WHAT EARLY HUSSERL CAN TELL US ABOUT MATHEMATICAL INTUITION

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Submitted to

Central European University

Department of Philosophy

In partial fulfilment of the requirements for the degree of Master of Arts

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Budapest, Hungary

2014

Abstract

The work of early Husserl occupies an uneasy place in the history of philosophy. His first published monograph, *Philosophy of Arithmetic*, is often perceived as an immature work of an amateur psychologist, which was rightly criticized by Frege. The main objective of my thesis is to demonstrate that the utter rejection of Husserl's early project has been unjustified. More specifically, I argue that he anticipates both the contemporary definition and application of mathematical intuition. In order to establish this, I firstly show that Frege might have misinterpreted Husserl's definition of the concept of number, and so his influential characterization of Husserl's early thought may be misleading. I then proceed to the analysis of the definition and role of intuition in early Husserl's philosophy of mathematics and show that it highly resembles the use of intuition in the work of Charles Parsons, who is credited as the flag-bearer of contemporary proponents of mathematical intuition. Finally I address the question whether Husserl's use of intuition in *Philosophy of Arithmetic* can provide an answer to the access problem.

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1. INTRODUCTION

The common reading of Husserl's early work tends to be faultfinding: it is an immature attempt to marry psychology and the definition of the concept of number, in which he is reproducing the ideas of his teachers, Carl Stumpf and Franz Brentano, rather than offering original insights. What seems to have contributed largely to the negative reception of Husserl's early work was Frege's critical review of Husserl's *Philosophy of Arithmetic,* which has by now come close to having the status of a manifesto of anti-psychologism in mathematics. It seems to be a common way of thinking amongst analytic philosophers that since Husserl himself turns his back on psychologism in his later work, *Philosophy of Arithmetic* is not worth paying much attention to. Those who nevertheless focus on the correspondence of Husserl and Frege tend to conclude that it was precisely Frege's criticism that influenced Husserl to reject psychologism and turn to Platonism.

In my thesis I will argue that such a quick dismissal of Husserl's early work is largely unwarranted. I will claim that Husserl, in his definition and application of mathematical intuition, anticipated one of the contemporary uses of the concept. To this end I will focus on the work of Charles Parsons, for it is in his writings that the notion of mathematical intuition has been recently most thoroughly developed. Husserl applies the concept to the same cases – concrete perceptions – and for the same reasons – to explain how we have immediate knowledge of a body of mathematical truths. Finally I will show that Husserl's motivation for introducing intuition in his epistemology resembles that of contemporary mathematical realists –it may be seen as a means to answer the access problem.

All this said, I do not intend to claim that all of the criticisms that have been voiced both by Husserl's contemporaries and the more recent critics are entirely misguided. One of the biggest obstacles to appreciating Husserl's earliest work is the at times impenetrable obscurity of his writing, which often leaves one guessing just what Husserl's actual position on one or the other issue was. This becomes especially counterproductive when the issues in question are of central importance to the project of the book, as is the case with Husserl's ontological commitments. Thus, the reading of early Husserl can be an arduous investigative process the results of which are, of course, subject to interpretations and disagreements.

However, as I already mentioned, what I think is much less subject to disagreement is the answer to the question whether his work could be interesting to the contemporary philosopher. In order to establish my point I will first show how Frege largely misinterpreted Husserl's early position, and thus unquestioned reliance on his criticisms is not only counterproductive but also misleading. The third chapter will open with a historical introduction to the use of intuition in philosophy, both of mathematics and more generally. This will be followed by the analysis of the concept of mathematical intuition in *Philosophy of Arithmetic* along with the historical background of both end of the XIX century psychologism and where Husserl's position parts ways with it. In the fourth chapter I will center my attention on reconstructing contemporary uses of mathematical intuition, focusing primarily on the work of Charles Parsons.

The fifth, and final, part of the thesis deals with the question of whether and how could Husserl be seen as answering the access problem. Mathematical intuition is most commonly evoked as an account for the epistemological issues stemming from Benacerraf's Dilemma –how do we, beings located entirely in space-time, could have any contact, and thus, knowledge, of the ideal objects in the Platonic realm. As Husserl seems to be also committed to some form of realism, his use of mathematical intuition resembles that of contemporary realists not only in the way it is defined, but also in the way it is used to combat the afore mentioned epistemological problem.

Just what role intuition is to play in this will be determined by Husserl's ontological commitments. I address two plausible interpretations, that of a Platonist ontology and that of a critical realist one, in the vein of Kant and Brentano. I conclude by claiming that the issue of Husserl's ontological commitments remains an open question, as decisive reasons to favor one interpretation over the other have yet to be found.

2. HUSSERL'S EARLY PHILOSOPHY OF MATHEMATICS

2.1. A Philosophical No-Man's Land

Michael Dummett once observed that Frege and Husserl "may be compared with the Rhine and the Danube, which rise quite close to one another and for a time pursue roughly parallel courses, only to diverge in utterly different directions and flow into different seas" (Dummett: 1994, 26). The quote accurately captures the way Husserl is perceived by most contemporary philosophers– while his philosophical roots are akin to those of Frege, eventually he takes a radically different course, which is seen to be the most significant and fruitful time of his career. One need not look far for reasons. His later work[1] played a pivotal role in the development of both German and French existentialism, further developments in phenomenology and post-structuralism, in turn making him one of the "godfathers" of continental philosophy. Perhaps as a result, few analytic philosophers see much interest in Husserl's later work, and those who do take an interest in little else than finding parallels between his phenomenology and the contemporary theories of consciousness.

Husserl's earliest work, however, fails to arouse much interest in either of the two camps. Continental philosophers tend to see this period as essentially immature and thus not worthy of much attention. The very few who do read *Philosophy of Arithmetic* (henceforth *PA*) primarily focus on finding the roots of Husserl's phenomenological method in general, rather than his views on the philosophy of mathematics. This way his early mathematical and logical investigations are discarded as irrelevant to anything that followed them and *PA* is generally perceived as a failed attempt to salvage psychologism in philosophy of mathematics, one final digression before the truly relevant philosophical work.

[1] Here I will take the early period to be Husserl's writings from the preparation of his Habilitation Thesis to the publication of *Logical Investigations* (1887-1900/01).

Given that Husserl's early interests are exclusively in philosophy of mathematics, and that he corresponded with and commented on Frege, it would not be unreasonable to expect that his early period is more popular among analytic philosophers. However, the evaluation of his early writings seems to be one of the few things continental and analytic philosophers tend to agree upon – the period before *LI* and the revolutionary method of phenomenology is the time of apprenticeship, rather than original insights.

There are many reasons for this dismissal, not the least of which is the fact that Husserl himself viewed the majority of his writings before *LI* as misguided and immature. He remarks in his *Tagebuch* from 1921, June 4th:

> How immature, how naïve, how almost childish this work seems to me; and not without reason did its publication trouble my conscience… I was a mere beginner, without proper knowledge of philosophical problems, and with insufficiently trained philosophical skills" (Husserl –quoted from Bell—1990, 40).

At least part of the naïveté that Husserl is referring to has to do with his defense of psychologism that was a relevant influence on his early work. Indeed, by the time of writing *LI* Husserl himself has become an ardent critic of such an approach. The objective of his criticism is to show that the introduction of psychologism into apriori disciplines such as logic and other apriori domains will yield in unsolvable contradictions and is thus unfounded. The main target the thought of scholars such as John Stuart Mill,[2] Theodor Lipps, Gerardus Heymans, and Wilhelm Jerusalem.

What is characteristic for all of these thinkers is the contention that logic is in one way or another a part of psychology and thus should be studied appropriately. Husserl's has three prominent arguments against such views, outlined in Part Two of *LI*: (1) If logical rules

[2] Here it ought to be noted that it is not entirely clear what was Mill's position on the relationship between logic and psychology, however Husserl certainly takes him to be one to claim that logic is a part of psychology.

were modeled by psychological rules, the former would have to be as vague as the latter. Surely not all logical rules are vague, thus they cannot all be modeled upon the psychological laws (Husserl: 2001, 46). (2) The laws of psychology are not known a priori, contrary to those of logic, therefore the laws of logic are not equivalent to those of psychology (ibid., 47-48). (3) The laws of psychology refer to psychological entities, the laws of logic refer to eternal truths that are not psychological, therefore the laws of psychology are not the laws of logic (ibid., 51-54).

Nevertheless, it is not clear whether the criticisms in *LI* necessarily apply to everything in *PA.* I will elaborate more on this in the following chapters, however, here it will suffice to say that if we consider Husserl's main task in *PA* to be an account of how we gain access to abstract objects such as numbers, the psychologistic nature of the work may translate to nothing else but an epistemological orientation, rather than an inquiry into the nature of numbers. Furthermore, as I will show, Husserl certainly does not assume that numbers are subjective representations crafter through counting.

However, this was Frege's reading of *PA*; and as the majority of contemporary philosophers, especially those in the analytic tradition, come to know of Husserl's early work through Frege's criticism, it is not difficult to see why it is common to assume that Husserl's early writings are a essentially a failure. In order to argue that this is not necessarily the case, in the proceeding section I will show how Frege might have misinterpreted Husserl. Therefore the rejection of his work, based solely on Frege's criticism may be unwarranted.

2.2. Frege's Curse

There is an old tale amongst analytic philosophers that it was precisely Frege's critical review of Husserl's *PA* coupled with the latter's ardent study and the consequent embracement of Frege's views on logic and mathematics that led him to denounce psychologism in the first volume of *LI*.[3] Given this common conviction, namely, that it was Frege who was responsible for any meaningful claims about philosophy of mathematics Husserl has ever made, it is not at all surprising that Husserl's early work has been viewed so unfavorably. After all, if Husserl is essentially claiming the same things as Frege claimed before him, why not simply look at what Frege had to say?

Frege's anti-psychologism is a characteristic mark of his philosophy. He provided various counterarguments to psychologism, however the ones that reoccur most often and are the most prominent in his "Review of Dr. E. Husserl's *Philosophy of Arithmetic"* are the following. (i) Mathematics is exact, while psychology is vague and imprecise, and it is implausible to assume that one could be explained by means of the other. (ii) The subject matter of psychology are representations (*Vorstellungen*) that are inherently subjective, while the subject matter of logic and mathematics are ideal objects, that are universal and objective. If we reduce mathematics to psychology, we end up assuming that mathematical objects are mere representations, and thus are subjective. However, if they are subjective, how are we to account for their universal applicability? These and similar considerations lead Frege to conclude that Husserl's psychologistic philosophy of mathematics was a doomed venture.

