# Disentangling the Impact of Eurozone Interest Rate Movements on CEECs' Business Cycle Fluctuations: The Role of Country Spread

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## Abstract

This paper investigates the relationship between the Eurozone interest rate, country spread and business cycle fluctuations in a sample of five CEECs: Bulgaria, the Czech Republic, Hungary, Poland and Romania. Henceforth, I propose two extensions to the Chang and Fernandez (2009) model. The first builds on the idea that both households and firms face the same interest rate, by assuming that country spread is not only the function of Solow residual but it also depends non-linearly on the external debt position of the country. The second extension merges two distinct approaches used in the emerging economy business cycle literature (i.e. endogenous versus exogenous country spread). Thus, I assume that country spread is a function of its own lagged values (i.e. country spread itself follows a persistent AR(1) process) which is augmented by domestic economic fundamentals (i.e. Solow residual and a nonlinear function of external debt position) and by the Eurozone interest rate. I estimate the model by using Bayesian techniques. By comparing the theoretical business cycle moments and impulse responses with those computed and estimated from the data of the five sampled CEECs, I show that the second extension performs better in replicating both business cycle moments and historical impulse responses than the original model or the first extension (i.e. the theoretical response of output to Eurozone interest rate shock is about twice as large as those derived from the original model). This result suggests that persistent and endogenous country spread could serve as an amplification mechanism of the impact of Eurozone interest rate shocks on the small open economy and improves the performances of the original model to replicate CEECs' business cycle moments.

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## 1 Introduction

This paper investigates the relationship between the Eurozone interest rate, country spread and business cycle fluctuations in a sample of five CEECs: Bulgaria, the Czech Republic, Hungary, Poland and Romania, by looking for the answers to the following questions: (i) Is there any causal relationship between Eurozone interest rate and business cycle fluctuations in CEECs?, and (ii) What is the role of country spread in the transmission mechanism of the Eurozone interest rate shocks to CEECs? In this way, it aims to contribute to the RBC literature which studies the source of business cycle fluctuations in CEECs.

The quantitative international real business cycle (RBC) literature provides the foundations for answering the previously posed questions since it has widely analyzed what are the main driving forces of emerging market business cycles. Typically, technology, terms of trade and interest rate shocks have been identified by many authors (Neumeyer and Perri,2005; Uribe and Yue, 2006; Aguiar and Gopinath, 2007b; Garcia-Cicco et al., 2010) as the primary sources of business cycle fluctuations. However, these authors agree that even if the importance of each shock in shaping business cycles differs across countries, the emerging market business cycles are largely driven by external interest rate shocks. (i.e. interest rate that these countries face on international financial markets).<sup>1</sup> Moreover, they state, by most frequently using empirical evidences from Latin American countries, that country spread, as a component of external interest rate, plays a crucial role in the transmission process of external financial shocks and acts as an amplification mechanism of the impact of these shocks on the domestic real economy.

Earlier studies assumed that the cost of external borrowing (i.e. external interest rate) can be decomposed into world interest rate and country spread, both being exogenous to the small open economy and modeled as exogenous stochastic processes. For example, Neumeyer and Perri (1999, 2001) or Chari et al. (2005) show that the baseline small open economy RBC model (Mendoza, 1991), augmented by the decomposed external interest rate, does a fairly good job in replicating the importance of world interest rate in explaining output fluctuation in Latin America. However, this approach was mainly criticized by advocates<sup>2</sup> of the idea that there is a feedback relationship between country spread and the real economy, by arguing that country spread is an indicator of sovereign default risk. Thus, they propose the modeling of country spread as an endogenous variable (i.e. it is the function of productivity shock) by building upon the idea that country spread may have an important informational content

<sup>&</sup>lt;sup>1</sup>Uribe and Yue (2006) show that country spread and US interest rate jointly explain about 20% of the variance of output in seven Latin American countries.

<sup>&</sup>lt;sup>2</sup>For example: Oviedo (2005), Uribe and Yue (2006) Aguiar and Gopinath (2007b) Chang and Fernandez (2009)

regarding the general economic conditions<sup>3</sup> in a country. They show that the calibrated or estimated model which assumes endogenous country spread can better replicate the strong countercyclical nature of country spread, as highlighted by the data, and it performs better in identifying the sources of business fluctuations in emerging economies than a model which assumes exogenous country spread.

By heavily relying on this extensive theoretical RBC literature developed for the Latin American countries, in this paper I propose the combination of the two previously described approaches. Basically, I assume that the external interest rate that the small open economy faces on international financial markets can be decomposed into world interest rate (i.e. the Eurozone interest rate) and country spread. The world interest rate is assumed to be exogenous and modeled as an AR(1) mean-reverting stochastic process. I suppose that country spread has an endogenous component (1) and, thus, it is the function of country specific fundamentals (e.g. Solow residual, the external debt position), global factors (e.g. Eurozone interest rate) and its own lags (i.e. I document that country spread is a highly persistent variable). Moreover, it has an exogenous component (2) modeled by an i.i.d. stochastic variable which captures the impact of exogenous factors, like news or internal political climate, on the cost of external financing. The main motivation for this combined approach is provided by the empirical literature which investigates the determinants of country spread (Cantor and Packer, 1996; Eichengreen and Mody, 1998; Hilscher and Nosbusch, 2010) and argues that it is misleading to assume that country spread is exogenous, by pointing out that there is not only a certain feedback relationship between country spread and world interest rate, but country spread also reacts to movements in economic fundamentals.

I integrate this combined definition of country spread into the small open economy RBC model developed by Chang and Fernandez (2009), by proposing two extensions to this model: (1) I assume that all the agents face uniform interest rate by defining country spread as a nonlinear function of households' external debt position and technology shocks, contrarily to the original version of the model which assumes that the two categories of agents face differentiated interest rates, and (2) I integrate the previously described combined definition of country spread into the original small open economy framework proposed by Chang and Fernandez (2009). I estimate the original model and the two proposed extensions by using Bayesian techniques. I evaluate the goodness of fit of the models by comparing the model implied business cycle moments against those computed from Hodrick-Prescott (HP) filtered time series and theoretical impulse responses against those derived from an estimated

<sup>&</sup>lt;sup>3</sup>The cost of external borrowing is lower when the small open economy has better economic performances.

structural vector autoregressive (SVAR) model<sup>4</sup>, as reported in the first part of this paper.

The model evaluation exercise suggests that the second extension of the Chang and Fernandez (2009) model, i.e. the country spread has both exogenous and endogenous component, performs the best in replicating the CEECs' business cycle stylized facts reported in the first part of the paper (i.e. consumption is more volatile than output; trade balance to output ratio is strongly countercyclical; country spread is countercyclical and persistent). Moreover, the theoretical impulse responses implied by the second extension, are about twice as large as those obtained from the first extension or the Chang and Fernandez (2009) model, and they are the closest to the historical impulse responses. This suggests that endogenous and persistent country spread serves as an amplification mechanism of the impact of Eurozone interest rate shocks on the domestic economy. Thus, the model could be further developed by giving more micro-foundation for country spread and by attributing more structure to the modeling of agents' financial decisions in small open economy RBC.

The paper is organized as follows. Section 2 contains a short review of the related literature. Section 3 aims to document and report CEECs business cycle stylized facts and the implications of Eurozone interest rate shocks on CEECs' output fluctuations based on an estimated SVAR model. In Section 4 I present the model, the estimation method and estimation results. Then I conclude the paper in Section 5.

### 2 Related Literature

It is widely documented in the quantitative business cycle literature that external interest rate<sup>5</sup> (i.e. the interest rate that country faces on international financial markets) shocks are one of the sources of business cycle fluctuations but their importance in shaping business cycles depends significantly on whether the country is a developed or an emerging one. Concerning developed countries, Mendoza (1991) shows that external interest rate shocks do not play considerable role in explaining business cycle fluctuations in Canada, while interest rate is rather acyclical variable lagging the business cycles. Stock and Watson (1998) report that the contemporaneous correlation of output with federal fund rates was 0.38 in the USA, and Neumeyer and Perri (2005) argue that the same indicator, computed for a sample of five

<sup>&</sup>lt;sup>4</sup>I estimated the impact of Eurozone interest rate and country spread shocks on output and investment, in the case of Bulgaria, Hungary and Poland, in a structural vector autoregressive framework (SVAR), by using historical macroeconomic and financial data for the period 1995-2009. The main results suggest: Eurozone interest rate shock has unimportant impact on output, country spread, as the other component of external interest rate, explains large part of the output variance, while there is a strong interconnection between output and country spread.

<sup>&</sup>lt;sup>5</sup>This is most frequently defined as the sum of world interest rate and country spread (i.e. country risk premium) in the case of emerging countries.

developed countries, is on average approximately 0.2. However, many quantitative business cycle papers, focusing on Latin American countries, have highlighted that the opposite is true in the case of emerging economies where external interest rate turned out to be strongly countercyclical, leading the cycle. For example, Neumeyer and Perri (1999) observe strong negative correlation between external interest rate and gross domestic product (about -.45) in Argentina, Brazil and Mexico. By extending the sample to twelve emerging economies, Neumeyer and Perri (2001) arrive at the conclusion that increases in the interest rate that emerging economies face on international financial markets are followed by drops in domestic output. This is in contradiction with the findings reported in the case of developed countries where interest rate seems to be acyclical or procyclical.

Other papers (Oviedo, 2005; Uribe and Yue, 2006; Aguiar and Gopinath, 2007b; Chang and Fernandez, 2009) proceeded by decomposing the external interest rate into external financing premium (i.e. country spread) and world interest rate. In this way, it has been shown that, essentially, country spread is that component of the external interest rate which is responsible for the fact that business cycles in emerging economies are driven by world interest rate shocks. They argue that this is because country spread serves as an amplification mechanism of the effect of the world interest rate shock on the domestic real economy. Thus, Uribe and Yue (2006) infer from historical impulse response functions estimated on a panel of seven Latin American countries that US interest rate and country spread shocks jointly explain about 20% of the aggregate domestic fluctuation in Latin American countries, while country spread shocks individually account for 12% of the domestic output variance on a four years horizon.

The puzzles presented in the last two paragraphs, gave birth to a new line of theoretical business cycle research dealing with the implications of interest rate movements on business cycle fluctuations in emerging economies<sup>6</sup>. This literature mainly focuses on developing models that can overcome the shortcomings of the baseline small open economy RBC model and can both qualitatively and quantitatively reproduce the above mentioned stylized facts, while trying to answer two types of questions: Is there any causal relationship between world interest rate and business cycle fluctuations in emerging economies? What is the role of country spread in the transmission mechanism of the world interest rate shocks to emerging countries?

In order to solve the previously described puzzles and properly answer the above mentioned questions, the new line of literature aimed to identify imperfections of the standard neoclassical model, which performs well in the case of developed countries where external

<sup>&</sup>lt;sup>6</sup>The most influential papers are Neumeyer and Perri (1999, 2002, 2005), Oviedo (2005), Uribe and Yue (2006), Aguiar and Gopinath (2007b), Chang and Fernandez (2009) or Garcia-Cicco et al. (2010)

interest rate fluctuations turn out not to have significant role in driving business cycle fluctuations (Mendoza, 1991), but which cannot replicate the countercyclical interest rate suggested by the data in the case of emerging economies. As Neumeyer and Perri (1999) point out, the baseline RBC model calibrated to the Argentine economy generates a 0.97 correlation between output and the interest rate, while the same statistics computed from the data is -0.7.

They reason that this is because within this framework fluctuations in external interest rate affect economic activity through two channels which act in the opposite directions. The direct effect acts through the investment channel, such that an upward jump in the world interest rate lowers investments in physical capital, which subsequently leads to a drop in output. However, the indirect effect works through the labor market, such that when interest rate increases both substitution and income effects raise labor supply in the domestic economy which, consequently, leads to higher output. Neumeyer and Perri (1991, 2001) argue that, under certain preference specifications, the indirect effect overcomes the negative direct effect and the model generates procyclical interest rate.

In order to reconcile this shortcoming of the baseline model, two modifications of the baseline small open economy RBC model have been proposed: (1) modeling the external interest rate as function of the world interest rate and country spread (2) imposing working capital constraint on firms' decision making concerning production factor procurement. In other words, these papers assume that incomes and expenditures of the firms are not perfectly synchronized i.e. firms have to pay for the production factors in advance, before receiving their income. This induces a wedge between marginal factor productivity and factor price distorting in this way firms' labor demand decisions and generating an inverse relationship between demand for labor and interest rate. Thus, for example, an unexpected positive interest rate shock makes borrowing more expensive inducing firms to demand less labor and to contract their activity. Hence, the presence of working capital constraint brings into the model an additional channel through which external interest rate alter the level of domestic output in addition to the above mentioned two effects, amplifying in this way the effect of the external interest rate shocks and boosting the magnitude of direct effect above the indirect one. Actually, this is the main mechanism through which the baseline model augmented by working capital constraint can generate countercyclical external interest rate.

The approaches through which country spread is integrated into the baseline model can be grouped into two categories depending on whether country spread is assumed to be exogenous or endogenous. The first category contains papers (Neumeyer and Perri, 1999, 2002; Chari, Kehoe and MacGrattan, 2005; Garcia-Cicco et al., 2009) which assume that the interest rate that small open economies face on international financial markets is a function of the world interest rate and country spread, both being modeled as exogenous stochastic variables, while firms faces working capital constraint. For example, Neumeyer and Perri (1999) extend the baseline small open economy RBC model proposed by Mendoza (1991) and assume that domestic agents' borrowing decisions are subject to a stochastic exogenously given external interest rate (i.e. they model this interest rate as a mean reverting independent autoregressive stochastic process) which interacts with firms'working capital constraint. They show that the calibrated model can replicate the countercyclical Argentine interest rate and implies that external interest rate shocks explain a significant part (about 55%) of domestic output fluctuations.

However, as Aguiar and Gopinath (2007b) point out, the model implied contemporaneous correlation between output and interest rate is too low relative to the data in the above described setup since external interest rate shocks are orthogonal to productivity shocks. By using Argentine time series, they show that higher productivity is followed by lower external interest rate and this is why the contemporaneous correlation between output and external interest rate is much stronger in the data than that implied by the Neumeyer and Perri (1991) model. They thus allow for correlation between country spread and technology shock, while world interest rate is exogenously given and show that this version of the model can replicate the strong countercyclical interest rate observed in the Argentine data. The main intuition behind this is simple. For example, a positive technology shock hitting the economy have a double impact on output in the Aguiar and Gopinath (2007b) framework, i.e. the actual increase in productivity and the lower interest rate which makes consumption and investment cheaper, generating a higher aggregate demand and consequently output than in the benchmark case.

As a consequence, a series of papers (Neumeyer and Perri, 2005; Oviedo, 2005; Uribe and Yue, 2006; Aguiar and Gopinath, 2007b; Chang and Fernandez, 2009) propose the modeling of country spread as an endogenous variable which responds negatively to productivity improvements, i.e. interest rate that countries face on international financial markets is a function of productivity shocks. This assumption is basically borrowed from the sovereign debt literature.<sup>7</sup> The basic mechanism works in the following way: positive productivity shock induces a rise in consumption and investment which is enhanced by the contemporaneous decline in the interest rate. This coupled with the working capital constraint acts as an amplification mechanism of the shocks, thus generating highly volatile consumption, a negative correlation between net exports and output, and countercyclical

<sup>&</sup>lt;sup>7</sup>For example, Arellano and Mendoza (2002) or Arellano and Ramanarayanan (2005) models sovereign default by assuming that the probability of default increases when negative productivity shocks hit the economy.

interest rate.

In the spirit of these ideas, Neumeyer and Perri (2005) propose a small open economy RBC model with three structural shocks: technology shock, world interest rate and country risk shock. They define the interest rate faced by domestic agents on international financial markets as a function of country spread (i.e. defined as a function of technology shocks) and world interest rate (i.e. defined as an exogeous stochastic variable) They show that endogenous country spread generate an amplification mechanism necessary to reconcile data and the small open economy RBC model. They also point out that augmenting this theoretical framework by working capital constraint, as a friction which distorts the labor demand decision of firms, the model does a good job in replicating business cycle moments observed from Argentine data, especially the countercyclical country risk and external interest rate. Their findings indicate a negative but rather delayed impact<sup>8</sup> of 1% world interest rate shock on the domestic output, i.e. the highest deviation of output from its trend is 2% and it is attained after half a year.

Similarly, Uribe and Yue (2006) propose a small open economy real business cycle model with several frictions and feed into the model an external interest rate rule as a function of world interest rate and macroeconomic variables estimated in a VAR framework. They calibrate and estimate the model by using impulse response matching. They show that working capital constraint and endogenous interest rate are indispensable ingredients of the small open economy RBC in order to generate countercyclical country spread and to obtain simulated moments and theoretical impulse responses both qualitatively and quantitatively in line with the stylized facts obtained from the data of a panel of eight emerging economies.

Chang and Fernandez (2009), by using Bayesian techniques, estimate an encompassing model which embodies both financial frictions (i.e. working capital constraint) and endogenous country spread (i.e. country spread is a function of expected future permanent and transitory technology shocks). Based on their simulation results, they conclude that this new version of the model performs better in replicating Mexican business cycle stylized facts than a simple financial friction model (i.e. only working capital constraint) or the Aguiar and Gopinath (2007a) stochastic trend model, by arguing that financial frictions coupled with endogenous country spread serves as an amplification mechanism of external financial shocks.

However, both of these two approaches exhibit imperfections. For example, Oviedo (2005), by building on the idea that business cycles in emerging economies are driven by interest rate shocks, points out that the extent to which interest rate shocks can drive business

<sup>&</sup>lt;sup>8</sup>Because of the persistence of the world interest rate and potential spillover of this type of shock on country spread.

cycles depends heavily on the statistical properties of the shock rather than the nature of the financial frictions which interacts with interest rate or country spread shocks in the model. Likewise, Aguiar and Gopinath (2007b) propose a small open economy RBC model which is augmented by total factor productivity that has a stationary and an integrated component, while intertemporal decisions concerning consumption and labor are subject to interest rate shocks. They consider three versions of this model by assuming: (1) exogenous interest rate shocks which are independent of the productivity shocks (2) interest rate respond to transitory productivity shocks (3) interest rate respond to permanent productivity shocks. They estimate the model on Mexican and Canadian data and conclude that business cycles in Mexico are driven by large technology shocks which are correlated with the interest rate (i.e. this version of the model performs the best in replicating Mexican business cycle stylized facts).

Finally, empirical evidences can be used to challenge the way country spread is modeled in the RBC literature. These evidences suggest that it is misleading to assume that country spread is exogenous or it only depends on the domestic productivity by pointing out that there is not only a certain feedback relationship between country spread and world interest rate, but country spread also reacts to movements in economic fundamentals. This makes extremely difficult to disentangle the impact of world interest rate on business cycle fluctuations. In line with these ideas, empirical investigations like those conducted by Cantor and Packer (1996), Eichengreen and Mody (1998) and recently Hilscher and Nosbusch (2010) conclude that country spread is endogenously determined by domestic economic fundamentals. By estimating different panel regression models they find two main categories of factors which determine the level of country spread in emerging economies: country specific fundamentals (e.g. terms of trade, the volatility of terms of trade, debt to GDP ratio, reserves to GDP ratio, credit ratings)<sup>9</sup> and global factors (e.g. S&P 500 index, US default yield spread, the 10-year US Treasury yield). Thus, these results provide an empirical support for merging the two distinct modeling approaches of country spread (i.e. exogenous versus endogenous) into one unified theoretical framework, as it is presented in Section 4.

<sup>&</sup>lt;sup>9</sup>I mention here only those factors which are the most frequently reported as main fundamental determinant of the country spread. For example Hilscher and Hosbusch (2010) by using a panel of 31 emerging countries found that a one percentage point terms of trade volatility augment country spread by 0.3718 percentage point, while the same increase in debt to GDP ratio push up the spread by 0.04 percentage point. Without considering the terms of trade, similar results concerning the impact of the debt to GDP ratio on the spread are reported by Eichengreen and Mody (1998) for a larger panel of countries.

### 3 Empirical Approach or "What do the data tell us?"

The reason for this empirical analysis is twofold. On the one hand, it focuses on the description of the main business cycle moments of the most important macroeconomic variables like output, consumption, investment, trade balance to output ratio and one financial variable, the country spread in the case of Bulgarian, the Czech Republic, Hungary, Poland and Romania. On the other hand, it looks for the answer to the question: To what extent are business cycles in CEECs driven by interest rates that countries face on international financial markets? I approach this question by using a VAR framework which allows for identifying the Euro Zone interest rate and country spread shocks and provides quantitative assessments of the question in two dimensions. Firstly, since the VAR setup permits the identification of possible feedback effects between variables, a series of information can be extracted from the estimated impulse response functions of the real macroeconomic variables to country spread and Eurozone interest rate shocks. Secondly, based on forecast error variance decomposition, it can be evaluated to what extent country spread and Eurozone interest rate explain fluctuations in the main domestic macroeconomic variables. The main advantage of this kind of country by country analysis relative to adopting a panel framework is that it allows for detecting possible heterogeneities in the characteristics of business cycles across countries or in the way business cycles are driven by Euro Zone interest rate or country spread shocks.

The results of the business cycle investigation must be treated with some prudence since the definition of the underlying national account data was subject to several methodological changes over the sample period, especially in the case of Bulgaria and Romania, and it might also contain measurement errors. Another potential issue refers to the sensitivity of HP filter to short samples and the poor performance of this filter at the end-points of the time series. However, by abstracting from the potential cross country heterogeneities and the previously presented methodological issues, the forthcoming business cycle moments investigation reports five common stylized features concerning CEECs' business cycles: (i) consumption is more volatile than output (ii) investment is the most volatile component of output (iii) trade balance to output ratio is countercyclical (iv) country spread is countercyclical but the computed correlation coefficient is significant only in the case of Hungary (v) not only the components of output but also the country spread is characterized by relatively high persistence.

In this section, I also document the following qualitative implications of the estimates concerning the interaction between domestic real macroeconomic variables, country spread and Eurozone interest rate derived from an estimated SVAR model: (1) Eurozone interest rate shocks dampen real domestic economic activity and boost the level of country spread according to the impulse response analysis<sup>10</sup>, however the forecast error variance decomposition suggests that the Eurozone interest rate explains a negligible proportion of output and country spread variance, and (2) there is a strong feedback relationship between the real domestic economy and country spread in the CEECs, which has to be interpreted with caution because of the potential bias created by omitted determinants of country spread from the VAR model.

### 3.1 Business cycle moments

The first part of the empirical investigation focuses on defining common patterns of the main business cycle moments in the five sampled CEECs. As highlighted in the literature, emerging market business cycles have two specific characteristics in comparison with business cycles of developed countries: (i) the trade balance to output ratio and the interest rate that small open economies (e.g. emerging or developing countries) face on international financial markets are strongly countercyclical (ii) consumption is more volatile than output. Moreover, since many authors (Neumeyer and Perri, 1999; Uribe and Yue, 2006) show that country spread is that component of the cost of borrowing that creates the strong countercyclical nature of the interest rate, it becomes important to verify whether the country spread of CEECs moves pro- or countercyclically over the business cycles.

In order to provide a complete characterization of the business cycle moments in these countries, I compute the absolute and relative volatility of the quarterly HP filtered output (y), consumption (c), investment (i) and trade balance to output ratio (tby)<sup>11</sup>, over 1995Q1-2009Q4 sample period. I report the persistence of each variable measured by the first order autocorrelation coefficient and their contemporaneous correlation with output.<sup>12</sup> Unfortunately, country spread data with a consistent definition across countries (i.e. EMBI spread) is available only in the case of Bulgaria (1995Q1-2009Q4), Hungary (1996Q1-2009Q4) and Poland (1996Q1-2009Q4), and it does not cover the whole sample period restricting in this way the completeness of the empirical investigation.

