FISCAL POLICY MULTIPLIERS IN A NEW KEYNESIAN MODEL UNDER POSITIVE AND ZERO NOMINAL INTEREST RATE

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Submitted to

Central European University

Department of Economics

In partial fulfilment of the requirements for the degree of Masters of Arts

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Budapest, Hungary 2010

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Abstract

This thesis calculates short and long-run fiscal policy multipliers of three types (government spending, sales tax and payroll tax) in a standard New Keynesian model. Each of them is conducted separately and assumed to be financed by lump-sum taxes. When solving for the multipliers analytically we use the method of undetermined coefficients. Otherwise, the models are solved numerically using Dynare. When nominal interest rate is positive government spending multiplier for non-separable preferences is higher than the one for separable preferences with the opposite being true for payroll and sales tax cut. However, when calculating long-run multipliers the difference between the size of multipliers coming from preference specifications disappears. In line with Christiano et al. (2009) and Eggertsson (2009) we found that government spending multiplier can be very high when the zero lower bound on the nominal interest rate binds. We also managed to reconcile the most important finding of Eggertsson (2009) who uses separable preferences that the payroll tax multiplier in case of zero nominal interest rate is negative for non-separable preferences as well. However, in contrast to the finding of Eggertsson (2009) who uses separable preferences and assumes that the nominal rate is zero, we show that the sales tax cut is not as good as the increase in government spending for stimulating the economy when we use non-separable preferences and holding his other assumptions. In the same type of model extended with capital we found by fixing the nominal rate on constant level firstly for one and secondly for two years that the government spending multiplier is close or slightly above one but is definitely lower than the ones reported by Bernstein and Romer (2009).

1 Introduction

The American Recovery and Reinvestment Act was passed at the beginning of 2009 in order to help the US economy recover from the financial crises started in 2008. Bernstein and Romer (2009) provided a document that gives a detailed picture of the estimated effects of this stimulus package¹. However, there is wide disagreement in the economics profession on the value of the fiscal multipliers² listed in their paper³. As Cogan *et al.* (2009) assert, it is not straightforward what kind of model Bernstein and Romer (2009) used to obtain multipliers above one⁴ for a permanent increase in government spending under the assumption that the nominal interest rate is held constant⁵ for the time interval of their simulation. Furthermore, Cogan *et al.* (2009) argues that the Bernstein and Romer (2009) model can't be a New Keynesian model as the model's setup would imply explosive dynamics.

This thesis proposes a standard New Keynesian dynamic stochatic general equilibrium (DSGE) model (with and without capital) used widely in academic literature and in central banks for supporting decision-making to investigate into the effects of various fiscal stimuli on output both for zero and non-zero nominal interest rate. The DSGE model used here is basically a stochastic growth (or RBC) model enriched with monopolistic competition on the product market and staggered price setting in Calvo-style (Calvo, 1983). Being aware that fiscal policy is not constrained to variations only in spending but can also operate with various taxes to achieve its goal, we consider three possible sources of a fiscal stimulus separately: an increase in non-productive (that is not creating investment opportunities in the economy) government spending, a sales tax cut and a cut in payroll tax.

Of course, this is not the first paper using a New Keynesian model that studies fiscal multipliers. Two recent contributions of the topic are Christiano *et al.* (2009) and Eggertsson (2009). Christiano *et al.* (2009) discuss government spending both for a model with and without capital when the zero bound on interest rate is binding and not-binding by using non-separable preferences in consumption and leisure. Their most interesting finding is that the spending multiplier is more than three times higher when the nominal interest rate is zero compared to the case when it is positive. Eggertsson (2009) calculates fiscal multipliers (payroll tax cut, profit tax cut, sales tax cut, capital tax cut and an increase in government

¹For example, in their Table 5, they provide numbers on the jobs created in each industry in 2010Q4 as a result of the Recovery Package.

²This is the change in output due to a change in government spending, dY_{t+k}/dG_t . For k = 0 we get back the impact multiplier.

³In particular, in their Appendix 1 they consider "output effects of a permanent stimulus of 1% of GDP (percent)"

⁴That is, a dollar spent by the government increases output by more than one dollar.

⁵Note that at the time of the introduction of the recovery package the federal funds rate was almost zero and this is a fact that a model has to take into consideration.

spending) for the case of separable preferences with special attention to the case of binding zero bound on nominal interest rate. His most interesting finding is that the multiplier associated with a payroll tax cut is **negative**.

Here we describe the extensions made to the above papers and, thereby, the new results obtained. Firstly, we study not only government spending but also payroll and sales tax cut in the Christiano *et al.* (2009) setting (that is using non-separable preferences) and secondly, in contrast to Eggertsson (2009) who uses separable preferences and five types of multipliers (for zero and non-zero policy rate), we employ here both separable and non-separable preferences with three types of multipliers both for zero and non-zero nominal interest rate. Besides being successfull in reconciling the above papers' findings⁶ we point out three new results *not* discussed by the above papers: when we use non-separable preferences the multipliers associated *with labour and sales tax cuts* are lower than the ones in the separable case for positive nominal rate. Also for the non-separable case: when zero lower bound on nominal rate becomes binding the sales tax multiplier declines⁷ in contrast to the government spending one which rises. The payroll tax multiplier is negative in accordance with the finding of Eggertsson (2009) for *both* type of preferences for zero policy rate⁸.

After extending the New Keynesian model with capital we formulate *two* more results. Firstly, the difference between separable and non-separable preferences concerning the size of a temporary spending shock on the impact disappears if we consider the long-run multiplier (calculated similarly to the one in Campolmi *et al.*, 2010). Secondly, we present in the same model with capital and non-separable preferences that the *short* and *long-run* multipliers associated with a *permanent* increase in government spending, confirming the findings of the baseline model, can prove to generate an increase in output that is equal or slightly higher than one. However, these multipliers are still *definitely lower* than the ones reported by Bernstein and Romer (2009).

This thesis shows by using non-separable preferences that we can match the stylised fact of rising consumption in response to a positive government spending shock. As it is wellknown, when Ricardian equivalence holds, the use of non-separable preferences mitigates the negative wealth effect associated with the fact that consumers expect a rise in future taxes when there is an increase in government spending (or a tax cut) in the present. In the

 $^{^{6}}$ The success of reconciling the results of Eggertsson (2009) is not straightforward because we use here the calibration of Christiano et al. (2009).

⁷Note that Eggertsson (2009) shows the opposite for separable preferences.

⁸Furthermore, we point out a technical finding that is connected to but not much emphasized by Christiano *et al.* (2009). In order to obtain a spending multiplier that is larger than one for non-separable preferences and to ensure that the zero bound binds for a discount factor shock we have to assume that those firms who cannot adjust prices fix them for more than a year that is not in line with some of the empirical evidence (see, e.g., Bils and Klenow (2004) who report an average price stickiness that is shorter than a year)

following we provide some empirical evidence on the weakness of this negative wealth effect (Monacelli and Perotti, 2008).

There is extensive empirical literature on the effect and size of fiscal multipliers (see, e.g., Blanchard and Perotti, 2002 and Gali *et al.*, 2007). For example, Gali *et al.* (2007) reports some VAR evidence that government spending multiplier i.e. the change in output with respect to a change in government spending, is 0.78 on impact, and 1.74 at the end of the second year. Interestingly they also found that **consumption**, **working hours and wages** respond to increased government purchases **positively** in small and large (including a complete list of explanatory variables) VAR models on many subsamples. Is is also important that the magnitude of the response in consumption, working hours and wages are quantitatively large. In case of consumption the change is usually close to or larger than one in the 4th and the 8th quarter but definitely not on impact after a rise in government spending. However, not all the empiricial VAR literature is consistent with the positive connection between consumption and government spending. For example, the identification strategy applied by Ramey (2008) implies that shortly after increases in government spending consumption declines. The latter one is based on capturing news about government spending hikes, instead of relying on the delayed effect as in standard VAR.

The outline of the thesis is as follows. In Section 1, we formulate the simple New-Keynesian model (firstly with separable and, then, with non-separable preferences), derive analytical short-run (or impact) multipliers of three cases (for both separable and non-separable preferences in section 2 and 3, respectively) and discuss their sensitivity to the underlying parameters. In section 4, we modify the baseline model with restriction only to non-separable preferences to investigate into the case of zero nominal interest rate. Section 5 contains the baseline model augmented with capital to assess the robustness of the findings of the models without capital. Finally, we conclude with the main results.

2 A simple DSGE model without capital

The setup of the model used here builds strongly upon Christiano *et al.* (2009). The idea of tax rates (labour and sales tax) are introduced into the model following Eggertsson (2009). However, Eggertsson (2009) use only separable preferences, while here **both** separable and non-separable preferences are used and discussed. Christiano *et al.* (2009) use non-separable preferences and refers to their results — without reporting them — on separable preferences. As we will see, the optimality conditions can always be characterised by the intratemporal condition, the intertemporal Euler equation (or, Aggregate Demand, AD), the New Keynesian Phillips curve (NKPC or aggregate supply, AS) and the exogenous shock process.

2.1 The household's problem

The household maximises the following utility that is separable in consumption and leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + v(G_t^N) \right]$$

with respect to its budget constraint

$$(1+R_t)B_t + \int_0^1 profit_t(i)di + (1-\tau_t^W)P_tW_tN_t = B_{t+1} + (1+\tau_t^S)P_tC_t + T_t$$

where τ_t^W denotes the payroll tax and τ_t^S denotes the sales tax. B_t denotes the amount of one-period riskless bonds, R_t is the net nominal one-period rate of interest that pays off in period t. N_t is the sum of all labour types i, that is, $N_t \equiv \int N_t(i)di$ and $P_tW_tN_t$ denotes the 'mass' of nominal wages (with the real wage rate W_t). T_t denotes lump-sum taxes net of transfers. $profit_i$ denotes the profit of firm i. The transversality condition, $\lim_{t\to\infty} B_{t+1}/[(1+R_0)(1+R_1)...(1+R_t)] \ge 0$, is also satisfied.

The household has separable preferences in consumption (C_t) , leisure $(1 - N_t)$ and government spending $(v(G_t))$. We do not specify v here as it is not needed for the optimality conditions. Throughout the whole paper we assume that $\sigma > 1$ and $\varphi \ge 0$.

2.2 The firms' problem

2.2.1 Final good sector

The competitive firms produce a single final good using the following technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1$$

where $Y_t(i)$, $i \in [0, 1]$ denotes the intermediate good *i*. The profit-maximisation problem of competitive firms results in the demand equation for $Y_t(i)$:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t \tag{1}$$

where $P_t(i)$ denotes the price of the intermediate good *i* and P_t is the price of the homogenous final good.