It is true that *PA* is plagued with obscurities in argument, poor choice of terminology and many other issues that contribute to the perception of the work as a misguided attempt of an amateur psychologist. However, it is both a mistake and a large misinterpretation of Husserl's work to assume that all that is noteworthy in his philosophy of mathematics is there thanks to Frege.

Most importantly, Frege mistakenly takes the general aim of Husserl's philosophy of mathematics to provide a psychologistic account of the nature of numbers, in the vein of Mill. It rather was, as Tieszen accurately puts it, "an investigation into the a priori conditions for

[3] See Beth (1965, 353), Dummett (1973, 158), Follesdal (1994, 3-47), Sluga (1980, 39-40) and Sluga(1986, 3-47)

the possibility of the consciousness and knowledge of number" (Tieszen: 1994, 108). Mill's conviction was that numbers were nothing but our subjective representations of aggregates of units, and that such representations, and in turn our concepts, are crafted by the psychological processes involved in, say, counting. Nothing of this sort can be found in *PA.* If one pays careful attention and is patient enough with his obscure prose, it is clear that Husserl is attempting to give an epistemological account of our concepts of numbers rather than define their ontology. There is no talk of the nature of numbers or number representations and definitely no mention that they are in any sense of the term subjective. In this respect, as David Bell notes, "adoption of the methodological constrains which in part determine the nature of descriptive psychology is entirely motivated" (Bell: 1990, 61).

Husserl studied the works of Frege extensively –this is evident not only from his notes that can be found in the *Nachlass,*[4] remarks in his letters to Stumpf,[5] but also from the lengthy passage in *PA* devoted to the criticisms of Frege's theory. Indeed if one reads PA carefully, one of the prominent impressions that she may have is its contra-Fregean nature. Husserl, contesting the logicist approach to the definition of number, states: "no concept can be thought without foundation in a concrete intuition." (Husserl: 2003, 83). That is to say, in order to demonstrate the soundness of arithmetical knowledge, one must show how its concepts can be traced to concrete cases, or phenomenal experiences in which the number concepts appear. A definition of number as the extension of the "concept equinumerous with the concept F" is too artificial, that is, not what we mean by number in ordinary language.

However we might worry that both Husserl and Frege missed each other's respective points. While they are both attempting to give an account of the foundations of arithmetic, they seem to be answering two utterly different questions in philosophy of mathematics. Frege is more concerned with semantical issues and the truth conditions of the propositions in

[4] See (Husserl: 1970)
[5] See (Husserl: 1990)

which number terms appear. Husserl, on the other hand, seems to be convinced that no foundations can be provided without first giving an account of how knowledge of numbers is possible. What is interesting is that neither of them gave an all-encompassing account of numbers–Frege famously struggled in explaining how we come to have knowledge of ideal objects, if we are to disregard any experiential input; Husserl, in turn, without addressing semantical issues, does not have a proper account of the truth conditions of mathematical propositions. In some respect their dispute continues today, the only difference being that Frege's position is well known, while Husserl's is neglected in comparison.

At the risk of overgeneralizing, we may make a distinction between two contemporary approaches to the foundations of mathematics. One, in the vein of Frege's work, primarily focuses on the semantical issues concerning mathematics, say, the questions what makes mathematical propositions true. A currently very popular example of such an approach is neo-logicism, advocated, among others, by Bob Hale and Crispin Wright. As the name suggests, the theory has direct links to Frege, the essential difference between his work and that of Hale and Wright's being that that the latter replace Frege's ill-fated Basic Law V with Hume's principle to account for the issues arising from Russell's paradox. However, just as Frege did, neo-logicists struggle with explaining how we come to have knowledge of purely abstract mathematical entities. Which is to say, that the epistemological questions remain largely unanswered.

The second approach focuses primarily on epistemology, setting semantical questions aside or simply claiming that mathematical propositions are true in virtue of them referring to objects in the Platonic Heaven. Proponents of such an approach will often rely heavily on our everyday experiences in which mathematical propositions are involved. These theories primarily tend to focus on the simplest mathematical objects, such as small numbers or elementary mathematical operations such as, for example, addition of small numbers. In short, mathematical objects that could be accessible by way of direct experience. Examples of such theories are those of, among others, Charles Parsons (Parsons: 1980, 2007), Mark Steiner (Steiner: 1975) and Penelope Maddy (Maddy: 1980, 1997).[6] What is characteristic to these accounts is that a significant role is ascribed to intuition, which is supposed to account for our epistemic ability to immediately connect concrete instances with the corresponding abstract concepts.

Surely Husserl's work in *PA* did not influence the work of Parsons, Steiner, Maddy and others, to the extent that Frege's work influenced that of Wright and Hale. However we can evidently see the same strategies in Husserl's work as in that of the proponents of the epistemological approach to philosophy of mathematics. This makes *PA* not only a piece worth studying from the perspective of the history of philosophy, but also something that could potentially contribute to the ongoing debates in mathematical epistemology.

Before we move on the role of intuition in Husserl's project, some preliminary remarks ought to be made on how intuition is going to be used here. Due to its common employment in ordinary language, as something of a "mysterious sixth sense", the term and especially its application to epistemology tends to raise suspicion. To avoid possible misunderstandings and misinterpretations, in the following two sections I will provide a brief historical consideration of the philosophical application of the term and the definitions that followed it.

[6] Maddy's theory primarily deals with sets, rather than elementary numbers, but her account of knowledge of these mathematical objects is, nevertheless, based on everyday experiences.

3. MATHEMATICAL INTUITION IN *PHILOSOPHY OF ARITHMETIC*

3.1. Historical Introduction to the Notion of Intuition[7]

The Latin term *'intuitio'* first gained mainstream philosophical attention in the works of the scholastics. As Jaakko Hintikka notes, what is remarkable about the scholastics' use of *intuition* is its broad scope. In the work of Scotus "intuitive cognitions are those which (i) are of the object as existing and present and (ii) are caused in the perceiver directly by the existing and present object" (Adams: 1987, 501). In Ockham's thought "an intuitive cognition of a thing is that in virtue of which one can have evident knowledge of whether or not a thing exists, or more broadly, of whether or not a contingent proposition about the present is true." (ibid., 502).

What is characteristic to all of these uses of the term is that knowledge arrived to by way of intuition is taken to be immediate and spontaneous, therefore many scholastic philosophers came to equate intuitive knowledge with perceptual knowledge. However the broad scholastic notion of intuition was short-lived. As Hintikka notes, "The geometrical and mechanical vision of the world of early modern science showed that what looks like a direct perceptions is in reality a complex process involving all sorts of inferences, albeit often unspoken and even unconscious ones." (Hintikka: 2003, 170). In the light of this, the subject's relation to the perceived objects could no longer be thought as an immediate one, and thus the attribution of intuition to objects of perception lost its force.

Although this severely limited the scope of intuition, it was generally taken that the domain to which it could nevertheless be applied is "the world of our ideas and mental acts" (*ibid.*), as we do have direct and unmediated access to it, or so it was argued. This position is

[7] The historical reconstruction of the use of intuition from the scholastics to Locke here is in large part based on Hintikka (2003, 169-173).

well summarized by Locke, who claims that “*Intuitive knowledge* is the perception of the *certain* agreement or disagreements of two ideas immediately compared together” (Locke: 1999, 681).

Another philosopher of the Early Modern period in whose views intuition plays a significant role and who, at least in part, influenced the contemporary views of intuition was Descartes. In this brief summary it will suffice to mention two essential features of Cartesian intuition: (i) Descartes takes objects of our intuition to be independent of the mind:

> When, for example, I imagine a triangle, even if perhaps no such figure exists, or has ever existed anywhere outside of my thought, there is still a determinate nature, or essence, or form of the triangle which is immutable and eternal, and not invented by me or dependent on my mind (Descartes: 1985, 45).

Thus, if we have an intuition of some mathematical property, we have it in virtue of our minds somehow managing to forge contact with the mathematical objects or properties. It is important to note, however, that even though intuition is supposed to have the ability to put the agent in contact with the mind-independent objects, Descartes does not provide any novel conception of it and relies on its characterization of “immediate knowledge”, as did those before him.

(ii) While Descartes often mentions the analogy with perception when defining it, he takes intuition to be fully distinct from sensation:

> But if I want to think of a chiliagon, although I understand that it is a figure consisting of a thousand sides just as well as I understand the triangle to be a three-sided figure, I do not in the same way imagine the thousand sides or see them as if they were present before me. It is true that since I am in the habit of imaging something whenever I am in the habit of imaging something whenever I think of a corporeal thing, I may construct in my mind a confused representation of some figure. (Descartes: 1985, 50/ AT 72)

Alas, views of those as Descartes’ raises a further issues –how can we ever be sure that such intuitions would represent anything substantial about the world, if all they can show us is our ideas that may as well be considered to be subjective representations. Attempts to ground such justification, for example by way of innate ideas, which was central to

Descartes' philosophy, were heavily damaged by empiricist criticism and eventually fell out of fashion. This way, as Hintikka observes, these and similar criticisms "eroded the content of the idea of intuition until it became little more than a synonym for immediacy" (Hintikka: 2003, 171).

3.2. A Kantian definition of Intuition

The use of mathematical intuition gains its prominence after Kant's work; and, as Parsons notes, in the late nineteenth and early twentieth centuries it was precisely "Kant [who] offered a kind of a paradigm of a philosophical conception of intuition applied to mathematics" (Parsons: 2007, 193). A philosopher who falls decisively under this paradigm is Edmund Husserl, even though his application of intuition deviates considerably from that of Kant's.

In the first chapter of the *Critique of Pure Reason* (henceforth CPR) Kant writes:

> Our cognition arises from two fundamental sources in the mind, the first of which is the reception of representations (the receptivity of impressions), the second the faculty for cognizing an object by means of these representations (spontaneity of concepts); through the former an object is given to us, through the latter it is thought in relation to that representation.[…] Intuition and concepts therefore constitute the elements of all our cognition, so that neither concepts without intuition corresponding to them in some way, nor intuition without concepts can yield a cognition. (Kant: 1999, 193)

The passage introduces two central notions of Kant's epistemology –intuition (*Anschauung)* and concept (*Begriff*). The most relevant distinction between the two is that while concepts are mediate and general, intuitions are immediate and singular. As Janiak points out, each represent objects, properties or states of affairs but both do so distinctly (Janiak, 2009). While the precise relation between these two terms is a contested question between Kant scholars, the following are the most common ways to characterize it: intuition is (i) the epistemic ability of "picking out" a singular concrete X with all its perceptible properties (whether directly, or through imagination); (ii) the epistemic ability of immediately connecting the said particular to a concept, which attributes a distinct meaning to it.