Results are reported in Table 1 and they suggest an important heterogeneity across CEECs concerning the size of the business cycle moments, but minor differences in the signs of certain business cycle statistics. By analyzing each variable separately, one can easily

<sup>&</sup>lt;sup>10</sup>The estimated confidence interval seemed to be way too large, suggesting in this way low statistical significance of the estimated impulse responses.

<sup>&</sup>lt;sup>11</sup>See Data Appendix for more information concerning the data definition and the filtering technique used.

 $<sup>^{12}</sup>$  As Benczur and Ratfai (2005) advocate, I also check the significance of the contemporaneous correlation of each variable with output by comparing it with the 95% benchmark significance level computed as  $2/\sqrt{T} \simeq 2/\sqrt{59} \simeq 0.2603$ 

notice that the volatility of output is strikingly higher in Bulgaria and Romania than in the other three CEECs. A meaningful explanation for these would be the more powerful and delayed structural reforms which these two economies faced in the late 90s.

	Bulgaria	Czech Rep.	Hungary	Poland	Romania
Absolute	volatility				
$\sigma_y$	3.7791	2.0578	1.7082	1.1787	3.8469
$\sigma_c$	4.3989	2.0035	2.3043	1.2187	5.0139
$\sigma_i$	12.8541	3.9518	2.6316	6.3480	8.1564
$\sigma_{tby}$	3.1273	1.6134	1.9853	1.4826	4.4321
$\sigma_s$	3.7682	NA	1.0698	0.7677	NA
Relative	volatility				
$\sigma_c/\sigma_y$	1.1640	0.9736	1.3489	1.0338	1.3033
$\sigma_i/\sigma_y$	3.4014	1.9204	1.5405	5.3853	2.1202
$\sigma_{tby}/\sigma_y$	0.8275	0.7840	1.1621	0.8872	1.1521
$\sigma_s/\sigma_y$	0.9971	NA	0.6262	0.4494	NA
First	order	autocorr.			
$\rho_y$	0.5901	0.7222	0.7234	0.8162	0.6435
$ ho_c$	0.7574	0.6381	0.7572	0.4538	0.7281
$ ho_i$	0.6578	0.7623	0.5421	0.7542	0.6251
$ ho_{tby}$	0.8615	0.6175	0.5080	0.8311	0.7952
$ ho_s$	0.8235	NA	0.7698	0.698	NA
Contemp.	correaltion				
$\rho(c,y)$	0.7779	0.5968	0.6394	0.5571	0.6872
$\rho(i,y)$	0.4012	0.7116	0.3870	0.8581	0.3699
$\rho(tby,y)$	-0.1853	-0.1543	-0.3913	-0.6884	-0.4364
$\rho(s,y)$	-0.081	NA	-0.3937	-0.0785	NA

Table 1. Historical Business Cycle Moments

A similar conclusion can be drawn concerning the persistence of output which is lower in Romania and Bulgaria than in the other three countries included in the sample. Thus, the presence of noise or measurement error in the output series of these two countries could also be the explanation for the joint occurrence of high volatility and low autocorrelation coefficient.

Consumption is more volatile than output, excepting the Czech Republic, where it is about as volatile as output. Benczur and Ratfai (2005) report similar findings regarding business cycle moments in twelve CEECs or those presented by many other authors in the case of Latin American countries (Aguiar and Gopinath ,2004; Neumeyer and Perri, 2005; Garcia-Cicco, Pancrazzi and Uribe, 2006; Chang and Fernandez, 2010). Thus, the large volatility of consumption relative to output accepted as a stylized fact of emerging economies' business cycles is also present in the case of CEECs. This is due to the fact that these countries generally face stronger constraints on financing their consumption smoothing process than the developed countries, which leads to a high sensitivity of consumption to fluctuations in income. However, this result contradicts the evidences documented in the case of developed countries, where, usually, consumption is less volatile than output. For example, Benczur and Ratfai (2008) document that large industrial countries (G7) are typically characterized by relative volatility of consumption less than one.<sup>13</sup> Moreover, consumption has positive and statistically significant contemporaneous correlation with output in all of the five countries (i.e. it is pro-cyclical) and it has high persistence especially in Bulgaria and Romania.

As emphasized in the business cycle literature, investment is the most volatile component of output, i.e. it is at least twice as volatile as output in all the sampled countries. This stylized fact is in line with the findings carried out in the case of developed countries. As can be inferred from the business cycle statistics reported in Table 1, the volatility of investment is strikingly higher in Bulgaria and Romania than in the other countries. Investment is strongly procyclical<sup>14</sup> and it has relatively high persistence in all of the five countries.

The trade balance to output ratio is almost as volatile as output in Hungary, Poland and Romania, while it is less volatile than output in Bulgaria and in the Czech Republic. This external trade indicator is less volatile than consumption and investment in all of the five countries. It exhibits the highest persistence in Bulgaria and Poland, while it is the least persistent in Hungary. The trade balance to output ratio is the other component of the output, besides consumption, which has been shown to behave differently in emerging economies. Its contemporaneous correlation with output is negative and has a large size in all of the countries, with the exception of Bulgaria and the Czech Republic, where this

<sup>&</sup>lt;sup>13</sup>They document, by using second order business cycle moments computed from first differenced and HP filtered time series in the case of 60 countries, that consumption is less volatile than output in the large industrial, G7, countries.

<sup>&</sup>lt;sup>14</sup>Investment has statistically significant contemporaneous correlation with output, i.e. the correlation coefficient is higher than 0.26 (the 95% significance level).

indicator takes a relatively low value in comparison with the other sampled countries and it is not statistically significant.<sup>15</sup>

Deriving any conclusion concerning the moments of the EMBI spread is limited by the lack of available time series for the Czech Republic and Romania. Based on the existing data and moments reported in Table 1, one can notice that the Bulgarian EMBI spread is significantly more volatile than that observed in the case of Hungary or Poland (i.e. the volatility of EMBI spread is three times larger in Bulgaria than in Hungary). The explanation for this could be the difficulties faced by the Bulgarian economy in the 90s, which materialized in a higher sovereign risk, and which coupled with the presence of the currency board, boosted the premium demanded by investors to invest in Bulgarian foreign currency denominated sovereign bonds (i.e. higher EMBI spread). In addition, the EMBI spread has negative contemporaneous correlation with output (i.e. it is countercyclical) and it is statistically significant only in the case of Hungary. This finding is not in line with the rather acyclical nature of different government issued bond spreads documented in the case of developed countries (Stock and Watson, 1998; Neumeyer and Perri, 2005).

### 3.2 Empirical impulse responses and variance decomposition

To assess the response of the domestic aggregate economy to Euro Zone interest rate shocks and to quantify the extent to which fluctuations in the aggregate domestic economic activity in CEECs are driven by world interest rate and country spread shocks, I define a vector autoregressive (VAR) model by including in the vector of endogenous variables a domestic macroeconomic variable, the real Euro Zone interest rate and country spread. There are two major barriers imposed by the lack of the data on this type of analysis: (i) The number of variables that can be included in the VAR is limited. Because of the short sample available in the case of all five countries, one has to be careful while selecting the number of variables included in the VAR model such that to save degree of freedom. Thus, I estimate two specifications of a three dimensional VAR model by including the HP filtered GDP series as a measure of aggregate economic activity (i.e. output), Euro Zone real interest rate and EMBI spread in the first specification. I replace output by investment in the second specification, but the other two variables remain the same. (ii) Country spread data is not available for the whole sample of countries. Hence, empirical impulse responses can be estimated only in the case of Bulgaria, Hungary and Poland.

Written in structural form, the VAR model which is to be estimated is the following:

 $<sup>^{15}</sup>$ The reported contemporaneous correlation of tby with output (Table 1) is less than 2.60, the 95% benchmark significance level.

$$Ay_t = A_0^* + A_1^* y_{t-1} + \dots + A_p^* y_{t-p} + v_t$$
(1)

where:  $(y_t)_{(1\times3)}^T = [r_t^*, x_t, s_t]$  is a vector of endogenous I(0) variables and it contains the Euro Zone real interest rate series, a domestic macroeconomic variable (i.e. HP filtered<sup>16</sup> logged real output  $(y_t)$  when the first specification is estimated or the HP filtered logged real investment  $(i_t)$  in the case of the second specification) and the country spread series. I performed Augmented Dickey-Fuller and Phillips-Perron unit root tests in order to check whether the previously described time series are stationary. The tests rejected the null hypothesis in the case of all variables (i.e. the time series has a unit root) allowing me to conclude that all the series included in the VAR model are stationary.  $(y_{t-1})_{(3\times1)}$  is the vector of lagged endogenous variables,  $(A_0^*)_{(3\times3)}$  is the matrix of constants,  $(A_i^*)_{(3\times3)}$ ,  $\forall i = \overline{1:p}$  is the matrix of structural coefficients and  $(\nu_t)_{(3\times1)}$  is a vector of structural residuals with  $E[v_t] = 0$  and variance-covariance matrix  $\Sigma_v$ . These structural errors or residuals can be defined as linear combinations of structural shocks which cannot be observed directly but can be identified by imposing different restriction on the VAR model. Thus the structural residuals can be written as:

$$v_t = B\epsilon_t \tag{2}$$

where  $\epsilon_t$  is the structural shock defined as a random variable with zero mean and variancecovariance matrix  $I_3$ .

The reduced form of the same VAR model is:

$$y_t = A^{-1}A_0^* + A^{-1}A_1^*y_{t-1} + \dots + A^{-1}A_p^*y_{t-p} + A^{-1}v_t$$
(3)

$$y_t = A_0 + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \tag{4}$$

where the reduced form residual is  $u_t = A^{-1}B\epsilon_t$  or  $Au_t = B\epsilon_t$  and  $(A_i)_{(3\times 3)}, \forall i = \overline{1:p}$  is the matrix of reduced form coefficients.

In order to determine the number of lags necessary to include in the VAR model when estimating it, I applied three lag length selection or information criteria: Akaike, Schwarz and Hannan-Quinn. These selection criteria suggested that a first order VAR model fits best

<sup>&</sup>lt;sup>16</sup>Before HP filtering the GDP and investment series taken in logarithm, I performed ADF and PP unit root tests on these level series. The results of these tests indicated that both time series have a deterministic trend component, i.e. the time trend had statistically significant coefficient in the regression of each variable on its own lags. This suggested that in order to stationarize the output and investment detrending, by using a filter is better than first differencing. Discussing in more details the results of the tests or reporting them is beyond the scope of this paper, but they are available upon request.

the data in all of the three countries.

Since a Euro Zone interest rate shock cannot be directly observed, it has to be identified in order to derive impulse response functions and conduct variance decomposition analysis. This identification has to be realized such that the structural shocks to be orthogonal, in this way, assessing the dynamic impact of an isolated shock. This identification is equivalent to imposing restrictions on A and B matrices, in  $Au_t = B\epsilon_t$ . The intuition for these restrictions is provided by economic theory and they are as follows: the real part of the economy does not react instantaneously to financial shocks i.e.  $\epsilon_{r^*}$  or  $\epsilon_s$ ), financial markets react contemporaneously to innovations (e.g. news) coming from the real domestic economy (i.e.  $\epsilon_y$ ) and international financial markets (i.e.  $\epsilon_{r^*}$ ), shocks originating from the domestic economy (i.e.  $\epsilon_y$ ) cannot have any impact on international financial markets since all of the countries included in the analysis are small open economies. Therefore a natural choice of the restrictions on A and B matrices, by keeping the above presented ordering of variables, is: matrix A is lower-triangular with ones on the main diagonal (i.e.  $a_{12} = a_{13} = a_{23} = 0; a_{11} = a_{22} = a_{33} = 1$  and  $a_{21}$  coefficient equal to zero, while matrix B is restricted to be a diagonal one  $(b_{ij} = 0, \forall i \neq j)$ .

# 3.2.1 Impulse response analysis: Euro Zone interest rate shocks and business cycle fluctuations in CEECs

Figure 1 comprises the response of the HP filtered real gross domestic product series to one standard deviation (S.D.) Euro Zone interest rate shock.



Figure 1. The Response of Output to One S.D. Eurozone Interest Rate Shock

According to the estimated impulse responses, output does not react instantaneously to the Eurozone interest rate shock but its response has a one period delay in all of the countries (i.e. this originates from the previously assumed identification strategy).

Since a positive Eurozone interest rate shock has an adverse effect on the domestic economy, by making external borrowing more expensive, domestic output decreases in the subsequent period of the occurrence of this shock and, then, it slowly converges back to its initial level. However, the estimated two standard deviation error bands<sup>17</sup> of the impulse responses seem to be pretty wide indicating significant uncertainty regarding the quantitative plausibility of the estimated impulse responses. A slight difference can also be noticed in the shape of the impulse response functions. While output sharply drops in the first period right after the occurrence of the shock and then it starts to converge back to its original value in Poland and Bulgaria, the same cannot be concluded about the output dynamics in Hungary: it systematically decreases and starts to adjust back to its initial value just after five quarters. The Bulgarian output has the strongest reaction to Euro Zone interest rate shock followed by Poland and Hungary.<sup>18</sup>

If one compare these results with the response of output to country spread shocks (Figure 3, included in the Figure appendix) somewhat similar conclusions to the former ones can be drawn: output drops relative to its value at the moment of the occurrence of country spread shock. It attains its lowest level after about four quarters in all of the three countries and then it goes slowly back to its initial value. Intuitively this can be explained by the fact that since country spread, by its information content, signals the overall performances of an economy and because it is a component of the cost of external borrowing, its increase will deteriorate the external financing possibilities of the domestic economy, leading to decline in domestic output. Moreover, by its nature, country spread is a persistent variable, which explains its slow adjustment after the occurrence of the shock. The Hungarian output is the most sensitive to country spread shock and it exhibits the largest drop, i.e. about twice as large as the shrinkage in Polish and Bulgarian output. By comparing the estimated responses of output to country spread shock with those to external interest rate shock, it can be seen that in Poland and Bulgaria output is more sensitive to Euro Zone interest rate shocks than to country spread shocks, while in Hungary one S.D. country spread shock leads to a larger drop in output than the external interest rate shock.

By having these results, the next exercise seems to be quite natural: it asks whether

<sup>&</sup>lt;sup>17</sup>These are not reported here but they are available upon request.

<sup>&</sup>lt;sup>18</sup>In order to asses the robustness of the estimated impulse response functions I changed the ordering of the variables by putting output as the first one and Euro Zone interest rate the second one in the vector of endogenous variables. The estimated impulse response functions turned out to be similar to those presented in Figure 1.

country spread responds to unexpected innovations in external interest rate or to domestic output shocks. All these kind of investigations would provide empirical motivation for the plausibility of assuming endogenous country spread in the theoretical framework. Figure 2 illustrates the estimated responses of country spread to one S.D. Euro Zone interest rate shock.



Figure 2. The Response of Country Spread to one S.D. Eurozone Interest Rate Shock

This suggests that country spread increases in all countries as a response to higher Eurozone interest rate and it slowly adjusts back to its initial value as the effect of the shock dies away. Put differently, the increase of the external interest rate tightens the constraints on the external financing possibilities of the small open economy, thus indicating potential future deterioration of the domestic economic performances. Since investors incorporate expectations about future dynamics of the economic fundamentals into their expected returns they will ask for higher compensation in exchange for any additional unit of risk assumed. This materializes in higher country spread. However the magnitude of the impact of external interest rate shock on country spread is lower than that on domestic output.

Concerning the impact of one S.D output shock on country spread, by inspecting Figure 3 (Figure Appendix) one can observe that the plotted impulse response function is hump shaped, i.e. as a positive output shock hits the economy, country spread decreases, than it increases till it attains the highest deviation from its original level and adjusts back to its trend level, as domestic output converges back to its initial level. This type of dynamics is a natural one since positive output shocks improve domestic economic performances and consequently create better external financing conditions. But, significant heterogeneity in the size of the responses of country spread to output shock can be noticed across counties, while the large two standard deviation error band creates doubt concerning the quantitative

implications of these results. This result is in line with Uribe and Yue's (2006) findings in a panel of Latin American countries which point out a similar behavior of country spread

I conducted a similar exercise to the previously presented one by replacing output with investment in the VAR model. Because of the impossibility to estimate a stable VAR for Bulgaria and thus to obtain meaningful impulse response estimates, only results in the case of Hungary and Poland are reported. Figure 4 depicts the estimated impulse response functions and suggests that qualitatively the results are similar to those obtained in the previous case, but there are minor quantitative differences which are meaningless to be further investigated given the low statistical significance of the estimated impulse responses, indicated by the wide two standard deviation error bands.

# 3.2.2 Variance decomposition: what explains the variance of output and that of country spread?

Table 2 contains the results of the forecast error variance decomposition on a sixteen quarter horizon.

		Output	variance		Spread	varaince	
	Quarters	y <sub>t</sub>	$r_t^*$	$S_t$	y <sub>t</sub>	$r_t^*$	$S_t$
Bulgaria	4	95.3751	2.2368	2.3880	10.0210	3.0033	86.9755
	8	91.7310	2.2802	5.9887	15.479	2.2384	82.2816
	12	90.3206	2.2476	7.4316	17.0513	2.0493	80.8992
	16	89.8322	2.2339	7.9338	17.5459	1.9913	80.4626
Hungary	4	72.2284	1.3961	26.3754	35.2288	4.1821	60.5889
	8	57.4539	2.3600	40.1860	33.4862	4.3081	62.2055
	12	54.5535	2.5703	42.8760	34.8090	4.2063	60.9846
	16	54.7461	2.5595	42.6942	34.9736	4.1850	60.8412
Poland	4	93.9323	5.0588	1.0088	8.4722	1.3590	90.1686
	8	90.6107	5.4847	3.9045	28.7680	1.8831	69.3488
	12	88.6454	5.4679	5.8865	39.0853	2.5149	58.3996
	16	88.0392	5.4279	6.5327	41.8629	2.7584	55.3785

 Table 2. Forecast Error Variance Decomposition Predicted by the

 Estimated VAR

By examining the figures, a series of common qualitative characteristics can be noticed across countries besides the quantitative differences. Firstly, Euro Zone interest rate explains roughly a constant and relatively low proportion of output and country spread variance, both across countries and over time, i.e. there is no evidence for possible amplification of the explanatory power of  $r^*$  over time. This might suggest that Eurozone interest rate has no importance in explaining output variance in the sampled CEECs countries.

Secondly, as time passes more and more variation of output is explained by country spread, and a higher and higher proportion of country spread variance is explained by output, suggesting a strong feedback relationship between country spread and output. For example, in the case of Hungary, country spread explains about 26% of the variation of output after one year and this proportion becomes stable at 42% after three years. A similar tendency can be noticed in the case of the variation of country spread: output explains about 34% of the country spread variance and this value remains approximately constant over time. A comparable qualitative conclusion can be drawn about the forecast error variance decomposition results in the case of Bulgaria and Poland, but quantitatively the figures are somewhat lower.

If one rigorously consider the definition of country spread (i.e. it is an indicator of default risk) and takes into account that it has a series of determinants - like terms of trade, debt to GDP ratio, reserves to GDP ratio, credit ratings or global factors as documented by Hilscher and Nosbusch (2010) - she can conclude that the strong interconnection between country spread and output might suffer of omitted variable bias (i.e. it is overestimated relative to its true value). However, by adding some of the previously listed variables to the VAR model, the number of parameters to be estimated would increase. This would reduce the credibility of the point estimates because more parameters should be estimated by using the same small number of observations.

Finally, financial variables (i.e. country spread and Euro Zone interest rate jointly) explain different proportion of output variance across countries: about 11% of the Bulgarian and Polish output variance and about 45% percent of the Hungarian output variance. This result, however, indicates that external interest rate (i.e. the interest rate that small CEECs faces on international financial markets), which can be decomposed into country premium and world interest rate, would have some role in driving business cycle fluctuations in the CEECs.

The last empirical exercise consists of the forecast error variance decomposition of investment and country spread based on the estimation of the second VAR specification. Table 3 shows the results.

		Investment	variance		Spread	variance	
	Quarters	$i_t$	$r_t^*$	$S_t$	$i_t$	$r_t^*$	$S_t$
Hungary	4	96.0456	1.3781	2.5761	5.8163	11.0142	83.1693
	8	94.8883	1.6409	3.4707	7.5870	11.1998	81.2131
	12	94.7926	1.6611	3.5461	7.7346	11.2086	81.0568
	16	94.7857	1.6626	3.5516	7.7453	11.2091	81.0454
Poland	4	98.8926	0.3374	0.7699	7.9451	1.9794	90.0753
	8	96.9358	0.4601	2.6039	32.1722	1.2766	66.5511
	12	95.2457	0.5264	4.2277	48.9496	1.0080	50.0423
	16	94.1357	0.5599	5.3042	56.4726	0.9300	42.5972

Table 3. Forecast Error Variance Decomposition Predicted by theEstimated VAR

The figures highlight general characteristics comparable with those obtained from the previous exercise, like the Eurozone interest rate has unimportant role in explaining investment and country spread variance, while there is a strong feedback relationship between output and country spread. However, in this case investment explains a substantially lower proportion of country spread variance than output does in the case of Hungary, i.e. about 7% instead of the 34% observed in the previous case. Overall, financial variables explain a smaller part of the variance of investment both in Hungary and Poland (i.e. about 5% of the Hungarian and 6% of the Polish investment variance is explained jointly by country spread and external interest rate) than it resulted from the previous exercise.

Concerning the overall quantitative performance of the previously presented results, two possible weaknesses created by the quality of data must be mentioned: (i) the length of the time series is limited by the availability of macroeconomic data for CEECs before 1995, and (ii) the presence of possible measurement errors in the data. Both of them might alter the consistency of the point estimates by creating biases in different directions, while some asymptotic properties of the estimator might not hold because of the fact that too many coefficients are estimated by using few numbers of observations.

## 4 Theoretical Approach or "What kind of RBC model could replicate the stylized facts?"

In order to create a theoretical framework consistent with the stylized facts assessed in the previous section I build heavily on the encompassing model developed by Chang and Fernandez (2009) since it contains the modeling tools needed to replicate emerging market business cycle stylized facts.<sup>19</sup> I propose two extensions to this model: (1) since Chang and Fernandez (2009) assume that households and firms pay different interest rates, a fairly natural extension of the model considers that the two types of agents faces the same interest rate (i.e. uniform interest rate model), and (2) I assume that country spread can be decomposed in two components: an endogenous component determined by economic fundamentals (i.e. Solow residual, world interest rate and the debt elastic component and the lags of country spread) and an exogenous component which accounts for the fact that country spread can be influenced by exogenous shocks (e.g. news shocks, investors sentiment, political climate in a given country).

The main motivations of these two extensions are manifold, being strongly related to economic intuition, to empirical literature focusing on the determinants of country spread and to the stylized facts suggested by the data in the case of CEECs. First of all, in a small open economy RBC model based on the assumption that households are the owners of the firms and have access to the international financial markets, it is counterintuitive to assume that households, when they have positive external debt position, borrow at a higher interest rate from international financial markets than the rate at which they provide loans to firms. Hence, by introducing minor modifications into the Chang and Fernandez (2009) framework, this issue can be eliminated and the theoretical framework becomes more consistent with the stylized facts, i.e. the country spread depends on the external debt position in a non-linear way.

Secondly, the EMBI data suggest that country spread is a persistent variable, while the estimated impulse response functions and the forecast error variance decomposition suggest a feedback relationship between country spread and Euro Zone interest rate, on the one hand, and between domestic output and Euro Zone interest rate on the other hand. Thus, the theoretical framework would become more consistent with the stylized facts by assuming that the current level of country spread depends in an autoregressive way on its past realizations and it is a function of Euro Zone interest rate besides the Solow residual. Finally, the exogenous component of the country spread can be modeled as an exogenous stochastic

<sup>&</sup>lt;sup>19</sup>For example, permanent and transitory technology shocks, working capital constraint coupled with endogenous country spread and capital adjustment cost.

process (e.g. an i.i.d. process with zero mean and  $\sigma_s$  variance) which captures the impact of the above mentioned exogenous factors on the country spread.

