HERDING IN MARKETS WITH NETWORK EXTERNALITIES: THE IMPACT OF ADVERTISEMENTS AND AUTONOMY ON THE BEHAVIOR OF CONSUMERS

by

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Abstract

In this paper I develop a herding model to examine the behavior of consumers in markets with network externalities. I build a sequential game theoretic model with incomplete information about product quality. Consumers enjoy positive network externalities from a new product only if at least half of them purchases the product; otherwise their expected payoff is smaller than status quo. The results indicate that herding on purchasing the product occurs with smaller probability than without payoff externalities; early players face the risk of being the only consumers of the new product, hence hesitate to purchase. The model provides an alternative, herding based explanation why firms face significant entry barriers in markets with network externalities. Public advertisements that signal product quality are an efficient way to convince early consumers to enter the market; it increases their belief about product quality and about the probability that subsequent consumers purchse the product as well. Therefore, the impact of advertisements is stronger with this specific form of payoff externality. Consumers always welcome more precise advertising signals, however, increasing the precision of the advertising signal might hurt consumers by preventing social learning. Introcuding heterogeneity to the extent to which consumers takes into consideration the behavior of others leads to enhanced social learning: autonomous players always follow their own signal and sophisticated players learn the true value of product quality.

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1 Introduction

Economically relevant decisions of certain individuals are often influenced by observed behavior of other individuals. Research in social learning has shown that in these kinds of environments it might be optimal even for rational agents to follow the behavior of preceding individuals leading to rational herding. In the motivating example of Banerjee (1992), consumers choose between two nearby restaurants based on a common prior, their own private signal and the behavior of other customers. Consumers arrive at the restaurants in an ex-ante known sequence and observe every action their predecessors took. After a while everyone chooses the same restaurant. Imagine the decision of the third player, after observing two identical actions: she probably ignores her own information and follows her predecessors. This basic intuition works for all of the subsequent players, hence everyone chooses the same restaurant: herding occurs.

Imagine the following quite similar situation. There is a new restaurant in town with unknown quality. Consumers are certain that if the number of others visiting this restaurant does not reach a given threshold the restaurant can not provide quality service, whereas they know the quality of the old restaurant for sure. Some *sophisticated* consumers might observe the behavior of others while *autonomous* consumers could totally ignore it. Moreover, the new restaurant uses advertisement efforts to convince consumers to switch. Alternatively, imagine a new product introduced in a market with network externalities, for instance a new smartphone or a mobile operating system. Early consumers are in a tough situation: if they purchase the product there is a chance they remain the only consumers and suffer significant losses. How should consumers behave? How and to what extent does advertisement alter consumer's behavior in this environment? What is the impact of strategic complementarities and autonomous consumers?

In this paper I develop a herding model based on Bikhchandani et al. (1992) to examine these questions. First, I build a sequential game theoretic model with strategic complementarites in payoffs and incomplete information. More than half of the consumers have to choose the new product in order to enjoy positive payoff complementarities, otherwise they get a lower expected payoff than in status quo. Consumers are uncertain about product quality (the underlying economic fundamental) and observe a private signal that provides some information about it. I assume that these private signals are generated conditionally on product quality, and therefore are correlated across players. Compared to herding models the type of payoff externality requires consumers not just to observe the action of their predecessors, but to predict the behavior of the subsequent players as well.

Second, I introduce public advertisement that works both as a signal of product quality and indirectly as a coordination device as well. Both views focus on the positive side of advertisement, and suggest welfare gains which I will examine further in the model. It is important to highlight that the model focuses on the game across different types of consumers and therefore the advertising decision of the firm is not endogenous. Based on the results of Milgrom and Roberts (1986) I will assume that advertisement partially signals product quality.

Third, I consider heterogeneity to the extent to which consumers care about the actions of others: *autonomous* consumers do not take effort to observe the behavior of others, their decision is based only on their own information.

This paper is most closely related to the small literature on herding with payoff externalities. There are several ways to introduce payoff externalities in a Bikhchandani et al. (1992) setup. In the model of Chari and Kehoe (1997) investors choose between a risky project with uncertain payoffs and a safe alternative. The risky project gets funded only if the number of investors exceed a certain threshold. Otherwise all of the money goes to the safe project and even investors choosing the risky project get the payoff of the safe alternative. Therefore, in this model investors face only the risk related to a negative payoff when the project is funded. The authors point out that even small amounts of new information can lead to huge capital flows (similarly to usual herding models). Dasgupta (2000) introduces strong strategic complementarities: only if all of the players choose the same action do they enjoy the benefits of externalities, otherwise they suffer losses. The form of payoff externality creates a strong incentive to coordinate and herding occurs even in a rich signal space.

Drehmann et al. (2007) empirically test several modifications of the Bikhchandani et al. (1992) model including payoff externalities: the payoff of players depend positively on how many others choose the same action. The authors find that the probability of herding is higher with positive payoff externalities and that cascades are shorter and less frequent than theory predicts. Eyster and

Rabin (2010) critize rational herding models and offer an alternative way players might behave: they assume that others are naive and follow their own signals. If all agents behave this way herding might occur in richer signal spaces as well. However, the possibility that players are naive or that there is heterogeneity in the extent to which players infer signals from actions is not taken into consideration.

