Disadvantages of Derivative Pricing in Scope of the Global Financial Crisis

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Abstract

This thesis investigates possible causes of the Global Financial Crisis. It finds that excessive use of derivatives and "shaky" mathematics behind it played major role in burst of the crisis. It uses tests that check the Black-Scholes model, the most widely used option pricing model and finds that this model contradicts with reality in a number of ways. Special consideration is given to assumptions used in the Black-Scholes model. Nevertheless thesis considers other possible causes that could also lead to the crisis. Among them is the Sub-prime Mortgage Crisis. Thesis uses evidence provided by published papers and offers plausible solution, when possible.

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Introduction

The Global Financial Crisis of 2008 and the economic recession, which followed it, became to be the most powerful crisis seen since the Great Depression. It showed itself in sharp drop of main economic indices and rise of unemployment throughout the world. It is not surprising that an event of such a great scale and importance became to be major subject of recent researches in finance and economics. Most of them are concerned with the causes of this crisis and finding ways to mitigate its consequences.

Researches devoted to investigation of causes revealed a huge number of offered explanations. They vary significantly from each other. Some of these explanations go deep into basics of modern financial structure of the world, while others are concerned with bad performance and failure of some US companies or excessive losses incurred by USA's war in Iraq.

This thesis is aimed at finding the most realistic among the offered explanations. For this purpose it tries to summarize those of explanations that go deeper than others. Later it will be shown that one of such explanations is *excessive use of derivatives* and *disadvantages of option pricing models* that were used to price them. Among these models Black-Scholes model, the most widely used option pricing model, will be given priority.

Published in 1973 by Fischer Black and Myron Scholes in *Journal of Political Economy*, the Black-Scholes model later became to be the most widely used option pricing model until nowadays. It was the first model that provided strict mathematical basis for option pricing. Rubinstein (1994) states that the Black-Scholes option pricing model is the most widely used formula, with embedded probabilities, in human history.

Along with its advantages, it was noticed from the very beginning, that the model contradicts with reality in a number of ways. The main disadvantage of the model is the

assumptions made to derive the famous Black-Scholes formula. Among them most serious one is the assumption that stock prices are log-normally distributed. It was believed long before the Black-Scholes model was published that stock prices do not behave strictly like log-normal (Fama (1965)). They were exposed to higher risk of large changes in prices, the so called "fat tails" problem. As another example of contradicting assumption I can mention assumptions of efficient market, no transaction costs, no arbitrage possibility, limitless borrowing and lending assumptions.

The model also have very surprising outcome: option price does not depend on local mean rate of return of underlying asset. To understand it imagine that two traders are trying to price an option. They agree on all parameters of this option, except one of them: one trader strongly believes that stock price will fall tomorrow and another one is pretty sure that it will rise. Although they do not agree n this item they price the option equally, which a little bit surprising.

Instead of expectation about future direction of stock prices the variable of greater importance is volatility of stock prices. There model faces another problem in form of unavailability of this variable. Instead of this variable model uses its estimation which also gives rise to several difficulties.

Although problems dealing with the Black-Scholes model are given priority in this thesis, they are not the only considered problems as a cause of Global Financial Crisis. Thesis also deals with agency problem in mortgage market, failure of risk models which led to excessive risk being taken by mortgage companies.

The rest of the thesis is organized as follows. Chapter 1 gives brief introduction to derivative pricing and Black-Scholes model. Chapter 2 provides quick overview of Global Financial Crisis. These two chapters aimed at preparing readers, those that are not familiar with the topic, to the same level of understand the model and the crisis. Nevertheless it is not

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possible to provide all needed material in two short chapters. For more detailed information readers should look up in related literature. These chapters are used in Chapter 3, which discusses possible causes of the Global Financial Crisis. The same chapter tries to provide possible solutions, when it is possible, to erected problems. Thesis ends with conclusion that summarizes all the implications and suggestions provided in this research.

Chapter 1: Principles of Derivatives Pricing

1.1 Introduction

The main purpose of this chapter is to provide a brief theory of financial assets called *financial derivatives*, which will serve as a basis throughout this thesis. Readers that are not quite familiar with the topic thus will be prepared to understand main results of this research. For now it will be useful to mention one particular example of a European call option.

Suppose that two companies, Hungarian company HC and American company AC, agreed today over the following contract: company AC will deliver in three months ten cars to company HC. In return company HC will pay for each car 10,000 dollars (USD). The spot currency rate today between USD and HUF is 200 HUF/USD. It is obvious from the statement of the problem that HC is exposed now to a *currency risk:* what if currency rate changes in three months? If it stays 200 HUF/USD then company HC pays 20,000,000 HUF for ten cars, as was originally planned, but if it increases to 250 HUF/USD company HC pays 25,000,000 HUF and is worse off. What should company HC do in this case to secure itself against this risk? There are several possible strategies:

1. Buy today 100,000 USD at the price of 20,000,000 HUF and keep it until delivery date. In this case currency risk completely disappears, but it can happen that this money is not available today for the Hungarian company or even if it is available then after exchange this money will be useless for the following three months. So this strategy is not the best one.

2. Arrange a forward contract with some financial institution, e.g. bank, to buy 100,000 USD after three months at a price of 200 HUF/USD. Now in case currency rate increases to 250 HUF/USD HC will be better off by 5,000,000

HUF. But if it falls to 190 HUF/USD HC will be worse off by 1,000,000 HUF. The risk is eliminated, but only partly.

3.   There exists third, much more efficient way of solving this kind of problems, namely HC should buy a *European call option.*

<u>Definition 1</u>: A *European call option* on the amount of X USD, with strike price K HUF/USD and the *exercise date T* is a contract written at $t = 0$ with the following properties:

- The holder of the contract has, exactly at the time $t = T$, the right to buy X USD at the price K HUF/USD.

- The holder of the option has no obligation to buy the dollars.

In this case company HC secures itself against currency risk in the most efficient way. If currency rate increases then the Hungarian company uses the call option and is better off. But if it falls HC just forgoes from using the call option.

However, in contrast to a forward contract, which was costless to enter in, there exists price for European call options, which is not obvious immediately. So the main problem of Derivative Pricing is to determine the "fair" price for such kind of financial assets. Solution of this problem is not so obvious as it may seem. This chapter will be completely devoted to this problem. Results obtained and their consequences will mainly be discussed later.

I conclude this introduction providing main principles of Derivative Pricing which will lead us through derivations of this chapter. First of all, financial derivatives are defined in terms some underlying assets. As a consequence prices of derivatives should be related to prices of underlying assets. Second, prices of derivatives should be consistent with prices of underlying assets determined by the market. Finally, summarizing all said above, we can say *price of derivative is defined in terms of the market prices of the underlying assets.*

1.2 Portfolio Dynamics

Consider a financial market containing stocks, bonds (with different maturities) and various kinds of financial derivatives. It is our aim to derive price dynamics of *self-financing portfolios*, which will be used later on, taking as given price dynamics of all assets described above. We shall derive this formula in discreet time case.

Time is divided into intervals of equal sizes and trade takes place only at the points dividing these time intervals. We shall need the following designations:

N - The number of different types of stocks.

$h_i(t)$ - Number of shares of type i held during the period $[t, t + \Delta t]$.

