

# HUNGARIAN GREEN INVESTMENT SCHEME: A GAME THEORETIC APPROACH

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## Abstract

Game theoretic concepts are widely used in the environmental area. However, application of game theory to the field of energy efficiency, particularly in the buildings sector, remains largely unexplored. This thesis aims at proving that game theory might be instrumental in finding the strategies to turn panel block constructions into energy efficient buildings. The research is based on the idea that a well-designed Green Investment Scheme (to be applied in Hungary), or a GIS, is an effective funding mechanism that can motivate a building's dwellers to opt for an energy efficient renovation of their house. To support this hypothesis, two game theoretic models are developed that present interactions, first, between dwellers within one building and, second, between the government and households. Results suggest that a household invests into a retrofit when the benefit it obtains from it surpasses a certain threshold. Additionally, the investment increases with neighbor's benefit if the latter is high enough. It is also shown that a GIS is an efficient instrument of national policy only if households face audits with a certain probability.

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I owe special thanks to Prof Diana Urge-Vorsatz who inspired me to undertake research in the area of environment, namely, energy efficiency in the building sector, with a special application to Hungary.

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I would like to dedicate this work to my dear friend Anna and to her child, my goddaughter, Polina.

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## Introduction

Game theory is broadly defined as the analysis performed by decision makers, or players, “from a rational rather than a psychological or sociological viewpoint” (Aumann 1987). The key concept in game theoretical approaches is a strategic interdependence, i.e. the payoff of every player depends not only on his choice, but also on strategies of other players. Since its official establishment in 1944 by John von Neumann and Oskar Morgenstern, game theory has become a powerful tool in many academic fields. Being used initially only in natural sciences, it later became an important application in social sciences as well. Nowadays, game theoretic concepts are widely used in the environmental area, predominantly with regard to open-access fisheries, interboundary pollution, and energy resources. However, application of game theory to the field of energy efficiency, particularly in the buildings sector, remains largely unexplored. As energy conservation projects in multifamily buildings involve numerous households with interdependent self-interest strategies, lack of coordination between these agents might put the projects on hold. Game theory might be instrumental in changing the households’ behavior towards a cooperative solution and pointing out the strategic importance of turning panel block constructions into energy efficient buildings.

The following background information on the development of a Green Investment Scheme in Hungary would facilitate understanding of the thesis’ underlying ideas. Involvement of the local or federal authorities into households’ decision about retrofit proved to be necessary after energy efficiency programs in the buildings sector in Hungary ceased to be attractive for households due to decreased financial support for refurbishments. It became clear that until the grant support exceeds the 20% VAT in

Hungary, the construction sector and, consequently, their clients will consider the black labor market more advantageous and money-saving. Moreover, the households might opt for postponing or canceling renovations they had planned. To encourage the implementation of energy efficiency projects, and not least importantly, to make use of the so called “hot air”<sup>1</sup>, the government is planning to introduce Green Investment Schemes (GIS)<sup>2</sup>. These might serve as a stimulus for households to cooperate with the government, as the payoff of participating in a GIS would exceed that of the deviation in favor of black market or of non-participation.

Another problem arising in this context is refurbishments of multi-family buildings with many agents involved. Implementation of an energy conservation project requires interaction between households within one building. As the renovation is set to being planned, one of the formidable tasks is to convince households to participate in the project. As a rule, there are agents that do not share in the project's expenses and, thus, free-ride on the benefits of the renovation. Green Investment Schemes are supposed to offer a larger financial support, which would reduce the project's costs per household and motivate a higher participation rate.

My research hypothesis is that a well-designed Green Investment Scheme is an effective funding mechanism that will motivate a building's dwellers to opt for an energy efficient renovation of their house. In order to prove the hypothesis, I will develop two game theoretical models that will present interactions, first, between dwellers within one

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<sup>1</sup> “Hot air” – surplus greenhouse gas emissions (Assigned Amount Units, or AAUs) in the Eastern European countries and the former Soviet Union originating from a major economic downturn in these regions in the 1990s. The “hot air” is expected to help these countries to be below their Kyoto targets in 2008-2012 and, moreover, to allow for selling the surplus AAUs to other countries (Source: Frontier Economics 2002).

<sup>2</sup> Green Investment Scheme (GIS) – a mechanism introduced to facilitate the AAU trade by way of ensuring that the revenues received by seller countries will be channeled to environmental purposes (Source: Blyth and Baron 2003).



building and, second, between the government and households, and their strategies will be discussed with regard to an energy efficient refurbishment of a residential house. As regards methodology, simple one-stage non-zero-sum games with three agents (a government and two households) for the first model and with two agents (a government and a household) for the second model will be developed in the thesis.

The thesis is structured as follows. Chapter 1 describes a case of practical application of a Green Investment Scheme that is being planned in Hungary. Chapter 2 gives an account of relevant academic literature and outlines models applicable to the described framework in case those have not yet been developed for energy efficiency issues. Chapter 3 deals with modeling of “household vs. household” interaction, with the degree of governmental authority used as a proxy for a GIS award and for non-compliance punishment. Chapter 4 relates a one-stage game between the government and a household. In addition to an investment variable, a variable representing energy savings is introduced that indicates a very important feature of retrofitting activities.

# Chapter 1 - Realization of a Green Investment Scheme in Hungary<sup>3</sup>

An idea of introducing Green Investment Schemes (GIS) into a national climate change policy emerged in 2000 (Korppoo and Gassan-zade 2008). It has been experiencing rapid development lately and attracted high attention in 2008 with Hungary passing a GIS law. Against the backdrop of these circumstances, the chapter shortly examines the situation in Hungary with respect to a GIS.

As it was mentioned above, a GIS is to be funded from selling “hot air”, or Assigned Amount Units (AAUs), to the countries that are striving to comply with mandatory Kyoto Protocol limits on CO<sub>2</sub> emissions. The Scheme was named “green” since the revenues from AAU sales are supposed to finance only environment-improving projects, preferably those mitigating climate change, i.e., related to reducing greenhouse gas emissions. Therefore, an overarching question is the disbursement of revenues from sales of AAUs. In Hungary, the primary proposition is to channel the AAU money into the buildings sector due to several reasons. First of all, this sector represents one of the priority areas to be addressed in the context of climate change mitigation as it accounts for around 30% of total CO<sub>2</sub> emissions<sup>4</sup> (IPCC 2007), which demonstrates a high mitigation potential of this sector. In addition, this potential can be tapped at a very low cost (*ibid.*), that is, the measures that result in large CO<sub>2</sub> reductions are sufficiently cheap in the buildings sector.

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<sup>3</sup> This Chapter draws on Sharmina et al. 2008

<sup>4</sup> “Total CO<sub>2</sub> emissions” generally include emissions from all sectors of the economy

The second reason for a GIS to be implemented in this sector is related to difficulties with capturing the potential described above. There are a number of barriers that prevent addressing this issue effectively. For example, numerous households within one building pose a challenge to be persuaded to apply any of climate mitigation and energy saving measures to their building. Another barrier is related to rented floor area. Tenants are not motivated to improve their apartments/offices as, due to their temporary status, they are not interested in long-term benefits of that. In addition, long payback periods of investing into retrofit projects in buildings discourage even owners of the built property. Thus, governmental support appears an advantageous approach to this problem.

The Hungarian GIS builds on several national programs supporting energy efficiency activities in the residential and tertiary sectors of the economy. After financial support of retrofits in buildings was reduced to less than 20% of the retrofit costs due to the growing budget deficit, it became less attractive for households and companies to engage into refurbishment activities. Therefore, AAU revenues are to reinforce the funding of energy saving and climate mitigation projects. The Hungarian GIS will target the following types of initiatives:

1. Complex measures that are supposed to result in the retrofitted building reaching a higher category within a special certification system (see Appendix C). This implies that the new category should be assigned to the entire building, and not only to one apartment.
2. Individual measures that are not financed either by other national energy efficiency programs or by the EU structural funds (Feiler 2008 in Sharmina et al. 2008).