This paper also connects to the industrial organization (IO) literature on advertising and network externalities. In a sequential game theoretic model with two players, periods and goods, Farrell and Saloner (1985) show that consumers might not switch from an inferior product if they are uncertain of the quality of the new product. *Excess inertia* emerges and companies face a significant entry barrier due to network externalities. Milgrom and Roberts (1986) find that even uninformative advertisement is able to signal product quality in case of *experience goods* and repeat purchases. Nevertheless, in these cases the IO literature focuses on the game between the company and the consumer.

The paper has several contributions. First of all, it models consumer behavior in markets with network externalities; therefore the form of payoff externality is special and differs from the most closely related herding models. These herding models were designed to examine certain properties of investor's behavior in financial markets. The large number of players allows me to model the specific network externality that replicates a *winner takes it all* phenomenon: at least half of the consumers have to choose the new product in order to benefit positive externalities. This form of network externality captures the idea that consumers should take into consideration the behavior of subsequent consumers as well, and hence, it is closer to reality. In contrast to the IO literature, the herding approach introduces social learning about product quality. The basic model provides an alternative, herding based explanation for why it is difficult for companies to enter markets with network externalities.

Second, due to the flexibility of information signals the model is able to capture the effect of public advertisements on consumer behavior. The behavior of consumers responds significantly to advertisements (and public information in general). Furthermore, it is possibly to compare the effect of advertisement with and without payoff externalities: advertisements have a stonger effect with this specific form of network externality. This result is due to the phenomenon that advertisements alter the beliefs of players about the behavior of subsequent players as well. Every player welcomes more precise advertisements, however increasing the precision of advertisements might hurt the welfare of consumers by preventing social learning.

Third, the modeling approach makes it possible to introduce heterogeneity of players to the extent to which they take into consideration the actions of others. The assumption that some players decide independently of behavior of others matches reality better; for instance, consumers might have to take efforts to observe the behavior of others which some of the consumers might not take. Autonomous players stop information cascades which provides an explanation for the shorter and less frequent herds found by Drehmann et al. (2007).

In Section 2 I discuss the related literature in more detail. Section 3 describes the basic model with strategic complementarities and presents the optimal behavior of consumers. In Section 4 I model advertisements as public signal of product quality and examine how it changes the outcome of the game. In Section 5 I introduce autonomous players, analyze their behavior and present the consequences on the outcome of the game and on the effect of advertisements. Finally, Section 6 concludes and discusses further considerations and possible extentions.

2 Relevant literature

This paper relates to several strands of literature. In this section I discuss the most important approaches and findings in the economics of herding, advertisements and markets with strategic complementarities.

2.1 Herding in economics

Bikhchandani et al. (1992), and Banerjee (1992), the two canonical paper of sequential herding, proved that information cascades might occur even if agents are rational. Agents observing their predecessors acting similarly have a tendency to ignore their own signals and follow the herd if it provides stronger information than their own signals. After a herd is initiated agents do not follow their own information, information revelation is blocked, and agents can not learn the true state of the world. Because herds are not confident they might be stopped by an agent with more precise signal; an expert with more precise information might decide not to follow the herd which provides new information and stops herding. Similarly, as Bikhchandani et al. (1992) emphasizes, public information migh have huge impact on the outcomes and can even reveal a false cascade suggesting the possible positive effect of governmental intervention. Based on these papers a significant literature emerged on rational[1] and naive herding with applications especially relevant for financial markets.

Eyster and Rabin (2010) critize rational herding models for several reasons. With rational agents herding does not hurt agents in expectations. Moreover, agents are not confident in the state they are herding on. The authors introduce *best response trailing naive inference (BRTNI)*, agents best respond to the belief that all their predecessors followed their own signal. Eyster and Rabin (2010) show that with BRTNI agents if agents herding can occur even in rich information settings and it also can be confident. Anderson and Holt (1997) and Kübler and Weizsacker (2004) provide experimental evidence supporting their assumption that players follow best response trailing naive inference. In both experiments agents observe the action of others for free, and Kübler and Weizsacker (2004) give players the opportunity to buy private signals. Many agents buy private signals

[1]Chamley (2004) gives a wide review of rational herding models in economics.

indicating that they do not totally trust the behavior of their predecessors. However, these experiments offer a clear and free way of observing the behavior of others, as they are all participating in an experiment. Real life sitations may be different in several ways. Agents might also have to take efforts to observe the behavior of others and they might differ in their willingness to pay or efforts to take for information, especially if this information is not easy to acquire. Eyster and Rabin (2010) ignore the possibility that players behave heterogenously, some of them might be totally rational, others might be BRTNI and some of them might be totally naive just in the way BRTNI players think of others.

In several markets the purchasing decisions made by a consumer might influence the payoffs of other consumers as well. I will refer to this interdependence as *strategic complementarity*. Drehmann et al. (2007) claim that there is a surprisingly small theoretical literature on herding with payoff externalities and the experimental work is also scarce. There are many ways to introduce payoff externalities, however existing papers usually model these differently from the setup of Bikhchandani et al. (1992). Chari and Kehoe (1997) models externalities in a setup closely related to Bikhchandani et al. (1992) with binary signals; players may choose to invest in a risky project or to choose the safe option. In order an investment to get funded at least half of the investors have to invest. In the model of Chari and Kehoe (1997) investors are not punished if they choose to invest which simplifies the solution of the model. The authors focus on the consequences of the herding behavior of investors: even tiny bits of information may lead to huge capital flow. Dasgupta (2000) presents a model in which consumers enjoy externalities only if all of them chooses the same action, otherwise they suffer losses. This creates a strong incentive to coordinate and herding occurs even in a rich signal space.

