EUROZONE AS COMMON CURRENCY AREA:

OPTIMAL CURRENCY BASKET PERSPECTIVE

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Abstract

The study aims at contribution to the literature of the theory of optimal currency basket and optimal currency area. We derive analytical solution of the model of Zhang et al. (2011) and use it to compute structure of individual optimal currency baskets for 5 European countries – Spain, Italy, France, Germany and Portugal for year 1994. Based on the empirical results we conclude that creation of common currency union in the late nineties was justified from the perspective of the optimal currency basket approach. Moreover, signs of presence of possible synchronization in external sector against shocks to the exchange rate create room for efficient use of common monetary policy.

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1. Introduction

In the year 1999 eleven European countries took an unprecedented step forward to deeper economical and political integration by creating a common monetary union – the Economic and Monetary Union (EMU, sometimes referred to as the Eurozone). Since that moment, the benefits and costs of such integration have been deeply studied. This thesis would like to contribute to the fruitful debate related to this subject by adopting a new approach based on the theory of the optimal currency basket.

In this thesis we use the theory of the optimal currency basket in order to be able to shed some light on the process of creation of the common currency union in Europe from the economic point of view. To be able to do this we will adopt the concept of the optimal currency basket that will serve as a tool for assessing economic suitability for creation of a monetary union with selected currencies from the perspective of the denomination of the external trade and international capital flows.

Before going into methodological details, it is important to clarify the distinction between the structure of the *optimal common currency basket* and the structure of the *individual optimal currency basket* from a perspective of a single country. While the former one determines the structure of an optimal currency basket for a group of countries that eventually may enter a common currency union, the latter one is used by a country that is planning to operate in some form of a fixed exchange rate regime.

The link between those two different concepts is straightforward: we consider the optimal structure of an *individual currency basket* as a possible indicator in favor or against the adoption of a common currency with other countries. Countries are likely to be suitable for creating a common currency are in case of high interdependence in individual optimal currency baskets.

The common currency used in a currency union should then be created according to the optimal structure of a *common currency basket*. This basket may take into consideration key factors different to those ones that were in the center of analysis of individual optimal currency baskets for single countries.¹ The differentiation between individual and common

¹ We may use some European countries as a practical example. According to the model of the optimal currency basket, Slovakia, Estonia and Slovenia in the late eighties should have kept fixing their domestic currencies to the Deutsche Mark as the majority of their foreign production was exported to Germany. Similar structure of their optimal currency baskets indicated that by creating a common currency union among themselves, with Germany as their main trading partner, they should not have been expected to experience significant problems in

currency basket is in line with Ogawa and Shimizu (2006), yet the usage of an individual currency basket as an indicator is our own decision based on Mori et al. (2002) approach.

Strictly speaking, we assume that *high share* of currencies involved in a currency union in a domestic optimal currency basket is a *favorable factor speaking in favor* of monetary integration and reversely. More precisely, if a country's domestic optimal currency basket is mostly created by currencies of countries that are already involved in a currency union or are planning to create a currency union together with the domestic country the decision to become a part of the currency union should not bring tensions into external trade and financial flows due to adoption of a common currency.

Higher share of currencies involved in the optimal currency basket indicates existence of strong link between domestic economy and the foreign economies, thus their high interdependence. According to the theory of the optimal currency area (OCA) proposed by Mundell (1961) and contributed by many other authors², it is beneficial to form a common currency area in presence of one or few dominant trading (Broz, p. 72). In this case we assume that high share of currencies of countries in the optimal currency basket represents dominant trading partners in the external trade sector or in international financial flows. Therefore there are possible gains from a common currency union.

Moreover, domestic export, import and international flows previously denominated in domestic currencies become independent on the evolution of the euro against currencies of countries outside the EMU by creating common currency union with trading partners from the EMU.³ This brings us to other interesting point.

Not only high share of currencies involved in the optimal currency basket has to be viewed as a positive factor; the *similar structure* of the optimal currency basket against countries outside the EMU is even more important because it may indicate synchronization in the external sector of individual countries. *Common monetary policy* is expected to be *highly efficient* in case of highly synchronized countries in external trade and international flow which is to be viewed as a positive factor in favor of common monetary union.

the foreign trade sector because of different requirements on denomination of foreign export prior to creation of a common currency area.

² For comprehensive literature review regarding theory of optimal currency area see, for example, Broz (2005).

³ We are aware of the fact that this situation may not be true in some cases. Let us assume that due to positive evolution of euro against the currencies of countries outside the EMU the import from those countries becomes relatively cheaper to the import from countries inside the EMU. In case of high substitutability of imported goods the evolution of a common currency may change preferences and the relative value of trade between countries inside the EMU and outside countries. Yet, for the sake of simplicity we do not further elaborate this issue.

The main contribution of this thesis is threefold: firstly, we apply the theory of the optimal currency basket to the problem of a common monetary union, EMU. Secondly, we calculate the structure of the optimal currency baskets for the year 1994 of 5 selected European countries that were considering joining the Eurozone at that time – Germany, France, Italy, Portugal and Spain, in order to be able to assess possible positive and negative effects of the common monetary union in the external sector. In the year 1994 the second stage of the monetary integration of the European Union began by establishment of the European Monetary Institute as a predecessor of the European Central Bank. As this year represent significant milestone of the process of the European monetary integration we find it interesting to calculate optimal currency weights relevant for this year.

Lastly, we derive analytical solution to the current model of optimal currency basket proposed by Zhang et al. (2011) that serves as a tool for computation of the optimal currency weights in individual currency baskets.

Our empirical results suggests that from, the perspective of the optimal currency basket, some of the countries (Portugal and partially Spain and Italy) *should have benefited* from the adoption of the single currency euro as most of their external trade and financial flows would not have been affected by exchange rate fluctuations any more. Other countries, that were highly open toward the countries outside the EMU (Germany and France) at that time, will still be exposed to negative effects in their external sector due to variability of common currency, euro. Yet, those negative effects may have been minimized by creation of the countries.

By saying that, we do not suggest that member countries of the Eurozone should have pegged their newly created currency against specific optimal combination of currencies of major trading partners in the late nineties. Or that this should be something to be done today or in the near future. Instead, we claim that *the existence* of such an optimal currency basket resulting from *similar structure of individual currency baskets* is just visible manifestation of synchronization in the external sector. Thus, in case of fluctuations in the common currency, euro, all countries will be affected in the same direction and that is to be viewed positively. Moreover, common exchange rate policy is the more effective the more member countries are synchronized in the external sector. Similar structure of individual currency baskets suggests that *common monetary policy* in the case of 5 countries is *expected to be highly effective* from the perspective of possible exchange rate policy.

The theory of the optimal currency basket structure for countries using various forms of fixed exchange rate regimes has been subject to academic discussion since the eighties. Academic literature related to this kind of research is broad and rich in terms of either geographical distribution or methodology used for calculation.

In the eighties, the concept of the optimal common currency basket was applied to the calculation of the European Currency Unit (ECU), the basket currency of the former European Economic Community (Edison, 1986). According to our best knowledge, the concept of an individual optimal currency basket has never been used before for analyzing the structure of the optimal currency basket of European countries that joined the EMU one decade later.

Unsurprisingly, most of the current literature on the optimal currency basket covers countries from East Asia where debates about forming a common currency union are the most active. Countries in the East Asian region experienced troublesome period in years 1997 and 1998 caused by economic, banking and exchange rate crisis which showed that "a country that exports to all the major economies but targets stability only in its exchange rate with one major currency will experience variability in its effective exchange rate and its bilateral exchange rates with the other major currencies (Pontines, 2009)."

In recent years, the debate about creating a common currency area and probably a monetary union similar to the one created in Europe has come into center of academic attention (Adams and Chow, 2009). Thus, the experience of the current member countries of the Eurozone may help the Asian countries to better grasp the challenging task that lies before them before creating a fully functional common currency area.

As far as the methodological procedure is concerned, we may distinguish between the following approaches that have been developed in this area of research since the eighties:

• Solving the optimal currency basket structure by the new open economy macroeconomic model – presented by Shioji (2006) for countries of East Asia, the paper enhances the basic open macroeconomic model proposed Corsetti et al. (2000) which is in turn based on the basic "redux" model by Obstfeld and Rogoff (1995, 1996). On the contrary to the standard macroeconomic model by Corsetti et al. (2000), Shioji uses a three-country model with one type of a nontradable good and two types of tradable goods. Shioji (2006) is able to provide predictions about the responses of the current account balance and GDP rate to foreign exchange shocks. Yet, as it deals with a three-country model, the optimal structure

of an Asian currency basket used for pricing in country one consists from up to two currencies (combination of the US dollar and Japanese yen). An extension of the model to n-country model would be necessary in case of a more complex structure of an optimal currency basket.

• Concept of currency invariant index in terms of variance minimization framework – presented by Hovanov et al. (2004) and later used by Pontines (2009) for calculating an optimal structure of the currency basket. This approach deals with issues of choosing the best base currency used for calculation by creating a reduced normalized value in exchange rate of i-th currency in such a way that the selection of a base currency does not matter any more. Optimal weights of currencies are further calculated by solving a basic optimization problem of diversified portfolio similar to Markowitz (1959). Hence, as this model does not take into account incentive of countries to minimize their current account balances or external position it represents useful technical exercise but does not reflect decisions of economic agents in domestic countries furthermore.

• Integration of standard macroeconomic models with input-output tables – Yano and Kosaka (2003) combine two basic models for developing countries – skeleton model proposed by them combined with the official UNCTAD model extended for the foreign sector. This approach was used for computation of an optimal currency basket for Asian countries along with an analysis of changes in foreign trade patterns and their influence on domestic countries and the required change in exchange rate regimes. The model does not incorporate international flow of capital which may be viewed as a problematic feature. Inclusion of this structural block into the model is necessary due to the current state of world economy which may be characterized as highly dependent on the international capital flows.

The literature related to the composition of an optimal currency basket either from the perspective of an individual country or a group of countries is much broader. Models presented above cover basic approaches to the construction of such optimal baskets. However, majority of the models used in economic literature focuses solely on external trade sector. Thus, flow of international capital is a missing part in all of the models mentioned previously.

With respect to the current debt crisis in Euro Zone countries it seems that the inclusion of the foreign debt position to models examining the structure of an optimal currency basket should be required. Additionally, some models are appropriate only for small number of countries involved (Shioji, 2006) or do not include any economic reasoning, thus represent good mathematical exercise but without any economical background (Pontines, 2009).