In the rest of this section, I present a detailed description of the two main blocks of the small open economy RBC model proposed by Chang and Fernandez (2009). This basic setup amalgamates the main features of two leading types of models from the international emerging RBC literature: the Neumeyer and Perri (2005) model and the Aguiar and Gopinath (2007a) stochastic trend model. Then I proceed in describing the way in which the model is closed when the two different extensions are considered. Moreover, I present the estimation strategy applied to parameterize the model and, finally, a model evaluation exercise based on a comparative analysis of theoretical versus historical business cycle moments and impulse responses.

### 4.1 The Basic Setup

### 4.1.1 Households' behavior

The representative household chooses the sequence of values for  $\{C_t, h_t, I_t, K_{t+1}, D_{t+1}, \}_{t=0}^{\infty}$ in order to maximize the expected discounted sum of lifetime utility derived from the consumption of goods and leisure<sup>20</sup>:

$$\max_{\{C_t, h_t, K_{t+1}, D_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, h_t, \Gamma_{t-1}\right)$$
(5)

where  $C_t$  is time t consumption,  $h_t$  labor supplied by the household at time t,  $\beta$  is the discount factor,  $\Gamma_{t-1}$  allows for balanced growth path in utility<sup>21</sup>, U() is twice differentiable concave utility function, increasing in its first argument and decreasing in its second argument.

The household acquires income by supplying labor  $(h_t)$  to the firms and getting the wage  $w_t$  in exchange, by renting capital  $(K_t)$  to the firms at the rental rate of capital  $u_t$ , and by borrowing from  $(D_{t+1} > 0)$  or by investing  $(D_{t+1} < 0)$  in one period noncontingent bonds on the international financial market. Thus, one unit of foreign asset or debt costs  $q_t$ <sup>22</sup> in units of consumption goods. The household uses this income to purchase consumption goods,  $C_t$ , to invest in investment goods,  $I_t$  and to pay back previous loans  $(D_t > 0)$  contracted from or to sell international bonds  $(D_t < 0)$  purchased from international financial markets. In this way she accumulates two types of capital stock which she owns entirely: physical capital (K) and internationally traded noncontingent bond (D). Thus, the household's per period

 $<sup>^{20}</sup>$ The solution of the household's and firm's optimization problem is presented in detail in the Model Appendix

<sup>&</sup>lt;sup>21</sup>More details about the way balanced growth path is defined you can find in the next subsection.

<sup>&</sup>lt;sup>22</sup>The definition of  $q_t$  varies across extensions and is presented in the "Closing the model" section.

budget constraint is:

s.t. 
$$w_t h_t + u_t K_t + q_t D_{t+1} = D_t + C_t + I_t$$
 (6)

where the left-hand side of the identity characterizes the structure of the per period income while the right-hand side represent the per period expenditures.

Changing the capital stock is costly and in each period t it is realized according to a capital accumulation rule which states that the per period capital stock is nothing else than the sum of the existing capital stock net of depreciation and current investments minus the adjustment cost,  $\Phi(K_{t+1}, K_t)$ , paid for each unit of capital accumulated:

$$K_{t+1} = (1 - \delta) K_t + I_t - \Phi (K_{t+1}, K_t) K_t$$
(7)

This modeling technique is frequently used in the RBC literature because it improves the performances of the model to generate moderate investment volatility and increases the persistence of investment. By assumption,  $\Phi()$  is a strictly increasing, concave function.

### 4.1.2 Firms' behavior

The representative firm behaves in a perfectly competitive way, hires labor  $(h_t)$  and rents capital  $(k_t)$  in order to produce consumption goods  $(Y_t)$  based on a neoclassical production technology (F()):

$$Y_t = a_t F\left(K_t, \Gamma_t h_t\right) \tag{8}$$

where  $a_t$  models transitory technology improvement defined as an exogenous autoregressive stationary ( $|\rho_a| < 1$ ) process of order one

$$\log a_t = \rho_a \log a_{t-1} + \epsilon_t^a \tag{9}$$

it captures shocks to total factor productivity as one source of uncertainty through  $\epsilon_t^a$  *i.i.d.* process with mean zero and standard deviation  $\sigma_a$ .  $\Gamma_t$  models permanent technology improvements as a labor augmenting productivity growth. As Aguiar and Gopinath (2007a) showed, by assuming stochastic trend improvement in the neoclassical growth model, it does better job in replicating emerging market business cycle stylized facts like the countercyclical nature of trade balance to output ratio or higher volatility of consumption than output.

The main intuition is related to the fact that as a result of a positive permanent productivity shock, labor productivity increases permanently which generates a higher increase in permanent income and, consequently, in consumption than in current income which potentially is financed by issuing external debt. Thus, in line with Aguiar and Gopinath (2007a), permanent technology shocks act through an exogenous and mean reverting stationary process:

$$\Gamma_t = g_t \Gamma_{t-1} \tag{10}$$
$$\log\left(g_t/\mu\right) = \rho_g \log\left(g_{t-1}/\mu\right) + \epsilon_t^g$$

where  $|\rho_g| < 1$ ,  $\epsilon_t^g$  is an *i.i.d.* process with mean zero and standard deviation  $\sigma_g$  and it models shocks to labor productivity which are incorporated into  $\Gamma_t$  through  $g_t$ , resulting in trend growth improvements.  $\mu$  represents long-run labor productivity growth.

The representative firm, like households, is an optimizing agent having the primary objective to maximize the total discounted sum of all future profits  $(\Pi_t)$  subject to a working capital constraint:

$$\max_{\{a_t,h_t,k_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \Pi_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t F\left(K_t, \Gamma_t h_t\right) - w_t h_t \left[ 1 + \theta \left(R_{t-1} - 1\right) \right] - u_t K_t \right\}$$
(11)

In other words, the working capital constraint induces an additional friction into the model since the representative firm must borrow at  $R_{t-1}$  rate of interest from the representative household in each period in order to be able to finance  $\theta$  fraction of the wage bill at the beginning of each period, in advance of the realization of income. Neumeyer and Perri (2005) and Uribe and Yue (2006) argue that this financial friction creates a linkage between interest rate movements and real economic activity, which improves the ability of the baseline RBC model to generate strong countercyclical trade balance through the following mechanism: falling interest rate, on the one hand, reduces the cost of labor allowing firms to hire more labor and consequently to produce more output. On the other hand, it diminishes the cost of borrowing by boosting, in this way, aggregate demand which generates deterioration in trade balance. Thus output and trade balance move in the opposite direction when the cost of financing through borrowing changes.

### 4.1.3 Closing the Model

In order to solve the model (i.e. derive the equilibrium conditions which characterizes the representative agents'optimal decisions in the small open economy) a few more assumptions are needed concerning the dynamics of domestic and world interest rates, country spread, the price paid by the representative household for the foreign debt and functional forms of

the utility, production and capital adjustment cost function. Chang and Fernandez (2009) assume that whenever the optimal level of foreign debt issued by the representative household deviates from the steady state debt level the interest rate at which households borrow from international financial markets  $(1/q_t)$  is higher than the rate at which they lend money  $(R_t)$  to the representative firm. Put differently, they assume that the domestic interest rate at which households lend to firms is defined as the product of gross country spread  $(S_t)$  and gross foreign interest rate  $(R_t^*)$ :

$$R_t = S_t R_t^* \tag{12}$$

The dynamic of world interest rate is described by a mean reverting stationary ( $|\rho_{r^*}| < 1$ ) first order autoregressive process:

$$\log\left(R_t^*/\overline{R}^*\right) = \rho_{r^*} \log\left(R_{t-1}^*/\overline{R}^*\right) + \epsilon_t^{r^*}$$
(13)

where  $\epsilon_t^{r^*}$  is an *i.i.d.* process with zero mean and standard deviation  $\sigma_{r^*}$ . Moreover, they assume, in line with Neumeyer and Perri (2005) and Aguiar and Gopinath (2007b) that country spread is endogenous and it is inversely related ( $\eta > 0$ ) to future expected productivity improvements in the domestic economy, i.e. as a positive technology shock hits the economy, it is expected that the default risk of the country decreases dampening the spread and, consequently, the cost of foreign borrowing.

$$\log\left(S_t/\overline{S}\right) = -\eta E_t \left[\log a_{t+1} + \log\left(g_{t+1}^{\alpha}/\mu^{\alpha}\right)\right] \tag{14}$$

However, Chang and Fernandez (2009) state that the price of one unit of foreign debt that the representative household issues depends on the domestic interest rate  $(R_t)$  and their degree of indebtedness  $(D_t)$  fed into the cost of borrowing through an interest rate premium,  $\Psi()$ , defined as an increasing and convex function of the external debt position of domestic household:

$$\frac{1}{q_t} = R_t + \Psi \left( D_{t+1} / \Gamma_t \right) \tag{15}$$

$$\Psi\left(D_{t+1}/\Gamma_t\right) = \psi\left[\exp\left(\frac{D_{t+1}}{\Gamma_t} - \overline{d}\right) - 1\right]$$
(1)

Thus, whenever the households' external debt position exceeds the steady state one, they pays higher interest rate (i.e. this is because  $\Psi() > 0$ ) when borrowing from international financial markets than the rate at which they lend to the representative firm. This is the case

when  $1/q_t > R_t$ . When the external debt position is lower than the steady state one exactly the opposite to the previous case happens, i.e.  $1/q_t < R_t$ . Under the (14)and (15)assumptions the only case when households and firms face the same interest rate is when the external debt position is at its steady state level. At the same time, this convex debt adjustment cost function ensures the stationary behavior of the model as shown in Schmitt-Grohe and Uribe (2003).

Uniform interest rate and nonpersistent country spread In the first extension of the Chang and Fernandez (2009) model, I assume that households and firms face the same interest rate. Thus I rewrite equations (14) and (15) such that country spread is not only the function of the permanent and transitory technology shocks, but also it depends on the household's external debt position:

$$\log\left(S_t/\overline{S}\right) = -\eta\left[\log a_{t+1} + \log\left(g_{t+1}^{\alpha}/\mu^{\alpha}\right)\right] + \Psi\left(D_{t+1}/\Gamma_t\right)$$
(14')

In this way the price of the external debt issued by the representative household and, respectively, the interest rate becomes:

$$\frac{1}{q_t} = R_t \tag{15'}$$

Thus, since country spread is a component of domestic interest rate, changes in the external debt position alters both qualitatively and quantitatively in the same way the cost of borrowing faced by the two types of agents, i.e. they always pay the same interest rate under (14') and (15') assumptions, while (12) and (13) remain the same as in the previous case.

Uniform interest rate and persistent country spread In the second extension of the Chang and Fernandez (2009) model, I assume that (12), (13) and (15') hold while the country spread exhibits a certain degree of persistence ( $\rho_s$ ), it reacts to fluctuations in world interest rate and to exogenous country spread shocks besides being a function of technology shocks and the external debt position:

$$\log\left(S_t/\overline{S}\right) = -\eta_{SR}\left[\log a_t + \log\left(g_t^{\alpha}/\mu^{\alpha}\right)\right] + \Psi\left(D_{t+1}/\Gamma_t\right) + \eta_{r^*}\log\left(R_t^*/\overline{R}^*\right) + \rho_s\log\left(S_{t-1}/\overline{S}\right) + \epsilon_t^s$$
(14")

More specifically, I assume that country spread follows a mean reverting first order stationary ( $|\rho_s| < 1$ ) autoregressive process augmented by a nonlinear component,  $\Psi(D_{t+1}/\Gamma_t)$ , which captures the impact of changes in external debt position on country spread, and the Solow residual which controls for potential feedback relationship between country spread and the real economy. This definition of country spread is more consistent with the evidences provided by the empirical literature which studies the determinants of country spread (i.e. country specific fundamentals and global factors), it takes into account that country spread is a persistent variable and its level can also be altered by exogenous shocks.

In addition, from technical point of view, it is expected that the persistent endogenous country spread, as it is defined (14"), serves as an amplification mechanism of the impact of world interest rate shocks on the domestic real economy. Since when country spread is persistent, current changes in  $r^*$  are fed into the future realization of country spread through the autoregressive term, which ensures that a contemporaneous increase in the world interest rate will generate higher external interest rate also in the future and, consequently larger drops in domestic output. This is because an  $r^*$  shock coupled with persistent country spread distorts more the intertemporal rate of substitution faced by the household and the firms'marginal factor productivity ratio than was the case in the other two versions of the model. Since agents have perfect foresight in this model, they consume and invest less in order to build up larger savings relative to the non-persistent interest rate case, ensuring in this way smooth consumption over time. This implies lower domestic output and slower adjustment of output back to its initial level after the occurance of a world interest rate shock.

**Functional forms** Household's preferences are modeled according to Greenwood et al. (1988) (GHH) preferences<sup>23</sup> since many papers show that they improve the performances of international RBC models in replicating business cycle stylized facts.

$$U(C_t, h_t, \Gamma_{t-1}) = \frac{(C_t - \tau \Gamma_{t-1} h_t^{\omega})^{1-\sigma}}{1-\sigma}, \ \omega > 1, \ \tau > 0$$
(16)

As Mendoza (1991), Neumeyer and Perri (2005) or Chang and Fernandez (2009) argue GHH preferences play an important role in modeling emerging economy business cycles, since they generate labor supply which is independent of consumption and, consequently, of interest rate shocks. When an unexpected interest rate shock hits the economy and firms face working capital constraint, the change in labor demand induced by a higher or lower interest rate will generates an adjustment in the equilibrium employment level which magnitude will depend only on the nature of the labor supply unaltered by interest rate shocks. Thus, GHH preferences can generate larger shifts in equilibrium employment than Cobb-Douglas

<sup>&</sup>lt;sup>23</sup>Cobb-Douglas preferences are also widely used but, as Neumeyer and Perri (2005) show, they have poorer modeling performances in replicating emerging market stylized fact than GHH preferences.

preferences and respectively a larger adjustment in the aggregate output.

Capital adjustment cost is modeled as a quadratic function of the current and past capital stock level and it captures the fact that faster adjustments in the capital stock are more expensive:

$$\Phi(K_{t+1}, K_t) = \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - \mu\right)^2$$
(17)

The production function is assumed to be Cobb-Douglas, where  $\Gamma_t$  allows for labor augmenting productivity growth and  $\alpha$  represents the share of labor in total income:

$$F(K_t, \Gamma_t h_t) = K_t^{1-\alpha} \left(\Gamma_t h_t\right)^{\alpha}$$
(18)

Finally, the trade balance to output ratio is defined as:

$$TBY_t = \frac{Y_t - C_t - I_t}{Y_t} \tag{19}$$

In order to get the model to the data, a log-linear solution to the stationarized version of the model has to be derived. Therefore I proceed in the following way: (1) I stationarized the model by detrending all the variables which exhibit long run growth (2) then I derive the first order conditions, which characterize the optimal behavior of the two types of agents in the economy (3) finally, I log-linearized the optimality conditions around the steady state of the model. The resulting linear rational expectation system of difference equations which fully characterizes the optimal dynamics of the state and control variables was solved in Matlab.<sup>24</sup>

### 4.2 Calibration and estimation

In order to solve the linear rational expectation system of difference equations, which fully characterizes the optimal dynamics of the model economy, values to the model parameters must be assigned. I do this in two different ways: I estimate, by using Bayesian techniques, a set of parameters which are considered in the literature as being country specific or difficult to calibrate and I calibrate the other set of parameters.

#### 4.2.1 Calibrated parameters

I set the value of calibrated parameters based on long averages provided by macroeconomic data in the case of each country, or I set them to values commonly used in emerging market

<sup>&</sup>lt;sup>24</sup>More information about the steps followed and the algebra is included in Model appendix

	Bulgaria	The Czech Republic	Hungary	Poland	Romania
α	0.6742	0.6012	0.6097	0.6820	0.6428
δ	0.025	0.025	0.025	0.025	0.025
σ	2	2	2	2	2
τ	s.t. $\overline{h}$	s.t. $\overline{h}$	s.t. <i>h</i>	s.t. $\overline{h}$	s.t. $\overline{h}$
ω	1.6	1.6	1.6	1.6	1.6
β	0.98	0.98	0.98	0.98	0.98
μ	1.0130	1.0106	1.0071	1.0110	1.0151
$\overline{r}^*$	1.0039	1.0039	1.0039	1.0039	1.0039
$\overline{d}$	0.4457	0.1703	0.7617	0.3545	0.3145
$\overline{h}$	0.3877	0.4665	0.3912	0.3828	0.4385

business cycle literature. These are presented in Table 4 for each country.

 Table 4. Calibrated Model Parameters

The labor share of income ( $\alpha$ ) is set at the ten years average of the compensation of labor input to value added ratio in the case of each country. <sup>25</sup> The rate of capital depreciation,  $\delta$  is 0.025, which would imply a 10% annual depreciation rate and which is commonly used in emerging market business cycles papers. The preference parameters are set as follows: the coefficient of relative risk aversion ( $\sigma$ ) defining the curvature of the utility function is 2;  $\omega$ , the exponent of labor in the utility function is 1.6 and the weight of labor in the per period utility ( $\tau$ ) is set such that it implies that households allocate  $\overline{h}$  share of their total time to working. These definitions and parameterizations are in line with those assumed by Schmidt-Grohe and Uribe (2003), Neumeyer and Perri (2005), Garcia-Cicco et al. (2010) and others.  $\beta$  discount factor is set at 0.98 which would imply a 2.04% quarterly risk free real interest rate.

The long run productivity growth ( $\mu$ ) is defined as average real GDP growth rate over the period 2000-2009; <sup>26</sup> The steady state level of gross foreign interest rate ( $\bar{r}^*$ ) is computed as the quarterly average of the Euro Zone real interest rate over 2000-2009 period.  $\bar{d}$ , the long run external debt to GDP ratio, is proxied as the 1997-2007 average of the net foreign

<sup>&</sup>lt;sup>25</sup>Details concerning the data sources and definitions of different time series are included in Data Appendix.

<sup>&</sup>lt;sup>26</sup>This type of association between parameter and statistical definition is used by Aguiar and Gopinath (2007a) in the case of Argentina and than it was taken over by other papers, e.g. Aguiar and Gopinath (2007b), Chang and Fernandez (2009).

asset to GDP ratio. The steady state share of labor  $(\overline{h})$  is defined as the ten years average employment to total population ratio.

### 4.2.2 Estimated parameters

I use Bayesian techniques as proposed by Smets and Wouters (2003), Lubik and Schorfheide (2005) and An and Schorfheide (2007) over the sample period 1995Q1:2009Q4<sup>27</sup> in order to estimate those parameters which are country specific (e.g. working capital parameter, capital adjustment cost coefficient) or difficult to calibrate (e.g. the persistence and standard deviation of different shocks, the elasticity of country spread with respect to the Solow residual). These authors argue that the main advantage of the Bayesian estimation of DSGE models in comparison with moment or impulse response matching is that the parameters are seen as random variables, and a prior density which incorporate initial beliefs and information about these parameters is specified together with a likelihood function for the DSGE model. Based on these, the posterior mode of each parameter can be computed and by using Bayes theorem to update initial beliefs, the conditional distribution of each parameter (i.e. posterior distribution) is calculated given the observable variables (i.e. data) and the model. An and Schorfheide (2007) point out that the main benefit of this posterior distribution consists of the fact that it permits to perform inference concerning the parameters and to conduct a likelihood based checking of the goodness of fit.

Basically, this is a two step procedure: (1) in the first step the posterior mode is computed by using an optimization routine to find that value of the parameters which minimizes the negative log-likelihood given the model, the data based on which we perform the estimation and prior probability distribution of the parameters to be estimated.<sup>28</sup> The inverse of the Hessian of the log-likelihood function is evaluated at the optimal posterior mode. (2) The second step consists of running a Markov Chain Metropolis-Hastings (MCMH) algorithm which constructs a Gaussian approximation around the posterior mode in the following way: it draws a number from a normal distribution characterized by mean equal to the posterior mode obtained in the previous step and variance-covariance matrix set at the inverse of the log-likelihood function evaluated at the same posterior mode and scaled up by a constant (i.e. jumping distribution). Then it decides whether to accept this newly drawn value as the mean of the jumping distribution used for the next draw with a certain probability (i.e. acceptance ratio computed as the ratio between the log-likelihood of the model evaluated

<sup>&</sup>lt;sup>27</sup>This procedure is implemented in Dynare toolbox which I used to run the Metropolis-Hastings algorithm in order to trace the distribution around the posterior mode.

<sup>&</sup>lt;sup>28</sup>To find the optimal value of the posterior mode I use a Monte-Carlo based optimization routine, with 100000 simulations, implemented in Dynare. This turned out to be a more robust method than solving a simple constrained minimization problem.

at the newly drawn parameter and the same likelihood evaluated at the parameter value obtained from the previous draw) or to continue with the parameter value obtained from the previous draw, with probability one minus the acceptance ratio. The more these draws are repeated the more efficiently and effectively the algorithm can explore the posterior distribution in the neighborhood of the posterior mode.

Therefore three main ingredients are needed to implement the above described procedure: the model, the observable variables and the definition of the prior distribution of the estimated parameters. Firstly, the model has to be written in a linear rational expectation system form which is solved by using numerical methods.<sup>29</sup> Then the model is written in state space form together with the measurement equations while its likelihood is computed by using the Kalman filter.

Secondly, the observable variables must be defined and fed into the model. I use four observable variables because they are available in the case of all countries and the methodology based on which they are collected should not differ across countries. These are the HP filtered quarterly gross domestic product (obsY), consumption (obsC) and investment (obsINVE) taken in logarithm and HP filtered trade balance to output ratio (obsTBY) over the period 1995:1-2009:4.

I assume that these observables are subject to measurement errors because of two reasons. One reason is technical in nature, and it aims to overcome the stochastic singularity problem, i.e. the number of structural shocks in the model must be equal to the number of observables considered for the estimation. Since in the present models there are three, respectively four shocks, at least one measurement error must be included while estimating the uniform interest rate and non-persistent country spread version of the model or at least two measurement errors have to be defined when estimating the uniform interest rate and persistent country spread extension. I assume that the measurement errors  $\{\epsilon_{obsY}, \epsilon_{obsC}, \epsilon_{obsI}, \epsilon_{obsTBY}\}$ follow *i.i.d.* processes with zero mean and  $\sigma_{z2=\{\epsilon_{obsY},\epsilon_{obsC},\epsilon_{obsI},\epsilon_{obsTBY}\}}$  standard deviation. The other reason for defining measurement errors is related to the potential role they play in conducting robustness check of the results. Therefore, when estimating the model I distinguish two cases: estimation without measurement errors, when I augment the state space just by as many measurement errors as many are needed to overcome the stochastic singularity problem, versus estimation with measurement errors, when I consider that all the observables contain measurement error. By considering that macroeconomic data usually contain errors and omissions because of aggregation, changes in methodologies, definitions,

 $<sup>^{29}</sup>$ For this I use perturbation methods implemented in Dynare toolbox which provide as a solution a linear dynamic version of the model which contains the dynamics of the state and control variables, i.e. policy functions.

rules and regulations applied during collecting them the inclusion of measurement errors should improve the estimation results if the errors are indeed influential.

Finally, the prior distribution is usually subjective because it describes uncertainty and prior knowledge about the model and its parameters. I select the shape of the distribution based on those applied in earlier papers on emerging market business cycles and based on the restrictions regarding possible domain of definition of different parameters. The expected value and standard deviation of each distribution assumed are set at values provided by macroeconomic data. Overall, I estimate thirteen structural parameters, as reported in Table 5 which can be divided into two groups.