Drehmann et al. (2007) empirically test herding behavior in different conditions, one of which is designed to imitate network effects. Subjects participating in the online experiment have to choose between two investment options and they get some positive payoff after every other subject that decided similarly to them. The authors find that in line with the prediction of theory with positive strategic complementarities the probability of herding behavior is higher. However, compared to earlier results a lower percentage of subjects behaves according to theory suggesting that cascades may be shorter and less frequent.

2.2 Market entry with strategic complementarities across consumers

The most widely examined markets in the industrial organization literature with strategic complementarities are markets with network effects or goods. *Network effects* are present if the utility of each consumer is increasing in the number of others purchasing the same product (Belleflamme and Peitz, 2010) and therefore it represents positive strategic complementarities across consumers. Farrell and Klemperer (2007) claim that network effects provide incentives to coordinate with others which might lead to multiple equilibria. Because the actions of early adopters are observed they influence the decision of latecomers and thus the equilibrium of the game. Therefore, convincing early adopters and managing expectations are crucial.

Network effects constitute a significant entry barrier for companies. Farrell and Klemperer (2007) show that network effects and switching costs usually lead to *inertia*, consumers stick to the status quo. In models with simultaneous decisions and switching costs a *chicken and egg phenomenon* emerges, consumers would be better off if both of them would change from the status quo simultaneously. However, neither of the players is incentivized to change alone, therefore agents stick to the status quo leading to *excess inertia*.

Excess inertia might also be present in games with sequential movements. Intuitively, in an environment with incomplete information consumers can not be certain that their early purchasing decision will be followed by others. Thus, consumers might not want to take the risk of moving first. Farrell and Saloner (1985) show this phenomenon in a model with two agents, two network goods, incomplete information and sequential moves. Each agent has the opportunity to switch in either period 1 or 2 or not at all. Agents differ in their evaluation of network effects which is only known to themselves. The authors prove the presence of *excess inertia*: agents might not switch even from an inferior product.

Convincing early adopters is thus crucial for companies entering a new market with network goods. Farrell and Klemperer (2007) describe the most successful strategies firms may take in order to overcome *inertia*. Reduced entry prices and specific discounts to early adopters might help to make others purchase the product even in an increased price. With the same logic firms might want to decrease the costs early adopters face if they remain the only consumers by lowering the risks of early entry. Belleflamme and Peitz (2010) highlight that in managing expectations the role of commercial advertising might be crucial. However, the authors do not provide convincing evidence for why consumers should believe this kind of advertisements.

2.3 Advertisement in economics

Bagwell (2007) discusses the three views economists considered why consumers respond to advertisements. The first view claims that advertisement is *persuasive* in a sense that it changes the preferences of consumers, and leads to specific demand for the product of the advertising firm.

The second view emphasizes that advertising might be *informative*, i.e. it might convey directly or indirectly relevant information about the characteristics of a product. For this paper the indirect channel will be important, as advertisement might be interpreted as a signal of product quality even without actually referring to the quality of the product. The canonical model of advertising signals of product quality was developed by Milgrom and Roberts (1986). The authors build on Nelson's idea that even uninformative advertisement may alter consumers decision by signaling product quality. Nelson (1974) claimed that for *experience goods* whose quality can not be fully detected before purchase, high-quality firms can differentiate themselves by using advertisement. This insight is valid only if the firm with the better product would use more advertisement leading to a separating equilibrium. Repeat purchases offer a difference in gains by advertisement across high and low qualitiy producers making advertisement a possible signaling device. However, as Milgrom and Roberts (1986) argues, this result is only valid when the firm decides both the price and the level of advertisement. The most important contribution of Milgrom and Roberts (1986) is that they are able to prove theoretically that advertisement can serve as a signal of product quality in the case of repeat purchases. Future purchases matter even in markets where one consumer buys the good for her lifetime if information spreads across consumers. An unsatisfied consumer today might easily lead to no one buying the product tomorrow. Because almost every company should take into consideration future purchases the assumption of repeated purchases is widely satisfied.

According to the third approach emphasized by Becker and Murphy (1993) advertising directly enters the utility function as a *complementary* good to the advertised product. According to this approach consumers care about the social/self image that is offered by the product and by that advertisment increases consumer experience.

In this paper I will focus on the second view and offer a fourth that considers advertisement as a *coordination* device. This idea comes from the literature of global games [2]: publicly available information might serve as a coordination device. Bagwell and Ramey (1994) developes a static model in which uninformative advertisement serves as a coordination device [3]. The authors argue that due to economies of scale both consumers and retailers are interested in a concentrated market structure which gives rise to coordination economies. Even uninformative advertising might lead consumers to firms which expects to become dominant players on the market justifying their beliefs about future winners.

DellaVigna and Gentzkow (2010), a recent review, discusses the empirical evidence of persuasion on the behavior of consumers, voters, donors and investors. The authors conclude that the evidence on the effect of advertisements on sales is ambigous. The strong correlation between advertisement spending and sales is unquestionable, however, reverse causality is a prevalent problem especially if advertisement works as a signal of product quality. Nevertheless, several articles are able to show that the effect of advertising is stronger on consumers who had no experience with the product supporting the predictions of informative advertisements in markets with experience goods.

[2] Global games are coordination games of incomplete information, where players receive signals about the underlying state of world. The main contribution of this literature is that for specific assumptions the usual multiple equilibria result of coordination games disappears which makes it possible to forecast the outcomes of the game. Morris and Shin (2000) nicely summarize the results of global games.

