Term Structure of Interest Rates in Small Open Economy Model

by

Aleš Maršál

Submitted to

Central European University

Department of Department of Economics

In partial fulfulment of the requirements for the degree of Master of Science

Supervisor: Professor Alessia Campolmi

Budapest, Hungary

2010

I, the undersigned [Aleš Maršál], candidate for the degree of Master of Science at the Central European University Department of Economics, declare herewith that the present thesis is exclusively my own work, based on my researc and only such external informaion as properly credited in notes and bibliography. I declare that no unidentifies and illegitimate use was made of work of others, and no part the the thesis infrignes on any person's or intstitution's copyright. I also declare that no part the theisi has been submitted in this for to any other institution of higher education for an academic degree.

Budapest, 7 June 2010

Signature

Acknowledgments

I would like to thank the CEU faculty for a challenging program in Economics and particularly, Alessia Campolmi for supervising me. For providing me with data, I thank Debt Management Department of Czech Treasury.

Abstract

I lay out small open economy model with nominal rigidities to study the implication of model dynamics on the term structure of interest rates. It has been shown that in order to obtain at least moderate match simultaneously of the macro and finance data, one has to introduce long-memory habits in consumption together with a large number of highly persistent exogenous shocks. These elements of the model however worsen the fit of macro data. I find that in the open economy framework the foreign demand channel allows us to match some of the data features even without including habits and a large number of exogenous shocks.

Table of Contents

Chapter 1

Introduction

The term structure of interest rates is the key source of information in macroeconomics and finance. The yield curve has been established as an essential tool in predicting the business cycle; it is a fundamental input in asset pricing and debt management. However, macroeconomic models have had difficulties in matching the macro and financial data. For this reason estimates of the term structure are usually derived from the latent factor financial models. This dichotomous modeling approach leads to several problems.

First, it does not confirm mainstream economic theory. As emphasized by Rudenbush and Swanson (2008) the importance of joint modeling of both macroeconomic and finance variables within a DSGE framework is often underappreciated. Macroeconomics and the theory of asset pricing are closely related. This fact is nicely formulated by Cochrane (2001), who points out that asset markets are the mechanism by which consumption and investment are allocated across time and states of nature in such way that the marginal rates of substitution and transformation are equalized. Hordahl et al. (2007) argue that the inability of macro models to match asset prices could be, to some extent, justified since the expected future profitability of individual firms is unobservable and difficult to evaluate.

Equity prices may therefore be thought to be subject to fluctuations disconnected from the real economy. Yet this reasoning is not valid for bond prices. The term structure of interest rates incorporate expectations of future monetary policy decisions which have been relatively well predictable in recent two decades.

Second, financial models do not account for monetary policy and macroeconomics fundamentals as stressed by Rudebusch and Wu (2004). The short term interest rate is the basic building block of the yield curve which is under direct control of monetary authority. The long interest rates are nothing else than risk adjusted expectations about the short term interest rates, hence the behavior of the central bank is an important source of information in determining the shape of the yield curve.

Third, many interesting questions in economics are related exactly to the interaction between macroeconomics variables and asset prices. For example, recent problems of many countries to pay back their government debts and their excessive debt financing in general arise questions how does the implied increase in term premium affects the economy.

My work contributes to the discussion related to a modeling the term structure of interest rates in the DSGE framework. However, contrary to other authors e.g. (Rudenbush and Swanson, 2008), (Hordahl et al., 2007), (Andreasen, 2008) who rely entirely on nominal rigidities, habit formation and large persistence of shocks I focus on the open economy implications on the term structure of interest rates in the DSGE model. To my knowledge this question has been neglected by the macro-finance literature.

The main motivating ideas behind the exercise encouraging my research question are driven by the fact that there is basically no model reaching at least moderate success in matching the data which do not include habit formation. However, it is known Justiniano (2010) that the implications of habit formation are different in small open economy in comparison with the closed economy. In the closed economy, habits decrease the standard deviation of output and consumption contrary to the increase in small open economy. At the same time volatility of consumption is one of the key elements affecting the term premium. Habit formations also significantly alter the autocorrelation of some series and as it will be emphasized later, autocorrelation is important factor influencing the variance of bond prices. For this reason, the benchmark model does not contain habit formation. Moreover, the behavior of agents facing the shock is different in the small open economy than in the closed economy. Consumption smoothing households in closed economy react to positive shock (characteristic by increase in real wages) by decreasing hours worked. Yet in open economy households do not have to decrease hours worked in order to smooth consumption because of the foreign demand channel. They can keep consumption constant, increase numbers of hours worked and sell the extra production to the rest of the world. Nevertheless, eventually the accumulated wealth leads to rise in consumption. The different dynamics of consumption behavior in small open economy may be the second aspect modifying the evolution of the term structure of interest rates throughout the business cycle.

Introduction of foreign demand channel in the DSGE model has following consequences: *i*) the model calibrated to fit the Czech moments is capable of delivering the positive term premium and solve Backus, Gregory, and Zin (1989) puzzle without introducing the habit formation, nevertheless the model does not match the level of term premium, parameterization matching the level of term premium produce negative slope of yield curve *ii*) contrary to closed economy models, the small economy framework generate sufficiently hight volatility on the long tail of yield curve, *iii*) model is not able to generate high enough term premium simultaneously with the positive slope of an yield curve and sufficiently high volatility of long yields.

The methods how to derive a small open economy can be various. In the open economy literature one can often encounter the technique proposed by Galí (2002) where the small open economy is one among a continuum of infinitesimally small economies making up the world economy. Another way to derive the small open economy model from two country model is based on assuming approximately zero weight in price and consumption index of the foreign country e.g.(Monacelli, 2003). I use the third, less frequent option e.g. (De-Paoli, 2006) and (Sutherland, 2006) which is based on taking the limit of the size of one of country to zero. This method allows, as in Monacelli (2003), to derive the small open economy from the two country model but it is more intuitive and coherent. Nevertheless, three methods I have just mentioned are equivalent, they deliver the same equilibrium conditions.

I use two country model of Bergin and Tchakarov (2003) to derive small open economy model. The model is suitable because it offers relatively rich model representation of the economy with money in the utility function, intermediate and final markets and habits in consumption, moreover this model can be easily extended of currency substitution e.g. (Colantoni, 2010) Although, I simplify the Bergin and Tchakarov (2003) framework for my benchmark model it can be easily again extended for future studying of implications of particular model specifications of open economy model on the term structure of interest rates which I am going to address in my future work.

The conclusion of Backus, Gregory, and Zin (1989) and Den-Haan (1995) that the general equilibrium models cannot generate term premia of a magnitude comparable to what we can observe in actual data has triggered fast growing research in this area. Consequently, there have been several relatively successful attempts to fit macro and term structure data in DSGE model. Hordahl et al. (2007) use the stochastic discount factor to model term premium. They assume expectations hypothesis which implies that the term premium is constant over time. The success of their model to fit macro and finance data relies on relatively large number of exogenous shock, long memory and high degree of interest rate smoothing. The nominal rigidities have indirect effect; sticky prices imply monetary non-neutrality. Number of papers tries to match the data using third order approximation e.g. (Rudenbush and Swanson, 2008). This method allows for variable term premium. Nevertheless, Rudenbush and Swanson (2008) conclude that in order to match the finance date in DSGE model, one has to necessary seriously distort the ability to fit other macroeconomic variables. Caprioli and Gnocchi (2009) uses collocation method with Chebychev polynomials to investigate the impact of monetary policy credibility on the term structure of interest rates. Andreasen (2008) address the fact that stationary shocks to the economy have only moderate effects on interest rates with medium and long maturities. Hence, they introduce non-stationary shocks. They argue that whereas highly persistent stationary shock may also affect interest rates with longer maturities this shocks are likely to distort the dynamics of the macroeconomy and this is not the case of permanent shocks.

The rest of the thesis is organized as follows. Section 2 presents the macro part of the model which consists of a small open economy DSGE model. The section 3 discuses the calibration of the benchmark model. The finance part is presented in section 4 where I outline the general characteristics of the term structure of interest rates data and derive the yield curve implied by the DSGE model. In section 6 I evaluate the results of model simulations compare to the data from the Czech economy. The extensions to the benchmark model are presented in section 7. Section 8 concludes.

Chapter 2

Macro part: Model

This section presents a DSGE model which has three types of agents: *i*) households, *ii*) firms, *iii*) monetary authority. The economy is assumed to be driven by the foreigner output shock. The small economy framework is derived as a limiting case of the two country model similar to Bergin and Tchakarov (2003). The technique I employ to solve for small open economy model builds on the method developed by Obstfeld and Rogoff (1994) and used in Sutherland (2006) and De-Paoli (2006). The specification of the model allows us to produce deviations from purchasing power parity which arise from the existence of home bias in consumption. The benchmark model is specific by single foreign output shock and linear production function.

2.1 Households

The economy is populated by continuum of representative, infinitely - long living households which sum up to one. The representative household seeks to maximize the following intertemporal sum of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t)^{1-\sigma_1}}{1-\sigma_1} - \omega \frac{N_t^{1+\sigma_2}}{1+\sigma_2} \right\} \tag{2.1}$$

where $\beta \in (0,1)$ is the subjective discount factor of future stream of utilities. C is the aggregate consumption.

The representative household faces following budged constrain

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} \leq B_t + D + W_t N_t + T_t \tag{2.2}$$

where $Q_{t,t+1}$ is one period ahead stochastic discount factor at time t. Agents have access to a complete array of state-contingent claims, thus B_{t+1} can be understand as a single financial asset that pays a risk-free rate of return (one year risk free bond). D is the share of the aggregate profits. Firms are assumed to be owned by households therefore profits serve as a resource for households. T_t are lump-sum government transfers. All variables are expressed in units of domestic currency.

The representative household has to solve the following problem.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{(C_t)^{1-\sigma_1}}{1-\sigma_1} - \omega \frac{N_t^{1+\sigma_2}}{1+\sigma_2} \\ +\lambda_t \left[\begin{array}{c} B_t + D + W_t N_t + T_t - P_t C_t - Q_{t,t+1} B_{t+1} \end{array} \right] \end{array} \right\}$$

$\frac{\partial \mathcal{L}}{\partial C_t}$:

$$(C_t)^{-\sigma_1} = P_t \lambda_t \tag{2.3}$$

$\frac{\partial \mathcal{L}}{\partial B_{t+1}}$:

$$E_t \beta^{t+1} \lambda_{t+1} = \beta^t \lambda_t Q_{t,t+1} \tag{2.4}$$

$\frac{\partial \mathcal{L}}{\partial W_t(i)}$:

$$\frac{W_t}{P_t} = \frac{\omega N_t^{\sigma_2}}{C_t^{-\sigma_1}} \tag{2.5}$$

2.2 Preferences

The small open economy representation induces independence of the rest of the world from the domestic policy and therefore we can abstract from the strategic interaction between SOE and ROW.

Consumption C is represented by a Dixit-Stiglitz aggregator of home and foreign consumption.

$$C_t = \left[\gamma^{\frac{1}{\rho}} (C_{H,t})^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}} (C_{F,t})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{2.6}$$

where $\rho > 0$ is the elasticity of substitution between home and foreign goods and C_H and C_F refers to the aggregate of home produced and foreign produced final goods. The parameter γ represents home consumers' preference towards domestic and foreign goods, respectively. The preference parameter is as in De-Paoli (2006) function of the relative size of the foreign economy, $1-n$, and of the degree of openness, λ; more specifically $(1-\gamma) = (1-n)\lambda$.

$$C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n C_{H,t}(j)^{\frac{\phi-1}{\phi}}\,\mathrm{d}j\right]^{\frac{\phi}{\phi-1}}, \qquad C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\phi}} \int_n^1 C_{F,t}(l)^{\frac{\phi-1}{\phi}}\,\mathrm{d}j\right]^{\frac{\phi}{\phi-1}} \tag{2.7}$$

where ϕ is an elasticity of between particular goods.

