# EXCHANGE RATES, REGIME SWITCHES AND CURRENCY OPTIONS

 $\mathbf{B}\mathbf{y}$ 

Anna Naszodi

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY AT CENTRAL EUROPEAN UNIVERSITY BUDAPEST, HUNGARY JULY, 2008

© Copyright by Anna Naszodi, 2008

# CENTRAL EUROPEAN UNIVERSITY DEPARTMENT OF ECONOMICS

The undersigned hereby certify that they have read and recommend to the Department of Economics for acceptance a thesis entitled "Exchange Rates, Regime Switches and Currency Options" by Anna Naszodi.

Date: 22 September 2008

I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Supervisor:

László Mátyás I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Internal Examiner:

László Halpern

I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

External Examiner:

David Veredas

I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Committee Member:

Miklós Koren

I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Chair:

Julius Horvath

### CENTRAL EUROPEAN UNIVERSITY

Date: 22 September 2008

Author: Anna Naszodi

Title: Exchange Rates, Regime Switches and Currency Options

Department: **Department of Economics** 

Degree: Ph.D.

HEREBY I TESTIFY THAT THIS THESIS CONTAINS NO MATERIAL ACCEPTED FOR ANY OTHER DEGREE IN ANY OTHER INSTITUTION AND THAT IT CON-TAINS NO MATERIAL PREVIOUSLY WRITTEN AND/OR PUBLISHED BY AN-OTHER PERSON, EXCEPT WHERE APPROPRIATE ACKNOWLEDGEMENT IS MADE.

Signature of Author

Permission is herewith granted to Central European University to circulate and to have copied for non–commercial purposes, at its discretion, the above title upon the request of individuals or institutions.

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without author's written permission.

# Table of Contents

Table	of Contents	v			
$\mathbf{Abstr}$	Abstract v				
1 Ex State	change Rate Dynamics Under Contingent Stochastic Process Switching:				
An A	policition to the EMU Entry of New EU Members	1			
<b>ЛП Л</b>	Introduction	т 9			
1.1	Exchange Bate Model	$\frac{2}{4}$			
1.4	1.2.1       Dynamics       Dynamics       Dynamics         1.2.2       Functional Relationship Between the Exchange Rate and Underly-	6			
	ing Factors	8			
1.3	Option Pricing	11			
1.4	Stabilizing Effect	11			
	1.4.1 Stabilizing Effect in the Model	12			
	1.4.2 Stylized Facts on the Stabilizing Effect	14			
	1.4.3 Estimated Stabilizing Effect with Constant Locking Date and Fac-				
	tor Volatilities	15			
	1.4.4 Stabilizing Effect with Time-Varying Locking Date and Factor Volatil-				
	ities	16			
1.5	Conclusion	23			
1.6	Acknowledgements	24			
Biblic	graphy	28			
2 Fil Exten	2 Filtering the Market Expectation for the Euro Locking Rate in the Extended Agent Pricing Exchange Rate Model				
2.1	Introduction	36			
2.2	Exchange Rate Model	39			
	$2.2.1$ Dynamics $\ldots$	41			
	2.2.2 Functional Relationship Between the Exchange Rate and Underly-				
	ing Factors	45			
2.3	Option Pricing	46			
2.4	Filtering Factors	47			
	2.4.1 Filtering the Expected Locking Date	49			
	2.4.2 Filtering Problem of the Expected Locking Rate	51			
	2.4.3 Parameters	52			
	2.4.4 The Filtered Expected Locking Rate and the Stabilizing Effect of				
	Locking	55			
2.5	Conclusion	58			

	2.6	Acknowledgements	59	
Bi	ibliog	graphy	67	
3	Tar	get Zone Realignments and Exchange Rate Behavior in an Options-		
Ba	ased	Model	80	
	3.1	Introduction	81	
	3.2	The Options-Based Model	82	
	3.3	Option Pricing Method	84	
	3.4	Application of the Model	84	
		3.4.1 Assumed Process of the Floating Exchange Rate	85	
		3.4.2 Calibration	86	
		3.4.3 Decomposition of the Exchange Rate Changes	87	
		3.4.4 The Band Widening of the French Franc	88	
		3.4.5 The Band Shift of the Portuguese Escudo	89	
		3.4.6 The Band Shift of the Hungarian Forint	90	
	3.5	Critique of the Model	91	
	3.6	Conclusions	92	
	3.7	Acknowledgements	93	
	3.8	Appendix	94	
Bi	Bibliography			

# Abstract

The Ph.D. thesis focuses on three specific topics that are fully elaborated in three subsequent chapters.

The first Chapter derives a closed-form solution for the exchange rate dynamics before an anticipated regime switch, *i.e.*, before the adoption of the Euro. The problem is set for the most general case, where the switching date as well as the fixed exchange rate in the second regime are uncertain, while market participants form expectations on both. The empirical part of the first Chapter examines how the prospect of locking influences the exchange rate volatility in three EMU candidate countries.

The exchange rate model of the second Chapter is the same as that of the first one, but it devotes more space on modeling the Maastricht criteria. These criteria necessitate the modeling of the regime switching as state-contingent. In the empirical part of the Chapter, the expected locking rate and date are filtered from the historical exchange rate, interest rate and currency option data.

The road to the Euro consists of at least two regime switches. First, entering the ERM II and than locking the exchange rate. The model of the first two Chapters simplifies the institutional arrangement of EMU entry. It is assumed that countries switch from the floating regime directly to the fixed one. In contrast to the first two Chapters, the third Chapter focuses on the intermediate regime. Chapter three develops an options-based model of target zone arrangements. The model is used to decompose exchange rate changes after band realignment into the direct effect of realignment, changing expectations and changing uncertainty. It is applied to regime switches of France, Hungary and Portugal prior to their EMU entry.

The three Chapters have the following common features. First, the models of all three

chapters are based on the conventional asset pricing model of the exchange rate <sup>1</sup> that has become the cornerstone of every exchange rate models. In this model the exchange rate is a function of the fundamental and a forward looking element. The forward looking nature of the exchange rate is modeled explicitly in the first two Chapters, whereas it is captured by the option prices in the third Chapter.

Second, in all three Chapters, an important determinant of the processes of the exchange rates is the *prospect of locking*. In the model of the first two Chapters, market participants form expectations for the locking rate and date and their expectations alter the process of the exchange rate from that of the latent exchange rate. In this model, the latent exchange rate is the one that would prevail if there were no future locking. The third Chapter focuses on the expectations on the limiting effect of the target zone. The expectations discussed in this Chapter also alter the process of the exchange rate. However, here the regime of the latent exchange rate is characterized by lacking target zone but having future locking. Therefore, the expected locking rate is assumed to work as a dragging anchor directly on the latent exchange rate and not only on the observable exchange rate. Despite of the differences between the definitions and the assumed processes of the latent exchange rates in these Chapters, the exchange rates of both models converge to the expected locking rate in expected terms. The convergence in the models of the first two Chapters is ensured by having expectations for the locking rate, and these expectations become more and more important at determining the exchange rate as the locking date approaches. Whereas the convergence in the third Chapter is due to the process of the latent exchange rate assumed to be a Brownian bridge.

Third, the empirical parts of all three Chapters investigate the effect of regime switches

<sup>&</sup>lt;sup>1</sup>The asset pricing model of the exchange rate has different names in the literature. It is called "asset market view model" by Frenkel and Mussa (1980), the "canonical model" by Krugman (1992) and by Gardeazabal et al. (1997) and the "rational expectations present-value model" by Engel and West (2005).

on the exchange rates. The regime switch considered in the first two Chapters is a switch from a floating to a fixed exchange rate regime. By contrast, the regime switch in the third Chapter is the realignment of the target zone.

Fourth, all three Chapters use the *option pricing theory*, but in a different manner. The relationship between exchange rates and options can be considered in two different ways. First, currencies can be the underlying product of options; second, currencies may contain embedded<sup>2</sup> options. The first two Chapters are based on the first approach. In these papers data on currency option prices are used for estimation. The first Chapter uses historical option prices with different maturities to identify the volatilities of the short-term component and the long-term component of the exchange rate. In the second Chapter these estimated volatilities identify the market expectation for the locking rate through the variances. The third Chapter is based on the approach of embedded options. Here, the currency of a target zone system is taken to be a composite financial asset consisting of an underlying asset and two options.

 $<sup>^{2}</sup>$ An embedded option is an option that is an inseparable part of another instrument. The main difference between the normal or bare options and an embedded option is, that while the former can be traded separately from the underlying security, the latter can not. A common embedded option is the call provision in most corporate bonds.

# Chapter 1

# Exchange Rate Dynamics Under State-Contingent Stochastic Process Switching: An Application to the EMU Entry of New EU Members

This paper gives a closed-form solution of a process switching problem, *i.e.*, the adoption of Euro. Preceding the regime switch, the exchange rate is derived as a function of the following factors: fundamental, market expectations for both the Euro locking rate, and date. Similarly, the exchange rate volatility is a function of the covariances of these factors. The model is applied to examine how the prospect of locking influences the exchange rate volatility in three EMU candidate countries. The stabilizing effect, estimated from currency option prices, is found to be substantial despite of the highly uncertain locking date and rate.

JEL: F31, F36, G13.

*Keywords:* Stochastic process switching, Eurozone entry, exchange rate stabilization, asset-pricing exchange rate model.

## 1.1 Introduction

This paper gives a closed-form solution of a particular type of stochastic process switching problem for the exchange rate. The exchange rate regime is anticipated to change irrevocably from a floating to a fixed one. An example of such regime switch is the adoption of the Euro by some EU member states. The paper derives how the prospect of regime switch influences the process of the exchange rate in the floating regime. The problem is set for the most general case, where both the switching date and the fixed exchange rate in the second regime are uncertain, while market participants form expectations on both.

The process of the exchange rate is derived by extending the conventional asset-pricing exchange rate model with the final locking assumption. In the conventional asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks. Similarly, in the model with final locking the exchange rate is a convex combination of the fundamental and the market expectation for the Euro locking rate. The relative weights of the convex combination depend on the market expectation for the locking date.

The exchange rate dynamics in anticipation of switching to the fixed exchange rate regime has been studied by a number of papers. These differ in various aspects. First, the regime switch is modeled either as being time-contingent or state-contingent. Second, the switching date is either assumed to be announced and considered fully credible by the market, or possible delays are modeled. Third, the exchange rate after the regime switch is either assumed to be pre-specified in a credible manner, or subject to uncertainty.

Obstfeld and Stockman (1985) analyze a time-contingent transition from floating rate to a pre-specified fixed rate occurring at a pre-announced date. Djajic (1989) conducts his analysis also in a time-contingent setting with a credible switching date, but the exchange rate is fixed at a level to be chosen by the authorities no earlier than the date of fixing. Miller and Sutherland (1994) model Britain's return to gold standard in 1925 in a timecontingent regime switching framework with pre-announced transition date. Flood and Garber (1983) work on a state-contingent switching model where the country will switch from a floating to a fixed exchange rate system whenever the exchange rate reaches a pre-determined boundary. In their model the date of regime switch is stochastic, but the exchange rate in the second regime is not. Froot and Obstfeld (1991a,b) model the regime switch with the same absorbing barrier condition proposed by Flood and Garber (1983) and they derive a closed-form solution for the process of the exchange rate. De Grauwe et al. (1999a) set up a model for the Euro conversion, where the Euro locking rate is determined by different pricing rules and the locking date is fixed. They then extend the framework with stochastic locking date. In the extended model, agents attribute a positive probability to the scenario that there will be no regime switch at all. The complementary scenario is that the regime is switched at a pre-announced date. The probability of having a future regime switch is exogenous, consequently the expected locking date is independent of the state of the economy. Therefore, both the baseline and the extended models are time-contingent regime switching models. In contrast to De Grauwe et al. (1999a), it is the locking rate which is pre-determined and the expected

time until locking which is modeled as a random variable in Wilfling and Maennig (2001). Similarly to De Grauwe et al. (1999a), the regime switch is time-contingent in Wilfling and Maennig (2001), as the expected switching date is independent of the fundamental.

In the light of the above literature the theoretical contributions of this paper are the following. First, the regime switch is state-contingent here, like in Flood and Garber (1983) and Froot and Obstfeld (1991a,b). What distinguishes this paper from the above studies, is that the locking rate is not predetermined, but modeled as having stochastically changing expectations for it. This assumption is more realistic for the EMU candidate countries where the locking rate is not announced. The expected switching date is a stochastic function of two state variables. These are the fundamental and the expected inflation, assumed to correlate with the expected switching date. It is important to model the interdependence of the switching date and the fundamental on the one hand and the interdependence of the switching date and the expected inflation on the other hand in the context of Eurozone entry. The main point is that a country is not eligible to adopt the Euro unless it fulfills the Maastricht criteria. Stronger fundamentals or a lower expected inflation rate improve the chances of a country to join the Eurozone by a given date. Therefore, the Euro locking date should be modeled as being state dependent.

Second, this model allows not only for the possible delays of the Eurozone entry, but also, for the possibility of earlier entry dates too. This generalization has been already made by Wilfling and Maennig (2001) but in a more restrictive framework with a predetermined locking rate.

Third, the paper derives a closed-form solution for the exchange rate as a function of three stochastic factors: the fundamental, the expected locking rate and the date. De Grauwe et al. (1999a) also derive a closed-form solution for the exchange rate but for a simpler case with constant locking date. They remark that one potential extension of their model is to consider the uncertainty of the locking date. This paper not only generalizes the result of De Grauwe et al. (1999a), but also presents the condition under which the solution for the extended model is the same as that of De Grauwe et al. (1999a).

Forth, the paper analyzes the stabilizing effect of the prospect of locking in a framework where the locking date is uncertain. De Grauwe et al. (1999b) analyze the same effect of establishing the Euro in 1999. However, in their sample period covering a few years directly before the locking, the uncertainty concerning the locking date was less of an issue. This paper shows that the uncertainty concerning the locking date is important, although this is due to its interdepence with the locking rate. The existence of the stabilizing effect hinges mainly on the uncertainty concerning the expected locking rate, and somewhat on the uncertainty concerning the locking date. Even if the locking date is announced credibly, a highly uncertain locking rate may increase the volatility of the exchange rate. In contrast to the locking date, the credible announcement of the locking rate always stabilizes the exchange rate no matter what is the expected locking date. However, the role of uncertainty concerning the locking date should not be underestimated as a highly uncertain locking date is likely to be associated with a highly volatile expected locking rate.

This paper is unique in the sense that it uses historical currency option prices in order

to estimate the magnitude of the stabilizing effect. The data on options enables us to make inferences on the presence of stabilizing effect even if both the locking date and rate are uncertain. In the empirical part of the paper, the model is applied to estimate the stabilizing effect of three new EU members prior to their adoption of the Euro. These countries are the Czech Republic, Hungary and Poland, which will join the Eurozone and will lock their exchange rates irrevocably at the final conversion rate. In the case of these three countries, neither the locking rate nor the locking date are credibly announced by the authorities. However, market participants have already started to form expectations on both. These expectations are in the focus of our interest, because they are likely to influence the exchange rate even some years before the locking. Intuitively, the more stable these expectations are, the lower the volatility of the exchange rate is.

The volatilities of the latent exchange rate and the expected locking rate are estimated by using a theoretical option pricing model, and cross-sectional data on option prices with different maturities. The identification is based on the following. The volatility of the latent exchange rate has higher relative weight in short options, then in longer ones. In contrast to the volatility of the latent exchange rate, the volatility of the expected locking rate influences relatively more the long end of the option term structure.

The expected Euro locking rate is found to be less volatile than either the historical exchange rate or the latent exchange rate. This result has the important implication that the prospect of locking has a stabilizing effect on the exchange rate in all three analyzed countries. Moreover, this stabilizing effect is substantial despite the fact that locking in these countries will take place in the relatively far future and the locking date is yet highly uncertain.<sup>1</sup>

The paper is structured as follows. Section 1.2 presents the exchange rate model. Section 1.3 derives an option pricing formula, which is used for parameter estimation in the empirical part of the paper. Section 1.4 investigates the stabilizing effect of the prospect of locking both theoretically and empirically. Finally, Section 1.5 concludes.

## 1.2 Exchange Rate Model

The exchange rate model is the conventional asset-pricing exchange rate model extended with the assumption of final locking. In the conventional asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted

<sup>&</sup>lt;sup>1</sup>By comparing the historical volatilities of different periods, De Grauwe et al. (1999b) find that the exchange rate stabilizing effect was substantial for the currencies that participated in the first wave of EMU. The high stabilizing effect a few years before 1999 can be partly due to the relatively high confidence in the announced date of regime switch, the high relative weight of the expected locking rate in the exchange rate due to the vicinity of regime switch, and finally to the high chance that the central parities will be taken as the conversion rates to the Euro. As it is noted by Hagen and Traistaru-Siedschlag (2006), the ECOFIN has announced "the fixed-conversion rule at its Summit in Mondorf, 13-14 September 1997, more than seven months prior to the EU Summit in Brussels, 2-3 May 1998, where the members of EMU were decided. Although this was not made explicit, markets widely interpreted this decision as taking the existing central parities as the internal conversion rates to the euro."

value of future shocks. Another extended version of the asset-pricing model is the target zone model of Krugman (1991), which shows some similarities to this model.

First, in Krugman's model, the exchange rate would be equal to the fundamental if there was no target zone. Similarly, in this model, the log exchange rate would be equal to the fundamental in the absence of future locking. Given this relationship, the fundamental is referred to as the latent exchange rate, *i.e.*, the exchange rate that would prevail if the currency was never going to be locked against the Euro. While Krugman investigates the stabilizing feature of the target zone with a floating regime as a bench-mark, we explore the stabilizing effect of future locking with the "no locking" as a bench-mark.

Also, the implicit relationship between the exchange rate subject to future locking and the latent exchange rate in this model is the same as the relationship between the target zone exchange rate and the fundamental in Krugman's model. In general, this relationship between the exchange rate and the fundamental is common across all asset-pricing based representative agent models. In a reduced form, this relationship is

$$s_t = v_t + c \frac{E_t(ds_t)}{dt} , \qquad (1.1)$$

where, s is the log exchange rate, and v is the log latent exchange rate. Both s and v are measured as the log price of one Euro in terms of the domestic currency. The constant c is the time scale. Engel and West (2005) and Svensson (1991) present the Money Income Model  $^{2}$  as one possible structural model that rationalizes the reduced form (1.1). In the Money Income Model c is the interest rate semi-elasticity of the money demand. The term  $\frac{E_t(ds_t)}{dt}$  is the expected<sup>3</sup> instantaneous change of the exchange rate. As we will see later, the expected instantaneous change of the exchange rate depends on the log latent exchange rate  $v_t$ , the market expectation for the log final conversion rate  $x_t$  and the locking date  $T_t$ .

By following the Money Income Model, the latent exchange rate is defined as a function of some macro variables:

$$v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + c\dot{i}_t^* , \qquad (1.2)$$

where y denotes the domestic real output, q is the real log exchange rate,  $\psi$  is the risk premium,  $p^*$  is the Eurozone log price, m denotes the domestic nominal money supply and  $i^*$  is the Euro interest rate.

It is assumed that the exchange rates will be locked at their equilibrium value. Here, the concept of equilibrium exchange rate is used under which the strong law of purchasing

(2)  $q_t = s_t + p_t^* - p_t$  real exchange rate (3)  $\psi_t = i_t - i_t^* - \frac{E(ds_t)}{dt}$  risk premium (4)  $v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + ci_t^*$ fundamental/latent exchange rate.

<sup>&</sup>lt;sup>2</sup>The Money Income Model is the following.

<sup>(1)</sup>  $m_t - p_t = \alpha y_t - ci_t \quad \alpha > 0 \quad c > 0$ money market equilibrium

<sup>&</sup>lt;sup>3</sup>In this paper there are two different types of expectations. One is the subjective market expectation, and the other is the mathematical expected value of a random variable. Here, it is referred to the latter one. However, under rational expectation the two are the same.

power parity (PPP) holds for the locking rate, *i.e.*, the log nominal exchange rate at the date of locking  $T^*$  is equal to the difference between the domestic and Eurozone log prices  $s_{T^*} = p_{T^*} - p_{T^*}^*$ . Under rational expectation the market expects the log final conversion rate at time t to be  $x_t = E_t(s_{T^*})$ , which gives

$$x_t = p_t - p_t^* + \int_t^{T_t} E_t(\pi_\tau - \pi_\tau^*) d\tau , \qquad (1.3)$$

where  $\pi$  and  $\pi^*$  denote domestic and Eurozone inflation rates respectively.

Neither the definitions (1.2) and (1.3) nor the corresponding macro data are used directly in the empirical part of this paper. Mainly because of the low frequency of these data, but, also, because of a possible misspecification of the underlying macro models. For instance, the examples of the current EMU countries show that the locking rate can deviate from its PPP value. Still, it is the equilibrium real exchange rate that should be the key determinant of the locking rate chosen on the basis of economic considerations. Therefore, while not used directly in the rest of the paper, these definitions motivate the choice of the processes of the underlying factors and, also, the interpretation of the results.

#### 1.2.1 Dynamics

This Section specifies the processes of the factors. These processes will be used to derive the process of the exchange rate.

The factors are assumed to follow Brownian motions. This assumption can be decomposed into an assumption on the martingale property of the processes and into the Gaussian distribution of the innovations. The Gaussian distribution of the innovations is assumed for technical reason. The martingale property of these processes, however, can be easily explained:

- Under rational expectation, the expectation of the market participants for the log final conversion rate is the expected value of the log final conversion rate given all the information available at the time the expectation is formed  $x_t = E_t(s_{T^*})$ . And, also, the market expectation for the locking date is the expected value of the true locking date  $T^*$ given all the information available at the time the expectation is formed  $T_t = E_t(T^*)$ . The law of iterated expectations implies that the process of both  $T_t$  and  $x_t$  are martingales, since  $E_t(E_{t+1}(s_{T^*})) = E_t(s_{T^*})$  and  $E_t(E_{t+1}(T^*)) = E_t(T^*)$ .

- The martingale property of the process of the *log latent exchange rate* can be derived from the Money Income Model under the assumption that the right hand side variables of Equation (1.2) have martingale processes. The martingale property of  $v_t$  allows us to focus entirely on the dynamics caused by the future final locking, as opposed to the effects of predictable future changes in the latent exchange rate. The process <sup>4</sup> of the market expectation for the log Euro locking rate  $x_t$  is

$$dx_t = \begin{cases} \sigma_{x,t} dz_{x,t} , & \text{if } t < T^* \\ 0 , & \text{otherwise} \end{cases}$$
(1.4)

Where  $dz_{x,t}$  is a Wiener process. The parameter  $\sigma_{x,t}$  is the time-varying volatility of the expected locking rate. The reason for being modeled as a time-varying parameter is that the volatility should be zero at the locking  $\sigma_{x,T^*} = 0$ , when no uncertainty remains concerning  $x_{T^*}$ , and the volatility is likely to decrease over time as the locking date is getting closer and closer. For obvious reasons  $\sigma_{x,t}$  is assumed to be bounded.

When the right hand side variables of Equation (1.2) follow Brownian motion, so does  $v_t$ . Hence, its process can be written as

$$dv_t = \sigma_{v,t} dz_{v,t} \qquad 0 < \sigma_{v,t} < \infty , \qquad (1.5)$$

where  $\sigma_{v,t}$  is the time-varying volatility of the fundamental. In contrast to  $\sigma_{x,t}$ , the volatility  $\sigma_{v,t}$  should not necessarily approach zero at the regime switch. Still, it may change in the run up period to the EMU. The Wiener process  $dz_{v,t}$  is not necessarily independent of  $dz_{x,t}$ . We assume linear, contemporaneous relationship between the two shocks that can be captured by their time-varying correlation denoted by  $\rho(dz_{v,t}, dz_{x,t})$ .

The third factor, the market expectation for locking date  $T_t$  is modeled as a stochastic variable. The expected time until locking can not be negative, or in other words, the market can not expect before the locking that the regime has already been switched. Therefore, the process of  $T_t$ , similar to a geometric Brownian motion, is assumed to have a multiplicative error term

$$dT_t = \begin{cases} (T_t - t)\sigma_{T,t}dz_{T,t}, & \text{if } t < T^* \\ 0, & \text{otherwise,} \end{cases}$$
(1.6)

where  $0 < \sigma_{T,t} < \infty$ . And  $dz_{T,t}$  is a Wiener process which may correlate with the other two shocks  $dz_{v,t}$  and  $dz_{x,t}$ .

The following restrictions are imposed on the correlations:

$$\rho(dz_{x,t}, dz_{v,t}) = \rho(dz_{T,t}, dz_{v,t})\rho(dz_{T,t}, dz_{x,t}) , \qquad (1.7)$$

$$\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t} - \rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t} = (v_t - x_t)\frac{\sigma_{T,t}(T_t - t)}{c} .$$
(1.8)

<sup>4</sup>In a discrete time framework the process can be derived from Equation (1.3):

$$\Delta x_t = [\pi_{t+1} - E_t(\pi_{t+1})] + \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i) - E_t(\pi_i)] - [\pi_{t+1}^* - E_t(\pi_{t+1}^*)] - \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i^*) - E_t(\pi_i^*)].$$

If both the expectation errors and the change of expectations are independent and normally distributed with zero mean, then  $x_t$  follows Brownian motion in a continuous time framework.

Equations (1.7) and (1.8) are less restrictive than the assumption that one of the factors is constant. In the literature of exchange rate locking, De Grauwe et al. (1999a) assume constant locking date and Wilfling and Maennig (2001) assume constant locking rate and both assume independent factors. By that they implicitly assume that all the correlations  $\rho(dz_{x,t}, dz_{v,t})$ ,  $\rho(dz_{T,t}, dz_{v,t})$  and  $\rho(dz_{T,t}, dz_{v,t})$  are zero, and Equations (1.7) is fulfilled in both papers by assumption.

In reality, the expected locking rate  $x_t$  is likely to be influenced by the productivity shocks affecting the fundamental  $v_t$ . This link between  $x_t$  and  $v_t$  is supported by the Balassa-Samuelson effect. The Balassa-Samuelson effect is substantial in the transition economies; therefore it is more realistic to allow non-zero correlation between the expected locking rate and the fundamental. Chapter 2 of the thesis explains in more detail the economic considerations that rationalize the restrictions on the correlations.

De Grauwe et al. (1999a) derive a closed-form functional relationship between the exchange rate on one hand and the constant locking date, the fundamental and the expected locking rate on the other hand. As shown next, restriction (1.8) ensures to have the same function for the general model with stochastic locking.

### 1.2.2 Functional Relationship Between the Exchange Rate and Underlying Factors

This section derives the functional relationship between the exchange rate on the one hand and the latent exchange rate, the market expectations for the locking date and rate on the other hand. The next Section presents the derivation under the assumption of having constant locking date. Then, this assumption is relaxed in order to derive the function in the general case with stochastic locking date.

The derivation has the following two steps in both cases. First, the process of the log exchange rate  $s_t$  is derived from the processes of the factors by using Ito's stochastic change-of-variable formula. Second, we get that the function satisfying the derived process, the terminal condition  $s_{T^*} = x_{T^*}$  and Equation (1.1) is

$$s_t = f(t, v_t, x_t, T_t) = \left(1 - e^{-\frac{T_t - t}{c}}\right) v_t + e^{-\frac{T_t - t}{c}} x_t .$$
(1.9)

Equation (1.9) shows that the log exchange rate is the weighted average of the log latent exchange rate and the expected log final conversion rate. The weights are changing over time; if the time until locking is infinite, or in other words, if the currency will never be locked, the weight of the latent exchange rate is one, and the weight of the expected conversion rate is zero. As the time until locking decreases, the weight of the expected conversion rate increases. Finally, as the time until locking approaches zero, the weight of the expected conversion rate approaches one.

#### Constant Locking Date

In this Section, the expected locking date is assumed to be constant  $T_t = T$ . According to Ito's formula, the function  $f(t, v_t, x_t, T)$  should satisfy the following expression

$$df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_t}\mu_{x,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\sigma_{x,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t\partial v_t}\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t}\right]dt + \frac{\partial f}{\partial v_t}\sigma_{v,t}dz_{v,t} + \frac{\partial f}{\partial x_t}\sigma_{x,t}dz_{x,t} . \quad (1.10)$$

The different  $\mu$ 's denote the drift terms, whose values are zero in Equations (1.4) and (1.5).

At time  $T^*$  the exchange rate  $s_{T^*}$  is equal to the market expectation for the final conversion rate  $x_{T^*}$ , because at the time of locking the market already knows the conversion rate. Consequently, the function  $f(t, v_t, x_t, T)$  should satisfy the terminal condition

$$f(T^*, v_{T^*}, x_{T^*}, T^*) = x_{T^*} . (1.11)$$

The solution for  $f(t, v_t, x_t, T)$  that satisfies (1.1), (1.10) and (1.11) is given by (1.9). Appendix A presents the proof for the general case with stochastic locking date.