[3] To my knowledge, Bagwell and Ramey (1994) is the only study which viewed advertising as a coordination device for consumers; however, their approach is different from mine. This denying of coordination issues in the theory of advertising reflects the interest in the field in the game between the firm and its consumers. Nevertheless, in some situations the game the consumers play with each other has important consequences on the behavior of the firm. The need for tractability of the analysis usually does not allow for handling both games endogenously.

3 Basic model of market entry without advertisement

The basic model is designed to examine whether a new product could be successful in a market with strategic complementarities across consumers with a setup based on Bikhchandani et al. (1992). Consumers want to buy the new product only if more than half of them purchase it; otherwise they would stick to the status quo. Chari and Kehoe (1997) model investors' behavior using a closely related setup. In their model players decide whether to invest in a risky project which only gets funded if a previously set amount N of them invest. Players who do not invest or invest but the project does not get funded receives the smaller, status quo yield. Hence, investors who decided to invest while the project had not been funded are not punished, therefore they do not care about the risk of belonging to the minority. Exercise (13.4) in Chamley (2004) is a simplified version of the main modeling setup of Chari and Kehoe (1997), where the risky project gets funded only if at least half of the consumers invest. These models were designed to match the observations on financial markets and not to model the purchasing decision of a new product. Hence, my model differs systematically. First, I use a different payoff structure designed for the specific game in interest, with punisment for belonging to the minority. Second, based on the phenomena of industries with network externalities or strategic complementarities consumers enjoy benefits only when the new product is the winner, meaning more than half of the consumers purchase it.

3.1 Setup

A large (N) number of consumers play a sequential game with incomplete information and binary signals about product quality. Their order of apperance is determined exogeneously and similarly to usual signaling games; players observe only the action of other players before them and this is the only way they communicate. There is an underlying state of nature θ which determines the distritution of the signals θ_i (i=1,2,...N) that players observe and the outcomes they get. Thus, every player has a private signal indicating her type.

The timing of the game is the following:

1. Nature determines the state of the world θ and the private signals θ_i of the players.

2. Player 1 observes θ_1 and decides on her action a_1.

3. Player 2 observes θ_2 and the action a_1 of Player 1. Then chooses her action a_2.

4. Player i (i=3, 4, ..., N) observes θ_i and a_j (j=1,2, ..., i-1) the action of every player before her. Then chooses her action a_i.

5. Finally, payoffs are distributed

3.1.1 Product quality and signals

Through the model I will follow the standard notation of Chamley (2004). The underlying state of the world θ represents product quality. Players have a common prior on θ which can take values 1 and -1 with ex-ante probabilities:

$$\theta = \begin{cases} 1 & \text{with} Pr(\theta = 1) = \mu_0 \\ -1 & \text{with} Pr(\theta = -1) = 1 - \mu_0 \end{cases}$$

θ determines the conditional distribution of the private signals. For every (i, j), θ_i and θ_j is drawn independently from the same conditional distribution and might also take values 1 and -1 according to Table 1[4],

$\forall i$	$\theta_i = 1$	$\theta_i = -1$
$\theta = 1$	q	$1 - q$
$\theta = -1$	$1 - q$	q

Table 1: Signal structure of the game

where for example $Pr(\theta_1 = 1|\theta = 1)$ equals q. In the most natural interpretation the number q stands for the precision of the signals. Because $Pr(\theta_i = 1|\theta_j = 1) = \mu_0 q + (1 - \mu_0)(1 - q)$, this signal structure reflects the fact that private signals are correlated as they are generated from the same underlying distribution. For instance, costumer reviews and expert opinions depend on

[4]The binary signal structure I use makes calculations easier, however it might seem too restrictive. As I will argue in Section 6 with the payoff externalities I introduce my results would probably be at least partially valid for richer signal structures as well.

the general product quality with some noise. Therefore, if a product is of high quality ($\theta = 1$) consumers face information that the product is good with higher probability.

Assumption 3.1. *The quality of the new product equals the status quo in expectations:*

$$\mu_0 = \frac{1}{2}$$

and the private signal carries some information about product quality:

$$q \geq \frac{1}{2}$$

3.1.2 Strategies and the payoff structure

Players choose whether to enter the market *(In)* or not *(Out)*. Depending on θ_1 Player 1 has 4 strategies. For instance, *{In, Out}* means the following strategy of Player 1:

$$\{In, Out\} = \begin{cases} In & \text{Player 1 observed } \theta_1 = 1 \\ Out & \text{Player 1 observed} \theta_1 = -1 \end{cases}$$

Player 2 has 16 strategies as she also observes Player 1's choice. For instance, *{In, Out, In, Out}* means the following strategy of Player 2:

$$\{In, Out, In, Out\} = \begin{cases} In & \text{Player 2 observed} a_1 = In \text{and} \theta_2 = 1 \\ Out & \text{Player 2 observed} a_1 = In \text{and} \theta_2 = -1 \\ In & \text{Player 2 observed} a_1 = Out \text{and} \theta_2 = 1 \\ Out & \text{Player 2 observed} a_1 = Out \text{and} \theta_2 = -1 \end{cases}$$

The number of strategies for players is increasing quickly; therefore it makes sense to introduce the following notations for the history of the game.