2.2.2 Intermediary sector

The intermediate good $i, Y_t(i)$, is produced by a monopolist i^{th} using a linear technology:

$$Y_t(i) = N_t(i)$$

where $N_t(i)$ denotes the hours used by monopolist *i* to produce intermediate good *i*. To be able to calculate multipliers analytically, we later abstract from capital formation in this section. However, in Section 5 we introduce capital into the production function as well. We assume that there is no entry or exit into the industry that produces the *i*th intermediate good. Furthermore, we have Calvo-price setting that means that a *random* fraction of firms are allowed to re-optimise its price every period with probability $1 - \xi$. With probability ξ a fraction of firms cannot re-optimise their price and uses their previous period price:

$$P_t(i) = P_{t-1}(i).$$

The discounted profit of the i^{th} intermediary firm can be written as:

$$E_t \sum_{T=0}^{\infty} \beta^{t+T} v_{t+T} \left[P_{t+T}(i) Y_{t+T}(i) - (1-\nu) W_{t+T} N_{t+T}(i) \right],$$

where we assume that the subsidy is set such $(\nu = \frac{1}{\varepsilon})$ that corrects for the steady-state distortion induced by the presence of monopoly power and v_{t+T} is the Lagrange multiplier on the budget contraint in the household's optimisation problem.

2.3 Monetary Policy

The monetary policy is assumed to follow the following simple rule:

$$R_{t+1} = \max(Z_{t+1}, 0) \tag{2}$$

where

$$Z_{t+1} = (1/\beta)(1+\pi_t)^{\phi_1(1-\rho_R)}(Y_t/Y)^{\phi_2(1-\rho_R)}[\beta(1+R_t)]^{\rho_R} - 1$$
(3)

where Y denotes the steady-state value of Y_t^9 . π_t is the time-t rate of inflation. As usual, we assume that $\phi_1 > 1$ and $\phi_2 \in (0, 1)$. The main implication of the rule in equation (2) is that whenever the nominal interest rate becomes negative, the monetary policy set it equal to zero, otherwise it is set by the Taylor rule specified in equation (3). The parameter ρ_R measures

⁹And for the rest of the paper, a variable without a time subscript denotes steady-state value.

how quickly monetary policy reacts to changes in inflation and output and we assume that $0 < \rho_R < 1$. Furthermore, we also assume that the inflation in steady state is zero which implies that steady-state net nominal interest rate is $1/\beta - 1$.

2.4 Fiscal policy

We have an exogenous AR(1) process for government spending (and the same could be written for labour tax and sales tax as well):

$$G_{t+1} = (G_t)^{\rho_G} \exp(\varepsilon_{t+1}^G) \tag{4}$$

where ρ_G measures persistence of government spending process and ε^G is an *i.i.d.* shock with zero mean and constant variance. We assume in this simple model that the government spending, the labour tax cut, the sales tax cut and the employment subsidy to restore efficiency in steady-state is financed through *lump-sum* taxes. That is, the Ricardian equivalence holds under our assumptions and the exact timing of taxes is irrelevant and we don't have to take into consideration the government budget constraint. The implications of fiscal policy when the nominal rate is zero is discussed in Section 4.

2.5 Equilibrium

The equilibrium can be characterised by four equations. The Intratemporal, Euler, NKPC equations are listed here and the shock process is in equation (4). The real marginal cost that appears in the NKPC coincides with W_t due to the linear technology and is model-specific. Variables with a hat, $\hat{}$, denote percentage deviations from steady-state.

Intratemporal condition (in linearised form)

$$\widehat{MC}_t = \widehat{W}_t = \varphi \widehat{N}_t + \sigma \widehat{C}_t + \frac{\tau^W}{1 - \tau^W} \widehat{\tau}_t^W + \frac{\tau^S}{1 + \tau^S} \widehat{\tau}_t^S$$

Euler equation (in linearised form)

$$-\sigma \hat{C}_{t+1} - \frac{\tau^{S}}{1+\tau^{S}} \hat{\tau}_{t+1}^{S} + \beta (R_{t+1} - R) = -\sigma \hat{C}_{t} - \frac{\tau^{S}}{1+\tau^{S}} \hat{\tau}_{t}^{S} + E_{t} \pi_{t+1}$$

The New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{MC}_t \tag{5}$$

where $\kappa \equiv (1 - \xi)(1 - \beta \xi)/\xi$ and ξ is the Calvo parameter.

2.6 Parametrisation

Parameters of the model are given in Table 1 for separable and non-separable preferences separately. Most of the parameters, like β , $\rho_G(=\rho_{\tau W}=\rho_{\tau S})$ and ϕ_1 are standard in economics literature. The value of φ is taken from Gali *et al.* (2007). The values of σ and γ are from Christiano *et al.* (2009). To guarantee stability, the inflation coefficient, ϕ_1 in the Taylor rule must be greater than one. The steady-state values of payroll tax, τ^W , sales tax, τ^S and the government spending to GDP ratio, g are taken from Uhlig (2009) who calibrated them to US data. The value of the Calvo parameter, ξ is usually chosen to be 0.67 (or 0.75) implying that firms that cannot determine prices optimally use their last price for three quarters (or for a year) on average. However, we choose here ξ somewhat larger (0.85) for reasons asserted in the following sections¹⁰. The standard deviation of the noise term (σ_{ε^G} , $\sigma_{\varepsilon^{\tau W}}$ and $\sigma_{\varepsilon^{\tau S}}$) of the shock process in equation (4) for all three types of stimulus is one percent.

Parameters	Separable	Non-separable
σ	2	2
φ	0.2	na
β	0.99	0.99
γ	na	0.29
$\rho_G = \rho_{\tau^W} = \rho_{\tau^S}$	0.8	0.8
ϕ_1	1.5	1.5
ϕ_2	0	0
ρ_R	0	0
ξ	0.85	0.85
$G/Y(\equiv g)$	0.15	0.15
$ au^W$	0.28	0.28
$ au^S$	0.05	0.05
$\sigma_{\varepsilon^G} = \sigma_{\varepsilon^{\tau^W}} = \sigma_{\varepsilon^{\tau^S}}$	0.01	0.01
Implied parameters		
ĸ	0.03	0.03
N	na	1/3

 Table 1: Parametrisation of the New Keynesian Model without Capital

Remark to Table 1: na=non applicable. The parameters φ and γ are present for separable and non-separable preferences, respectively.

¹⁰The Calvo parameter, ξ , should be greater than 0.82 for two reasons: (1) we can achieve a government spending multiplier that is larger than one for non-separable preferences and (2) we can meet a necessary requirement in the model of Section 4 for the zero bound to bind.

2.7 Multipliers for Separable Preferences

There are three important requirements for being able to solve the model analytically by methods of undetermined coefficients: (1) linear production function, (2) no interest rate smoothing in Taylor rule ($\rho_R = 0$) and (3) the assumption that government spending and changes in distortionary taxes are financed through lump-sum taxes (in other words Ricardian equivalence holds). The parametrisation for the separable case can be found in the first column of Table 1. The exact formulas for the multipliers (by assuming that the zero bound does not bind in this section) presented here are derived under the above three main assumptions.

We solve the model *analytically* by using the method of undetermined coefficients. That is, we guess that output and inflation is some function of \hat{G}_t (and similarly for $\hat{\tau}_t^W$ and $\hat{\tau}_t^S$) and can be expressed as:

$$\pi_t = A_\pi \widehat{G}_t \tag{6}$$

$$\widehat{Y}_t = A_Y \widehat{G}_t \tag{7}$$

Moreover, to be able to eliminate forward-looking variables, that is, e.g. E_tG_{t+1} , we assume an exogenous AR(1) process for government spending as it is in equation (4).

2.7.1 Government spending multiplier

First, we discuss when the government spending multiplier is larger than one:

$$\frac{dY_t}{dG_t} = \frac{1}{g} \frac{d\widehat{Y}_t}{d\widehat{G}_t} = \frac{1}{g} \frac{d\left\lfloor (1-g)\widehat{C}_t + g\widehat{G}_t \right\rfloor}{d\widehat{G}_t} = 1 + \frac{1-g}{g} \frac{d\widehat{C}_t}{d\widehat{G}_t}$$
(8)

This formula implies that the size of the spending multiplier depends on how consumption reacts to government spending. For separable preferences the latter one in equation (8) is negative: $d\hat{C}_t/d\hat{G}_t < 0$. Thus, the spending multiplier is smaller than one and this can also be seen on Figure 1 where consumption falls and the multiplier is smaller than one on impact (0.97).

The government spending multiplier — with the complete derivation in Appendix A — can be expressed from equation (7):

$$\frac{dY_t}{dG_t} = \frac{1}{g} \frac{d\hat{Y}_t}{d\hat{G}_t} = \sigma \frac{(1-\rho) + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho}}{\sigma(1-\rho) + \phi_2(1-g) + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho} \left(\varphi + \frac{\sigma}{1-g}\right)}$$

Figure 1: The effects of a 1% temporary shock to government spending in the model with separable preferences. Note that dC/dG < 0.



As we have discussed it in detail it is always smaller than one. It can be also noted that with certain parametrisation it can be very close to one but never goes beyond one.

2.7.2 Labour tax multiplier

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W} = \frac{(\rho - \phi_1)\frac{\kappa}{1 - \beta\rho}\frac{\tau^W}{1 - \tau^W}}{\sigma - (\rho - \phi_1)\frac{\kappa}{1 - \beta\rho}\left(\varphi + \sigma\right) - (\sigma\rho - \phi_2)}$$

The value of the multiplier (with the baseline calibration) is 0.19, which is somewhat larger than the one in Eggertsson (2009). The difference comes from the fact that Eggertsson (2009) has different calibration than the one here in Table 1. He calibrates his model parameters to data prevailing under the Great Depression by maximising the posterior distribution of his model to match a 30 percent decline in output and a 10 percent deflation at the first quarter of 1933 when the zero lower bound became to be binding on the nominal interest rate. Estimation of the models' parameters used here is subject to further research.

2.7.3 Sales tax multiplier

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S} = \frac{\frac{\tau^S}{1+\tau^S} \left[(\rho-1) - (\phi_1 - \rho) \frac{\kappa}{1-\beta\rho} \right]}{(\sigma+\phi_2) + (\phi_1 - \rho) \frac{\kappa}{1-\beta\rho} \left(\varphi + \sigma\right) - \sigma\rho}$$

The value of the multiplier (0.4) is broadly in line with the corresponding one in Eggertsson (2009). We also have to note that the sales tax multiplier is seemingly lower than the one of government spending because the coefficient on the sales tax term $(\hat{\tau}_t^S)$ in the Euler equation, $\frac{\tau^S}{1+\tau^S}$ is lower than the one of the spending term, \hat{G}_t ($\frac{\sigma g}{1-g}$ which can be seen from equation (28) in Appendix A) and this is why sales tax has smaller expansionary effect.

2.8 Sensitivity analysis of the fiscal multipliers (separable case)

Government spending

For the sensitivity analysis first I linearise equilibrium conditions and then solve the model *numerically* by using Dynare. The benchmark value of the multipliers (for the parameter values in Table 1) is denoted by \Box on the following graphs.

In (1,1) element of Figure 2 we can see that the multiplier is increasing with σ in a concave manner. When $\sigma > 1$ the negative wealth effect on labour supply dominates, that is, the household substitutes consumption for hours after an increase in spending because he/she wants to make up for the loss in consumption used by government. When σ is higher the substitution effect is higher (the labour supply of the household shifts out to the right even more decreasing real wages) and the multiplier is also higher. This is the aggregate supply channel. However, there is an aggregate demand channel as well. The increase in government purchases can be interpreted as an "autonomous" increase in spending that stimulates aggregate demand which is satisfied by those firms that — due to price stickiness — cannot increase their prices but can raise their labour demand. When σ is higher the stimulus effect of spending and the multiplier is also higher.