P_t is the overall price index of the final good, $P_{H,t}$ depicts the price index of home goods and $P_{F,t}$ of foreign goods denominated in home currency.

$$P_t = \left\{\gamma[P_{H,t}]^{1-\rho} + (1-\gamma)[P_{F,t}]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \tag{2.8}$$

$$P_{H,t} = \left[\left(\frac{1}{n}\right)\int_0^n [P_{H,t}(j)]^{1-\phi}\,\mathrm{d}j\right]^{\frac{1}{1-\phi}}, \qquad P_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 [P_{F,t}(l)]^{1-\phi}\,\mathrm{d}j\right]^{\frac{1}{1-\phi}} \quad (2.9)$$

2.2.1 Intra-basked demands for final good

The firm has to solve the optimal composition of the basket of the home and foreign goods.

$$\min_{C_{H,t}(j)} \int_0^n P_{H,t}(j) C_{H,t}(j)\,\mathrm{d}j$$

$$\text{s. t.} \qquad C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n C_{H,t}(j)^{\frac{\phi-1}{\phi}}\,\mathrm{d}j\right]^{\frac{\phi}{\phi-1}}$$

$$P_{H,t}(j) = \lambda C_{H,t}^{\frac{1}{\phi}} C_{H,t}(j)^{-\frac{1}{\phi}} \left(\frac{1}{n}\right)^{\frac{1}{\phi}} \quad (2.10)$$

$$P_{H,t}(j)^{1-\phi} = \lambda^{1-\phi} C_{H,t}^{\frac{1-\phi}{\phi}} C_{H,t}(j)^{\frac{\phi-1}{\phi}} \left(\frac{1}{n}\right)^{\frac{1-\phi}{\phi}}$$

$$\left[\int_0^n P_{H,t}(j)^{1-\phi}\,\mathrm{d}j\right]^{\frac{\phi}{\phi-1}} = \lambda^{-\phi} C_{H,t}^{-1} \left[\int_0^n C_{H,t}(j)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \left(\frac{1}{n}\right)^{-1}$$

$$\left(\frac{1}{n}\right)^{1+\frac{1}{\phi-1}} \left[\int_0^n P_{H,t}(j)^{1-\phi}\,\mathrm{d}j\right]^{\frac{\phi}{\phi-1}} = \lambda^{-\phi} C_{H,t}^{-1} \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n C_{H,t}(j)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

$$\lambda = P_{H,t} \quad (2.11)$$

Substituting for λ in 2.10 we derive domestic demand for home produced good j.

$$C_{H,t}(j) = \frac{1}{n}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} C_{H,t} \quad (2.12)$$

In similar way one can derive home demand for imported good j.

$$C_{F,t}(l) = \frac{1}{1-n}\left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\phi} C_{F,t} \tag{2.13}$$

2.2.2 Demands for input

Firms choose inputs in order to maximize their profit.

$$\max_{C_{H,t},C_{F,t}} P_t C_t - P_{H,t}C_{H,t} - P_{F,t}C_{F,t}$$
$$\text{s. t.} \qquad C_t = \left[\gamma^{\frac{1}{\rho}}\left(C_{H,t}\right)^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}}\left(C_{F,t}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

$\partial C_{H,t}$:

$$\begin{aligned} P_t \underbrace{\left[\gamma^{\frac{1}{\rho}}(C_{H,t})^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}}(C_{F,t})^{\frac{\rho-1}{\rho}}\right]^{\frac{1}{\rho-1}}}_{C_t^{\frac{1}{\rho}}} \gamma^{\frac{1}{\rho}}(C_{H,t})^{\frac{-1}{\rho}} - P_{H,t} &= 0 \\ P_t\left[C_t^{\frac{1}{\rho}}\gamma^{\frac{1}{\rho}}(C_{H,t})^{\frac{-1}{\rho}}\right] - P_{H,t} &= 0 \end{aligned}$$

$$C_{H,t} = \gamma\left(\frac{P_{H,t}}{P_t}\right)^{-\rho} C_t \tag{2.14}$$

$$C_{F,t} = (1-\gamma)\left(\frac{P_{F,t}}{P_t}\right)^{-\rho} C_t \tag{2.15}$$

2.2.3 Foreign sector

The variables representing the rest of the world (ROW) relative to the Czech Republic (SOE) are denoted with an asterisk. The foreign economy has to solve the same problem as the SOE, therefore:

- The aggregation technology for producing final good

$$C_t^* = \left[{\gamma^*}^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}} + (1-\gamma^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} \right] \tag{2.16}$$

- ROW demand for particular good from SOE & foreign demand for their own good

$$C_{H,t}^*(j) = \frac{1}{n} \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\phi} C_{H,t}^* \qquad C_{F,t}^*(l) = \frac{1}{1-n} \left(\frac{P_{F,t}^*(l)}{P_{F,t}^*} \right)^{-\phi} C_{F,t}^* \tag{2.17}$$

- ROW demand for the Czech exports & ROW demand for the goods produced in the rest of the world

$$C_{H,t}^* = \gamma^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\rho} C_t^* \qquad C_{F,t}^* = (1-\gamma^*) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\rho} C_t^* \tag{2.18}$$

Similarly to De-Paoli (2006) $\gamma^* = n\lambda$, therefore as $n \to 0$ rest of the world version of the equation 2.8 implies that $P_t^* = P_{F,t}^*$ and $\pi_t^* = \pi_{F,t}^*$

2.2.4 Total demand for a generic good j an l

Using consumers's demands, and market clearing condition for good j and l we can derive the total demand for a generic good j, produced in SOE, and the demand for a good l produced in country F. The real exchange rate is defined as $RS = \frac{\varepsilon_t P_t^*}{P_t}$.

$$Y_t(j) = nC_{H,t}(j) + (1-n)C_{H,t}^*(j) \tag{2.19}$$

$$Y_t(l) = nC_{F,t}(l) + (1-n)C_{F,t}^*(l) \tag{2.20}$$

$$Y_t(j) = n\frac{1}{n}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} C_{H,t} + (1-n)\frac{1}{n}\left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\phi} C^*_{H,t}$$

Using the demand for $C_{H,t}$ and $C^*_{H,t}$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} \gamma \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} C_t + \frac{(1-n)\gamma^*}{n}\left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\phi}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\rho} C^*_t$$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi}\left\{\left(\frac{P_{H,t}}{P_t}\right)^{-\rho}\left[\gamma C_t + \frac{\gamma^*(1-n)}{n}\left(\frac{1}{RS}\right)^{-\rho} C^*_t\right]\right\} \tag{2.21}$$

$$Y_t(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\phi}\left\{\left(\frac{P_{F,t}}{P_t}\right)^{-\rho}\left[\frac{(1-\gamma)n}{1-n}C_t + (1-\gamma)^*\left(\frac{1}{RS}\right)^{-\rho} C^*_t\right]\right\} \tag{2.22}$$

Applying the definition of γ and γ^* and taking the limit for $n \to 0$ as in De-Paoli (2006) we can see that external changes in demand affect the small open economy, but the reverse is not true. In addition, exchange rate fluctuation does not influence the ROW's demand. Thus, the demand of the rest of the world is exogenous for the small open economy.

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi}\left\{\left(\frac{P_{H,t}}{P_t}\right)^{-\rho}\left[(1-\lambda)C_t + \lambda\left(\frac{1}{RS}\right)^{-\rho} C^*_t\right]\right\} \tag{2.23}$$

$$Y_t(l) = \left(\frac{P^*_{F,t}(l)}{P^*_{F,t}}\right)^{-\phi}\left\{\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\rho} C^*_t\right\} \tag{2.24}$$

2.3 Pass-through and Deviations from PPP

I assume that there are no trade barriers and no market segmentation and thus law of one price holds. This means that the price of Czech apples in CZK is the same at the Czech market and world market in CZK. Formally,

$$P_{F,t}(l) = \varepsilon_t P^*_{F,t}(l) \qquad P_{H,t}(j) = \varepsilon_t P^*_{H,t}(j) \tag{2.25}$$

$$P_{F,t} = \varepsilon_t P^*_{F,t} \qquad P_{H,t} = \varepsilon_t P^*_{H,t} \tag{2.26}$$

where ε_t is nominal exchange rate (i.e. how much cost one unit of foreign currency in terms of CZK)

However, on the aggregate level the low of one price fails to hold in our model specification. In other words, the economy is characterized by deviations from purchasing power parity $P_t \neq \varepsilon_t P^*_t$.

In order to track the sources of deviation from the aggregate PPP in this framework it is useful to rewrite real exchange rate[1]

$$\begin{aligned} RS_t &= \frac{\varepsilon_t P^*_t}{P_t} \\ &= \frac{\varepsilon_t P^*_t S_t}{g(S_t) P_{F,t}} \\ &= \Upsilon_{F,t} \frac{S_t}{g(S_t)} \end{aligned} \tag{2.27}$$

where $g(S_t)$ is defined in equation 2.42, since $P_{F,t} = \varepsilon_t P^*_t$ we know that $\Upsilon_{F,t} = 1$ for all t , thus the distortion of PPP comes from the heterogeneity of consumption baskets between the small open economy and the rest of the world.

[1]this can be find also in Monacelli (2003) for log-linearized system

2.4 Goods sector

Goods are imperfect substitutes and continuum of firms hiring labor operates at the market. A firm has control over its price, nevertheless it has to face quadratic adjustment cost when changing the price.

The production function is given by:

$$Y_t(j) = N_t(j) \tag{2.28}$$

The total cost of the firm are:

$$TC = W_t N_t \tag{2.29}$$

Using the production function we can write:

$$TC = W_t Y_t \tag{2.30}$$

$\frac{\partial TC}{\partial Y_t(j)}$:

$$MC_t = W_t \tag{2.31}$$

All firms face the same marginal costs, therefore $MC_t = MC_t(j)$

Next, I use market clearing condition $Y_t(j) = nC_H(j) + (1-n)C_H^*(j)$ and previous definitions from this section to set up the profit maximization problem of monopolistic competitive firm. After we plug equation 2.17 and 2.12 into the market clearing condition we get:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} C_{H,t} + \frac{(1-n)}{n}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\phi} C_{H,t}^* \tag{2.32}$$

2.4.1 Profit maximization

$$\max_{P_{H,t}(j)} E_0 \sum_{t=0}^{\infty} Q_{t,t+1} \left\{ \begin{array}{c} \left(P_{H,t}(j) - (1-\tau_p)MC_t - P_{H,t}\frac{\varphi_p}{2}\left[\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2\right)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} C_{H,t} \\ + \left(\varepsilon_t P^*_{H,t}(j) - (1-\tau_p)MC_t - P_{H,t}\frac{\varphi_p}{2}\left[\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2\right)\left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\phi^*} C^*_{H,t}\frac{(1-n)}{n} \end{array} \right\} \tag{2.33}$$

where

$$W_t N_t(j) = MC_t[nC_H(j) + (1-n)C^*_H(j)] \tag{2.34}$$

$$Q_{t,t+1} = \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma_1} \frac{P_t}{P_{t+1}} \tag{2.35}$$

$$AC_{P,t}(j) = \frac{\varphi_p}{2}\left[\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 Y_t(j) \tag{2.36}$$

and τ_p is is a subsidy the government can use to offset the steady state distortions due to monopolistic competition.