By substituting (1.4), (1.5), (1.6), and (1.9) into Equation (1.10), we obtain the dynamics of the exchange rate:

$$ds_{t} = \frac{1}{c} \frac{e^{-\frac{T_{t}-t}{c}}}{1-e^{-\frac{T_{t}-t}{c}}} \left(x_{t}-s_{t}\right) dt + \left(1-e^{-\frac{T_{t}-t}{c}}\right) \sigma_{v,t} dz_{v,t} + e^{-\frac{T_{t}-t}{c}} \sigma_{x,t} dz_{x,t} .$$
(1.12)

Equation (1.12) shows that the dynamics of the exchange rate is such that it converges to the actual market expectation for the final conversion rate. Moreover, the closer the locking date, the faster the convergence is. The deviation from this trend is due to the stochastic innovations  $(dz_{v,t}, dz_{x,t})$  of the factors; consequently, the instantaneous volatility of the exchange rate depends on the joint distribution of these innovations. In the constant locking date model,  $dz_{v,t}$  and  $dz_{x,t}$  are independent under (1.7). Therefore, the instantaneous variance of returns at time t is simply

$$\sigma_{s,t}^2 = \left(1 - e^{-\frac{T_t - t}{c}}\right)^2 \sigma_{v,t}^2 + e^{-2\frac{T_t - t}{c}} \sigma_{x,t}^2 \quad . \tag{1.13}$$

#### Stochastic Locking Date

Here, it is assumed that the expected locking date  $T_t$  is stochastic and its process is given by (1.6). The function  $f(t, v_t, x_t, T_t)$  is derived under the assumption of stochastic locking date similarly to the deterministic case. The solution is again given by (1.9), however, this finding depends on restriction (1.8).

The Ito calculus can be used again to find the function  $f(t, v_t, x_t, T_t)$ . By using Ito's stochastic change-of-variable formula, we obtain a similar expression for df as previously

with constant locking date, however, some new terms also appear in the formula:

$$df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_t}\mu_{x,t} + \frac{\partial f}{\partial T_t}\mu_{T,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\sigma_{x,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial T_t^2}\sigma_{T,t}^2(T_t - t)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial T_t\partial x_t}\rho\left(dz_{T,t}, dz_{x,t}\right)\left(T_t - t\right)\sigma_{T,t}\sigma_{x,t} + \frac{1}{2}\frac{\partial^2 f}{\partial T_t\partial v_t}\rho\left(dz_{T,t}, dz_{v,t}\right)\left(T_t - t\right)\sigma_{T,t}\sigma_{v,t} + \frac{1}{2}\frac{\partial^2 f}{\partial x_t\partial v_t}\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t}\right]dt + \frac{\partial f}{\partial v_t}\sigma_{v,t}dz_{v,t} + \frac{\partial f}{\partial x_t}\sigma_{x,t}dz_{x,t} + \frac{\partial f}{\partial T_t}\left(T_t - t\right)\sigma_{T,t}dz_{T,t}.$$
 (1.14)

The  $\mu$ 's are zero again, because the process of the factors are driftless.

We obtain again that the function satisfying the derived process (1.14), the terminal condition (1.11) and Equation (1.1) borrowed from Krugman  $(1991)^5$  is given by (1.9). The proof can be found in Appendix A.

In order to examine the exchange rate dynamics of the model, we substitute (1.4), (1.5), (1.6), (1.7), (1.8) and (1.9) into Equation (1.14).

$$ds_{t} = \frac{1}{c} \frac{e^{-\frac{T_{t}-t}{c}}}{1-e^{-\frac{T_{t}-t}{c}}} \left(x_{t}-s_{t}\right) dt + \left(1-e^{-\frac{T_{t}-t}{c}}\right) \sigma_{v,t} dz_{v,t} +$$

$$+e^{-\frac{T_{t}-t}{c}} \sigma_{x,t} dz_{x,t} - \frac{1}{c} \frac{e^{-\frac{T_{t}-t}{c}}}{1-e^{-\frac{T_{t}-t}{c}}} \left(x_{t}-s_{t}\right) \left(T_{t}-t\right) \sigma_{T,t} dz_{T,t} .$$

$$(1.15)$$

Similarly to the constant locking date model, the dynamics of the exchange rate is such that it converges to the actual market expectation for the final conversion rate. Moreover, the closer the expected time of locking, the faster the convergence is. The deviation from trend is not only due to the stochastic innovations  $dz_{v,t}$  and  $dz_{x,t}$ , but also to  $dz_{T,t}$ . The instantaneous volatility of the exchange rate is a function of the variances and covariances of these normally distributed innovations.

The instantaneous variance of returns can be derived in the general framework with stochastic locking date from Equations (1.7), (1.8), (1.9), and (1.15). The instantaneous variance of returns at time t is then

$$\sigma_{s,t}^{2} = \left[ 1 - 2e^{-\frac{T_{t}-t}{c}} \left( 1 - \rho^{2}(dz_{T,t}, dz_{v,t}) \right) + e^{-2\frac{T_{t}-t}{c}} \left( 1 - \rho^{2}(dz_{T,t}, dz_{v,t}) \right) \right] \quad \sigma_{v,t}^{2} + e^{-2\frac{T_{t}-t}{c}} \left( 1 - \rho^{2}(dz_{T,t}, dz_{x,t}) \right) \quad \sigma_{x,t}^{2} \quad (1.16)$$

<sup>&</sup>lt;sup>5</sup>Similarly to the solution in the Krugman (1991) paper, Equation (1.9) also satisfies a smooth pasting condition  $\frac{ds_{T^*}}{dv_{T^*}} = 0.$ 

It is easy to check that if both  $\rho(dz_{T,t}, dz_{v,t})$  and  $\rho(dz_{T,t}, dz_{x,t})$  are nill, like in the specification with constant locking date, Equation (1.16) is reduced to (1.13).

## **1.3** Option Pricing

This section derives a simple closed form option pricing formula for European-type options. The empirical part of the paper uses the derived pricing formula to estimate the magnitude of the stabilizing effect. This option valuation formula can be used under the restrictive assumptions of having constant locking date and factor volatilities. Under these assumptions, the process of the instantaneous volatility of the exchange rate is independent of the exchange rate, therefore the price of the option in terms of volatility can be calculated as the square root of the integrated instantaneous variance:

$$g(t,m,\sigma_{s,\tau}) = \left[\int_t^{t+m} \sigma_{s,\tau}^2 d\tau\right]^{\frac{1}{2}}.$$
(1.17)

The option is sold at time t and the maturity is m.

By using Equation (1.13), the following closed form pricing formula can be derived from Equations (1.17):

$$g(t, m, \sigma_x, \sigma_v) = \left( (m + \Gamma_1 - 2\Gamma_2) \ \sigma_v^2 + \Gamma_1 \ \sigma_x^2 \right)^{\frac{1}{2}}, \qquad (1.18)$$

where  $\Gamma_1 = \frac{c}{2}e^{-\frac{2}{c}(T-t-m)} - \frac{c}{2}e^{-\frac{2}{c}(T-t)}$  and  $\Gamma_2 = ce^{-\frac{1}{c}(T-t-m)} - ce^{-\frac{1}{c}(T-t)}$ .

If the restrictive assumptions of having constant locking date and factor volatilities are not fulfilled, the process of the instantaneous volatility of the exchange rate is not independent of the underlying asset of the options. Therefore, Equation (1.17) mis-prices the options and alternative methods, like simulation method should be used instead.

Here, the simple option valuation formula (1.18) is used for parameter estimation, because it is much faster to compute than the simulation based option price. The formula (1.18) is used not only when the locking date and factor volatility are assumed to be constant, but also in the specification with time-varying parameters. In the latter case, the theoretical price of option is approximated by (1.18). Therefore, the estimation is in fact a pseudo estimation. The validity of this pseudo estimation of course needs to be checked. Section 1.4.4 provides a robustness check that justifies it.

## 1.4 Stabilizing Effect

Here we investigate the stabilizing effect of the prospect of locking both theoretically and empirically.

#### 1.4.1 Stabilizing Effect in the Model

The stabilizing effect of locking can be detected by comparing the volatility of the exchange rate with future locking  $\sigma_s$  on the one hand and the volatility of exchange rate without locking  $\sigma_v$  on the other hand. Obviously, if the former is smaller than the latter, we can infer that the prospect of locking stabilizes the exchange rate. If the former happen to exceed the latter, the future regime switch destabilizes the exchange rate.

We define the measure of the stabilizing effect as the ratio of the instantaneous volatility of the exchange rate and the instantaneous volatility of the latent exchange rate. If the ratio is higher than unity, the exchange rate is destabilized, otherwise it is stabilized by the locking. Here, the measure of the stabilizing effect is derived for four different model specifications.

- In the first both the locking rate x and date T are deterministic, but not the latent exchange rate v.

- In the second the locking rate x is deterministic, but not the other two factors.

- In the third the locking date T is deterministic, but not the other two factors.

- The fourth is the most general one, where all three factors are stochastic.

In the first specification  $T_t = T$ ,  $\rho(dz_{T,t}, dz_{x,t}) = \rho(dz_{T,t}, dz_{v,t}) = \rho(dz_{v,t}, dz_{x,t}) = 0$ and  $\sigma_{x,t} = \sigma_{T,t} = 0$ . By rearranging Equation (1.16) under these zero restrictions, the stabilizing effect simplifies to

$$\left. \frac{\sigma_{s,t}}{\sigma_{v,t}} \right|_{\sigma_{x,t}=\sigma_{T,t}=0} = 1 - e^{-\frac{T-t}{c}} . \tag{1.19}$$

Equation (1.19) shows that if the only source of uncertainty is the fundamental, the stabilizing effect is a deterministic decreasing function of the time until locking. Or in other words, the stabilizing effect is getting more and more substantial as the expected regime switch approaches. Moreover, even if the locking is expected in the very far future, the exchange rate is not destabilized, because  $1 - e^{-\frac{T-t}{c}} \leq 1$ .

In the second specification the locking rate x is deterministic, but not the other two factors. Here,  $\sigma_{x,t} = \rho (dz_{T,t}, dz_{x,t}) = \rho (dz_{v,t}, dz_{x,t}) = 0$ , therefore the stabilizing effect is

$$\frac{\sigma_{s,t}}{\sigma_{v,t}}\Big|_{\sigma_{x,t}=0} = \left[1 - 2e^{-\frac{T_t - t}{c}} \left(1 - \rho^2(dz_{T,t}, dz_{v,t})\right) + e^{-2\frac{T_t - t}{c}} \left(1 - \rho^2(dz_{T,t}, dz_{v,t})\right)\right]^{\frac{1}{2}}.$$
 (1.20)

One can easily show that the relative volatility  $\frac{\sigma_{s,t}}{\sigma_{v,t}}$  under deterministic locking rate never exceeds unity. Previously, we found the same under the first specification with constant locking rate and date. Therefore, one conclusion we can make is the following. If there is no uncertainty concerning the locking rate, the expectations on the future locking will mitigate the volatility of the exchange rate no matter whether the locking date T is already credibly announced or highly uncertain. In contrast to the qualitative finding on the presence of stabilization, the magnitude of the stabilizing effect does depend on the locking date. By comparing Equations (1.19) and (1.20), we obtain that the higher is the correlation between the expected locking date and the fundamental  $\rho(dz_{T,t}, dz_{v,t})$  in absolute terms, the smaller the stabilizing effect is.

Let us now derive the formula for the *third specification* with deterministic locking date. Under this specification  $T_t = T$ ,  $\sigma_{T,t} = \rho (dz_{T,t}, dz_{x,t}) = \rho (dz_{T,t}, dz_{v,t}) = \rho (dz_{v,t}, dz_{x,t}) = 0$ , therefore the relative volatility is

$$\frac{\sigma_{s,t}}{\sigma_{v,t}}\Big|_{\sigma_{T,t}=0} = \left[ \left(1 - e^{-\frac{T-t}{c}}\right)^2 + e^{-2\frac{T-t}{c}} \frac{\sigma_{x,t}^2}{\sigma_{v,t}^2} \right]^{\frac{1}{2}}.$$
(1.21)

Figure 1.1 shows the magnitude of the stabilizing effect measured by Equation (1.21) as a function of the time until locking T - t and the relative volatility  $\frac{\sigma_x}{\sigma_v}$ . One can see that the prospect of locking can even destabilize the exchange rate. We have destabilization, when the uncertainty concerning the locking rate  $\sigma_{x,t}$  is sufficiently high relative to the uncertainty concerning the fundamental.

The *fourth specification* with three stochastic factors can be considered as a generalization of any of the previous three specifications. Here, the ratio of volatilities is

$$\frac{\sigma_{s,t}}{\sigma_{v,t}} = \left\{ \left[ 1 - 2e^{-\frac{T_t - t}{c}} \left( 1 - \rho^2 (dz_{T,t}, dz_{v,t}) \right) + e^{-2\frac{T_t - t}{c}} \left( 1 - \rho^2 (dz_{T,t}, dz_{v,t}) \right) \right] + e^{-2\frac{T_t - t}{c}} \left( 1 - \rho^2 (dz_{T,t}, dz_{x,t}) \right) \left\{ \frac{\sigma_{x,t}^2}{\sigma_{v,t}^2} \right\}^{\frac{1}{2}} \right\}^{\frac{1}{2}} . \quad (1.22)$$

Obviously, destabilization is possible if all three factors are stochastic just like in the third specification. The magnitude of the stabilization or destabilization hinges on the covariance matrix of the factors. The second specification demonstrates that the higher is the correlation between the expected locking date and the fundamental  $\rho(dz_{T,t}, dz_{v,t})$  in absolute terms, the smaller is the stabilizing effect. Here, the magnitude of stabilization is decreasing not only in  $|\rho(dz_{T,t}, dz_{v,t})|$ , but also in  $|\rho(dz_{T,t}, dz_{v,t})|$ . Moreover, the stabilizing effect is decreasing also in the product of  $|\rho(dz_{T,t}, dz_{v,t})|$  and  $|\rho(dz_{T,t}, dz_{x,t})|$  that is  $|\rho(dz_{x,t}, dz_{v,t})|$ . Intuitively, the currency can be thought as a portfolio of assets x and v. Portfolio theory suggests that the higher the correlation between the assets, the smaller the diversification. Analogously, the higher  $|\rho(dz_{x,t}, dz_{v,t})|$ , the smaller the stabilizing effect.

All of the findings derived under these four different specifications can be demonstrated by a numerical example. Figure 1.2 shows the stabilizing effect of the future locking under all four specifications. The lines of Figure 1.2 are downward sloping, or in other words, the relative volatility is decreasing in the time until locking. The closer the locking date, the higher the stabilizing effect is. The lines of the first two specifications are below the constant line at unity. This can be interpreted as follows. If the locking rate is predetermined, then the prospect of locking always stabilizes the exchange rate no matter how far the regime switch is. Whereas in the last two specifications destabilization is possible. Accordingly, the lines of the third and fourth specifications cross the line of constant unity.

The difference between the lines of the first two specifications and the lines of the third and forth specifications demonstrate the idea that the magnitude of stabilization depends on the uncertainty both concerning the locking rate and date. Whether the exchange rate is stabilized or destabilized by the prospect of locking hinges mainly on the uncertainty concerning the expected locking rate, and only indirectly on the uncertainty concerning the locking date. Even if the locking date is announced credibly, the volatility of the exchange rate with future locking can exceed the volatility of the latent exchange rate without future locking. In contrast to the locking date, the credible announcement of the locking rate always stabilizes the exchange rate no matter what is the expected locking date. Still, the role of uncertainty concerning the locking date may be very important at determining the exchange rate volatility, because it is not independent from the uncertainty concerning the locking rate. If the locking is postponed due to an adverse real shock to the economy, the monetary authority is likely to ease its targeted inflation path and therefore the access inflation rate cumulated until locking increases as well. As the expected locking rate x is modeled as being the cumulated access inflation rate, it should increase as well. The second moments of the distributions of the expected locking rate and date are not independent either. Obviously, if the uncertainty concerning the locking date increases, then the uncertainty concerning the locking rate increases as well. The only exception is, when the monetary authority does stick to the inflation path targeted before the real shock. But this policy is not optimal if the central bank has both the inflation rate and output in its objective function.

Some further interesting ideas are also demonstrated on Figure 1.2 by a shift from the line of the first specification to the line of the second specification and back.

In this model not only monetary policy, but also fiscal policy can influence the exchange rate volatility. In point A, the central bank is perfectly credible and so effective that it is able to perfectly eliminate the uncertainty concerning the future inflation path  $\sigma_{x,t} = 0$ . An unexpected fiscal shock can increase the uncertainty concerning the locking date and therefore it can increase the exchange rate volatility by shifting the economy from point A to B. Such an unfavorable shift can not be avoided by the central bank.

What is also demonstrated by Figure 1.2 is that the EMU candidate country can mitigate the exchange rate volatility by credibly announcing the locking date. This date should be later than what is expected by the market before the announcement, otherwise market participants will not believe that fulfilling all the Maastricht criteria by the announced date is feasible for sure. If the market considers the announcement to be credible, the economy shifts from point B of the line of the second specification to point C of the line of the first specification. By that shift, the stabilizing effect gets more substantial.

#### 1.4.2 Stylized Facts on the Stabilizing Effect

In this section we investigate the stabilizing effect of locking by looking at some stylized facts on the term structure of options.

Throughout this section, it is assumed to have constant locking date and independent factors with constant volatilities. The advantage of this framework, is that it is consistent with the simple option pricing formula (1.18). Moreover, even the term structure of implied volatilities can provide insight into the stabilizing feature of locking.

The option pricing formula (1.18), but also our intuitions suggest that longer options are more exposed to shocks occurring in the far future than options with shorter maturities. Or in other words,  $\sigma_x$  has higher relative weight in a longer option, then in a shorter one. And the opposite holds for  $\sigma_v$ . Consequently, if the term structure of options is downward sloping, then  $\sigma_v > \sigma_x$ . Moreover, if  $\sigma_v > \sigma_x$ , then the exchange rate is stabilized, because than  $\frac{\sigma_s}{\sigma_v} < 1$  as it is trivially follows from Equation (1.21). Therefore, a decreasing term structure can be interpreted as evidence for the stabilizing feature of locking. This relationship between the term structure and the stabilizing effect is not an if and only if relationship, because even if the term structure is somewhat upward sloping, it is possible to have  $\sigma_v > \sigma_x$  and stabilization.

Table 1.1 shows the average implied volatilities for each of the six maturities for the three countries. The six options are at-the-money options. In case of Czech Koruna and Polish Złoty the maturities are one-month, two-months, three-months, six-months, nine-months and one-year. Whereas in case of the Hungarian Forint the currency options have one-week, one-month, two-months, three-months, six-months and one-year maturities. For more details on the data see Csávás and Gereben (2005).

The average term structure of options is clearly downward sloping for Czech Koruna and Polish Złoty, but not for Hungarian Forint in the sample period between January 4, 2005, and March 6, 2007. In order to have a more detailed picture on the Hungarian term structure, the sample is divided into two equal sized subsamples and the average implied volatilities are calculated for the two subsamples separately. In the second half of the sample, the Hungarian term structure is downward sloping as well, but not in the first one.

All in all, the average term structures of these countries suggest that the prospect of locking has stabilized the Czech Koruna and Polish Złoty, and also, the Hungarian Forint in the second half of the sample. Purely based on the investigation of the term structure, we can not rule out that locking have stabilized the Forint even in the first half on the sample. In order to judge the stabilization in that period and to make inferences on the magnitude of the stabilizing effects, we estimate the factor volatilities in the next sections.

# 1.4.3 Estimated Stabilizing Effect with Constant Locking Date and Factor Volatilities

The option pricing formula (1.18) and historical option prices can be used to estimate the constant volatilities  $\sigma_v$ ,  $\sigma_x$  of the factors. We assume that the pricing errors, *i.e.*, the difference between the historical prices and the theoretical prices, are independent identically distributed with Gaussian distribution. Under this assumption, the volatilities  $\sigma_v$ ,  $\sigma_x$  can be estimated by non-linear least squares (NLLS) by solving the following minimization problem:

$$\min_{\sigma_{v},\sigma_{x}} \sum_{i=1}^{6} \left[ g(t, m(i), \sigma_{x}, \sigma_{v}) - \sigma_{t,i}^{imp} \right]^{2} .$$
(1.23)

The constant market expectation for the locking date T is calibrated to 1.Feb.2008, 13.Feb.2010, and 8.Jun.2010 for Czech Republic, Hungary and Poland respectively. The calibration is based on the filtered  $\hat{T}_{t_0}$  where  $t_0 = 4$ .Jan.2005 is the beginning of the sample. (See Section 1.4.4 on the filtering of  $T_t$ .)

The parameter c is calibrated to 10.75. By following Engel and West (2005), the calibration of c is based on estimates on the interest rate semi-elasticity of money demand. Frankel (1979), Stock and Watson (1993, 802, Table 2, Panel 1) and Bilson (1978) estimate the semi-elasticity to 29, 40, 60 respectively. These estimates are either on quarterly data or rescaled to quarterly basis. By dividing the quarterly semi-elasticity parameters by 4 we get the annualized semi-elasticities. The annualized parameter c is set to the arithmetic average ( $\hat{c} = 10.75$ ) of the annualized estimates of the three cited studies.

Table 1.2 presents the estimated factor volatilities. The stabilizing effect for the whole sample is measured as the average of the daily values of  $\frac{\sigma_{s,t}}{\sigma_v}$  that are calculated from the estimated constant volatilities  $\hat{\sigma}_v$  and  $\hat{\sigma}_x$  by using Equation (1.21). <sup>6</sup> Both the point estimates and the calculated average volatility ratios  $\frac{\sigma_{s,t}}{\sigma_v}$  support the existence of the stabilizing effect in all three countries as having  $\hat{\sigma}_v > \hat{\sigma}_x$  and  $\frac{\sigma_{s,t}}{\sigma_v} < 1$ . Moreover, the differences between the volatilities are significant in most of the cases based on the likelihood ratio test. <sup>7</sup> These results, however, are not very convincing on the presence of the stabilizing effect given that the goodness of fit of the constant volatility model is poor in all cases. The only exception may be the Czech Republic. In contrast to the other two countries, the time series of implied volatilities of Czech Republic have a clear decreasing pattern in our sample period, what explains the relatively good model performance. But even for Czech Republic the  $R^2$  can not be considered to be very high given that a rich panel data of historical option prices is used for estimation.

### 1.4.4 Stabilizing Effect with Time-Varying Locking Date and Factor Volatilities

This Section extends the model of Section 1.4.3. In order to improve the model performance, the volatilities are now considered to be time-varying. Moreover, the expected

<sup>&</sup>lt;sup>6</sup>It is worth noting that Equation (1.21) does not automatically ensure stabilization. In contrast to Equations (1.19) and (1.20), destabilization is not ruled out by construction of the measure of the stabilizing effect in (1.21). This feature of Equation (1.21) is discussed in Section 1.4.1.

<sup>&</sup>lt;sup>7</sup>The null hypothesis for testing the presence of stabilization should be the inequality  $\sigma_x \leq \sigma_v$  instead of the tested equality  $\sigma_v = \sigma_x$ . The test statistics has a chi-square distribution in the latter case, whereas it has a mixture of chi-square distributions under the null of inequality. As it is discussed by Gourieroux et al. (1982), the power of testing the inequality is substantially higher than that of the conventional likelihood ratio test for the equality.

locking date is allowed to change as well. Estimating the stabilizing effect with timevarying locking date is important, because of two reasons. First, the estimated stabilizing effect is sensitive to the expected locking date. Second, the expected locking dates of all three countries have been changed substantially in the sample period.

Until now, the paper has not shown any evidence on the fact that expectation on the locking date has been changed. The next Section fills in the gap by filtering the highly time-varying expected locking dates. Then, the stabilizing effect in the time-varying locking date framework is estimated by utilizing the filtered locking date. Finally, the paper shows that the estimated stabilizing effect is robust.

#### Filtering the Expected Locking Date

This section filters the expected locking date from the Euro and domestic yield curves. In order to demonstrate the large changes in the filtered locking date, its variance is estimated by applying the Kalman filter technique.

The yield curve method has been popularized by Bates (1999) and it has been applied for Hungary by Csajbók and Rezessy (2006). The method is based on the fact that after adopting the Euro, the interest rate differential will become marginal, but not of course before the adoption. The higher probability is attached by the market participants to the scenario that a country is already a full-fledged member of the Eurozone by a given year, the lower is the absolute forward differential for that particular year.

Along these lines the market expectation on the locking date can be estimated from the forward differentials. Appendix B describes the method in details.

Among the analyzed countries, Czech Republic has negative forward premium with an upward sloping term structure in the sample period. It is important to emphases that it is the *absolute* forward premium that is indicative about the EMU probabilities. Both the negative and the positive forward premiums should disappear after the regime switch, so the yield-curve method works even for countries with negative premium.

The yield curve based estimates of the market expectation are noise, therefore different smoothing methods are applied in the literature. Csajbók and Rezessy (2006) use HP filter. Here, the Kalman filter is applied, because it does not only smooth the yield curve based estimated  $\tilde{T}_t$ , but it also provides estimates for  $\sigma_{T,t}$  by maximizing the filtering likelihood. In the empirical part of the paper  $\sigma_{T,t}$  is restricted to be constant  $\sigma_{T,t} = \sigma_T$ . This assumption is still in line with our intuitions. The variance  $\sigma_{T,t}^2$  does not have to decrease over time in order to make the uncertainty disappear by the locking date. Even if  $\sigma_{T,t}$  is constant, the variance of the changes of the expected locking date is an increasing function of the expected time until locking as being  $\sigma_{dT_t}^2 = (T_t - t)^2 \sigma_T^2$  (see Equation (1.6)). Moreover, if the country is just about to lock its exchange rate, no uncertainty remains concerning the locking date.

The discrete time model used to Kalman filter the underlying time series of  $T_t$  is the following:

$$T_{t+1} = T_t + u_{t+1}$$
  $u_{t+1} \sim N\left(0, (T_t - t)^2 \sigma_T^2\right),$  (1.24)

$$\widetilde{T}_t = T_t + w_t \qquad w_t \sim N\left(0, \sigma_w^2\right),$$
(1.25)

where  $\widetilde{T}_t$  is the yield curve based noise estimates on the expected locking date. The system error  $u_{t+1}$  and the observation error  $w_t$  are assumed to be independent.

The estimates on  $\sigma_T$  is presented in Table 1.3 together with the estimated standard deviation of the observation error  $\sigma_w$ . The estimated  $\sigma_T$  is much higher for Czech Republic, than for the other two countries. But the variances of the changes of the expected locking date  $\sigma_{dT_t}^2$  are almost the same for the three countries as being calculated as  $\sigma_{dT_t}^2 = (T_t - t)^2 \sigma_T^2$ , and the time until locking  $T_t - t$  is the smallest for Czech Republic. Or in other words, Czech Republic is likely to be the first among the three countries that will be eligible to join the EMU.

Figure 1.3 shows both the original filtered market expectation for the locking dates  $T_t$ , and the smoothed  $T_t$ . Moreover, Figure 1.3 shows also survey data on the expected locking date. The Reuters polls queries the analysts about their opinion on the most likely year of EMU and ERM II entry of the three analyzed countries. These are the only references for the filtered  $T_t$  to be compared with. The filtered time series and the survey-based one show similar trends. The filtered locking dates suggest that expectations were subject to both large positive and negative shocks during 2006 and the first quarter of 2007 in all three countries. The filtered locking date is the most stable for Czech Republic in the sense that it has fluctuated in a much narrower band than for the other two countries. But even the expected locking date of Czech Republic is dominated by an upward trend pointing toward the postponement of the EMU entry. The hypothesis of having constant expected locking dates in these countries can easily be rejected.

# Estimated Stabilizing Effect with Time-Varying Locking Date and Factor Volatilities

This Section estimates a more flexible model, than that of Section 1.4.3. Here, the expected locking date is time-varying and it is equal to its filtered time series.

Moreover, the constant volatility assumption is relaxed in order to improve the model performance in terms of goodness of fit. There are two alternatives to make the volatilities of x and v time-varying.

One possible way is to assume that the volatilities are increasing in the expected time until locking. This assumption is plausible, because the future cumulated inflation rate becomes less and less uncertain and it is likely that more and more information are released on the preferred locking rate over time. Moreover, the macro stabilization of the EMU candidate country makes it plausible to have decreasing volatility of v as well.

The other way to relax the constant volatility assumption does not put any structure on the time series of the volatilities. It estimates the daily volatilities from cross-sectional data of option prices of each day.