The (1,2) element Figure 2 shows that the multiplier is in positive relationship with the Calvo parameter, ξ . When government spending increases, total demand as well as marginal cost increases. As prices are sticky, the price over marginal cost falls due to a rise in demand. In the presence of monopolistic competition a fall in the markup lead to rise in labour demand, a corresponding rise in production and a surge in output. When price stickiness is higher (i.e. the ξ is higher), the *markup-effect* as well as the multiplier is also higher.

The (1,3) element of Figure 2 shows the essence of inflation targeting: the rise in marginal cost leads to higher inflation which, due to the Taylor rule, is counterbalanced by a rise in nominal interest. A high coefficient on inflation (ϕ_1) in the Taylor rule implies more strict feedback to inflation by implying a higher real rate that decrease consumption and

accordingly, the multiplier.

The (2,1) element of Figure 2 shows how strictly monetary policy responds to an increase in output gap. If ϕ_2 is higher than the interest rate response to a change in output gap is higher based on the Taylor rule. The higher interest rate induce people to consume less today.

The (2,2) element of Figure 2 shows that as ρ_R increases the spending multiplier rises. When ρ_R is high the monetary policy responds less rapidly to a rise in spending (that materialises in the form of higher output gap and inflation) by an increase in nominal interest rate and the multiplier can stay to be high for a longer time. This practice is often noted as the traditional view of accomodative monetary policy (Christiano *et al.*, 2009).

The (2,3) element of Figure 2 displays that the multiplier is decreasing function of persistence parameter, ρ_G of the government spending AR (1) process. The parameter indicates that the present value of taxes connected to a rise in government spending is higher when ρ_G is higher. That is, the corresponding negative wealth effect of government spending is higher if ρ_G is higher and the multiplier is lower.



Figure 2: Sensitivity of government spending multiplier, separable preferences

Payroll tax cut

When there is a fall in labour tax people are willing to work more as they got more money after each hour worked. More hours worked induce an outward shift in labour supply that decreases real wages which in turn makes firms able to supply more goods at a lower price. A fall in prices leads to a deflationary spiral which, due to the Taylor rule, leads to a decline in nominal interest rate to curb deflation. However, in case of labour tax cut there is no such autonomous increase in spending as it is in case of government spending. Clearly, in case of payroll tax cut, it is only the AS curve that shifts out creating a decent rise in output and a fall in prices as the *AD curve remains still* due to the lack of an element that would directly induce spending (Eggertsson, 2009).

On the (1,1) element of Figure 3 we can see that the multiplier is decreasing in σ . With separable preferences consumption and hours are independent. With a decrease in labour tax, firms are willing to employ more people, increase production and finally output. However, we know that Ricardian equivalence holds in our model, and the household focuses on the total discounted value of his/her income stream. That is, the consumer knows that a tax reduction today is equivalent to a tax increase in the future. And this is why the consumer reduces leisure and consumption. A higher σ means higher sensitivity to a movement in consumption and amplifies the reduction in consumption as a result of the negative wealth effect and works against the increase in output (and the multiplier).

On the (1,2) element of Figure 3 we can see that the multiplier is decreasing in the Calvo parameter (ξ). Higher price stickiness implies lower multiplier. A lower wage tax implies lower marginal cost of production. As prices are sticky, the price over marginal cost rise that implies a higher markup. As we have monopolistic competition in the model, a rise in the markup counterbalance the increase in labour demand implied by smaller labour cost¹¹. The higher is price stickiness the stronger is the *markup-effect*. Thus, the higher is price stickiness the larger is the fall in output due to a *rise in markup* which works against the *cost-induced* expansion in output and implies a smaller multiplier. All the other elements of Figure 3 with the only exception of element (2,1) show exactly the opposite of Figure 2.

The (1,3) element of Figure 3 shows that the multiplier in case of a labour tax cut is increasing with inflation coefficient (ϕ_1) in the Taylor rule. The deflationary spiral, induced by the tax cut is mitigated more with a larger ϕ_1 in the Taylor rule. That is, with deflation a larger ϕ_1 means larger cut in the nominal rate that stimulates demand and act against deflation. As we have no capital in the model the lower interest rate means that agents do not delay their purchases to future dates and the latter contributes to a higher multiplier.

 $^{^{11}}$ This counterbalancing effect has to be true, otherwise the increase in labour demand would result in higher real wages that would create an incentive for workers to rise their consumption.

The (2,1) element of Figure 3 is in line with the result for government spending. Here the expansionary effect of a tax cut on output is mitigated if we have a higher coefficient (ϕ_2) on output gap in the Taylor rule because higher ϕ_2 means a bigger increase in interest rate response to output and acts contractionary.

The (2,2) element of Figure 3 shows that ρ_R works exactly the opposite way as it is with government spending. In case of a payroll tax cut, there is deflation. Thus, the quicker is the response of monetary policy to deflation (i.e. it is less accomodative operating with a lower value of ρ_R) by a decrease in interest rate, the higher will be the multiplier. In case of a labour tax cut — contrary to a rise of government spending — less accomodative monetary policy (i.e. a lower value of ρ_R) implies higher multiplier.

The (2,3) element of Figure 3 shows the higher is the persistence of payroll tax shock process the higher is the payroll tax multiplier. If $\rho_{\tau W}$ is higher the aggregate supply effect is stronger and the multiplier is higher because the stronger negative wealth effect that makes people substitute consumption more for working hours which increases output and the multiplier. When $\sigma > 1$ we know that the wealth effect on labour supply dominates.



Figure 3: Sensitivity of the payroll tax multiplier, separable preferences

Sales tax

The sales tax cut works very similar to government spending as it directly stimulates private spending but to a less extent because the sales tax multiplier is reduced by a steadystate tax term $\left(\frac{\tau^S}{1+\tau^S}\right)$ that is smaller than one. The only difference between government spending and sales tax multiplier except for their size can be captured by the element (1,1) of the corresponding graphs: the sales tax multiplier (on Figure 4) is decreasing in σ (while it is the opposite for dY/dG shown on Figure 2). To explain why this is the case, remember that there is a negative wealth effect associated with an increase in spending. In the Euler equation G_t appears directly and indirectly (by substituting the budget contraint for C_t , see equation (25) in Appendix A). After substituting the budget constraint for C_t the coefficient that originally multiplied C_t is multiplying G_t now. When σ is higher the effect of a rise in G_t is stronger. However, sales tax, τ_t^S stimulate spending only directly but not indirectly because it is not to be found in the budget constraint. Thus, a higher σ (multiplying C_t that falls due to the wealth effect) in the AD equation implies a stronger (i.e. more negative) wealth effect (i.e. magnifies the fall in consumption) and results in a lower multiplier.



Figure 4: Sensitivity of the sales tax multiplier, separable preferences

3 Non-separable preferences

The household maximises the following utility that is non-separable in consumption (C_t) and leisure $(1 - N_t)$:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma} (1 - N_t)^{1 - \gamma} \right]^{1 - \sigma} - 1}{1 - \sigma} + v(G_t^N) \right]$$

with respect to its budget constraint

$$(1+R_t)B_t + \int_0^1 profit_t(i)di + (1-\tau_t^W)P_tW_tN_t = T_t + B_{t+1} + (1+\tau_t^S)P_tC_t$$

3.1 Equilibrium

The intratemporal condition (in linearised form)

$$\widehat{C}_t + \frac{N}{1-N}\widehat{N}_t = \widehat{W}_t - \frac{\tau^W}{1-\tau^W}\widehat{\tau}_t^W - \frac{\tau^S}{1+\tau^S}\widehat{\tau}_t^S$$
(9)

The Euler equation (in linearised form):

$$E_t \left\{ \beta \left(R_{t+1} - R \right) + \left[(1 - \sigma)\gamma - 1 \right] \widehat{C}_{t+1} - (1 - \gamma)(1 - \sigma) \frac{N}{1 - N} \widehat{N}_{t+1} - \frac{\tau^S}{1 + \tau^S} \widehat{\tau}_{t+1}^S \right\} \\ = \left[(1 - \sigma)\gamma - 1 \right] \widehat{C}_t - (1 - \gamma)(1 - \sigma) \frac{N}{1 - N} \widehat{N}_t - \frac{\tau^S}{1 + \tau^S} \widehat{\tau}_t^S + E_t \pi_{t+1}$$
(10)

The New Keynesian Phillips curve (in linearised form)

In case of non-separable preferences the \widehat{MC}_t term that is included in the NKPC is different from the one for separable case. To show this, first, recall intratemporal condition:

$$\widehat{C}_t + \frac{N}{(1-N)}\widehat{Y}_t = \widehat{W}_t - \frac{\tau^W}{1-\tau^W}\widehat{\tau}_t^W - \frac{\tau^S}{1+\tau^S}\widehat{\tau}_t^S,$$

and the budget constraint in linear form:

$$\hat{Y}_t = (1-g)\hat{C}_t + g\hat{G}_t$$
 (11)

with $g \equiv G/Y$. In the next, we can express equation (11) for consumption and substitute back into the intratemporal condition to obtain:

$$\left[\frac{1}{1-g} + \frac{N}{1-N}\right]\widehat{Y}_t - \frac{g}{1-g}\widehat{G}_t = \widehat{W}_t - \frac{\tau^W}{1-\tau^W}\widehat{\tau}_t^W - \frac{\tau^S}{1+\tau^S}\widehat{\tau}_t^S.$$

If the production function is linear the real marginal cost and real wage coincides:

$$\widehat{MC}_t = \widehat{W}_t$$

The general form of NKPC from the intermediary firm's price-setting problem is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{MC}_t$$

which after substituting for the model-specific \widehat{MC}_t can be written as:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \widehat{Y}_t - \frac{g}{1-g} \widehat{G}_t + \frac{\tau^W}{1-\tau^W} \widehat{\tau}_t^W + \frac{\tau^S}{1+\tau^S} \widehat{\tau}_t^S \right]$$

3.2 The role of non-separable preferences

In the New Keynesian model used here we have infinitely-lived agents, complete asset markets, monopolistic competition, lump-sum taxation and sticky prices. One of our major finding is that the size of the government spending multiplier depends largely on the preference specification of the representative household. In order to generate a government spending multiplier that is larger than one we have to assume complementarity between consumption and hours worked, that is, non-separable preferences in consumption and leisure has to be used.

In the New Keynesian model with separable preferences, a rise in government spending induce a negative wealth effect as the consumer expects a rise in future lump-sum taxes and, as a consequence, he/she consumes less and works more. The negative wealth effect implies an outward shift in the labour supply curve leading to higher hours worked and *lower* real wages while the labour demand curve remains unchanged. The negative Hicksian wealth effect induced by government spending leads to a rise in output and a fall in consumption and real wages.

However, there is little empirical evidence on the strength of this negative wealth effect (see, e.g., Gali *et al.*, 2007). Monacelli and Perotti (2008) revisits the so-called Greenwood-Hercowitz-Huffmann (GHH) preferences which implies a very low Hicksian wealth effect and concludes by using non-separable preferences of GHH type that we can generate a case when the labour supply curve does not shift, but stays still, in reaction to a rise in government spending (that is, the wealth effect is zero).

