A dissertation submitted to the Department of Environmental Sciences and Policy of Central European University in part fulfillment of the

Degree of Doctor of Philosophy

Scaling in Cities as an Indicator of Energy Consumption:

What Fractal Analysis Could Tell Us About Resilience and Disparity in Complex Systems

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ABSTRACT OF DISSERTATION submitted by:

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Climate change and global resource extraction peaks are problems that may require a complete overhaul of the fundamental paradigms of our civilization in order to be solved. Paradigm changes are implemented in societies first and foremost through a redefinition of indicators. Our current indicator systems do not take into account the complexity of human social, economic and material systems and prioritize "efficiency" and "growth". The objective of this research was to develop an indicator for one type of anthropogenic complex systems, i.e. cities that took into account the complex nature of the system and provided a quantitative way to prioritize alternative values such as "resilience" and "sustainability". Scaling has been identified as one measure of complexity in a system, which is relatively easy to compute and comprehend. Here, I have developed a scaling indicator for cities based on fractal dimension. US block wise census data was used to calculate the exponent of the power-law distribution of population density across different census blocks in a city. The power-law, or scaling indicator, herein referred to as the fractal dimension was then compared to parameters such as population, area, population density, gasoline sales, gasoline sales per capita and area, and carbon emissions and carbon emissions per capita. It was noted that the fractal dimension had a power-law correlation with gasoline sales per unit area in the cities. The analysis was then extended a second complex system, i.e. national economies. Fractal dimension or scaling of percentages of incomes across the highest earning to lowest earning twenty percent segments of the population was calculated using World Bank economic data for 2004 (the year for which most extensive dataset was available). The relationship between this scaling indicator and energy usage per capita in countries was again found to be a power-law with an r-square value of more than 0.35 (similar to the correlation between urban fractal dimension and gasoline sales per area in cities). A new planning tool is developed to allow incorporation of consideration of these complexity indicators in development planning for cities and national economies. The planning-plane allows for visualization of the impacts of particular interventions in cities (e.g. housing scheme) and economies (e.g. changes in tax-code) on energy consumption parameters across two independent variables (e.g. population and fractal dimension) instead of the usual practice of using one indicator (e.g. population density). The similar nature of the correlation between scaling indicators and energy consumption indicators in two completely different anthropogenic complex systems hints at some underlying similarity in the mechanism through which these complex systems develop. It is hypothesized here that steeper scaling (e.g. higher income differences in economies) in complex systems makes good system regulation more energy intensive, thereby affecting the energy consumption parameters as observed in this study. Steeper scaling also thus negatively affects the effectiveness of regulation in complex systems and makes the system more prone to internal shocks. Planning system evolution for resilience would thus benefit from consideration of scaling indicators in the planning process.

Keywords: Complexity, resilience, scaling, sustainability, environmental indicators, fractal dimension, cities, national economies

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Table of Contents

List of Tables

List of Figures

List of Appendices

List of Abbreviations

BCM	Box Counting Method
EPI	Environmental Performance Index
ESI	Environmental Sustainability Index
EVI	Environmental Vulnerability Index
GDP	Gross Domestic Product
HDI	Human Development Index
IPCC	Inter-governmental Panel on Climate Change
IUCN	International Union for Conservation of Nature
SAMI	Scale Adjusted Metropolitan Indicator
UCIMLR	UC Irving Machine Learning Repository
USGBC	United States Green Building Council

1. Introduction

Climate change and peak resource –especially energy- extraction have been recognized scientifically as existential problems for our civilization (International Energy Agency 2013; IPCC 2007). On the one hand energy constraints continue to impose a lean energy diet on future development activity while on the other hand, increasing need to remain within safe greenhouse gas levels dictate that even accessible fossil fuel reserves may need to be left un-accessed to avoid catastrophic climate change. Energy constraints thus define key operating parameters within which social and human development should be achieved. Extensive recent body of literature though suggests that technological solutions may exist (Delucchi M. A. and Jacobson M. Z. 2011; Jacobson M. Z. and Delucchi M. A. 2011) to ensure we do not exceed limitations imposed on our activity by climate change and peak-oil. However, political response to these grave crises continues to be slow at best, and dangerously oblivious at worst (Meadows D. H. *et al.* 1992; Mills J. I. and Emmi P. C. 2006; Monbiot G. 2009, 2010; Murray J. and King D. 2012). The primary problem with the political and economic system appears to be that the system may still be mired in a paradigm that values economic growth and efficiency above all else and thus makes a virtue out of the growth of production and consumption that is a key driver of unsustainability. Though opposition to this paradigm is now visible in the area of political discourse and even scientific analysis (Piketty T. 2014), work still needs to be done in the area of translation of alternative paradigms into practical policy frameworks and instruments (Costanza R. *et al.* 2009). It can be argued that the social, cultural and economic changes needed to transform societies to effectively address problems such as climate change and peak oil, amount to a phase change in modern industrial civilization. From a societal perspective, these phase changes can only be triggered from a higher level leverage point (Meadows D. 1998). The highest level leverage point for affecting such changes in society is the power to change paradigms. Right now we live in a world where this power is perhaps more distributed than at any other time in the history of human civilization. No one institution or individual can claim to exercise this power today though there can be identified a group of apex institutions that define the consensus on the operating paradigm of our civilization; which is focused on growth. However the paradigm is now being challenged on multiple fronts by a number of emerging alternative institutions as well as by reformists from inside the status-quo institutions, specifically academics.

For policy level adoption paradigms need to be translated into indicators. The key development indicator of the current operational paradigm i.e., Gross Domestic Product (GDP), has been the focus of intensive critique for over a decade, yet alternative indicators that take into account the complexities of the systems, especially measuring scaling within the system, seem to have only gained little ground in policy development process. The indicator framework generally used in policy analysis for monitoring and planning human development emerges from a paradigm that has the following weaknesses;

a) It does not take into account the complexity of the systems being measured, especially scaling within the system. Most anthropogenic systems of human civilization such as cities and economies are complex and the relationship between indicators and the values being measured or optimized is often neither linear nor continuous. Current indicator frameworks often assume a linear relationship between the indicator and the value, for instance GDP is taken to be a direct proxy for well-being. Additionally natural discontinuities in functions are not taken into account because they cannot be modeled using historical data. From the perspective of the dominant models thus, abrupt changes often happen in systems which may appear as black swans (e.g.

financial collapses). To summarize, indicator systems do not capture non-linearities in the systems being measured.

b) Since the paradigm values things like growth and economic efficiency, the resulting indicator system also measure progress on these fronts.

So while the paradigm needs to be challenged at all levels of discourse, one aspect of the presentation of alternatives is the development of practical and implementable indicators that address the above discussed weaknesses of the indicators emerging from the current paradigm. This work aims at taking a first step in that direction by developing an indicator for urban development that takes into account the non-linearities of the urban system and that helps optimize development for resilience and sustainability instead only of economic efficiency and growth. The specific aspect of complexity that this work will explore deals with scaling within the system and tries to study disparity of distribution within the system from a scaling perspective.

The research has been facilitated by the availability of large, high resolution datasets and extensive cheap memory and processing powers. As such the ideas and theories pursued here have been in circulation in literature in theoretical form for the last three decades, but have only recently started to find empirical justification in peer reviewed publications. Scaling in complex systems has been identified both as an indicator of complexity, aesthetic value, sustainability and resilience. This has been true of cities, buildings, ecosystems, biological organs and organisms and even in some cases economies and corporations. I will focus my research on cities. The primary data used for the analysis here comes from US Census 2010 block wise. I will be using this higher resolution (census-block instead of city) data to establish and calculate one scaling indicator for each city. This scaling indicator or similar indicator for urban systems has so far not been calculated for this data at the resolution I am using, in the available literature. The high resolution and quantitative (as opposed to image based) nature of scaling indicator calculation also enhances the repeatability of this methodology compared to other methods of studying scaling. The veracity or utility of this scaling indicator shall then be established by studying its correlation with established environmental parameters such as gasoline usage or carbon emissions in the city. I now formally define a problem statement and research questions for this project.

1.1 Problem Statement

In light of the above discussion, the problem statement seeding this investigation can be elaborated as follows. The current indicator framework for measuring and monitoring sustainable development has the following weaknesses;

a) It does not take into account system non-linearities.

b) It promotes the optimization for efficiency or growth instead of for resilience or sustainability (Gallopín G. C. 2006).

To address the above mentioned problems I will be developing an indicator for a complex anthropogenic system (city) which takes into account system non-linearities and optimizes for system resilience. The primary reason for using cities as a complex system to study these indicator problems is availability of high resolution, high accuracy, extensive dataset.

In literature, the complexity of a city has not been explored and quantified using application of fractal analysis to the kind of high resolution dataset that shall be used in this research. As such both methodologically and potentially in terms of results, this research would contribute to the existing knowledge in the disciplines of a) urban sustainability, b) environment, c) complexity and d) resilience.

1.2 Aims and Objectives

To work towards addressing the above problem set I need to meet the following aims and objectives.

a) Develop an indicator system that does take into account system non-linearities and optimizes for resilience instead of growth or efficiency.
b) Establish the veracity of the indicator developed by studying its correlation with directly measured environmental or energy consumption parameters/indicators.

There are two reasons for selection of energy consumption indicators for this analysis. First is the availability of data for similar urban denomination (metropolitan statistical area) as the one being used for calculation of complexity indicator. The other is the significance of energy limits towards building a sustainable civilization. One can argue based on available literature that energy is one of the key, if not the key limiting factor for environmental sustainability of systems such as cities.

Further, one other complex system besides cities will be studied to analyze the generalization of results to complex systems in general. Also, to explore the utility of this research I will propose tools to incorporate the results in policy analysis.

1.3 Research Questions

Based on the above discussion, we can identify the following research questions that need to be explored once a complexity based indicator has been developed;

a) What if any, is the correlation between a complexity based indicator and direct and well established energy consumption and at least one other environmental pressure indicator?
b) What if anything, does this correlation tell us about the relationship between resilience/sustainability and complexity as expressed in scaling within the system?

It should be noted that this research shall explore these questions for the case of cities as complex systems. The environmental pressure indicators identified will be considered the chosen environmental sustainability indicators for this analysis, and to establish the veracity and utility of complexity indicators as environmental sustainability indicators. The complexity based indicator to be developed and used for this analysis would be a scaling indicator based on fractal dimension of cities; the most commonly used urban complexity indicator. To address the second research question I will also look at a second complex system, i.e. national economies to ascertain if the results can apply to other complex systems.

1.4 Definitions

In this section I will explain what some key terms mean in the context of this thesis. More elaborate discussions on the definition of these terms in literature can be found in the Literature Review chapter. That section will discuss the evolution of these terms in literature and their current usage. The objective here is to merely identify a first formulation of concept for key terms for this document, so that complications in further reading can be avoided.

1.4.1 Resilience

Resilience for the purposes of this dissertation is defined as the ability of a system to continue its operations without undergoing a phase-change or crossing over a tipping point. Resilience is a property that manifests itself in response to environmental or other external stimuli, some of them predictable stochastically while others not so much. In that sense, resilience can be especially contextualized as the ability of the system to avoid a tipping point in response to external shocks which can't be predicted.

1.4.2 Sustainability

Sustainability is the ability of the system to continue to exist in a certain form and continue certain operations in the foreseeable future. This dissertation argues that resilience is an important condition for sustainability and non-resilient systems cannot be considered sustainable.

1.4.3 Fractals

Fractals are algorithmically generated geometries, emerging out of repetition of patterns and exhibiting certain properties such as self-similarity across scales and power law distribution of elements.

1.4.4 Fractal Dimension

Fractal dimension is the dimension of a geometry in fractal space. Fractal dimension is a measure of space filling within the system as well as the disparity in distribution of sizes across different scales.

1.4.5 Scaling

Scaling is the manner in which populations of different elements of a subsystem are distributed across different scales, simply speaking what is the distribution of smaller elements versus larger elements. Fractal dimension can also be a measure of scaling. Scale free networks for instance exhibit similar scaling at all scales.

1.4.6 Complexity

Complexity is the ability of the system to exhibit emergent phenomena; i.e. phenomena that can't be modeled using a model which is a sum of models of all the different parts of a system. Scale free networks and fractals are used to model complexity.

1.4.7 Paradigm

The dominant, overarching and underlying narrative that governs the various mechanisms of a system.

1.5 Document Structure

This document is divided into six chapters. The second chapter provides a comprehensive survey of the relevant literature and references related to the arguments already discussed in this introduction, arriving at a focal theory. The third chapter explains the research methodology in detail. The results are then reported in chapter four. The results are discussed and underlying mechanisms as well as policy implications elaborated upon in chapter five. The final chapter summarizes the conclusions of this research.

2. Literature Review

Based on the research questions identified, the literature review has to explore the literature in the following general areas of inquiry.

- How has urban planning literature so far incorporated complexity analysis in its studies?
- The mathematics of complexity, especially related to fractal based modeling of complexity.
- The application of fractal based modeling of systems to cities and calculation and use of fractal dimension of cities.
- Theory of resilience in complex systems especially as it pertains to high impact low probability events (Black Swans or Normal Accidents).
- The application of Black Swan studies to resilience in complex systems.
- Sustainable development literature and considerations of resilience in sustainable development.
- Indicator systems developed for sustainable development; any focusing especially on complexity.

Through evaluation of the above literature, a focus on a focal theory of urban sustainability shall be developed, quantified by calculation and analysis of the fractality of urban systems.

One of the longer-term solutions for the problem of climate change must be a major realignment of our way of life, and our ideas of prosperity, progress and wealth to better reflect the realities of an energy scarce future (The Royal Swedish Academy of Sciences 2011). One of the most important places to start doing that, is by changing the way we envision and build our habitat, most importantly our cities (Bettencourt L. and West G.

2010). This research focuses on studying structure and scaling within cities to find patterns of sustainable, more efficient energy use. More specifically, it focuses on how these attributes are measured, as measurement is an opportunity for high-leverage intervention into the operations of a system. In order to do that, the literature review will cover the fields of urban planning, fractal mathematics, risk estimation in fat tailed systems and sustainable development, specifically measurement and indicators related to sustainable development.

2.1 Scoping

The research has to be grounded in theoretical work from the areas of urban planning, fractal mathematics, risk estimation in fat tailed systems and sustainable development, specifically, measurement and indicators related to sustainable development as shown in Figure 1. In this chapter some of the areas of scientific investigation this literature review will cover are listed.

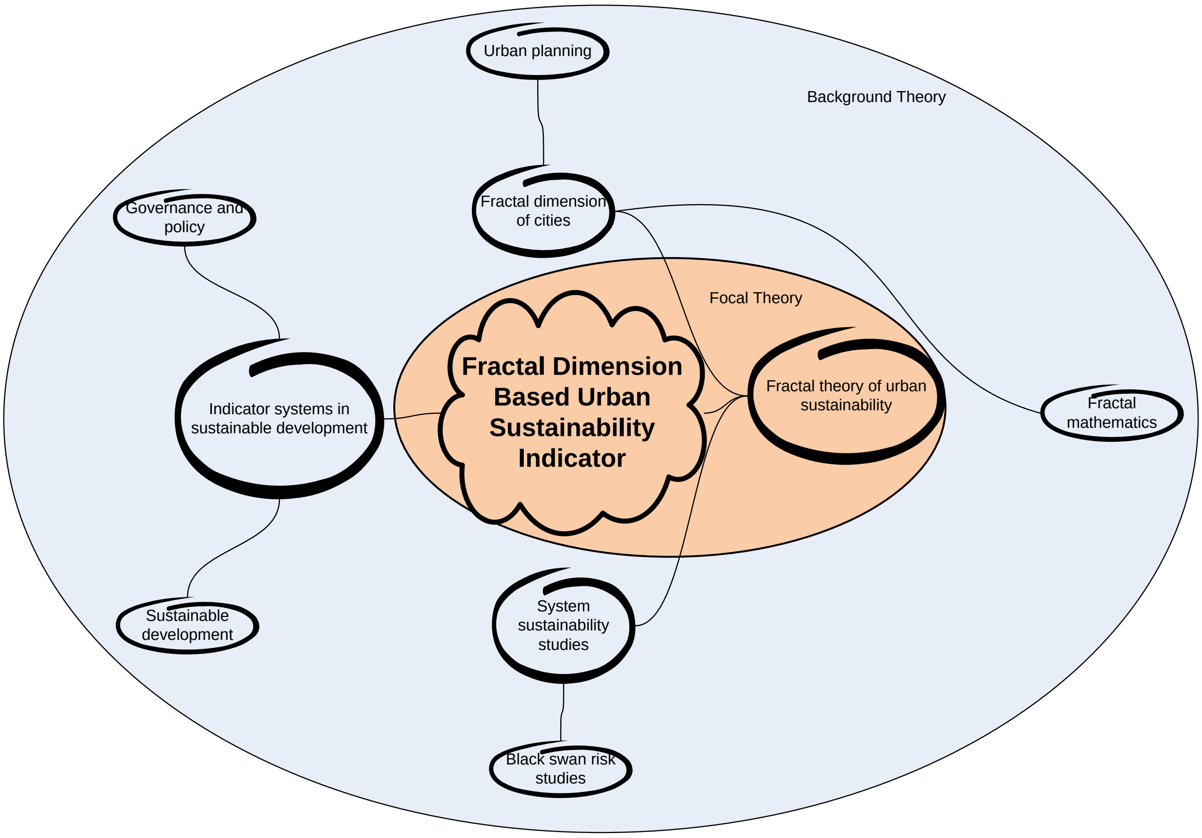


Figure 1: Theoretical Framework

2.2 Background Theory

The global energy conundrum is expressing itself in terms of two conjoined problems. On the one hand the specter of peak oil, now admitted by even some conservative estimates to have occurred around 2006 (Kerr R. A. 2011) is dampening prospects of continued global economic growth; while on the other hand, manmade climate change is demanding that we burn no more than 500 billion tonnes of the Earth's carbon reserves (equivalent to 1830 billion tonnes of CO_2); roughly 60% of the currently discovered fossil fuel reserves (capable of producing roughly 3000 billion tonnes of CO_2 equivalent green house gas emissions) (World Energy Council 2007), if we are to avoid a cataclysmic two degrees centigrade plus change in temperature by the end of the century (Allen M. R. *et al.* 2009).

Despite academic and research ventures that indicate the viability of a renewables based energy system, markets continue to remain skeptical of the ability of renewable technologies to replace fossil fuels as a profitable, or even viable energy source. While the investments in green energy went up by 32% in the year 2011 (Frankfurt School *et al.* 2011), the rise came on the back of nearly 70% increase in green energy subsidies between 2007 and 2010 (International Energy Agency 2011). Further the crucial venture capital investments needed to fuel innovation in the sector actually went down in 2011 (Freed J. and Stevens M. 2011). In the future, realizing the potential in renewable energy sources will require significantly more subsidies (from $66 Billion in 2011 to $250 billion in 2035) in order to compete with coal and natural gas as a potentially profitable venue for future private investments, and yet investments in coal and natural gas and their share in global energy consumption are expected to rise much more steeply (International Energy Agency 2011). Even the most optimistic estimates for replacement of global energy supply from fossil fuel to renewable, do not foresee the transformation happening before 2050 (Delucchi M. A. and Jacobson M. Z. 2011). According to one estimate preparation for peak-oil will take at least twenty years (Hirsch R. L. *et al.* 2005). If we continue the current trend of fossil fuel consumption we would have gone through enough fossil fuel in forty years, to raise the temperature of the earth by two degrees centigrade if burned within 500 years (Allen M. R. *et al.* 2009). Renewable energy sources alone do not seem viable and the transition to renewable sources will take more time than we have. Meanwhile incidents like the Fukushima disaster in Japan cast a pall of popular uncertainty over nuclear technology as one of the main non-fossil alternatives, leaving even technologically advanced nations such as Germany to turn the clock back on nuclear power generation. Such events, which have low probability of occurrence but high impacts, also expose the vulnerability of the hyper-complex, global, industrialized economy to localized, unpredictable shock events or “Black Swans” (Taleb N. 2008b). It appears that our inability to foresee events like stock market crashes, localized food shortages, higher number of higher intensity extreme weather events or industrial accidents etc. is profound and that makes us ill prepared for whatever an uncertain future will throw at us. In summary, the ability to solve the problem through the supply side alone by transitioning to low emission, non-fossil fuel energy resources such as nuclear, wind and solar continues to be questionable. Further, human beings feel powerless to predict with any degree of certainty how human societies will react to climate change and the scarcity of a fundamental resource such as fossil fuels, and therefore, are almost paralyzed to inaction in the face such historic vicissitudes.

One of the longer-term solutions for sustainability of the human project thus, must be a major realignment of our way of life, and our ideas of prosperity, progress and wealth to prepare us for events we cannot predict and to better reflect the realities of an energy scarce future (Lovins A. B. 1976). Human beings need to change the way they live and create a holistic efficiency revolution in energy consumption (The Royal Swedish Academy of Sciences 2011). One significant way of doing that is by changing the way we envision and build our habitat, most importantly our cities. Researchers are now calling for a new theory of cities that defines human development along more sustainable lines (Bettencourt L. and West G. 2010). The solution may in a significant part lie is the reimagining of cities so that they are no longer just organisms for exponentially increasing consumption, but are sustainable systems robust to unpredictable events as well as nourishing human habitats in equilibrium with their natural support systems as if, part of an ecology.

2.2.1 Sustainable Development

Already by the early nineties the buzzword credentials of 'sustainable development' as a term had been inarguably well established (Sharachchandra M L. 1991). As soon as the status of sustainable development as a legitimate area of scientific inquiry was established, critical voices denouncing the supposed 'emptiness' of the concept had started to emerge (Fortune J.

and Hughes J. 1997). The Agenda 21 agreed upon at Rio went a long way towards building consensus on the need for establishing 'indicators for sustainable development' giving United Nations the mandate to move forward in this regard (Sitarz D. 1993).

Initially sustainability in practice was concerned mostly with monitoring of environmental indices. The evolution of 'sustainability paradigm' can be traced to six different strains of thought (Kidd C. 1992); biosphere concern, environmental concern, carrying capacity concern, critique of technology, no/low growth concern and eco-development concern. Of these, the carrying capacity concern is specifically relevant to this research. Central to the carrying capacity concern is the idea that if the maximum sustainable yield of any resource intrinsic to a system is exceeded by the rate of consumption of that resource by the system, the system is not sustainable (Botsford L. W. *et al.* 1997; Link J. S. *et al.* 2002).

Although the carrying capacity idea emerged initially from fish stock assessments where depletion was most clearly measurable, it really is essential to all the other strains of sustainability thought (Meadows D. H. *et al.* 2004). All of the other five concerns can be said to have their roots in the concern for the survival of human species in the face of limiting resources and diminishing carrying capacity of a finite planet. This was the concept adopted by IUCN in their definition for sustainable development; "[Sustainable development is] development that improves the quality of human life while living within the carrying capacity of supporting ecosystems" (IUCN *et al.* 1991).

This idea of the carrying capacity of the system is fundamental to the visualization of sustainability in the context of this research. For when we have assumed sustainability as limited by the limiting resource within the carrying capacity of the system, two fundamental questions about the nature of this carrying capacity still remain. These are as follows;

- What is the spatial extant of this carrying capacity?
- What is the temporal extant of this carrying capacity? (Bell S. and Morse S. 2008)

In short, if a system is to be defined sustainable, till what time and within what spatial domain should it be able to sustain itself. These questions have been answered in a context specific domains for instance most Environmental Impact Assessment studies define the zone of impact for which studies have to conducted and experts have suggested time zones for the potential impacts of activities in various industries, however universal answers to these questions continue to evade us.

It may be instructive in this case to go back to the basics and to look at the very meaning of the word sustainable. Essentially, any process or system can be deemed sustainable if it can be seen or predicted to continue its operations uninterrupted over the foreseeable future. The definition of foreseeable can be a murky issue when dealing with complex systems where predictability breaks down. In such systems it is not merely enough to establish sustainability through an observation of the directly predictable phenomena but to keep an eye on system evolution through systemic matrices that tell of the internal health of the system.

Now as the discussion moves on to literature in sustainable development on indicators for measurement, one of the key ideas to take forward is the role of indicators in the chain of codes that govern any system. Most indicators are expressions of paradigms and values and as those values and paradigms shift to accommodate demands of a changing operating environment, the indicators must be revised as well (Meadows D. 1998).

a. Indicator Systems in Sustainable Development

Although sustainability indicators continue to be numerous and varied from those measuring environmental indicators to those documenting the evolution of socioeconomic parameters, there now seems to be an emerging consensus on fundamental indicator development principle frameworks such as Bellagio STAMP (Pintér L. *et al.* 2012). This theoretical development and framework can be significant in designing any indicator systems. The literature in this area will be reviewed.

Several countries, regions, national and international institutions have been working on developing their own indicator systems for sustainability assessments (Hak T. *et al.* 2007). The need for indicator development have evolved from the need essentially to answer the question whether things are getting better or worse (Lawrence G. 1997). Even with the evolution of a number of composite indices early on, the need for guidelines for index development seemed eminent. The Bellagio principles were one of the earliest attempts at developing such a guideline (Hodge R. A. and Hardi P. 1997). Guidelines for national level sustainable development policy were also developed such as those early on for the Canadian government that took account of the carrying capacity of the system (Hardi P. and Pinter L. 1995). Carrying capacity concerns were getting recognized specifically in the context of ecosystem sustainability indicators development (Ullsten O. *et al.* 2004).

Reviews have identified the limitations of many national strategies for sustainable development as these continue to be governed by regional politics (Swanson D. and Pintér L. 2004). The need for innovative policy instruments to bring together concerns for budgetary balance and environmental sustainability was recognized early on (Volkery A. *et al.* 2006). Significant progress was made towards development of guidelines for composite indicators or indices with the International Institute for Sustainable Development report for United Nations Sustainable Developments Division that highlighted the important role composite indicators could play in measuring progress towards any end in a complex system (Pintér L. *et al.* 2005). More recently, the Bellagio STAMP principles have laid down simple guidelines for indicator development that are essential to be followed if the indicator has to have a meaningful relationship with the state of the system it is reporting on (Pintér L. *et al.* 2012).

From national to international and sub-national levels, development of indicators and indicator sets for environmental and sustainable considerations has mushroomed into an industry of sorts with now, literally hundreds of indicator systems available for consideration.

The indicators reviewed here though are specifically ‘sustainable development’ indicators in that they try to measure some aspect of development and its linkage to environment and sustainability and are not mere measures or sets of measures of the state of the quality of environment. The distinction between environmental and sustainability indicators has been discussed in detail elsewhere (Kidd C. 1992) and is not relevant to the research at hand. These are also indicators that seemed to achieve at least some level of international recognition and at least for a while were distinctly monitored by national and international policy making bodies. These are also in many cases, indicators that had evolved at the cusp of economic and sustainability concerns, striking or trying to strike some sort of consensus between two divergent paradigms.

The Green National Product (Cobb C. W. and Cobb J. B. 1994) or the Genuine Progress Indicator first proposed in 1989 as the Index for sustainable economic welfare (Cobb C. W. 1989) took account of the damage caused by the environmental activity towards calculating what essentially was a green alternative for GDP. While the index was not perfect it presented one of the first examples of a quantitative attempt at introducing sustainability concerns in the development discourse. The index was plagued by all the typical problems associated with monetization of ecological and environmental resources. In hindsight it can be seen that it is in fact the attempt to monetize, and not the methodology employed that is the problem. The solution to quantifying sustainability concerns cannot be a reduction of environmental resources to dollar figures.

Marine ecosystems as a science has been responsible for the generation of the some of the most relevant examples of sustainable indicator systems, when it comes to looking at the problem from a carrying capacity perspective. A good early example here is AMOEBA (Ten Brink B. J. E. *et al.* 1991). It was a methodology published in 1991 to maintain not just fish stocks at a sustainable level but to ensure the preservation of marine ecosystem in all its glorious complexity and variation for generations to come. Though the monitoring has been consistent, AMOEBA has been unable to bring observable change in the practices or stakeholders or in the policies governing stakeholders such as large scale industrial fisheries.

The year 1997 saw the publication and popularization of the term ecological footprint (Wackernagel M. and Rees W. 1997). The indicator was based on the land and water requirements required to maintain national standards of living to infinity. The required adjustment to per capita consumption was based on the ratio of the required resources to infinity, to the current consumption. Any ratio value above one was considered to be unsustainable. While the index hasn't really caught on as a policy tool, the concept behind it continues to engage the general public and dominate many a discourses about sustainability. One of its greatest advantages is in how elegantly and simply it allows the presentation of information needed to assess the general sustainability of a national economy. As a species our civilization for instance can be deemed unsustainable if we are consuming resources at a pace more than earth as a system is capable of replenishing them. The index however cannot provide specific guidance on the areas in which resource consumption ought to be reduced.

1997 also saw the publication of the Genuine Savings Index (Atkinson G. *et al.* 1997). The genuine savings index sought to measure the reinvestment from the 'rent' derived from resources back into the regeneration of capital stock, so that the capital stock never declines. The capital stock in this case included human resources as well as natural capital. The index has proven to be popular in policy development, though it still utilizes a monetization approach towards natural capital depreciation.

The Living Planet Index published next year (WWF 1998) saw for the first time, the publication of an index that was an indirect, quantitative comment on the state of the system that is the biosphere and human civilization (Loh J. *et al.* 2005). Biodiversity in itself may have many reasons for its conservation, the discussion of which is beyond the scope of this research, but it also serves as a significant index on the state of the ecosystem that humans as a species inhabit. While the alarm on biodiversity loss has gone up ever since, the actual translation into policy actions has remained limited. This could be considered one of the cons of all indirect indices; that while consensus may be easily achieved on the state of the system they are reporting on through monitoring of these indices, the need and areas of action to rectify the problems are not immediately apparent. This theme will be revisited later in detail as the indicator system being proposed in this research is an indirect indicator and its propagation and effectiveness may be faced with similar daunting challenges.

Several modifications have been proposed to the Gross Domestic Product (GDP) to include some consideration of sustainability starting with the one proposed by Nick Hanley in 2000 (Hanley N. 2000). Such measures have the advantage of simplicity as they provide an immediate figure of comparison with GDP; however their use in serious policy formulation continues to be limited.

A significant stride was made in evaluating sustainability from a systems perspective with the publication of the City Development Index for the first time in 2001 (UN-HABITAT 2001). The report and the index stressed to certain extent the importance of cities as complex adaptive systems in monitoring the sustainability of growth. The index had five dimensions; infrastructure, waste, health, education and city product. This index however does not look at sustainability from a carrying capacity perspective and since most sustainability policy is still driven at the national level, has found it hard to affect policy discourse at the level of implementation.

The Human Wellbeing Index was proposed in 2001 (Prescott-Allen R. 2001) to present a unified picture of the country wise human and environmental wellbeing. The index took into account health, population, welfare, knowledge, culture and society and equity as well as for the environment, air, water, land, species and genetic resources. The index is a very instructive alternative gauge for measuring the success of societies and civilizations in a holistic context. The index does not necessarily provide commentary on the sustainability of a society or nation state but it is assumed that environmental resource status and concerns for future will reflect the sustainability of the continued operation of the society. A similar alternative paradigm for evaluation of national performance is provided by the famous happiness index (Bates W. 2009). Again, although these indices do not provide direct commentary on sustainability, by providing an alternative means to measuring success in the operations of state, they pave the way for indirect sustainability indicators to achieve greater recognition and acceptance.

The most popular of GDP alternatives, the Human Development Index (HDI) was adopted in 2005 by United Nations Development Program (Sudhir A. and Amartya S. 1994). The HDI takes into account life expectancy, gross national product and education index. The HDI has achieved significant success in diverting development investment towards human wellbeing goals. In its current form the HDI is not a comment on the system sustainability even from a purely environmental perspective. Being however the most widely quoted and used alternative index for policy development, significant lessons are to be learned from the process and development mechanism and propagation of HDI. The HDI also continues to be a fairly indicative representation of the standard of living and wellbeing in nations relative to each other, on a year by year basis.

It would be essential here to mention three environmental indicators which have made an attempt to give some weight to sustainability concerns albeit not essentially from a systems perspective. The Environmental Sustainability Index (ESI) published as a pilot project in 2005 measured sustainability across five dimensions (Esty D. C. *et al.* 2005); environmental Systems, environmental stresses, human vulnerability to environmental stresses, societal capacity to respond to environmental challenges and global stewardship. These five dimensions in turn were based on 21 indices derived from 76 variables. While a very comprehensive environmental indicator system, the ESI does not present a measure of sustainability in terms of natural capital reserves. In 2006, the Yale Center for Environmental Law and Policy also came up with the Environmental Performance Index (EPI) to measure the policy performance of different nations towards meeting environmental targets. Together with the ESI, the EPI could be used to guide environment policy for nation states though not necessarily providing guidance towards long term sustainability goals.

The Environmental Vulnerability Index (EVI) (SOPAC 2005) used an equally weighted composite of 50 indicators of hazards, resilience and damage to arrive at a national vulnerability index to environmental hazards. The range was from 1 to 7. The index provides a significant measure of the state of preparation of each country to the increased frequency of higher intensity extreme weather events that may result from a changing climate. In that manner, the index captures directly a very important measure of the sustainability of a society to climate change.

The systems perspective in measuring sustainability is only just getting acknowledged in the development of indices. The Economic Complexity Index developed by Harvard Kennedy School and MIT Media Lab measures the sustainability and potential growth of economies in terms of their complexity (Hausmann R. *et al.* 2011b). Complexity here is measured as the ability of the economy to produce largest number of goods. Such a measure of complexity can not only provide information about the long term growth potential of economies but also the resilience of economies to local or international, large scale, unpredictable shocks. Though this index does not have a direct relation to sustainability defined in environmental terms and does not measure environmental variables, it has significant development policy implications, even in an environmental context. Many indicators have been proposed an alternative to GDP based development policy however in economic planning and policy circles, GDP continues to enjoy the status not afforded to any other indices or indicators. The reason for that is the assumption that maximization of GDP is the road to maximization of economic growth and resilience. The surest way to maximize GDP was considered to be specialization in specific traits to gain the surest competitive advantage possible (Leamer E. E. 2007). With the Economic Complexity Index now turning out the be a better predictor of growth than GDP and other related indices (Hausmann R. *et al.* 2011b), it should be evident that a narrow minded focus on GDP through specialization is not a determinant of the resilience of an economy. The systems perspective requires that countries optimize their productive capability through closed economic policies and development of a varied production base and human resource capital. This may require investments in education and development sector which are not justifiable by the doctrine of immediate economic maximization of revenue. The Economic Complexity Index shows the potential for paradigm shift in the pursuit of sustainability research from the systems perspective.

An important indicator that employs systems' studies to arrive at a measure of urban performance that is irrespective of the scale of the city is the Scale Adjusted Metropolitan Indicator (SAMI) (Bettencourt L. M. A. *et al.* 2010). SAMI shows the economic, crime and innovation performance of various cities after adjusting them for the improvement or decrease that comes merely as a factor of the scale of the cities, irrespective of the development policies. Again, this is significant because it helps highlights policies that actually do have an impact on the performance of urban centers as opposed to those that don't. Many of the currently existing sustainable development indicators do not meet the fundamental requirements for good indices formation, i.e. normalization weighting and aggregation (Böhringer C. and Jochem P. E. P. 2007). It has been summarized that the development of a city is a balancing act between the friction of urban life and optimization of serendipitous opportunity (Bettencourt L. M. 2013).

The ideas of system complexity and the identification of critical leverage points should hold significance in any discussion of indicators for sustainable development, going forward. We must also understand that our conceptions of systems are paradigms and as such should be flexible and we should be open to multiple interpretations, realizing the significance of indicators in governance but appreciating that their relevance is bounded by the context in which they are conceptualized, formalized and measured (Meadows D. 1998).

Several bottom-up urban indicator systems have been proposed which measure indices related to urban form at a low granularity and keep track of them. These include urban indicators developed by UN Habitat (UN HABITAT 2003) and ICLEI for example. The indicators define rural community development and take into account quantitative as well as qualitative measures and provide a framework for sustainable community development (North Central Regional Center for Rural Development 1999).

Several indicators have been developed that incorporate GDP externalities by estimating shadow costs for things in the commons or things otherwise not being valued. Examples include the comprehensive wealth accounts published by the World Bank (The World Bank 2013) and more recently the Inclusive Wealth Index developed by the United Nations' International Human Development Program (IHDP) and Environment Program (UNEP). The Inclusive Wealth Index (IWI) estimates the increase in a nation's inclusive wealth by subtracting the cost of exhaustible natural resource consumption and adding the human capital among other holistic considerations (UNU-IHDP and UNEP 2012).

have been efforts such as the Economic Complexity Index (Hausmann R. *et al.* 2011a) and "Virtual Sustainability" (UNU-IHDP and UNEP 2012) to take into account the complex web of interactions that form economies, we are far from comprehending the beast that is complexity. The Economic Complexity Index measures the diversity of a national economy in terms of things it can produce as well as the ability of countries to produce unique goods and services. The higher the number of more unique goods produced by a country, the most resilient its economy would be. Economic Complexity Index has been shown to be a better predictor of long term growth than GDP and may potentially be used in the future to identify fragilities.