Table 1 and Table 2 show the structure of a GIS support in the Hungarian building sector. The design of the Scheme is almost identical for both renovation projects and new constructions. Note that for both project categories, the GIS money is planned to finance the measures that are applied to 100 m<sup>2</sup> of living area per dwelling maximum. That would eliminate excessively costly projects that might otherwise overburden the budget.

**Table 1. Financial support for retrofitting activities under a Green Investment Scheme in Hungary, HUF/m<sup>2</sup>**

<b>Original State</b>	<b>Renovated State</b>				
	<b>Category C (1000 m<sup>2</sup> or less)</b>	<b>Category B</b>	<b>Category A</b>	<b>Category A+</b>	<b>Category A++</b>
<b>I</b>	2,000	4,000	6,000	9,000	12,000
<b>H</b>	1,500	3,500	5,500	7,500	10,500
<b>G</b>	1,000	3,000	5,000	7,000	9,500
<b>F</b>	Not supported	Not supported	4,000	6,000	8,000
<b>E</b>	Not supported	Not supported	3,000	5,000	7,500
<b>D</b>	Not supported	Not supported	3,000	4,500	7,000

Source: Csoknyai and Szalay 2008; Csoknyai pers. comm.

**Table 2. Financial support for new buildings under a Green Investment Scheme in Hungary, HUF/m<sup>2</sup>**

<b>Level reached after constructing is over</b>				
	<b>A</b>	<b>A+</b>	<b>A++</b>	<b>Passive house</b>
<b>HUF/sq. m</b>	10,000	13,000	16,000	20,000
<b>Maximum sum of funding, HUF</b>	1 million	1.3 million	1.6 million	2 million

Source: Csoknyai and Szalay 2008; Csoknyai pers. comm.

The cost of putting a Green Investment Scheme into practice can be evaluated only approximately as it is conditional on a number of factors, such as, demand on AAUs from buying countries and AAU supply from the sellers that determinint the price of an Assigned Amount Unit; initial energy efficiency of targeted dwellings; and public awareness about energy conservation in general and a GIS in particular. To assess potential costs of enforcing a GIS (see Table 3), several assumptions were made. First, it was assumed that about 50% of potential GIS participants would be apartment houses with the other half being single-family buildings. Second, around one-third of the GIS-involved living units were estimated to be new constructions and two-thirds – refurbishments. Finally, the aggregate GIS funding was assessed to amount to HUF 48 billion (Csoknyai and Szalay 2008).

**Table 3. Average expenses of putting a GIS into practice and an approximate number of potentially involved households**

	Renovation	New constructions	
		Single-family house	Apartment
Average GIS grant, million HUF/living unit	0.4	1.5	1.2
Average investment, million HUF/living unit	1.5	28	22
Calculated number of GIS projects	58,882	9,313	9,313

Source: adopted from Csoknyai and Szalay 2008

## Chapter 2 - Historical and Literature Overview

After 1944, when John von Neumann and Oskar Morgenstern published the first book on the subject, game theory gradually became an important tool not only in mathematics, economics, genetics and other disciplines related to numbers and their combinations, but also in social sciences including environmental science. Within the latter, initial studies were mostly related to fisheries and “acid rain games” with an insignificant share of other applications. Gordon (1954) was the one who pioneered the work on economics of fisheries as a common resource developing a model on optimal fishing efforts. Another way of describing fisheries games appeared in Munro’s study (1979) where he combined Nash’s two-person cooperative game with a dynamic-model time setting. Subsequently, most of the games in fisheries economics involved, as a rule, a temporal dimension, thus having a dynamic set-up.

“Acid rain game” was first developed by Maeler (1989) and aimed at internalizing reciprocal externalities cost-effectively and equitably. He described acid rain as a negative environmental externality and modeled it into a two-country game with the players seeking for selfishly efficient payoffs. The game would generally result in non-cooperative outcomes in the absence of international coordination, such as an enforceable binding treaty. Typically, such a business-as-usual scenario, i.e. without any policies implemented, in “acid rain games” is contrasted with the Nash-equilibrium and with the Pareto-dominance scenarios.

With respect to energy issues, game theoretic concepts have been mostly limited to the studies either on electric networks and transmission capacity games (see, for example, Haurie and Breton 1985, Ahmad et al. 2008, and Wang et al. 2009) or on

coalitions between energy suppliers and on other market failures of power industry (for instance, Exelby and Lucas 1993, Neimane et al. 2008). The former topic involves fairly complicated game-theoretic models such as, for instance, max-max-min optimization problems for computing systems (Ahmad et al. 2008) or dynamic programmatic approaches to network flow models (Haurie and Breton 1985). The games describing power industry and fossil fuel manufacturers represent supply-side solutions to energy environment problems. The game-theoretical approaches range from simple non-cooperative non-zero sum games that illustrate a country's fuel-switching strategies (Magirou 1984) to software-based models that describe oligopolistic competition in electricity markets (Bompard et al. 2006).

A narrower branch of environmental science is related to energy efficiency in various sectors of the economy. The application of game theory to efficient energy use in buildings is particularly under-researched, though this field represents a wealth of opportunities to apply game theoretic concepts to the body of knowledge on changing behavior of households. Energy efficiency programs in the buildings sector are not attractive for dwellers unless the grant support outweighs the disadvantages of being involved into retrofit activities <sup>5</sup>. To encourage the implementation of energy efficiency projects, the government might want to introduce a stimulus for households to cooperate with the government so that the payoff of it exceeds that of the deviation in favor of non-participation.

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<sup>5</sup> To retrofit – “to install or fit (a device or system, for example) for use in or on an existing structure, especially an older dwelling”. The noun “retrofit” means “an instance of modernizing or expanding with new or modified parts, devices, systems, or equipment: e.g., a retrofit for the heating system”. Source: Minter Ellison 2009

A problem that might arise in this context is generally referred to as the Samaritan's Dilemma first developed by Buchanan (1977). Schmidtchen (1999) describes it as a situation when financial help brings about shirking and improper use of budget resources. He takes Buchanan's matrix diagram (see Table 4) illustrating the players' payoffs in a simultaneous game and presents it in an extensive form while changing the set of the game into a dynamic one (see Figure 1 and Figure 2).

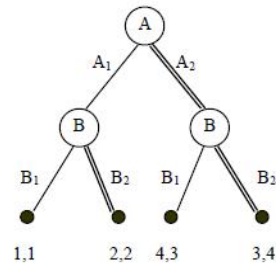
**Table 4. Matrix representation of the Samaritan's Dilemma game**

		Player B	
		Work	Not Work
Player A	Help	<u>4</u> , 3	<u>3</u> , <u>4</u>
	Not Help	2, <u>2</u>	1, 1

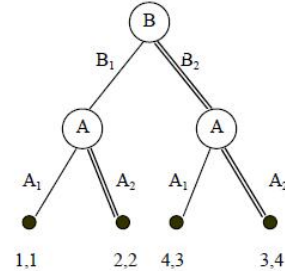
Note that Player A is a Samaritan and Player B is a person receiving help (e.g., a money transfer) from Player A. The latter, according to Buchanan (1977), has altruistic preferences and his utility increases when he helps his opponent, i.e., Player A's payoffs are higher if he chooses strategy Help than otherwise, regardless Player B's choice. The second player prefers working when he is not helped by Player A and prefers shirking if he is helped. There exists a unique Nash Equilibrium when Player A and Player B choose (Help, Not Work) respectively. Hence, the Samaritan's dilemma.



**Figure 1.** Samaritan's dilemma as a one-stage dynamic game with the Samaritan playing first



**Figure 2.** Samaritan's dilemma as a one-stage dynamic game with the Samaritan playing second



Schmidtchen shows that in both cases the Samaritan's dilemma remains unresolved, i.e. the Samaritan chooses «help» regardless the second player's strategy. For that reason, the author suggests to extend Buchanan's game by way of including a third party who is supposed to watch a potential scrounger B and reward him only if he works. Schmidtchen arrives at the conclusion that the only solution to this problem is to give the Samaritan (the government) the authority to decide, provided that courts are efficient. The situation is somewhat similar to the one in the buildings sector when the government intends to finance households' energy conservation activities. To avoid the Samaritan's dilemma, a third party monitoring and verification are performed with a litigation process and/or a fine as a punishment.

