Bringing Logic Closer to Natural Language:
A Non-model-theoretic Completeness Proof for Hanoch Ben-Yami's Logic of Quantification with Plural Subjects

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Abstract

The topic of this thesis is Hanoch Ben-Yami's book *Logic and Natrural Language*, and the natural deduction system for quantified logic with plural subjects developed therein. Unlike standard modern logical systems which take the terms in the subject position as predicative, Ben-Yami treats them as referring. I will examine his and others' takes on the topic. The goal of this thesis is to present a formalization and develop a completeness proof for Ben-Yami's system. Instead of the standard model-theoretic completeness proof I use, following Ben-Yami, a substitutional approach.

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Introduction

The topic of my thesis Hanoch Ben-Yami's *Logic of Quantification with Plural Subjects* (LQPS for short), introduced in his 2004 book *Logic and Natural Language* (although the label LQPS is introduced later). More specifically, my goal is to provide a non-model-theoretic completeness proof for it. Instead of a model theory, the approach I will take, following Ben-Yami's suggestion for the predicate calculus in his manuscript *Truth and Proof without Models,* is a substitutional one.

Ben-Yami's logical system departs from the standard modern logical tradition by introducing a distinction between a subject and the predicate into his logic - unlike the standard Fregean logic, also called the *predicate calculus*, which views both as predicative – and furthermore, as the name of the system suggests, making the subjects refer to a plurality of objects (taken loosely) as well. The first three chapters will therefore deal with the specificities of Ben-Yami's system by examining the background of other historical and modern approaches to the issues Ben-Yami discusses. These constitute the first part of my thesis, while the second part is the fourth chapter, in which the completeness proof is laid out.

To motivate my research, I will turn my attention in the first chapter of my thesis to the predicate calculus. As will be obvious from the first part, there was a long-standing tradition in logic of treating the subject and the predicate of a sentence as serving logically distinct functions. However, an approach suggested by Gottlob Frege and advocated by, among others, Bertrand Russell, broke away from this tradition. The aim of the first part of my thesis is to show how Ben-Yami undermines their arguments for doing so and, thus, demonstrate that this shift was undermotivated. However, the predicate calculus is ubiquitous in its use in philosophy and logic today. One of the reasons for that is its simplicity and utility. Therefore, in advocating an alternative approach, one needs to demonstrate that it can match and excel those properties of Frege's system. For this reason, in the closing sections of the second part, I will demonstrate how certain intuitively valid inferences, most notably those from Aristotle syllogistic logic, can be proven in Ben-Yami's system (but not in the predicate calculus).

Some authors, most notably Peter Geach, Peter Strawson and Gyula Klima have taken an approach which has some overlaps and divergences from Ben-Yami, and, of course, Aristotle's syllogistic logic followed the same vein. In the following two chapters of my thesis, I will examine all of these different approaches and demonstrate how they are similar or different from Ben-Yami's.

In the central part of my thesis I present a formalization of Ben-Yami's system (which relies on the natural linguistic capacities of reader rather than a formal set of necessary and sufficient condition). I will first present a formalization of Ben-Yami's language into a logical form, and then use that language to formulate a natural deduction system for it. This system differs from the standard predicate calculus in introducing such operations (which mimic the surface structure of natural languages better) as copula negation and anaphora. I will then proceed to demonstrate that this system possesses the desirable metatheoretical properties of soundness and completeness. While soundness is already presented in Ben-Yami's book *Logic and Natural Language*, there is no completeness proof. This proof has later been published in an article by Ben-Yami and Ran Lanzet, in which they use a model-theoretic framework. There are, however, reason to be dissatisfied with that approach, some of which are broth forth by Ben-Yami himself in a forthcoming article. Therefore, after voicing those concerns, I will present a completeness proof which is *substitutional* rather than model-theoretic – it connects the truth values of quantified sentences with those of their substitution-instances, rather than with properties of objects in a constructed models. Other than this difference, the proof is standard Henkin-style proof, with adjustment made for the peculiarities of the system.

1. The Predicate Calculus

In this chapter I will first briefly present the logical system that I compare Ben-Yami's LQPS with – the predicate calculus. The father of this system is Gottlob Frege, but it owes its universal adoption to, among others, Bertrand Russell. For that reason, the exposition of that system will be followed by a discussion of their reasons and arguments for accepting it. Next, I will provide Ben-Yami's counter-arguments to these reasons, with the goal of demonstrating that the acceptance of the system was under-motivated. This, in turn, will be a reason for taking an alternative approach of LQPS. Although the argumentation here is purely negative, I will show in the next chapter that the preceding system, that of Aristotle, could address some objections raised against it, and that the predicate calculus was therefore not essential, at least with regards to its application on natural language. Then I will demonstrate that LQPS has the desirable properties of a logical system, while at the same time staying true to both the Aristotelian tradition and the surface structure of natural language.

1.1 The Language of Predicate Calculus

The language of the Predicate Calculus (PC) consists of a set of terms T, a set of predicate variables P, a set of truth-functional connectives $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$, a set of quantifiers $\{\exists, \forall\}$ and a set of parentheses {(,)}. The set T consists of individual constants $\{a,b,c,\ldots\}$, a set of variables $\{x,y,z,..\}$ and a set of function symbols $\{f,g,\ldots\}$ which take terms as their arguments. The set P consists of *n*-place predicates $\{P,Q,R,\ldots\}$, where *n* is a number of arguments a predicate takes (and which is fixed for each predicate).

1.1.1 Sentence Formation Rules

If *t1,…,tn* are terms and P is an *n*-place predicate, then $P(t_1,\ldots,t_n)$ is an atomic *well-formed formula* (*wff*). If P and Q are *wff*''s, then so are $(\neg P)$,$(P \wedge Q)$, $(P \vee Q)$, $(P \rightarrow Q)$ and $(P \leftrightarrow Q)$, where the outermost parentheses can be omitted. If P is a *wff* and v is a variable, then $\forall vP$ and $\exists vP$ are *wff*'s (and any occurrence of v in $\forall vP$ or $\exists vP$ is said to be bound).[1]

A *wff* where all the variables are bound is a sentence, and no formula that is not generated by these rules is a sentence.

1.1.2 Assignment of Truth Values to Sentences

In this section only a provisional definition of the assignments of truth values will be given. Consequently, I will use some expression without giving their formal definitions. It will, however, suffice for our present purposes.

An atomic sentence (i.e. and atomic *wff* with no variables) $P(t_1,\ldots,t_n)$ is true just in case the ordered *n*-tuple $< t_1,\ldots,t_n >$ is a member of the extension of the predicate P (where the extension of an *n*-place predicate is a subset of a Cartesian product D^n, with D being the domain), and false otherwise.

The rules for the truth-functional connectives state that $\neg P$ is true just in case P is false, $P \wedge Q$ is true just in case P and Q are both true, $P \vee Q$ is true just in case P or Q are true, $P \rightarrow Q$ is true just in case P is false or Q is true, and $P \leftrightarrow Q$ is true just in case P and Q are both true, or P and Q are both false.

If $\varphi(v)$ is a formula where v is the only unbound variable, then $\forall v \varphi(v)$ is true just in case every element of D satisfies $\varphi(v)$, and $\exists v \varphi(v)$ is true just in case some element of D satisfies $\varphi(v)$.

[1] J. Barwise, J. Etchemendy, *Language, Proof and Logic*, CSLI Publications, New York, London, 1999, p.232

This short exposition of the *PC* will be enough to make the remaining part of this chapter legible. Further detail will be introduced as the discussion warrants (for instance, how these sentences are read). We now turn our attention to the reasons for adopting the predicate calculus in the first place and Ben-Yami objections to it.

1.2 Referring Nouns as Predicates

Ben-Yami analyzes common nouns in quantified noun phrases as referring expressions.[2] However, in the standard predicate calculus, they are treated as predicates attributing some property (understood broadly) of an individual constant or a variable (which then falls under a scope of a quantifier). For example, the sentence "All men are mortal" would be formalized as:

$$\forall x(Man(x) \rightarrow Mortal(x))$$

In a somewhat contrived way, this can be read as "All *x* are such that, if *x* is a man, then *x* is mortal." Obviously, "man" here does not refer to anything, but simply attributes a property of being a man to every instance of a substitution of the variable by an element of the domain. Given the prevalence of the predicate calculus, this is standard and widely accepted way of treating common nouns. However, it was not commonplace when Frege first suggested it. Let us observe what Ben-Yami suggests were his motivations for doing so.[3]

First, the idea that the common nouns serve the same logical function in the subject and the predicate place might originate in Aristotle, where the latter are treated identically with any other word type (e.g. adjective) occurring in the same position. Combined with the notion that the common nouns always serve a similar purpose, it is easy to see how the

[2]H. Ben-Yami, *Logic & Natural Language: On Plural Reference and Its Semantic and Logical Significance*, revised edition, Ashgate, Aldershot, 2004, pg.45
[3] The following sections are a summary of the section 4.1 of Ben-Yami,2004, beginning at the page 45

conflation of the logical roles of the subject and the predicate occurs. Until the nineteenth century, this resulted with the predicate being treated as a referring term. Frege's understanding of concept words as predicative led to him reversing the trend – both subject and predicate were still treated as having the same logical role, but it was now akin to predication.

1.2.1 Frege

Frege approached the treatment of natural language from a mathematical point of view. He understood concepts as functions whose value is a truth value. Consequently, Frege conflates the subject-predicate distinction with the relation of an argument and a function. Since only singular-referring expressions are the arguments of mathematical functions, they were the only ones treated as denotative. Conversely, given that the general concepts were treated as predicative, and that common nouns are general terms, they were assigned the predicative role as well.

Frege argues that the sentence

All whales are mammals

doesn't talk about whales, but rather the concept of whales, since when uttering it we cannot indicate even a single animal we are referring to. Furthermore, even if there was a whale around, we couldn't infer anything from this sentence without a further proposition that it is a whale. Therefore, this sentence is about the relation of concepts, not about whales.

Yet, as Ben-Yami points out, the same line of reasoning could applied to a sentence

Peter is ill.

Even if Peter were present at the time of the uttering of this sentence, one still could not conclude anything about the man present without a further proposition to the effect that the man present was Peter. And Frege would concede that ‘Peter’ does, in fact, have a referential role. Therefore, Frege’s argument fails to clearly delineate the predicative use of concepts. One could equally well argue that ‘all whales’ refers to the plurality of whales the same way ‘Peter’ refers to Peter.

Another argument Frege offers for his analysis concerns negations of sentences with apparently plural-referring subjects. Consider the sentence

All mammals are land-dwellers

If the expression ’all mammals’ were a subject, its negation would be ‘All mammals are not land-dwellers’, which it is not. Ben-Yami objects that this example only shows that the negations of sentences with plural subjects differ from the negations of the ones with a singular subjects. The negation of the sentence ‘Peter and Mary are painters” is not “Peter and Mary are not painters,” but rather “Peter and Mary are not *both* painters,” and ‘Peter and Mary’ is the subject of that sentence. So, the fact that the negations work differently does not commit us to believe that both are not cases of a common noun playing a role of a subject with plural reference rather than serving a predicative function.

These considerations suggest Frege’s account, as Ben-Yami concludes, does not provide sufficient reason for treating the common nouns in the subject position as a predicate.

1.2.2 Russell and Bradley

While Frege was the initiator of the new approach, its most significant early proponent who caused it to be widely accepted was Bertrand Russell, who himself was influenced by

Bradley.[4] Bradley himself offers two arguments against viewing common nouns as referring. First, he argues that, since in saying that (all) animals are mortal we do not talk about merely the presently existing animals, but about *any* past, present or future animal. Now, since future animals do not yet exist, it is impossible to refer to them. Therefore, we can not view the common noun "animal" as referring to animals. The second argument states that in speaking about animals we cannot have the complete collection of animals in mind, and thus cannot refer to them. In either case, what we are saying is that "'Whatever is an animal will die,' but that is the same as *If* anything is an animal *then* it is mortal. The assertion really is about mere hypothesis; it is not about fact."[5]

Ben-Yami provides three answers to the former argument. First, we can grant that referring to future entities would not be a paradigmatic case of reference. But, even if we disallow it altogether, this does not constitute an argument against cases devoid of such a difficulty, such as with the expression 'my children'.[6] Next, Russell's analysis encounters a similar problem – even if we reconstruct 'All animals are mortal' as 'for any *x*, if *x* is an animal then *x* is mortal', the variable *x* still needs to range over or, in a sense, refer to the future animals (on the pain of losing the initial step of the argument). However, how a variable should refer to future entities is no clearer than how a common noun should do so.[7] Ben-Yami's final response to the first argument of Bradley's is that "there are good reasons for allowing the possibility of reference to future individuals. Consider the sentences

All children born last year got the flu.

All these children got the flu.