2.2.2 Urban Planning

In architecture and urban planning there has been for the past three decades an emerging body of work rediscovering the significance of scaling in design (Salingaros N. A. and West B. J. 1999). Through a general review of some of the general literature in the area, I intend to focus on the specific examples of works studying cities on the basis of their fractal dimensions.

In quarters of the urban planning discipline, a strong albeit somewhat marginalized resistance to suburban sprawl started to emerge at the beginning of the emergence of sprawl itself (Katz P. *et al.* 1994). Without doubt the most significant challenge facing this movement was the quantification and verbalization of what they thought was wrong with sprawl in terms that were objective and concrete. There were urban planners who could see that there was something wrong with the sprawl started by the Interstate Highway Development program initiated during U.S. President Eisenhower’s term, they just couldn’t verbalize what it was. The New Urbanist movement initially started as an almost obsessive drive to measure in detail urban elements such as the length of road and curbs and their relation to ‘good’ or ‘bad’ urban design. The idea was to identify quantitatively what was so wrong with suburbia. The focus of the movement soon shifted to scouring the annals of historical architectural design in search of principles that made traditional architecture so appealing at a deep instinctive level. What the movement soon discovered was the relationship of urban and architectural design to fractals. It was noticed that traditional historical architecture almost everywhere from Alhambra to the Sistine Chapel was biophilic and biomimic in nature, in that it mimicked the fractal nature of life. This lent the designed environment the vitality of the living environment making it that much less alien to inhabit. On a greater scale, the New Urbanist movement noted that these principles were followed in the evolution of design of historical cities (Katz P. *et al.* 1994). Since then, the complex-adaptive nature of the urban system has been explored in detail in comparison to other complex-adaptive systems commonly found in nature (Portugali J. 2011). While the contribution of New Urbanism to critical analysis of urban design theory have been significant, solutions for practical application of the principles to urban design in a fossil fuel powered or changing climate have yet to emerge. Famous New Urbanist model cities such as Seaside, Florida have either been rendered unproductive tourist attractions or have generally failed to cultivate, healthy, organic urban interactivity (Katz P. *et al.* 1994). Efforts to include New Urbanist principles in standards for sustainable urban design (USGBC 2007) have largely focused on direct measurements of direct elements such as the percentage of area devoted to parking, and not on measuring, assessing and tweaking systemic parameters that define the city at an intrinsic level.

2.2.3 Fractal Mathematics

With growing interest in them since their designation as “fractals” (Mandelbrot B. B. 1983), systems exhibiting power law distributions have been shown to underlie a number of natural phenomena from the distribution of widths of tree trunks in forests to the distribution of wealth and other socioeconomic measures in markets and economies (Bettencourt L. M. A. *et al.* 2010; Taleb N. 2008b; West G. B. and Brown J. H. 2004). Moving on from a general review of literature in fractal mathematics I will focus on the studies using fractal geometry to study cities and research that comments on systems sustainability based on fractal dimensions and structuring.

There has been growing interest in the structure of fractal systems ever since Benoit Mandelbrot coined the term "fractal" in the early sixties (Seuront L. 2011). Living organisms and many other similarly complex adaptive systems have been shown to obey a power law in scaling of the sizes of their various elements and are therefore fractal in nature, with the exponent of the power law being the fractal dimension (Mandelbrot B. B. 1983; West G. B. and Brown J. H. 2004; West G. B. *et al.* 1997). If we look at life for instance as a system, the distribution of many fundamental properties across species, such as metabolic rates follows a power law with respect to size (West G. B. and Brown J. H. 1997). The scaling within such systems, measured as the fractal dimension of the system is a good indicator of the health of such systems with aberrations skewing the distribution and hence fractal dimension in one direction or the other. Aberrant growths such as malignancy in living cells can be observed as having distinct fractal dimensions (Hern W. M. 2008). In architecture and urban planning there has been an emerging body of work rediscovering the significance of scaling in design especially within the new urbanism movement (Batty M. and Longley P. 1994; Benguigui L. *et al.* 2000; Bettencourt L. and West G. 2010; Coward L. A. and Salingaros N. A. 2004; Salingaros N. A. and West B. J. 1999; Shen G. 2002). It has also been shown that on a greater scale, similar properties as fractal systems can be attributed to the distribution of human population in general with cities having predictable socioeconomic and infrastructural parameter values based on their size (Bettencourt L. M. A. *et al.* 2010; Chen Y. 2011; Hern W. M. 2008). Further, in closely placed cities, a cascading effect has been observed which diminishes according to a power law distribution (Chen Y. 2010b).

2.2.4 Fractal Dimensions of Cities

There have been a number of studies that have estimated the fractal dimension of cities, mostly using box-counting mechanisms on maps of different resolutions. The literature and their findings will be reviewed (Batty M. and Longley P. 1994; Benguigui L. *et al.* 2000;

Shen G. 2002), though the methodology to be employed in this research is much more sophisticated.

Traditionally, the Box Counting Mechanism (BCM) has been used for estimating the fractal dimension of cities (Batty M. and Longley P. 1994; Benguigui L. *et al.* 2000; Hern W. M. 2008; Shen G. 2002). This usually involves implementing a grid on a map or satellite image of the city and then counting or estimating the covered area or populated area within each box. The count is then binned into classes according to increasing size or increasing number of boxes (having count within the class range) within each class. The fractal dimension is then estimated by plotting a log-log graph of the count range against the number of boxes falling within that count range; the slope of the resulting trend-line is the exponent of the power law or the fractal dimension of the distribution of sizes of elements.

Some of the earliest explorations of the fractal nature of cities included studies of transportation networks such as railway systems. A clear power law distribution of elements hinted at the fractal nature of urban systems (Benguigui L. and Daoud M. 1991). Qualitative analyses even earlier were painting a picture of the city as a complex adaptive system with non-linear, unpredictable processes (Wong D. W. S. and Fotheringham A. S. 1990). With the development of more rigorous implementations of BCM the study of the fractal nature of city expanded into different disciplines (Clarke K. C. and Schweizer D. M. 1991). Initial investigations in systems analysis using cellular automata highlighted the similarities between the evolution of cities and the progression of fractal systems such as cellular automata (White R. and Engelen G. 1993). With increasing computing power fractal based descriptions of urban form grew in the accuracy of their consistency with historical data (Batty M. and Longley P. A. 1987). The generation of city-like structures through cellular automata presented further evidence for the fractal nature of cities (Batty M. 1997). Direct modeling experiments also generated positive results (White R. *et al.* 1997). The evidence for the fractal nature of cities piled up with advances in computer analysis and simulation techniques and increasing computing power (Batty M. and Xie Y. 1996). The fractal nature of cities was soon taken to be uncontested in literature (Batty M. and Longley P. 1997).

A methodology for estimating fractal dimension for 3-D objects envisions the values of a third non-spatial variable being utilized for the estimation of the fractal dimension (Ge C. and Le-shan Z. 2010). In essence the methodology uses the value of a third variable and its distribution over space to estimate the fractal dimension. Similar technique in my estimation of fractal dimension of distribution of a third variable such as population density over space will be employed.

Researchers have also been exploring the relationship between fractal dimension and other geometric measures of the urban form, however the BCM for estimating fractal dimension has not been improved on (Yanguang C. 2011a). Higher resolution remote sensing images have also been utilized to study the evolution of the fractal dimension of cities (Ge M. and Lin Q. 2009). BCM has been frequently used to analyze the evolution of land use (Hua L. *et al.* 2010). BCM was again used to measure environmental degradation in terms of loss of green space for instance for Lijiang City in China (Wang H. *et al.* 2011). The spatio-temporal evolution of urban systems has been studied using BCM-based fractal dimension confirming again the complex adaptive nature of urban systems (Chen Y. and Jiang S. 2009). Urban sprawl in Istanbul was studied using BCM and it was found that the fractal dimension is positively correlated to city growth when the sprawl is 'concentrated' (Terzi F. and Kaya H. S. 2011). The fractal nature of European cities has also been explored using BCM to calculate the fractal dimension (Thomas I. *et al.* 2010). The relationship between fractal dimension as a measure of space filling and urban spaces has been explored and the idea of intermittency has been introduced to explain less than optimum space filling (Yanguang C. 2011b). However, chance and intermittency fail to explain less than optimum space filling in highly planned urban areas especially in North America. While there have been significant improvements and deployment of new technology in fractal dimension estimation using BCM, such as the use of wave spectrum methodology for image analysis (Chen Y. 2010a), the central application of fractal mathematics remains unchanged in the form of BCM analysis.

A more computation based approach with an automated module was developed to estimate the fractal dimension of cityscape skyline using BCM. In this case the height of the skyscrapers was used as the third dimension for estimating fractal dimension (Chalup S. K. *et al.* 2009). BCM has been used to study the urban-rural delineation (Zhaoxian G. 2011). The urban boundary problem has also been explored using fractal analysis for several European cities and it was found that the distance separating the urban conglomeration and land use type from surrounding areas and a distance threshold for urban boundary calculated using the dilation curve, were positively correlated (Tannier C. *et al.* 2011). Fractal dimension has also been identified as an indicator to study the evolution of estuaries and deltas (Edmonds D. A. *et al.* 2011). In the Tian Shan mountains of Central Asia, fractal growth was mapped onto oasis city structures for the study of regional structure and spatial morphology. The growth in the presence of minimum urban planning was found to be distinctly fractal in nature (Wang H. *et al.* 2011; Zhang Y. *et al.* 2009). BCM-based fractal dimension has been used to study the intra-urban diversity of Brussels (De Keersmaecker M.-L. *et al.* 2003). A correlation has also been found in the fractal dimension of built structures such as the shape and size of windows and population density distribution on the peri-urban fringe (Thomas I. *et al.* 2007). The transportation networks in US cities have also been shown to be fractal with excessive sprawl having a negative impact on the fractal nature of the evolution of a city (Lu Y. and Tang J. 2004). BCM based fractal studies of various Asian cities also find power law scaling (Carvalho R. and Penn A. 2004). The urban transport system in Seoul is found to be distinctly fractal in nature (Kim K. S. *et al.* 2003). Fractal characteristics have also been identified as a features of urban street patterns (Cardillo A. *et al.* 2006). Census data has also previously been used in fractal analysis. An analysis of fractal dimension of European cities shows that national contexts matter little in terms of evolution of cityscape (Thomas I. *et al.* 2011). In the US, there have been calls for studying population distribution using fractal analysis (Wu J. *et al.* 2011) utilizing the extensive US Census geospatial data, though few studies have made extensive use of the data at its highest resolution.

There have also been calls for the inclusion of fractal analysis in the city design and hence planning process (Batty M. 2009) though these have taken more polemic, qualitative forms with quantitative recommendations missing from the actual guidelines or design code. Analysis and standardization guidelines such as LEED Neighborhood (USGBC 2007) have focused primarily on direct measurements of city structure elements and not the systemic properties that define the city structure.

The following section detail all the different areas of urban analysis where fractal dimension calculation and analysis has been employed. Fractal dimension analysis has been used to help understand the city as cellular automata (agent based modeling), in disaster risk resilience planning and other planning processes. There is also significant literature available that criticizes the use of fractal analysis and discusses its limitations. A large number of studies focus on explaining why city behaves and evolves like a fractal. All of these are discussed in the following section.

a. City as a Cellular Automata

Experiments with cellular automata to model urban forms have generated promising results (Barredo J. I. *et al.* 2003). Cellular automata are fractal mathematical structures that create complex patterns out of repetition of simple algorithms. The minor difference between natural processes and cellular automata is the presence of random mutations in natural processes. Successfully modeling applications of cellular automata to urban development are further evidence of the fractal nature of the city.

One of the commonalities between cellular automata and fractal structures such as cities is self-similarity. Self-similarity is the property of the structure to repeat a simple algorithm again and again to generate complexity. Self-similarity is a law of nature and can be observed at different scales in all natural phenomena. For instance, self-similarity and mutation are the foundations of evolutionary processes. Fractal dimensions can be used as a means to identify the presence of self-similarity in the evolution of a complex system. In a recent analysis of the city of Tel Aviv it was noted that the fractal characteristics observed in the development of the city hints at 'leap-frogging' or stop-gap development in the evolution of the city (Benguigui L. and Czamanski D. 2004). A cellular automata based simulation of the growth of the city concluded that this mechanism of growth is primary responsible for the fractal nature of city.

b. Disaster Risk Resilience

BCM based fractal dimension analysis is also being used to comment on the resilience of cities to disaster risk (Wang W. *et al.* 2011). Fractal analysis has been used in seismic hazard assessment in some cities (Spada M. *et al.* 2011) though the process has yet to emerge as a standardized methodology. Large scale Census based data has only recently been used to analyze self-similarity and fractal distribution in evolution of city landscape (Bettencourt L. M. A. *et al.* 2009). In some analyses though, population density models have been shown to display a latent fractal distribution (Chen Y. 2010c).

c. Why City is Fractal

More complex multi-fractal models have been proposed to explain the fractal nature of cities. Such models have tried to evolve urban fractal patterns through algorithms based on central place theory or entropy maximizing principle and have significantly advanced the understanding of evolution of urban form (Chen Y. and Zhou Y. 2004). Certain city hierarchy models have been reduced to scaling laws as well in order to computationally study their evolution and the evolution of fractal cities governed by these laws. All of these have contributed to the development of multi-fractal city models (Chen Y. and Zhou Y. 2003).

Central place theory tries to explain the evolution and structure of cities by envisioning the mushrooming of the urban environment around a central place which exists to provide goods and services (Berry B. J. L. and Pred A. 1961). The theory then goes on to define and rank the nature of interactions that exist between this center place and the urban areas that develop around it. These interactions vary from marketing to administration and can be imagined as simple algorithm analogous to steps in a cellular automaton. In this manner, the evolution of city can be seen as the growth of a cellular automata governed by simple rules. However, when we are considering a complex system such as a city where the rules of the algorithm are not well defined, the evolution of the system state cannot be predicted; which is not to say that that can always be done in the case of cellular automata either. Just that at least in computing environments cellular automata can be generated and regenerated, while cities cannot be created or recreated for purposes of research in a laboratory. The development of cities is governed by certain Bayesian probability axioms just like the development of any other complex-adaptive systems. Entropy maximizing principle is just such an axiom. According to the principle the state of a complex system will evolve to maximize information entropy given initial state and data are defined (Shore J. E. and Johnson R. W. 2002). In terms of the evolution of city, this principle can be said to mean that the evolution of the city will most likely follow the path of least resistance. This least resistance can be in terms of for instance least consumption of energy or in terms of a transportation of people and materials, so the city will evolve around a major highway. The least resistance can also be the? least administrative resistance whereby a city can grow in a way that conforms to all policies laid down by administrative bodies. If city authorities dictate that there should be a hundred square meters of parking for each hundred square meters of built area, the path of least resistance would be to follow this policy. The path of least resistance thus can be defined in terms of each of the rules or interactions of the central place theory model. Several other rules in fact can be sketched out which may influence the evolution of the city along the path of least resistance. Through studies of cellular automata and other natural systems it has been deduced that the evolution of fractals in a result of the system operating in a manner to be as conservative in its evolution and consumption of energy during operations as possible. The evolution of city is no exception. Cities may be fractal because they evolve along the paths of least resistance while following the rules of certain basic societal interactions.

The evolution of fractal structure of cities has also been studied in light of percolation phenomenon (movement of liquids through porous media) providing yet another lens on the complex dynamics of the growth of urban areas (Stanley H. E. *et al.* 1999). Comparisons and analogies have long been made to other chemical processes such as diffusion limited aggregation and percolation (Fotheringham A. S. *et al.* 1989; Makse H. A. *et al.* 1998). The conclusions here again point to the fundamental laws of energy conservation and entropy maximization as governing the growth of cities.

With fractal dimension already being used to study the quality of habitat of other species (Imre A. R. and Bogaert J. 2004), it is only the next logical step to use fractal analysis to comment on the quality of human habitats.

d. Reservations and Criticisms

Some computationally rigorous studies have however shed light on the practice of drawing systemic conclusions from case based studies of city dynamics using fractal mathematics (Vaughan J. and Ostwald M. J. 2010). It is important thus to limit generalizations in this promising area of scientific investigation to a minimum while connecting numerically derived indicators such as fractal dimension and its status within the geometry of theoretical concepts as opposed to living concepts such as cities. Fractal dimension may tell us a lot about the characteristics of for instance a Sierpinski gasket – which is a very widely known fractal set with an overall shape of an equilateral triangle- but that does not necessarily mean that a fractal dimension should be an indicator of similarly significant note when studying cities. In any study thus, especially one where novel methodologies are employed, it is essential that fractal dimension as a parameter be defined rigorously before further links ought to be established. My research is one step forward in that direction.

2.2.5 Black Swan Risk Studies

Black Swans are low probability, high risk events that can happen in fat tailed systems (systems where the distribution of risk is highly skewed towards the tail). Aberrant growths observable by unexpected fractal dimensions in power-law distributed systems are a source of Black Swans. As such the study of sustainability in terms of fractal dimensions is the study of sustainability in terms of resilience to Black Swans. In Black Swan literature, I will focus on the mathematics of resilience or “anti-fragility” (Taleb N. 2008b).

In his book, Normal Accidents (Perrow C. 1984), Perrow tries to make sense of the accident at Three Mile Island and why thousand-year-events were much more frequent than one every thousand years in the nuclear industry. After reviewing risk in various enterprises from aviation to aircraft carrier operations, he identifies a type of systems where two things are at play. Firstly, there is tight coupling within constituent systems of the composite system, which means that whatever happens in a constituent system affects the operations of other connected constituent systems directly. Typical risk analysis, Perrow found out looked only at one degree of event risk, so for instance if a shaft breaks and causes boiler pressure to leak, there will be an estimate of risk of the shaft breaking and the boiler pressure falling, but no account will be taken of the composite risk of boiler pressure falling in estimating the risk of shaft failure. If the boiler explodes for instance and causes a meltdown, that meltdown would be a second-degree or indirect result of the shaft failure. However in the nuclear industry, Perrow observed, the failure of shaft risk estimate did not take into account the cascading effect of the event and the compounding risk. In tightly-coupled systems where there's no buffer between elements, events can cascade and risks compound.

The second problem was what Perrow termed "Interactive Complexity" where there were elements within the system that formed constituent elements of more than one sub-systems; for instance, a shaft that both heated the control room and served as a heat sink pipe for the boiler. Failure of such a shaft would not only destabilize the boiler but may render any continued work in the control room impossible. This again had an effect of compounding risk. Perrow saw that tight coupling and interactive complexity lead to accidents which not only were impossible to assign a realistic risk value to, but also, almost impossible to predict. In that sense, such events were "accidents", but on the other hand, they were also systemic and borne of the complex nature of the system and were in a way inevitable because of the tight-coupling and interactive-complexity, and hence, were "normal".

It is important to note here that both, tight coupling and interactive complexity are design characteristics that come mainly out of a drive for economic efficiency in design. Buffers are redundant and hence not economically efficient; similarly, utilizing one constituent element for two or more purposes can be economically efficient. Similar analysis has also been recommended in cumulative impact assessment literature (Löwgren M. 1999).

In the financial industry, these 'normal accidents', are now known by the popular name "Black Swans" (Taleb N. 2008b). Nassim Nicholas Taleb in his book, "The Black Swan", discusses the impact of the low probability, high consequence event on the history of mankind. Taleb divides scientific inquiry into four quadrants as shown in Figure 2;

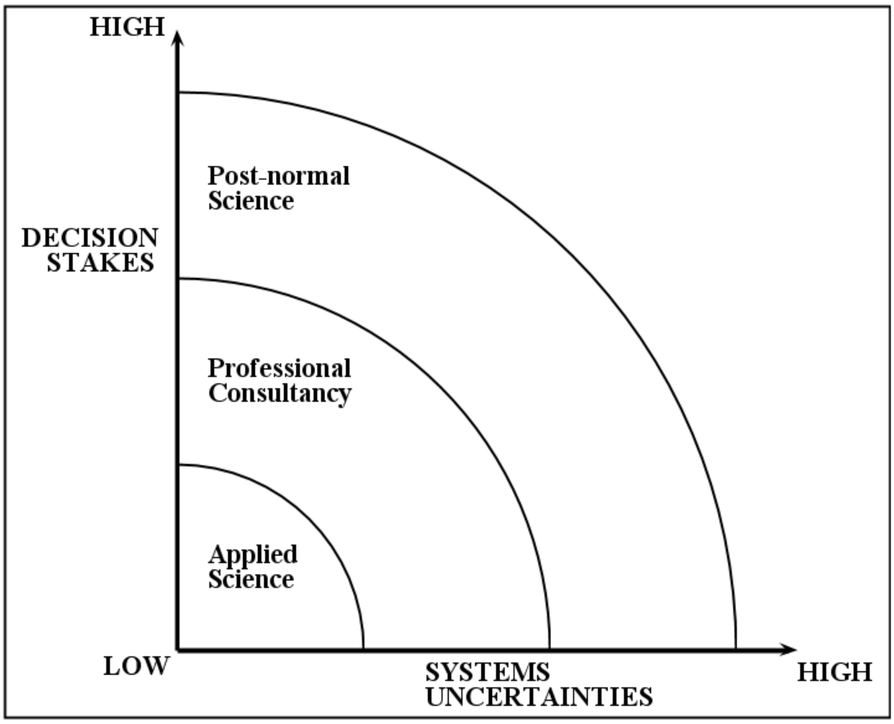
APPLICATION DOMAIN	Simple payoffs	Complex payoffs
Distribution 1 ("thin tailed")	Extremely robust to Black Swans	Quite robust to Black Swans
Distribution 2 ("heavy" and/or unknown tails, no or unknown characteristic scale)	Quite robust to Black Swans	**LIMITS of Statistics – extreme fragility to Black Swans**

Source: (Taleb N. 2008a)

Figure 2: The Fourth Quadrant; Where Predictability Breaks Down

On the x-axis is increasing complexity and therefore a decrease in predictability, while the y-axis shows increasing impacts or high consequentiality. In systems where both are present, high complexity and potentially high consequence events, Black Swans are bound to happen; and are "normal accidents".

A similar classification, though in a different paradigm has been identified by the post-normal science literature to delineate areas where "normal" science may no longer be applicable (Funtowicz S. O. and Ravetz J. R. 1994). As shown in Figure 3, when uncertainty and decision stakes are both high, normal predictive sciences should not be the only decision making factor to consider.



Source: (Funtowicz S. O. and Ravetz J. R. 1994)

Figure 3: Post-Normal Science; the Domain Where Predictability Breaks Down

Climate change is another area where we are faced with high impact scenarios emerging from the dynamics of a complex, unpredictable system (Patt A. G. 1997). Due to its limited predictive capabilities, science at the current state may not be able to produce results having enough certainty to convince everyone. The future, beyond the two-degrees centigrade warming, post peak-oil may only appear hazy. But that is no reason not to transform the systems of civilization in a way that makes them robust, resilient, even "anti-fragile" (Taleb N. 2008a) to the unforeseen and unpredictable but highly costly. And that ought to be considered in any definition of "sustainability" heretofore.

a. Self-organized Criticality in Real Systems

Self organized criticality is an important function of all complex-adaptive systems and the failure of systems to observe and implement this function is one of the primary reasons for system failure. The role of self-organized criticality in urban systems has long been recognized (Michael B. and Yichun X. 1998), though not necessarily as a function of urban sustainability. In terms of system success and failure this characteristic can be defined as the ability of the system to restrict the emergence of too-big-to-fail sub-systems or elements. If this simple mechanism is seen to be failing in the urban planning process than the process can conclusively said to be leading to unsustainable development.

The role of critical limiting resources in systems evolution has been studied extensively in laboratory (Cavailhès J. *et al.* 2009) as well as in practice. The consequences of such a limit analysis on the economics for instance of the land markets has also been studied and has been shown to be of significance for further research (Caruso G. *et al.* 2011).

Studies have suggested that fractal laws in social systems may be an expression of the maximization of stability of system in response to increasing entropy (Yanguang C. 2012).

2.2.6 Information in Governance and Policy

Decision making in the face of unpredictable events and planning for the development of social units be they towns, cities or organizations is a niche subject in governance that has gained prominence during the last ten years. It is now widely recognized that service delivery perspective of governance is not only not enough, it is also unsustainable by the very virtue of its definitive lack of vision (Bovaird T. and Löffler E. 2002). This realization also has lead to realignment in measurement practices and the definition of indices for assessment of governance performance. Measurement in governance is not only dependent now upon improvements in public policy outcomes but on implementation by all stakeholders on a set of agreed upon principle for an integrated, long term vision. The second objective should also include a vision for the quality of life.

It has indeed been this stress on the quality of life which has informed and evolved the science of information flow and management in gauging governance performance. From local level initiatives to global projects like the Human Development Index, the focus has been increasingly on measuring the immeasurable; from human contentment to sustainability of development projects (Bovaird T. and Löffler E. 2003).

It had long been recognized that the flow information and its usability by complex system components was not a linear process (Chandler D. 1994), however the application of these ideas in theories of governance took a little time coming. The debate in some ways harkened back to the age old debate about the role of information, science and research in politics (Weiss C. H. 1973, 1977; Weiss C. H. 1993).

In order to consider the role of information in governance, the complex nature of the societies being governed needs to be considered. The emerging consensus in this regard seems to be that merely the communication of information is not enough but it is also necessary to make the information accessible to stakeholders and decision makers and to place the information in a context and a form where it can influence the direction in which the system will evolve. Key concepts that have evolved out of the exploration of these ideas are concepts such as leverage points; which are significant points in the system where information can be fed to produce rapid and demonstrable change in system composition. The significance of conveying the message in a form that resonates with system agents is also being recognized. To that extent, even institutions such as the United Nations Environment Program stress the significance of narratives in inspiring change, as opposed to a mere publication of data (UNEP 2005).

Institutional explorations of the significance of information in systems have been expanding with the creation and growth of such institutions are the ICLEI; Local Governments for Sustainability. The ICLEI is focused on the documentation and sharing of knowledge between research organizations, local governments and other practitioners and scholars in the field. ICLEI organizes conferences and provides training and development consultancies for local government bodies.

Another such institution is the Community Indicator Consortium which is engaged in knowledge sharing on community indicators within North American communities mostly. Another such institution is the Canadian Sustainability Network Indicator that is working on bridging the gap between the science on indicators and the practitioners and their use for indicators. Developments in the science of indicator development have started to identify the significance of transparency and 'process' integration in indicator development. Integration in the indicator development process may be just as important as scientific credence especially when the indicators are being developed to measure complex systems (Kaufmann D. and Kraay A. 2008).

3. Focal Theory of this Research

Through a review of the literature in all the aforementioned and surveyed fields, a very specific orientation for the future progress of this research is now starting to emerge. Literature suggests that cities are complex adaptive systems. Fractal dimensions of such systems such as those observed in nature as living organisms and ecosystems are a good indication of the health of the system in many ways. They are an essential indication of the 'sustainability' of the system defined as the carrying capacity of the limiting resource to growth. They are also a measure of the resilience of the system to 'Black Swan' events. While cities are now considered complex-adaptive systems, theories that provide practical guidance to policy on urban development from that perspective have not been an area of focused research. A fractal theory of urban sustainability is needed.

3.1 Fractal Theory of Urban Sustainability

This is not the first attempt to link fractal analysis to environmental or sustainability concerns directly. There have been suggestions of use of fractal analysis in Environmental Impacts Assessment process (Triantakonstantis D. and Barr S. 2009), however such methods have not really caught on.

The Fractal Theory of Urban Sustainability starts off by acknowledging that the city is a complex-adaptive system. Like for all other complex-adaptive systems arriving at a detailed predictive model of the city behavior based on changes in all the relevant directly measureable variables is a near impossible task; not much different from arriving at a model of human body based on variables like temperature, platelet counts etc. that can accurately predict the onset of disease. Direct variables provide important information but some indirect structural? measures are also needed to gauge the health of the system. The theory also defines sustainability essentially as the ability of a system to continue its operation without any change in the governing equations, for any foreseeable future. A civilization that is dependent on fossil fuels in a post-peak oil world in this scenario is essentially unsustainable; and so is a city dependent on extensive inputs of cheap energy for its continued operation. However, one of the problems with scaling from the universal to the specific, in this case, from the 'unsustainable civilization' to the 'unsustainable city' is that such analyses are always post-factum, observational or critical, and not predictive. Going from the large to small in detecting phenomena is essentially academic post-mortem, paleo-analysis, historical research. This is because phenomena take much longer to become apparent at larger scales; the peak production of an oil well can be identified much earlier than that of an entire field, which in turn can be identified much earlier than that of a country, or eventually of the whole world. Accumulation of phenomena over spatial and temporal scale makes them clearer but takes time to show their hand.

What is needed is a theory of urban sustainability that identifies when and where a city will cross the line from sustainable to unsustainable, and does so at the pre-factum, planning stage. Fractal analysis tells us about the health of systems such as human body or an ecosystem when anomalous growths first start to threaten system sustainability. Healthy ecosystems maintain their fractal dimension within a very specific range. The Fractal Theory of Urban Sustainability aims to identify and propose such a range for cities.

3.2 Fractal Dimension based Urban Development Sustainability Indicator

The links between New Urbanist principles and urban development planning in a quantitative manner have been explored in theoretical studies (Joo J. 2009). However, any theory in order to be effective needs to identify key variables which can be monitored to observe not only the success of the theory in predicting system state but also in the case of any sustainability theory, the health of the system itself. For the Fractal Theory of Urban Sustainability, this indicator is obviously the fractal dimension.

The urban system however is not only a complex system, it is a composite of many complex systems layered upon each other and interwoven in intricate ways. Mathematically, the fractal dimension is a measure of the spread of a certain variable over the entire distribution within the system. The fractal dimension can be measured for the distribution of any number of variables within the city. A city can have many fractal dimensions. In order to study the city using the Fractal Theory of Urban Sustainability thus not one fractal dimension based indicator, but a system of indicators, aggregate indicators and indices would be needed. In order to establish scientific credibility, statistical linkages between these indicators and indices and direct measures of health, wealth, environment, fuel efficiency would need to be established. This specific piece of research is not expected to accomplish all of these tasks, but it is intended to provide the basis for further exploration of the study of urban sustainability from the perspective of complex-adaptive systems. The hope is that fractal dimension based urban sustainability indicator systems will continue to be a field of evolving intricacy and breadth.

3.3 Scaling Down from Sustainable Development to Fractal Nature of Cities – A Summary

The following section summarizes the findings of the literature review. While it repeats some of the ideas, the objective is to demonstrate in summary how sustainable development is related to fractal nature of cities.

The global energy conundrum is expressing itself in terms of two conjoined problems. On the one hand the specter of peak oil, now admitted by even some conservative estimates to have occurred around 2006 (Kerr R. A. 2011) is dampening prospects of continued global economic growth; while on the other hand, manmade climate change is demanding that we burn no more than 500 billion tonnes of the Earth's carbon reserves (equivalent to 1830 billion tonnes of CO_2); roughly 60% of the currently discovered fossil fuel reserves (capable of producing roughly 3000 billion tonnes of CO_2 equivalent green house gas emissions) (World Energy Council 2007), if we are to avoid a cataclysmic two degrees centigrade plus change in temperature by the end of the century (Allen M. R. *et al.* 2009).

Despite academic and research ventures that indicate the viability of a renewables based global civilization, markets continue to remain skeptical of the ability of renewable technologies to replace fossil fuels as a profitable, or even viable energy source. While the investments in green energy went up by 32% in the year 2011 (Frankfurt School *et al.* 2011), the rise came on the back of nearly 70% increase in green energy subsidies between 2007 and 2010 (International Energy Agency 2011). Further the crucial venture capital investments needed to fuel innovation in the sector actually went down in 2011 (Freed J. and Stevens M. 2011). In the future, realizing the potential in renewable energy sources will require significantly more subsidies (from $66 Billion in 2011 to $250 billion in 2035) in order to compete with coal and natural gas as a potentially profitable venue for future private investments, and yet investments in coal and natural gas and their share in global energy consumption are expected to rise much more steeply (International Energy Agency 2011). Even the most optimistic estimates for replacement of global energy supply from fossil fuel to renewable, do not foresee the transformation happening before 2050 (Delucchi M. A. and Jacobson M. Z. 2011). If we continue the current trend of fossil fuel consumption we would have gone through enough fossil fuel in forty years, to raise the temperature of the earth by two degrees centigrade if burned within 500 years (Allen M. R. *et al.* 2009). Renewable energy sources alone do not seem viable and the transition to renewable sources will take more time than we have. Meanwhile incidents like the Fukushima disaster in Japan cast a pall of popular uncertainty over nuclear technology as one of the main non-fossil alternatives, leaving even technologically advanced nations such as Germany to turn the clock back on nuclear power generation. Such events, which have low probability of occurrence but high impacts, also expose the vulnerability of the hyper-complex, global, industrialized economy to localized, unpredictable shock events or “Black Swans” (Taleb N. 2008b). It appears that our inability to foresee events like stock market crashes, localized food shortages, higher number of higher intensity extreme weather events or industrial accidents etc. is profound and that makes us ill prepared for whatever an uncertain future will throw at us. In summary, the ability to solve the problem through the supply side alone by transitioning to low emission, non-fossil fuel energy resources such as nuclear, wind and solar continues to be a question mark. Further, we feel powerless to predict with any degree of certainty how human societies will react to climate change and the scarcity of a fundamental resource such as fossil fuels, and therefore, are almost paralyzed to inaction in the face such historic vicissitudes.

One of the longer-term solutions for sustainability of the human project thus, must be a major realignment of our way of life, and our ideas of prosperity, progress and wealth to prepare us for events we cannot predict and to better reflect the realities of an energy scarce future (Lovins A. B. 1976). We need to change the way we live and create a holistic efficiency revolution in energy consumption (The Royal Swedish Academy of Sciences 2011). One significant way of doing that is by changing the way we envision and build our habitat, most importantly our cities. Researchers are now calling for a new theory of cities that defines human development along more sustainable lines (Bettencourt L. and West G. 2010). The predominant problem domain where the solution in this regard may lie is the reimagining of cities so that they are no longer just organisms for exponentially increasing consumption, but are sustainable systems robust to unpredictable events as well as nourishing human habitats in equilibrium with their natural support systems as if, part of an ecology.

In his excellent book, Normal Accidents (Perrow C. 1984), Perrow tries to make sense of the accident at Three Mile Island and why thousand-year-events were much more frequent than one every thousand years in the nuclear industry. After reviewing risk in various enterprises from aviation to aircraft carrier operations, he identifies a type of systems where two things are at play. Firstly, there is tight coupling within constituent systems of the composite system, which means that whatever happens in a constituent system affects the operations of other connected constituent systems directly. Typical risk analysis, Perrow found out looked only at one degree of event risk, so for instance if a shaft breaks and causes boiler pressure to leak, there will be an estimate of risk of the shaft breaking and the boiler pressure falling, but no account will be taken of the composite risk of boiler pressure falling in estimating the risk of shaft failure. If the boiler explodes for instance and causes a meltdown, that meltdown would be a second-degree or indirect result of the shaft failure. However in the nuclear industry, Perrow observed, the failure of shaft risk estimate did not take into account the cascading effect of the event and the compounding risk. In tightly-coupled systems where there's no buffer between elements, events can cascade and risks compound.

The second problem was what Perrow termed "Interactive Complexity" where there were elements within the system that formed constituent elements of more than one sub-systems; for instance, a shaft that both heated the control room and served as a heat sink pipe for the boiler. Failure of such a shaft would not only destabilize the boiler but may render any continued work in the control room impossible. This again had an effect of compounding risk. Perrow saw that tight coupling and interactive complexity lead to accidents which not only were impossible to assign a realistic risk value to, but also, almost impossible to predict. In that sense, such events were "accidents", but on the other hand, they were also systemic and borne of the complex nature of the system and were in a way inevitable because of the tight-coupling and interactive-complexity, and hence, were "normal".

It is important to note here that both, tight coupling and interactive complexity are design characteristics that come mainly out of a drive for economic efficiency in design. Buffers are redundant and hence not economically efficient; similarly, utilizing one constituent element for two or more purposes can be economically efficient.

What Perrow didn't foresee what that decades later, a Levantine hedge fund manager would discover his "normal accidents" in the financial markets, study them, name them "Black Swans" (Taleb N. 2008b) and would end up writing a bestselling book about them. Nassim Nicholas Taleb in his book, "The Black Swan", discusses the impact of the low probability, high consequence event on the history of mankind. In systems where both are present, high complexity and potentially high consequence events, Black Swans are bound to happen; and are "normal accidents".

A similar classification, though in a different paradigm has been identified by the post-normal science literature to delineate areas where "normal" science may no longer be applicable (Funtowicz S. O. and Ravetz J. R. 1994).