Zhang et al. (2011) partially addresses the issues raised above by solving a problem of optimal currency weights by minimizing the volatility of country's external account. We use this approach for specifying optimal currency weights of selected individual countries that joined the common currency union in Europe in the late nineties.

This diploma thesis is divided into the following subsections. Firstly, we describe the basic features of the model proposed by Zhang et al. (2011) used for computation of the optimal currency weights. Analytical derivation of the model proposed by Zhang et al. (2011) is developed in the third chapter using the concept of Karush-Kuhn-Tucker nonlinear programming. Additionally, we briefly discuss the concept of the currency invariant index that is used for computation of value of currencies included into the potential currency basket as proposed in Hovanov et al. (2004).

Results obtained by the analytical approach to modeling of optimal currency weights for selected European countries are discussed in the final chapter of this thesis. Conclusion briefly summarizes the results of our analysis.

2. Theoretical model

The theoretical model created by Zhang et al. (2011) serves for modeling the composition of the optimal currency basket in China taking into account specific requirements of Chinese exchange rate regime. As this model incorporates both external trade sector and international capital flows we find this model suitable for modeling the structure of the optimal currency basket for selected European countries.

The model proposed by Zhang et al. (2011) includes evolution of the net international investment position into the models of optimal currency baskets. We share the opinion of the authors that not only real economy is important for computation of the optimal currency weights but capital flows may affect evolution in external sector even more significantly.

Authors assume that domestic country would like to stabilize development of their trade balance and international investment position in an environment of flexible exchange rates. In order to achieve this goal country tries to minimize costs related to variability of exchange rates by fixing its own currency to a currency basket which is optimally structured.

In this model authors use an asymmetric pricing of import and export trade - local currency pricing (LCP) for export and producer currency pricing (PCP) for import. Additionally, the local currency pricing (LCP) is used in computation of IIP variation due to exchange rate shocks and shocks to IIP. Furthermore, authors decompose net IIP position according to the currency denomination of the foreign debt.

We stick to the model proposed by Zhang et al. (2011) with the asymmetric pricing for our computation with data for selected European countries. We are aware of the fact that this may be not fully plausible in the case of European countries, especially in the short run, but our results may still provide fruitful insights regarding the efficiency of the common currency union from the perspective of the external sector.⁴

⁴ There are various approaches to how to measure the effect of the changes in the exchange rate on import price, or the exchange rate pass-through effect (ERPT), thus how to support usage of LCP or PCP pricing in economic models. Empirical results for developed European countries do not provide a clear picture about the level of ERPT in those countries. For instance, Campa et al. (2002) estimates short-run ERPT in France on 0.56 and Germany on 0.50 and long-run ERPT on 1.60/0.70. On the contrary, estimates for short-run for United Kingdom shows values of 0.32.

There is a compelling evidence of partial pass-through effect in the short run, thus rejecting both LCP and PCP pricing. Yet, the PCP pricing is more prevalent for many types of imported goods in the long run (Campa et al., 2005; Campa et al., 2002).

Therefore, based on empirical evidence, the application of LCP pricing on export sector and PCP pricing on import sector might be plausible, especially from the long run perspective.

Zhang et al. (2011) solve their model numerically. In order to calculate the optimal currency basket structure of selected European countries we have developed our own numerical algorithm.⁵ Yet, due to high requirements on computational strength and its low precision, the results obtained by these calculations are highly inaccurate. Furthermore, we observe that many optimal vectors are not found due to low precision as they do not fall within the area covered by the algorithm. Our findings thus question empirical results obtained by Zhang et al. (2011) by their numerical algorithm whose structure is not further elaborated in their paper.

Hence, one of the main contributions of this paper lies in developing and deriving an analytical solution to the model proposed by Zhang et al. (2011) that will provide us with the proper global minimum.

2.1. Basic structure of the model by Zhang et al. (2011)

In this section we describe basic features of the optimization problem proposed by Zhang et al. (2011). A complete derivation of the model is given in the Appendix I.

This theoretical model consists of the following system of equations:

1. total cost subject to minimization

 $Z = \Delta NA + \Delta NX ,$

where ΔNA represents the change in international investment position (IIP) and ΔNX the change in trade balance.

[1]

2. initial conditions for the ratio of trade balance and IIP on GDP

⁵ We use data of 17 countries for modeling the optimal structure of currency basket of selected European countries. Thus, the total possible vectors of different currency weights is s^{17} where s represents specification of precision. For example, the total amount of combination is 10^{17} with precision to the one decimal place

which is immensely demanding for computation. This is the reason why we use algorithm developed for our specific needs. This algorithm is based on the following principles: (1) as the sum of weights in a currency basket must equal unity and each component of the currency basket must be nonnegative and lower or equal to one, unity will be decomposed to specific elements sum of which is equal one (e.g. 0.1+0.1+0.8) while the precision of decomposition is a changeable variable; (2) each combination of elements represents a possible vector of optimal currency weights in the currency basket; thus, the variance Var(z) will be computed for each vector; (3) the vector ω with the lowest variance

Var(z) represents optimal currency weights in the currency basket. Hence, we do not need to consider all possible combinations of elements of the unity; we use only the combinations where the sum of elements is equal to one.

Hereby I would like to thank to Martin Mytny and Peter Kopac for the consultations regarding numerical algorithm as described above.

$$NA_0 = \alpha_1 Y_0; \qquad NX_0 = \alpha_2 Y_0,$$
 [2]

where α_1, α_2 represents coefficients related to the ratios of trade balance and IIP on initial value of domestic product Y_0 .

3. dynamic equations for total cost as a share of total product

$$Z = \Delta NA + \Delta NX$$

$$z = \frac{Z}{Y_0} = \alpha_1 \Delta Y + \alpha_2 \Delta Y$$

$$z = \alpha_1 nx + \alpha_2 na$$
[3]

where the corresponding lower case characters nx, na indicate change in variables NX, NA in logarithmic version.

4. optimization problem

$$\min_{\omega_1,\omega_2,\ldots,\omega_N} VAR(z), \quad s.t. \sum_{j=1}^N \omega_j = 1,$$
[4]

where ω_j represents the share of currency j in the optimal currency basket with j = 1, 2, ..., N.

5. trade balance

- export evolution -> $x = \sum_{j=1}^{N} \gamma_j e(j)$, where γ_j represents the share of the export to country j on total export, e(j) represents the change in the exchange rate of domestic currency to the currency of country j;

- import evolution -> $i = \sum_{j=1}^{N} \delta_j i(j) = \sum_{j=1}^{N} \left[(1-\eta) \delta_j e(j) \right]$, where η represents the elasticity of substitution,⁶ δ_j represents share of import from country j on total import, e(j) represents the change in exchange rate of domestic currency to currency of country j;

- net export ->
$$nx = \sum_{j=1}^{N} \left[\frac{\beta}{\beta - 1} \gamma_j - \frac{\delta_j (1 - \eta)}{\beta - 1} \right] e(j)$$
 [5]

6. net international investment position

$$na = \sum_{j=1}^{N} \left[\lambda_j \left(\mathcal{E}^a(j) + e(j) \right) \right]$$
[6]

⁶ We assume that consumers have constant elasticity of substitution among imported goods.

where λ_j represents share of IIP towards the j-th country on total value of net IIP in t=0and $\varepsilon^a(j)$ represents the stochastic process⁷ describing the changes in IIP towards country j.

7. total cost equation

$$z = \alpha_{1}nx + \alpha_{2}na,$$

$$z = \alpha_{1}\sum_{j=1}^{N} \left[\frac{\beta}{\beta-1}\gamma_{j} - \frac{\delta_{j}(1-\eta)}{\beta-1} \right] e(j) + \alpha_{2}\sum_{j=1}^{N} \left[\lambda_{j} \left(\varepsilon^{a}(j) + e(j) \right) \right],$$

$$z = \sum_{j=1}^{N} \left\{ \alpha_{1} \left[\frac{\beta}{\beta-1}\gamma_{j} - \frac{\delta_{j}(1-\eta)}{\beta-1} \right] e(j) + \alpha_{2}\lambda_{j} \left(\varepsilon^{a}(j) + e(j) \right) \right\},$$

$$z = \sum_{j=1}^{N} \left\{ \left(\alpha_{1} \left[\frac{\beta}{\beta-1}\gamma_{j} - \frac{\delta_{j}(1-\eta)}{\beta-1} \right] + \alpha_{2}\lambda_{j} \right) e(j) + \alpha_{2}\lambda_{j}\varepsilon^{a}(j) \right\}.$$
[7]

8. evolution of exchange rate

$$e(j) = -\mathcal{E}^{e}(j) + \sum_{i=1}^{N} \omega_{i} \mathcal{E}^{e}(i),$$
[8]

where $\varepsilon^{e}(j)$ represents the stochastic process⁸ of shocks to exchange rate.

9. total cost equation with shocks

$$z = \left(\Psi^T \quad \Xi^T\right) \begin{pmatrix} \boldsymbol{\varepsilon}^e \\ \boldsymbol{\varepsilon}^a \end{pmatrix},$$
[9]

where $\varepsilon^{e} = \{\varepsilon^{e}(j)\}_{Nx1}$ represents the vector of shocks to exchange rate, $\varepsilon^{a} = \{\varepsilon^{a}(j)\}_{Nx1}$ represents the vector of shocks to international investment position, Ψ a Ξ are vectors with coefficients given by the following formulas:

$$\Psi = \left\{ \omega_{j} \sum_{i=1}^{N} \left\{ \alpha_{1} \left[\frac{\beta}{\beta - 1} \gamma_{i} - \frac{\delta_{i} (1 - \eta)}{\beta - 1} \right] + \alpha_{2} \lambda_{i} \right\} - \left\{ \alpha_{1} \left[\frac{\beta}{\beta - 1} \gamma_{j} - \frac{\delta_{j} (1 - \eta)}{\beta - 1} \right] + \alpha_{2} \lambda_{j} \right\} \right\}_{Nx1}, \quad [10]$$

$$\Xi = \left\{ \alpha_{2} \lambda_{j} \right\}_{Nx1}.$$

10. variance of total cost

⁷ We assume that a stochastic process has a zero mean and constant variance $(\sigma_j^a)^2$ correlated with a change of exchange rate.