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[4] Ben-Yami, 2004, p.48
[5] F.H. Bradley, *The Principles of Logic, corrected impression of the 2nd edition,* Oxford University Press, Oxford, 1922, p.47
[6] Ben-Yami, 2004, p.50
[7] Ben-Yami, 2004, p.50

All children that will be born next year will get the flu."[8]

As Ben-Yami points out, all these sentences are very similar in both their grammatical form and in their method of verification (find the children born last year and see if they got the flu, check if the children pointed at got the flu, and wait to find out which children were born the following year and whether they got the flu, respectively). Since "these children" is a paradigmatic case of reference, this analogy should suggest we should grant the same status to the noun phrases in the first and the third sentences. Ben-Yami goes into a more elaborate discussion of reference, but we will omit it here. Instead, let us now observe his objections to Bradley's second argument.

Ben-Yami sees this argument as question begging. Presumably, what Bradley thinks by having a complete collection in mind is thinking of each and every individual falling under that description (as would be the case with the aforementioned phrase 'my children' when uttered by a mother, as opposed to, say, a priest). But there is no reason to see having a collection in mind this way, as opposed to having in it mind in terms of an expression that refers to all those and only those individuals that form that collection.[9]

1.3 Final Remarks

As with the Frege's account above, we can see that the adoption of the view that the expressions in the subject position are predicative in their deep logical structure was undermotivated, at least with regards to the natural language (mathematical considerations aside). On the other hand, it broke away from a longstanding tradition in philosophy, dating back to Aristotle. However, there might have been other problems with Aristotle's approach

[8] Ben-Yami, 2004, p.50
[9] Ben-Yami, 2004, p.50

that Frege's approach successfully coped with. Therefore, in the next chapter we will take a look at Aristotle's logical system. After a brief overview, we will observe one such difficulty and then see that there is a way of avoiding it, dating back to the logic of Middle Ages (and obviously predating Frege and Russell). While the constraints of space do not allow a thorough examinations of every issue Aristotelian logic faces, showing how it can address one substantial difficulty will serve to further undermine the reasons for the switch to a new approach.

2.Referential Import

The first part of this chapter will serve as a brief overview of the main characteristics of Aristotelian logic. I will first present a formalization of sentences of the type relevant for our present discussion (subject-predicate ones), and then see some of the entailments that hold between them. One of the characteristics of the Aristotelian logic is the principle of referential import (using Ben-Yami's notation). I will than bring out a modern objection to this principle, and show how Gyula Klima, using a formalization of medieval Aristotelian logic, addresses it. This, in addition to defending referential import, shows another way in which the introduction of the predicate calculus can be seen as undermotivated. Finally, in the last section of the chapter I will present Ben-Yami's take on the same issue.

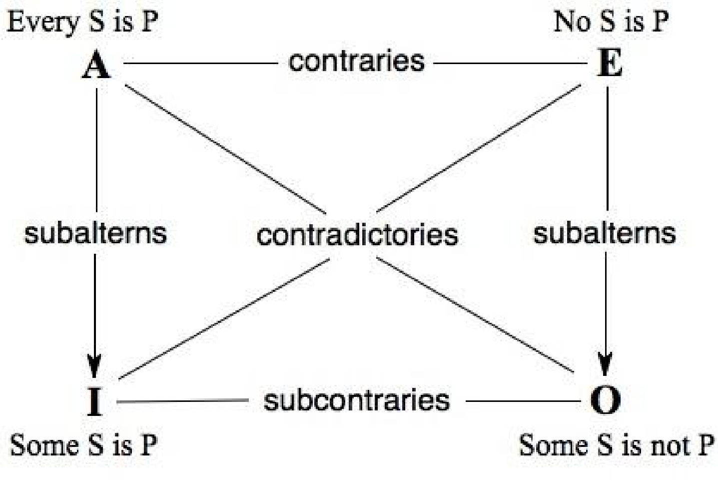
2.1 The Square of Oppositions

In the logic of Aristotle, four types of categorical propositions – a subject-predicate propositions[10] – are taken to be standard[11]: *A* type – *All S are P*, *E* type – *No S are P*, *I* type – *Some S are P* and *O* type – *Some S are not P*. To illustrate, an *A* type proposition would be *All*

[10] Subject term is labeled *S* and the predicate term is labeled *P* throughout this exposition
[11] E. A Hacker, W. T Parry, *Aristotelian Logic*, SUNY Press, 1991, p.145

men are mortal; an *E* type would be *No men are mortal* etc. The four differ from each other according to two parameters – quantity (all/no-some) and quality (is/are – isn't/aren't).

The relations that hold between these types of propositions are usually sketched in a useful diagram called the Square of Oppositions. Before laying out this diagram, let us introduce the terminology for labeling these relations. First, two propositions which share the subject and predicate terms but differ in type are called *opposed*[12]. A pair of opposed propositions can be: *contradictories, contraries, subcontraries* and *alterns.* Contradictories are those propositions which differ in both quantity and quality (*A* and *O*, *E* and *I*), contraries are universal propositions ("all"/"no") which differ in quality (*A* and *E*), subcontraries are particulars ("some") which differ in quality (*I* and *O*), and alterns are those propositions which differ in quantity alone (*A* and *I*, *E* and *O*, where the particular propositions are called "subaltern," and the universal ones "superaltern")[13]. Finally, we can sketch out the square of oppositions[14]:



[12] Hacker, Parry, 1991, p. 155
[13] Hacker, Parry, 1991, p. 157
[14]T. Parsons, 'The Traditional Square of Oppositions', in *Stanford Encyclopedia of Philosophy*. Oct 1, 2006, http://plato.stanford.edu/entries/square/, viewed Apr 13, 2012

2.1.1 Immediate Inferences

The types of opposition relation which hold in a pair of propositions carry with them a connection of their truth values as well. Contradictories cannot have the same truth value – they can neither both be true nor both be false.[15] Contrary propositions cannot both be true,[16] subcontrary ones cannot both be false.[17] Finally, universal propositions entail, but are not entailed by, their respective subalterns.[18]

Since the truth values of some types of propositions are interdependent, they form a basis for a number of valid arguments called *immediate inferences* (an immediate inference is an argument with one premise and one conclusion[19]). These immediate inferences are as follows: first, A and $\neg O$ entail one another (since they always have the opposite truth value). Consequently, $\neg A$ and O will entail one another as well. The other pair of contradictories, E and I share the same type of a relation – a mutual entailment relation will hold between E and $\neg I$. As was obvious from the definition of alterns, A will entail I and E will entail O. And finally, the inferences among (sub)contraries are: A entails $\neg E$ and $\neg I$ entails O. Formally, these are the relations that need to hold in the square of oppositions: $A \Rightarrow I$, $E \Rightarrow O$, $A \Leftrightarrow \neg O$, $E \Leftrightarrow \neg I$, $A \Rightarrow \neg E$ and $\neg I \Rightarrow O$[20]. A logical system will be considered to be Aristotelian if it retains these entailments.

2.2 Klima: Existence and Reference in Medieval Logic

In his paper *Existence and Reference in Medieval Logic*, Gyula Klima provides "account of how it was possible for medieval logicians to maintain Aristotle's theory of the four

[15] Hacker, Parry,1991, p.159
[16] Hacker, Parry, 1991, p.161
[17] Hacker, Parry, 1991, p.162
[18] Hacker, Parry, 1991, p. 163
[19] Hacker, Parry, 1991, p. 155
[20] G. Klima, 'Existence and Reference in Medieval Logic' in A. Hieke, E. Morscher (eds.), *New Essays in Free Logic*, Kluwer Academic Publishers, 2001, 197–226, p.198

categoricals and to dispense with these existential assumptions in the framework of their theory of reference, the *theory of supposition*."[21] The semantic theory in question states that the role the subject terms serve is a referring one – they stand for the objects falling under them, or stand for *nothing* if no such objects exist. In case they stand for nothing, the affirmative sentences will be false, whereas their contradictories will therefore be true – negative categorical propositions carry no presupposition of existence. Thus all the entailments of the square of oppositions are maintained.[22]

2.2.1 One Objection to the Medieval Analysis

Klima discusses two interesting objections to the medieval analysis. However, as the second one is relatively complex for the sake of brevity only the first one will be discussed in this paper. It should, however, suffice for our present purposes.

One worry that arises with this kind of analysis is that, should the affirmative universal proposition be understood as false when the subject term is empty, then its contradictory should be true. This is problematic, and the predicate calculus avoids it. Take the sentence of the type

$$\forall x(Unicorn(x) \rightarrow Mortal(x)).$$

As we have seen in the preceding chapter, this is true just in case every element of the domain satisfies the formula with one unbound variable $Unicorn(x) \rightarrow Mortal(x)$. Now, since no element of the domain satisfies the formula *Unicorn(x)*, the conditional is true for every element of the domain, and therefore the universally quantified sentence is as well. We will

[21] Klima, 2001, p.197
[22] Klima, 2001, p.198

see that the medieval logicians can address this problem, and that it therefore does not constitute a reason for abandoning Aristotelian logic.

Klima addresses this problem by distinguishing two ways in which a proposition can be negated – *negating* (propositional negation) and *infinitizing* (term negation).[23] Klima illustrates the point on the famous example of Bertrand Russell – negating that the king of France is bald. This can be understood either as saying that it is not true that the king of France is bald (propositional negation), which is true since there is no king of France, or as a claim that the king of France is non-bald (term negation), which is false as there is no such person.

In a similar manner, the negation of a sentence *Every winged horse is a horse* (which is true as its subject term is empty) can be interpreted either as

1) Some winged horse is not a horse,

i.e. as saying that it is not the case that some winged horse is a horse, or as

2) Some winged horse is a non-horse.

To formalize the two we need to introduce "*restricted variables,* representing general terms in their referring function, as they occur in the subject-positions of these sentences."[24] Restricted variables range over the extension of an open sentence, if the said extension is not an empty set, and take zero-entity (which is not an element of the universe of discourse) as their value otherwise. If Wx is an open sentence with a free variable x, then the restricted variable formed from it will be written as $x.Wx$. Now, the formalization of the above sentences will be as follows:

[23] Klima, 2001, p.200
[24] Klima, 2001, p.202

1') $(\exists x.Wx)\neg(H(x.Wx))$,

which can be read as "It doesn't hold of zero-entity that it is a horse," which is true. The formalization of the second sentence would be

2') $(\exists x.Wx)([\neg H](x.Wx))$,

which would say that it holds of the zero-entity that it is a non-horse. Since the extension of a term negation of a predicate parameter ('$\neg H$') are all those entities in the universe of discourse which are not horses, and the zero-entity is not in the universe of discourse, this is false.[25]

Therefore, one can address the problem raised at the beginning of this section by stating that it wrongly construes the negation of the sentence *Every winged horse is a horse* as (2) and (2'), and that the proper ways of understanding and formalizing it are those offered in (1) and (1'), in direct analogy with the example of the king of France.

This analysis demonstrates that it is possible to have meaningful (i.e. either true or false) categorical propositions even when they concern things that are outside of the natural world, while at the same time retaining all the entailments that Aristotelian logic requires. Therefore, the talk of entities that are not real in a sense that tables and chairs are need not commit us to a different brand of logic.

[25] Klima, 2001, p.202

2.3 Referential Import in LQPS

One notable difference from the predicate calculus becomes obvious if we consider that universal quantifiers have referential (but not existential) import in LQPS. This is not the case in the predicate calculus – consider once again the sentence that all men are mortal:

(1) $\forall x(Man(x) \to Mortal(x))$

Since it requires only that every assignment of values to variables satisfies the open formula $Man(x) \to Mortal(x)$, if there are no elements in the domain which satisfy the open formula *Man(x)* this conditional will turn out to be vacuously true, and therefore (1) will be true as well. On the other hand, the sentence that some men are mortal

(2) $\exists x(Man(x) \wedge Mortal(x))$

requires that there be an element of the domain that is a man (who is, naturally, mortal), it will be false when (1) is vacuously true. Therefore, (1) does not entail (2) in predicate calculus. So, the principle of Aristotelian logic that the superalterns entail their respective subaltern sentences is not retained in the predicate calculus.

LQPS, on the other hand, preserves these entailments due to the derivation rule of *Referential Import* (a formal specification will be provided in a later chapter). Let us consider their proofs:

1	(1)	*All Men are Mortal*	Premise
2	(2)	*a is a man*	Premise
3	(3)	*a is Mortal*	Premise

2,3 (4) *Some Men are Mortal* RI: 2,3

As the only premises which contain *a* that (4) relies on are (2) and (3), and (4) itself does not contain *a*, we can apply the rule of *Referential Import* and conclude:

1 (5) *Some Men are Mortal* RI: 1,2,3,4

So, the universal-affirmative sentence entails the particular-affirmative one with the same subject and predicate terms, as in the Aristotelian logic.

