to the state of the world. This deviation will bring payoffs of zero, as long as second-period out-of-equilibrium beliefs are as specified above. Thus, we can state that a pure differentiated equilibrium strategy must result in nonnegative payoffs. It is perhaps in itself useful to establish such a condition, but also, it is this fact that rules out the possibility of a differentiated equilibrium strategy in the forum choice stage in the No Communication case (proof of this statement can be found in Appendix A.1).

What do the examples above and in Appendix A.3 have in common? In each case there is a state that the Sender would *like* to credibly reveal in private, but cannot. That is, the Sender's and the Receivers' interests are aligned in a given state, but because of the values of the parameters in the other states, they cannot credibly communicate with each other. The additional stage of the game sometimes enables the players to avoid this problem, which is what happens in the examples presented. In this model, in private such a situation can arise only in the case of a pooling equilibrium. Under separation credible communication is always possible. When no separation is possible, but there is a partially revealing equilibrium in public, then in private there is a state that the Sender would like to reveal but cannot. But although the state is not revealed, the Receiver's partially revealing action will ensure that the Sender receives the payoffs he wishes to. Under pooling, however, consider a case when v_1 is positive, v_2 and v_3 are negative, but the Receiver's pooling action is q_2. In this case, the Sender would like to reveal state one, but since it is in his interest in every state to induce the Receiver to choose q_1, his statement that "$s = s_1$" is not credible, and will not be believed even if it is true. In each of the examples provided there is only a pooling equilibrium available with at least one of the Receivers in public, and there is a case which the Sender would like to reveal credibly, but cannot.

3.1.1 Equilibrium Selection

The introduction of a further stage into the game leads to an even greater number of possible pure strategy equilibria. In the case detailed above where forum choice leads to superior payoffs for the Sender, there are many—however, only two equilibria are neologism-proof, and only the equilibrium detailed in the example survives a generalised version of neologism-proofness. Thus, equilibrium refinement criteria eliminate all but the equilibrium which gives the Sender the highest payoffs.

Example 3.3 (*Continued from Example 3.1.*) In the example, the partially revealing and pooling actions of the Receivers coincide and are also coherent, therefore any equilibrium which involves using a less informative equilibrium in the second, cheap talk stage than the most informative equilibrium available will not be neologism-proof, based on Propositions 2.1.9 and 2.1.10. This leaves only two pure strategy equilibria[2], the one presented in the example and its "mirror image", in which the Sender requests private talk in state one and public talk otherwise, the Receivers choose their first actions upon hearing a request for private talk and their partially revealing actions upon hearing a request for public talk. They thus ignore all the cheap talk of the second stage. The Sender's payoffs are $v_1 + w_1$ in state one, $v_2 + w_2$ in state two and zero in state three. All the payoffs are nonnegative and the Sender has no incentive to deviate in the forum choice stage (and is ignored in the second stage). ∎

The concept of neologism-proofness is defined (implicitly in Farrell (1993) and formally in Matthews, Okuno-Fujiwara and Postlewaite (1991)) for games in which one or more rounds of cheap talk take place before the Receiver(s) choose actions—the only actions that take place during the game. The forum choice game, however, introduces an action phase, the Sender's action, before the cheap talk begins. In the above example, when examining neologism-proofness, this action stage was ignored from the point of view of equilibrium refinement: the beliefs and the forum of communication induced by the Sender's action were taken as given, and the cheap talk phase was brought under scrutiny in the same manner as in Chapter 2. This approach eliminated all but two equilibria. But which of these two is more likely to take place?

A further refinement is needed in order to be able to make a prediction. The fact that the previous approach did not explicitly take into account was that not only may

[2] I am abstracting away from the fact that the strategies used in these equilibria could be supported by various out-of-equilibrium beliefs, resulting in a multitude of possible equilibria. Throughout, I assume the Receivers ignore out-of-equilibrium messages, that is, do not change their standing beliefs upon hearing them, and act in the best way they can given these unchanged beliefs.

neologisms be stated in both private and public, but different neologisms are credible in different fora. The Sender may take this into account, and *deviate* in the forum choice stage in order to be able to deliver an unexpected speech in his desired forum. Once the possibility of making unexpected speeches occurs to the Sender, he suddenly acquires the incentive to deviate in the forum choice stage and make such an unexpected speech. Observe the case of the "mirror image" equilibrium described above in Example 3.1.1:

Example 3.4 (*Continued from Example 3.1 and 3.1.1.*) Assume that the Sender, in state two, requests private communication instead of the public specified in his strategy. In this case, the Receivers would assume the Sender is following the equilibrium strategy, believe that the state of the world is state one, and ignoring further communication from the Sender choose their first actions. This would result in an inferior payoff of zero to the Receiver in state two. However, what if the Sender makes the following speech to Receiver **R** (with whom he can separate in private) in his chosen private forum before **R** takes his action?

> "Wait a moment, **R**. The state of the world is in fact state two. I only implied it was state one by choosing a private forum to get **Q** out of the conversation. But with you, we can talk! In a private forum, I have no incentive to tell you that the state of the world is state two, unless it really is—then, however, I *do* have an incentive to do so. I am sorry I misled you with my choice of forum, but this conversation could never have taken place in public. Your interests and mine coincide, and you must understand that we are better off without **Q**."

The above speech implies that once the Sender has made his deviation, his state s_2 becomes self-signalling. A very similar speech can be made in state three, if the Sender also deviates from equilibrium by choosing a private forum.

In the equilibrium presented in Example 3.1 no such neologism is credible. The only state in which the Sender could possibly be better off is state two, where the payoff w_2 would be superior to his payoff of $v_2 + w_2$. For this payoff to be achievable, however, the Sender would have to dissuade Receiver **Q** from choosing her pooling action—something he cannot do, due to the difference in their interests. There is no self-signalling set in this case. ∎

In the above example, the use of an extension of the concept of neologism-proofness has proved useful in offering a plausible prediction as to how such a game may unravel if actually played. Notice that both the "simple" strategies of (private, private, private) and (public, public, public) have been eliminated. It would be interesting to characterise the full set of cases when the forum choice stage makes superior payoffs achievable to the Sender, and check whether these equilibria are (the only) plausible equilibria according to the generalised concept of neologism-proofness. This question may be answered in further work.

Chapter 4

Further Extensions

If everybody thought before they spoke, the silence would be deafening.

— George Barzan

This chapter offers a few observations on how the results of the basic model of Chapter 2 change when further extensions to the model are made. Firstly, I investigate the case of the k-state, k-action, two-audience game, describing the new types of pure strategy equilibria that arise, and drawing some inferences regarding the players' welfare. Secondly, I analyse a two-state game with *three* audiences, describe a new scenario, Two-Sided Discipline, and touch again upon the issue of forum choice, now from among many possible fora.

4.1 The k-state, Two-Audience Model

As seen in Chapter 2, the three-state, two-audience model with three possible actions for each Receiver has three possible pure strategy equilibria, the pooling equilibrium, the partially revealing equilibrium and the separating equilibrium. The partially revealing equilibrium was not available when there were only two possible states of the world. It is worth briefly investigating what other pure strategy equilibria may arise if the number of states is increased further.

Firstly, in order to further preserve the symmetry of the model, assume that the number of states always corresponds to the number of possible actions for each Receiver. The payoff structure is thus, following that of Table 2.1, as shown here in Table 4.1. The x_i and the y_i are positive, and the Receivers have prior beliefs about the state of the world s_i, specifically, they believe that the probability of s_i is π_i, where $\sum_{i=1}^{k} \pi_i = 1$.

Table 4.1: The payoff structure

Q	s_1	s_2	$\cdots$	s_k	**S from Q**	s_1	s_2	$\cdots$	s_3
q_1	x_1	0	$\cdots$	0		v_1	0	$\cdots$	0
q_2	0	x_2	$\cdots$	0		0	v_2	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$	$\vdots$	$\ddots$	$\vdots$
q_k	0	0	$\cdots$	x_k		0	0	$\cdots$	v_k
R	s_1	s_2	$\cdots$	s_3	**S from R**	s_1	s_2	$\cdots$	s_3
r_1	y_1	0	$\cdots$	0		w_1	0	$\cdots$	0
r_2	0	y_2	$\cdots$	0		0	w_2	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$	$\vdots$	$\ddots$	$\vdots$
r_k	0	0	$\cdots$	y_k		0	0	$\cdots$	w_k

In this k-state model, as before, pooling equilibria always and separating equilibria sometimes exist (exact conditions for their existence can be found in Appendix A.4). There are also other pure strategy equilibria, however. In a k-state game, there are $k-2$ partially revealing equilibria, each revealing a different number of states, from one to $k-2$.

Definition A *partially revealing equilibrium* in the k-state game is a pure strategy perfect Bayesian equilibrium where 1 to $k-2$ cases are individually revealed, while the rest are pooled together: the Sender gives no information as to which of these is the true state of the world.

The conditions for a partially revealing equilibrium to exist can easily be generalised from the three-state case. Take, for example, the case of private communication with Receiver **Q**, in a k-state game. I will now describe the conditions for a partially revealing equilibrium revealing $m \in \{1, 2, \ldots, k-2\}$ states to exist. Without loss of generality, assume these revealed states are the first m states. Firstly, the Sender has to have incentives to reveal these m states. This will be the case if $v_i \geq 0, i = 1, 2, \ldots, m$, or expressed by utilities, if:

$$U(s_i, \text{“}s = s_i\text{”}) \geq U(s_i, \text{“}s = s_j\text{”}); j \neq i;\ i, j = 1, 2, \ldots, m. \tag{4.1}$$

Furthermore, the v parameter connected to the Receiver's partially revealing action must also be nonnegative (otherwise the Sender could profitably deviate and receive a payoff of zero by pretending to be in one of the revealed states). The situation is slightly more complex in the case of public communication, but again, can be directly related to the conditions in the three-state model. In the revealed states, $v_i + w_i$ must be positive. Furthermore, if the Receivers' partially revealing actions are coherent (in the sense defined in Chapter 2), for example, $(q_{pr}, r_{pr}) = (q_l, r_l)$, $l \in \{m+1, \ldots, k\}$, then $v_l + w_l$ must be nonnegative. If the partially revealing actions are incoherent, for example, $(q_{pr}, r_{pr}) = (q_a, r_b)$, $a, b \in \{m+1, \ldots, k\}$, $a \neq b$, then both v_a and w_b must be nonnegative for the partially revealing equilibrium to exist.