Climate change is another area where we are faced with high impact scenarios emerging from the dynamics of a complex, unpredictable system (Patt A. G. 1997). Due to its limited predictive capabilities, science at the current state may not be able to produce results having enough certainty to convince everyone. The future, beyond the two-degrees centigrade warming, post peak-oil may only appear hazy. But that is no reason not to transform the systems of civilization in a way that makes them robust, resilient, even "anti-fragile" (Taleb N. 2008a) to the unforeseen and unpredictable but highly costly. And that ought to be considered in any definition of "sustainability" heretofore, in the land of unknown-unknowns.

If we could see everything till the end of time, it would be easy to identify systems which are "sustainable", which will not sustain and which will sustain for a given time period. In this fundamental conception of "sustainability", a sustainable system is simply one which can be foreseen to continue its existence and operation within a normal range of variability until something unforeseeable or unpredictable happens and puts an end to business-as-usual. In this context, we can define "sustainability" of a city or system as the property of having no identifiable limiting resource in terms of the continued operation of the system as a whole, in the foreseeable, predictable future; whether for instance it's the heat bearing capacity of the biosphere or availability of minerals like oil or rare earth metals necessary for the "sustainability" of industrial civilization. Based on this I recognize cities as open systems whose sustainability is an outcome of dynamically interacting external and internal factors.

Of course for practical purposes the definition of sustainability is a debate mired in conflict, uncertainty and controversy (Johnston P. *et al.* 2007). There are questions about what and how far we can see, i.e. our ability to predict and about the exact probability of occurrence and intensity and nature of whatever it is that we even do agree that we can see i.e., have a scientific consensus on. There is a growing scientific consensus for instance that we can see (or predict) unprecedented climate change in the near future. It is much harder to establish consensus on predictions of how exactly this climate change will affect our civilization and its "sustainability", and whether anything we can do can have any meaningful impact on the nature, extent and intensity of this climate change.

What lends this problem its proverbial "glorious" complexity of course is the complex nature of the system that is being studied, i.e. global climate. Predictability is an essential goal of science, inherent to the scientific process in the positivist approach, however climate change may be one of those zones of intellectual inquiry falling within the land of black swans and unknown-unknowns, where predictability essentially breaks down. In such areas, the definition of sustainability should also include robustness, resilience and "anti-fragility" to unpredictably low probability, potentially high impact events.

We can now identify some salient characteristics of systems that can be defined sustainable as discussed above. For one thing, it is mathematically obvious that exponential growth of any kind is inherently unsustainable because it eventually outruns the capacity of any host system to keep it supplied with the essential resource base. This is why businesses, the special type of complex-adaptive systems built upon the principle of pursuit of fastest growth possible, are much less sustainable compared to other complex-adaptive systems such as cities, eco-systems and the tree of life (West G. 2011). Mathematically, any system development and growth that introduces buffers to reduce tight coupling and decouples connected sub-systems to reduce interactive complexity, is essentially designed against the principles of narrowly interpreted economic efficiency, because they introduce functional redundancies in the system. Such systems will not express their growth as exponential but will exhibit a power-law growth with an increase in size, requiring greater energy for further increase of similar proportions.

Secondly, not only is exponential growth of the entire system unsustainable, but if one constituent element or sub-system of a system starts to grow exponentially, that can pose a threat to the sustainability of the entire system. I put forward the hypothesis that such aberrant growth, analogous to cancerous growth in living cells (Hern W. M. 2008), can be identified by analyzing system structure and studying the scaling within the system. Aberrant growth will appear "out of scale" and will skew the entire distribution generating identifiably "un-sustainable" patterns. Cities are examples of complex-adaptive systems and with increasing urbanization responsible for an increasing amount of global material and energy consumption and greenhouse gas emissions. By analyzing the structure and scaling of various elements of urban systems, their fractal dimension can be identified as a potential sustainability indicator and its relationship with various sustainability attributes of the urban system studied.

Sustainable systems observed in nature display very specific scaling characteristics in the distribution of sizes of their constituents. What this means is that in such systems, the design elements are distributed at various scales or sizes such that the number of elements *p*, at each size *x* are related according to the equation px^m = constant (Salingaros N. A. and West B. J. 1999). Like the teeth along the edge of a toothed leaf or the orbits of moons and planets, similar design elements repeat themselves at different scales and also on the same scale. Natural complexity emerges out of a repetition of design algorithms with slight variations or anomalies or mutations for each repetition and at each varying scale. In other words, these systems do not have aberrantly sized elements within them and the number of component elements decreases as the scale to which the element belongs increases in size. The bigger an element is, the lesser its population in the system.

There has been growing interest in the structure of such systems ever since Benoit Mandelbrot coined the term “fractal” in the early sixties. Living organisms and many other similarly complex adaptive systems have been shown to obey a power law in scaling of the sizes of their various elements and are therefore fractal in nature, with the exponent of the power law being the fractal dimension (Mandelbrot B. B. 1983; West G. B. and Brown J. H. 2004; West G. B. *et al.* 1997). If we look at life for instance as a system, the distribution of many fundamental properties across species, such as metabolic rates follows a power law with respect to size (West G. B. and Brown J. H. 1997). The scaling within such systems, measured as the fractal dimension of the system is a good indicator of the health of such systems with aberrations skewing the distribution and hence fractal dimension in one direction or the other. Aberrant growths such as malignancy in living cells can be observed as having distinct fractal dimensions (Hern W. M. 2008). In architecture and urban planning there has been an emerging body of work rediscovering the significance of scaling in design especially within the new urbanism movement (Batty M. and Longley P. 1994; Benguigui L.

et al. 2000; Bettencourt L. and West G. 2010; Coward L. A. and Salingaros N. A. 2004; Salingaros N. A. and West B. J. 1999; Shen G. 2002). It has also been shown that on a greater scale, similar properties as fractal systems can be attributed to the distribution of human population in general with cities having predictable socioeconomic and infrastructural parameter values based on their size (Bettencourt L. M. A. *et al.* 2010; Hern W. M. 2008). So if cities are also complex adaptive systems with information and commodity distribution networks akin to distribution networks within living organisms, the scaling of distribution of sizes of elements like population and length of roads for instance should be an indicator of use in the analysis of the health of the system. And, as I will try to show, such an indicator also has a fundamental relationship to the "sustainability" of the system from the point of view of material or energy metabolism, and therefore also an important variable to be considered in ex-ante urban sustainability analysis and planning.

4. Methodology

To address the first research question concerning correlation between complexity based and energy consumption indicators, the methodology of analysis include the following;

- Design and calculation of a complexity indicator i.e., fractal dimension-based scaling indicator for cities, including the development and use of a clustering algorithm for large geospatial datasets
- Study of correlation between fractal dimension and energy consumption indicators

In addition, to answer the second question and to try to generalize the results, similar scaling indicators were developed for national economies and the results correlated with energy consumption indicators at the national level. One more indicator of scaling in cities i.e., area covered by 20% least dense housing was also included in the analysis.

This methodology chapter also includes discussion and results of analytics on the methods developed including analysis of utility of algorithms (time savings) and sensitivity analysis where needed.

Further I propose here a methodology and a planning plane as a tool to incorporate the results in the analysis. The methodology of drawing planning planes is explained.

4.1 Design and Calculation of a Fractal Dimension based Scaling Indicator for Cities

This study included the design of a novel fractal dimension-based indicator as a complexity-based metric of sustainability and the demonstration of its use at the level of cities and national economies.

The fractal dimension based scaling indicator was calculated by plotting inverse of population density against the area covered by housing of that density. Once plotted on log-log scales the resulting slope of the line would be the fractal dimension based scaling indicator of the distribution of population densities within the city, as expressed by Equation 1 (Salingaros N. A. and West B. J. 1999).

$$D = \frac{\log N_x}{\log\left(\frac{1}{x}\right)} \qquad \textit{Equation } 1$$

Where,

D = indicator for how the parameter scales (analogous to fractal dimension)

x = certain population density

N_x = Total area covered by that population density housing

This scaling indicator is a measure of how the city is spread and how human population fills the three dimensional space of the city.

US cities, given the availability of consistent and reliable data from the US Census Bureau required for this analysis. A map showing the cities selected for the study is presented in Figure 4.

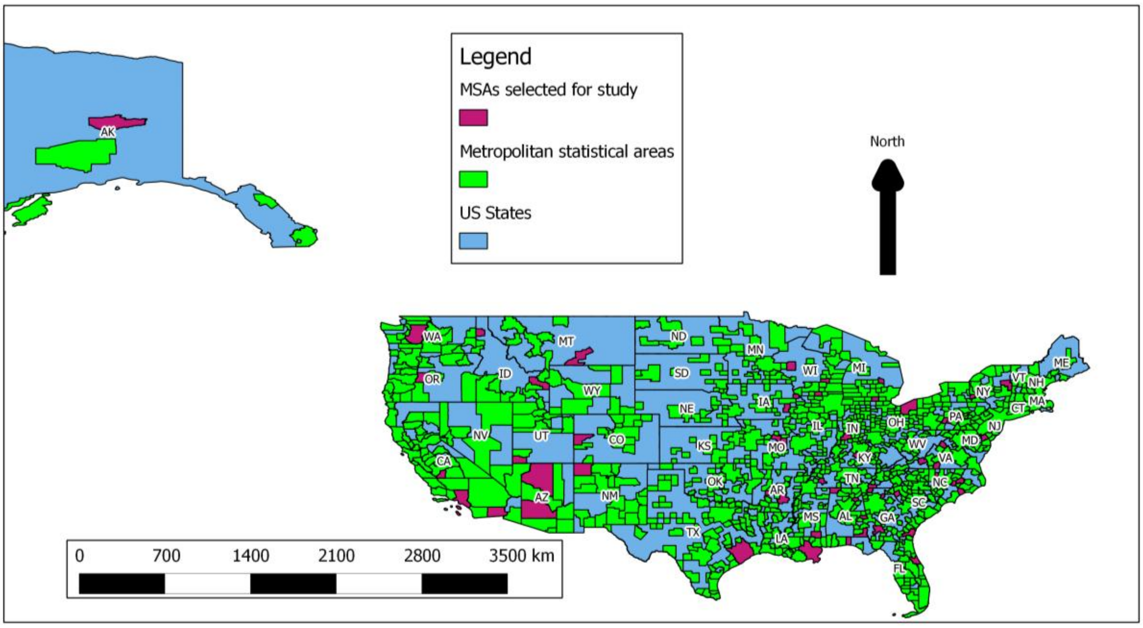


Figure 4: Final Cities Selected for the Study

4.1.1 Data

For complexity analysis and scaling indicator calculation we need high resolution data for cities. To compare these numbers with energy consumption indicators and carbon emissions indicators, the values for these indicators has to also be calculated for the same analytic unit, i.e. MSAs. Based on these considerations the selection of datasets was finalized for further analysis.

Data on US population by census blocks is downloaded from the US Census Bureau website (US Census Bureau 2010). A census block is a small unit roughly congruent to a neighbourhood block. As such, the assumption that the housing type within the census block is largely homogenous should hold. The data is downloaded for Metropolitan Statistical Areas (MSAs) which are census designated places that take into account the network of economic, industrial and commercial activity. So if a suburb has most of its financial linkages to a metropolitan area, the corresponding MSA would include the suburb as part of the MSA. Although cities were selected randomly, care was taken to ensure that a broad range of sizes (in terms of population and covered area), percentage change in population over the last ten years, urban topography, climate and states was captured.

Data on sales at gasoline stations within the MSAs was downloaded from the US Economic Census 2007 website (US Census Bureau 2007). Data on income is also US Census data, though the income data used is for year 2006 and downloaded from a secondary source (Santa Fe Institute - Cities Group 2010). The data on CO_2 emissions is for the year 2008 and downloaded from the Arizona State University's Vulcan Project (The Vulcan Project 2012). The emissions only for road transport were considered for the analysis.

The gasoline station sales data is from 2007 however, for the year 2010 data is available for gasoline station attendant salaries. The sales data is extrapolated for 2010 using the percentage change in total salaries from 2007 to 2010.

4.1.2 Analysis for Cities

In order to arrive at the scaling indicator values in a manner that is replicable I used extensive US Census datasets with hundreds of thousands of numerical values; a data regime that is much more quantitatively specific compared to satellite images of varying resolution. The first order of business was to select the cities for analysis. The following heuristic was followed for selection of cities;

a. City Selection

1. A set of ten cities were initially selected to run a pilot test study. These cities were selected randomly, though it was ensured that the cities came from different states, geological and climatic zones and represented various scales (population sizes) from the smallest to the largest.
2. The remaining cities were listed alphabetically to arrive at a certain pseudo-randomness. The first seventy cities were selected for analysis, though cities lying in multiple states or sharing counties with different cities were ignored.

The census blocks for each city were sorted according to increasing population density and then binned in fifty classes using k-means clustering (Lloyd's algorithm) along the population density spectrum (Khan F. 2012).

b. Specialized Clustering Algorithm for Geospatial Application

Clustering or classification of data into groups that represent some measure of homogeneity across a given variable range or values of multiple variables, is a much analyzed and studied problem in pattern recognition. K-means clustering is one of the most widely used methods for implementing a solution to this problem and for assigning data into clusters. The method in its initial formulation was first proposed by Mac Queen in 1967 (Mac Queen J. 1967) though the approximation developed by Lloyd (Lloyd S. 1982) has proven to be most popular in application. The method assumes a priori knowledge of the number of clusters *k* and requires seeding with initial values of centers of these clusters in order to be implemented. These initial seed values have been shown to be an important determinant of the eventual assignment of data to clusters. In other words, k-means clustering is highly sensitive to the initial seed selection for the value of cluster centers (Peña J. M. *et al.* 1999).

K-means++ has been proposed to overcome this problem and has been shown to produce a scale improvement in algorithm accuracy and computational efficiency or speed (Arthur D. and Vassilvitskii S. 2007; Ostrovsky R. *et al.* 2006). The algorithm assesses the performance of the initial seed selection based on the sum of square difference between members of a cluster and the cluster center, normalized to data size. While this is a worthwhile means of assessing method performance, it may be noted that in many clustering applications, the replicability of the resultant cluster assignment can be much more desirable than the homogeneity of the cluster perceived through an objective measure.

I encountered one such application of the clustering problem while trying to cluster georeferenced data into classes for mapping and visualization using ArcGIS, a Geographic

Information System (GIS) software. ArcGIS utilizes a proprietary modification of Jenks' natural breaks algorithm (Jenks G. F. 1967) to classify values of a variable for visualization in maps (ArcGIS 2009). The classification this method obtains seems to reproduce itself with remarkable accuracy for each run. The clustering bounds do not vary from run to run, even with variable values in eleven significant figures.

Jenks' algorithm differs only slightly from k-means clustering. K-means using Lloyd's algorithm aims to minimize the following cost function *C* defined in Equation 2;

$$C = \sum_{\substack{1 \leq i \leq n \\ 1 < j < k}} dist(d_i, c_j) \qquad \text{Equation 2}$$

Where *n* is the data size of number of data points, *k* is the number of clusters and $dist(d_i, c_j)$ computes the Euclidean distance between point d_i and its closest center c_j. The algorithm runs as follows;

a) Select centers $c_1, \ldots, c_k$ at random from the data.
b) Calculate the minimum cost function *C*, assigning data points $d_1, \ldots, d_n$ to their respective clusters having the closest mean.
c) Calculate new centers $c_1, \ldots, c_k$ as means of the clusters assigned in step 2.
d) Repeats steps *b* and *c* until no change is observed in center values $c_1, \ldots, c_k$.

Jenks' algorithm differs in that instead of *C* it minimizes the cost function *J*, defined in Equation 3;

$$J = C - \sum_{1 \leq j \leq (k-1)} dist(c_{j+1}, c_j) \qquad \text{Equation 3}$$

As seen in Equation 3, Jenks' algorithm not only searches for minimum distance between data points and centers of clusters they belong to but for maximum difference between cluster centers themselves (Jenks G. F. 1967).

If we are trying to develop a methodology for geo-processing - say a utility that studies the scaling characteristic of a city and models the distribution of sizes of housing within different size clusters - it can be essential to have a clustering mechanism that produces almost exactly similar results each time. Drawing inspiration from Jenks' algorithm, I propose an initial seed selection algorithm for k-means clustering that produces similar clusters on each run. I compare the results to those obtained by k-means as well as the widely used k-means++ initial seed selection methodology. K-means++ selects the initial centers as follows;

a) Select one center at random from the dataset.
b) Calculate squared distance of each point from the nearest of all selected centers and sum the squared distances.
c) Choose the next center at random. Calculate sum of squared distances. Re-select this center and calculate the sum of squared distances again. Repeat a given number of trials and select the center with the minimum sum of squared distance as the next center.
d) Repeat steps *b* and *c* until *k* centers are selected.

The methodology is novel in that unlike other initial seed selection algorithms, it does not introduce any new parameters (such as number of trials for k-means++) in the clustering algorithm thereby avoiding additional degrees of freedom. By clustering along the deepest valleys or highest gaps in the data series, the method introduces a measure of distance between cluster centers augmenting the k-means optimization for minimum distance between cluster center and cluster members. Additionally, unlike initialization algorithms like k-means++ there is no randomness involved in the algorithm and the initial clusters obtained are always the same.

1. New Initialization Algorithm

I developed the following method for calculating initial seed centers of k-means clustering along one attribute.

a) Sort the data points in terms of increasing magnitude $d_1,\ldots,d_n$ such that d_1 has the minimum and d_n has the maximum magnitude.

b) Calculate the Euclidean distances D_i between consecutive points d_i and d_{i+1} as shown in Equation 4;

$$D_i = d_{i+1} - d_i; \quad \text{where } i = 1,\ldots,(n\text{-}1) \qquad \text{Equation 4}$$

c) Sort D in descending order without changing the index i of each D_i. Identify k-1 index i values $(i_1,\ldots,i_{(k\text{-}1)})$ that correspond to the k-1 highest Di values.

d) Sort $i_1,\ldots i_{(k\text{-}1)}$ in ascending order. The set $(i_1,\ldots,i_{(k\text{-}1)},i_k)$ now forms the set of indices of data values d_i, which serve as the upper bounds of clusters $1,\ldots,k$; where; $i_k = n$.

e) The corresponding set of indices of data values d_i which serve as the lower bounds of clusters $1,\ldots,k$ would simply be defined as $(i_0, i_1+1,\ldots,i_{(k\text{-}1)}+1)$, where $i_0 = 1$.

f) The values of cluster centers c will now simply be calculated as the mean of d_i values falling within the upper and lower bounds calculated above. This set of cluster centers $(c_1,\ldots,c_k)$ will form the initial seed centers.

The methodology discussed above simply draws the cluster boundaries around points in the data where the gap between consecutive data values is the highest or the data has deepest ‘valleys’. In this way, a measure of distance is brought between consecutive cluster centers.

The method can be easily implemented for small to medium size datasets by using the spreadsheet freely available for download at *http://ge.tt/api/1/files/7FON8KH/0/blob?download*.

To test the replicability of cluster assignments produced using this methodology, the same data was clustered using this methodology ten times. The variance observed in cluster centers for these ten runs was calculated and averaged over the number of cluster centers. For comparison similar analysis was performed employing k-means and widely used k-means++ initial seeding methodology and the variance averaged over the number of cluster centers was calculated.

The analysis was run for five different datasets. The first is the popular Iris dataset from UC Irvine Machine Learning Repository (UCIMLR) (Fisher R. A. 1936). Attribute one of the data was used for clustering. The data having 150 points was classed into 5 clusters. The second data is US census block wise population data for the Metropolitan Statistical Area (MSA) of St. George, Utah. The population, land area and water body area data was downloaded from the US Census Bureau website (US Census Bureau 2010). The area was calculated by summing and water and land areas for the census block. The population density for each census block was estimated by dividing population for the block with the area for the block. The data having 1450 points was clustered along population density into 10 clusters. The third data was the Abalone dataset from UC Irving Machine Learning Repository (UCIMLR) (Nash W. J. *et al.* 1994). Attribute 5 was used for clustering. The data has 4177 instances and was clustered into 25 classes. The fourth set of data was cloud cover data downloaded from Phillipe Collard (Collard P. 1989). Data in column 3 was used for cluster analysis. The data having 1024 points was clustered in 50 clusters. The fifth data set was randomly generated normally distributed data with a mean of 10 and standard deviation of 1. The data having 10,000 points was clustered into 100 clusters.

2. Verification of Utility of the Method

While the objective of the development of this method is to produce more replicable results, the sums of squared differences between cluster members and cluster centers between the proposed method and k-means++ were compared and are juxtaposed in Table 1. As seen in Table 1, k-means++ in general continues to produce more accurate clustering using this methodology, though for two of the five datasets, the proposed method produced better results.

Table 1: Sum of squared differences between cluster members and their closest Centers (Normalized to Data size)

Dataset	k-means++	Proposed method	Reduction%
Iris	0.042243916	0.037471719	11.30%
St. George	2.39419E-07	1.76868E-07	26.13%
Abalone	0.000817549	0.001229598	-50.40%
Cloud	2.379979794	5.22916047	-119.71%
Normal	0.000644885	0.001465068	-127.18%

As shown in Table 2, my proposed method is also significantly faster than k-means++, clustering as much as 89% faster than k-means++ in some cases. The advantage in clustering speed is obtained over the initial seed selection, where k-means++ takes significantly longer comparative to both, my proposed method and k-means (Arthur D. and Vassilvitskii S. 2007).

Table 2: Algorithm running time (Seconds)

Dataset	k-means++	Proposed method	Reduction%
Iris	0.101	0.011	89.11%
St. George	2.312999994	0.438000001	81.06%
Abalone	19.79400002	16.191	18.20%
Cloud	7.771000001	1.886000005	75.73%
Normal	207.8150008	145.3870012	30.04%

The premier advantage of my proposed method over k-means and k-means++ though is in improving method replicability. The results are presented in Table 3. As seen in all three cases, the variance was virtually reduced to zero using proposed method, which was at least a 90% improvement on k-means++ and k-means.

Table 3: Variance of centers over ten (10) runs averaged to the number of clusters

Dataset	Proposed method	k-means++	Reduction%	k-means	Reduction%
Iris	4.73317E-31	0.046361574	100.00%	0.499704	100.00%
St. George	1.12847E-37	1.22722E-36	90.80%	1.23E-36	90.80%
Abalone	2.37968E-32	0.003285155	100.00%	0.005395	100.00%
Cloud	1.72981E-28	31.54401321	100.00%	22.24461	100.00%
Normal	5.75868E-31	0.009478013	100.00%	0.054631	100.00%

3. Justification for Usage of Proposed Clustering Algorithm

The method for initial seed selection of algorithm I propose reduces the variance of clustering to zero, accurate up to eleven significant figures, for clustering along one attribute or dimension. The further advantage of the proposed initialization method is that unlike k-means++ it does not introduce any new variables within the analysis, such as the number of trials. Almost perfect replicability and avoidance of additional degrees of freedom make the method especially suited for inclusion as part in a protocol or standard methodology or algorithm. Further, the method also produces results faster than k-means++ and hence is more computationally efficient at least in two-dimensional space.

The method has applications in all areas of data analysis where a Jenks style ‘natural’ classification, with a high level of replicability may be needed. It has the following distinct advantages over other initialization methods and naked k-means implementation:

- The results are highly replicable
- The method is fast and easy to implement

- No additional degrees of freedom or modifiable parameters are introduced that may need expert input for getting replicable results

The clustering may be more ‘natural’ in the manner of Jenks’ algorithm considering that a measure of distance between cluster centers is introduced to augment the k-means optimization of minimum distance between cluster members and cluster center.

Above advantages can render the initialization method highly useful in all areas where large datasets have to be handled or a ‘natural’ classification of data is sought. This includes areas like bioanalysis for instance where density based clustering is commonly deployed; the method can be made part of a more detailed analysis regime with confidence that the replicability of the results will not be negatively affected by the clustering algorithm. In the area of market segmentation and computer vision, the method can be used to standardize clustering results. This makes the method especially suited to utility development for GIS applications and has been used in further research here. The Visual Basic macro script used for clustering is shown in **Appendix1**.

c. Algorithm for Fractal Dimension Calculation

The classes calculated as above are congruous to the ‘boxes’ in the box-counting mechanism. As in the box counting mechanism, the number of elements in each box is counted; the area covered by the housing type falling within each class was summed. The population density of the area within each class was calculated. The inverse of these population densities were then plotted against the total area covered by housing of that population density on a log-log scale. The scaling indicator was estimated as the slope of the trend line for this plot. The spreadsheet used to calculate the indicator is available for download from the web address <http://ge.tt/7flihAg/v/0?c>. The visual basic macro script used for fractal dimension calculation is shown in **Appendix 2**. Figure 5 shows city map for St. George, Utah as a sample to demonstrate the spread of fractal dimension across the cityscape. It should be noted that the fractal dimension as shown in this figure is not how it is defined in the rest of this dissertation. Fractal dimension is calculated in this image for each block instead of for the whole city as in the rest of the document. Figure 6 shows the calculation of fractal dimension for St. George, Utah.

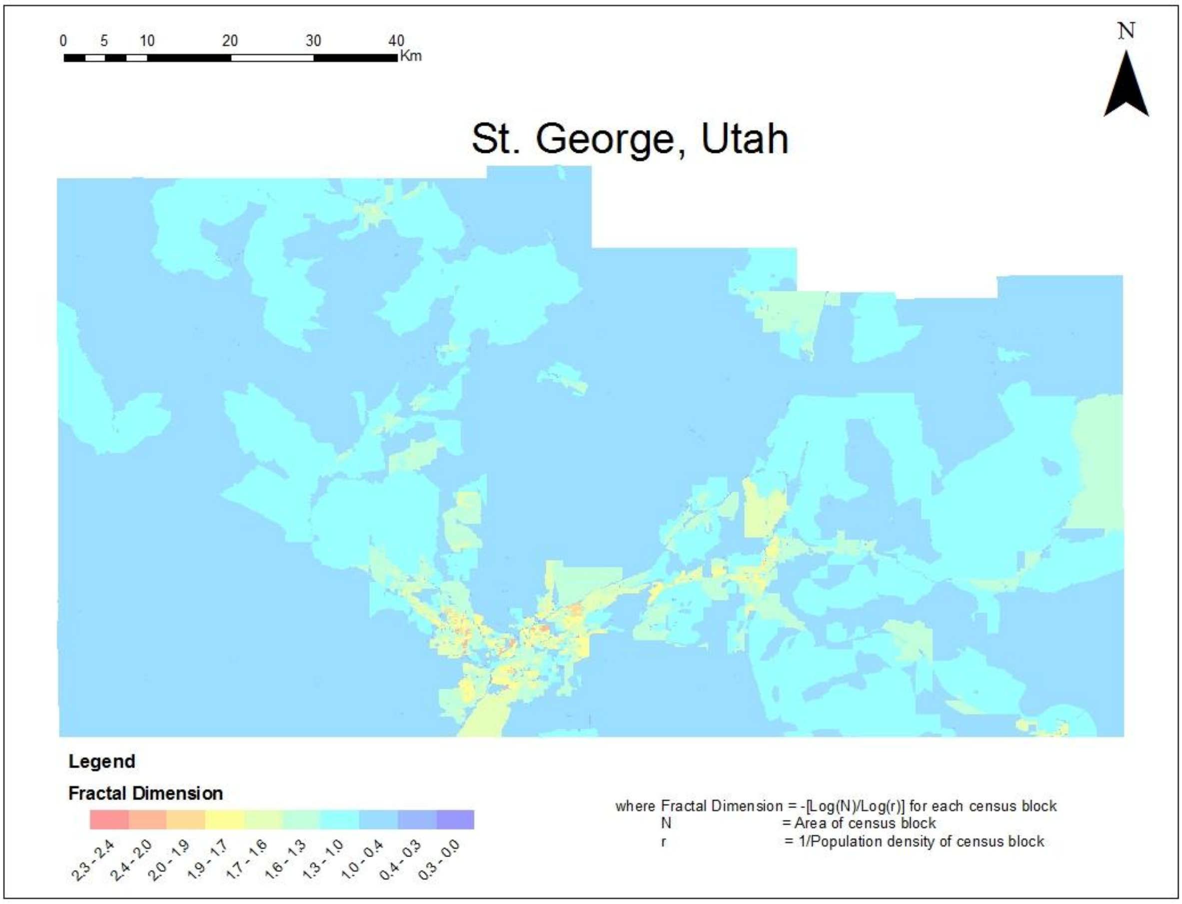


Figure 5: Spread of [Log(N)/Log(r)] over Cityscape for St. George, Utah (referred to as Fractal Dimension in this Figure)

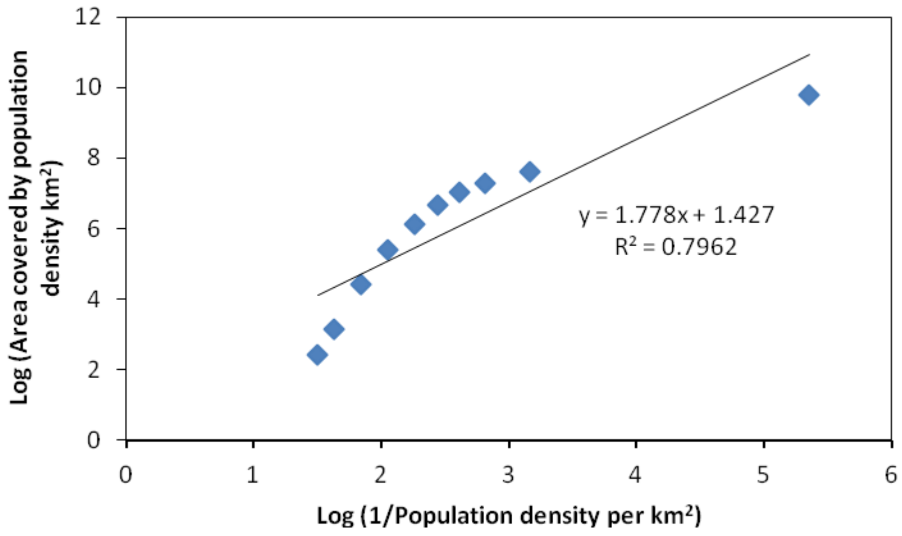


Figure 6: Calculation of Fractal Dimension for St. George, Utah

This methodology is novel in the sense that it avoids any deduction of numerical values from any form of visual, image or raster analysis thereby eliminating an additional source of potential variability in results upon repetition.

d. What Does This Fractal Dimension Mean?

The greater the fractal dimension the greater the disparity within the system. So if there is a large area covered by low density housing and little area covered by high density housing then in Figure 7 the point on the right is going to be higher and the point on the left is going to be lower, consequently we will get high fractal dimension. In this way fractal dimension is a measure of disparity within the system. Compare the two cities in Figure 7 for instance; Houston, Texas has a higher area covered by its lowest density housing and lesser area covered by its highest density housing compared to Pine Bluff, Arizona. Thus there is greater disparity between extremes in Houston, Texas compared to Pine Bluff, Arizona and thus it has much higher fractal dimension compared to Pine Bluff, Arizona.

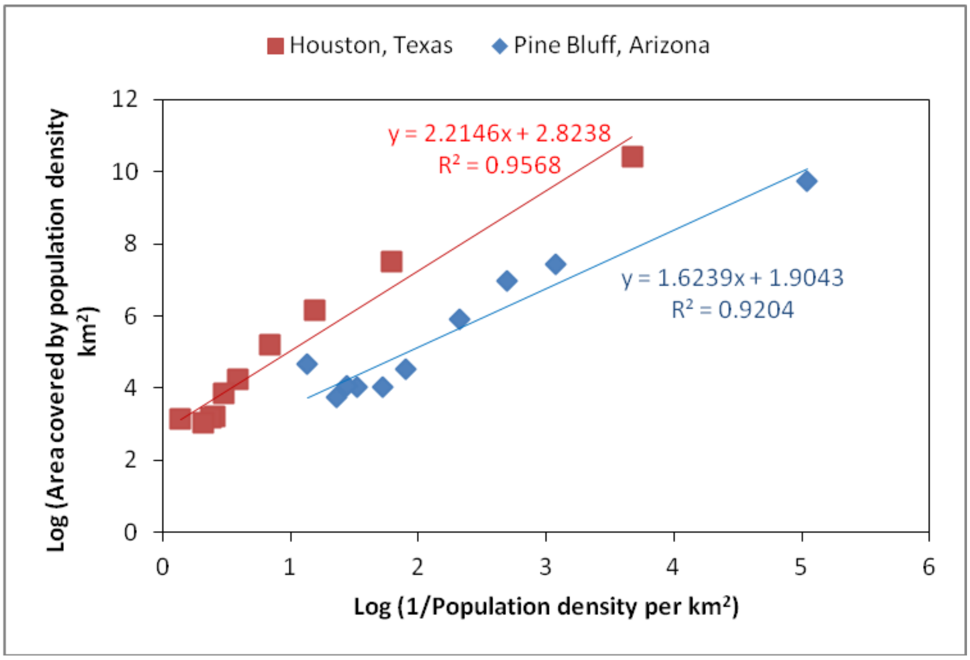


Figure 7: Fractal Dimension as a Measure of Disparity of Distribution

e. Sensitivity Analysis

In the interest of standardization a sensitivity analysis was also done on ten sample cities for the number of classes, the one parameter that could be varied in my proposed method. The scaling indicator values for each of the cities was calculated by changing the classes to 5 (-50%), 7, 50 and 100 (900%) from 10; the base number of classes used for this study. For a change in the number of classes of -50% to 900%, the maximum change observed in the scaling indicator was 31.66%. No correlation could be detected between the change in class size and the change in the scaling indicator. For all classes, the scaling indicator for any of the cities did not fall below 1.47 or above 2.92. For all number of classes, Pine Bluff, AR had the lowest scaling indicator while Cleveland-Elyria-Mentor, OH exhibited the highest scaling indicator for all number of classes, except 5, for which it had the fifth lowest scaling indicator value. This suggests that changing the number of classes has little impact on the scaling indicator value of city in relation to scaling indicator values of other cities. The ranking of cities in terms of scaling indicator remains largely unchanged. While the sensitivity analysis was done with cities, the results may be applicable to other systems as well for the methodology developed. The results of the sensitivity analysis are shown in Table 4.

Table 4: Percentage change in scaling indicator value with change in number of classes

Number of Classes	**5**	**7**	**50**	**100**
% Change in no. of Classes from Base Case(10)	**-50.00%**	**-30.00%**	**400.00%**	**900.00%**
Carson City, Nevada	4.77%	0.67%	0.15%	0.24%
Pine Bluff, Arkansas	9.39%	-1.37%	9.09%	10.49%
St. George, Utah	-1.13%	3.31%	18.04%	23.08%
Deltona-Daytona Beach-Ormand Beach, Florida	-0.31%	5.61%	20.53%	24.67%
New Orleans-Metairie-Kenner, Louisiana	-0.93%	-1.10%	17.74%	20.64%
Cleveland-Elyria-Mentor, Ohio	24.75%	3.22%	18.90%	18.82%
Seattle-Tacoma-Bellevue, Washington	15.06%	-6.96%	26.81%	31.66%
Phoenix-Mesa-Glendale, Arizona	1.07%	-9.35%	5.75%	9.19%
Houston-Sugar Land-Baytown, Texas	10.06%	-7.12%	5.48%	6.61%
Los Angeles-Long Beach-Santa Ana, California	-7.66%	-2.51%	21.68%	26.55%
Average Change (%)	**5.51%**	**-1.56%**	**14.42%**	**17.20%**

4.2 Percentage of Area Covered by 20% Least Density Population

This is the second scaling indicator that has been selected for analysis besides fractal dimension. The indicator is merely percentage of area covered by 20% least densely populated housing and the value is calculated by dividing area covered by 20% of the population having the least population density by total area.

If the system obeys Pareto distribution or a rough power-law distribution then the value of this indicator should be around 80%.

4.3 Calculation of Total Gasoline Sales for 2010

Gasoline sales data was taken from the US economic census for 2007. This data was updated for some of the parameters in 2010 including the 2010 payroll estimates for gasoline sales. 2010 gasoline sales were estimated by extrapolating linearly the 2007 gasoline sales based on the change in payroll estimates for gasoline stations between 2007 and 2010. The results are shown in Table 5.