$h(t)$ - The portfolio $[h_1(t, ..., h_N(t))]$ held during period t.

$c(t)$ - Money spent on consumption per unit time during the period $[t, t + \Delta t]$.

$S_i(t)$ - The price of one share of type i during the period $[t, t + \Delta t]$.

$V(t)$ - The value of the portfolio h at time t.

We consider no dividend paying portfolios. Note also that, as the name says (self-financing), there is no exogenous influence on portfolio - everything is financed by portfolio itself. It is obvious that at the start of period t value of portfolio is equal to the value from the previous period. So using inner product designation:

$$V(t) = \sum_{i-1}^{N} h_i(t - \Delta t) S_i(t) = h(t - \Delta t) S(t) \qquad (1)$$

There we have two possible ways: reinvest in a new portfolio $h(t)$ or consume at the rate $c(t)$ over the period t. After we observe prices $S(t)$ we decide which portfolio $h(t)$ to hold and how much to consume $c(t)$, then our budget constraint is:

$$h(t - \Delta t) S(t) = h(t) S(t) + c(t) \Delta t \qquad (2)$$

Designating $\Delta X(t) = X(t) - X(t - \Delta t)$ we can write (2) as:

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$S(t)\Delta h(t) + c(t)\Delta t = 0$ (3)

To proceed we add and subtract $S(t-\Delta t)\Delta h(t)$ to (3), which leads to:

$S(t)\Delta h(t) + S(t-\Delta t)\Delta h(t) - S(t-\Delta t)\Delta h(t) + c(t)\Delta t = 0$ (4)

Rearranging and applying $\Delta t \to 0$: $S(t)dh(t) + dh(t)dS(t) + c(t) = 0$ (5)

Now let $\Delta t \to 0$ in (1) then $V(t) = h(t)S(t)$ (6)

Taking Ito differential: $dV(t) = h(t)dS(t) + S(t)dh(t) + dS(t)dh(t)$ (7)

And substituting (5) into (7) we get:

$dV(t) = h(t)dS(t) - c(t)dt$ (8)

This is the main result of this section. It can be interpreted as follows: in absence of any exogenous influence on portfolio, its value is affected only by change in stock price and consumption size. Equation (8) gives intuitive understanding of self-financing portfolio dynamics. Note also that (8) can be written as:

$$dV(t) = \sum_{i=1}^{N} h_i(t)dS_i(t) - c(t)dt \quad (9)$$

Using notion of *relative portfolio*: $u_i(t) = \dfrac{h_i S_i(t)}{V(t)}, i = 1,...,N$ (10), where $\sum_{i=1}^{N} u_i(t) = 1$ we

can make the following statement: A portfolio-consumption pair (h,c) will be self-financing

if and only if $dV(t) = V(t)\sum_{i=1}^{N} u_i(t)\dfrac{dS_i(t)}{S_i(t)} - c(t)dt$ (11)

To prove it we just need to use (10) in (11).

1.3 Arbitrage Pricing

Let us again consider financial market with two types of assets: risk free asset with price process B and stock with price process S. For the first of them we shall put:

$dB(t) = r(t)B(t)dt$ (1)

Which can be interpreted as a bank with short rate of interest r (in case r is constant B can be interpreted as a bond). For the second type of assets we put:

$$dS(t) = S(t)\alpha(t, S(t))dt + S(t)\sigma(t, S(t))d\overline{W}(t) \quad (2)$$

Here $\overline{W}$ - is a Wiener process, α (local mean rate of return of S) and σ (volatility of S) are given deterministic functions. In case r, α, σ are deterministic constants we get the so called *Black-Scholes model*:

$$dB(t) = rB(t)dt \quad (3)$$

$$dS(t) = \alpha S(t)dt + \sigma S(t)d\overline{W}(t) \quad (4)$$

In the introduction to this chapter I gave an intuitive definition of European call option. In this subsection this definition, together with definition of contingent claim, will be of such importance that I provide them once more in a more precise way.

Definition 1: A *European call option* with *exercise price (strike price) K* and *time to maturity (exercise date) T* on the underlying asset S is a contract defined by the following clauses:

- The holder of the option has, at time T, the right to buy one share of the underlying stock at the price K from the underwriter of the option.

- The holder of the option is in no way obliged to buy the underlying stock.

- The right to buy the underlying stock at the price K can only be exercised at the precise time T

We see that European call option (together with other similar contracts) is defined by means of underlying asset S, that is why it is usually referred to as *contingent claim*.

Definition 2: Consider a financial market with vector price process S. A *contingent claim* with date of maturity T is a stochastic variable $X \in F_T^S$ (i.e. the value of X can be completely determined given observations of the trajectory $\{S(t); 0 \leq t \leq T\}$). A contingent claim X is

called a *simple claim* if it is of the form $X = \Phi(S(T))$ and the function Φ is called the *contract function.*

As was already mentioned in the introduction to this chapter, the main problem of derivative pricing consists of determining price for such contingent claims. I provide now intuition of how it can be done.

First of all note that a European call option is a simple claim. Price of contingent claim X at time t will be denoted as $\Pi(t, X)$. Obviously at time T the only reasonable price for European call option could be $S(T) - K$ (if we exercise the option) or 0 if we do not, so:

$$\Pi(T) = \max[S(T) - K, 0] \quad (5)$$

For more general contingent claim X we have: $\Pi(T, X) = X \quad (6)$

and particularly for a simple claim: $\Pi(T, X) = \Phi(S(T)) \quad (7)$

Now, what could be the reasonable price for contingent claims at periods of time different from T? At first it seems that this price can only be determined by a market. But accepting several assumptions, it is possible to derive formula describing price of contingent claims (*Black-Scholes formula*). The first and most important assumption is absence of *arbitrage possibility.*

Definition 3: An arbitrage possibility on a financial market is a self-financed portfolio h such that $V^h(0) = 0 \quad (8)$ and $V^h(T) > 0$ a.s. $\quad (9)$

If there is no arbitrage possibility, then market is arbitrage free.

To avoid the arbitrage possibility we use the following result:

Proposition 1: Suppose that there exists a self-financed portfolio h such that the value V^h has the dynamics: $dV^h(t) = k(t)V^h(T)dt \quad (10)$

Then it must hold that $k(t) = r(t)$ for all t to eliminate arbitrage possibility.

The rest assumptions are:

- The derivative investment in question can be bought and sold on a market.

- The price process for the derivative asset is of the form $\Pi(t; X) = F(t, S(t))$, (11)

  where F is some smooth function.

Having made all these assumptions let us derive the Black-Scholes equation. We start from:

$dB(t) = rB(t)dt$ (12)

$dS(t) = S(t)\alpha(t, S(t))dt + S(t)\sigma(t, S(t))d\overline{W}(t)$ (13)

$X = \Phi(S(T))$ (14)

$\Pi(t) = F(t, S(t))$ (15)

We shall find such smooth function F that the market will be free of arbitrage. Applying Ito

formula to (15) and (13) we shall get: $d\Pi(t) = \alpha_\pi(t)\Pi(t)dt + \sigma_\pi(t)\Pi(t)d\overline{W}(t)$ (16)

Where $\alpha_\pi(t) = \dfrac{F_t + \alpha S F_s + \dfrac{1}{2}\sigma^2 S^2 F_{ss}}{F}$ (17) and $\sigma_\pi(t) = \dfrac{\sigma S F_s}{F}$ (18)

Now we form a portfolio consisting of two assets: underlying stock and the derivative asset.