A different type of principal-agent problem in housing retrofit is illustrated by Qingmao et al. (2008). They emphasize the significance of dwellers' efforts in a cooperative game for the government and households, under informational asymmetries. The authors describe an optimization problem for both agent and principal, subject to Incentive Compatibility (IC) and Individual Rationality (IR) constraints, where the government chooses the level of incentives and risk-taking and the households choose the efforts they put into retrofitting. Unfortunately, it would be difficult to replicate the

model in a concise form, since the researchers introduce more than a dozen variables; in addition, citing IC and IR constraints without derivations would be meaningless in this case.

Regarding refurbishments of multifamily buildings, another issue emerges due to a large number of agents involved. As the renovation is set to being planned, one of the formidable tasks is to convince households to participate in the project. As a rule, there are agents that do not share in the project's expenses and, thus, free-ride on the benefits of the renovation: since these benefits represent a public good<sup>6</sup>, nobody can be excluded from it. An energy efficiency project is a typical example of a collective action where households decide on how much they want to contribute to the retrofitting. A standard result here is that unregulated collective action leads to the underprovision of public good from social point of view.

A game theoretic concept that might help turn a non-cooperative outcome of a «households vs households» game into a cooperative one is the Groves-Clarke demand revealing mechanism (Clarke 1971; Groves and Loeb 1975). In case of private provision of a public good, such as an energy efficiency project, this scheme induces agents to report truthfully their willingness to co-finance the project. The start conditions are that agents report their willingness-to-pay (WTP)  $w_i$ , but their reports  $r_i$  about WTP are not necessarily true. The public good is supplied only if the sum of their reports  $r_i$  is nonnegative and is not supplied otherwise. If the public good is supplied, every agent is paid an amount equal to the sum of other reports, or “bids” (Varian 1992). Therefore, potential payoffs of agent  $i$  are as follows:

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<sup>6</sup> Olson (1971) defines a public good as “any good such that if person X [...] in a group [...] consumes it, it cannot feasibly be withheld from others in that group”

$$(1.1) \quad \begin{cases} w_i + \sum_{j \neq i} r_j & \text{if } r_i + \sum_{j \neq i} r_j \geq 0 \\ 0 & \text{if } r_i + \sum_{j \neq i} r_j < 0 \end{cases}$$

If the sum of the agent  $i$ 's WTP and his side-payment is positive, it is profitable for the agent to report truthfully so that the public good is supplied. If, on the contrary, the sum of the agent  $i$ 's WTP and his side-payment is negative, then he would prefer to thwart the public good provision by reporting, again, truthfully. Thus truthful disclosure of one's WTP emerges a dominant strategy in any case. One of the problems with this mechanism is that it does not guarantee full voluntary participation unless the benefits of it surpass the agents' initial endowments (Sager 2007). In spite of this fact, the Groves-Clarke scheme might provide useful insights into mapping strategic behavior of dwellers who are faced with energy efficiency projects.

Another approach that can be used for modeling interactions between dwellers (as well as between the government and households) is studied by, *inter alia*, O'Donoghue and Rabin (1999, 2001), Akin (2007), Grenadier and Wang (2007), Settle and Shogren (2004). They all suggest using hyperbolic and quasihyperbolic discounting in order to model time inconsistency of agents. For example, Akin (2007) models bargaining between players who demonstrate unpredictable behavior. He endows players with certain characteristics – naivete, cunningness, and ability to learn. The author assumes that players possess perfect information about opponents' movements; he uses a dynamic setup, and solves the model utilizing a concept of naive backwards induction. The results explain delays in bargaining process and largely depend on the agents' types. A discounting factor predictably plays a major role in the model's outcomes, which is similar to a situation in the buildings' retrofit projects. Grenadier and Wang (2007)

approximate energy-related investment conditions even better. Despite the fact that they model decisions of entrepreneurs, their framework fits well into our problem. For example, the authors abandon a notion of a single-payment payoff in favor of a cash-flow sequence. This is exactly what households face after a retrofit, since the payback is calculated as a future flow of monetized energy savings.

## Chapter 3 - Households vs. households

The interaction between households will be described as a one-stage non-zero-sum game with shared interests and with perfect monitoring (i.e. the government possesses information about households' decisions). The problem represents an N-agent game where payoffs depend mostly on the players' incentives to deviate. There is a finite set of households  $N$  in a high-rise building. To simplify the model, assume that the number of players is equal to three (government and two households), where the households are denoted as  $i$  and  $j$ . First, the government and households negotiate a contract where they define how much each household will invest into energy efficiency retrofit in their building. Then, after households make actual investments, the government's auditors inspect their adherence to the agreement and decide on a reward or fine depending on the results of the energy audit. It is assumed that this is a game of complete information and the government knows the actual amount  $I_i$  household  $i$  has invested into retrofitting activities. The reward/fine  $F_i$  of household  $i$  by the government encompasses difference between the household's attested investment  $I_i$  into an energy saving project and an agreed amount  $A$  that the household pledged to invest when signing a contract with the government; it is represented by

$$(3.1) \quad F_i = (I_i - A) \cdot \theta$$

As it can be seen from (3.1), the reward/fine  $F_i$  is a weighed difference with  $\theta \geq 0$  representing the degree of authority of the government. The higher  $\theta$ , the higher power the state has over the households. For example, when a municipality or its auditors, who inspect a household's compliance with the contract, are sufficiently influential,

punishment for non-compliance is more severe as well as reward for higher investment is more generous. Hence,  $\theta$  has an impact on a household's utility function  $U_i$

$$(3.2) \quad U_i(x_i, P, B_i) = u(x_i) + v(P, B_i)$$

with  $u(\cdot)$  and  $v(\cdot)$  strictly increasing and concave in  $x_i$  and  $P$  respectively. Function  $v(\cdot)$  indicates utility of household  $i$  from taking part in an energy saving project, while  $u(\cdot)$  represents utility from consuming all other goods. Benefits obtained by households as a result of a retrofit project are available to any household in the refurbished building, that is, no dweller can be excluded from enjoying these benefits. Moreover, the latter are not subject to congestion problem as the number of households in a building is limited. Hence, these benefits are non-rivalled and non-excludable, which is characteristic of a public good. It is assumed that function  $v(\cdot)$  has the form

$$(3.3) \quad v(P, B_i) = B_i \cdot \ln(P)$$

where  $P = I_i + I_j$  is the sum of contractual investments of both households in an energy efficiency project;  $B_i > 0$  measures importance of a retrofit for household  $i$  relatively to other goods. The form of function  $u(\cdot)$  is

$$(3.4) \quad u(x_i) = \ln(x_i)$$

Thus, household  $i$  faces the following optimization problem:

$$(3.5) \quad \max_{I_i} U_i(x_i, P, B_i) = \ln(x_i) + B_i \cdot \ln(P)$$

where

$$(3.6) \quad P = I_i + I_j$$

Moreover, the household's utility is subject to the budget constraint

$$(3.7) \quad x_i + I_i = m_i + F_i$$

where  $m_i$  is a household's budget that is partly invested into public good and partly spent on all other goods  $x_i$ . In addition,

$$(3.8) \quad I_i, I_j, x_i \geq 0$$

Substituting (3.6) and (3.7) into (3.5) gives

$$(3.9) \quad \max_{I_i} U_i = \ln[m_i - I_i \cdot (1 - \theta) - A \cdot \theta] + B_i \cdot \ln(P)$$

assuming that  $\theta < 1$ .