To demonstrate the same entailment for the negative alterns some additional terminology is required. Ben-Yami interprets the universal-negative statements as “Every *S* isn’t a *P*” instead of “No *S* is a *P*.” While this is slightly unnatural, predicate calculus faces the same problem – the formalization

(3) $\forall x(S(x) \rightarrow \neg P(x))$

would be read more naturally as “Every *S* is a *non-P*,” or “Every *S* isn’t a *P*.” If we therefore deem this acceptable, the proof would go through, *mutatis mutandis*, same as the one above. However, if we want to be more natural in our formalization, Ben-Yami offers the rules for null-quantifier “*No*” as well. The introduction rule mirrors *Universal Introduction,*

> “Suppose sentence (i) is the premise ‘a is an A’. Suppose further that sentence (j) contains a single appearance of ‘a’, and does not rely on any premise which contains a apart from (i). Suppose further that if we substitute‘a’ by ‘u A’, then that appearance of ‘u A’ governs sentence (j). Then in any following line (k) one can write the sentence identical to (j) apart from the fact that in it ‘u A’ has been substituted for ‘a’.

(k) relies on all the premises on which (j) relies, apart from (i). Its justification is written 'UI, j, i'",[26]

,with the extra requirement that the line (j) is of the form *(np_1,…,np_n) isn't P*, where np_i is a definite noun phrase. With this rule, the proof for negative alterns would proceed as follows:

1	(1)	*No Men are Mortal*	Premise
2	(2)	*a is a Man*	Premise
3	(3)	*a isn't Mortal*[27]	Premise
2,3	(4)	*Some Men aren't Mortal*	PI:2,3
1	(5)	*Some Men aren't Mortal*	RI:1,2,3,4

2.4 Final Remarks

The first part of this chapter has shown that there is a further way that the adoption of the predicate calculus is undermotivated – Aristotelina logic can cope with some of the problems *PC* is supposed to solve. In many ways, the solution Klima offers is similar to Ben-Yami's project – for example, he introduces the sentence and term negation (what Ben-Yami would call copula negation), and distinguishes between reference and existence. However, an advantage of LQPS is that it needs not resort to an unnatural zero-entity to maintain the Aristotelian entailments.

The observations in the third section of this chapter serve to show that *LQPS* differs from the predicate calculus, and is not, therefore, simply a different notational system. In fact, Ben-Yami's system stays true to a logical system with a longstanding tradition while at the same time it retains all the advantages, at least as the natural language is concerned, of the

[26] Ben-Yami, 2004, p.146
[27] This mirrors the sentence (i) from the introduction rule

predicate calculus. To demonstrate it stands on equal grounds with it, some metatheoretical properties of *LQPS* need to be demonstrated, but first we will move the debate to the twentieth century and examine some modern approaches to the topics of our discussion.

3. Subject-Predicate Distinction and Quantification

After considering the historical positions on the debate at hand in the previous chapter, we now turn our attention to the more contemporary contributions to the discussion. As a lead-in to the exploration of Ben-Yami's own contribution, we will compare and contrast it with two significant authors that tackled some of the issues relevant to our present discussion.

In *Logic and Natural Language,* Ben-Yami comments on the works of Peter Geach and Peter Strawson related to the issues of the subject-predicate distinction, plural reference and quantification. After in turn presenting the positions of each of the two authors in this chapter, I will provide Ben-Yami's comments on the segments he accepts and provide reasons for rejecting others. This will serve to place the discussion of LQPS into a wider framework of the contemporary philosophical debate.

3.1 Geach on Subject and Predicate

In *Reference and Generality,* Peter Geach uses the terms 'subject' and 'predicate' as purely linguistic terms – a man is not a subject of a sentence, but rather his name. For example, the name 'Peter' is the subject of a sentence 'Peter is an Apostle', and not the apostle himself.[28] Likewise, the predicate of this sentence is the "verbal expression"[29] 'Apostle', and not the property of being such. However, what the predicate is predicated of is the man, and not the name. Given these stipulations, Geach provides explanations of the terms 'subject' and 'predicate':

> "A *predicate* is an expression that gives us an assertion about something if we attach it to another expression that stands for what we are making the assertion about. A

[28] P. T, Geach, *Reference and Generality*, third edition, Cornell University Press, Ithaca and London, 1980,p.49
[29] Geach, 1980, p.49

> *subject* of a sentence S is an expression standing for something that S is about, S itself being formed by attaching a predicate to that expression."[30]

These explanations are provisory ones, however, and Geach discusses some amendments to them. Geach first points out that there is a distinction between these two explanations – we talk of a "subject of a sentence", but do not talk of predicates in the same way. The reason for this difference is that a predicate *can* be attached to an expression that stands for something, and not that it actually always is. For this reason Geach substitutes, in the above explanation, the term '*predicable'* for the term 'predicate'. So, a predicable can be attached to a name, whereas if it actually is, it is called a 'predicate'. Next in line is the clarification of the expression "making an assertion about." This he substitutes by a clearer expression "forming the proposition about," (the term 'proposition' being understood here as a merely linguistic entity).[31] Therefore, the final explanation of the terms in question comes out as.

> "A *predicable* is an expression that gives us a proposition about something if we attach it to another expression that stands for what we are forming the proposition about; the predicable then becomes a *predicate*, and the other expression becomes its *subject*; I call such a proposition a predication."[32]

Next, let us observe more closely the role of the subject. Geach holds that the names have a use – he calls it an *independent* use – in which they do "not require any immediate context of words, uttered or understood"[33]. In this use, they can simply acknowledge the existence of whatever they name, as in simply calling someone out. This goes against Frege and early

[30] Geach, 1980, p.50
[31] Geach, 1980, p.52
[32] Geach ,1980, p.52
[33] Geach, 1980, p.52

Wittgenstein who held that names can stand for something only within a proposition. Granting this way of using names, Geach notes that there is no difference between proper and common nouns – I might equally call someone out as either “Mary!” or “Madam!”

Geach defines a *bearer* of a name as the object that it names. Then, if we remove a name from a proposition, the remaining part, according to Geach, will be what the proposition states about the bearer of the name, i.e. its predicate. The names denote their bearers, and can stand on their own (in the act of naming) - they have a “complete sense.”[34] On the other hand, predicables apply to things (they are true of them), but do not have a complete sense. As the predicable (or, a predicate if we are discussing their use in a sentence) is what we obtain when we remove the name from a sentence, they are on their own essentially incomplete and contain an empty place that is to be filled by a subject. Therefore, Geach concludes, the subject and the predicate are essentially different. Consequently, Geach sees a situation where an expression is used interchangeably as a subject or as a predicate as an ambiguity within a language.[35] Furthermore, if we view the predicate as everything that remains in a sentence when the subject is removed, it follows, as Geach maintains, that identifying the distinctive role of a copula is superfluous – it is simply part of a predicable.

3.1.1 Ben-Yami on Geach

Geach holds, and Ben-Yami agrees, that in a sentence of the form ‘*F(qA)’*, in which ‘*F’* is a predicate, ‘*q’* is a quantifier and ‘*A’* is a substantival general term, ‘*A’* functions as a name. However, Geach further concludes, given his understanding of a subject and a predicate explained above, that “we should read ‘*F(every A)*’ and ‘*F(some A)*’ as got by attaching the different predicates ‘*F(every _)*’ and ‘*F(some _)*’ to ‘A’, not by attaching the predicable ‘*F( )*’

[34] Geach, 1980, p.57
[35] Geach, 1980, p.57

to two different quasi subjects '*every A*' and '*some A*', which refer to the things called '*A*' in two different ways."[36]

Ben-Yami concedes that the second option should be rejected, but sees little merit in the semantical analysis of the sentences of the form '*F(qA)*' in just a subject and a predicate part. Rather, he suggests that it should be viewed as having at least three parts – the expression '*A*' referring to particulars, the quantifier '*q*' stating how many of the *A*'s the referring expression refers to, and the predicate '*P*' predicating something of those particulars.[37]

Furthermore, whereas Geach sees no distinctive role of a copula, Ben-Yami believes it is essential in determining the mode of predication. Since predication in natural language has the form $(np_1,\ldots,np_n)P$ (where $np_1,\ldots,np_n$ are noun phrases and P is a predicate), and each of the noun phrases can be quantified, quantifiers are syntactically a part of the subject. Therefore, (unlike in the predicate calculus), a negation cannot appear between the quantifier and the predicate in the natural language. Consequently, to keep track of different modes of predication, the natural language needs to introduce two different copulas – is (are) and isn't (aren't).

Some further differences between Ben-Yami and Geach are that, whereas Geach sees the use of common nouns in both subject and predicate places, Ben-Yami's account of predication encounters no difficulty there and that Geach treats the variables of the predicate calculus as semantically equivalent to pronouns in the natural language.[38] Despite these differences, Ben-Yami states that their basic view on the semantic role of common nouns and quantifiers are substantially similar, and that therefore his analysis of quantification can be seen as an elaboration of Geach's.[39] There is, nonetheless, one last difference, which will be

[36] Geach, 1980, p.200
[37] Ben-Yami, 2004, p.72
[38] Ben-Yami, 2004, p.72
[39] Ben-Yami, 2004, p.72

of importance to this thesis, and that is that Ben-Yami develops a natural deduction on the basis of his analysis.

3.2 Strawson on Common Nouns and Predication

Another author that made a significant contribution to the subject-predicate debate is Peter Strawson. In his 1974 book *Subject and Predicate in Logic and Grammar*, he touches upon some of the topics Ben-Yami also covers. On the common nouns, while he maintains they are basically predicative, he does state that they have a secondary role he labels *substantiation* in the chapter titled *Substantiation and its Modes.*

The sentences Strawson is discussing prior to that chapter function such "that each specifies a type-of substance-involving situation or state of affairs and is apt for the expression of a proposition to the effect that such a situation or state of affairs obtains."[40] Within this broad function, several specific functions can be distinguished, one of which is "the function of identifying specification of individual substantial particulars."[41] At this point Strawson suggests a broadening of this function, but in such a way that would still maintain the overarching function of the sentence above. This broadening treats the particular-specifying function as a special case of a larger function he calls *substantiation* (with the particular-specification labeled *individually identifying substantiation*), in which no particular needs to be specified. So the sentence of the kind that Strawson now introduces is "a sentence apt for expressing a proposition to the effect that a situation of (…) a general type obtained, *without* specifying a particular…"[42]

The situation of a general type here needs further clarification. Let us therefore, consider some examples. Take particular specifying sentences S_1: '*John pursues Mary*' and

[40] P. F. Strawson, *Subject and Predicate in Logic and Grammar,* Methuen & Co. Ltd., London, 1974, p.99
[41] Strawson, 1974, p.99
[42] Strawson, 1974, p.101

S_2: '*Tom is drowning*'. Grasping of this sentences will include, Strawson states, grasping of something general at the same time, expressed by sentences S_3: '*A man is pursuing a woman*' and S_4: '*A cat is drowning*', respectively. The general type of the situation is what is expressed by S_3 and S_4 with regards to S_1 and S_2, and, as Strawson points out, there is no need to introduce a new function for those types of general situations to enter into the picture – one needs only to view the previous particular-specifying function as its special case.[43] An approach similar to what Strawson does here will be taken in the formalization of Ben-Yami's logical system, when the introduction of a plural subject terms into a language will be performed by substituting them for a singular one.

In contrast, Ben-Yami does not see the referential role of common nouns as derived or secondary to the predicative one, but treats it as their basic role instead. On the other hand, Strawson treats the referential role of common nouns as on par with the same role being served by adjectives and verbs, or to be more precise, the purely semantic categories of *nominals*, *adjectivals* and *verbals* corresponding to the three grammatical categories.[44] Ben-Yami, on the contrary, draws a distinction between the functioning of these categories. For example, the sentence with a common noun, '*This animal is an elephant*' does not, according to Ben-Yami, attribute any property of *this animal*, but rather states its kind. On the other hand, the sentence '*This animal is dangerous*', which contains an adjective, does attribute a property to the animal. Ben-Yami therefore distinguishes the functions of these expressions which Strawson treats as the same.[45]

[43] Strawson, 1974, p.101
[44] Strawson, 1974, p.103
[45] Ben-Yami, 2004, p.73

3.3 Strawson on Quantification

Strawson briefly touches upon quantification when he discusses as further expansion of the modes of substantiation, by way of pluralization. He identifies three ways this is usually done in languages – with bare pluralization, by various degrees of pluralization (e.g. few, some, several, many etc.) and by numerization (e.g. one, twice, etc.). What is specified or identified by pluralization is “some particular group or set.” These, he suggests, are pluralized analogues of individually identifying substantiation. [46]

However, as Ben-Yami points out as well, these considerations are limited and offer no indication whether, for example, he would treat the sentence ‘*Some horses are brown*’ as referring to some horses and predicating of them that they are brown, or as referring to horses and predicating that some of them are brown. In any case, as has been shown by Geach that the first option is not viable, Ben-Yami opts for the second.