Using notion of relative portfolio and equation (11) from section 1.2 we get the dynamics of

this portfolio (u_s, u_π): $dV = V\{u_s[\alpha dt + \sigma d\overline{W}] + u_\pi[\alpha_\pi dt + \sigma_\pi d\overline{W}]\}$ (19). Rearranging

$dV = V[u_s\alpha + u_\pi a_\pi]dt + V[u_s\sigma + u_\pi\sigma_\pi]d\overline{W}$ (20)

Now having condition $u_s + u_\pi = 1$ (21)

we also choose these variables to satisfy $u_s\sigma + u_\pi\sigma_\pi = 0$ (22).

Then (22) reduces (20) into $dV = V[u_s\alpha + u_\pi\alpha_\pi]dt$ (23)

Now recalling Proposition 1 we must have $u_s\alpha + u_\pi\alpha_\pi = r$ (24)

to avoid arbitrage pricing. Solving (21) and (22) we get for u_s and u_π:

$u_s = \dfrac{\sigma_\pi}{\sigma_\pi - \sigma}$ (25)

$$u_\pi = \frac{-\sigma}{\sigma_\pi - \sigma} \quad (26)$$

Using the expression for σ_π in (25), (26) and substituting them together with (17) into (24) we obtain:

$$F_t(t,s) + rS(t)F_s(t,S(t)) + \frac{1}{2}\sigma^2(t,S(t))S^2(t)F_{ss}(t,S(t)) - rF(t,S(t)) = 0 \quad (27)$$

and $\Pi(T) = \Phi(S(T)) \quad (28)$

must hold. Here few comments can be made:

- Price of a contingent claim was obtained as a function of the underlying asset's price.

- It was assumed that price of a contingent claim is a function F of t and $S(t)$, after this I showed that in arbitrage free market F must satisfy Black-Scholes equation. But what if we cannot make this assumption?

- It was also assumed that there exists a market for derivative assets. If this market does not exist, then we cannot construct a portfolio and previous derivations are not valid. For European call options it is acceptable assumption, but for OTC instruments ("over the counter") it is not applicable.

- By (27) price of a derivative does not depend on local mean rate of return $\alpha(t,s)$ of underlying asset. It depends only on volatility $\sigma(t,s)$. This is the most surprising result of Derivative Pricing Theory, which will be discussed in more detail in consecutive chapters.

We now turn to the solution of (27) and (28). These equations can be solved by Feynman-Kac stochastic representation formula and solution will be:

$$F(t,s) = e^{-r(T-t)}E_{t,s}[\Phi(X(T))] \quad (29)$$

The process X is defined by:

$$dX(u) = rX(u)du + X(u)\sigma(u,X(u))dW(u) \quad (30)$$

$X(t) = s$ (31)

W is a Wiener process. The only difference between (30) and (13) is that r - short rate of interest used instead of α - local mean or return of underlying asset. So if we change probability measure from original P for (13) to probability measure Q defined for:

$dS(t) = rS(t)dt + S(t)\sigma(t, S(t))dW(t)$ (31)

W -Wiener process in Q -dynamics.

Now we can use (31) instead of (13) in this new measure.

<u>Theorem 1:</u> The arbitrage free price of the claim $\Phi(S(T))$ is given by $\Pi(t;\Phi) = F(t, S(t))$, where F is defined from the formula:

$$F(t, s) = e^{-r(T-t)} E^Q_{t,s}[\Phi(S(T))]$$ (32)

And (31) is the Q -dynamics of S.

Formula (32) usually referred to as *risk neutral valuation* formula and probability measure Q is called *martingale measure* or *risk adjusted measure*. Note also that we changed $\overline{W}$ with W.

Let us now apply (32) to Black-Scholes model:

$dB(t) = rB(t)dt$ (33)

$dS(t) = \alpha S(t)dt + \sigma S(t)d\overline{W}(t)$ (34)

α, σ are constants. Then we get: $F(t, s) = e^{-r(T-t)} E^Q_{t,s}[\Phi(S(T))]$ (35)

And Q -dynamics of S are given by:

$dS(u) = rS(u)du + \sigma S(u)dW(u)$ (36)

$S(t) = s$ (37)

This is equation of Geometric Brownian Motion, which in this case will have the following solution: $S(T) = s \exp\{(r - \frac{1}{2}\sigma^2)(T-t) + \sigma(W(t) - W(t))\}$ (38).

Shortly it can be written as: $S(T) = se^Y$, Y is a stochastic variable with distribution $N[(r - \frac{1}{2}\sigma^2)(T - t), \sigma\sqrt{T - t}]$, from which we obtain the following pricing formula

$$F(t, s) = e^{-r(T-t)} \int_{-\infty}^{\infty} \Phi(se^Y) f(y) dy \quad (39)$$

Where f is the density function of Y. To proceed we need the exact form of function Φ. For example, in case of European call option $\Phi(x) = \max[x - K, 0]$. Let us do calculations for this case. First note that $S(T)$ can be represented in a more convenient form:

$S(T) = se^{\tilde{r}\tau + \sigma\sqrt{\tau}Z}$, where $\tilde{r} = (r - \frac{1}{2}\sigma^2)$, $\tau = T - t$ and Z is a standardized normal variable.

Using these designations (39) becomes now: $\int_{-\infty}^{\infty} \max[se^{\tilde{r}\tau + \sigma\sqrt{\tau}z} - K, 0]\varphi(z) dz$, where φ is the density of the $N(0,1)$: $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$. Note also that function under integral vanishes when $z < z_0$, where $z_0 = \dfrac{\ln(\frac{K}{s}) - \tilde{r}\tau}{\sigma\sqrt{\tau}}$. So it can be written as:

$\int_{z_0}^{\infty} (se^{\tilde{r}\tau + \sigma\sqrt{\tau}z} - K)\varphi(z) dz = \int_{z_0}^{\infty} se^{\tilde{r}\tau + \sigma\sqrt{\tau}z}\varphi(z) dz - \int_{z_0}^{\infty} K\varphi(z) dz = A - B$. Integral B can be written as $K \cdot \Pr ob(Z \geq z_0)$, but distribution $N(0,1)$ is symmetric, thus $K \cdot \Pr ob(Z \leq -z_0)$. Using notation: $N[x] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$ integral B can be written also as $B = K \cdot N[-z_0]$. For integral A we make the following:

$A = \dfrac{se^{\tilde{r}\tau}}{\sqrt{2\pi}} \int_{z_0}^{\infty} e^{\sigma\sqrt{\tau}z - \frac{1}{2}z^2} dz = \dfrac{se^{\tilde{r}\tau}}{\sqrt{2\pi}} \int_{z_0}^{\infty} e^{-\frac{1}{2}(z - \sigma\sqrt{\tau})^2 + \frac{1}{2}\sigma^2\tau} dz = \dfrac{se^{\tilde{r}\tau}}{\sqrt{2\pi}} \int_{z_0}^{\infty} e^{-\frac{1}{2}(z - \sigma\sqrt{\tau})^2} dz$.