If there was a shift in the labour supply, the real wage would decrease and the consumer would substitute consumption for hours worked (negative substitution effect). Thus, to generate a rise in consumption we need the real wage to increase that can be only achieved by a positive outward shift in the labour demand curve. To make this happen we have to introduce sticky prices into the model. Under the presence of sticky prices, not all the firms can change its prices when the demand for their products, due to an increase in government purchases, increase. Thus, those firms who cannot change price will satisfy new demand by an increase in production which can be achieved by hiring extra workers. When hiring extra workers, labour demand shifts out and the rising real wage as a necessary condition for rising consumption after a spending spree is satisfied (Monacelli and Perotti, 2008).

3.3 Multipliers for Non-separable Preferences

It remains true also in case of non-separable preferences that we can solve for the multipliers (see necessary assumptions at the separable case) analytically by the methods of undetermined coefficients. Figure 5 shows the response of variables (and the multiplier) to a temporary 1% spending shock under non-separable preferences. We can observe two things: (1) the multiplier is slightly larger than one (1.05) on impact and (2) dC/dG > 0.

Figure 5: The effects of a 1% temporary government spending shock in the model with non-separable preferences. Note that dC/dG > 0.



3.3.1 Government spending multiplier

$$\frac{dY_t}{dG_t} = \frac{(\rho - \phi_1)\kappa - [\gamma(\sigma - 1) + 1](1 - \rho)(1 - \beta\rho)}{(1 - \beta\rho)[\rho - 1 - (1 - g)\phi_2] + (1 - g)(\rho - \phi_1)\kappa\left(\frac{1}{1 - g} + \frac{N}{1 - N}\right)}$$

As previously argued in detail, the government spending multiplier is generally larger than the one corresponding to separable preferences (see Figure 5). However, it is important to note that a multiplier that is larger than one can be obtained by assuming a high value for average price stickiness, that is a value of at least $\xi = 0.8$ (firms that cannot change price holding their last price for longer than a year) or larger which means that κ is around at most 0.03.

3.3.2 Labour tax cut

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W} = \frac{\left(\phi_1 - \rho\right)\frac{\tau^W}{1 - \tau^W}\frac{\kappa}{1 - \beta\rho}}{\left[\left(1 - \sigma\right)\gamma\tau^W - 1\right]\left(1 - \rho\right) - \phi_2 - \left(\phi_1 - \rho\right)\frac{\kappa}{1 - \beta\rho}\left(1 + \frac{N}{1 - N}\right)}$$

Note again that a labour tax cut has only indirect effect on output (that is modifying only the economy's AS curve leaving the AD unaffected) as it modifies the household's labour supply decision which is given implicitly by the intratemporal condition. A labour tax cut has smaller effect in case of non-separable preferences because the output coefficient in the Euler equation are multiplied by the steady-state of payroll tax, τ^W which latter is smaller than one. In case of separable preferences there is no such "discount term" on output (see more on this term at the sensitivity analysis). Based on this fact, the labour tax multiplier is extremely small (roughly 0.1) in case of non-separable preferences.

3.3.3 Sales tax cut

$$\frac{d\hat{Y}_t}{-d\hat{\tau}_t^S} = \frac{\tau^S}{1+\tau^S} \left[\frac{(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho} - (\rho - 1)}{\frac{\gamma(1-\sigma)\tau^S - (1+\tau^S)}{1+\tau^S}(1-\rho) - \phi_2 - (\phi_1 - \rho)\frac{\kappa}{1-\beta\rho}\frac{1}{1-N}} \right]$$

We have argued in the separable case that the sales tax multiplier is lower than the one of government spending because the *direct effect* of sales tax cut on output (that is increasing aggregate spending) is generally *lower* than the one of government spending. In case of non-separable preferences the sales tax multiplier is even lower than the one corresponding to separable preferences because the direct effect is even weaker. That is, output is "discounted" even more due to a term multiplying output that is lower than one. See for more details the sensitivity analysis. Next we study the sensitivity of multipliers to various parameter values.

3.4 Sensitivity analysis of the fiscal multipliers (non-separable case)

Government spending

As discussed previously we need sufficient price stickiness and non-separable preferences to generate spending multiplier that is larger than one. Non-separable preferences imply complementarity between consumption and leisure in the model. The (1,1) element of Figure 6 shows how government spending changes with σ . The intuition provided at element (1,1) of Figure 2 remains applicable here, however, there is one more effect we have to consider now: the higher σ implies higher complementarity between consumption and leisure and a correspondingly higher multiplier. When there is an increase in demand, then not only employment but the marginal utility of consumption is also higher. When the increase in marginal utility is high enough, then there is scope for consumption to rise in response to an increase in government spending. Evidence on rising consumption is also provided on Figure 5 concerning a temporary shock to government spending (dC/dG > 0). The interpretations of the other elements of Figure 6 is very similar to the ones on Figure 2. For example, the (1,2) element of Figure 6 shows, as previously argued, that the multiplier can be above one for a Calvo parameter, ξ , that is equal or larger than 0.8.





Payroll tax cut

As we emphasized at the separable case the wage tax can stimulate the economy only indirectly as it affects the houshold's labour supply decision. There is no such direct "autonomous" spending effect that is present for government spending and sales tax. Consequently, the payroll tax multiplier has to be smaller than the government and sales tax ones. The behaviour of the labour tax multiplier in case of non-separable preferences for six different parameters (on Figure 7) is very similar to the corresponding one with separable preferences (see Figure 3). Furthermore, the payroll tax multiplier for non-separable preferences is lower than the one in the separable case because the possible increase in output after a cut in labour tax is muted not only by the steady-state tax term ($\tau^W < 1$) but also by another term that contains steady-state hours.



Figure 7: Sensitivity of payroll tax multiplier, non-separable preferences

Sales tax cut

In case of separable preferences government spending and sales tax cut behaved quite similarly to the parameter values (except for parameter σ^{12}) as they both stimulated spending in some way. The arguments asserted in the separable case remains true here as well. That is, the sales tax multiplier (shown on Figure 8) is less than one and less than the one for government spending. In other words, due to the non-separability between consumption and leisure, the consumption term (or, using the market clearing the latter is equivalent to output, Y_t and hours, N_t) is multiplied by a term containing the steady state of hours, $\frac{N}{1-N}$, which, depends — through intratemporal condition — on the steady state level of sales tax (which is less than one) and the latter dampens the increase in output after a sales tax cut even more than it is in the separable case. Thus, steady-state level of taxes play a key role in determining the value of the multipliers.



Figure 8: Sensitivity of sales tax multiplier, non-separable preferences

¹²The negative connection between σ and the multiplier is explained in detail for separable preferences and remains valid here as well.

4 When zero bound on interest rate binds

In accordance with Christiano *et al.* (2009) we assume that the zero bound on nominal interest rate binds due to an exogenous increase in the discount rate (people's propensity toward savings increases). To be able to model zero bound we modify the discount factor in the household's problem to become time dependent and is given by the cumulative product of interest rates. Thus, following the notations of Christiano *et al.* (2009), the household maximises its utility which is non-separable in consumption and leisure:

$$U = E_0 \sum_{t=0}^{\infty} d_t \left[\frac{\left[C_t^{\gamma} (1 - N_t)^{1 - \gamma} \right]^{1 - \sigma} - 1}{1 - \sigma} + v(G_t^N) \right]$$

with respect to its budget constraint:

$$(1+R_t)B_t + \int_0^1 profit_t(i)di + (1-\tau_t^W) \int_0^1 P_t W_t(i)N_t(i)di = T_t + B_{t+1} + (1+\tau_t^S)P_t C_t$$

where the discount factor, d_t is given by $(r_{t+1} \text{ denotes the real rate of interest at time } t$ that will be actual in t + 1):

$$d_t = \begin{cases} \frac{1}{1+r_1} \frac{1}{1+r_2} \dots \frac{1}{1+r_t}, t \ge 1\\ 1, t = 0 \end{cases}$$

The time-t discount factor can be characterised by two values: r and r_l where r > 0 and $r_l < 0$. The steady-state value of r_{t+1} is denoted as r. Also, the following holds in steady-state:

$$\beta(1+r) = 1$$

Initially the economy is in the steady state. Then, in the first period $r_1 = r_l$. Thereafter, r_t follows the process described below by the second row of the following matrix:

where each of the rows sums up to one and therefore the property of a transition matrix is satisfied¹³. If we are already in the zero bound then the discount factor remain high with probability p, i.e.

$$\Pr(r_{t+1} = r_l | r_t = r_l) = p,$$

¹³Note that this is practically a two state Markov process: there is a steady-state and another state in which the real interest is negative (or can be called deflationary state).

or returns to its steady-state value with probability 1 - p, i.e.

$$\Pr(r_{t+1} = r | r_t = r_l) = 1 - p.$$

The first row of the matrix shows two further cases which are not of our interest:

$$\Pr(r_{t+1} = r | r_t = r) = 1$$
 and $\Pr(r_{t+1} = r_l | r_t = r) = 0.$

Moreover, we assume that the shock to the discount factor is high enough to make the zero bound binding. Following Christiano *et al.* (2009), we assume that the following are true in the zero bound state:

$$\widehat{G}_t = 0, \ E_t(\widehat{G}_{t+1}) = p\widehat{G}_l, \ E_t(\pi_{t+1}) = p\pi_l, \ E_t(\widehat{Y}_{t+1}) = p\widehat{Y}_l$$

that is, when we are out of the zero bound, $\hat{G}_t = 0$ and otherwise $\hat{G}_l > 0$. If we are initially in the zero bound we remain there with probability p, i.e. $p\hat{G}_l$ or exit with 1-p, i.e. $(1-p)\hat{G}_l$.

4.1 Solution and calibration of the model

The equilibrium is characterised by two values for each variable: one value when the zero bound binds (denoted by lower case l) and one when it does not. That is, when zero bound binds inflation and output is denoted by π_l and \hat{Y}_l , respectively.

In case of the time-dependent discount factor, the linearised Euler equation in (10) modifies to:

$$E_{t}\left\{\beta\left(R_{t+1}-r_{t+1}\right)+\left[(1-\sigma)\gamma-1\right]\widehat{C}_{t+1}-(1-\gamma)(1-\sigma)\frac{N}{1-N}\widehat{N}_{t+1}-\frac{\tau^{S}}{1+\tau^{S}}\widehat{\tau}_{t+1}^{S}\right\}$$

= $\left[(1-\sigma)\gamma-1\right]\widehat{C}_{t}-(1-\gamma)(1-\sigma)\frac{N}{1-N}\widehat{N}_{t}-\frac{\tau^{S}}{1+\tau^{S}}\widehat{\tau}_{t}^{S}+E_{t}\widehat{\pi}_{t+1}$ (12)

where note that the only change is that R drops out and a new term, the real interest rate, r_{t+1} appears on the LHS. When the zero bound on nominal interest rate binds, $R_{t+1} = 0$. The derivation of the sales tax multiplier (the derivation of the labour tax multiplier is very similar and not presented here) when the zero bound binds can be found in Appendix B and are only provided for the case of non-separable preferences. In Christiano *et al.* (2009) we can find the steps for derivation of the government spending multiplier.