Parameter	Prior shape	Expected value	Std. dev.
$\sigma_{z1}$	Gamma	0.2	0.1
	Inv Gamma	0.2	Inf
$ ho_a$	Beta	0.9	0.01
$ ho_g$	Beta	0.72	0.01
$ ho_{r^*}$	Beta	0.8	0.01
$ ho_s$	Beta	0.65	0.01
$\eta_{SR}$	Gamma	0.5	0.1
$\eta_{r^*}$	Gamma	0.5	0.1
Ψ	Gamma	0.15	0.05
$\phi$	Gamma	6	2
$\theta$	Beta	0.5	0.1
$\sigma_{z2}$	Gamma	0.02	0.01
	Inv Gamma	0.02	Inf

Table 5. Prior Distribution of the EstimatedModel Parameters

The first group contains the persistence and the standard deviation of the shock processes. In line with previous studies, I assume that the persistence of the AR(1) shock processes follow *Beta* distribution because this distribution is the most suitable for parameters taking values between zero and one. The standard deviation of this distribution is harmonized across different shocks and it is set at 0.01 while the mean of the distribution differs across shocks. Since the transitory technology shock is documented in the literature as
persistent (e.g. Smets and Wouters, 2007; Chang and Fernandez, 2009; Garcia-Cicco et al., 2010), I set the mean of the distribution of  $\rho_a$  to 0.9; the mean of the persistence of permanent technology shock ( $\rho_g$ ) is set at 0.72 since previous estimation results reported in the RBC literature suggests that it is less persistent than the transitory one; quantitative macroeconomic literature provides a series of evidences that US and Euro Zone real interest rates are both highly persistent or they follow an almost random walk process. Uribe and Yue (2006) estimate an AR(1) process on the 3-month real gross Treasury bill rate and they obtain a significant point estimate of the persistence parameter equal to 0.83. Similarly, Lubik and Schorfheide (2005) estimate the same parameter and get 0.84 in the case of the US and 0.83 in the Eurozone. Thus I set the mean of the distribution of  $\rho_{r^*}$  at 0.8 while the distribution of the persistence of country spread is assumed to have mean 0.65 in line with the autocorrelation coefficient of the EMBI spread reported in the first section of the paper.

The distribution of the standard deviation of the shock processes  $(\sigma_{z1=\{\epsilon_a,\epsilon_g,\epsilon_r,\epsilon_s\}})$  is considered being the same across different types of shocks. Generally, in the quantitative macro literature dealing with Bayesian estimation of DSGE models, standard errors of shocks are assumed to follow inverse gamma distribution mainly because this is a sensible distribution to relative shock sizes<sup>30</sup>. Moreover, gamma distribution could be another good candidate for the distribution followed by these standard deviations because it is used in the case of parameters which take positive values. Thus I estimate each specification of the model by considering two cases: first, I assume that the standard deviation of the shocks follows gamma distribution with mean 0.2 and standard deviation 0.1, while, in the other case, I consider inverse gamma distribution with mean 0.1 and scale parameter infinity.

The second group of structural parameters consists of those parameters which characterize financial decisions in the model. These are the elasticities of country spread with respect to Euro Zone interest rate  $(\eta_{r^*})$  and Solow residual  $(\eta_{SR})$ . Since with the exception of  $\eta_{SR}$  in the case of Argentina there are no previous evidences concerning the Bayesian estimation of these parameter I assume that they follow Gamma distribution (i.e. they take positive values because it is intuitive to assume that country spread increases as Euro Zone interest rate gets higher or when a negative productivity shock hits the economy) with mean 0.5 and standard deviation 0.1. The mean of the distribution was set based on the observed correlation coefficient between EMBI spread and Euro Zone interest rate, on the one hand, and EMBI spread and output, on the other hand.

Another financial parameter of the model is the debt-elastic interest rate parameter  $(\psi)$  which measures the sensitivity of interest rate/country spread to fluctuations in external

<sup>&</sup>lt;sup>30</sup>For a detailed description of the most frequently used distributions in DSGE Bayesian estimation, advantages and shortcomings of them consult Ermolaev et al. (2008): Estimating GPM with Dynare mimeo.

debt position. This parameter is usually not estimated and is calibrated at 0.001, value which was proposed by Mendoza (1991) in the case of the Canadian economy. Garcia-Cicco et al. (2010) are the only authors who estimate this parameter, assuming that it follows uniform distribution. Thus it is worth to pay particular attention to this parameter and estimate it because it could easily happen that in the case of emerging economies the data would attribute a higher value to it. Logically it can be expected that country spread is more sensitive to movements in debt position in emerging economies than in developed countries; hence calibrating it to 0.001 would be erroneous. I assume that this parameter follows a gamma distribution with mean 0.15 and standard deviation 0.05. This is a rather loose prior but since there is no previous evidence on estimating it, the prior should take into account this uncertainty (i.e. this is why the standard deviation of the distribution takes relatively high value).

Because the capital adjustment cost parameter ( $\phi$ ) is a positive parameter it is assumed to follow a gamma distribution with mean 6 and standard deviation 2.<sup>31</sup> This is a somewhat tighter prior than those assumed by Chang and Fernandez (2009) who estimate  $\phi$  by considering that it follows a gamma (3,2) distribution in the case of Mexico or Smets and Wouters (2007) who assume normal (1.25, 0.24) in the case of Eurozone. However, this is in line with Garcia-Cicco et al. (2010) who consider gamma(8,4) distribution. The working capital constraint ( $\theta$ ) parameter takes values between zero and one, thus I assume that it follows Beta distribution with mean 0.5 and standard deviation 0.1. The main motivation for setting the mean of this parameter to 0.5 is provided by the data which suggests that the cross country average of the short run private credit of non-financial corporations to value added ratio is around 0.45. This assumption is similar to those made by Chang and Fernandez (2009).

Finally there is a set of nonstructural parameters which consists of the standard deviation of measurement errors  $(\sigma_{z2=\{\epsilon_{obsY},\epsilon_{obsC},\epsilon_{obsI},\epsilon_{obsTBY}\}})$ . I estimate the model by assuming that these parameters follow: inverse gamma with mean 0.02 and scale parameter set to infinity or gamma distribution with mean 0.02 and standard deviation 0.01. This is a wide prior, but in this way uncertainty concerning the values of these standard deviations can be incorporated into the prior knowledge.

I estimate four specifications of the three versions of the model: (i) the Chang and Fernandez (2009) model with their original specification that the two types of agents pay

<sup>&</sup>lt;sup>31</sup>I estimated the model by assuming a loose prior for the distribution of  $\phi$  like gamma (3,2) for the first time and then I systematically tightened the prior by increasing the mean of the distribution. It turned out that indifferently of how tight or loose the prior is the data and the model had the tendency to push up the posterior mode of  $\phi$  to values around 6 across all the countries. Hence I set the mean at 6 in order to obtain a better fit.

differentiated interest rates (ii) one extension of the model which assumes that the interest rate paid is uniform across the types of agents (iii) another extension which assumes that country spread has both exogenous and endogenous components (i.e. country spread is modeled as an AR(1) process augmented by Euro Zone real interest rate, Solow residual and the external debt indicator). The four specifications differ by the prior distributions considered for the estimation, by the number of measurement errors included in the estimation and by the parameters estimated.

These different specifications aim to answer the following questions: Does inverse gamma distribution fit better the distribution of the standard deviation of the shock processes than the gamma distribution? Does adding measurement errors to the observables improve the goodness of fit of the model? What happens to other estimated parameters when  $\psi$  is calibrated and not estimated? This is because, it would be interesting to check which parameter estimates are changing in the small open economy RBC setup when  $\psi$  is calibrated to such a low value as reported by Mendoza (1991). Thus the four specifications in the case of each model are: (1) standard errors of the shock processes are assumed to follow inverse gamma distribution (I-Gamma), all the observables contain measurement errors and  $\psi$  is estimated, (2) assumes that standard errors of the shock processes follow gamma distribution (Gamma), all the observables contain measurement errors and  $\psi$  is estimated, (3) only as many variables contain measurement errors are gamma and  $\psi$  is estimated, and (4)  $\psi$  is calibrated (no PSI), standard errors are gamma and all the variables contain measurement error.

I set the algorithm to make one hundred thousand draws<sup>32</sup>; the initial 50% of draws were dropped out to ensure that the results do not depend on the initial value, and the scaling parameter in the jumping distribution was fine tuned (around 0.4) such that to obtain approximately one third acceptance ratio. Moreover, in order to conduct convergence diagnostic (i.e. ensure that two different chains converge to the same stationary distribution) I run two parallel Markov Chains and check the presence of convergence by comparing recursively computed second order moments<sup>33</sup> of the distributions constructed under each and every draw per chain.

<sup>&</sup>lt;sup>32</sup>Initially, I set the number of draws to half a million in the MCMH algorithm but due to the robustness of the optimization method used in the first step the two chains started to converge fast after 20000 draws. I checked if this convergence is present in the case of all the five countries by estimating the Chang and Fernandez (2009) version of the model with both 100000 and half a million draws. The estimation results and the convergence statistics confirmed that 100000 draws are enough to achieve convergence

<sup>&</sup>lt;sup>33</sup>These second order moments are the variance of the distribution, skewness and confidence interval constructed around the parameter mean.

#### 4.3 Estimation results

Tables A1-A5 reports the parameter estimates (i.e. posterior mean), their confidence intervals concerning the 90% high probability region and average acceptance ratios per chain in the case of the three model across the five countries by considering different prior, estimated parameter set and measurement error specifications as robustness check.<sup>34</sup> Goodness of fit statistics like marginal likelihood<sup>35</sup> and log data density are also reported in order to assess the performance of each model in fitting the data under different prior and parameter specification.

The main results can be summarized as follows: (1) The goodness of fit statistics indicate that the specification which fits the best the data assumes gamma distribution for the standard deviations of the shocks, all the observable variables contain measurement errors and the parameter  $\psi$  is estimated in the case of all the five sampled countries, (2) The point estimates of  $\psi$  suggest that it is erroneous to calibrate the debt elastic interest rate parameter to 0.001, as proposed by Mendoza (1991) and as it has been used so far in the literature. The data suggests that, given the considered model setup,  $\psi$  varies in [0.02, 0.23] interval across countries and across different model specifications, (3) The posterior means of the persistence and standard deviation parameters of the shocks are relatively stable across different model specifications, and (4) Financial parameters (like the elasticity of country spread, working capital constraint or capital adjustment cost parameter) vary slightly across different model versions. In addition, a certain degree of heterogeneity in the point estimates of these financial parameters can be observed across countries. The rest of this subsection discusses these results in more details.

Focusing firstly on the four different specifications in the case of each model, it can be inferred, based on the goodness of fit statistics, that the "Gamma" specification is always superior to the other three. The specification with inverse gamma distribution of the standard errors of the shocks has the poorest performance. (i.e. both the log data densities and marginal likelihoods are lower than in the case of "gamma specification").<sup>36</sup> By including measurement errors for all the observables, the goodness of fit statistics increase

 $<sup>^{34}</sup>$ The statistical significance of the estimated posterior mode was also verified by using t-statistics and it resulted that all the estimates were statistically significantly different of zero. The results of the tests are available upon request.

<sup>&</sup>lt;sup>35</sup>Laplace approximation is used to calculate marginal likelihood,  $ML(\theta) = \int_{\theta} p(\theta/M) p(Y/\theta, M) d\theta$ , as suggested by Geweke (1998), Schorfheide (2005) or An and Schorfheide (2005) which consist of approximating the marginal likelihood by applying a standard correction to the posterior mode ( $\theta$ ).

<sup>&</sup>lt;sup>36</sup>As well, convergence problems have been observed in the case of the two parallel chains, i.e. the convergence diagnostic signaled significant divergence in all three higher order moments especially in the case of the shock persistence and standard error parameters. This led to strange "spiny" shapes of the posterior distribution instead of the expected bell shape.

by almost 20%, signaling the fact that the observables might contain measurement errors. This means that the gamma prior distribution and the presence of the measurement errors in all the observables improve significantly the goodness of fit of the model. This result confirms the initial suspicions that macroeconomic data might be subject to measurement errors originating from the previously mentioned sources, while the sample is short which weakens the performance of the HP filter at the endpoints of the time series.<sup>37</sup>

When the debt elastic interest rate parameter  $(\psi)$  is not estimated, the estimates of financial parameters<sup>38</sup>, especially that of the capital adjustment cost parameter changes significantly, as reported in Table 6. Independently of the model version considered, by calibrating  $\psi$  to 0.001, as proposed by Mendoza (1991), the estimate of the capital adjustment cost parameter becomes more than double in comparison with the case when  $\psi$  is estimated. In addition, the elasticity of country spread and the working capital constraint parameter increase.

		Bulgaria	Czech Rep.	Hungary	Poland	Romania
C&F (2009)	ψ	0.0880	0.2653	0.1810	0.1694	0.2467
$\psi$ estimated	$\phi$	6.9844	4.5536	5.1354	3.4059	6.2795
$\psi$ calibrated	φ	17.1166	10.6866	13.7247	14.2256	14.4391
Uniform r	ψ	0.0447	0.0601	0.0721	0.0778	0.0806
$\psi$ estimated	φ	7.9387	7.7906	6.4808	4.9972	7.6186
$\psi$ calibrated	φ	17.2981	13.1446	13.6264	14.2024	14.6835
Persistenst s	ψ	0.0592	0.0937	0.0947	0.0620	0.1174
$\psi$ estimated	$\phi$	6.3813	6.4968	5.762	4.2809	8.2747
$\psi$ calibrated	$\phi$	14.1538	12.5412	13.0356	14.0662	14.8136

Table 6. The Posterior Mean of  $\psi$  and  $\phi$  under Different Model Specifications

This result is not counterintuitive at all and it actually underlines the role of capital adjustment cost in the model. When  $\psi$  is calibrated to a low value relative to what the data would imply the interest rate is less sensible to large movements in external debt, i.e.

<sup>&</sup>lt;sup>37</sup>Kaiser and Maravall (1999) show that the HP filter performs poorly at the endpoints of the time series and propose ARIMA type forecast and backcast of the level series before applying the filter to them.

<sup>&</sup>lt;sup>38</sup>For example, the elasticity of country spread with respect to the Solow residual and Euro Zone interest rate or the working capital constraint parameter.

external financing is relatively cheap. Consequently, agents would like to adjust even more their capital stock by using besides their own funds more external financing, generating in this way higher investment volatility. In order to limit this investment volatility, the cost of an additional unit of capital accumulated must be higher, i.e. the capital adjustment cost parameter must increase.

For example, Chang and Fernandez (2009) calibrate  $\psi$  instead of estimating it and they obtain a point estimate of  $\phi$  equal to 14.72 in the case of Argentina. This result is similar to those I report in Table 6 suggesting two digit numbers for  $\phi$  across all the countries when  $\psi$ is calibrated. However, when  $\psi$  is estimated its point estimates vary in [0.02, 0.23] interval depending on country, model and prior specification. These are somewhat lower estimates than the  $\psi$  equal to 2.8 reported by Garcia-Cicco et al. (2010). This indicates that the data shifts the posterior mode of the conditional distribution upper, to the right in the distribution of the parameter, suggesting that the external debt elasticity of country spread or interest rate in the case of CEECs might be higher than 0.001, i.e. country spread or interest rate is more sensitive to changes in the external debt position in CEECs than in developed ones, like Canada.

Regarding the estimates of  $\phi$  reported in Table 6, one can observe that  $\phi$  takes values between 3.4 and 8.2 when  $\psi$  is estimated. These results are not at odds neither with the previous evidences in RBC literature nor with the existing empirical evidences obtained from research on investment behavior. For example Cummins et al. (2006) estimate  $\phi$  by using cross section data of firms from the US and conclude that the estimated  $\phi$  is about 7.2 in the US. Thus, the estimates of  $\phi$  obtained both in the case of the uniform interest rate model and the persistent spread model are quite plausible. To motivate lower estimates of  $\phi$ , one can use the figures reported by Aguiar and Gopinath (2007a) who obtain estimates for  $\phi$  between 2.82 and 3.79 by using moment matching, or those recently documented by Garcia-Cicco et al. (2010) (i.e. they estimate  $\phi$  at 4.6 in the case of Argentina by using Bayesian techniques).

The estimated  $\eta_{SR}$ , according to the prior specification with the best fit, varies with the nature of the country spread and interest rate. Across countries, the point estimates of  $\eta_{SR}$  decrease as interest rate becomes uniform and it gets even lower when country spread responds to movements in the Euro Zone interest rate and it is assumed to be persistent. This can be explained by the fact that when country spread is persistent current technology shocks are fed into the future realizations of country spread through the autoregressive term which allows for adjustment of the spread over time instead of the one time jump, i.e. the adjustment in  $s_t$  is distributed over time. Thus, country spread decreases by about 0.25 percentage point as a response to one percentage point technology improvement in Poland

when firms and households face different interest rate, while this figure becomes 0.16 when country spread is assumed to have some persistence. The results are similar in the case of Bulgaria and the Czech Republic, while, in the case of Hungary and Romania, the estimated  $\eta_{SR}$  suggests that country spread is more sensitive to changes in technology (i.e. the point estimate is 0.33 under differentiated interest rate and it drops to 0.24 under uniform interest rate in the case of Hungary).

Excepting Poland, the estimates of the working capital constraint parameter,  $\theta$ , reported in Table 7, stay relatively stable across different model specifications suggesting that firms' decisions concerning the share of the wage bill paid in advance is not significantly influenced by the fact whether all the agents pay the same interest rate or not.

	Bulgaria	Czech Rep.	Hungary	Poland	Romania
C&F (2009)	0.4251	0.3708	0.4930	0.3179	0.5755
Uniform r	0.4843	0.4757	0.4907	0.5563	0.5381
Persistent s	0.4388	0.4422	0.4589	0.4871	0.4908
Data	NA	0.3276	0.5532	0.3128	0.2231

Table 7. The Posterior Mean of  $\theta$  under Different ModelSpecifications

I also report the statistical counterpart of  $\theta$  proxied by the five years average of the working capital loans of non-financial private corporations to compensation for labor ratio<sup>39</sup>. By comparing the figures one can admit that the actual and estimated  $\theta$  are relatively similar, with the exception of Romania. Concerning cross country differences in the estimates of  $\theta$ , one can notice that it varies in a relatively tight interval, [0.3,0.6]. However, when country spread is persistent, this parameter becomes even more uniform across countries, i.e. it varies in [0.43, 0.5] interval.

The estimate of the elasticity of country spread with respect to the Euro Zone interest rate  $(\eta_{r^*})$  is almost the same across countries, i.e. it is 0.46 in Bulgaria, Hungary and Romania, while it is a slightly higher, 0.48 in the Czech Republic and 0.47 in Poland. This indicates the fact that country spread moves by approximately the same amount across

<sup>&</sup>lt;sup>39</sup>Since historical time series concerning working capital loans to non-financial private corporations are only available for Poland, I use as proxy to this variable the short term loans to non-financial private corporations series in the case of the Czech Republic, Hungary and Romania. Unfortunately, there is no data available for Bulgaria.

countries as Eurozone interest rate changes by one percentage point in the model economy. This result perfectly matches the contemporaneous correlation of country spread and real Eurozone interest rate in Poland which is equal to 0.47 according to the data. However, the above mentioned correlation implied by the data is only 0.19 in Hungary, respectively 0.32 in Bulgaria, by indicating that  $\eta_{r^*}$  is overestimated in the case of this countries.

The estimated persistence and standard errors of the shock processes vary slightly across different specifications and they always exhibit the same pattern: transitory technology shock is the most persistent followed by the Eurozone interest rate shock, while the permanent technology shock has the lowest estimated persistence. Hence,  $\rho_{r^*}$  is estimated being approximately 0.8 across all the countries and specifications. The persistence of the permanent and transitory technology shocks is relatively stable across all within country specifications. Thus, on average the estimate of the first order autoregressive coefficient of  $a_t$  and respectively  $g_t$  is 0.91 and 0.72. According to the estimates of the standard deviation of the shock processes, it can be concluded that the permanent technology shock is the most volatile, followed by the transitory technology shock and respectively the Euro Zone interest rate shock. This result is in agreement with the findings of Chang and Fernandez (2009) or Garcia-Cicco et al. (2010). Country spread shock seems to be the least persistent since the point estimate of  $\rho_s$  is about 0.64, while its volatility varies across countries.

# 4.4 Model evaluation: theoretical business cycle moments and impulse responses

Based on the analysis conducted in the previous section, I concluded that the specification which fits the data best is characterized by gamma distribution of the standard errors of the shocks while measurement errors are defined for each observable variable. However, in order to argue about the goodness of fit of each version of the model (i.e. the Chang and Fernandez (2009) model, the uniform interest rate model and the uniform interest rate model coupled with endogenous persistent country spread), I assess the way in which the models can replicate business cycle statistics and compare the historical impulse response functions to those implied by the models.

The main conclusions of this analysis can be summarized as follows: (1) All the three versions of the model generate a slightly more volatile consumption than the volatility of output, while none of them can replicate the high volatility of investment suggested by the data, (2) Both extensions of the Chang and Fernandez (2009) model, as well as the original model imply countercyclical trade balance and country spread, and underpredict the persistence of trade balance to output ratio. Generally, the original model and the

uniform interest rate extension overpredict the persistence of country spread, (3) The sum of squared differences (SSD)<sup>40</sup> between eighteen theoretical and historical business cycle moments suggests that the second extension (i.e. it assumes that country spread has both exogenous and endogenous components, and it is persistent) does a better job in replicating business cycle moments than the other two versions of the model, and (4) The second extension replicates fairly well, both qualitatively and quantitatively, the historical impulse responses estimated from the data. The other two versions of the model generate too low responses of the main model variables to different structural shocks in comparison to what was derived from the SVAR model. The rest of this subsection presents these results in more details.

Table 8 reports second order moments of the theoretical time series implied by different versions of the model together with those computed from the data, and the SSD. These business cycle statistics allow for verifying whether the model can replicate particular features of emerging market business cycles: more volatile consumption than output, highly volatile investment in comparison to output, countercyclical trade balance and country spread, non-random walk behavior of trade balance.

The relative volatility of consumption implied by the model suggests that all versions of the model generate a somewhat more volatile consumption than output with the exception of the Czech Republic, but quantitatively the results are inferior to what one can see from the data. By relaxing the differentiated interest rate assumption and assuming that country spread has some persistence, the dynamics of consumption does not change significantly, indicating that assumptions concerning the nature of interest rate do not affect the consumption side of the economy. This could be related to the fact that when the interest rate becomes uniform across agents, firms are those who face higher interest rate when households have positive external debt position. Consequently, their investment and working capital financing decisions are affected by the new interest rate which alters the dynamics of investment.

The evidence reported in the previous paragraph is confirmed by the theoretical relative volatility of investment which increases when interest rate becomes uniform and it gets even higher when country spread is assumed to be persistent. The intuition behind this goes in the following way: as the interest rate becomes uniform, firms face higher cost of financing their investment activity, because interest rate also depends on the external debt position.

 $<sup>^{40}</sup>$ I compute the sum of squared differences between the model implied and historical absolute volatility, relative volatility, contemporaneous correlation and persitence of output, consumption, investment and trade balance to output ratio.