The question whether intuition should be considered to be a content or an act, has been a matter of dispute among Kant scholars.[8] To the best of my knowledge, Kant does not concern himself much with this issue. Throughout *CPR* he often uses intuition to designate both a singular representation,[9] as opposed to a concept, and the act of attributing concrete singular representations to the general concept, making intuition a certain epistemic ability that allows the recognition that a particular concept belongs to this singular representation, an "immediate cognition" of kinds.[10] This ambiguity is also present in the works of both Husserl and Parsons, but the use of intuition as immediate cognition is dominant.

It must be noted that Kant is careful not to fully equate intuition with perception, even though intuition can be applied "only insofar an object is given to us by means of sensibility" (Kant: 1999, 155). Intuition is distinct from sensation alone, although Kant considers it to be "something akin to perception". Intuition, as opposed to mere sensation, is supposed to represent something about the object in question, while sensations alone can only convey information about the subject's states. However, intuition is not entirely distinct from sensation: "Objects are given to us by means of sensibility, and it alone affords us intuitions" (Kant: 1999, 172).

This leads to one of the central claims of Kant's epistemology, namely that sensation, through which intuitions are given, imposes certain forms on the intuitable objects –space being the form of all objects outside of the subject, and time being the form of all possible objects of intuition. This obtains to all knowable objects and is supposed to be the answer to the question of how are we to know anything *a priori* about the world, if all our knowledge stems from experience. In the introduction to the *CPR* Kant writes: "If intuition has to

[8] Among others see Hintikka (1969), Parsons (1982), Thomson (1972), Wilson (1975), Falkenstein (1991).
[9] See Kant (1999, 639/A731-B741), Kant (1819, 52-57)
[10] Kant (1999, 155/A19-B33).

conform to the constitution of the objects, then I do not see how we can know anything of them *a priori*; but if the object conforms to the constitution of our faculty of intuition, then I can very well represent this possibility to myself" (Kant: 1999, 110). What is meant by this is that *a priori* knowledge is possible because "the sensory world is constructed by the human mind from a combination of sensory matter that we receive passively and *a priori* forms [of time and space] that are supplied by our cognitive faculties" (Janiak: 2009).

Here we might ask just how can we have knowledge of mathematical objects then, if cognition first and foremost is grounded in intuition which functions similarly to perception. Mathematical objects in themselves, after all, are taken to exist in the abstract realm, rather than the concrete and perceivable form. Kant's answer to this is the claim that "we are intuitively aware of mathematical subject matter via illustrations that draw on our capacity of sensation" (Chudnoff: forthcoming, 15). This is to say that in order to cognize a mathematical proposition I have to have an intuition of a certain object that would correspond to it, either concrete or imaginary. To illustrate this claim, Kant discusses the cognition of a 'triangle': "Thus I construct a triangle by exhibiting an object corresponding to this concept, either through mere imagination, in pure intuition, or on paper, in empirical intuition…" (Kant: 1999, 630). This brings us to an essential feature of Kant's philosophy of mathematics. Intuition does not provide access to mathematical objects, it can only yield in knowledge of appearances and not about *things in themselves* (*Dinge an sich*). In this way the mind is attributed a creative role, providing access only to objects that are interpreted through the prism of human cognition.

Kant's ideas have been very important to later philosophy of mathematics and logic, both as a direct influence and as an object of criticism. The early twentieth century developments in the foundations of mathematics were directly influenced by Kant's thought, with both Hilbert and Brouwer mentioning their Kantian roots on more than one occasion.

Frege's logicism, at least in part, takes Kant's use of intuition in philosophy of mathematics to be one of the main ideas to be refuted by his logicism. What is more, towards the end of his life, convinced that his logicist program had failed, Frege turned precisely to the Kantian notion of intuition to establish the new foundations of mathematics.

The relation between Kant's ideas and those of the scholars' belonging to the psychologistic trend is ambivalent. The radical empiricism of Mill, at least in part, was a reaction to Kantian views. On the other hand, Husserl's teacher, Franz Brentano, as we will see, while critical of Kant's general project, incorporated crucial aspects of it into his own system. Other psychologistic oriented philosophers such as Paul Natorp and Friedrich Paulsen are frequently labeled as neo-Kantian. Both Paulsen, who was Husserl's teacher in Berlin, and Natorp, Husserl's colleague, had an influence on his early work.

However, Husserl's conception of intuition, while heavily influenced by Kant's work, should not be taken to be equivalent to Kant's. Firstly, it is not clear whether Husserl had the same ontological commitments as Kant did, or was closer to someone like Descartes, claiming that intuition provides direct access to mathematical objects. The latter interpretation seems dominant between early Husserl scholars. Most prominently it is defended by Richard Tieszen, who seems to take much of its supposed plausibility from the fact that Husserl does use intuition in the quasi-Cartesian way in his later works, dating from the publication of *LI.* However, Husserl's background and influences at the time of writing *PA* and his own criticism of realism, at least that in the vein of Frege, may suggest a different interpretation of his ontology. I will have more to say on this in the chapters to follow, but here it will suffice to note that it is in no way obvious just what Husserl's exact position was and one should be cautious not to attribute him views that were not his, either entirely Kantian or otherwise.

The most prominent difference between Kant and Husserl is, as I already noted above, that intuition for Husserl is primarily a form of immediate cognition, rather than something that designates a concrete particular (which in Husserl's terminology is simply a representation (*Representazion)*), and in this respect he is much closer to the pre-Kantian uses of the term. The same will hold to the contemporary views of intuition that I will address in here. Summarizing we may say that the following three aspects of Kantian intuition are common to both Husserl and the contemporary uses of intuition, as that of Parsons:

(i) Intuition is an epistemic capacity that is primarily involved in connecting concrete instances of objects with the abstract (or quasi-abstract) objects/concepts to which they correspond;

(ii) Intuitive knowledge that yields from intuition is immediate as opposed to mediate, inferential knowledge;

(iii) Intuition is primarily understood as intuition *of* objects, rather than intuition *that* a proposition is true.

This third point should be clarified. It is common to differentiate between intuition *of* and intuition *that*, where the former is part of a cognitive process that is involved in grasping a distinct object and the latter one is a process involved in coming to know – or seeing the intrinsic plausibility – of some proposition. Contemporary epistemology pays much more attention to propositional intuition, though this need not be all there is to it. Intuition *of* is preferred by those philosophers who emphasize the analogy between intuition and perceptual knowledge in the process of grasping mathematical concepts. On such a view, a necessary condition for an intuition to arise is the agent's direct awareness of the object. This is true not only of Husserl but also of philosophers such as Parsons, Maddy and, to a lesser extent, Steiner, for all of their conceptions of intuition involve direct connection to the perceived object.[11] What is characteristic of all of these views is that despite the differences in their proposed ontologies of mathematical entities, intuition is the crucial element in the agent's coming to have knowledge of a certain mathematical object.

Finally, before turning to Husserl's particular conception of intuition, I should consider more generally the epistemic status of the propositions that result from intuitions. On the classical analysis of knowledge, one knows that *p* just in case one has a justified, true belief that *p*. Do these propositions that we arrive to by way of intuition satisfy these criteria? In other words, does intuition result in knowledge? This question deserves a much more thorough and careful treatment than I can provide her. It will have to suffice to say that, at least, in the contemporary conceptions intuition is non-factive –intuition does not guarantee truth. Intuition is generally taken to be at play in cases of immediate cognition, but it need not necessarily yield in knowledge.[12] The degrees of reliability of intuition will vary greatly from one conception to another. However, at least in the part of contemporary philosophy of mathematics that will interest me here, namely the mathematical epistemology of Charles Parsons, it is taken that the degree of reliability of intuition is dependent on the competence of the cognizer. But the non-factiveness thesis remains: intuition, however competent the cognizer, may yield in faulty propositions,[13] and thus is does not, in itself, guarantee knowledge.

Here Husserl parts ways with the contemporary approaches considerably, for he considers intuitions to be factive. This, of course, is understandable, for Husserl is attempting to give an account of how we arrive at knowledge of number-concepts. Given the role that intuition plays in that process, it is desirable that intuition be reliable.

[11] This is not to say that Husserl or Parsons' theories do not allow for intuition of propositions, but the general form of intuition that is applied is primarily intuition of objects.

[12] Intuition that is modeled on perception may, for example, yield in faulty propositions for my perception may be flawed.

The biggest problem with this is that intuitions, even if they are defined merely as immediate cognition of sorts, are evidently very often fallible. To put it in other words, even if I clearly and distinctly perceive that *p* it does not seem to guarantee the truth of *p*. In his later career Husserl pays much attention to the issues arising from knowledge based on perception, such as the fallibility of our perceptions. However no talk of it is present in PA. Thus all I can do is provide a *guess* that he, following Descartes, Kant and Brentano, assumes that if one's intuition turns out to be wrong, one never had a genuine intuition in the first place.

3.3. The Role of Intuition in Husserl's *Philosophy of Arithmetic*

As we will see Husserl's analysis of the concept of number is first aimed at answering the question of how number concepts are known. The focus on epistemology rather than, say ontology or semantics, in Husserl's early writings was highly influenced by his work with Carl Stumpf, under whom he studied in Halle from 1887 to 1901. It was Stumpf's idea that the question of the knowledge of the concept has priority over the question of the content or essence of the concept, for it is only through the careful attendance to the origin of our knowledge of the concept that we are able to bring about the content of it.[14] Guided by such methodological provisions, Husserl thus claims that the major task of *PA* is to deal with the 'origin of the concept of number' (Husserl: 2003, 311) and is convinced that once the origin of our knowledge of the concept of number is illuminated, the essence of this concept will be obvious and will require no further considerations[15].

[14] In *On the Psychological Origin of Space Representations,* a book studied closely by Husserl, Stumpf writes: "The question 'Whence arises a representation?' is of course […] to be clearly distinguished from the other question, 'What is its knowledge content, once we have it?'. However these two questions are methodologically related, insofar as the question of the origin of a representation leads us to the separate parts of which it is composed, and therefore yields a more precise grasp of its content" (Stumpf: 1873, 3-4)

[15] What is meant here by 'origin of the concept' should be understood as a question about the origin of the knowledge of the concept, rather than the origin of the concept itself. Husserl main aim is to analyze our knowledge of number-concepts, rather than how concepts are created.