Table 5: Total gasoline station sales for 2010 (1,000 USD)

Metropolitan Statistical Area	State	2007 Payroll for gasoline stations	2007 Gasoline station sales	2010 Payroll for gasoline stations	2010 Gasoline station sales estimate
Albany, GA Metro Area	GA	8,564	278,858	9,559	311,257
Altoona, PA Metro Area	PA	10,939	233,160	12,218	260,421
Ames, IA Metro Area	IA	5,574	140,852	5,558	140,448
Anderson, SC Metro Area	SC	9,921	338,744	9,870	337,003
Auburn-Opelika, AL Metro Area	AL	7,862	219,227	6,658	185,654
Bay City, MI Metro Area	MI	5,280	223,006	6,383	269,592
Bend, OR Metro Area	OR	9,793	207,729	8,805	186,772
Billings, MT Metro Area	MT	12,899	424,568	15,018	494,314
Blacksburg-Christiansburg-Radford, VA Metro Area	VA	9,489	247,967	11,063	289,099
Bloomington, IN Metro Area	IN	8,354	251,561	7,361	221,659
Brunswick, GA Metro Area	GA	10,071	326,670	9,546	309,641
Burlington, NC Metro Area	NC	12,146	446,193	10,879	399,649
Carson City, NV Metro Area	NV	2,231	80,769	2,029	73,456
Cleveland, TN Metro Area	TN	7,225	193,351	7,766	207,829
Cleveland-Elyria-Mentor, OH Metro Area	OH	85,089	2,700,457	79,785	2,532,125
Coeur d'Alene, ID Metro Area	ID	7,424	307,199	7,020	290,482
Columbia, MO Metro Area	MO	10,195	313,946	9,522	293,222
Crestview-Fort Walton Beach-Destin, FL Metro Area	FL	8,333	298,411	9,928	355,529
Dalton, GA Metro Area	GA	10,436	340,225	9,935	323,892
Danville, VA Metro Area	VA	8,273	223,910	7,267	196,682
Deltona-Daytona Beach-Ormond Beach, FL Metro Area	FL	23,561	735,538	22,032	687,805
Dothan, AL Metro Area	AL	11,092	243,046	12,941	283,561
Dover, DE Metro Area	DE	8,012	240,486	8,650	259,636
Dubuque, IA Metro Area	IA	6,819	169,059	7,563	187,505
Eau Claire, WI Metro Area	WI	13,863	438,577	13,481	426,492
El Centro, CA Metro Area	CA	6,913	212,872	6,727	207,145
Elizabethtown, KY Metro Area	KY	8,600	272,467	9,564	303,009
Fairbanks, AK Metro Area	AK	7,448	115,997	6,754	105,188
Farmington, NM Metro Area	NM	15,150	264,722	12,877	225,005
Flagstaff, AZ Metro Area	AZ	11,450	284,095	13,049	323,769
Florence-Muscle Shoals, AL Metro Area	AL	8,200	241,647	9,094	267,992
Fond du Lac, WI Metro Area	WI	7,284	212,852	7,560	220,917
Gadsden, AL Metro Area	AL	3,641	112,624	4,213	130,317

Metropolitan Statistical Area	State	2007 Payroll for gasoline stations	2007 Gasoline station sales	2010 Payroll for gasoline stations	2010 Gasoline station sales estimate
Gainesville, GA Metro Area	GA	8,519	261,910	8,418	258,805
Glens Falls, NY Metro Area	NY	8,985	253,685	11,241	317,382
Goldsboro, NC Metro Area	NC	6,026	179,857	6,286	187,617
Grand Junction, CO Metro Area	CO	8,759	278,733	11,405	362,935
Greenville, NC Metro Area	NC	11,613	337,435	10,016	291,032
Hanford-Corcoran, CA Metro Area	CA	3,714	138,699	4,260	159,089
Harrisonburg, VA Metro Area	VA	7,730	197,062	8,493	216,513
Hattiesburg, MS Metro Area	MS	14,546	320,432	16,867	371,561
Hot Springs, AR Metro Area	AR	5,651	183,007	5,278	170,927
Houston-Sugar Land-Baytown, TX Metro Area	TX	200,208	7,737,272	235,160	9,088,033
Idaho Falls, ID Metro Area	ID	7,404	220,112	7,296	216,901
Iowa City, IA Metro Area	IA	9,020	202,219	10,174	228,090
Ithaca, NY Metro Area	NY	5,026	114,448	5,629	128,179
Jackson, MI Metro Area	MI	8,471	210,841	9,672	240,734
Jackson, TN Metro Area	TN	7,367	226,105	9,551	293,135
Jacksonville, NC Metro Area	NC	7,178	242,112	7,852	264,846
Janesville, WI Metro Area	WI	9,297	328,879	9,414	333,018
Jefferson City, MO Metro Area	MO	10,559	330,926	11,555	362,141
Johnstown, PA Metro Area	PA	8,412	225,190	10,473	280,363
Los Angeles-Long Beach-Santa Ana, CA Metro Area	CA	293,158	12,850,576	313,473	13,741,084
New Orleans-Metairie-Kenner, LA Metro Area	LA	54,499	1,733,293	54,605	1,736,664
Phoenix-Mesa-Glendale, AZ Metro Area	AZ	180,782	5,886,205	187,774	6,113,862
Pine Bluff, AR Metro Area	AR	4,991	141,083	5,052	142,807
Seattle-Tacoma-Bellevue, WA Metro Area	WA	113,904	3,714,472	117,807	3,841,751
St. George, UT Metro Area	UT	8,483	239,914	7,073	200,037

Source: (US Census Bureau 2007)

4.4 Correlation Between Scaling Indicators and Energy Consumption Indicators

In the final run I used the following independent variables;

- Fractal dimension
- Area covered by 20% least dense housing

- Population
- Area

The correlation of these independent variables with the following dependent variables were studied using linear regression. The plots of these regressions are presented in the results section.

- Gasoline sales
- Carbon emissions

4.5 Expanding Analysis to Another System Besides Cities to Study Generalization of Results

To answer the second research question it is imperative to also incorporate some degree of generalization of results. To do that, the research was expanded to include national economic statistics with national economies being considered as complex systems.

4.5.1 National Economic Statistics

Just like cities, national economies are complex adaptive systems and should also exhibit similar scaling properties as other complex systems. In order to see how scaling in economic systems affect environmental indicators I looked at fractal dimension of distribution of income. The environmental or direct sustainability indicator studied was per capita energy usage. The data was obtained from the World Bank open data platform (World Bank 2004). Data from the year 2004 was used as that provided us with the biggest set of countries for which data was available. In this case the primary limitation was income distribution data which was available for only a small number of countries. The countries selected for the analysis and respective data is shown in Table 6.

Table 6: Economic and energy use data (2004)

Country Name	GDP per capita (at constant 2005 USD values)	Energy use per capita (kg oil equivalent)	% of Income share of highest 20% earners	% of Income share of the 2nd highest 20% earners	% of Income share of the middle 20% earners	% of Income share of the 2nd lowest 20% earners	% of Income share of the lowest 20% earners
Albania	2469	675	39.5	22.6	17	12.7	8.18
Argentina	4380	1757	53.8	22	13.2	7.83	3.21
Armenia	1422	692	45.9	20.2	15	11.4	7.54
Bosnia and Herzegovina	2683	1238	43.1	22.3	16.1	11.5	6.98
Belarus	2837	2763	35.8	22.8	18	14.1	9.4
Brazil	4648	1141	60.9	19.3	11.1	6.31	2.51
Colombia	3290	611	62	18.2	11	6.62	2.16
Comoros	634	58.1	68	15.1	8.94	5.35	2.55
Costa Rica	4440	914	53.4	20.8	13.5	8.51	3.81
Dominican Republic	3377	709	57	19.2	12.1	7.79	3.93
Estonia	9468	3915	43.2	22.2	16.2	11.6	6.8
Guatemala	2131	629	57.5	20.9	12.4	7.05	2.08
Honduras	1349	570	62	19	10.8	5.87	2.24
Croatia	9683	1989	38	22.6	17.4	13.3	8.73
Hungary	10499	2588	38.9	22.3	17.2	13.1	8.56
Kazakhstan	3469	3378	40.6	22.4	16.6	12.3	8.03
Kyrgyz Republic	483	498	42.9	22.1	15.8	11.5	7.69
Cambodia	423	258	49.4	19.9	13.9	10	6.89
Lithuania	7010	2732	43	22.4	16.3	11.6	6.79
Latvia	6271	1919	42.9	22.4	16.3	11.7	6.79
Moldova	771	937	43.6	21.7	15.9	11.5	7.26
Maldives	3715	855	44.2	22.7	15.7	10.9	6.51
Mexico	7722	1457	51.2	21	14.1	9.16	4.55
Macedonia, FYR	2750	1321	45	22.6	15.7	10.7	6.01
Malaysia	5372	2314	44.8	22.4	15.6	10.8	6.46
Namibia	3535	581	68.6	15	8.24	5.03	3.15
Nigeria	798	748	48.6	21.9	14.7	9.67	5.13
Panama	4367	791	59	20.1	11.7	6.62	2.64
Peru	2712	470	55.2	20.4	12.6	7.81	3.97
Poland	7682	2393	43.2	22.5	16	11.4	6.92
Paraguay	1476	691	58.3	19.1	11.7	7.41	3.41
Romania	4379	1784	39.7	22.6	17	12.7	7.99
Russian Federation	4993	4500	44.1	22.5	15.8	11	6.59
El Salvador	2728	724	52.9	21.8	13.6	8.54	3.21
Serbia	3208	2424	41.1	22.3	16.6	12.3	7.8
Slovak Republic	10683	3410	38.4	21.9	17.1	13.4	9.16

Country Name	GDP per capita (at constant 2005 USD values)	Energy use per capita (kg oil equivalent)	% of Income share of highest 20% earners	% of Income share of the 2nd highest 20% earners	% of Income share of the middle 20% earners	% of Income share of the 2nd lowest 20% earners	% of Income share of the lowest 20% earners
Slovenia	17196	3571	39.4	22.6	17	12.8	8.22
Syrian Arab Republic	1537	1015	43.9	21.4	15.5	11.4	7.68
Tajikistan	325	352	41.7	22.1	16.4	12.1	7.75
Turkey	6665	1210	48.2	22	14.8	9.79	5.25
Ukraine	1768	3031	37.3	22.6	17.6	13.6	8.99
Uruguay	4861	863	52.2	21.4	13.5	8.48	4.43
Venezuela, RB	5023	2149	51.6	22	14.2	8.87	3.38
Vietnam	658	478	44.5	21.8	15.5	11.1	7.2
Zambia	610	627	55.2	20.6	12.8	7.76	3.63

Source: (World Bank 2004)

4.5.2 Fractal Dimension based Scaling Indicator of Income Distribution (National Economies)

Fractal dimension based scaling indicator of national income distribution was calculated by plotting cumulative income share against the cumulative population percentage. Once plotted on log-log scales the resulting slope of the line would be the fractal dimension based scaling indicator of the distribution of income within the country.

4.5.3 Correlation Analysis for National Economies

The correlation between fractal dimension based scaling indicator of income distribution and energy usage per capita in the economies was studied using linear and non-linear regression. The relevant correlations are plotted and presented in the results section. The results of national level analysis will be further discussed from the perspective of robustness in the discussions section.

4.6 Planning Planes

I propose a new planning tool here in situations where more than one variable needs to be considered in order to optimize the value of a third value. The tool is being proposed as an easy to use, though only indicative tool for incorporating multivariate concerns in the planning process. Typically in planning processes one variable is considered at a time. This is understandable when considering the complexities involved in building multi-stakeholder consensus usually needed in most planning tasks. It is difficult enough to get that consensus when one variable or indicator is being considered. Bringing multiple variables or indicators simultaneously in the process has the potential to complicate it exponentially. However, new visualization tools can ease communication of complex ideas in a way that would facilitate consideration of multiple variables. The planning plane is just such a tool. Basically it is a plane that shows how a dependent variable changes in values based on two independent variables. The x and y axes are independent variables and color or contour can represent the dependent variable. The plane is built using empirical datasets and spatial interpolation. The values of the dependent variable is interpolated from empirical values using spatial interpolation over a certain range of x and y, independent variable values. Statistical diagnostics should of course be run on the interpolation to ensure that the data available is sufficient for the construction of the planning plane.

A planning plane as proposed here consists of an interpolated surface of a dependent variable over two independent variables plotted along the x and y scales. The interpolated surface can be developed using a litany of interpolation mechanisms. The planes presented in this research are developed using ordinary kriging. R statistical package was used to do the kriging operations using the gstat library. The R script used for drawing planning planes is shown in Appendix 3.

5. Results

The following section presents results of the correlation analysis between the independent and dependent variables (and various combination thereof) listed below.

Independent variables for cities;

- Fractal dimension
- Area covered by 20% least dense housing
- Population
- Area

Dependent variable for cities;

- Gasoline sales
- Carbon emissions

In addition similar analysis was done for the national economic indicators with the following variables;

- Independent variable for national economies: Scaling indicator of income distribution.
- Dependent variable for national economies: Energy consumption in the country

While interesting results have been observed in national level analysis as well, it should be noted that the primary idea of exploring national data was to understand and comment on the universality of the observations that were to be made about the mechanisms underpinning the phenomena observed in urban results.

5.1 Urban Analysis Results

Fractal dimension was calculated for 76 US cities in total. Out of these thirteen (13) were excluded from the analysis because the data showed a difference between the geographically calculated population numbers and those reported in US Census data for the metro area on a cumulative basis. For three (3) cities, the data for gasoline sales was not available due to privacy protection of those surveyed. This left a dataset of fifty eight (58) cities for further analysis. The complete list of cities and basic data (population, area, fractal dimension, gasoline sales, carbon emissions) are shown in Table 7.

Table 7: List of cities for final analysis and basic data

Metropolitan Statistical Area	State	2010 Population [1]	Area (square kilometers) [1]	2010 Gasoline station sales (1,000 USD) [2]	2008 Annual Carbon Emissions (million tonnes) [3]	Fractal Dimension	Percentage of area covered by 20% least density population
Albany, GA Metro Area	GA	157,308	5,071.10	311,257	0.26	2.04	74.47%
Altoona, PA Metro Area	PA	127,089	1,365.10	260,421	0.15	1.88	82.48%
Ames, IA Metro Area	IA	89,542	1,485.75	140,448	0.11	1.77	88.06%
Anderson, SC Metro Area	SC	187,126	1,961.75	337,003	0.27	2.69	63.75%
Auburn-Opelika, AL Metro Area	AL	140,247	1,595.04	185,654	0.15	2.05	73.82%
Bay City, MI Metro Area	MI	107,771	1,633.55	269,592	0.19	2.20	57.81%
Bend, OR Metro Area	OR	157,733	7,911.81	186,772	0.17	1.66	26.76%
Billings, MT Metro Area	MT	158,050	12,201.94	494,314	0.19	1.65	64.66%
Blacksburg-Christiansburg-Radford, VA Metro Area	VA	162,958	2,821.76	289,099	0.25	1.93	77.13%
Bloomington, IN Metro Area	IN	192,714	3,483.70	221,659	0.23	1.70	64.59%
Brunswick, GA Metro Area	GA	112,370	4,160.41	309,641	0.24	1.92	44.85%
Burlington, NC Metro Area	NC	151,131	1,125.97	399,649	0.19	1.91	74.11%
Carson City, NV Metro Area	NV	55,274	407.26	73,456	0.04	1.86	33.55%
Cleveland, TN Metro Area	TN	115,788	2,004.09	207,829	0.19	2.02	58.59%
Cleveland-Elyria-Mentor, OH Metro Area	OH	2,077,240	6,550.75	2,532,125	2.48	2.45	60.93%
Coeur d'Alene, ID Metro Area	ID	138,494	3,407.45	290,482	0.23	1.97	59.81%
Columbia, MO Metro Area	MO	172,786	3,010.89	293,222	0.24	1.74	82.02%

Metropolitan Statistical Area	State	2010 Population [1]	Area (square kilometers)[1]	2010 Gasoline station sales (1,000 USD)[2]	2008 Annual Carbon Emissions (million tonnes)[3]	Fractal Dimension	Percentage of area covered by 20% least density population
Crestview-Fort Walton Beach-Destin, FL Metro Area	FL	180,822	2,802.57	355,529	0.32	2.08	39.82%
Dalton, GA Metro Area	GA	142,227	1,651.68	323,892	0.26	2.57	71.04%
Danville, VA Metro Area	VA	106,561	2,647.29	196,682	0.14	2.31	69.09%
Deltona-Daytona Beach-Ormond Beach, FL Metro Area	FL	494,593	3,306.93	687,805	0.78	2.03	56.03%
Dothan, AL Metro Area	AL	145,639	4,477.87	283,561	0.25	2.26	75.73%
Dover, DE Metro Area	DE	162,310	2,067.67	259,636	0.26	2.03	56.53%
Dubuque, IA Metro Area	IA	93,653	1,597.07	187,505	0.10	1.70	90.60%
Eau Claire, WI Metro Area	WI	161,151	4,368.09	426,492	0.26	1.88	85.73%
El Centro, CA Metro Area	CA	174,528	11,607.60	207,145	0.32	1.38	24.47%
Elizabethtown, KY Metro Area	KY	119,736	2,314.84	303,009	0.13	2.09	72.29%
Fairbanks, AK Metro Area	AK	97,581	19,279.06	105,188	0.05	1.65	48.48%
Farmington, NM Metro Area	NM	130,044	14,344.45	225,005	0.18	1.85	35.03%
Flagstaff, AZ Metro Area	AZ	134,421	48,332.65	323,769	0.34	1.51	39.23%
Florence-Muscle Shoals, AL Metro Area	AL	147,137	3,478.66	267,992	0.20	2.02	67.45%
Fond du Lac, WI Metro Area	WI	101,633	1,983.52	220,917	0.12	1.75	52.98%
Gadsden, AL Metro Area	AL	104,430	1,420.94	130,317	0.16	2.10	69.24%
Gainesville, GA Metro Area	GA	179,684	1,111.87	258,805	0.26	2.26	58.21%
Glens Falls, NY Metro Area	NY	128,923	4,603.68	317,382	0.21	2.09	74.92%
Goldsboro, NC Metro Area	NC	122,623	1,442.24	187,617	0.12	2.52	70.99%
Grand Junction, CO Metro Area	CO	146,723	8,653.47	362,935	0.16	1.86	58.68%

Metropolitan Statistical Area	State	2010 Population [1]	Area (square kilometers)[1]	2010 Gasoline station sales (1,000 USD)[2]	2008 Annual Carbon Emissions (million tonnes)[3]	Fractal Dimension	Percentage of area covered by 20% least density population
Greenville, NC Metro Area	NC	189,510	2,385.96	291,032	0.19	1.95	81.65%
Hanford-Corcoran, CA Metro Area	CA	152,982	3,604.05	159,089	0.17	1.71	26.93%
Harrisonburg, VA Metro Area	VA	125,228	2,255.45	216,513	0.16	1.93	65.02%
Hattiesburg, MS Metro Area	MS	142,842	4,198.03	371,561	0.24	1.99	68.30%
Hot Springs, AR Metro Area	AR	96,024	1,902.66	170,927	0.11	2.27	60.40%
Houston-Sugar Land-Baytown, TX Metro Area	TX	5,946,800	25,061.89	9,088,033	6.67	2.21	63.97%
Idaho Falls, ID Metro Area	ID	130,374	7,786.05	216,901	0.18	1.78	46.84%
Iowa City, IA Metro Area	IA	152,586	3,092.63	228,090	0.22	1.74	89.03%
Ithaca, NY Metro Area	NY	101,564	1,273.13	128,179	0.11	1.76	72.79%
Jackson, MI Metro Area	MI	160,248	1,873.85	240,734	0.22	2.02	73.16%
Jackson, TN Metro Area	TN	115,425	2,187.46	293,135	0.24	1.64	57.37%
Jacksonville, NC Metro Area	NC	177,772	2,346.30	264,846	0.16	2.18	42.03%
Janesville, WI Metro Area	WI	160,331	1,880.61	333,018	0.24	2.26	89.02%
Jefferson City, MO Metro Area	MO	149,807	5,901.56	362,141	0.24	1.99	76.88%
Johnstown, PA Metro Area	PA	143,679	1,796.50	280,363	0.16	1.93	80.20%
Los Angeles-Long Beach-Santa Ana, CA Metro Area	CA	12,828,837	13,637.56	13,741,084	10.80	2.17	44.67%
New Orleans-Metairie-Kenner, LA Metro Area	LA	1,167,764	15,368.78	1,736,664	1.23	1.85	30.02%
Phoenix-Mesa-Glendale, AZ Metro Area	AZ	4,192,887	37,810.26	6,113,862	5.42	2.15	30.16%
Pine Bluff, AR Metro Area	AR	100,258	5,399.37	142,807	0.16	1.62	64.61%

Metropolitan Statistical Area	**State**	**2010 Population [1]**	**Area (square kilometers)[1]**	**2010 Gasoline station sales (1,000 USD)[2]**	**2008 Annual Carbon Emissions (million tonnes)[3]**	**Fractal Dimension**	**Percentage of area covered by 20% least density population**
Seattle-Tacoma-Bellevue, WA Metro Area	WA	3,439,809	15,955.55	3,841,751	4.13	2.00	55.30%
St. George, UT Metro Area	UT	138,115	6,293.54	200,037	0.19	1.78	33.75%

[1]Source: (US Census Bureau 2010)
[2]Source: (US Census Bureau 2007)
[3]Source: (The Vulcan Project 2012)

The minimum fractal dimension calculated for the cities was 1.38 while the maximum calculated was 2.69. Cities had an average fractal dimension of 1.97. The fractal dimensions were normally distributed with a median of 1.96 and a standard deviation of 0.26. The distribution of fractal dimension is shown in Figure 8.

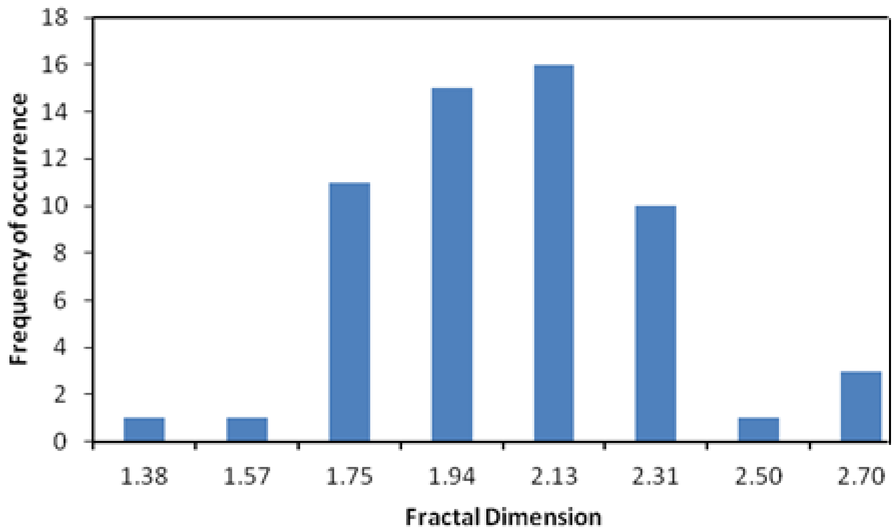


Figure 8: Distribution of Fractal Dimension for 58 Cities

To start with I have studied some correlations between the basic variables to understand the underlying correlations in the findings.

The following section summarizes correlation study results for the urban indicators selected in my research. Initially the correlations explore the connections between some fundamental parameters such as population and area and population density and the fractal dimension as well as the second scaling indicator. The objective is to identify underlying correlations between fundamental variables that may influence the correlation between fractal dimension and energy indicators. The correlations between the scaling indicators and energy consumption indicators are then explored in detail.

5.1.1 The Two Scaling Indicators

Fractal dimension and the percentage of area covered by 20% of the population living in the least densely populated blocks are independent of each other as shown in Figure 9. This means that the overall scaling indicator that is fractal dimension is not being affected by a 'long taill', or the profile of one extreme.

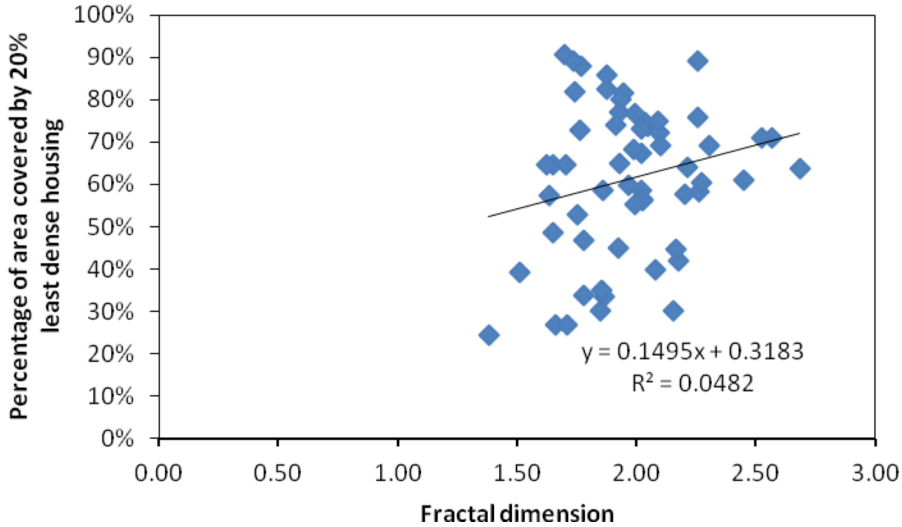


Figure 9: Fractal Dimension and Percentage of Area Covered by 20% of the Least Densely Populating Habitants

5.1.2 Total Gasoline Sales and Carbon Emissions

As expected, gasoline sales is strongly linearly correlated with carbon emissions, shown in Figure 10. It should be noted here that the carbon emissions and gasoline sales come from two different data sources and the strong correlation is indication that they can be considered to be indicative of the same city unit for the purposes of this analysis.

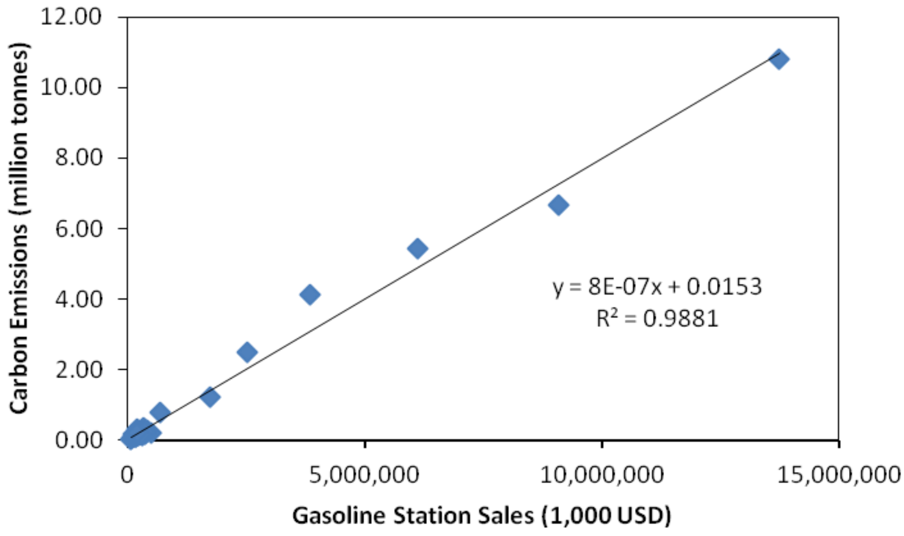


Figure 10: Gasoline Sales Correlate Linearly with Carbon Emissions

5.1.3 Population and Other Parameters

As expected population and gasoline sales have a strong linear relationship as shown in Figure 11. Gasoline station sales in a city increase with the city size.

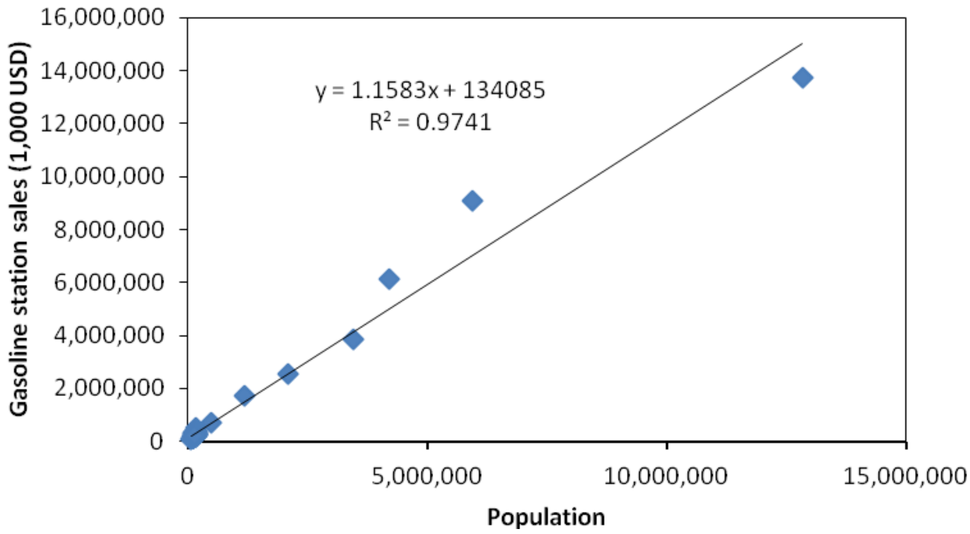


Figure 11: Population Correlates Linearly with Gasoline Sales

Once again, we get a strong linear correlation between population and carbon emissions as shown in Figure 12. This is because carbon emissions have a strong correlation with gasoline sales. This also shows that population or city size continues to be a strong predictor of the overall emissions in a city.

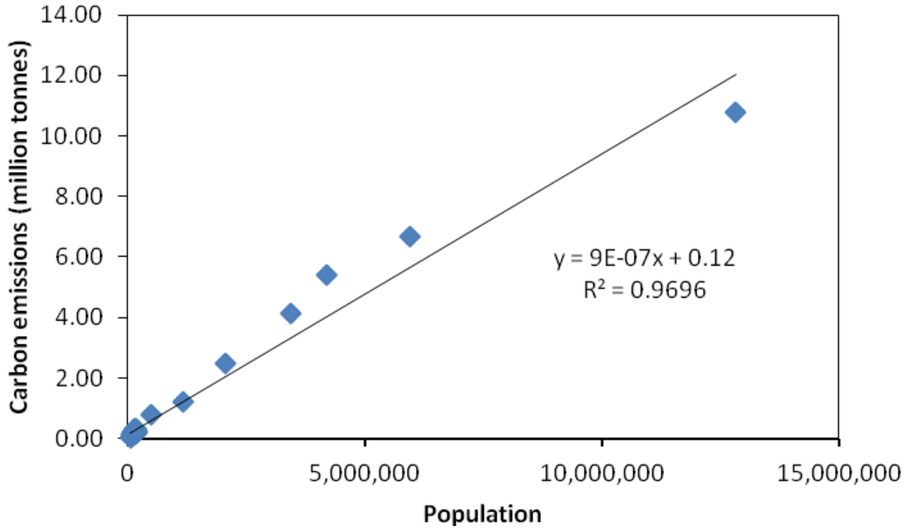


Figure 12: Population Correlates Linearly with Carbon Emissions

Surprisingly enough in the data, population does not seem to have a strong correlation with the city area, shown in Figure 13. The distribution of population density across different cities is such that the area covered does not correlate strongly with the population.

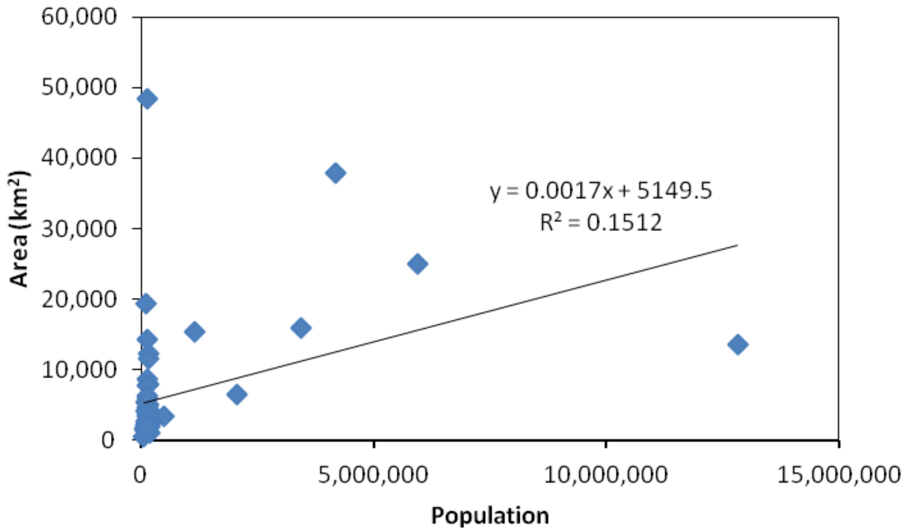


Figure 13: Population and Area are not Strongly Correlated

Fractal dimension does not display a strong dependence on population as shown in Figure 14. This means that fractal dimension is an indicator independent of the size of city in terms of population. The form of the city as measured using a scaling indicator such as fractal dimension is independent of scale.

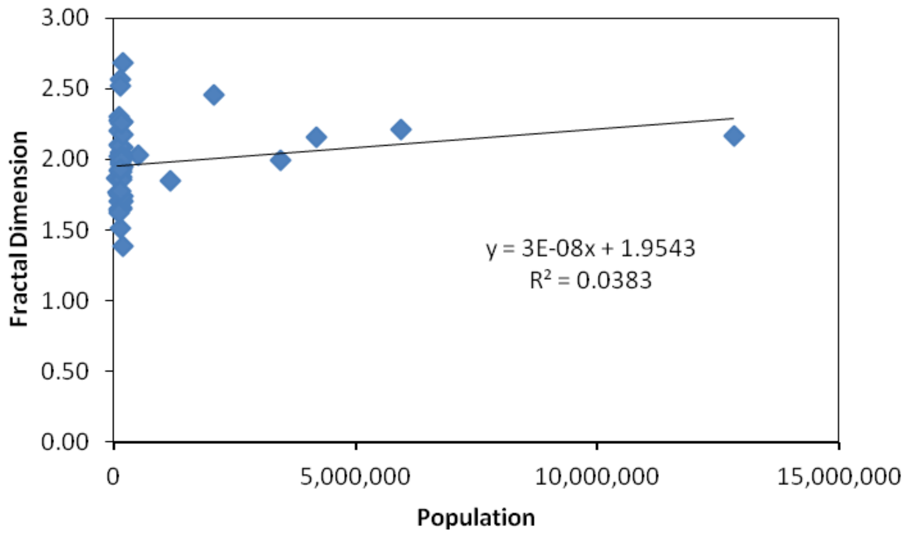


Figure 14: Population and Fractal Dimension are not Strongly Correlated

Just like fractal dimension the second scaling indicator of my choosing is also not influenced strongly by population or size of the city as shown in Figure 15.

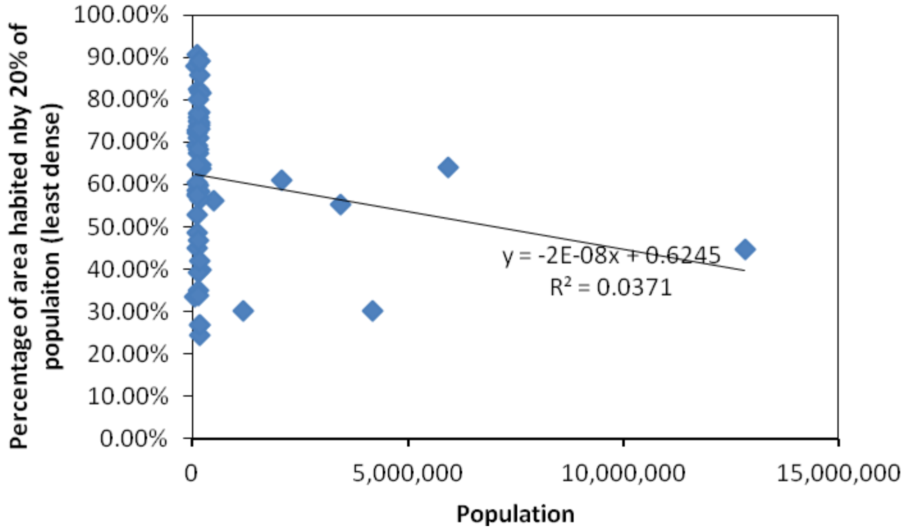


Figure 15: Population does not affect the Percentage of Area Occupied by 20% Least Densely Populating Habitants

5.1.4 Area and Other Parameters

The total gasoline sales are just as unaffected by the area covered by the city as the population as shown in Figure 16. This is once again a surprising find because one would expect the gasoline sales to be correlated and dependent upon the actual physical space covered by the city.

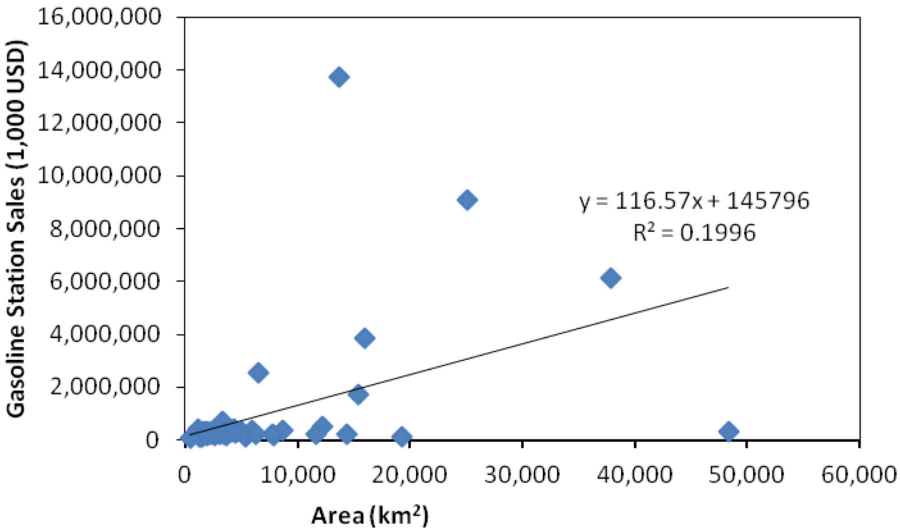


Figure 16: Area and Total Gasoline Sales are not Strongly Correlated

Although there is a weak trend showing that the gasoline sales overall may be rising, the r-square values are low, indicating that the correlation is not very strong. This is in general indicative of the detachment from physical space, of the layout or orientation of the American city.