⁸ We assume that a stochastic process has a zero mean and constant variance $(\sigma_j^e)^2$ correlated with a change of exchange rate.

$$Var(z) = \Phi^T . \rho . \Phi , \qquad [11]$$

where ρ represents the correlation matrix between shocks to exchange rate and shocks to international investment position in the following form:

$$\rho = \begin{pmatrix} \rho^{ee} & \rho^{ea} \\ \rho^{ae} & \rho^{aa} \end{pmatrix},$$
[12]

and Φ represents the vector of shocks weighted by their variances in the following form:

$$\Phi = \begin{pmatrix} \Phi^{e} \\ \Phi^{a} \end{pmatrix} = \begin{pmatrix} \Psi_{1} \sigma_{1}^{e} \\ \vdots \\ \Psi_{N} \sigma_{N}^{e} \\ \Xi_{1} \sigma_{1}^{a} \\ \vdots \\ \Xi_{N} \sigma_{N}^{a} \end{pmatrix}.$$
[13]

The optimal solution to the optimization problem described above is derived analytically in Section 3. The optimal solution is later used for modeling structure of optimal currency baskets for 5 European countries with empirical data on their trade balances and net international investment positions.

2.2. Currency invariant index

As shown in Hovanov et al. (2004, 2007), the choice of base currency may alter results obtained by numerical computation of the optimal currency baskets. Thus, we would like to apply some procedure that allows us to compute value of each single currency independently on the choice of base currency.

Usually, the concept of effective exchange rate is applied. According to this concept, value of domestic currency is weighted with shares of main trading partners. Yet, this concept is not suitable for our needs as one entire building block of the model proposed by Zhang et al. (2011) strongly relies on computation of import and export shares towards all trade partners.⁹

The use of effective exchange rate would introduce bias into our computation as the shares of main trading partners would be assigned higher priority as they would be used twice in the computational procedure.

⁹ See the derivation of the trade balance equation described in [5] in Section 2.1.

For this reason we apply procedure proposed by Hovanov et al. (2004) that enable us to compute value of every single currency independently on the choice of base currency no matter what the shares of major trade partners in external sector are.

Let us now assume that there are N currencies in the sample where i = 1, 2, ..., N. The ratio of currency *i* to currency *j* at time *t*, or the cross rate in economic terms, is denoted as $Val_{ij}(t)$ where i, j = 1, 2, ..., N and currency *i* is called quote currency and *j* is called the base currency.

Let us define the scaling factor β by:

$$\beta = \left[GeoMean(Val_{1j}(t), ..., Val_{nj}(t))\right]^{-1} = \left[\left(\prod_{r=1}^{n} Val_{rj}(t)\right)^{1/n}\right]^{-1} = \left[\sqrt[n]{\prod_{r=1}^{n} Val_{rj}(t)}\right]^{-1}$$
[14]

The scaling factor β is computed as an inverse of the geometric mean of all currency cross rates with fixed base currency. Without loss of generality we assume that $Val_{rj}(t)$ for r = j equals the unity.¹⁰

Then a *normalized value in exchange* (normalized index of value in exchange) is computed in the following way:

$$NVal_{ij}(t) = \beta Val_{ij}(t) = \frac{Val_{ij}(t)}{GeoMean(Val_{1j}(t), \dots, Val_{nj}(t))} = \frac{Val_{ij}(t)}{\sqrt[n]{\prod_{r=1}^{n} Val_{rj}(t)}}$$
[15]

According to Proposition 2 derived and proved in Hovanov et al. (2004), "for a positive homogeneous transformation $NVal_{ij}(t) = \varphi(Val_{ij}(t)) = \beta Val_{ij}(t)$, $\beta > 0$, to be independent of the standard good (currency) c_j choice, it is sufficient to fix β as the inverse of the geometric mean GeoMean($Val_{1j}(t), ..., Val_{nj}(t)$) of the values in exchange $Val_{ij}(t)$, i = 1, 2, ..., N."

The normalized value in exchange $NVal_{ij}(t)$ when used for computation of invariant currency index for given currency *i* and selected base currency *j* will be called *invariant currency* value index (ICVI).

¹⁰ For example, the cross rate of USD/USD is equal to one in the case when the USD is chosen as a base currency in some sample.

3. Analytical solution of the model by the Karush-Kuhn-Tucker method

The basic theoretical model proposed by Zhang et al. (2011) represents the optimization problem in the framework of nonlinear programming with linear equality constraints. In contrast with this, we impose two types of constraints: linear equality and linear inequality constraints. For such a type of optimization problem the Karush-Kuhn-Tucker (KKT) method seems to be the most proper and is therefore used in this thesis.

In this section we will heavily draw on the KKT method as described in Proposition 3.3.1. in Bertsekas (1999). This will enable us to derive optimal values of currency basket with respect to set of equality and inequality constraints.

The general form of KKT may be rewritten as an optimization problem with two types of constraints -(1) inequality constraints and (2) equality constraints. The general form of optimization problem in KKT framework may be therefore expressed in the following way:

$$\min_{x} f(x), \quad s.t. \ g_i(x) \le 0, \quad h_j(x) = 0$$
[16]

In our optimization problem we would like to minimize Var(z) as expressed in equation [11] with respect to following set of equality and inequality constraints:

$$\begin{array}{ll} \underset{\omega}{\min} & Var(z(\omega)), \quad for \quad i=1,...,N \\
s.t. & \omega_i \leq 1 \Rightarrow \underbrace{\omega_i - 1}_{g_i(\omega)} \leq 0 \Rightarrow g_i(\omega) \leq 0, \\
& \omega_i \geq 0 \Rightarrow \underbrace{-\omega_i}_{g_{i+N}(\omega)} \leq 0 \Rightarrow g_{i+N}(\omega) \leq 0, \\
& \sum_{i=1}^{N} \omega_i = 1 \Rightarrow \underbrace{\sum_{i=1}^{N} \omega_i - 1}_{h_i(\omega)} = 0. \\
\end{array}$$
[17]

Firstly, we shall impose basic equality constraint on the sum of all weights in the basket. Logically, the total share of all currencies in one basket should equal one. Secondly, we shall consider only those currency weights that are nonnegative and smaller than or equal to one. Thus, we do not allow for a short selling as in the case of optimal currency basket this is simply not plausible. By observation we may conclude that the regularity condition of KKT is satisfied as all constraints are affine.¹¹ By compactness of the set $\left\{(\omega_1,...,\omega_N), 0 \le \omega_i \le 1, \sum_{i=1}^N \omega_i = 1, i = 1,...,N\right\}$ and the continuity of the function $Var(z(\omega))$ there exists and optimal $\omega^* = (\omega_1^*,...,\omega_N^*)$ such that the function $Var(z(\omega))$ has a minimum $Var(z(\omega^*))$. Furthermore, since all the prerequisites for KKT method are satisfied then there exists $\eta_1,...,\eta_N,\eta_{N+1},...,\eta_{2N},\lambda$ and the optimal solution must satisfy following stationarity conditions:

$$\nabla Var(z(\omega^*)) + \sum_{i=1}^{2N} \eta_i \nabla g_i(\omega^*) + \lambda \nabla h(\omega^*) = 0,$$

which is a vector notation for the following coordinates:

$$\frac{\partial Var(z(\omega^*))}{\partial \omega_j} + \sum_{i=1}^N \eta_i \frac{\partial(\omega_i - 1)}{\partial \omega_j} + \sum_{i=1}^N \eta_{i+N} \frac{\partial(-\omega_i)}{\partial \omega_j} + \lambda \frac{\partial\left(\sum_{i=1}^N \omega_i - 1\right)}{\partial \omega_j} = 0$$
[18]

This stationarity condition is represented by system of N equations with 2N+1 KKT multipliers $\eta_1, ..., \eta_N, \eta_{N+1}, ..., \eta_{2N}, \lambda$ and N coordinates of vector ω^* :

Thus, we need to solve this system of N equations with 3N+1 variables. In order to be able to do this, we will follow impose the additional necessary conditions prescribed by the KKT method. According to the KKT, the following set of three types of necessary conditions needs to be satisfied *simultaneously* for the vector ω^* to be optimal solution:

¹¹ In order for a vector ω^* to be optimal solution, specific regularity conditions need to be satisfied. By *linearity* constraint qualification if g_i and h_j are affine functions, then no other condition is needed to be satisfied for KKT to hold. In our optimization problem all 2N+1 linear constraints are linear in ω_i with constant term. Thus, regularity conditions are satisfied and we may use KKT method for finding vector of optimum weights ω^* .

(1) primal feasibility condition

$$0 \le \omega_i^* \le 1, \quad \sum_{i=1}^N \omega_i^* - 1 = 0, \quad \text{for} \quad i = 1, ..., N,$$
[20]

(2) dual feasibility conditions

$$\eta_1, \dots, \eta_N, \eta_{N+1}, \dots, \eta_{2N} \ge 0$$
 [21]

(3) complementary slackness

[1]
$$\forall i = 1,...N, \quad \eta_i g_i(\omega^*) = 0,$$

 $\eta_i(\omega_i^* - 1) = 0,$
[2] $\forall i = 1,...N, \quad \eta_{i+N} g_{i+N}(\omega^*) = 0,$
 $\eta_{i+N}(-\omega_i^*) = 0.$
[22]

The primal and dual feasibility conditions will be taken care of in the last steps of our algorithm. We will firstly focus on complementary slackness conditions because they allow us to reduce system of N equations with 3N + 1 variables to the system of N equations with N variables. Such a system can by solved by standard linear algebra.

Case [1] in complementary slackness condition

Let us first assume that $\eta_i \neq 0$ for at least one η_i . Then in order for [1] in the complementary slackness condition in [22] to hold, the *i*-*th* coordinate of the optimal omega must equal unity, thus $\omega_i^* = 1$. The optimal solution in this case would be the vector omega with all components being zero except the *i*-*th* coordinate because the sum of all omega coordinates

must equal one,
$$\sum_{i=1}^{N} \omega_i = 1$$
.

This observation leads to the point that either the minimum is in the "corners" or else the condition $\eta_i = 0$ must be satisfied for all i = 1,...N simultaneously. Thus, we will need to test N possible vectors of with N-1 coordinates being zeros and one coordinate being unity and find the optimal one for which the variance $Var(z(\omega^*))$ is minimal.

Let us now assume that $\eta_i = 0$ for i = 1, ..., N. Again, based on the reasoning used in previous section, condition $\eta_i = 0$ must be satisfied for all i = 1, ..., N simultaneously in order to

 $\sum_{i=1}^{N} \omega_i = 1$ holds. Then the system of the stationarity conditions is reduced to the system of N equations with 2N + 1 variables:

Case [2] in complementary slackness condition

In this case we have $\eta_{i+N}(-\omega_i^*) = 0$, so that for each i = 1, ..., N either $\eta_{i+N} = 0$ or $(-\omega_i^*) = 0$ holds. An analytically plausible method in this case is to compute all possible combination of zeros and non-ones for all i = 1, ...N combinations. With this approach we investigate only those combinations where exactly one variable is equal zero and other variable may be assigned different values. Thus, we will need to solve 2^N combinations of certain system of linear equations and find the solution to this system of N linear equations with 2N+1variables being $\omega_1, ..., \omega_N, \eta_{N+1}, ..., \eta_{2N}, \lambda$.