As it is the density of a $N[\sigma\sqrt{\tau}, 1]$ distribution, we have $A = se^{r\tau} \cdot \Pr ob(Z' \geq z_0)$, where Z' is $N[\sigma\sqrt{\tau}, 1]$. If we normalize Z' to a $N[0,1]$ variable, then A can be written as:

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$A = se^{r\tau} \cdot N[-z_0 + \sigma\sqrt{\tau}]$. So, finally having defined integrals A and B, we thus completely defined integral in equation (39). This gives us famous Black-Scholes formula.

<u>Proposition 2</u>: The price of a European call option with strike price K and time to maturity T is given by the formula $\Pi(t) = F(t, S(t))$, where

$$F(t,s) = s \cdot N[d_1(t,s)] - e^{-r(T-t)} \cdot K \cdot N[d_2(t,s)] \qquad (40)$$

Here N - cumulative distribution function for the $N[0,1]$ distribution and

$$d_1(t,s) = \frac{1}{\sigma\sqrt{T-t}}\{\ln(\frac{s}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)\} \qquad (41)$$

$$d_2(t,s) = d_1(t,s) - \sigma\sqrt{T-t}.$$

To apply this formula in a particular case we should have numerical values of the following five inputs: s, r, T, t, σ. First four of them do not cause any problems, but σ - volatility of S, should be somehow estimated, as it is not explicitly observable. There exist two main ways of estimating σ: use "historic volatility" or "implied volatility".

The first one, as the name says, uses "historical" data on stock prices for the period of time equal to that for which we are estimating σ. Then using the "fact" that S is log-normally distributed and standard statistical methods we calculate estimate of σ.

Although we can use the method described earlier to find σ, one can argue that this method actually finds volatility for the *past* period of the observed stock prices and is not consistent with prices of assets which have already been priced by the market. The second method is aimed to avoid these problems, using *market expectation* of the volatility for the next period. To do it we consider price of another similar option, in our case European call option, with the same time to maturity and the same underlying asset, usually called "benchmark" option. Using standard designation, denoting price of this option by p and the same by Black-Scholes formula as $c(s, t, T, r, \sigma, K)$ we find σ from:

$$p = c(s, t, T, r, \sigma, K).$$

This value of σ is called *implied volatility.*

At the end it worth mentioning that the method described above can be applied to test the Black-Scholes formula. Suppose we have prices of several European calls, with the same exercise date on the same underlying asset. By this formula we should obtain the same σ for different exercise prices and graph of σ as a function of exercise price must be straight line. However, it will be shown in Chapter 3, that empirical data does not prove it. Options that are "out of the money" $(S < K)$ and "in the money" $(S > K)$ have higher implied volatility than options "at the money" $(S = K)$. Graph of volatility σ does not look like straight line, but a curve. For this reason this graph is called the *volatility smile.*

Having introduced readers to the principles of derivative pricing I have thus provided a basis, relying on which chapter 3 will investigate disadvantages of derivative pricing in scope of the Global Financial Crisis. But before doing this it will be useful to mention some facts about the crisis itself. The next chapter is completely devoted to this purpose.

Chapter 2: Global Financial Crisis of 2008

In this chapter I give brief introduction to the Global Financial Crisis of 2008: how it started, grew and which consequences it caused. The purpose of this chapter is to introduce readers to some facts about the crisis and events that could cause it, because some of them will be discussed in the next chapter. Discussion of reasons is left for the next chapter.

The Global Financial Crisis of 2008 ant Late 2000th Recession is a financial and economic crisis which showed itself in 2008 with deterioration of the main economic indicators in the most countries and was followed by the Global Recession at the end of the same year. Its predecessor the Mortgage Crisis in USA revealed itself in 2006 with drop of sales in house market and developed into Sub-prime Mortgage Crisis in 2007. But soon even credible debtors faced problems with crediting. The Mortgage Crisis started to grow into a financial crisis and affected other countries. By the beginning of 2008 it started to spread all over the world and caused sharp decrease in production, demand and prices of raw materials and led to increase of a unemployment rate.

It is better to start a discussion of how the crisis started with the events and processes that happened shortly before it. They will be considered in turn in consecutive sections. One of them is high prices. In the first decade of the twenty-first century there was a worldwide consumption boom which led to growth in prices. In 2008 prices, especially oil and food prices, achieved such a high level that it could lead to stagflation and threatened globalization. In January 2008 oil prices reached 100 USD per barrel and continued to grow. In July 11, 2008 it reached highest level ever seen – 147.27 USD per barrel, but started to decrease, reaching 61 USD/barrel in October 24 and 51 USD/barrel in November of the same year. Price of sulfuric acid (very important factor in steel industry) increased six times during a year! High oil prices caused sharp decrease of sales in car market: in August 2008 sales in Europe decreased by 16%, in September 2008 sales in the USA decreased by 26% and 5.3%

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in Japan. This in turn led to drop in steel and car production, worsening situation in these and related industries.

An immediate predecessor of the Global Financial Crisis, as was already mentioned, is the Sub-prime Mortgage Crisis in the USA of 2007. It happened as a consequence of a large number of delinquencies by sub-prime borrowers. Although it started in 2006, it showed its scale fully only in 2007. The first victims of this crisis were New Century Financial Corporation, USA second largest sub-prime mortgage lender and American Home Mortgage Investment Corporation, the tenth largest retail mortgage lender in the United States. These companies stopped crediting, reduced their staff and their shares dropped. As a consequence of a 20% drop in real estate prices American house owners lost nearly five trillion dollars! Nevertheless it hardly can be accepted as a real reason for the Crisis. Famous American financier George Soros in "Die Welt" of October 14, 2008 defined United States Housing Bubble as a "trigger" for a much more dangerous and bigger bubble.

An essential factor which influenced beginning of the Crisis was widespread use of financial derivatives starting from the beginning of 1990th, which were used to increase profits at the expense of increased risk. But there is no consensus between experts about derivatives in scope of this problem. Some argue that they are "guilty" for this crisis and some say that the crisis was inevitable even in the absence of the "derivatives factor". This problem will be discussed in more detail in the next chapter.

The Sub-prime Mortgage Crisis of 2007 provoked the Liquidity Crisis in September of 2008. As a consequence banks stopped crediting, especially crediting of car purchases, which led to huge losses in the car industry. Three car industry giants Opel, Daimler and Ford announced the reduction of their production in Germany. Thus the crisis moved from the real estate sector into the economy, the official recession and decrease of production started. The LIBOR-OIS spread (the difference between three month London Inter Bank Offered Rate and

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Overnight Index Swap – index of availability of credit in the market) reached by the end of September 2008 for dollar credits 200 base point and 250 in the beginning of October 2008 (making credits less available). Lehman Brothers' bankruptcy in September 15, 2008 led to loss of confidence in insurance and CDS (Credit Default Swap) market. Moreover existence of CDS as a financial instrument was under question.