The correlation between carbon emissions and area is similarly weak, just as the correlation between area and population or gasoline sales. This is shown in Figure 17.

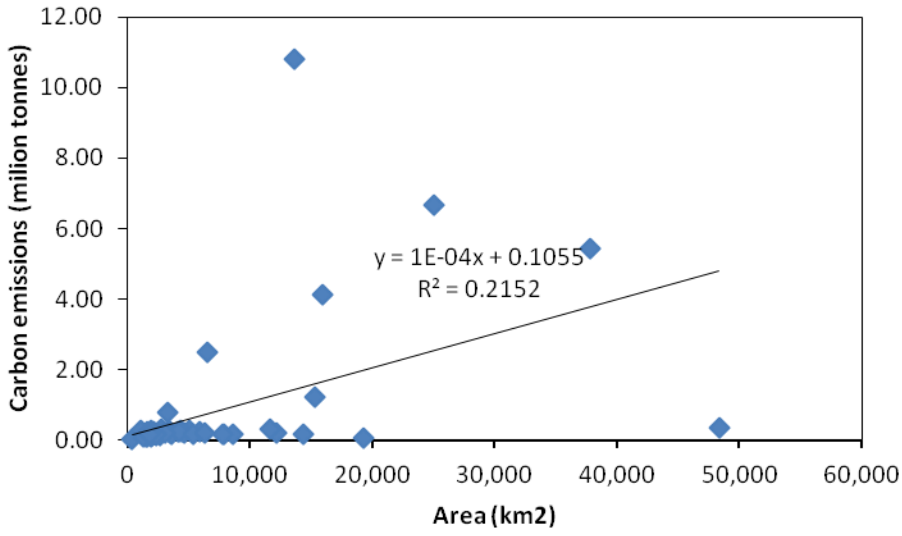


Figure 17: Area and Total Carbon Emissions are not Strongly Correlated

Just as with population, the fractal dimension is unaffected by area as shown in Figure 18. The scaling in cities is unaffected by the area just as well as by the population.

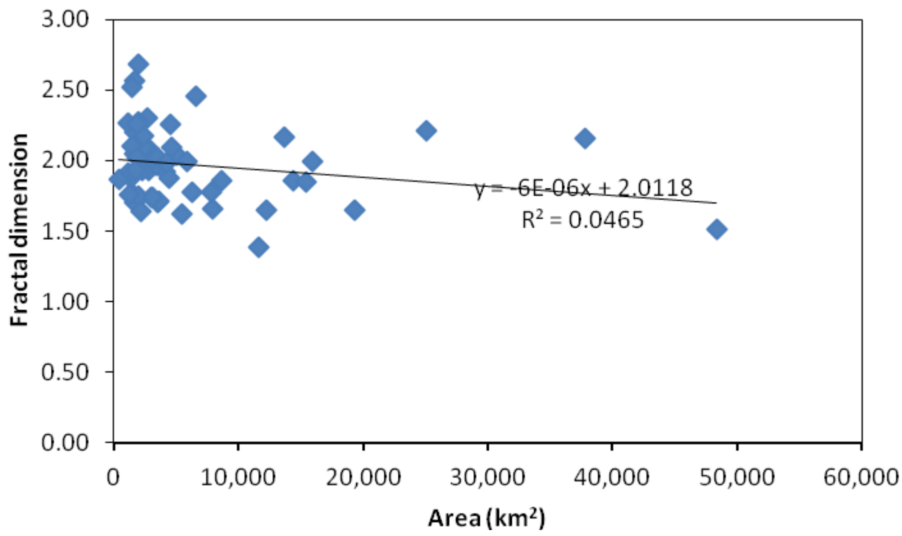


Figure 18: Area and Fractal Dimension are not Strongly Correlated

For percentage of area covered by 20% least densely populated housing once again we see that the scaling indicator is almost completely independent of a basic variable, i.e. the area of the city as shown in Figure 19.

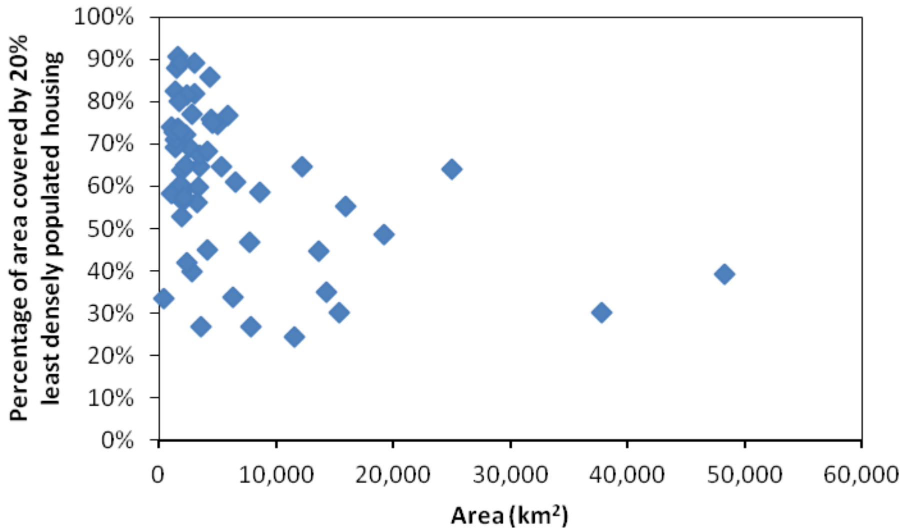


Figure 19: Area Does Not Influence the Percentage of Area Occupied by 20% Least Densely Populating Habitants

5.1.5 Population Density and Other Parameters

The second level of analysis involved mean indicators. Three mean indicators were calculated, namely population density, gasoline sales per capita and carbon emissions per capita. 2010 population, area and gasoline sales values were used while carbon emissions were calculated using 2008 values from the Vulcan project. The mean indicator values are presented in Table 8.

Table 8: Mean indicator values for cities

Metropolitan Statistical Area	State	Population density (per km square)	Gasoline station sales per capita (1,000 USD)	Carbon emissions per capita (tonnes)
Albany, GA Metro Area	GA	31	1.979	1.65
Altoona, PA Metro Area	PA	93	2.049	1.22
Ames, IA Metro Area	IA	60	1.569	1.21
Anderson, SC Metro Area	SC	95	1.801	1.46
Auburn-Opelika, AL Metro Area	AL	88	1.324	1.07
Bay City, MI Metro Area	MI	66	2.502	1.77
Bend, OR Metro Area	OR	20	1.184	1.09
Billings, MT Metro Area	MT	13	3.128	1.19
Blacksburg-Christiansburg-Radford, VA Metro Area	VA	58	1.774	1.52
Bloomington, IN Metro Area	IN	55	1.150	1.22

Metropolitan Statistical Area	State	Population density (per km square)	Gasoline station sales per capita (1,000 USD)	Carbon emissions per capita (tonnes)
Brunswick, GA Metro Area	GA	27	2.756	2.17
Burlington, NC Metro Area	NC	134	2.644	1.27
Carson City, NV Metro Area	NV	136	1.329	0.70
Cleveland, TN Metro Area	TN	58	1.795	1.64
Cleveland-Elyria-Mentor, OH Metro Area	OH	317	1.219	1.19
Coeur d'Alene, ID Metro Area	ID	41	2.097	1.64
Columbia, MO Metro Area	MO	57	1.697	1.41
Crestview-Fort Walton Beach-Destin, FL Metro Area	FL	65	1.966	1.79
Dalton, GA Metro Area	GA	86	2.277	1.79
Danville, VA Metro Area	VA	40	1.846	1.32
Deltona-Daytona Beach-Ormond Beach, FL Metro Area	FL	150	1.391	1.57
Dothan, AL Metro Area	AL	33	1.947	1.69
Dover, DE Metro Area	DE	78	1.600	1.58
Dubuque, IA Metro Area	IA	59	2.002	1.08
Eau Claire, WI Metro Area	WI	37	2.647	1.63
El Centro, CA Metro Area	CA	15	1.187	1.82
Elizabethtown, KY Metro Area	KY	52	2.531	1.09
Fairbanks, AK Metro Area	AK	5	1.078	0.55
Farmington, NM Metro Area	NM	9	1.730	1.40
Flagstaff, AZ Metro Area	AZ	3	2.409	2.54
Florence-Muscle Shoals, AL Metro Area	AL	42	1.821	1.38
Fond du Lac, WI Metro Area	WI	51	2.174	1.16
Gadsden, AL Metro Area	AL	73	1.248	1.58
Gainesville, GA Metro Area	GA	162	1.440	1.46
Glens Falls, NY Metro Area	NY	28	2.462	1.62
Goldsboro, NC Metro Area	NC	85	1.530	0.98
Grand Junction, CO Metro Area	CO	17	2.474	1.11
Greenville, NC Metro Area	NC	79	1.536	0.99
Hanford-Corcoran, CA Metro Area	CA	42	1.040	1.12
Harrisonburg, VA Metro Area	VA	56	1.729	1.28
Hattiesburg, MS Metro Area	MS	34	2.601	1.69
Hot Springs, AR Metro Area	AR	50	1.780	1.17
Houston-Sugar Land-Baytown, TX Metro Area	TX	237	1.528	1.12
Idaho Falls, ID Metro Area	ID	17	1.664	1.39
Iowa City, IA Metro Area	IA	49	1.495	1.46
Ithaca, NY Metro Area	NY	80	1.262	1.06
Jackson, MI Metro Area	MI	86	1.502	1.37
Jackson, TN Metro Area	TN	53	2.540	2.06
Jacksonville, NC Metro Area	NC	76	1.490	0.91

Metropolitan Statistical Area	State	Population density (per km square)	Gasoline station sales per capita (1,000 USD)	Carbon emissions per capita (tonnes)
Janesville, WI Metro Area	WI	85	2.077	1.47
Jefferson City, MO Metro Area	MO	25	2.417	1.57
Johnstown, PA Metro Area	PA	80	1.951	1.10
Los Angeles-Long Beach-Santa Ana, CA Metro Area	CA	941	1.071	0.84
New Orleans-Metairie-Kenner, LA Metro Area	LA	76	1.487	1.05
Phoenix-Mesa-Glendale, AZ Metro Area	AZ	111	1.458	1.29
Pine Bluff, AR Metro Area	AR	19	1.424	1.60
Seattle-Tacoma-Bellevue, WA Metro Area	WA	216	1.117	1.20
St. George, UT Metro Area	UT	22	1.448	1.36

As can be expected based on the strong correlation between population and gasoline sales and carbon emissions, there exists a strong linear correlation between population density and gasoline sales and population density and carbon emissions as shown in Figure 20 and Figure 21. What this seems to suggest is that as the city gets denser on average, the potential efficiency savings are wiped out by the concurrent increase in scale, or population of the city. However as we will see in the next section, this correlation falls apart when we normalize the three variables with respect to area, for population and with respect to population for gasoline sales and carbon emissions.

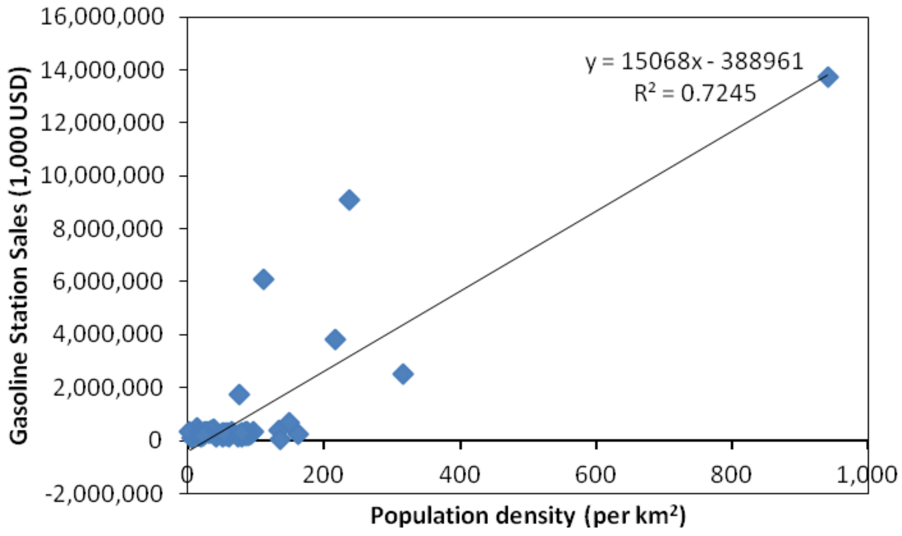


Figure 20: Population Density and Gasoline Station Sales are Correlated

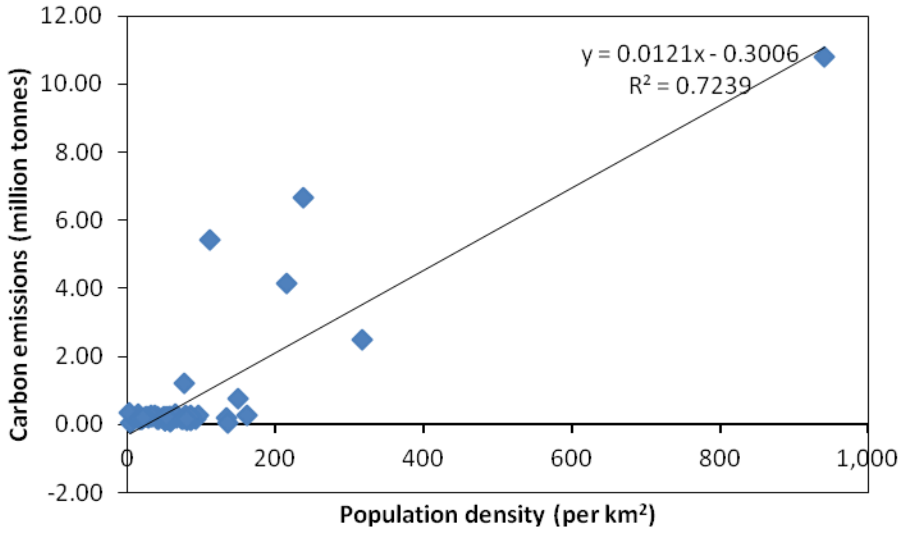


Figure 21: Population Density and Total Carbon Emissions are Correlated

The correlation between population density and gasoline sales per capita does not seem very strong, further supporting the hypothesis that the efficiency gains made through denser arrangement of population are lost due to increase in scale, even at the per capita level. The correlation is shown in Figure 22.

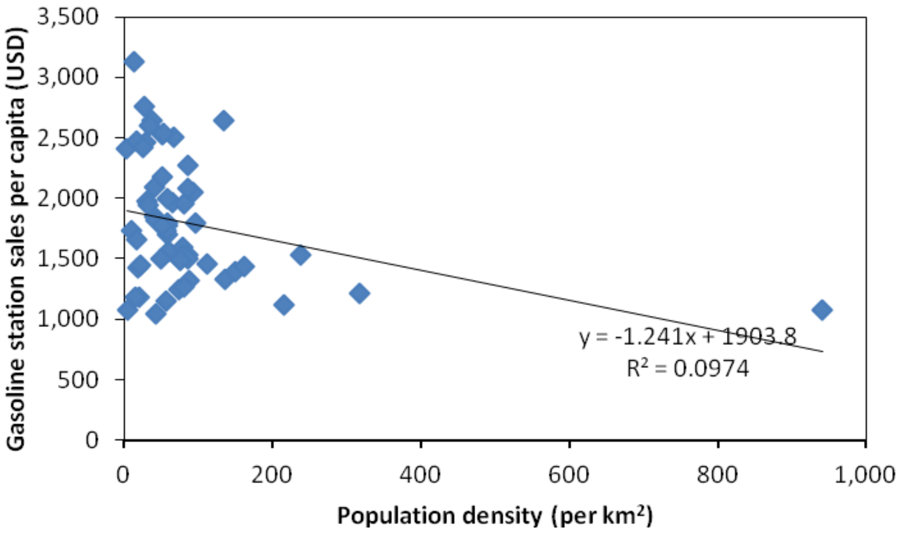


Figure 22: Population Density and Gasoline Sales per Capita are not Strongly Correlated

Population density and carbon emissions per capita do not have a strong correlation either as shown in Figure 23. This is again expected because of the lack of correlation between population density and gasoline sales per capita.

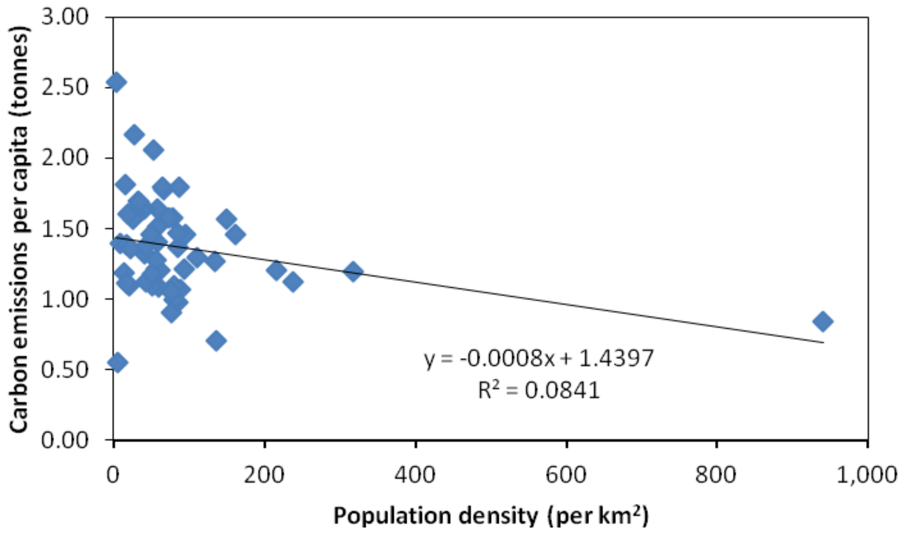


Figure 23: Population Density and Carbon Emissions per Capita are not Strongly Correlated

It is observed that fractal dimension is very weakly power law dependent on population density as shown in Figure 24.

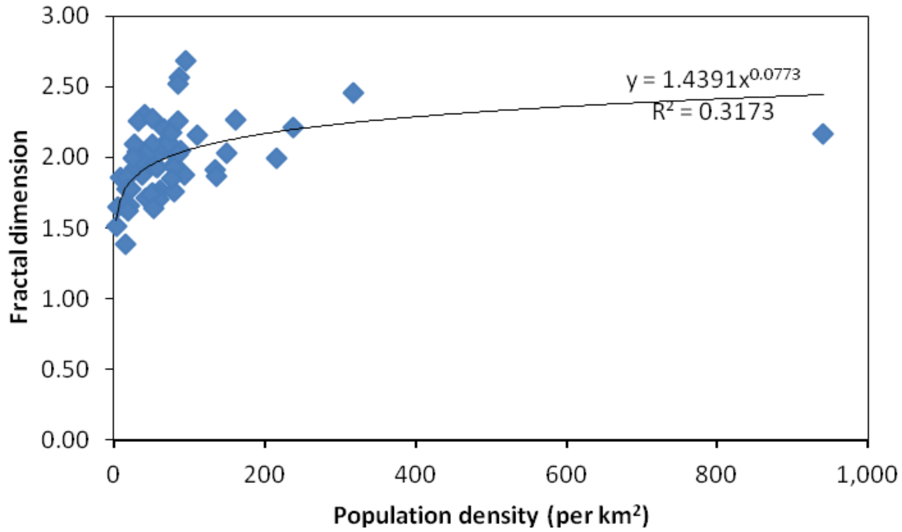


Figure 24: Population Density and Fractal Dimension are Weakly Correlated

The percentage of area covered by 20% of the least densely populating habitants is largely independent of the population density for the entire city as shown in Figure 25.

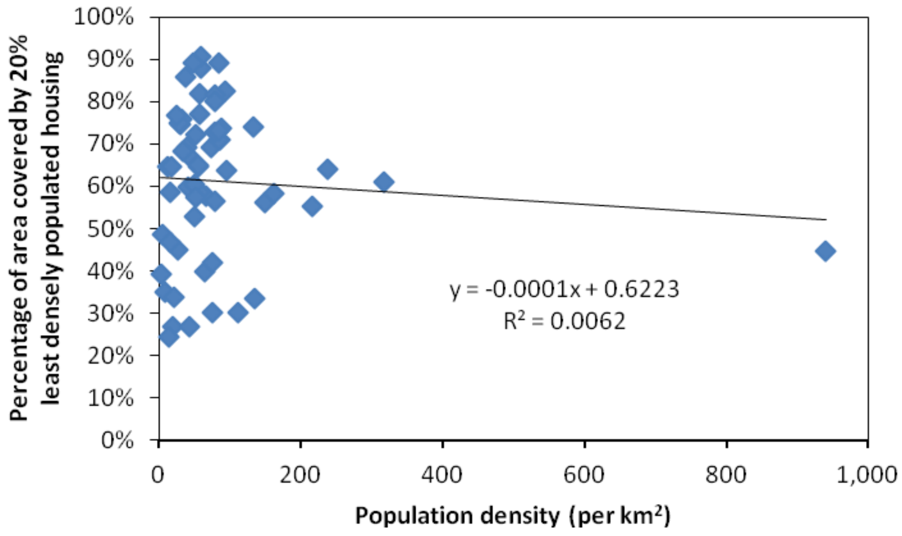


Figure 25: Population Density and Percentage of Area Covered by 20% of the Least Densely Populating Habitants are not Correlated

5.1.6 Fractal Dimension and Other Parameters

Fractal dimension does not seem to have a strong influence on the gasoline station sales for the cities as shown in Figure 26. However, a visual analysis of the resulting graph suggests that gasoline usage changes with changing fractal dimension in two very distinct ways. For high gasoline consumption cities (cities with higher population), the rise in fractal dimension is almost exponential. This compared to a weak linear trend of change in gasoline sales with changing fractal dimension for cities of smaller population indicates potential for differentiation or identification of criticalities using fractal dimension.

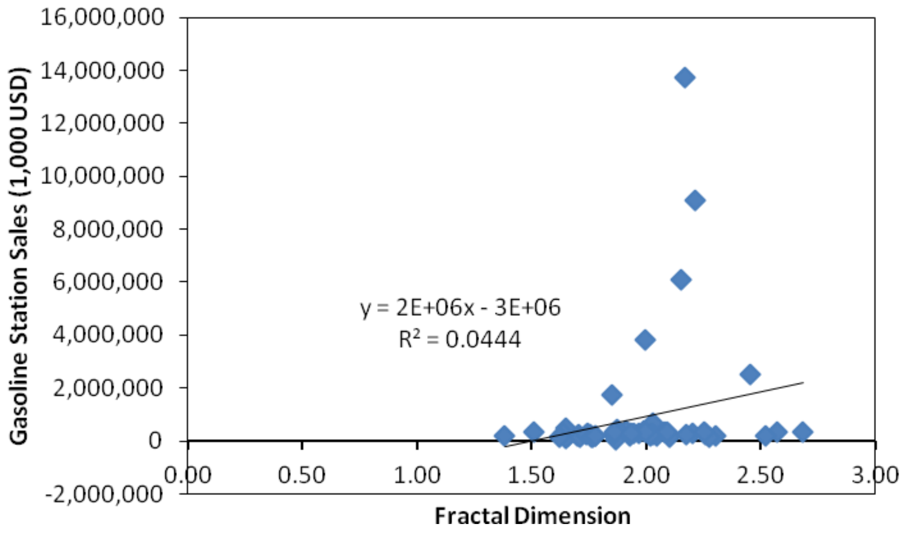


Figure 26: Fractal Dimension does not Strongly Impact Gasoline Station Sales

The correlation between fractal dimension and carbon emissions for cities is similar to the correlation between fractal dimension and total gasoline sales as shown in Figure 27. In each case there is no strong predictive correlation, though a pattern seems to be visually detectable between larger and smaller cities.

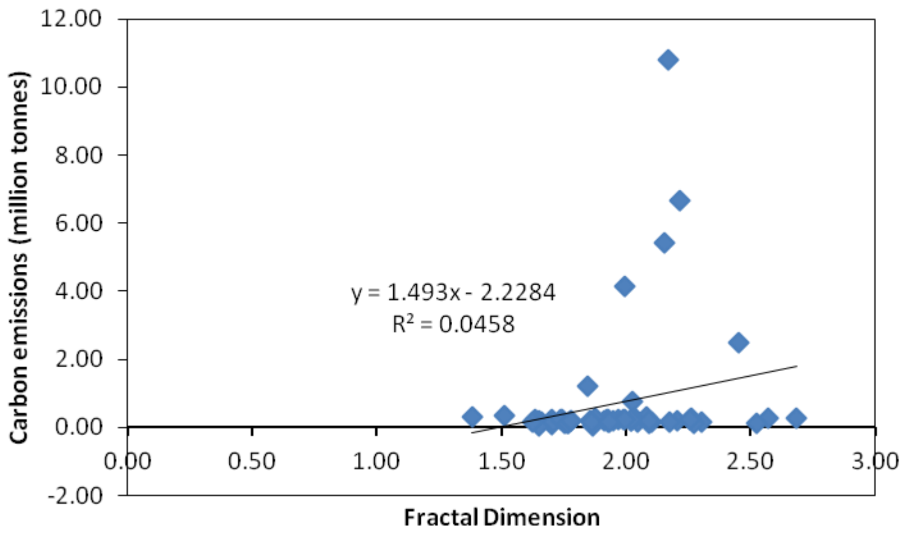


Figure 27: Fractal Dimension does Not Strongly Correlate with Carbon Emissions

This is the first of the original results of the research. As shown in Figure 28 fractal dimension is not a strong predictor of gasoline sales per capita for the city. Scaling and resource consumption efficiency using gasoline sales per capita as a proxy indicator cannot be considered to be correlated based on these measures.

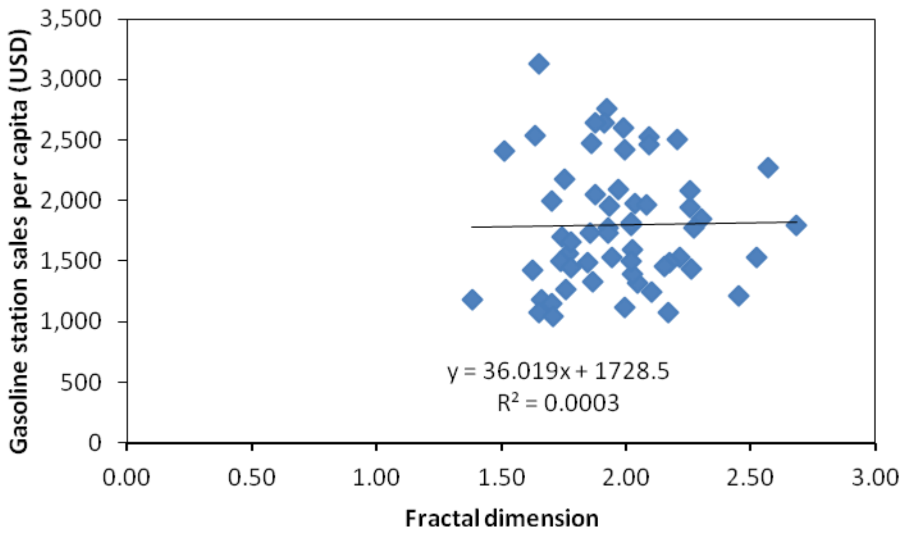


Figure 28: Fractal Dimension and Gasoline Station Sales per Capita are not Correlated

Just as with gasoline sales per capita, carbon emissions per capita are not strongly influenced by fractal dimension. The correlation is shown in Figure 29.

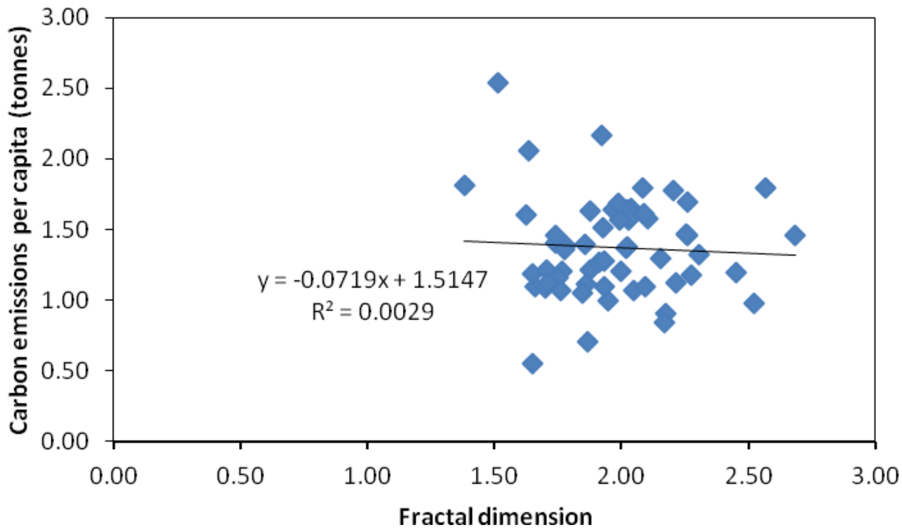


Figure 29: Fractal Dimension and Carbon Emissions per Capita are not Correlated

5.1.7 Percentage of Area Covered by 20% of Least Dense Housing and Other Parameters

No correlation is observed between percentage of area covered by housing for least densely populating 20% of the residents either. The correlation can be seen in Figure 30.

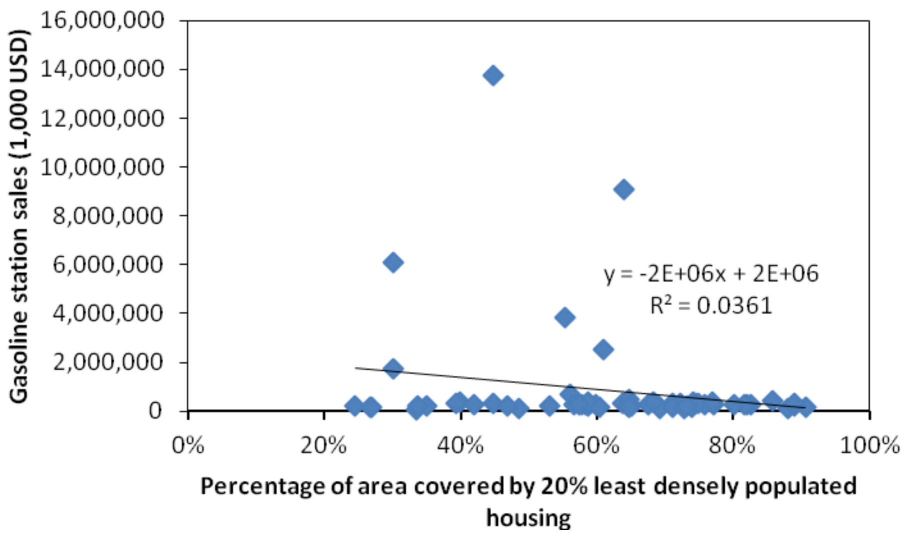


Figure 30: Percentage of Area Covered by 20% of Least Dense Housing and Total Gasoline Sales are Not Correlated

Carbon emissions in a city are not influenced by the percentage of the city area covered by the 20% of the housing having least density as shown in Figure 31.

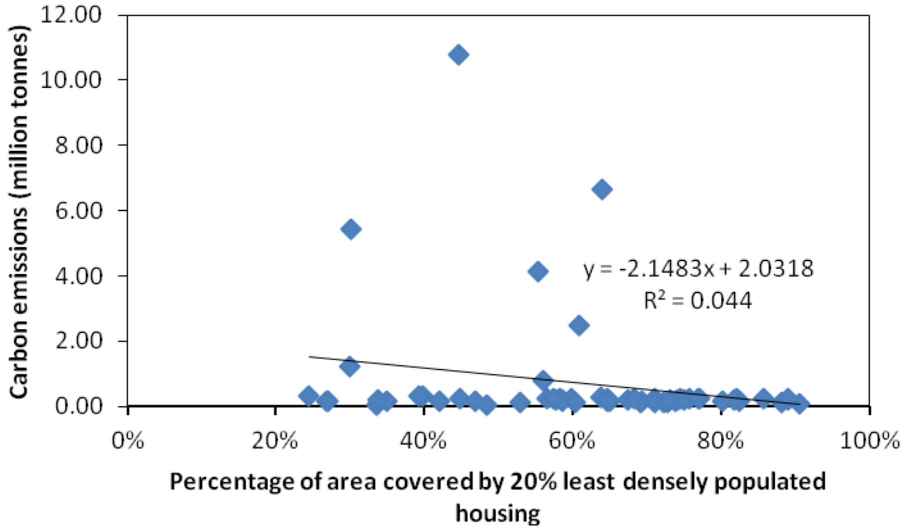


Figure 31: Percentage of Area Covered by 20% of Least Dense Housing and Carbon Emissions

Percentage of Area Covered by 20% of Least Dense Housing and Gasoline Sales per Capita

Gasoline sales per capita, being used in this study as primary direct resource consumption indicator is not influenced by the percentage of urban area covered by the least dense housing as shown in Figure 32.

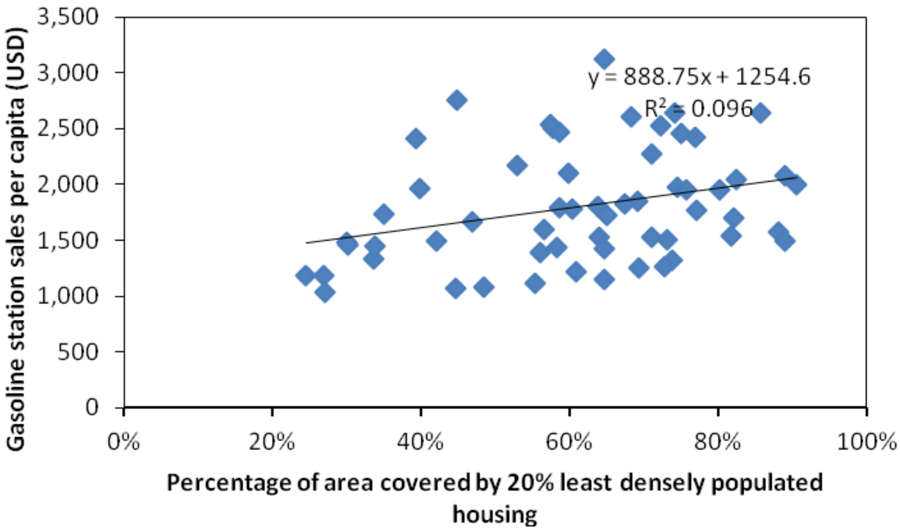


Figure 32: Percentage of Area Covered by 20% of Least Dense Housing and Gasoline Sales per Capita are Not Correlated

Even after having been normalized by city size (population) the carbon emissions observed continue to be unaffected by the percentage of area covered by 20% of least dense housing as shown in Figure 33.

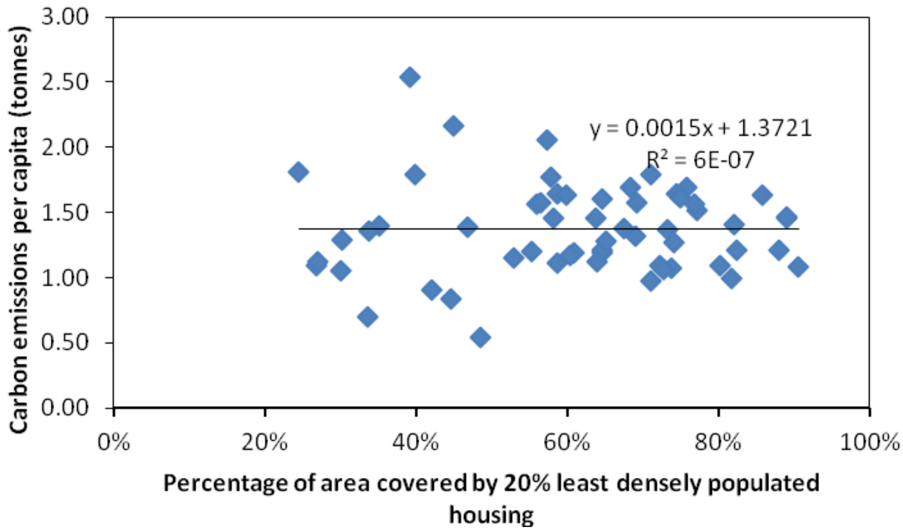


Figure 33: Percentage of Area Covered by 20% of Least Dense Housing and Carbon Emissions per Capita are Not Correlated

5.1.8 Area Normalized Consumption Indicators

The consumption indicators in my study or the direct environmental indicators (gasoline sales and carbon emissions) were also analyzed in density terms (per unit area). The gasoline station sales per unit area and carbon emissions per unit area are reported in Table 9. These shall be used in further correlation analysis.