In order to find a reaction function of household  $i$ , assume that investment  $I_j$  of household  $j$  is constant. This poses a maximization problem with respect to  $I_i$  that has the following first order condition

$$(3.10) \quad \frac{\partial U_i}{\partial I_i} = \frac{1 - \theta}{m_i - I_i \cdot (1 - \theta) - A \cdot \theta} - \frac{B_i}{I_i + I_j} = 0$$

which gives household  $i$ 's reaction function (see Appendix A 1 for derivations)

$$(3.11) \quad I_i = \frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)} - \frac{I_j}{1 + B_i}$$

provided the expression on the right-hand side is non-negative (if it is negative, we have  $I_i = 0$ ). Equation (3.11) demonstrates that the actual investment  $I_i$  of household  $i$  depends on the government's power  $\theta$  over the households, the  $i$ 's pledged investment  $A$ , household  $i$ 's income  $m_i$ , and attested investment  $I_j$  of household  $j$ . Comparative statics analysis with respect to  $B_i$

$$(3.12) \quad \frac{\partial I_i}{\partial B_i} = \frac{(1-\theta) \cdot (m_i - A \cdot \theta) + I_j \cdot (1-\theta)^2}{[(1-\theta) \cdot (1+B_i)]^2} > 0$$

shows that a household's investment  $I_i$  is positively correlated with its benefit  $B_i$  from a retrofit (more detailed calculations are presented in Appendix A 2), i.e., the higher benefit the household can extract from a retrofit project, the more it will invest into this project.

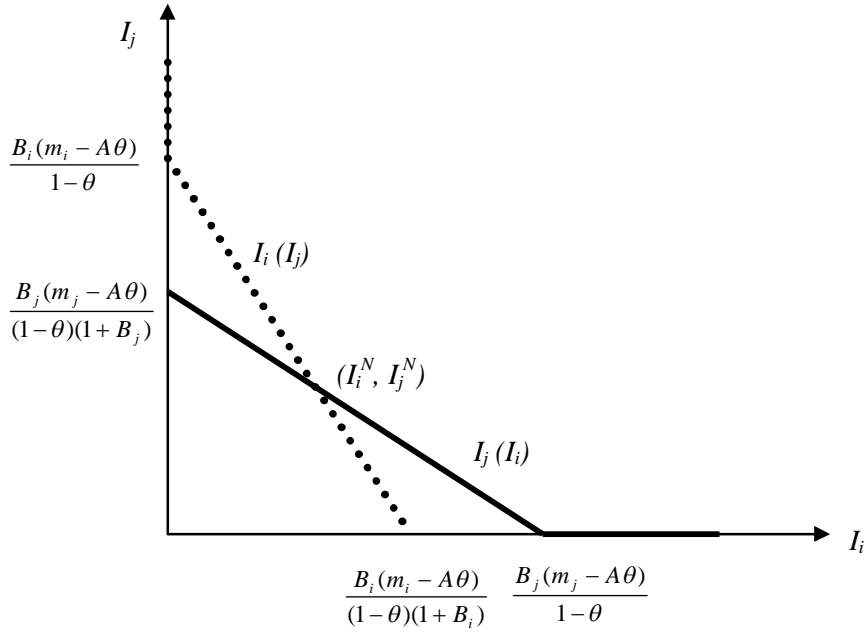
We will find now Nash equilibrium values  $I_i^N$  and  $I_j^N$ . Since the optimal response functions for both  $i$  and  $j$  are symmetric, (3.11) shows an optimal reaction function of household  $j$  as well, though with altered subscripts as in (3.13)

$$(3.13) \quad I_j = \frac{B_j \cdot (m_j - A \cdot \theta)}{(1-\theta) \cdot (1+B_j)} - \frac{I_i}{1+B_j}$$

Both (3.11) and (3.13) have negative slopes, with the slopes' absolute values less than one. Hence, the two households' strategies are strategic substitutes, i.e., when one player increases her investment, another player responds by decreasing his. The reactions functions are illustrated in Figure 3.



**Figure 3. Best response functions of household  $i$  and household  $j$**



To find Nash Equilibrium of the game, the following system of equations should be solved after reorganizing (3.11) and (3.13) (see Appendix A 3 for derivations):

$$(3.14) \quad \begin{cases} (1+B_i) \cdot I_i + I_j = \frac{(m_i - A \cdot \theta) \cdot B_i}{1-\theta} \\ I_i + (1+B_j) \cdot I_j = \frac{(m_j - A \cdot \theta) \cdot B_j}{1-\theta} \end{cases}$$

From (3.14), the Nash Equilibrium investments of the household are

$$(3.15) \quad I_i^N = \frac{B_i \cdot (m_i - A \cdot \theta) \cdot (1+B_j) - B_j \cdot (m_j - A \cdot \theta)}{(1-\theta) \cdot [(1+B_i) \cdot (1+B_j) - 1]}$$

and

$$(3.16) \quad I_j^N = \frac{B_j \cdot (m_j - A \cdot \theta) \cdot (1+B_i) - B_i \cdot (m_i - A \cdot \theta)}{(1-\theta) \cdot [(1+B_i) \cdot (1+B_j) - 1]}$$

Comparative statics analysis shows that, at the optimum, a household's investment is positively correlated with the benefit it extracts from a retrofit implementation. Furthermore,  $I_i$  increases with the neighbor's benefit  $B_j$  if the latter is high enough (the proofs of these propositions can be found in Appendix A 4).

From (3.11) and (3.13), intersections of the response functions with the axes can be found. Equation (3.11) implies that when  $I_j = 0$

$$(3.17) \quad I_i(I_j = 0) = \frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)}$$

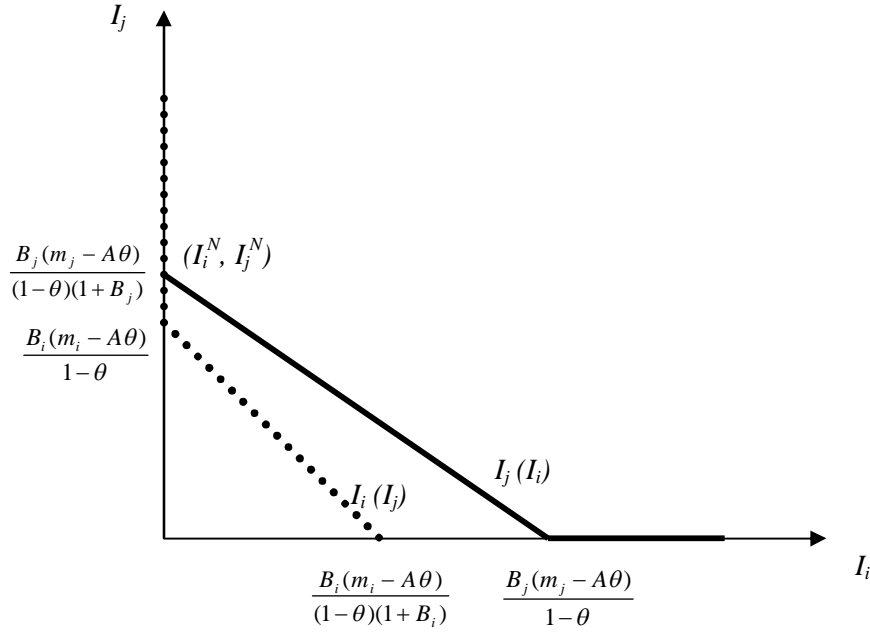
and when  $I_i = 0$

$$(3.18) \quad I_j \geq \frac{B_i \cdot (m_i - A \cdot \theta)}{1 - \theta}$$

Intersections of the  $j$ 's response function with the axes are derived in a similar way.

There exist two corner solutions when either  $I_i^N$  or  $I_j^N$  takes on a value of zero. In order for the  $i$  to invest nothing into an energy conservation project, its reaction function should cross the  $I_j$ -axis in the interval  $[0, \frac{B_j \cdot (m_j - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_j)}]$  as demonstrated in Figure 4.

**Figure 4. Response functions of the two households with household  $i$  free-riding**



This requirement is met as long as

$$(3.19) \quad \frac{B_j \cdot (m_j - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_j)} \geq \frac{B_i(m_i - A \theta)}{1 - \theta}$$

Solving (3.19) for  $B_i$  gives

$$(3.20) \quad B_i \leq \frac{B_j \cdot (m_j - A \cdot \theta)}{(m_i - \theta) \cdot (1 + B_j)}$$

Inequality (3.20) indicates a range of values of household  $i$ 's benefit from the project when it does not make any investment and its neighbor  $j$  does.