Husserl begins his analysis by distinguishing between authentic and inauthentic concepts of numbers. In short, we form an *authentic* concept by directly perceiving countable objects and by way of intuition get to the concept of number. In contrast, an *inauthentic* concept of number is generated in the mind without direct perceptual evidence – for example through symbolic representation. Since authentic representations require direct perceptual experience, we can approach only the elementary truths of arithmetic in this way. To surpass this limitation, Husserl builds more complex concepts of number from the elementary concepts and in so doing he can accommodate more sophisticated arithmetic.

In addition to what has been mentioned about Husserl's use of intuition in the previous section, one further clarification must be made. Since Husserl's goal is to define the concept of number, or, more precisely, how do we come to know numbers, he is interested in cases where we immediately perceive/experience an instance of it. In many cases the mind has to actively "purify" the perceptual representations of the irrelevant content in order to grasp a basic concept of number. As an example of this, let us consider the agent's perception of two cups on the table. Husserl claims that in order to arrive at the numerical concept 'two' the agent has to be able to get rid of all the irrelevant features, such as, say, where the cups are, what color they are, that one of them has a chipped edge, and so on, until she arrives at the simple intuition of two objects. The mind could arrive at such an intuition by, for example, ignoring or disregarding parts of a given complex whole; or combining or unifying singular parts of a given complex whole.

What is specific to Husserl's early views on the sort of intuition involved in the generation of elementary arithmetic concepts is that these primitive mental activities appear parallel to intuition. Since Husserl grounds his analysis of the concept of number in everyday perceptual experiences, he assumes that aside from being able to intuit, the mind must have a certain apparatus to simplify the intuition. And while the mental activities involved in simplifying the intuition can be conceptually distinguished from intuition, they nevertheless do not appear separately and thus are an integral part of the process through which we come to know number concepts.

Though perceptual awareness is crucial in defining a concept of number, it is not enough for the formation of the concept of, say, ‘the number three’, that I am able to perceive three books stacked by my bedside, for I can perceive the objects without actually grouping them and thus recognizing that a certain number can be attributed to them. Thus the first step in Husserl’s analysis is noting that the agent has a perceptual representation[16] of a determinate collection of objects including all of its specific features, such as its context and the particular properties of the perceived objects. In order to arrive at the numerical concept, however, I must be able to grasp the collection of objects as a unified whole having a distinct numerical value, and this is precisely where intuition plays a crucial role. By perceiving a concrete instance of objects, I intuit that a certain abstract numerical property can be ascribed to them, and it is in this way that we arrive at knowledge of distinct numbers.

As noted above, Husserl’s early conception of intuition goes hand in hand with a number of primitive mental activities, and the one that plays a central role in the generation of number concepts is *collective connection* (*kollektive Verbindung*). Collective connection is a way in which our mind reduces the initial perceptual representations to more basic intuitions. What is crucial here is that while I grasp a certain collection of distinct objects I am able to see them as belonging to one sort[17]. As Willard puts it, “in such a case, a new and distinctive type of whole is […] present to me with my field of consciousness: a totality or a multiplicity – a concrete unity of x number of objects” (Willard: 2003, xviii). Once I have grasped the collection of objects as a unified whole I am able to intuit the numerical property

[16] This need not be limited to actual perceptual experience i.e. Husserl also considers cases of counting where the objects involved are, say, imaginary or merely in our mind and not materially presented to us. Direct perception here is replaced with imagination. See (Husserl: 2003, 17)

[17] “Disregarding the properties that are different, we retain those that are common to all, as those which may belong to the concept in question” (Husserl: 2003, 19).

that ought to be ascribed to them. The role of collective combination along with intuition is thus to present the consciousness with an objective representation of a numerical property that can be applied to a multiplicity of perceived units.

One arrives at a fully abstract object (i.e. a quantity stripped of even its distinct numerical property) by performing yet another mental procedure, that of *abstraction*, by which one bracket all the remaining concrete parts of the collection of concrete objects perceived until he arrives at a concept of an indeterminate multiplicity. What follows from this is that a number in itself is primarily a certain multiplicity of units, a mere featureless 'something' (Husserl: 2003, 123). A distinct numerical quantity, on the other hand, is given to us by intuition and collective combination and is viewed by Husserl as property of that multiplicity. It is important to note here that Husserl is not describing how number concepts are created. His analysis requires that one already be equipped with a certain concept of number, intuition merely connects the particulars with such concepts.

A perceptual account of knowledge of number is surely incapable of dealing with all numerical concepts, since there is a limit to the number of distinct items we can explicitly notice or focus our attention on. In order to tackle this issue, Husserl introduces inauthentic concepts of number, which he defines as presentations via signs.[18] As Bell notes, Husserl's theory of symbolic representation has three essential features: (1) The signs themselves are perceptible items; whether written or spoken they can be the content of concrete representations. (2) They comprise a recursive progression, so that any possible combination of numerals has a unique place in this series, and so that the series of numerals is generated recursively, in that there is an effective procedure, which enables us to generate any later term in the series on the basis of earlier terms. (3) One or more of the earliest signs in the series must be correlated with the authentic concept of number (Bell: 1990, 56).

[18] "If a content is not directly given to us, as what it in fact is, but only given via signs that uniquely characterize it, then our presentation of that content is not an authentic, but rather, a symbolic one" (Husserl: 193).

We may summarize Husserl's definition of number as follows. Both authentic concepts of numbers and determinate numerical quantities are arrived at by perceptual intuition: I have direct contact with them in that I can immediately and assuredly say both that this aggregate of objects is a multiplicity and that the property 'having n units' applies to this multiplicity. On the other hand my ability to grasp larger numbers is based on my ability to recognize their place in the arithmetical system of signs and, as Bell notes, my understanding of large numbers reaches no further than my ability to correctly identify numerals and their arithmetical signs (Bell: 1990, 57).

What Husserl's analysis of the concept of number points to is that we cannot define number separately from the cognitive processes in which we come to grasp it. As noted above, such a definition becomes more of a specification of a priori conditions in which we come to grasp it, rather than a formal description of the concept, like the one that Frege proposes. In this sense, the non-inferential and immediate knowledge of number concepts that we arrive to by way of intuition is of crucial importance.

It is important to note that Husserl's use of intuition is not limited to mathematics alone. Indeed, he stresses that it is applicable in any case of *immediate cognition* and it surely should not be confined to the context of our knowledge of numbers. This observation will be especially important to his later work. In *LI* and onwards intuition gets developed into an independent faculty and, one of its forms, namely, categorical intuition, is explicitly intended to account for our knowledge of all ideal objects, whether mathematical or otherwise.

Because Husserl (perhaps due to his insistence upon the priority of epistemology) does not touch upon ontological issues concerning numbers, that is, it is unclear whether he takes them to be ideal objects, we cannot fully equate his conception of intuition in *PA* with that in *LI.* However, I think it is plausible to state that we can see the headwaters of this orientation already in the *PA.* It is defined in a similar fashion, namely as an immediate cognition of sorts, and it is as essential to our knowledge of numbers in *PA* as it is in *LI.*

3.4. Varieties of Psychologism: Mill, Wundt and Brentano

Before we leave early Husserl for the time being, one more feature of his project needs to be mentioned, namely psychologism. His teachers Franz Brentano and Carl Stumpf were one of the leading figures of the trend, thus In order to see what motivated Husserl's thought and, in turn, to understand his position better, it will be helpful to look a closer into just how psychologism was defined and what version of it, if any, did Husserl commit himself to.

The term 'psychologism' (*Psychologismus*) was coined by Johan Erdmann as a characterization of the work of Eduard Beneke (Erdman 1870). In a way Beneke sets the scene for psychologism, as it is in his works that we find the first explicit insistence on the reduction of philosophy to psychology and the claim that genetic analysis of mental phenomena is the only way to properly analyze concepts: an attitude shared almost univocally by those sympathetic to psychologism. However, while we are at it, it must be noted that there was no agreement neither over the definition of psychologism, nor of the list of scholars who can be truly considered to belong exclusively to this position. Providing a thorough and attentive analysis of the variety of approaches to psychologism is unfortunately beyond the scope of this thesis, thus here we will have to be satisfied with merely a broad-stroked characterization of the main features of it, which, of course, is at the risk of being over-generalized. However our task is to reconstruct the motivation and background of Husserl's thought and we should keep it front of us.

It is common to start one's characterization of psychologism, especially that of mathematical variety, with the discussion of the work of John Stuart Mill, as his *System of*

Logic (1843) was one of the main inspirations of the German psychologism. Mill's psychologism is well characterized by his views on geometry and arithmetic. What is essential to his position is that the truths of mathematics are not considered to be necessary. According to Mill, the only necessity is verbal necessity; as the truths of mathematics clearly have more than just verbal input, they are thus not necessary. Thus the truths of geometry and arithmetic are first and foremost empirical truths, the premises of which are established inductively. Mathematical truths and principles are not mind-independent objects existing outside space-time, but inductive empirical generalizations of the world around us. This way, the most direct way of defining the, say, the concept of number, would be to look into the psychological procedures that are responsible for the establishment of the such a concept. A number, according to Mill, is thus nothing other but a collection of objects, that we generalize over and name as being, say, "two" (Mill: 1843, 56).

Mill's approach was important both to the more general further developments of psychologism in the German speaking academia and to Husserl's position in particular. Husserl quotes and criticizes Mill's account on more than one occasion and takes it as one of the extreme ends of a faulty definition of the concept of number (the other is Frege's) (Husserl: 2003, 169-171). His central critical remarks bring to mind that of Frege's, namely if we consider numbers to be created through by an empirical observation of the numerical predicates of multiplicities, we are left without a proper account of neither the objectivity of numbers, nor their necessity. Here Husserl fully agrees with Frege in saying that "numbers are not heaps of objects" (Frege:1970, 8).

Husserl's criticism of Mill may be quite telling of his own position and more than that, it again shows how vastly Frege misinterpreted him by claiming that Husserl intends "everything to be shunted off into the subjective" (ibid., 9). Husserl's views of Mill's treatment of number concepts shows that he is not only aware of this but that he agrees with Frege and that he clearly intends number concepts to belong to the realm of objectivity.

Another significant influence on Husserl's early thought was the teachings of Wilhelm Wundt, with whom he worked in Leipzig and Berlin. Wundt is often credited to be the father of experimental psychology and the publication of his *Principles of Physiological Psychology* (1902) marks the birth of psychology as an independent discipline. What is mentioned more rarely is that Wundt primarily held a chair in philosophy. This should come as no shock as the majority of the end of the XIX century psychologists originally had positions in philosophy departments. It was often seen, that merging philosophy with psychology will give philosophy authority as it will make it more "scientific", this way making philosophical research more credible.