Once $k > 3$, a further type of pure strategy equilibrium emerges. Imagine, for example, a four-state case of private communication with Receiver **Q**, in which $q_{pool} = q_1$, $\pi_3 x_3 \geq \pi_4 x_4$, $v_1 \geq 0$ and $v_3 \geq 0$. In this case there exists an equilibrium in which the Sender sends a message with the meaning "I am of type one or type two" when $s = s_1$ or $s = s_2$ and a message meaning "I am of type three or type four" when $s = s_3$ or $s = s_4$. In this case the Sender partitions the set of types into two sets, therefore this type of equilibrium may be called a partition equilibrium (the expression "partition equilibrium" features in Crawford and Sobel (1982)'s seminal article, where the message space is partitioned by the Sender, albeit in a setting where the state of the world and the message space form a one-dimensional continuous subset of the real line and the ordering of the states is important; thus, this concept is *not* equivalent to Crawford and Sobel (1982)'s concept). The Sender only informs the Receivers of the partition he is in, but not which state within that partition.

Definition A *partition equilibrium* is the k-state game ($k > 3$) is a pure strategy perfect Bayesian equilibrium in which the Sender truthfully tells the Receivers which set of types he belongs to (each set has at least two elements), but not his exact type.

When does such an equilibrium exist? In each partition, the pooling action(s) of the Receiver(s) must afford the Sender a nonnegative payoff (otherwise, the Sender could, in the state that would give him a negative payoff, profitably deviate and receive a payoff of zero by pretending to be in another partition). Expressed using the parameters of the model, the above mean that for example, in private communication with Receiver **Q**, the v parameter corresponding to the Receiver's pooling action within each partition must be nonnegative. In public, in any partition, if the Receivers' pooling actions within that partition are coherent, then the corresponding $v_i + w_i$ must be nonnegative. If they are incoherent, for example $(q_{pr}, r_{pr}) = (q_a, r_b)$, $a \neq b$, then both v_a and w_b must be nonnegative.

Once there are five possible states of the world, combinations of partition- and partially revealing equilibria often occur, which may called *partially revealing partition equilibria.* For example, imagine a situation of private communication with Receiver **Q**, where $v_1 \geq 0$, $v_2 \geq 0$, $v_3 \leq 0$. $v_4 \geq 0$, $v_5 \leq 0$, $\pi_2 x_2 \geq \pi_3 x_3$ and $\pi_4 x_4 \geq \pi_5 x_5$. In this case, there exists an equilibrium in which the Sender reveals state one, pools state two and state three, and separately pools state four and state five.

Definition A *partially revealing partition equilibrium* in the k-state game $(k > 4)$ is a pure strategy perfect Bayesian equilibrium in which the Sender would truthfully reveal at least one of his types, but uses a partition equilibrium in the other states of the world.

At least five states are needed for an equilibrium of this type to emerge: in which there is an element of a partially revealing equilibrium, through the revealing of state one, but there are elements of a partition equilibrium, in that the Sender partitions the message space into three parts, and within two of them, uses a pooling strategy.

Notice that once there are more than five possible states, no new types of pure strategy equilibria emerge. Under two possible states, the generic pooling equilibrium and the possible separating equilibrium are the only two pure strategy equilibrium possibilities. Under three states of the world, a partially revealing equilibrium becomes possible if the parameters of the model fulfill some conditions. Under four states of the world, a partition equilibrium, where the Sender reveals which partition his type is in, but not what his exact type is, may emerge. And under five possible states of the world, combinations of the partially revealing and the partition equilibria may also occur.

It is clear from the above that there is no essential difference between a partially revealing equilibrium and a partition equilibrium. Indeed, a partially revealing equilibrium revealing, say, the first $k-2$ states can be thought of as a partition equilibrium where the Sender partitions the message space into $k-1$ parts, the first $k-2$ of which contain only one possible state of the world, while the last contains two, within which pooling occurs.

In fact, under private communication, certain partially revealing equilibria and partition equilibria are *outcome- and payoff-equivalent* (they lead to the same actions by and payoffs for the players). This statement is made more exactly in Proposition 4.1.1:

Proposition 4.1.1 *Under private communication between a Sender and a Receiver, for every partition equilibrium and partially revealing partition equilibrium, there exists an outcome- and payoff-equivalent partially revealing equilibrium. Under public communication, no such statement holds.*

The proof of Proposition 4.1.1 can be found in Appendix A.1. One useful consequence of Proposition 4.1.1 is that from the point of view of welfare ranking, under private communication partition equilibria can be ignored: they are equilvalent to certain partially revealing equilibria. Thus, the results of the welfare analysis in Section 2.1.3 extend to the k-state game. Specifically, the Sender prefers the separating equilibrium (if it exists) to all other equilibria, and his least preferred type of equilibrium is the pooling equilibrium. The ranking of the various partially revealing equilibria is straightforward: the Sender prefers equilibria that reveal more states to equilibria which reveal less, since the $v_i + w_i$ corresponding to revealed states is always nonnegative.

4.2 The Two-State, Three-Audience Model

Another possible extension of the model of Farrell and Gibbons (1989) is to allow more than two Receivers. In some examples, for instance in the example where a politician is talking to his constituents, the audience is made up of a very large amount of individuals. In the two-Receiver model, these individuals are treated as belonging to two homogeneous groups. However, even if certain types of voters can be assumed to have similar preferences, a politician speaks not only to voters, but also to lobby groups, to his own party, to unions etc. It is therefore worth investigating whether any significant changes occur in the structure and predictions of the model, if there are more than two audiences.

In this section, a two-state, three-audience model will be briefly sketched. I will present the possible equilibria and the relationships between public and private talk. The presence of three audiences increases the number of possible fora of speech from three to seven. The Sender may talk in private with Receiver **P**, with Receiver **Q** or with Receiver **R**, may talk in public to Receivers **P** and **Q**, to Receivers **P** and **R** or Receivers **Q** and **R**, or he may talk to the grand public, to all three Receivers at once. However, it is shown that this multiplicity of possible fora of communication in itself does not enable the Sender to profit from being able to make the choice of forum himself *ex post*. The presence of at least three states of the world is required for such a possibility.

In the two-state, three-audience model, the Sender first observes the state of the world $s \in \{s_1, s_2\}$. There are three Receivers, **P**, **R** and **Q**. The Receivers have prior beliefs about the state of the world, specifically, the prior probability that the state of the world is s_1 is π, and thus the probability that it is s_2 is $1 - \pi$. The payoff structure is presented in Table 4.2.

I assume that the x_i, y_i and z_i $i = 1, 2$ are positive. This means that if the state of the world s_i were known to the Receivers, **P**, **Q** and **R**, they would choose actions p_i, q_i and r_i respectively. This ensures that the Receivers would like to find out the Sender's type, whatever that type is. The Sender's payoff is the *sum* of the payoffs received from **P**, **Q** and **R**, for example, if the Sender's type is s_1 and the Receiver's choose actions p_1,

Table 4.2: The payoff structure

P	s_1	s_2	**S** from **Q**	s_1	s_2
p_1	z_1	0		u_1	0
p_2	0	z_2		0	u_2
Q	s_1	s_2	**S** from **Q**	s_1	s_2
q_1	x_1	0		v_1	0
q_2	0	x_2		0	v_2
R	s_1	s_2	**S** from **R**	s_1	s_2
r_1	y_1	0		w_1	0
r_2	0	y_2		0	w_2

q_1 and r_2, the Sender's payoff will be $u_1 + v_1 + 0 = u_1 + v_1$. The u_i, v_i and w_i, $i = 1, 2$, can be either positive or negative (or zero).

In every scenario, there exists a pooling equilibrium, and under certain conditions (similar to the case of the two-audience models) a separating equilibrium. Exact conditions for its existence can be found in Appendix A.4, where it is also shown that the following proposition holds:

Proposition 4.2.1 *Incentives for complete honesty in two relationships in private imply incentives for complete honesty in public with those two Receivers; incentives for complete honesty in all three relationships in private imply incentives for complete honesty in communication with the grand public (all three Receivers), and in public with either two Receivers. In each case, the converse is not true: no form of public honesty necessarily implies incentives for honesty in private.*

4.2.1 Scenarios

Building on Proposition 4.2.1 it is possible to analyse the possible scenarios that may arise depending on the type of equilibria available with each Receiver and in the various public fora. The scenarios arising in communication with any two of the Receivers together or separately can be adequately described in the same way as in the two-state, two-audience model of Farrell and Gibbons (1989), see Table 4.3.

Furthermore, the same can be said about the relationships between two Receivers in

Table 4.3: Scenarios with Any Two Receivers

No.	Name	**Q**	**R**	**Public**
1	Full Communication	s	s	s
2	No Communication	p	p	p
3	One-Sided Discipline	p	s	s
4	Subversion	p	s	p
5	Mutual Discipline	p	p	s

The roles of **Q** and **R** can be reversed.

public, the third in private, and all three of them in grand public. For example, assume that there is only a pooling equilibrium with Receiver **P** in private and with Receivers **Q** and **R** in public. Then, in grand public, there may be separating equilibrium, which would describe a case of Mutual Discipline, or not, which would be a case of No Communication. The other cases can be described similarly.

But what relationship is there between communication with each of the Receivers in private and communication in grand public? Some of the well-known scenarios remain. For example, Mutual Discipline occurs when there are only pooling equilibria with the Receivers in private, but there is a separating equilibrium in grand public. When there is no separating equilibrium even in public, the case of that of No Communication. Also, when there are separating equilibria in both grand public and the private relationships, the situation is that of Full Communication. The rest of the cases contain some degree of Subversion or of Non-Mutual Discipline. Non-Mutual Discipline is a similar situation to One-Sided Discipline in the two-audience models. Although there is only a pooling equilibrium with at least one of the Receivers in private, separation is possible in grand public. Table 4.4 shows the cases described in this paragraph. As before, "p" signifies a case with only a pooling equilibrium available, while "s" a case where separation is also a possibility.