$\partial P_{H,t}(j)$:

$$\begin{aligned} & Q_{t,t+1}\left(1 - P_{H,t}\varphi_p\left[\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]\frac{1}{P_{H,t-1}(j)}\right)\left(\begin{array}{c} \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} C_H \\ + \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\phi^*} C^*_{H,t}\frac{(1-n)}{n} \end{array}\right) \\ & + Q_{t,t+1}\left(P_{H,t}(j) - (1-\tau_p)MC_t - P_{H,t}\frac{\varphi_p}{2}\left[\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2\right)\left(\begin{array}{c} -\frac{\phi}{P_{Ht}}\left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\phi-1} C_{H,t} \\ -\frac{\phi^*}{P_{H,t}}\left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\phi^*-1} C^*_{H,t}\frac{(1-n)}{n} \end{array}\right) \\ & + E_t Q_{t+1,t+2}\left(\frac{P_{H,t+1}}{P^2_{H,t}(j)} P_{H,t+1}(j)\varphi_p\left[\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]\right)\left(\begin{array}{c} \left(\frac{P_{H,t+1}(j)}{P_{H,t+1}}\right)^{-\phi} C_{H,t+1} \\ + \left(\frac{P_{H,t+1}(j)}{P_{H,t+1}}\right)^{-\phi^*} C^*_{H,t+1}\frac{(1-n)}{n} \end{array}\right) \\ & = 0 \end{aligned} \tag{2.37}$$

where I use $P_{H,t}(j) = \varepsilon_t P^*_{H,t}(j) \to \frac{P^*_{H,t}(j)}{P^*_{H,t}} = \frac{P_{H,t}(j)\frac{1}{\varepsilon_t}}{P_{H,t}\frac{1}{\varepsilon_t}} = \frac{P_{H,t}(j)}{P_{H,t}}$

We know that all firms and households solve the same problem therefore they must behave the same way in equilibrium, therefore after choosing optimal prices we can impose

$C_{H,t} = C_{H,t}(j) \qquad K_t(j) = K_t \qquad N_t(j) = N_t \qquad P_{H,t}(j) = P_{H,t}$

$$Q_{t,t+1}\left(1-\varphi_p\left[\pi_{H,t}-1\right]\pi_{H,t}-\phi+(1-\tau_p)\phi\frac{MC_t}{P_{H,t}}+\phi\frac{\varphi_p}{2}\left[\pi_{H,t}-1\right]^2\right)\left(C_{H,t}+C^*_{H,t}\frac{(1-n)}{n}\right)$$
$$+E_t\left\{Q_{t+1,t+2}\varphi_p\left[\pi_{H,t+1}-1\right]\pi^2_{H,t+1}\left(C_{H,t+1}+C^*_{H,t+1}\frac{(1-n)}{n}\right)\right\}=0 \tag{2.38}$$

and $\phi = \phi^*$ which means that the rest of the world has the same elasticity of demand as the Czech Republic.

Further, I plug 2.18 , 2.14 and $\gamma = (1-\lambda)$ into 2.38

$$\left(C_{H,t}+C^*_{H,t}\frac{(1-n)}{n}\right)=\left(\frac{1}{g(S_t)}\right)^{-\rho}\left[(1-\lambda)C_t+\lambda\left(\frac{1}{RS}\right)^{-\rho}C^*_t\right]=Y_t \tag{2.39}$$

$$\left(1-\varphi_p\left[\pi_{H,t}-1\right]\pi_{H,t}-\phi+(1-\tau_p)\phi\frac{MC_t}{P_{H,t}}+\phi\frac{\varphi_p}{2}\left[\pi_{H,t}-1\right]^2\right)(Y_t)$$
$$+E_t\left\{\frac{R_t}{R_{t+1}}\varphi_p\left[\pi_{H,t+1}-1\right]\pi^2_{H,t+1}\right\}(Y_{t+1})=0 \tag{2.40}$$

After some algebraic operations and using $R_t = \frac{1}{E_t Q_{t,t+1}}$ the Phillips Curve can be written in following form:

$$P_{H,t} = \frac{\phi}{(\phi - 1)}\left((1-\tau_p)MC_t + P_{H,t}\frac{\varphi_p}{2}\left[\pi_{H,t}-1\right]^2\right) + P_{H,t}\frac{\varphi_p}{(\phi-1)}\left[1-\pi_{H,t}\right]\pi_{H,t} + P_{H,t}\frac{\varphi_p}{(\phi-1)}E_t\left[\frac{R_t}{R_{t+1}}\left[\pi_{H,t+1}-1\right]\pi^2_{H,t+1}\right]\frac{Y_{t+1}}{Y_t}$$

where the last equation is *Phillips Curve* and $\frac{\phi}{(\phi-1)}$ the constant price mark up coming from the monopolistic competition on the market. The firm can choose a price which is higher than marginal cost. As $\phi \to \infty$ and $\varphi_p = 0$ we are approaching the competitive output market, where $P_{H,t} = MC_t$. Nevertheless, in the presence of the Rotemberg quadratic adjustment cost Rotemberg (1982), price settings deviate from the monopolistic competition without price stickiness. Marginal cost are now augmented with price adjustment costs on the unit of output. The second term in the previous equation depicts the fact that firms are unwilling to make significant price changes because it is costly, for example firms' customers are unhappy with recurrent price changes as it decreases the reputation of the firm. Those changes are much more apparent when large changes occur, thus quadratic cost seems to be good approximation The second term is nothing else than marginal adjustment cost on the unit of output (note that the term is actually negative). The last term represents the forward looking part of price setting. If the firm expects large price changes in the future, it will tend to change the prices more already today. Thus, a firm operating in monopolistic competition will set a higher price in order to be hedged against future price changes. Compare to Calvo prices, Rotemberg adjustment costs have an advantage that firms do not have to wait and they can change prices when the price stickiness becomes large.

2.5 Terms of trade

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{2.41}$$

we can rewrite price index using definition for terms of trade 2.41

$$\frac{P_t}{P_{H,t}} = \left\{\gamma + (1-\gamma)[S_t]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \equiv g(S_t) \tag{2.42}$$

$$\frac{P_t}{P_{F,t}} = \left\{\gamma[S_t]^{\rho-1} + (1-\gamma)\right\}^{\frac{1}{1-\rho}} \equiv \frac{g(S_t)}{S_t} \tag{2.43}$$

$$\begin{aligned} \frac{\frac{P_t}{P_{H,t}}}{\frac{P_{t-1}}{P_{H,t-1}}} &= \frac{\{\gamma + (1-\gamma)[S_t]^{1-\rho}\}^{\frac{1}{1-\rho}}}{\{\gamma + (1-\gamma)[S_{t-1}]^{1-\rho}\}^{\frac{1}{1-\rho}}} \\ \pi_t^{1-\rho} &= \frac{\{\gamma + (1-\gamma)[S_t]^{1-\rho}\}^{\frac{1}{1-\rho}}}{\{\gamma + (1-\gamma)[S_{t-1}]^{1-\rho}\}^{\frac{1}{1-\rho}}}\pi_{H,t-1} \end{aligned} \tag{2.44}$$

2.6 Financial Markets

It has been shown for example in Cochrane (2001), De-Paoli (2006) or Uribe (May 4, 2009) that in complete markets the contingent claim price ratio is the same for all investors. Thus, at domestically and internationally complete markets with perfect capital mobility, the expected nominal return from the complete portfolio of state contingent claims (risk-free bond paying one in every state of the world) is equal to the expected domestic-currency return from foreign bonds $E_t Q_{t,t+1} = E_t(Q^*_{t,t+1}\frac{\varepsilon_{t+1}}{\epsilon_t})$

In order to determine the relationship between the real exchange rate and marginal utilities of consumption, I use the first order condition with respect to bond holdings for the "rest of the world economy" (ROW). μ is the marginal rate of consumption substitution.

$$\beta\left(\frac{\mu^{\star}_{t+1}}{\mu^{\star}_{t}}\right)\left(\frac{P^{\star}_{t}}{P^{\star}_{t+1}}\right)\left(\frac{\epsilon_{t}}{\epsilon_{t+1}}\right)=Q_{t,t+1} \tag{2.45}$$

Then I use the first order condition 2.4 together with the definition of the real exchange rate $RER_t \equiv \frac{\varepsilon_t P^*_t}{P_t}$; it follows that

$$\left(\frac{C^*_t}{C^*_{t+1}}\right)^{\sigma_1}\left(\frac{P^{\star}_t}{P^{\star}_{t+1}}\right)\left(\frac{\varepsilon_t}{\varepsilon_{t+1}}\right)=\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}_{1}\left(\frac{P_t}{P_{t+1}}\right) \tag{2.46}$$

This expression holds at all dates and under all contingencies. The assumption of complete financial markets implies that arbitrage will force the marginal utility of consumption of the residents from the ROW economy to be proportional to the marginal utility of domestic residents multiplied by the real exchange rate.

$$C_t = \vartheta C^*_t RS_t^{\frac{1}{\sigma_1}} \tag{2.47}$$

ϑ is a constant consisting of the initial conditions. Since countries are perfectly symmetric one can assume that at time zero they start from the same initial conditions.

2.6.1 UIP

The equilibrium price of the risk-less bond denominated in foreign currency is given as in Galí (2002) by $\varepsilon_t(R^*_t)^{-1} = E_t\{Q_{t,t+1}\varepsilon_{t+1}\}$. Combining previous with the domestic pricing equation $R_t^{-1} = E_t\{Q_{t,t+1}\}$, one can obtain a version of the uncovered interest parity condition:

$$E_t\{Q_{t,t+1}[R_t - R^*_t(\varepsilon_{t+1}/\varepsilon_t]\} = 0 \tag{2.48}$$

Further, as all prices are expressed in terms of trade we need to substitute for nominal exchange rate in the equation 2.48. Using low of one price and 2.41 the UIP takes following form:

$$R_t = R_t^* \triangle S_t \frac{\pi_{t+1,H}}{\pi_{t+1}^*} \tag{2.49}$$

2.7 Trade Balance

Trade balance is in general defined as export minus import. Because $C_{t,H}^*$ and $C_{t,F}$ are defined as per-capita demand one has to multiply demands by the size of domestic economy, analogically as in market clearing condition case.

$$NX_{H,t}(j,l) = nC_{H,t}^*(j) + \left(\frac{P_{F,t}}{P_{H,t}}\right) nC_{F,t}(l) \tag{2.50}$$

Using the equations 2.17, 2.18, 2.15, 2.13 and aggregating over j, l we can write net export as follows:

$$NX_{H,t} = \lambda \left[\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\rho} C_t^* - S_t \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} C_t \right] \tag{2.51}$$

Further, we can use definition of real exchange rate and equation 2.42 to write:

$$NX_{H,t} = \lambda \left(\frac{1}{g(S_t)}\right)^{-\rho} \left[\left(\frac{1}{RS_t}\right)^{-\rho} C_t^* - S_t C_t \right] \tag{2.52}$$

2.8 General Equilibrium

The equilibrium requires that all markets clear and all households and all firms behave identically. In particular, the equilibrium is characterized by the following system of stochastic differential equations:

2.8.1 Goods market equilibrium

Goods market clearing condition 2.19 and aggregate demand for generic good j give aggregate demand[2]

$$Y_t = \left(\frac{1}{g(S_t)}\right)^{-\rho}\left[(1-\lambda)C_t + \lambda\left(\frac{1}{RS}\right)^{-\rho} C_t^*\right] \tag{2.53}$$

Using international risk sharing equation 2.47 we can write:

$$Y_t = g(S_t)^{\rho} C_t^* \left[(1-\lambda) + \lambda RS_t^{\rho - \frac{1}{\sigma_1}}\right] \tag{2.54}$$

Next, if we use Euler equation 2.64 we would be able to derive dynamic IS equation. This is analytically tractable, however, only in log-linearized form.