	$\sigma_c/\sigma_y$	$\sigma_i/\sigma_y$	$\rho(tby,y)$	$\rho(s,y)$	$ ho_s$	$ ho_{tby}$	SSD
Bulgaria							
C&F (2009)	1.2026	1.2528	-0.1454	-0.3473	0.8368	0.6859	149.39
Uniform int. rate	1.0917	1.6485	-0.2056	-0.7189	0.9034	0.7243	132.01
Pers. Spread	1.1103	1.9331	-0.1707	-0.5182	0.7983	0.6359	116.06
Data	1.1640	3.4014	-0.1853	-0.081	0.8235	0.8615	
The Czech Rep.							
C&F (2009)	0.9605	1.1337	-0.1904	-0.4074	0.9104	0.3973	5.67
Uniform int. rate	0.9741	1.2741	-0.1603	-0.6082	0.9329	0.5742	4.96
Pers. Spread	0.9669	1.3401	-0.1419	-0.4065	0.6818	0.4085	3.50
Data	0.9736	1.9204	-0.1543	NA	NA	0.7632	
Hungary							
C&F (2009)	1.0052	1.1746	-0.2397	-0.3529	0.8991	0.6255	6.53
Uniform int. rate	1.0231	1.1299	-0.2511	-0.4671	0.8948	0.6655	6.49
Pers. Spread	1.0187	1.1485	-0.2369	-0.4301	0.7423	0.5432	5.15
Data	1.3489	1.5405	-0.3913	-0.3937	0.7698	0.8874	
Poland							
C&F (2009)	1.0123	1.5941	-0.1341	-0.5273	0.8942	0.4425	31.95
Uniform int. rate	1.0201	1.6677	-0.1426	-0.3704	0.8434	0.5850	30.69
Pers. Spread	1.0189	2.4491	-0.1854	-0.3821	0.8065	0.5562	24.62
Data	1.0338	5.3853	-0.6884	-0.0785	0.698	0.8843	
Romania	-						
C&F (2009)	1.0204	1.4011	-0.2601	-0.4847	0.8809	0.5688	60.52
Uniform int. rate	1.0312	1.4507	-0.2121	-0.4963	0.8912	0.6340	55.50
Pers. Spread	1.0230	1.7436	-0.2700	-0.4102	0.7608	0.5902	43.87
Data	1.3033	2.1202	-0.4364	NA	NA	0.7952	

Table 8. Theoretical and Historical Business Cycle Moments

Thus investment becomes more sensitive to fluctuations in interest rate and, accordingly, it exhibits higher volatility. When country spread is assumed to have some persistence, the interest rate sensitivity of investment increases even more since current changes in the cost of financing are carried into the future through the autoregressive term of country spread, boosting in this way the volatility of investment. However, all three versions of the model underpredicts the relative volatility of investment, and this difference becomes striking in the case of Bulgaria and Poland where the volatility of investment suggested by the data is way too high in comparison with other countries<sup>41</sup>.

All versions of the model considered can replicate the countercyclical nature of trade balance and country spread observed from historical data. Even if the theoretical contemporaneous correlation of output with the trade balance to output ratio is close to that suggested by the Bulgarian, Hungarian and Czeck data, the same cannot be concluded about the correlation of output with the trade of Poland and Romania, where the model significantly underpredict this statistics.

Similarly, the model does poorly in quantitatively replicating the relatively low contemporaneous correlation between country spread and output in the case of Bulgaria and Poland, while it almost perfectly matches the correlation suggested by the Hungarian data. The Chang and Fernandez (2009) and the uniform interest rate version of the model overestimate the persistence of country spread. However, when country spread is assumed to have both exogenous and endogenous components, the model implied first order autocorrelation of country spread is almost the same as that suggested by the EMBI data in the case of Bulgaria and Hungary.

The theoretical autocorrelation of trade balance to output ratio gets lower and lower as more and more financial frictions are assumed, i.e. the theoretical framework allows for stronger and stronger amplification mechanism of different structural shocks. This result is in line with those obtained by Garcia-Cicco et al. (2010), who show that when financial frictions (i.e. working capital constraint coupled with debt dependent interest rate) are introduced into the baseline small open economy RBC model the autocorrelation of trade balance drops from near random walk to 0.53.

The main intuition for this originates from the interpretation of debt dependent interest rate which makes the debt financed external trade deficit more sensitive to movements in interest rate. Put differently, when interest rate depends directly on the external debt position of a country, an increase in trade deficit financed by external funds pushes interest rate up which discourages domestic investment and consumption, generating a correction in external trade deficit. As a consequence, the more sensitive the interest rate is to movements in external debt position the less persistent the trade balance to output ratio is (i.e. the stronger the amplification mechanism of the shock is the lower the autocorrelation of tby is). Nevertheless, quantitatively all model specifications underpredict the autocorrelation of

 $<sup>^{41}</sup>$ The volatility of investment computed from the data is 6. 34% in Poland and 12.85 % in Bulgaria.

trade balance to output ratio in all the five countries.

In order to evaluate the overall business cycle moments replicating performance of the three versions of the model, I report the sum of squared difference (SSD) between the theoretical and historical absolute volatility, relative volatility, contemporaneous correlation with output and autocorrelation coefficient of output, consumption, investment, trade balance to output ratio and country spread. This indicator suggests that theoretical business cycle moments deviate the least from their historical counterpart when interest rate is assumed to be uniform and country spread has both exogenous and endogenous components. Hence the third model specification provides theoretical business cycle statistics closest to those observed from the data.

Turning to the theoretical impulse response functions, Figures 5 to 8 depict the responses of output and country spread to various structural shocks in the five CEECs under different versions of the small open economy RBC model, along with those estimated from historical time series. The aim of this type of analysis is to verify how well the model can replicate the dynamics of different macroeconomic variables inferred from historical data.

Figure 5 shows that, following a positive Euro Zone interest rate shock, output declines in CEECs under all the three versions of the model. This suggests that when external financing becomes more expensive, agents reduce their consumption and restrict their investment activity in order to save more, which leads to lower domestic output. (i.e. higher interest rate distorts households' intertemporal rate of substitution between consumption and leisure and firms' marginal factor productivity). Qualitatively, this result is in line with the economic intuition and with the impulse response functions derived from the SVAR model. However there are significant quantitative differences across different model specifications. These differences indicate that assumptions concerning interest rate and country spread do influence the way the model economy reacts to structural shocks. As more and more structure is imposed on country spread, output becomes more and more sensitive to  $r^*$  shocks. By relaxing the assumption of Chang and Fernandez (2009), that firms and households pay different interest rates does not alter the size of the response of output to one S.D.  $r^*$  shock and consequently the estimated impulse response functions nearly coincide with those obtained from the Chang and Fernandez (2009) model.

When country spread is supposed to have both exogenous and endogenous components, the response of output significantly deviates from those estimated in the other two cases, i.e. it more than doubles, especially in the case of the Czech Republic, Poland and Romania. This signals the fact that persistent endogenous country spread serves as an amplification mechanism of the impact of Eurozone interest rate shocks on the domestic real economy. This mechanism works as follows. When  $r^*$  shocks hit the domestic economy, the cost of external financing is boosted through the two components of the interest rate, i.e. country spread and Eurozone interest rate. As country spread is persistent, current changes in  $r^*$ are fed into the future realization of country spread through the autoregressive term which ensures that a contemporaneous increase in the cost of financing will generate higher interest rate and slow down adjustment back to the steady state. Since agents have perfect foresight in this model, they consume and invest less in order to build up larger savings relative to the non-persistent interest rate case, which implies lower domestic output.

The intuition behind this mechanism is similar to that of the financial accelerator mechanism proposed by Bernanke and Gertler (1986) which generates endogenous external financing premium. Recently, this mechanism was integrated into DSGE models by Gertler et al. (2007) and others showing that the endogenous financing premium amplifies the impact of the shocks on the real economic activity and it can generate significant and sharp responses of the main macroeconomic variables to an unanticipated increase in the country external borrowing cost.

Figure 6 depicts the response of country spread to one S.D. positive Euro Zone interest rate shock. As in the Chang and Fernandez (2009) version of the model, country spread is the function of the Solow residual, external interest rate shocks do not have any impact on country spread, i.e. the estimated impulse responses are zero. Moreover, when interest rate is assumed to be uniform the estimated theoretical impulse responses are at odds with the historical ones predicting that country spread decreases as a response to positive external interest rate shock. This result contradicts the economic intuition supported by the stylized facts derived from the estimated SVAR; when external financing becomes more expensive we should expect that the small open economy faces more risk of financing domestic economic activity, which has to materialize in higher country spread.

The third version of the model does a better job in capturing this aspect of world interest rate shock transmission mechanism, predicting that country spread jumps contemporaneously as Euro Zone interest rate shock hits the economy. Then the spread slowly adjusts back to its equilibrium level as the effect of the shock dies away. These results confirm that persistent endogenous country spread serves as an amplification mechanism of structural shocks. However, quantitatively the results differ slightly from those implied by the SVAR, such that the model underpredicts the response of country spread relative to that suggested by the data in the case on Bulgaria and Hungary, while it overpredicts the contemporaneous jump of country spread as a response to  $r^*$  in the case of Poland.

One S.D. positive productivity shock induces a contemporaneous drop in country spread regardless of the considered theoretical framework (Figure 7). Historical impulse responses predict that country spread decreases when a positive output shock hits the economy and the impulse response function has a humped shape. Qualitatively, this stylized fact is not replicated by the first two versions of the model, indicating that the non-persistent country spread adjust too smoothly relative to what we observe from the data. But when country spread is assumed to be persistent, the theoretical impulse responses become more curved relative to those implied by the other two versions of the model. However, none of the models can replicate the hump shaped responses of country spread.

Finally, the third version of the model is the only theoretical framework in which the impact of an exogenous country spread shock on the domestic real economy can be investigated. This extension of the Chang and Fernandez (2009) model assumes that country spread fluctuations have some exogenous sources. As it can be noticed from Figure 8, when a positive country spread shock hits the economy, domestic real economic activity suffers a contraction since external borrowing becomes more expensive. Consequently firms invest less and households limit their consumption in order to avoid depressing output production. As the effect of the shock dies away the domestic output slowly adjusts back to its initial level.

### 5 Conclusions

This paper aimed to answer two questions based on a mixed, empirical and theoretical, approach: (i) Is there any causal relationship between Eurozone interest rate and business cycle fluctuations in CEECs?, and (ii) What is the role of country spread in the transmission mechanism of the Eurozone interest rate shocks to CEECs? The empirical strategy focused, first of all, on the investigation of the main characteristics of business cycle moments in CEECs. Secondly, it involved an impulse response analysis, as well as forecast error variance decomposition. These were derived from an estimated SVAR model, in which shocks were identified as suggested by economic intuition. Overall, the second order moments of the HP filtered consumption, investment, output and trade balance to output ratio series suggest that consumption is more volatile than output in the five sampled CEECs, the investment is the most volatile component of output, and trade balance to output ratio is countercyclical. Moreover, by using EMBI spread data in the case of Bulgaria, Hungary and Poland, I showed that country spread is characterized by relatively high persistence and it has a statistically significant negative correlation with output only in Hungary.

Unfortunately, the results of the impulse response analysis and variance decomposition exhibit quantitative limits because of the quality of data and the short sample used for the estimation on the SVAR model. However, these results have a series of qualitative implications concerning the interaction between domestic real macroeconomic variables, country spread and Euro Zone interest rate. Thus, the variance decomposition suggests that there is a feedback relationship between output and country spread in the CEECs, which could be overpredicted because of the omitted determinants of countries spread from the VAR model. In addition, according to the estimated impulse responses, positive Euro Zone interest rate shocks have a negative impact on output and boost the level of country spread, but the large two standard error confidence intervals limits the quantitative implications of these results. Finally, both variance decomposition and the estimated impulse responses indicate that country spread is that component of the external interest rate that is strongly interconnected with domestic macroeconomic variables, while Eurozone interest rate has no relevant role in explaining output, investment or country spread variance.

The foundations for the theoretical approach were provided by the Chang and Fernandez (2009) small open economy RBC model. I proposed two extension of this model: (1) The first one was based on the assumption that households and firms face the same interest rate on financial markets (2) In the second extension I proposed a combined definition of country spread by joining the two different definitions of country spread (i.e. exogenous versus endogenous country spread) used in the literature so far. Basically, I assumed that country spread has an endogenous component driven by macroeconomic fundamentals (i.e. the country's external debt position, domestic productivity) and global factors (i.e. Eurozone interest rate); it has some persistence as it depends on its own lags and it has an exogenous component, modeled as an i.i.d. stochastic process.

The theoretical business cycle moments and impulse responses implied by the estimated model suggest that the second extension not only does a better job in replicating CEECs' business cycle moments than the first extension or the original Chang and Fernandez (2009) model, but it also implies theoretical responses of output to Eurozone interest rate shocks which are closer, in comparison with the other two versions of the model, to the historical impulse responses. This result signals the fact that persistent endogenous country spread serves as an amplification mechanism of the impact of Eurozone interest rate shocks on the domestic real economy. I argue that, most likely, this is because an  $r^*$  shock coupled with persistent country spread distorts more the households' and the firms' intertemporal decisions than it is the case in the original versions of the model. Since agents have perfect foresight in this model, they learn that the adjustment of the interest rate back to its original value will be slow. Consequently, they consume and invest less in order to build up larger savings when country spread is persistent relative to the non-persistent case, ensuring in this way smooth consumption over time. These results provide ground for further investigations of the above mentioned research questions, by giving more micro-foundation for country spread and by integrating different financial market structures in small open economy RBC.

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### 6 Data Appendix - Sources and definitons

1. National account components from Eurostat (quarterly frequency): the following components measured in millions of national currency chain-linked volumes (reference year 2000) are used in the paper (with the corresponding identification code in the brackets):

- 1.1. Output: gross domestic product at market prices (B1GM) series
- 1.2. Consumption: household and NPISH final consumption expenditure (P31\_S14\_S15):
- 1.3. Investment: gross fixed capital formation (P51)
- 1.4. Exports: exports of goods and services (P6)
- 1.5. Imports: imports of goods and services (P7)

1.6. Trade balance to output ratio: is the ratio between net exports, computed as the difference between exports of goods and services (P6) and imports of goods and services (P7), and gross domestic product at market prices (B1GM)

I seasonally adjusted of all the above presented series by using Tramo-Seats method implemented in Demetra 2.1. software package.

Cyclical components (logarithmic deviation from the long run trend) of the seasonally adjusted output, consumption investment and trade balance to GDP ratio: are computed in three steps (i) firstly, the above described level series are taken in logarithm (ii) the time series, obtained in this way, are filtered by using Hodrick-Prescott filter with  $\lambda$ =1600 scale parameter (iii) the cyclical component of each series is computed as a difference between the logged level series and its trend component.

2. Real Euro Zone interest rate (quarterly frequency): is computed as the difference between the overnight German money market rate and annualized quarterly German inflation rate. The source of the interest rate is the Online Historical Financial Statistics published by The Deutsche Bundesbank and it is defined as Frankfurt banks / Overnight money / Monthly average: SU0101, from which the quarterly series is constructed by taking the corresponding monthly rate at the end of each quarter. The annualized quarter-to-quarter inflation rate is computed based on the quarterly consumer price index (13464ZF)(2000=100) from International Financial Statistics of IMF according to the following formula:

$$\inf = 400 * \ln\left(\frac{CPI_{t,2000=100}}{CPI_{t-1,2000=100}}\right)$$
(20)

3. Country spread (quarterly frequency): is quantified by using JP Morgan EMBI Global Divers-Stripped spread in Poland (JPMGPOC (SSPRD)) covering the period 1995Q1:2009Q4, in Hungary (JPMGHNC (SSPRD)) over the period 1999Q1:2009Q4 and in Bulgaria (JPMP-BUL (BSPRD)), 1994Q3:2009Q4; In order to compute EMBI spread US-dollar denominated Brady bonds, Eurobonds, and traded loans issued or guaranteed by sovereign entities are considered. Only issues with a current face amount outstanding of \$500 million or more and a remaining life of greater than 2 1/2 years are eligible for inclusion in the index. <sup>42</sup> Moreover the yield spread used in the case of Hungary is computed as the absolute difference between yield on long term Eurobonds issued by the Hungarian government (568915(RYAN)) and yield on long term US Treasury bonds. (993766(RYAN)) <sup>43</sup>.

4. Compensation of labor input (annual frequency): (i) total compensation of labor (lab\_tot) is obtained from the EU KLEMS database for The Czech Republic, Poland and Romania. This is computed as the sum of compensation for employees and that part of the operating surplus/ mixed income which represents the compensation for self-employed <sup>44</sup> Data concerning mixed income is not available in the case of Bulgaria and Romania (ii) compensation of employees at industry level published by Eurostat in the National Accounts by 6 branches at current prices (nama\_nace06\_c) and defined as the total remuneration, in cash or in kind, payable by an employer to an employee in return for work done by the latter. Social contributions paid by the employer are also included in this indicator.

5. Value added (annual frequency): (i) total value added in the economy (all industries) at current prices, in millions of euro, is obtained from the EU KLEMS database (va\_tot) for Hungary, the Czech Republic and Poland; in the case of Bulgaria and Romania the source of value added is Eurostat (B1G) (ii) industry level value added series, at current prices, in millions of national currency, are from the National Accounts aggregates and employment by branch tables (NACE) (nama\_nace) published by Eurostat.

6. Private credit of non-financial corporations (annual frequency): since there are no

<sup>&</sup>lt;sup>42</sup>Source: http://www.jpmorgan.com/pages/jpmorgan/investbk/solutions/research/EMBI

<sup>&</sup>lt;sup>43</sup>Source: Datastream

 $<sup>^{44}\</sup>mathrm{For}$  more details see EU KLEMS Growth and Productivity Accounts Version 1.0 Methodology, March 2007 http://www.euklems.net/

publicly available figures concerning the type of loans besides their maturity for the Czech Republic, Hungary and Romania, short term financial liabilities (stocks) of non-financial corporations in millions of national currency, from the Czech National Bank's, the National Bank's of Hungary and the Romanian National Bank's Monetary and financial statistics, are taken as proxy for working capital loans; working capital loans to non-financial private corporations (stocks) in millions of national currency is considered in the case of Poland published by the National Bank of Poland in the Monetary and financial statistics: Assets and liabilities of monetary financial institutions; Bulgaria: there is no publicly available data.

7. Real GDP growth rate (quarterly frequency): measured as the percentage change of the gross domestic product at constant prices relative to the previous period from the GDP and main components-volumes (namq\_gdp\_k) table published by Eurostat

8. Net Foreign Asset (NFA) position (annual frequency): in millions of US dollars is from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007) in the "External Wealth of Nations" Dataset, 1970-2007

9. Gross Domestic Product (annual frequency): is at current prices, in millions of US dollars from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007) in the "External Wealth of Nations" Dataset, 1970-2007<sup>45</sup>

10. Employment: (annual frequency): measured as annual average total employment (15-64 years, resident population concept - LFS) in thousand of persons from the Employment (main characteristics and rates) (lfsi emp a ) table published by Eurostat.

11. Population: is in thousand of persons and it is obtained from Population, activity and inactivity - Annual averages (lfsi act a) table, Eurostat.

<sup>&</sup>lt;sup>45</sup>More details about the source of the data at http://www.philiplane.org/EWN.html

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	Chang and	l Fernandez (20	109) model		Unif	orm interest ra	ite			Uniform intere	st rate and per-	sistent spread
	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI
η <sub>SR</sub>	0.2933	0.2631	0.2231	0.3881	0.3117	0.2249	0.2165	0.3501	0.3622	0.1622	0.1425	0.2007
	[0.2317, 0.3441]	[0.1909, 0.3157]	[0.1731, 0.2679]	[0.2832, 0.4943]	[0.2578, 0.3875]	[0.1540, 0.2946]	[0.1977, 0.2375]	[0.2585, 0.4416]	[0.2533, 0.4759]	[0.1015, 0.2169]	[0.0977, 0.1673]	[0.1854, 0.2231]
JL*									0.3642	0.4642	0.4675	0.4522
									[0.2453, 0.4830]	[0.3106, 0.6086]	[0.3154, 0.6157]	[0.3984, 0.5221]
≯	0.1534	0.0880	0.0621		0.1591	0.0447	0.0224		0.2018	0.0692	0.0448	
	[0.1022, 0.1897]	[0.0415, 0.1375]	[0.0473, 0.0917]		[0.0921, 0.2189]	[0.0134, 0.0736]	[0.0136, 0.0302]		[0.1191, 0.2881]	[0.0249, 0.0917]	[0.0150, 0.0724]	
9	8.2782	6.9844	12.6789	17.1166	7.8754	7.9387	13.7039	17.2981	10.5361	6.3813	6.5836	14.1538
	[4.1549, 13.3467]	[5.3589, 8.2863]	[10.5691, 13. 9823]	[14.4647, 19.4817]	[6.0234, 9.8751]	[6.1183, 9.7694]	[11.4487, 15.7233]	[14.5728, 19.7721]	[7.5349, 13.7071]	[5.0443, 7.6682]	[4.9914, 8.0831]	[12.0314, 16.8912]
θ	0.3245	0.4251	0.2278	0.6090	0.3505	0.4843	0.3680	0.5945	0.3384	0.4388	0.4464	0.5273
	[0.2398, 0,4781]	[0.3432, 0.5146]	[0.1879, 0.2678]	[0.4535, 0.7570]	[0.2305, 0.4527]	[0.3260, 0.6515]	[0.2210, 0.5185]	[0.4425, 0.7471]	[0.1970, 0.4879]	[0.2682, 0.5965]	[0.2788, 0.6082]	[0.3467, 0.6123]
β	0.8921	0.9089	0.9091	0.8989	0.9072	0.9075	0.9084	0.8983	0.9026	0.9115	0.9145	0.8969
	[0.8712, 0.9178]	[0.8956, 0.9245]	[0.8891, 0.9189]	[0.8821, 0.9149]	[0.8723, 0.9327]	[0.8923, 0.9228]	[0.8964, 0.9209]	[0.8815, 0.9144]	[0.8861, 0.9189]	[0.8970, 0.9269]	0.8990, 0.9306]	[0.8642, 0.9217]
Ρ	0.7187	0.7228	0.7198	0.7182	0.7253	0.7245	0.7433	0.7186	0.7226	0.7265	0.7230	0.7201
	[0.7024, 0.7293]	[0.7098, 0.7355]	[0.7011, 0.7317]	[0.7017, 0.7347]	[0.6976, 0.7467]	[0.7076, 0.7406]	[0.7274, 0.7593]	[0.7016, 0.7347]	[0.7067, 0.7389]	[0.7104, 0.7432]	[0.7063, 0.7394]	[0.7045, 0.7478]
Pr*	0.8017	0.7990	0.8013	0.8003	0.7955	0.7994	0.8044	0.8003	0.7978	0.7989	0.7999	0.7997
	[0.7356, 0.8204]	[0.7866, 0.8118]	[0.7891, 0.8156]	[0.7840, 0.8170]	[0.7823, 0.8245]	[0.7835, 0.8156]	[0.7881, 0.8205]	[0.7830, 0.8166]	[0.7812, 0.8136]	[0.7822, 0.8155]	[0.7833, 0.8164]	[0.7721, 0.8189
ρ									0.6478	0.6470	0.6462	0.6465
									[0.6313, 0.6644]	[0.6305, 0.6627]	[0.6297, 0.6623]	[0.6245, 0.6591]
a <sub>a</sub>	0.0441	0.0242	0.0491	0.0163	0.0366	0.0227	0.0130	0.0165	0.0381	0.0190	0.0263	0.0103
	[0.00267, 0.0602]	[0.0186, 0.0301]	[0.0312, 0.0521]	[0.0089, 0.0235]	[0.0257, 0.0489]	[0.0164, 0.0290]	[0.01099, 0.01533]	[0.0092, 0.0240]	[0.0308, 0.0457]	[0.0141, 0.0235]	[0.0203, 0.0324]	[0.0076,0.0249]
ъ в	0.0462	0.0331	0.0501	0.0264	0.0416	0.0314	0.0142	0.0257	0.0448	0.0294	0.0417	0.0234
	[0.0312, 0.0527]	[0.0259, 0.0398]	[0.0417, 0.0628]	[0.0200, 0.0324]	[0.0321, 0.0521]	[0.0241, 0.0380]	[0.1191, 0.1662]	[0.0197, 0.0317]	[0.0355, 0.0535]	[0.0230, 0.0355]	[0.0338, 0.0496]	[0.0186, 0.0289]
0 <sup>r</sup> *	0.0147	0.0049	0.0072	0.0052	0.0284	0.0041	0.0103	0.0055	0.0354	0.0042	0.0051	0.0039
	[0.0104, 0.0213]	[0.0025, 0.0072]	[0.0051, 0.0087]	[0.0028, 0.0076]	[0.0112, 0.0389]	[0.0019, 0.0061]	[0.0082, 0.0121]	[0.0031, 0.0082]	[0.0272, 0.0430]	[0.0028, 0.0055]	[0.0030, 0.0135]	[0.0017, 0.0052]
a									0.0466	0.0044	0.0054	0.0049
									[0.0345, 0.0583]	[0.0015, 0.0072]	[0.0030, 0.0135]	[0.0021, 0.0069]
ML(0)	87.3812	126.7938	92.5612	118.5772	52.2084	104.9236	88.3649	95.1910	27.5929	78.3346	57.9575	63.9026
LDD	86.7632	110.3366	91.8972	117.5184	51.3487	103.8797	89.8415	96.2312	27.2437	76.5283	56.2569	61.4587
Acc.R.	0.3859 / 0.3780	0.3459 / 0.3480	0.3741 / 0.3722	0.3354 / 0.3323	0.3458 / 0.3578	0.3246 / 0.3253	0.3410 / 0.3458	0.3328 / 0.3312	0.3180 / 0.3252	0.3091 / 0.3088	0.3516 / 0.3504	0.3421 / 0.3398