As Martin Kusch notes, the most characteristic feature of Wundt's views was that "the days when philosophy would figure as the foundation of the natural and of the human sciences were gone; instead philosophy was to be based on the results of these sciences" (Kusch: 1995, 127). However, it is important to note, that philosophy was not seen as something that should lose its meaning or should be considered a doomed venture all together. Instead Wundt saw it as a union of the various branches of science: "[whereas the various approaches] separate knowledge into a great number of individual objects of knowledge, the eye of philosophy is directed from the start towards the interrelation between all these objects of knowledge" (Wundt: 1902, 48).

Along with Wundt's *Principles,* another essential text of German psychologism was Brentano's *Psychology from and Empirical Standpoint* (1874). However, it is interesting to note, that Brentano, though a self-proclaimed psychologist, was much more interested in philosophical issues of epistemology, metaphysics and philosophy of mind than empirical psychology, which was much more Wundt's focus. In deed, by 1886 in his letter to Stumpf

Brentano notes: “I am now completely a metaphysician and I must confess that after having been a psychologist for a couple of years I’m glad of the change” (Stumpf: 1976, 16). And, as David Bell notes, although Husserl was undoubtedly interested in Brentano’s work in psychology, what influenced him most was “distinctively philosophical discipline which Brentano liked to call ‘descriptive psychology’, or ‘psychognosy’, or sometimes ‘descriptive phenomenology’ (Bell: 1990, 6).

It is important to note that neither Brentano nor Husserl considered descriptive psychology to be an empirical research. It is much better characterized as belonging to philosophy of mind, for it primarily is concerned not with contingent generalizations but with a priori truths and this sets the tone of Husserl’s thought. In deed, hints of Husserl’s dissatisfaction of introduction of Mill’s radical empiricism to the definition of the essence of number can already be found in Brentano, who claims on more than one occasion that the laws of psychology yield in inductively based generalizations which will be contingent and only probabilistic:

> [There are] two factors which prevent us from acquiring the exact conception of the highest laws of mental succession: first, they are only empirical laws dependent upon the variable influences of unexplored physiological presences; secondly, the intensity of mental phenomena cannot be subjected to measurement. (Brentano: 1874, 70).

Brentano’s approach, on the other hand, is aimed at the *a priori,* “it’s results are ‘exact’, ‘apodictic’ and ‘self-evident’, and it arrives at the laws immediately: ‘at one stroke’, ‘without induction’ (Bell: 1990, 6). Thus Brentano’s psychologism, much in the same way as Husserl’s, does not exclude the existence of ideal objects, knowledge of which may be *a priori.* This is well illustrated by Brentano’s famous dictum: “My psychological standpoint is empirical; experience alone is my teacher. Yet I share with other thinkers the conviction that this is entirely compatible with a certain ideal point of view” (Brentano: 1874, 3).

4. CONTEMPORARY APPLICATIONS OF MATHEMATICAL INTUITION

4.1. Mathematical Intuition Today: New Horizons, Old Distinctions

Contemporary philosophical approaches to intuition differ to the extent that "it is not at all clear that those who defend the idea of mathematical intuition, and those who attack it, have the same concept in mind" (Parsons: 2008, 139). However it is interesting to note, that the main divergence, at least in the way intuition's role has been perceived in the last one hundred years of philosophy of mathematics, can already be found in the Descartes and Kant's differing conceptions, which have been sketched above.

The crucial differences may be summarized in the following fashion: those who belong the Cartesian tradition (i) see intuition as an ability that puts the cognizer in direct contact to mathematical objects; (ii) intuition is defined as the "mind's eye", namely as something that allows us to see clearly and distinctly 'see' that, say, a certain mathematical proposition is true; (iii) the Cartesian conception of intuition need not be grounded in sensation, it need not have any empirical input, say as a concrete perception or imagination. This use of intuition is present in the work of, among others, Kurt Gödel and, to some extent, Penelope Maddy.

The Kantian tradition is most thoroughly argued for in the works of Charles Parsons. These views can be characterized by negating the first and the third theses of the Cartesians: (i) intuition does not put the cognizer into a direct contact with mathematical objects; (ii) intuition is primarily grounded in perception, either direct or imaginary. However, these characterizations do not necessarily contradict each other. Someone who defends the Kantian view, may as well accept the Cartesian definition of intuition, however insisting that these types of intuition answer to, say, different mathematical propositions or have generally different tasks.

The context in which mathematical intuition is applied is however different from that of Descartes' or Kant's times. While it has always figured in issues dealing with immediate and non-discursive knowledge whatever ontology the proponent has accepted, today it is primarily seen in the context of mathematical Platonism. More precisely, it is used as means in answering the epistemic issues arising from Benacerraf's dilemma. The dilemma first sketched in Benacerraf's influential article "Mathematical Truth", roughly summarized, states that no interpretation of mathematical truth encompasses both a coherent semantics and epistemology. Put in other words, if the interpretation of mathematical truth satisfies the requirements of a homogenous semantical theory, "in which the semantics for the statements of mathematics parallel the semantics for the rest of the language" (Hale, Wright: 2006, 1), it will clash with a reasonable epistemology.

What Benacerraf takes to be "a coherent semantical theory" is the classical correspondence theory of truth, best formulated in the works of Alfred Tarski. According to Tarskian interpretation, propositions are true in virtue of their reference to the corresponding objects. Therefore the semantical horn of the dilemma is formulated as follows:

> (S) Any theory of mathematical truth [ought to] be in conformity with a general theory of truth […] which certifies that the property of sentences that the account calls 'truth' is indeed truth. (Benacerraf: 1973, 408)

However just how should we think of the mathematical objects that such propositions are to refer to? It is generally taken that if there exist any abstract mathematical objects, they are ideal, existing outside space-time and thus causally inert. How then are we, physical beings existing entirely in space-time, supposed to merge any contact with these objects? Namely, if what guarantees the truth of mathematical propositions is utterly inaccessible to us, how can we say that we know any of these propositions? Thus Benacerraf formulates the epistemological horn of the dilemma:

> (E) A satisfactory account of mathematical truth […] must fit into an over-all account of knowledge in a way that makes it intelligible how we have the mathematical knowledge that we have. An acceptable semantics of mathematics must fit and acceptable epistemology (Benacerraf: 1973, 409)

It is not difficult to see how this dilemma threatens both Nominalist and Platonist approaches. Proponents of Nominalism will have no problem explaining how we may have an unproblematic epistemology, since distinct, mind-independent mathematical objects are banned from their ontology. However, in order to answer the semantic horn of the dilemma, they will have to reject Tarskian semantics and come up with a new way of establishing the truth conditions of mathematical propositions that would also be satisfactory outside the scope of mathematical language.

A mathematical Platonist then has to deal with a problem of explaining just how are we supposed to access and thus have knowledge of such mathematical objects. One way to account for this access has been by way of the application of mathematical intuition. What this strategy will involve is either the claim that (i) human beings have a special cognitive faculty that allows for the grasping of abstract mathematical objects and in this way puts the cognizer in direct contact with such objects (Cartesian intuition); or, somewhat more modestly, (ii) that intuition enables the recognition of concepts, however it does not put the cognizer in direct contact with the objects themselves (Kantian intuition).

It is generally taken that the first instance on the application of intuition to the access problem is found in the work of Kurt Gödel. Gödel claims that we gain knowledge of mathematical objects in much the same way as we gain knowledge of concrete objects, that is, by experiencing them in a certain way. This 'experience of mathematical objects' is precisely what Gödel considers to be mathematical intuition. Just as we perceive and experience concrete objects as actually present and true, we intuit mathematical objects as both actually present and true. Gödel writes:

> But, despite their remoteness from sense-experience, we do have something like a perception of the objects of set theory, as is seen from the fact that the

> axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense-perception. (Gödel: 1964, 271).

How does intuition have access to such objects remains a controversial aspect of Gödelian exegesis, though it is generally taken that Gödel considered our minds to be of the same nature as the abstract mathematical objects, namely immaterial. Thus the claim that mathematical objects are somehow epistemically suspicious simply rests on a faulty and overly naturalistic conception of the mind.

Gödel's view has been criticized by many, to the extent that it has now become more of a piece of philosophical mythology than a legitimate position. Among the many issues one may have with Gödel's use of intuition, perhaps the most prominent ones are the following. Firstly, Gödel's proposal is committed to an radical version of mind-body dualism. Since Gödel is defending a version of mathematical intuition where the contact with the objects in question is direct, he assumes that our minds are such that they have the ability to come into contact with entities of the Platonic realm, and thus themselves must be immaterial and located outside space-time. It seems extremely difficult to defend such a picture of the mind, and a very few Platonist are ready to pay such a great price for the answer of the access problem. The second problem is that even if one agreed with the dualism Gödel's view entails, his proposal does not seem to solve the access problem but rather postpone it. One may be fully justified in asking just how it is the case that our immaterial mind comes into contact with mathematical objects. Merely postulating the faculty of intuition does not seem to be helpful, for the lack of access problem still remains unanswered.

In the light of these and other criticisms, the contemporary use of mathematical intuition has taken a different turn. The new approach my be best characterized by claiming that "we posses a psychological apparatus whose only ultimate sources of information are the naturalistic sources of perception and introspection, but that nevertheless generates intuitive beliefs and thoughts about mathematical objects (or structures or patterns)" (Balaguer: 2001, 37).

The most prominent defender of such a conception of mathematical intuition is Charles Parsons, though similar accounts have been given by Jerold Katz and Mark Steiner. In addition to the latter, Stewart Shapiro and Michael Resnik have been proposing something along the lines of a naturalized intuition, even though they prefer the term 'abstraction' over 'intuition'. This naturalized version of intuition is generally considered to be taking inspiration from Kant. However in what follows I will show that in its definition and application it is closer to that of Husserl's.

4.2. Parsons' Application of Mathematical Intuition

I now turn the discussion of the parallels between Husserl and Parsons' applications of intuition. Insofar as Husserl anticipated an application of intuition, in the vein of Parsons, his theory should be of interest to contemporary philosophy, and both the continuities and the discontinuities between his and Parsons' views may be quite telling.[19]

Parsons' begins his article "Mathematical Intuition" with the claim that if mathematical intuition is to be relevant to philosophy of mathematics, "it should play a role like that of sense-perception in our knowledge in every day world and physics" (Parsons: 1980, 145). This way stating that the central feature of intuition is the analogy between sense perception as a cognitive relation to the physical world, and "something like a perception" giving a similar relation to mathematical objects (Parsons: 1980, 145). Such intuition, Parsons is convinced, will be very limited in scope, in the sense that it will only cover the

[19] It must however be mentioned that drawing parallels between Husserl and Parsons views on mathematical intuition has been done before – most notably by Richard Tieszen, who has argued that Husserl's notion of intuition can help Parsons account for the difficulties his theory encounters when faced with potential infinities (Tieszen: 1984). However no work has been done, either by Tieszen, Parsons or others to investigate the relation that Parsons' view of intuition could bear to that of early Husserl's put forth in the *PA*, his only work devoted solely to philosophy of mathematics.

simplest cases of elementary geometry and arithmetic. Here I will focus mainly on his conception of intuition of elementary arithmetical concepts.