By analogy, the second corner solution where  $I_i^N = \frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)}$  and  $I_j^N = 0$  can be

found taking into account the fact that household  $j$ 's reaction function should cross the  $I_i$ -axis in the interval  $[0, \frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)}]$ . By analogy with (3.20),

$$(3.21) \quad B_j \leq \frac{B_i \cdot (m_i - A \cdot \theta)}{(m_j - \theta) \cdot (1 + B_i)}$$

Therefore, household  $i$  will invest  $\frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)}$  and household  $j$  will refrain from

financing a project if (3.21) holds, which gives the second corner solution. In addition, it follows from (3.21) that household  $i$  invests the described amount when

$$(3.22) \quad B_i \geq \frac{B_j \cdot (m_j - A \cdot \theta)}{m_i - A \cdot \theta + B_j \cdot (m_j - A \cdot \theta)}$$

Inequalities (3.20) and (3.22) allow to derive Nash equilibrium values of household  $i$ 's investment depending on the benefit  $B_i$  it extracts from a retrofit implementation, as shown in

(3.23)

$$I_i^N(B_i) = \begin{cases} \frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)} & \text{if } B_i \geq \frac{B_j \cdot (m_j - A \cdot \theta)}{m_i - A \cdot \theta + B_j \cdot (m_j - A \cdot \theta)} \\ \frac{B_i \cdot (m_i - A \cdot \theta) \cdot (1 + B_j) - B_j \cdot (m_j - A \cdot \theta)}{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]} & \text{if } B_i \in \left( \frac{B_j \cdot (m_j - A \cdot \theta)}{(m_i - \theta) \cdot (1 + B_j)}, \frac{B_j \cdot (m_j - A \cdot \theta)}{m_i - A \cdot \theta + B_j \cdot (m_j - A \cdot \theta)} \right) \\ 0 & \text{if } B_i \leq \frac{B_j \cdot (m_j - A \cdot \theta)}{(m_i - \theta) \cdot (1 + B_j)} \end{cases}$$

Household  $j$ 's investment as a function of its benefit  $B_j$  has a representation analogous to (3.23) due to symmetry.

To summarize, the three cases described above – one interior and two corner solutions – give three sets of Nash equilibrium strategies of the households

$$(3.24) \quad \left( \begin{aligned} I_i^N &= \frac{B_i \cdot (m_i - A \cdot \theta) \cdot (1 + B_j) - B_j \cdot (m_j - A \cdot \theta)}{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]}, \\ I_j^N &= \frac{B_j \cdot (m_j - A \cdot \theta) \cdot (1 + B_i) - B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]} \end{aligned} \right)$$

$$\left( I_i^N = 0, I_j^N = \frac{B_j \cdot (m_j - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_j)} \right)$$

$$\left( I_i^N = \frac{B_i \cdot (m_i - A \cdot \theta)}{(1 - \theta) \cdot (1 + B_i)}, I_j^N = 0 \right)$$

In other words, (3.24) indicates the households' investments that are best responses to the opponent's strategy, and a household does not have an incentive to deviate from that provided that the other household chooses its best response.

Furthermore, households' investments into a retrofit project depend on power  $\theta$  that the state holds over the households. Earlier it was assumed that the state's authority  $\theta$  means either punishment or reward for a household's activities. However, in application to a GIS, it is rather an award than a fine that matters. Thus, the three Nash equilibrium strategies in (3.24) might pertain to this scheme in spite of the model's stylized framework.

The developed model has certain inherent limitations. First of all, in practice, higher investment is not an objective as such, since it results in higher energy savings up to a certain threshold. After this threshold is passed, the funds received under a GIS might be put to a wrong use. In addition, it may eventually overburden the state budget. The latter problem is of particular importance to Hungary, notorious for its decade-long budget deficit. However, the Hungarian government accounted for this potential problem while developing the current Green Investment Scheme by limiting the financing of

refurbishment projects and new buildings to 100 m<sup>2</sup> of area per dwelling (see Chapter 1 for more detailed information about the Hungarian GIS).

In terms of game theory, the threshold described above can be taken into account by limiting a household's investment to a certain interval. Another way of ensuring proper use of GIS money is to introduce into the model an additional variable that represents monetized energy savings that result from making an investment. Such a variable might be most useful in determining efficiency of investment into energy conservation projects. Apart from game theory, there exists an effective mechanism, namely a labeling system of buildings' certification that can partially solve the problem (see Appendix C).

## Chapter 4 - Government vs. households

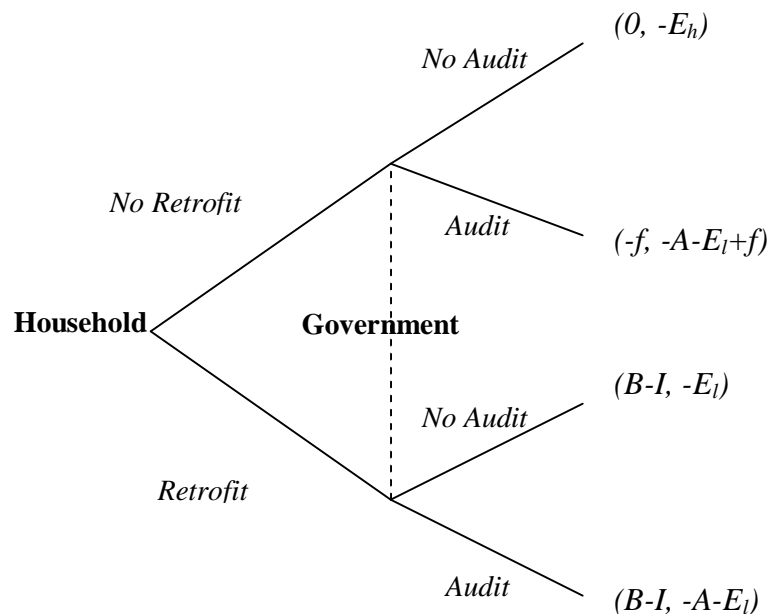
The model developed in the previous chapter encompasses households' interaction and is a simplified version of an  $N$ -agent game. In addition, it accounts for the government's power over households, as the state's stimulating interference in the form of a GIS is an underlying idea of the thesis. In the current chapter, interaction between the state and households is modeled. In addition, the chapter describes how the government can influence investment decisions of dwellers with regard to energy efficiency projects. The key assumption in case there is no governmental control is the incompleteness of information that the authorities possess, which reflects a real situation in the buildings sector accurately enough and has the following two implications. On the one hand, perfect monitoring of households' preferences and decisions is prohibitively costly. On the other hand, lack of information encourages the government to look for cheaper alternatives, such as random checks of dwellers' compliance and various enforcement mechanisms including coercion, taxes, and subsidies. To this end, governmental audits of households are introduced into the model in order to remove the incomplete information problem. Another assumption is that there is only one household in this model and it does not communicate with other households, or, at least, their interaction does not influence their decisions to finance a retrofit. This assumption is plausible as long as the retrofit concerns single-family houses.

A general case of an  $N$ -agent game is simplified to a game with two players: a state and a household. Initially, the government implements a GIS and distributes money to the dwellers to encourage them to invest into a retrofit. Then, the agents play a simultaneous move game of complete information with the following strategies and

payoffs. The household can either invest into energy efficiency of its apartment/house or refrain from it. It will spend  $I > 0$  and get benefit  $B > 0$  in the former case and zero in the latter. Assume that, in the short run,  $B < I$  since a retrofit investment's payback period is sufficiently long to discourage the investment.

The government chooses between performing an audit, which incurs costs  $A$ , and taking the household on trust. If the audit reveals that the household has not invested the money received under the GIS appropriately, the state imposes a fine  $f > 0$  on the household. Furthermore, if the household does not finance a retrofit, it consumes much higher amount of energy than otherwise. Monetized energy consumption is  $E_h$  ("high") before retrofit and  $E_l$  ("low") after retrofit. Energy consumption in monetary terms adds up to the government's costs since the energy is supplied by a state-owned utility. Therefore, the government can learn from energy bills about the household's decision. Figure 5 demonstrates an extensive form of the game between the government and the household with the payoffs indicating the agents' costs.