Aggregating equation 2.24 over l we can see that the small open economy can treat C_t^* as exogenous.

$$Y_t^* = C_t^* \tag{2.55}$$

2.8.2 Aggregate Demand and Supply

In equilibrium, aggregate supply must be equal to consumption and resources spent on adjusting prices.

$$g(S_t)Y_t = g(S_t)C_t + \frac{\varphi_p}{2}\left[\pi_{H,t} - 1\right]^2 Y_t \tag{2.56}$$

Production function:

$$Y_t = N_t \tag{2.57}$$

[2]plug equation 2.23 into equation $Y_t = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n Y_t(j)^{\frac{\phi-1}{\phi}}\,\mathrm{d}j\right]^{\frac{\phi}{\phi-1}}$

2.8.3 Labor market equilibrium

Real wage is defined $\frac{W_t}{P_t} = w_t$.

$$\omega N_t^{\sigma_2} = C_t^{-\sigma_1} w_t \tag{2.58}$$

ω is the scaling parameter equal to $\bar{C}^{-\sigma_1}$

2.8.4 Monetary Policy

Monetary authority follows interest a rate rule, so that the nominal interest rate is determined by past interest rates and responds to the current CPI inflation rate.

$$log(R_t) = log\left(\frac{1}{\beta}\right) + (\Phi_\pi \pi_t + \Phi_y Y_t) \tag{2.59}$$

2.8.5 Phillips Curve

First, I derive the relationship between domestic PPI and CPI inflation.

$$\pi_t = \frac{g(S_t)}{g(S_{t-1})} \pi_{H,t} \tag{2.60}$$

by using equations 2.38 and 2.54 we derive:

$$\begin{aligned} &\left(1 - \varphi_p \left[\pi_{H,t} - 1\right] \pi_{H,t} - \phi + (1-\tau_p)\phi mc_t + \phi \frac{\varphi_p}{2} \left[\pi_{H,t} - 1\right]^2\right)(Y_t) \\ &+ E_t \left\{ \frac{R_t}{R_{t+1}} \varphi_p \left[\pi_{H,t+1} - 1\right] \pi_{H,t+1}^2 \right\} (Y_{t+1}) = 0 \end{aligned} \tag{2.61}$$

We can rewrite marginal costs as follows $\bar{mc}_t \frac{P_t}{P_{H,t}} = mc_t g(S_t) = \frac{MC_t}{P_{H,t}}$ Further, from cost minimization, we know that $MC_t = W_t$

$$\frac{MC_t}{P_H} = \frac{W_t}{P_t}\frac{P_t}{P_H}$$
$$mc_t = w \times g(S_t) \tag{2.62}$$

Marginal cost can be decomposed by using equation 2.5, 2.47 and 2.64

$$\begin{aligned} mc_t &= \omega N_t^{\sigma_2} C^{\sigma_1} g(S_t) \\ &= \omega Y_t^{\sigma_2} (Y^*)^{\sigma_1} S_t \end{aligned} \tag{2.63}$$

This is convincing way to show that marginal costs are growing with positive foreign output shock, increase in home output and decrease in improvement in terms of trade.

2.8.6 Euler Equation

$$1 = \beta E_t R_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma_1} \frac{P_t}{P_{t+1}} \tag{2.64}$$

A stationary rational expectation equilibrium is set of stationary stochastic processes $\{S_t, C_t, Y_t, N_t, \pi_t, \pi_{H,t}, R_t, w_t\}_0^\infty$

And exogenous processes $\{Y_t^*\}_0^\infty$

2.9 Steady State

As proved by Galí (2002) analytically [3] terms of trade are $\bar{S} = 1$ and $\bar{Y} = \bar{Y}^*$in steady state. It follows that $g(\bar{S}) = 1$, $\bar{\pi} = \bar{\pi}_H = \bar{\pi^*} = 1$ and real exchange rate $\bar{RS} = 1$. From

[3] I solve for the steady state also numerically, to confirm the proof since my model slightly differs from the one of Galí (2002)

the equilibrium conditions in steady state one can derive remaining perfect foresight initial conditions.

International risk sharing 2.47 delivers following:

$$\bar{C} = \bar{C}^* \tag{2.65}$$

Euler equation 2.64 gives us steady state

$$\bar{R} = \frac{1}{\beta} \tag{2.66}$$

The labor market equilibrium in steady state, using the fact that $\omega = \bar{C}^{-\sigma_1}$.

$$\bar{N}^{\sigma_2} = w \tag{2.67}$$

From equation 2.54 we get that:

$$\bar{Y} = 1 \tag{2.68}$$

$$1 - \phi + (1 - \tau_p)\phi \frac{\bar{MC}}{\bar{P_H}} = 0 \tag{2.69}$$

$$1 - \phi + (1 - \tau_p)\phi \bar{mc} = 0 \tag{2.70}$$

Hence,

$$\bar{mc} = \frac{1}{(1 - \tau_p)} \frac{\phi - 1}{\phi} \rightarrow \bar{MC} = \frac{\phi - 1}{\phi} \frac{1}{(1 - \tau_p)} \bar{P_H} \tag{2.71}$$

So, one can see that the nominal wage is constant mark-up over domestic prices. Setting τ_p to $\frac{1}{\phi}$, marginal costs collapse to one in steady state.

Chapter 3

Calibration

The model is calibrated using data for the Czech Republic obtained from the Czech Statistical Office [1] and World Bank [2]. Further, I follow Natalucci and Ravenna (2002) and Vasicek and Musil (2006) in choosing values for parameters. However values of parameters which are not easy to estimate are not taken as granted and are used to adjust the simulated data of the model to the real data for Czech economy.

3.1 Preferences

The quarterly discount factor β is fixed at 0.99, which means that households have high degree of patience with respect to their future consumption and it implies real interest rate of 4 percent in steady state. To calibrate the elasticity of intertemporal substitution I follow approximately Vasicek and Musil (2006) estimates and set the value $\sigma_1 = 0.45$ which

[1] http://www.czso.cz/

[2] http://data.worldbank.org/indicator/NE.EXP.GNFS.ZS

means that the elasticity of intertemporal substitution is 2.22. Intertemporal elasticity of substitution can be interpreted as a willingness of households to agree with deviation from their current consumption path. In other words, with higher elasticity households smooth consumption more over time and they are willing to give up larger amount of consumption today to consume a little more in the future. Elasticity of labor supply is chosen to be 2 in baseline calibration implying $\sigma_2 = 0.5$. The increase of the real wage by 1% brings 2 percentage increase of the labor supply, which indicates that the labor supply is elastic.

3.2 Technology

The degree of monopolistic competition, $\phi = 4$ brings a markup of 33%. The elasticity between imported goods and domestic goods is set to 5. The exact rate is hard to compute, but in general the elasticity has increased in the Czech Republic recently with the development of the economy. Thus, I do not follow Vasicek and Musil (2006) who use Bayesian estimation to back up this parameter. They find the value 0.38 for the data from 90s. Natalucci and Ravenna (2002) uses $\rho = 0.5$. The degree of openness, λ, is assumed to be 0.75, implying a 75% import share of the GDP and determining the parameter γ (share of domestic good in consumption basket) to be 0.25. The degree of openness is calibrated based on the time series of import to GDP share data for the Czech Republic. I set price adjustment costs to the standard value $\varphi_p = 5-$ Bergin and Tchakarov (2003) implying that 95 percent of the price has adjusted 4 periods after a shock.

3.3 Shocks

The only shock in the benchmark model comes from the world economy and is characterized by degree of persistence. The foreign output inertia is estimated in Vasicek and Musil

(2006) to $\rho_y = 0.8$. Nevertheless, since this is the only source of variability in the model I increase the autocorrelation of modeled variables by setting $\rho_y = 0.9$. The standard deviation of foreign output shock is estimated of 0.05.

3.4 Monetary Policy

A monetary authority is set to follow simple form of Taylor rule. A weight connected to inflation is set in such away that the ratio between inflation and output is about 7. The central bank in the regime of the inflation targeting prefers to keep the current inflation at the steady state value seven times more than the output.

Chapter 4

The finance part

In this section I borrow from Hordahl et al. (2007) to summarize some stylized facts on the term structure of interest rates. I present well known facts from previous studies and add my brief analysis of data for the Czech Republic. In the second part, I derive the yield curve implied by the DSGE model outlined in the section 2.

4.1 Finance related data

From the table 4.1 it is apparent that the yield curve is, on average, upward sloping. The mean of 10 year zero-coupon bond yield exceeds the mean of three year bond yield by 13 percentage points over the period 1961Q2 - 2007Q2. The mean of three months yield is

Maturity	3m	6m	1Y	3Y	5Y	10Y
mean	1.47	1.49	1.53	1.63	1.68	1.76
Std.Dev.	0.72	0.71	0.70	0.66	0.64	0.60

Table 4.1: Summary statistics for US Yield Curve, 1961Q2 - 2007Q2. Quartely US data, in percent. Source: Hordahl et al. (2007)

29 percentage points less than the mean of a 10 year bond. On the other hand, volatilities has tended to be slightly downward-sloping. The volatility of a 10 year zero coupon bond was 12 percentage points higher than volatility of a three month zero coupon bond.

The availability of data for the Czech Republic is limited, thus the picture about the yield curve behavior presented here can be only approximate. The data for zero coupon bond provided by the Debt Management Office of Czech Ministry of Finance are daily closing values. Due to the fact that Reuters stores daily data only for two years, I am forced to work with only a two year period (April 2008 to May 27, 2010). For this reason, I also present the quarterly averages of Government coupon bonds for 10 year period (2000 - 2010). Together with data for US one can gain sufficiently good intuition about behavior of Czech term structure of interest rates.

In the table 4.2 one can see that the mean of both zero coupon bond daily data and government quarterly coupon bonds are very similar in spite of different character of the data. Hence, we can conclude that if the simulated time series will generate a mean of the yield somewhere close to 3.5 for the 3 year zero coupon bond and 4.7 yield for the 10 years bond, the model will be very good at fitting the mean of the yield curve data. The standard deviations, however, differ substantially. If we take a look at the volatility of the US data we can see that the values in percent for Government coupon bonds are very close to US zero coupon bond standard deviations. It is likely that the time series of zero coupon bond standard deviations for the short period of time is not good in describing the population standard deviation. What can the intuition tell us? The US market is characterized by higher liquidity, thus it should be more volatile. On the other hand, the US market is less risky and more predictable compare to the Czech market which eliminates, to some extent, the fluctuations due to the mis-pricing. Hence, it is not straightforward what are the true standard deviations, but they should be somewhere close to the Government coupon bond standard deviations. From this reason, I consider as a good fit, if my model is able to

	Zero coupon bond		Government coupon bond	
Maturity	3Y	10Y	3Y	10Y
Mean	3.36	4.85	3.69	4.68
Std.Dev.	0.79	0.50	1.48	1.04

Table 4.2: The Term Structure of Interest rates for Czech Republic. Source: Reuters and Czech National bank. The mean and Standard deviation of the zero coupon bond is calculated from daily data from April 2008 to May 2010, the data for Government coupon bond are for 2000Q1 - 2010Q1

replicate standard deviation of 40 percent to the mean for 3 years zero coupon bond and 20 percent to the mean for 10 years zero coupon bond.

The Czech term structure of interest rates does not differ in its characteristics from the US one. It is, on average, upward sloping and more volatile at the long tail of the curve.

4.2 Term structure of interest rates

The complete markets and no-arbitrage assumption in the DSGE model implies that we can price all financial assets in the economy. Once we specify a time-series process for one period discount factor $Q_{t,t+i}$ we can determine price of any bonds by chaining the discount factors $P_t^{(i)} = E_t\{Q_{t,t+1}, Q_{t+1,t+2} \dots Q_{t+i-1,t+i}\}$.[1] I solve the discount factors forward to get particular maturities. Hence, the price of zero-coupon bond paying 1 dollar at the maturity date i is:

$$P_t^{(i)} = E_t\left[\beta^i \left(\frac{C_t}{C_{t+i}}\right)^{\sigma_1} \prod_{j=1}^{i} \frac{1}{\pi_{t+j}}\right] \tag{4.1}$$

where the price of a default-free one period zero-coupon bond that pays one dollar at maturity $P_t^{(1)} \equiv R_t^{-1}$, R_t is the gross interest rate and $P_t^{(1)} \equiv 1$ (*i.e.* the time t price of one dollar delivered at time t is one dollar). One can see that the price of the bonds is defined

[1]see for example Cochrane (2001)

by the behavior of consumption and inflation.