Table 9: Gasoline station sales and carbon emissions per unit area

Metropolitan Statistical Area	State	Gasoline sales per unit area (USD per square meters)	Carbon emission per unit area (tonnes per km^2)
Albany, GA Metro Area	GA	0.0614	51.09
Altoona, PA Metro Area	PA	0.1908	113.10
Ames, IA Metro Area	IA	0.0945	72.82
Anderson, SC Metro Area	SC	0.1718	139.20
Auburn-Opelika, AL Metro Area	AL	0.1164	94.23
Bay City, MI Metro Area	MI	0.165	117.00
Bend, OR Metro Area	OR	0.0236	21.79
Billings, MT Metro Area	MT	0.0405	15.40
Blacksburg-Christiansburg-Radford, VA Metro Area	VA	0.1025	87.54
Bloomington, IN Metro Area	IN	0.0636	67.25
Brunswick, GA Metro Area	GA	0.0744	58.53

Metropolitan Statistical Area	State	Gasoline sales per unit area (USD per square meters)	Carbon emission per unit area (tonnes per km^2)
Burlington, NC Metro Area	NC	0.3549	170.20
Carson City, NV Metro Area	NV	0.1804	95.03
Cleveland, TN Metro Area	TN	0.1037	94.89
Cleveland-Elyria-Mentor, OH Metro Area	OH	0.3865	378.30
Coeur d'Alene, ID Metro Area	ID	0.0852	66.52
Columbia, MO Metro Area	MO	0.0974	80.71
Crestview-Fort Walton Beach-Destin, FL Metro Area	FL	0.1269	115.50
Dalton, GA Metro Area	GA	0.1961	154.50
Danville, VA Metro Area	VA	0.0743	53.20
Deltona-Daytona Beach-Ormond Beach, FL Metro Area	FL	0.208	234.80
Dothan, AL Metro Area	AL	0.0633	55.02
Dover, DE Metro Area	DE	0.1256	124.00
Dubuque, IA Metro Area	IA	0.1174	63.50
Eau Claire, WI Metro Area	WI	0.0976	60.26
El Centro, CA Metro Area	CA	0.0178	27.29
Elizabethtown, KY Metro Area	KY	0.1309	56.56
Fairbanks, AK Metro Area	AK	0.0055	2.77
Farmington, NM Metro Area	NM	0.0157	12.64
Flagstaff, AZ Metro Area	AZ	0.0067	7.05
Florence-Muscle Shoals, AL Metro Area	AL	0.077	58.24
Fond du Lac, WI Metro Area	WI	0.1114	59.22
Gadsden, AL Metro Area	AL	0.0917	115.80
Gainesville, GA Metro Area	GA	0.2328	235.90
Glens Falls, NY Metro Area	NY	0.0689	45.24
Goldsboro, NC Metro Area	NC	0.1301	83.05
Grand Junction, CO Metro Area	CO	0.0419	18.89
Greenville, NC Metro Area	NC	0.122	78.87
Hanford-Corcoran, CA Metro Area	CA	0.0441	47.69
Harrisonburg, VA Metro Area	VA	0.096	71.08
Hattiesburg, MS Metro Area	MS	0.0885	57.46
Hot Springs, AR Metro Area	AR	0.0898	59.18
Houston-Sugar Land-Baytown, TX Metro Area	TX	0.3626	266.20
Idaho Falls, ID Metro Area	ID	0.0279	23.23
Iowa City, IA Metro Area	IA	0.0738	71.98
Ithaca, NY Metro Area	NY	0.1007	84.91
Jackson, MI Metro Area	MI	0.1285	117.00
Jackson, TN Metro Area	TN	0.134	108.60
Jacksonville, NC Metro Area	NC	0.1129	68.71
Janesville, WI Metro Area	WI	0.1771	125.50

Metropolitan Statistical Area	**State**	**Gasoline sales per unit area (USD per square meters)**	**Carbon emission per unit area (tonnes per km^2)**
Jefferson City, MO Metro Area	MO	0.0614	39.83
Johnstown, PA Metro Area	PA	0.1561	87.66
Los Angeles-Long Beach-Santa Ana, CA Metro Area	CA	1.0076	791.90
New Orleans-Metairie-Kenner, LA Metro Area	LA	0.113	79.75
Phoenix-Mesa-Glendale, AZ Metro Area	AZ	0.1617	143.50
Pine Bluff, AR Metro Area	AR	0.0264	29.75
Seattle-Tacoma-Bellevue, WA Metro Area	WA	0.2408	258.90
St. George, UT Metro Area	UT	0.0318	29.88

We will now see how density measures of consumption relate to the scaling indicators. Gasoline station sales per unit area and carbon emissions per unit area do not depend strongly on the percentage of area covered by 20% of least densely populated housing as shown in Figure 34 and Figure 35 respectively.

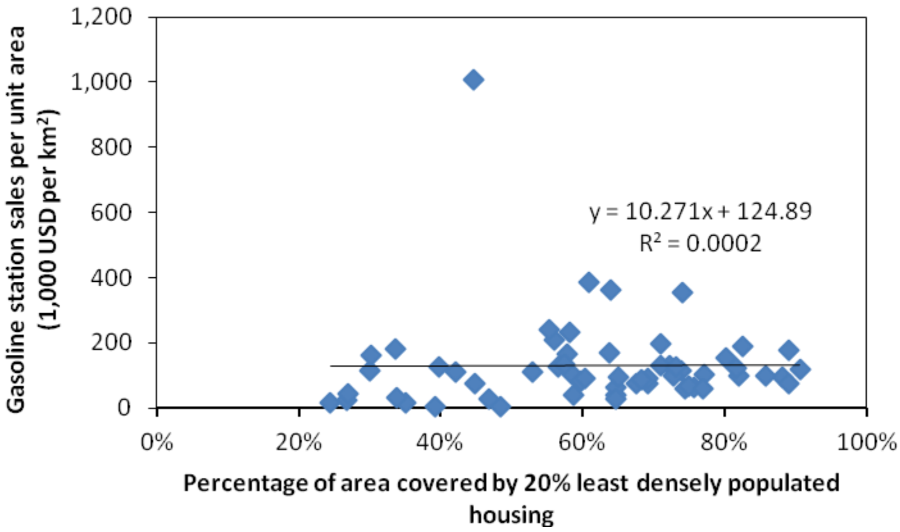


Figure 34: Percentage of Area Covered by 20% of Least Densely Populated Housing does not Affect Gasoline Station Sales per unit Area

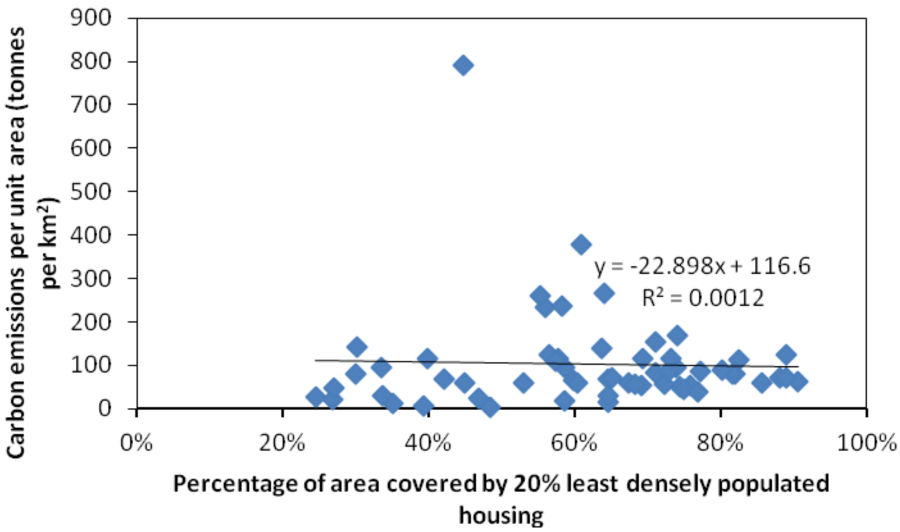


Figure 35: Percentage of Area Covered by 20% of Least Densely Populated Housing does not Affect Carbon Emissions per unit Area

There is a weak power-law correlation between fractal dimension and gasoline sales per unit area as shown in Figure 36.

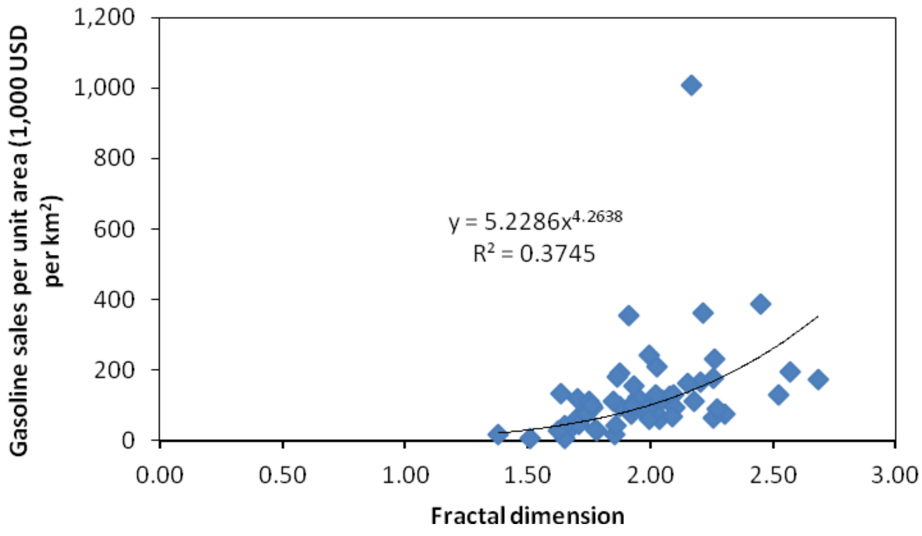


Figure 36: Gasoline Station Sales per Unit Area Correlate with Fractal Dimension

Once again we see that a weak power law correlation affects how carbon emissions per unit area change with changing fractal dimensions. The correlation is shown in Figure 37.

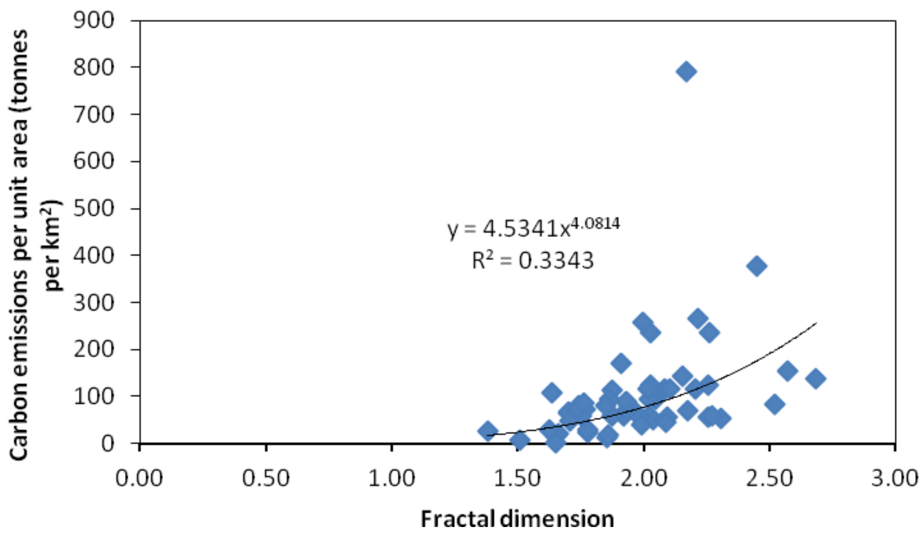


Figure 37: Fractal Dimension and Carbon Emissions per Unit Area are Weakly Correlated

5.2 Planning Planes for Urban Indicators

During the analysis it was observed that gasoline sales density depends on fractal dimension and population density. I will now present a tool for incorporating these two variables in policy analysis. The planning plane allows for consideration of fractal dimension and population density into planning for optimization of energy use. The planning plane for minimizing gasoline sales per unit area is shown in Figure 38. The variance for the planning plane is shown in Figure 39.

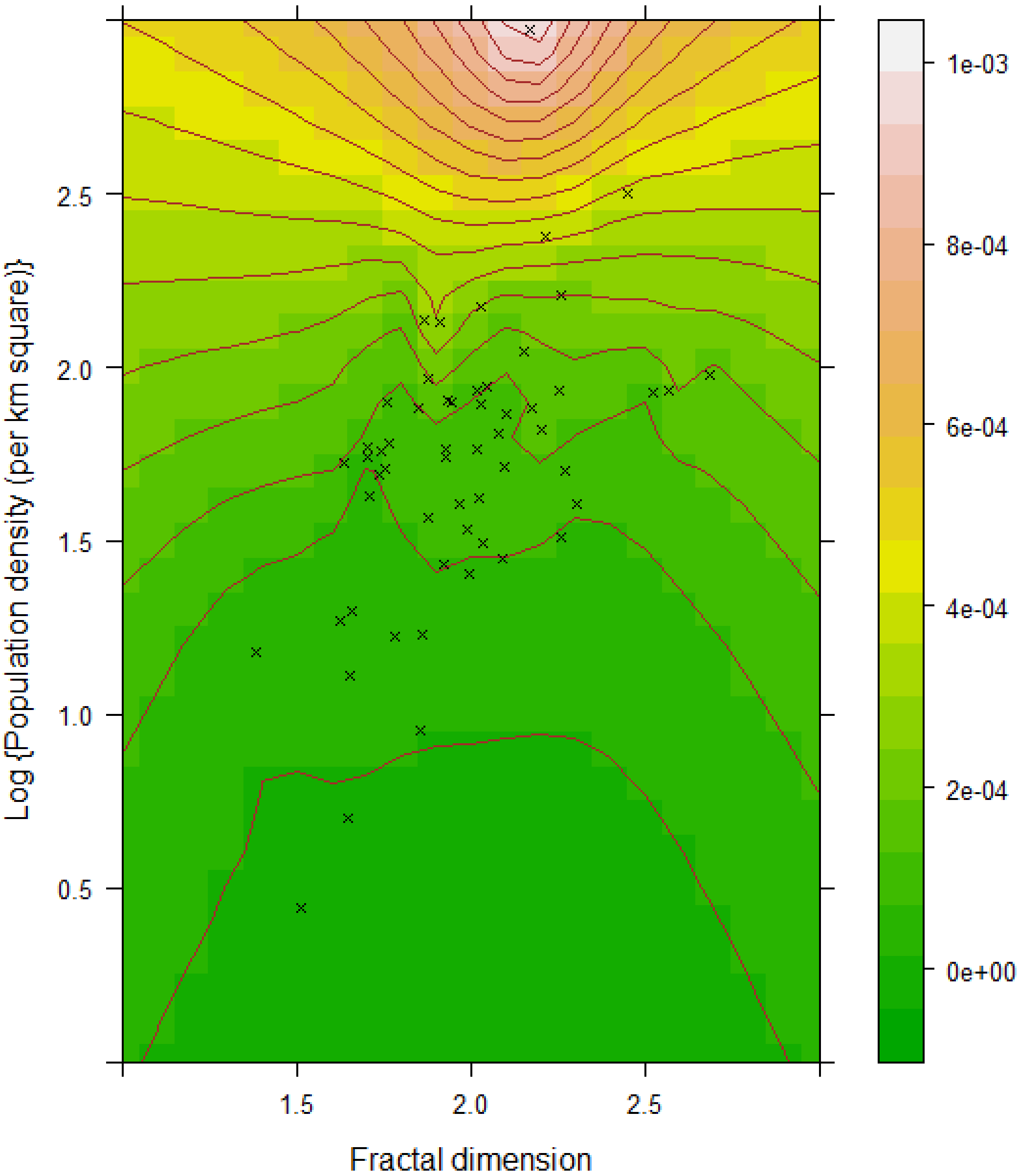


Figure 38: Fractal Dimension-Population Density Planning Plane for Gasoline Sales per Unit Area

What the plane shows is that in order to minimize gasoline usage within growth in area and population density, the fractal dimension should be planned to be minimized till a value of around 2.1. After that the fractal dimension should be maximized, though fewer data points beyond that point indicate that the conservative choice is to keep urban fractal dimension less than 2.1 for minimizing gasoline sales per unit area. The variance for the kriging plot is shown in Figure 39 and is generally within acceptable limits.

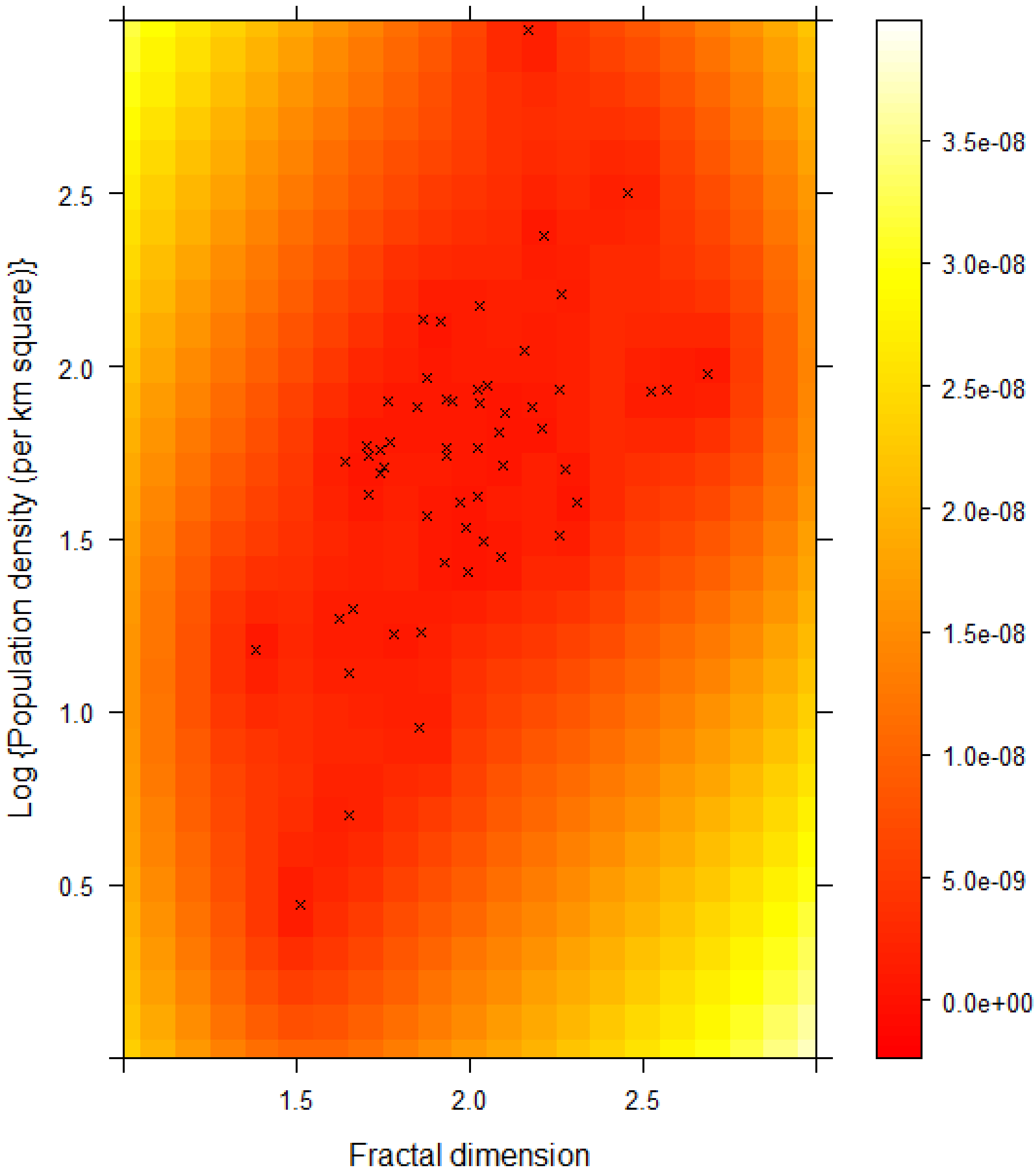


Figure 39: Variance for Fractal Dimension-Population Density Planning Plane for Gasoline Sales per Unit Area

The planning plane for carbon emissions per unit area are shown in Figure 40. The variance of the plane is shown in Figure 41. The topography is similar to the one for gasoline sales with a maxima of carbon emissions around fractal dimension of 2.1. The variance values are close to negligible.

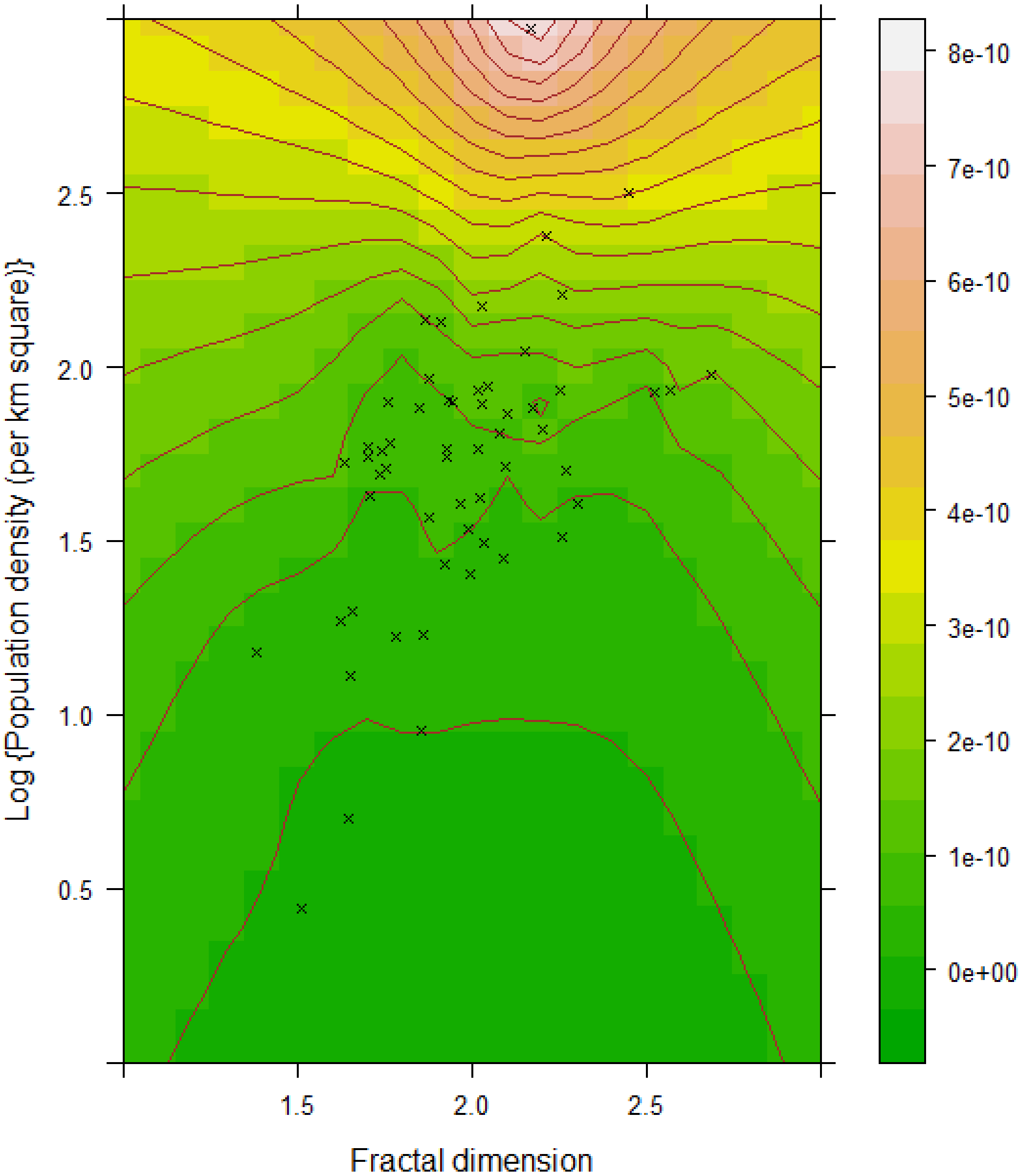


Figure 40: Fractal Dimension-Population Density Planning Plane for Carbon Emissions per Unit Area

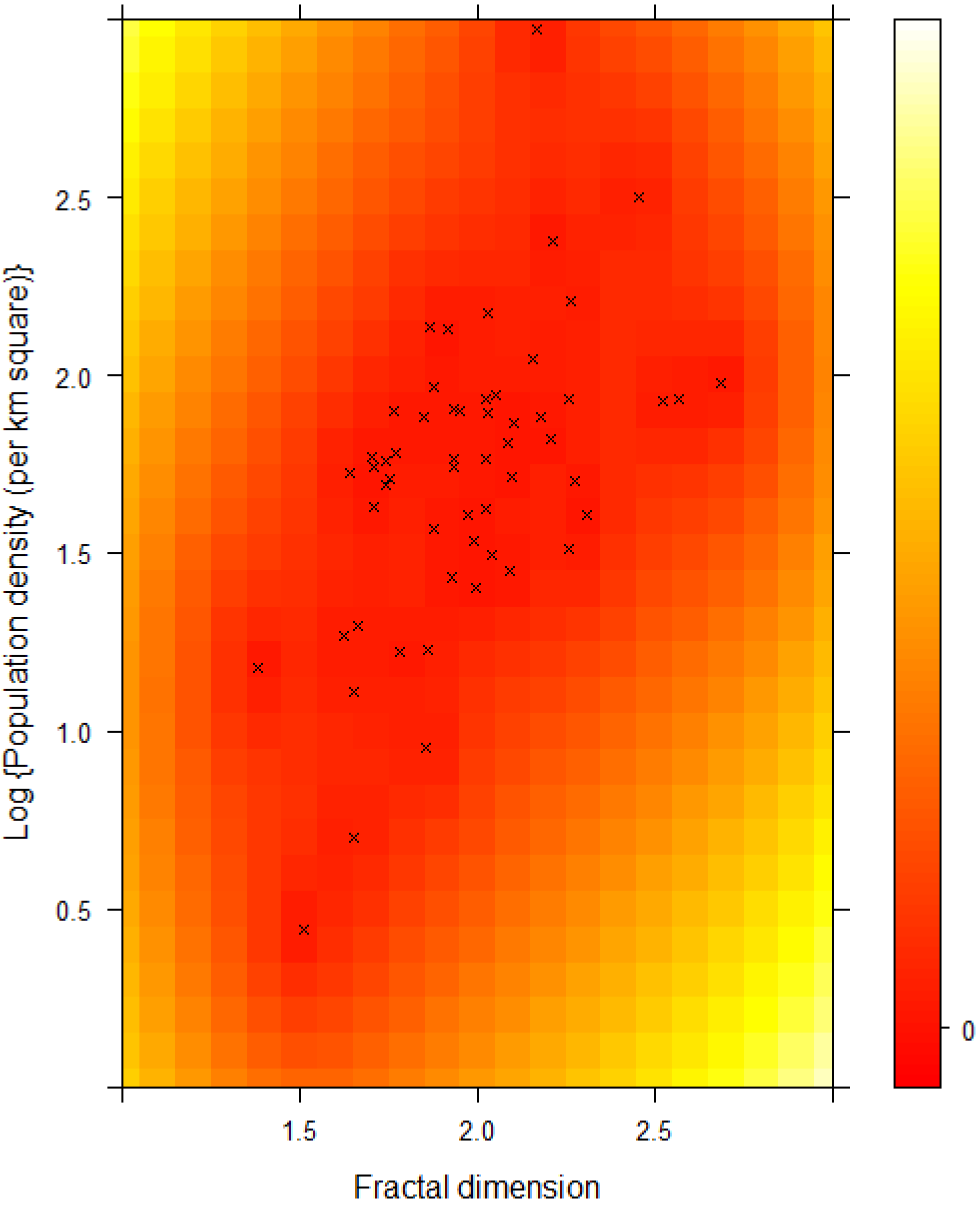


Figure 41: Variance of Fractal Dimension-Population Density Planning Plane for Carbon Emissions per Unit Area

5.3 National Economic Indicators, Correlations and Planning Planes

The fractal dimension based scaling indicator for income distribution in national economies was calculated by plotting on log-log scale the cumulative income distribution and cumulative population percentage and taking the slope of the regression line. The r-squared value was greater than 0.95 for all the linear fits for all countries, indicating strong power law distribution. The values are shown in Table 10.

Table 10: Fractal dimension based scaling indicator for income distribution

Country Name	**Fractal dimension based scaling indicator for income distribution**
Albania	0.59
Argentina	0.39
Armenia	0.49
Bosnia and Herzegovina	0.53
Belarus	0.65
Brazil	0.32
Colombia	0.31
Comoros	0.24
Costa Rica	0.40
Dominican Republic	0.36
Estonia	0.53
Guatemala	0.35
Honduras	0.30
Croatia	0.61
Hungary	0.60
Kazakhstan	0.57
Kyrgyz Republic	0.53
Cambodia	0.44
Lithuania	0.53
Latvia	0.54
Moldova	0.52
Maldives	0.51
Mexico	0.43
Macedonia, FYR	0.51
Malaysia	0.51
Namibia	0.24
Nigeria	0.46
Panama	0.34
Peru	0.38
Poland	0.53
Paraguay	0.34
Romania	0.58
Russian Federation	0.52
El Salvador	0.41
Serbia	0.56
Slovak Republic	0.60

Country Name	Fractal dimension based scaling indicator for income distribution
Slovenia	0.59
Syrian Arab Republic	0.52
Tajikistan	0.55
Turkey	0.46
Ukraine	0.62
Uruguay	0.41
Venezuela, RB	0.42
Vietnam	0.51
Zambia	0.38

Fractal Dimension and GDP per capita are found to be largely not related to each other as shown in Figure 42.

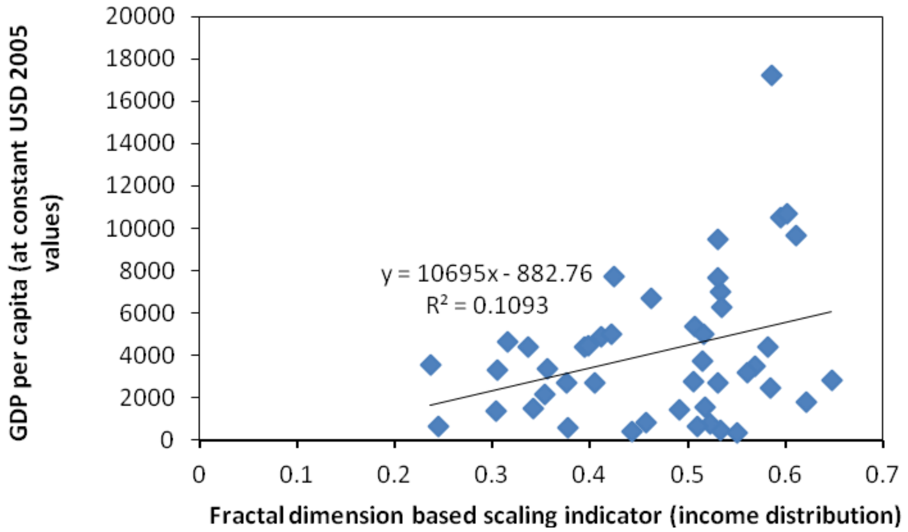


Figure 42: Fractal Dimension and GDP per capita are Not Correlated

Unlike often hypothesized, energy use per capita and GDP per capita do not have a strong linear correlation, though an upward trend is clearly visible. The data is visualized in Figure 43.

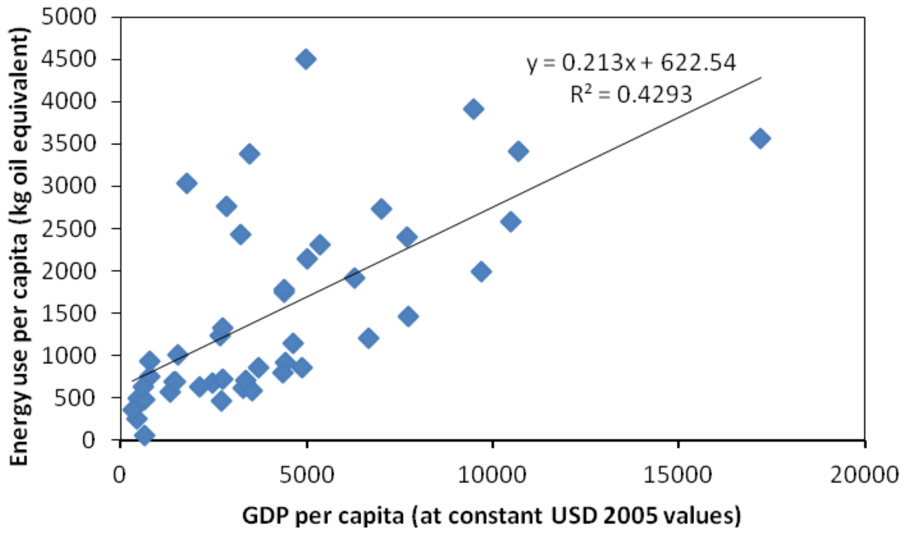


Figure 43: GDP per Capita and Energy Use per Capita Correlate only Weakly

The correlation between fractal dimension based scaling indicator for income distribution and energy use per capita for countries is very similar to the similar correlation between fractal dimension and gasoline sales for cities as shown in Figure 44.

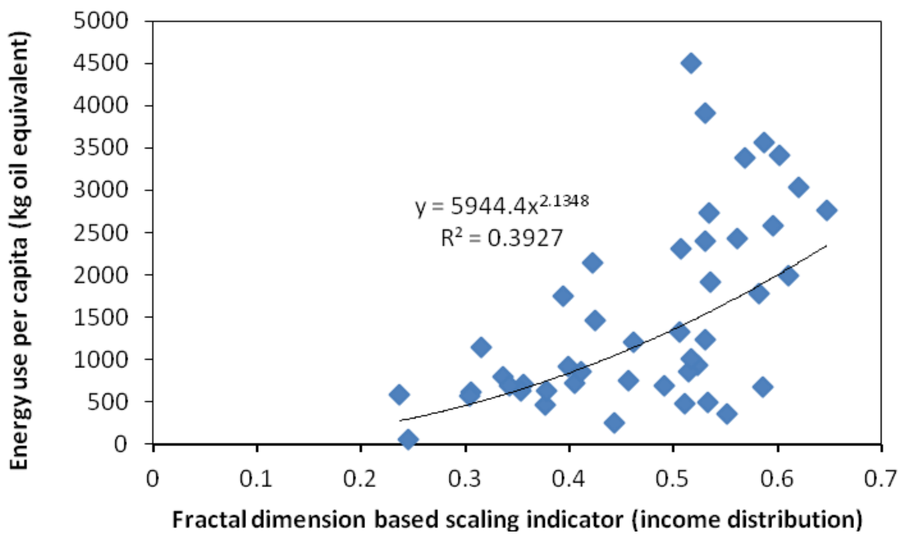


Figure 44: Fractal Dimension based Scaling Indicator and Energy Use Per Capita Correlate Weakly (Power Law)

This is a remarkable result. Two completely different complex systems i.e. national economies and urban population distribution show remarkably similar effect on a mean or density based measure of energy consumption. This hints at similarities between the nature of the systems which transcend the actual manifestation of the systemic dynamics.

The planning plane for energy consumption planning based on fractal dimension and GDP per capita is shown in Figure 45. The variance is shown in Figure 46.