Parametrisation of the model is the same as in Table 1 for non-separable preferences with the only distinction that the persistence of the government spending process, ρ_G is equivalent to p in the model when the zero bound binds. This is true because the AR(1) shock process is equivalent to a two-state Markov process. The interpretation, of course, changes somewhat. The higher is the value of p the longer we are in the zero bound state.

Government spending multiplier

Similarly to the case of positive nominal interest rate we assume here that $\rho_R = 0$ to be able to derive multipliers analytically.

The output and inflation in the zero bound is given by:

$$\widehat{Y}_{l} = \frac{(1-g)(1-\beta p)\beta}{\Delta}r_{l} + \frac{\{[(\sigma-1)\gamma+1](1-p)(1-\beta p) - p\kappa\}}{\Delta}g\widehat{G}_{l}$$
(13)

and

$$\pi_{l} = \frac{(1-g)\beta\kappa\left[\left(\frac{1}{1-g} + \frac{N}{(1-N)}\right)\right]}{\Delta}r_{l} + \frac{\kappa g(1-p)\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)(\sigma-1)\gamma - \frac{N}{1-N}\right]}{\Delta}\widehat{G}_{l} \qquad (14)$$

where

$$\Delta \equiv (1 - \beta p)(1 - p) - \kappa p \left[1 + \frac{N}{1 - N}(1 - g) \right]$$

and make sure that $\Delta > 0$ as $(1 - g)\beta\kappa \left[\left(\frac{1}{1-g} + \frac{N}{(1-N)}\right)\right]r_l < 0$ is for sure as the r_l is negative when there is an increase in the discount factor (household decides to save more). The coefficient on the r_l in both of the equations above cannot be positive because both output and inflation would be larger than their steady-state values and this would require an increase in the nominal interest rate due to the Taylor rule and zero bound on the nominal interest rate would not bind. Therefore, $\Delta > 0$ is necessary for the zero bound to bind. In case of sales tax and labour tax the system collapses into two equations for Y_l , π_l as well similarly to equation (13) and (14)— when the zero bound binds.

Why does the zero bound bind in equilibrium?

Christiano *et al.* (2009) has an appealing interpretation for market clearing when zero bound binds. In this simple model without investment the savings has to be zero in equilibrium. A possible way to curb peoples' desire to save more is through a reduction in the real interest rate. According to the Fisher rule we know there are two possible ways to decrease real interest rate: a decrease in the nominal rate or an increase in expected inflation. However, we know that the decrease in the nominal rate is limited by its natural zero lower bound. We also know that the inflation cannot accelerate when there is a discount factor shock (if we look at equation (14), and, at the same time, assuming that G_l does not change, we can see there is deflation due to $r_l < 0$). Otherwise, positive inflation in our sticky prices model is accompanied by increasing output that can induce people to save more. Thus, the reduction in real interest rate may not be enough to deter people from further saving. If the discount rate shock is high enough the real interest rate cannot fall by enough to reduce savings because the zero bound becomes binding prior to the point that would re-establish equilibrium. Therefore, the only possible way for savings to become zero in equilibrium is a **large** transitory fall in output and an accompanying deflation as it can be seen on the element (1,2) and (1,3) of Figure 9, respectively.

The government spending multiplier when the zero bound binds can be expressed from equation (13) as:

$$\frac{d\hat{Y}^l}{d\hat{G}^l} = \frac{(1-\beta p)(1-p)[\gamma(\sigma-1)+1] - \kappa p}{\Delta}$$

The zero bound in case of government spending binds — in accordance with the $\Delta > 0$ requirement — for $0.02 \le \kappa \le 0.036$ and $0.75 \le p \le 0.82$. This range of values of κ implies a Calvo parameter that is $\xi \ge 0.82$ (and this is true for each of the multipliers considered here).

Why is the spending multiplier is so high when the nominal rate is zero?

When there is an increase in spending the marginal cost, the inflation and the output rises and the markup falls. If the zero bound binds, the nominal interest is zero and the Taylor rule is inact. Because of the zero nominal rate, the rise in inflation will not coincide with an increase in the nominal rate (which in normal circumstances would react to inflation by larger than one due to the coefficient on inflation in the Taylor rule) and therefore lead to a fall in the real interest rate that encourages people to consume more today (note that we have no investment channel in this model). Higher consumption implies higher output, higher inflation and even lower real rate that again leads to a rise in output and the process replicates. The result is a large multiplier. The (1,1) element of 9 shows the government spending multiplier (where \Box indicates the benchmark value based on the parameter configuration in Table 1). As κ rises, we have more flexible prices (i.e. the Calvo parameter, ξ , is lower) and the value of the multiplier rises. The (1,2) and (1,3) elements of Figure 9 show the value inflation and output, respectively for zero nominal interest rate in the absence of a change in government spending. It can be inferred that the more flexible prices are (i.e. the higher is κ) the larger transitory fall in output (and a corresponding deflation) is needed to restore savings to zero in equilibrium. The second row of Figure 9 shows the longer the economy is in the zero bound state (i.e. a higher is p), the higher is the value of the multiplier and the bigger is the deflation and contraction in the economy to restore equilibrium level of savings.

Figure 9: Sensitivity of government spending multiplier, inflation and output to parameters κ and p when the zero bound binds



4.2 The case of negative labour tax multiplier

A cut in labour tax makes the AS curve shift to the right as one additional unit of hours worked incurs less taxes that creates the incentive for people to work more (i.e. providing more labour). As Eggertsson (2009) argues the outward shift in labour supply reduce real wages, firms are willing to supply more goods at a lower price leading to deflationary pressures. However, when the zero bound becomes binding the negative slope of AD in the *output-inflation* space changes to positive. This seems to be counterintuitive but let us discuss what happens. After solving AS and AD curves together we can express for \hat{Y}_l and π_l respectively by¹⁴:

 $^{^{14}}$ Note, again, that we have a two-state Markov process: the steady state and the state in which the real interest rate is negative.

$$\widehat{Y}_{l} = \frac{(1-N)(1-\beta p)p}{(1-p)\left[1-(1-\sigma)\gamma\tau^{W}\right](1-N)(1-\beta p)-\kappa p}\beta r_{l} \\
+ \frac{p\kappa(1-N)}{(1-p)\left[1-(1-\sigma)\gamma\tau^{W}\right](1-N)(1-\beta p)-\kappa p}\frac{\tau^{W}}{1-\tau^{W}}\widehat{\tau}_{l}^{W}$$
(15)

$$\pi_{l} = \frac{\kappa p}{(1-p) \left[1-(1-\sigma)\gamma\tau^{W}\right](1-N)(1-\beta p)-\kappa p}\beta r_{l} + \frac{(1-p) \left[1-(1-\sigma)\gamma\tau^{W}\right](1-N)(1-\beta p)}{(1-p) \left[1-(1-\sigma)\gamma\tau^{W}\right](1-N)(1-\beta p)-\kappa p} \frac{\tau^{W}}{1-\tau^{W}}\widehat{\tau}_{l}^{W}$$
(16)

where $(1-p)\left[1-(1-\sigma)\gamma\tau^W\right](1-N)(1-\beta p)-\kappa p>0$ has to be satisfied for the zero bound to bind. Again, the value of κ can be at most 0.0365 (implying a Calvo parameter (ξ) of 0.82). Technically speaking, we can infer from the equations we got for \hat{Y}_l and π_l that a cut in payroll tax in the deflationary state l leads to a fall both in output and inflation. Now let us discuss the intuition behind this. To gain insight we start with the case of positive nominal rate.

In the absence of zero nominal interest, the reaction of the central bank to deflation is a cut in the nominal interest rate by more than one-to-one with inflation (this is the famous $\phi_{\pi} > 1$ requirement in the Taylor rule). If the inflation speeds up then the answer of the central bank is an increase in the nominal rate by more than one-to-one with inflation. Thus, in case of deflationary pressures the real interest rate will decline as the central bank will cut nominal interest rate by more than one in proportion to inflation.

However, this is no longer true when the zero bound binds and the central bank cannot cut interest rates to mitigate deflationary shock. As the zero bound becomes binding the deflationary spiral will induce a rise in the real rate which, as a consequence, lead to a fall in output. That is, the downward-sloping AD curve in the inflation-output space becomes upward sloping when the zero bound becomes binding. Accordingly, we can say that a simple New Keynesian model with Calvo pricing implies that labour tax is contractionary in an environment of zero policy rate (Eggertsson, 2009).

The payroll tax multiplier for non-separable preferences and under the assumption that the nominal rate is zero is given by:

$$\frac{d\hat{Y}^{l}}{-d\hat{\tau}_{l}^{W}} = \frac{p\kappa(1-N)}{(1-p)\left[1-(1-\sigma)\gamma\tau^{W}\right](1-N)(1-\beta p) - \kappa p} \frac{\tau^{W}}{1-\tau^{W}}$$

The payroll tax multiplier is depicted on the element (1,1) of Figure 10 for a range of κ . The elements (1,2) and (1,3) of Figure 10 show the deflation and contraction in output associated with the zero bound state is increasing in κ in the absence of a change in payroll tax. As we can see on (2,1) element of Figure 10 the longer the economy is in the zero bound state (i.e. the higher is p) the smaller is the payroll tax multiplier (i.e. it is more negative) and the bigger is the associated deflation and contraction in output needed to decrease savings to zero level, shown, respectively on (2,2) and (2,3) elements of Figure 10.





Sales tax cut

The sales tax multiplier is given by taking the total derivative of equation (32) in Appendix B with respect to $\hat{\tau}_l^S$:

$$\frac{d\hat{Y}^{l}}{-d\hat{\tau}_{l}^{S}} = \frac{(1-N)\tau^{S} \left\{p\kappa + (1-\beta p)(p-1)\right\}}{(p-1)\left[(1-\sigma)\gamma\tau^{S} - (1+\tau^{S})\right](1-\beta p)(1-N) - (1+\tau^{S})\kappa p}$$

The requirement for a binding zero bound is:

$$(p-1)\left[(1-\sigma)\gamma\tau^{S} - (1+\tau^{S})\right](1-\beta p)(1-N) - (1+\tau^{S})\kappa p > 0,$$

which is — similarly to government spending and labour tax — true for $0.02 \le \kappa \le 0.036$ and $0.75 \le p \le 0.82$. The value of the sales tax cut multiplier in case of zero nominal interest rate (for the benchmark values it is 0.16), shown on the (1,1) and (2,1) elements of Figure 11 for benchmark κ and p, respectively, is smaller than the one of the non-separable case when the zero bound does not bind (see Figure 8). The range of κ stands for a Calvo parameter that is $\xi \ge 0.82$, implying a price stickiness of a year or longer. As κ rises (which implies a lower ξ , i.e. a lower level of price stickiness) the multiplier rises. The more flexible prices (i.e. the higher is κ) are the larger transitory fall in prices and output is needed to restore savings to zero level as shown on elements (1,2) and (1,3) of Figure 11. As can be seen in the second row of Figure 11, the longer is the economy in the zero bound state (i.e. a higher p) the higher is value of the sales tax cut multiplier and the higher is the deflation and contraction in the economy.