The first approach does not seem to be appealing empirically, because the hypothesis of having significantly decreasing volatilities in this sample period is rejected by estimating a model with one specific structure on the time series of the volatilities. By following the second approach, the option pricing formula (1.18) is used again to estimate the daily volatilities  $\sigma_{v,t}$ ,  $\sigma_{x,t}$  of the factors. The parameter c is set to the same value as in Section 1.4.3. The parameters  $\sigma_{v,t}$  and  $\sigma_{x,t}$  are estimated only for a subsample, because not all 6 option data are available for each day, moreover, the first order condition of the minimization is not always satisfied. If either having missing daily observations or the first order condition of the minimization on daily data is failed to be fulfilled, no daily estimates could be made. The available estimated daily volatilities  $\hat{\sigma}_{v,t}$  and  $\hat{\sigma}_{x,t}$  are substituted into Equation (1.21) in order to calculate the daily values of  $\frac{\sigma_{s,t}}{\sigma_{v,t}}$ . The stabilizing effect for the whole sample is measured as the average of the available daily values of  $\frac{\sigma_{s,t}}{\sigma_{v,t}}$ .

the scarcity of data.

Table 1.4 presents the averaged estimated daily volatilities and the calculated stabilizing effect. In contrast to the  $R^2$ 's of Table 1.2, the  $R^2$ 's of Table 1.4 show that the model fits the data very well. This is not surprising, since in this time-varying specification not only two parameters ensure the fit of the model, but two times the size of the time series dimension. The stabilizing effect is substantial: the volatility of the exchange rate dropped approximately to its two third, *i.e.*, the average ratio of volatilities (average  $\frac{\widehat{\sigma}_{s,t}}{\sigma_{v,t}}$ ) are between 65% and 89% in the analyzed countries.

The estimated average stabilizing effect is significant, as it is suggested by the tstatistics. <sup>8</sup> The t-statistics is calculated on the following way. First, by investigating the sensitivity of the stabilizing effect to the fundamental volatility, we get that  $\sigma_{v,t}$  should be 70 %, 45 %, 20 % and 50 % lower for Czech Republic, the first subsample of Hungary, the second subsample of Hungary, and Poland respectively in order to have no stabilization. If for each time t the fundamental volatility  $\sigma_{v,t}$  would have been that small ( $= \sigma_{v,t}^*$ ) while having all the other parameters unchanged then the average ratio of the volatilities of the exchange rate and that of the latent exchange rate was one. Second, we test, whether the magnitude of the option pricing error can account for such a huge deviation of the estimated  $\sigma_{v,t}$  from the theoretical value of  $\sigma_{v,t}^*$ . Under the null  $\sigma_{v,t} = \sigma_{v,t}^*$ , the option pricing errors calculated with average parameter values should be 0.58 %, 0.82%, 0.81 %, and 1.05 % respectively in terms of implied volatilities. The t-tests reject the null at all commonly used significance levels.

Intuitively, the magnitude of the stabilizing effect should depend on two main determinants: the stability of market expectations for the locking rate, and the importance of expectations in determining the exchange rate. In case of an earlier Euro area entry, the stabilizing effect is likely to be more substantial, because market expectations for the

<sup>&</sup>lt;sup>8</sup>The applied test is different from the standard test on parameter restrictions. The standard tests, like the likelihood ratio test, Wald test, and Lagrange multiplier test are impossible to use because of the high dimension of the parameter set ( $2 \times$  the size of the time series dimension).

locking rate are themselves more stable. Moreover, the relative weight of the expectations in the exchange rate is also higher. Based on this argument, the prospect of locking should contribute the most to the stabilization of the Koruna. Actually, the estimated stabilizing effect of locking is found to be the highest in Czech Republic (65%).

#### **Robustness Check**

The previous Section estimates the stabilizing effect in the general framework, where both the locking date and the factor volatilities are time-varying. In this framework the theoretical option prices are just approximated by the simple option valuation formula (1.18), but their precise values may be different. The approximation error is due to the fact that the simple option valuation formula accounts neither for the stochastic exchange rate volatility caused by the time-varying factor volatilities nor for the jumps in the exchange rate volatility caused by the changes in the expected time until locking.

This Section justifies the pseudo estimation. It shows by simulation that the pseudo estimation method is conservative, *i.e.*, it underestimates the magnitude of the stabilizing effect. First, parameters of the model are calibrated for the simulation exercise. Second, this parameter set is used to generate option prices with 6 different maturities by applying Monte Carlo method. (See Appendix C on option valuation with Monte Carlo simulation). Third, the stabilizing effect is estimated like in the previous Section on the historical data. Finally, the estimated stabilizing effect is compared with its theoretical value.

The primary goal of this Section is to show that even if both the expected locking date and the volatilities of x and v are stochastic, the pseudo estimation is valid. Therefore, those parameters need to be calibrated the most carefully that determine the process of the stochastic locking date and the factor volatilities. The other parameters are randomly generated from a reasonable range.

In order to allow  $T_t$  to change substantially over time, the annualized volatility of  $\frac{dT}{T_t-t}$  is set equal to  $\sigma_T = 0.3$ . Imagine the event of postponing the regime switch substantially, by 3 years during a year while the originally expected time until locking is 5 years. Even this highly unlikely event may occur with more than 5% chance if  $\sigma_T$  is as large as 0.3.

The process of the instantaneous variances of the factors x and v are assumed to follow Cox-Ingersoll-Ross (CIR) process:

$$d\sigma_{v,t}^2 = \alpha_1(\alpha_2 - \sigma_{v,t}^2) + \alpha_3 \sigma_{v,t} \epsilon_{\sigma_v} \qquad \epsilon_{\sigma_v} \sim N(0,1)$$
(1.26)

$$d\sigma_{x,t}^2 = \beta_1(\beta_2 - \sigma_{x,t}^2) + \beta_3 \sigma_{x,t} \epsilon_{\sigma_x} \qquad \epsilon_{\sigma_x} \sim N(0,1) . \tag{1.27}$$

The calibration of the parameters of the processes of  $\sigma_{v,t}$  and  $\sigma_{x,t}$  are motivated by their time series filtered in the previous Section. Table 1.5 summarizes the choice of some parameters. All the other parameters of the model are calculated from the calibrated ones by using Equations (1.4), (1.5), (1.6), (1.7), (1.8), (1.9), (1.26), and (1.27).

For each set of randomly generated parameters that satisfies the restrictions in Table 1.5, option prices are simulated. The stabilizing effect is estimated on the simulated data by applying the pseudo estimation. By comparing the theoretical and the estimated stabilizing effect, we obtain that the relative volatility  $\frac{\sigma_s}{\sigma_v}$  is always overestimated. This can be interpreted as evidence for the conservative nature of the pseudo estimation.

This result is not surprising, because by not taking into account the stochastic nature of the volatility, the options are undervalued and the pricing error is higher at options with longer maturities.<sup>9</sup> So, the pseudo estimates on both  $\sigma_x$  and  $\sigma_v$  are upward biased, and the bias in  $\hat{\sigma}_x$  is higher.

<sup>&</sup>lt;sup>9</sup>As it is pointed out by Hull (1997, p. 620): "For options that last less than a year, the pricing impact of a stochastic volatility is fairly small in absolute terms. It becomes progressively larger as the life of option increases."

## 1.5 Conclusion

This paper introduced a theoretical model for exchange rates subject to future locking. The model is the conventional asset-pricing exchange rate model extended with the future locking assumption. The asset-pricing model with final locking is a three-factor model, where the factors are the latent exchange rate and the market expectations for the locking rate and date. The locking is modeled as being state-contingent motivated by the Maastricht criteria.

In this regime switching framework the volatility of the exchange rate is a simple function of the variances and covariances of the three factors. When the factors are uncorrelated, the variance of the returns of the exchange rate is the weighted sum of the variances of the expected locking rate and the latent exchange rate. The closer is the expected locking date, the higher is the relative weight of the first term. When the expectation for the locking rate is relatively stable, its increasing importance at determining the exchange rate volatility has two implications. The time series one is that the exchange rate gets more and more stable as the regime switch is approaching. The cross sectional implication is that the slope of the term structure of currency options decreases over time.

The policy conclusion of the paper is that the authorities can mitigate the exchange rate volatility by credibly announcing the targeted locking date. The exchange rate stabilization works not only through the direct channel of decreasing the uncertainty concerning the locking date, but also through the indirect one. As the confidence in the locking date increases, the variance of the expected access inflation rate cumulated until locking decreases. And so does the volatility of the expected locking rate given that the final conversion rate will be determined partly by the purchasing power parity. Obviously, a more stable anchor for the exchange rate, the locking rate, can mitigate the volatility.

The empirical part of the paper shows that there was no credibly announced locking date in the analyzed countries, *i.e.*, the expected locking dates filtered from the forward curves have an upward trend in these countries. The upward trend can be interpreted as the continuous postponements of the locking date in the sample period between January 2005 and March 2007. The most important empirical finding is that the stabilizing effect of the future locking is substantial despite of the highly uncertain EMU entry dates. Moreover, this qualitative result is in line with our view based on the downward sloping term structure of option prices.

The model simplifies the institutional arrangement of EMU entry. It is assumed that countries switch from the floating regime directly to the fixed one. In reality, the road to the Euro consists of at least two regime switches. First, entering the ERM II and than locking the exchange rate. The exchange rate is likely to be stabilized in the intermediate regime, the ERM II, as it is suggested both by theoretical and empirical papers (See: Krugman (1991), Artis (1993)). However, this stabilizing effect starts only after the exchange rate band is introduced. The main point is that in contrast to the exchange rate, the volatility is not determined in a forward looking way. *I.e.*, expecting to have controlled volatility in a future regime does not reduce by itself the volatility in the current regime. Therefore, the intermediate target zone regime can be disregarded when

**CEU eTD Collection** 

the volatility in the regime preceding the ERM II entry is examined. For the same reason, if the exchange rate is stabilized in the floating regime, then it can be fully attributed to the locking and not to the honeymoon effect. The stabilizing effect estimated from currency option prices is not biased either, because even the options with the longest maturity expire before the expected ERM II entry date. Therefore, neither the historical option prices, nor the estimated factor volatilities, nor the estimates on the stabilizing effect are influenced by the intermediate regime.

This regime switching framework with stochastic terminal date and rate is very likely to create interest in the future because of its broad applicability. First, the model can easily be applied to stock markets. The prospect of mergers affects the volatility of the potential target companies even if neither the date of mergers nor the price offered to stock holders are known. Then, the current process of EMU enlargement, the possibility of the creation a single currency in Asia and the pegging to the US dollar in Latin America will make the exchange rate regime switches an interesting future topic of research. Finally, the same framework can even be applied to countries with no plan to lock their exchange rates, but with convergence to the purchasing power parity.

## 1.6 Acknowledgements

The author gratefully acknowledges comments and suggestions from Péter Benczúr, Giuseppe Bertola, Olivier Blanchard, Attila Csajbók, Zsolt Darvas, Paul De Grauwe, Hans Dewachter, Balázs Égert, András Fülöp, Christian Gourieroux, László Halpern, Csilla Horváth, Péter Karádi, Júlia Király, Tamás Kollányi, Péter Kondor, István Kónya, Péter Medvegyev, László Mátyás, Ádam Reiff, Jean-Marc Robin, Ákos Valentinyi and David Veredas.

The paper has been presented on the following conferences:

- Econometric Society European Winter Meeting, November 16-17, 2007, Brussels;
- 8th Annual Missouri Economics Conference, March 28-29, 2008, Columbia, MO;
- 1st Conference of the Hungarian Economics Association (MKE), December 19-20, 2007, Budapest;
- Doctoral Seminar Series at Corvinus University, April 4, 2008, Budapest;
- Seminar at Katholieke Universiteit Leuven, June 4, 2008, Leuven.

Thanks are extended to the organizers and participants of these conferences and to the participants of discussions at the Magyar Nemzeti Bank and Central European University.

The paper has been submitted to the Journal of International Money and Finance.

#### Appendix A

This Appendix proves that the derived function  $s_t = f(t, v_t, x_t, T_t)$  of (1.9) satisfies the implicit relationship (1.1) between the exchange rate and fundamental.

By calculating the partial derivatives of (1.9) and by substituting these derivatives and  $\mu_{v,t} = \mu_{x,t} = \mu_{T,t} = 0$  into (1.14), we obtain

$$ds_{t} = \left[\frac{1}{c}e^{-\frac{T_{t}-t}{c}}\left(x_{t}-v_{t}\right) + \frac{1}{2}\frac{1}{c^{2}}e^{-\frac{T_{t}-t}{c}}\left(x_{t}-v_{t}\right)\sigma_{T,t}^{2}\left(T_{t}-t\right)^{2} + \frac{1}{2}\frac{1}{c}e^{-\frac{T_{t}-t}{c}}\rho\left(dz_{T,t},dz_{x,t}\right)\left(T_{t}-t\right)\sigma_{T,t}\sigma_{x,t} + \frac{1}{2}\frac{1}{c}e^{-\frac{T_{t}-t}{c}}\rho\left(dz_{T,t},dz_{v,t}\right)\left(T_{t}-t\right)\sigma_{T,t}\sigma_{v,t}\right]dt + \left(1-e^{-\frac{T_{t}-t}{c}}\right)\sigma_{v,t}dz_{v,t} + e^{-\frac{T_{t}-t}{c}}\sigma_{x,t}dz_{x,t} - \frac{1}{c}e^{-\frac{T_{t}-t}{c}}\left(x_{t}-v_{t}\right)\left(T_{t}-t\right)\sigma_{T,t}dz_{T,t}.$$
 (1.28)

By using (1.28), the expected instantaneous change of the exchange rate can be expressed as

$$\frac{E_t(ds_t)}{dt} = \left[ \frac{1}{c} e^{-\frac{T_t - t}{c}} (x_t - v_t) + \frac{1}{2} \frac{1}{c^2} e^{-\frac{T_t - t}{c}} (x_t - v_t) \sigma_{T,t}^2 (T_t - t)^2 + \frac{1}{2} \frac{1}{c} e^{-\frac{T_t - t}{c}} \rho \left( dz_{T,t}, dz_{x,t} \right) (T_t - t) \sigma_{T,t} \sigma_{x,t} + \frac{1}{2} \frac{1}{c} e^{-\frac{T_t - t}{c}} \rho \left( dz_{T,t}, dz_{v,t} \right) (T_t - t) \sigma_{T,t} \sigma_{v,t} \right].$$
(1.29)

The implicit function (1.1) can be rewritten as

$$\frac{E_t(ds_t)}{dt} = \frac{1}{c} \left( s_t - v_t \right) \ . \tag{1.30}$$

Consequently, if the right-hand-side (RHS) of Equation (1.29) is equal to the RHS of Equation (1.30), then the implicit function (1.1) is satisfied by (1.9). In order to prove the equality, it is sufficient to show that the first term of the RHS of (1.29) is equal to the RHS of (1.30), whereas the other three terms of (1.29) sum up to zero.

By rearranging (1.9), we obtain that the first term of the RHS of (1.29) is equal to the RHS of (1.30).

$$\frac{1}{c}(s_t - v_t) = \frac{1}{c}e^{-\frac{T_t - t}{c}}(x_t - v_t) \quad . \tag{1.31}$$

What remains to prove is that the other three terms of (1.29) sum up to zero. This follows easily from Equation (1.8).

#### Appendix B

This Appendix shortly introduces the yield curve method applied to estimate market expectation for the locking date. The paper by Bates (1999) and Csajbók and Rezessy (2006) provide a more detailed description on this method.

The market expectation, formed at time t, on the date of locking  $T_t$  can be estimated from the forward differentials as follows. The expected value is calculated from the subjective probability distribution of the year the country enters the EMU.

$$T_{t} = \sum_{i=t}^{T} p_{t}(EMU_{i}) \ i , \qquad (1.32)$$

where  $p_t(EMU_i)$  is the probability that the market attaches at time t to the scenario in which the country becomes a full member of Eurozone in the *i*th year. The distribution is assumed to have finite support, the country will join the Eurozone in the year  $\overline{T}$  at the latest. The marginal probability  $p_t(EMU_i)$  can be calculated from the cumulative probability distribution  $P_t(EMU_i)$ . The interpretation of  $P_t(EMU_i)$  is straightforward: it is the probability that the market attaches at time t to the scenario in which the country is in the EMU by the *i*th year. The cumulative probability can be derived from the pricing equation of the one-year forward interest differential.

$$FS_{t,i} = (1 - P(EMU_i))FS_{t,i}^{non-EMU_i} + P(EMU_i)FS_{t,i}^{EMU_i}, \qquad (1.33)$$

where  $FS_{t,i}$  is the one-year forward interest rate differential for year *i* observed at time *t*. The  $FS_{t,i}^{non-EMU_i}$  and  $FS_{t,i}^{EMU_i}$  are the expected interest rate differentials under the two alternative scenarios, *i.e.*, the accession country is either in or out the Eurozone by year *i*. By rearranging (1.33) we obtain

$$P(EMU_i) = \frac{FS_{t,i}^{non-EMU_i} - FS_{t,i}}{FS_{t,i}^{non-EMU_i} - FS_{t,i}^{EMU_i}}.$$
(1.34)

Among the RHS variables of (1.34) only  $FS_{t,i}$  is observable. However, by assuming that the analyzed countries will surly enter the EMU in nine years, and have almost zero chance to become an EMU member in one year, one can set  $FS_{t,i}^{non-EMU_i} = FS_{t,t+1}$  and  $FS_{t,i}^{EMU_i} = FS_{t,t+9}$ .

#### Appendix C

This Appendix introduces the Monte Carlo simulation method used for valuing options. It is used to generate the distribution of the exchange rate at the maturity of the option. Once the distribution of the exchange rate is given by simulation, options can be priced by

$$g(t, m, K, i_{t,m}) = E^Q(max(S_{t+m} - K, 0))e^{-i_{t,m}m}, \qquad (1.35)$$

where  $S_{t+m}$  is the exchange rate at maturity,  $i_{t,m}$  is the domestic risk free interest rate at time t for maturity m. The expected value is calculated under the risk neutral measure Q. Parameter K is the strike price of the option. In our case options are at-the-money, therefore the strike price is the forward exchange rate  $K = S_t e^{(i_{t,m}-i_{t,m}^*)m}$ .

The distribution of  $S_{t+m}$  is given by the system of Equations (1.4), (1.5), (1.6), (1.7), (1.8), (1.9), (1.26), (1.27), and by the initial values of  $s_{t0}$ ,  $T_{t0}$ ,  $x_{t0}$ , and by the constant parameters c,  $\sigma_T$ , and  $\frac{dx}{dT}$ . Section 1.4.4 discusses in details the calibration of these parameters. All other parameters are functions of the known ones.

The initial value of all three state variables, and the covariance matrix of the innovations of the next period are used to simulate the next period state. This simulated state is used to calculate the covariance matrix of the innovations of the next period. This covariance matrix is then used to simulate the next period state.

To some up, the simulation of one exchange rate path consist of the following steps: Step 1. Calculate  $v_t$  from  $s_t$ ,  $T_t$ ,  $x_t$ , c by using Equation (1.9).

**Step 2.** Calculate the covariance matrix  $P_t$  of the vector  $(dz_{v,t}, dz_{x,t}, dz_{T,t})$  of normally distributed shocks by using the previous state vector  $(v_t, x_t, T_t)$ , and the parameters  $\sigma_T$ ,  $\frac{dx_t}{dT_t}$ , and Equations (1.4), (1.5), (1.6), (1.7), (1.8), (1.9), (1.26), (1.27).

**Step 3.** Generate the vector  $dz_t = (dz_{v,t}, dz_{T,t}, dz_{T,t})$  of normally distributed innovations with the correlation matrix  $P_t$ , by using the Cholevsky decomposition.

**Step 4.** Calculate the state vector  $(v_{t+1}, x_{t+1}, T_{t+1})$  by using  $dz_t$  and  $\sigma_{v,t}$ ,  $\sigma_{x,t}$ ,  $\sigma_T$  and Equations (1.4), (1.5), (1.6).

**Step 5.** Calculate  $s_{t+1}$  from the state vector  $(v_{t+1}, x_{t+1}, T_{t+1})$  by using Equation (1.9).

**Step 6.** Update t to t + 1 and repeat steps between 2 and 5 N times.

By simulating M number of independent exchange rate paths, we obtain the simulated distribution of the exchange rate.

# Bibliography

- Artis, M. J., 1993. The Stabilizing Effect of the ERM on Exchange Rates and Interest Rates: An Empirical Investigation. *IMF Working Paper* No. 93/67. http://ssrn.com/abstract=883549
- [2] Bates, D. S., 1999: Financial Markets' Assessment of Euro. Carnegie-Rochester Conference Series on Public Policy 51 (1), 229–269.
- [3] Bilson, J. F. O., 1978: The Monetary Approach to the Exchange Rate: Some Empirical Evidence. *IMF Staff Paper* 25, 48–75.
- [4] Csajbók, A., Rezessy, A., 2006: Hungary's Eurozone Entry Date: What do the Markets Think and What If They Change Their Minds? *Contemporary Economic Policy* 24 (3), 343–356.
- [5] Csávás, Cs., Gereben, Á., 2005: Hagyományos és egzotikus opciók a magyar devizapiacon. MNB Working paper 35.
- [6] Djajic, S., 1989: Dynamics of the exchange rate in anticipation of pegging. *Journal* of International Money and Finance 8 (4), 559–571.
- [7] De Grauwe, P., Dewachter, H., Veestraeten, D., 1999a: Price dynamics under stochastic process switching: some extensions and an application to EMU. *Journal of International Money and Finance* 18 (2), 195–224.
- [8] De Grauwe, P., Dewachter, H., Veestraeten, D., 1999b: Explaining Recent European Exchange-Rate Stability. *International Finance* 1 (2), 1–31.
- [9] Engel, C., West, K. D., 2005: Exchange Rates and Fundamentals. Journal of Political Economy 113 (3), 485–517.
- [10] Flood, R. P., Garber, P. M., 1983: A model of stochastic process switching. *Econo-metrica* 51 (3), 537–551.
- [11] Frankel, J. A., 1979: On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials. *American Economic Review* 69, 610–22.
- [12] Froot, K. A., Obstfeld, M., 1991a: Stochastic process switching: some simple solutions. *Econometrica* 59, 241–250.
- [13] Froot, K. A., Obstfeld, M., 1991b: Exchange rate dynamics under stochastic regime shifts: a unified approach. *Journal of International Economics* 31 (1/2), 203–229.
- [14] Gourieroux, C., Holly, A., Monfort, A., 1982: Likelihood Ratio Test, Wald Test, and Kuhn-Tucker Test in Linear Models with Inequality Constraints on the Regression Parameters. *Econometrica* 50, 63–80.
- [15] Gourieroux, C., Jasiak, J., 2001: Financial Econometrics: Problems, Models, and Methods. Princeton University Press, Princeton.
- [16] Hagen, J., Traistaru-Siedschlag I., 2006: Macroeconomic Adjustment in the New EU Member States. In: Balling, M. (Eds.), The European Money and Finance Forum, *SUERF Studies* 2006/4.
- [17] Hamilton, J. D., 1990: Time Series Analysis. Princeton University Press, Princeton.
- [18] Harvey, A. C., 1990: Forecasting, structural time series models and the Kalman filter. Cambridge University Press, Cambridge.
- [19] Hull, J. C., 1997: Option, futures and other derivatives. Prentice Hall.
- [20] Krugman, P., 1991: Target Zones and Exchange Rate Dynamics. The Quarterly Journal of Economics 106 (3), 669–682.
- [21] Miller, M., Sutherland, A., 1994: Speculative anticipations of sterling's return to gold: was Keynes wrong? *Economic Journal* 104 (4), 804–812.
- [22] Naszódi, A., 2004: Target Zone Rearrangements and Exchange Rate Behavior in an Option-Based Model. *MNB Working Papers* 2.
- [23] Naszodi, A., 2007: Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?, Working Paper In: Focus on European Economic Integration 07/1 Vienna: Oesterreichische Nationalbank, pp. 115–134.
- [24] Naszodi, A., 2008: Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?, *MNB Working Papers* 1.
- [25] Obstfeld, M., Stockman, A. C., 1985: Exchange-rate dynamics. In: Kenen, P. B., Jones, R. W. (Eds.), Handbook of International Economics, North-Holland, Amsterdam, pp. 917–977.
- [26] Stock, J. H., Watson, M., 1993: A Simple Estimator of Cointegrating Vectors in Higher Order Integrating Systems. *Econometrica* 61 (July), 783–820.
- [27] Wilfling, B., Maennig, W., 2001: Exchange rate dynamics in anticipation of timecontingent regime switching: modelling the effects of a possible delay. *Journal of International Money and Finance* 20, 91–113.

## Tables and Figures

	CZ	HU			PL
Sample	full	full	1st half	2nd half	full
$\sigma^{imp}_{1W}$	-	7.71%	6.37%	9.06%	-
$\sigma_{1M}^{imp}$	4.8%	7.69%	6.48%	8.9%	8.64%
$\sigma^{imp}_{2M}$	4.76%	7.71%	6.72%	8.7%	8.5%
$\sigma^{imp}_{3M}$	4.72%	7.78%	6.99%	8.57%	8.41%
$\sigma^{imp}_{6M}$	4.71%	7.9%	7.35%	8.46%	8.3%
$\sigma^{imp}_{9M}$	4.75%	-	-	-	8.29%
$\sigma_{1Y}^{imp}$	4.71%	8.04%	7.65%	8.43%	8.25%
Num. obs.	3168	3012	1506	1506	3162

 Table 1.1: The term structure of implied volatilities (Source: Reuters)

	CZ	HU		PL
Sample	full	1st half	2nd half	full
$\widehat{\sigma}_v$	19.53%	12.73%	14.15%	19.43%
(t-stat)	(200.22)	(11.82)	(8.97)	(145.91)
$\widehat{\sigma}_x$	4.12%	8.21%	10.6%	7.72%
(t-stat)	(175.42)	(19.02)	(35.06)	(81.04)
$\textbf{LR-test} \ \widehat{\sigma}_x = \widehat{\sigma}_v$	4300.08	16.71	3.1	1285.53
Critical value - $5\%$ signif level	3.84	3.84	3.84	3.84
Average $\frac{\widehat{\sigma_{s,t}}}{\sigma_v}$	24.96%	54.4%	60.6%	43.82%
$R^2$	57.42%	0.24%	-0.67%	16.94%
Num. obs.	3168	1506	1506	3162

Table 1.2: The estimated constant volatilities and average stabilizing effect – with constant locking date

	$\mathbf{CZ}$	HU	$\mathrm{PL}$
$\widehat{\sigma}_T$	5.02%	1.57%	1.93%
$\widehat{\sigma}_w$	56.86%	26.9%	53.42%

Table 1.3: The estimated parameters of the system covariance and the observation covariance of the Kalman filtered  $T_t$ 

	CZ	H	[U	$\mathbf{PL}$
Sample	full	1st half	2nd half	full
Average $\widehat{\sigma}_{v,t}$	12.66%	11.52%	13.54%	13.22%
Average $\hat{\sigma}_{x,t}$	4.57%	9.13%	10.66%	9.84%
Average $\frac{\widehat{\sigma_{s,t}}}{\sigma_v}$	65.17%	76.97%	88.85%	76.41%
Multiplicator $rac{\widehat{\sigma}_{v,t}}{\sigma_{v,t}^*}$ for no stabilization,	170%	145%	120%	150%
<i>i.e.</i> , with $\sigma_{v,t}^*$ the average $\frac{\widehat{\sigma_{s,t}}}{\sigma^*_v} = 1$				
Theoretical option price	4.51%	7.16%	8.39%	7.93%
with average parameters				
$\mathbf{g}(\mathbf{t}, 1\mathbf{Y}, \mathbf{average} \ \widehat{\sigma_x}, \mathbf{average} \ \widehat{\sigma_v}, \ \mathbf{average} \ \widehat{\sigma_t})$				
Theoretical option price	3.93%	6.34%	7.57%	6.88%
with $\sigma_{v,t}^*$ (no stabilization)				
$\mathbf{g}(\mathbf{t}, 1\mathbf{Y}, \mathbf{a}\mathbf{v}\mathbf{e}\mathbf{r}\mathbf{a}\mathbf{g}\mathbf{e}\;\widehat{\sigma_{x}}, \mathbf{a}\mathbf{v}\mathbf{e}\mathbf{r}\mathbf{a}\mathbf{g}\mathbf{e}\;\widehat{\sigma_{v}}^{*}, \; \mathbf{a}\mathbf{v}\mathbf{e}\mathbf{r}\mathbf{a}\mathbf{g}\mathbf{e}\;\widehat{T_{t}})$				
Difference between the option prices	0.58%	0.82%	0.81%	1.05%
(t-stat)	(5.39)	(13.37)	(3.83)	(10.41)
$R^2$	97.18%	98.85%	93.93%	$9\overline{6.91\%}$
Num. obs.	1764	390	318	1374

 Table 1.4:
 The average estimated time-varying volatilities and stabilizing effect – with time-varying locking date

$\sigma_T$	0.3
$\alpha_1$	0.3452
$\alpha_2$	0.0347
$lpha_3$	0.0409
$\beta_1$	0.3773
$eta_2$	0.0157
$eta_3$	0.0341
$\sigma_{v,t0}$	uniform [5%;20%]
$ \rho(dz_{T,t0}, dz_{x,t0}) $	uniform $[0;1]$
$\frac{dx}{dT}$	uniform $[0;3\%]$
$T_{t0} - t0$	5yrs
$ \rho(dz_{T,t0}, dz_{v,t0}) $	uniform $[0;1]$
$x_{t0}$	0
c	10.75
<b>Restriction:</b>	$abs(dv_{t0} - ds_{t0}) < 0.1$
<b>Restriction:</b>	$abs(dx_{t0} - \overline{ds_{t0}}) < 0.1$
<b>Restriction:</b>	$\sigma_{v,t} > \sigma_{x,t} > .04$

Table 1.5: The parameters of the simulated option prices



**Figure 1.1**: Stylized stabilization and destabilization as a function of time until locking and relative volatilities of x and v.  $\frac{\sigma_s}{\sigma_v} = \left[ \left( 1 - e^{-\frac{T-t}{c}} \right)^2 + e^{-2\frac{T-t}{c}} \frac{\sigma_x^2}{\sigma_v^2} \right]^{\frac{1}{2}}$ , with c = 10.75. If  $\frac{\sigma_s}{\sigma_v} < 1$  then the exchange rate is stabilized by the prospect of locking, otherwise it is destabilized.