3.4 Final Remarks

This chapter served to distinguish LQPS from other positions concerning the relation between the subject and the predicate, plural reference and quantification by giving an overview of those other positions. Naturally, the debate at hand is much more comprehensive than what is presented here, but it will suffice for the present purposes. One important note, which was mentioned at the end of the section 3.1.1 is that unlike the others, on the basis of his analysis of the topics of this chapter, Ben-Yami provides a natural deduction system. This is what we turn to in the following chapters.

[46] Strawson, 1974, p.112

4. The Metatheoretic Properties of LQPS – Soundness and Completeness

In this chapter we will focus on the formalization of Ben-Yami's system and the metatheoretical properties of it. In the first part of this chapter, we will present first the formal language LQPS, and then in the second part we will provide the rules of inference for its system of natural deduction. Next, in the third we will briefly sketch out the proof of soundness for LQPS, provided by Ben-Yami in *Logic and Natural Language*.

Finally, in the fourth part we turn our attention to the very center of this thesis, and that is the non-model-theoretic proof of completeness of LQPS. We start by explaining the motivation for such an approach – first by pointing out some deficiencies of non-model theoretic approach, and then by offering some positive reasons for laying out our proof without resorting to models. This is followed by a discussion of Universal Reduction - a principle that allows us to omit the universal quantifier from our considerations. The reason for doing so is familiar from the predicate calculus – the interdefinability of the universal and the existential quantifiers, namely that we can substituted '$\forall$' for '$\neg\exists\neg$.' This proof will not be examined in its entirety, but just in its most essential part. These initial remarks will set up the scene for the completeness proof itself – we will first present an alternative to the model theory – the substitutional approach – and then present the Henkin-style proof, adjusted for LQPS. The Henkin style proofs proceed as follows: we first extend our valid arguments by a set of axioms and then define an assignment of truth values (in place of a model – this is where the proof differs from the standard ones) that makes our extended valid arguments propositionally valid. As the completeness of propositional logic has been previously established, we then know that the extended arguments are propositionally provable. What remains to be done then is to show that if our extended arguments are propositionally provable, then the non-extended version is provable in LQPS. We do this in the closing sections of the chapter by proving the Elimination Theorem, which, in a nutshell, shows how we can do without each of the axioms we added.

Before proceeding a remark on notation is in order. For eligibility, all theorems, definitions and proofs are indented. The terms defined are in bold and italics, the names of theorems are in bold letters, and proofs (and sometimes their segments) are in italics. The symbols φ and ψ is used to signify any formula, $\varphi(st_i)$ is used to signify a formula which contains the subject term st_i and $\varphi(\ldots,\psi,\ldots)$is used to signify a formula φ which contains the formula ψ .

4.1 The Formal Language of LQPS

The language LQPS consists of sets of *subject terms* S, *predicates* P, truth functional connectives $\{\neg,\wedge,\vee,\rightarrow\}$, *quantifiers* {*some, all*}, and parentheses {(,)}. The set S contains *singular subject terms* (*SST*'s) $\{a,a_1,\ldots, a_n,b,b_1,\ldots,b_n \ldots\}$, *plural subject terms* (*PST*'s) of the form qA_i, where $q \in Q$, $A \in P$, i=1,…,n (and this index can be omitted), and *anaphoric expressions* $\{\alpha_1,\ldots,\alpha_n\}$. The set P contains n-place predicates {P,Q,R,…}, where n is the number of subject terms the predicate takes as arguments, indicated with parenthesis in front of the predicate, e.g. "$(st_1,\ldots,st_n)$P" (with the exception of a 2-place predicate of identity, '=', which uses an infix notation, e.g. "$st_1=st_2$").

4.1.1 Sentence formation

If P is an n-place predicate and $st_1,\ldots,st_n$ are (not necessarily different) SST's, then $(st_1,\ldots,st_n)$P is a sentence (where any of the SST's may or may not have an index number, but no different SST's can share the same one), called an *elementary sentence*. If A is a sentence, then the formula A*, obtained by substituting some or all of the occurrences of an SST with a

PST in A is also a sentence (bearing, once again, in mind that if the PST has an index, it differs from any other). If A is a sentence, then a formula A**, obtained by substituting some or all occurrences of an SST or a PST with an anaphoric expression α_i, such that in A** there exists to the left of a_i an occurrence of an SST or a PST which bears the same index *i* as the anaphoric expression, is also a sentence. Sentences which contain only predicates and subject terms are called *atomic sentences.*

If A and B are sentences, then $(\neg A)$, $(A \wedge B)$, $(A \vee B)$, and $(A \rightarrow B)$ are also sentences (the pair of outermost parentheses can be omitted). The sentences which contain truth-functional connectives are called *molecular sentences.*

4.1.2. Negative Predication

Another mode of predication in LQPS is *negative predication* – saying that something isn't such-and-such. The formation rule is as follows: if $(st_1,\ldots,st_n)$P is an elementary sentence, then $(st_1,\ldots,st_n)\neg$P is a sentence.

Finally, no expression that cannot be generated using these rules is a sentence.

4.1.3 Governing

Another notion we need to keep in mind in our language is that of governing:

> (***Governing***): In case pst_i is the left most plural subject term in sentence *C*, and *C* does not contain any sentence *C*' containing 'pst_i' and all the anaphors of any quantified noun phrase appearing in *C*', we shall say that pst_i governs *C*.[47]

[47] Ben-Yami, 2004, pg. 126

4.2 Rules of Inference of LQPS

The proof system that will be considered in this paper contains the propositional calculus (any one of several classical systems in use will do for our purposes) enriched by the rules of *Universal Elimination* (*UE*), *Universal Introduction* (*UI*), *Particular Introduction (PI), Referential Import (RI), Identity Introduction (=I), Identity Elimination (=E), Copula Negation to Sentence Negation (CNSN), Anaphora Introduction (AI)* and *Anaphora Elimination (AE)*. All the derivations are presented in a method similar to those of Lemmon and Newton-Smith.[48] Any standard natural deduction system for propositional logic will suffice, with an additional provision that in sentences of the forms $P \wedge Q$, $P \vee Q$ and $P \rightarrow Q$ to which the rules of a propositional calculus are being applied, all anaphoric expressions in *P* must be anaphoric on subject terms occurring within *P* (and likewise for *Q*). In other words, *P* and *Q* must be sentences for the rules of the propositional calculus to apply to them. We now proceed to enrich the natural deduction with the abovementioned rules.

4.2.1 Universal Elimination

> Suppose the *PST* '*all A*' governs sentence (i). Suppose further that sentence (j) is '*(a)A*'. Then, in any line (k), one can write the sentence identical to sentence (i) apart from the fact that in it '*a*' has been substituted for '*all A*'. Line (k) relies on the lines on which lines (i) and (j) rely. Its justification is written 'UE: i, j'.[49]

Schematically, the general form of this rule is:

Γ	(i)	$\varphi\,(all\ A)$	*

[48] Ben-Yami, 2004, pg. 138
[49] Ben-Yami, 2004, pg. 143

Δ	(j)	*(a) A*	
$\Gamma \cup \Delta$	(k)	$\varphi\,(a)$	UE: i, j

*where *all A* governs $\varphi\,(all\ A)$

4.2.2 Universal Introduction

Suppose sentence (i) is the premise '*(a)A*'. Suppose further that sentence (j), which contains '*a'*, does not rely on any premise which contains '*a*' apart from (i). Suppose further that if we substitute '*a*' by a *PST* '*all A'* in (j), then that appearance of '*all A*' governs sentence (j). Then in any following line (k) one can write the sentence identical to (j) apart from the fact that in it '*all A*'has been substituted for '*a*'. (k) relies on all the premises on which (j) relies, apart from (i). Its justification is written 'UI: j, i'.[50]

It should be noted that if there are multiple occurrences of an *SST* in (j), one occurrence is substituted by the *PST*, while others are substituted by the appropriate anaphoric expressions, similar to the natural tendencies in a language. This familiar procedure appears frequently in translating predicate calculus sentences. For example, from 'John is a man' and 'John loves John' we derive (given that all other requirements are satisfied) 'Every man loves himself', rather than 'Every man loves every man' (which would have a different meaning).

The general scheme for this rule is as follows:

i	(i)	*(a)A*	Premise
Γ	(j)	$\varphi\,(a)$	*

[50]Ben-Yami, 2004, pg. 144

Γ -{i}	(k)	φ *(all A)*	UE: j,i

*does not rely on any premise which contains *a* other than (i)

4.2.3 Particular Introduction

Suppose sentence (i) contains an *SST* ‘*a*’, and that if we substitute a *PST* ‘*some A*’ for ‘*a*’ then this appearance of ‘*some A*’ governs sentence (i). Suppose further that sentence (j) is ‘*(a)A’*. Then in any subsequent line (k) one can write the sentence identical to (i) apart from the fact that in it ‘*some A*’ has been substituted for ‘*a*’. Line (k) relies on the lines on which lines (i) and (j) rely. Its justification is written ‘PI: i, j’.[51]

The general scheme for this rule is:

Γ	(i)	φ *(a)*	
Δ	(j)	*(a) A*	
$\Gamma \cup \Delta$	(k)	φ *(some A)*	PI: i,j

4.2.4 Referential Import

Suppose sentence (i), which does not rely on sentences (j) or (k) and does not contain ‘*a*’, contains a *PST* ‘*qA*’, where ‘*q*’ is either the particular or the universal quantifier, which governs it. Suppose further that sentence (j) is the premise ‘*(a) A’*, and sentence (k) the premise which is identical to sentence (i) apart for the fact that ‘*a*’ has been substituted for ‘*qA*’. Now suppose that sentence (l) does not contain ‘*a*’, and does not rely on any sentence which contains *a* apart from (j) and (k). Then in line (m) sentence

[51] Ben-Yami, 2004, p.148

(l) can be rewritten, relying on whatever sentences that sentences (i) and (l) rely on, apart from (j) and (k). Its justification is written 'RI: i, j, k, l'.

This rule schematically looks as follows:

Γ	(i)	$\varphi\,(qA)$	*
j	(j)	$(a)A$	Premise
k	(k)	$\varphi\,(a)$	Premise
		. . .	
Δ	(l)	ψ	**
$\Gamma \cup \Delta$ - {j,k}	(m)	ψ	RI: i,j,k,l

* where $j \notin \Gamma$, $k \notin \Gamma$, (i) does not contain '*a*', and '*qA*' governs (i)

** where no member of Δ-{j,k} contains '*a*', and (l) does not contain '*a*'

As Ben-Yami points out, "*Referential Import* relies on the fact that the use of '*qA*' presupposes reference to A's (this is the source of its name), namely that some sentence of the form 'c is A' is true."[52] This does not, however, mean that we presuppose existence – in using the common nouns we assume they refer to something, and not that those things we refer to actually exist (we can, for example, talk about the characters from the Bible without agreeing which of them, if any, are real people).

[52] Ben-Yami, 2004, p.150, italics added

4.2.5 Identity Introduction

In any line (i) any sentence of the form ‘a = a’, where ‘a’ is a singular subject term, can be written, not relying on any line. Its justification is written ‘=I’. [53]

The general form of this rule is:

(i)	$a = a$	=I

4.2.6 Identity Elimination

Suppose that sentence (i) is ‘a = b’, where ‘a’ and ‘b’ are singular subject terms, and that ‘a’ appears in sentence (j) too. Then in any line (k) one can write the sentence identical to sentence (j) apart from the fact that in it ‘a’ has been substituted by ‘b’ in some or all of its appearances. Line (k) relies on the lines on which lines (i) and (j) rely. Its justification is written ‘=E: i, j’.[54]

Formally, this rule looks as follows:

Γ	(i)	$a = b$	
Δ	(j)	$\varphi(a)$	
$\Gamma \cup \Delta$	(k)	$\varphi(b)$	=E: i,j

[53] Ben-Yami, 2004, p.178
[54] Ben-Yami, 2004, p.178

4.2.7 Copula Negation to Sentence Negation

This rule covers two possibilities – proceeding from a sentence negation to negative predication, or vice versa. Let us consider them in turn, starting with sentence negation to negative predication:

> If sentence (i) is or contains the sentence '$\neg(st_1,\ldots,st_n)$ P', where every 'st_i' is an SST, then in any following line (j) the sentence identical to sentence (i), but with '$(st_1,\ldots,st_n)\neg$P' substituted for '$\neg(st_1, \ldots st_n)$ P', can be written. Sentence (j) relies on the same premises as sentence (i), and its justification is written 'CNSN: i'.[55]

Schematically, this rule takes on the following form:

Γ	(i)	$\varphi(\ldots,\neg(st_1,\ldots,st_n)\ P,\ldots)$	
Γ	(j)	$\varphi(\ldots,(st_1,\ldots,st_n)\neg P,\ldots)$	CNSN: i

Next, we observe the other possible direction of the rule – from negative predication to sentence negation:

> If sentence (i) is or contains the sentence '$(st_1,\ldots,st_n)\neg$P', where every 'st_i' is an SST, then in any following line (j) the sentence identical to sentence (i), but with '$\neg(st_1,\ldots,st_n)$ P' substituted for '$(st_1,\ldots,st_n)\neg$P', can be written. Sentence (j) relies on the same premises as sentence (i), and its justification is written 'CNSN: i'.[56]

The general form of this rule is as follows:

[55] Ben-Yami, 2004, p.143, text adjusted to present notation
[56] Ben-Yami, 2004, p.143, text adjusted to present notation

Γ (i) $\varphi(\ldots,(st_1,\ldots,st_n)\neg P,\ldots)$

Γ (j) $\varphi(\ldots,\neg(st_1,\ldots,st_n)\ P,\ldots)$ CNSN: i

4.2.8 Anaphora Introduction

We now turn our attention to the rules for anaphora. First, Anaphora Introduction:

> Suppose the sentence (i) contains multiple appearances of an SST 'st_i'. Then, in any subsequent line (j) we can write a sentence identical to (i) except that some or all of the appearances of 'st_i', other than the leftmost one, have been substituted by an anaphoric expression 'α_n', where n is the same index that 'st_i' has, or a new index assigned to all the appearances of both the 'st_i' and 'α_n'. The sentence (j) relies on whatever premises the sentence (i) relies on, and its justification is written AI: i.