It is worth further describing the case of Non-Mutual Discipline, taking into account the possibility of public speech to two Receivers. Observe, for example, the case of Non-Mutual Discipline highlighted in Table 4.5, which will be referred to as Two-Sided Discipline.

Table 4.4: Scenarios with Three Receivers

No.	Name	**P**	**Q**	**R**	**Grand Public**
1	Full Communication	s	s	s	s
2	No Communication	p	p	p	p
3 (a)	Non-Mutual Discipline	p	p	s	s
3 (b)		p	s	s	s
4 (a)	Subversion	p	p	s	p
4 (b)		p	s	s	p
5	Mutual Discipline	p	p	p	s

The roles of **P**, **Q** and **R** can be permutated.

Table 4.5: Two-Sided Discipline

P	**Q**	**R**	**P** and **Q**	**P** and **R**	**Q** and **R**	**Grand Public**
p	s	s	p	p	s	s

To understand what prompts the name of Two-Sided Discipline, consider the situation of Receiver **P**. In private, no credible communication is feasible between her and the Sender. When another Receiver is added to the conversation, be it either Receiver **Q** or Receiver **R**, the situation does not change: no credible talk can take place. However, once the remaining, third Receiver is also present, conversation is disciplined, and a separating equilibrium becomes possible. That is, to discipline communication with Receiver **P**, the presence of both other Receivers is needed. Not counting permutations of the case described in Table 4.5, there is only one more case of Two-Sided Discipline, and it is shown in Table 4.6.

Table 4.6: Two-Sided Discipline

P	**Q**	**R**	**P** and **Q**	**P** and **R**	**Q** and **R**	**Grand Public**
p	p	s	p	p	s	s

In the scenario in Table 4.6, Two-Sided Discipline exists from the point of view of Receiver **P**. Credible communication with her is not possible in private, nor in the presence of either one of the other two Receivers. However, separation becomes a possibility once all three Receivers are present. Notice that from the point of view of Receiver **Q**, the situation is less clear-cut, since communication with her can be disciplined by the presence of only Receiver **R**, but not by the presence of only Receiver **P**. It can be shown (I will not do so now) that no case of Two-Sided Discipline exists that can be described as such from the point of view of two Receivers. This would require that there be no separating equilibrium with either *pair* of Receivers, but that there exist a separating equilibrium with one Receiver in private, and also in grand public. This is not possible.

Example As an example of Two-Sided Discipline, take the case mentioned at the beginning of the section: that of a politician talking to his constituents. Assume that the politician is an incumbent (he is currently in power), but is campaigning for reelection. He may have one of two types: he may plan to increase government spending on social welfare (state one), or he may plan to curtail it (state two). The Sender-politician's payoffs from communication with three audiences can be seen in Tables 4.7 and 4.8. The identity and preferences of the audiences will now be described.

Table 4.7: Two-Sided Discipline

State	**P**	**Q**	**R**
	p	s	s
s_1	$u_1 = 10$	$v_1 = 1$	$w_1 = 1$
s_2	$u_2 = -10$	$v_2 = 6$	$w_2 = 6$

Table 4.8: Two-Sided Discipline

State	**P** and **Q**	**P** and **R**	**Q** and **R**	**Grand Public**
	p	p	s	s
s_1	$u_1 + v_1 = 11$	$u_1 + w_1 = 11$	$v_1 + w_1 = 2$	$u_1 + v_1 + w_1 = 12$
s_2	$u_2 + v_2 = -4$	$u_2 + w_2 = -4$	$v_2 + w_2 = 12$	$u_2 + v_2 + w_2 = 2$

The first audience (Audience **P**) is made up of the voters, the majority of whom are short-sighted in the sense that they prefer larger social spending to smaller, regardless of possible subsequent adverse effects on the economy or tax increases needed to finance the spending. Therefore is they knew the politician indeed intended to increase social spending, enough of them would vote for him to achieve reelection (Action One), and if they knew he did not, they would not (Action Two). The politician of course wishes to gain votes and when talking to the voters alone, will always claim to plan to increase social spending (this can be seen from his payoffs: u_1=10 and $u_2 = -10$). Knowing this, the voters will not believe his claims and there will be only a pooling equilibrium in private. However, the politician's claims may also be heard by two other audiences: possible foreign investors (Audience **Q**) and the European Union (Audience **R**). Foreign investors and the European Union all prefer a tight budget, and the investors would plan smaller investments (Action One) if spending was going to be increased than if it was not (larger investments: Action Two), while the European Union may impose harsher directives on the country if it increased spending (Action One) or not if it did not (Action Two). The politician in theory would always prefer large investments and EU support, but in order to maintain good foreign relations and a reputation, he nevertheless has incentives for truth-telling to both these audiences separately in private and thus also to the two of them together. This is represented in Table 4.7 by the payoffs of 1 in state one and 6 in state two.

Altogether, the politician is unable to credibly communicate with his voters, and his communication is not disciplined by the presence of either of the two other audiences alone (see Table 4.8, where there is only a pooling equilibrium both with the voters and the investors, and the voters and the EU in public). However, if his statements are made to the grand public, communication is disciplined and truth-telling becomes possible. The politician can now credibly communicate his plans since his incentives to correctly inform the investors and the EU about them overrides his wish to possibly misinform his voters. Notice that this may also mean that the politician will not be reelected. Whether the politician actually chooses to separate in grand public or pool depends on the audiences' pooling actions. However, the possibility to credibly communicate is nonetheless present. ∎

Taking a final look at Table 4.4, observe that no situation resembling Mutual Subversion is present. In order for Mutual Subversion to arise, at least three states of the world are necessary. A similar statement can be made in relation to the forum choice issue, which I will analyse in the next section.

4.2.2 Forum Choice

Now consider the game where, just as in the Forum Choice chapter, the Sender makes the choice of the forum of communication prior to his communication regarding his type, but after having discovered his type (that is, *ex post*). If the Sender is under obligation to talk in some manner to all of his audiences, he has five possible constellations at his disposal: he can talk to each Receiver in private, talk to any two of them in public, and one of them in private (three distinct situations), or talk to all three in grand public. In many cases, the Sender strictly prefers a different forum depending on his type, for example, observe again Tables 4.7 and 4.8, a case of Two-Sided Discipline, and assume pooling actions are the following: $(p_{pool}, q_{pool}, r_{pool}) = (p_1, q_2, r_2)$. In this case the Sender's payoffs in various fora can be seen in Table 4.9.

Table 4.9: Payoffs in Various Fora

Forum		State One	State Two
1	**P**; **Q**; **R**	12	12
2	**P** and **Q**; **R**	11	12
3	**P** and **R**; **Q**	11	12
4	**Q** and **R**; **P**	12	12
5	**P** and **Q** and **R**	12	2

According to Table 4.9, the Sender prefers Forum 1, 4 or 5 in state one and Forum 1, 2, 3 or 4 in state two, thus his preferences do not completely overlap.[1] In this model it is typical that the Sender is indifferent among some of the fora in each state of the world, but that these preferences differ across his types.

However, despite the multitude of possibilities for the Sender's forum choice preferences, it turns out that the main idea of Proposition A.2.1 in the Forum Choice chapter holds:

Proposition 4.2.2 *In the two-state, three-audience model the Sender cannot increase his*

[1]For completeness observe that when talking to Receivers **Q** and **R**, the Sender prefers to separate. When talking in grand public, in state one the Sender would rather separate (and receive a payoff of 12 rather than 10), but in state two, the Sender would rather pool (and receive a payoff of 12 rather than 2). If a pooling equilibrium is played, the Sender prefers Forum 1 or 4 in state one and is completely indifferent in state two.

payoffs by making his choice of forum ex post*, rather than* ex ante*, that is, he cannot gain from using a differentiated strategy.*

The proof of Proposition 4.2.2 can be found in Appendix A.1.

Chapter 5

Conclusions

The real art of conversation is not only to say the right thing at the right place but to leave unsaid the wrong thing at the tempting moment.

— Dorothy Nevill

This thesis has developed a cheap talk model in which one agent (the Sender) who is in possession of some private information, communicates with two or more audiences (Receivers). The structure of the basic model (where the Sender has two possible types) is provided by Farrell and Gibbons (1989).

A large degree of everyday (strategic) communication can be described by scenarios depicted in the extended model detailed in this thesis. Any piece of information worth noting is usually important to more than one agent: if someone is prepared to share it, audiences may be numerous. Also, it is safe to say that most communication flowing between people, firms, organizations can be best characterised as cheap talk. People continually, unceasingly talk, e-mail, question, conjecture, inquire, and tentatively suggest courses of action to each other, implying something of their private information, attempting to exercise persuasion, and paint a picture of credibility. All this they can usually do without direct consequences, through unverifiable claims. These conversations may take place with one or more people present, and may well go differently depending on exactly who is there. Farrell and Gibbons (1989) make a crucial contribution to the cheap talk literature by taking into account the possibility of more than one listener to whom the Sender may address his words. In their short paper, they present a baseline model and mention several directions for possible further research. Some questions left open in the paper have not been addressed since. This thesis is written with this fact in mind and fills some of the holes, to provide a deeper understanding of how such cheap talk games unravel—in a broader sense, how people strategically communicate.

I first extended the baseline model to allow the Sender's private information to be more varied, as represented by the number of possible states of the world. In the three-state model, I identified the pure strategy perfect Bayesian equilibria, the ever-existent pooling equilibrium, and depending on the degree of coincidence of the players' interests, the more informative separating and partially revealing equilibria. I identified the possible scenarios based on how much information could be credibly conveyed in the various fora: with each Receiver in private and with both Receivers present in public. A result of interest is the scenario of Mutual Discipline, in which some information can be credibly transmitted between the Sender and each of the Receivers separately in private, but no communication is credible in public. Such a case does not arise in Farrell and Gibbons (1989)'s model and points to the importance of allowing for the fact that the Sender's information in real life can be quite varied. Under Mutual Subversion, as the name suggests, the presence of one person *subverts* communication with the other, and *vice versa*. Ample examples for this kind of situation arise in everyday life. In one of them, for example, the Sender needs to explain his plan of action to two potential sponsors, who both agree with the general aim of the Sender, but have diametrically opposed views as to the mode of execution.