One can rewrite the nominal default-free bond [2] with maturity i as follows:

$$P_t^{(i)} = E_t\{Q_{t,t+1}P_{t+1}^{(i-1)}\} \tag{4.2}$$

Next, using the definition of covariance:

$$P_t^{(i)} = P_t^{(i-1)}E_tP_{t+i-1}^{(1)} + Cov_t\{Q_{t,t+i-1}P_{t+i-1}^{(1)}\} \tag{4.3}$$

The last equation, 4.3, says that price of the risk-free bond is equal to the expected price of one period bond at time $t+i-1$ discounted by the discount factor for the period $i-1$. Yet note that although the bond is default free, it is still risky in the sense that its price can covary with the households' marginal utility of consumption. For instance, if the negative world output shock hits the economy in our baseline model, it pushes up the CPI index and domestic output. In this case, households perceive the nominal zero-coupon bond as being very risky, because it loses its value exactly when households values consumption the most. In our baseline model, the correlation of CPI to output is high about 98 percent although for PPI index reaches only about 3 percent, thus if households expect the economy to be exposed to the foreign output shock, they will consider the bond very risky and its price will fall. Another way of thinking about he covariance term is through precautionary savings motive. As I elaborate bellow, if the bond price and consumption fall at the same time, consumption smoothing households wish to save some of their consumption for the unfortunate time when the economy is hit by shock and price of bonds fall with consumption. However, this is not possible in the equilibrium, thus price of bonds must increase in order to distract the demand. We can see that the covariance

[2] the derivation of real default-free bond is analogical, see Caprioli and Gnocchi (2009)

term is the approximation for the risk premium.[3]

I follow the term structure literature and I denote $ytm_t^{(i)} = log(P_t^{(i)})$. The logarithm of price has convenient interpretation. If the price of one year-zero coupon bond is 0.98, the log price is $ln(0.98) = -0.0202$, which means that the bonds sells at 2 percent discount. Further, I define the nominal interest rates as yields to maturity.

$$\begin{aligned} P^{(i)} &= \frac{1}{\left[Y^{(i)}\right]^{(i)}} \\ ytm_t^{(i)} &= -\frac{1}{i} log(P_t^{(i)}) \end{aligned} \tag{4.4}$$

The equations 4.4 states that if the yield of 10 years bond is 40 percent, the yield to maturity is 4 percent per year.

In order to understand better the term premium, it is useful to derive the second order approximation of the yield to maturity around the log-steady state.[4]

$$\widehat{ytm_t}^{(i)} = \frac{1}{i} \left\{ \begin{array}{c} \sigma_1 E_t[\Delta^{(i)}\hat{c}_{t+i}] + \sum_{n=1}^{i} E_t[\hat{\pi}_{t+n}] - \frac{1}{2}\sigma_1^2 Var_t\left[\Delta^{(i)}\hat{c}_{t+i}\right] \\ -\frac{1}{2} Var_t\left[\Delta^{(i)}\hat{\pi}_{t+i}\right] - \sigma_1 Cov_t\left[\sum_{n=1}^{i}\hat{\pi}_{t+n}, \Delta^{(i)}\hat{c}_{t+i}\right] \end{array} \right\} \tag{4.5}$$

Equation 4.5 illustrates that risk averse agents make precautionary savings if there is uncertainty about future consumption. The higher supply of savings decreases yield to maturity. The high level of expected consumption increases the yield to maturity because of income effect and inflation pushes yield to maturity up because households care about real variables. The last term of equation 4.5 supports the previous example, in the economy

[3]possible extension is to add preference shock to capture the fact, that households perceive risk differently in time, for example in recession the foreigner output shock is much more painful and households would demand higher compensation for holding the bond

[4]Steps of derivations are presented in the appendix

with high inflation and low consumption the households require higher compensation for holding the bond since it loses its value when households need resources the most.

Further, I present the second order approximation of the slope of the term structure of interest rates around the log-steady state. This exercise provides insight on the factors determining the term premium and consequently provide intuiting for the calibration of the model.

$$
\begin{aligned}
E[\widehat{ytm_t}^{(i)}] - \hat{i}_t &= -\frac{1}{2}\sigma_1^2 \left(\frac{E[Var_t(\Delta^{(i)}\hat{c}_{t+i})]}{i} - E[Var_t(\Delta\hat{c}_{t+1})] \right) \\
&- \frac{1}{2}\left(\frac{E[Var_t(\sum_{n=1}^{i}\hat{\pi}_{t+n})]}{i} - E[Var_t(\hat{\pi}_{t+1})] \right) \\
&- \sigma_1 \frac{E[Cov_t(\sum_{n=1}^{i}\hat{\pi}_{t+n}, \Delta^{(i)}\hat{c}_{t+i})]}{i} + \sigma_1 E[Cov_t(\hat{\pi}_{t+1}, \Delta\hat{c}_{t+1})]
\end{aligned}
\tag{4.6}
$$

First two terms of equation 4.6 represents the so called Backus, Gregory, and Zin (1989) puzzle. In data the first-order autocorrelation of consumption growth is positive. Intuitively, aggregate consumption varies more over 10 years period than 3 months. Hence, the variance of the consumption growth over longer period should be higher than the the variance of one period consumption growth. From this reason the difference of first two terms should be positive. This, however, implies that the yield curve should have negative slope, which is not supported by data. As a result, it appears that the model is not able to generate a positive slope of the yield curve together with a positive serial correlation.

Hordahl et al. (2007) points out that the variance of consumption growth arises from the property of simple models which connects term premia with precautionary savings. In DSGE models the economy is exposed to uncertainty due to the various shocks hitting the system. The uncertain consumption stream and concave character of the utility function implies that expected consumption is always smaller than certain consumption. From this reason, consumption smoothing households tends to save more to transfer the consumption to the future. Yet this is not feasible in equilibrium, therefore the return on real bonds must fall in order to discourage savings. In other words, everybody wants to save now for the future, so the demand for bonds pushes the yields down. Assuming that the economy is hit by stationary shocks the consumption in the more distance future is actually less risky for households because the effect of the shocks continuously vanishes. The effect of precautionary savings must be weaker far out in the future. Hence, the short term rates fall more than long term yields.

The inflation part of the equation 4.6 is affected by monetary policy. Central bank fights against the inflation more intensive if the coefficient on inflation in Taylor rule is hight. In this case, inflation reacts to shock only in the first periods because of the monetary policy action, therefore the inflation is more volatile in short run rather than in long run. The reaction of monetary policy pushes up interest rate which drive the bond price down. The last term implies that if the consumption growth and inflation are negatively correlated the model will generate positive risk premium because of the persistence in inflation and consumption. Note, that the level of the slope is directly affect by the parameter σ_1.

Chapter 5

Solution method

To solve the model I rely on the perturbation method applied to the second order approximation of the nonlinear relationships which links all endogenous variables to the predetermined variables. The point around which the approximation is computed is the non-stochastic steady state. The second order approximation is necessary since first-order approximation of the model eliminates the term premium entirely, the covariance term from equation 4.3 is zero. This property is known as certainty equivalence in linearized models,[1] when agents in equilibrium behave as they were risk neutral.

The model is a highly nonlinear system of equation without closed form solution, therefore it has to be solved numerically. I use Matlab, in particular Dynare package.[2] The approach of second order approximation is described by Schmitt-Grohe and Uribe (2004). To solve for the bond price and yield curve I construct an algorithm depicted in the appendix.

[1] Alternatively, one can use log-linear/log-normal approach, see Emiris (2006)

[2] see Julliard (2010)

Once I compute an approximate solution to the model, I compare the model and the data using macro and finance simulated moments and data for Czech Republic. The focus is on matching means and standard deviations of consumption, inflation and output.

Chapter 6

Comparing the Model to the Data

In this section, I present results based on a calibrated version of my benchmark model and argue that although it fits relatively well the unconditional moments of macro data, it confirms the previous research for closed economy models in a inability to match finance data.

6.1 Macro Data

Table 6.1 illustrates that the model does fit the Czech data relatively well, taking into account: *i*) the specification of the model with only one exogenous shock and trivial production function, *ii*) limited tunning of calibration. Standard deviations in percent corresponds to the real data almost perfectly. The correlations achieve a poor match of the data, though this is partly given by the character of the shock. Richer model with shock in production function is able to fit the correlations with domestic product somewhat better, although the high correlation of output with consumption persist due to the equation 2.54. Nevertheless, we can conclude that even the simplified benchmark model is capable

Statistics	**Simulation**	**Czech Republic**	G-7
Standard Deviation %			
Output	2.20	2.469	1.85
Consumption	2.23	2.694	1.59
Nominal Interest Rate	0.782	0.783	0.45
CPI inflation	0.6	1.057	1.35
Contemporaneous correlation with domestic output			
Consumption	0.99	0.66	0.75
Nominal Interest Rate	-0.93	0.28	0.03
CPI inflation	-0.98	0.44	-0.57

Table 6.1: Model Simulations of Moments. The data sample for Czech Republic is 1993Q4 - 2002Q1; for G-7 1973-1996. Source: Natalucci and Ravenna (2002) and Stock and Watson (2000) for data on CPI inflation. Note: the data for CPI inflation in the column G-7 are values for US from 1956-1996

of matching the driving forces of Czech economy to some extent.

Figure 6.1 presents the impulse response function to a persistent foreign output orthogonalized shock for a baseline model. The impact of a temporary positive foreign output shock is divided between higher foreign and domestic consumption. The foreign producers decrease their prices in order to make their goods more attractive for agents in the SOE. This can be seen in the drop in terms of trade (S). The higher and cheaper foreign output allows to increase domestic production by 1.5 percent. However, the increase of the total world output pushes the domestic output even higher, but since there is not room for further increase in production, the shock projects to the PPI inflation (due to the increase in real wages). The effect of PPI inflation growth overweights the drop in import prices after 3 periods and leads consequently to the increase in CPI inflation. To the increase in the CPI inflation, a monetary authority responds by increase in interest rates. Higher real wages lead to an output decrease, moreover, the growing interest rates pushes output under the steady state level . The increase in marginal costs given by stronger growth in real wage than drop in terms of trade can be seen also in marginal cost decomposition equation 2.63. The effect of the shock is much stronger for bond prices with longer maturity as they