This rule schematically looks as follows:

Γ (i) $\varphi(st_i)$

Γ (j) $\varphi(st_{in},\ldots,\alpha_n,)$ AI: i

4.2.9 Anaphora Elimination

The last rule we will introduce is Anaphora Elimination:

> Suppose the sentence (i) contains one or more occurrences of an SST st_{in} and one or more occurrences of an anaphoric expression α_n, both with the same index n. Then, in any subsequent line (j) we can write a sentence identical to (i), except that some or all

of the occurrences of α_n have been substituted by st_{in} (if no anaphoric expressions remain, the index can be omitted). The line (j) relies on all the premises that the line (i) relies on, and its justification is written AE: i.

The general form of this rule is:

Γ	(i)	$\varphi (st_{in},\ldots, \alpha_n)$	
Γ	(j)	$\varphi (st_i)$	AI: i

This concludes the presentation of the rules of LQPS. Bear in mind that, in addition to these rules, the natural deduction system of LQPS also contains the rules of derivation of the propositional calculus.

Before proceeding to prove that this system is complete, in the next section we will first offer a few considerations which demonstrate that these rules preserve validity, i.e. that the system is sound.

4.3 The Soundness of LQPS

First of the desirable metatheoretical properties, that of soundness (that all the deductions preserve validity), is demonstrated in Ben-Yami's book. The proof is by mathematical induction – first, a one-line argument is valid, since its only step is a premise. Next, Ben-Yami demonstrates that every rule of derivation will likewise preserve validity. Let us consider just the examples of *Referential Import*, since it is the only one that differs from the standard rules for quantifiers:

> "Suppose the sentences on which sentence (m) relies are true. In that case, all sentences on which sentence (i) relies are true, and sentence (i) is true too. If sentences (j) and (k) were true, then all the sentences on which sentence (l) relies were true, and it were true too. Since sentence (i) is true, then according to the substitution rule given above (§ 8.6, p. 126), there is a 'c' so that 'c is an A' is true, and if we substitute the governing appearance of 'q A' by 'c' in (i), we get a true sentence. Let us substitute 'a' by 'c' in our argument. Since all derivation rules rely only on sameness of definite singular noun phrases and not on the specific definite noun phrase used, the argument up to line (m-1) remains valid. Now since 'c is an A' is true, premise (j) is now true. Moreover, since 'a' did not appear in (i), (i) remained unchanged after the substitution, and it is still true. But (k) is now the result of substituting 'c' for the governing appearance of 'q A' in (i), and is therefore true. And since (j) and (k) are the only premises containing 'a' on which sentence (l) relies, all other premises on which (l) relies remain true after the substitution. So sentence (l) relies only on true premises, and so sentence (l), that is, sentence (m), is true, and RI preserves validity."[57]

Ben-Yami provides a corresponding proof for all of the remaining rules in his system and thus demonstrates that it preserves validity, and is therefore sound. We next turn our attention to the central topic of my thesis, the completeness proof for *LQPS*.

4.4 The Completeness of LQPS

While Ben-Yami does not provide a completeness proof in his book, it has nonetheless been demonstrated in the 2004 article by Lanzet and Ben-Yami. However, as was noted earlier,

[57] Ben-Yami, 2004, p.151

that proof is model theoretic, and there are certain reasons to be dissatisfied with such proofs in general. Let us consider those in order to motivate an alternative approach.

A logical proof system has the metatheoretical property of completeness just in case every valid argument is provable. Validity represents a certain relation of the truth values of premises to that of the conclusion (if the premises are all true, so is the conclusion), while provability concerns merely the ways of deriving some sentences from others according to a set of rules. Since provability is not concerned with the truth values of sentences, a theory of truth is needed to connect validity and provability. This is the role model theory plays in supplying a completeness proof for a given system.

4.4.1 Objections to Model Theory[58]

The model theory is not without its problems, however. The first concerns the model theory as a theory of meaning. Since it is merely concerned with form of sentences, it will regard widely different sets of predicates as uniform, and treat their connections to a vast array of particulars in the same manner. For instance, model theory makes no distinction between persons, their character traits, and events. It is hard to grasp any sense of a "domain" in which it could straightforwardly be applied to all these and many other kinds of things that normally fall under it.

An additional argument for restraint in use of a model theory in a completeness proof is purely logical. Since validity is defined as a conditional relation between the premises and the conclusion of an argument, and completeness is a conditional holding between validity and provability, it would suffice to provide a theory of the *relations* of truth values. Model theory goes one step further, however, in developing a theory of truth. It would be a desirable result if a completeness proof could be provided without needlessly resorting to that stronger

[58] H. Ben-Yami, *Tuth and Proof without Models*, manuscript, p.1

commitment. It is my goal to provide such a proof in my thesis. To do so, a substitutional approach, first advocated by Ruth Barcan Marcus[59], will be used instead – "Instead of the Tarskian rules of Model Theory, which by relying on interpretations in a domain of the non-logical symbols of a language specify the truth value of its sentences, including quantified sentences (objectual semantics), we give rules that relate the truth value of a quantified sentence to those of its substitution instances."[60] As an illustration, a substitutional account would handle the relation of truth values of quantified sentences to those of their instances via the following rules:

> *Universal rule*: the sentence of the form 'φ(*all A*)', where '*all A*' governs the sentence, is true just in case so are all the instances of substitution, in that sentence, of '*all A'* by an '*a'*, for which "(*a) A"* is true.

> *Particular rule*: the sentence of the form 'φ(*some A*)', where '*some A*' governs the sentence, is true just in case so is an instance of substitution, in that sentence, of '*some A'* by an '*a'*, for which "(*a*)A" is true.

4.4.2 Universal Reduction

In the completeness proof of this thesis, I will be concerned only with the particular quantifier. The justification for this is familiar from the predicate calculus – the universal quantifier $\forall$ can be defined, and substituted in sentences, by $\neg\exists\neg$. Therefore, for any formula of the form $\forall x\varphi(x)$, there is an equivalent formula of the form $\neg\exists x\neg\varphi(x)$.

[59] Ben-Yami, manuscript, p.4
[60] Ben-Yami, manuscript, p.3

Consequently, in standard Henkin-style completeness proofs, the universal quantifier is, roughly speaking, reduced to the existential one, and only the latter is tackled.

For similar reasons I will accept the principle of *Universal Reduction*:

(**Universal Reduction**): For any sentence of LQPS φ which contains any number of universal quantifiers, there is an equivalent sentence φ' which contains no universal quantifiers.

Like above, the sentence φ' will likely contain a number of particular quantifiers combined with negations. However, the proof of Universal Reduction is not a straightforward matter and is more complicated (as will be obvious from the remainder of this section) than the proof of the corresponding principle for the Predicate Calculus. In particular, the introduction of a new mode of predication, namely the negative predication, seems to complicate matters.

For this reason I will leave the proof of Universal Reduction for further research, and will use it here as an assumption. However, in order to demonstrate the plausibility of this assumption, I will demonstrate in this section a limited case of Universal Reduction for sentences which contain the universal quantifiers, are governed by a PST, and are of complexity not greater than 2. Let us define complexity first:

(***Complexity***): complexity of a sentence A ('*comp(A)'*) is 0 if A is an elementary sentence; if A$\equiv\neg\varphi$ then comp(A)=comp(φ)+1, and if A$\equiv(\varphi\wedge\psi)$, A$\equiv(\varphi\vee\psi)$ or A$\equiv(\varphi\rightarrow\psi)$, the comp(A)=max(comp($\varphi$),comp($\psi$))+1, where the maximum function ('*max*') picks out the greater of the two numbers. Furthermore, if '*q*' is a quantifier, '*a*' is an SST, '$\varphi(qA)$' is a sentence governed by the PST '*qA*' and $\varphi(a)$ is a sentence obtained by substituting '*a*' for the PST in '$\varphi(qA)$', then comp($\varphi(qA)$)=comp($\varphi(a)$)+1. Lastly, substituting one SST by another does not change the complexity of a sentence (the proof of this, which will not be supplied here, is inductive).

From the definition of complexity above we can see that the set of sentences we are concerned with, which contain the universal quantifier and have complexity no greater than 2, will consist of the sentences of the following forms:

1) *(All S)P*

2) *(All S)*$\neg$*P*

3) *(All* S_1*)P*$\wedge(\alpha_1)$*Q*

4) *((All* S_1*)P*$\vee(\alpha_1)$*Q)*

5) *((All* S_1*)P*$\rightarrow(\alpha_1)$*Q)*

6) *(All S,Some P)R*

7) *(Some S,All P)R*

8) *(All S,All P)R*

I will next provide a proof for each of these types sentences, which will demonstrate that each of them is equivalent to some sentence which contains no universal quantifiers. The convolutedness of some of the equivalent sentences need not concerns us – these are just a tool for simplification of the completeness proof itself, and I do not aim to produce natural-sounding sentences.

Each of the equivalences will be presented as a pair of proofs, the first demonstrating that the equivalence holds in the direction from left to right, and the second proving that it holds from right to left.

Proof 1: (All S)P $\Leftrightarrow \neg$ *(Some S)* $\neg$ *P*

Proof 1.1: (All S)P $\Rightarrow \neg$ *(Some S)* $\neg$ *P*

1	(1)	*(All S)P*	Premise
2	(2)	*(Some S)* $\neg$ *P*	Premise
3	(3)	*(c)S*	Premise
4	(4)	*(c)* $\neg$ *P*	Premise
1,3	(5)	*(c)P*	UE:1,3
4	(6)	$\neg$ *(c)P*	CNSN:4
1,3,4	(7)	$\bot$	$\bot$ I:5,6
1,2	(8)	$\bot$	RI:2,3,4,7
1	(9)	$\neg$ *(Some S)* $\neg$ *P*	$\neg$ I:2,8

Proof 1.2: $\neg$ *(Some S)* $\neg$ *P* $\Rightarrow$ *(All S)P*

1	(1)	$\neg$ *(Some S)* $\neg$ *P*	Premise
2	(2)	*(c)S*	Premise
3	(3)	$\neg$ *(c)P*	Premise
3	(4)	*(c)* $\neg$ *P*	CNSN:3
2,3	(5)	*(Some S)* $\neg$ *P*	PI:2,4
1,2,3	(6)	$\bot$	$\bot$ I:1,5
1,2	(7)	$\neg\neg$ *(c)P*	$\neg$ I:3,6
1,2	(8)	*(c)P*	$\neg$ E:7
1	(9)	*(All S)P*	UI:2,8

Proof 2: (All S) $\neg$ *P* $\Leftrightarrow \neg$ *(Some S) P*

Proof 2.1: (All S) $\neg$ *P* $\Rightarrow \neg$ *(Some S) P*

1	(1)	*(All S)* $\neg$ *P*	Premise
2	(2)	*(Some S)P*	Premise
3	(3)	*(c)S*	Premise
4	(4)	*(c)P*	Premise
1,3	(5)	*(c)* $\neg$ *P*	UE:1,3

1,3	(6)	$\neg(c)P$	CNSN:5
1,3,4	(7)	$\bot$	$\bot$I:5,6
1,2	(8)	$\bot$	RI:2,3,4,7
1	(9)	$\neg(Some\ S)\ P$	$\neg$I:2,8

Proof 2.2: $\neg(Some\ S)\ P \Rightarrow (All\ S)\neg P$

1	(1)	$\neg(Some\ S)\ P$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$\neg(c)\neg P$	Premise
3	(4)	$\neg\neg(c)\ P$	CNSN:3
3	(5)	$(c)P$	$\neg$E:4
2,3	(6)	$(Some\ S)\ P$	PI:2,5
1,2,3	(7)	$\bot$	$\bot$I:1,6
1,2	(8)	$\neg\neg(c)\neg P$	$\neg$I:3,7
1,2	(9)	$(c)\ \neg P$	$\neg$E:8
1	(10)	$(All\ S)\neg P$	UI:2,9