A subsequent welfare analysis of the possible pure strategy perfect Bayesian equilibria gave indications as to which equilibrium the players would prefer to play, when there is more than one available. While in private communication with one Sender and one Receiver, the players all preferred the more informative equilibria to the less informative, this was no longer true in public: the Sender sometimes wished to remain inscrutable. Again, this is a situation which often arises in real life: we often do better by staying silent. The example I described in detail featured a boss who needed to decide which of his employees to promote, while the candidates would rather have left the company than stay in their then positions. The boss was able to induce them both to stay, in line with his wishes, by not giving away anything about his plans.

To further address the question of equilibrium selection, an important one due to the multitude of pure strategy perfect Bayesian equilibria in most cases, I applied an equilibrium refinement criterion (after having reviewed the literature on the topic), Farrell (1993)'s neologism proofness, in an attempt to provide predictions as to which equilibrium was likely to be played. The criterion proved useful in certain cases, in others, however, it resulted in excluding *all* the equilibria. This is a problematic case and it is useful to draw attention to such cases, since they seem to leave us with no good answer to the simple question: how are people likely to communicate in such a situation? In reality, something must take place—but what will it be? Farrell (1993) mentions the possibility of applying an evolutionary approach to the problem, in which dynamic equilibria may exist. The evolutionary approach incorporates learning and sudden changes (mutations) into the model: plausible extensions to a theory of communication and speech.

The next section addressed a question that Farrell and Gibbons (1989) left open but mentioned as worthy of further work, the issue of how the forum in which communication was to take place was selected. I presented the case where the Sender, after having established the state of the world, decided himself how to communicate with whom. His goal was to design a mechanism in which he could achieve the best possible payoffs through influencing the Receivers actions. In certain cases, he was able to do so. I described such cases (using the parameters of the model and a real life example), and showed that the Sender manages to communicate at least one of his types credibly to a Receiver with whom he otherwise can only play a pooling equilibrium. A possible direction of further work would be to more precisely characterise the conditions under which the Sender can improve his payoffs via forum choice in this three-state, two-audience model.

The issue of equilibrium selection arose in this case, too: there were often many possible equilibria which may conceivably have been played. I began by applying Farrell

(1993)'s concept of neologism-proofness to an example described in the chapter, but found that the concept needed to be extended in order to successfully deal with the forum choice game. I described a generalisation of the concept of neologism-proofness, which succeeded in selecting one plausible equilibrium from among the many available equilibria. More precisely exploring the results of such a generalised equilibrium refinement criterion will be the subject of further work.

In the final chapter, I presented two further extensions of the model which lead to some insights. The first was a model where the Sender had $k > 2$ possible types, where I examined the players' preferences over the various equilibria, finding that in private communication, all the statements of the three-state model remained unchanged, despite the fact that there was a greater number of possible pure strategy perfect Bayesian equilibria. The second extension showed the Sender talking to three audiences, in several possible fora. A scenario named Two-Sided Discipline materialised, in which the presence of all three Receivers was needed to discipline communication with one of the audiences. A real-life example underscored the parameters, in which an incumbent politician's incentives to always tell his voters what they would like to hear could only be disciplined by his drive to tell the truth to *both* European officials and prospective foreign investors. The issue of forum choice with more than two audiences is worth further investigating, since as the number of audiences increases, the number of possible constellations in which they can communicate with the Sender increases also, and more rapidly so. The Sender's forum choice set thus becomes larger, possibly allowing him to further increase his payoffs.

Chapter A

Appendix

A.1 Appendix 1: Proofs

Proof of Proposition 2.1.3

Proof Assume, without loss of generality, that the revealed state is state three, that is, $v_3 \geq 0$; further assume that $q_{pr} = q_2$, and thus to sustain the partially revealing equilibrium, $v_2 \geq 0$. The Sender is indifferent between the pooling and the partially revealing equilibrium as long as $q_{pool} = q_{pr} = q_2$:

$$\text{if } \pi_2 x_2 \geq \pi_1 x_1 \text{ and } \pi_2 x_2 \geq \pi_3 x_3 \text{ then } q_{pool} = q_{pr} = q_2. \tag{A1.1}$$

Because of the existence of the partially revealing equilibrium it is not possible that $q_{pool} = q_1$, but there is a third possibility. Let us examine the case when $q_{pool} = q_3$:

$$\text{if } \pi_3 x_3 \geq \pi_1 x_1 \text{ and } \pi_3 x_3 \geq \pi_2 x_2 \text{ then } q_{pool} = q_3, \tag{A1.2}$$

$$\text{however, by assumption } \pi_2 x_2 \geq \pi_1 x_1 \text{, thus } q_{pr} = q_2. \tag{A1.3}$$

In this case, the Sender receives different payoffs in the two equilibria. The following are his expected payoffs *ex ante*, that is before the state of the world becomes known to him:

$$\text{EU}_p^S = \pi_1 \times 0 + \pi_2 \times 0 + \pi_3 \times v_3 = \pi_3 v_3 \tag{A1.4}$$

$$\text{EU}_{pr}^S = \pi_1 \times 0 + \pi_2 \times v_2 + \pi_3 \times v_3 = \pi_2 v_2 + \pi_3 v_3. \tag{A1.5}$$

Since $\pi_2 v_2 \geq 0$, the Sender cannot strictly prefer pooling to partially revealing *ex ante*. The situation is similar *ex post*:

$$U_p^S(s = s_1) = 0, \quad (A1.6)$$
$$U_{pr}^S(s = s_1) = 0, \quad (A1.7)$$
$$U_p^S(s = s_2) = 0, \quad (A1.8)$$
$$U_{pr}^S(s = s_2) = v_2, \quad (A1.9)$$
$$U_p^S(s = s_3) = v_3, \quad (A1.10)$$
$$U_{pr}^S(s = s_3) = v_3. \quad (A1.11)$$

Thus, in states one and three, the Sender is still indifferent, but in state two, he cannot strictly prefer pooling to partially revealing, since, by assumption, $v_2 \geq 0$. ∎

Proof of Proposition 2.1.4

Proof Since the game is coherent, pooling can result in the action pairs (q_1, r_1), (q_2, r_2) or (q_3, r_3). In a separating equilibrium the Sender can induce either of these action pairs. Thus, the Sender's choice set is larger under separating than under pooling and the larger choice set contains the smaller. Consequently, the Sender at least weakly prefers separating to pooling *ex post* and therefore also *ex ante*. ∎

Proof of Proposition 2.1.5

Proof The possible pooling actions of the Receivers in the incoherent case can be (q_1, r_2), (q_1, r_3), (q_2, r_1), (q_2, r_3), (q_3, r_1) or (q_3, r_2). Let us assume, without loss of generality, that pooling results in (q_1, r_2). First, we look at the Sender's preferences *ex ante*. The Sender's utilities under separation and under pooling are the following:

$$EU_s^S = \pi_1(v_1 + w_1) + \pi_2(v_2 + w_2) + \pi_3(v_3 + w_3) \quad (A1.12)$$
$$EU_p^S = \pi_1 v_1 + \pi_2 w_2. \quad (A1.13)$$

We know that $v_3 + w_3$ is nonnegative, since a public separating equilibrium exists. Therefore, for pooling to be preferred by the Sender, one or both of v_2 and w_1 must be large and negative, to achieve

$$\pi_1 w_1 + \pi_2 v_2 + \pi_3(v_3 + w_3) < 0. \quad (A1.14)$$

Second, we look at the Sender's preferences *ex post*. Recall that $v_i + w_i \geq 0 \; \forall i = 1, 2, 3$. The Sender's utilities under separation and under pooling if $s = s_1$ are the following:

$$U_s^S = v_1 + w_1 \quad (A1.15)$$
$$U_p^S = v_1. \quad (A1.16)$$

Thus, pooling is strictly preferred if $w_1 < 0$. The Sender's utilities under separation and under pooling if $s = s_2$ are the following:

$$\mathrm{U}_s^S = v_2 + w_2 \tag{A1.17}$$
$$\mathrm{U}_p^S = w_2. \tag{A1.18}$$

Thus, pooling is strictly preferred if $v_2 < 0$. The Sender's utilities under separation and under pooling if $s = s_3$ are the following:

$$\mathrm{U}_s^S = v_3 + w_3 \tag{A1.19}$$
$$\mathrm{U}_p^S = 0. \tag{A1.20}$$

Since $v_3 + w_3 \geq 0$, in the third state of the world pooling can be only weakly preferred, if $v_3 + w_3 = 0$. To sum up, pooling is weakly preferred *ex post* if $w_1 < 0$, $v_2 < 0$ and $v_3 + w_3 = 0$. The conditions can be calculated similarly for the other pooling action pairs of the Receivers.