**Figure 1.2**: Stylized stabilization and destabilization in the last 5 years before the expected locking. In the specification with three stochastic factors the parameters are  $\sigma_{T,t} = 1.8(T_t - t)^{-\frac{1}{1.2}}$ , c = 10.75,  $\sigma_{x,t} = 5\%\sqrt{T_t - t}$ ,  $\sigma_{v,t} = 6\%$ ,  $\rho (dz_{T,t}, dz_{x,t}) = 0.31\%(min(T_t - t, 5yrs))^{2-\frac{1}{1.2}}$ ,  $\rho (dz_{T,t}, dz_{v,t}) = 2\%(min(T_t - t, 5yrs))^2$ . The parameters are calibrated to the same values in the other specifications with some deterministic factors, except that the corresponding variances and correlations are restricted to zero.









Poland



**Figure 1.3**: The average expectation of the analysts for the EMU and the ERM II entry date and the filtered locking date  $T_t$ 

## Chapter 2

# Filtering the Market Expectation for the Euro Locking Rate in the Extended Asset-Pricing Exchange Rate Model

This paper investigates the market expectation for the Euro locking rate in a regime switching model. This regime switch is the locking of the exchange rates of the EMU candidate countries at the final conversion rate. First, the process of the exchange rate is derived in the extended asset-pricing exchange rate model as a function of the processes of three factors, the latent exchange rate and the market expectation for the Euro locking rate and date. Then, the model is applied to analyze the exchange rates of the Czech Koruna, the Hungarian Forint, and the Polish Złoty versus the Euro in the period between January 4, 2005, and March 6, 2007. The latent exchange rate, the expected locking rate and date are filtered from the historical exchange rate, interest rate and currency option data. The expected locking rate is found to be an important component of the exchange rate.

JEL: F31, F36, G13.

*Keywords:* Eurozone entry, asset-pricing exchange rate model, stochastic process switching, factor model, Kalman filter.

## 2.1 Introduction

This paper investigates how the expected Euro locking rate influences the exchange rates of EMU candidate countries. This is studied in a regime switching model, where the regime switch is the adoption of Euro. A theoretical exchange rate model and a rich data set are used to filter the market expectation for the Euro locking rate. Besides the locking rate, the paper filters also the expected locking date, and the latent exchange rate that would prevail if the currency was never going to be locked.

The exchange rate model used in this paper is developed by Naszodi (2008b). It extends the conventional asset-pricing exchange rate model with the final locking assumption in order to examine how the prospect of locking influences the exchange rate volatility. The exchange rate is derived to be the linear combination of the latent exchange rate and the expected Euro locking rate in the extended asset-pricing model. The relative weights of the linear combination depend on the expected locking date. This paper reviews the model and further develops it. Naszodi (2008b) models the regime switch as being state contingent motivated by the Maastricht criteria. The novelty of this paper is that it models the Maastricht criteria in a more explicit way.

The exchange rate dynamics in anticipation of a switch from flexible to fixed exchange rate regimes is studied by a number of papers. These formalize the regime switch either as being time-contingent or state-contingent. Time-contingent approaches are used by Obstfeld and Stockman (1985), Djajic (1989), Miller and Sutherland (1994), De Grauwe et al. (1999a), and Wilfling and Maennig (2001). While state-contingent approaches are used by Flood and Garber (1983), Froot and Obstfeld (1991a,b). In these state-contingent models the exchange rate system is changed whenever the exchange rate reaches a predetermined boundary. This paper and Naszodi (2008b) are distinguished from the above studies by modelling the time until regime switch as a stochastic function of the expected inflation rate and the fundamental. This type of state contingency is more relevant for EMU candidate countries that have to fulfill the Maastricht criteria to become eligible to enter the Monetary Union. For these countries, the better the fundamentals and the lower the inflation rate, the higher is the chance of meeting all the Maastricht criteria at an early date.

In the empirical part of the paper, the model is applied to analyze the exchange rate dynamics of three new EU members prior to the adoption of the Euro: the Czech Republic, Hungary, and Poland, which will join to the Eurozone and will lock their exchange rates irrevocably at the final conversion rate. In the case of these three countries neither the locking rate nor the locking date are yet set. However, market participants have already started to form expectations on both. These expectations are in the focus of our interest, because they are likely to influence the exchange rate even a couple of years before the locking. The expected locking date and rate and also the latent exchange rate are filtered from the historical exchange rate, interest rate and currency option data. The expected locking date is estimated from the Euro and domestic forward curves by following the method suggested by Bates (1999).<sup>1</sup> The other two factors are filtered from the historical exchange rate by the Kalman filter technique. The Kalman filter decomposes the exchange rate changes into changes in the remaining two factors by using the identification through the variances.

De Grauwe et al. (1999b) also uses the Kalman filter in order to access the weights of the expected conversion rates in the exchange rates of countries that participated in the first wave of EMU. Their sample is from the period just before the Euro was established, when there was almost no uncertainty for the locking date and the uncertainties for the locking rates were moderate. Whereas in case of the currencies analyzed by this paper, the expected time until locking is more than 3 years and both the locking rate and date are highly uncertain. Moreover, the degree of uncertainty is likely to be time-varying that motivates us to to allow for hereroscedasticity.

These differences necessitate some deviations from the metodology of De Grauwe et al.. First, a more complex model is used that captures the stochastic changes in the expected locking date. Second, the factor variances are estimated more carefully, rather than estimating constant variances simply from the time series of exchange rates.

The contribution of the paper to the econometric literature on filtering heteroscedastic latent factors is that it identifies the factor variances from a special dataset. This dataset consists of option prices with different maturities, where the price of the underlying asset of these options is the observable variable in the filtering problem. The paper combines a non-linear filtering method for the variance process with a linear filtering method for the latent factors.

For those assets, where option prices are not available, alternative approaches can be applied. Harvey et al (1992) filter both the factors and their variance processes exclusively from the time series of the asset price. As it is shown by Fiorentini et al (2004), the Kalman filter-based Gaussian approximation introduced by Harvey et al (1992) produces reasonable results when the signal to noise ratio is high, and the variation of the volatility of the factor is low <sup>2</sup>, but it may lead to substantial biases in other cases. Fiorentini et al (2004) propose to use a state of the art method to solve the previous problem and to carry out likelihood inference on latent models. This method is based on the Markov chain Monte Carlo (MCMC) algorithm and it is developed to filter the latent factor within the context of the same conditional heteroscedastic model used by Harvey et al (1992).

The filtering method applied in this paper has the following advantages over those of Harvey et al (1992) and Fiorentini et al (2004). First, in contrast to the GARCH model, it does not assume that there is only one shock driving innovations to the level of the latent factors and their variances. Therefore, the uncertainty for the locking rate is allowed to increase substantially even without any change in the expected locking rate. Second, the proposed method of this paper is likely to provide more efficient estimates, simply because

<sup>&</sup>lt;sup>1</sup>The yield curve based EMU probability calculator approach has been implemented inter alia by Lund (1999) and Favero et al. (2000). An alternative of the yield curve based EMU probability calculator is the currency options based calculator suggested by Driessen and Perotti (2004).

<sup>&</sup>lt;sup>2</sup>The extreme case of having no variation of the volatility, *i.e.*, the conditionally homoscedastic Gaussian case is discussed by Geweke and Zhou (1996).

of using more information captured by the option prices. Third, this method reduces the dimension of the state vector by using direct information on the volatility processes.

The identification in the option-based method of the time-varying variances is based on the following. Option prices both with long and short maturities are functions of the variances of the latent factors. However, the long end of the option term structure is influenced relatively more by the variance of one of the factors (the expected locking rate) than the short end. At the same time, the relative weight of the variance of the other factor (the latent exchange rate) in the option price is decreasing in the maturity.

Rather than filtering market expectations, one could, in principle, have obtained the expectations from an alternative source (Reuters polls) or with an alternative method (estimating equilibrium exchange rates). However, filtering has some advantages over its alternatives, for the reasons outlined below.

Reuters regularly<sup>3</sup> surveys the expectations of analysts for the dates of EMU and ERM II entries and for the central parities in ERM II of all three analyzed countries. The central parities expected by respondents may be considered as the market expectations for the final conversion rate. And the reported expectations for either the date of EMU or ERM II entries may be considered as the market expectations for the locking date. Yet extracting market expectations from daily historical exchange rate and interest rate data may yield more accurate and more up-to-date information than the monthly or quarterly Reuters polls. Moreover, a higher frequency of the filtered expectations enables us to investigate how the process of the exchange rate is influenced by the future EMU entry.

As the EMU candidate countries aim at having their irrevocable conversion rates set equal to their equilibrium exchange rates, reliable estimates on the latter also reflect market expectations for the final conversion rate. In the context of our research this concept poses, however, at least three problems.<sup>4</sup> First, economists use a number of different concepts to define, and a number of different methods<sup>5</sup> to estimate the equilibrium real exchange rate. Second, these estimates refer to the real, rather than the nominal, exchange rate. Third, market expectations might deviate from the estimated nominal equilibrium exchange rate, especially if the choice of the final conversion rate is based not only on observable fundamentals, but also on unobservable factors. For instance, if not only economic, but also political considerations play a role in the negotiations over the locking rates. Another example is when the locking rate is determined in a backward looking manner, *i.e.*, by a kind of average of past exchange rates.<sup>6</sup> In that case, speculative price pressure<sup>7</sup> can push away the exchange rate from its equilibrium level and the

<sup>&</sup>lt;sup>3</sup>The frequency of these surveys is monthly for Hungary and quarterly for Czech Republic and Poland.

<sup>&</sup>lt;sup>4</sup>Egert, Halpern and MacDonald (2006) survey a number of issues related to the equilibrium exchange rates of transition economies. They conclude that "deriving a precise figure for the equilibrium real exchange rates in general and also for the transition economies is close to mission impossible as there is a great deal of model uncertainty related to the theoretical background and to the set of fundamentals chosen."

 $<sup>^{5}</sup>$ Williamson (1994) gives an overview of the widely used FEER, BEER, NATREX methods.

<sup>&</sup>lt;sup>6</sup>This rule for setting the locking rate is better known as the Lamfalussy rule, named after the former president of the European Monetary Institute.

<sup>&</sup>lt;sup>7</sup>The personal view of the author is that the chances for the analyzed currencies to be subject to

bubble will never burst due to the locking.

One important contribution of the paper to the literature is that it filters the subjective expectation from market prices, which should mirror all the information that the market uses at forming expectations and also at pricing bonds, options and currencies. This set of information is likely to be wider than the one used in the literature of estimating equilibrium exchange rate.

Another important contributions of the paper to the exchange rate literature is that it finds empirical support for the conventional asset-pricing exchange rate model. By comparing the filtered expectations for the locking rate with the survey-based expectations the difference between the two is found to be small for all three countries. Furthermore, if we consider the survey-based expectations to be unbiased estimates<sup>8</sup> on the general view of the markets, then the model used for filtering should be taken to be successful provided that the filtered expectations are close to the reported expectations of the surveys. This test of the model is different from the usually applied one proposed by Meese and Rogoff (1983) which is based on the forecasting ability of the exchange rate. It is similar in spirit to the one applied by Engel and West (2005). They use the functional relationship between the current fundamental and the discounted future fundamental on one hand and the exchange rate on the other hand to forecast future fundamental. They find evidence for forecasting ability that can be interpreted as a general support for the conventional asset-pricing exchange rate models and also for this specific model.

This paper finds that the prospect of locking has a substantial stabilizing effect on the exchange rate in all three analyzed countries. This finding is based on the comparison of the volatilities of the filtered expected locking rate and the filtered latent exchange rate. The filtered expected Euro locking rate is less volatile than either the historical exchange rate or the filtered latent exchange rate. Naszodi (2008b) obtains the same qualitative finding on the stabilizing effect by directly comparing the estimated factor volatilities.

The paper is structured as follows. Section 2.2 presents the exchange rate model developed in Naszodi (2008b), and ellaborates more on modelling the Maastricht criteria. Section 2.3 derives an option pricing formula, which is used for parameter estimation in the empirical part of the paper. Section 2.4 presents the filtering methods and their applications. Finally, Section 2.5 offers some concluding remarks.

## 2.2 Exchange Rate Model

The exchange rate model of this paper is the conventional asset-pricing exchange rate model extended with the assumption of final locking. Similarly to any other asset, the currency is priced by taking into account information not only about the current state of the economy, but also expectations on future events. Therefore, in the conventional

speculative appreciation before they enter the ERM II or even in the ERM II system have increased by experiencing the Slovakian ERM II band shifts on 16th of March 2007 and on 29 May 2008.

<sup>&</sup>lt;sup>8</sup>Our confidence in this result should depend, however, on how reliable are the survey data. The survey data usually tend to show systematic bias in the reported expectations as it is documented by the early papers of Frankel and Froot (1987) and Froot and Frankel (1989).

asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks.

$$s_t = v_t + c \frac{E_t(ds_t)}{dt} , \qquad (2.1)$$

where s is the log exchange rate, and v is the fundamental. The constant c is the time scale. As we will see later on, the log exchange rate would be equal to the fundamental in the absence of future locking. Given this relationship, the fundamental is referred to as the latent exchange rate, *i.e.*, the exchange rate that would prevail if the currency was never going to be locked against the Euro. Both s and v are measured as the log price of one Euro in terms of the domestic currency. Engel and West (2005) and Svensson (1991) present the Money Income Model as one possible structural model that rationalizes the reduced form (2.1). The Money Income Model is one of the simplest models, where Equation (2.1) can be derived from no more than four structural equations. In the Money Income Model c is the interest rate semi-elasticity of the money demand. The term  $\frac{E_t(ds_t)}{dt}$ is the expected instantaneous change of the exchange rate. It is assumed that expectations are formed rationally, therefore the expectation operator  $E_t$  referes to both the subjective market expectation formed at time t and the mathematical expected value conditional on the information available at time t. In this regime switching model, the expected instantaneous change of the exchange rate depends on the log latent exchange rate  $v_t$ , the market expectation for the log final conversion rate  $x_t$  and the locking date  $T_t$ . The exact functional relationship is introduced in Section 2.2.2.

Our assumption that the exchange rate is driven by the fundamental raises the issue of the disconnect puzzle, *i.e.*, the lack of link between the nominal exchange rate and economic fundamentals. Luckily, the disconnect puzzle is less relevant for exchange rates subject to future locking due to the following reasons. The time horizons of shocks to the exchange rate are different in the cases with or without locking. Obviously, it is infinite without locking, but finite with a future irrevocable regime switch. The relative importance of the current state of the economy captured by the fundamental is less uncertain when the horizon is finite. And large changes in the discount factor c are less likely on a shorter horizon.

The fundamental, or in other words the latent exchange rate is defined as a function of some macro variables by following the Money Income Model:

$$v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + ci_t^* , \qquad (2.2)$$

where y denotes the domestic real output, q is the real log exchange rate,  $\psi$  is the risk premium,  $p^*$  is the Eurozone log price, m denotes the domestic nominal money supply and  $i^*$  is the Euro interest rate.

It is assumed that the exchange rates will be locked at the equilibrium value. Here, the concept of equilibrium exchange rate is used under which the strong law of purchasing power parity (PPP) holds for the locking rate, *i.e.*, the log nominal exchange rate at the date of locking  $T^*$  is equal to the difference between the domestic and Eurozone log prices  $s_{T^*} = p_{T^*} - p_{T^*}^*$ . Under rational expectation the market expects the log final conversion rate at time t to be  $x_t = E_t(s_{T^*})$ , which gives

$$x_t = p_t - p_t^* + \int_t^{T_t} E_t(\pi_\tau - \pi_\tau^*) d\tau , \qquad (2.3)$$

where  $\pi$  and  $\pi^*$  denote domestic and Eurozone inflation rates respectively.

Neither definitions (2.2) and (2.3) nor the corresponding macro data are used directly in the empirical part of this paper. Mainly because of the low frequency of these data, but, also, because of a possible misspecification of the underlying macro models. For instance, the examples of the current EMU countries show that the locking rate can deviate from its PPP value. Still, it is the equilibrium real exchange rate that should be the key determinant of the locking rate chosen on the basis of economic considerations. Therefore, while not used directly in the rest of the paper, these definitions motivate the choice of the processes of the underlying factors and, also, the interpretation of the results.

#### 2.2.1 Dynamics

In this Section we specify the processes of the factors. These will then be used to derive the process of the exchange rate. The factors  $x_t$  and  $v_t$  are assumed to follow arithmetic Brownian motions before the locking date. The process of  $T_t$ , similar to a geometric Brownian motion, is assumed to have a multiplicative error term. The geometric Brownian motion has the attractive property that it can not be negative. The assumed process of  $T_t$  is such that it is the expected time until locking  $T_t - t$  that is avoided to be negative and not  $T_t$  itself.

The process of the market expectation for the log Euro locking rate  $x_t$  is

$$dx_t = \begin{cases} \sigma_{x,t} dz_{x,t} , & \text{if } t < T^* \\ 0 , & \text{otherwise }, \end{cases}$$
(2.4)

where  $dz_{x,t}$  is a Wiener process. The parameter  $\sigma_{x,t}$  is the time-varying volatility of the expected locking rate. For obvious reasons  $\sigma_{x,t}$  is assumed to be bounded.

The process of the latent exchange rate can be written as

$$dv_t = \sigma_{v,t} dz_{v,t} \qquad 0 < \sigma_{v,t} < \infty \quad , \tag{2.5}$$

where  $\sigma_{v,t}$  is the time-varying volatility of the latent exchange rate. The Wiener process  $dz_{v,t}$  is not necessarily independent of  $dz_{x,t}$ . A linear, contemporaneous relationship is assumed between the two shocks that can be captured by their time-varying correlation  $\rho(dz_{v,t}, dz_{x,t})$ .

The process of  $T_t$  is

$$dT_t = \begin{cases} (T_t - t)\sigma_{T,t}dz_{T,t} & \text{if } t < T^* \\ 0 & \text{otherwise} \end{cases}$$
(2.6)

where  $dz_{T,t}$  is a Wiener process which may correlate with the other two shocks  $dz_{v,t}$  and  $dz_{x,t}$ . The following restriction is imposed on the correlations.

$$\rho(dz_{x,t}, dz_{v,t}) = \rho(dz_{T,t}, dz_{v,t})\rho(dz_{T,t}, dz_{x,t}) .$$
(2.7)

Restriction (2.7) can be rationalized by the assumption that  $v_t$  and  $x_t$  are interrelated through  $T_t$ . Under this assumption, there is no such shock that affects both  $v_t$  and  $x_t$ but not  $T_t$ . Accordingly, we can make restrictions on the regression Equations (2.8), and (2.9):

$$dz_{x,t} = \rho(dz_{T,t}, dz_{x,t})dz_{T,t} + u_{T,x,t} .$$
(2.8)

$$dz_{T,t} = \rho(dz_{T,t}, dz_{v,t})dz_{v,t} + u_{T,v,t}.$$
(2.9)

The shock  $u_{T,x,t}$  of Equation (2.8) is independent of  $dz_{T,t}$ , but effects  $dz_{x,t}$ . If there is no such shock that affects both  $v_t$  and  $x_t$  but not  $T_t$ , then  $u_{T,x,t}$  should be independent of  $dz_{v,t}$ . By substituting Equation (2.9) into Equation (2.8), we obtain

$$dz_{x,t} = \rho(dz_{x,t}, dz_{v,t})dz_{v,t} + u_{x,v,t} .$$
(2.10)

Under the independence of  $u_{T,x,t}$  and  $dz_{v,t}$ ,  $u_{x,v,t} = u_{T,x,t} + \rho(dz_{T,t}, dz_{x,t})u_{T,v,t}$  and  $u_{x,v,t}$  is orthogonal to  $dz_{v,t}$ . Moreover, the relationship between the three correlations is given by (2.7).

Equation (2.7) is restrictive in the sense that it makes the innovation of one of the factors to be a deterministic function of the innovations of the other two factors and the time-varying covariance matrix of the three innovations. However, Equation (2.7) is less restrictive than the assumption that one of the factors is constant which is used in the literature of exchange rate locking. De Grauwe et al. (1999a) assumes constant locking date and Wilfling and Maennig (2001) assumes constant locking rate.

Given that the EMU candidate countries are not eligible to join the Eurozone unless the Maastricht criteria are fulfilled, the market expectations for the time of locking should depend on the followings. First, the expected future inflation rate due to the inflationary criterion. Second, the expected long-term interest rate that reflects the market view on the long run sustainability of the price stability. Third, the debt-to-GDP and deficit-to-GDP ratios, both being strongly influenced by their common nominator.

Finally, the criterion on the ERM II entry should be discussed, *i.e.*, applicant countries should join the ERM II for two consecutive years and should not devaluate their currencies during that period. The model simplifies the institutional arrangement of EMU entry by assuming that countries switch from the floating regime directly to the fixed one. Luckyly, the intermediate regime disregarded in the model is not relevant for the exchange rate in the floating regime under an implicit assumption. This implicit assumption has been made explicit by the choice on the process for the expected locking rate. It is ruled out to have jumps in the exchange rate process, *i.e.*, the transition from the floating regime to the target zone is assumed to be smooth. Or in other words, the announcement on the ERM II modalities is not expected to surprise the market.

The asymmetric nature of the ERM II, *i.e.*, devaluations are not allowed, can be captured by the time varying correlation between the expected locking date and any of the other two factors. If there is a depreciation pressure on the currency of the EMU candidate country due to weak fundamentals, for instance, then it can have a higher impact on the expected locking date. Whereas the correlations can be smaller before entering the ERM II, because than the exchange rate can depreciate without risking to start again the two years period in the ERM II.

For the sake of simplicity, it is assumed that the Maastricht criteria can be simplified to one nominal and one real criterion. The nominal criterion reflects the criterion both on the past inflation and on the expected future inflation rate. The real criterion is motivated by those criteria that can be met by having sufficiently high growth rate.

The nominal criterion is assumed to be captured by the expected locking rate which is driven by the future expected inflation rate according to Equation (2.3). The higher is the cumulated expected excess inflation rate, the later will the locking take place. The real criteria are captured by the fundamental, which is a decreasing function of the log output according to Equation (2.2). The lower the log output and therefore the higher the fundamental, the later will the EMU candidate country become eligible to join the Euro area.

This simplified view of the Maastricht criteria can be described by the following Taylorrule-type functional relationship between the expected time until locking on one hand and the expected locking rate and fundamental on the other hand:

$$T_t - t = \left(\frac{b_t^2}{2c\lambda_t \sigma_{v,t}^2} \ v_t^2 + \frac{b_t^2}{2c\lambda_t \sigma_{x,t}^2} \ x_t^2\right)^{\lambda_t} \quad \lambda_t > 1 .$$
 (2.11)

The parameters  $b_t$  and  $\lambda_t$  together with the expected time until locking  $T_t - t$  are assumed to determine the instantaneous volatility of the expected locking date:

$$\sigma_{T,t}^2 = b_t^2 (T_t - t)^{-1 - \frac{1}{\lambda_t}} . (2.12)$$

If both  $b_t$  and  $\lambda_t$  are constant,  $\sigma_{T,t}$  is a decreasing function of the time until locking  $T_t - t$ . This is not counterintuitive, because the variance of the changes of the expected locking date is still an increasing function of the expected time until locking as being  $\sigma_{dT_t}^2 = (T_t - t)^2 \sigma_{T,t}^2 = b_t^2 (T_t - t)^{1 - \frac{1}{\lambda_t}}$  (see Equation (2.6)). Moreover, if the country is just about to lock its exchange rate, no uncertainty remains concerning the locking date.

Figure 2.1 demonstrates the Maastricht criteria with a simple constant parametrization. Point A corresponds to a country with relatively strong fundamental v and low expected cumulated inflation rate x. In comparison with point A, the fundamental vis weaker in point B, but the expected inflation path is the same. In point C the expected cumulated inflation rate x is higher, than in point A, but the fundamentals are the same. Therefore, country A is expected to join the Euro area earlier than the other two countries. Country B and C are expected to adopt the Euro at the same time as both are on the same iso-time-until-locking curve. In country B, it is the real criterion that is more binding, whereas in country C it is the inflationary criterion. The slope of the iso-time-until-locking curve determines the marginal rate of substitution between the expected cumulated inflation rate and the fundamental.

It is worth noting that the model is more flexible in some aspects, than suggested by Figure 2.1 with constant parametrization. First, in Figure 2.1, the real cost of one percentage point desinflation is increasing in the expected cumulated inflation rate. However, the real cost of desinflation can be decreasing as well under some alternative parametrizations. Second, under constant parametrization, locking takes place only if both v and xare zero. However, Equations (2.11) and (2.12) are not that restrictive. It is sufficient to have  $b_t > 0$  before locking  $t < T^*$ , and  $b_t = 0$  at the locking date  $t = T^*$ . While  $v_{T^*}$  and  $x_{T^*}$  can be different from zero. Third, the modeled Maastricht criteria can be used even for countries that do not prefer to enter the EMU although they have become eligible to do so. These incentives are likely to be state-contingent, *i.e.*, they are expected to change once the economy moves to some other states. In that case, the same model can be applied, however instead of using simply the objective Maastricht criteria, the most binding criteria should be used among the inventive compatibility constraint and the Maastricht criteria.

If  $b_t$ ,  $\lambda_t$ ,  $\sigma_{x,t}$ , and  $\sigma_{v,t}$  are not constant, the following restrictions are made on their relative sensitivities to the changes in the factors:

$$\frac{E(\frac{\partial h_t}{\partial x_t})}{E(\frac{\partial h_t}{\partial v_t})} = \frac{\sigma_{v,t}^2}{\sigma_{x,t}^2} , \qquad (2.13)$$

where  $h_t$  can denote any of the time-varying parameters of  $b_t$ ,  $\lambda_t$ ,  $\sigma_{x,t}$ , and  $\sigma_{v,t}$ .

The four-equations restriction (2.7), (2.11), (2.12), and (2.13) with  $\lambda_t$  and  $b_t$  can be used to derive the following single-equation restriction on the correlations without  $\lambda_t$  and  $b_t$  (see Appendix A on the derivation),

$$\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t} - \rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t} = (v_t - x_t)\frac{\sigma_{T,t}(T_t - t)}{c} .$$
(2.14)

Equation (2.14) can be motivated by some economic considerations along the following lines. If  $\sigma_{T,t}(T_t - t)$  is zero, there is no uncertainty concerning the locking time, either because the country is just about to switch regime  $T_t = t$ , or no future shock can modify the expectations  $\sigma_{T,t} = 0$ . If  $\sigma_{T,t}$  is positive, then there is chance for any of the Maastricht criteria to be binding in the future, *i.e.*, it can not be ruled out that the locking will be postponed due to the failure of the country to meet the criteria in time. Moreover, the higher is the uncertainty related to the time of locking  $\sigma_{T,t}$  or the longer is the expected time until locking  $(T_t - t)$ , the Maastricht criteria are more likely to be binding at a given horizon. If  $\sigma_{T,t}(T_t - t) > 0$  and  $v_t > x_t$  ( $v_t < x_t$ ), the real (inflationary) criteria are more difficult for the country to meet. In that case, the shocks to  $v_t$  ( $x_t$ ) should have a stronger effect on the market expectation for the locking date, than the shocks to  $x_t$ ( $v_t$ ). Moreover, the higher c, *i.e.*, the interest rate semi-elasticity of money demand, the more effective is the monetary policy at boosting the economy at the expense of higher inflation or at disinflating by sacrificing real growth. A highly effective monetary policy can contribute to the most binding criterion to be meet easier.