It is possible that both the KKT multipliers and omega coordinates are zero at one point. Yet we do not need to consider these possibilities separately because such combinations may result as the optimum solution of our optimization problem.

The system of stationarity conditions in [23] is further expanded with the equality condition

of
$$\sum_{i=1}^{N} \omega_i - 1 = 0$$
. Such an expansion creates system of $N+1$ equations with $2N+1$ variables.

The second set of N equations which allow us to check all 2^N combinations of ones and zeros will expand the system of stationarity conditions in [23]. We will return to this point later in this chapter.

$$\begin{bmatrix} \frac{\partial Var(z(\omega^*))}{\partial \omega_1} - \eta_{1+N} + \lambda = 0\\ \dots\\ \frac{\partial Var(z(\omega^*))}{\partial \omega_N} - \eta_{2N} + \lambda = 0\\ \sum_{i=1}^{N} \omega_i - 1 = 0 \end{bmatrix}_{(N+1)\times 1}$$
[24]

Based on the system of equation derived in [24] we will further reformulate the system of equations in order to be able to solve it by linear algebra in Matlab environment. Additionally, we will derive analytical solution for specific optimization problem as stated in [11].

3.1.1. First order conditions

In this section we analytically derive first order conditions of variance with respect to the coordinates of the vector omega ω . Firstly, let us rewrite the formula for the variance of the total cost equation.

$$Var(z) = \Phi^{T} \cdot \rho \cdot \Phi$$

$$Var(z) = \left(\Psi_{1}\sigma_{1}^{e}, ..., \Psi_{N}\sigma_{N}^{e}, \Xi_{1}\sigma_{1}^{a}, ..., \Xi_{N}\sigma_{N}^{a}\right) \cdot \begin{pmatrix}\rho^{ee} & \rho^{ea} \\ \rho^{ae} & \rho^{aa} \end{pmatrix} \cdot \begin{pmatrix}\Psi_{1}\sigma_{1}^{e} \\ \cdot \\ \Psi_{N}\sigma_{N}^{e} \\ \Xi_{1}\sigma_{1}^{a} \\ \cdot \\ \Xi_{N}\sigma_{N}^{a} \end{pmatrix}$$

$$Var(z) = \left((\omega_{1}\xi - \xi_{1})\sigma_{1}^{e}, ..., (\omega_{N}\xi - \xi_{N})\sigma_{N}^{e}, \alpha_{2}\lambda_{1}\sigma_{1}^{a}, ..., \alpha_{2}\lambda_{N}\sigma_{N}^{a}\right) \begin{pmatrix}\rho^{ee} & \rho^{ea} \\ \rho^{ae} & \rho^{aa} \end{pmatrix} \cdot \begin{pmatrix}(\omega_{1}\xi - \xi_{1})\sigma_{1}^{e} \\ \cdot \\ (\omega_{N}\xi - \xi_{N})\sigma_{N}^{e} \\ \alpha_{2}\lambda_{1}\sigma_{1}^{a} \\ \cdot \\ \alpha_{2}\lambda_{N}\sigma_{N}^{a} \end{pmatrix} [25]$$

Based on the conditions for values of the vector Ψ as in [10] we may reformulate equation for variance of z as in [25]. At this point it is necessary to notice that the vector Ψ can be expressed using the following variables: the first variable denoted by ξ is invariant to the selection of the currency j and is computed as the sum over all possible i currencies, thus $\xi = \sum_{i=1}^{N} \xi_i$; the second variable denoted as ξ_j depends on the choice of currency *j*. More precisely:

$$\Psi = \left(\omega_{j} \underbrace{\sum_{i=1}^{N} \left\{ \alpha_{1} \left[\frac{\beta}{\beta - 1} \gamma_{i} - \frac{\delta_{i}(1 - \eta)}{\beta - 1} \right] + \alpha_{2} \lambda_{i} \right\}}_{\xi} - \underbrace{\left\{ \alpha_{1} \left[\frac{\beta}{\beta - 1} \gamma_{j} - \frac{\delta_{j}(1 - \eta)}{\beta - 1} \right] + \alpha_{2} \lambda_{j} \right\}}_{\xi} \right), \qquad [26]$$
$$\Xi = \left(\alpha_{2} \lambda_{j} \right).$$

Then the variance of z may be expressed in the following way:

$$Var(z) = \left((\omega_{1}\xi - \xi_{1})\sigma_{1}^{e}, ..., (\omega_{N}\xi - \xi_{N})\sigma_{N}^{e}, \alpha_{2}\lambda_{1}\sigma_{1}^{a}, ..., \alpha_{2}\lambda_{N}\sigma_{N}^{a}\right) \begin{pmatrix}\rho^{ee} & \rho^{ea}\\\rho^{ae} & \rho^{aa}\end{pmatrix} \begin{pmatrix} (\omega_{1}\xi - \xi_{1})\sigma_{1}^{e}\\ .\\ (\omega_{N}\xi - \xi_{N})\sigma_{N}^{e}\\ \alpha_{2}\lambda_{1}\sigma_{1}^{a}\\ .\\ \alpha_{2}\lambda_{N}\sigma_{N}^{a} \end{pmatrix} [27]$$

In the next step we would like to minimize $Var(z(\omega))$ with respect to the vector ω in order to find the optimal values of the currency basket that minimize variations of the external trade and IIP caused by shocks to exchange rate and shocks to IIP.

$$\left(\frac{\frac{\partial Var(z(\omega^{*}))}{\partial \omega_{1}}}{\vdots \\ \vdots \\ \frac{\partial Var(z(\omega^{*}))}{\partial \omega_{N}}\right) = 2\xi^{2} \underbrace{\left(\begin{array}{ccc} \sigma_{1}^{e}\sigma_{1}^{e}\rho_{1,1} & \sigma_{1}^{e}\sigma_{2}^{e}\rho_{1,2} & \dots & \sigma_{1}^{e}\sigma_{N}^{e}\rho_{1,N} \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{N}^{e}\sigma_{1}^{e}\rho_{N,1} & \sigma_{N}^{e}\sigma_{2}^{e}\rho_{N,2} & \dots & \sigma_{N}^{e}\sigma_{N}^{e}\rho_{N,N} \end{array}\right)}_{\Theta} \left(\begin{array}{c} \omega_{1} \\ \omega_{2} \\ \vdots \\ \vdots \\ \vdots \\ \omega_{N} \end{array}\right) - 2\xi \left(\begin{array}{c} \sigma_{1}^{e}\sum_{k=1}^{N}\left(\xi_{k}\sigma_{k}^{e}\rho_{1,k} - \lambda_{k}\sigma_{k}^{a}\alpha_{2}\rho_{1,k+N}\right) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{N}^{e}\sum_{k=1}^{N}\left(\xi_{k}\sigma_{k}^{e}\rho_{N,k} - \lambda_{k}\sigma_{k}^{a}\alpha_{2}\rho_{N,k+N}\right) \\ \varepsilon \end{array}\right) \left[28\right]$$

First order conditions derived in the previous section will be used further for finding optimal values of currency basket weights by KKT method.

3.1.2. System of equations that satisfy complementary slackness condition

In order to find optimal value of vector ω^* we implement complementary slackness conditions directly into the system of linear equations as derived in [28].

Recall the case 2 in the complementary slackness condition:

$$\left[\eta_{i+N}g_{i+N}(\omega^*)=0\to\eta_{i+N}(-\omega_i^*)=0\right] \quad for \ \forall \ i=1,...N;$$

In order to satisfy the complementary slackness condition described above we would like to test all possible combinations where either $\eta_{i+N} = 0$ or $(-\omega_i^*) = 0$ for i = 1, ..., N.¹² We implement this condition into our system of linear equations with help of two square matrices that are created as follows.

The matrix χ_1 is a diagonal matrix with possible combination of -1 and 0 on diagonal and zeros off diagonal. The diagonal of one realization of the matrix χ_1 represents *exactly one* possible solution to the case [2] of the KKT slackness condition. The matrix χ_2 is a diagonal matrix with possible combinations of 1 and 0 on diagonal and zeros off diagonal in such a way that χ_2 complements the choice of 0 and 1 of the matrix χ_1 .

Joining these matrices together with a single zero column we create the following system of equations:

$$\begin{pmatrix} -1 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & & & & \\ 0 & \dots & 0 & 0 & \dots & 1 & 0 \\ \vdots & & & & & \\ 0 & \dots & 0 & \dots & 1 & 0 \\ \chi_1 & & & \chi_2 & & \\ \lambda & & & & \\ \lambda & & & \\ 2N+1)_{X1} & & \\ 0 & & \\ 0 & & \\ Nx1 \end{pmatrix}$$
 [29]

The possible choices of the values on the diagonals of the matrices χ_1, χ_2 are specified by a very simple algorithm:

for all
$$i = 1, ..., N$$

if $\chi_1(i,i) = -1$
then $\chi_2(i,i) = 0$
else $\chi_2(i,i) = 1$
end
[30]

In the next step we would like to rewrite one of the initial constraints, $\sum_{i=1}^{N} \omega_i - 1 = 0$, into the form compatible with the system of 2N + 1 equation with 2N + 1 variables:

¹² As discussed previously, we do not need to take into consideration such cases where $\eta_{i+N} = 0 \wedge (-\omega_i^*) = 0$. Such combinations may occur during the numerical computation as results of the minimization algorithm.

$$(1 \dots 1 \quad 0 \dots 0 \quad 0)_{1x(2N+1)} \begin{pmatrix} \omega_{1} \\ \cdot \\ \omega_{N} \\ \eta_{N+1} \\ \cdot \\ \eta_{2N} \\ \lambda \end{pmatrix}_{(2N+1)x1} = 1$$
 [31]

By joining systems of equations derived in [28], [29] and [31] the final system of 2N+1 equation with 2N+1 variables may be formally rewritten as:

There are 2^N possible systems of equation for different combinations of 0 and -1 for η_{N+i} and ω_i that satisfy KKT slackness condition in the case [2] of [22]. In our next step we will consider primal and dual feasibility conditions that allow us to narrow down the set of all possible optimal vectors that satisfy all three KKT conditions simultaneously.