Proof 3: $((All\ S_1)P \wedge (\alpha_1)Q) \Leftrightarrow (\neg((Some\ S_1)\neg P \vee (\alpha_1)\neg Q))$

Proof 3.1: $((All\ S_1)P \wedge (\alpha_1)Q) \Rightarrow (\neg((Some\ S_1)\neg P \vee (\alpha_1)\neg Q))$

1	(1)	$(All\ S_1)P \wedge (\alpha_1)Q$	Premise
2	(2)	$(Some\ S_1)\neg P \vee (\alpha_1)\neg Q$	Premise
3	(3)	$(c)S$	Premise
4	(4)	$(c_1)\neg P \vee (\alpha_1)\neg Q$	Premise
4	(5)	$(c)\neg P \vee (c)\neg Q$	AE:4
1,3	(6)	$(c_1)P \wedge (\alpha_1)Q$	UE:1,3
1,3	(7)	$(c)P \wedge (c)Q$	AE:6
8	(8)	$(c)\neg P$	Premise
8	(9)	$\neg(c)P$	CNSN:8
1,3	(10)	$(c)P$	$\wedge$E:7
1,3,8	(11)	$\bot$	$\bot$I:9,10
12	(12)	$(c)\neg Q$	Premise
12	(13)	$\neg(c)Q$	CNSN:12

1,3	(14)	$(c)Q$	$\wedge$E:7
1,3,12	(15)	$\bot$	$\bot$I:13,14
1,3,4	(16)	$\bot$	$\vee$E:5,8,11,12,15
1,2	(17)	$\bot$	RI:2,3,4,16
1	(18)	$\neg((Some\ S_1)\neg P \vee (\alpha_1)\neg Q)$	$\neg$I:2,17

Proof 3.2: $(\neg((Some\ S_1)\neg P \vee (\alpha_1)\neg Q)) \Rightarrow ((All\ S_1)P \wedge (\alpha_1)Q)$

1	(1)	$\neg((Some\ S_1)\neg P \vee (\alpha_1)\neg Q)$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$\neg((c_1)P \wedge (\alpha_1)Q)$	Premise
3	(4)	$\neg((c)P \wedge (c)Q)$	AE:3
3	(5)	$\neg(c)P \vee \neg(c)Q$	Prop. Calc.:4
6	(6)	$\neg(c)P$	Premise
6	(7)	$(c)\neg P$	CNSN:6
6	(8)	$(c)\neg P \vee (c)\neg Q$	$\vee$I:7
9	(9)	$\neg(c)Q$	Premise
9	(10)	$(c)\neg Q$	CNSN:9
9	(11)	$(c)\neg P \vee (c)\neg Q$	$\vee$I:10
3	(12)	$(c)\neg P \vee (c)\neg Q$	$\vee$E:5,6,8,9,11
3	(13)	$(c_1)\neg P \vee (\alpha_1)\neg Q$	AI:12
2,3	(14)	$(Some\ S_1)\neg P \vee (\alpha_1)\neg Q$	PI:2,13
1,2,3	(15)	$\bot$	$\bot$I:1,14
1,2	(16)	$\neg\neg((c_1)P \wedge (\alpha_1)Q)$	$\neg$I:3,15
1,2	(17)	$(c_1)P \wedge (\alpha_1)Q$	$\neg$E:16
1	(18)	$(All\ S_1)P \wedge (\alpha_1)Q$	UI:2,17

Proof 4: $((All\ S_1)P \vee (\alpha_1)Q) \Leftrightarrow (\neg((Some\ S_1)\neg P \wedge (\alpha_1)\neg Q))$

Proof 4.1: $((All\ S_1)P \vee (\alpha_1)Q) \Rightarrow (\neg((Some\ S_1)\neg P \wedge (\alpha_1)\neg Q))$

1	(1)	$(All\ S_1)P \vee (\alpha_1)Q$	Premise
2	(2)	$(Some\ S_1)\neg P \wedge (\alpha_1)\neg Q$	Premise
3	(3)	$(c)S$	Premise

4	(4)	$(c_1)\neg P \wedge (\alpha_1)\neg Q$	Premise
4	(5)	$(c)\neg P \wedge (c)\neg Q$	AE:4
1,3	(6)	$(c_1)P \vee (\alpha_1)Q$	UE:1,3
1,3	(7)	$(c)P \vee (c)Q$	AE:6
8	(8)	$(c)P$	Premise
4	(9)	$(c)\neg P$	$\wedge$E:5
4	(10)	$\neg(c) P$	CNSN:9
4,8	(11)	$\bot$	$\bot$I:8,10
12	(12)	$(c)Q$	Premise
4	(13)	$(c)\neg Q$	$\wedge$E:5
4	(14)	$\neg(c) Q$	CNSN:13
4,12	(15)	$\bot$	$\bot$I:12,14
1,3,4	(16)	$\bot$	$\vee$E:7,8,11,12,15
1,2	(17)	$\bot$	RI:2,3,4,16
1	(18)	$\neg((\text{Some } S_1)\neg P \wedge (\alpha_1)\neg Q)$	$\neg$I:2,17

Proof 4.2: $(\neg((\text{Some } S_1)\neg P \wedge (\alpha_1)\neg Q)) \Rightarrow ((\text{All } S_1)P \vee (\alpha_1)Q)$

1	(1)	$\neg((\text{Some } S_1)\neg P \wedge (\alpha_1)\neg Q)$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$\neg((c_1)P \vee (\alpha_1)Q)$	Premise
3	(4)	$\neg((c)P \vee (c)Q)$	AE:3
3	(5)	$\neg(c)P \wedge \neg(c)Q$	Prop. Calc.:4
3	(6)	$\neg(c)P$	$\wedge$E:5
3	(7)	$(c)\neg P$	CNSN:6
3	(8)	$\neg(c)Q$	$\wedge$E:5
3	(9)	$(c)\neg Q$	CNSN:8
3	(10)	$(c)\neg P \wedge (c)\neg Q$	$\wedge$I:7,9
3	(11)	$(c_1)\neg P \wedge (\alpha_1)\neg Q$	AI:10
2,3	(12)	$(\text{Some } S_1)\neg P \wedge (\alpha_1)\neg Q$	PI:2,11
1,2,3	(13)	$\bot$	$\bot$I:1,12
1,2	(14)	$\neg\neg((c_1)P \vee (\alpha_1)Q)$	$\neg$I:3,13
1,2	(15)	$(c_1)P \vee (\alpha_1)Q$	$\neg$E:14

1	(16)	$(All\ S_1)P \vee (\alpha_1)Q$	UI:2,16

Note that in the previous two proofs (namely, 3.2 and 4.2) a shortcut has been taken to avoid the proofs being excessively long. As this is a well known equivalence that relies merely on the propositional calculus, this shortcut does not affect the quantificational part in any way.

Proof 5: $((All\ S_1)P \rightarrow (\alpha_1)Q) \Leftrightarrow (\neg((Some\ S_1)P \wedge (\alpha_1) \neg Q))$

Proof 5.1: $((All\ S_1)P \rightarrow (\alpha_1)Q) \Rightarrow (\neg((Some\ S_1)P \wedge (\alpha_1) \neg Q))$

1	(1)	$(All\ S_1)P \rightarrow (\alpha_1)Q$	Premise
2	(2)	$(Some\ S_1)P \wedge (\alpha_1) \neg Q$	Premise
3	(3)	$(c)S$	Premise
4	(4)	$(c_1)P \wedge (\alpha_1) \neg Q$	Premise
4	(5)	$(c)P \wedge (c) \neg Q$	AE:4
1,3	(6)	$(c_1)P \rightarrow (\alpha_1)Q$	UE:1,3
1,3	(7)	$(c)P \rightarrow (c)Q$	AE:6
4	(8)	$(c)P$	$\wedge$E:5
1,3,4	(9)	$(c)Q$	$\rightarrow$E:7,8
4	(10)	$(c) \neg Q$	$\wedge$E:5
4	(11)	$\neg (c)\ Q$	CNSN:10
1,3,4	(12)	$\perp$	$\perp$I:9,11
1,2	(13)	$\perp$	RI:2,3,4,12
1	(14)	$\neg((Some\ S_1)P \wedge (\alpha_1) \neg Q)$	$\neg$I:2,13

Proof 5.2: $(\neg((Some\ S_1)P \wedge (\alpha_1) \neg Q)) \Rightarrow ((All\ S_1)P \rightarrow (\alpha_1)Q)$

1	(1)	$\neg((Some\ S_1)P \wedge (\alpha_1) \neg Q)$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$\neg((c_1)P \rightarrow (\alpha_1)Q)$	Premise
3	(4)	$\neg((c)P \rightarrow (c)Q)$	AE:3
5	(5)	$\neg (c)P$	Premise
6	(6)	$(c)P$	Premise

5,6	(7)	$\bot$	$\bot$I:5,6
5,6	(8)	*(c)Q*	$\bot$E:7
5	(9)	*(c)P*$\rightarrow$*(c)Q*	$\rightarrow$I:6,8
3,5	(10)	$\bot$	$\bot$I:4,9
3	(11)	$\neg\neg$*(c)P*	$\neg$I:5,10
3	(12)	*(c)P*	$\neg$E:11
13	(13)	*(c)Q*	Premise
14	(14)	*(c)P*	Premise
13,14	(15)	*(c)P*$\wedge$*(c)Q*	$\wedge$I:13,14
13,14	(16)	*(c)Q*	$\wedge$E:15
13	(17)	*(c)P*$\rightarrow$*(c)Q*	$\rightarrow$I:14,16
3,13	(18)	$\bot$	$\bot$I:4,17
3	(19)	$\neg$*(c)Q*	$\neg$I:13,18
3	(20)	*(c)*$\neg$*Q*	CNSN:19
3	(21)	*(c)P*$\wedge$ *(c)* $\neg$*Q*	$\wedge$I:12,20
3	(22)	$(c_1)P\wedge(\alpha_1)\neg Q$	AI:21
2,3	(23)	$(Some\ S_1)P\wedge(\alpha_1)\neg Q$	PI:2,22
1,2,3	(24)	$\bot$	$\bot$I:1,23
1,2	(25)	$\neg\neg((c_1)P\rightarrow(\alpha_1)Q)$	$\neg$I:3,24
1,2	(26)	$(c_1)P\rightarrow(\alpha_1)Q$	$\neg$E:25
1	(27)	$(All\ S_1)P\rightarrow(\alpha_1)Q$	UI:2,26

Proof 6: $(All\ S,Some\ P)R\Leftrightarrow\neg((Some\ S_1)S\wedge\neg(\alpha_1,Some\ P)R)$

Proof 6.1: $(All\ S,Some\ P)R\Rightarrow\neg((Some\ S_1)S\wedge\neg(\alpha_1,Some\ P)R)$

1	(1)	*(All S,Some P)R*	Premise
2	(2)	$(Some\ S_1)S\wedge\neg(\alpha_1,Some\ P)R$	Premise
3	(3)	*(c)S*	Premise
4	(4)	$(c_1)S\wedge\neg(\alpha_1,Some\ P)R$	Premise
4	(5)	*(c)S*$\wedge\neg$*(c,Some P)R*	AE:4
1,3	(6)	*(c,Some P)R*	UE:1,3
4	(7)	$\neg$*(c,Some P)R*	$\wedge$E:5
1,3,4	(8)	$\bot$	$\bot$I:6,7

1,2	(9)	$\bot$	RI:2,3,4,8
1	(10)	$\neg((Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)R)$	$\neg$I:2,9

Proof 6.2: $\neg((Some\ S1)S \wedge \neg(\alpha\ 1, Some\ P)R) \Rightarrow (All\ S, Some\ P)R$

1	(1)	$\neg((Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)R)$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$\neg(c, Some\ P)R$	Premise
2,3	(4)	$(c)S \wedge \neg(c, Some\ P)R$	$\wedge$I:2,3
2,3	(5)	$(c_1)S \wedge \neg(\alpha_1, Some\ P)R$	AI:4
2,3	(6)	$(Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)R$	PI:2,5
1,2,3	(7)	$\bot$	$\bot$I:1,6
1,2	(8)	$\neg\neg(c, Some\ P)R$	$\neg$I:3,7
1,2	(9)	$(c, Some\ P)R$	$\neg$E:8
1	(10)	$(All\ S, Some\ P)R$	UI:2,9

Proof 7: $(Some\ S, All\ P)R \Leftrightarrow ((Some\ S1)S \wedge \neg(\alpha\ 1, Some\ P)\neg R$

Proof 7.1: $(Some\ S, All\ P)R \Rightarrow ((Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)\neg R$