Figure 11: Sensitivity of sales tax multiplier, inflation and output to parameters κ and p when zero bound binds



4.3 Summary of the models without capital

Before extending the baseline New Keynesian model with capital, we summarise multipliers of the models discussed above. For the purpose of comparison I collected the 9 multipliers in Table 2. As it can be seen the government spending multiplier is more than three times higher when the zero bound binds (R = 0) compared to the case of 'normal' times when nominal interest rate is positive ($R \neq 0$). Also interestingly, we managed to reproduce Eggertsson's (2009) most interesting finding of the negative payroll tax cut multiplier not only for separable but also for non-separable preferences as well for a zero nominal rate. Consequently, the fiscal policy which aims to reduce tax on wages does not stimulate (but even depress) the economy when the federal funds rate is zero. The intuitive proof for this result can be found in the main text.

Eggertsson (2009) finds that the sales tax multiplier at the zero nominal rate is as high as the spending multiplier in case of separable preferences. However, this is not true any more if we have non-separable preferences as the multiplier reduces from 0.22 to 0.16 as the zero bound becomes binding. Accordingly, the policy recommendation of Eggertsson (2009) for a sales-tax cut is debateable because it fails robustness for preferences.

Multipliers	Separable	Non-separable		
	$R \neq 0$	$R \neq 0$	R = 0	
Gov. spending, $\frac{d\hat{Y}_t}{d\hat{G}_t}$	0.98	1.05	3.7	
Payroll tax, $\frac{d\hat{Y}_t}{-d\hat{\tau}_t^W}$	0.19	0.09	-0.71	
Sales tax, $\frac{d\hat{Y}_t}{-d\hat{\tau}_t^S}$	0.42	0.22	0.16	

 Table 2: Summary of Multipliers

Remarks to Table 2: when nominal interest rate is not zero, $R \neq 0$, the impact multipliers are calculated numerically, and for the case when the zero bound binds it is simply given by the coefficient on the shock term $(G_l, \tau_l^W \text{ or } \tau_l^S)$ in the equation we derived for equilibrium Y_l .

It can also be important to re-emphasize that the Christiano *et al.* (2009) model for studying multipliers under the zero nominal rate works for very high values of the Calvo price stickiness parameter implying that the condition for a binding zero lower bound can be satisfied by assuming very long time of price inertia (that is, more than a year) for those firms who cannot choose their prices optimally. That is, the condition of a binding zero bound is satisfied only for a low range of κ parameters which is the parameter multiplying the marginal cost in the NKPC (that is, the Calvo parameter, ξ , has to be at least 0.82 which is more than one-year price stickiness).

5 Adding capital to the New Keynesian model

In this section we assume that both households and firms can also use some of their resources to invest into capital. The government is still supposed to make purchases that are financed by lump-sum taxes. Moreover, we also include capital adjustment costs into the model to be able to match the observed slugishness of real variables to shocks. Accordingly, both household's and intermediate goods firms' problems change after the inclusion of capital. Again, following the notations of Christiano *et al.* (2009), we start with the household's optimisation problem.

5.1 The household's problem

The household maximises the following non-separable utility in consumption (C_t) and leisure $(1 - N_t)$:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma} (1 - N_t)^{1 - \gamma} \right]^{1 - \sigma} - 1}{1 - \sigma} + v(G_t^N) \right]$$
(17)

with respect to its budget constraint

=

$$(1+R_t)B_t + \int_0^1 profit_t(i)di + (1-\tau_t^W) \int_0^1 P_t W_t N_t(i)di + \int_0^1 R_t^k K_t(i)di$$

= $T_t + B_{t+1} + (1+\tau_t^S)P_t C_t + P_t I_t$ (18)

where R_t^k denotes the real rental rate of capital which serves as an income for the household and I_t denotes investment as a further way of spending.

There is an equation that describes the accumulation of capital. According to this equation, investment is the change in capital stock from time t to time t + 1:

$$K_{t+1} = I_t + (1-\delta)K_t - \frac{\sigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$
(19)

where the last term on the RHS is the capital adjustment cost that is now specified as quadratic. The parameter $\sigma_I > 0$ governs the magnitude of adjustment costs to capital accumulation. That is, the household's problem is to maximise its utility in equation (17) subject to its budget constraint in equation (18) and the capital accumulation equation (19).

5.2 The final and intermediary goods' producers problem

The final good producers' problem remains the same while the intermediary firms' problem can be written as follows. Intermediaries set their prices in Calvo manner as it is in model in section one. The i^{th} intermediary maximises its discounted profit:

$$E_t \sum_{T=0}^{\infty} \beta^{t+T} v_{t+T} \left[P_{t+T}(i) Y_{t+T}(i) - (1-\nu) \left[P_{t+T} W_{t+T} N_{t+T}(i) + P_{t+T} R_{t+T}^k K_{t+T}(i) \right] \right], \quad (20)$$

where $N_t(i)$ and $K_t(i)$ denotes the value of labour and capital used by i^{th} intermediary, respectively. As we can see from the above formulation the costs are made up of two parts: labour and capital rental costs, respectively. The output of the i^{th} is produced by:

$$Y_t(i) = [K_t(i)]^{\alpha} [N_t(i)]^{1-\alpha}.$$
(21)

Similarly to the model in the first section we assume that the monopolist markup in the steady-state is eliminated by a fiscal subsidy, that is, $\nu = 1/\varepsilon$. Also note that v_{t+T} corresponds to the Lagrange multiplier on the budget constraint in the household's optimisation problem. Accordingly, the i^{th} intermediary maximises the expression in equation 20 with respect to the production function in equation (21) and the demand function for $Y_t(i)$ in equation (1).

The conduct of monetary and fiscal policy is not affected by the inclusion of capital into the baseline model.

5.3 Equilibrium

The Intratemporal condition in equation (9) and the Euler equation $(10)^{15}$ are exactly the same in the model with capital and they are not listed again here. However, after taking the derivative of the households' problem with respect to I_t and K_{t+1} we get two more new equilibrium conditions.

Firstly, there is a connection between capital and investment that also involves Tobin's q i.e. the consumption value of an additional unit of capital¹⁶:

$$1 = q_t \left[1 - \sigma_I \left(\frac{I_t}{K_t} - \delta \right) \right], \tag{22}$$

which formula implies the mean reversion of I_t/K_t toward its steady-state value, δ . In Christiano *et al.* (2009) interpretation, the latter equation implies that an increase in investment by one unit raises K_{t+1} by $1 - \sigma_I \left(\frac{I_t}{K_t} - \delta\right)$ unit i.e., due to capital adjustment cost K_{t+1} rises by less than one unit.

¹⁵However, the NKPC should be written in recursive form instead of the loglinear one. And, of course, the real marginal cost changes after including capital into the model.

¹⁶This first order condition is obtained by taking the derivative of the Lagrangian associated with the houshold's problem with respect to I_t .

Secondly, there is another equilibrium condition describing the dynamics of Tobin-q that can be derived from the household's problem¹⁷:

$$\frac{\vartheta_t}{\beta\vartheta_{t+1}} = \frac{1}{q_t} \left\{ R_t^k + q_{t+1} \left[(1-\delta) + \frac{\sigma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 - \frac{\sigma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right] \right\}$$
(23)

where $\vartheta_t \equiv [C_t^{\gamma}(1-N_t)^{1-\gamma}]^{-\sigma} C_t^{\gamma-1}(1-N_t)^{1-\gamma}$. As there is no money in the model we measure capital in consumption units as well. One unit of consumption good worth $1/q_t$ units of installed capital. In order to understand the intuition behind equation (23) we have to observe that the LHS equals the real interest rate based on the Euler equation in $(10)^{18}$. Thus, the LHS equals real return on one-period bonds. The real return of installed capital (the RHS of equation 23) is composed of the following terms: the first term on the RHS of equation 23 is the marginal product of capital, the second term is the undepreciated capital in consumption units, $(1-\delta)q_{t+1}$ and the third and fourth terms capture the reduction in adjustment costs as the value of installed capital increase from time t to time t + 1. As a result, equation (23) can be interpreted as a no-arbitrage condition (Christiano *et al.*, 2009).

When we analyse the case of binding zero bound we assume that the monetary authority holds the interest rate at constant level. In order to be able to analyse the zero lower bound in Dynare we have to compute the equations in their original form (that is, not in log-linear form)¹⁹. However, the NKPC is a log-linear equilibrium condition. Alternatively, we can express NKPC in recursive form. For this purpose let us express the ratio of optimal price, P_t^* and the economy-wide price index, P_t recursively as:

$$\frac{P_t^*}{P_t} = \frac{M_t}{F_t}$$

where M_t and F_t are given, recursively, by:

$$M_t = \vartheta_t M C_t^* + \xi \beta E_t \left\{ \pi_{t+1}^{\theta} M_{t+1} \right\}$$

and

$$F_t = \vartheta_t + \xi \beta E_t \left\{ \pi_{t+1}^{\theta - 1} F_{t+1} \right\}.$$

¹⁷Note that, in equilibrium, the R_t^k equals the real marginal product of capital, i.e. $R_t^k = E_t \alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}$. ¹⁸The Euler is given by: $1 = E_t \left(\beta \frac{\vartheta_{t+1}}{\vartheta_t} \frac{1+R_{t+1}}{P_{t+1}/P_t} \right)$ with corresponding stochastic discount factor, ϑ_t .

¹⁹When equations are computed into the Dynare in log-linear form they are not allowed to contain constants. When zero bound on nominal interest rate binds, nominal interest rate in the Taylor rule is held at constant level and this is the reason why the log-linear setup in Dynare is not suitable for the analysis of the model when the nominal interest rate is zero.

Accordingly, the expression for the real marginal cost changes to:

$$MC_t^* = \Upsilon \left(R_t^k \right)^{\alpha} W_t^{1-\alpha},$$

with $\Upsilon \equiv \alpha^{-\alpha} (1 - \alpha)^{-(1 - \alpha)}$.

The capital and labour demand of intermediate goods firms are given, respectively, by

$$K_t = \alpha \frac{MC_t}{R_t^k} Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di \equiv \alpha \frac{MC_t}{R_t^k} Y_t \Delta_t,$$

and

$$N_t = (1-\alpha) \frac{MC_t}{W_t} Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di \equiv (1-\alpha) \frac{MC_t}{W_t} Y_t \Delta_t,$$

where we have $N_t \equiv \int N_t(i) di$ and $K_t \equiv \int K_t(i) di$ in both cases above with $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di$. Accordingly, we can simply write the integral of optimal price ratio Δ_t recursively as:

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di = (1-\xi) \left(\frac{P_t^*}{P_t}\right)^{-\theta} + \xi \pi_t^{-\theta} \Delta_{t-1}.$$

The economy's resource constraint after including investment, I_t , modifies to:

$$Y_t = C_t + I_t + G_t.$$

The government spending shock in the economy is the same as specified by equation (4).