Notes: ML(B) is the marginal density computed by Laplace approximation, LLD: log data density, Acc.R.: Average acceptance ratio per chain; [.]: confidence intervals concerning the 90% high probability region

Table A1: Posterior means and confidence intervals of the model parameters: Bulgaria

# 7 Table Appendix

**CEU eTD Collection** 

	Chang and	Fernandez (2	2009) model		Unif	orm interest r	ate.		Ū	niform intere	st rate and per	sistent spread
	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI
$\eta_{SR}$	0.2891	0.2103	0.1883	0.3053	0.3145	0.1718	0.1329	0.1578	0.3603	0.1822	0.1400	0.1231
	[0.1917, 0.3804]	[0.1379, 0.2820]	[0.1288, 0.2445]	[0.1915, 0.4168]	[0.2312, 0.3781]	[0.1038, 0.2320]	[0.0977, 0.1650]	[0.1025, 0.2048]	[0.2342, 0.4816]	[0.1080, 0.2505]	[0.0977, 0.1826]	[0.0977, 0.1494]
ղ։*									0.3382	0.4823	0.4887	0.4844
									[0.2225, 0.4564]	[0.3220, 0.6478]	[0.3264, 0.6357]	[0.3388, 0.6280]
≯	0.4814	0.2653	0.0977		0.1675	0.0601	0.0250		0.2405	0.0937	0.0541	
	[0.367, 0.5976]	[0.1460, 0.3920]	[0.0332, 0.1549]		[0.1267, 0.2012]	[0.0231, 0.0942]	[0.0150, 0.0353]		[0.1422, 0.3325]	[0.0424, 0.1451]	[0.0208, 0.0850]	
0	6.5999	4.5536	11.0181	10.6866	7.8775	7.7906	12.8395	13.1446	7.5162	6.4968	6.6447	12.5412
	[4.6815, 8.5330]	[2.8598, 6.2093]	[8.8866, 13.2417]	[8.9774, 12.3382]	[6.0134, 9.1235]	[6.0949, 9.4699]	[10.8482, 14.7640]	[10.9761, 14.9713]	[5.7182, 9.0596]	[5.0786, 7.8952]	[4.8404, 8.3421]	[10.3203, 14.6116]
θ	0.2490	0.3708	0.1722	0.5991	0.3045	0.4757	0.3522	0.6610	0.2479	0.4422	0.4088	0.6651
	[0.1260, 0.3692]	[0.2392, 0.4943]	[0.0926, 0.2488]	[0.4405, 0.7406]	[0.2267, 0.3741]	[0.3141, 0.6485]	[0.2045, 0.4927]	[0.5274, 0.7966]	[0.1274, 0.3673]	[0.2727, 0.6065]	[0.2476, 0.5723]	[0.5280, 0.8059]
ρ	0.9055	0.9112	0.9242	0.8976	0.9160	0.9062	0.9134	0.8871	0.9119	0.9140	0.9135	0.8837
	[0.8901, 0.9208]	[0.8973, 0.9260]	[0.9118, 0.9376]	[0.8808, 0.9138]	[0.8991, 0.9235]	[0.8911, 0.9220]	[0.9013, 0.9258]	[0.8728, 0.9022]	[0.8944, 0.9275]	[0.9002, 0.9283]	[0.8992, 0.9277]	[0.8679, 0.9011]
ρ	0.7278	0.7287	0.7279	0.7233	0.7298	0.7309	0.7343	0.7244	0.7298	0.7303	0.7324	0.7291
	[0.7116, 0.7440]	[0.7131, 0.7451]	[0.7112, 0.7432]	[0.7074, 0.7402]	[0.7025, 0.7412]	[0.7139, 0.7469]	[0.7178, 0.7514]	[0.7073, 0.7408]	[0.7141, 0.7467]	[0.7142, 0.7459]	[0.7154, 0.7480]	[0.7144, 0.7446]
ρ.*	0.8008	0.8005	0.8019	0.8018	0.7975	0.8010	0.8014	0.8120	0.8003	0.8003	0.8019	0.8065
	[0.7840, 0.8186]	[0.7840, 0.8167]	[0.7854, 0.8178]	[0.7849, 0.8176]	[0.7725, 0.8216]	[0.7849, 0.8170]	[0.7846, 0.8177]	[0.7961, 0.8296]	[0.7853, 0.8184]	[0.7843, 0.8164]	[0.7849, 0.8184]	[0.7899, 0.8216]
Ps									0.6487	0.6476	0.6457	0.6462
									[0.6338, 0.6648]	[0.6309, 0.6641]	[0.6296, 0.6634]	[0.6288, 0.6634]
σ <sub>a</sub>	0.0426	0.0286	0.0697	0.0080	0.0403	0.0334	0.0615	0.0234	0.0411	0.0280	0.0320	0.0216
	[0.0343, 0.0504]	[0.0216, 0.0350]	[0.0494, 0.0705]	[0.0029, 0.0131]	[0.0234, 0.0571]	[0.0253, 0.0412]	[0.0502, 0.0724]	[0.0174, 0.0296]	[0.0336, 0.0487]	[0.0217, 0.0336]	[0.0248, 0.0386]	[0.0151, 0.0282]
ď	0.0420	0.0310	0.0618	0.0270	0.0419	0.0351	0.0606	0.0262	0.0424	0.0333	0.0361	0.0280
	[0.0346, 0.0492]	[0.0254, 0.0375]	[0.0514, 0.0721]	[0.0219, 0.0318]	[0.0211, 0.0514]	[0.0282, 0.0419]	[0.0502, 0.0709]	[0.0213, 0.0313]	[0.0350, 0.0497]	[0.0268, 0.0397]	[0.0290, 0.0429]	[0.0220, 0.0341]
<b>0</b> ^*	0.0275	0.0060	0.0109	0.0055	0.0259	0.0078	0.0074	0.0106	0.0318	0.0071	0.0058	0.0052
	[0.0235, 0.0309]	[0.0031, 0.0091]	[0.0078, 0.0138]	[0.0032, 0.0078]	[0.0113, 0.0379]	[0.0057, 0.0091]	[0:0057, 0:0090]	[0.0077, 0.0131]	[0.0254, 0.0375]	[0.0047, 0.0093]	[0.0036, 0.0081]	[0.0034, 0.0069]
٩									0.0399	0.0063	0.0068	0.0069
									[0.0308, 0.0488]	[0.0025, 0.0101]	[0.0025, 0.0109]	[0.0024, 0.0114]
MIL(0)	50.6237	98.9591	68.1926	54.6993	31.2075	78.7371	62.3799	71.4402	8.4293	49.6749	40.9374	47.8186
LDD	60.2127	97.8387	66.7991	52.5840	30.1278	77.1843	61.2478	77.0267	8.1576	48.5795	39.5448	52.7362
Acc.R.	0.3642 / 0.3575	0.3128 / 0.3149	0.3654 / 0.3580	0.2947 / 0.2927	0.3623 / 0.3678	0.3700 / 0.3753	0.3467 / 0.3533	0.3035 / 0.3116	0.3769 / 0.3773	0.3110 / 0.3135	0.3286 / 0.3304	0.3600 / 0.3579
	Notes: ML(0) is :	the marginal dens	sity computed by La	aplace approximati	on, LLD: log data	density, Acc.R.:	Average acceptanc	e ratio per chain; [,]	: confidence inter	vals concerning th	he 90% high proba	bility region

Table A2: Posterior means and confidence intervals of the model parameters: the Czech Republic

	C&F (2009	)): differentiated i	interest rate		Un	niform interest ra	Ite			Uniform inte	rest rate and persi	stent spread
	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI
$\eta_{SR}$	0.5278	0.3366	0.3256	0.3665	0.3641	0.2488	0.1599	0.3256	0.3460	0.2242	0.1544	0.2500
	[0.4637, 0.5998]	[0.2362, 0.4349]	[0.2142, 0.4283]	[0.2486, 0.4843]	[0.2567, 0.4187]	[0.1599, 0.3298]	[0.1107, 0.2070]	[0.2434, 0.3998]	[0.2377, 0.4617]	[0.1405, 0.3061]	[0.0977, 0.2206]	[0.1945, 0.3234]
ľ,									0.3358	0.4646	0.4616	0.4521
									[0.2245, 0.4412]	[0.3164, 0.6209]	[0.3132, 0.6142]	[0.3082 0.6233]
≯	0.2983	0.1810			0.1932	0.0721	0.0195		0.1890	0.0947	0.0725	
	[0.2276, 0.3713]	[0.0898, 0.2725]			[0.1357, 0.2678]	[0.0297, 0.1122]	[0.0075, 0.0317]		[0.1159, 0.2624]	[0.0441, 0.1418]	[0.0314, 0.1132]	
9	8.8012	5.1354	11.6264	13.7247	7.4378	6.4808	13.3033	13.6264	9.0857	5.7620	7.3991	13.0356
	[7.0367, 10.3511]	[3.6557, 6.6448]	[7.1237, 13.5423]	[11.7081, 15.7282]	[5.0918, 10.2937]	[5.1149, 7.8095]	[10.4736, 16.1384]	[12.0032, 14.9282]	[5.3581, 11.6154]	[4.4262, 7.0254]	[4.5856, 10.2231]	[10.00126, 16.4132]
θ	0.4712	0.4930	0.5757	0.5828	0.5179	0.4907	0.3869	0.5757	0.3554	0.4589	0.6183	0.5952
	[0.3480, 0.6080]	[0.3500, 0.6506]	[0.4257, 0.8215]	[0.4327, 0.7403]	[0.4387, 0.6089]	[0.3281, 0.6514]	[0.2280, 0.5375]	[0.4112, 0.7247]	[0.1833, 0.5112]	[0.3026, 0.6276]	[0.4787, 0.7656]	[0.4971, 0.7752]
βa	0.8874	0.9111	0.9016	0.9011	0.9087	0.9103	0.9194	0.9016	0.9065	0.9121	0.8886	0.8969
	[0.8756, 0.9003]	[0.8969, 0.9259]	[0.8781, 0.9189]	[0.8840, 0.9176]	[0.8978, 0.9206]	[0.8959, 0.9255]	[0.9062, 0.9319]	[0.8795, 0.9204]	[0.8915, 0.9206]	[0.8971, 0.9273]	[0.8703, 0.9068]	[0.8725, 0.9113]
β	0.7427	0.7296	0.7219	0.7212	0.7159	0.7297	0.7301	0.7219	0.7278	0.7314	0.7242	0.7248
	[0.7270, 0.7540]	[0.7135, 0.7465]	[0.7102, 0.7413]	[0.7052, 0.7373]	[0.7034, 0.7387]	[0.7139, 0.7458]	[0.7150, 0.7467]	[0.7102, 0.7387]	[0.7107, 0.7450]	[0.7153, 0.7473]	[0.7076, 0.7400]	[0.7089, 0.7418]
ρ.*	0.7857	0.7999	0.8020	0.8013	0.8023	0.7990	0.8044	0.8020	0.7987	0.7996	0.8005	0.7999
	[0.7756, 0.7961]	[0.7833, 0.8162]	[0.7821, 0.8209]	[0.7838, 0.8174]	[0.7811, 0.8167]	[0.7827, 0.8159]	[0.7885, 0.8207]	[0.7847, 0.8192]	[0.7842, 0.8140]	[0.7828, 0.8156]	[0.7838, 0.8179]	[0.7822, 0.8201]
ρ									0.6447	0.6478	0.6441	0.6488
									[0.6276, 0.6622]	[0.6310, 0.6639]	[0.6270, 0.6599]	[0.6281, 0.6612]
ďa	0.0351	0.0178	0.0077	0.0077	0.0345	0.0175	0.0547	0.0077	0.0376	0.0160	0.0230	0.0079
	[0.0283, 0.0416]	[0.0138, 0.0216]	[0.0041, 0.0112]	[0.0039, 0.0114]	[0.0246, 0.0419]	[0.0136, 0.0214]	[0.0447, 0.0640]	[0.00408, 0.0121]	[0.0298, 0.0445]	[0.0126, 0.0194]	[0.0128, 0.0334]	[0.0056, 0.0083]
ъ <sup>в</sup>	0.0340	0.0208	0.0186	0.0194	0.0327	0.0223	0.0617	0.0186	0.0392	0.0215	0.0313	0.0184
	[0.0283, 0.0401]	[0.0169, 0.0245]	[0.0147, 0.0227]	[0.0158, 0.0230]	[0.0218, 0.0391]	[0.0181, 0.0264]	[0.0514, 0.0720]	[0.0154, 0.0241]	[0.0316, 0.0465]	[0.0174, 0.0254]	[0.0239, 0.0386]	[0.0101, 0.0232]
<sup>ل</sup> *	0.0275	0.0029	0.0047	0.0045	0.0029	0.0034	0.0120	0.0047	0.0320	0.0037	0.0058	0.0018
	[0.0235, 0.0319]	[0.0011, 0.0045]	[0.0025, 0.0071]	[0.0024, 0.0066]	[0.0017, 0.0041]	[0.0019, 0.0049]	[0.0093, 0.0145]	[0.0026, 0.0068]	[0.0252, 0.0379]	[0.0016, 0.0048]	[0.0033, 0.0082]	[0.0007, 0.0026]
ď									0.0410	0.0033	0.0148	0.0042
									[0.0311, 0.0513]	[0.0015, 0.0053]	[0.0061, 0.0231]	[0.0011, 0.0073]
ML(0)	86.5470	171.2116	155.3505	156.5853	110.2457	159.9558	94.8838	155.3505	42.6430	131.9482	97.3427	128.6799
LDD	82.7986	171.2575	152.3417	155.5171	108.7645	158.5496	93.2757	153.1254	45.6949	130.8725	99.3673	127.3156
Acc.R.	0.3085 / 0.3093	0.3655 / 0.3716	0.3417 / 0.3502	0.3469 / 0.3441	0.3608 / 0.3589	0.3461 / 0.3414	0.3096 / 0.3084	0.3478 / 0.3501	0.3366 / 0.3233	0.3296 / 0.3356	0.3575 / 0.3524	0.3417 / 0.3456
	Notes: ML(0) is th	ne marginal densit,	y computed by Lap	place approximation,	, LLD: log data den	sity, Acc.R.: Aver	rage acceptance ra	tio per chain; [,]: cor	nfidence intervals o	concerning the 90	% high probability	region

Table A3: Posterior means and confidence intervals of the model parameters: Hungary

**CEU eTD Collection** 

	C&F (2009	): differentiated i	interest rate		U	iform interest ra	ıte			Uniform inte	rest rate and pers	istent spread
	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI
$\eta_{SR}$	0.3515	0.2501	0.1464	0.4568	0.2931	0.2362	0.3205	0.4641	0.2641	0.1595	0.1231	0.4889
	[0.2439, 0.4504]	[0.1682, 0.3268]	[0.0991, 0.1842]	[0.3357, 0.5743]	[0.1943, 0.3891]	[0.1392, 0.3314]	[0.2001, 0.4414]	[0.3419, 0.5834]	[0.1496, 0.3341]	[0.0978, 0.2115]	[0.0977, 0.1521]	[0.3498, 0.6270]
ľ									0.3521	0.4665	0.4686	0.4610
									[0.2378, 0.4731]	[0.3103, 0.6128]	[0.3153, 0.6199]	[0.3109, 0.6097]
≯	1.0517	0.1694	0.0728		0.1456	0.0778	0.1838		0.2138	0.0620	0.0458	
	[0.9125, 1.1887]	[0.0868, 0.2488]	[0.0168, 0.1319]		[0.0789, 0.2014]	[0.0306, 0.1244]	[0.1100, 0.2602]		[0.1375, 0.2878]	[0.0246, 0.0968]	[0.0211, 0.0709]	
9	4.0118	3.4059	10.0403	14.2256	5.7891	4.9972	9.2339	14.2024	6.4791	4.2809	5.0313	14.0662
	[2.5062, 5.5031]	[2.2180, 4.6065]	[6.8756, 13.4176]	[12.3974, 16.0351]	[3.0382, 6.9451]	[3.5580 6.3511]	[8.0093, 10.5641]	[12.3955, 16.0500]	[2.8971, 8.5679]	[3.2545, 5.2708]	[3.7959, 6.1682]	[10.9640, 17.1010]
θ	0.3334	0.3179	0.2190	0.6260	0.5812	0.5563	0.6472	0.6125	0.3947	0.5071	0.4871	0.5666
	[0.1774, 0.4827]	[0.2049, 0.4150]	[0.1055, 0.3210]	[0.4703, 0.7733]	[0.2917, 0.7831]	[0.4308, 0.7698]	[0.4959, 0.8046]	[0.4594, 0.7621]	[0.2972, 0.4238]	[0.3372, 0.6728]	[0.3229, 0.6510]	[0.4030, 0.7197]
ρ	0.9050	0.9132	0.9361	0.9096	0.9061	0.9086	0.9327	2606.0	0.9121	0.9142	0.9167	0.9084
	[0.8897, 0.9194]	[0.8988, 0.9268]	[0.9244, 0.9493]	[0.8935, 0.9247]	[0.8902, 0.9137]	[0.8915, 0.9261]	[0.9209, 0.9456]	[0.8946, 0.9250]	[0.8971, 0.9286]	[0.8992, 0.9303]	[0.9031, 0.9318]	[0.8920, 0.9248]
ρ	0.7254	0.7325	0.7363	0.7245	0.7289	0.7372	0.7337	0.7251	0.7287	0.7371	0.7299	0.7385
	[0.7096, 0.7409]	[0.7169, 0.7484]	[0.7217, 0.7521]	[0.7087, 0.7404]	[0.7132, 0.7426]	[0.7211, 0.7530]	[0.7188, 0.7497]	[0.7092, 0.7410]	[0.7158, 0.7459]	[0.7215, 0.7527]	[0.7132, 0.7452]	[0.7224, 0.7546]
Pr*	0.8012	0.8016	0.8010	0.7993	0.8037	0.8007	0.8015	0.7986	0.8007	0.8000	0.8004	0.7985
	[0.7849, 0.8165]	[0.7854, 0.8188]	[0.7843, 0.8171]	[0.7824, 0.8158]	[0.7872, 0.8207]	[0.7848, 0.8168]	[0.7853, 0.8176]	[0.7822, 0.8149]	[0.7881, 0.8167]	[0.7840, 0.8157]	[0.7845, 0.8170]	[0.7826, 0.8154]
ρ									0.6457	0.6464	0.6460	0.6487
									[0.6307, 0.6679]	[0.6298, 0.6626]	[0.6287, 0.6620]	[0.6318, 0.6646]
σ <sub>a</sub>	0.0330	0.0176	0.0603	0.0067	0.0357	0.0175	0.0547	0.0057	0.0203	0.0158	0.0192	0.0077
	[0.0274, 0.0385]	[0.0138, 0.0215]	[0.0475, 0.0725]	[0.0034, 0.0079]	[0.0216, 0.0478]	[0.0131, 0.0216]	[0.0441, 0.0661]	[0.0035, 0.0079]	[0.0134, 0.0279]	[0.0125, 0.0191]	[0.0151, 0.0228]	[0.0047, 0.0105]
đ	0.0353	0.0233	0.0677	0.0170	0.0371	0.0238	0.0636	0.0167	0.0314	0.0230	0.0295	0.0215
	[0.0290, 0.0410]	[0.0187, 0.0276]	[0.0548, 0.0800]	[0.0142, 0.0200]	[0.0257, 0.0436]	[0.0196, 0.0282]	[0.0511, 0.0758]	[0.0137, 0.0196]	[0.0246, 0.3870]	[0.0188, 0.0270]	[0.0240, 0.0347]	[0.0176, 0.0254]
q_*	0.0266	0.0170	0.0050	0.0170	0.0211	0.0140	0.0079	0.0017	0.0257	0.0170	0.0210	0.0015
	[0.0235, 0.0296]	[0.007, 0.027]	[0.0034, 0.0066]	[0.008, 0.026]	[0.0128, 0.0342]	[0.006, 0.021]	[0.0055, 0.0103]	[0.0008, 0.0026]	[0.0102, 0.0367]	[0.004, 0.019	[0.012, 0.030]	[0.0006, 0.0025]
α									0.0192	0.0018	0.0032	0.0040
									[0.0098, 0,0277]	[0.0006, 0.0028	[0.0012, 0.0051]	[0.0015, 0.0064]
ML(0)	159.2609	234.2473	160.1679	204.8078	149.3345	217.7287	176.0702	204.0570	112.3578	178.4159	164.6034	98.5900
LDD	163.6091	232.6512	158.3041	202.7328	151.3847	216.4342	174.9264	201.4715	114.2756	179.8336	162.9067	98.0810
Acc.R.	0.3274 / 0.3255	0.3441/0.3515	0.3071 / 0.3060	0.3769 / 0.3755	0.3317 / 0.3422	0.3509 / 0.3517	0.3314 / 0.3285	0.3295 / 0.3243	0.3669 / 0.3591	0.3281 / 0.3317	0.3218 / 0.3207	0.3417 0.3433
	Notes: ML(0) is t	he marginal densi	ity computed by La	place approximation	n, LLD: log data d	ensity, Acc.R.: Av	verage acceptance	e ratio per chain; [,]:	confidence interv	als concerning the	e 90% high proba	bility region

Table A4: Posterior means and confidence intervals of the model parameters: Poland