All these events preceding or accompanying the Crisis forced government officials and international organizations to undertake some actions to prevent the Crisis from worsening. On September 18, 2008 the United States Treasury Secretary, Henry Paulson, announced that he is preparing, with aid of Federal Reserve System and several Congress representatives, a plan ("Paulson financial rescue plan") to solve problems concerning systemic risk (the risk of collapse of an entire financial system or entire market) and toxic assets (financial assets which value significantly dropped and there is no functioning market for them, so that they cannot be sold). The Plan was ready by the end of September 2008. The main points of this plan were to use 700 billion USD to buy toxic assets and make capital injection into banks. But on September 29, 2008 the United States House of Representatives rejected this Plan and only on October 1, 2008 Senate accepted it (Emergency Economic Stabilization Act of 2008). Within hours it was signed by President Bush and the Troubled Assets Relief Program, with 700 billion USD, was thus created to purchase failing bank assets. Later, on November 25 of 2008, FRS offered to inject additional 800 billion USD into banking system, to make more lending available to consumers. 600 billion USD of this amount were planned to buy mortgage-backed securities (Fannie Mae, Freddie Mac, Ginnie Mae) and 200 billion USD to unfreeze consumer credit market.

But all these arrangements did not help much. A year before all these happened, in October most of stock exchange indices all over the world reached their maximum and started do decline. On Friday, October 10 of 2008, stock markets crashed across Europe and

Asia. London, Paris and Frankfurt dropped 10% within an hour of trading and again when Wall Street opened for trading. Global markets experienced their worst weeks since 1987 and some indices, S&P 500, since the Wall Street Crash of 1929.

The five largest investment banks in USA stopped functioning as investment banks. Bear Sterns was acquired by JP Morgan Chase in March 2008. In September 2008 Lehman Brothers went into bankruptcy, Merrill Lynch was acquired by Bank of America, Goldman Sachs and Morgan Stanley transformed into bank holding companies.

It can be seen that Global Financial Crisis led to dramatic losses in world economy. Cost of the Crisis was evaluated to be about five trillion USD (until now). The United States and most of the OECD (Organization for Economic Co-operation and Development) countries entered into official recession since World War 2. Unemployment reached highest level during the last twenty years. International Trade reduced. Global economy was predicted to start growing only from 2010.

In scope of such dramatic consequences it is not surprising that the Crisis became the major subject of research in economics and finance. The greatest minds of nowadays started to talk about the reasons of this crisis and to make forecasts for its perspective. This problem will be considered in more detail in the next chapter.

This chapter introduced readers to some facts about the current crisis. It was not aimed to describe everything about the event of such a scale, because it will just take hundreds of pages to do that. But instead readers that are not informed about all these events will not get lost in the next chapter, because some cases briefly mentioned here will be discussed in more detail in chapter 3 (especially the case of Fannie Mae and Freddie Mac).

Chapter 3: Possible causes of the Global Financial Crisis

Previous chapter makes us ask: "After all what caused the current financial crisis?" As an answer to this question a whole spectrum of explanations were offered, starting from very naïve and simple explanations, concerning with underperformance of some US companies and war in Iraq, and ending with fundamental explanations concerning with the basics of modern financial structure of the world. In this chapter I try to reveal causes that could lead to this Crisis. For this purpose some evidence from several papers will be provided. First I start with an immediate predecessor of the Global Financial Crisis – the Sub-prime Mortgage crisis in USA. In the section below we consider possible causes of the latter.

3.1 Agency problem and securitization as a cause of the Sub-prime Mortgage Crisis

Along with the problems caused by implications the Black-Scholes model, agency problem, the mispricing of risk, and the failure of securitization to distribute risks across the financial system, that caused housing bubble, are among the main reasons of the current Global Financial Crisis. There is a long chain between the home buyers, the mortgage broker and controversial securities in the housing market. Let's take one unit in the chain: mortgage brokers. Suppose they are paid by the number of mortgages that they serve each month (as they were in real life). The more mortgages they handle the higher their income. Although they are required to meet certain guidelines as they do this, so long as their income depends upon the number of mortgages passing through their hands and not what happens to the mortgages later on they have incentives to push the guidelines as hard as they can to increase the number of mortgages, even at the cost of safety of business.

But even in this scope of the problem, if mortgage brokers had done their job properly, i.e. made loans to people who could return money at required dates, probably we would not see this Crisis. So we should clarify why they were so eager to make loans to people with poor credit history. Probably the problem is in agency issue. The brokers had no interest in the outcome after the mortgages passed their "stage". The same with banks, all they had to do was to process the mortgages, sell them and collect their fee.

Let's try to simulate behavior of an ordinary mortgage broker. Assume that at some level we clearly see approaching end of this wonderful business. What will be our reasonable behavior? Reasonably we should stop providing questionable loans, start to warn everybody to do the same and perform additional steps to protect ourselves against disaster. But this will not happen, as the real life showed us recently. Instead we shall foresee the effect of "panicked herd" and will make even more loans, to earn as many as possible before the end. We are just one of the many, what shall we do? Bubble is going to burst anyway! Everybody starts to think like that and the mania begins.

Perhaps solution to this will be to give each person in the chain a stake depending on the future outcome of the mortgage. If mortgage brokers' income had been connected to a financial instrument that pays off according to the future performance of the mortgages they write, probably they would have behaved differently?

Natural question arises along with the explanation given above. We considered only one unit of the chain, at the lower or middle part of it. But why people at higher part of this chain did not do anything against these risky mortgage loans, after all they had stakes and not small stakes in this business. The best answer to this question is that mispricing and mal-distribution of risk played a key role here, along with poor management decisions in cases where alarms were raised. The agency issues above and the consequences of the failure to predict and distribute risk are very important at this stage.

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To proceed further we need to involve notion of the *shadow banking system*. This consists of non-bank financial institutions that were playing an increasingly critical role in lending businesses the money necessary to operate. They usually play a role of intermediary between investors and borrowers and they do not take deposits, so they are not under the same regulations as banks. As an example famous Lehman Brothers can be mentioned.

Institutions in the shadow banking system were also eager to take a lot of risky loans. Their leaders did not understand fully possible consequences of risk when they engaged in sub-prime business or when they took on securities derived from it. If the risk assessment models they relied upon had been correct and securitization had distributed risk evenly, the bubble burst should not be such a big problem.

The famous American Economist, Brad DeLong (Deputy Assistant Secretary of the United States Department of the Treasury in the Clinton Administration), argued that by the time of the crisis there were: 11 trillions USD in US mortgages, 60 trillions USD of global financial assets and loss of 2 trillions USD on mortgage-backed securities should not pose a big problem for Wall Street. So if the risks were distributed evenly there would not be such a crisis. Thus misprediction was only half of the problem; otherwise losses could have been absorbed. Failure of securitization which led to concentration of risk was another half of the problem.

Summarizing all said above it was the agency problem, the failure of risk prediction and distribution models that increased bubble and caused larger problems when it popped.