Notation 3.2. *Let $h_i(a_1, a_2, ..., a_{i-1})$ denote the actions observed by Player i in a specific realization of the game.*

Notation 3.3. *Let $P_i(k)$ denotes the difference between the number of actions In and Out observed by Player i. Let denote $P_i'(k)$ denote the difference between the number of signals In and Out Player i could infer from the actions she observed.*

Notation 3.4. *Let μ_i denote the belief of Player i that $\theta = 1$ given the information she knows. This notation is consistent with μ_0 being the common prior.*

During the model I will examine the best response of each player for certain kinds of histories of the game which will make the analysis easier to follow. $P_i(k)$ and $P_i'(k)$ will be a useful indexes that group certain kind of histories as they will determine the best response of Player i in equilibrium which makes the analysis much simpler.

Table 2 shows the payoff matrix of the game. Payoffs are normalized to 0 and 1, only the choice of $\frac{1}{2}$ is arbitrarily which measures the incentive for coordination. Because I assumed that the prior μ_0 equals $\frac{1}{2}$ the expected product quality of the new good equals the status quo product quality. Players receive strategic complementarities if and only if more than half of them buys the new product. The status quo might be interpreted as an existing product of a company for which the network is settled and consumers are certain that they receive strategic complementarities. This setup partially represents the *winner takes all* phenomenon in markets with network externalities: the company that could build the bigger network is the only one successful. Consumer experience of the new product when the number of consumers who purchase it does not reach the threshold is below its product quality ($\theta - \frac{1}{2}$) because the advantages from complementarity can not emerge.

Player i	Number of In$>\frac{N}{2}$	Number of In $\leq \frac{N}{2}$
In	θ	$\theta - \frac{1}{2}$
Out	0	0

Table 2: The payoff matrix of the game

In order to avoid complications in timing I assume that payoffs are distributed only at the end of the game. Another possible assumption is that consumers who purchased the product get $\theta - \frac{1}{2}$ until the number of consumers reaches the threshold.

3.2 Equilibrium of the basic model

3.2.1 Assumptions and the behavior of the first few players

The appropriate equilibrium concept for incomplete information games is Perfect Bayesian Equilibrium (PBE), and especially its variation defined for signaling games. Basically this concept requires two conditions to be satisfied: first, that for given beliefs in every information set every player best responds to the strategy of the other (in expected terms); second, that consumers use Bayes' rule to update their beliefs about uncertain events. Throughout the paper I assume that players maximize expected payoffs and update their beliefs according to Bayes' rule. Moreover, in order to exclude unnatural equilibria due to off path beliefs I assume that every player uses best respond strategies off equilibrium path as well and this is common knowledge.

It is important to highlight that the form of the equilibrium concept which is introduced for simple signaling games can be used here; however, finding the equilibria requires more effort. That is why throughout the paper I focus on best responses of players given certain conditions and examine whether these conditions are satisfied in equilibrium. First, I examine the behavior of early players which will allow to infer the behavior of subsequent players. Then I will show that the beliefs and behavior of early players is consistent in equilibrium with the behavior of subsequent players.

Without uncertainty in product quality the subgame perfect Nash equilibrium of the game would be unique depending on the state. Solving the game by backward induction shows that for $\theta = 1$ every consumer would buy the product, whereas for $\theta = -1$ no one would switch from status quo. For $\theta = -1$ strategy *In* is dominated for all players leading to an equilibrium when no one enters. For $\theta = 1$ strategy *Out* is dominated if the number of consumers buying the new product exceeds the threshold. Every player infers correctly that this is the case in equilibrium, hence every player enters the market and the beliefs are justified.

Player i wants to enter the market if her expected payoff by entering is higher than sticking to status quo given the information she has:

$$Pr\left(In > \frac{N}{2}|h_i,\theta_i\right)\left[\sum_{\theta=1,-1}\theta * Pr(\theta|h_i,\theta_i)\right] + Pr\left(In \leq \frac{N}{2}|h_i,\theta_i\right)\left[\sum_{\theta=1,-1}(\theta-\frac{1}{2}) * Pr(\theta|,h_i,\theta_i)\right] > 0 \tag{1}$$

where $In > \frac{N}{2}$ refers to the case when the number of consumers buying the new product exceeds $\frac{N}{2}$, h_i is the history and θ_i is the signal observed by Player i. Player i infers the signal of previous players from the history of the game. It is important to recognize that by entering the market all players face significant risks from two sources: first that they are uncertain about product quality, and second that not enough consumers after them decide to enter the market. This means that they must believe that the new product is good with probability at least $\frac{1}{2}$ in order to purchase it.

Due to practical reasons I want to focus on equilibria in which Player 1 enters the market for $\theta_1 = 1$, whereas she does not for $\theta_1 = -1$. Otherwise, Player 2 would ultimately follow Player 1's decision leading to perfect herding immediately. If Player 1 always plays *In* so does Player 2 because she faces the same decision with even more incentives to enter the market. Since Player 2 observed an action *In*, the probability of reaching the threshold increases making her decision towards *In*. Other players also follow the herd due to the same argumentation. The case for Player 1 always using strategy *Out* is similar.

Therefore, I start by showing what assumptions have to be satisfied in order for Player 1 to enter the market only if $\theta_1 = 1$. I will show that these assumptions are satisfied in equilibrium in Theorem 3.14.