**Figure 5. A tree form of the game between government and household during energy refurbishment of a building**





We can see from Figure 5 that the government will always prefer to refrain from auditing the household if the latter invests into a retrofit because the government's payoff in this case is higher than otherwise (i.e.,  $-E_l > -A-E_l$ ). Choice of the household will be always not to retrofit when its opponent does not perform an audit since  $B - I < 0$ . Other payoffs are less straightforward to compare and need certain assumptions to determine dominant strategies. For this purpose, it is useful to give a matrix representation of this simultaneous move game, which is demonstrated in Table 5 (where the best responses are underlined).

**Table 5. Matrix form of the game between government and household during energy refurbishment of a building**

		Government	
		Audit	No Audit
Household	Retrofit	$B-I, -A-E_l$	$B-I, \underline{-E_l}$
	No Retrofit	$-f, -A-E_h+f$	$\underline{0}, -E_h$

When the household shirks from retrofitting, the government's decision to audit will depend on whether its payoff from auditing ( $-A-E_h+f$ ) is larger than that from non-auditing ( $-E_h$ ). If this condition holds, i.e., if the cost of audit  $A$  is lower than the fine  $f$ , the government's dominant strategy will be always to perform an audit regardless the household's choice. Otherwise, if either auditing is too expensive or the fine  $f$  is low enough, there will be a unique Nash Equilibrium with the household choosing No Retrofit (since  $0 > -f$  always holds) and the government choosing No Audit.

When the government opts for Audit and, additionally,  $B - I > -f$ , or  $f > I - B$ , the household's response will be to Retrofit. However, since  $-E_l > -A-E_l$ , the government will prefer to refrain from auditing, and Nash Equilibria will not exist in this case. A similar

situation arises in case the government chooses No Audit. Then the household will always respond with No Retrofit as  $(B - I)$  is assumed to be less than zero in the short run, which gives no Nash Equilibria.

If the government prefers to audit, the other player's payoff from retrofitting will surpass the one from refusing to retrofit if and only if  $B - I + f > 0$ . In other words, knowing that  $B - I < 0$ , if either the penalty  $f$  is mild enough, or the benefit  $B$  is low, or the required investment  $I$  is too high, so that  $f > I - B$  does not hold, the household will choose to shirk from investing, and 'No Retrofit' will be its dominant strategy. In addition, if  $-A - E_h + f > -E_h$ , or  $f > A$ , that is, if the fine is high enough or if auditing is cheap, then the only Nash Equilibrium will be for the government to Audit and for the household to choose No Retrofit.

In summary, there are two situations when a unique Nash Equilibrium exists: if  $f < A$ , a unique NE is (No Retrofit, No Audit); if  $f > A$  and  $f > I - B$ , a unique NE is (No Retrofit, Audit). As in both cases the households opts for shirking from a retrofit, it implies that a Green Investment Scheme is not an efficient motivation for retrofitting in the described environment. Moreover, if the discussed conditions are not met, the performed analysis shows that there are no Nash Equilibria in pure strategies.

In that case, it is suggested to find the equilibrium of the game in mixed strategies. Assume that the household and the government have the following respective probability profiles:  $P = (p, 1-p)$  and  $P_{gov} = (p_{gov}, 1-p_{gov})$ . In particular, the household chooses to make energy efficient investment with probability  $p$  and chooses to shirk with probability  $(1-p)$ . In the latter case, it has to pay fine  $f$  if the state has audited it. The

probability of a governmental audit is  $p_{gov}$ . For example, in case of noncompliance the household's payoff is

$$(4.1) \quad (1-p) \cdot p_{gov} \cdot (-f)$$

if it is audited and penalized by the authorities.

From here, the household's expected payoff function  $F_{HH}$  acquires the following form

$$(4.2) \quad E[F_{HH}] = p \cdot p_{gov} \cdot (B-I) + p \cdot (1-p_{gov}) \cdot (B-I) + (1-p) \cdot p_{gov} \cdot (-f) + 0$$

This, after some algebra (see Appendix B 1), can be simplified to

$$(4.3) \quad E[F_{HH}] = p \cdot (B-I) - f \cdot (1-p) \cdot p_{gov}$$

It is also necessary to consider the payoff function that the government faces in the mixed strategies game when it either performs an energy audit or refrains from it

$$(4.4) \quad \begin{aligned} E[F_{gov}] &= (1-p_{gov}) \cdot [(1-p) \cdot (-E_h) + p \cdot (-E_l)] \\ &\quad + p_{gov} \cdot [(1-p) \cdot (-A-E_h+f) + p \cdot (-A-E_l)] \end{aligned}$$

that simplifies to

$$(4.5) \quad E[F_{gov}] = p_{gov} \cdot (f-A-p \cdot f) - p \cdot (E_h-E_l) - E_h$$

(see Appendix B 2 for calculations).

To find a mixed strategy Nash Equilibrium values  $p^N$  and  $p_{gov}^N$ , it is necessary to minimize the expected payoffs of the household and the government with respect to their respective probabilities of participating in a retrofit and of auditing.

$$(4.6) \quad \frac{\partial E[F_{HH}]}{\partial p} = B-I + f \cdot p_{gov} = 0$$

from where

$$(4.7) \quad p_{gov}^N = \frac{I - B}{f}, \quad p_{gov}^N \in [0,1]$$

According to (4.7), the probability of the government's audit, first, depends inversely on the size of the fine  $f$ . The intuition behind this result is that higher fines make the household keep alert to the possibility of facing a severe penalty and they become more careful in deciding to shirk. This fact renders it less necessary to supervise the household when the fine is large enough. Second,  $p_{gov}^N$  decreases as the benefit from implementing a retrofit increases. Intuitively, if an energy efficiency project turns out to be a profitable investment, the project is more attractive for the household. Therefore, the government will be willing to cut on unnecessary auditing. Finally,  $p_{gov}^N$  increases in the household's investment  $I$ , which can be explained by the government's interest in supervising large projects that require a substantial investment.

The equilibrium probability  $p^N$  can be found similarly. The derivative of (4.5) with respect to  $p_{gov}$  is

$$(4.8) \quad \frac{\partial E[F_{gov}]}{\partial p_{gov}} = f - A - p \cdot f = 0$$

from where

$$(4.9) \quad p^N = 1 - \frac{A}{f}, \quad p^N \in [0,1]$$

Comparative statics analysis of (4.9) shows that  $p^N$  depends directly on the size of fine  $f$ . That is, the more severe the penalty, the higher probability of the household's compliance. Furthermore, there is an indirect relationship between  $p^N$  and cost  $A$  of

performing an audit, i.e., the household will be more likely to invest into an energy efficiency project if it is cheaper for the government to audit the household's compliance with a GIS agreement.

The model developed in this chapter demonstrates that a Green Investment Scheme is an efficient instrument of national policy if and only if households that receive money under the Scheme face potential audits with a certain probability. Furthermore, as it was shown, auditing is more efficient in encouraging households' compliance with the GIS contract in case the penalty for noncompliance is severe enough. The process of penalizing was assumed to be costless, i.e., transaction costs are zero, which might be considered a minor limitation of the model, since this assumption can be easily relaxed. In fact, the Hungarian government does not use penalties as it deems that random checks and rewards for compliance are sufficient to discipline the households participating in a GIS.

Another potential limitation concerns our presupposition that there is only one household that does not interact with other households. In reality, this is hardly true unless the household in question is a single family building. This assumption can be eased by assuming that dwellers' investment decisions are dependent on their neighbors' participation in a retrofit. In addition, the model might be enhanced if we take into account the fact that incentives for cooperation / deviation are influenced by the agents' income levels and that their contributions are proportional to their wealth.