1	(1)	$(Some\ S, All\ P)R$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$(c, All\ P)R$	Premise
4	(4)	$(c, Some\ P)\neg R$	Premise
5	(5)	$(d)P$	Premise
6	(6)	$(c,d)\neg R$	Premise
6	(7)	$\neg(c,d)\ R$	CNSN:6
3,5	(8)	$(c,d)R$	UE:3,5
3,5,6	(9)	$\bot$	$\bot$I:7,8
3,4	(10)	$\bot$	RI:4,5,6,9
3	(11)	$\neg(c, Some\ P)\neg R$	$\neg$I:4,10
2,3	(12)	$(c)S \wedge \neg(c, Some\ P)\neg R$	$\wedge$I:2,11
2,3	(13)	$(c_1)S \wedge \neg(\alpha_1, Some\ P)\neg R$	AI:12
2,3	(14)	$(Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)\neg R$	PI:2,13

1	(15)	$(Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)\neg R$	RI:1,2,3,14

Proof 7.2: $((Some\ S1)S \wedge \neg(\alpha\ 1, Some\ P)\neg R) \Rightarrow (Some\ S, All\ P)R$

1	(1)	$(Some\ S_1)S \wedge \neg(\alpha_1, Some\ P)\neg R$	Premise
2	(2)	$(c)S$	Premise
3	(3)	$(c_1)S \wedge \neg(\alpha_1, Some\ P)\neg R$	Premise
3	(4)	$(c)S \wedge \neg(c, Some\ P)\neg R$	AE:3
5	(5)	$(d)P$	Premise
6	(6)	$\neg(c,d)R$	Premise
6	(7)	$(c,d)\neg R$	CNSN:6
5,6	(8)	$(c, Some\ P)\neg R$	PI:5,7
3	(9)	$\neg(c, Some\ P)\neg R$	$\wedge$E:4
3,5,6	(10)	$\perp$	$\perp$I:8,9
3,5	(11)	$\neg\neg(c,d)R$	$\neg$I:6,10
3,5	(12)	$(c,d)R$	$\neg$E:11
3	(13)	$(c, All\ P)R$	UI:5,12
2,3	(14)	$(Some\ S, All\ P)R$	PI:2,13
1	(15)	$(Some\ S, All\ P)R$	RI:1,2,3,14

Note that the previous two proofs contain unnatural formulations "*Some S is S*." As stated earlier, these sentences are just an auxiliary step in establishing the completeness of LQPS. We can freely use the more natural sentences containing the universal quantifiers when dealing with actual proofs in the system.

Proof 8: $(All\ S, All\ P)R \Leftrightarrow \neg(Some\ S, Some\ P)\neg R$

Proof 8.1: $(All\ S, All\ P)R \Rightarrow \neg(Some\ S, Some\ P)\neg R$

1	(1)	$(All\ S, All\ P)R$	Premise
2	(2)	$(Some\ S, Some\ P)\neg R$	Premise
3	(3)	$(c)S$	Premise
4	(4)	$(c, Some\ P)\neg R$	Premise

5	(5)	*(d)P*	Premise
6	(6)	*(c,d)¬R*	Premise
6	(7)	*¬(c,d) R*	CNSN:6
1,3	(8)	*(c,All P)R*	UE:1,3
1,3,5	(9)	*(c,d)R*	UE:5,8
1,3,5,6	(10)	⊥	⊥I:7,9
1,3,4	(11)	⊥	RI:4,5,6,10
1,2	(12)	⊥	RI:2,3,4,11
1	(13)	*¬(Some S,Some P)¬R*	¬I:2,12

Proof 8.2: ¬(Some S,Some P)¬R⇒(All S,All P)R

1	(1)	*¬(Some S,Some P)¬R*	Premise
2	(2)	*(c)S*	Premise
3	(3)	*(d)P*	Premise
4	(4)	*¬(c,d)R*	Premise
4	(5)	*(c,d)¬R*	CNSN:4
3,4	(6)	*(c,Some P)¬R*	PI:3,5
2,3,4	(7)	*(Some S,Some P)¬R*	PI:2,6
1,2,3,4	(8)	⊥	⊥I:1,7
1,2,3	(9)	*¬¬(c,d)R*	¬I:4,8
1,2,3	(10)	*(c,d)R*	¬E:9
1,2	(11)	*(c,All P)R*	UI:3,10
1	(12)	*(All S,All P)R*	UI:2,11

These proofs will serve to establish the plausibility of our assumption of Universal Reduction. They can be used as an initial step of a much larger inductive proof of that principle, but as this digression has proven to be a long one as it is, this is left for further research. Note, however, that these results straightforwardly lead to the principle of Universal Reduction for sentences of greater complexity. First, by contraposing these equivalences we can establish how to eliminate the universal quantifier for negations of any sentence of the type 1-8.

Second, we can use these to eliminate the universal quantifier in any subsentence (of the type 1-8) of a larger sentence.

4.4.3 Substitutional Approach to Completeness Proof

In the substitutional approach, every elementary sentence is assigned a truth value regardless of the assignments of truth values to any other elementary sentence. This is similar to how truth values are assigned to propositional variables is the propositional logic. Next, we connect the truth values of other sentence to the assignments of truth values to the elementary sentences. Next, the atomic sentences, the rules for which we have already seen in arguing the advantages of the substitutional approach to the model-theoretic one (of which we will be using only the one for the particular quantifier, due to the principle of universal reduction):

> (***Particular Rule***): the sentence of the form 'φ (*some P*)', where '*some P'* governs the sentence, is true just in case so is an instance of substitution, in that sentence, of '*some P'* by an '*a'*, for which "(*a*) *P"* is true.

Molecular sentences are the next in line. These are straightforward and familiar – an assignment of truth values assigns 'true' to $\neg A$ just in case it assigns 'false' to A, to $A \wedge B$ just in case it assigns 'true' to both A and B, to $A \vee B$ just in case it assigns 'true' to A or B, and to $A \rightarrow B$ just in case it assigns 'false' to A or 'true' to B.

Next, we define the rules for negative predication and anaphora:

> (***Negative Predication Rule***): the sentence of the form $(st_1,\ldots,st_n)\neg P$ is true just in case a sentence $(st_1,\ldots,st_n)P$ is false.

(***Anaphora Rule***): a sentence of the form $(\ldots, st_{in}, \ldots, \alpha_n, \ldots)\varphi$, where '$st_{in}$' is a singular or plural subject term, α_n is an anaphoric expression and they both (or all) share the same index *n*, is true just in case a sentence of the form $(\ldots, st_{in}, \ldots)\varphi$, where all occurrences of 'α_n'have been substituted by 'st_{in}', is true.

Before proceeding we still need to define the truth values for sentences containing identity:

(***Law of Identity***): '*a* = *a*' is true (namely, all sentences of this form are true on any assignment of truth values)[61]

(***Indiscernibility of Identicals***): If '*a* = *b*' is true and '(…, *a*, …) *P*' an elementary formula which contains '*a*', then if '(… ,*a*, …) *P*' is true then so is '(…,*b*, …) *P*' (where '*b*' has replaced all or some occurrences of '*a*' in (…, *a*, …)*P*).[62]

Notice that sentences of the form '*a*=*b*' are elementary sentences, and therefore whether they are in fact true will depend on the particular assignment of truth values to elementary sentences. We now need to show that the Indiscernibility of Identicals generalizes:

(**Indiscernibility of Identicals Generalization**): for any pair of sentences $\varphi(a)$ and $\varphi(b)$, such that $\varphi(a)$ is a sentence that contains '*a*', and $\varphi(b)$ is a sentence obtained by substituting some or all instances of '*a*' with '*b*' in $\varphi(a)$, it holds that if '*a*=*b*' is true then, if '$\varphi(a)$'is true so is '$\varphi(b)$'.

[61] Ben-Yami, manuscript, p.8
[62] Ben-Yami, manuscript, p.8

The proof for Indiscernibility of Identicals Generalization (IIG) is by induction on the complexity of the formula. As before, the complexity is defined as:

(***Complexity***): complexity of a sentence A (‘*comp(A)’*) is 0 if A is an elementary sentence; if $A \equiv \neg\varphi$ then comp(A)=comp(φ)+1, and if $A \equiv (\varphi \wedge \psi)$, $A \equiv (\varphi \vee \psi)$ or $A \equiv (\varphi \rightarrow \psi)$, the comp(A)=max(comp($\varphi$),comp($\psi$))+1, where the maximum function (‘*max*’) picks out the greater of the two numbers. Furthermore, if ‘*q’* is a quantifier, ‘*a*’ is an SST, ‘$\varphi(qA)$’ is a sentence governed by the PST ‘*qA’* and $\varphi(a)$ is a sentence obtained by substituting ‘*a’* for the PST in ‘$\varphi(qA)$’, then comp($\varphi(qA)$)=comp($\varphi(a)$)+1. Lastly, substituting one SST by another does not change the complexity of a sentence.

Now let us proceed with the inductive proof of Indiscernibility of Identicals Generalization:

IIG – Basic step
The sentences of complexity 0 are, by the definition of complexity, elementary sentences. As Indiscernibility of Identicals was defined for elementary sentences, its generalization holds for the basic step.

Next, we need to demonstrate that if IIG holds for the sentences of complexity *n* or less, it holds for sentences of complexity *n*+1, for any *n*.

IIG – Inductive step
Suppose that *a*=*b* holds. Suppose further that *A* is a sentence of complexity *n*+1. If $A \equiv \neg\psi$(a), then the complexity of ψ(a) is *n*. By inductive hypothesis, it is true just in case ψ(b) is. But *A* is true just in case ψ(a) is false. Therefore, *A* is true just in case

ψ (b) is false. Since ψ (b) is false just in case $\neg\psi$ (b) is true (by definition of truth value assignments for negation), *A* is true just in case $\neg\psi$ (b) is. The proofs for the other truth-functional connectives proceed along similar lines.

If A $\equiv$ (ψ (*some B*))(*a*), then, by Particular Rule, *A* is true just in case so is some sentence (ψ (*c*))(*a*) (where *(c)B* is true). But, since the complexity of (ψ (*c*))(*a*) is *n*, IIG holds for it by inductive hypothesis. Therefore, it is true just in case (ψ (*c*))(*b*) is. This, in turn, is true, once again by the Particular Rule, just in case (ψ (*some B*))(*b*)is. Therefore, *A* is true just in case (ψ (*some B*)) (*b*) is as well. This concludes the inductive step of the proof of IIG.

4.4.4 Henkin Theory

Henkin Theory is a set of axiomatic schemas which we will use to establish a connection between LQPS and the propositional calculus, which we know is complete. In the standard procedure for Henkin-style proofs we first define the members of the Henkin Theory. But, to do this, we must first define a language L_H, which represents an expansion of our language meant to accommodate the Henkin Theory, by way of *Witnessing SST's*.

4.4.4.1 Witnessing SST's

For any formula (*Some P*)φ of LQPS, we define a witnessing singular subject term $w_{(P)}\varphi$[63]. Adding this witnessing SST to our language expands it into the language L_1, which will contain some new sentences, themselves of the form (*Some P*)φ. We repeat the process for these, thereby generating L_2, and go on until we generate the *Henkin language* L_H, which

[63] This can be understood as as definite description *P*-that-is-φ

expands LQPS with symbols belonging to any expanded language L_n, for any n. The stage of generation of the symbol ("n") is called its *date of birth*. No witnessing SST will occur in a language with lower n than its date of birth.

Now that we have defined L_H, we can define Henkin theory - a set of sentences of L_H, with one of the four following forms:

(***Henkin Theory***):

(**H1**): $(Some\ P)\varphi \rightarrow ((w_{(P)\varphi})\varphi \wedge (w_{(P)\varphi})P)$

(**H2**): $((c)P \wedge (c)\varphi) \rightarrow (Some\ P)\varphi$

(**H3**): $c=c$

(**H4**): $c=d \rightarrow (\varphi(c) \rightarrow \varphi(d))$

(**H5.1**): $(st_1,\ldots,st_n)\neg P \rightarrow \neg(st_1,\ldots,st_n)P$

(**H5.2**): $\neg(st_1,\ldots,st_n)P \rightarrow (st_1,\ldots,st_n)\neg P$

(**H6.1**): $(\ldots,st_i,\ldots)\varphi \rightarrow (\ldots,st_{in},\ldots,\alpha_n,\ldots)\varphi$

(**H6.2**): $(\ldots,st_{in},\ldots,\alpha_n,\ldots)\varphi \rightarrow (\ldots,st_i,\ldots)\varphi$

The first axiom can be understood as saying that is some P is φ, the that-P-which-is-φ is φ (and also P). The second axiom corresponds to the rule of Particular introduction, the third to Identity Introduction (and to the Law of Identity), and the fourth to the Identity Elimination (and Indiscernibility of Identicals Generalization). Axioms H5.1 and H5.2 correspond to the two directions of the rule CNSN, and the axioms H6.1 and H6.2 to the rules for Anaphora Introduction and elimination, respectively. Therefore, the axioms H2-H6.2 are theorems of LQPS.