Assumption 3.5. *Let denote $\bar{p} = Pr(In > 0.5N|\theta_1 = 1)$ and $\underline{p} = Pr(In > 0.5N|\theta_1 = -1)$. μ_0 stands for the common prior and q for the precision of the private signal.*

$$\frac{1+\bar{p}}{3-\bar{p}} \geq \frac{1-q}{q}\frac{1-\mu_0}{\mu_0} \tag{2}$$

$$\frac{1+\underline{p}}{3-\underline{p}} \leq \frac{q}{1-q}\frac{1-\mu_0}{\mu_0} \tag{3}$$

and for $\mu_0 = \frac{1}{2}$ (2) becomes $\bar{p} \geq 3 - 4q$ and (3) is always satisfied.

Proposition 3.6. *Under Assumption 3.5 Player 1 plays In for $\theta_1 = 1$.*

Proof. Player 1 starts from $P_1(0)$. She computes the expected value of playing *In* and compares it to the expected value of playing *Out* according to (1). Let $\bar{q}$ denote $Pr(\theta = 1|\theta_1 = 1)$ given prior μ_0.

$$
\begin{aligned}
E[In|\theta_1 = 1] = Pr(In > 0.5N|\theta_1 = 1)[1 * Pr(\theta = 1|\theta_1 = 1) - 1 * Pr(\theta = -1|\theta_1 = 1)]+ \\
[1 - Pr(In > 0.5N|\theta_1 = 1)][0.5 * Pr(\theta = -1|\theta_1 = 1) - 1.5 * Pr(\theta = -1|\theta_1 = 1)] \\
= \bar{p}[\bar{q} - (1 - \bar{q})] + (1 - \bar{p})[0.5\bar{q} - 1.5(1 - \bar{q})] \geq 0
\end{aligned}
$$

$$
\bar{p} \geq 3 - 4\bar{q} \tag{4}
$$

For $\mu_0 = 0.5$ this expression simplifies as $\bar{q} = q$. Otherwise

$$
\bar{q} = Pr(\theta = 1|\theta_1 = 1) = \frac{Pr(\theta_1 = 1|\theta = 1) * Pr(\theta = 1)}{Pr(\theta_1 = 1)} = \frac{q * \mu_0}{q * \mu_0 + (1 - q) * (1 - \mu_0)} \tag{5}
$$

Substituing back (5) to (4) and after some algebra we may get the following expression:

$$
\frac{1 + \bar{p}}{3 - \bar{p}} \geq \frac{1 - q}{q} \frac{1 - \mu_0}{\mu_0}
$$

Therefore, Player 1 plays *In* if Assumption 3.5 is satisfied. ■

In order to make interpretation easier it makes sense to compute some threshold values for $\bar{p}$ and see whether they are satisfied. For example, for $q = 0.75$ and $\mu_0 = 0.5$ the expression becomes $\bar{p} \geq 0$ indicating that Player 1 would always enter. This expression also shows that if the private signal is precise Player 1 enters without taking into consideration the risk that the network does not reach its treshold size.

Proposition 3.7. *Under Assumption 3.5 Player 1 plays Out for* $\theta_1 = -1$.

Proof. This proof is quite similar to Proposition 3.6, and so, it is presented in Appendix 8.1.1. ■

In order to make interpretation easier it makes sense to compute some threshold values for $\underline{p}$ and see whether they are satisfied. For example, for $q = 0.5$ and $\mu_0 = 0.5$ the expression becomes $\underline{p} \leq 1$ indicating that Player 1 would always play *Out*. This expression shows that Player 1 under Assumption 3.1 would never want to enter the market for $\theta_1 = -1$.

Assumption 3.8. *Players initiate a herd if and only if it is strictly beneficial for them.*

Proposition 3.9. *Under Assumption 3.8 Player 2's best response is the following:*

$$a_2 = \begin{cases} In & \textit{if Player 2 observed In}(P_2(1))\textit{and}\theta_2 = 1 \\ Out & \textit{if Player 2 observed In and}\theta_2 = -1 \\ Out & \textit{if Player 2 observed Out}(P_2(-1)\textit{regardless}\theta_2 \end{cases}$$

Proof. Player 2 infers θ_1 as Player 1's action totally reveals her signal. When the two signals coincide (i.e. either $\theta_{1,2} = 1$ or $\theta_{1,2} = -1$) it is easy to see that Player 2 follows Player 1. She evaluates the same expression (1). For $\theta_{1,2} = 1$ her belief about θ and about the probability that enough consumer chooses the new product is bigger than these belief of Player 1, therefore Player 2 plays *In*. With the same argumentation, for $\theta_{1,2} = -1$ she plays *Out*.

The interesting cases are when θ_1 and θ_2 are different. In these cases, Player 2's belief about $\theta = 1$ equals the common prior $\mu_0 = 0.5$ as the two signals cancels each other out. Player 2 does not purchase the product if there is the smallest chance that she will belong to the minority, because the expected value of playing *In* for the prior 0.5 is smaller than 0 due to the punishment if she belongs to the minority:

$$\begin{aligned} E[In|\mu_i = 0.5] &= Pr(In > 0.5N|\mu_i = 0.5, a_i = In)(1 * 0.5 - 1 * 0.5) \\ &\quad + [1 - Pr(In > 0.5N|\mu_i = 0.5, a_i = In)](0.5 * 0.5 - 1.5 * 0.5) \leq 0 \\ &= [1 - Pr(In > 0.5N|\mu_i = 0.5, a_i = In)] * (-0.5) \leq 0 \end{aligned}$$

where μ_i denotes the belief of Player 1 that $\theta = 1$ given her information. When Player 2 initiates a herd by playing *In*, the belief $Pr(In > 0.5N|\mu_i = 0.5, a_i = In) = 0$; therefore Player 2 is indifferent of the two actions. Assumption 3.8 makes best response of Player 2 (and as we see later every Player j when j is an even number) unique. This assumption is very useful from a practical perspective and is easily justified in reality[5]. ∎

[5] In Theorem 3.14 I discuss the consequences of this assumption in more detail. I will show that playing *In* leads to a different equilibria which is not favoured from a practical perspective. This assumption is also necessary for the uniqueness of the equilibrium.