## Summary and conclusions

Game theory is used universally for modeling interactions between agents in various research areas, including environmental sciences. However, only few branches of the latter are actually subject to game theory application, including studies on open-access fisheries, interboundary pollution, and energy resources. As regards energy efficiency issues, especially in residential and tertiary buildings sectors, there are virtually no comprehensive pieces of research that would contribute to the current body of knowledge in this respect. This major lack of research is a serious omission all the more so as the importance of energy efficiency in the buildings sector cannot be overestimated. To start with, this sector accounts for around 30% of total CO<sub>2</sub> emissions (IPCC 2007) and, thus, represents one of the priority areas to be addressed in the context of climate change mitigation. Moreover, this potential can be captured at a very low cost (*ibid.*), that is, the measures that result in large CO<sub>2</sub> reductions are sufficiently cheap in the buildings sector. However, there are many barriers to tapping this potential. One of those is that numerous households within one building pose a challenge to be persuaded to apply any of climate mitigation and energy saving measures to their building. To this end, game theory that links behavioral and economic aspects of various agents' responses might be instrumental in finding cooperative strategies to turn panel block constructions into energy efficient buildings.

Hungary, among other countries of Central and Eastern Europe, has embarked upon pursuing energy efficiency targets quite recently. However, it has already pioneered one of the innovative approaches to addressing this issue in the buildings sector, a Green Investment Scheme. This policy tool is planned to encourage energy efficiency

refurbishments in residential buildings through a special subsidy. Consequently, complex interactions between the government and households are inevitable and can be also modeled by means of game theory.

This thesis builds on the idea that a well-crafted Green Investment Scheme planned to be enforced in Hungary is an effective financial mechanism that can encourage a building's dwellers to take part in an energy efficient renovation of their house. To support the hypothesis, two game theoretic models are developed that present interactions, first, between dwellers within one building and, second, between the government and households. The first interaction is modeled as a one-stage non-zero-sum game that involve three agents - a government and two households. The government enjoys perfect monitoring, that is, it has complete information about the households' investment decisions through the energy bills that households submit to a state-owned utility. The second interaction is presented by a simplified one-stage game with two players: a state and a household. The problem of imperfect information is removed by introducing governmental audits of a household's activities.

Results suggest that a household invests into a retrofit project when the benefit it obtains from the project surpasses a certain threshold. Additionally, the investment depends on the neighbor's benefit from the refurbishment; namely, the former increases with the other household's benefit if the latter is high enough. It is also shown that a GIS is an efficient instrument of national policy if and only if households that receive money under the Scheme face audits with a certain probability. Furthermore, auditing is more efficient in encouraging households' compliance with the GIS contract in case the penalty for noncompliance is severe enough. Nonetheless, the government will be willing to cut

on unnecessary auditing when the households' benefit from the project implementation increases. Lastly, the probability of the governmental audits is positively correlated with the household's investment  $I$ , which can be explained by the government's interest in supervising large projects that require a substantial investment.

The developed models represent highly stylized frameworks and have certain intrinsic limitations. To begin with, the number of agents is limited to either two or three players, which is hardly credible unless the household under consideration is a single-family building. Another shortcoming is that, in practice, maximization of retrofit investment is not an objective as such, since it results in higher energy savings up to a certain extent. Finally, the results of the models are highly sensitive to the assumptions made.

Further research is needed that will not only account for the described limitations, but will also consider the models' extensions as well as calibration of the results. Moreover, as literature search has attested, there exist various game theoretic concepts that can be applied to modeling energy efficiency in the buildings sector, such as the Groves-Clarke demand revealing mechanism (Clarke 1971, Groves and Loeb 1975) and hyperbolic and quasihyperbolic discounting (Akin 2007, Settle and Shogren 2004) that model time inconsistency of agents.



## Appendix A

### A 1. Proof of (3.11):

$$\begin{aligned}
 \frac{\partial U_i}{\partial I_i} &= \frac{1-\theta}{m_i - I_i \cdot (1-\theta) - A \cdot \theta} - \frac{B_i}{I_i + I_j} = 0 \\
 \frac{(1-\theta) \cdot (I_i + I_j) - B_i \cdot [m_i - I_i \cdot (1-\theta) - A \cdot \theta]}{[m_i - I_i \cdot (1-\theta) - A \cdot \theta] \cdot (I_i + I_j)} &= 0 \\
 I_i + I_j - I_i \cdot \theta - I_j \cdot \theta - B_i \cdot m_i + B_i \cdot I_i \cdot (1-\theta) + A \cdot B_i \cdot \theta &= 0 \\
 I_i \cdot [1-\theta + B_i \cdot (1-\theta)] + I_j \cdot (1-\theta) - B_i \cdot (m_i - A \cdot \theta) &= 0 \\
 I_i &= \frac{B_i \cdot (m_i - A \cdot \theta)}{(1-\theta) \cdot (1+B_i)} - \frac{I_j}{1+B_i}
 \end{aligned}$$

### A 2. Proof of (3.12):

$$\begin{aligned}
 \frac{\partial I_i}{\partial B_i} &= \frac{(m_i - A \cdot \theta) \cdot (1+B_i - \theta - B_i \cdot \theta) - (1-\theta) \cdot (B_i \cdot m_i - A \cdot B_i \cdot \theta - I_j + I_j \cdot \theta)}{[(1-\theta) \cdot (1+B_i)]^2} \\
 &= \frac{m_i + B_i \cdot m_i - m_i \cdot \theta - B_i \cdot m_i \cdot \theta - A \cdot \theta - A \cdot B_i \cdot \theta + A \cdot \theta^2 + A \cdot B_i \cdot \theta^2 - B_i \cdot m_i + \dots}{[(1-\theta) \cdot (1+B_i)]^2} \\
 &\quad \dots \frac{+ A \cdot B_i \cdot \theta + I_j - I_j \cdot \theta + B_i \cdot m_i \cdot \theta - A \cdot B_i \cdot \theta^2 - I_j \cdot \theta + I_j \cdot \theta^2}{\dots} \\
 &= \frac{m_i - m_i \cdot \theta - A \cdot \theta + A \cdot \theta^2 + I_j - 2I_j \cdot \theta + I_j \cdot \theta^2}{[(1-\theta) \cdot (1+B_i)]^2} \\
 &= \frac{(1-\theta) \cdot (m_i - A \cdot \theta) + I_j \cdot (1-\theta)^2}{[(1-\theta) \cdot (1+B_i)]^2} > 0
 \end{aligned}$$

### A 3. Proof of (3.15) and (3.16):

The system of equations can be solved using the Cramer Rule. The matrix form of (3.14)

is:

$$\begin{vmatrix} (1+B_i) & 1 \\ 1 & (1+B_j) \end{vmatrix} \times \begin{vmatrix} I_i \\ I_j \end{vmatrix} = \begin{vmatrix} \frac{(m_i - A \cdot \theta) \cdot B_i}{1-\theta} \\ \frac{(m_j - A \cdot \theta) \cdot B_j}{1-\theta} \end{vmatrix}$$

From here the main determinant is

$$\Delta = (1+B_i) \cdot (1+B_j) - 1$$

The determinant of  $I_i$  variable is

$$\Delta_{I_i} = B_i \cdot (m_i - A \cdot \theta) \cdot (1+B_j) - B_j \cdot (m_j - A \cdot \theta)$$

The determinant of  $I_j$  variable is

$$\Delta_{I_j} = B_j \cdot (m_j - A \cdot \theta) \cdot (1+B_i) - B_i \cdot (m_i - A \cdot \theta)$$

From here, Nash Equilibrium values of the households' investments can be found:

$$I_i^N = \frac{\Delta_{I_j}}{\Delta} = \frac{B_i \cdot (m_i - A \cdot \theta) \cdot (1+B_j) - B_j \cdot (m_j - A \cdot \theta)}{(1-\theta) \cdot [(1+B_i) \cdot (1+B_j) - 1]}$$

By analogy,

$$I_j^N = \frac{\Delta_{I_i}}{\Delta} = \frac{B_j \cdot (m_j - A \cdot \theta) \cdot (1+B_i) - B_i \cdot (m_i - A \cdot \theta)}{(1-\theta) \cdot [(1+B_i) \cdot (1+B_j) - 1]}$$