But among the reasons that caused sub-prime crisis one of the most popular is bad performance and unwise decisions made by leaders of Fannie Mae (Federal National Mortgage Association) and Freddie Mac (Federal Home Loan Mortgage Corporation). Let's look at their prehistory. By September 2008 two corporations Fannie Mae and Freddie Mac owned or guaranteed about half of the $11-12 trillion mortgage market of USA. In 1999,

Fannie Mae came under pressure from the Clinton Administration to expand mortgage loans to low and moderate income borrowers. The same year The New York Times reported that with the corporation's move towards the sub-prime market "Fannie Mae is taking on significantly more risk, which may not pose any difficulties during flush economic times. But the government-subsidized corporation may run into trouble in an economic downturn, prompting a government rescue similar to that of the savings and loan industry in the 1980s." Nassim Taleb wrote in his *"The Black Swan"* (written in 2003-2006): "The government-sponsored institution Fannie Mae, when I look at its risks, seems to be sitting on a barrel of dynamite, vulnerable to the slightest hiccup. But not to worry: their large staff of scientists deems these events 'unlikely'". Indeed when, in 2007, sub-prime crisis started it led to large and growing losses of these companies. Although in July 2008 the government attempted to decrease market fears by reiterating that: "Fannie Mae and Freddie Mac play a central role in the US housing finance system", by August 2008 shares of both Fannie Mae and Freddie Mac had dropped more than 90% from their one-year prior levels. On September 7, 2008, James Lockhart, director of the Federal Housing Finance Agency (FHFA), announced that Fannie Mae and Freddie Mac were being placed into conservatorship (i.e. to be under legal control of an external entity or organization) of the FHFA, the action which was called "one of the most sweeping government interventions in private financial markets in decades".

But did really these two companies contribute to development of the crisis so much, as it is believed? Certainly management of Fannie and Freddie made unwise decision in following shadow banking system, when the latter continued to take on risky assets and still pay investors a relatively high return. There were a lot of faults in their behavior and also in the government oversight of their decisions. The crisis was showing itself in falling value of two corporations and they continued to take on more risky securities. But most probably the agency problem and the failures of risk models and securitization would have created

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problems in the largely unregulated shadow banking sector even if these two institutions behaved properly and had taken the safest of mortgages. The bubble still would have been developing in the shadow banking system, maybe to a smaller one, but it still would have been large enough to cause big problems when it burst. Even the best behavior of Fannie and Freddie could not stop the bubble from expansion in other parts of the financial sector and then transforming into global financial crisis when the housing prices dropped.

3.2 Derivatives

As another offered explanation of the current crisis was excessive use of derivatives during the last decades. By the end of 1996 there were 16 trillion USD of derivatives of top ten US bank. By June of 2008 the Bank of International Settlements estimated that the notional value of all outstanding derivatives is $1,114 trillion. It is a huge number compared to US mortgage market just before the crisis hit, not to mention losses of this market. Despite of its popularity, the mechanisms and "shaky" mathematics behind the derivatives are not fully understood even by the specialists of this sector! Calculation of value for even the simplest derivatives involves very popular, but complex and doubtful Black-Scoles formula. For more complex derivatives much more difficult formulas were introduced by financial engineers. George Soros, widely accepted to be one of the most talented persons of this sector, declared that he hardly used derivatives, because he had difficulties in understanding them. In 1994, Soros argued in the U.S. Congress that: "There are so many [derivatives], and some of them are so esoteric that the risks involved may not be properly understood even by the most sophisticated of investors, and I'm supposed to be one of them. Some of these instruments appear to be specifically designed to enable institutional investors to take gambles which they would otherwise not be permitted to take". In his opinion derivatives were used as a trick to overreach margin requirements on bank's investments.

But despite of all said above designers of derivatives were very smart people, who were highly paid for their contributions and here more careful readers can see resemblance with the agency problem of the previous section. As the mortgage brokers did not care a lot about mortgages once they have passed their hands, maybe those designers of complex and fancy mathematical formulas were more concerned with their profits than with the hidden risk contained in the models created by them.

So what could be the best response for such a situation? Derivatives set a unique problem. If even George Soros can't understand derivatives, so it will be more difficult for government officials to understand and make proper decisions about them. Best proposal maybe is that derivatives should be subject to the same margin requirements as conventional stock purchases. Possible restrictions on financial engineers' ability to design highly leveraged instruments would probably reduce the risk of financial crisis.

3.3 Test of the Black-Scholes Model

In this section the famous Black-Scholes model will be tested on how it works in real life. For this purpose the major results of Fortune (1996) will be used and discussed.

Description and derivations of the Black-Scholes model were generally given in the Chapter 1. Note that there are generally two ways of testing such models. First we can check how the outcomes of the model fit the observed real data. Second we can check the assumptions, on which the model is based, i.e. how real or at least close to reality they are. We shall proceed in both directions.

Before we start investigation it will be useful to briefly mention the key features of the Black-Scholes model, as all this section will be based on them. First of them is that options can be replicated, i.e. for any option there can be constructed a portfolio, consisting of stocks and bond, with the same value. Second is the absence of arbitrage possibilities. As

the model was first constructed for European options, it is directly applicable for them, even in case of dividend paying options. However the model is not applicable to American options, unless there are no dividend payments. Finally we mention other assumptions made by the model:

- Time considered to be continuous

- Agents can lend or borrow at risk free interest rate as much as they wish

- Traders share the same probability and beliefs about the relevant parameters

- Price of stock prices are log-normally distributed

- Prices of underlying assets are continuous

- No or negligible small transaction costs

Test 1: Volatility test. The first test is devoted to volatility. As was mentioned in Chapter 1, the Black-Scholes formula requires a set of inputs to return option price. Most of them are directly observable, except volatility of stock prices σ. To apply the Black-Scholes formula we should use an estimate of σ. For this purpose an estimate called "implied volatility" is used. Shortly it uses data on the market so that to return an estimate coinciding with market predictions for options. As a consequence it is assumed by traders that actual and implied volatility diverge only in few, random and small intervals, i.e. generally prediction is true. But using data of S&P 500 for 1992-1994 it was shown by Fortune (1996) that this not true. While having roughly the same average, actual and implied volatility have significant divergence.
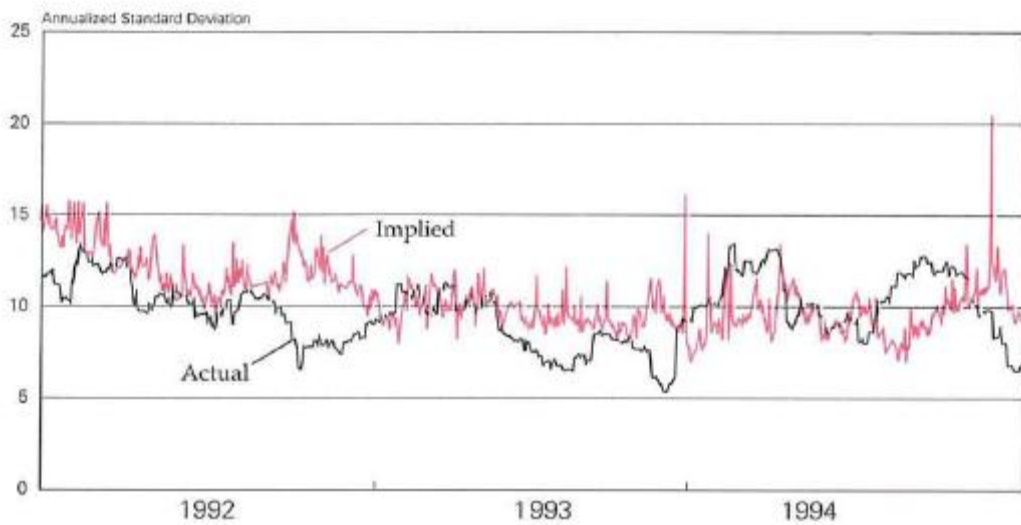
Volatility of 60-Day S&P 500 Returns

Figure 1 (Source: Fortune (1996))

Similar results were achieved by other researchers using different methods and data. For example, Canina and Figlewski (1993), using data of S&P 100 for 1983-1987 found that implied volatility is a poor forecast for actual volatility.