The manner in which Parsons present the notion of intuition is quite akin to that of Husserl's, namely through providing a genetic analysis of the concept of number. However, while in *PA* the use of intuition was evoked as a necessary condition for arriving to the concept of number, in Parsons' work we may note the reverse –the conditions in which we grasp the concept of number are reconstructed as means to define and illuminate the use mathematical intuition. However differing the motivations, in both of their works we may find something of an explication of a priori conditions for our knowledge of elementary mathematical concepts.

Contrary to Husserl, Parsons, while focusing mainly on epistemological issues, does not leave ontological considerations aside. At the very least, they are required to see what exactly does intuition provide access to–concrete objects, abstract or some sort of intermediary between the two?

Thus, a critical feature of his project is a three-level ontology, the constituents of which are purely concrete (physical level), quasi-abstract (conceptual level) and purely abstract objects (the objects in the Platonic Heaven). The objects of mathematical intuition are the quasi-concrete, while abstract objects are taken to be causally inert and thus not accessible by any of our epistemic faculties. This is why we cannot have an intuition of natural numbers, however we can have intuitions of quasi-abstract structures that represent the numbers.

A way to understand the distinction better is to see the quasi-concrete objects as a certain structure of concepts and concrete objects as instantiations of parts of that structure. A good illustrative example of this is Hilbert's conception of finitary mathematics. Hilbert, similarly to Husserl and Parsons, asserts that there is a body of elementary mathematical objects that are known to us intuitively, namely, immediately. Furthermore, Hilbert insists that the existence of such primitive and intuitively known objects is necessary, for they are the underlying conditions of any sort of higher-order reasoning (see Hilbert: 2002, 376).

Parsons, following Hilbert, considers such objects to be strings of strokes that he calls types, while the concrete instances of the types are tokens and are understood as objects given to us in concrete perception. What makes up a type is a special geometrical form composed of strokes. According to Parsons, purely abstract entities, such as numbers, are defined by types. Thus if I wanted to define elementary natural numbers in this language, I could say that 1 is defined by a type that has the form of |, 2 is a type that has the form of ||, 3 is a type that has the form of |||, etc. I can recursively generate greater and greater numbers by the process of repetition.

By perceiving a concrete token of the type (say two cats on the fence) the cognizer intuits its type (||). Therefore the role of intuition is, roughly put, to immediately connect some object of perception, a concrete token, with a concept of a type[20] – to make the form of the type immediately clear.

Though Husserl discusses the role of intuition in the formation of the concept of number in much more detail than Parsons, paying careful attention to the processes that come parallel to intuition (as collective combination and alike), in both of their approaches intuition is defined in the same fashion. It is a cognitive process that is essential for the acquisition of elementary mathematical objects. In both of their uses of the term intuition puts the agent into a direct cognitive relationship with an intuited object or, in the case of Husserl, a property of a certain object.

Furthermore, Parsons' use of intuition is much closer to Husserl's than that of Kant's. While, as was indicated previously, Kant uses intuition to designate both immediate

[20] See (Parsons: 1980, 103)

cognition and a singular representation, the latter is much more prominent in his epistemology. We may note the reverse in Husserl and Parsons –intuition is understood almost exclusively as immediate cognition, and very rarely used to designate a singular representation.

For Kant intuition is a necessary condition for knowledge of any mathematical objects. Husserl and Parsons, on the other hand, apply it only to a very limited body of mathematical objects. Both Husserl and Parsons are convinced that it simply is implausible to assume that intuition will be applicable to objects that are too complex to be perceived. Therefore, Husserl evokes the notion of inauthentic concepts of number, namely those that are not directly experienced and thus not given by perceptual intuition. Parsons makes this point very clear with his criticism of Maddy's application of intuition to set theory (Parsons: 2007, 167) where he states that set theory is too complex and relies too heavily on non-empirical observations to be an object of mathematical intuition.

All that said, I do not intend to claim that Husserl and Parsons' use of intuition is identical. Parsons' evocation of the term is much more nuanced and does not commit itself to such counterintuitive claims as Husserl's insistence that the only true mathematical knowledge we have is that of elementary mathematical concepts and those that can be traced back to such concepts. In this respect, Husserl can be seen as advocating a crude and early version of causal theory of knowledge. The main assumption behind such a view is that in order for some belief to be considered knowledge it has to be properly caused by a truly existing object or state of affairs which the belief addresses. The only proper mathematical knowledge that we can have is that which is caused by the perceivable objects that our beliefs are about, such as a collection of countable items and alike.

Many contemporary epistemologists and philosophers of mathematics are skeptical towards this view[21]. Among other reasons, it cannot account for a body *a priori* propositions that we plausibly consider to be knowledge. Parsons is thus cautious not to commit himself to such views, stating explicitly that his evocation of intuition does not imply an account for all mathematical knowledge (Parsons: 2007, 152).

However, setting this difference aside, I think it is plausible to assume that Husserl nevertheless anticipated the conception and application of mathematical intuition, in the vein of Parsons. It is not in Kant's or Gödel's but in Husserl's early work that we find the first explicit articulation of the relevance of mathematical intuition to the knowledge of elementary mathematical concepts in a sufficiently similar way to the contemporary use of the term.

[21] Among others see Collier (1973, 350-352).

5. Husserl's Conception of Mathematical Intuition and the Contemporary Debates

5.1. Husserl and the Access Problem

It is often asserted that Husserl's main philosophical objective throughout his career was to unite two competing assumptions about knowledge: (i) knowledge involves ideal objects; (ii) knowledge can only arise from experience.[22] This resonates well with the access problem, which, as we have seen, can be put in similar terms. If Husserl is addressing this issue already in *PA*, relating his use of intuition to the contemporary debates becomes even more plausible, as we can then claim that Husserl, in part, applies intuition to answer the access challenge similarly to some contemporary platonists.

To see whether Husserl's application of intuition could be at all useful to the access challenge, we must begin with the question of his ontological commitments in PA. This is a burdensome task, especially given that nowhere does Husserl fully state his stance on the issue; one rather has to reconstruct it from controversial hints found all throughout the book. The primary question here is: did Husserl consider numbers to be ideal in any sense of the term?

It is tempting to interpret him as stating that numbers are crafted through counting, and thus are abstract only in the sense that they are generalizations over physical objects. This is a popular interpretation for several reasons. Firstly, it is given in Frege's influential review of *PA,* which I discussed in section two of this paper. Secondly, the very nature of Husserl's inquiry would suggest this kind of reading. After all, the inquiry into the concept of number should address the essence of number, a supposedly abstract and objective entity, not merely a description of the conditions in which we grasp the concept. Therefore, Husserl's

[22] Among others see Bell (1990, 23), De Boer (1978, 12), Miller (1982, 89-100).

definition, at least on the face of it, suggests that the concept of number is a *psychical* entity, as he calls it following Brentano, without any further connections to the objective realm. The closest he comes to objectivity, as we have already seen, is by calling a number “something”, an object that we have abstracted away from all of its particulars:

> If this designation [i.e. number] is to a have a genuine basis, then it must reside in the characteristic common to each and every one of those contents. But there is only one all-encompassing concept - that of the something. (Husserl: 2003, 123)

However the vast majority of early Husserl scholars tend to see his early work in a different light.[23] According to, for example, Dallas Willard, despite “the many confusions of thought and language [in Husserl’s] earlier works, he never thought that number and the laws of number were in any usual sense of the word “psychical”’ (Willard: xxvii). According to Willard, Husserl’s formation of the concept of number points at “on-reaching, higher order intentionality carrying over to the totality of whatever things are being counted, that totality not being the part of the act or dependent for its nature or existence upon the act” (Willard: 2003. xvii). Thus Willard holds that the term ‘psychic’ was rather an “incredibly misfortunate choice of phrasing” (ibid.) than an actual conviction of Husserl’s.

It may also be the case that Husserl simply takes it to be a matter of fact that our knowledge of numbers is knowledge of abstract objects, in one or the other sense of the term. Husserl’s prose can be frustratingly obscure, as he seldom offers clear and sharp distinctions and definitions, however if we are careful to distinguish mental acts and objects towards which they are directed, it is clear that he does not intend number concepts to be subjective representations. Neither does he consider mathematics to be an extension of psychology, what Frege has famously accused him of.

Furthermore, that Husserl was taking some sort of a realist stance can be also inferred from his harsh criticisms of nominalist approaches to mathematics. For example, he writes:

[23] Among others see, Bell (1990, 59), Tieszen (1994, 97-99), Miller (1982, 89-100).

“nominalism loses both concept and essence, and has no way of elucidating the relationship of symbols to that which they represent” (Husserl: 2003, 179). However what sort of realism was Husserl actually advocating by the time of writing *PA* remains a mystery.

The vast majority of early Husserl scholars are inclined to claim that already by the time of *PA* Husserl was a full blown Platonist. However this interpretation seems to take its force not from actual textual evidence from either *PA,* nor texts written around that time, neither from the positions of those who influences Husserl, but from the fact that Husserl later in his career argues for radical Platonism, thus, for some reason we should also assume that he is a Platonist in *PA.* This, among others, is the position of Claire Ortiz-Hill (Ortiz-Hill: 2000, 95), Guillermo Rosaddo Haddock (Haddock: 200, 199) and Richard Tieszen’s (Tieszen: 1994, 99). In what follows I will discuss two possible interpretations of early Husserl’s ontological commitments in relation to the access problem –the first one being that of full-blown Platonism, and the other that of critical realism, in the vein of Kant and Brentano.