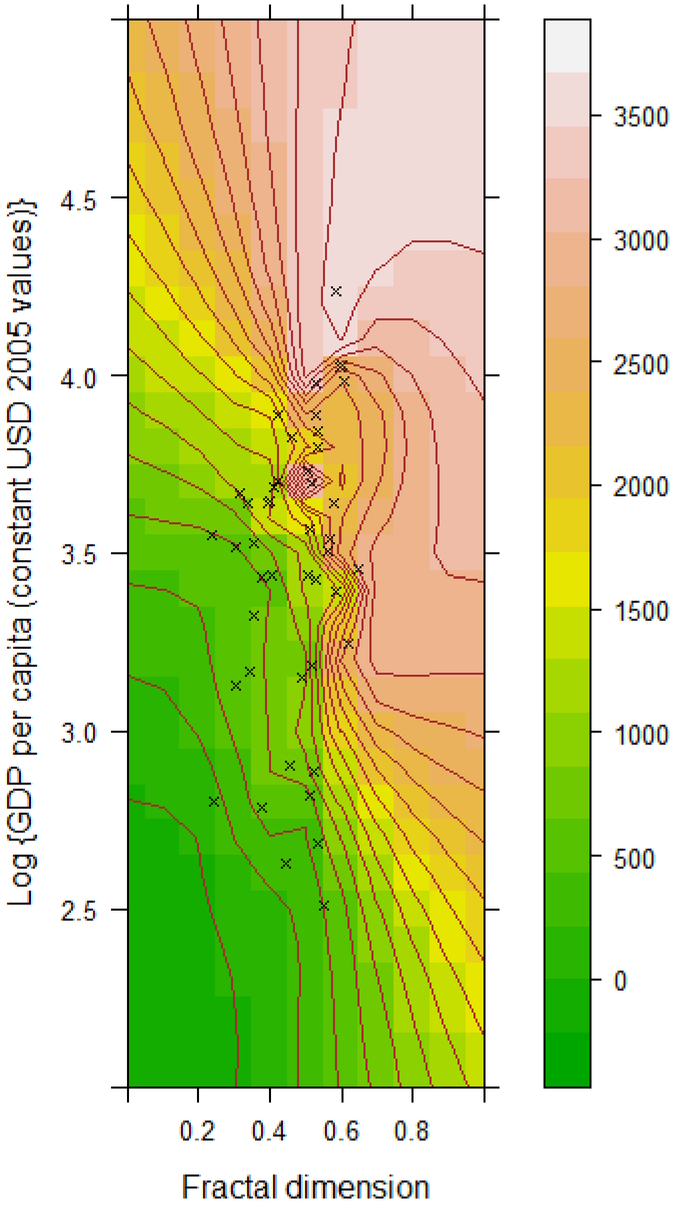


Figure 45: Fractal Dimension and GDP per Capita Planning-Plane for Energy Use per Capita

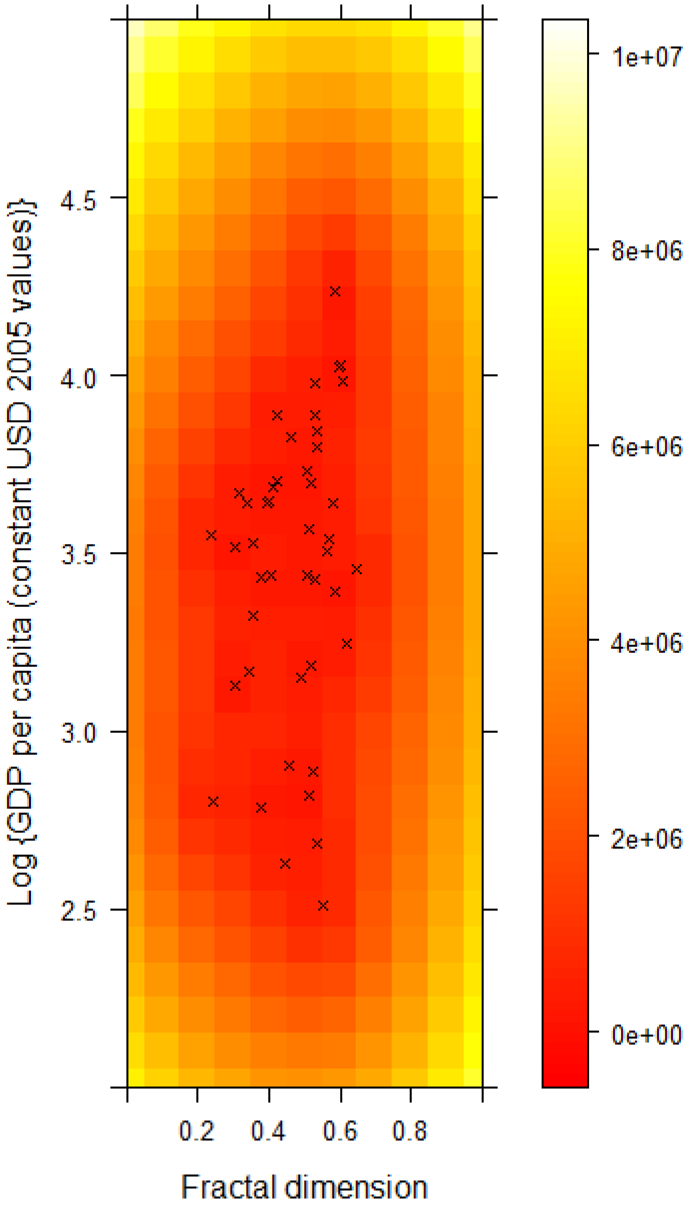


Figure 46: Variance for Fractal Dimension and GDP per Capita Planning-Plane for Energy Use per Capita

6. Discussion

In the results section I looked at a number of correlations, summarized in Table 11. First I checked was whether the total gasoline station sales based on US census data and total carbon emissions from the Vulcan dataset had any correlation; which they did. They were strongly linearly correlated as expected despite coming from different sources, showing that the two datasets could be compared. Surprisingly, population and area were not correlated. The size had no impact on the form in general; a result which shows the extent of top-down planning intervention in American cities. Fractal dimension also did not depend on population showing that size and form were unrelated. Fractal dimension also did not depend on area, once again establishing that any correlations between fractal dimension and energy indicators would have nothing to do with these fundamental correlations. Neither gasoline sales, nor carbon emissions were dependent on the area. Population density also did not seem to effect either gasoline sales or carbon emissions, total or on a per capita basis. Fractal dimension depended in a cumulative power law way on population density, albeit very weakly. Now both are indicators of urban form, however what this correlation shows is that beyond a certain form there is a change in evolution of form which is non-linear and thus not captured by population density. Neither gasoline sales nor carbon emission, total or per capita had any dependence on fractal dimension. This is further evidence of divorce of form and scale in modern American cities. Both gasoline sales and carbon emission per unit area depended on fractal dimension, albeit weakly. Up to a certain point the effect was minimal, but if the fractal dimension increased beyond that point, the per unit area gasoline sales and carbon emissions increased rapidly. To study generalization of these phenomena, I looked at national economies. The national economic scaling indicator was calculated using income distribution for consecutive 20th percentiles. A very similar correlation between fractal dimension and energy consumption at both city and country scale was observed.

Table 11: Summary of Correlations Studied

Independent	Dependent	Correlate?	Type	Strength
Fractal Dimension	%age of Area Covered by 20% of the Least Densely Populating Habitants	No	N/A	N/A
Total gasoline sale	Total carbon emissions	Yes	Linear	Strong
Population	Total gasoline sales	Yes	Linear	Strong
Population	Total carbon emissions	Yes	Linear	Strong
Population	Area	No	N/A	N/A
Population	Fractal dimension	No	N/A	N/A
Population	%age of Area Covered by 20% of the Least Densely Populating Habitants	No	N/A	N/A
Area	Fractal dimension	No	N/A	N/A
Area	Total gasoline sales	No	N/A	N/A
Area	Total carbon emissions	No	N/A	N/A
Area	%age of Area Covered by 20% of the Least Densely Populating Habitants	No	N/A	N/A
Population density	Total gasoline sales	Yes	Linear	Weak
Population density	Total carbon emissions	Yes	Linear	Weak
Population density	Gasoline sales/capita	No	N/A	N/A
Population density	Carbon emissions/capita	No	N/A	N/A
Population density	Fractal dimension	Yes	Power law	Weak
Population density	%age of Area Covered by 20% of the Least Densely Populating Habitants	No	N/A	N/A
Fractal Dimension	Total gasoline sales	No	N/A	N/A
Fractal Dimension	Total carbon emissions	No	N/A	N/A
Fractal Dimension	Gasoline sales/capita	No	N/A	N/A
Fractal Dimension	Carbon emissions/capita	No	N/A	N/A
%age of Area Covered by 20% of the Least Densely Populating Habitants	Total gasoline sales	No	N/A	N/A
%age of Area Covered by 20% of the Least Densely Populating Habitants	Total carbon emissions	No	N/A	N/A
%age of Area Covered by 20% of the Least Densely Populating Habitants	Gasoline sales/capita	No	N/A	N/A
%age of Area Covered by 20% of the Least Densely Populating Habitants	Carbon emissions/capita	No	N/A	N/A
%age of Area Covered by 20% of the Least Densely Populating Habitants	Gasoline sales/area	No	N/A	N/A
%age of Area Covered by 20% of the Least Densely Populating Habitants	Carbon emissions/area	No	N/A	N/A
Fractal Dimension	Gasoline sales/area	Yes	Power law	Weak

Independent	Dependent	Correlate?	Type	Strength
Fractal Dimension	Carbon emissions/area	Yes	Power law	Weak
National economic scaling indicator	GDP/capita	No	N/A	N/A
GDP/capita	Energy use/capita	Yes	Linear	Weak
National economic scaling indicator	Energy use/capita	Yes	Power law	Weak

Looking back at the research questions one can see that the question concerning the correlation between scaling indicator and energy consumption in cities has been answered in affirmative to some extent with a power law correlation between the two variables. The second question concerning the relation between sustainability and complexity will be addressed as we explore the greater mechanisms through which scaling comes to affect energy consumption in cities. This chapter will focus on two things: a) the mechanisms that may explain some of the findings of this research; and, b) practical policy implications and utility of the findings; and how this work can be incorporated into planning for complex systems like cities.

6.1 The Mechanism

There are two correlations that have been observed in this research which need to be analyzed for presence or absence of causation. The first result is presented in Figure 36 and the second result in presented in Figure 44. At a higher level analysis both figures represent the same finding though for two completely different complex systems. That primary finding is that energy usage (or some measure of it) in complex systems correlates with scaling within the complex system according to a power-law. The steeper the scaling or the greater the disparity in distribution between different segments of the complex system, the higher will be the energy consumption. While power law relationships indicate that there may be a minima beyond which the energy consumption might actually increase with flatter scaling, the empirical data both for cities and economies does not show scaling indicator values approaching or lower than the supposed minima. It should also be noted that the power law correlations are weak with R values of less than 0.5 indicating that the only assertion that can be made with any certainty about this result concerns that trend of change in energy consumption with changing scaling indicator value. Conclusions beyond this may be indicative but not entirely supported by the data.

6.1.1 Scaling Indicator Value and Disparity of Distribution

To understand why the trend exists, I analyze first what scaling means. A higher value of the scaling indicator means that the change in distribution of certain properties across different scales is steeper. For instance for cities, a higher fractal dimension would mean that greater urban area is occupied by lower density housing compared to a city with lower fractal dimension. For a national economy, a higher scaling indicator would mean that a greater percentage of the income has been concentrated in the richer percentiles of the population. Higher scaling indicator values would also mean that for cities, the spread of population density (difference between minimum and maximum) is lesser or the difference between areas covered by minimum and maximum density housing is higher or both, and for economies it would mean that disparity in distribution of incomes (difference between minimum and maximum) is higher. In general, higher scaling indicator values are indicative of higher disparity in distribution of elements across the system, i.e. greater wealth disparity for economies and for cities, anomalous land use pattern with a high percentage of the land occupied by very few people and an excessively large part of the population living in overcrowded conditions.

Now the question is, why does this higher disparity cause higher energy consumption? One obvious answer that one can reject based on available data for cities, is the idea that higher fractal dimension may mean higher percentage of area covered by lower density housing; and since that may result in greater travel distances, the gasoline usage should be higher. The reason we can reject this idea is because the results show that greater fractal dimension does not always mean greater area covered by lower density housing; there is no correlation between fractal dimension and area covered by top 20% of population living in least densely populated areas. Secondly, my results show a correlation between fractal dimension and gasoline sales per unit area; the normalization by area implying that the area covered (and potentially distances to be traversed) is not necessarily a contributing factor.

I cannot reject this hypothesis for national economies though as data for energy consumption by percentiles of population based on income is not available. It may be that higher scaling indicator corresponds to higher energy consumption because the richest 20% end up consuming a disproportionally large percentage of energy per capita. Though based on the results from cities we cannot generalize this mechanism to all complex systems. To explore this issue in further detail I looked at the mathematics of regulation in complex systems.

6.1.2 Regulation and Disparity in Complex Systems

One of the by-products of the revolution in information technology over the last three decades has been our enhanced capacity to visualize, model and understand complex phenomena. This has allowed science to identify and visualize key traits associated with complexity such as self-similarity (Mandelbrot B. 1967) and recursion (Hofstadter D. R. 1979), interconnectedness of elements (Barabási A.-L. *et al.* 1999), high sensitivity to initial conditions (Wolfram S. 2001), and theorize about the sources of these traits (Bettencourt L. M. 2013; Mandelbrot B. B. 1983; West G. B. and Brown J. H. 1997; West G. B. *et al.* 1999; Wissner-Gross A. D. and Freer C. E. 2013) and evolution of complex systems (Chaisson E. 2001). These developments though have not brought us much closer to eliminating widespread skepticism about either our ability to build predictive models of complex phenomena (Taleb N. 2008b) or arrive at feasible mechanisms to describe the emergence and selection of such phenomena associated with complexity as human cognition (Nagel T. 2012), though some of the findings are already being incorporated in systems analysis, design and architecting (Dagli C. H. *et al.* 2009).

Recently however, it was demonstrated that traits associated with the human cognitive niche such as tool use and social cooperation can naturally emerge under the action of causal entropic forces (Wissner-Gross A. D. and Freer C. E. 2013). Here, through a simple model, I demonstrate that even more rudimentary complex phenomena associated with human cognition such as ‘self-awareness’, can naturally emerge in systems in response to ‘internal stimuli’ as these internal stimuli eliminate less ‘self-aware’ systems.

a. Model Construction

To construct the model I start with a system which is a ‘good regulator’ of itself (Conant R. C. and Ross Ashby W. 1970). It has been shown that any good regulator of a system is also a model of the system (Wissner-Gross A. D. and Freer C. E. 2013). So if R is a good regulator of System S, then it is both a) internal to the system and b) a model of the system. Also for every ‘real world’ state the system S assumes, R (being a model of S) assumes a corresponding ‘model’ state. For the purposes of development of this model ‘self awareness’ (to be denoted by Δ) now is defined as the change in internal model R with change in system S.

$$\Delta = \frac{dR}{dS} \qquad \textit{Equation 5}$$

Defined in this manner, self-awareness stops being a binary property but instead can be represented by a continuous bounded function (with values between 0 and 1). Instead of just either having or not having ‘self-awareness’, systems can have varying degrees of self-awareness; self-similarity for instance being one of the cruder forms (lower degree) of self-awareness. Every system can be imagined to have an internal model of itself within it, the question remains only of quantifying the degree of accuracy of that model.

Imagine now that starting from a state S_o, the system goes to a critical state S_c at which the system ceases to exist due to internal stimuli. At state S_o, the internal model of the system is in state R_o. However, the internal model (which is also a good regulator) also has a state R_c at which the system realizes the threat posed by the internal stimuli and adjusts its state before it reaches the critical state S_c. Any system for which the time T_R taken for R to reach R_c is smaller than the time T_S taken for S to reach S_c would have a longer time of existence compared to a system where $T_S<T_R$. This is the survival advantage that systems with higher Δ would have, given all else is equal. So, for a regulator to be good enough to provide survival advantage;

$$T_R<T_S$$

Where;

$$T_S = \frac{S_c - S_o}{\frac{dS}{dt}} \qquad \textit{Equation } 6$$

And

$$T_R = \frac{R_c - R_o}{\frac{dR}{dt}} \qquad \textit{Equation } 7$$

Substituting in Equation 5, for an internal model to be good enough to provide survival advantage;

$$\frac{R_c - R_o}{\frac{dR}{dt}} < \frac{S_c - S_o}{\frac{dS}{dt}} \qquad \textit{Equation } 8$$

Given that dR = ΔdS;

$$\frac{R_c - R_o}{S_c - S_o} < \Delta \qquad \textit{Equation } 9$$

The probability of condition specified in equation 9 being true increases with increasing Δ (where Δ is some function of the internal state variable/s of S with a range between 0 and 1) or 'self-awareness'. What this results seems to imply is that not only is a good regulator one which is a model of the system being regulated, but the better this internal model of the system is -or the higher the self-awareness of the system- the more probable it is to survive (in response to internal threats to its existence).

The description and details of the model that follows concern mostly with understanding of regulation within disparate systems. This theoretical work may not seem immediately relevant to the results obtained from cities but the relevance of the findings is explained at a later stage.

A simple numerical model consisting of a universe with hundred systems of varying self-awareness was built to further demonstrate how this mechanism naturally selects for systems with higher self-awareness. A binary property ρ to be called 'agency' was also introduced in the model. When R equaled R_c for any system, the system readjusted only if ρ equaled 1. Overtime, I expected to see more systems with the agency switch 'on' ($\rho = 1$) survive as opposed to those where ρ was equal to 0. The magnitude of the readjustment depended upon the 'plasticity' of the system. Plasticity was defined as the deformation in S, per unit of available energy E, normalized to the initial value of S. Plasticity, denoted by ϵ can be expressed as;

$$\epsilon = \frac{dS}{ES} \quad \text{Equation 10}$$

Further, R_c depended on how quickly the system was able to identify the need for a readjustment. This property was termed 'agility'; defined as the difference between the system critical value (S_c) and internal model critical value (R_c), normalized to the system critical value S_c. Agility, denoted by τ can be expressed as;

$$\tau = \frac{(S_c - R_c)}{S_c} \quad \text{Equation 11}$$

Four parameters are monitored across the set of 'living' systems as the universe evolved and some systems were eliminated due to S having reached critical value S_c; i) the average self-awareness Δ_{ave}; ii) ratio of number of systems with 0 agency against number of systems with agency equal to 1, ρ_R; iii) average agility τ_{ave} and iv) average plasticity ϵ_{ave}. Model details are described below.

1. I start by building a universe consisting of a hundred systems. The initial R values are normally distributed with a mean of 50 and standard deviation of 1.75. The difference between initial R and S values is normally distributed with a mean of 0.5 and standard deviation of 0.0346. Normally distributed Δ are assigned to the systems with mean of 0.5 and standard deviation of 0.178. Normally distributed values of τ, ϵ and E are also assigned to the systems.
2. Given the mechanism proposed, what f(R) actually is should not have an impact on the results of the experiment. For the purposes of this analysis R is taken to increase exponentially with an exponent of 0.017. S_c is set at 700.

$$f(R) = R\ e^{0.017}$$

3. For consequent time-steps R is calculated using equation 11.
4. S_2 is calculated using equation 7 for consequent time-steps.
5. The model is run for 1000 time-steps.

b. Model goodness of fit, complexity and disparity

One immediately observable fact was that all these properties across the universe evolved in bursts (spasmodically) in a manner reminiscent of scale-free networks (Barabási A.-L. *et al.* 1999).

Average self-awareness for the set of living systems was indeed seen to increase with elimination of less self-aware systems, though it was observed that the maximum attainable self-awareness for any system was limited by the product of self-awareness, plasticity and energy for that system typology. I term this product the adaptive capacity. Figure 47 shows the elimination process at four time steps during the model run of the universe with hundred random systems. Bubbles with dotted fill are systems with agency (ρ) = 0, while bubbles with solid fill are systems with agency (ρ) = 1. Bubble size indicates value of one system state variable X. Size of the dotted outlined bubble inside bigger bubbles indicates internal model value x for the same variable X in the internal model R. As can be seen in d at time-step 163, the surviving systems are ones with very high self-awareness (dotted outline is closest to solid outline).

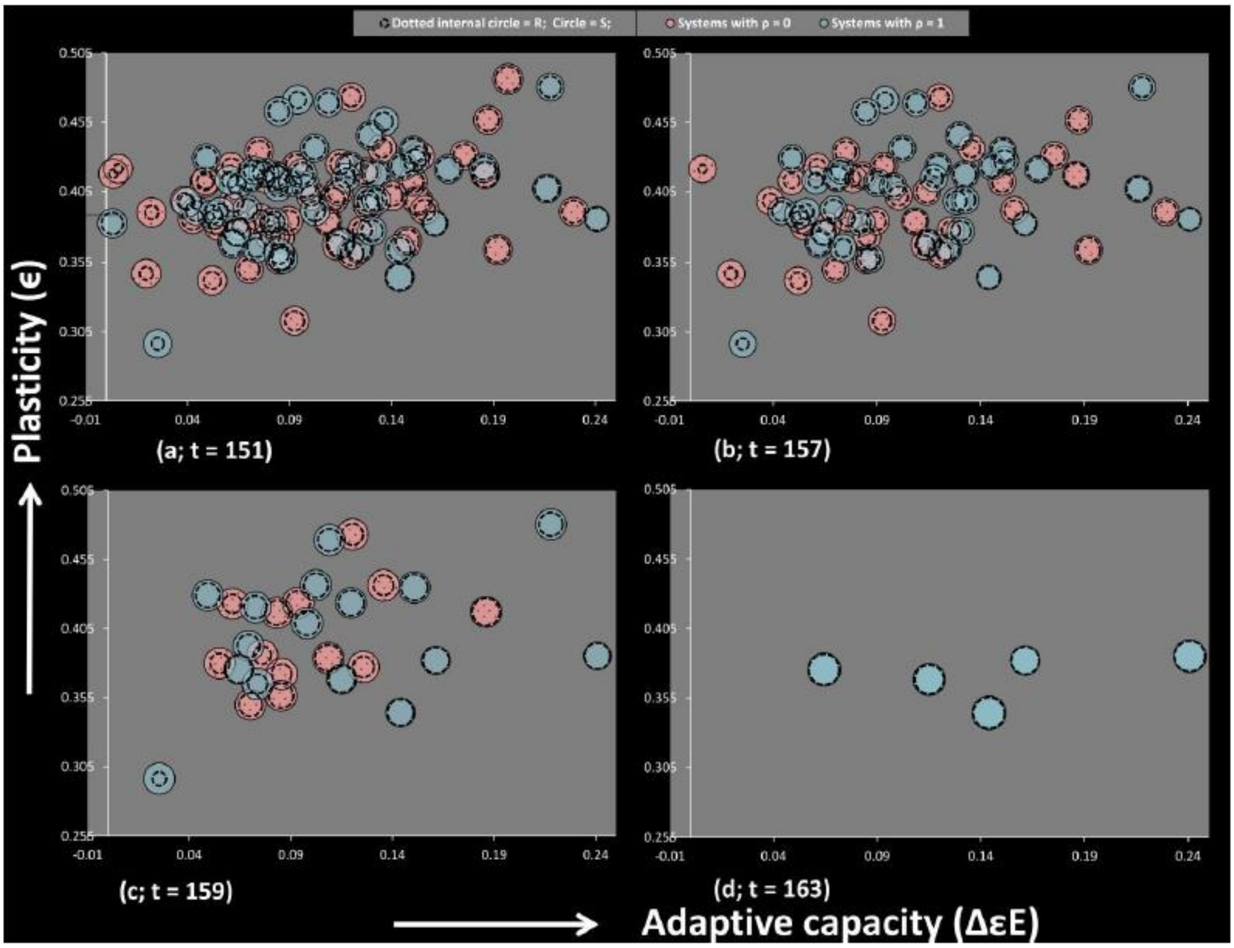


Figure 47: Systems with lower adaptive capacity (ΔεE) die-off under adaptive selection as universe evolves over time-steps a) 151, b) 157, c) 159, d) 163

The model set up with a hundred systems was run for a thousand time-steps. Figure 48 shows how the monitored properties evolved over time for the universe of living systems with average self-awareness and agility increasing and ratio of positive agency over null agency systems decreasing as expected, and the average plasticity decreasing. The rise in plasticity is somewhat surprising. One should expect that the more plastic a system is, the more adaptable it should be, and hence the more resilient. What we see instead is that the systems that survive are the ones with lower plasticity.

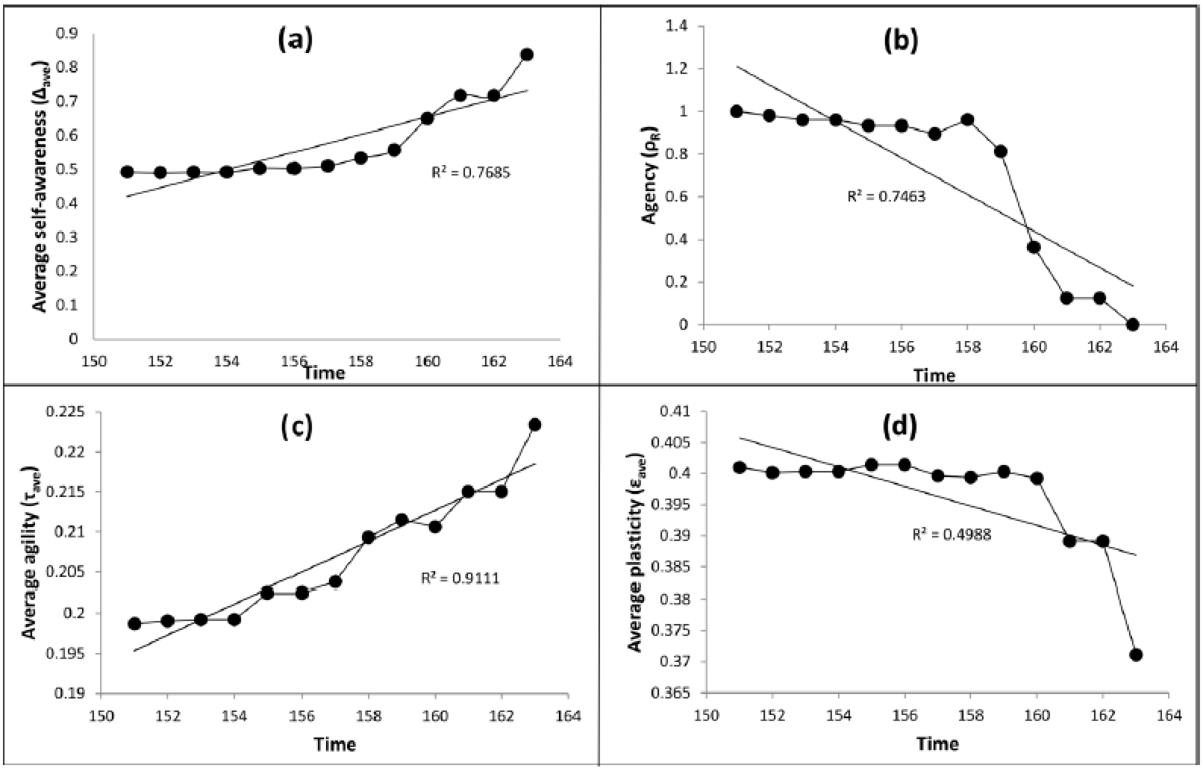


Figure 48: Average Self-Awareness of the Set of Living Systems Increases Over Time; b) Non-reactive Systems Die-off as the Ratio of Non-reactive to Reactive systems Decreases Over Time; c) Average Agility of the Set of Living Systems Increases Over Time; d) Average Plasticity of the Set of Living Systems Decreases Over Time

However, from equations 5 and 10 I deduce that the change in model normalized to the original system state is equal to the product of self-awareness, plasticity and energy availability.

$$\frac{dR}{S} = \Delta\epsilon E \qquad Equation\ 12$$

From Equation 12 one can see that plasticity (ϵ) and self-awareness (Δ) are inversely related. Upon consideration this result does appear to make intuitive sense. Plasticity is a measure of how much change R can incur in S, while self-awareness is a measure of how R changes with changes in S. For any given system, the internal model can be made of either energy or matter, however in most cases, the internal model substitutes information for what is material in a system; actual quantities are replaced by say, a number representing that quantity. A state variable in the internal model say R though is more likely to either be 'information' or energy, while S, the corresponding system state variable, can be expected to have more of a material component. Imagine for instance a refrigerator, say S to a model of the refrigerator as it exists in your mind, say R. The former has a lot more material content compared to the latter. Self-awareness thus can be conceptualized as the amount of change incurred in informational content with change in real world material counterpart. Plasticity then is a measure of how that change in information comes back and affects a change in its real world material counterpart. This loop –system affecting model affecting system- is the essence of sentience and consciousness. The term $\Delta\epsilon E$ arrived at in Equation 12 defines the upper bounds for this property for any given system. For any given system 'typology' (all systems with the same plasticity and energy availability), the product ϵE determines the upper bounds of adaptive capacity.

This model demonstrates not only how systems naturally tend towards greater self-awareness but also how the potential for self-awareness is restricted by the plasticity of the system and the energy availability. For any given typology (here defined by the product of plasticity and energy) thus, we will see more self-aware systems survive over longer runs, but no system can rise above the limitations imposed upon it by its typology. For planetary systems for instance, the energy available as electromagnetic forces is very weak as electromagnetic forces are weak at that scale. Energy available as gravitational force, though stronger is still comparatively weaker in terms of its ability to cause strain in the system (hence lower plasticity). This means that $\Delta\epsilon E$ has a low value compared to organic systems where electromagnetic forces act on organic matter (much more malleable hence susceptible to higher strain and having higher plasticity). Since both ϵ and E are quantifiable terms, establishing indicative values of ϵE for different system typologies should be trivial. It could be easy to show why the organic brain with its high material malleability and energy availability offers such a generous nursery for the rise of self-awareness.

Now, we can also try to imagine what happens when the scaling of distribution of sizes within a system becomes more disparate (e.g., high income inequality in economies). As disparity increases, the segment of the system that is meant to act as a regulator, becomes less and less of an accurate model of the system, unless the regulator size (as percentage of system) and energy cost of regulation is increased proportionally. The intuitive conjecture that can be drawn here is that a more disparate system needs higher energy to be effectively regulated.

6.1.3 Disparity affecting Transport Governance and Energy Usage in Cities

So now that I have shown how in complex systems in general, disparity requires system regulation to be more energy intensive by requiring the construction of a system model that is more energy intensive, I am going to discuss this result in the light of my findings for cities.

Regulation from a transportation point of view may be imagined as establishing a network of transport systems that can effectively allow residents to traverse the city. Such a transport network thus would be a network 'model' of the complex system that is the city (albeit a very simplified one); the model that would allow for regulation of the city, and whose goodness determines the effectiveness of the regulation (or the effectiveness of the ability of residents to travel around the city). Great disparity in population density would require provision of different types or modes of transport. A largely 'urban' city (with more mixed use medium to high density living areas) can be connected with public transport alone. A city that also has suburbs and high rises would require provision of highways and roads for suburban traffic; traffic which cannot be catered by public transport as low population density would mean higher operational costs. From a transport system design point one can already imagine that the more disparate the land use in the city, the more complicated and energy intensive the transport network would have to become. No single solution would allow for addressing the regulation problem (effective transportation) on its own, doing away with the benefits of economies of scale for one thing. Similarly, greater variation just for one type of transportation i.e. personal vehicle would mean decentralization of residential quarters into separate suburbs, an arrangement that would require greater travel distances and thus greater fuel usage. This mechanism illustrates that higher scaling indicator values do in fact affect gasoline usage in cities.

6.1.4 Disparity affecting Regulation and Energy Consumption in National Economies

In this case regulation would mean provision of energy to the citizens. Greater disparity in incomes would have two effects. One, it would disproportionately increase the share of energy consumed by the higher percentiles. Secondly, it would make the task of regulation, i.e. efficiently providing energy in different forms to all citizens that much more complicated. At a national level, the higher energy cost of regulating a more disparate system shows up in the cost of politics. It becomes more difficult to have a regulatory body that is also a good 'model' of the system, thereby increasing the energy cost of regulation.

In the long run, if systems continue to become more disparate, the regulating models start to become either more energy consuming or 'bad' models. Both scenarios open the doors to potential system collapse. In transportation in cities this manifests in the form of urban sprawl and eventually the rise of phenomena such as 'food deserts'. In nations, failure of regulation is both, a political and eventually a social failure and may manifest in the form of political and social strife or conflict.

6.2 Significance and Applications

Climate change and peak-oil are transformations that not only threaten to impose a lean energy diet upon systems of human civilization such as cities (The Royal Swedish Academy of Sciences 2011), but increase the potential of low probability, high impact shocks through increased frequency of, for instance, extreme weather events (IPCC 2007) and contagions in market systems caused by acute, localized energy and other resources shortages (Taleb N. 2008b). Preparing for climate change and peak oil should thus include reorienting cities to consume less energy and be resilient to unpredictable Black Swan events. Eco-systems and many other incrementally growing complex adaptive systems naturally favor these properties in their development. Such systems also exhibit very specific scaling characteristics, captured partially in the measure of their fractal dimensions (Hern W. M. 2008; Mandelbrot B. B. 1983; Salingaros N. A. and West B. J. 1999; West G. B. and Brown J. H. 1997, 2004). Fractal dimension can thus serve as a useful indicator to guide development of complex adaptive systems such as cities so that they grow to favor low energy consumption and high resilience to Black Swan events –essentially by being better regulators and facilitating better connectedness between regulator and the system. I partially demonstrate that here by showing that fractal dimension has a strong relationship to one measure of energy consumption, namely gasoline usage.

6.2.1 A Complexity Based Index for a Complex System

As we begin to understand the nature of complex-adaptive systems, the calls for trying to avoid over-simplification in the study of systems such as cities, continue to pour in. In industrial risk assessments consideration of holistic, systemic risks has gained significance since Three Mile Island (Perrow C. 1984). The post-normal science literature has long been calling for new analysis techniques that take into account risks emerging from system complexity (Funtowicz S. O. and Ravetz J. R. 1994), and post 2008, even in the financial world, the impact of Black Swans is studied and considered with interest (Taleb N. 2008b). While recognizing the fallibility of linear analysis in trying to model and predict the behavior of complex systems, these branches of scientific inquiry have yet to propose rigorous methodology for formalizing complex systems analysis within their disciplines. Fractal dimension analysis proposed here, aims to lay the groundwork for providing that rigor in at least one aspect of the area of urban sustainable development planning.

Weighted average aggregation of indicators to arrive at indices is a process that uses linear mathematics to study non-linear systems. Since fractal dimension is a mathematical property of the system, exploring its relationship to directly measureable indicators such as gasoline consumption and formalizing and documenting these linkages, can transform fractal dimension into a composite indicator that provides for much more intuitive observations and commentary on the sustainability of the system. It should be noted though that many concepts associated with this discussion are unfamiliar to a general audience. For wider application among policy-makers, capacity building may be needed.

6.2.2 Policy Applications

Fractal dimension as a systemic indicator of urban sustainability can influence planning in at least the following three manners.

a. Disaster Risk Resilience

Tackling the increased intensity and frequency of extreme weather events will require designing cities which are resilient to Black Swan events. In such cities, infrastructure, utility, health and other urban facilities will be distributed in a manner which is most convex to Black Swans and thus anti-fragile (Taleb N. 2008a). Fractal dimension of the distribution of such urban services can provide a measure of anti-fragility. The calculation of fractal dimension can be incorporated during strategic planning phases.

b. Sustainable Neighbourhood Design

While designing cities for sustainability, development projects can be assessed based on how they impact the fractal dimension of the city. For this purpose, pre-project and post-project fractal dimensions can be calculated.

Another manner of implementing this method is through inclusion of a fractal dimension-based requirement in sustainable neighborhood certification standards such as LEED-Neighborhood (USGBC 2007). Certifications can be used as an incentive for urban planners and developers to make their cities more sustainable.

c. An Alternative Development Investment Paradigm

By providing a resilience and sustainability based justification for strategic urban planning, the fractal dimension as an indicator system can evolve into an alternate paradigm for infrastructural and urban development investment that complements the otherwise purely economic considerations that usually underpin such planning processes.

The use of fractal dimension as an indicator of city structures can provide an objective higher-level matrix to assess the structural patterns that may be indicative of the overall system sustainability. Such higher level matrices can, through research, be linked to a number of direct measures such as per capita gasoline sales to establish how the matrix; for instance fractal dimension, is an indicator of what's happening at the level of individual variables within the city. Once such relationships are established the higher-level matrices can be used to provide guidance on policy decision-making. In this way I feel, the fractal dimension of the distribution of sizes of various parameters within the city can be a significant higher-level matrix guiding policy for sustainable urban development.

Direct policy implications of this research can include, among others, an improvement of sustainability indicator and standards systems such as LEED-Neighborhood (USGBC 2007). However, as discussed in detail earlier, I see this research as part of a greater ongoing, multidisciplinary, multi-sector effort to develop a science of sustainability which caters to people making decisions in areas of low certainty and high stakes, such as climate policy makers and urban planners. The fractal dimension as calculated using a highly replicable and standardized methodology here has the potential to provide an alternate paradigm for infrastructural and urban development investment that favors resilience and sustainability as opposed to economic maximization only. Such a systemic indicator is needed because we are dealing with complex systems here and direct indicators cannot always be a good predictor of system behavior.

d. Planning Plane Application Case Study

To visualize application imagine that the city of St. George, Utah receives an application for a new housing development. To incorporate fractal dimension in the analysis the planner would input the new housing development plan in his map of the city and calculate the new fractal dimension and population density for it. Using these two variables she will then plot how the energy use in the city will be affected by the new housing development overall, by plotting the new fractal dimension and population density values on the planning plane and comparing with previous results as shown in Figure 49. If there is a potential increase in energy use expected, she can factor this result in the decision for letting the housing development proceed as proposed or develop alternatives.