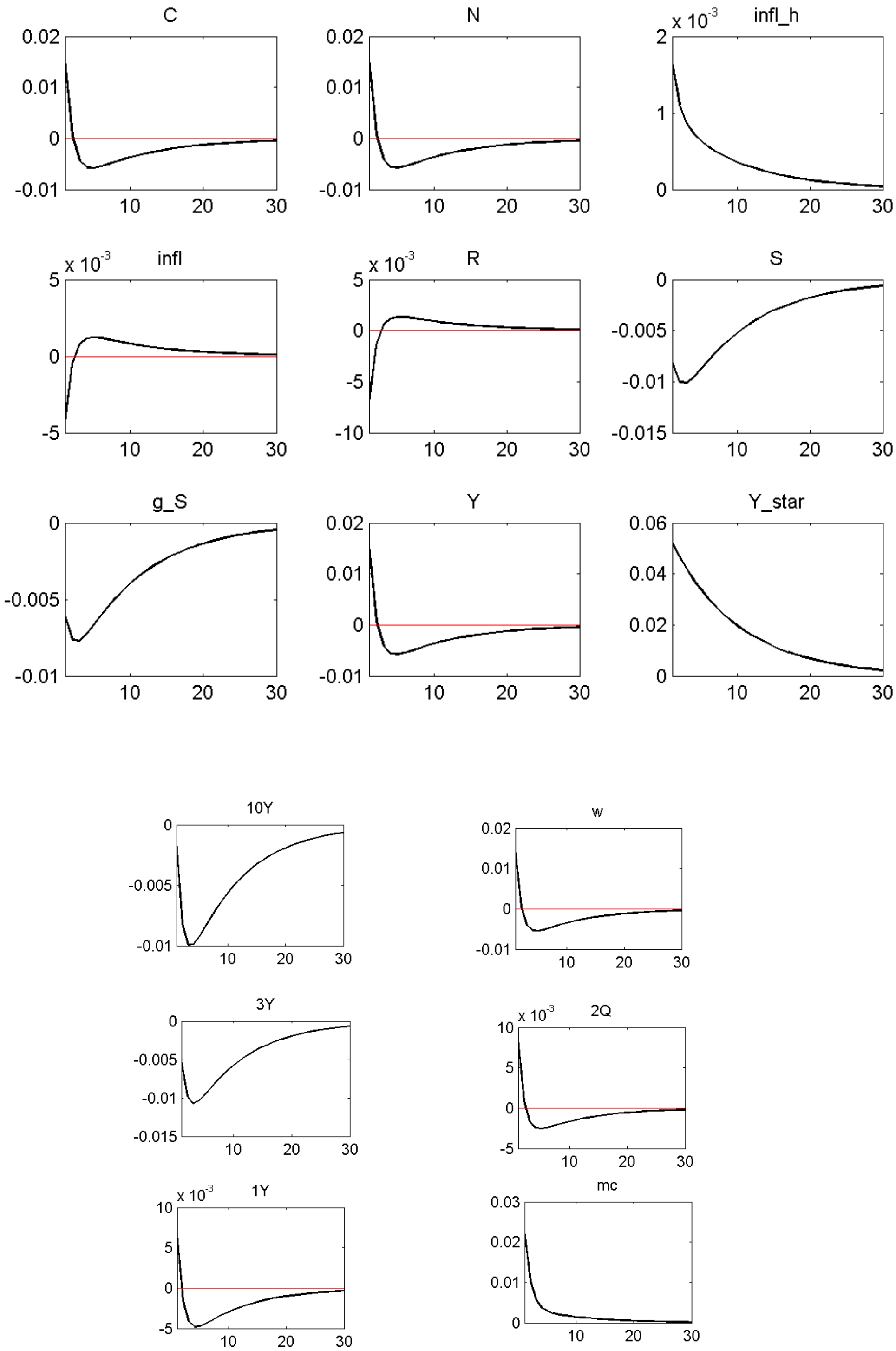

Figure 6.1: Impulse Response Function to a persistent foreign output orthogonalized shock in a baseline model

	Simulations				Czech Republic		US	
Maturity	6M	1Y	3Y	10Y	3Y	10Y	3Y	10Y
Mean	4.16	4.15	4.16	4.19	3.50	4.70	1.63	1.76
Std.Dev.%	59.40	44.60	25.76	14.42	40	20	40	34

Table 6.2: Simulation of The Term Structure of Interest Rates. Source: Own calculations based on model simulation (for 26 100 periods) and data from Reuters and Czech National bank. *Results for standard deviation are presented in % to the mean value*

are more risky.

6.2 Finance data

The focus is on matching the features of the term structure described in the section 4 and fitting the mean and standard deviation of 3 and 10 years zero coupon bonds.

The model delivers flat term structure of interest rates if we do not include term premium in the model, thus we can say that the expectation hypothesis holds in the model. Table 6.2 presents the results for the model simulations of yield curve. The simulated term premium (second term of equation 4.3) generated by the model is constant in time.[1] This corresponds to findings for DSGE models with term structure in the literature. Nevertheless, the term premium is too small to significantly influence the moments of bonds yield to maturity. The term premium, difference between a 10 year zero coupon bond and a two quarters bond is 1.6×10^{-4}. The model can solve Backus, Gregory, and Zin (1989) puzzle and generate positive slope of the term structure of interest rates, nevertheless the level of term premium too low.

It can be seen that the model is capable to fit the volatilities relatively well. This result is valid also because the term premium influences only first moments of the bond prices.

[1]except for the first approximately 500 periods when the time series of bond prices is not long enough and thus the covariance of short series fluctuate a lot.

Maturity	1Q	2Q	1Y	3Y	10Y	Y
Autocorrelation						
model	0.32	0.42	0.71	0.96	0.97	0.495
US data	0.92	0.93	0.94	0.95	0.97	0.86
Czech data				0.95	0.965	0.730

Table 6.3: First order autocorrelations of Bond princes in the baseline model. Source: US data are taken from Hordahl et al. (2007) and Stock and Watson (2000), Czech data are based on my calculations from time series from Czech Central Bank for. Data for Czech output are from 1994 to 2004, US 1970 to 1990 and Czech bonds from 2000 to 2010

The term premium is constant in time, therefore independent from second moments of the term structure of interest rates.[2] For this reason, the ability of model to fit volatilities is *not* induce by the fact that the model does not fit level of term premium.

The ability to reproduce volatility of bond prices is specific to the small open economy model and could be considered as a good achievement of the model. It has been argued in the literature e.g. (Hordahl et al., 2007) and (Rudenbush and Swanson, 2008) that in the simple model without habits in consumption and large and persistent shocks, it is particularly difficult to generate sufficient persistence in the short term rates to ensure that its variability is transmitted almost one-to-one to long term rates. It has been shown in Hordahl et al. (2007) that even with large persistence in shocks the propagation of variation to long term bonds is not sufficient. Table 6.3 presents the first order autocorrelations, we can see that the autocorrelation of the ten year bond is close to unity and matches real data. The fact, that the model generates significantly higher autocorrelation on the long tail of the yield curve is the source of our successful bond volatility fit. Based on sensitivity analysis of the parameter ρ, elasticity of substitution between home and foreign goods, one can see that it is the main parameter driving the autocorrelation in bond prices.

Figure 6.2 represents the random draw from the simulated term structures of interest

[2]Models solved up to the second order approximation all includes a constant term premium

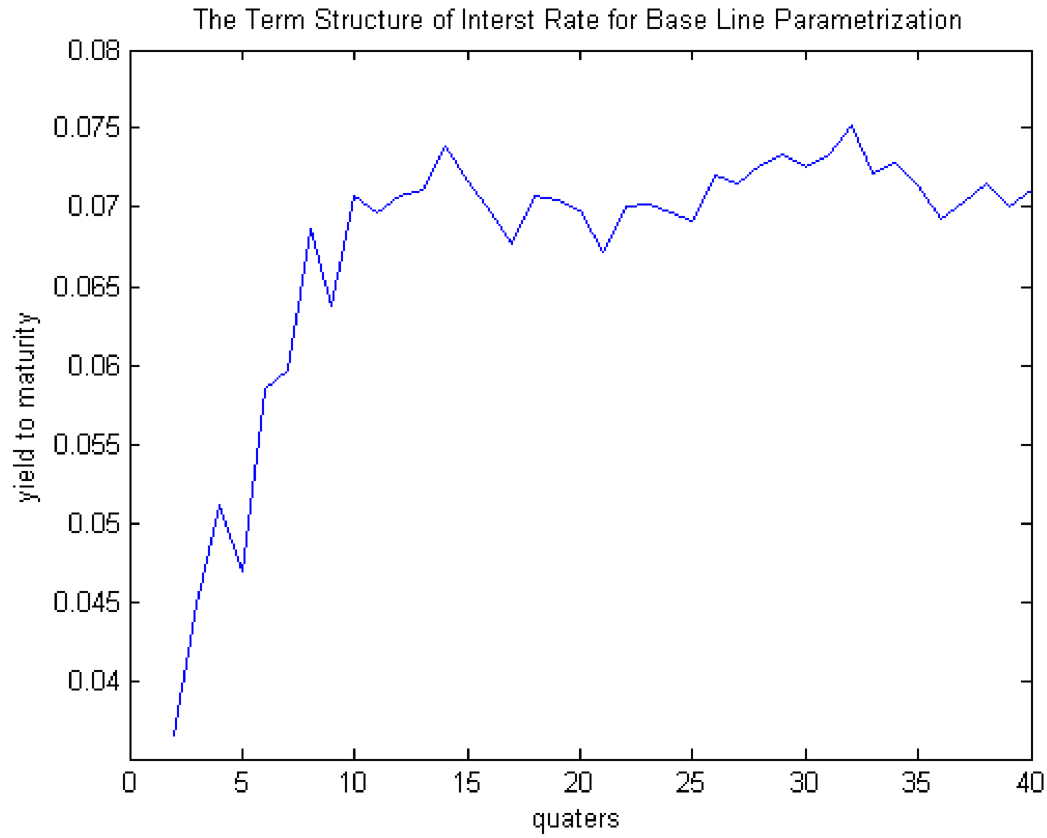

Figure 6.2: The random draw of the Yield Curve from the benchmark model at time $t = 1$

rates at time $t = 1$ generated by the benchmark model. The yield curve is increasing but, depending on starting time, the term structure can be also decreasing. In fact, on average the model produces a constant yield curve as emphasized above. The plot embodies the effect of foreign output shock, which is on average zero.

To conclude, the model fits the macro data relatively well, however because of a too small premium it is not able to reproduce the finance level data. Nevertheless, the volatility of zero coupon bond matches the real data qualitatively and also approximately quantitatively.

6.3 Best Fit of Finance Data Parametrization

As pointed out above, the relatively standard macroeconomic parametrization produces a term premium which is too small. The purpose of this section is to find such parameters which will be capable to generate as large term premium as possible. In order to increase the chances of the model to succeed in this exercise I introduce technology shock into the model which is mutually dependent with the foreign output shock, the correlation is set to 0.3. In this exercise I employ a following strategy: *i*) I use parameters from benchmark model but run loops over the grid of open economy parameters, *ii*) I fix small open economy parameters at the benchmark model values and run loops over the vectors of parameters which are assumed to have highest impact on the term premium *iii*) I use parameters found in Rudenbush and Swanson (2008) as a best fit parameters and run loops over the grid of open economy specific parameters.

6.3.1 Case 1: Open Economy parameters smoothing

I focus here on tunning the parameter ρ, elasticity of substitution between home and foreign goods, ρ_A, the autoregressive coefficient on productivity shock, and Φ_π, weight of monetary policy on inflation in such way that I maximize the slope of the term structure (i.e. the difference between the yield of a 10 year bond and a two quarter zero coupon bond.

I run the loop over the coarse grid $\rho \in \{0.8, 2.8, 4.8, 6.8, 7.8\}$ for $\Phi_\pi \in \{1, 21, 41, 61, 81, 101\}$ and $\rho_A \in \{0.6, 0.7, 0.8, 0.9\}$. After finding the best fit $\rho = 6.8$, $\Phi_\pi = 21$ and $\rho_y = 0.9$ I refine the grid to $\rho \in \{6.7, 7, 7.3, 7.6\}$ for $\Phi_\pi \in \{2, 7, 12, 17, 22, 25\}$ and $\rho_A \in \{0.88, 0.92, 0.96, 1\}$. The final combination bringing the highest term premium is $\rho = 7.5$, $\Phi_\pi = 2.1$ and $\rho_A = 0.99$.