#### A 4. Comparative statics of (3.15) w.r.t. $B_i$ and $B_j$ :

$$\begin{aligned}
 \frac{\partial I_i^N}{\partial B_i} &= \frac{(m_i - A \cdot \theta) \cdot (1 + B_j) \cdot (1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1] - \dots}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \dots \\
 &\quad \dots \frac{(1 - \theta) \cdot (1 + B_j) \cdot [B_i \cdot (m_i - A \cdot \theta) \cdot (1 + B_j) - B_j \cdot (m_j - A \cdot \theta)]}{\dots} \\
 &= \frac{(1 - \theta) \cdot (1 + B_j) \cdot [(m_i - A \cdot \theta) \cdot (1 + B_i) \cdot (1 + B_j) - \dots}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \dots \\
 &\quad \dots \frac{-(m_i - A \cdot \theta) - B_i \cdot (m_i - A \cdot \theta) \cdot (1 + B_j) + B_j \cdot (m_j - A \cdot \theta)}{\dots} \\
 &= \frac{(1 - \theta) \cdot (1 + B_j) \cdot [(m_i - A \cdot \theta) \cdot (1 + B_j + B_i + B_i \cdot B_j - 1 - \dots}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \dots \\
 &\quad \dots \frac{-B_i - B_i \cdot B_j + B_j \cdot (m_j - A \cdot \theta)]}{\dots} \\
 &= \frac{(1 - \theta) \cdot (1 + B_j) \cdot [B_j \cdot (m_i - A \cdot \theta) + B_j \cdot (m_j - A \cdot \theta)]}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \\
 &= \frac{(1 - \theta) \cdot (1 + B_j) \cdot B_j \cdot (m_i - m_j - 2A \cdot \theta)}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} > 0
 \end{aligned}$$

$$\begin{aligned}
\frac{\partial I_i^N}{\partial B_j} &= \frac{B_i \cdot (m_i - A \cdot \theta) - (m_j - A \cdot \theta) \cdot (1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1] - \dots}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \dots \\
&\quad \dots \frac{-(1 - \theta) \cdot (1 + B_i) \cdot B_i \cdot (m_i - A \cdot \theta) \cdot (1 + B_j) - B_j \cdot (m_j - A \cdot \theta)}{\dots} \\
&= \frac{(B_i \cdot m_i - A \cdot B_i \cdot \theta - m_j) \cdot (1 - \theta) \cdot (B_i + B_j + B_i \cdot B_j) - (1 - \theta) \cdot \dots}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \dots \\
&\quad \dots \frac{\cdot (1 + B_i) \cdot (B_i \cdot m_i - A \cdot B_i \cdot \theta + B_i \cdot B_j \cdot m_i - A \cdot B_i \cdot B_j \cdot \theta - B_j \cdot m_j - A \cdot B_j \cdot \theta)}{\dots} \\
&= \frac{(1 - \theta) \cdot (B_i^2 \cdot m_i - A \cdot B_i^2 \cdot \theta - B_i \cdot m_j + B_i \cdot B_j \cdot m_i - A \cdot B_i \cdot B_j \cdot \theta - B_j \cdot m_j + \dots}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \dots \\
&\quad \dots \frac{+ B_i^2 \cdot B_j \cdot m_i - A \cdot B_i^2 \cdot B_j \cdot \theta - B_i \cdot B_j \cdot m_j - B_i \cdot m_i - A \cdot B_i \cdot \theta + B_i \cdot B_j \cdot m_i + \dots}{\dots} \\
&\quad \dots \frac{+ A \cdot B_i \cdot B_j \cdot \theta + B_j \cdot m_j + A \cdot B_j \cdot \theta - B_i^2 \cdot m_i + A \cdot B_i^2 \cdot \theta - B_i^2 \cdot B_j \cdot m_i + \dots}{\dots} \\
&\quad \dots \frac{+ A \cdot B_i^2 \cdot B_j \cdot \theta + B_i \cdot B_j \cdot m_j + A \cdot B_i \cdot B_j \cdot \theta}{\dots} \\
&= \frac{(1 - \theta) \cdot (-B_i \cdot m_j - B_i \cdot m_i + A \cdot B_i \cdot \theta + A \cdot B_i \cdot B_j \cdot \theta)}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2} \\
&= \frac{(1 - \theta) \cdot B_i \cdot (-m_j - m_i + A \cdot \theta + A \cdot B_j \cdot \theta)}{\{(1 - \theta) \cdot [(1 + B_i) \cdot (1 + B_j) - 1]\}^2}
\end{aligned}$$

In order for this result to be more than zero, the following should hold:

$$-m_j - m_i + A \cdot \theta + A \cdot B_j \cdot \theta > 0$$

OR

$$B_j > \frac{m_i + m_j}{A \cdot \theta} - 1$$

That is,  $I_i$  increases with the neighbor's benefit  $B_j$  if the latter is high enough.

Calculations for comparative statics of (3.16) are analogous.

## Appendix B

### B 1. Proof of (4.3):

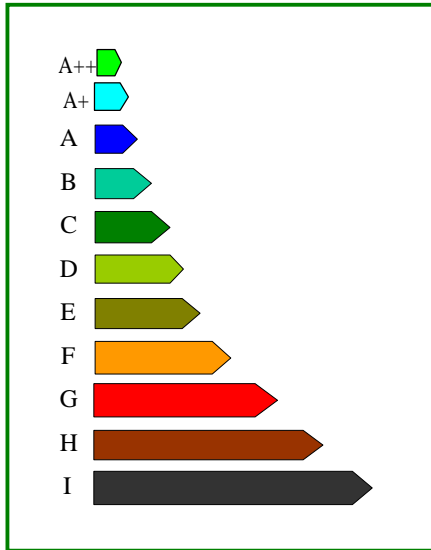
$$\begin{aligned} E[F_{HH}] &= p \cdot p_{gov} \cdot (B - I) + p \cdot (1 - p_{gov}) \cdot (B - I) + (1 - p) \cdot p_{gov} \cdot (-f) + 0 \\ &= p \cdot p_{gov} \cdot B - p \cdot p_{gov} \cdot I + p \cdot B - p \cdot I - p \cdot p_{gov} \cdot B + p \cdot p_{gov} \cdot I - f \cdot (1 - p) \cdot p_{gov} \\ &= p \cdot (B - I) - f \cdot (1 - p) \cdot p_{gov} \end{aligned}$$

### B 2. Proof of (4.5):

$$\begin{aligned} E[F_{gov}] &= (1 - p_{gov}) \cdot [(1 - p) \cdot (-E_h) + p \cdot (-E_l)] \\ &\quad + p_{gov} \cdot [(1 - p) \cdot (-A - E_h + f) + p \cdot (-A - E_l)] \\ &= p \cdot E_h - E_h - p_{gov} \cdot p \cdot E_h + p_{gov} \cdot E_h - p \cdot E_l + p_{gov} \cdot p \cdot E_l \\ &\quad + p_{gov} \cdot f - p_{gov} \cdot A - p \cdot E_h - p_{gov} \cdot p \cdot f + p_{gov} \cdot p \cdot A \\ &\quad + p_{gov} \cdot p \cdot E_h - p_{gov} \cdot p \cdot A - p_{gov} \cdot p \cdot E_l \\ &= p \cdot E_h - E_h - p \cdot E_l + p_{gov} \cdot f - p_{gov} \cdot A - p_{gov} \cdot p \cdot f \\ &= p_{gov} \cdot (f - A - p \cdot f) - p \cdot (E_h - E_l) - E_h \end{aligned}$$

## Appendix C

**Figure 6. Energy labeling of buildings in Hungary**



Source: the 176/2008 (VI.30) Decree in Csoknyai and Szalay 2008; Zöld 2008

**Table 6. Energy performance of residential and commercial constructions – energy labeling break-down**

Category	Energy consumption, kWh/m <sup>2</sup> /yr.	Characteristic
<b>A++</b>	<45	Ultra-low energy consumption
<b>A+</b>	<55	Low energy consumption
<b>A</b>	56-75	Energy efficient
<b>B</b>	76 – 95	Exceeds requirements
<b>C</b>	96-100	Meets requirements
<b>D</b>	101-120	Close to requirements
<b>E</b>	121-150	Better than average
<b>F</b>	151-190	Average
<b>G</b>	191-250	Close to average
<b>H</b>	251-340	Poor
<b>I</b>	341 <	Bad

Source: the 176/2008 (VI.30) Decree in Csoknyai and Szalay 2008

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