The exact functional form of restriction (2.14) is motivated purily by its nice analytical properties. As it is shown by the next Section, the log exchange rate can be derived as a closed-form function  $s_t = f(t, v_t, x_t, T_t)$  of the factors under the restriction of Equation (2.14).

## 2.2.2 Functional Relationship Between the Exchange Rate and Underlying Factors

In this Section we introduce the functional relationship between the exchange rate on the one hand and the latent exchange rate, the market expectations for the locking rate and date on the other hand. The derivation has the following two steps. First, the process of the log exchange rate  $s_t$  is derived from the processes of the factors by using Ito's stochastic change-of-variable formula. Second, we obtain that the function satisfying the derived process and the terminal condition

$$f(T^*, v_{T^*}, x_{T^*}, T^*) = x_{T^*} , \qquad (2.15)$$

and Equation (2.1) is given by

$$s_t = f(t, v_t, x_t, T_t) = \left(1 - e^{-\frac{T_t - t}{c}}\right) v_t + e^{-\frac{T_t - t}{c}} x_t .$$
(2.16)

Using Ito's stochastic change-of-variable formula, we obtain the following

$$df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_t}\mu_{x,t} + \frac{\partial f}{\partial T_t}\mu_{T,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\sigma_{x,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial T_t^2}\sigma_{T,t}^2(T_t - t)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial T_t\partial x_t}\rho\left(dz_{T,t}, dz_{x,t}\right)\left(T_t - t\right)\sigma_{T,t}\sigma_{x,t} + \frac{1}{2}\frac{\partial^2 f}{\partial T_t\partial v_t}\rho\left(dz_{T,t}, dz_{v,t}\right)\left(T_t - t\right)\sigma_{T,t}\sigma_{v,t} + \frac{1}{2}\frac{\partial^2 f}{\partial x_t\partial v_t}\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t}\right]dt + \frac{\partial f}{\partial v_t}\sigma_{v,t}dz_{v,t} + \frac{\partial f}{\partial x_t}\sigma_{x,t}dz_{x,t} + \frac{\partial f}{\partial T_t}(T_t - t)\sigma_{T,t}dz_{T,t}.$$
 (2.17)

The different  $\mu$ 's denote the drift terms, whose values are zero in Equations (2.4), (2.5) and (2.6).

The function satisfying the derived process (2.17), the terminal condition (2.15) and the Equation (2.1) is given by (2.16). The proof can be found in Naszodi (2008b) for the general model with stochastic locking date. For a restricted model with constant locking date, the proof is provided by De Grauwe et al. (1999a).

Equation (2.16) shows that the log exchange rate is the weighted average of the log latent exchange rate and the expected log final conversion rate. The weights are changing over time; if the time until locking is infinite, or in other words, the currency will never be locked, then the weight of the latent exchange rate is one, and the weight of the expected conversion rate is zero. As the time until locking decreases, the weight of the expected conversion rate increases. Finally, as the time until locking approaches zero, the weight of the expected conversion rate approaches one. Therefore, the expected locking rate has increasing importance at determining the exchange rate as the locking date is approaching, and at the locking date the exchange rate is exactly equal to the expected final conversion rate.

In order to examine the exchange rate dynamics of the model, we substitute (2.4), (2.5), (2.6), and (2.16) into Equation (2.17).

$$ds_{t} = \frac{1}{c} \frac{e^{-\frac{T_{t}-t}{c}}}{1-e^{-\frac{T_{t}-t}{c}}} \left(x_{t}-s_{t}\right) dt + \left(1-e^{-\frac{T_{t}-t}{c}}\right) \sigma_{v,t} dz_{v,t} +$$
(2.18)

$$+e^{-\frac{T_t-t}{c}}\sigma_{x,t}dz_{x,t}-\frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}\left(x_t-s_t\right)\left(T_t-t\right)\sigma_{T,t}dz_{T,t}\;.$$

The dynamics of the exchange rate is such that it converges to the actual market expectation for the final conversion rate. Moreover, the closer the expected time of locking, the faster the convergence is.

Equations (2.4), (2.5), (2.6) and (2.16) define a three-factor model. One factor is the market expectation for the final conversion rate; another is the market expectation for the locking date; and the third is the latent exchange rate.

## 2.3 Option Pricing

In this Section a pricing formula for European-type options is derived. The pricing formula is consistent with the exchange rate model with constant locking time and factor volatilities. Although not being fully consistent with the stochastic locking time, the pricing formula is also used in the general framework with stochastic locking time to estimate parameters. The parameters to be estimated are the variances of the innovations of the factors  $v_t$  and  $x_t$ . The historical option prices are given in terms of implied volatility; consequently, the pricing formula is derived in terms of volatility as well.

In the theoretical model the uncertainty is present due to the stochastic innovations  $(dz_{v,t}, dz_{x,t}, dz_{T,t})$  of the factors; consequently, the price of an option is a function of the variances and covariances of these normally distributed innovations. From Equations

(2.16), and (2.18) we can derive that the instantaneous variance of returns at time t

$$\sigma_{s,t}^{2} = \left(1 - e^{-\frac{T_{t}-t}{c}}\right)^{2} \sigma_{v,t}^{2} + \left(e^{-\frac{T_{t}-t}{c}}\right)^{2} \sigma_{x,t}^{2} + \left(\frac{1}{c}e^{-\frac{T_{t}-t}{c}}\right)^{2} (x_{t} - v_{t})^{2} (T_{t} - t)^{2} \sigma_{T,t}^{2} + 2\left(1 - e^{-\frac{T_{t}-t}{c}}\right) \left(e^{-\frac{T_{t}-t}{c}}\right) \sigma_{v,t} \sigma_{x,t} \rho \left(dz_{v,t}, dz_{x,t}\right) + 2\frac{1}{c}e^{-\frac{T_{t}-t}{c}} (x_{t} - v_{t}) (T_{t} - t) \sigma_{T,t} \left(1 - e^{-\frac{T_{t}-t}{c}}\right) \sigma_{v,t} \rho \left(dz_{T,t}, dz_{v,t}\right) + 2\frac{1}{c}e^{-\frac{T_{t}-t}{c}} (x_{t} - v_{t}) (T_{t} - t) \sigma_{T,t} \left(1 - e^{-\frac{T_{t}-t}{c}}\right) \sigma_{v,t} \rho \left(dz_{T,t}, dz_{v,t}\right) + 2\frac{1}{c}e^{-\frac{T_{t}-t}{c}} (x_{t} - v_{t}) (T_{t} - t) \sigma_{T,t} e^{-\frac{T_{t}-t}{c}} \sigma_{x,t} \rho \left(dz_{T,t}, dz_{x,t}\right) . \quad (2.19)$$

In order to derive a closed form option pricing formula, the simple constant volatility model is used. This approach is similar to the one applied in the Black-Scholes model. The covariance matrix of the three shocks is assumed to be constant over the life of the option. However, options sold at different times can be priced with different covariance matrices. No time series type of restrictions are imposed on these time-varying covariance. Obviously, the price of options in the stochastic volatility framework is different from the one of the constant volatility framework, however the latter is a good approximation for the theoretical value of at-the-money options with a maximum of one-year maturity.<sup>9</sup>

In order to have a closed form option pricing formula, not only the covariance matrix is kept constant over the life of the options, but also the expected locking date  $T_t$ . However,  $\sigma_{T,t}$  is not restricted to zero. By applying this final simplification and by calculating the integral, we obtain the option pricing formula

$$g(t, m, \sigma_{x,t}, \sigma_{v,t}, \sigma_{T,t}, \rho \left( dz_{v,t}, dz_{x,t} \right), \rho \left( dz_{T,t}, dz_{x,t} \right), \rho \left( dz_{T,t}, dz_{v,t} \right) \right) = \left[ \int_{t}^{t+m} \sigma_{s,\tau}^{2} d\tau \right]^{\frac{1}{2}} = \left[ \sigma_{v,t}^{2}m + \Gamma_{1} \left( \sigma_{v,t}^{2} + \sigma_{x,t}^{2} - \rho^{2} \left( dz_{T,t}, dz_{v,t} \right) \sigma_{v,t}^{2} - \rho^{2} \left( dz_{T,t}, dz_{x,t} \right) \sigma_{x,t}^{2} \right) + \Gamma_{2} \left( -2\sigma_{v,t}^{2} + 2\rho^{2} \left( dz_{T,t}, dz_{v,t} \right) \sigma_{v,t}^{2} \right) \right]^{\frac{1}{2}}, \quad (2.20)$$

where  $\Gamma_1 = \frac{c}{2}e^{-\frac{2}{c}(T_t-t-m)} - \frac{c}{2}e^{-\frac{2}{c}(T_t-t)}$  and  $\Gamma_2 = ce^{-\frac{1}{c}(T_t-t-m)} - ce^{-\frac{1}{c}(T_t-t)}$ . The option is sold at time t and the maturity is m.

### 2.4 Filtering Factors

The Kalman filter technique is applied to extract the time series of the factors from the time series of some observable variables. Filtering all three factors from only the time

<sup>&</sup>lt;sup>9</sup>As pointed out by Hull (1997, p. 620): "For options that last less than a year, the pricing impact of a stochastic volatility is fairly small in absolute terms. It becomes progressively larger as the life of option increases."

series of the exchange rate would be overambitious and unlikely to provide robust results. Moreover, the Kalman filter technique can be applied only if the model is linear<sup>10</sup> in all the factors to be filtered. In fact, the log exchange rate  $s_t$  is linear in two of the factors, namely the log latent exchange rate  $v_t$  and the expected log final conversion rate  $x_t$ , but not in the expected locking date  $T_t$ . However, the expected locking date can be filtered independently from the other two factors from the forward curves. The applied yield curve method is introduced in the next Section.

With exogenous  $T_t$ , the Kalman filter can be interpreted as a method for decomposing the exchange rate changes ds into changes in the state variables v and x. The decomposition is such that the higher is the relative weight of v in s, and the higher is its relative volatility  $\frac{\sigma_v}{\sigma_x}$ , the higher change will be attributed to v by the Kalman filter. (See Appendix D on the Kalman filter in general and also on this specific feature.)

De Grauwe et al. (1999b) also uses the Kalman filter in order to access the weight of the expected conversion rate in the exchange rates of countries that participated in the first wave of EMU. Their sample is from the period just before the locking of the currencies, *i.e.*, from January 1, 1998 to September 2, 1998. In this period there was almost no uncertainty for the locking date and the uncertainties for the locking rates were moderate. Whereas in case of the currencies analyzed by this paper, the expected time until locking is more than 3 years and both the locking rate and date are highly uncertain. Therefore, the model of this paper is not as simple as that of De Grauwe et al... First, the expected locking date is not constant, but allowed to be stochastic. Second, De Grauwe et al. assume that the market expectation for the locking rate is centered around the central parities of the ERM II that latter were used as conversion rates. Here, the expected locking rate is allowed to deviate substantially from a constant by assuming it follows a Brownian motion. Third, the variancies of shocks to the latent exchange rate and the expected locking rate are constant in De Grauwe et al. (1999b), whereas these are time-varying in this paper. Moreover, they restrict the shocks to be independent, while here the shocks are allowed to correlate. Fourth, the parameter c is estimated<sup>11</sup> by De Grauwe et al. and the estimates for most of the countries they analyze are of such magnitude that they are difficult to interpret as the interest rate semi-elasticity of money demand. Here, the *c* parameter is calibrated instead.

The market expectation concerning the final conversion rate has a similar role in this model as the exchange rate targeted by the central bank. The target exchange rate of the National Bank of Hungary was explicitly announced between June 2001 and December 2003. Karádi (2005) analyzes the weight of the medium term target in the exchange rate in this period. The main difference between the model of this paper and that of Karádi (2005) is that the target is a medium term target in the latter, whereas the market

<sup>&</sup>lt;sup>10</sup>To filter all three factors from the time series of the exchange rate, one should apply a different technique than the Kalman filter, because the model is not linear in  $T_t$ . The Extended Kalman filter and the particle filter would be possible candidates.

<sup>&</sup>lt;sup>11</sup>Dewachter and Veestraeten (2001) provides also estimates on the c parameter in order to measure the speed of convergence of the exchange rates toward their pre-announced conversion rates in the Monetary Union.

expectation concerning the locking is a long term anchor in this model. Consequently, the relative importances of the two forward looking elements of the exchange rate are different. The exchange rate elasticity with respect to the target is estimated by Karádi (2005) and found to be close to one in the long run. Whereas in our model it never exceed 80%. Another important difference between the two models is that the time to reach the medium term target exchange rate is constant over time, while the time until the locking is time-varying. Hence, the exchange rate elasticity with respect to the target exchange rate is time-invariant, whereas the corresponding elasticity with respect to the market expectation concerning the final conversion rate is time-varying.

Section 2.4.1 introduces the applied method and the results of filtering  $T_t$ . Then, the filtering problem is set up for the remaining two factors in Section 2.4.2. Section 2.4.3 describes how the parameters of the filtering problem are set. Finally, Section 2.4.4 presents the filtered market expectation for the locking rate  $x_t$  and its estimated stabilizing effect on the exchange rate.

#### 2.4.1 Filtering the Expected Locking Date

The expected locking date can be estimated independently from the other two factors from the Euro and domestic yield curves. The yield curve method has been popularized by Bates (1999) and it has been applied for Hungary by Csajbók and Rezessy (2006). The method is based on the fact that after adopting the Euro, the interest rate differential will become marginal, but not of course before the adoption. The higher the probability attached by the market participant to the scenario that a country is already a full-fledged member of the Eurozone by a given year, the lower the absolute forward differential is for that particular year. Along these lines the market expectation on the date of locking can be estimated from the forward differentials. Appendix B describes the method in details.

Figure 2.2 shows the market expectations for the locking dates filtered from the forward differentials. These yield curve based estimates of the market expectation are noise, therefore different smoothing methods are applied in the literature. Csajbók and Rezessy (2006) use HP filter. Here, the Kalman filter is used, because it does not only smooths the yield curve based estimated  $\tilde{T}_t$ , but it also provides estimates for  $\sigma_{T,t}$  by maximizing the filtering likelihood. In the empirical part of the paper  $\sigma_{T,t}$  is restricted to be constant  $\sigma_T$ .

The discrete time model used to Kalman filter the underlying time series of  $T_t$  from the observable yield curve based estimated  $\tilde{T}_t$  is the following:

$$T_{t+1} = T_t + u_{t+1}$$
  $u_{t+1} \sim N\left(0, (T_t - t)^2 \sigma_T^2\right),$  (2.21)

$$\widetilde{T}_t = T_t + w_t \qquad w_t \sim N\left(0, \sigma_w^2\right),$$
(2.22)

where the system error  $u_{t+1}$  and the observation error  $w_t$  are assumed to be independent.

The estimates of the parameter of the system covariance  $\sigma_T$  and the standard deviation of the observation error  $\sigma_w$  are presented in Table 2.1.

Figure 2.2 shows not only the original filtered market expectations for the locking dates  $T_t$ , but also the smoothed  $T_t$ . Moreover, it also shows survey data on the expected locking date. The Reuters polls queries the analysts about their opinion on the most likely year of EMU and ERM II entry of the three analyzed countries. These are the only references for the filtered  $T_t$  to be compared with. The filtered time series and the survey-based one show similar trends. One can see that analysts expectations were relatively stable until the autumn of 2005 and have changed subsequently between the quarterly polls of August and November for the Czech Republic and Poland, and between September and October 2005 for Hungary. Until the autumn of 2005 the three countries had been expected to enter ERM II in the year 2007. Thereafter, the expectations changed dramatically, as reflected by the monthly and quarterly Reuters polls, pointing to a postponement of ERM II entry to 2008 for the Czech Republic, 2009 for Poland, and 2010 for Hungary. Similar trend can be detected in the expected EMU entry date. The filtered locking dates suggest that expectations were subject to both positive and negative shocks during 2006 and the first quarter of 2007 in all three countries. The positive shocks shifting the expected locking date of Hungary and Poland substantially upwards have started in July 2005 according to the filtered locking date, whereas the survey-based expectations have changed only a few months later. This finding suggests that the estimates on the expected locking date provide earlier warnings on the changes in the market sentiment than the surveys. In the first half of 2006 the filtered expected locking dates have downward trends in Hungary and Poland, which is either due to some favorable news on the EMU outlook, or due to a correction of the previous overshooting of the expectations.

The locking of the exchange rate can precede the EMU entry, but not vice versa. For instance, the exchange rates of almost all the countries that entered the ERM II system in recent years<sup>12</sup> are almost fixed: the volatility of the Estonian kroon, the Lithuanian lita, the Slovenian tolar, the Cyprus pound and the Maltese lira dropped below 1% after entering the ERM II regime. These facts imply that practically locking does not necessarily take place when a country enters the Monetary Union, but rather when it enters the ERM II regime. The filtered locking date of the Czech Republic has fluctuated between the narrow band given by the survey-based expected EMU and ERM II entry dates, which is in accordance with the sequence of locking and EMU entry. Whereas in case of Hungary and Poland the survey-based expected EMU entry dates precede sometimes the yield curve based locking dates. Or in other words, the survey-based expectations are more optimistic relative to the yield curve based estimates. Since we can have more confidence in the filtered expectations then in the survey-based expectations, the difference is attributed to systematic bias in the survey data. This systematic bias can be explained, for instance, with the skewed distribution of the entry date. If market analysts report the most likely entry date of a country, *i.e.*, the mode of the distribution, instead of the

<sup>&</sup>lt;sup>12</sup>The Estonian kroon, the Lithuanian lita and the Slovenian tolar joined ERM II on 27 June 2004. On 2 May 2005 three other Member States joined ERM II: Cyprus, Latvia and Malta.

expected value and if the subjective distribution of the entry date is skewed towards a later entry date, then the survey-based expectations are downward biased estimates on the expected locking dates.

Once we have estimates on  $T_t$ , we can filter the other two factors from the historical exchange rate. While filtering  $v_t$  and  $x_t$  the  $T_t$  is treated as being exogenously given.

#### 2.4.2 Filtering Problem of the Expected Locking Rate

This Section sets up the filtering problem for  $x_t$  and  $v_t$  in the general model with stochastic expected locking date  $T_t$ .

Since the third factor  $T_t$  is exogenous and it is not independent of the other two factors, the conditional distributions of  $x_t$  and  $v_t$  have to be used, where we condition on the realization of  $T_t$ . The conditional expected innovations of  $x_t$  and  $v_t$  are  $\sigma_{x,t}\rho(dz_{T,t}, dz_{x,t})dz_{T,t}$ and  $\sigma_{v,t}\rho(dz_{T,t}, dz_{v,t})dz_{T,t}$  respectively. The expected values of the innovations are taken into account in the model by having a constant as a third state variable. Moreover, the system covariance matrix Q(t) is also conditional on  $T_t$ .

The filtering problem can be written in the usual form:

$$\Lambda(t+1) = A(t)\Lambda(t) + w_1(t+1)$$
(2.23)

$$\Omega(t) = C(t)\Lambda(t) + w_2(t) \tag{2.24}$$

$$E\left[\left(\begin{array}{c}w_1(t+1)\\w_2(t)\end{array}\right)(w_1(t+1)\ w_2(t))\right] = \left(\begin{array}{c}Q(t)\ 0\\0\ R\end{array}\right).$$
(2.25)

In our case, the vector of states is  $\Lambda'(t) = \begin{pmatrix} v_t & x_t & 1 \end{pmatrix}$ . The system matrix is

$$A(t) = \begin{pmatrix} 1 & 0 & \sigma_{v,t}\rho(dz_{T,t}, dz_{v,t})\frac{dT_t}{\sigma_{T,t}(T_t-t)} \\ 0 & 1 & \sigma_{x,t}\rho(dz_{T,t}, dz_{x,t})\frac{dT_t}{\sigma_{T,t}(T_t-t)} \\ 0 & 0 & 1 \end{pmatrix}$$

The vector  $w_1(t)$  is assumed to be a Gaussian white noise. The observable variable is the log exchange rate  $\Omega(t) = s_t$ . Equation (2.16) implies that the observation matrix is  $C(t) = \left( \begin{array}{cc} 1 - e^{-\frac{T_t - t}{c}} & e^{-\frac{T_t - t}{c}} & 0 \end{array} \right).$ 

The system covariance matrix can be written as

$$Q(t) = \begin{pmatrix} Q_{1,1}(t) & Q_{1,2}(t) & 0\\ Q_{1,2}(t) & Q_{2,2}(t) & 0\\ 0 & 0 & 0 \end{pmatrix}.$$

Where the covariance is conditional on the observed  $T_t$ , therefore

$$Q_{1,1}(t) = \sigma_{v,t}^2 \left[ 1 - \rho^2 (dz_{T,t}, dz_{v,t}) \right],$$
$$Q_{1,2}(t) = \sigma_{v,t} \sigma_{x,t} \left[ \rho(dz_{x,t}, dz_{v,t}) - \rho(dz_{T,t}, dz_{v,t}) \rho(dz_{T,t}, dz_{x,t}) \right]$$

$$Q_{2,2}(t) = \sigma_{x,t}^2 \left[ 1 - \rho^2 (dz_{T,t}, dz_{x,t}) \right]$$

The conditional covariance  $Q_{1,2}$  between  $dz_{x,t}$  and  $dz_{v,t}$  is zero due to the restriction (2.7). In contrast to the conditional covariance, the unconditional covariance between  $dz_{x,t}$  and  $dz_{v,t}$  is not zero, which is captured by the system matrix A(t).

The error term  $w_2(t)$  is assumed be deterministic and nill. In other words, the exchange rate is assumed to be observed without error and the model (2.16) perfectly describes the relationship between the factors and the exchange rate. Hence, the variance of the observation error term R is set to zero. The Kalman filter remains valid even in this case.<sup>13</sup>

In our case, the observation matrix C(t), the system matrix A(t) and the system covariance Q(t) are changing over time.<sup>14</sup>

The parameters c,  $\sigma_{v,t}$ ,  $\sigma_{x,t}$ ,  $\sigma_{T,t}$ ,  $\rho(dz_{v,t}, dz_{x,t})$ ,  $\rho(dz_{T,t}, dz_{x,t})$  and  $\rho(dz_{T,t}, dz_{v,t})$  of the system covariance Q(t) and of the system matrix A(t) need to be either calibrated or estimated. Moreover, the initial values  $x_{t_0}$  and  $v_{t_0}$  of the factors belonging to the beginning of the sample period,  $t_0 = \text{Jan } 4$ , 2005, need to be set as well. The next Section describes how these parameters are estimated or calibrated.

#### 2.4.3 Parameters

This Section shows how the parameters c,  $\sigma_{T,t}$ ,  $x_{t_0}$ ,  $v_{t_0}$ ,  $\rho(dz_{v,t}, dz_{x,t})$ ,  $\rho(dz_{T,t}, dz_{x,t})$ ,  $\rho(dz_{T,t}, dz_{x,t})$ ,  $\sigma_{v,t}$  and  $\sigma_{x,t}$  are calibrated or estimated.

As part of the robustness check, the factors are filtered not only by using the model with stochastic locking time, but also by the one with constant locking date. The calibration of parameters is the same with the constant locking date, except that the constant market expectation for the locking date T is set equal to the initial value of the filtered time series  $T_{t_0}$  and the parameters  $\sigma_{T,t}$ ,  $\rho(dz_{v,t}, dz_{x,t})$ ,  $\rho(dz_{T,t}, dz_{x,t})$  and  $\rho(dz_{T,t}, dz_{v,t})$  are zero.

By following Engel and West (2005), the parameter c is calibrated based on estimates on the interest rate semi-elasticity of money demand. Frankel (1979), Stock and Watson (1993, 802, Table 2, Panel 1) and Bilson (1978) estimate the semi-elasticity approximately 29, 40, 60 respectively. These estimates are either on quarterly data or rescaled to quarterly basis. By dividing the quarterly semi-elasticity parameter by 4 we get the annualized c parameter. It should be in the range of 7.25–15. Therefore, c is calibrated to the average ( $\hat{c} = 10.75$ ) of the estimates of the three cited studies and the two extreme values of the range are used for sensitivity analyses.

The market expectation for the locking date  $T_t$  is estimated from the yield curves and smoothed by the Kalman filter as discussed in Section 2.4.1. Estimates on its volatility parameter  $\sigma_T$  are presented in Table 2.1.

 $<sup>^{13}</sup>$ See Harvey (1990) page 108 for a detailed discussion.

<sup>&</sup>lt;sup>14</sup>The Kalman filter toolbox for Matlab written by Kevin Murphy is used. Its advantage is that it allows for time-varying parameters. (See http://www.ai.mit.edu/ murphyk/Software/kalman.html for details.)

The Reuters polls are used for the calibration of the *initial states*,  $x_{t_0}$  and  $v_{t_0}$ . The  $x_{t_0}$  is calibrated to be equal to the log of averaged expectations on the central parity reported by the last Reuters polls of 2004. The initial value of  $v_{t_0}$  is calculated by plugging  $s_{t_0}$ ,  $\hat{x}_{t_0}$ ,  $\hat{c}$  and  $\hat{T}_{t_0}$  into Equations (2.16).

The calibrated initial values are used in the Kalman filter not only as the initial state variables, but also to estimate the *correlations*. These estimated correlations are plugged into the system matrix and the system covariance matrix used to filter the next period state variables. And these filtered state variables are used again for estimating the correlations between the next period's shocks. These steps of estimation and filtering follow each other until the end of the Universe or until the end of the sample.

By rearranging (2.4) and (2.6) we obtain that the correlation between the shocks to market expectation for the locking rate and locking date is equal to  $\rho(dz_{T,t}, dz_{x,t}) = \frac{dx_t}{dT_t} \frac{\sigma_{T,t}(T_t-t)}{\sigma_{x,t}}$ . This formula suggests that it is sufficient to calibrate or estimate  $T_t$ ,  $\sigma_{T,t}$ ,  $\frac{dx_t}{dT_t}$  and  $\sigma_{x,t}$  in order to obtain the correlation.

Among these terms  $\frac{dx_t}{dT_t}$  is calibrated based on the inflation dynamics. Equation (2.3) of the exchange rate model assumes that the market expectation for the log locking rate  $x_t$  equals to the difference between the current domestic and Eurozone log prices plus the cumulated expected excess domestic inflation rate over the Eurozone inflation rate. Based on a plausible view on the inflation dynamics until locking, we can calibrate  $\frac{dx_t}{dT_t}$ . The calibration of  $\frac{dx_t}{dT_t}$  depends on whether the change in  $T_t$  is due to a real or an inflationary shock.

If the expected locking date is changing due to a *real shocks*,  $\frac{dx_t}{dT_t}$  depends largely on the reaction of the monetary policy. The targeted path of inflation is assumed to be adjusted by the central bank so that the inflationary Maastricht criterion will be met only at the new expected locking date. Further, the targeted inflation at the locking date is assumed to be 2 % no matter when the locking will be realized and whether the country had higher or even lower inflation rate before. Moreover, the Eurozone inflation rate is expected to be constant 2 % as well.

This strategy of the central bank is illustrated in Figure 2.3. The initial inflation rate  $\pi$  in Figure 2.3 is expected to decrease linearly to the targeted inflation rate  $\pi^*$  until time T. Hence, the initial cumulated excess inflation rate is the checkboard area. If the expected locking date shifts to  $T^s$  due to a *real shock*, the cumulated inflation rate will increase by the dotted area. Under this scenario the term  $\frac{dx_t}{dT_t}$  is half of the excess inflation  $\frac{\pi - \pi^*}{2}$ .

If the locking is postponed due to an inflationary shock, the rate of disinflation is assumed to be unchanged. This scenario is also shown by Figure 2.3. If the expected locking date shifts to  $T^s$  while the inflation rate increases to  $\pi^s$ , the cumulated inflation rate will increase not only by the dotted area, but also by the striped one. Under this scenario, the term  $\frac{dx_t}{dT_t}$  is the product of the time until locking and the constant rate of disinflation  $\nu$ , which can be approximated by the excess inflation  $\pi - \pi^*$ .

*Real shocks are likely to be more relevant*, because EMU entry delays were mainly due to fiscal slippages in Hungary. Moreover, in Czech Republic and Poland the inflation rates

were already moderate in the analyzed period. Table 2.2 presents the inflation rates by country and by year and also the average inflation rate used for calibration. The total derivative  $\frac{dx_t}{dT_t}$  is calibrated to half of the average excess inflation rate over 2%: -0.085 %, 1.15% and -0.125 % for Czech Republic, Hungary and Poland respectively.

As part of the sensitivity analysis, the alternative approach of calibration is used, where changes in  $T_t$  are due to an inflationry shock. Then, the calibrated values of  $\frac{dx_t}{dT_t}$  are twice as big as in the baseline case.