5.2. Husserl as a Full-Blown Platonist: Between Maddy and Parsons

Tieszen has argued that not only does Husserl intend number concepts to be objective by the time of writing PA, he means them to be purely abstract entities existing outside space-time (Tieszen: 1994, 98-100). He grounds his interpretation by claiming that we should see Husserl’s work in PA as an attempt to provide a genetic analysis of the concept of number. This sort of analysis takes up a crucial part of Husserl’s later philosophy, and he famously mentions the relevance of it in “The Origin of Geometry” (Husserl: 1973), stating that task of a genetic account of arithmetic will be to explain how we may have knowledge of ideal objects, namely those located outside space time. Thus, Tieszen’s claims, since the method in *PA* is sufficiently similar to his later works, where the notion of genetic analysis is fully formulated, we ought to conclude that he indeed considers numbers to be ideal objects and that he is attempting to give an answer to the question of how we come to know them. Therefore, Tieszen continues, Frege's criticism was utterly misguided since Husserl was already a full-blown Platonist by the time of writing PA, as he clearly intends numbers to be "outside space-time, immutable and acausal" (ibid, 98).

This certainly was Husserl's position in *LI,* where he insists that there is a crucial difference between concrete instances of numbers and numbers as ideal objects:

> It is… *evident* that when I say 'Four' in the generic sense, as, e.g., in the statement 'Four is a prime number relatively to seven', I am meaning the species *Four,* I have *it* before my logical regard, and am passing judgment on it, and not on anything individual. I am not judging about any individual group of four things (Husserl: 2001, 239)

Numbers are not localized in space and time, they are 'timeless unities' (ibid. 240), however we who *intend* them and have them intuitively present are located in space and time (Miller: 1982, 90). Husserl goes on to argue that the being of such ideal objects and knowledge of them can be illuminated only through a phenomenological inquiry, the project of which indeed has its headwaters in *PA.* Having this in mind it is not difficult to see how an interpretation such as Tieszen's may seem plausible. If the only substantial difference between his early and later period is the changing of the term descriptive psychology to phenomenological analysis, why not assume that he was a full blown Platonist already by the time of writing *PA.*

If Husserl indeed considers number to be ideal objects, his views on intuition might bring to mind not only the views of Charles Parsons but also those of Penelope Maddy. Maddy's theory in a nutshell is this: we have knowledge of abstract mathematical entities because we are causally connected to them by direct sense perception. If Gödel's strategy was to show how our minds could ascend to the Platonic realm, Maddy attempts to show how the abstract entities could descend to the physical realm. Maddy's main focus is on sets, and her claim is that sets are spatio-temporally located and that knowledge of them is given to us by perceptual intuition.

A spatio-temporally located set is any aggregate of objects that I can perceive (either by direct perception or by imagination); by perceiving a stack of three books I grasp by intuition that it is a set containing three elements. Intuition here is understood as a hybrid of that of Kant and Gödel's: perception is necessary for it, but it puts us in direct contact to mathematical object, not merely our representations of them. We could see Husserl as painting a similar picture: by perceiving a concrete collection of objects I come to know, by way of intuition, that a certain numerical property, which is also considered as a mathematical object of sorts, can be attributed to them.

The most urgent objection that can be raised to Maddy's theory and to which she does not seem to have a clear answer is this: once we bring the objects of mathematics into space-time, we seem to be left without a natural account of what certain mathematical objects are supposed to be. A good example of this problem is an infinite set. If Maddy's views are to have any plausibility, she should be able to demonstrate how we have knowledge of sets that we cannot perceive or which cannot exist in a finite universe.

Furthermore one may ask whether she needs to be a Platonist in the first place. She writes: "On some terminological conventions, this means that sets no longer count as 'abstract'. So be it; I attach no importance to them"(Maddy: 1980, 59). If, as Maddy claims, we can have unproblematic knowledge of entities such as sets via perception, why take a Platonist stance in the first place? However if Maddy's view omits Platonism, it is in danger of collapsing into John Stuart Mill's proposal to see sets as nothing but aggregates of physical matter. Mill's view has been famously criticized for many reasons, but the two which apply most readily to Maddy are the following: a set has a determinate number of members, an aggregate does not; second, it is impossible to distinguish an aggregate of 3 elements from the “aggregate” containing that aggregate, but it is very important to distinguish a set from a set containing that set.

At first glance, it seems that these criticisms would be equally applicable to Husserl. Intuition of numbers understood as aggregates of units does seem to come very close to intuition of sets in Maddy’s sense. However here again we are left in confusion, for nowhere does Husserl explicitly connect set theory with the concept of number. If it happens to be the case that Husserl is merely talking about cardinalities of aggregates, then the problems that Maddy has to deal with are not applicable to his view, for such problems largely stem from axioms of set theory.

On the other hand, if it happens that Husserl is modeling his account of numbers on set theory, he can at least account for the first challenge, namely that we cannot perceive an infinite set. Husserl’s account seems to be preferable to Maddy’s, in that it does not hold that direct perceptual experience is always a necessary condition for the knowledge of numbers. As we have seen from Husserl’s discussion of symbolic representation, direct perceptual knowledge of numbers is strictly limited, and knowledge of number concepts that are more complex is arrived at by my recognition that the number belongs to the algebraic system that rests on the authentic number concepts. This allows for symbolic representations of authentically acquired concepts of numbers and manipulations of them. Thus Husserl’s answer to the challenge would simply be that it is impossible to grasp an infinite set through perceptual intuition.

Thus, if Husserl is a full-blown Platonist, he could be seen as answering the access problem in a vaguely Maddian way –namely, by postulating that we are causally connected to abstract entities –without succumbing to all of the problems which plague Maddy’s theory. Seen in this light, Husserl’s answer to the access problem is that we are causally connected at least with the abstract formal properties of multiplicities, and thus we have knowledge of elementary arithmetical concepts via perceptual intuition. All the greater numbers are built out of these authentic representations, and we know them in virtue of knowing the elementary numbers.

However how sufficiently does this answer the access problem? Husserl may successfully avoid the issues that Gödel and Maddy struggled with –he neither has to defend a strong version of mind-body dualism, nor does his theory presuppose an intuition-based account of large numbers that simply cannot be given to us by way of perceptual intuition. But, if he indeed is a Platonist, he needs to explain how numbers as formal properties truthfully describe the aggregates to which they correspond. If we consult the rules of standard Platonist views, this ought to be done by reference to some purely abstract objects or properties that correspond, however how do we access them, remains as unclear.

This should not be taken as merely an issue with Husserl's approach. Charles Parsons has to deal with a similar problem. As we have seen, there is one crucial difference which makes Parsons' theory more plausible than that of Gödel: there is no direct link between the physical and the Platonic realm, only between concrete and quasi-abstract entities. This way Parsons, among other things, does not have to commit himself to strong dualism, as Gödel would.

But if this is the end of the story, we may be left with a lingering worry, namely that the access to the Platonic realm is still left unaccounted for. Parsons theory of mathematical intuition shows in what way we could have knowledge of quasi-abstract entities, but that is where the application of intuition stops. What about the abstract entities? The furthest Parsons goes in defining natural numbers qua abstract objects is to say that they are "a structure satisfying the Dedekind-Peano axioms" (Parsons: 2007, 188), but just how the abstract structure is related to the quasi-abstract mathematical objects remains unclear.

Here Parsons' ingenious strategy of introducing quasi-abstract objects might backfire. If no adequate explanation of more direct access is given, then the faculty of intuition is no help at all in answering the question of how is contact with the Platonic Heaven forged. All that the application of intuition does is show how we can have immediate knowledge of fairly unproblematic elementary mathematical concepts.

I think what has been discussed points to much larger problems of applying intuition to Platonism than just inadequacies in the approaches discussed. What is common to the views of Husserl (interpreted as a full-blown Platonist), Maddy and Parsons is that it cannot be shown how it would be even remotely plausible that intuition has contact to the third realm. At most they demonstrate how we manage to connect concrete particulars with their corresponding concepts.

5.4. Critical Realism

However, if Husserl is a critical realist, as opposed to a full blown-Platonist, we may see him as addressing the access problem in a different light. Despite the wide spread conviction that Husserl was a Platonist by the time of writing *PA*, this interpretation is not supported by much textual evidence. While it is true that Husserl turns to Platonism in *Logical Investigations* and spells it our very clearly, there is no mention of his ontological stance in PA.

In this respect, Tieszen's reference to "The Origins of Geometry" may turn out to be misguiding. First of all, it is a piece written long after the publication of *Logical Investigations*, namely after Husserl's embrace of Platonism. While the issue of the possibility of arithmetical knowledge discussed in "The Origins of Geometry" resonates with the general topical direction of *PA*, why should we conclude that Husserl had the same position in both of the works, especially given that there is no mention of PA in "The Origins of Geometry"?

In the light of these suspicions, in what follows I will outline another possible interpretation of Husserl's ontological commitments, one that is in the vein of critical realism of Kant and Brentano. I do not mean to suggest that it is necessarily the right one. Just as there is no mention of full-blown Platonism in *PA*, there is no mention of critical realism. However, as it fits the project equally well as the Platonist interpretation it deserves a mention.

The first and perhaps most important similarity between Kant and Husserl's thought is the role intuition plays in both of their epistemology and their respective views in philosophy of mathematics. As I have already mentioned earlier on, central to Kant's epistemology is the claim that our ability of sensation imposes the forms of space (the form of everything that is outside us) and time (the form of all intuitable objects) on the objects of our intuitive awareness. This is to say that we cannot ever perceive the pure objects in themselves, because our cognitive capacities are such that they will always impose, among other things, the forms of time and space on our perceptions.

In relation to this, another relevant feature of Kant's epistemology, as we have seen in the previous chapter, is the view that any concept is given to us by means of sensation, thus, in order to give a thorough analysis of a certain concept we will inevitably have to touch upon the conditions under which knowledge of such a concept is arrived to. What follows from these two ideas is that talk of these concepts outside the scope of our representations of them is impossible.

These two claims are the kernel of Kantian constructivism, which, roughly summarized, is the claim that the concepts that we attribute to the objects in our experience cannot describe them directly. Their meanings are rather determined by the nature of our cognitive faculties. What someone who defends such a thesis will inevitably have to address is just how are concepts then constructed. Kant's answer to this is the theory of the schematism of pure concepts of understanding (*Schematismus der reine Verstandsbegriffe*).

Following Klaus Jorgensen, we may summarize Kant's analysis of our knowledge of concepts by two theses: (i) there is a harmony between the pure concepts of understanding and the way appearances are given to us. This harmony ensures that appearances can be cognized under categories; (ii) it is due to the schematism that such categorization of appearances is possible (Jorgensen: 2006, 2). The schematism of the intellect may thus be understood as the rule that determines how a particular is related to a concept. Intuition here could be interpreted as the process that immediately relates the particular to such a rule.