3.1.3 Primal and dual feasibility conditions

Primal and Dual Feasibility conditions allow us to find only those vectors ω which satisfy all KKT conditions simultaneously. Firstly, let us consider the dual feasibility conditions. These state that all KKT coefficients $\omega_1, ..., \omega_N, \eta_{N+1}, ..., \eta_{2N}, \lambda$ must be non-negative.

In our case, KKT coefficients $\eta_1, ..., \eta_N$ are not included into the final system of 2N+1 equations. Based on the reasoning in Section 3 of this paper they equal zero. The case of the "corners"¹³ is to be checked separately.

For the 2^N systems of linear equations as described in [32] we will get 2^N possible conditions for the optimal vector ω^* , the KKT multipliers $\eta_{N+1},...,\eta_{2N}$ and the KKT (or Lagrangean)

¹³ Optimal solution in the case [1] of [22] as described previously.

multiplier λ . The condition $\sum_{i=1}^{N} \omega_i - 1 = 0$ is automatically satisfied for all 2^N possible solutions as this condition has been included into the system in the previous step in [32].

Thus, the optimal solution to the optimization problem as described in [11] will be given by the subset of the set of 2^N optimal vectors ω^* for which, in addition, condition $\forall \eta_{N+1}, ..., \eta_{2N} \ge 0$ is satisfied.

In addition to the general dual feasibility condition we impose an additional condition on the coordinates of vector ω^* . As the elements represent optimal values of currencies in the currency basket we require them to be non-negative and smaller or equal than one. Thus, we will consider only those ω^* where this condition is satisfied.

The final step of the optimization process will be to take all vectors ω^* that satisfy all of the previous conditions and compute the value of their variance, $Var(z(\omega^*))$. Vector ω^* with the lowest variance represents the optimal currency weights in the currency basket, thus ω^* is the solution to our optimization problem.

3.1.4. Comments on the numerical solution of the model

In our analytical derivation of the optimization problem we use the KKT method for finding optimal vector ω^* . In order to be able to satisfy complementary slackness conditions imposed by the KKT method we need to solve 2^{17} systems of equations as specified by the [32] and select those vectors that satisfy primal and dual feasibility conditions. Generally, for each and every possible system of equations as specified by the [32] to achieve unique solution matrix Θ needs to have to be of full rank.¹⁴

Unfortunately, this can not always be the case when using real data for external trade and net IIP positions for various countries. If the matrix Θ filled with real economic data happens to be almost singular (determinant of the matrix Θ is almost zero) numerical computation may result in treating this matrix as a singular one, therefore given incorrect results.

¹⁴ Let us recall the Frobenius criterion: given a system of linear equations $\Theta \omega = \mathbb{Z}$, there exists a solution if and only if $rank(\Theta) = rank(\Theta | \mathbb{Z})$. In our case, since Θ is (2N+1)x(2N+1) square matrix, this can be further elaborated as follows: either (1) $rank(\Theta) = 2N + 1$ which is equal to the condition $det(\Theta) \neq 0$, then there exists an unique solution given by $\omega = \Theta^{-1}\mathbb{Z}$; or (2i) $rank(\Theta) < 2N+1$ which is equal to the condition $det(\Theta) = 0$ and the case that $rank(\Theta) < rank(\Theta | \mathbb{Z})$, then there are no solutions; and finally the case of (2ii) $rank(\Theta) < 2N+1$, which is equal to the condition $det(\Theta) = 0$ and $rank(\Theta) = rank(\Theta | \mathbb{Z})$, then there exist multiple solutions.

Computationally, as such matrix is treated as a singular one two possible outcomes of the numerical computation will be considered: (1) there is no solution to the system of linear equations or (2) there are infinitely many solutions to the system of linear equations. In the first case, computational software will not provide us with the optimal solution for our vector ω in that single system of linear equations. Due to small precision of the computational software some of the possible solutions may be thrown away as the matrix Θ seems to be almost singular. Problems may occur when some of those solutions represent also the solution to the KKT method, yet due to the numerical imprecision they are discarded.

Unfortunately, without the possibility to compute with higher precision we do not see at this point way how to avoid this problem.

4. Data description and calibration

The model proposed by Zhang et al. (2011) is used for computing the optimal currency basket for 5 current member countries of the EMU – Spain, Italy, France, Germany and Portugal. The data used in our computation are drawn from following sources.

The external trade position of the European countries is publicly available from the official Eurostat database. We use data classified in form of Standard International Trade Classification (SITC) that are suitable for comparison on a worldwide basis. Data are denominated in ECU equivalents. Data are calculated on a yearly basis for the year 1994 as a base year.

Data denominated in ECU equivalents for total GDP are drawn from the Eurostat database and expressed in nominal terms. Data are calculated on a yearly basis for year 1994 as a base year.

Bilateral exchange rates for the selected countries are drawn from the database of the OECD on a yearly basis and calculated as a period average.

Data for the net international investment position in the geographical breakdown are not publicly available, according to our knowledge. Therefore we will use data computed by Kubelec and Sá (2010) that provide us with a detailed geographical composition of national external balance sheets during the period of 1980-2005. As those data are denominated in the current US dollar value we recalculate the original data from Kubelec and Sá (2010) with the annualized bilateral ECU/USD exchange rate published by OECD.

We use data for France, Germany, Italy, Portugal and Spain since these are the countries that are subject to calculation of the optimal currency basket composition. Bilateral positions of national external balance sheets are available for the set of 17 countries. Thus, the optimal currency basket for selected countries may consist from up to 17 currencies.¹⁵ Detailed description of the procedure used for estimation of external position of selected currencies is available in Kubelec and Sá (2010).

Although the United Kingdom does not currently belong to the Eurozone Area, due to a strong connection between members of the Eurozone and the United Kingdom we would like to closely analyze the share of the United Kingdom in the optimal currency baskets along with other countries of the European Union.

¹⁵ For tractability purposes we assume that domestic currency may be part of the currency basket, however the weight for domestic currency will always be zero.

As far as the export and import is concerned, usage of bilateral data for only 17 countries limits our analysis on circa 60 percent of the total export and import of the selected countries. Therefore we must be aware of all restrictions imposed on interpreting results of our analysis. Moreover, due to a lack of data on the total IIP position on bilateral basis we only use data relevant for 17 countries. Thus, the only conclusion that may be drawn is that our results are empirically valid for 60 percent of the total external trade of selected countries and the share of the net IIP position relevant for countries included into the sample.

Detailed description of data used for the analysis is available in Appendix II, Appendix III and Appendix IV. Based on the data available we compute the optimal structure of the individual currency baskets for the year 1994.

The standard value of 2 is used for the coefficient of elasticity of substitution η .

5. Empirical results

Based on the analytical solution derived in the section 3 we have created an algorithm for computation of optimal currency weights in the currency basket for 5 countries, current members of the Eurozone – Spain, Portugal, Germany, France and Italy. Percentage shares of optimal currency weights in currency baskets for those countries are available in Appendix V.

The empirical results show that the optimal share of currencies of countries joining Eurozone varies strongly among our sample countries; 37 percent for France and Germany, approximately 50 percent for Spain and Italy and up to 70 percent for Portugal. Those five countries may be viewed as a representative example of three different types of countries that have joined the Eurozone since 1999. On the one hand, we have countries such as France and Germany that are highly open toward countries outside the Eurozone in terms of external trade and international financial flows. On the other hand, countries such as Portugal with a strong connection in external sector to its neighbors and trade partners from the Eurozone represent second group. Finally, there is a group of countries that are as open towards external partners as close to the Eurozone members.

The high diversity in external sector suggests that adoption of the common currency, euro, has different effects on different groups of countries.

By adopting the common currency, capital flows and external trade do not dependent any more on the fluctuations of the exchange rate within the currency area. As observable in the data this is the situation especially in the case of Portugal and a bit less for Spain and Italy.

However, in the case of France and Germany most of the external sector is highly exposed toward the evolution of the exchange rate of partners from outside the common currency area. Thus, those countries will be affected by the variability of the common currency even after their joining.

Based on the empirical results we may conclude that major trading partners in terms of external trade and flow of international capital from third countries represent the US, Japan and Singapur, surprisingly. The case of Singapur is an interesting one because the high share of this country in the optimal basket of France or Italy is not a consequence of the significant position of Singapur as a trading partner in external trade.¹⁶ Its significant position in the optimal currency basket is rather resulting from its role as an international finance center.

¹⁶ According to the import data for the year 1994 in Appendix IV, share of Singapore on the total import of France or Italy is almost insignificant (0.5% for France and 0.3%).

Thanks to the theoretical model by Zhang et al. (2011) the optimal basket structure truly reflects the *dominant position of trading partners* not only in real economy, but also in the *financial sector*.

Note however, that the results regarding Singapore should be taken with caution due to the fact that we are not able to specify the true denomination of the net IIP position in a more detailed way. The results presented here are relevant only for the case where the total net IIP is denominated in the Singapore dollar, which is not likely to be fully satisfied in real economic conditions.

Another interesting result refers to the role of the British pound in the optimal currency baskets. United Kingdom surely represents a major trading partner not only in the sector of external trade but especially in the sector of financial services thus influencing the flow of international capital.

At a first sight, a very heterogeneous position of the United Kingdom or Singapore in optimal currency baskets of selected countries might speak against the possible efficiency gains from the common monetary union in the external sector. Secondly, the differences in optimal currency baskets of 5 countries are likely to speak against the idea of common monetary union.

In order to either confirm or reject this hypothesis we analyze the optimal structure of the common currency basket consisting only from countries outside the EMU.¹⁷ Results of this analysis are summarized in the Appendix VI.

From the overall perspective, the currency basket structure for Portugal, France, Germany, Italy and Spain is homogeneous to some extent. Three major economic partners: Japan, United Kingdom and United States are dominant in every single basket and are accompanied with a group of countries that accounts for one third of total shares on average. There are some irregularities in every single basket,¹⁸ yet the main message remains untouched. By creation of a common currency union there is a possibility to minimize shocks to the external sector caused by fluctuations of the common currency in the way that *will not harm* the external sector of *any member country*.

¹⁷ Argentian, Australia, Brazil, Canada, China, Hong Kong, Japan, Korea, Mexico, Singapore, United Kingdom, United States.

¹⁸ High share of Mexico in the Spanish currency basket or already discussed strong position of Singapore in French or German currency basket.

We do not claim that there will not be any pressure put on the external sector of every member country due to an adoption of a single currency. But there exists such a structure of the common currency basket that would minimize those fluctuations and this structure is *optimal for every* member country. In addition, this structure is optimal not only from the perspective of external trade but also from the perspective of international financial flows.