**CEU eTD Collection** 

	C&F (2009	): differentiated	interest rate		U	niform interest ra	fe		Uniform inte	rest rate and pers	istent spread	
	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI	I-Gamma	Gamma	No ME	no PSI
η <sub>sr</sub>	0.8710	0.3796	0.3264	0.3742	0.3778	0.3147	0.1347	0.3550	0.5813	0.2784	0.1670	0.2904
	[0.6431, 1.0737]	[0.2624, 0.4951]	[0.2092, 0.4425]	[0.2234, 0.5284]	[0.2503, 0.5026]	[0.2175, 0.4164]	[0.0977, 0.1668]	[0.2067, 0.5024]	[0.4992, 0.6821]	[0.1739, 0.3768]	[0.1040, 0.2258]	[0.1965, 0.3811]
ŋr,									0.4006	0.4648	0.4905	0.4574
									[0.3275, 0.4717]	[0.3040, 0.6067]	[0.3371, 0.6407]	[0.3047, 0.6025]
≯	0.2141	0.2467			0.1509	0.0806	0.0310		0.1978	0.1174	0.0599	
	[0.0623, 0.3453]	[0.1381, 0.3564]			[0.0757, 0.2268]	[0.0314, 0.130]	[0.0169, 0.0454]		[0.1530, 0.2810]	[0.0541, 0.1791]	[0.0237, 0.0963]	
9	20.2767	6.2795	18.2020	14.4391	11.8863	7.6186	13.1684	14.6835	13.7127	8.2747	6.7509	14.8136
	[19.0410, 21.4372]	[4.3413, 8.3631]	[14.9409, 21.1320]	[11.6665, 17.0866]	[9.0577, 14.6034]	[6.1269, 9.1298]	[11.0887, 15.2389]	[12.0135, 17.4769]	[12.4326, 15.0461]	[6.0012, 10.5447]	[5.2015, 8.2638]	[12.2772, 17.4269]
θ	0.7749	0.5755	0.5994	0.6087	0.4817	0.5381	0.4078	0.5959	0.4662	0.4908	0.4773	0.6051
	[0.7062, 0.8502]	[0.4292, 0.7104]	[0.4428, 0.7564]	[0.4484, 0.7718]	[0.3117, 0.6452]	[0.3709, 0.6882]	[0.2447, 0.5682]	[0.4237, 0.7611]	[0.3902, 0.5486]	[0.3283, 0.6493]	[0.3111, 0.6497]	[0.4598, 0.7508]
ρ	0.9173	0.9086	0.8914	0.8985	0.9014	0.9074	0.9054	0.8983	0.8919	0.9066	0.9120	0.8984
	[0.9023, 0.9319]	[0.8931, 0.9236]	[0.8756, 0.9068]	[0.8814, 0.9155]	[0.8861, 0.9176]	[0.8922, 0.9222]	[0.8909, 0.9203]	[0.8821, 0.9151]	[0.8797, 0.9036]	[0.8911, 0.9213]	[0.8975, 0.9267]	[0.8821, 0.9151]
ρ	0.7330	0.7251	0.7207	0.7207	0.7236	0.7261	0.7400	0.7214	0.7271	0.7249	0.7288	0.7228
	[0.7206, 0.7466]	[0.7084, 0.7412]	[0.7049, 0.7361]	[0.7040, 0.7382]	[0.7069, 0.7396]	[0.7098, 0.7428]	[0.7242, 0.7563]	[0.7045, 0.7379]	[0.7158, 0.7404]	[0.7088, 0.7411]	[0.7130, 0.7451]	[0.7070, 0.7396]
ρ.*	0.7949	0.8001	0.7911	0.8009	0.7981	0.8008	0.8038	0.8010	0.7980	0.7998	0.8014	0.7981
	[0.7800, 0.8103]	[0.7835, 0.8161]	[0.7754, 0.8078]	[0.7851, 0.8180]	[0.7813, 0.8147]	[0.7841, 0.8171]	[0.7877, 0.8206]	[0.7845, 0.8170]	[0.7901, 0.8070]	[0.7832, 0.8166]	[0.7849, 0.8177]	[0.7821, 0.8144]
ρs									0.6485	0.6486	0.6471	0.6457
									[0.6355, 0.6620]	[0.6328, 0.6656]	[0.630, 0.6639]	[0.6298, 0.6626]
g <sub>a</sub>	0.0643	0.0307	0.0380	0.0106	0.0454	0.0252	0.1117	0.0110	0.0425	0.0280	0.0356	0.0093
	[0.0464, 0.0821]	[0.0227, 0.0387]	[0.0287, 0.0461]	[0.0041, 0.0171]	[0.0363, 0.0652]	[0.0182, 0.0320]	[0.0926, 0.1304]	[0.0042, 0.0171]	[0.0352, 0.0499]	[0.0199, 0.0361]	[0.0269, 0.0438]	[0.0044, 0.0140]
ď	0.0743	0.0376	0.0426	0.0285	0.0526	0.0348	0.1191	0.0279	0.0527	0.0385	0.0431	0.0270
	[0.0603, 0.0888]	[0.0297, 0.0458]	[0.0331, 0.0517]	0.0213, 0.0356]	[0.0413, 0.0628]	[0.0274, 0.0422]	[0.0987, 0.1384]	[0.0211, 0.0348]	[0.0427, 0.0625]	[0.0283, 0.0478]	[0.0345, 0.0513]	[0.0217, 0.0321]
¢ <sup>r</sup> *	0.0407	0.0124	0.0303	0.0119	0.0375	0.0104	0.0182	0.0125	0.0388	0.0171	0.0113	0.0047
	[0.0310, 0.0502]	[0.0063, 0.0193]	[0.0245, 0.0358]	[0.0076, 0.0164]	[0.0283, 0.0460]	[0.0062, 0.0147]	[0.0142, 0.0219]	[0.0078, 0.0171]	[0.0308, 0.0469]	[0.0123, 0.0221]	[0.0069, 0.0156]	[0.0021, 0.0071]
٩									0.0507	0.0135	0.0144	0.0150
									[0.0373, 0.0631]	[0.0039, 0.0225]	[0.0055, 0.0235]	[0.0064, 0.0229]
ML(0)	35.8747	49.9269	30.1335	45.8679	18.9446	41.7890	18.9446	37.2807	6.3516	32.4791	7.5052	29.6077
LDD	38.7579	48.2168	232.9188	45.9524	17.8216	41.9782	17.8216	34.2306	6.0340	28.8735	6.8752	30.8394
Acc.R.	0.3668 / 0.4025	0.3245 / 0.3267	0.3092 / 0.3336	0.2753 / 0.1513	0.3289 / 0.3249	0.3345 / 0.3329	0.3289 / 0.3249	0.3309 / 0.3206	0.2989 / 0.2921	0.3402 / 0.3435	0.3441 / 0.3388	0.2939 / 0.2931
	Notes: ML(0) is the	e marginal density	computed by Lapla	ce approximation, L	LD: log data densi	ty, Acc.R.: Avera	ge acceptance ration	per chain; [,]: confi	dence intervals con	cerning the 90% h	ligh probability reg	on

Table A5: Posterior means and confidence intervals of the model parameters: Romania

# 8 Figure Appendix



Figure 3. Historical impulse responses obtained from a VAR with endogenous variables  $x'_t = [r^*_t, y_t, s_t]$ 



Figure 4. Historical impulse responses obtained from a VAR with endogenous variables  $x'_t = [r^*_t, i_t, s_t]$ 



Figure 5. The response of output to one S.D. Euro Zone interest rate shock



Figure 6. The response of country spread to one S.D. Euro Zone interest rate shock



Figure 7. The response of country spread to one S.D. output shock



Figure 8. The response of output to one S.D. country spread shock

# 9 Model Appendix

### 9.1 The stationarised model

### Firms' behavior

Firms are optimizing agents, their primary objective is to maximize the total discounted sum of all future profits:

$$\max_{\{a_t,h_t,k_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \Pi_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t F\left(K_t, \Gamma_t h_t\right) - w_t h_t \left[1 + \theta \left(R_{t-1} - 1\right)\right] - u_t K_t \right\}$$

The resulting first order conditions describe the optimal way in which firms demand capital and labor:

$$\frac{\partial}{\partial k_t}: \quad a_t F_K \left( K_t, \Gamma_t h_t \right) = u_t \tag{21}$$

$$\frac{\partial}{\partial h_t} : a_t F_h(K_t, \Gamma_t h_t) \Gamma_t = w_t \left[ 1 + \theta \left( R_{t-1} - 1 \right) \right]$$
(22)

It can be noticed that because of the working capital constraint the marginal product of labor will be a mark-up over the wage. This mark-up is drived by the level of interest rate at which domestic agents can borrow from the financial markets and the parameter  $\theta$  from the working capital constraint.

Because a realization of g permanently influences  $\Gamma_t$  the output and its components are nonstationary with a stochastic trend. Thus, the model must be stationarized by detrending the variables (i.e. by normalizing each trending variable by trend productivity through period t-1):

$$x_t = \frac{X_t}{\Gamma_{t-1}}$$

The solution to the model is invariant to this choice of normalization but this ensures that if  $X_t$  is in the agent's information set then is also  $x_t$ .

Therefore, the firm's equilibrium conditions are:

$$a_t F_h\left(K_t, \Gamma_t h_t\right) \Gamma_t = w_t \left[1 + \theta \left(R_{t-1} - 1\right)\right] \stackrel{:\Gamma_{t-1}}{\Longrightarrow}$$

$$\implies \alpha \ a_t K_t^{1-\alpha} \left(\Gamma_t h_t\right)^{\alpha-1} \frac{\Gamma_t}{\Gamma_{t-1}} \frac{\Gamma_{t-1}^{-\alpha}}{\Gamma_{t-1}^{-\alpha}} = \frac{w_t}{\Gamma_{t-1}} \left[1 + \theta \left(R_{t-1} - 1\right)\right]$$
$$\implies \frac{\alpha \ a_t k_t^{1-\alpha} h_t^{\alpha-1} g_t^{\alpha}}{\left[1 + \theta \left(R_{t-1} - 1\right)\right]} = \frac{w_t}{\Gamma_{t-1}} \tag{23}$$

$$a_t F_K \left( K_t, \Gamma_t h_t \right) = u_t \stackrel{: \Gamma_{t-1}}{\Longrightarrow} a_t \left( 1 - \alpha \right) K_t^{-\alpha} \left( \Gamma_t h_t \right)^{\alpha} \frac{1}{\Gamma_{t-1}} = \frac{u_t}{\Gamma_{t-1}}$$

$$\implies (1 - \alpha) a_t k_t^{-\alpha} h_t^{\alpha} g_t^{\alpha} = u_t \tag{24}$$

The stationarized production function is:

$$Y_t = a_t K_t^{1-\alpha} \left(\Gamma_t h_t\right)^{\alpha} \stackrel{:\Gamma_{t-1}}{\Longrightarrow} \frac{Y_t}{\Gamma_{t-1}} = a_t K_t^{1-\alpha} \left(\Gamma_t h_t\right)^{\alpha} \frac{1}{\Gamma_{t-1}}$$
(25)

 $\Longrightarrow$ 

$$y_t = a_t \left(\frac{K_t}{\Gamma_{t-1}}\right)^{1-\alpha} \left(\frac{\Gamma_t}{\Gamma_{t-1}}\right)^{\alpha} h_t^{\alpha} \Longrightarrow y_t = a_t k_t^{1-\alpha} g_t^{\alpha} h_t^{\alpha}$$
(26)

## Households' behavior

By stationarizing the household's objective function and resource constraint:

$$\max_{\{C_{t},h_{t},K_{t+1},D_{t+1}\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{\left(C_{t} - \tau\Gamma_{t-1}h_{t}^{\omega}\right)^{1-\sigma}}{1-\sigma} \stackrel{:\Gamma_{t-1}}{\Longrightarrow} \max_{\{C_{t},h_{t},K_{t+1},D_{t+1}\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{\left(\frac{C_{t}}{\Gamma_{t-1}} - \tau h_{t}^{\omega}\right)^{1-\sigma}}{1-\sigma}$$

$$s.t. \ w_{t}h_{t} + u_{t}K_{t} + q_{t}D_{t+1} = D_{t} + C_{t} + K_{t+1} - (1-\delta) K_{t} + \frac{\phi}{2} \left(\frac{K_{t+1}}{K_{t}} - \mu\right)^{2} K_{t} \stackrel{:\Gamma_{t-1}}{\Longrightarrow}$$

$$s.t. \ \frac{w_{t}}{\Gamma_{t-1}}h_{t} + u_{t}\frac{K_{t}}{\Gamma_{t-1}} + q_{t}\frac{D_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_{t}}{\Gamma_{t}} = \frac{D_{t}}{\Gamma_{t-1}} + \frac{C_{t}}{\Gamma_{t-1}} + \frac{K_{t+1}}{\Gamma_{t-1}}\frac{\Gamma_{t}}{\Gamma_{t}} - (1-\delta)\frac{K_{t}}{\Gamma_{t-1}} +$$

$$\Gamma_{t-1} \stackrel{h_t \to a_t}{\Gamma} \Gamma_{t-1} \stackrel{h_t \to q_t}{\Gamma} \Gamma_{t-1} \Gamma_t \quad \Gamma_{t-1} \stackrel{h_t \to r_{t-1}}{\Gamma} \Gamma_t \Gamma_t \quad \Gamma_{t-1} \Gamma_t + \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} \frac{\Gamma_{t-1}}{\Gamma_{t-1}} \frac{\Gamma_t}{\Gamma_t} - \mu \right)^2 \frac{K_t}{\Gamma_{t-1}}$$

Thus we have:

$$\max_{\{c_t,h_t,k_{t+1},d_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t - \tau h_t^{\omega})^{1-\sigma}}{1-\sigma}$$
(27)

s.t. 
$$\frac{w_t}{\Gamma_{t-1}}h_t + u_tk_t + q_td_{t+1}g_t = d_t + c_t + k_{t+1}g_t - (1-\delta)k_t + \frac{\phi}{2}\left(\frac{k_{t+1}}{k_t}g_t - \mu\right)^2 k_t \quad (28)$$

The Lagrangian corresponding to the problem presented in (1') and (2') is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_t - \tau h_t^{\omega})^{1-\sigma}}{1 - \sigma} + \lambda_t \left[ \frac{w_t}{\Gamma_{t-1}} h_t + u_t k_t + q_t d_{t+1} g_t - d_t - c_t - k_{t+1} g_t + (1 - \delta) k_t - \frac{\phi}{2} \left( \frac{k_{t+1}}{k_t} g_t - \mu \right)^2 k_t \right] \right\}$$

where  $\lambda_t$  is the Lagrange multiplier.

The resulting first order conditions characterize the households' optimal decision con-

cerning consumption, labor supply, capital stock and external debt position.

Equation (7) characterizes the trade-off between leisure and consumption choices for period t such that in the optimum marginal utility of one additional unit consumed must be equal to the shadow price of households' resource constraint.

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \quad \Rightarrow \ (c_t - \tau h_t^{\omega})^{-\sigma} = \lambda_t \tag{29}$$

Households' optimal labor supply decision is described by equation (18) and it states that they supply that quantity of labor which ensures an equality between utility loss from working and marginal revenue:

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \quad \Rightarrow \tau \omega h_t^{\omega - 1} \ \left( c_t - \tau h_t^{\omega} \right)^{-\sigma} = \lambda_t \frac{w_t}{\Gamma_{t-1}} = \lambda_t \frac{\alpha \ a_t k_t^{1-\alpha} h_t^{\alpha - 1} g_t^{\alpha}}{\left[ 1 + \theta \left( R_{t-1} - 1 \right) \right]} \tag{30}$$

The optimal decision of households concerning investment in capital stock is characterized by:

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \implies \lambda_t \left[ g_t + \phi \left( \frac{k_{t+1}}{k_t} g_t - \mu \right) g_t \right]$$
(31)

$$= \beta E_t \lambda_{t+1} \left[ u_{t+1} + 1 - \delta + \phi \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \mu \right) \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \frac{\phi}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \mu \right)^2 \right]$$
  
$$\implies \lambda_t \left[ g_t + \phi \left( \frac{k_{t+1}}{k_t} g_t - \mu \right) g_t \right]$$
  
$$\beta E_t \lambda_{t+1} \left[ (1 - \alpha) a_{t+1} k_{t+1}^{-\alpha} h_{t+1}^{\alpha} g_{t+1}^{\alpha} + 1 - \delta + \phi \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \mu \right) \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \frac{\phi}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \mu \right)^2 \right]$$

Optimal debt decision is defined based on the following equation:

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0 \quad \Rightarrow \quad \lambda_t q_t g_t = \beta E_t \lambda_{t+1} \tag{32}$$

Based on equations (17) - (20) and (4') households derive their contingent plan :  $\{c_t, h_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}$  by taking as given:  $\{q_t, w_t, u_t\}_{t=0}^{\infty}$  where  $w_t$  and  $u_t$  are the prices of labor, respectively capital which clear the market under perfect competition.  $q_t$  is the price of one unit external debt and its definition varies across model specification:

1. Under Chang and Fernandez (2009) differenciated interest rate specification  $q_t$  is given by:

$$\frac{1}{q_t} = R_t + \Psi\left(D_{t+1}/\Gamma_t\right) \Longrightarrow \frac{1}{q_t} = R_t + \Psi\left(d_{t+1}\right)$$

where:

=

$$\Psi\left(D_{t+1}/\Gamma_t\right) = \Psi\left(d_{t+1}\right) = \psi\left[\exp\left(d_{t+1} - \overline{d}\right) - 1\right]$$

Therefore, the stationarized  $q_t$  is:

$$\frac{1}{q_t} = R_t + \psi \left[ \exp \left( d_{t+1} - \overline{d} \right) - 1 \right]$$
(33)

2. Under uniform interest rate specification (i.e. households and firms pay the same interest rate):

$$\frac{1}{q_t} = R_t \tag{33'}$$

### 9.2 Nonlinear equilibrium conditions

Based on the algebra presented in the previous section, the following 16 nonlinear equations describe the optimal dynamics of the model variables

 $\{y_t, a_t, k_{t+1}, h_t, g_t, i_t, d_{t+1}, c_t, R_t, S_t, R_t, q_t, tb_t, tby_t, \lambda_t\}:$ 

$$(c_{t} - \tau h_{t}^{\omega})^{-\sigma} = \lambda_{t}$$

$$\tau \omega h_{t}^{\omega-1} (c_{t} - \tau h_{t}^{\omega})^{-\sigma} = \lambda_{t} \frac{\alpha \ a_{t} k_{t}^{1-\alpha} h_{t}^{\alpha-1} g_{t}^{\alpha}}{[1 + \theta (R_{t-1} - 1)]}$$

$$\lambda_{t} q_{t} g_{t} = \beta E_{t} \lambda_{t+1}$$

$$\frac{1}{q_{t}} = R_{t} + \psi \left[ \exp \left( d_{t+1} - \overline{d} \right) - 1 \right]$$

$$\frac{\alpha \ a_{t} k_{t}^{1-\alpha} h_{t}^{\alpha-1} g_{t}^{\alpha}}{[1 + \theta (R_{t-1} - 1)]} h_{t} + (1 - \alpha) \ a_{t} k_{t}^{-\alpha} h_{t}^{\alpha} \ g_{t}^{\alpha} k_{t} + q_{t} d_{t+1} g_{t} = d_{t} + c_{t} + k_{t+1} g_{t} - (1 - \delta) \ k_{t} + \frac{\phi}{2} \left( \frac{k_{t+1}}{k_{t}} g_{t} - \mu \right)^{2} k_{t}$$

$$g_{t} k_{t+1} = (1 - \delta) \ k_{t} + i_{t} - \frac{\phi}{2} \left( \frac{k_{t+1}}{k_{t}} g_{t} - \mu \right)^{2} k_{t}$$

$$\log a_{t} = \rho_{a} \log a_{t-1} + \epsilon_{t}^{a}$$

$$\log (g_{t+1}/\mu) = \rho_{g} \log (g_{t}/\mu) + \epsilon_{t+1}^{g}$$

$$R_{t} = S_{t} R_{t}^{*}$$

$$\log \left( R_{t}^{*}/R^{*} \right) = \rho_{R^{*}} \log \left( R_{t-1}^{*}/\overline{R}^{*} \right) + \epsilon_{t}^{R^{*}}$$

$$tb_{t} = y_{t} - c_{t} - i_{t}$$

 $tby_t = \frac{y_t - c_t - i_t}{y_t}$ 

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The country spread specification varies across models which are as follows:

1. Under Chang and Fernandez (2009) differenciated interest rate specification

$$\log\left(S_t/\overline{S}\right) = -\eta\left[\log a_{t+1} + \log\left(g_{t+1}^{\alpha}/\mu^{\alpha}\right)\right]$$
(34)

2. Under uniform interest rate specification (i.e. households and firms pay the same interest rate) when country spread is assumed not to be persistent:

$$\log\left(S_t/\overline{S}\right) = -\eta\left[\log a_{t+1} + \log\left(g_{t+1}^{\alpha}/\mu^{\alpha}\right)\right] + \Psi\left(d_{t+1}\right)$$
(34')

where  $\Psi(d_{t+1}) = \psi \left[ \exp \left( d_{t+1} - \overline{d} \right) - 1 \right]$  is

3. Under uniform interest rate specification when country spread has some persistence and it reacts to fluctuations in the Eurozone interest rate:

$$\log\left(S_t/\overline{S}\right) = -\eta_{SR}\left[\log a_t + \log\left(g_t^{\alpha}/\mu^{\alpha}\right)\right] + \Psi\left(d_{t+1}\right) + \eta_{r^*}\log\left(R_t^*/\overline{R}^*\right) + \rho_s\log\left(S_{t-1}/\overline{S}\right) + \epsilon_t^s$$
(34")

## 9.3 Steady state of the model

In steady state, the dynamics of all the model variables  $(x_t)$  can be characterized by the identity:  $x_t = x_{t+1} = \overline{x}$ . Thus, the system of equations which fully characterizes the steady state of the model is nothing else than the nonlinear rational expectation equation system which gives the optimal dynamic of the model written in  $\overline{x}$ :

$$\left(\overline{c} - \tau \overline{h}^{\omega}\right)^{-\sigma} = \overline{\lambda} \tag{35}$$

$$\tau \omega \overline{h}^{\omega - 1} \left( \overline{c} - \tau \overline{h}^{\omega} \right)^{-\sigma} = \overline{\lambda} \frac{\alpha \ \overline{a} \overline{k}^{1 - \alpha} \overline{h}^{\alpha - 1} \overline{g}^{\alpha}}{\left[ 1 + \theta \left( \overline{R} - 1 \right) \right]}$$
(36)

$$\left[\overline{g} + \phi\left(\frac{\overline{k}}{\overline{k}}\overline{g} - \mu\right)\overline{g}\right] = \beta\left[(1-\alpha)\overline{a}\overline{k}^{-\alpha}\overline{h}^{\alpha}\overline{g}^{\alpha} + 1 - \delta + \phi\left(\frac{\overline{k}}{\overline{k}}\overline{g} - \mu\right)\frac{\overline{k}}{\overline{k}}\overline{g} - \frac{\phi}{2}\left(\frac{\overline{k}}{\overline{k}}\overline{g} - \mu\right)^{2}\right]$$
(37)

$$\overline{qg} = \beta \tag{38}$$

$$\frac{1}{\overline{q}} = \overline{R} + \psi \left[ \exp \left( \overline{d} - \overline{d} \right) - 1 \right]$$
(39)

$$\frac{\alpha \,\overline{a}\overline{k}^{1-\alpha}\overline{h}^{\alpha-1}\overline{g}^{\alpha}}{\left[1+\theta\left(\overline{R}-1\right)\right]}\overline{h}+\left(1-\alpha\right)\overline{a}\overline{k}^{-\alpha}\overline{h}^{\alpha}\,\overline{g}^{\alpha}\overline{k}+\overline{q}\overline{d}\overline{g}=\overline{d}+\overline{c}+\overline{k}\overline{g}-\left(1-\delta\right)\overline{k}+\frac{\phi}{2}\left(\frac{\overline{k}}{\overline{k}}\overline{g}-\mu\right)^{2}\overline{k}\quad(40)$$

$$\overline{g}\overline{k} = (1-\delta)\overline{k} + i - \frac{\phi}{2} \left(\frac{\overline{k}}{\overline{k}}\overline{g} - \mu\right)^2 \overline{k}$$
(41)

$$\log \overline{a} = \rho_a \log \overline{a} \tag{42}$$

$$\log\left(\overline{g}/\mu\right) = \rho_g \log\left(\overline{g}/\mu\right) \tag{43}$$

$$\overline{y} = \overline{a}\overline{k}^{1-\alpha}\overline{g}^{\alpha}\overline{h}^{\alpha} \tag{44}$$

$$\overline{R} = \overline{SR}^* \tag{45}$$

$$\log\left(\overline{R}^*/\overline{R}^*\right) = \rho_{R^*} \log\left(\overline{R}^*/\overline{R}^*\right) \tag{46}$$

$$\log\left(\overline{S}/\overline{S}\right) = -\eta\left[\log\overline{a} + \log\left(\overline{g}^{\alpha}/\overline{\mu}^{\alpha}\right)\right] \tag{47}$$

$$\overline{tb} = \overline{y} - \overline{c} - \overline{i} \tag{48}$$

$$\overline{tby} = \frac{\overline{y} - \overline{c} - \overline{i}}{\overline{y}} \tag{49}$$