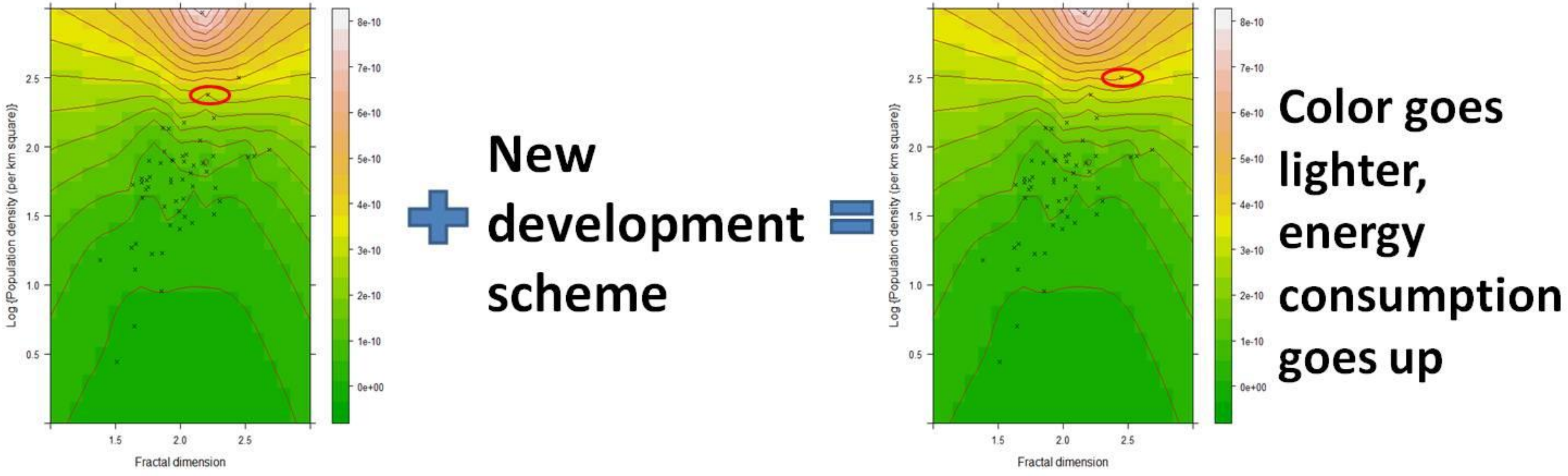


Figure 49: Planning Plane Application

e. *National Level Planning*

At national levels, this work can segue into policy development for urban and rural development. National population distribution and land use patterns can be identified and the scaling of city sizes in countries can be correlated with energy consumption in the country, economic performance or other indicators. This can allow for countries to incorporate complexity and diversity as a consideration in their development planning programs, moving away from the focus on 'growth poles' and addressing some of the criticisms of that development framework, including inequity of benefit distribution.

f. *Implications for Theory and Practice of Sustainable Indicator Systems Development*

It needs to be acknowledged that the work presented here contributes to the understanding complex systems in a manner that will require much more additional research. The research will have to come from both empirical studies of big data collected on complex anthropogenic systems as well as from the mathematics of complexity. The work presented here demonstrates the potential for use of these ideas in the planning process and the utility in terms of both preparing for and reducing the risk of high impact low probability effects. In essence what has been discussed here can be seen as an methodology to rigorously analyze an important, but neglected aspect of the sustainability of complex systems. This aspect can be included in concerns for planning and design of systems through development of new indicators which look at scaling within the systems as well as through establishing the veracity of these indicators as composite indices.

Scaling based indicators can contribute to the science of indicator development for complex systems. Typically index development usually involves weighted summation of different indicators or parameters which are selected based on specific criteria and represent key dimensions of a complex system or phenomenon. If it can be shown that complexity based scaling indicators correlate with a wide range of parameters, then the scaling indicator can be used as an aggregate index (without the need for aggregation).

The other implication is the consideration for scaling of certain parameters in analysis besides just averages. Many planning processes are concerned primarily with indicator averages which do not provide any assessment of how the indicator changes in distribution over the population. This is of course not a comment on systems where design process considers some measure of peak parameter value for analysis (e.g. water treatment plant designs). Such scaling indicators can be calculated for any parameters which are averaged over a large population or sample and should be used in conjunction with average values to temper or put in context the findings from average values.

g. Analysis for the Identification of Historical Patterns

There is some potential for the use of scaling based indicators or indices in ex-post anthropological and historical studies to identify patterns of growth and development that lead to societal collapse. However application in such scenarios continues limited by the accuracy and extent of data availability. Any such results would need to be seen in the context of the accuracy and extent of data available.

h. Implications for Urban, Social and Educational Theory

In urban theory this research could point out what kind of development process leads towards more sustainable cities. There are two opposing strains of thought already emerging in this area of analysis (Romero-Lankao P. and Dodman D. 2011). On the one hand efficiency is seen to be optimized by maximizing the scale of development unit and centralizing governance and decision making. On the other hand resilience seems to be optimized by scaling down and localizing many of the processes of modern civic life. The hope is that this kind of analysis could provide in the future, more credible numerical and quantitative arguments to support the resilience based paradigm.

In social dynamics the work is an attempt at pointing out and identifying tipping points where small changes can affect large outcomes. These outcomes can be both positive and negative and by considering non-linearity the planning process could become cognizant of the potential for negative outcomes and be on-guard before such tipping points, while positioning the system to benefit from any positive tipping points or Black Swans. The underlying theme of this work is to identify ways in which quality of life can be improved without increasing the entropic footprint of our civilization.

Some of the ideas discussed earlier in this thesis regarding the significance of post normal science in dealing with complex systems such as societies and cities continue to be open questions for philosophical debate (Turnpenny J. *et al.* 2011). While complex systems may not be explored the way physics explores mechanical systems, certain elements may still be able to be isolated. What is important is to remember that in complex systems for every significant correlation modeled there are probably at least ten others which may not have been modeled.

i. What Kind of System is a City?

What do these results tell us about what kind of system a city is? Is it like a star or a brain or a black hole or a social system with competing dynamics, or merely an energy consumption engine? How much is it like other complex systems and how and where does it differ? One thing that these results suggest is the friction between the bottom up emergence of the city and the top down intervention from higher-level e.g., national policies. The city is a system that is borne out of the interaction between these two (besides other stimuli). From the bottom up is the extraordinary complexity of the city, the awe inspiring dynamics of having real human beings as agents constructing a system bit by bit; the top down interventions are often an attempt to rationalize some of the complexities emerging from bottom up development (Barca F. *et al.* 2012; Hall P. 2014). The hope is that this work will show the optimal point at which these two dynamics need to strike a bargain. There's a growing feeling in the domain of urban planning that there is too much top down intervention for the city to be a resilient or sustainable system (Barber B. R. 2013). This kind of analysis may, by pointing at a lower scale of development as optimum, provide a quantitative justification for decentralizing the governance of cities.

6.3 Limitations and Constraints

Although cognizant of the complex nature of urban systems, fractal dimension based indicator systems share most of the limitations of any other indicator systems. An indicator by nature is a compromise between science-based understanding of complex phenomena and the need for distilling complex information into bite-size summary form that is necessary in most decision making processes. The important thing to realize is that the structural development that is necessary to conceive indicators is in essence a tiered compilation of assumptions, choices and informed decisions. Starting from the fundamental paradigm that dictates the creation of the indicator right down to an eventual number, the structures that inform decision making are held together by the scaffolding of conditionality and exceptions (Meadows D. 1998). These must never be forgotten while putting an indicator to use in the service of decision making.

6.3.1 Limitations

Fractal based indicators can have the following limitations.

a. Lack of Universality

There is some evidence to suggest that any fractal indicator based optimums identified using analysis of cities may only be applicable to cities within a country and may not carryover well to other countries (Bettencourt L. M. A. *et al.* 2010). While the general principles hold, the exponents identified may differ from country to country. This suggests that urban development, although having some universal characteristics the world over is also dependent to a large extent on external policies such as national policies that may have no relation to urban structure or topography at all. While trying to interpret relationships expressed by fractal analysis, such limitations should be kept in mind.

b. Extensive Data Requirement

While the data required for implementation of this indicator is usually available in most regions of the world in the form of city maps or census data, in case additional temporal resolution is needed than the usual census time step of a decade, this can become an expensive indicator to calculate.

c. Use of Complexity Mathematics

One of the necessary requirements for an effective indicator is that it should be easy to understand so as to be able to create 'buy-in' amongst a wide stakeholder base. While the fundamentals of this indicator system are not necessarily complicated, any wide scale adoption of the indicator would have to be coupled with informational media campaigns to communicate some of the ideas behind the indicator system within the public sphere. This is largely due to the fact that many of the concepts behind this indicator system would be new for the general public even in some of the cities with highest education rates in the world. Adoption of new indicators is often as much a politics and policy issue as it is an issue of scientific debate and if the primary stakeholders, i.e. the city dwellers don't have a general understanding and acceptance of the ideas behind important decision influencing indicators, that can render their adoption uncertain and vulnerable to political criticisms and dismantling.

6.3.2 Limits to Optimization based on One Consideration Alone

In light of the above limitations, the following minimum steps should be taken along with the adoption of the fractal dimension based indicator.

a. Use of Other Metrics

It is not sufficient to use fractal dimension in relation to one metric such as gasoline consumption, rather the relationship of fractal dimension to other urban sustainability metrics especially at least one economic metric such as Gross Metropolitan Product and one environmental metrics such as CO_2 emissions, should be explored. Fractal dimension as a structural complexity indicator is related to all urban sustainability issues where spatial scale matters; it adds a fundamentally new element to metric development. As such, it should be developed as a stand-alone index to stand above and complement other structural indicators. This should involve establishing fractal dimension optima for multiple environmental and sustainability indicators by exploring the statistical linkages between fractal dimension and these indicators. Fractal dimension based indices are best utilized as alternative aggregate indicators to be used in conjunction with other indicators.

b. Scenario Modeling

For most decision makers and stakeholders a number may not always carry information of actionable significance. Using fuzzy modeling techniques, scenario sets should be developed to visually demonstrate what cities with specific fractal properties could look like and how changing fractal dimension would change the city and life in the city, including various urban sustainability attributes. These scenarios should be based on the results of the Phase I study and should model resulting cities based on how fractal dimensions affect city structure, topography and sense of place. These should be not just mathematical models documenting how fractal dimension affects say gasoline consumption but visual models with strong architectural input that demonstrate how cities within certain fractal dimension range look like. The objective is to put a picture on specific numbers and develop a narrative which is more compelling than mere numbers.

c. Media and Educational Campaign

Extensive media campaign should be conducted during pilot studies both at the city and national level to inform stakeholders and city dwellers about the ideas underlying the fractal dimension based indicator systems. These should include press releases and publication of online brochures, development of online presentations and production of at least one documentary film for TV and DVD release. The educational and media campaign is essential to introduce the ideas underlying the indicator development in the public sphere, where these would be essentially be considered novel.

d. Stakeholder Consultation

In the end it should be realized that an indicator is just a number and only as good as the general knowledge of the decision maker about the background, science and information content of that number (Pintér L. *et al.* 2012). This is the reason why any decision making based on indicators should be complemented by a comprehensive multi-tiered stakeholder consultation process. This not only facilitates creation of buy-in but brings in information to the table which may have been ignored by the limited process that leads to indicator development. It should be ensured that a vast majority of the stakeholders concerned and consulted are as well informed about the indicator and the information it conveys as possible. The impact of indicator variability should be explained to stakeholder groups and tiers using well visualized scenarios.

Even while evolving our indicator systems to reflect some of the complexity of systems such as cities, one must not forget that the primary lesson to be derived from complexity science is not one of additional trust in numbers, but less trust, no matter how rigorous the derivations of those numbers are. As we begin to understand the nature of complex adaptive systems, the calls for trying to avoid over-simplification in the study of such systems, continue to pour in. In industrial risk assessments consideration of holistic, systemic risks has gained significance since Three Mile Island (Perrow C. 1984). The post-normal science literature has long been calling for new analysis techniques that take into account risks emerging from system complexity (Funtowicz S. O. and Ravetz J. R. 1994), and post 2008, even in the financial world, the impact of Black Swans is studied and considered with interest (Taleb N. 2008b).

It must be realized that even focusing more on indicators as compared to comprehensive models with assured predictabilities is a new way of scientific decision making, one in which administrators and policy makers must see themselves as permaculture gardeners, tending to a complex system they must let evolve organically with as little interruption as possible. Even when interruption is necessary, it should be realized that the only way to handle decision making in the face of daunting complexity is to attempt to understand complex systems with humility and with due respect for all forms of knowledge, scientific, meta-scientific, numeric, fuzzy, narrative and expert.

7. Summary and Conclusions

This research started off with two primary questions; what if any, is the correlation between a complexity based indicator and direct and well established energy consumption and other environmental pressure indicators? And what if anything, does this correlation tell us about the relationship between resilience/sustainability and complexity? There is evidence from the research that the answer to both questions is in affirmative; that there is a correlation between complexity based indicator and direct and well established energy consumption and other environmental pressure indicators. And that this correlation tells us that holistic analyses of resilience/sustainability should take into account system complexity quantitatively.

To quantify urban complexity, a scaling indicator was developed based on the fractal dimension, and calculated using large block wise census dataset for US cities. To explore the relationship with energy and environment indicators, the relationship between this scaling indicator and gasoline sales per unit area was studied (among many other correlations between a number of other variables). A weak power law correlation (but a strong trend) was observed between the fractal dimension based scaling indicator and gasoline sales per unit area in cities.

In order to be able to generalize the findings for all anthropogenic complex systems, I decided to also do a similar analysis for national economies. Based on data for income distribution between consecutive 20^{th} percentiles of populations, the scaling indicator was calculated for national economies. This scaling indicator had a correlation with energy usage per capita in the countries, very similar to the correlation between fractal dimension in cities and gasoline sales per unit area; a weak power law with r-squared value between 0.35 and 0.4. This similarity suggested that there was some underlying mechanism at work in these complex systems that resulted in complexity (measured as a scaling indicator) affecting energy consumption.

To understand the mechanism through which complexity affects energy consumption in complex systems, I looked at the process of regulation in systems and how complexity affected that. Building on existing results that showed that a good regulator of a system must also be a model of the system, I showed that the better this model was, the more effective the function of regulations. Greater disparity within the system, or steeper scaling increased the energy cost of 'goodness' of model, thereby making the function of regulation costlier in energy terms. In cities, this phenomenon manifested itself in the shape of the complications and energy costs associated with providing effective transportation for a city with disparate land use patterns. In national economies, the energy consumption increased with the increase in disparity, and the representativeness or fitness of the model (regulator) decreased, thereby needing for increased input of energy in various forms. Further research may be needed in this area to explore these concepts.

As a corollary this has consequences for aspects of sustainability concerned with system resilience. Study, analysis, and planning for development of complex systems such as cities and economies, as a scientific and practical disciplines, has been affected by the specter of low probability, high impact events sometimes referred to as Black Swans. In economics specifically the impact of black swans –events which cannot be predicted by governing models- has been significant. In development sciences the focus on such events continues to strengthen with every financial crisis and Fukushima. The primary challenge in climate change adaptation continues to be preparing communities for higher intensity, more frequent extreme weather events (essentially black swans). The question is, how can systems be developed to be optimized to exhibit resilience to these black swan events and how can absence of resilience be identified ex-ante.

From this research I have learned that greater disparity or steepness in scaling in complex systems causes for system models to consume more energy in order to be more reflective of reality and to aid in the function of regulation. Black swans are essentially model failures; emerging from the inability of the model to identify some significant causal relationship or variable. By making it more energy expensive to allow development of effective and accurate models (and therefore regulators), greater disparity in complex systems make model failures more likely and hence the systems more prone to black swan events. In this manner scaling affects resilience and sustainability.

Though it is recognized that some of the ideas pursued in this work may be classified as “blue sky”, particularly the proposed generalization of the findings from urban systems to complex systems in general, there is also strong indication that the methodologies and findings discussed can have direct short-term policy implications. Nevertheless the research has been conceived and executed with the understanding that this work forms only part of a bigger research program into complexity, the relationship between complexity and resilience and the question of decision making in the face of complexity.

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Appendix 1: VB Script for k-means clustering

```
Sub centercalculator2()

'declarations
Dim numlocaltries, numiterations, i, j, k, l, n, centerindex(), bestindex, classsize(), tempcenterindex, index, datasize, numclasses As Long
Dim bestnewsumclosestdistsq, bestnewclosestdistsq, datasorter(3), center(), sumdifferror, sumdifferrorave, errorfinal, sumclasserror, data(), classerror(), tempcenter, sumpop(), sumarea(), popdensity(), logpopdensity(), logarea() As Double
Dim closestdistsq(), sumclosestdistsq, newsumclosestdistsq, classsum(), gaps(), tempgap, newclosestdistsq, randsum, difference(), difsorter(2) As Double
Dim bounds(), boundsk(), tempdist, boundeddata(), boundeddatak(), classassignmentk(), finaldifsq, finaldifsum, classassignmentl(), distsq() As Double
Dim timestart, timeend, time1, time2 As Single



'Reading parameter values

datasize = Range("D2").Value
numclasses = Range("D4").Value
numiterations = Range("D6").Value
numlocaltries = Range("D10").Value

'Declaring array sizes
ReDim center(numclasses), data(datasize, 3), gaps(numclasses + 1), classerror(numclasses), logpopdensity(numclasses), difference(datasize, 2), logarea(numclasses), closestdistsq(datasize), classsum(numclasses), sumpop(numclasses), sumarea(numclasses), popdensity(numclasses)
ReDim centerindex(numclasses), bestindex(datasize), classsize(numclasses)
ReDim distsq(datasize, numclasses), boundsk(datasize), boundeddatak(numclasses, datasize), classassignmentl(datasize), classassignmentk(datasize)

'Clearing previous results
Range("E2:K33000").ClearContents

'Reading data
For i = 0 To 2
   k = i + 1
   For j = 1 To datasize
       data(j, k) = Range("A1").Offset(j, i).Value
   Next j
Next i

'Sorting data
For j = 1 To (datasize - 1)
  For i = j + 1 To datasize
      If data(i, 1) < data(j, 1) Then
          datasorter(1) = data(j, 1)
          datasorter(2) = data(j, 2)
          datasorter(3) = data(j, 3)
          data(j, 1) = data(i, 1)
```

```
            data(j, 2) = data(i, 2)
            data(j, 3) = data(i, 3)
            data(i, 1) = datasorter(1)
            data(i, 2) = datasorter(2)
            data(i, 3) = datasorter(3)
        End If
    Next i
Next j

timestart = Timer

'creating a matrix of data differences
For i = 1 To (datasize - 1)
    difference(i, 1) = i
    difference(i, 2) = data((i + 1), 1) - data(i, 1)
Next i

'sorting difference matrix
difference(datasize, 2) = 0
For j = 1 To (datasize - 1)
   For i = j + 1 To datasize
        If difference(i, 2) > difference(j, 2) Then
            difsorter(2) = difference(j, 2)
            difsorter(1) = difference(j, 1)
            difference(j, 1) = difference(i, 1)
            difference(j, 2) = difference(i, 2)
            difference(i, 1) = difsorter(1)
            difference(i, 2) = difsorter(2)
        End If
    Next i
Next j
gaps(1) = 0

'picking index of highest gaps
For i = 2 To numclasses
    gaps(i) = difference((i - 1), 1)
Next i
gaps(numclasses + 1) = datasize

'sorting indexes of highest gaps
For j = 1 To (numclasses - 1)
    For i = j + 1 To numclasses
        If gaps(i) < gaps(j) Then
            tempgap = gaps(j)
            gaps(j) = gaps(i)
            gaps(i) = tempgap
        End If
    Next i
Next j

'Assigning data to classes
k = 1
For i = 1 To numclasses
    For j = (gaps(i) + 1) To gaps(i + 1)
        classassignmentk(k) = i
        k = k + 1
```

```
    Next j
Next i

'calculating class size and class sum
For j = 1 To numclasses
    classsize(j) = 0
    classsum(j) = 0
    For i = 1 To datasize
        If classassignmentk(i) = j Then
            classsize(j) = classsize(j) + 1
            classsum(j) = classsum(j) + data(i, 1)
        End If
    Next i
Next j

'calculating initial centers
For j = 1 To numclasses
    If classsize(j) = 0 Then
        center(j) = 0
        Else
        center(j) = classsum(j) / classsize(j)
    End If
Next j

'sorting initial centers
For j = 1 To (numclasses - 1)
    For i = j + 1 To numclasses
        If center(i) < center(j) Then
            tempcenter = center(j)
            center(j) = center(i)
            center(i) = tempcenter
        End If
    Next i
Next j
timeend = Timer

time1 = Format(timeend - timestart, "Fixed")
'Prints initial centers
For j = 1 To numclasses
   Range("E1").Offset(j, 0).Value = center(j)
Next j

Range("D20").Value = time1


'clear number of iterations cell
Range("D12").ClearContents

timestart = Timer
'run kmeans
For n = 1 To numiterations

    'recalculate centers
    For j = 1 To numclasses
        classsize(j) = 0
        classsum(j) = 0
```

```
        For i = 1 To datasize
            If classassignmentk(i) = j Then
                classsize(j) = classsize(j) + 1
                classsum(j) = classsum(j) + data(i, 1)
            End If
        Next i
    Next j
    For j = 1 To numclasses
        If classsize(j) = 0 Then
            center(j) = 0
            Else
            center(j) = classsum(j) / classsize(j)
        End If
    Next j
    'sort centers
    For j = 1 To (numclasses - 1)
        For i = j + 1 To numclasses
            If center(i) < center(j) Then
                tempcenter = center(j)
                center(j) = center(i)
                center(i) = tempcenter
            End If
        Next i
    Next j

    'back up old class assignment
    classassignmentl = classassignmentk

    'reassign classes
    For j = 1 To datasize
        bestnewclosestdistsq = -1
        newclosestdistsq = 0
        For l = 1 To numclasses
            newclosestdistsq = (data(j, 1) - center(l)) * (data(j, 1) - center(l))
            If (bestnewclosestdistsq < 0 Or newclosestdistsq < bestnewclosestdistsq) Then
                bestnewclosestdistsq = newclosestdistsq
                bestindex = l
            End If
        Next l
        classassignmentk(j) = bestindex
    Next j

    'check if kmeans has converged ie compare last two class assignments
    finaldifsum = 0
    For i = 1 To datasize
        finaldifsq = (classassignmentl(i) - classassignmentk(i)) * (classassignmentl(i) - classassignmentk(i))
        finaldifsum = finaldifsum + finaldifsq
    Next i

    If finaldifsum = 0 Then
        Range("D12").Value = n
        n = numiterations
    End If

Next n
timeend = Timer
```

```
time2 = Format(timeend - timestart, "Fixed")

Range("D22").Value = time2
Range("D24").Value = Range("D20").Value + Range("D22").Value

'Prints final centers
For j = 1 To numclasses
  Range("F1").Offset(j, 0).Value = center(j)
Next j

'assign data to class wize array
For j = 1 To numclasses
  k = 0
  For i = 1 To datasize
      If classassignmentk(i) = j Then
         k = k + 1
         boundeddatak(j, k) = data(i, 1)
      End If
  Next i
Next j

'calculate squared errors between data points and their closest center
For j = 1 To numclasses
   classerror(j) = 0
   For i = 1 To datasize
      If classassignmentk(i) = j Then
         classerror(j) = classerror(j) + ((data(i, 1) - center(j)) * (data(i, 1) - center(j)))
      End If
   Next i
Next j

'sum squared errors
sumclasserror = 0
For j = 1 To numclasses
   sumclasserror = sumclasserror + classerror(j)
Next j
'print sum of squared errors
Range("D14").Value = (sumclasserror / datasize)

'calculate sum of squared differences between class centers
sumdifferror = 0
For j = 1 To (numclasses - 1)
   sumdifferror = sumdifferror + ((center(j + 1) - center(j)) * (center(j + 1) - center(j)))
Next j
sumdifferrorave = sumdifferror / (numclasses - 1)

'calculate X-Y error
errorfinal = (sumclasserror / datasize) - sumdifferrorave

'print errors
Range("D16").Value = sumdifferrorave
Range("D18").Value = errorfinal

'calculate bounds
For j = 1 To numclasses
   boundsk(j) = boundeddatak(j, 1)
```

```
Next j
boundsk(numclasses + 1) = data(datasize, 1)

'print bounds
For j = 1 To (numclasses + 1)
  Range("G1").Offset(j, 0).Value = boundsk(j)
Next j

'calculate variable a and b sum and centers
For j = 1 To numclasses
   sumpop(j) = 0
   sumarea(j) = 0
   classsize(j) = 0
   For i = 1 To datasize
        If classassignmentk(i) = j Then
           sumpop(j) = sumpop(j) + data(i, 2)
           sumarea(j) = sumarea(j) + data(i, 3)
           classsize(j) = classsize(j) + 1
        End If
   Next i
Next j
For j = 1 To numclasses
    If (sumarea(j) = 0 Or sumpop(j) = 0) Then
        popdensity(j) = 0
        logpopdensity(j) = 0
        logarea(j) = 0
    Else
        popdensity(j) = sumpop(j) / sumarea(j)
        logpopdensity(j) = sumpop(j) / classsize(j)
        logarea(j) = sumarea(j) / classsize(j)
    End If
Next j

'print variable a and b sum and centers
For j = 1 To numclasses
  Range("H1").Offset(j, 0).Value = sumpop(j)
  Range("I1").Offset(j, 0).Value = sumarea(j)
  Range("J1").Offset(j, 0).Value = logpopdensity(j)
  Range("K1").Offset(j, 0).Value = logarea(j)
Next j


End Sub
```

Appendix 2: VB Script for fractal dimension calculation of cities

```
Sub centercalculator2()

Dim numlocaltries, numiterations, i, j, k, l, n, centerindex(), bestindex, classsize(), tempcenterindex, index,
datasize, numclasses As Long
Dim bestnewsumclosestdistsq, bestnewclosestdistsq, datasorter(3), center(), sumdifferror, sumdifferrorave,
errorfinal, sumclasserror, data(), classerror(), tempcenter, sumpop(), sumarea(), popdensity(), logpopdensity(),
logarea() As Double
Dim closestdistsq(), sumclosestdistsq, newsumclosestdistsq, classsum(), gaps(), tempgap, newclosestdistsq,
randsum, difference(), difsorter(2) As Double
Dim bounds(), boundsk(), tempdist, boundeddata(), boundeddatak(), classassignmentk(), finaldifsq, finaldifsum,
classassignmentl(), distsq() As Double
Dim vstat As Variant
'Dim timestart, timeend, timeelapsed As Single

'Dim classassignment() As Double

datasize = Range("D2").Value
numclasses = Range("D4").Value
numiterations = Range("D6").Value
numlocaltries = Range("D10").Value

ReDim center(numclasses), data(datasize, 3), gaps(numclasses + 1), classerror(numclasses),
logpopdensity(numclasses), difference(datasize, 2), logarea(numclasses), closestdistsq(datasize),
classsum(numclasses), sumpop(numclasses), sumarea(numclasses), popdensity(numclasses)
ReDim centerindex(numclasses), bestindex(datasize), classsize(numclasses)
ReDim distsq(datasize, numclasses), boundsk(datasize), boundeddatak(numclasses, datasize),
classassignmentl(datasize), classassignmentk(datasize)
'ReDim boundsk(datasize), boundeddatak(numclasses, datasize), classassignment(datasize)

Range("E2:L33000").ClearContents

For i = 0 To 2
   k = i + 1
   For j = 1 To datasize
      data(j, k) = Range("A1").Offset(j, i).Value
   Next j
Next i

'Sorter
For j = 1 To (datasize - 1)
   For i = j + 1 To datasize
      If data(i, 1) < data(j, 1) Then
         datasorter(1) = data(j, 1)
         datasorter(2) = data(j, 2)
         datasorter(3) = data(j, 3)
         data(j, 1) = data(i, 1)
         data(j, 2) = data(i, 2)
         data(j, 3) = data(i, 3)
         data(i, 1) = datasorter(1)
```

```
            data(i, 2) = datasorter(2)
            data(i, 3) = datasorter(3)
        End If
    Next i
Next j


'Data Printer
'For i = 0 To 2
'    k = i + 1
'    For j = 1 To datasize
'        Range("F1").Offset(j, i).Value = data(j, k)
'    Next j
'Next i

'working area
For i = 1 To (datasize - 1)
    difference(i, 1) = i
    difference(i, 2) = data((i + 1), 1) - data(i, 1)
Next i
difference(datasize, 2) = 0
For j = 1 To (datasize - 1)
   For i = j + 1 To datasize
        If difference(i, 2) > difference(j, 2) Then
            difsorter(2) = difference(j, 2)
            difsorter(1) = difference(j, 1)
            difference(j, 1) = difference(i, 1)
            difference(j, 2) = difference(i, 2)
            difference(i, 1) = difsorter(1)
            difference(i, 2) = difsorter(2)
        End If
    Next i
Next j
gaps(1) = 0

For i = 2 To numclasses
    gaps(i) = difference((i - 1), 1)
Next i
gaps(numclasses + 1) = datasize

For j = 1 To (numclasses - 1)
    For i = j + 1 To numclasses
        If gaps(i) < gaps(j) Then
            tempgap = gaps(j)
            gaps(j) = gaps(i)
            gaps(i) = tempgap
        End If
    Next i
Next j
k = 1
For i = 1 To numclasses
    For j = (gaps(i) + 1) To gaps(i + 1)
        classassignmentk(k) = i
        k = k + 1
    Next j
Next i
```

```
'Print classassignments
'For j = 1 To datasize
'  Range("N1").Offset(j, 0).Value = classassignmentk(j)
'Next j


For j = 1 To numclasses
    classsize(j) = 0
    classsum(j) = 0
    For i = 1 To datasize
        If classassignmentk(i) = j Then
            classsize(j) = classsize(j) + 1
            classsum(j) = classsum(j) + data(i, 1)
        End If
    Next i
Next j
For j = 1 To numclasses
    If classsize(j) = 0 Then
        center(j) = 0
        Else
        center(j) = classsum(j) / classsize(j)
    End If
Next j

For j = 1 To (numclasses - 1)
    For i = j + 1 To numclasses
        If center(i) < center(j) Then
            tempcenter = center(j)
            center(j) = center(i)
            center(i) = tempcenter
        End If
    Next i
Next j
'Prints initial centers
For j = 1 To numclasses
   Range("E1").Offset(j, 0).Value = center(j)
   'Range("F1").Offset(j, 0).Value = centerindex(j)
Next j

Range("D12").ClearContents
For n = 1 To numiterations
    For j = 1 To numclasses
        classsize(j) = 0
        classsum(j) = 0
        For i = 1 To datasize
            If classassignmentk(i) = j Then
                classsize(j) = classsize(j) + 1
                classsum(j) = classsum(j) + data(i, 1)
            End If
        Next i
    Next j
    For j = 1 To numclasses
        If classsize(j) = 0 Then
            center(j) = 0
            Else
            center(j) = classsum(j) / classsize(j)
```

```
            End If
        Next j

        For j = 1 To (numclasses - 1)
            For i = j + 1 To numclasses
                If center(i) < center(j) Then
                    tempcenter = center(j)
                    center(j) = center(i)
                    center(i) = tempcenter
                End If
            Next i
        Next j

        classassignmentl = classassignmentk

        For j = 1 To datasize
            bestnewclosestdistsq = -1
            newclosestdistsq = 0
            For l = 1 To numclasses
                newclosestdistsq = (data(j, 1) - center(l)) * (data(j, 1) - center(l))
                If (bestnewclosestdistsq < 0 Or newclosestdistsq < bestnewclosestdistsq) Then
                    bestnewclosestdistsq = newclosestdistsq
                    bestindex = l
                End If
            Next l
            classassignmentk(j) = bestindex
        Next j

        finaldifsum = 0
        For i = 1 To datasize
            finaldifsq = (classassignmentl(i) - classassignmentk(i)) * (classassignmentl(i) - classassignmentk(i))
            finaldifsum = finaldifsum + finaldifsq
        Next i

        If finaldifsum = 0 Then
            Range("D12").Value = n
            n = numiterations
        End If

    Next n



    'Prints final centers
    For j = 1 To numclasses
       Range("F1").Offset(j, 0).Value = center(j)
    Next j

    For j = 1 To numclasses
       k = 0
       For i = 1 To datasize
            If classassignmentk(i) = j Then
               k = k + 1
               boundeddatak(j, k) = data(i, 1)
            End If
       Next i
```

```
Next j

For j = 1 To numclasses
    classerror(j) = 0
    For i = 1 To datasize
        If classassignmentk(i) = j Then
            classerror(j) = classerror(j) + ((data(i, 1) - center(j)) * (data(i, 1) - center(j)))
        End If
    Next i
Next j

sumclasserror = 0
For j = 1 To numclasses
    sumclasserror = sumclasserror + classerror(j)
Next j
Range("D14").Value = (sumclasserror / datasize)

sumdifferror = 0
For j = 1 To (numclasses - 1)
    sumdifferror = sumdifferror + ((center(j + 1) - center(j)) * (center(j + 1) - center(j)))
Next j

sumdifferrorave = sumdifferror / (numclasses - 1)

errorfinal = (sumclasserror / datasize) - sumdifferrorave
Range("D16").Value = sumdifferrorave
Range("D18").Value = errorfinal

For j = 1 To numclasses
    boundsk(j) = boundeddatak(j, 1)
Next j
boundsk(numclasses + 1) = data(datasize, 1)

For j = 1 To (numclasses + 1)
  Range("G1").Offset(j, 0).Value = boundsk(j)
Next j

For j = 1 To numclasses
   sumpop(j) = 0
   sumarea(j) = 0
   For i = 1 To datasize
        If classassignmentk(i) = j Then
           sumpop(j) = sumpop(j) + data(i, 2)
           sumarea(j) = sumarea(j) + data(i, 3)
        End If
   Next i
Next j
For j = 1 To numclasses
    If (sumarea(j) = 0 Or sumpop(j) = 0) Then
        popdensity(j) = 0
        logpopdensity(j) = 0
        logarea(j) = 0
    Else
        popdensity(j) = sumpop(j) / sumarea(j)
        logpopdensity(j) = Log(1 / popdensity(j)) / Log(10)
        logarea(j) = Log(sumarea(j)) / Log(10)
```

```
    End If
Next j

For j = 1 To numclasses
  Range("H1").Offset(j, 0).Value = sumpop(j)
  Range("I1").Offset(j, 0).Value = sumarea(j)
  Range("J1").Offset(j, 0).Value = popdensity(j)
  Range("K1").Offset(j, 0).Value = logpopdensity(j)
  Range("L1").Offset(j, 0).Value = logarea(j)
Next j

'Printing class assignment and class size
'For j = 1 To numclasses
'    Range("I1").Offset(j, 0).Value = classsize(j)
'Next j
'For j = 1 To datasize
'  Range("H1").Offset(j, 0).Value = classassignmentk(j)
'Next j

vstat = Application.WorksheetFunction.LinEst(Range("L2:L" & (numclasses + 1)), Range("K2:K" &
(numclasses + 1)), True, True)
Range("M2").Value = vstat(1, 1)
Range("M4").Value = vstat(3, 1)


End Sub
```

Appendix 3: R script for drawing planning planes

```
function(d,ytext)
{
 e <- na.omit(d)
   coordinates(e) <- ~x + y
   x.range <- as.integer(range(e@coords[, 1]))
   y.range <- as.integer(range(e@coords[, 2]))
   x.range[2] <- x.range[2] + 1
   y.range[2] <- y.range[2] + 1
   grd <- expand.grid(x = seq(from = x.range[1], to = x.range[2],
      by = 0.1), y = seq(from = y.range[1], to = y.range[2],
      by = 0.1))
   coordinates(grd) <- ~x + y
   gridded(grd) <- TRUE
   g <- gstat(id = "elev", formula = elev ~ 1, data = e)
   plot(variogram(g, map = TRUE, cutoff = 0.8, width = 0.04),
      threshold = 0.002)
   v <- variogram(g, alpha = c(0, 45, 90, 135))
   v.fit <- fit.variogram(v, model = vgm(model = "Lin", anis = c(0,
      0.5)))
   plot(v, model = v.fit, as.table = TRUE)
   g <- gstat(g, id = "elev", model = v.fit)
   p <- predict(g, model = v.fit, newdata = grd)
   par(mar = c(2, 2, 2, 2))
   image(p, col = terrain.colors(20))
   contour(p, add = TRUE, drawlabels = FALSE, col = "brown")
   points(e, pch = 4, cex = 0.5)
   pts <- list("sp.points", e, pch = 4, col = "black", cex = 0.5)
   spplot(p, zcol = "elev.pred", col.regions = terrain.colors(20),
      cuts = 19, sp.layout = list(pts), contour = TRUE, col = "brown",
      scales = list(draw = T), xlab = "Fractal dimension",
      ylab = ytext)
}
```