Moreover, the similar structure of the individual currency baskets points out possible synchronization of the external sector in case of shocks to the common currency, euro. Thus, there still will be shocks to the external trade and flow of international capital, yet they should have similar effects on the external sector of member countries and should not create distortions to the structure of the external sector among the member countries. Common monetary policy with respect to exchange rate should therefore be *fully effective*. From this point of view, the creation of the common monetary union among 5 European countries is justified.

Most of the irregularities in the structure of individual currency baskets are likely to be associated with the inclusion of the net IIP position into the theoretical model. Countries such as Hong Kong or Singapur do not represent the major trading partners in external import or export sector of Italy, Spain or Portugal, although they play a relatively significant role in the individual currency baskets of those three countries (see Appendix II and III). Yet, it is plausible to assume that financial flows between Hong Kong and Singapur are denominated in currencies of Italy, Spain or some third country at least to some extent. Thus, by relaxing the assumption of the LCP pricing for the net IIP position the optimal weight for Hong Kong or Singapur will be significantly lower.

Our conclusions should be interpreted very carefully. As mentioned in the theoretical part of this thesis, we heavily rely on the assumption of the full LCP pricing in the export sector and PCP pricing in the import sector. However, these assumptions may be valid in the long run but not necessarily in the short run. Moreover, due to the lack of data on bilateral positions in the net IIP we are not able to include other member countries of the Eurozone into our analysis. Different structure of either external trade or a high exposure towards different creditors in the net IIP may therefore alter our results significantly.

We strongly advice adjusting the basic model for possible variations of the pass-through effect by incorporating the possibility of different pricing methods both in the export and import side of the model. This will change the basic equation for export pricing by making

demand for domestic goods dependable on exchange rate fluctuations and varying with respect to different values of the elasticity coefficient. Similarly, domestic import will depend on variations in the exchange rate with respect to the elasticity coefficient incorporated into the model.

The same reasoning should be applied for the net IIP position. Due to the fact that the international financial assets and international foreign reserves have been denominated mostly in the US dollar since the Second World War, it is plausible to assume that the share of the US dollar in individual currency baskets would jump significantly.

Lastly, the model proposed by Zhang et al. (2011) carefully investigates variations in the external trade by differentiating between export and import side and modeling them separately. However, variations in the IIP side are not scrutinized carefully and only total value of the net IIP position is taken into consideration. Yet, as in the case of export and import, the IIP net position should be modeled separately with respect to the total sum of debit and credit entries. This approach would allow us to analyze dynamics of the IIP more precisely.

6. Conclusion

The main goal of this paper is to model an optimal structure of the currency basket for selected European countries that formed a common currency union in the late nineties. We assume that *a high share* of currencies involved in a currency union in the domestic optimal currency basket may serve as an indicator of positive effects of joining the common currency union and vice versa. Secondly, *similar structure* of individual currency baskets against third countries speaks in favor of creating or joining the common currency area.

Computation of optimal currency weights in the currency basket was based on the model derived in Zhang et al. (2011). Because the numerical procedure of finding optimal solution described in Zhang et al. (2011) was unsatisfactory for our needs, we have derived an analytical solution to the optimization problem using the Karush-Kuhn-Tucker method for nonlinear programming. With help of the analytical solution we have programmed an algorithm for computing the optimal currency weights in the Matlab programming language.

According to the analysis of optimal individual currency baskets of 5 current members of the Eurozone – Italy, Spain, Portugal, Germany and France, we may conclude that the creation of the common currency union in the late nineties *was justified* from the perspective of the optimal currency basket approach. Moreover, signs of presence of possible *synchronization in the external sector* against shocks to the exchange rate create room for efficient use of common monetary policy.

However, the results of our analysis should be interpreted carefully due to various reasons. Firstly, future possible research in this area should adjust the model for a possible incomplete pass through effect in the export and import sector. Secondly, by analyzing the net IIP position not only the exposure towards different countries should be taken into consideration, the denomination of the net IIP should also play a significant role. Additionally, the possible pass through effect of exchange rates on the value of credit and debit entries in the IIP position should be incorporated into the model.

Appendix I

Let us assume that the optimal currency basket consists of N currencies. Movements of exchange rate affect domestic economy through demand for export and import. Moreover, changes in exchange rate that affect international flows between domestic and foreign country are accounted in net international investment position (IIP). We assume that domestic country would like to minimize fluctuations to external trade and net IIP position caused by exchange rate shocks and shocks to international investment position. In this environment we assume, that in the short-run prices of export and import are fixed and sticky due to nominal rigidities.

Thus, total costs to domestic economy are expressed by costs function in the following way:

$$Z = \Delta NA + \Delta NX , \qquad [A1.1]$$

where ΔNA represents change in international investment position (IIP), ΔNX change in trade balance and variable Z total costs. Total effects are normalized by GDP in the initial period. Coefficients α_1, α_2 represent ratios of balance of CA and IIP on initial value of domestic product Y_0 .

$$NA_0 = \alpha_1 Y_0; \qquad NX_0 = \alpha_2 Y_0, \tag{A1.2}$$

The costs function may be rewritten in dynamic environment with logarithmic transformation where low characters indicate logarithmic version of change of selected variables, e.g. $d \log NX = nx$:

$$Z = \Delta NA + \Delta NX$$

$$z = \alpha_1 nx + \alpha_2 na$$
 [A1.3]

We assume that domestic country would like to minimize variance of total costs function with respect to the optimal currency basket weights or with respect to the shocks to exchange rate respectively. The general form of the model described in Zhang et al. (2011) allows negative optimal currency weights, yet we will further impose condition on non-negative values of currency weights in the optimal currency basket.

$$\min_{\omega_1,\omega_2,\dots,\omega_N} VAR(z), \quad s.t.\sum_{j=1}^N \omega_j = 1,$$
[A1.4]

In the optimization problem described above, ω_j represents share of currency j in optimal currency basket and j = 1, 2, ..., N.

Trade balance

Evolution of the trade balance depends on two economic variables that are modeled separately: export side and import side. Exchange rate variability will affect both of them in different way.

Export evolution may be rewritten in the form of equation, where γ_j represents share of export to country j on total export, e(j) represents change in exchange rate of domestic currency to currency of country j. Thus, we assume that total value of change in export due to changes in exchange rate becomes the average of movements in exchange rates, each weighted by the country's share in total export:

$$x = \sum_{j=1}^{N} \gamma_j e(j)$$
[A1.5]

Coefficient γ_j is calculated in base year as the share of export to country j on total export

where $X_0(j) = \gamma_j X_0$ and conditions $X_0 = \sum_{j=1}^N X_0(j)$ and $\sum_{j=1}^N \gamma_j = 1$ must be satisfied. By assumption, we use LCP approach on export side, thus value of total export to country *j* may be expressed as:

$$X(j) = E(j)P^{X}(j) * D(j)$$
 [A1.6]

where E(j) represents exchange rate of country j, $P^{X}(j)^{*}$ price of exported good denominated in foreign currency and D(j) represents demand for exported good in country j. Demand D(j) is not affected by changes in exchange rate as the price of export is denominated in foreign price by assumption of LCP pricing. In case of full PCP pricing, value of export would depend on domestic price and foreign demand would be elastic to the changes in exchange rate.

Import evolution may be rewritten in the form of equation, where η represents elasticity of substitution¹⁹, δ_j represents share of import from country j on total import, e(j) represents change in exchange rate of domestic currency to currency of country j and i(j) represents changes in import from country j.

$$i = \sum_{j=1}^{N} \delta_{j} i(j) = \sum_{j=1}^{N} \left[(1 - \eta) \delta_{j} e(j) \right],$$
[A1.7]

¹⁹ We assume that consumers have constant elasticity of substitution among imported goods.

Coefficient δ_j is calculated in base year as the share of import from country j on total import where $I_0(j) = \delta_j I_0$ and conditions $I_0 = \sum_{j=1}^N I_0(j)$ and $\sum_{j=1}^N \delta_j = 1$ must be satisfied. By assumption, we use PCP approach on import side which means that imported goods are denominated in the producer (foreign) country and evaluated by current exchange rate. Thus, home country demand for goods of country j may be expressed as:

$$C(j) = \left(\frac{E(j)P^{I}(j)^{*}}{P^{I}}\right)^{-\eta} C$$
[A1.8]

where $P^{I}(j)^{*}$ represents foreign price of imported goods, P^{I} is the aggregation of prices of imports from each country and *C* represents consumption index of imported goods in Dixit-Stiglitz (1977) form. Aggregation of prices of imports are expressed in the following way:

$$P^{I} = \left(\sum_{j=1}^{N} \left(E(j)P^{I}(j)^{*}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$
 Consumption index of imported goods is expressed in the

following way: $C = \left(\sum_{j=1}^{N} C(j)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$. Constant η in consumption index represents elasticity

of substitution among imported goods.

Net export is expressed as the difference between total value of export and import. Thus, NX = X - I. As we are interest in the changes in total net export, those changes are given by following equation:

$$nx = \frac{\beta}{\beta - 1}x - \frac{1}{\beta - 1}i$$
[A1.9]

where coefficient β represents initial ratio of total export to total import in base period, or $\beta = \frac{X_0}{I_0}$ respectively. Based on the previous derivations, evolution of net export expressed through changes in exchange rate may be derived as following:

$$nx = \sum_{j=1}^{N} \left[\frac{\beta}{\beta - 1} \gamma_j - \frac{\delta_j (1 - \eta)}{\beta - 1} \right] e(j)$$
[A1.10]

Net international investment position

For tractability we assume that all assets and liabilities are denominated in the currency of partner j. Thus, net international investment position which is calculated as the difference between assets and liabilities of home country against country j may be expressed as following:

$$NA(j) = \exp(\varepsilon^{a}(j))E(j)NA(j)_{0}^{*}$$
[A1.11]

where $\varepsilon^{a}(j)$ represents stochastic process²⁰ describing changes in IIP toward country j and $NA(j)_{0}^{*}$ represents initial value of net IIP denominated in currency of country j in the base year. The correlation matrix of all shocks to the IIP is assumed to be matrix of NxN dimensions with following properties: $\rho^{aa} = \{\rho_{ij}^{aa}\}_{NxN}$; $\rho_{ii}^{aa} = 1$; $\rho_{ij}^{aa} = \rho_{ji}^{aa}$.

In dynamic framework and logarithmic version, changes in IIP may be expressed by following equation.

$$na = \sum_{j=1}^{N} \left[\lambda_j \left(\varepsilon^a(j) + e(j) \right) \right]$$
[A1.12]

where λ_j represents share of IIP toward country j on total value of net IIP in t=0, $\varepsilon^a(j)$ represents stochastic process²¹ describing changes in IIP toward country j.