In our completeness proof, we need to demonstrate that every valid argument is provable. In order to do that, let us first define validity:

(***Validity***): an argument $\psi_1,\ldots,\psi_n \models \varphi$ is valid just in case every assignment of truth values that assigns truth to all the sentences $\psi_1,\ldots,\psi_n$ also assigns truth to φ even if we add names to our language.

Let us now take a valid argument of LQPS, T $\models$ S (where T is a set of sentences of LQPS, and S is a sentence of LQPS). Since L_H differs from LQPS only in having additional names, T $\models$ S is valid in L_H as well. We need to make this argument propositionally valid, and this we will do by defining a Henkin assignment:

(***Henkin Assignment***): Henkin Assignment *A* "is an assignment that assigns truth values to the sentences of L_H while respecting the rules for the connectives of the propositional calculus, and that A also makes all the Henkin axioms true." [64]

It can be shown that

(**Lemma 2**): "*A* also respects the rules for the relation of the truth value of a quantified sentence to those of its instances and the rules for truth assignments that involve identity,"[65] anaphora and negative predication.

L2 -Proof:

To prove this, let us first examine the particular quantifier. Since *A* follows the rules of the propositional calculus and makes all the sentences of the Henkin Theory true, by H_1 it will follow that if any sentence of the form "(*Some P*)φ" which is

[64] Ben-Yami, manuscript, p.11
[65] Ben-Yami, manuscript, p.11

governed by "*Some P*" is true, so is the sentence of the form "$(a)\varphi$" (namely, "$(w_{(P)}\varphi)\varphi$"), for which $(a)P$ holds. So the Particular rule is satisfied in the direction from left to right. Likewise, if a sentence of the form $(a)\varphi$, such that $(a)P$ holds, is true, then by H_2, which *A* satisfies, so is the sentence of the form $(Some\ P)\varphi$. Therefore, the Henkin Assignment satisfies the Particular rule.

Next, let us observe the rules for identity. Since *A* satisfies all the instances of H_3, the Law of Identity straightforwardly holds – every sentence of the form "$a=a$" is true. Next, since *A* satisfies H4, if '$a=b$' holds, then, if $(\ldots,a,\ldots)\varphi$ is true, the sentence $(\ldots,b,\ldots)\varphi$ will be true as well (where the latter has been obtained, as before, by substituting some or all instances of '*a*' by '*b*' in the former one). Therefore, the Indiscernibility of Identicals Generalised holds under *A* as well.

Finally, let us consider the rules for negative predication and anaphora. Since *A* satisfies H5.1 and H5.2, sentence of the form $(st_1,\ldots,st_n)\neg P$ will be true just in case so is the sentence $\neg(st_1,\ldots,st_n)P$. But this is true just in case $(st_1,\ldots,st_n)P$ is false, so the Negative Predication rule is satisfied. Since *A* satisfies axioms H6.1 and H6.2, sentence of the form $(\ldots,st_{in},\ldots,\alpha_n,\ldots)\varphi$ will be true just in case so is the sentence $(\ldots,st_i,\ldots)\varphi$, and therefore the Anaphora Rule holds. This concludes our proof.

Let us observe our valid inference T ⊨ S again - as it was stated earlier, it is also valid in L_H. Therefore, the inference T, H ⊨ S (where H is the Henkin theory) is valid in L_H as well – adding premises to a valid argument does not change its validity, and L_H contains the names necessary for H. Now, notice that every assignment of truth values that makes this argument valid will be a Henkin assignment – it will make all the Henkin axioms true. Therefore, by Lemma 2, that assignment makes T, H ⊨ S propositionally valid. By the completeness of propositional logic, it follows that S is provable from T and H: T, H ⊢ S. What remains to be shown now is that, if T, H ├ S is propositionally provable, then T ├ S is provable in LQPS – the *Elimination Theorem*. We need to bear in mind that in LQPS, sentences of the form H2 – H6.2 are theorems and that, since LQPS is an extension of the propositional calculus with some additional rules, whatever is propositionally valid will be valid in LQPS.

4.4.5 Elimination Theorem

(**Elimination Theorem**): If S is a sentence of LQPS provable from sentences $\psi_1,\ldots,\psi_n$ together with the sentences of Henkin theory, then S is provable from $\psi_1,\ldots,\psi_n$ alone.

To demonstrate this theorem, we will need to prove some propositions and lemmas first. Let T be a set of sentences of our language, and *p*, *q* and *r* sentences of our language.

(**Deduction Theorem**): If T, *p* ├ *q*, then T ├ $p \to q$.

Deduction Theorem – Proof:

This theorem follows from the definition of $\to$I rule of the propositional calculus – with T as a list of premises, assume *p*. By T, *p* ├ *q* we know we can now infer *q*. Therefore, by $\to$I we can infer $p \to q$, and therefore T ├ $p \to q$.

(**Proposition 1**): If T, $p_1,\ldots,p_n$ ├ *q*, and for every $i=1,\ldots,n$, T ├ p_i, then T ├ *q*.

Proposition 1 – Proof:

If T, $p_1,\ldots,p_n \vdash q$, then by the repeated application of the Deduction Theorem we know that T $\vdash p_1 \to (p_2 \to (p_3 \ldots \to (p_n \to q)\ldots)$. Since T $\vdash p_1$, by the application of $\to$E we obtain T $\vdash (p_2 \to (p_3 \ldots \to (p_n \to q)\ldots)$. By the repeated application of the same procedure for every $i=1,\ldots,n$, we finally obtain T $\vdash q$.

(**Lemma 3.1**): If T $\vdash p \to q$ and T $\vdash \neg p \to q$, then T $\vdash q$.

Lemma 3.1 – Proof:

This lemma follows from the fact that '$p \vee \neg p$' is a theorem of our system, T $\vdash p \vee \neg p$. From there and the assumptions T $\vdash p \to q$ and T $\vdash \neg p \to q$, by the application of $\vee$E, it follows that T $\vdash q$.

(**Lemma 3.2**): If T $\vdash (p \to q) \to r$, then T $\vdash \neg p \to r$ and T $\vdash q \to r$

Lemma 3.2 – Proof:

Once again, this lemma follows by elementary propositional calculus. By assuming either $\neg p$ or q we can obtain $p \to q$, and r from there, in either case, by $\to$E. We then obtain the respective conditionals by $\to$I.

(**Lemma 4**): If c is an SST that does not occur anywhere in T, φ(*Some P*), or q, then if T $\vdash (\varphi(c) \wedge (\mathrm{c})P) \to q$, then T $\vdash \varphi(\textit{Some } P) \to q$.

Lemma 4 – Proof:

Let T be the set of premises, and introduce as a premise, in some line (i), that φ(*Some P*). Furthermore introduce as a premise (for RI) in some line (j) that $\varphi(c)$, and (c)P in some line (k). Combining the last two assumptions with a conjunction, and by T $\vdash$ $(\varphi(c) \wedge (c)P) \rightarrow q$, we know we can derive q in some line (m). Now, since q does not contain c, we can conclude that q from the steps (i), (j), (k) and (m). This line will depend, among others, on the line (i), which means we can now derive, by $\rightarrow$I, $\varphi(Some\ P) \rightarrow q$ (with premises T).

The scheme of this proof is the following:

T	(1)-(i-1)	*T*	Premise(s)
i	(i)	φ *(Some P)*	Premise
j	(j)	φ *(c)*	Premise
k	(k)	*(c)P*	Premise
j,k	(k+1)	$\varphi(c) \wedge (c)P$	$\wedge$I:j,k
T	(m-1)	$(\varphi(c) \wedge (c)P) \rightarrow q$	
T,j,k	(m)	q	$\rightarrow$E:k+1,m-1
T,i	(m+1)	q	RI:i,j,k,m
T	(m+2)	$\varphi(Some\ P) \rightarrow q$	$\rightarrow$I:i,m+1

This proof satisfies all the requirements for RI – as, by assumption, φ *(Some P)* does not contain '*c*', and this is the only premise (i) relies on, the introduction of premises (j) and (k) conforms to the restraints laid out by RI – it holds that $j \notin \{i\}$, $k \notin \{i\}$, and (i) does not contain '*c*'. Likewise for the step (m) – as neither T nor q contain '*c*', the only premises that (m) relies on that contain '*c*' are j and k, and (m) does not contain '*c*'. Thus, the move to (m+1) is warranted, and this in turn demonstrates in (m+2) that T $\vdash \varphi(Some\ P) \rightarrow q$.

The last lemma we need to demonstrate before proceeding with the proof of the Deduction theorem is Lemma 5, which states we can eliminate the sentences containing witnessing singular subject terms:

(**Lemma 5**): Suppose c is an SST that does not occur anywhere in T, φ(*Some P*), or q. Then, if T, $\varphi(\textit{Some } P) \rightarrow (\varphi(c) \wedge (c)P) \vdash q$, then T $\vdash q$.

Lemma 5 – Proof:

By applying the deduction theorem on the argument T, $\varphi(\textit{Some } P) \rightarrow (\varphi(c) \wedge (c)P) \vdash q$, we obtain T $\vdash (\varphi(\textit{Some } P) \rightarrow (\varphi(c) \wedge (c)P)) \rightarrow q$. Then, by Lemma 3.2 we get the statements that (1) T $\vdash \neg(\varphi(\textit{Some } P)) \rightarrow q$ and (2) T $\vdash (\varphi(c) \wedge (c)P) \rightarrow q$. Applying Lemma 4 on (2) (and given our assumptions) we get (3) T $\vdash \varphi(\textit{Some } P) \rightarrow q$. Finally, applying Lemma 3.1 on (1) and (3), we obtain T $\vdash q$.

After these initial considerations, we can proceed with the proof of the Elimination Theorem itself. We demonstrate it by induction on the number of sentences of H, call it k, that a proof contains.

Elimination Theorem – Proof:

Elimination theorem - Basic step

If k = 0, the elimination theorem vacuously holds as there are no sentences of H to eliminate.

Elimination Theorem - Inductive Step

We now wish to show that, if Elimination theorem holds for sentences whose proofs contain *k* or less sentences of H, it also holds for any sentence S the (minimal) proof of which contain *k*+1 sentences of H. There are two cases to consider here. First, if one of the sentences of H in the proof is of the form H2-H6.2. As all the sentences of those forms are theorems, it follows that this sentence is provable from $\psi_1,\ldots,\psi_n$ and the other *k* Henkin axioms. Therefore, by Proposition 1, the sentence S is provable from $\psi_1,\ldots,\psi_n$ and the other *k* Henkin axioms alone. Then it follows, by the inductive hypothesis, that it is provable just from $\psi_1,\ldots,\psi_n$. The second case is if all the Henkin axioms in the (minimal) proof of S are of the form H1. In that case we choose one instance of H1 the witnessing SST of which is of the same or greater date of birth than any witnessing constant of any other instance of H1 within the proof. Since this witnessing constant does not appear in any of the other axiom instances, and neither does it appear in $\psi_1,\ldots,\psi_n$ or S (as they are also sentences of the non-extended language LQPS), by Lemma 5 it can be eliminated. We now have a proof with *k* instances of the axioms of H, and by the inductive hypothesis these can be eliminated. This concludes the demonstration of the Elimination Theorem.

If we now apply the Elimination theorem to the propositionally provable argument T, H $\vdash$ S, it follows that T $\vdash$ S is provable in LQPS. Therefore, if an argument T $\models$ S is valid in LQPS, it is also provable in LQPS. This concludes our non-model-theoretic completeness proof of LQPS.

Conclusion

This thesis has served to demonstrate three important points. First, as ubiquitous as it may be, Frege’s logic is not the only way of formalizing the natural language. In fact, many other systems are able to perform the function, while at the same time maintaining the surface structure of the natural language in their respective logical forms. Therefore, the predicate calculus represents an unnecessary mathematization of natural language. This is a fault that LQPS avoids.

The second point demonstrated is that LQPS retains, and in fact exceeds, all the expressive power of the predicate calculus. It exceeds it in at least two ways. First, it is able to formalize such linguistic procedures as copula negation and anaphors, for which the user of Frege’s system must rely solely on her linguistic capabilities. Second, it allows for formal demonstrations of certain valid inferences with a long-standing tradition in logic and philosophy – including, but not limited to, some immediate inferences found in Aristotle’s work, as well as the works of medieval logicians.

The third and main point of this thesis was to show that LQPS possesses the metatheoretical properties expected of a modern logical natural deduction system. While the soundness of Ben-Yami’s system has been previously established, the completeness proof presented here represents a novel contribution in keeping with Ben-Yami’s suggestion for the predicate calculus. It does away with standard model-theoretic proofs and rather opts for an approach that does not straddle itself with so strong logical commitments.

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