From (35) and (36) it results:

$$\tau \omega \overline{h}^{\omega - 1} = \frac{\alpha \ \overline{a} \overline{k}^{1 - \alpha} \overline{h}^{\alpha - 1} \overline{g}^{\alpha}}{\left[1 + \theta \left(\overline{R} - 1\right)\right]} \Longrightarrow \overline{h} = \left\{ \left[ \frac{\alpha \ \overline{a} \overline{g}^{\alpha} \left(\overline{h} / \overline{k}\right)^{\alpha - 1}}{1 + \theta \left(\overline{R} - 1\right)} \right] \frac{1}{\tau \omega} \right\}^{\frac{1}{\omega - 1}}$$
(50)

From (37):

$$\overline{g} = \beta \left[ (1-\alpha) \,\overline{a} \overline{k}^{-\alpha} \overline{h}^{\alpha} \,\overline{g}^{\alpha} + 1 - \delta \right] \Longrightarrow \frac{\overline{h}}{\overline{k}} = \left\{ \left[ \frac{\overline{g}}{\beta} - (1-\delta) \right] \frac{1}{(1-\alpha) \,\overline{a} \overline{g}^{\alpha}} \right\}^{\frac{1}{\alpha}} \tag{51}$$

From (38):

$$\overline{q} = \frac{\beta}{\overline{g}} \tag{52}$$

From (39) and (52):

$$\overline{R} = \frac{1}{\overline{q}} = \frac{\overline{g}}{\beta} \tag{53}$$

From (40), (44), (52) and (53):

$$\overline{c} = \left[\frac{\alpha}{\left[1 + \theta\left(\frac{\overline{g}}{\beta} - 1\right)\right]} + (1 - \alpha)\right]\overline{y} + (\beta - 1)\overline{d} - \overline{i}$$
(54)

From (41):

$$\overline{i} = [\overline{g} - (1 - \delta)] \overline{k} \tag{55}$$

From (45) and (53):

$$\overline{S} = \frac{\overline{R}}{\overline{R}^*} \Longrightarrow \overline{S} = \frac{\overline{g}}{\beta \overline{R}^*}$$
(56)

Thus, by calibrating the model parameters and  $\{\overline{a}, \overline{g}, \overline{R}^*, \overline{d}\}$ , we have the steady state values of the model variables as follows:

$$\overline{R} = \frac{\overline{g}}{\beta}$$

$$\frac{\overline{h}}{\overline{k}} = \left\{ \left[\overline{R} - (1 - \delta)\right] \frac{1}{(1 - \alpha)\overline{a}\overline{g}^{\alpha}} \right\}^{\frac{1}{\alpha}}$$

$$\overline{h} = \left\{ \left[ \frac{\alpha \ \overline{a}\overline{g}^{\alpha}(\overline{h}/\overline{k})^{\alpha-1}}{1 + \theta(\overline{R} - 1)} \right] \frac{1}{\tau \omega} \right\}^{\frac{1}{\omega - 1}}$$

$$\overline{k} = \overline{h} / \left( \frac{\overline{h}}{\overline{k}} \right)$$

$$\overline{y} = \overline{a}\overline{k}^{1 - \alpha}\overline{g}^{\alpha}\overline{h}^{\alpha}$$

$$\overline{i} = \left[ \overline{g} - (1 - \delta) \right] \overline{k}$$

$$\overline{c} = \left[ \frac{\alpha}{\left[ 1 + \theta\left( \frac{\overline{g}}{\beta} - 1 \right) \right]} + (1 - \alpha) \right] \overline{y} + (\beta - 1) \overline{d} - \overline{i}$$

$$\overline{S} = \frac{\overline{R}^{*}}{\overline{R}}$$

$$\overline{q} = \frac{\beta}{\overline{g}}$$

$$\overline{\lambda} = \left( \overline{c} - \tau \overline{h}^{\omega} \right)^{-\sigma}$$

$$\overline{tb} = \overline{y} - \overline{c} - \overline{i}$$

$$\overline{tby} = \frac{\overline{y} - \overline{c} - \overline{i}}{\overline{y}}$$

## 9.4 Log-linearized equilibrium conditions

The last part of the model appendix presents the derivation of the log-linearized equilibrium conditions around the steady state as follows:

The equation describing leisure-consumption trade-off:

$$(c_t - \tau h_t^{\omega})^{-\sigma} = \lambda_t \iff \left(\overline{c}e^{\widehat{c}_t} - \tau \left(\overline{h}e^{\widehat{h}_t}\right)^{\omega}\right)^{-\sigma} = \overline{\lambda}e^{\widehat{\lambda}_t}$$
$$-\sigma \left(\overline{c} - \tau \left(\overline{h}\right)^{\omega}\right)^{-\sigma-1} \overline{c}\widehat{c}_t + \sigma \left(\overline{c} - \tau \left(\overline{h}\right)^{\omega}\right)^{-\sigma-1} \tau \left(\overline{h}\right)^{\omega} \omega \widehat{h}_t = \overline{\lambda}\widehat{\lambda}_t$$

$$\frac{\sigma}{\overline{c} - \tau \overline{h}^{\omega}} \left[ \tau \omega \overline{h}^{\omega} \widehat{h}_t - \overline{c} \widehat{c}_t \right] = \widehat{\lambda}_t \tag{57}$$

Labor supply equation:

 $\implies$ 

 $\Rightarrow$ 

$$\tau \omega h_t^{\omega - 1} \left( c_t - \tau h_t^{\omega} \right)^{-\sigma} = \lambda_t \frac{\alpha \ a_t k_t^{1 - \alpha} h_t^{\alpha - 1} g_t^{\alpha}}{\left[ 1 + \theta \left( R_{t-1} - 1 \right) \right]}$$

$$\tau\omega\overline{h}^{\omega-1} \ (-\sigma)\left(\overline{c}-\tau\overline{h}^{\omega}\right)^{-\sigma-1}\left(\overline{c}\widehat{c}_t-\tau\omega\overline{h}^{\omega}\widehat{h}_t\right)+\left(\overline{c}-\tau\overline{h}^{\omega}\right)^{-\sigma}\tau\omega\left(\omega-1\right)\overline{h}^{\omega-1}\widehat{h}_t=$$

$$\begin{split} \overline{\lambda} \frac{\alpha \ \overline{a} \overline{k}^{1-\alpha} \overline{h}^{\alpha-1} \overline{g}^{\alpha}}{1+\theta \left(\overline{R}-1\right)} \left(\widehat{\lambda}_{t}+\widehat{a}_{t}\right) + \overline{\lambda} \frac{\alpha^{2} \overline{a} \overline{k}^{1-\alpha} \overline{h}^{\alpha-1} \overline{g}^{\alpha-1}}{1+\theta \left(\overline{R}-1\right)} \overline{g} \widehat{g}_{t} + \overline{\lambda} \frac{\alpha \left(1-\alpha\right) \overline{a} \overline{k}^{-\alpha} \overline{h}^{\alpha-1} \overline{g}^{\alpha}}{1+\theta \left(\overline{R}-1\right)} \overline{k} \widehat{k}_{t} \\ + \overline{\lambda} \frac{\alpha \left(\alpha-1\right) \overline{a} \overline{k}^{1-\alpha} \overline{h}^{\alpha-1} \overline{g}^{\alpha}}{1+\theta \left(\overline{R}-1\right)} \overline{h} \widehat{h}_{t} - \overline{\lambda} \frac{\alpha \overline{a} \overline{k}^{1-\alpha} \overline{h}^{\alpha-1} \overline{g}^{\alpha}}{\left[1+\theta \left(\overline{R}-1\right)\right]^{2}} \theta \overline{R} \widehat{R}_{t-1} \\ \Longrightarrow \end{split}$$

$$-\frac{\sigma}{\overline{c}-\tau\overline{h}^{\omega}}\left(\overline{c}\widehat{c}_{t}-\tau\omega\overline{h}^{\omega}\widehat{h}_{t}\right)+(\omega-1)\widehat{h}_{t}=\widehat{\lambda}_{t}+\widehat{a}_{t}+\alpha\widehat{g}_{t}+(1-\alpha)\widehat{k}_{t}+(\alpha-1)\widehat{h}_{t}-\frac{\theta\overline{R}}{1+\theta\left(\overline{R}-1\right)}\widehat{R}_{t-1}$$

By combining this with (57) it yields:

$$(\omega - 1)\widehat{h}_t = \widehat{a}_t + \alpha \widehat{g}_t + (1 - \alpha)\widehat{k}_t + (\alpha - 1)\widehat{h}_t - \frac{\theta R}{1 + \theta \left(\overline{R} - 1\right)}\widehat{R}_{t-1}$$
(58)

Optimal capital goods investment decision:

$$\lambda_{t} \left[ g_{t} + \phi \left( \frac{k_{t+1}}{k_{t}} g_{t} - \mu \right) g_{t} \right]$$

$$= \beta E_{t} \lambda_{t+1} \left[ (1 - \alpha) a_{t+1} k_{t+1}^{-\alpha} h_{t+1}^{\alpha} g_{t+1}^{\alpha} + 1 - \delta + \phi \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \mu \right) \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \frac{\phi}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \mu \right)^{2} \right]$$

$$\Longrightarrow$$

$$+\beta\overline{\lambda}\left\{(1-\alpha)\,\overline{a}\overline{k}^{-\alpha}\overline{h}^{\alpha}\,\overline{g}^{\alpha}\left[\widehat{a}_{t+1}-\alpha\widehat{k}_{t+1}+\alpha\left(\widehat{h}_{t+1}+\widehat{g}_{t+1}\right)\right]+\phi\frac{\overline{k}}{\overline{k}^{2}}\overline{g}^{2}\overline{k}\widehat{k}_{t+2}+\phi\left(\frac{\overline{k}}{\overline{k}}\overline{g}-\mu\right)\frac{\overline{k}}{\overline{k}}\overline{g}\widehat{k}_{t+2}\right\}+$$

$$+\beta\overline{\lambda}\left\{\phi\frac{\overline{k}^{2}}{\overline{k}^{2}}\overline{g}^{2}\widehat{g}_{t+1}+\phi\left(\frac{\overline{k}}{\overline{k}}\overline{g}-\mu\right)\frac{\overline{k}}{\overline{k}}\overline{g}\widehat{g}_{t+1}-\phi\left(\frac{\overline{k}}{\overline{k}}\overline{g}-\mu\right)\frac{\overline{k}^{2}}{\overline{k}^{2}}\overline{g}\widehat{k}_{t+1}-\frac{\overline{k}}{\overline{k}}\overline{g}^{2}\phi\frac{\overline{k}^{2}}{\overline{k}^{2}}\widehat{k}_{t+1}\right\}$$

$$\Longrightarrow$$

$$\widehat{\lambda}_{t} + (1 + \phi \overline{g}) \,\widehat{g}_{t} + \phi \overline{g} \left(\widehat{k}_{t+1} - \widehat{k}_{t}\right) = \frac{\beta}{\overline{g}} \left[ (1 - \alpha) \,\overline{a} \overline{k}^{-\alpha} \overline{h}^{\alpha} \,\overline{g}^{\alpha} + 1 - \delta \right] E_{t} \widehat{\lambda}_{t+1} + \beta \left\{ (1 - \alpha) \,\overline{a} \overline{k}^{-\alpha} \overline{h}^{\alpha} \,\overline{g}^{\alpha-1} \left[ \widehat{a}_{t+1} - \alpha \widehat{k}_{t+1} + \alpha \left( \widehat{h}_{t+1} + \widehat{g}_{t+1} \right) \right] + \phi \overline{g} \left( \widehat{k}_{t+2} + \widehat{g}_{t+1} - \widehat{k}_{t+1} \right) \right\}$$
(59)  
Household's optimal decision concerning their external debt position:

Household's optimal decision concerning their external debt position:

$$\lambda_t q_t g_t = \beta E_t \lambda_{t+1} \Longrightarrow \overline{\lambda} \overline{q} \overline{g} \left( \widehat{\lambda}_t + \widehat{q}_t + \widehat{g}_t \right) = \beta \overline{\lambda} E_t \widehat{\lambda}_{t+1}$$
$$\widehat{\lambda}_t + \widehat{q}_t + \widehat{g}_t = E_t \widehat{\lambda}_{t+1}$$
(60)

The price of one unit external debt:

 $\Rightarrow$ 

 $\implies$ 

1. Under Chang and Fernandez (2009) differenciated interest rate specification:

$$\frac{1}{q_t} = R_t + \psi \left[ \exp \left( d_{t+1} - \overline{d} \right) - 1 \right] \Longrightarrow -\frac{1}{\overline{q}^2} \overline{q} \widehat{q}_t = \overline{R} \widehat{R}_t + \psi \overline{d} \widehat{d}_{t+1} -\frac{1}{\overline{q}} \widehat{q}_t = \overline{R} \widehat{R}_t + \psi \overline{d} \widehat{d}_{t+1}$$

$$(61)$$

2. Under uniform interest rate specification:

$$-\widehat{q}_t = \widehat{R}_t \tag{61'}$$

The household's budget constraint:

$$\frac{\alpha \ a_t k_t^{1-\alpha} h_t^{\alpha} g_t^{\alpha}}{\left[1+\theta \left(R_{t-1}-1\right)\right]} + (1-\alpha) \ a_t k_t^{1-\alpha} h_t^{\alpha} \ g_t^{\alpha} + q_t d_{t+1} g_t = d_t + c_t + k_{t+1} g_t - (1-\delta) \ k_t + \frac{\phi}{2} \left(\frac{k_{t+1}}{k_t} g_t - \mu\right)^2 k_t$$

$$\implies \frac{\alpha \ \overline{ak}^{1-\alpha} \overline{h}^{\alpha} \overline{g}^{\alpha}}{\left[1+\theta \left(\overline{R}-1\right)\right]} \left[\widehat{a}_t + \alpha \widehat{g}_t + (1-\alpha) \ \widehat{k}_t + \alpha \widehat{h}_t - \frac{\theta \overline{R}}{\left[1+\theta \left(\overline{R}-1\right)\right]} \widehat{R}_{t-1}\right] + (1-\alpha) \ \overline{ak}^{1-\alpha} \overline{h}^{\alpha} \ \overline{g}^{\alpha} \left[\widehat{a}_t + (1-\alpha) \ \widehat{k}_t + \alpha \left(\widehat{h}_t + \widehat{g}_t\right)\right] + \overline{qd} \overline{g} (\widehat{q}_t + \widehat{d}_{t+1} + \widehat{g}_t) =$$

$$= \overline{dd}_t + \overline{cc}_t + \overline{kg} \left(\widehat{k}_{t+1} + \widehat{g}_t\right) - (1-\delta) \ \overline{kk}_t \qquad (62)$$

The capital accumulation equation:

$$g_t k_{t+1} = (1 - \delta) k_t + i_t - \frac{\phi}{2} \left(\frac{k_{t+1}}{k_t} g_t - \mu\right)^2 k_t$$
$$\implies \overline{k}\overline{g} \left(\widehat{k}_{t+1} + \widehat{g}_t\right) = (1 - \delta) \overline{k}\widehat{k}_t + \overline{i}\widehat{i}_t$$
(63)

The stochastic processes which describe the dynamics of exogenous transitory and permanent technology growth:

$$\log a_t = \rho_a \log a_{t-1} + \epsilon_t^a \iff \hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t^a \tag{64}$$

$$\log\left(g_{t+1}/\mu\right) = \rho_g \log\left(g_t/\mu\right) + \epsilon_{t+1}^g \iff \widehat{g}_{t+1} = \rho_g \widehat{g}_t + \epsilon_{t+1}^g \tag{65}$$

The firm's production technology:

$$y_t = a_t k_t^{1-\alpha} g_t^{\alpha} h_t^{\alpha} \Longrightarrow \widehat{y}_t = \widehat{a}_t + (1-\alpha) \,\widehat{k}_t + \alpha \left(\widehat{g}_t + \widehat{h}_t\right) \tag{66}$$

The domestic real interest rate equation:

$$R_t = S_t R_t^* \Longrightarrow \widehat{R}_t = \widehat{S}_t + \widehat{R}_t^* \tag{67}$$

The stochastic process which describe the dynamics of Eurozone interest rate:

$$\log\left(R_t^*/\overline{R}^*\right) = \rho_{R^*}\log\left(R_{t-1}^*/\overline{R}^*\right) + \epsilon_t^{R^*} \iff \widehat{R}_t^* = \rho_{R^*}\widehat{R}_{t-1}^* + \epsilon_t^{R^*} \tag{68}$$

Trade balance identity:

$$tb_t = y_t - c_t - i_t \Longrightarrow \widehat{tb}_t = \overline{y}\widehat{y}_t - \overline{c}\widehat{c}_t - \overline{i}\widehat{i}_t$$
(69)

The trade balance to output ratio:

$$tby_t = \frac{y_t - c_t - i_t}{y_t} \Longrightarrow \widehat{tby}_t = -\frac{\overline{c}}{\overline{y}} \left(\widehat{c}_t - \widehat{y}_t\right) - \frac{\overline{i}}{\overline{y}} \left(\widehat{i}_t - \widehat{y}_t\right)$$
(70)

The country spread equation:

1. Under Chang and Fernandez (2009) differenciated interest rate specification

$$\log\left(S_t/\overline{S}\right) = -\eta\left[\log a_{t+1} + \log\left(g_{t+1}^{\alpha}/\mu^{\alpha}\right)\right] \Longrightarrow \widehat{S}_t = -\eta\left[\widehat{a}_{t+1} + \alpha\widehat{g}_{t+1}\right]$$
(71)

2. Under uniform interest rate specification (i.e. households and firms pay the same interest rate) when country spread is assumed not to be persistent:

$$\log\left(S_t/\overline{S}\right) = -\eta\left[\log a_{t+1} + \log\left(g_{t+1}^{\alpha}/\mu^{\alpha}\right)\right] + \Psi\left(d_{t+1}\right) \Longrightarrow \widehat{S}_t = -\eta\left[\widehat{a}_{t+1} + \alpha\widehat{g}_{t+1}\right] + \psi\overline{d}\widehat{d}_{t+1}$$
(71')

3. Under uniform interest rate specification when country spread has some persistence

and it reacts to fluctuations in the Eurozone interest rate:

$$\log\left(S_t/\overline{S}\right) = -\eta_{SR}\left[\log a_t + \log\left(g_t^{\alpha}/\mu^{\alpha}\right)\right] + \Psi\left(d_{t+1}\right) + \eta_{r^*}\log\left(R_t^*/\overline{R}^*\right) + \rho_s\log\left(S_{t-1}/\overline{S}\right) + \epsilon_t^s$$

$$(71")$$

$$\Longrightarrow \widehat{S}_t = -\eta\left[\widehat{a}_{t+1} + \alpha\widehat{g}_{t+1}\right] + \psi\overline{d}\widehat{d}_{t+1} + \eta_{r^*}\widehat{R}_t^* + \rho_s\widehat{S}_{t-1} + \epsilon_t^s$$

Consequently, the log-linenearized rational expectation system of equations which describes the optimal dynamics of  $\left\{\widehat{y}_t, \widehat{a}_t, \widehat{k}_{t+1}, \widehat{h}_t, \widehat{g}_t, \widehat{i}_t, \widehat{d}_{t+1}, \widehat{c}_t, \widehat{R}_t, \widehat{S}_t, \widehat{R}_t^*, \widehat{q}_t, \widehat{tb}_t, \widehat{tby}_t, \widehat{\lambda}_t\right\}$  is:

$$\begin{split} \frac{\sigma}{\overline{c}-\tau h^{\omega}} \left[ \tau \omega \overline{h}^{\omega} \widehat{h}_{t} - \overline{c} \widehat{c}_{t} \right] &= \widehat{\lambda}_{t} \\ \frac{\sigma \tau \omega \overline{h}^{\omega}}{\overline{c}-\tau h^{\omega}} \widehat{h}_{t} + (\omega - 1) \widehat{h}_{t} &= \widehat{a}_{t} + \alpha \widehat{g}_{t} + (1 - \alpha) \widehat{k}_{t} + (\alpha - 1) \widehat{h}_{t} - \frac{\theta \overline{R}}{1 + \theta (\overline{R} - 1)} \widehat{R}_{t-1} \\ &= \frac{\beta}{\overline{g}} \left[ (1 - \alpha) \overline{a} \overline{k}^{-\alpha} \overline{h}^{\alpha} \overline{g}^{\alpha} + 1 - \delta \right] E_{t} \widehat{\lambda}_{t+1} + \\ + \beta \left\{ (1 - \alpha) \overline{a} \overline{k}^{-\alpha} \overline{h}^{\alpha} \overline{g}^{\alpha - 1} \left[ \widehat{a}_{t+1} - \alpha \widehat{k}_{t+1} + \alpha \left( \widehat{h}_{t+1} + \widehat{g}_{t+1} \right) \right] + \theta \overline{g} \left( \widehat{k}_{t+2} + \widehat{g}_{t+1} - \widehat{k}_{t+1} \right) \right\} \\ &= \widehat{\lambda}_{t} + \widehat{q}_{t} + \widehat{g}_{t} = E_{t} \widehat{\lambda}_{t+1} \\ \frac{\alpha}{1 + \theta (\overline{R} - 1)} \left[ \widehat{a}_{t} + \alpha \widehat{g}_{t} + (1 - \alpha) \widehat{k}_{t} + \alpha \widehat{h}_{t} - \frac{\theta \overline{R}}{1 + \theta (\overline{R} - 1)} \widehat{R}_{t-1} \right] + \\ &+ (1 - \alpha) \overline{a} \overline{k}^{1 - \alpha} \overline{h}^{\alpha} \overline{g}^{\alpha} \left[ \widehat{a}_{t} + (1 - \alpha) \widehat{k}_{t} + \alpha \left( \widehat{h}_{t} + \widehat{g}_{t} \right) \right] + \\ &+ \overline{q} d \overline{g} (\widehat{q}_{t} + \widehat{d}_{t+1} + \widehat{g}_{t}) \\ &= \overline{d} \widehat{d}_{t} + \overline{c} \widehat{c}_{t} + \overline{k} \overline{g} \left( \widehat{k}_{t+1} + \widehat{g}_{t} \right) - (1 - \delta) \overline{k} \widehat{k}_{t} \\ &= \overline{k} \widehat{d}_{t} + \overline{c} \widehat{c}_{t} + \overline{k} \overline{g} \left( \widehat{k}_{t+1} + \widehat{g}_{t} \right) - (1 - \delta) \overline{k} \widehat{k}_{t} \\ &= \overline{d} \widehat{d}_{t} + \overline{c} \widehat{c}_{t} + \overline{k} \overline{g} \left( \widehat{k}_{t+1} + \widehat{q}_{t} \right) - (1 - \delta) \overline{k} \widehat{k}_{t} \\ &= \widehat{d} \widehat{d}_{t} + \overline{c} \widehat{c}_{t} + \overline{k} \widehat{g} \left( \widehat{k}_{t+1} + \widehat{g}_{t} \right) \\ &= \overline{d} \widehat{d}_{t} + \overline{c} \widehat{c}_{t} + \overline{k} \widehat{g} \left( \widehat{k}_{t+1} + \widehat{g}_{t} \right) \\ &= \widehat{d} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + \widehat{c}_{t}^{*} \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + \widehat{c}_{t}^{*} \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + \widehat{k}_{t}^{*} \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + 1 \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + 1 \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + 1 \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 0 \widehat{g} \widehat{k}_{t} + 1 \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} - \widehat{g} \widehat{k}_{t} + 0 \widehat{k} \\ &= \widehat{g} \widehat{g} \widehat{k}_{t} + 1 \\ &= \widehat{g} \widehat{k} \widehat{k}_{t} + 1 \\ &= \widehat{g} \widehat{k} +$$