Total cost equation

Based on the previous reasoning we may express changes in total cost function with the following equation:

 $z = \alpha_{1}nx + \alpha_{2}na$ $z = \alpha_{1}\sum_{j=1}^{N} \left[\frac{\beta}{\beta - 1} \gamma_{j} - \frac{\delta_{j}(1 - \eta)}{\beta - 1} \right] e(j) + \alpha_{2}\sum_{j=1}^{N} \left[\lambda_{j} \left(\varepsilon^{a}(j) + e(j) \right) \right]$ $z = \sum_{j=1}^{N} \left\{ \alpha_{1} \left[\frac{\beta}{\beta - 1} \gamma_{j} - \frac{\delta_{j}(1 - \eta)}{\beta - 1} \right] e(j) + \alpha_{2}\lambda_{j} \left(\varepsilon^{a}(j) + e(j) \right) \right\}$

²⁰ We assume that a stochastic process has a zero mean and constant variance $(\sigma_j^a)^2$ and is correlated with change of exchange rate.

²¹ We assume that a stochastic process has a zero mean and constant variance $(\sigma_j^a)^2$ correlated with a change of exchange rate.

$$z = \sum_{j=1}^{N} \left\{ \left(\alpha_1 \left[\frac{\beta}{\beta - 1} \gamma_j - \frac{\delta_j (1 - \eta)}{\beta - 1} \right] + \alpha_2 \lambda_j \right) e(j) + \alpha_2 \lambda_j \varepsilon^a(j) \right\}$$
[A1.13]

Let us now derive evolution changes in exchange rates as a function of shocks to the exchange rate. We assume that evolution of exchange rate against currency j depends on the shocks to the exchange rate j and weighted sum of exchange rate shocks to other currencies.

$$e(j) = -\varepsilon^{e}(j) + \sum_{i=1}^{N} \omega_{i} \varepsilon^{e}(i)$$
[A1.14]

where $\varepsilon^{e}(j)$ represents stochastic process²² of shocks to exchange rate and ω_{i} represents weight of currency *i* in the optimal currency basket. Optimal weights in the currency basket should minimize variation in exchange rate caused by shocks to the foreign exchange rates. The correlation matrix of all shocks to the exchange rate is assumed to be matrix of *NxN* dimensions with following properties: $\rho^{ee} = \left\{ \rho_{ij}^{ee} \right\}_{NxN}$; $\rho_{ii}^{ee} = 1$; $\rho_{ij}^{ee} = \rho_{ji}^{ee}$.

Total cost equation with shocks

Using expression for changes in exchange rate together with the equation for changes in total costs we may derive total cost equation with shocks to exchange rate and shocks to international investment position in the following way:

$$z = \sum_{j=1}^{N} \left\{ \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{j} - \frac{\delta_{j} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{j} \right) \left(-\varepsilon^{e}(j) + \sum_{i=1}^{N} \omega_{i} \varepsilon^{e}(i) \right) + \alpha_{2} \lambda_{j} \varepsilon^{a}(j) \right\}$$

$$z = \sum_{j=1}^{N} \left\{ \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{j} - \frac{\delta_{j} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{j} \right) \sum_{i=1}^{N} \omega_{i} \varepsilon^{e}(i) - \varepsilon^{e}(j) \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{j} - \frac{\delta_{j} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{j} \varepsilon^{a}(j) \right\}$$

$$z = \sum_{j=1}^{N} \left\{ \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{j} - \frac{\delta_{j} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{j} \right) \sum_{i=1}^{N} \omega_{i} \varepsilon^{e}(i) \right\} - \sum_{j=1}^{N} \varepsilon^{e}(j) \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{j} - \frac{\delta_{j} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{j} \varepsilon^{a}(j) \right\}$$

$$z = \sum_{j=1}^{N} \left\{ \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{j} - \frac{\delta_{j} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{j} \right) \sum_{i=1}^{N} \omega_{i} \varepsilon^{e}(i) \right\} + \left(\psi^{T} \quad \Xi^{T} \right) \left(\varepsilon^{e} \\ \varepsilon^{a} \right)$$

$$z = \sum_{j=1}^{N} \left\{ \omega_{j} \varepsilon^{e}(j) \sum_{i=1}^{N} \left(\alpha_{1} \left[\frac{\beta}{\beta-1} \gamma_{i} - \frac{\delta_{i} (1-\eta)}{\beta-1} \right] + \alpha_{2} \lambda_{i} \right) \right\} + \left(\psi^{T} \quad \Xi^{T} \right) \left(\varepsilon^{e} \\ \varepsilon^{a} \right)$$

$$z = \left(\Psi^{T} \quad \Xi^{T} \right) \left(\varepsilon^{e} \\ \varepsilon^{a} \right)$$

²² We assume that a stochastic process has a zero mean and constant variance $(\sigma_j^e)^2$ correlated with a change of exchange rate.

where $\varepsilon^{e} = \{(\varepsilon^{e})_{Nx1}\}$ represents vector of shocks to exchange rate, $\varepsilon^{a} = \{(\varepsilon^{a})_{Nx1}\}$ represents vector of shocks to international investment position, Ψ a Ξ are vectors of coefficients with following formulas:

$$\Psi = \left\{ \omega_{j} \sum_{i=1}^{N} \left\{ \alpha_{i} \left[\frac{\beta}{\beta - 1} \gamma_{i} - \frac{\delta_{i} (1 - \eta)}{\beta - 1} \right] + \alpha_{2} \lambda_{i} \right\} - \left\{ \alpha_{i} \left[\frac{\beta}{\beta - 1} \gamma_{j} - \frac{\delta_{j} (1 - \eta)}{\beta - 1} \right] + \alpha_{2} \lambda_{j} \right\} \right\}_{Nx1} \quad [A1.16]$$
$$\Xi = \left\{ \alpha_{2} \lambda_{j} \right\}_{Nx1}$$

Variance of total cost

In our final step we would like to minimize variance of changes of total costs equation with respect to the weights of optimal currency basket. Firstly, variance of changes in total costs equation is expressed in the following form:

$$Var(z) = \Phi^T \cdot \rho \cdot \Phi , \qquad [A1.17]$$

where ρ represents correlation matrix between shocks to exchange rate and shocks to international investment position in following form $\rho = \begin{pmatrix} \rho^{ee} & \rho^{ea} \\ \rho^{ae} & \rho^{aa} \end{pmatrix}$, and Φ represents vector

of shocks weighted by their variances in following form:

$$\Phi = \begin{pmatrix} \Phi^{e} \\ \Phi^{a} \end{pmatrix} = \begin{pmatrix} \Psi_{1} \sigma_{1}^{e} \\ \cdot \\ \Psi_{N} \sigma_{N}^{e} \\ \Xi_{1} \sigma_{1}^{a} \\ \cdot \\ \Xi_{N} \sigma_{N}^{a} \end{pmatrix}$$

PARTNER/REPORTER	GERMANY (incl DD from 1991)	SPAIN	FRANCE	UNITED KINGDOM	ITALY	PORTUGAL
ARGENTINA	1 060	759	1 117	289	1 180	54
AUSTRALIA	2 447	228	893	2 459	1 096	44
BRAZIL	2 637	299	835	665	1 542	76
CANADA	2 284	357	1 421	2 451	1 463	103
CHINA (PEOPLE'S REPUBLIC OF)	5 350	656	1 831	1 085	1 921	15
HONG KONG	3 009	352	1 980	2 814	2 637	34
JAPAN	9 315	755	3 949	3 826	3 407	114
KOREA, REPUBLIC OF (SOUTH KOREA)	3 836	462	1 461	1 239	1 496	17
MEXICO	2 340	1 148	1 253	500	778	14
SINGAPORE	2 356	286	1 242	2 240	1 197	23
UNITED STATES	28 146	2 981	14 627	21 724	12 372	785
Non Eurozone Countries	62 780	8 281	30 609	39 291	29 089	1 279
FRANCE	43 173	12 355	-	17 347	21 184	2 223
GERMANY (incl DD from 1991)	-	8 608	37 259	22 080	30 781	2 858
ITALY	27 261	5 644	20 618	8 661	-	507
PORTUGAL	3 071	4 811	3 064	1 625	2 192	-
SPAIN 0	11 363	-	14 590	6 451	7 507	2 187
Eurozone Countries	84 868	31 417	75 532	56 163	61 665	7 774
	28 786	4 914	20 730	-	10 521	1 753
Subtotal	176 435	44 612	126 871	95 454	101 274	10 807

Appendix II Export Dataset Description for Year 1994 (mil. EUR)

Total	358 914	64 477	210 649	172 405	160 873	15 123
% covered by 17 countries	49.16%	69.19%	60.23%	55.37%	62.95%	71.46%
"Old Eurozone Countries"						
AUSTRIA	20 653	507	2 177	1 251	3 925	167
BELGIUM (and LUXBG -> 1998)	24 315	1 854	18 009	9 448	4 837	550
FINLAND	2 799	211	776	1 559	637	157
GREECE	3 001	565	1 497	1 197	2 907	67
IRELAND	1 647	263	1 231	9 188	561	75
NETHERLANDS	27 419	2 378	9 437	12 059	4 662	813
% from total	22.24%	8.96%	15.73%	20.13%	10.90%	12.09%
CYPRUS	524	58	393	309	257	13
ESTONIA	139	6	14	19	24	1
MALTA	255	41	158	255	888	9
SLOVAKIA	1 059	32	108	57	319	3
SLOVENIA	1 457	78	538	108	1 194	2
% from total	0.55%	0.21%	0.32%	0.37%	0.93%	0.17%
New subtotal	71.95%	78.37%	76.27%	75.86%	74.77%	83.72%