5.4 Calibration

The parametrisation of the model with capital can be found in Table 3. After including capital, the resource constraint will contain investment as well. Accordingly, we have to account for the investment-output ratio (I/Y) that is taken from Uhlig (2009). The value of δ are standard in economics literature. The share of capital, α , in the production function is calibrated by using the value of INV/Y, δ and β . The parameter of the convex adjustment cost of capital, σ_I , can be found in Christiano *et al.* (2009)²⁰.

²⁰It can be interesting to note that if we use the linearised version of the equation that describes the dynamics of Tobin Q — which is not the case here — then the σ_I parameter is not needed as we do not need to specify the capital adjustment cost function with a certain functional form that contains σ_I .

Parameters	Separable $+$ capital	Non-separable $+$ capital
σ	2	2
φ	0.2	na
β	0.99	0.99
γ	na	0.29
ρ_G	0.8	0.8
δ	0.025	0.025
ϕ_1	1.5	1.5
ϕ_2	0	0
ρ_R	0	0
ξ	2/3	2/3
$G/Y(\equiv g)$	0.15	0.15
I/Y	0.25	0.25
C/Y	0.6	0.6
σ_I	na	17
σ_{ε^G}	0.01	0.01
Implied parameters		
κ	0.03	0.03
N	na	1/3

Table 3: Parametrisation of the New Keynesian Model with Capital

5.5 Experiments

As we already said in the Introduction the Bernstein and Romer (BR) (2009) numbers are based on a permanent fiscal stimulus. In the following we restrict the analysis to government spending multiplier only and study the effects of a permanent, anticipated increase in spending to be able to test the robustness of the numbers of BR (2009). As we have a forward looking model we have to specify explicitly the assumptions about firms' and households' expectations. Here the main assumption is that people expect a permanent increase in spending that is initially financed by issuing debt. Later, the debt is reduced by levying lump-sum taxes that do lower the after tax income earnings and thereby wealth (Cogan *et al.*, 2009). In the following we consider three types of multipliers under the assumption of fixing the nominal interest rate at a constant level for one and two years in line with recent empirical evidence on US.

When nominal interest rate is positive

In Table 4 we can see short and long run multipliers from the model with capital. The impact multipliers are calculated similarly to the ones in section 2 and 3. The idea of long-run multiplier is borrowed from Campolmi *et al.* (2009). It is calculated as the sum of discounted output changes divided at each time t by the sum of discounted spending changes. The impact multipliers of spending in the model with capital are generally lower than the corresponding ones in the model without capital because in the former one an increase in spending leads to a rise in real interest rate that crowds out private investment (please compare the findings in Table 2 and Table 4). As we can also observe that the impact multiplier is a bit lower for separable preferences that is in line with our findings of section 2. However, the long-run multipliers, which are generally lower than the impact ones, tell that the distinction coming from the assumption on preferences disappear on the long run.

Table 4:	Impact ar	ad Long-Run	Multipliers	of a tem	porary 1	% spei	nding s	hock fo	r sepa	arable
and non	-separable	preferences								

	Impact Multiplier $(dY/dG _{t=1})$	Long-run Multiplier
Separable	0.94	0.91
Non-separable	0.96	0.91

Remark to Table 4: the long-run multiplier is defined as dividing the discounted output changes by the discounted changes in spending at each time t.

When nominal interest rate is held $constant^{21}$

Table 5 shows the response of real GDP to a permanent, anticipated²² increase in government purchases of 1 per cent of steady-state GDP assuming that the nominal interest rate is held constant for a duration of two years starting in the first quarter of 2009. The latter means that the Fed can start to increase interest rate in 2012Q1 at earliest (technically, it means that the Taylor rule will be put back into practice after 2012Q1). The first and second row shows the findings of BR (2009) and this thesis, respectively. As we can see the impact multiplier of the BR (2009) are generally in line with the finding of ours. However, the latter is not true for longer horizons. As we can see our multipliers are generally around one at one, two or even at three years horizon. However, the BR (2009) numbers are much larger than the ones of ours. Here we can confirm the findings of Cogan *et al.* (2009) who find using a more elaborate (i.e. containing more frictions) version of the type of model used here that

 $^{^{21}}$ Here we assume in line with the most elaborate model of Christiano *et al.* 2009 that the nominal rate is held constant at the natural rate of interest.

 $^{^{22}}$ Here we assumed that people anticipate the recovery package by two quarters before it is taken into action. However, for example Uhlig (2009) assumed four quarters.

the spending multiplier should decline with the horizon. However, in contrast to Cogan et al. (2009) who find that multipliers decline sharply with the horizon, we show here that the multipliers can remain around one even for longer horizons.

Figure 12 show a type of long run multiplier used in Uhlig (2009). This multiplier is defined as the discounted sum of output changes until each horizon is divided by the sum of discounted spending changes until the same horizon (Uhlig, 2009). As we can see on very long horizons (e.g. after one hundred periods), the multiplier is still around one.

Table 5: Spending multiplier calculated by assuming that the nominal rate is held constant for two-year duration from 2009Q1 on

	Percentage increase in real GDP						
	2009Q1 2009Q4 2010Q4 2011Q4 2012Q4 Long run						
BR (2009)	1.05	1.44	1.57	1.57	1.55	na	
Our findings	1.07	1.03	1.02	1.00	0.97	1.03	
\mathbf{P} \mathbf{I} \mathbf{I} \mathbf{F} \mathbf{D} \mathbf{P} (2000) \mathbf{P} (2000)							

Remarks to Table 5: BR (2009) = Bernstein-Romer (2009) paper.

We re-did our calculations assuming that nominal rate is held constant for a duration of one year. The result can be observed in Table 6 and Figure 13. As we can see in the second row of Table 6, the multipliers are generally lower if nominal interest rate is fixed at a constant number for a shorter period of time (here it is one instead of two years). The impact multiplier coincides with the one in BR (2009). However, the longer horizon findings of ours depart a bit more far from the BR (2009) results while at the same time approach more the ones of Cogan *et al.* (2009). However, it has to be pointed out that our results concerning the multipliers of more than one-year time horizon are higher than the ones reported by Cogan *et al.* (2009). Figure 13 shows that the long run multiplier is surely lower than one when the nominal rate is fixed at constant level for one year.

Table 6: Spending multiplier calculated by assuming that the nominal rate is held constant for one-year duration from 2009Q1 on

	Percentage increase in real GDP						
	2009Q1 2009Q4 2010Q4 2011Q4 2012Q4 Long rur						
BR (2009)	1.05	1.44	1.57	1.57	1.55	na	
Our findings	1.05	0.99	0.98	0.96	0.95	0.99	

Figure 12: Long run multiplier of a permanent government spending shock calculated as Uhlig (2009) and assuming that the nominal rate is constant for two years from 2009Q1 on.



Figure 13: Long run multiplier of a permanent government spending calculated as Uhlig (2009) and assuming that the nominal interest rate is constant for one year from 2009Q1 on.



5.6 Further extensions of the models

The models considered thus far assume that the government deficit is financed through lumpsum taxes. However, in a recent paper, Uhlig (2009) considers a simple RBC model with capital, distortionary taxation (that is, labour, capital and sales tax) and a budget rule. In the budget rule of Uhlig (2009) a certain part of the deficit plus a random amount is financed by current labour taxes. In the latter case the value of the multiplier is influenced meaningfully by a parameter called budget balance speed defining the share of the deficit financed through labour taxes. If the budget balance speed is sufficiently low then the positive effect of a government spending on output will last longer. However, the most important finding of Uhlig (2009) is that the long run multiplier associated with a spending shock is always negative even if its impact on GDP is positive in the first couple of years. Future research may test the robustness of the findings of Uhlig (2009) in a reacher structure including frictions²³ that are popular now in leading New Keynesian models such as the Smets and Wouters (2007) model. The estimation of the parameters of the models used here is, of course, also a matter of future research²⁴.

²³Such frictions include but not restricted to sticky prices and wages, variable capital utilisation, investment adjustment costs, habit formation and non-Ricardian (i.e. credit constrained) households.

 $^{^{24}}$ See, e.g., Denes and Eggertsson (2009) who estimated the model of Eggertsson (2009) with Bayesian methods.

6 Conclusions

Even if the models presented here are too simple (i.e. they contain only few frictions) for providing sufficient background for policy decisions, still we can obtain a *fair* picture on the outcome of possible fiscal policy measures. Initially, in a model without capital, we consider the effects of three different fiscal policy measures that can be used for stimulating the economy: a temporary and unexpected rise in non-productive government spending, a cut in sales tax or a cut in payroll tax. Each of them are considered separately (that is, when government spending increases there is no change in sales or payroll taxes) and financed by lump-sum taxes. Thus, Ricardian equivalence holds. The paper used separable and nonseparable preferences in consumption and leisure as well. Most of VAR evidence point out that increased government spending lead to a rise in consumption. To model this stylised fact, we use non-separable preferences in consumption and leisure which imply low negative Hicksian wealth effect emerging after a rise in government spending in contrast to separable preferences where negative wealth effect on consumption is high. However, in the same model with capital we show that the long-run multiplier we borrowed here from Campolmi et al. (2010) to calculate the long-run effects of a government spending shock produces the same result for both type of preference specification.

During the recent financial crises the nominal interest rate in the U.S. was almost zero, that is the zero lower bound on nominal interest rate was binding. Based on this stylised fact we consider the above multipliers in a model with non-separable preferences when the zero bound binds. In case of zero nominal interest rate I restrict my analysis to non-separable preferences only because, as previously argued, this is the case mainly supported by empirics. In line with Christiano *et al.* (2009) and Eggertsson (2009) we found that the government spending multiplier can be very high when holding the nominal interest rate at zero level.

Another remarkable finding of this thesis is related to Eggertsson (2009). He finds that the sales tax multiplier derived by using separable preferences can be very high in case of zero nominal interest rate just as in the case of government spending multiplier. However, here, I found that the sales tax multiplier is even lower in case of zero nominal rate than the one of non-zero policy rate for non-separable preferences.

Finally, we augmented our model with capital and used three types of multipliers (impact, long run and Uhlig (2009) type) to compare our findings to the ones of Bernstein and Romer (BR) (2009) and draw the following conclusions. Firstly, it is possible to obtain multipliers around one but they are not as high as the ones in BR (2009). Secondly, the multipliers decline with the horizon similarly to the findings of Cogan *et al.* (2009) and in contrast with the results of BR (2009).

7 Appendix A

In Appendix A we can find the analitic derivation of the government spending multiplier for separable preferences. Here, the focus is on the change in government spending with respect to output with the implicit assumption that other fiscal variables do not change.