Test 2: Volatility smile test. One of the consequences of the Black-Scholes model is that options differing only in strike prices will have the same implied volatility. So if we draw graph of implied volatility as a function of strike price we shall get straight line. But it was mentioned in Chapter 1 that empirical evidence does not support this. It is believed that options far out of the money $(S < K)$ and far in the money $(S > K)$ have higher implied volatilities than options at the money $(S = K)$, i.e. graph will look like a smile. Indeed by Fortune (1996) there is strong evidence of smile in volatilities, although of a strange form, as shown in figure below.
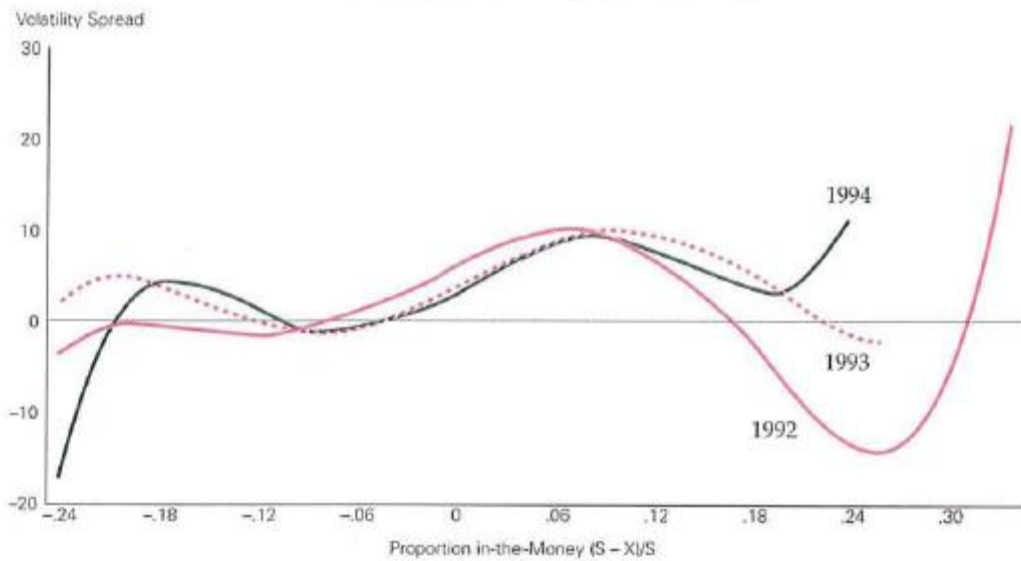
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Figure 2 (Source: Fortune (1996))

This test was done for call options.

Test 3: Put-Call parity test. Test checks another prediction of the Black-Scholes model: put and call options having the same characteristics should also have the same price. Generally put and call options, having the same strike price and term to maturity, will have the same price if they are at the money in present value sense. Recall the put-call parity $P_t + e^{-q(T-t)} S_t = C_t + K \cdot e^{-r(T-t)}$ (we consider dividend paying case with constant rate q). In case of current value sense for at the value status, i.e. $S = K$ instead of $S = K \cdot e^{-(r-q)(T-t)}$, put options should have slightly lower price than calls, which means their implied volatility should be less than for calls. Using the same data as in the previous tests Fortune (1996) shows this is not true. Difference between puts and calls volatilities is depicted in figure below.

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Difference between Implied Volatilities on SPX Puts and Calls
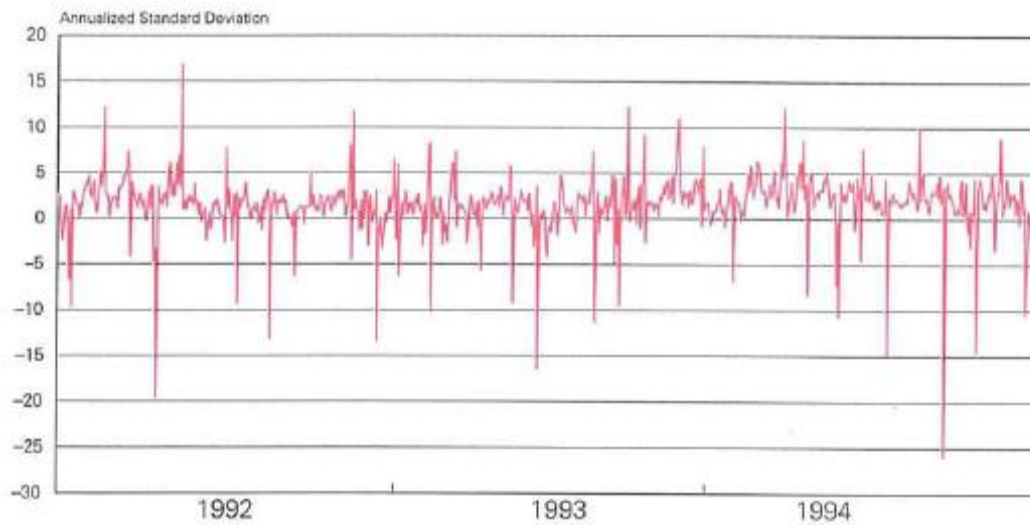
Figure 3 (Source: Fortune (1996))

Figure shows that generally difference is positive, which shows that volatilities for puts are higher than for calls. This contradicts with the prediction of the Black-Scholes model.

Test 4 checks errors implied by the Black-Scholes model. Shortly it says that put options are more accurately priced than calls, which is a little bit surprising, as the model was first constructed for call options.

Test 5 checks the assumption that stock prices are log-normally distributed. It has been long believed that this assumption is not entirely true and rather made for convenience. Probably it started with Fama (1965), when he realized that distribution of changes in logarithm of stock prices has more "fat tails" than the normal distribution, i.e. it was more exposed to large changes than normal distribution. Furthermore the observed distribution had a small skewness and was more bunched in the middle than the normal distribution. The phenomenon of "fat tails" was used to explain the "volatility smile": options far in the money or far out of the money have higher prices, than the model predicts. This will show up as higher implied volatilities for off-the-money options than for at-the-money options.

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Fortune (1996) also shows that logarithm of S&P 500 does not behave strictly as the normal. Its distribution thicker in the middle and shows more large changes than the normal distribution.

Summarizing all five tests we found that implied volatility is a weak estimate for actual volatility. It shows a smile form, in contrast to what the Black-Scholes model predicts: implied volatility is the same for options having the same underlying assets and time to maturity. Also we found that implied volatility for at the money puts is greater than for at the money calls, again contradicting with the models prediction that put-call parity will make them equal. The Black-Scholes model works better for put options than for call options, which is expressed in accuracy of their computed prices. And finally we found that logarithm of S&P 500 does not behave as with normal distribution.

Explanations: Fortune (1996) provides possible explanations to the Black-Scholes model's failures. Systematic overpricing of put options compared to call options can be explained by restrictions on arbitrage. These restrictions are generally of two forms: transaction costs and risk exposure. They are greater for short selling of stock, the way arbitrageurs would take advantage of put overpricing, than for buying stock, the mechanism for correcting put underpricing.