Corollary 3.10. *When Player 1 plays Out* $(\theta_1 = -1)$ *a herd is already initiated as Player 2 does not follow her own signal. After one player observed* $P_i(-1)$ *every player herds on action Out.*

Proof. Proposition 3.9 shows the best response of Player 2 in this case. As she does not follow her own signal, subsequent players can not infer her signal. Therefore, Player i for $i > 2$ will not enter the market, since every Player i will have the same information set as Player 2. Moreover, these players are even more incentivized not to enter the market since belonging to the minority is more probable (if the number of consumers is finite). Therefore, after one player observed $P_i(-1)$ every player herds on *Out*. ■

Proposition 3.11. *Player 3's best response is the following:*

$$a_3 = \begin{cases} In & \textit{if Player 3 observed } h_3(In, In), P_3(2) \textit{ regardless } \theta_3 \\ In & \textit{if Player 3 observed } h_3(In, Out), P_3(0) \textit{ and } \theta_3 = 1 \\ Out & \textit{if Player 3 observed } h_3(In, Out), P_3(0) \textit{ and } \theta_3 = -1 \\ Out & \textit{if Player 3 observed } h_3(Out, Out), P_i(-2) \textit{ regardless } \theta_3 \end{cases}$$

Proof. After observing $h_3(In, In)$ and $\theta_3 = 1$ Player 3 obviously enters the market. For $h_3(In, In)$ with $\theta_3 = -1$ and $h_3(In, Out)$ with $\theta_3 = 1$ Player 3 faces the same situation as for Player 1 with $\theta_1 = 1$ $(\mu_1 = \mu_3)$ with the difference that the probability that consumers reach the threshold number $0.5N$ is higher. Proposition 3.5 shows that Player 1 enters for $\theta_1 = 1$, therefore so does Player 3. With the same argumentation for $h_3(In, Out)$ and $\theta_3 = -1$ Player 3 does not enter the market, whereas Corollaly 3.10 shows that Player 3 follows the herd for $h_3(Out, Out)$. ■

Corollary 3.12. *When Player 1 and Player 2 plays In, a herd is initiated as Player 3 does not follow her own signal. After one player observed* $P_j(2)$ *every player herds on In.*

Proof. Proposition 3.11 shows the best response of Player 3 in this case. As she does not follow her own signal, subsequent players can not infer her signal. Therefore, Player i for $i > 3$ will enter the market, since every Player i will have the same information set as Player 3. Moreover, these players are even more incentivized to purchase the product since belonging to the minority is less probable. Therefore, after one player observed $P_i(2)$ every player herds on *In*. ■

Proposition 3.13. *Player 4 follows the herd for histories* $h_4(In, In, In), h_4(In, Out, Out)$*,* $h_4(Out, Out, Out)$*, whereas* θ_4 *determines* a_4 *for* $h_4(In, Out, In)$*.*

Proof. The first part of the statement is a consequence of Corollaries 3.10 and 3.12. If $h_4(In, Out, In)$ Player 4 faces the same situation as Player 2 after observing $h_2(In)$, again with higher belief of the probability of belonging to the majority. Therefore, Player 4 behaves the same for $h_4(In, Out, In)$ as Player 2, follows her own signal. ∎

3.2.2 Equilibrium

Theorem 3.14. *The following strategies constitute a Perfect Bayesian Equilibrium under Assumptions 3.1, 3.5 and 3.8 if* $\bar{q}$ *satisfies* $\frac{\bar{q}}{1-\bar{q}+\bar{q}^2} \geq 3 - 4\bar{q}$ *or equivalently for* $\mu_0 = \frac{1}{2}$ *if* $q \geq 0.563$*. The best response strategy of Player i if i is an odd number in equilibrium is the following:*

$$a_i = \begin{cases} In & \textit{if Player i observed } P_i(2 \leq) \textit{ regardless} \theta_i \\ In & \textit{if Player i observed } P_i(0) \textit{ and} \theta_i = 1 \\ Out & \textit{if Player i observed } P_i(0) \textit{ and} \theta_i = -1 \\ Out & \textit{if Player i observed } P_i(\geq -2) \textit{ regardless} \theta_i \end{cases}$$

and the best response strategy of Player j if j is even odd number in equilibrium is the following:

$$a_j = \begin{cases} In & \textit{if Player j observed } P_j(3 \leq) \textit{ regardless} \theta_j \\ In & \textit{if Player j observed } P_j(1) \textit{ and} \theta_j = 1 \\ Out & \textit{if Player j observed } P_j(\geq -1) \textit{ regardless} \theta_j \end{cases}$$

Proof. From the behavior of Player 1 and Player 3 it is easy to infer the best response strategies for Player i if i is an odd number in equilibrium. Corollaries 3.10 and 3.12 justify that players follow the herd after it is initiated. For $P_i(0)$ players follow their own signal due to the same argumentation as in Proposition 3.11. They face the same situation as Player 1 in the beginning of the game with the only difference in their beliefs about the probability of the number of consumer buying the product exceeding the threshold. From the behavior of Player 2 and 4 it is easy to infer the best response strategies for Player j if j is an even number in equilibrium. Corollaries 3.10 and 3.12 justify that players follow the herd after it is initiated. After $P_i(1)$ players face the same situation as Player 2 to after observing $h_2(In)$ and therefore they follow their signal.