Recall that the linearised Euler equation in case of separable preferences is given by

$$-\sigma\widehat{C}_{t+1} - \frac{\tau^S}{1+\tau^S}\widehat{\tau}_{t+1}^S + \beta(R_{t+1}-R) = -\sigma\widehat{C}_t - \frac{\tau^S}{1+\tau^S}\widehat{\tau}_t^S + E_t\pi_{t+1}$$

and then let us subsitute for \widehat{C}_{t+1} and \widehat{C}_t the resource constraint in linear form,

$$\widehat{C}_t = \frac{1}{1-g} \left[\widehat{Y}_t - g \widehat{G}_t \right], \qquad (24)$$

to obtain:

$$-\sigma \frac{1}{1-g} \left[\widehat{Y}_{t+1} - g\widehat{G}_{t+1} \right] - \frac{\tau^S}{1+\tau^S} \widehat{\tau}_{t+1}^S + \beta (R_{t+1} - R) = -\sigma \frac{1}{1-g} \left[\widehat{Y}_t - g\widehat{G}_t \right] - \frac{\tau^S}{1+\tau^S} \widehat{\tau}_t^S + E_t \pi_{t+1}$$
(25)

The linearised version of monetary policy rule (see equation (3)) is given by:

$$R_{t+1} - R = \rho_R(R_t - R) + \frac{1 - \rho_R}{\beta}(\phi_1 \pi_t + \phi_2 \widehat{Y}_t)$$
(26)

Whenever the zero bound binds: $R_{t+1} = 0$. Using the method of undetermined coefficients given by equation (6) and (7) and the government shock process in equation (4) we can express for A_{π} and A_Y as follows.

Calculating A_{π} :

Let us first recall the linearised version of the intratemporal condition:

$$\varphi \widehat{N}_t + \sigma \widehat{C}_t + \frac{\tau^W}{1 - \tau^W} \widehat{\tau}_t^W + \frac{\tau^S}{1 + \tau^S} \widehat{\tau}_t^S = \widehat{W}_t = \widehat{MC}_t.$$

If we use market clearing and substitute for \hat{C}_t the linear form of the resource constraint from equation (24), we obtain:

$$\widehat{MC}_t = \varphi \widehat{Y}_t + \sigma \frac{1}{1-g} \left[\widehat{Y}_t - g \widehat{G}_t \right],$$

which is inserted into the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[\left(\varphi + \frac{\sigma}{1-g} \right) \widehat{Y}_t - \frac{\sigma g}{1-g} \widehat{G}_t \right].$$

Now, let us substitute for $\hat{\pi}_{t+1}$ and \hat{Y}_t the guesses in equation (6) and (7), respectively:

$$\pi_t = \beta E_t A_\pi \widehat{G}_{t+1} + \kappa \left[\left(\varphi + \frac{\sigma}{1-g} \right) A_Y \widehat{G}_t - \frac{\sigma g}{1-g} \widehat{G}_t \right].$$

Using equation (6) and the fact that $E_t \widehat{G}_{t+1} = \rho \widehat{G}_t$ we can express for A_{π} :

$$A_{\pi} = \frac{\kappa}{1 - \beta \rho} \left[\left(\varphi + \frac{\sigma}{1 - g} \right) A_Y - \frac{\sigma g}{1 - g} \right].$$
(27)

Calculating A_Y

Recall the linearised Euler, substitute for R_{t+1} the linear Taylor rule from equation (26) and use again the zero mean property, $E_t \hat{G}_{t+1} = \rho \hat{G}_t$:

$$-\sigma \frac{1}{1-g} \left[A_Y \rho \widehat{G}_t - g \rho \widehat{G}_t \right] + \left(\phi_1 A_\pi \widehat{G}_t + \phi_2 \widehat{Y}_t \right) = -\sigma \frac{1}{1-g} \left[\widehat{Y}_t - g \widehat{G}_t \right] + A_\pi \rho \widehat{G}_t, \quad (28)$$

Now let us substitute for A_{π} equation (27):

$$(\sigma + \phi_2(1-g))\widehat{Y}_t + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho} \left[\left(\varphi + \frac{\sigma}{1-g}\right)A_Y - \frac{\sigma g}{1-g} \right] \widehat{G}_t = \sigma \left(A_Y\rho - g\rho + g\right)\widehat{G}_t,$$

and calculate the multiplier:

$$\frac{d\widehat{Y}_t}{d\widehat{G}_t} = \frac{\left[\sigma\left(A_Y\rho - g\rho + g\right) - (1 - g)(\phi_1 - \rho)\frac{\kappa}{1 - \beta\rho}\left[\left(\varphi + \frac{\sigma}{1 - g}\right)A_Y - \frac{\sigma g}{1 - g}\right]\right]}{(\sigma + \phi_2(1 - g))}.$$

Realise that the RHS equals to A_Y and express for A_Y as:

$$A_Y = \frac{\sigma(g - g\rho) + (1 - g)(\phi_1 - \rho)\frac{\kappa}{1 - \beta\rho}\frac{\sigma g}{1 - g}}{\sigma + \phi_2(1 - g) - \sigma\rho + (1 - g)(\phi_1 - \rho)\frac{\kappa}{1 - \beta\rho}\left(\varphi + \frac{\sigma}{1 - g}\right)}.$$

And the multiplier we are interested in is given by :

$$\begin{aligned} \frac{dY_t}{dG_t} &= \frac{A_Y}{g} = \frac{1}{g} \left[g\sigma \frac{(1-\rho) + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho}}{\sigma(1-\rho) + \phi_2(1-g) + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho} \left(\varphi + \frac{\sigma}{1-g}\right)} \right] \\ &= \sigma \frac{(1-\rho) + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho}}{\sigma(1-\rho) + \phi_2(1-g) + (1-g)(\phi_1 - \rho)\frac{\kappa}{1-\beta\rho} \left(\varphi + \frac{\sigma}{1-g}\right)}. \end{aligned}$$

8 Appendix B

In Appendix B we can find the derivation of sales tax multiplier for non-separable preferences when the nominal interest rate is zero due to a discount rate shock.

Recall Euler equation in linear form and make use of market clearing $(\hat{Y}_t = \hat{C}_t)$ and the steady-steady of intratemporal condition to subsitute for $\frac{N}{1-N}$. After collecting similar terms we obtain:

$$E_{t}\left\{\beta\left(R_{t+1} - r_{t+1}\right) + \frac{(1 - \sigma)\gamma\tau^{S} - (1 + \tau^{S})}{1 + \tau^{S}}\widehat{Y}_{t+1} - \frac{\tau^{S}}{1 + \tau^{S}}\widehat{\tau}_{t+1}^{S}\right\}$$

= $\frac{(1 - \sigma)\gamma\tau^{S} - (1 + \tau^{S})}{1 + \tau^{S}}\widehat{Y}_{t} - \frac{\tau^{S}}{1 + \tau^{S}}\widehat{\tau}_{t}^{S} + E_{t}\widehat{\pi}_{t+1}$ (29)

Calculating π_l and \widehat{Y}_l

Firstly, when the zero bound binds at time t, we have $R_{t+1} = 0$ and (29) changes to:

$$-\beta pr_{l} + \left[\frac{(1-\sigma)\gamma\tau^{S} - (1+\tau^{S})}{1+\tau^{S}}\right]p\hat{Y}_{l} - p\pi_{l} - \frac{\tau^{S}}{1+\tau^{S}}p\hat{\tau}_{l}^{S} = \frac{(1-\sigma)\gamma\tau^{S} - (1+\tau^{S})}{1+\tau^{S}}\hat{Y}_{l} - \frac{\tau^{S}}{1+\tau^{S}}\hat{\tau}_{l}^{S},$$

and after some rearranging we express for \widehat{Y}_l :

$$\widehat{Y}_{l} = \frac{p}{(p-1)\left[\frac{(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})}{1+\tau^{S}}\right]} [\beta r_{l} + \pi_{l}] + \frac{\tau^{S}}{[(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})]} \widehat{\tau}_{l}^{S}.$$
(30)

The NKPC in case of binding zero bound can be rewritten as:

$$\pi_l = \frac{\kappa}{1 - \beta p} \left[\left[1 + \frac{N}{1 - N} \right] \widehat{Y}_l + \frac{\tau^S}{1 + \tau^S} \widehat{\tau}_l^S \right].$$
(31)

Calculating π when zero bound binds

Calculating π_l as a function of $\hat{\tau}_l^S$ and r_l , that is substituting (30) into (31),

$$\begin{aligned} \pi_l &= \frac{\kappa}{1-\beta p} \left[1 + \frac{N}{(1-N)} \right] \widehat{Y}_l + \frac{\kappa}{1-\beta p} \frac{\tau^S}{1+\tau^S} \widehat{\tau}_l^S \\ &= \frac{\kappa}{1-\beta p} \left[1 + \frac{N}{(1-N)} \right] \left[\frac{p}{(p-1) \left[\frac{(1-\sigma)\gamma\tau^S - (1+\tau^S)}{1+\tau^S} \right]} [\beta r_l + \pi_l] + \frac{\tau^S}{[1-\sigma)\gamma\tau^S - (1+\tau^S)]} \widehat{\tau}_l^S \right] \\ &+ \frac{\kappa}{1-\beta p} \frac{\tau^S}{1+\tau^S} \widehat{\tau}_l^S \end{aligned}$$

and make some further manipulations to express it for π_l :

$$\pi_{l} = \frac{\kappa p \beta (1 + \tau^{S})}{(1 - \beta p)(1 - N)(p - 1) \left[(1 - \sigma)\gamma\tau^{S} - (1 + \tau^{S}) \right] - \kappa p (1 + \tau^{S})} r_{l} + \frac{\kappa\tau^{S}}{(1 - \beta p)(1 - N)(p - 1) \left[(1 - \sigma)\gamma\tau^{S} - (1 + \tau^{S}) \right] - \kappa p (1 + \tau^{S})} \hat{\tau}_{l}^{S}$$

Calculating \hat{Y} when the zero bound binds

By substituting (31) into (30) we obtain:

$$\begin{split} \widehat{Y}_{l} &= \frac{p}{(p-1)\left[\frac{(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})}{1+\tau^{S}}\right]}\beta r_{l} + \frac{p}{(p-1)\left[\frac{(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})}{1+\tau^{S}}\right]}\frac{\kappa}{1-\beta p}\left[\frac{1}{1-N}\widehat{Y}_{l} + \frac{\tau^{S}}{1+\tau^{S}}\widehat{\tau}_{l}^{S}\right] \\ &+ \frac{\tau^{S}}{\left[(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})\right]}\widehat{\tau}_{l}^{S} \end{split}$$

which we express for \widehat{Y}_l as a function of r_l and $\widehat{\tau}_l^S$.

$$\widehat{Y}_{l} = \frac{p(1-\beta p)(1-N)}{(p-1)\left[\frac{(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})}{1+\tau^{S}}\right](1-\beta p)(1-N)-\kappa p}\beta r_{l} \\
+ \frac{(1-N)\left\{p\kappa\tau^{S}+\tau^{S}(1-\beta p)(p-1)\right\}}{(1+\tau^{S})\left\{(p-1)\left[\frac{(1-\sigma)\gamma\tau^{S}-(1+\tau^{S})}{1+\tau^{S}}\right](1-\beta p)(1-N)-\kappa p\right\}}\widehat{\tau}_{l}^{S}$$
(32)

The multiplier itself is:

$$\frac{dY^{l}}{-d\tau_{l}^{S}} = \frac{(1-N)\left\{p\kappa\tau^{S} + \tau^{S}(1-\beta p)(p-1)\right\}}{\left[(1-\sigma)\gamma\tau^{S} - (1+\tau^{S})\right](p-1)(1-\beta p)(1-N) - (1+\tau^{S})\kappa p}$$

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