In the case of Full Communication, all the v_i and w_i are nonnegative, which implies that pooling can not be preferred by the Sender neither *ex ante* nor *ex post*. Under One-Sided Discipline, the Sender may prefer pooling *ex ante*, for example, in our (q_1, r_2) case, when $\pi_1 w_1 + \pi_2 v_2 + \pi_3(v_3 + w_3) < 0$, but cannot prefer pooling ex post, since there must be a separating equilibrium with one of the Receivers, implying that either all v_i or all w_i are nonnegative. Under Mutual Discipline, however, all the conditions for pooling to be weakly preferred may be satisfied, both *ex ante* and *ex post*. ∎

Proof of Proposition 2.1.6

Proof Since the game is coherent, partially revealing can result in the action pairs (q_1, r_1), (q_2, r_2) or (q_3, r_3) (one of these is the case when the state of the world is revealed). In a separating equilibrium the Sender can induce either of these action pairs. Thus, the Sender's choice set is larger under separating than under partially revealing and the larger choice set contains the smaller. Consequently, the Sender at least weakly prefers separating to partially revealing *ex post* and therefore also *ex ante*. ∎

Proof of Proposition 2.1.7

Proof Let us assume, without loss of generality, that partially revealing results in (q_1, r_2). First, we look at the Sender's preferences *ex ante*. The Sender's utility under separation can be seen in Equation A1.12. His utility under partially revealing is the following:

$$\mathrm{EU}_{pr}^S = \pi_1 v_1 + \pi_2 w_2 + \pi_3(v_3 + w_3), \tag{A1.21}$$

which implies that for partially revealing to be preferred, one or both of v_2 and w_1 must be negative to achieve $\pi_1 w_1 + \pi_2 v_2 < 0$. Second, we look at the Sender's preferences *ex post*. The Sender's utility under separation in the three possible states of the world can be seen in Equations A1.15, A1.17 and A1.19. The Sender's utilities under partially revealing in the three possible states of the world are, in order:

$$\mathrm{U}^S_{pr}(s = s_1) = v_1 \quad \text{(A1.22)}$$
$$\mathrm{U}^S_{pr}(s = s_2) = w_2 \quad \text{(A1.23)}$$
$$\mathrm{U}^S_{pr}(s = s_3) = v_3 + w_3, \quad \text{(A1.24)}$$

which means that partially revealing is preferred in state one if $w_1 < 0$, in state two if $v_2 < 0$, and the Sender is indifferent between separating and partially revealing in state three. Thus, partially revealing is weakly preferred to separating as long as w_1 and v_2 are nonpositive.

In the case of Full Communication, all the v_i and w_i are nonnegative, which implies that partially revealing can not be preferred by the Sender neither *ex ante* nor *ex post*. Under One-Sided Discipline, the Sender may prefer partially revealing *ex ante*, for example, in our (q_1, r_2) case, when $\pi_1 w_1 + \pi_2 v_2 < 0$, but cannot prefer partially revealing ex post, since there must be a separating equilibrium with one of the Receivers, implying that either all v_i or all w_i are nonnegative. Under Mutual Discipline, however, all the conditions for partially revealing to be weakly preferred may be satisfied, both *ex ante* and *ex post*. ∎

Proof of Proposition 2.1.8

Proof 1. Notice that when the Receivers' pooling and partially revealing actions coincide, that is $(q_{pool}, r_{pool}) = (q_{pr}, r_{pr})$, then the Sender is indifferent between the two equilibria in states one and two, since they are outcome- and payoff-equivalent. This happens exactly when $\pi_3 x_3 \leq \pi_1 x_1$, $\pi_3 x_3 \leq \pi_2 x_2$, $\pi_3 y_3 \leq \pi_1 y_1$ and $\pi_3 y_3 \leq \pi_2 y_2$. However, in state three, the Sender receives $v_3 + w_3 \geq 0$ under partially revealing and zero under pooling. He thus at least weakly prefers partially revealing to pooling.

2. When $\pi_3 x_3 \geq \pi_1 x_1$, $\pi_3 x_3 \geq \pi_2 x_2$, $\pi_3 y_3 \geq \pi_1 y_1$ and $\pi_3 y_3 \geq \pi_2 y_2$, then $(q_{pool}, r_{pool}) = (q_3, r_3)$. Therefore, if the Sender is of type three, pooling and partially revealing again result in the same outcome. However, if the Sender is of type one or two, pooling results in payoffs of zero to the Sender, whereas in the case of partially revealing, the payoffs are nonnegative, since, in order for a partially revealing equilibrium to exist, the players' partially revealing actions must correspond to nonnegative payoffs. Thus partially revealing is at least weakly preferred both *ex post* and *ex ante*.

3. From the point of view of the Receivers' pooling actions, there are four cases left which the above two points do not cover. These are the following: $(q_{pool}, r_{pool}) = (q_1, r_3)$, (q_2, r_3), (q_3, r_1) or (q_3, r_2). Without loss of generality assume that $(q_{pool}, r_{pool}) = (q_1, r_3)$. The partially revealing action of Receiver **Q** is thus also q_1, therefore, for the partially revealing actions to be coherent, it must be that $r_{pr} = r_1$. This implies that $v_1 + w_1 \geq 0$. Recall also that since s_3 is the revealed state, $v_3 + w_3 \geq 0$. *Ex ante*, the Sender's expected utility under pooling is

$$\mathrm{EU}^S_{pool} = \pi_1 v_1 + \pi_3 w_3, \quad \text{(A1.25)}$$

and his utility *ex post* is

$$\mathrm{U}_{pool}(s = s_1) = v_1 \tag{A1.26}$$
$$\mathrm{U}_{pool}(s = s_2) = 0 \tag{A1.27}$$
$$\mathrm{U}_{pool}(s = s_3) = w_3. \tag{A1.28}$$

The Sender's expected utility under partially revealing *ex ante* is

$$\mathrm{EU}^S_{pr} = \pi_1(v_1 + w_1), \tag{A1.29}$$

thus pooling is strictly preferred if $\pi_3 w_3 > \pi_1 w_1$. *Ex post*, the Sender's utilities under partially revealing are

$$\mathrm{U}_{pr}(s = s_1) = v_1 + w_1 \tag{A1.30}$$
$$\mathrm{U}_{pr}(s = s_2) = 0 \tag{A1.31}$$
$$\mathrm{U}_{pr}(s = s_3) = v_3 + w_3, \tag{A1.32}$$

thus pooling is strictly preferred if both w_1 and v_3 are negative. The other three cases can be analysed in a similar manner.

4. Again, without loss of generality assume that from among the four cases remaining, $(q_p, r_p) = (q_1, r_3)$. The partially revealing action of Receiver **Q** is thus also q_1, therefore, for the partially revealing actions to be incoherent, it must be that $r_{pr} = r_2$. This implies that $v_1 \geq 0$ and $w_2 \geq 0$. Recall also that since s_3 is the revealed state, $v_3 + w_3 \geq 0$. The Sender's expected utility under pooling can be seen in Equation A1.25 *ex ante* and his utilities under pooling *ex post* are in Equations A1.26–A1.27. The Sender's expected utility under partially revealing *ex ante* is:

$$\mathrm{EU}^S_{pr} = \pi_1 v_1 + \pi_2 w_2, \tag{A1.33}$$

and pooling is strictly preferred if $\pi_3 w_3 > \pi_2 w_2$. *Ex post*, the Sender's utilities under partially revealing are

$$\mathrm{U}_{pr}(s = s_1) = v_1 \tag{A1.34}$$
$$\mathrm{U}_{pr}(s = s_2) = w_2 \tag{A1.35}$$
$$\mathrm{U}_{pr}(s = s_3) = v_3 + w_3, \tag{A1.36}$$

which means that pooling may only be *weakly* preferred if $w_2 = 0$ and $v_3 \leq 0$. The reason that pooling can, in this second case, not be strictly preferred by every type of the Sender lies in the fact that the Receivers' partially revealing actions are incoherent: $(q_{pr}, r_{pr}) = (q_1, r_2)$ instead of the coherent (q_1, r_1). In the incoherent case, the conditions needed for a public partially revealing equilibrium to exist are more severe: $v_1 \geq 0$ and $w_2 \geq 0$, as opposed to $v_1 + w_1 \geq 0$ (which does *not* necessarily imply $v_1 \geq 0$ and $w_1 \geq 0$) in the coherent case. ∎

Proof of Proposition 2.1.9

Proof 1. In private, the separating equilibrium achieves the best possible payoffs for the Sender (see the Proof of Lemma 3.1.1 here in Appendix A.1), thus he cannot improve his situation in any way—the equilibrium is thus neologism-proof. When the pooling equilibrium is played, however, there are two self-signalling sets. If the pooling action is Action i, $i \in \{1, 2, 3\}$, then the two states other than s_i are each a self-signalling set. The Sender, in one these states s_j, $j \neq i$, could make a credible speech along these lines:

> "Although you were not expecting to receive any meaningful information from me, listen. The state is s_j. Check that I would not say this if the state were not s_j, since it would result in a payoff of zero for me. Also, I *do* have an incentive to tell you this piece of information, since otherwise you would choose your pooling action, Action i, which would again give me a payoff of zero. However, if you believe me and choose Action j, we will both get nonnegative payoffs."

The partially revealing equilibrium also contains a self-signalling set: the state which is neither the revealed state, nor the state to which the Receiver's partially revealing action is connected. The Sender would like to reveal that state and only has an incentive to do so if his statement is true. Thus, neither the pooling, nor the partially revealing equilibrium is neologism-proof.

2. In private, when no separating equilibrium exists, the partially revealing equilibrium achieves the best possible payoffs for the Sender (see the Proof of Lemma 3.1.1 here in Appendix A.1), and is thus neologism-proof. When the pooling equilibrium is played, there is always a self-signalling state, however, which renders the equilibrium not neologism-proof. If the Receiver's pooling action coincides with her partially revealing action, then this state is the revealed state, and if her pooling action is the action corresponding to the revealed state, then this state is the state corresponding to her partially revealing action. In either case, in the self-signalling state, the Sender has an incentive to reveal that state, and has such an incentive only if he is in that state.

3. When there is only a pooling equilibrium, then at most two of the v parameters (assuming the Sender is talking to Receiver **Q**) are nonnegative. However, neither of these states are self-signalling, since the Sender has an incentive in the state with a negative v parameter payoff to pretend to be in either of the states with a nonnegative v. Thus the equilibrium is neologism-proof. ∎

Proof of Proposition 2.1.10

Proof Because of the coherence of both the pooling and the partially revealing actions of the players, the proof is entirely similar to the proof of Proposition 2.1.9. The two Receivers effectively act as one, since in all the available equilibria, each chooses the same action as the other. I now briefly refer to the self-signalling sets in each case:

When a separating equilibrium exists, it has no self-signalling sets, the partially revealing equilibrium has one (the state belonging to the actions never chosen by the Receivers), while in the pooling equilibrium, there are two self-signalling sets: both the states other than the state belonging to the actions chosen by the Receivers.