Once the parameters c,  $T_t$ ,  $\sigma_{T,t}$ ,  $\frac{dx_t}{dT_t}$  are calibrated or estimated, and the factors x and v have already been filtered until time t, the only terms that need to be estimated are the volatilities  $\sigma_{x,t}$  and  $\sigma_{v,t}$ . To see this, we express the correlations as functions of some already calibrated or estimated parameters and the volatilities  $\sigma_{x,t}$  and  $\sigma_{v,t}$ . The calibrated or estimated variables are denoted by hats.

$$\rho(dz_{T,t}, dz_{x,t}) = \frac{\widehat{dx_t}}{dT_t} \frac{\widehat{\sigma}_{T,t}(\widehat{T}_t - t)}{\sigma_{x,t}} .$$
(2.26)

By using (2.7), (2.11), (2.12), and (2.26), we obtain

$$\rho(dz_{T,t}, dz_{v,t}) = \frac{(\widehat{v}_t - \widehat{x}_t)\widehat{\sigma}_{T,t}(\widehat{T}_t - t)}{\sigma_{v,t}\widehat{c}} + \frac{\widehat{dx_t}}{dT_t}\frac{\widehat{\sigma}_{T,t}(\widehat{T}_t - t)}{\sigma_{v,t}} .$$
(2.27)

Finally, by substituting (2.26) and (2.27) into Equation (2.7), we obtain

$$\rho(dz_{v,t}, dz_{x,t}) = \frac{\widehat{dx_t}}{dT_t} \frac{\widehat{\sigma}_{T,t}(\widehat{T}_t - t)}{\sigma_{x,t}} \left[ \frac{(\widehat{v}_t - \widehat{x}_t)\widehat{\sigma}_{T,t}(\widehat{T}_t - t)}{\sigma_{v,t}\widehat{c}} + \frac{\widehat{dx_t}}{dT_t} \frac{\widehat{\sigma}_{T,t}(\widehat{T}_t - t)}{\sigma_{v,t}} \right].$$
(2.28)

The time-varying volatilities  $\sigma_{x,t}$  and  $\sigma_{v,t}$  are estimated from six implied volatilities  $\sigma_{t,i}^{imp}$  for each time t by non-linear least squares (NLLS). The six options have different maturities m(i). Unfortunately, there are missing observations. When not all 6 option data are available for a day, the estimates for the previous day are used. The basic idea of the estimation is to minimize the distance between the theoretical and the historical option prices.

The NLLS estimates of  $\sigma_{v,t}$  and  $\sigma_{x,t}$  satisfy the minimization problem

$$\min_{\sigma_{v,t},\sigma_{x,t}} \sum_{i=1}^{6} \left[ g(t, m(i), \sigma_{x,t}, \sigma_{v,t}, \sigma_{T,t}, \rho\left(dz_{v,t}, dz_{x,t}\right), \rho\left(dz_{T,t}, dz_{x,t}\right), \rho\left(dz_{T,t}, dz_{v,t}\right)\right) - \sigma_{t,i}^{imp} \right]^{2} .$$
(2.29)

The option pricing formula for g(.) is given by (2.20). All the input parameters should be substituted by their estimated or calibrated values. The only exceptions are the correlations, which are substituted by the expressions of (2.26), (2.27), and (2.28), where the correlations are functions of the volatilities  $\sigma_{v,t}$  and  $\sigma_{x,t}$ .

Figure 2.4 shows the historical option prices with the shortest and longest maturities and also the fitted values. The implied volatilities of the options with the shortest maturity fluctuate around the implied volatilities of the options with the longest maturity. In those periods, when the short implied volatility exceeds the long one, the uncertainty of the short horizon is relatively higher than that of the long horizon. Moreover,  $\sigma_{v,t}$  can be thought of as a proxy for the short-run uncertainty, whereas  $\sigma_{x,t}$  reflects more the expected magnitude of shocks occurring in the far future. Therefore,  $\sigma_{v,t}$  tend to be higher than  $\sigma_{x,t}$ , when the term structure of options is downward sloping.

## 2.4.4 The Filtered Expected Locking Rate and the Stabilizing Effect of Locking

This Section introduces the results of the filtering method and provides some evidence for the exchange rate stabilizing effect of the prospect of locking.

The filtered expected locking rate depends mainly on the relative weight of the locking rate in the exchange rate  $e^{-\frac{T_t-t}{c}}$ . Parameter c is one of the most important determinants of the relative weight that is calibrated to 10.75. One can interpret the relative weight with that value of c as follows. If a country will lock its exchange rate to the Euro in four years, the elasticity of the exchange rate with respect to market expectations for the final conversion rate  $e^{-\frac{4}{10.75}}$  is almost 70%. If locking is expected to occur in two years, this elasticity  $e^{-\frac{2}{10.75}}$  is more than 80%.

Figure 2.5 shows the relative weight of the locking rate component of the log exchange rate in the investigated period. Under the assumption of constant locking date, it is exponentially increasing in time. More interesting is the case with stochastic locking date, where positive shocks to  $T_t$  can even decrease the relative weight. The relative weight is stable for the Czech Republic, just like the expected time until locking. For Hungary and Poland, the largest change in the relative weights took place after July 2005, when the market sentiment on the EMU entry chances has started to become more and more pessimistic. This trend has been changed in 2006 turning the relative weight of x to increase again. What is important to note is that the relative weight of the expected locking rate has been always higher than 45% for each countries for the entire sample period. Therefore, the locking rate should be considered as an important component of the exchange rate in this period despite of the fact that locking is not expected to take place in less than three years.

Figure 2.6 shows the historical exchange rates of the Koruna, the Forint and the Złoty against the Euro, the filtered states and analysts' average expectations for the central parity as polled by Reuters. Market expectations about the final conversion rate may be thought to be close to the expected central parity of the ERM II regime. In that case, the expected central parity is a good reference for the filtered expected final conversion rate to be compared with. Here, we compare the filtered market expectations with the average expectations of the respondents in the Reuters polls.

Respondents' views on central ERM II parities vary a lot in each poll (see: Figure 2.7). There is at least 6% difference between the two extreme views, and even differences of more than 20% are not rare. These differences indicate that uncertainty concerning the central parity is likely to be high, which one needs to bear in mind when taking the

average reported expectations as the general view of the market.

As is evident from Figure 2.6, the patterns of the filtered expected final conversion rate and of the reported expected central parity are similar for each of the three countries. The difference between the filtered and the survey-based expectation is always less than 5% for Czech Republic. For Hungary and Poland this difference is more than 5% in a few times, but never exceeds 15%. Although being higher than for Czech Republic, still the difference can be considered to be moderate for Hungary and Poland as well. The relatively small deviation of the filtered expectation from the survey-based one can be interpreted as empirical support for the exchange rate model. This test of the model is similar in spirit to the one applied by Engel and West (2005).

For many years, the standard criterion for judging exchange rate models has been based on the forecasting power relative to the random-walk model. This criterion was first proposed by the seminal paper of Meese and Rogoff (1983). Recently, Engel and West (2005) has questioned the standard criterion. They have demonstrated on the asset-pricing exchange rate model that if fundamentals are integrated of order one and the discount factor is close to unity, the exchange rate will approximately follow randomwalk. The important implication of this result is that the standard criterion is not useful at judging the performance of exchange rate models. They have proposed an alternative criterion, which is based on the forecasting ability of the future fundamental. They provide also some empirical support for the conventional asset-pricing exchange rate model based on their test.

It is worth comparing the filtered locking rate  $\hat{x}_t$  not only to the survey-based expectation, but also to the filtered latent exchange rate  $\hat{v}_t$ . In this model higher  $\hat{v}_t$  than  $\hat{x}_t$ suggest that inflation is less of a problem for the economy. Whereas in the opposite case, a later EMU entry is partly due to the high current and future expected inflation.

The filtered locking rate is weaker than the filtered latent exchange rate in Hungary, supporting the view that not only the criteria on the real variables, but also the inflationary Maastricht criterion is binding. In contrast to Hungary, the filtered locking rate is stronger than the filtered latent exchange rate in Poland, what is due to the low inflation rate in the analyzed period. For Czech Republic, there is no significant difference between the filtered locking rate and the latent exchange rate.

This model is rich in dynamics: we have time-varying volatilities  $\sigma_{v,t} \sigma_{x,t}$ , and stochastic  $T_t$ . Each of these dynamics may have important effect on the filtered time series of the factors. In order to see what drives the dynamics of the filtered states we shut down each of these effects one by one. First, the states with deterministic  $T_t$  are filtered, *i.e.*, with zero  $\sigma_{T,t}$ , but with time-varying  $T_t$ . Second, the states with constant  $T_t$  are filtered, by shutting down not only the dynamics due to  $\sigma_{T,t}$ , but also the one due to time-varying  $T_t$ . Finally, we look at the effect of time-varying volatilities on the dynamics of the filtered  $x_t$  by filtering with zero  $\sigma_{T,t}$  and with constant volatilities  $\sigma_{v,t}$  and  $\sigma_{x,t}$ . These constant volatilities are set equal to the averages of their time-varying estimates. Figure 2.8 clearly shows that most of the dynamics do not seem to change the patterns of the filtered factors substantially. We have some important findings on the volatility of the expected final conversion rate as well. The volatility of market expectations for the final conversion rate is important, because relatively stable market expectations can stabilize the exchange rate. The locking rate is often referred to as the nominal anchor of the exchange rate that can decrease the volatility of the exchange rate through the expectation channel.

Regarding the volatilities, one can see from Table 2.3 that the filtered x is more volatile than the survey-based expectation both calculated from data on the same frequencies. This finding might adversely modify our previous view based purely on the Reuters polls on the stabilizing feature of locking. Still, if the volatility of x is lower than that of s, market expectations for the final conversion rate might have some stabilizing effect on the exchange rate. The advantage of filtering the market expectations over using the survey-based expectations is that no reliable estimate can be given on the volatility of x, and on its relative weight in the exchange rate from the low frequency Reuters polls data, therefore no estimate can be given for the stabilizing effect of the locking either.

The big picture on the stabilizing feature of locking in the entire sample period is provided by Table 2.4. The stabilizing effect of locking is calculated as the relative difference between the volatilities of the historical exchange rate and the filtered latent exchange rate. Across the sample period, the stabilizing effect of locking is found to be the highest in the Czech Republic. And somewhat lower, but still high in Poland and Hungary. Even in these two countries, the stabilizing effect is substantial, the volatility without future locking would be approximately twice as large. The estimated stabilizing effect is robust to the calibration of the parameters. See Appendix C on the robustness check.

The result of Naszodi (2008b) on the stabilizing effect is the same qualitatively, but somewhat different quantitatively. The difference between the estimates on the stabilizing effects is due to the following. First, Naszodi (2008b) calculates the stabilizing effect directly from the factor volatilities estimated from the option prices. While this paper uses the estimated factor volatilities to filter out the factors from the exchange rates and then calculates the stabilizing effect as the relative volatilities of the filtered factors. The advantage of this approach is that it better reflects the magnitude of the ex post stabilization as being calculated from the exchange rate. For instance, it suggests that we have stabilization when the exchange rate itself is stable, even if the forward looking measure of stabilization, *i.e.*, the one calculated directly from option prices, suggests just the opposit. Second, Naszodi (2008b) provides only a conservative estimates on the stabilizing effect, whereas here a more precise estimates is given that explicitely takes into account the correlations between the factors.

## 2.5 Conclusion

This paper investigated market expectations for the locking rate of EMU candidate countries based on a theoretical model for exchange rates subject to future locking. The model is the conventional asset-pricing exchange rate model extended with the future locking assumption. In this model, the exchange rate converges to the actual market expectation for the locking rate in expected term. The asset-pricing model with final locking is a three-factor model, where the factors are the latent exchange rate, and the market expectation for the locking rate and date. The expected locking date is modeled as being state-contingent motivated by the Maastricht criteria.

In the empirical part of the paper, the model is used to filter the subjective expectation of market participants for the locking rate for Czech Republic, Hungary and Poland. The expected locking rates in these countries should be considered as important components of the exchange rates as having more than 45 % relative weights. This result is surprising, because locking has not been expected to take place in three years in the sample period.

One of the important contributions of the paper to the theoretical exchange rate literature is that it finds empirical support for the conventional asset-pricing exchange rate model. If we consider the survey-based expectation to be unbiased estimates on the expectation of the markets, then the exchange rate model used to filter the market expectation should be taken to be successful provided the filtered expectations are close to the reported ones of the surveys. This test of the model is akin to the one applied by Engel and West (2005).

Another important finding of the paper is that the stabilizing effect of the future locking is substantial. This qualitative result is also supported by Naszodi (2008b). Moreover, this finding is robust to the calibration of the parameters. The estimated effect is that the volatility without future locking would be approximately twice as large as its current level in all three countries. The magnitude of the stabilizing effect depends mainly on the stability of market expectations for the locking rate, and on the importance of expectations in determining the exchange rate. In the case of an earlier EMU entry, the stabilizing effect is likely to be more substantial, because market expectations for the locking rate are themselves more stable. Moreover, the relative weight of the expectations in the exchange rate is also higher. Based on this intuitive argument, the prospect of locking should contribute the most to the stabilization of the Koruna, compared to the other two currencies. Actually, the estimated stabilizing effect of locking is found to be the highest in the Czech Republic.

The model simplifies the institutional arrangement of EMU entry. It is assumed that countries switch from the floating regime directly to the fixed one. In reality, the road to the Euro consists of at least two regime switches. First, entering the ERM II and than locking the exchange rate. The exchange rate is likely to be stabilized in the intermediate regime by the so called honeymoon effect. However, this stabilizing effect starts only after the exchange rate band is introduced. The main point is that in contrast to the exchange rate, the volatility is not determined in a forward looking way. *I.e.*, expecting to have controlled volatility in a future regime does not reduce by itself the volatility in the current regime. Therefore, the intermediate target zone regime can be disregarded when the volatility in the regime preceding the ERM II entry is examined. Regarding the level of the exchange rate before the ERM II entry, the intermediate regime is not relevant either. This is a direct consequence of an implicit assumption that the transition from the floating regime to the target zone regime is smooth, *i.e.*, it is ruled out to have jumps in the exchange rate. Or in other words, the announcement on the ERM II modalities does not surprise the market, because market participants are expected to learn the central parity preferred by the authorities preceding the ERM II entry. We have no reason to believe in an alternative scenario. First, central banks do not like to be the sources of huge exchange rate shocks (unless there are jumps in the fundamentals what they have to react to). Therefore, the authorities have the incentive to make the mechanism used to determine both the central parity in the ERM II and the locking rate as transparent as possible. Second, the authorities have no incentives either to hide information on the state of the economy. Therefore, all the input parameters of the mechanism will likely to be made available to the market.

Past examples of switches from floating to fixed exchange rate regime include the recent establishment of the Euro, the return of the British pound to gold standard in 1925, the resumption of Specie Payments in the USA after the Civil War, some European currencies following the Napoleonic Wars as well as the dollarization of some currencies or the introduction of currency boards in the Baltic countries. The regime switching framework introduced in this paper is very likely to create interest in the future not only because of the current process of EMU enlargement, but also because of its applicability to the main currency pairs. As a next step, future research will aim at examining these exchange rates by using the same idea presented in this paper. The exchange rates of the main currency pairs can also be expressed as a weighted average of a short and a long term component in accordance with the asset-pricing view. The long term component of the exchange rate can be justified not only by the possibility of final locking, but also by the long run reversion of the exchange rate to its PPP level. To complete the analogy between locking and convergence to PPP, the uncertainty concerning the half-lives of exchange rate deviations from PPP plays a similar role in case of the main exchange rates as the uncertainty concerning the locking dates.

## 2.6 Acknowledgements

The author gratefully acknowledges comments and suggestions from Péter Benczúr, Giuseppe Bertola, Olivier Blanchard, Attila Csajbók, Zsolt Darvas, Balázs Égert, András Fülöp, Christian Gourieroux, László Halpern, Csilla Horváth, Péter Karádi, Júlia Király, Tamás Kollányi, Péter Kondor, István Kónya, Péter Medvegyev, László Mátyás, Ádam Reiff, Jean-Marc Robin, Ákos Valentinyi and David Veredas.

The paper has been presented on the following conferences:

• 21st Meeting of the European Economic Association, August 24-28, 2006, Vienna;

- Thirteenth International Conference on 'Forecasting Financial Markets', May 31-June
- 2, 2006, Aix-en-Provence;
- XIth Spring Meeting of Young Economists (SMYE), May 26-28, 2006, Seville.

Thanks are extended to the organizers and participants of these conferences and to the participants of discussions at the Magyar Nemzeti Bank and Central European University.

A previous version of this paper received the *Olga Radzyner Award* from the National Bank of Austria in 2006 and has been *published* in Focus on European Economic Integration 1/2007 Vienna: Oesterreichische Nationalbank. A more recent, but work-in-progress version has been published as a MNB Working Paper (2008/1).

#### Appendix A

This Appendix derives Equation (2.14) from Equations (2.7), (2.11), (2.12), and (2.13). First, Equations (2.7), (2.11), and (2.12) are rearranged to obtain  $\rho(dz_{T,t}, dz_{x,t}) \sigma_{x,t}$ and  $\rho(dz_{T,t}, dz_{v,t}) \sigma_{v,t}$ .

In order to save space, we introduce the notation  $\chi_{x,t}$  and  $\chi_{v,t}$  for the sum of the following terms

$$\chi_{x,t} = \frac{\partial T_t}{\partial v_t} E\left(\frac{\partial v_t}{\partial x_t}\right) + \frac{\partial T_t}{\partial b_t} E\left(\frac{\partial b_t}{\partial x_t}\right) + \frac{\partial T_t}{\partial \lambda_t} E\left(\frac{\partial \lambda_t}{\partial x_t}\right) + \frac{\partial T_t}{\partial \sigma_{x,t}} E\left(\frac{\partial \sigma_{x,t}}{\partial x_t}\right) + \frac{\partial T_t}{\partial \sigma_{v,t}} E\left(\frac{\partial \sigma_{v,t}}{\partial x_t}\right),$$
(2.30)

$$\chi_{v,t} = \frac{\partial T_t}{\partial x_t} E\left(\frac{\partial x_t}{\partial v_t}\right) + \frac{\partial T_t}{\partial b_t} E\left(\frac{\partial b_t}{\partial v_t}\right) + \frac{\partial T_t}{\partial \lambda_t} E\left(\frac{\partial \lambda_t}{\partial v_t}\right) + \frac{\partial T_t}{\partial \sigma_{x,t}} E\left(\frac{\partial \sigma_{x,t}}{\partial v_t}\right) + \frac{\partial T_t}{\partial \sigma_{v,t}} E\left(\frac{\partial \sigma_{v,t}}{\partial v_t}\right).$$
(2.31)

With the notation of  $\chi_{x,t}$  and  $\chi_{v,t}$ , we obtain

$$\rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t} = E\left(\frac{dT_t}{dx^2}\right)\frac{\sigma_{x,t}^2}{\sigma_{T,t}(T_t - t)} = \left[E\left(\frac{\partial T_t}{\partial x_t}\right) + \chi_{x,t}\right]\frac{\sigma_{x,t}^2}{\sigma_{T,t}(T_t - t)} = \\
= \left[\lambda_t (T_t - t)^{1 - \frac{1}{\lambda_t}}\frac{1}{2\lambda_t}\frac{b_t^2}{c\sigma_{x,t}^2}2x_t + \chi_{x,t}\right]\frac{\sigma_{x,t}^2}{\sigma_{T,t}(T_t - t)} = \frac{\sigma_{T,t}(T_t - t)}{c}x_t + \frac{\chi_{x,t}\sigma_{x,t}^2}{\sigma_{T,t}(T_t - t)},$$
(2.32)

$$\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t} = E\left(\frac{dT_t}{dv2}\right)\frac{\sigma_{v,t}^2}{\sigma_{T,t}(T_t - t)} = \left[E\left(\frac{\partial T_t}{\partial v_t}\right) + \chi_{v,t}\right]\frac{\sigma_{v,t}^2}{\sigma_{T,t}(T_t - t)} = \left[\lambda_t(T_t - t)^{1 - \frac{1}{\lambda_t}}\frac{1}{2\lambda_t}\frac{b_t^2}{c\sigma_{v,t}^2}2v_t + \chi_{v,t}\right]\frac{\sigma_{v,t}^2}{\sigma_{T,t}(T_t - t)} = \frac{\sigma_{T,t}(T_t - t)}{c}v_t + \frac{\chi_{v,t}\sigma_{v,t}^2}{\sigma_{T,t}(T_t - t)} . \quad (2.33)$$

By substracting Equation (2.32) from (2.33), we get

$$\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t} - \rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t} = (v_t - x_t)\frac{\sigma_{T,t}(T_t - t)}{c} + \frac{\chi_{v,t}\sigma_{v,t}^2 - \chi_{x,t}\sigma_{x,t}^2}{\sigma_{T,t}(T_t - t)} .$$
(2.34)

What remains to prove is that the second term of the RHS of (2.34) is zero. This follows easily from Equations (2.13), and (2.7). The latter restricts the partial effect of  $x_t$  on  $v_t$  and vice versa to be zero in expected terms  $E(\frac{\partial v_t}{\partial x_t}) = E(\frac{\partial x_t}{\partial v_t}) = 0$ .

#### Appendix B

This appendix shortly introduces the yield curve method applied to estimate the market expectation for the locking date. The paper by Bates (1999) and Csajbók and Rezessy (2006) provide a more detailed description on this method.

The market expectation, formed at time t, on the date of locking  $T_t$  can be estimated from the forward differentials as follows. The expected value is calculated from the subjective probability distribution of the year the country enters the EMU.

$$T_t = \sum_{i=t}^{T} p_t(EMU_i) \ i \ , \tag{2.35}$$

where  $p_t(EMU_i)$  is the probability that the market attaches at time t to the scenario in which the country becomes a full member of Eurozone in the *i*th year. The distribution is assumed to have finite support, the country will join the Eurozone in the year  $\overline{T}$  at the latest. The marginal probability  $p_t(EMU_i)$  can be calculated from the cumulative probability distribution  $P_t(EMU_i)$ . The interpretation of  $P_t(EMU_i)$  is straightforward: it is the probability that the market attaches at time t to the scenario in which the country is in the EMU by the *i*th year. The cumulative probability can be derived from the pricing equation of the one-year forward interest differential.

$$FS_{t,i} = (1 - P(EMU_i))FS_{t,i}^{non-EMU_i} + P(EMU_i)FS_{t,i}^{EMU_i}, \qquad (2.36)$$

where  $FS_{t,i}$  is the one-year forward interest rate differential for year *i* observed at time *t*. The  $FS_{t,i}^{non-EMU_i}$  and  $FS_{t,i}^{EMU_i}$  are the expected interest rate differentials under the two alternative scenarios, *i.e.*, the accession country is either in or out the Eurozone by year *i*. By rearranging (2.36) we obtain

$$P(EMU_i) = \frac{FS_{t,i}^{non-EMU_i} - FS_{t,i}}{FS_{t,i}^{non-EMU_i} - FS_{t,i}^{EMU_i}}.$$
(2.37)

Among the right-hand-side variables of (2.37) only  $FS_{t,i}$  is observable. However, by assuming that the analyzed countries will surly enter the EMU in nine years, and have almost zero chance to become an EMU member in one year, we can set  $FS_{t,i}^{mon-EMU_i} = FS_{t,t+1}$  and  $FS_{t,i}^{EMU_i} = FS_{t,t+9}$ .

#### Appendix C

This Appendix presents the results of some sensitivity analysis on the stabilizing effect of locking. The parameters which are not subject of the analysis are calibrated exactly the same as in the baseline case presented in Table 2.4.

The estimates for different values of c are presented by Tables 2.5 and 2.6. The expectations for the locking decrease the volatility at least by 30% even under a smaller parameter of 7.25 for each of the countries. The stabilizing effect is increasing in c, for c = 15 it is close 60%.

The estimates are robust not only to the calibration of c, but also to the calibration of  $\frac{dx_t}{dT_t}$ . Table 2.7) shows that the volatility is decreased at least by 45% by the prospect of locking.

The sensitivity of the stabilizing effect to the locking date is also investigated. Instead of assuming stochastic locking date, one may assume to have either constant or deterministic one. In the constant case the market expectation for the locking date is set equal to the initial value of the filtered locking time ( $T_t = T_{t_0}$  where  $t_0 = \text{Jan 4}$ , 2005.). Whereas in the deterministic case, we have time-varying locking time with zero volatility, *i.e.*, the market expectation for the locking date is set equal to the filtered one and it is assumed to be independent of the other factors. In order to see, to what extent does the stabilizing effect depend on the dynamics of the volatilities  $\sigma_{v,t}$  and  $\sigma_{x,t}$ , the stabilizing effect is calculated also with constant volatilities. These constant volatilities are set equal to the averages of their time-varying estimates. Tables 2.8, 2.9 and 2.10 show that the magnitude of the stabilizing effect is robust even to the locking date and the dynamics of the volatilities.

#### Appendix D

This appendix shortly introduces the Kalman filter. It derives the estimator of the state vector  $\Lambda(t)$  for the general model and also for the restricted model with constant  $T_t$ . This appendix proves that the estimator of the restricted model can be interpreted as follows. The Kalman filter decomposes the exchange rate changes ds into changes in the state variables v and x. The higher is the relative weight of v in s, and the higher is its volatility relative to that of the other factor  $\frac{\sigma_v}{\sigma_x}$ , the higher change is attributed to v by the Kalman filter.

The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector  $\Lambda(t)$ , based on information available at time t. The state vector of time t is estimated from the previous periods' estimated state  $\widehat{\Lambda}(t-1)$  and the estimated covariance matrix of the estimation error  $\widehat{Cov}(v(t-1))$ .

The predicted value of state  $\Lambda(t)$  is  $A(t-1)\widehat{\Lambda}(t-1)$  based on Equation (2.23) and the prediction error is  $v(t|t-1) = \Lambda(t) - A(t-1)\widehat{\Lambda}(t-1) = A(t-1)(\Lambda(t-1)-\widehat{\Lambda}(t-1)) + w_1(t-1)$ . The predicted state is different from the estimated state, because the former is calculated purely from the estimated state of the previous periods and does not use the time t realization of the observable variable  $\Omega(t)$ . Therefore, the prediction error is different from the estimation error that is  $v(t-1) = \Lambda(t-1) - \widehat{\Lambda}(t-1)$  for the previous period. The prediction error can be expressed as a function of the previous periods' estimation error  $v(t|t-1) = A(t-1)v(t-1) + w_1(t-1)$  and its covariance matrix can be written as

$$Cov(v(t|t-1)) = A(t-1)Cov(v(t-1))A'(t-1) + Q(t-1).$$
(2.38)

Now, the estimator of the state vector of time t is conditional not only on the estimates of the previous state vector, but also on the contemporaneous observable variable  $\Omega(t)$ . In our system  $\Lambda(t)$  and  $\Omega(t)$  are jointly normally distributed, consequently, the expected value of the vector of states  $\Lambda(t)$  conditional on the observation  $\Omega(t)$  is

$$E(\Lambda(t)|\Omega(t)) = E(\Lambda(t)) + Cov(\Lambda(t), \Omega(t))Var^{-1}(\Omega(t))(\Omega(t) - E(\Omega(t))) .$$
(2.39)

Equation (2.39) is the key for the estimator of the state vector. The estimator of the state vector of time t can be obtained by substituting the estimators of the terms on the right-hand-side into Equation (2.39). First, let us see what are the estimators for these terms, then we can put together the estimator for  $\Lambda(t)$ . Equation (2.23) shows that the unconditional expected value

$$E(\Lambda(t)) = A(t-1)\Lambda(t-1) . \qquad (2.40)$$

The covariance is  $Cov(\Lambda(t), \Omega(t)) = E[(\Lambda(t) - E(\Lambda(t)))(\Omega(t) - E(\Omega(t)))']$ , therefore, one can estimate it by

$$\widehat{Cov}(\Lambda(t), \Omega(t)) = E[(\Lambda(t) - A(t-1)\widehat{\Lambda}(t-1))(C(t)\Lambda(t) + w_2(t) - C(t)\widehat{\Lambda}(t))'] = E[(\Lambda(t) - A(t-1)\widehat{\Lambda}(t-1))(\Lambda(t) - A(t-1)\widehat{\Lambda}(t-1))'C'(t)] = \widehat{Cov}(v(t|t-1))C'(t) .$$
(2.41)
The observable  $\Omega(t) = C(t)\Lambda(t) + w_2(t)$ , therefore its variance is

$$Var(\Omega(t)) = C(t)Cov(v(t|t-1))C'(t) + R(t) .$$
(2.42)

By substituting Equations (2.40), (2.41), and (2.42) into Equation (2.39) we get that the estimator for the state vector at time t is

$$\widehat{\Lambda}(t) = A(t-1)\widehat{\Lambda}(t-1) + \\ + \widehat{Cov}(v(t|t-1))C'(t)(C(t)\widehat{Cov}(v(t|t-1))C'(t) + R(t))^{-1}(\Omega(t) - C(t)A(t-1)\widehat{\Lambda}(t-1)) .$$
(2.43)

Where  $\widehat{Cov}(v(t|t-1)) = A(t-1)\widehat{Cov}(v(t-1))A'(t-1) + Q(t-1)$ . What we are left with is to provide estimates for Cov(v(t)) that

What we are left with is to provide estimates for Cov(v(t)) that will be used in the next step of the recursive process of the Kalman filter. The estimator is given by

 $Cov(v(t)) = Cov(v(t|t-1)) - Cov(v(t|t-1))C'(t)(C(t)Cov(v(t|t-1))C'(t) + R(t))^{-1}C(t)Cov(v(t|t-1)).$ 

After deriving the Kalman estimator for the state vector in the general case, we show that the estimator reduces to a simple, easily interpretable formula in a special case. This special case is the one, where we have exogenous  $T_t$ , the noise  $w_2$  is restricted to zero. For the sake of simplicity, it is assumed that Cov(v(t-1)) = 0, or in other words, the state of the previous period is known. Under these assumptions the variance of the observation error R is zero, the system matrix A is the identity matrix; and Cov(v(t|t-1)) = Q(t-1)is diagonal. In this special case the general formula (2.43) simplifies to

$$\widehat{\Lambda}(t) = \widehat{\Lambda}(t-1) + Q(t-1)C'(t)(C(t)Q(t-1)C'(t))^{-1}(\Omega(t) - \Omega(t-1)).$$
(2.44)

By substituting  $\Lambda'(t) = \begin{pmatrix} v_t & x_t \end{pmatrix}$ ,  $\Omega(t) = s_t$ ,  $C(t) = \begin{pmatrix} 1 - e^{-\frac{T_t - t}{c}} & e^{-\frac{T_t - t}{c}} \end{pmatrix}$ , and  $Q(t-1) = \begin{pmatrix} \sigma_{v,t-1}^2 & 0 \\ 0 & \sigma_{x,t-1}^2 \end{pmatrix}$  into (2.44) we get the following estimator for the two state variables of time t:

$$\widehat{v}_{t} = \widehat{v}_{t-1} + \frac{1 - e^{-\frac{T_{t} - t}{c}}}{\left(1 - e^{-\frac{T_{t} - t}{c}}\right)^{2}} \sigma_{v,t-1}^{2} + \left(e^{-\frac{T_{t} - t}{c}}\right)^{2} \sigma_{x,t-1}^{2} \qquad (2.45)$$

$$\widehat{x}_{t} = \widehat{x}_{t-1} + \frac{e^{-\frac{T_{t}-t}{c}}}{\left(1 - e^{-\frac{T_{t}-t}{c}}\right)^{2}} \sigma_{v,t-1}^{2} + \left(e^{-\frac{T_{t}-t}{c}}\right)^{2}} \sigma_{x,t-1}^{2} \quad (s_{t} - s_{t-1}) \quad (2.46)$$

These estimators provide an intuitive interpretation of the Kalman filter in this exchange rate model. It decomposes the exchange rate changes ds into changes in v and x. The higher is the relative weight of v in s, *i.e.*, the higher is  $1 - e^{-\frac{T_t - t}{c}}$ , and the higher is its volatility relative to that of the other factor  $\frac{\sigma_v}{\sigma_x}$ , the higher is the estimated innovation in v. Similarly, the higher is the relative weight of x in s, and the higher is its volatility relative to that of the other factor  $\frac{\sigma_x}{\sigma_v}$ , the higher is the estimated innovation in x.