The results are presented in the table 6.4. One can see that changing the open economy parameters did not bring any big progress in increasing the level of the term premium. On

Maturity	slope	2Q bond	10Y bond	C	output	infl
mean	0.0011	0.0990	0.0998	1	1	1
std	0.0366	0.0658	0.0603	0.046	0.037	0.016517

Table 6.4: Best fit of maximal slope, case 1

the other hand, the macro variables are just mildly destroyed compare to benchmark model results. However, what is important to note is that the model is able to break the Backus, Gregory, and Zin (1989) puzzle and create positive slope of term structure of interest rates.

Somewhat against the intuition outlined in the section 4, it is lower weight on inflation in monetary policy rule leading to a positive term premium. In section 4 I argued that if a monetary authority reacts strongly against the inflation growth, the interest rate is more volatile and consequently the variance bond price is pushed up. How is it then possible that in our model it is actually lower weight on inflation parameter leading to the positive term premia? If we take one more look at the equation 4.6 we can see that if central bank let inflation converge to the steady state slower it will affect households expectations. In the rational equilibrium central bank cannot fool households. From Phillips Curve we know that todays inflation is function of tomorrow expected inflation and tomorrow expected inflation is function of expected inflation day after tomorrow. Agents in the economy know that there will be higher inflation for several periods, therefore increase prices already today. Thus, the variance of inflation in near future is going to be higher then the average inflation in distance future. In distance future agents know that the shock is just temporary and inflation will die out. The parameter ρ amplify the magnitude of the volatilities of output and consumption (see Justiniano (2010).

6.3.2 Case 2: Parameters directly related to consumption

In order to generate positive term premium one would have to able to make consumption growth persistent. The intuition is straightforward, if we see that the impulse response function for consumption is first growing and only then decreasing (it is hump-shaped), we know that the consumption growth is higher in future than now. Consequently, positively correlated consumption growth implies positive term premia. This shape of impulse response function is difficult to get in closed economy without habit formation. In small open economy framework one can hope that the different dynamics of consumption due to the foreign demand channel will be able to generate positive term premia. So lets see if any parameter specification can generate positive slope of the term structure of interest rates.

I run the loop, as in the previous subsection, over the grid of parameters σ_1, σ_2, ϕ and Φ_π. Table 6.5 presents the results for parameter values: $\sigma_1 = 6.5$, $\sigma_2 = 0.4$, $\phi = 6$ and $\Phi_\pi = 25$.

The effect of the marginal utility of consumption is is in line with equation 4.6, with higher σ_1 we can observe higher slope of the yield curve. The impact of varying elasticity of labor supply, σ_2, is opposite to σ_1; lower elasticity of labor supply (higher σ_2) leads to lower slope of the term structure of interest rates. Lower elasticity of labor supply means that workers are less willing to increase the supply of labor if the real wage goes up. It means that it is more difficult for households to smooth consumption, since the shock can not be easily accommodated by adjusting labor supply. Hence, the consumption growth is in the first period after the shock and then evaporates. The mark-up over marginal cost magnify the slope of the yield curve, increasing ϕ makes the yields with longer maturity smaller. The monetary policy parameter increases the negative slope of the yield curve, however when reaching values about 25 it starts to have opposite effect and decrease the negative slope. The qualitative analysis from the section 4 implies that Φ_π should make

Maturity	slope	2Q bond	10Y bond	C	output	infl
mean	-0.077	0.1018	0.0246	1	1	1
std	0.0167	0.9041	0.1190	0.146476	0.146321	0.006552

Table 6.5: Best fit of maximal slope, case 2.

the slope more positive. The concave character of the function of slope of the yield curve on the Φ_π parameter stays unexplained. Note that the constrain to very large increase in analyzed parameters lies in the fact that from some point, too high parameters induce negative prices and thus non-real yield to maturity.

We can see that the benchmark small open economy is not capable of explaining the Backus, Gregory, and Zin (1989) puzzle. Although the the impulse response function for consumption is hump-shaped, the volatility of consumption growth is still higher far in the future and thus the different dynamics of evolution of consumption in small open economy does not change the level effect on term premium. In general, to create the larger slope of yield curve we had to enormously increase the variability of the model. The volatility of bond prices does not much data qualitatively nor quantitatively. The macro variables also are now match more volatile.

6.3.3 Case 3: Best fit building on Rudenbush and Swanson (2008)

In this section I use the best-fit parameterization found by Rudenbush and Swanson (2008). The best fit parameters are: $\sigma_1 = 6$, $\sigma_2 = 3$, $\rho_A = 0.95$ and $\sigma_A = 0.05$. I fix those parameters and run loop over the grid of following parameters: ρ, φ_p, Φ_p, ϕ, ρ_y. However, any combination of parameters is not able to produce simultaneously sufficiently high level of a term premia and positive slope of the yield curve.[3] For this reason I do not present

[3]I have also try to adjust Rudenbush and Swanson (2008) parameters but unsuccessfully

the results and only conclude that although the altered dynamics of consumption due to the foreign demand channel can contribute to solution of some features of DSGE models which the closed economy models are not able to reproduce. The open economy model can not, however, match levels and moments of data simultaneously.

Chapter 7

Extensions

In this section I introduce external habit to the benchmark model with productivity shock. First, I outline the main changes in equilibrium conditions. Next, I analyze the moments of simulated variables compare to the data similarly to the previous subsections.

7.1 Equilibrium condition changes

The equilibrium conditions change as follows:

International risk sharing

$$(C_t - \kappa C_{t-1}) = \vartheta(C_t^* - \kappa C_{t-1}^*) RS_t^{\frac{1}{\sigma_1}} \tag{7.1}$$

Euler Equation

$$1 = \beta E_t R_t \left(\frac{C_t - \kappa C_{t-1}}{C_{t+1} - \kappa C_t}\right)^{\sigma_1} \frac{P_t}{P_{t+1}} \tag{7.2}$$

Labor supply equation

$$\omega N_t^{\sigma_2} = (C_t - \kappa C_{t-1})^{-\sigma_1} w_t \tag{7.3}$$

Equations mentioned above are the main which alter the rest of the system. C stands again for the aggregate consumption and the first term in the international sharing function represents habit persistence, where κ denotes the intensity of habit formation and introduce the non-separability of preferences over time. Thus, the marginal utility from consumption is decreasing in current period, because of the concave character of the utility function, yet increasing in the next period. Intuitively, the more consumer eats today, the hungrier he or she is tomorrow.

7.2 Simulated moments and data analysis

First, I use the benchmark calibration will parameter $\kappa = 0.90$. The model delivers positive term premia but still underestimates the level. Moreover, the introduction of habits distorts slightly the macro variables. The results are presented in appendix. Hence, I repeat the exercise from last section and run loop on the grid of parameters to find the best fit of data. The result cast a pessimistic light on the ability of habit-based small open economy DSGE model to match the term premium. For any combination of parameters I have not found parameterization which delivers term premium between a 10 year bond and two quarter zero coupon bond higher than 0.5 percent.

Chapter 8

Conclusion

In the present thesis I have developed and analyzed impact of the small open economy dynamics on the term structure of interest rates. In particular, I have derived the small open economy model from the two country model by Bergin and Tchakarov (2003). I have simplified the model in order to be able to track the basic dynamics implied by foreign demand channel.

As the risk-free zero coupon bond is still risky in the sense that its price can covary with households' marginal utility of consumption, the expectation hypothesis does not hold and we can observe positive term premium in data. Another empirical regularity is connected to the variance of bond yields. In general, yields are less volatile on the long tail of the yield curve. Nevertheless, volatility decreases slowly with the maturity. It is common shortcomings of DSGE models, that the volatility decreases too fast due to low autocorrelation in bond prices. Further, DSGE models regularly produce negative slope of the yield curve because the volatility of consumption growth is usually bigger at present periods rather than in future periods.

The main purpose of this work was to test how the open economy model can address those puzzles. All in all, my results confirm the conclusions reached in case of the closed economy models. The fact, that households can adjust labor supply and insure themselves against consumption fluctuations leads to the smaller term premiums than we can find in data. The demand of the rest of the world for domestic goods boosts the labor supply effect even more than in the closed economy. On the other hand, the elasticity of substitution between home and foreign goods increases autocorrelation of bond prices and helps to match the volatility of bond yields. I also augment the model with production shock and external habit formation and show that in the open economy framework, even habit formation does not help to solve far too low level of term premium.

Appendix A

Slope of the term structure of interest rates

Following Hordahl et al. (2007):

The price of bond with maturity i is defined $P_t^{(i)} = E_t[Q_{t,t+i}]$; in the non-stochastic steady state $\bar{P} = \bar{Q}$. Lower case letters define logarithm of their upper case counterparts. Log-linearized stochastic discount factor looks $\hat{q}_{t,t+i} = \Delta^i \hat{\lambda}_{t+i} - \sum_{n=1}^{i} \hat{\pi}_{t+n}$ where λ is marginal utility of consumption.

$$\begin{aligned} \bar{p}(1 + \hat{p}_{t,i} + \frac{1}{2}\hat{p}_{t,i}^2) &= E_t\left[\bar{q}(1 + \hat{q}_{t+i} + \frac{1}{2}\hat{q}_{t+i}^2)\right] \\ &= \bar{q}_t\left[1 + \hat{q}_{t+i} + \frac{1}{2}\hat{q}_{t+i}^2\right] \end{aligned}$$

$$\hat{p}_{t,i} = E_t[\hat{q}_{t+i} + \frac{1}{2}\hat{q}_{t+i}^2] - \frac{1}{2}\hat{p}_{t,i}^2$$

Next, we know that $\hat{p}_{t,i}^2 = (E_t\hat{q}_{t+i})^2$. So we can write:

$$\hat{p}_{t,i} = E_t[\hat{q}_{t+i} + \frac{1}{2}\hat{q}^2_{t+i}] - \frac{1}{2}(E_t\hat{q}_{t+i})^2$$

From the last equation we can define price of one period bond.

$$\hat{p}_{t,1} = E_t[\hat{q}_{t+1}] + \frac{1}{2}Var[\hat{q}_{t+1}]$$

In general for any maturity:

$$\hat{p}_{t,i} = E_t[\Delta^i\hat{\lambda}_{t+i}] - \sum_{n=1}^{i}\hat{\pi}_{t+n} + \frac{1}{2}Var_t[\Delta^i\hat{\lambda}_{t+i} - \sum_{n=1}^{i}\hat{\pi}_{t+n}]$$

Using the definition of yield to maturity, $\widehat{ytm_t} = -(1/i)\hat{p}_{t,n}$

$$\widehat{ytm_t}^{(i)} = \frac{1}{i}\left\{\begin{array}{c} -E_t[\Delta^{(i)}\hat{\lambda}_{t+i}] + \sum_{n=1}^{i} E_t[\hat{\pi}_{t+n}] - \frac{1}{2}Var_t\left[\Delta^{(i)}\hat{\lambda}_{t+i}\right] \\ -\frac{1}{2}Var_t\left[\Delta^{(i)}\hat{\pi}_{t+i}\right] + Cov_t\left[\sum_{n=1}^{i}\hat{\pi}_{t+n}, \Delta^{(i)}\hat{\lambda}_{t+i}\right] \end{array}\right\}$$

Further, I subtract previous equation from the yield to maturity for one period bond to get:

$$\begin{aligned} E[\widehat{ytm_t}^{(i)}] - \hat{i}_t &= -\frac{1}{2}\left(\frac{E[Var_t(\Delta^{(i)}\hat{\lambda}_{t+i})]}{i} - E[Var_t(\Delta\hat{\lambda}_{t+1})]\right) \\ &- \frac{1}{2}\left(\frac{E[Var_t(\sum_{n=1}^{i}\hat{\pi}_{t+n})]}{i} - E[Var_t(\hat{\pi}_{t+1})]\right) \\ &+ \frac{E[Cov_t(\sum_{n=1}^{i}\hat{\pi}_{t+n}, \Delta^{(i)}\hat{c}_{t+i})]}{i} - E[Cov_t(\hat{\pi}_{t+1}, \Delta\hat{\lambda}_{t+1})] \end{aligned} \tag{A.1}$$