When there is no separating equilibrium, but there is a partially revealing equilibrium, then it has no self-signalling sets. If the Receivers' pooling and partially revealing action coincide, then the revealed state of the partially revealing equilibrium is a self-signalling state in the pooling equilibrium. If the Receivers' pooling and partially revealing actions do not coincide, then the state belonging to the players' partially revealing actions in the partially revealing equilibrium is a self-signalling state in the pooling equilibrium.

When there is only a pooling equilibrium, then it has no self-signalling sets. ∎

Proof of Lemma 3.1.1

Proof First, recall that according to the welfare analysis of private talk in Chapter 2, if a separating equilibrium exists, it is expected to be played, since it is preferred by all players to the partially revealing and the pooling equilibrium. Also, if a separating equilibrium does not exist, but a partially revealing equilibrium does, the partially revealing equilibrium is expected to be played, since it is preferred by all players to the pooling equilibrium.

A private separating equilibrium always achieves the maximum possible payoffs for a Sender of any type. For example, in a separating equilibrium with Receiver **Q**, the payoffs of the Sender are (v_1, v_2, v_3), which are all nonnegative, and are thus better or as least as good as the alternative achievable zero payoffs.

Now take the case when no private separating equilibrium exists, but a partially revealing equilibrium does. A partially revealing equilibrium also achieves the maximum possible payoffs for a Sender of any type. Without loss of generality, assume there is a partially revealing equilibrium with Receiver **R** where the revealed state is state one, and $r_{pr} = r_2$. For the equilibrium to exist, w_1 and w_2 must be nonnegative. For a separating equilibrium not to exist, w_3 must be negative. In any state s_i, the Sender can either receive a payoff of zero or a payoff of w_i. The payoffs of the Sender in the partially revealing equilibrium are $(w_1, w_2, 0)$, the maximum possible payoffs.

Thus, if there is either a partially revealing or a separating equilibrium with both Receivers in private, then the forum choice strategy (private, private, private) will give the best possible payoffs to the Sender, and is thus weakly dominant. ∎

Statement and proof of Lemma A.1.1

Lemma A.1.1 *In case 2 (No Communication), there is no differentiated equilibrium pure strategy in the forum choice stage.*

Proof The statement is equivalent to saying that any differentiated strategy leads to negative payoffs in at least one of the stages. In case 2 (No Communication) there is only a pooling equilibrium with the Receivers, both in public, and with each in private. In both fora, Receiver **Q** plays her pooling action q_{pool}, and Receiver **R** plays *her* pooling action r_{pool}. Notice that in any differentiated strategy, the revealed state will yield a payoff of

$v_i + w_i$. If all the $v_i + w_i$ are negative, then it is immediate that there is no differentiated equilibrium strategy. If one of the $v_i + w_i$ is positive, the Sender may wish to reveal that state. Then, the two players (irrespective of the forum) will play their partially revealing actions in the other two states. If these partially revealing actions are coherent, for example in state $j \neq i$, $(q_{pr}, r_{pr}) = (q_j, p_j)$, and $v_j + w_j$ is positive, then there is a partially revealing equilibrium in public, contrary to our assumption. If in the same coherent case, $v_j + w_j$ is negative, then there is no differentiated equilibrium. If the partially revealing actions are incoherent, for example in states $j, k \neq i$, $(q_{pr}, r_{pr}) = (q_j, p_k)$, and v_j, w_k are positive, then there is a partially revealing equilibrium in public, contrary to our assumption. If v_j and w_k are *not* both positive, then there is no differentiated equilibrium, since the Sender's payoff will be negative in at least one of the states. ∎

Proof of Proposition 4.1.1

Proof Assume the number of possible types for the Sender is k and assume there exists a partition equilibrium in which the Sender partitions the type-space into N parts. We have seen above that for there to exist such a partition equilibrium, the pooling actions of the Receiver (without loss of generality, assume the Sender talks to Receiver **Q**) *within* each partition must correspond to a nonnegative v-parameter. We shall now construct a partially revealing equilibrium. Take the set $\mathcal{N}$ of the Receiver's pooling actions within each partition. This set has N elements: $\mathcal{N} = \{q_{pool}(1), q_{pool}(2), \ldots, q_{pool}(N)\}$. The pooling action of the Receiver from among the *entire* set of possible actions k, q_{pool}, will be an element of this set, since $q_{pool} = q_i$ if $\pi_i x_i \geq \pi_j x_j$, $\forall i, j = 1, 2, \ldots, k$; $i \neq j$ while an element of $\mathcal{N}$, $q_{pool}(i) = q_a(i)$ if $\pi_a x_a \geq \pi_b x_b$, $\forall a, b \in \mathcal{P}_i$, where $\mathcal{P}_i$ is the ith partition. Assume without loss of generality that $q_{pool} = q_{pool}(i)$. Take the set $\mathcal{N} \backslash \{q_{pool}(i)\}$. This will be the set of revealed states in the partially revealing equilibrium. All the corresponding v parameters are by construction nonnegative. The rest of the states will be the unrevealed states, from among which the Receiver's pooling action will naturally be $q_{pool}(i)$. The v-parameter corresponding to $q_{pool}(i)$ is also by construction nonnegative, thus we have a partially revealing equilibrium.

The statement that such a correspondence between equilibria does not exist in public is shown through a counterexample. Observe the five-state case in Table 1. Also assume the following about the pooling actions of the Receivers: $\pi_1 x_1 > \pi_2 x_2 > \pi_3 x_3 > \pi_4 x_4 > \pi_5 x_5$ and $\pi_5 y_5 > \pi_4 y_4 > \pi_3 y_3 > \pi_2 y_2 > \pi_1 y_1$. Notice that the order of the Receivers' preferences over actions according to their prior beliefs are exactly reversed: Receiver **Q** prefers q_1 to q_2 to q_3 to q_4 to q_5 according to her *ex ante* beliefs, while Receiver **R** prefers r_5 to r_4 to r_3 to r_2 to r_1.

Table 1: Partition Equilibrium in Public

Q	**R**	**Public**
$v_1 > 0$	$w_1 < 0$	$v_1 + w_1 < 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 < 0$
$v_3 > 0$	$w_3 < 0$	$v_3 + w_3 < 0$
$v_4 < 0$	$w_4 < 0$	$v_4 + w_5 < 0$
$v_5 < 0$	$w_5 > 0$	$v_4 + w_5 < 0$

It is trivial from Table 1 that there is no partially revealing equilibrium in public, since all the $v_i + w_i$ are negative. However, there does exist a partition equilibrium, which partitions the type-space into 2 parts, one containing state one and two, the other, states three, four and five. The pooling actions of the Receivers in the first, smaller partition is (q_1, r_2), which correspond to v- and w-parameters v_1 and w_2, both of which are positive, ensuring that the Sender does not wish to deviate from the partition strategy is he is in states one or two. In the second, larger partition the Receivers' pooling actions are (q_3, r_5), which correspond to parameters v_3 and w_5, both of which are positive, ensuring that the Sender does not wish to deviate from the partition strategy if he is in states three, four or five, either. This concludes the counterexample and thus the proof of the proposition. ∎

Proof of Proposition 4.2.2

Proof The proof follows that of Proposition A.2.1. First, notice that any differentiated strategy fully reveals the state of the world to all Receivers, that is, it implies separation at the forum choice stage. This means that they both lead to payoffs of $(u_1+v_1+w_1, u_2+v_2+w_2)$.

Second, notice that if grand public separation exists in the second, cheap talk phase of the game—that is, in cases 1, 3 and 5 in Table 4.4: under Full Communication, Non-Mutual Discipline and Mutual Discipline, then the payoffs of $(u_1+v_1+w_1, u_2+v_2+w_2)$ can be achieved by the undifferentiated choice of grand public communication. This means that in these cases the Sender cannot gain by using a differentiated strategy which becomes available to him when the choice of forum is made *ex post*.

Third, in the cases where public separation in the second stage if not possible—cases 2 and 4 in Table 4.4: No Communication and Subversion, neither differentiated strategy is an equilibrium strategy. To see this, recall that since there is no public separation, the Sender receives negative payoffs in at least one of the states (either $u_1+v_1+w_1$ or $u_2+v_2+w_2$ is negative). However, by deviating in this state by announcing the forum that the *other* type of Sender is supposed to choose according to the differentiated strategy used, the Sender can achieve a superior payoff of zero. Thus, if the Receivers believe that the Sender is using the given differentiated strategy, the Sender will have incentives to deviate in (at least) one of the states. This concludes the proof. ∎

A.2 Appendix 2: Forum Choice in the Two-State Model

As conjectured in Farrell and Gibbons (1989), separating equilibria in the forum choice stage do exist (though not in all scenarios), but so do several other equilibria. Following Farrell and Gibbons, I here consider only pure strategy equilibria, partly for the sake of tractability, partly because taking the mixed startegy equilibria into account did not provide any new insights. To demonstrate, I will later describe the mixed strategy equilibria in one of the possible scenarios, One-Sided Discipline. The five scenarios that may occur in the two-state, two-audience model can be seen in Table 2. Recall that in the two-state model there is no partially revealing equilibrium, each Receiver has two possible actions and the Sender has two types. Furthermore, "p" signifies that only a pooling equilibrium exists and "s" that a separating equilibrium exists as well as a pooling equilibrium.

The beliefs of the Receivers may change following the Sender's choice of forum, and the updated beliefs can be calculated. In the general case, assume that if the state of the

Table 2: Scenarios

No.	Name	**Q**	**R**	**Public**
1	Full Communication	s	s	s
2	No Communication	p	p	p
3	One-Sided Discipline	p	s	s
4	Subversion	p	s	p
5	Mutual Discipline	p	p	s

The roles of **Q** and **R** can be reversed.

world is s_1, the Sender says "private" with probability p (and thus "public" with probability $(1-p)$), and if the state of the world if s_2, the Sender requests "private" communication with probability q (and consequently "public" talk with probability $(1-q)$). Let μ_e be the probability that the state of the world is s_1 if the Sender says "private" and μ_c the same probability if the Sender announces "public". Using Bayesian updating, these two probabilities can be expressed using p, q and the prior belief that the state of the world is s_1: π.

$$\mu_e = \Pr(s = s_1|\text{“private”}) = \frac{p\pi}{p\pi + q(1-\pi)} \tag{A1.37}$$

$$\mu_c = \Pr(s = s_1|\text{“public”}) = \frac{(1-p)\pi}{(1-p)\pi + (1-q)(1-\pi)}. \tag{A1.38}$$

The insight that forum choice has no important consequences in the two-state, two-audience model is summarised in the following statement:

Proposition A.2.1 *In the two-state, two-audience model the Sender cannot increase his payoffs by making his choice of forum* ex post*, rather than* ex ante*, that is, he cannot gain from using a differentiated strategy.*

Proof First, notice that there are two pure strategies available to the Sender in the forum choice stage that are unavailable if the decision cannot be contingent on the state of the world. These are to request public communication if the Sender's type is s_1, and private communication if it is s_2 (a strategy which I shall denote by (public, private)), and the converse: (private, public). Both of these differentiated strategies fully reveal the state of the world to both Receivers, that is, they imply separation at the forum choice stage. This means that they both lead to payoffs of $(v_1 + w_1, v_2 + w_2)$.