## Bibliography

- Bates, D. S., 1999. Financial Markets' Assessment of Euro. Carnegie-Rochester Conference Series on Public Policy 51 (1), 229–269.
- [2] Bilson, J. F. O., 1978. The Monetary Approach to the Exchange Rate: Some Empirical Evidence. *IMF Staff Paper* 25, 48–75.
- [3] Csajbók, A., Rezessy, A., 2006. Hungary's Eurozone Entry Date: What do the Markets Think and What If They Change Their Minds? *Contemporary Economic Policy* 24 (3), 343–356.
- [4] Dewachter, H., Veestraeten, D., 2001. Measuring convergence speed of asset prices toward a pre-announced target. *Applied Financial Economics* 11, 591–601.
- [5] Djajic, S., 1989. Dynamics of the exchange rate in anticipation of pegging. *Journal* of International Money and Finance 8 (4), 559–571.
- [6] Driessen, J., Perotti, E. C., 2004. Confidence Building on Euro Convergence: Theory and Evidence from Currency Options. *CEPR Discussion Paper* no. 4180.
- [7] De Grauwe, P., Dewachter, H., Veestraeten, D., 1999a Price dynamics under stochastic process switching: some extensions and an application to EMU. *Journal of International Money and Finance* 18 (2), 195–224.
- [8] De Grauwe, P., Dewachter, H., Veestraeten, D., 1999b. Explaining Recent European Exchange-Rate Stability. *International Finance* 1 (2), 1–31.
- [9] Égert, B., Halpern, L., MacDonald, R., 2006. Equilibrium Exchange Rates in Transition Economies: Taking Stock of the Issues. *Journal of Economic Surveys* 20 (2), 257–324.
- [10] Engel, C., West, K. D., 2005. Exchange Rates and Fundamentals. Journal of Political Economy 113 (3), 485–517.
- [11] Favero, C., Giavazzi, F., Iacone, F., Tabellini, G., 2000. Extracting Information from Asset Prices: the Methodology of EMU Calculators. *European Economic Review* 44, 1607–1632.
- [12] Fiorentini, G., Sentana E., Shephard, N., 2004. Likelihood-Based Estimation of Latent Generalized Arch Structures. *Econometrica* 72 (5), 1481–1517.
- [13] Flood, R. P., Garber, P. M., 1983. A model of stochastic process switching. *Econo*metrica 51 (3), 537–551.
- [14] Frankel, J. A., 1979. On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials. *American Economic Review* 69, 610–22.

- [15] Frankel, J. A., Froot, K. A., 1987. Using survey data to test standard propositions regarding exchange rate expectations. *American Economic Review* 77, 133–153.
- [16] Froot, K. A., Frankel, J. A., 1989. Forward discount bias: Is it an exchange risk premium? Quarterly Journal of Economics 104, 139–161.
- [17] Froot, K. A., Obstfeld, M., 1991a. Stochastic process switching: some simple solutions. *Econometrica* 59, 241–250.
- [18] Froot, K. A., Obstfeld, M., 1991b. Exchange rate dynamics under stochastic regime shifts: a unified approach. *Journal of International Economics* 31 (1/2), 203–229.
- [19] Geweke, J. F., Zhou, G., 1996. Measuring the pricing error of the arbitrage pricing theory. *Review of Financial Studies* 9, 557–587.
- [20] Gourieroux, C., Jasiak, J., 2001. Financial Econometrics: Problems, Models, and Methods. Princeton University Press, Princeton.
- [21] Hamilton, J. D., 1990. *Time Series Analysis*. Princeton University Press, Princeton.
- [22] Harvey, A. C., 1990. Forecasting, structural time series models and the Kalman filter. Cambridge University Press, Cambridge.
- [23] Harvey, A. C., Ruiz, E., Sentana, E., 1992. Unobserved component time series models with ARCH disturbances. *Journal of Econometrics* 52, 129–158.
- [24] Hull, J. C., 1997. Option, futures and other derivatives. Prentice Hall.
- [25] Karádi P., 2005. Exchange Rate Smoothing in Hungary. MNB Working Papers 6.
- [26] Lund, J., 1999. A Model for Studying the Effect of EMU on European Yield Curves. European Finance Review 2, 321–363.
- [27] Miller, M., Sutherland, A., 1994. Speculative anticipations of sterling's return to gold: was Keynes wrong? *Economic Journal* 104 (4), 804–812.
- [28] Naszódi, A., 2004. Target Zone Rearrangements and Exchange Rate Behavior in an Option-Based Model. MNB Working Papers 2.
- [29] Naszodi, A., 2007. Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?, Working Paper In: Focus on European Economic Integration 07/1 Vienna: Oesterreichische Nationalbank, pp. 115–134.
- [30] Naszodi, A., 2008a. Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?, *MNB Working Papers* 1.
- [31] Naszodi, A., 2008b. Exchange Rate Dynamics under State-Contingent Stochastic Process Switching: An Application to the EMU Entry of New EU Members, Chapter One of the Ph.D. Thesis.

- [32] Obstfeld, M., Stockman, A. C., 1985. Exchange-rate dynamics.. In: Kenen, P. B., Jones, R. W. (Eds.), Handbook of International Economics, North-Holland, Amsterdam, pp. 917–977.
- [33] Stock, J. H., Watson, M., 1993. A Simple Estimator of Cointegrating Vectors in Higher Order Integrating Systems. *Econometrica* 61 (July), 783–820.
- [34] Wilfling, B., Maennig, W., 2001. Exchange rate dynamics in anticipation of timecontingent regime switching: modelling the effects of a possible delay. *Journal of International Money and Finance* 20, 91–113.
- [35] Williamson, J. (Eds.) 1994. Estimates of FEERs.. In Estimating Equilibrium Exchange Rates, Washington DC

	CZ	HU	PL
$\widehat{\sigma}_T$	5.02%	1.57%	1.93%
$\widehat{\sigma}_w$	56.86%	26.9%	53.42%

**Table 2.1**: The estimated parameters of the Kalman filtered  $T_t$ 

	2005	2006	2007 Q1	$\mathbf{Average}^{15}$
	YoY	YoY	$\mathbf{QoQ}$	(2005-2007Q1)
CZ	1.60%	2.09%	1.71%	<b>1.83</b> %
HU	3.48%	4.04%	8.81%	<b>4.32</b> %
PL	2.18%	1.27%	1.96%	$\mathbf{1.75\%}$

 Table 2.2: Harmonized consumer price index HCPI (source IFS)

	CZ	HU	PL
$\sigma(X_{survey})$	2.4%	3.93%	4.31%
$\sigma(X_{filtered})$	4.29%	9.59%	7.07%
Frequency	Quarterly	Montly	Quarterly
Num. obs. of $X_{survey}, X_{filtered}, S$	7	19	7

**Table 2.3**: The volatility of filtered and survey-based locking rate (X) – all data are on the frequency of the survey data

<sup>&</sup>lt;sup>15</sup>The average inflation rates are calculated as the weighted average of the yearly and quarterly inflation rates, where the weights of the quarterly inflation rates are one-forth of that of the yearly rates.

	$\mathbf{CZ}$	HU	$\mathbf{PL}$
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	13.47%	16.86%	20.83%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-59.39%	-43.73%	-50.38%

**Table 2.4**: The volatilities of the exchange rate (S) and the latent exchange rate (V) – with stochastic  $T_t$  and time-varying volatilities, c=10.75.  $\frac{dx_t}{dT_t}$  equals to half of the average excess inflation rate over 2%: -0.085%, 1.15%, and -0.125% for Czech Republic, Hungary, and Poland respectively

	CZ	HU	PL
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	10.52%	13.61%	16.31%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-47.98%	-30.27%	-36.63%

**Table 2.5**: The volatility of the exchange rate (S) and the latent exchange rate (V) – Parameter c is calibrated to 7.25



**Figure 2.1**: Stylized Maastricht criteria - with parameters  $b_t = 1.8$ , c = 10.75,  $\sigma_{x,t} = 4\%$ ,  $\sigma_{v,t} = 6\%$ ,  $\lambda_t = 1.2$ 

	$\mathbf{CZ}$	HU	PL
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	17.07%	20.97%	26.45%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-67.95%	-54.74%	-60.92%

**Table 2.6**: The volatility of the exchange rate (S) and the latent exchange rate (V) – Parameter c is calibrated to 15

	CZ	HU	$\mathbf{PL}$
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	13.43%	17.35%	20.81%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-59.26%	-45.29%	-50.32%

**Table 2.7**: The volatility of the exchange rate (S) and the latent exchange rate (V) – Parameter  $\frac{dx_t}{dT_t}$  is calibrated under the assumption that changes in  $T_t$  are due to inflationary shocks: -0.17%, 2.3% and -0.25% for Czech Republic, Hungary and Poland respectively

	$\mathbf{CZ}$	HU	$\mathbf{PL}$
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	13.25%	18.12%	20.81%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-58.72%	-47.64%	-50.33%

**Table 2.8**: The volatility of the exchange rate (S) and of the latent exchange rate (V) – with deterministic locking time

	CZ	HU	$\operatorname{PL}$
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	12.47%	23.71%	21.89%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-56.13%	-59.97%	-52.77%

**Table 2.9**: The volatility of the exchange rate (S) and of the latent exchange rate (V) – with constant locking time  $(T_t = T_{t_0}$  where  $t_0 = \text{Jan}, 4\ 2005)$ 

	CZ	HU	$\mathrm{PL}$
$\sigma_S$	5.47%	9.49%	10.34%
$\sigma_V$	12.3%	15.06%	18.45%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-55.51%	-36.98%	-43.98%

**Table 2.10**: The volatility of the exchange rate (S) and of the latent exchange rate (V) – with deterministic locking time and constant volatilities  $\sigma_{x,t} = \overline{\hat{\sigma}}_{x,t}, \sigma_{v,t} = \overline{\hat{\sigma}}_{v,t}$ 









Poland



**Figure 2.2**: The average expectation of the analysts for the EMU and the ERM II entry date and the filtered locking date  $T_t$ 



Figure 2.3: Stylized inflation paths until locking









Poland



**Figure 2.4**: The option prices with the shortest and longest maturities and the fitted values in terms of volatility – with stochastic locking date

Czech Republic



Hungary



Poland



**Figure 2.5**: The relative weight of the expected log locking rate  $x_t$  in the log exchange rate  $s_t$  – with constant and stochastic locking date













Figure 2.6: The filtered expected locking rate, the latent exchange rate, the historical exchange rate and the survey-based expected locking rate – with stochastic locking date

## Czech Republic







Poland



**Figure 2.7**: Minimum, maximum and average expectation of the market analysts for the central parity in the ERM II

## Czech Republic











**Figure 2.8**: The filtered expected locking rate with stochastic, deterministic and constant  $T_t$  and with time-varying and constant  $\sigma_{x,t}$  and  $\sigma_{v,t}$ 

## Chapter 3

# Target Zone Realignments and Exchange Rate Behavior in an Options-Based Model

This paper develops an options-based model of target zone arrangements. The exchange rate in a target zone system is modeled as a fundamental term adjusted by the price of two options. The paper provides an accurate description of the options, which can capture the joint effect of the edges of the band. The model can be applied for a wide range of processes of the fundamental. The model is used to decompose exchange rate changes after band realignment into the direct effect of realignment, changing expectations and changing uncertainty. It is applied to realignments of France, Hungary and Portugal prior to their EMU entry.

JEL: F31, F33, G12, C63.

Keywords: target zone system, options, target zone realignment, EMU entry, EMS.

## 3.1 Introduction

In his seminal paper Krugman (1991) mentioned the possible link between the theories of the target zone system and that of option pricing. The idea is that the exchange rate in a target zone system is equivalent to the exchange rate of a currency in an underlying freely floating system adjusted by the price of two options.<sup>1</sup> This paper further develops Krugman's idea by exactly specifying the type of options and by providing the corresponding option pricing method.

The contribution of the paper to the existing literature is twofolds. First, a new optionbased model is set up, which takes into account the joint effects of the edges of the band. To model the simultaneous effects of the edges is highly relevant in the case of narrow exchange rate bands, like for example the French band prior to its widening in 1993. The novelty of the option-based model is that it can be applied for a wide range of processes of the underlying floating exchange rate. This flexibility of the model is important, when the processes of the exchange rate is mainly driven by the convergence to the expected euro locking rates and can not be modeled by a Brownian motion for instance.

Second, the model is applied to decompose and estimate the exchange rate changes after band realignments. The expected depreciations calculated by the model deviate only slightly from its historical level for the Hungarian Forint, whereas for the studied French and Portuguese realignments, the model somewhat overestimates the actual depreciation. One can expect the model to overestimate the exchange rate changes when the analyzed realignment is anticipated by the market. This shows that the French and Portuguese realignments were anticipated by the market at least a few days before the measure was taken, whereas the Hungarian realignment was mainly unexpected in the two weeks period preceding the band-shift.

There is only one paper in the literature, besides Krugman's, which touches the options-based approach of target zones. Copeland (2000) (Chapter 15.) models the effect of one edge of the band by one option which has only limited empirical relevance.

The options-based model developed in this paper is closely related to Krugman's target zone model. The target zone exchange rate is an S-shaped function of the fundamental in both models. In the options-based model the process of the target zone exchange rate is limited by two options. At the expiration date of the options the target zone exchange rate is a broken linear function of the underlying floating exchange rate (a flipped Zcurve), which is also the starting point of Krugman's model. In the Krugman model the forward looking nature of the exchange market leads to an S-curve instead of the flipped Z-curve, whereas in the options-based model the non-linear feature of the options explains the shape of the curve. The close relationship between the Krugman model and the options-based model is also indicated by the fact that the option's price as a function of the underlying asset has a curve shape caused by the expectations. Therefore, the options-based model can be thought of as a short cut to the process of the exchange rate,

<sup>&</sup>lt;sup>1</sup> "Now the actual exchange rate may be viewed as the price of a compound asset. This asset consists of the imaginary asset (...), plus the right to sell the asset at a price  $\underline{s}$ , plus the obligation to sell at the price  $\overline{s}$  on demand. "Krugman (1991), p. 677.

which utilizes the results of the option pricing literature.

Then, in both models the exchange rate would be equal to the fundamental if there were no target zone. Given this fact, I refer to the fundamental as the underlying floating exchange rate, *i.e.*, the exchange rate that would prevail in a floating system.<sup>2</sup>

The advantage of showing the analogy between the problems of option pricing and determining the effect of the target zone is that the option pricing literutre offers solution for a wide range of processes of the underlying asset. The underlying asset can have stochastic volatility, stochastic trend, non-gaussian returns, correlating process with the interest rate. In contrast to the flexibility of the options-based framework, the Krugman paper derives the process of the target zone exchange rate only in one particular case, when the process of the fundamental is Brownian motion.

The rest of the paper is organized as follows. Section 3.2 introduces the options-based model. Section 3.3 deals with the option pricing method. In Section 3.4 the model is applied to the band realignments of France (1993), Hungary (2003) and Portugal (1995). Section 3.5 deals with the possible objections to the options-based model, and Section 3.6 concludes.

## 3.2 The Options-Based Model

According to the options-based model developed here, a currency in a target zone system is nothing else than a currency in a freely floating system with two options. One is a long put option with the strike price equal to the weak edge of the band. The other is a short call option with the strike price equal to the strong edge of the band. The commitment to the exchange rate system is fully credible, so neither the edges of the band, nor the strike prices are stochastic.

The existence of the two options can be explained in the following way. If the central bank promises to keep the exchange rate in the predetermined band, then, on one hand, the bank assumes the obligation of repurchasing its currency at the rate equal to the weak edge of the band. This provides a long put option from the viewpoint of the currency holders. On the other hand, the central bank does not let the exchange rate strengthen beyond the strong edge of the band. From the viewpoint of the currency holders, it looks as if the central bank had a purchasing right at the rate equal to the strong edge of the band.

Since foreign exchange market participants can exercise their put option by trading with the central bank, the existence of the put option is obvious. The existence of the call option is less trivial, because the central bank can not force anybody to sell its strong domestic currency at the strong edge of the band. Instead, the central bank has the obligation to buy an unlimited amount of weak foreign currency at the strong edge, which

 $<sup>^{2}</sup>$ Rangvid and Sorensen (2001) applies the similar concept of the shadow exchange rate for a different problem. They filter out the shadow exchange rate of some ERM currencies and investigate which fundamental macroeconomic factors are able to explain the behavior of the filtered shadow exchange rate.

has the same effect on the exchange rate as a short call option. Therefore, the effect of the strong edge can be modelled indeed as a call option. The options are American-type, because they can be exercised at any time within the existing target zone system.

The underlying assets of the options are far from being obvious. If a currency holder exercises the put option, then he or she is free not only from the floating currency, but also from the obligation incorporated in the call option. In addition, if the central bank exercises its call option, then the central bank buys the floating currency and withdraws the currency holder's right incorporated in the put option. These options can be viewed as exchange options,<sup>3</sup> where the holder of the option has the right to exchange the underlying asset for money amounting to the strike price. Whatever is exchanged for the strike price, it is the underlying asset. Therefore, the underlying product of the call component of the target zone exchange rate is the floating currency along with the put option, and the underlying product of the put component of the target zone exchange rate is the floating currency along with the call option.<sup>4</sup> So each option is part of the underlying product of the other option:

$$S_t = F_t + P_{t,Kp,a}(F - C_{Kc,a}) - C_{t,Kc,a}(F + P_{Kp,a}) \quad , \tag{3.1}$$

where  $S_t$  is the exchange rate of the currency in the target zone system at time t,  $F_t$  is the exchange rate of the floating currency at time t. The value of the American-type put option at time t with the strike price Kp is  $P_{t,Kp,a}(F - C_{Kc,a})$ . Its underlying product, presented in parenthesis, is the floating currency along with the short call option. Kpequals the weak edge of the band.  $C_{t,Kc,a}(F + P_{Kp,a})$  is the value of the American-type call option at time t with the strike price Kc. Its underlying asset is the floating currency along with the long put option. Kc equals the strong edge of the band.

By taking Krugman's idea, we can formalize a similar but simpler options-based model

$$S_t = F_t + P_{t,Kp,a}(F) - C_{t,Kc,a}(F) \quad . \tag{3.2}$$

This expression differs from (3.1), as the underlying product of the options is simply the floating currency. The difference between the two models is insignificant if the band is wide enough, because in this case only one of the options is significant, while the value of the other is marginal. Consequently, the accurate and complex determination of the underlying product does not change the option prices significantly in a wide band, but might change it in a narrow band. Even if the simplified model (3.2) works well in practice, it might lead to theoretical problems, such as having an exchange rate outside the band. To show that possibility, suppose that the simple put option  $P_{t,Kp,a}(F)$  is exercised. Then this put option along with its underlying product is worth as much as the strike price:  $F_t + P_{t,Kp,a}(F) = Kp$ . By plugging this expression into (3.2), we get

 $F_t = F_t + P_{t,Kp,a}(F) - C_{t,Kc,a}(F) = Kp - C_{t,Kc,a}(F)$ . Having a positively valued call option, the exchange rate gets outside its band, being less than the weak edge of the band Kp.

<sup>&</sup>lt;sup>3</sup>See Margrabe (1978).

<sup>&</sup>lt;sup>4</sup>One can also think of these compound options as exotic knock out options. The knock out condition for both options is that the other option is exercised.

## 3.3 Option Pricing Method

The option pricing method is not straightforward for two reasons. First, these are American-type options, what can not be priced by a closed form pricing formula. Second, one part of the underlying assets of each of these options is the other option itself. Although the option pricing literature discusses the pricing of a compound option,<sup>5</sup> these results are not applicable here, because not only one of the underlying asset is an option, but both. Therefore, a suitable option pricing method needs to be developed here.

This is an iterative method developed within the basic binomial option pricing framework. Given the binomial tree of the floating exchange rate, this method prices the call and put components of the target zone exchange rate.

By applying the iterative method, I make a sequence of call and put options. The sequence of the put and call options are monotonously increasing, but they never exceed the put and call components of the target zone exchange rate (see Proposition 1 in the Appendix). These two characteristics, the monotony and the boundedness are sufficient conditions for the convergence of the sequences of put and call options (see Corollary 1 in the Appendix). The sequence of the put options converges to the put component of the target zone exchange rate and the sequence of the call options converges to the call component of the target zone exchange rate (see Proposition 2 in the Appendix).

Steps of the iterative method:

In the first step of the iterative method one determines the price of the options as if their underlying products were simply the floating part (F). We then get a  $P^1$  and a  $C^1$ option. Continuing the iterative method: in the *i*th step the underlying product of  $P^i$  is  $F - C^{i-1}$  and the underlying product of  $C^i$  is  $F + P^{i-1}$ .

The  $P^1$  and the  $C^1$  options, as well as every further options in the sequences, are priced on the usual way in the binomial framework.<sup>6</sup> First, one should determine the option prices at the final nodes in the binomial tree, and then the current price of the option is calculated by proceeding backwards. The price of an American type option at a given non-final node is the maximum of its intrinsic value and its discounted expected future value.

## 3.4 Application of the Model

The European Monetary System (EMS) was a multilateral target zone exchange rate system. There were fifty-six realignments during the 1979-1997 period, implemented in seventeen discrete adjustments.<sup>7</sup> Consequently, the EMS has witnessed plenty of devaluation episodes. Among these we particularly analyze two realignments which happened relatively close to the final lockings of the exchange rates: the French band widening in 1993, and the Portuguese band shift in 1995. We also study the 2003 Hungarian band

<sup>&</sup>lt;sup>5</sup>See, for example, Geske (1979) on valuing compound option. In Geske (1979), the underlying asset of the compound option is such an option that has a stock as an underlying asset.

<sup>&</sup>lt;sup>6</sup>See, for example Hull (1997) Chapter 9 on option pricing with the binomial tree. <sup>7</sup>See: Lederma Redríguez et al (2000)

 $<sup>^7\</sup>mathrm{See:}$  Le desma-Rodríguez et al (2000).

shift which is quite relevant as the locking date is approaching fast. Figures 3.2–3.4 show the time series of the exchange rates.

First, the assumed process of the floating exchange rate is introduced and then the method of decomposition of the exchange rate changes into the direct effect of realignment, changing expectations and changing uncertainty is discussed. The results of the French band widening and the Portuguese and Hungarian band shifts are discussed finally.

#### 3.4.1 Assumed Process of the Floating Exchange Rate

We choose the process of the floating exchange rate so as to fit an exchange rate with future locking. The only difference between the actual exchange rate and the floating exchange rate is that the first is in a target zone, but not the second. The floating exchange rate is assumed to be locked at the same time and at the same rate as the target zone exchange rate.

The French band widening took place approximately 5 years before its irrevocable locking while the Portuguese band shift was about 4 years prior to its locking. Hungary was expected to become a member of the EMU in 5-6 years at the time of the band shift, so the exchange rate of the local currency, the Forint was also expected to be locked in the vicinity of the realignment. The expected euro locking rate is surely having a strong influence on the spot exchange rate in the run-up period to currency union. Motivated by this, we assume that the floating exchange rate is mainly driven by the convergence to the expected euro locking rate.

Since the option pricing method developed here is based on the discrete binomial model, one has a great flexibility at choosing the process of the floating exchange rate. The assumed process is a special discretized Brownian Bridge process. The Brownian Bridge process is often used in the literature of fixed income for modeling the process of the bond price. Both the face value and the spot price of bonds are deterministic, but not the future price before maturity. Similarly, both the locking date and the spot exchange rate can be modeled as being deterministic, whereas the future exchange rate before the locking can be considered to be stochastic. This is captured by the Brownian Bridge process. The discretized process can be represented by a recombinant binomialtree. This process is illustrated in Figure 3.1. The initial value of the floating exchange rate is  $F_0$ ; the locking time and rate are T and  $S_T$  respectively. The time varying volatility of the floating exchange rate is determined by parameter h. The time interval [0,T] is divided into N number of intervals of length dt; consequently, parameter N determines the "fineness" of the binomial model showing the number of sub-periods until locking. At each sub-period the exchange rate can shift either upward or downward with equal probalities. The nodes are equidistantly distributed at the initial time around the initial value of the floating exchange rate  $(F_0)$ ; nodes are on the line segment or "radials" starting at the initial nodes and ending at the node representing the locking. The expected shift of the exchange rate at each node is along the "radial" starting from the given node.

The process of the floating exchange rate defined above is in line with our intuitions: as time passes, the ex-ante range of the process grows until the effect of the locking becomes dominant, turning the process to have decreasing range. Finally, the process should end up at its expected locking rate.

After having illustrated the process of the floating exchange rate, its algebraic definition is presented by determining its value for an arbitrary node. At time *idt* the floating exchange rate for the node representing k times "upward" shifts and i - k times "downward" shifts is

$$F_{t=idt,u=k} = \frac{i}{N}S_T + \frac{N-i}{N}F_0 + \left(\frac{N-i}{N}[k-(i-k)]\right)h \quad . \tag{3.3}$$

In order to determine the process of the exchange rate one has to set the  $S_T$ , T, h, N,  $F_0$  parameters of the process of the floating exchange rate. All the other parameters of the options are given: the strike prices are always equal to the actual edges of the band; foreign and domestic yield curves are also known.

#### 3.4.2 Calibration

In the case of the French Franc and the Portuguese Escudo it is assumed that the expected euro locking rate  $(S_T)$  is equal to the central parities, before and after the realignment as well. This is related to the fact that the central parity of the French Franc at the time of band widening, and the actual central parity, after the band shift, of the Portuguese Escudo, have become the locking rates some years later. It is also assumed that the expected time until locking (T) is the expost realized one, for the French Franc 5.4 years, and for the Portuguese Escudo 3.8 years. This simplification does not take into account the uncertainty related to the locking rate and time.