CEU eTD Collection

PARTNER/REPORTER	GERMANY (incl DD from 1991)	SPAIN	FRANCE	UNITED KINGDOM	ITALY	PORTUGAL
ARGENTINA	729	532	251	231	588	62
AUSTRALIA	790	230	681	1 341	767	28
BRAZIL	1 923	693	1 436	1 359	1 552	347
CANADA	1 946	322	1 327	2 426	1 224	60
CHINA (PEOPLE'S REPUBLIC OF)	7 641	1 169	2 649	4 280	2 627	145
HONG KONG	1 907	98	393	1 750	191	28
JAPAN	15 291	2 110	5 085	11 593	3 329	648
KOREA, REPUBLIC OF (SOUTH KOREA)	3 441	459	898	1 443	687	147
MEXICO	409	707	505	389	83	57
SINGAPORE	2 122	196	1 092	1 936	431	39
UNITED STATES	18 628	4 651	14 915	25 083	6 538	821
Non Eurozone Countries	54 828	11 167	29 232	51 831	18 017	2 383
FRANCE	36 334	13 449	-	19 307	19 388	2 915
GERMANY (incl DD from 1991)	-	11 365	42 113	28 023	27 463	3 161
ITALY	26 668	6 489	20 682	9 597	-	1 948
PORTUGAL	2 741	2 092	2 233	1 656	505	-
SPAIN ::;;	8 889	-	12 513	4 640	5 504	4 522
Eurozone Countries ଁ	74 632	33 396	77 541	63 223	52 860	12 546
	20 299	6 129	17 002	-	8 737	1 492
Subtotal	149 760	50 692	123 775	115 054	79 614	16 421
Total	320 624	74 705	206 807	196 782	142 214	22 749

Appendix III Import Dataset Description for Year 1994 (mil. EUR)

% covered by 17 countries	46.71%	67.86%	59.85%	58.47%	55.98%	72.18%
"Old Eurozone Countries"						
AUSTRIA	15 357	771	1 649	1 257	3 159	177
BELGIUM (and LUXBG -> 1998)	22 828	2 914	21 727	9 084	6 839	773
FINLAND	3 562	639	1 295	2 734	812	110
GREECE	1 623	190	450	462	1 131	27
IRELAND	3 714	686	2 782	7 484	1 345	154
NETHERLANDS	34 720	3 359	13 615	12 809	8 196	982
% from total	25.51%	11.46%	20.08%	17.19%	15.11%	9.77%
New subtotal	72.22%	79.31%	79.93%	75.66%	71.09%	81.95%
"New Eurozone Countries"						
CYPRUS	77	5	278	149	13	1
ESTONIA	98	4	10	48	14	8
MALTA	182	7	183	91	508	2
SLOVAKIA	1 138	36	104	85	307	5
SLOVENIA	1 737	32	543	136	767	7
% from total	1.01%	0.11%	0.54%	0.26%	1.13%	0.10%
New subtotal	73.23%	79.43%	80.47%	75.92%	72.22%	82.06%

CEU eTD Collection

	Annomilian	Australia	Duesti	Canada	China	France	0.0	lleng Keng	ltalu
	Argentina	Australia	Brazii	Canada	China	France	Germany	Hong Kong	italy
Argentina	1,00	-0,58	0,05	-0,75	-0,55	-0,37	-0,35	-0,69	-0,45
Australia	-0,58	1,00	-0,39	0,73	0,36	0,14	0,07	0,58	0,20
Brazil	0,05	-0,39	1,00	-0,13	0,10	-0,49	-0,36	-0,51	-0,23
Canada	-0,75	0,73	-0,13	1,00	0,70	0,22	0,25	0,73	0,32
China	-0,55	0,36	0,10	0,70	1,00	-0,06	-0,01	0,40	-0,03
France	-0,37	0,14	-0,49	0,22	-0,06	1,00	0,95	0,53	0,92
Germany	-0,35	0,07	-0,36	0,25	-0,01	0,95	1,00	0,42	0,94
Hong Kong	-0,69	0,58	-0,51	0,73	0,40	0,53	0,42	1,00	0,42
Italy	-0,45	0,20	-0,23	0,32	-0,03	0,92	0,94	0,42	1,00
Japan	-0,15	-0,03	-0,22	-0,06	-0,05	0,45	0,40	0,04	0,30
South Korea	-0,77	0,78	-0,25	0,89	0,59	0,38	0,37	0,80	0,41
Mexico	-0,30	0,54	-0,58	0,37	0,22	-0,03	-0,25	0,60	-0,19
Portugal	0,03	-0,05	-0,37	-0,30	-0,40	0,65	0,53	0,05	0,57
Singapore	-0,67	0,75	-0,44	0,90	0,56	0,25	0,22	0,82	0,26
Spain	-0,51	0,32	-0,29	0,37	0,02	0,90	0,84	0,59	0,92
UK	-0,50	0,44	-0,36	0,51	0,03	0,86	0,81	0,63	0,90
US	-0,67	0,60	-0,24	0,89	0,61	0,13	0,13	0,80	0,14

Appendix IV Correlations between Currencies (Based on Currency Invariant Index)

	Japan	South Korea	Mexico	Portugal	Singapore	Spain	UK	US
Argentina	-0,15	-0,77	-0,30	0,03	-0,67	-0,51	-0,50	-0,67
Australia	-0,03	0,78	0,54	-0,05	0,75	0,32	0,44	0,60
Brazil	-0,22	-0,25	-0,58	-0,37	-0,44	-0,29	-0,36	-0,24
Canada	-0,06	0,89	0,37	-0,30	0,90	0,37	0,51	0,89
China	-0,05	0,59	0,22	-0,40	0,56	0,02	0,03	0,61
France	0,45	0,38	-0,03	0,65	0,25	0,90	0,86	0,13
Germany	0,40	0,37	-0,25	0,53	0,22	0,84	0,81	0,13
Hong Kong	0,04	0,80	0,60	0,05	0,82	0,59	0,63	0,80
Italy	0,30	0,41	-0,19	0,57	0,26	0,92	0,90	0,14
Japan	1,00	0,16	-0,13	0,22	-0,13	0,22	0,21	-0,12
South Korea	0,16	1,00	0,33	-0,16	0,81	0,48	0,59	0,79
Mexico	-0,13	0,33	1,00	-0,07	0,65	0,00	0,06	0,55
Portugal	0,22	-0,16	-0,07	1,00	-0,26	0,65	0,55	-0,48
Singapore	-0,13	0,81	0,65	-0,26	1,00	0,31	0,46	0,95
Spain	0,22	0,48	0,00	0,65	0,31	1,00	0,94	0,20
UK	0,21	0,59	0,06	0,55	0,46	0,94	1,00	0,32
US	-0,12	0,79	0,55	-0,48	0,95	0,20	0,32	1,00

	Portugal	France	Germany	Italy	Spain
Argentina	0,30%	1,21%	1,44%	2,15%	2,32%
Australia	0,00%	0,00%	0,52%	0,00%	0,00%
Brazil	1,29%	2,63%	2,76%	2,37%	1,19%
Canada	0,46%	2,17%	2,70%	1,49%	0,01%
China	0,32%	0,99%	2,66%	1,60%	2,25%
France	27,26%	0,00%	15,88%	21,61%	36,40%
Germany	10,29%	24,83%	0,00%	24,03%	0,01%
Hong Kong	2,52%	0,31%	0,00%	4,88%	4,28%
Italy	7,02%	2,00%	15,03%	0,00%	15,23%
Japan	2,40%	8,58%	5,96%	4,66%	4,41%
Korea	0,27%	0,00%	3,45%	0,10%	0,00%
Mexico	0,00%	1,80%	1,58%	0,00%	1,13%
Portugal	0,00%	0,92%	2,71%	1,17%	9,67%
Singapore	2,01%	16,58%	0,00%	11,33%	0,86%
Spain	25,70%	11,26%	6,88%	6,84%	0,00%
United Kingdom	12,77%	18,51%	15,64%	9,75%	7,55%
United States	7,39%	8,22%	22,78%	8,03%	14,70%
Total	100,00%	100,00%	100,00%	100,00%	100,00%
From which					
Eurozone Countries	70,28%	38,09%	37,80%	52,48%	51,64%
UK	12,77%	18,51%	15,64%	9,75%	7,55%
Total Original Eurozone	83,05%	56,60%	53,44%	62,22%	<mark>59,18%</mark>
Total Outside Eurozone	16,95%	43,40%	46,56%	37,78%	40,82%
From which					
USA	7,39%	8,22%	22,78%	8,03%	14,70%
Japan	2,40%	8,58%	5,96%	4,66%	4,41%
Total USA + Japan	9,79%	16,80%	28,74%	12,69%	19,12%

Appendix V Optimal Currency Weights for Selected Countries for Year 1994

	Portugal	France	Germany	Italy	Spain	Average
Argenting	1.04%	1.98%	2.39%	4.79%	6.44%	3 3 3 9/
Aigentina	(-2.29%)	(-1.35%)	(-0.94%)	(1.46%)	(3.11%)	5.5570
Australia	0.59%	1.47%	2.06%	2.01%	0.95%	1 / 20/
Australia	(-0.83%)	(0.05%)	(0.64%)	(0.59%)	(-0.47%)	1.42 /0
Brazil	4.41%	4.20%	4.59%	5.32%	3.51%	1 11%
Diazii	(0.00%)	(-0.21%)	(0.18%)	(0.91%)	(-0.90%)	4.4170
Canada	1.49%	2.08%	3.82%	2.73%	1.69%	2 36%
Canada	(-0.87%)	(-0.28%)	(1.46%)	(0.37%)	(-0.67%)	2.30 /0
China	1.15%	3.16%	4.72%	4.12%	4.38%	3 51%
	(-2.36%)	(-0.35%)	(1.21%)	(0.61%)	(0.87%)	5.5170
Hong Kong	7.50%	0.87%	2.43%	8.50%	6.61%	5 18%
	(2.32%)	(-4.31%)	(-2.75%)	(3.32%)	(1.43%)	5.1070
lanan	7.58%	12.67%	10.00%	8.60%	11.33%	10.04%
Japan	(-2.46%)	(2.63%)	(-0.04%)	(-1.44%)	(1.29%)	10.0470
Korea	1.76%	1.02%	3.32%	2.21%	1.98%	2.06%
i lorea	(-0.30%)	(-1.04%)	(1.26%)	(0.15%)	(-0.08%)	2.0070
Mexico	0.84%	2.49%	2.48%	2.00%	7.80%	3 1 2%
WEXICO	(-1.37%)	(-0.63%)	(-0.64%)	(-1.12%)	(4.68%)	5.1270
Singanoro	0.52%	7.10%	3.49%	5.18%	1.79%	3 62%
olligapore	(-6.58%)	(3.38%)	(-0.13%)	(1.56%)	(-1.83%)	5.02 /0
United Kingdom	<mark>46.08%</mark>	28.65%	27.76%	27.99%	35.59%	33 21%
	(12.87%)	(-4.56%)	(-5.45%)	(-5.22%)	(2.38%)	00.2170
United States	27.06%	34.31%	32.95%	26.55%	17.93%	27 76%
onned Otales	(-0.70%)	(6.55%)	(5.19%)	(-1.21%)	(-9.83%)	21.1070

Appendix VI Optimal Currency Weights for Selected Countries for Year 1994 from the Perspective of the Common Currency Union

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