Second, notice that if public separation exists in the second, cheap talk phase of the game—that is, in cases 1, 3 and 5 in Table 2: under Full Communication, One-Sided Discipline and Mutual Discipline, then the payoffs of $(v_1 + w_1, v_2 + w_2)$ can be achieved by the undifferentiated choice of public communication. This means that in these cases the Sender cannot gain by using a differentiated strategy which becomes available to him when the choice of forum is made *ex post*.[1]

[1] The fact that the payoffs $(v_1 + w_1, v_2 + w_2)$ can be achieved using the strategy (public, public) does not mean that they *will* be achieved or even that this strategy would in fact be chosen by the Sender *ex ante*: there may be better possibilities. Consider, for example, the following case of Mutual Discipline:

Third, in the cases where public separation in the second stage if not possible—cases 2 and 4 in Table 2: No Communication and Subversion, neither (public, private) nor (private, public) is an equilibrium strategy. To see this, recall that since there is no public separation, the Sender receives negative payoffs in at least one of the states (either v_1+w_1 or v_2+w_2 is negative). However, by deviating in this state by announcing "private" instead of "public", the Sender can achieve a superior payoff of zero. Thus, if the Receivers believe that the Sender is using the (public, private) (or the (private, public)) strategy, the Sender will have incentives to deviate in (at least) one of the states. This concludes the proof. ■

The intuition behind the fact that the existence of *public* separation is crucial for the (private, public) or (public, private) strategies to be equilibrium strategies is the following: the forum choice announcement itself takes place in public. Thus, if there is no public separating equilibrium in the second stage, then there can be no credible public separating equilibrium in the first, forum choice stage either.

I will now make a brief detour and investigate the possibility of using mixed strategies in the forum choice stage. Take, for example, a case of One-Sided Discipline, with the parameters of the model described in Table 3. There is a separating equilibrium with Receiver **Q** and in public, and none with Receiver **R**.

Table 3: One-Sided Discipline

Q	**R**	**Public**
s	p	s
$v_1 > 0$	$w_1 < 0$	$v_1 + w_1 > 0$
$v_2 > 0$	$w_2 > 0$	$v_2 + w_2 > 0$

Recall that, just as in the three-state model, in the case of One-Sided Discipline separation is always preferred by the Sender to pooling *ex post*. Thus the preferences of the Sender and the Receivers are aligned, and the separating can be assumed to be played if it is available. Assuming as previously that the Sender requests private communication with probability p in state one and probability q in state two, his payoffs in state one and state two respectively are:

$$p(v_1 + \begin{cases} w_1 & \text{if } \mu_e y_1 \geq (1-\mu_e) y_2 \\ 0 & \text{otherwise} \end{cases} + (1-p)(v_1 + w_1)) \tag{A1.39}$$

$$q(v_2 + \begin{cases} w_2 & \text{if } \mu_e y_1 \geq (1-\mu_e) y_2 \\ 0 & \text{otherwise} \end{cases} + (1-q)(v_2 + w_2)). \tag{A1.40}$$

$$v_1 > 0,\ w_1 < 0,\ v_1 + w_1 > 0,$$
$$v_2 < 0,\ w_2 > 0,\ v_2 + w_2 > 0.$$

Here the strategy (public, public) in the first stage and separation in the second leads to payoffs of (v_1+w_1, v_2+w_2), but in fact, the Sender prefers to play the pooling equilibrium rather than the separating equilibrium in public, resulting in the superior payoffs of (v_1, w_2). Alternatively, the Sender could choose the strategy (private, private), which also leads to the superior payoffs of (v_1, w_2).

This means that if $\mu_e \geq \frac{y_2}{y_1+y_2}$, p is payoff-irrelevant and $q = 0$ (which in turn means that $\mu_e = 1$, satisfying the initial condition on the size of μ_e). That is, there is no mixed strategy that results in payoffs differing from the payoffs generated by the pure strategies. Also, if $\mu_e \leq \frac{y_2}{y_1+y_2}$, q is payoff-irrelevant and $p = 1$, and consequently $\mu_e \in [\pi, 1]$, and taking into account the initial condition, $\mu_e \in [\pi, \frac{y_2}{y_1+y_2}]$. Thus, again, there is no important mixed strategy.

The situation is similar in the cases of Subversion and Mutual Discipline. Under Full Communication and No Communication, payoffs are the same for all players regardless of the forum chosen, and thus the forum choice issue is uninteresting. The announcement made by the Sender leaves the Receivers' beliefs unchanged, and the game does not differ from the original game in any important way.

It is now clear that giving the Sender the possibility to choose the forum of communication *after* his type becomes known to him affords him no advantage: to differentiate his forum choice according to the state of the world is either not an equilibrium strategy, or can be replicated by a choice of (public, public), which was available already *ex ante*. Thus no new payoffs arise, neither for the Sender, nor for the Receivers. A more complex model is needed to analyse the effects a forum choice stage has on communication between a Sender and multiple Receivers. The three-state, two-audience model presented in Chapter 2 and extended in Chapter 3 is just such a model.

A.3 Appendix 3: Further Examples

In the three-state, two-audience model with *ex post* forum choice, three more examples are provided of cases when adding the forum choice stage enables the Sender to achieve superior payoffs to that of the model without such forum choice. The first and third examples concern cases of One-Sided Discipline (cases 3 (a) and 7 in Table 3.1) and the second, Mutual Discipline (case 5 (b)).

Example The signs of the parameters of the model are shown in Table 4. There is a separating equilibrium in public and with Receiver **R** in private, but only a pooling equilibrium exists with Receiver **Q** in private. Assume that Receiver **Q**'s pooling action is q_2.

Table 4: One-Sided Discipline

Q	**R**	**Public**
p	s	s
$v_1 > 0$	$w_1 > 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 > 0$
$v_3 < 0$	$w_3 > 0$	$v_3 + w_3 > 0$

The Sender's payoffs under private communication are:

$$\mathrm{U}(s = s_1,\ \text{private}) = w_1 \tag{A2.1}$$
$$\mathrm{U}(s = s_2,\ \text{private}) = v_2 + w_2 \tag{A2.2}$$
$$\mathrm{U}(s = s_3,\ \text{private}) = w_3, \tag{A2.3}$$

while under public communication they are:

$$U(s = s_1,\ \text{public}) = v_1 + w_1 \tag{A2.4}$$
$$U(s = s_2,\ \text{public}) = v_2 + w_2 \tag{A2.5}$$
$$U(s = s_3,\ \text{public}) = v_3 + w_3. \tag{A2.6}$$

Thus, the Sender prefers public communication in state one, private communication in state three, and is indifferent in state two. Using these facts as a point of departure, assume the Sender plays the differentiated strategy of (public, private, private). In this case, the best responses of the Receivers are the following: if they hear a request for public speech, they infer that the state of the world is state one, and accordingly both choose their first action, regardless of any communication in the second stage. In they hear a request for private speech, they infer that the state of the world is not state one, and update their beliefs accordingly. Their beliefs will become:

$$\mu_{e1} = 0 \tag{A2.7}$$
$$\mu_{e2} = \frac{\pi_2}{\pi_2 + \pi_3} \tag{A2.8}$$
$$\mu_{e3} = \frac{\pi_3}{\pi_2 + \pi_3}. \tag{A2.9}$$

The Sender will separate in the second stage with Receiver **R** who will thus choose action i is the state of the world is s_i. The Sender will pool in the second stage with Receiver **Q** who will thus choose her pooling action according to her updated beliefs. As can be seen from these beliefs, this action is q_2. Consequently, the Sender's payoffs will be:

$$U(s = s_1,\ \text{public}) = v_1 + w_1 \tag{A2.10}$$
$$U(s = s_2,\ \text{private}) = v_2 + w_2 \tag{A2.11}$$
$$U(s = s_3,\ \text{private}) = w_3. \tag{A2.12}$$

Here again, the Sender has the best of both worlds: in each state, he receives the payoff he prefers from among the payoffs in private and public communication. If the Receivers react in this way, the Sender has no incentive to deviate: In states one and three, he is receiving the best achievable payoff. In state two, announcing "public" would result in both Receivers choosing their first action (second period communication in state two in public requires the Sender to give no information), giving the Sender an inferior payoff of zero.

The facts that firstly, Receiver **Q**'s pooling action is connected to a state where v_i is negative and secondly, that one of the v_i is positive, is key. This is the fact that results in the Sender preferring public communication in one state and private communication in another. ∎

Now, I shall describe a second additional example for the case of Mutual Discipline (case 5 (b)).

Example The signs of the parameters of the model are shown in Table 5. There is a separating equilibrium in public, a partially revealing equilibrium with Receiver **R** in private, but only a pooling exists with Receiver **Q** in private. Assume that Receiver **Q**'s pooling action is q_1 and Receiver **R**'s partially revealing action is r_2, the revealed state being state one.