Appendix B

Algorithm solving the finance part

10 year zero coupon bond

forwarding inflation

for i=1:4090

$infl1(i,1) = infla(i+1,1);$

end

cumulative sum of inflation for bond price one year shorter than maturity

for i=1:4000

$cuminflcov(i,1) = infl1(i,1)*infl1(i+1,1)*infl1(i+2,1)*infl1(i+3,1)*infl1(i+4,1)*infl1(i+5,1)*infl1(i+6,1)*infl1(i+7,1)*infl1(i+8,1)*infl1(i+9,1)*infl1(i+10,1)*infl1(i+11,1)*infl1(i+12,1)*infl1(i+13,1)*infl1(i+14,1)*infl1(i+15,1)*infl1(i+16,1)*infl1(i+17,1)*infl1(i+18,1)*infl1(i+19,1)*infl1(i+20,1)*infl1(i+21,1)*infl1(i+22,1)*infl1(i+23,1)*infl1(i+24,1)*infl1(i+25,1)*infl1(i+26,1)*infl1(i+27,1)*infl1(i+28,1)*infl1(i+29,1)*infl1(i+30,1)*infl1(i+31,1)*infl1(i+32,1)*infl1(i+33,1)*infl1(i+34,1)*infl1(i+35,1)*infl1(i+36,1)*infl1(i+37,1)*infl1(i+38,1);$

$a1(i,1) = 1/cuminflcov(i,1);$

end

cumulative sum of inflation of bond price with maturity 10Y

for i=1:4000

$cuminfl(i,1) = infl1(i,1)*infl1(i+1,1)*infl1(i+2,1)*infl1(i+3,1)*infl1(i+4,1)*infl1(i+5,1)*infl1(i+6,1)*infl1(i+7,1)*infl1(i+8,1)*infl1(i+9,1)*infl1(i+10,1)*infl1(i+11,1)*infl1(i+12,1)*infl1(i+13,1)*infl1(i+14,1)*infl1(i+15,1)*infl1(i+16,1)*infl1(i+17,1)*infl1(i+18,1)*infl1(i+19,1)*infl1(i+20,1)*infl1(i+21,1)*infl1(i+22,1)*infl1(i+23,1)*infl1(i+24,1)*infl1(i+25,1)*infl1(i+26,1)*infl1(i+27,1)*infl1(i+28,1)*infl1(i+29,1)*infl1(i+30,1)*infl1(i+31,1)*infl1(i+32,1)*infl1(i+33,1)*infl1(i+34,1)*infl1(i+35,1)*infl1(i+36,1)*infl1(i+37,1)*infl1(i+38,1)*infl1(i+39,1);$

$aa1(i,1) = 1/cuminfl(i,1);$

end

price of 10Y bond

for i=1:3900

$bondmat(i,1) = (Caa(i,1)/Caa(i+40,1))^{(sigma_1)} * betta^{(40)} * aa1(i,1);$

end

price of 39Q bond

for i=1:3900

$bondcov10(i,1) = (Caa(i,1)/Caa(i+39,1))^{(sigma_1)} * betta^{(39)} * a1(i,1);$

end

stochastic discount factor at period 39

for i=1:4050

$Qahead(i,1) = Qa(i+39,1);$

end

term premium

for i=1:3800

$c = cov(-Qahead(1:i,1), bondcov10(1:i,1));$

$tp(i,1) = c(2,1);$

end

full price of bond

for i=1:3800

$bondfull(i,1) = bondmat(i,1) + tp(i,1);$

end

yield to maturity

for i=1:3800

$ytmbond(i,1) = -1/10 * reallog(bondfull(i,1));$

end

The term structure of interst rates at t=1

for i=1:1500

$infl1(i,1) = infl(i+1,1);$

end

$z = cumprod(infl1);$

for i=1:1000

$z1(i,1) = 1/z(i,1);$

end

for i=1:41

$price(i,1) = (C(1,1)/C(i,1))^(sigma_1) * betta^(i) * z1(i,1);$

end

plot(price)

for i=2:41

$price1(i-1,1) = price(i,1);$

end

for i=1:40

$c = cov(Q(1:i,1), price1(1:i,1));$

$tp(i,1) = c(2,1);$

end

plot(tp)

for i=1:40

$pricef(i,1) = price(i,1) + tp(i,1);$

end

for i=1:40

$ytm(i,1) = (-4/i) * reallog(pricef(i,1));$

end

plot(ytm)

Appendix C

results for benchmark model with habits

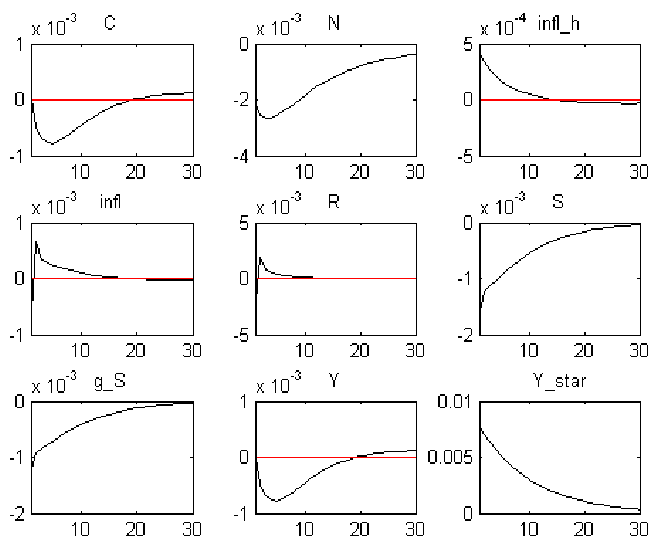

Figure C.1: Impulse Response for benchmark model with external habits 1

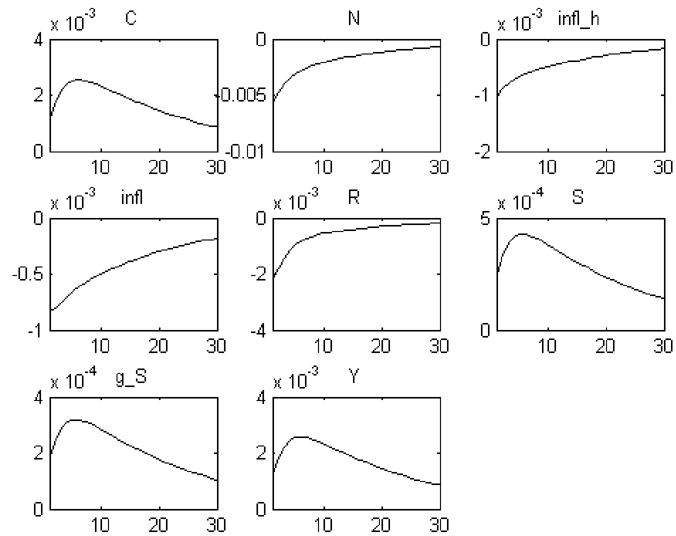

Figure C.2: Impulse Response for benchmark model with external habits 2

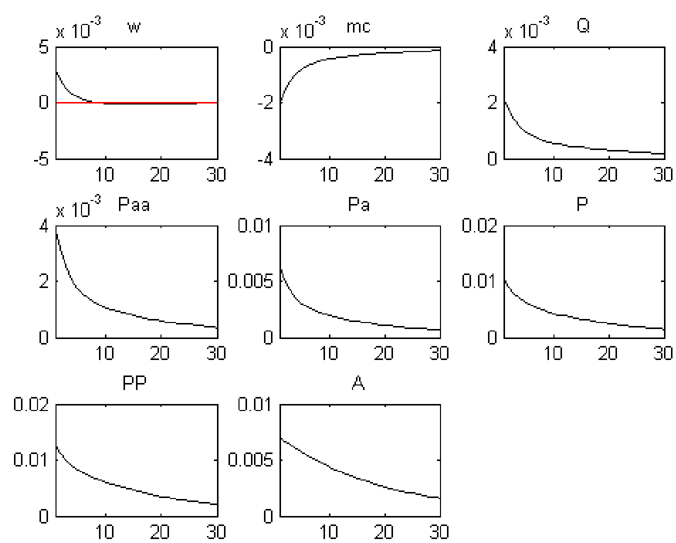

Figure C.3: Impulse Response for benchmark model with external habits 3

Maturity	slope	2Q bond	10Y bond	C	output	infl
mean	1.8138×10^{-5}	0.0415	0.0415	1	1	1
std		0.0230	0.0110	0.010070	0.010044	0.002813

Table C.1: benchmark model with external habits

Bibliography

Martin Moller Andreasen. Explaining macroeconomic and term structure dynamics jointly in non-linear dsge model. University of Aarhus and CREATES, Working Paper Series No. 236, 2008.

David Backus, Allan W. Gregory, and Stanley E. Zin. Risk premiums in the term structure. *Journal of Monetary Economics*, 1989.

Paul R. Bergin and Ivan Tchakarov. Does exchange rate risk matter for welfare? *NBER Working Paper No. W9900*, 2003.

Francesco Caprioli and Stefano Gnocchi. Monetary policy credibility and the term structure. Universitat Autonoma de Barcelona, 2009.

John H. Cochrane. *Asset pricing.* Princeton University Press, 2001.

Luca Colantoni. Exchange rate variability in the small open economy with currency substitution. Bocconi University, 2010.

Bianca De-Paoli. Monetary policy and welfare in a small open economcy. *CEP Discussion Paper No 639*, 2006.

Wouter Den-Haan. The term structure of interest rates in real and monetary economies. *Journal of Economic Dynamics and Control*, 1995.

Marina Emiris. The term structure of interest rates. Working paper research, 2006.

Jordi Galí. Monetary policy and exchange rate volatility in a small open economy. *NBER Working Paper*, 2002.

Peter Hordahl, Oreste Tristani, and Dvaid Vestin. The yield curve and macroeconomic dynamics. *Working paper series 832*, 2007. URL `http://www.ecb.europa.eu`.

Michael Julliard. *A program for simulating and Eestimating DSGE models*, 2010. URL `http:/www.cpremap.cnrs.fr`.

Alejandro Justiniano. Small open economy dsge models: Specification, estimation and model fit. Central Bank of Sweden Working paper series 179, 2010.

Tommaso Monacelli. Monetary policy in a low pass-through enviroment. *European Central Bank, Working Paper No, 227*, 2003. European Central Bank, Working Paper No, 227.

Fabio M. Natalucci and Federico Ravenna. The road to adoptiong the euro: Monetary policy and exchange rate regimes in eu candidate countries. *INternational Finance Discussion Papers*, December 2002.

Maurice Obstfeld and Kenneth Rogoff. Exchange rate dynamics redux. *NBER Working Paper No. 4693*, 1994.

Julio Rotemberg. Sticky prices in the united states. *Journal of Political Economy*, 1982.

Glenn D. Rudebusch and Tao Wu. Unknown title. *Journal of Economic Theory*, 2004.

Glenn Rudenbush and Eric T. Swanson. Examining the bond premium puzzle with a dsge model. *Journal of Monetary Economics*, 2008.

Stephanie Schmitt-Grohe and Martin Uribe. Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control*, 2004.

James Stock and Mark Watson. Has the business cycle changed and why. *NBER WP 9127*, 2000.

Alan Sutherland. The expenditure switching effect, welfare and monetary policy in a small open economy. *Journal of Economic Dynamics and Control*, 2006.

Martín Uribe. Lectures in open economy macroeconomics. available on-line, May 4, 2009. URL http://www.columbia.edu/~mu2166/GIM/lecture_notes.pdf.

Osvald Vasicek and Karel Musil. Behavior of the czech economy: New open economy macroeconomics dsge model. Working paper No. 23/2006, 2006.