Transcendental schemata are created through "imagination in relation to time", namely by perceiving certain regularities in time, the cognizer forms the rules by which sense impressions will get attributed to concepts. An illustrative example of this notions is number, which, according to Kant, is a transcendental schema of quantity. In *CPR* Kant writes:

> But the pure schema of magnitude [*quantitatis*], as the concept of understanding, is number. Which is a representation that binds together the successive addition of one thing to another (of the same kind). Thus, number is nothing other than the unity of the synthesis of the manifold of a homogenous intuition in general, a unity which comes about through the fact that I engender time itself in the apprehension of the intuition. (Kant: 1999, 273, 144/B180)

A possible Kantian interpretation of Husserl's work by now may have become apparent. Husserl's insistence on the claim that no concept is given to us without a basis in concrete representation resonates with Kant's idea that all possible concepts are given to us by means of sensibility which is governed by the forms of time and space, and thus it is impossible to refer to an object in its pure form. Therefore if we asked how knowledge of ideal objects, such as authentic number concepts, is possible, Husserl may respond by saying that it is possible only inasmuch we can trace it back to the concrete representations in which numerical judgments appear.

A critical realist would agree with the Platonist that mathematical objects are not only objective but wholly real, however they would disagree over the question of how we may have knowledge of them. Namely, a Platonist strategy will be targeted to showing how we come to direct contact with mathematical objects, while a critical realist would claim that we never have direct contact with objects themselves, just with their representations that are determined by our epistemological makeup.

However just how plausible would it be to state that Husserl was, if not a Kantian per se, then at least some sort of a critical realist? One potentially damaging consideration needs to be mentioned: Husserl is quite unsympathetic to Kant's treatment of number concepts throughout *PA*, which follows directly from Kant's ontological commitments.

While critically examining the approaches to the concept of number that are relying on Kant's temporal succession argument, he claims that the fact that such an approach is popular is more of a "consequence of the authority of his [Kant's] name than […] of the force of his argument" (Husserl: 22, 2003). Furthermore, Husserl harshly criticizes Kant's notion of number as a transcendental schema of quantity. He firstly points the equivocations in the use of the term 'schema' in Kant's work, and, more than anything, that the equation of number with schema is irreconcilable with the function of the schema. For example: "We wish to call […] the formal and pure condition of sensibility, to which the a concept of the understanding is restricted in its use, the schema of that concept of understanding" (Kant B180); and "This representation […] of a general procedure of the imagination in giving a concept its model [*Bild*] I call the schema [pertaining] to that concept" (ibid B 179-180). Husserl then goes on to argue that if we wanted to carry this last definition unto the concept of number then we would have to say "that a number is the general procedure of imagination in giving to the concept of quantity its model. However, by 'procedure' can only be meant the process of enumerating. But is it not clear that 'number' and 'representation of enumerating' are not the same?" (Husserl: 2003, 34).

However in order to be a critical realist Husserl need not accept Kant's philosophy of arithmetic, nor every twist and turn of his epistemology. Indeed many of Husserl's contemporaries have criticized various aspects of Kant's thought while still remaining sympathetic, if with certain provisos, to his proposed ontology. An good example of such views are those of Brentano, to whom *PA* is dedicated.

As Guillaume Frechette notes, in all of the criticisms Brentano has made against his predecessors and contemporaries, "Kant undoubtedly occupies a place of honor" (Frechette: 2012, 1). One of Brentano's most frequently stressed points is that Kant's postulation of synthetic *a priori* judgments is unjustified.[24] He parallels Kant's synthetic *a priori* judgments with Reid's judgments of common sense. Such judgments appear certain, though they are not evident, and, according to, at least Kant, they form a foundation for science. What Brentano could not agree with in Kant was not the *a priori* character of such statements, but the claim that one cannot 'see' their evidence. Accepting such 'blind' judgments as the foundation of all our knowledge is simply nonsensical, or so Brentano claims.

However this does not stop him from taking an ontological stance that draws inspiration from that of Kant's. The focus on Brentano's study of ontology, much like Kant's, is not on the things existing in themselves, but on things the way they are perceived by the human *eye*. Which is to say that Brentano's consideration will be of phenomenal objects, as opposed to noumenal ones, to use Kant's terminology. As Bell notes, a critical feature of Brentano's ontology is that the external, material world is bracketed, which is to say that

[24] "In Germany it was Kant who undertook to save knowledge from Hume's skepticism, and his method was in essence very similar to that of Reid. Kant claimed that science demands as its foundations a number of principles which he called synthetic *a priori* judgments. On close inspection of what he means by this, however, it turns out that the term *a priori* amounts for him to proposition that stand for us as true from the beginning without their being evident. The sum of *a priori* judgments have the same character as Reid's judgments of common sense" (Brentano: 1998, 99).

Brentano's treatment of ontology has a pronounced solipsistic tendency; namely, "there is no reference to, and no philosophical use of denizens of the natural world" (Bell: 1990, 9).

A critical distinction in Brentano's ontology is drawn between physical and mental phenomena. Mental phenomena are nothing but "acts which have content",[25] namely all mental acts and activities, and physical phenomena are the contents of the said acts. When introducing mental phenomena in *Psychology from an Empirical Point of View* Brentano writes, "what is characteristic to mental phenomena is them having something immanently as an object" (Brentano: 1874, 103). 'Physical phenomena' are thus used interchangeably with 'immanent objects', 'contents' or 'intentional correlates'. As Barry Smith and Arkadiusz Chrudzimski note, the most important aspect of the "immanence" of immanent objects is that they are inseparable from the corresponding mental act, and in this sense the immanent object is 'in' the mind (Chrudzimski, Smith: 2004, 205).

Physical phenomena thus, according to Brentano, 'intentionally in-exist' in mental phenomena. Intentionality is another relevant feature of Brentano's ontology, which can be simply understood as the mental act's necessary reference to an object. In *Psychology from an Empirical Standpoint* he famously writes:

> Every mental phenomenon is characterized by what the Scholastics of the Middle Ages called the intentional (or mental) in-existence of an object, and what we might call, though not wholly unambiguously, reference to a content, direction toward an object (which is not to be understood here as meaning a thing), or immanent objectivity. Every mental phenomenon includes something as object within itself (Brentano: 1874, 88)

Regrettably, Brentano does not provide an explicit account of this relation, instead merely giving examples and synonyms, such as 'mental in-existence', 'immanent objectivity', 'existence as an object in something' and alike. It is important to note, that Brentano often uses *object* and the *content of a mental act* interchangeably, for he uses both to designate immanent content present in a mental act. External objects therefore are not

[25] In Brentano's terminology all mental phenomena are acts –he uses 'mental phenomena' and 'mental act' interchangeably.

talked about, as they are not phenomena, rather noumena, and, following Kant, cannot be grasped in their original form.

We can thus safely say that Brentano's ontology is that of a critical realist one. While he departs considerably from Kant's analysis of knowledge, and Kant's partition of the constituents of cognition, he remains faithful to the claim that we have no direct contact to the objects themselves, save through the representations that are determined by our cognitive faculties. Brentano does not give an account of the concept of number, as did Kant, however if Husserl is a critical realist his account of number concepts would not go against the letter of Brentano's ontology. A brief sketch of such treatment of Husserl's *PA* could be the following: The grasping of the concept of number is an act/mental phenomenon, the number, either per se, or distinct numerical property, is the immanent object of such an act, to which the act is intentionally directed. Intuition may be understood as a fulfillment of an empty 'act-contents', namely as something that makes the immanent object immediately present to the mental phenomenon/act, this way yielding in our recognition of number concepts. By extension, we may suppose then that Husserl, along with Brentano's descriptive psychology, might have adopted his critical realism as well. He would then be stating that mathematical objects in whatever form they exist are real, however we may not reach them save through the representations that yield from our epistemic makeup and that are to be analyzed by categorizing them into mental and physical phenomena.

However, whether this interpretation is more plausible than the afore mentioned one remains an open question. Husserl's early project is heavily influenced by Brentano's work, but we should be careful not to attribute Brentano's ontology to *PA* without Husserl's consent. It is well known that Brentano was unsympathetic to Husserl's *LI* precisely because of Husserl's ontological stance, this time explicitly Platonist. As Rollinger notes, he saw it as "an affirmation of universals, sentences-in-themselves and other *Undinge*" (Rollinger: 1999, 43). However, from this we can hardly infer that Brentano agrees with Husserl's ontology in *PA,* without further elaborations on the matter, which, to the best of my knowledge, Husserl never provided.

The same must be said about the Platonist interpretation: if the strongest support that one can offer for a Platonist interpretation of early Husserl's ontological commitments is that he was a Platonist in *LI,* then we remain as much in the dark as we were before. Thus, both approaches to Husserl's early ontological commitments ought to treated as educated guesses, and it would be near-impossible to establish the legitimacy of one over the other, at least at this stage. Perhaps Husserl's correspondence with Brentano, or Brentano's own reflections on the matter, could shed more light on this issue. But it remains to be seen whether even this much could resolve the matter.

6. Conclusion

My intention in this thesis was to provide a thorough yet concise overview of Husserl's early philosophy of mathematics and argue that his early work can be interesting to the contemporary philosopher. I hope to have shown that not only does Husserl define and use mathematical intuition in a sufficiently similar way to the contemporary authors, he also anticipates the application of it to the access problem. However, in just what way does he apply intuition to the access problem remains yet to be determined. I have provided what I take to be two most plausible interpretations, that of a Platonist and of a critical realist ontology.

Needless to say not everything that deserved a mention found its way to this thesis. In order to provide a concise yet critical interpretation, I needed to set certain considerations aside for further research. For instance, a potentially relevant question might be what was Husserl's precise relation to the Neo-Kantian tradition and especially his teacher in Berlin, Friedrich Paulsen. In a similar way, it may be important to see what was Husserl's view on Kant's treatment of construction of concepts, namely what aspects does he agree with and which ones does he criticize. As we have seen, in Husserl's epistemological treatment of numbers the cognizer is already equipped with number concepts, as intuition is supposed to account for the connection for the particular with its corresponding concept. However we are very much left in the dark as to how the cognizer comes to have number concepts in the first place.

As a final remark it must be mentioned, that here I tried to steer away from a more critical assessment of whether intuition can be of much help for a Platonist (save from places where the interpretation required it). I did this because what I saw as my primary task was to show that there is more to early Husserl's philosophy of math than just a target of Frege's criticism and headwaters of his phenomenological project. Ideas that are being developed in *Philosophy of Arithmetic* are very much alive today and it is generally not recognized that Husserl anticipated them. Determining to what extent are these ideas plausible is, again, a matter of further research. However whatever the result of such an investigation, the fact remains: Husserl's early philosophy of mathematics not only deals with issues that are relevant to the contemporary philosophy of mathematics, it does so in much the same way.

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