In the Hungarian case, the monthly Reuters polls<sup>8</sup> are used for calibrating T and  $S_T$ . The polls survey the expectations of the market analysts concerning both the date of euro introduction and the two years ahead exchange rate. The latter is considered as the expected euro locking rate.

We determine the exchange rate before  $(S_b)$  and after the band realignments  $(S_a)$  as the arbitrarily chosen 15 days average exchange rates in order to get rid of microstructure noise, and other noises influencing the exchange rate in the short-run. The historical volatility of the exchange rate both before and after the realignments is estimated from the same 15 days data. The *h* parameter of the floating exchange rate is one of the most important determinants of the instantaneous volatility<sup>9</sup> of the target zone exchange rate besides the level of the target zone exchange rate. The *h* parameter is calibrated so, that the instantaneous volatility at a given level of the exchange rate equals the historical volatility. The strike prices of the options (Kp, Kc) are set to be equal the actual edges of the band both before and after the realignment. The parameter *N* determining the

<sup>&</sup>lt;sup>8</sup>Preceeding the Hungarian band shift of 4 June 2003, the Reuters poll interviewed market analysts on 22 May. The next Reuters poll was on 19 June. Consequently, the polls reports were not exactly about the expectations just before and just after the realignment. But unfortunately, this is our best source of information about the expectations.

<sup>&</sup>lt;sup>9</sup>The instantaneous volatility in the binomial framework equals to the standard deviation of the binary outcome in the next period.

"fineness" of the binomial model, is set to a reasonably high, but computationally feasible value: 52 times the number of years until the locking.

Once all the above parameters  $(T, N, S_T, h, Kp, Kc, yield curve)$  are known, the option pricing method can be applied to get a functional relationship between the floating and the target zone exchange rates. The inverse of this function provides the floating exchange rate  $(F_0)$  belonging to the observed spot exchange rate.

#### 3.4.3 Decomposition of the Exchange Rate Changes

The options-based model is used to decompose exchange rate changes after band realignment into the direct effect of realignment, changing expectations and changing uncertainty. The decomposition of the exchange rate changes is based on a comparative static analysis.

The direct effect of the changing band is calculated under the assumption that neither the floating exchange rate nor its volatility parameter (h) is affected by the realignment. These assumptions are relaxed, when the effect of changing expectations and growing uncertainty are calculated.

The effect of the changing band is calculated through the following steps. First, the unobservable parameter h is calibrated to the observed volatility and exchange rate before the realignment. Second, the functional relationship between the floating and the true exchange rates under the estimated h and under all the specified parameters  $(T, N, S_T, Kp, Kc)$  are determined. Third, by using the inverse of the functional relationship, I obtain the floating exchange rate belonging to the observed exchange rate before the realignment. Finally, the new functional relationship between the floating and target zone exchange rates is determined. This new functional relationship differes from the one before the realignment due to having different strike prices of the options that correspond to the exchange rate belonging to the unchanged floating exchange rate. This would be the exchange rate just after the realignment, if only the direct effect of the changing band was relevant.

The *effect of changing expectation* is calculated as an additional effect to the primary direct effect. The expected euro locking rate is assumed to change by the same rate as the central parity, if it changes at all. Moreover, it is also assumed that the spot floating exchange rate changes by the same rate as the expected locking rate. So, the possible change in both the spot floating exchange rate and in the expected euro locking rate are taken into account.

The effect of the changing expectation concerning the locking rate is calculated through the following steps. The changing locking rate changes the process of the floating exchange rate resulting in a new functional relationship between the floating and the target zone exchange rates. First, I this new functional relationship with the new locking rate and new strike prices is determined. Then, by using the new functional relationship, the exchange rate belonging to the new floating exchange rate is obtained. This exchange rate would characterize the period just after the realignment if only the direct effect and the changing expectations would affect the exchange rate. Another indirect *effect* besides the changing expectation is related to the *growing* uncertainty. An unexpected realignment not only motivates the market participants to revise their expectations about the locking rate, but it might also cause disturbance on the whole market. As it can be seen volatility increased in all the three analyzed countries. The changing volatility is captured by a new h parameter.

The additional effect of the changing uncertainty is calculated as follows. First, the new h parameter is set to characterize the period just after the realignment. The changing h parameter changes the process of the floating exchange rate resulting in a new functional relationship between the floating and the target zone exchange rates. Then, the target zone exchange rate belonging to the new floating exchange rate is obtained. This incorporates all three effects of the realignment.

#### 3.4.4 The Band Widening of the French Franc

On 3 August 1993, the exchange rate band of the French Franc widened from its original width  $\pm 2.25\%$  to  $\pm 15\%$ , while the central parity remained unchanged. The band widening had a similar effect on the French Franc, Belgian Franc and Danish Krone. Here only the case of French Franc is followed up.

Its exchange rate is measured as the price of one German Mark (DEM) in term of Franc (FRF). The exchange rate characterizing the period before  $(S_b)$  the widening was 3.416 FRF/DEM and the annualized volatility was 2.88%, whereas after the realignment it was 3.503 FRF/DEM and 6.78% respectively. Parameter h is calibrated to 18.2% before the band widening, in order to match the 2.88% volatility, while it is set to 3.41% after the realignment, to match the 6.78%. As indicated earlier, the euro locking rate  $(S_T)$ , as well as the time of locking (T) is set to the values historically developed later.

Figure 3.5 shows the relationships between the floating and the target zone exchange rates. Its line 0 demonstrates the relationship before the band shift. Since the exchange rate before the band widening  $(S_b)$  was approximately 3.416 FRF/DEM, the floating exchange rate should have been 3.92 FRF/DEM according to line 0. Line 1 deviates from line 0, because the strike prices of the options were changed. If the floating exchange rate remained unchanged, the target zone exchange rate should have weakened to 3.616 FRF/DEM as a result of the changing strike prices.

Line 2 shows not only the effect of the changing strike prices, but also of the changing volatility. According to line 1 the exchange rate should have weakened substantially to 3.77 FRF/DEM if the floating exchange rate remained unchanged. Consequently, the model attributes 5.9% weakening to the direct effect of the realignment and 4.4% depreciation to the growing volatility.

The model-based weakening (10.5%) does not coincide with the observed weakening, which was about 2.5%. The most likely reason is that the realignment was not completely unexpected. The deviation could also be partly explained by the changing short-term interest rate differential, which had risen from 0.92% to 1.37%. The almost 50 basis points increase might have had a short-term effect on the exchange rate, which is obviously not captured by the model.

#### 3.4.5 The Band Shift of the Portuguese Escudo

In 1995 the Portuguese Escudo depreciated by about 1.8% from 103.612 PTE/DEM  $(S_b)$  to 105.49 PTE/DEM  $(S_a)$  as an effect of the 3.7% devaluation of the band. Preceeding the band shift the annualized volatility was 2.1%, which had risen to 5.2% after the realignment. The volatility was relatively low despite the fact that the exchange rate was in the more volatile region of the band, *i.e.*, in the middle range, and the band was already widened allowing the exchange rate to fluctuate in a  $\pm 15\%$  band.

Figure 3.6 shows the relationships between the floating and the target zone exchange rates. Line 0 shows the relationship before the band shift. Line 1 deviates from line 0 because the strike prices of the options were changed. The exchange rate was approximately 104 PTE/DEM before the band shift  $(S_b)$ , and the adherent floating exchange rate should have been the same according to line 0.

Lines 0 and 1 coincide almost on their entire domain and also at the original floating exchange. Consequently, the band shift has no direct effect on the exchange rate. It is not surprising that according to the model the direct effect of the band shift is insignificant, since the exchange rate was near to the central parity, the band was relatively wide and the volatility low. In this case neither the band, nor a band-shift can significantly influence the exchange rate.

Next, line 2, differs from line 1 because the change of the expected euro locking rate was also taken into account. The new expected locking rate is set to the new central parity. Line 2 almost coincides with line 1 over the entire domain. Consequently, almost no effect can be attributed to the changing expectations under the assumption of having the floating exchange rate unchanged. We made the assumption that the floating exchange rate will be locked at the same rate and same time as the exchange rate. Based on this assumption one can argue that, as a result of the band shift, not only the expected locking rate should change, but also the floating exchange rate, because its expected locking rate changes as well. We can assume, for instance, that the floating exchange rate weakens by the same rate as its expected locking rate. As a result of the almost linear relationship between the floating exchange rate and the actual exchange rate inside the band, the latter should weaken by the same rate. As a consequence, the Escudo should have weakened by the magnitude of the band shift, namely 3.7% due to the changing expectations.

Line 3 embodies not only the changing strike prices and the changing expected euro locking rate, but also the changing volatility. This affects the shape of the line in the irrelevant range only, near to the edges of the band by bending it. Consequently, the changing volatility should not have any effect on the exchange rate either.

The observed weakening of the exchange rate was 1.8%, which is less than 3.7%. One reason for the deviation might be that the floating exchange rate weakens by less than the expected euro locking rate. Another possible explanation is provided by the changing interest rate differential, which had risen from 4.86% to 5.99%. The more than 100 basis points increase of the interest rate differential might have had an effect on the exchange rate, which is disregarded by the model. Finally, just like in the French case, one can not rule out the possibility that the band shift was anticipated by the market.

#### 3.4.6 The Band Shift of the Hungarian Forint

On 4 June 2003 the central parity of the Forint was shifted by 2.26%, while  $\pm 15\%$  intervention band remained unchanged. The exchange rate fell about 5.6%. This depreciation might be interpreted as being relatively large, after a small shift of the band. It is likely, that the band shift surprised the market, as a few months earlier, a decision to shift in the opposite direction was expected by some market participants. At that time the Forint's expected strengthening led to a speculative attack.<sup>10</sup> The exchange rate of the Forint begun to weaken somewhat before the realignment, which may be interpreted as some market participants have anticipated the band-shift, but the weakening may be attributed to other reasons as well.

The monthly Reuters poll informs us that the average expected exchange rate for the end of 2004 was 238.7 HUF/EUR before the realignment, and after it became weaker by 4%. At the time of realignment the expected locking rate was not surveyed by the Reuters poll, so it has to be set to its closest substitute available: the average expected exchange rate for end 2004, the furthest time reported. The parameter of the locking rate ( $S_T$ ) is set to 238.7 HUF/EUR before the realignment, and to 248.4 HUF/EUR after. The reported average expected date of Hungary's entry to the EMU was the middle of 2008 and did not change for the Reuters poll after the realignment. Consequently, T is set to 5 years according to the expectations.

The realignment caused a jump in the historical volatility: preceding the band shift the annualized volatility was 13.77%, which had risen to 18.4% after the realignment.

Figure 3.7 shows the relationships between the floating and the target zone exchange rates. Line 0 demonstrates the relationship before the band shift. The exchange rate was approximately 248 HUF/EUR before the band shift. Line 1 deviates from line 0 because the strike prices of the options were changed. For the unchanged floating exchange rate the exchange rate is 252.7 HUF/EUR according to line 1. Consequently, the direct effect of the band shift is 2%.

In order to take into account some further effects of the band shift, we plotted line 2 and line 3 as well, which show the effect of the changing expected euro locking rate and the growing volatility respectively. The floating exchange rate is assumed to weaken by the same rate as its expected locking rate. Since the expected euro locking rate of the floating exchange rate is equal to the expected euro locking rate of the exchange rate by assumption, the floating exchange rate should depreciate by 4% as well. (See the arrow in Figure 3.7) At that weaker floating exchange rate the target zone exchange rate is 252.24 HUF/EUR according to line 2, indicating an almost zero, 0.2% appreciation as an effect of the changing expectations. Line 3 differs from line 2 by taking into account the increased volatility: parameter h was increased from 14.9 to 25. Assuming that the changing volatility has no effect on the floating exchange rate, the exchange rate depreciates to 261 HUF/EUR.

Summarizing the decomposition: 2% depreciation can be explained by the direct effect, almost no effect (-0.2%) can be attributed to the changing expectations, and another 3.5%

<sup>&</sup>lt;sup>10</sup>See: Barabás (2003) on the speculative attack of the Forint in January 2003.

depreciation is due the growing volatility. The total effect of the band shift on the exchange rate is almost 5.4% according to the model, which is very close to the observed 5.6% depreciation. Since the exchange rates characterizing the before and after realignment states are calculated as the 15 days average exchange rates, all we can conclude about the expectations concerning the realignment is that the Hungarian band-shift was mainly unexpected in the two weeks period preceding the regime switch.

## 3.5 Critique of the Model

The model outlined here can be criticized both from theoretical and practical points of view. We discuss here the drawbacks of assuming a perfectly credible target zone system with intervention exclusively at the edges of the band and also the definition of the floating exchange rate.

The model introduced in the paper describes the target zone exchange rate as a combination of a floating exchange rate and two options. The floating exchange rate would be the true rate if the exchange rate system were a floating one, and all the real variables remained unchanged. This ceteris paribus way of thinking is not necessarily appropriate, if the target zone system influences the real variables by its very existence. <sup>11</sup>

As the target zone system reduces the volatility of the exchange rate and as it also moderates the uncertainty in the economy, it may have a favorable effect on real variables. Stockman (1999) challenges this assumption.<sup>12</sup> Baldwin–Krugman (1989) points out another connection between the real variables and the exchange rate system. They think that excessive exchange rate shocks, which do not occur in a target zone system, have a persistent effect on the trade and the equilibrium exchange rate.<sup>13</sup>

In order to maintain the target zone exchange rate system, a central bank does not exclusively have the opportunity to intervene in the foreign exchange market but it can

<sup>12</sup>Stockman argues that for most of the countries the floating exchange rate system is favorable. Although Stockman admits that the uncertainty could cause real effects, but the macro indicators in the last decades support the opposite. This fact can be explained by the improvement of the financial markets. The risk can be easily eliminated in an improved market by different hedging opportunities. Consequently, the real effects are significant only if the agents cannot hedge at a low price or if the practice of hedging is not widespread.

Albeit for the total elimination of the risk it would be also necessary to have the opportunity to hedge for whatever long term.

<sup>13</sup>Their reasoning is based on the fact that the entry and withdrawal of companies to/from the market and the beginning/halting of export activities depend on excessive changes in the exchange rates. Due to an overestimation of the domestic currency following an economic shock, foreign companies appear in the domestic market, and later, when the exchange rate returns to the original level, they will not withdraw from the market. Therefore the trade-balance along with the equilibrium exchange rate permanently changes.

<sup>&</sup>lt;sup>11</sup>Investigating the feedback effect from the exchange rate to the financial and macroeconomic variables or modeling them jointly is rearly done in the target zone literature. As it is discussed by Bekaert and Grey (1998): "Full information maximum likelihood requires that we model the joint density of exchange rate changes and the conditioning variables, which is beyond the scope of this paper. In empirical work in this area, it is customary to proceed by maximizing the conditional likelihood function."

also influence the exchange rate through its interest rate policy. Consequently, interest rate might have a different role in a target zone system than in a flexible one.

The above reasons compel us to take into consideration the endogenous nature of some real variables and the special role of the interest rate policy when determining the process of the exchange rate in the options-based model. But one should be cautious as well when applying models other than the options-based model, and determining the process of the fundamental, because similar problems might show up there too. It seems hardly possible to take into account this feedback of the exchange rate system concerning the main variables such as interest rates, and complete the models with them.

As to the assumed perfect credibility of the target zone system, it should be noted that in a realistic target zone system with the possibility of band realignments the exchange rate moves toward the direction of the band shift prior to the realignment, provided that the band shift is expected. Therefore, by assuming that no band-shift is expected and the target zone is perfectly credible, the model overestimates the effect of the band realignment on the exchange rate. In order to avoid this bias, one should increment the model with the possibility of band realignments and by modeling the expectations for realignments.

The assumption that the central bank intervenes only when the exchange rate already reaches the edge of the band might be unrealistic, especially when the central bank prefers more exchange rate stability than is declared by the official wider band. Most of the exchange rates in the EMS could fluctuate in a  $\pm 15\%$  range after the band widening in 1993, although they were restricted into a narrower band.<sup>14</sup> Since the interventions inside the band and the implicit bands are confidential, it is difficult to analyze the exchange rate in such systems.

### **3.6** Conclusions

In this paper we developed an options-based model of target zone arrangements. The exchange rate in a target zone system is equivalent to the exchange rate of a currency in an underlying freely floating system adjusted by the price of two options. This paper has determined the underlying assets of the options accurately and provided an option pricing method applicable for these options.

By applying the option-based approach, we analyzed the following band realignments: French band widening in 1993, Portuguese band shift in 1995, and the Hungarian band shift in 2003. We conclude that the French Franc and the Hungarian Forint depreciated partly due to the direct effect and partly due to the growing uncertainty, while the effect

<sup>&</sup>lt;sup>14</sup>In multilateral target zone systems the effective fluctuation bands should be narrower than the official bands. It can be illustrated by considering three currencies A, B, C, whoes pair-wise exchange rates can fluctuate in bands of the same size. It is impossible that A is maximally appreciated against B, while maximally depreciated against C, otherwise the exchange rate between B and C would be out of their band. Consequently, the intervention points do not coincide with the edges of the official bands. See Flandreau (1998) about modeling the effective bands in a multilateral target zone system. Here we do not distinguish between the effective and the official bands.

of changing expectation was minor. (See Table 3.1.). Whereas the Portuguese Escudo was hardly influenced by the direct effect and the growing uncertainty.

The depreciation calculated by the model deviate only slightly from the observed one for the Hungarian Forint; whereas for the studied French and Portuguese realignments, the model somewhat overestimates the actual depreciation. One can expect the model to overestimate the observed change in exchange rate when the analyzed realignment is anticipated by the market. This points towards interpreting the results as evidence for the French and Portuguese realignments were anticipated by the market at least a few days before the measure was taken, whereas the Hungarian realignment was mainly unexpected in the two weeks period preceding the band-shift.

The finding on the imperfect credibility of the target zone in France and Portugal before the realignments motivates a further line of research. The model framework introduced in this paper can be generalized to the limited credibility case that is highly relevant for target zone arrangements maintained by only one of the countries. In this case one edge of the band can be defended in a perfectly credible way, because the country can issue its own currency in unlimited amount on the demand of the market. In contrast, the other edge of the band can not be defended under all circumstances because of the lack of cooperation with the other country and the limited amount of reserves. With some modification, the option pricing model could shed light on the nature of this inherent asymmetry. The advantage of the options based approach is that it models the limiting effect of the two edges of the band as much separately as possible.

## 3.7 Acknowledgements

The author gratefully acknowledges comments and suggestions from Zsolt Darvas, Jin-Chuan Duan, András Fülöp, László Halpern, Tamás Kollányi, Tamás Makara, László Mátyás, Péter Medvegyev, and David Veredas.

The paper has been presented on the following conferences:

• Twelfth International Conference on 'Forecasting Financial Markets', June 1-3, 2005, Marseille;

- AEA Conference on Exchange Rates, April 6-8, 2005, Luxembourg;
- Xth Spring Meeting of Young Economists (SMYE), April 22-24, 2005, Geneva.

Thanks are extended to the organizers and participants of these conferences and to the participants of discussions at the Magyar Nemzeti Bank, Central European University and University of Toronto.

An earlier versions of the paper has been published as an MNB Working Paper (2004/2).

## 3.8 Appendix

<u>Proposition 1:</u> The sequence of the created put and call binomial trees are monotonously increasing on each node, but they never grow over the binomial trees of the put and call components of the target zone exchange rate.

#### Proof:

The proof of the boundedness and the monotony rely on the fact that the value of a put option is monotonously decreasing in its underlying asset, whereas a call option is monotonously increasing in its underlying asset.

First we check the *boundedness* of the sequences of the binomial trees of the options, by comparing their underlying assets with the underlying assets of the put and call component of the target zone exchange rate. The boundedness of the sequences of the binomial trees is proved by the logic of total induction: by showing that the binomial trees of  $P^1$  and  $C^1$  are bounded and the boundedness of  $P^{i-1}$  and  $C^{i-1}$  is inherited to  $P^i$  and  $C^i$ . The boundedness of the binomial trees of  $P^1$  and  $C^1$  is obvious, since the underlying asset of  $P^1$  is not less than the underlying asset of the put component of the target zone exchange rate  $(F - C_{Kc,a} \leq F)$  for each node, or in other words, at each state of the world. And the underlying asset of  $C^1$  is not higher than the underlying asset of the call component of the target zone exchange rate  $(F + P_{Kp,a} \ge F)$  at each state of the world. What we have to show next is that if the binomial trees of  $P^{i-1}$  and  $C^{i-1}$  are bounded, than the binomial trees of  $P^i$  and  $C^i$  are bounded as well. This comes from the fact that the boundedness of  $C^{i-1}$  implies that  $F - C^{i-1} \ge F - C_{Kc,a}$  at each state of the world. So the underlying product of the  $P^i$  is not less than the underlying product of the put component of the target zone exchange rate, consequently, no node value of the binomial tree of  $P^i$  is greater than the corresponding node value of the binomial tree of the put component. Similar argument holds for the  $C^i$ . So, we proved by total induction that for every  $i P^i$  and  $C^i$ are bounded.

Second we prove that the put and call sequences are monotonously increasing on each node by comparing the underlying assets again. It is obvious that  $P^2$  and  $C^2$  are not less than  $P^1$  and  $C^1$  respectively, since  $F - C^1 \leq F$  and  $F + P^1 \geq F$  at each state of the world. So, all we have to show is that if  $P^{i-1}$  and  $C^{i-1}$  are not less than  $P^{i-2}$  and  $C^{i-2}$ respectively, than  $P^i$  and  $C^i$  are not less than  $P^{i-1}$  and  $C^{i-1}$  respectively. The binomial tree of  $C^{i-1}$  is not less than the binomial tree of  $C^{i-2}$  is equivalent with  $C^{i-2} \leq C^{i-1}$  at each state of the world, which implies that  $F - C^{i-2} \geq F - C^{i-1}$ . So, the underlying product of the  $P^i$  is not greater than the underlying product of the  $P^{i-1}$ , consequently, no node value of the  $P^i$  binomial tree is less than the corresponding node value of  $P^{i-1}$ . Similar argument holds for the  $C^i$ .

#### Corollary 1:

According to a convergence theorem (See for example: Pownall (1994), p.162 Theorem 2.6.1 part (iii)) the sequences of the put and call binomial trees are convergent on each node, as they are both monotonous and bounded.

<u>Proposition 2</u>: The sequence of the put binomial trees converges to the binomial tree of the put component of the target zone exchange rate and the sequence of the call binomial trees converges to the binomial tree of the call component of the target zone exchange rate.

#### Proof:

We provide an indirect proof of Proposition 2: if either the put sequence or the call sequence or both sequences would converge to a lower upper bound, ie:  $C^{\infty} < C_{Kc,a}(F + P_{Kp,a})$  or  $P^{\infty} < P_{Kp,a}(F - C_{Kc,a})$  at least at one state of the world, then we get a contradiction. Let us assume that  $C^{\infty} < C_{Kc,a}(F + P_{Kp,a})$ . In this case the comparison of the underlying product of the limit of the put options with the underlying product of the put component indicates the following: the binomial tree of the limit of the sequence of the put options grows at least at one node over the binomial tree of the put component. The argument goes similarly if  $P^{\infty} < P_{Kp,a}(F - C_{Kc,a})$ .

## Bibliography

- Baldwin, R. P. Krugman (1998): Persistent Trade Effect of Large Exchange Rate Shocks. Quarterly Journal of Economics, 104(4), 635–54.
- [2] Ball, C. A. A. Roma (1993): A jump diffusion model for the European Monetary System. Journal of International Money and Finance, 12, 475–492.
- [3] Barabás, Gy. (eds.) (2003): Coping with the speculative attack against the forint's band. MNB Background Studies. http://english.mnb.hu/
- [4] Barone-Adesi, G. R. E. Whaley (1987): Efficient Analytic Approximation of American Option Values. *Journal of Finance*, 42, 301–320.
- [5] Bekaert, G. Grey, S. F. (1998): Target zones and exchange rates: An empirical investigation. Journal of International Economics 45 (1), 1–36.
- [6] Campa, J. M. P. H. K. Chang (1996): Arbitrage-Based Tests of Target Zone Credibility: Evidence from ERM Cross-Rate Options. *The American Economics Review*, 86(4), 726–740.
- [7] Campa, J. M. P. H. K. Chang J. F. Refalo (1999): An options-based analysis of emerging market exchange rate expectations: Brazil's Real plan, 1994, 1997. NBER Working Paper, No. 6929.
- [8] Copeland, L. S. (2000): *Exchange rates and international finance*. Pearson Education.
- [9] Cox, J. C. S. Ross M. Rubinstein (1979): Option Pricing: A Simplified Approach. Journal of Financial Economics, 7, 229–264.
- [10] Dumas, B. P. Jennergren B. Näslud (1993): Currency Option Pricing in Credible Target Zones. *Review of Futures Markets*, 12, 323–340.
- [11] Dumas, B. P. Jennergren B. Näslud (1995): Realignment Risk and Currency Option Pricing in Target Zones. *European Economic Review*, 39, 1523–1544.
- [12] Flandreau, M. (1998): The burden of intervention: externalities of multilateral exchange rates arrangements. *Journal of International Economics*, 45, 137–171.
- [13] Geske, R. (1979): The Valuation of Compound Options. Journal of Financial Economics, 7, 63-81.
- [14] Gros, D. N. Thygesen (1992): European Monetary Integration. Longman.
- [15] Hull, J. C. (1997): Option, futures and other derivatives. Prentice Hall.

- [16] Krugman, P. (1991): Target Zones and Exchange Rate Dynamics. The Quarterly Journal of Economics 106 (3), 669–682.
- [17] Ledesma-Rodríguez, F. M. Navarro-Ibánez –J. Pérez-Rodríguez –S. Sosvilla-Rivero (2002): On the Credibility of the Irish Pound in the EMS. *The Economic and Social Review*, 31 (2), 151-172.
- [18] MacMillan, L. W. (1986): Analytic Approximation for the American Put Option. Advances in Futures and Options Research, 1, 119–139.
- [19] Malz, A. M. (1996): Using options prices to estimate realignment probabilities in the European Monetary System: the case of sterling-mark. *Journal of International Money and Finance* 15 (5), 717–748.
- [20] Margrabe, W. (1978): The Value of an Option to Exchange One Asset for Another. Journal of Finance, 33, 177–186.
- [21] Mizrach, B. (1996): Did option prices predict the ERM crises? Working paper 96-10, Rutgers University, Department of Economics.
- [22] Pownall, M. W. (1994): *Real Analysis*. Brown Communications Inc.
- [23] Rangvid, J. C. Sorensen (2001): Determinants of the implied shadow exchange rates from a target zone. *European Economic Review*, 45, 1665–1696.
- [24] Rose, C. (1995): A statistical identity linking folded and censored distributions. Journal of Economics Dynamics and Control, 19, 1391–1403.
- [25] Stockman, A. C. (1999): Choosing an exchange-rate system. Journal of Banking & Finance, 23, 1483–1498.
- [26] Svensson, L. E. O. (1991): The term structure of interest rate differentials in a target zone. Journal of Monetary Economics, 28, 87–116.
- [27] Svensson, L. E. O. (1993): Stochastic Devaluation Risk and the Empirical Fit of Target-Zone Models. *Review of Economic Studies*, 60, 689–712.

# **Tables and Figures**

Table 3.1: Depreciation	after	target	zone	realignments	and	its	decomposition	with	the
options-based model									

	FRF 1993	PTE 1995	HUF 2003
	band widening: from ±2.25% to	band shift of	band shift of
Realignements	±15%	3.6%	2.26%
Historical depreciation	2.5%	1.8%	5.6%
Total depreciation according to the model Decomposition:	10.5%	3.7%	5.4%
direct effect	5.9%	0%	2.0%
changing expectation	-	3.7%	-0.2%
changing volatility	4.4%	0%	3.5%





**Figure 3.2:** Exchange rate of the French franc and its band in the EMS (13 March 1979–31 Dec 1998)



**Figure 3.3:** Exchange rate of the Portuguese escudo and its band in the EMS (2 Jan 1985–31 Dec 1998)



**Figure 3.4:** Exchange rate of the Hungarian forint and its band (4 Jan 2000–30 Apr 2004)





Figure 3.5: Depreciation of the French franc after the band widening (3 Aug 1993)

Figure 3.6: Depreciation of the Portuguese escudo after the band shift (6 March 1995)



N=198; ST,0=ST,1=98.918 PTE/DEM; ST,2=ST,3=102.505 PTE/DEM

Figure 3.7: Depreciation of the Hungarian forint after the band shift (4 June 2003)



sb=247.87 HUF/EUR; sa=261.81 HUF/EUR; h0=h1=h2=14.9; h3=25; r=6.57%; T=5 years; N=260; ST,0=ST,1=238.7 HUF/EUR; ST,2=ST,3=248.4 HUF/EUR