For explanation of divergence from normal distribution, Press (1967) supposed that stock prices are hit by occasional shocks (jumps). In the absence of these shocks the logarithm of stock prices will behave like normal. So the observed distribution of logarithm of stock prices is a mix of normal distributions, each having variance depending on the number of jumps. This was called "jump-diffusion" model. Figure below shows both models.
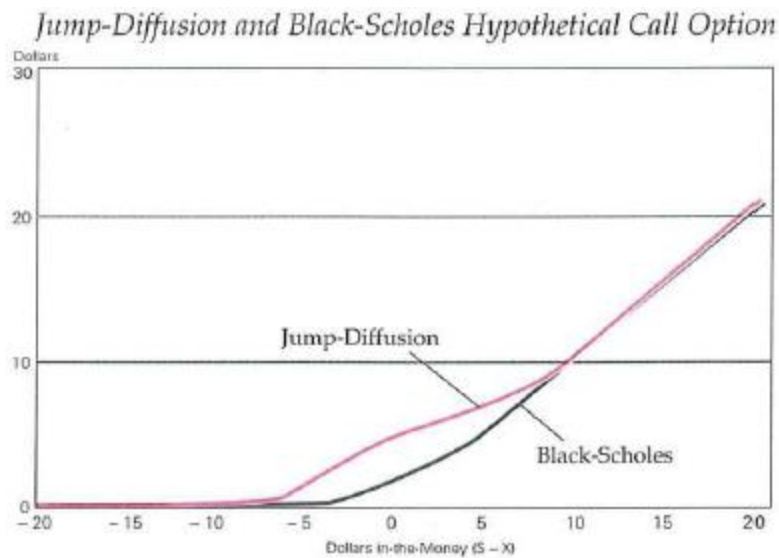
Figure 4 (Source: Fortune (1996))

Jump-diffusion model leads to an option pricing model that is a modification of the Black-Scholes model.

There are some papers comparing the original Black-Scholes and jump-diffusion models. For example McKenzie et al (2007) is devoted to the problem of how accurately Black-Scholes model predicts probability of European call option being exercised compared to jump-diffusion model. Using maximum likelihood method the paper finds that Black-Scholes model is accurate, even at 1% significance level. It also finds that all variables involved into the model are significant, except historical or implied volatility. Nevertheless that paper indicates that use of jump-diffusion approach increases statistical significance of the Black-Scholes model. This result supports major implication of Duan (1999).

3.4 The Black-Scholes model for options with longer term to maturity

Warren Buffet, world's one of the most successful investors, mentioned in the annual report to shareholders of Berkshire Hathaway (where he is the largest shareholder and CEO) in 2002 that "derivatives are weapons of financial mass destruction". But in the same report

he announced that he widely uses derivatives in his business. In his opinion the Black-Scholes model is wrong and as a result derivatives are mispriced. Indeed, by the model the longer the term to maturity the higher the price of an option. In short term it sounds reasonable, but what if the problem considered in long term. For example, assume that we want to price 100 year put option on S&P 500. Prediction of the model is that this option is very expensive. This means that strike price of put at the moment of exercise will be so higher than the stock price that the difference between them, even discounted to present time, will be a huge number. But it is very unlikely that S&P 500 will be much lower in 100 years from now.

Let's now try to investigate the possible reasons of the problem mentioned above and list possible remedies for it. First we should note that the Black-Scholes model is based on the stock prices not on the stock performance. Thus pricing of options does not take into account the average rate of growth in stocks, which really shows itself in reality for long time intervals. For example, after inflation S&P 500 tends to grow at an average rate of 7.5% per year over longer periods of time. The Black-Scholes model does not involve this into pricing formula, because it does not look on standard deviation of stock prices over whole time interval until maturity, but on standard deviation at given moment of time. Thus the longer the time to maturity the more wrong is the model.

What could be used to avoid this problem? Assume that instead of looking on stock prices we consider standard deviation of stock's average rate of return, as the prices are not static. Simply instead of S, in the Black-Scholes formula, use average rate of return on equity for the time period until maturity. For example, suppose we have a call option which enables us to buy one share of SPDR (Standard and Poor Depositary Receipts, also called Spiders or SPY) S&P 500 in fifty years with strike price equal to closing price for today. To price such an option we use another call, let's call it hypothetical, with strike price equal to

SPY value in fifty years, assuming it increasing with rate 7.5%per year from today. Then taking difference between hypothetical and actual strike prices, discounting this value for fifty years we shall account for the fact that SPY will be increasing over the period until maturity. This fact is not involved into the original Black-Scholes model.

Because the Black-Scholes model does not account for average rate of return on equity prices it cannot account for the law of averages. For instance, we expect SPY to grow at an average rate of 7.5% per year, but it need not to be true for shorter periods of time, e.g. month. So the longer the time period we consider SPY the most probable that its rate of return will be 7.5%.

As a possible remedy in this case we can involve new parameter, which will make the formula to account for the law of averages.

Conclusion

In this thesis I investigated possible causes of the Global Financial Crisis. To achieve this goal this thesis proceeded in several directions. Firstly Sub-prime Mortgage Crisis was discussed along with the possible causes of it. It was found that among the causes of housing bubble were agency problem, mispricing of risk and failure of securitization to distribute risk across the financial system. Mortgage brokers were more interested in the number of mortgages they handled, as their income depended on it, rather than in the outcomes of these mortgages. As a possible solution income of mortgage brokers should depend more on future outcome of mortgages. Another problem was erected concerning with unwise decisions made by the heads of this business. Here thesis found mis-pricing and mal-distribution of risk as plausible cause. Considering bad performance of Fannie Mae and Freddie Mac, thesis shows that they are not as guilty as it is widely believed. Despite these companies made poor decisions, the housing bubble was going to burst anyway, although in a smaller scale.

As another possible cause of the Global Financial Crisis were excessive use of derivatives and complex mathematics behind it, used to price them. It is believed by some famous economists, like George Soros, Warren Buffet and Nassim Taleb that "shaky" mathematics involved into the derivative pricing models was used to take on more risk to increase the profits. To reduce the ability of financial engineers to construct highly leveraged instrument, derivatives must be subject to margin requirements.

Thesis also provides evidence on how Black-Scholes model works in reality. It was found that the model contradicts with the real world in a number of ways. Implied volatility was found to be a weak estimate for actual volatility. Its graph shows a "smile" effect, in contrast to what the Black-Scholes model predicts: implied volatility is the same for options having the same underlying assets and time to maturity. Also thesis shows that implied volatility for at the money puts is greater than for at the money calls, again contradicting with

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the models prediction that put-call parity will make them equal. The Black-Scholes model works better for put options than for call options, which is expressed in accuracy of their computed prices. And finally thesis found that logarithm of stock price usually does not behave as with normal distribution.

As a possible improvement the "jump-diffusion" model was mentioned, created by Press. Investigation of papers dealing with comparison of the original Black-Scholes and "jump-diffusion" models shows that the latter has more significant results.

Thesis also checks how Black-Scholes model works in long term. It finds that the model does not account for the average rate of growth in stocks and thus the longer the time to maturity the more wrong is the model. As a plausible remedy one should use in the model stock prices that will grow until maturity date, instead of current stock price.

The model also does not satisfy the law of averages. To overcome this problem new parameter should be involved into the model, to take this issue into account.

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