Table 5: Mutual Discipline

Q	**R**	**Public**
p	pr	s
$v_1 < 0$	$w_1 > 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 > 0$
$v_3 > 0$	$w_3 < 0$	$v_3 + w_3 > 0$

The Sender's payoffs under private communication are:

$$\text{U}(s = s_1,\ \text{private}) = v_1 + w_1 \tag{A2.13}$$
$$\text{U}(s = s_2,\ \text{private}) = w_2 \tag{A2.14}$$
$$\text{U}(s = s_3,\ \text{private}) = 0, \tag{A2.15}$$

while under public communication they are:

$$\text{U}(s = s_1,\ \text{public}) = v_1 + w_1 \tag{A2.16}$$
$$\text{U}(s = s_2,\ \text{public}) = v_2 + w_2 \tag{A2.17}$$
$$\text{U}(s = s_3,\ \text{public}) = v_3 + w_3. \tag{A2.18}$$

Thus, the Sender is indifferent about the forum of communication in state one, prefers private communication in state two, and public communication in state three. Assume the Sender plays the differentiated strategy of (private, private, public). In this case, the best responses of the Receivers are the following: if they hear a request for public speech, they infer that the state of the world is state three, and accordingly both choose their third action, regardless of any communication in the second stage. In they hear a request for private speech, they infer that the state of the world is not state three, and update their beliefs accordingly. Their beliefs will become:

$$\mu_{e1} = \frac{\pi_1}{\pi_1 + \pi_2} \tag{A2.19}$$
$$\mu_{e2} = \frac{\pi_2}{\pi_1 + \pi_2} \tag{A2.20}$$
$$\mu_{e3} = 0. \tag{A2.21}$$

The Sender will separate in the second stage with Receiver **R** who will thus choose action i is the state of the world is s_i. The Sender will pool in the second stage with Receiver **Q** who will thus choose her pooling action according to her updated beliefs. As can be seen from these beliefs, this action is q_1. Consequently, the Sender's payoffs will be:

$$\mathrm{U}(s = s_1,\ \text{public}) = v_1 + w_1 \tag{A2.22}$$
$$\mathrm{U}(s = s_2,\ \text{private}) = w_2 \tag{A2.23}$$
$$\mathrm{U}(s = s_3,\ \text{private}) = v_3 + w_3. \tag{A2.24}$$

And so again, the Sender has the best of both worlds: in each state, he receives the payoff he prefers from among the payoffs in private and public communication. If the Receivers react in this way, the Sender has no incentive to deviate, since his payoff is positive in every state, and any deviation would result in a payoff of zero.

Again, the facts that firstly, Receiver **Q**'s pooling action is connected to a state where v_i is negative and secondly, that one of the v_i is positive, is crucial. This is the fact that results in the Sender preferring public communication in one state and private communication in another. ■

And finally, a case of PR—One-Sided Discipline.

Example The signs of the parameters of the model are shown in Table 6. There is a partially revealing equilibrium in public and with Receiver **R** in private, but only a pooling equilibrium exists with Receiver **Q** in private. Assume that Receiver **Q**'s pooling action is q_1 and Receiver **R**'s pooling action is r_1. The revealed state with Receiver **R** will be state one and her partially revealing action therefore r_2. The revealed state in public will be state three, the corresponding partially revealing actions thus (q_1, r_1).

Table 6: PR—One-Sided Discipline

Q	**R**	**Public**
p	pr	pr
$v_1 < 0$	$w_1 > 0$	$v_1 + w_1 > 0$
$v_2 < 0$	$w_2 > 0$	$v_2 + w_2 < 0$
$v_3 > 0$	$w_3 < 0$	$v_3 + w_3 > 0$

The Sender's payoffs under private communication are:

$$\mathrm{U}(s = s_1,\ \text{private}) = v_1 + w_1 \tag{A2.25}$$
$$\mathrm{U}(s = s_2,\ \text{private}) = w_2 \tag{A2.26}$$
$$\mathrm{U}(s = s_3,\ \text{private}) = 0, \tag{A2.27}$$

while under public communication they are:

$$U(s = s_1,\ \text{public}) = v_1 + w_1 \tag{A2.28}$$
$$U(s = s_2,\ \text{public}) = 0 \tag{A2.29}$$
$$U(s = s_3,\ \text{public}) = v_3 + w_3. \tag{A2.30}$$

Thus, the Sender is indifferent about the forum of communication in state one, prefers private communication in state two, and public communication in state three. Assume the Sender plays the differentiated strategy of (private, private, public). In this case, the best responses of the Receivers are the following: if they hear a request for public speech, they infer that the state of the world is state three, and accordingly both choose their third action, regardless of any communication in the second stage. In they hear a request for private speech, they infer that the state of the world is not state three, and update their beliefs accordingly. Their beliefs will become:

$$\mu_{e1} = \frac{\pi_1}{\pi_1 + \pi_2} \tag{A2.31}$$
$$\mu_{e2} = \frac{\pi_2}{\pi_1 + \pi_2} \tag{A2.32}$$
$$\mu_{e3} = 0. \tag{A2.33}$$

The Sender will separate in the second stage with Receiver **R** who will thus choose action i is the state of the world is s_i. The Sender will pool in the second stage with Receiver **Q** who will thus choose her pooling action according to her updated beliefs. As can be seen from these beliefs, this action is q_1. Consequently, the Sender's payoffs will be:

$$U(s = s_1,\ \text{public}) = v_1 + w_1 \tag{A2.34}$$
$$U(s = s_2,\ \text{private}) = w_2 \tag{A2.35}$$
$$U(s = s_3,\ \text{private}) = v_3 + w_3. \tag{A2.36}$$

And so yet again, the Sender has the best of both worlds: in each state, he receives the payoff he prefers from among the payoffs in private and public communication. If the Receivers react in this way, the Sender has no incentive to deviate, since his payoff is positive in every state, and any deviation would result in a payoff of zero. ∎

A.4 Appendix 4: Pooling- and Separating Equilibria in the Further Extensions

A.4.1 The k-state, Two-Audience Model

It is obvious that for any k number of states and any constellation of the parameters of the model v_i and w_i, there always exists a pooling equilibrium in which the Sender always sends the same message regardless of his type and is consequently ignored by the

Receivers, whose beliefs remain unchanged and who each choose their pooling actions, which are defined in an analogous way to those in Chapter 2:

$$q_{pool} = q_i \text{ if } \pi_i x_i \geq \pi_j x_j,\ \forall i,j = 1,2,\ldots,k;\ i \neq j \tag{A2.37}$$

and similarly,

$$r_{pool} = r_i \text{ if } \pi_i y_i \geq \pi_j y_j,\ \forall i,j = 1,2,\ldots,k;\ i \neq j. \tag{A2.38}$$

There also may exist a separating equilibrium. If all the v_i are nonnegative, then there exists a separating equilibrium with Receiver **Q** in private, and if all the w_i are nonnegative, then with Receiver **R** in private. If all the $v_i + w_i$ are nonnegative, then there exists a separating equilibrium in public, too.

A.4.2 The Two-State, Three-Audience Model

Equilibria

The following pure strategy perfect Bayesian equilibria are possible in the two-state, three-audience model:

The Pooling Equilibrium

Similarly to the two-audience models, there always exists a pooling equilibrium, in which the Sender sends the same message (which thus has the meaning "no information") in each of the possible states. The Receivers' beliefs remain unchanged following the cheap talk phase, and this their pooling actions are, similarly to the two-state, two-audience case, the following:

$$p_{pool} = \begin{cases} p_1 & \text{if } \pi z_1 \geq (1-\pi) z_2 \\ p_2 & \text{otherwise,} \end{cases} \tag{A2.39}$$

and similarly,

$$q_{pool} = \begin{cases} q_1 & \text{if } \pi x_1 \geq (1-\pi) x_2 \\ q_2 & \text{otherwise,} \end{cases} \tag{A2.40}$$

$$r_{pool} = \begin{cases} r_1 & \text{if } \pi y_1 \geq (1-\pi) y_2 \\ r_2 & \text{otherwise.} \end{cases} \tag{A2.41}$$

The Separating Equilibrium

While in the pooling equilibrium, the Sender's cheap talk message is effectively ignored by the Receivers, there may also exist an equilibrium when it is not. Specifically, there may exist an equilibrium in which the Sender truthfully reveals the state of the world to the Receivers, who believe him and take action accordingly. This is the separating equilibrium. The conditions for a separating equilibrium to exist are essentially unchanged relative to the two-audience model:

$$U(s_i, \text{“}s = s_i\text{”}) \geq U(s_i, \text{“}s = s_j\text{”}); i, j = 1, 2, i \neq j. \tag{A2.42}$$

The Sender must have no incentive to lie, which is guaranteed by Equations A2.42. Since the Sender's statements are credible, the Receivers believe them and their posterior beliefs become $\pi_i = 1$ is the meaning of message received is $s = s_i$, and $\pi_{-i} = 0$. The Receivers, now in possession of the information they are interested in, take action i if the meaning of the message heard is $s = s_i$, $i = 1, 2, 3$.

Using the parameters of the model, a separating equilibrium exists in private with Receiver **P** if the Sender has no incentive to lie in *any* of the states, which requires that $u_i \geq 0 \ \forall i \in \{1, 2, 3\}$. Similarly, in the case of private talk with Receivers **Q** and **R**, the conditions for a separating equilibrium to exist are $v_i \geq 0 \ \forall i \in \{1, 2, 3\}$ and $w_i \geq 0 \ \forall i \in \{1, 2, 3\}$ respectively. In the game where the Sender talks to two Receivers at once (in public), a separating equilibrium exists if the Sender has no incentive to lie in *any* state. For example if talk takes place to two Receivers, Receivers **P** and **Q**, then for this to be the case, $u_i + v_i \geq 0$ is needed $\forall i \in \{1, 2, 3\}$. A similar condition applies to other forms of public speech, including communication with the grand public, which requires $u_i + v_i + w_i \geq 0 \ \forall i \in \{1, 2, 3\}$. The above show that a version of Proposition 2.1.1 holds in this extended model as well:

Proposition A.4.1 *Incentives for complete honesty in two relationships in private imply incentives for complete honesty in public with those two Receivers; incentives for complete honesty in all three relationships in private imply incentives for complete honesty in communication with the grand public (all three Receivers), and in public with either two Receivers. In each case, the converse is not true: no form of public honesty necessarily implies incentives for honesty in private.*

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