In order to justify that the strategies described constitute an equilibrium it also has to be proved that Assumption 3.5 over $\bar{p} = Pr(In > 0.5N|\theta_1 = 1)$ and $\underline{p} = Pr(In > 0.5N|\theta_1 = -1)$ is satisfied in equilibrium. Start with the case for $\underline{p}$ which is always satisfied for $\mu_0 = \frac{1}{2}$ and $q \geq 0.5$. For the other part of the assumption I calculate the probability of herding on In given $\theta_1 = 1$. I assume that the number of players is infinite in order to avoid complications due to the number of players being even or odd. This estimation will be very close to the precise formula for large n and simplifies further calculations as well. The histories for which a herd is initiated on In are the followings:

$$(In, In), (In, Out, In, In), (In, Out, In, Out, In, In)...$$

and the probabilities of these histories respectively:

$$\bar{q}, (1-\bar{q})(\bar{q})^2, (1-\bar{q})\bar{q}(1-\bar{q})(\bar{q})^2, ...$$

Therefore, the probability of herding on In is the sum of the probabilities of these histories:

$$Pr(HerdIn|\theta = 1) = \bar{q} + (1-\bar{q})\bar{q}^2 + (1-\bar{q})\bar{q}(1-\bar{q})(\bar{q})^2 + ... = \frac{\bar{q}}{1-\bar{q}+\bar{q}^2}$$

As a herd is not necessary in order to exceed the threshold number of consumers $\frac{N}{2}$, $Pr(HerdIn|\theta = 1)$ is a lower bound on $\bar{p}$. Therefore, as for $\mu_0 = \frac{1}{2}$: $\bar{q} = q$, this expression combined with (4) yields:

$$\frac{\bar{q}}{1-\bar{q}+\bar{q}^2} \geq 3 - 4\bar{q}$$

$$4\bar{q}^3 - 7\bar{q}^2 + 8\bar{q} - 3 \geq 0$$

$$\bar{q} > 0.563$$

$$\bar{q} = \frac{q * \mu_0}{q * \mu_0 + (1-q) * (1-\mu_0)} > 0.563$$

Hence, for $\mu_0 = \frac{1}{2}$ and $\bar{q} \geq 0.563$ Assumption 3.5 is satisfied.

Players do not want to deviate from their best responses. There is one specific situation for which this statement is not obviously justified: when even players observe $P_j(1)$ and receive signal $\theta_j = -1$. In this situation, players are indifferent between deviation and the equilibrium strategy, so they would not want to deviate. ■

Theorem 3.14 supports the basic results for network goods: consumers switch to the new product only if they believe it is better than status quo and that early movers have more influence on the equilibrium outcome. The equilibrium depends on the sum of the common prior μ_0 and the precision of the signal q.

Corollary 3.15. *Under Assumption 3.8 the equilibrium in Theorem 3.14 is unique.*

Proof. The only situation where the best response of players in not unique is when even players observe $P_j(1)$ and receive signal $\theta_j = -1$. Assumption 3.8 ensures that players in this case choose Out. ■

It it important to highlight that without this assumption, the path where there is herding after $P_j(1)$ would also be equilibrium. However, I want to exclude this equilibrium for practical purposes. Consider the situation where this second equilibrium is allowed. For j being an even number Player j's would be still indifferent after $P_j(1)$ and signal $\theta_j = -1$; however, this equilibrium would only be valid if every Player j could be sure that the number of consumers purchasing the new product reaches the threshold. In this situation Player j generates a herd only if every player after her facing the same situation would follow the same equilibrium and make the same decision, $a_j = In$. As any subsequent player is indifferent, they might choose to play Out and breaking the herd which hurts Player j. The equilibrium in Proposition 3.14 does not require this criteria, hence it is favoured from a practical perspective.

3.3 Herding in the basic model

The equilibrium of the game has several important properties. Firms are especially interested in the expected number of consumers choosing their product and the probability that everyone herds on purchasing it. I follow the literature and call *herd behavior* or *herding* the phenomenon when players follow their predecessors independently of their own information[6]

Proposition 3.16. *The probability of herding goes to 1 as the number of players goes to infinity:*

$$\lim_{n\to\infty} Pr(Herding) = 1.$$

Proof. A herd is established if either of the players observe $P_i(2 \leq)$ or $P_i(\leq -1)$ which happens with probability 1. This statement is a direct consequence of the properties of the binomaial distribution. Once either of them is observed the herd can not be broken. ■

Proposition 3.17. *Assume that the number of players is large, the common prior is* $\mu_0 = \frac{1}{2}$ *and* $q \geq 0.563$. *Then*

$$Pr(HerdIn|\mu_0) \approx \frac{\mu_0^2}{1-\mu_0+\mu_0^2}$$

provides a close approximation of the probability of herding on action In.

Proof. Histories that lead to herding on In are the following:

$$(In, In), (In, Out, In, In), (In, Out, In, Out, In, In)...$$

and the probabilities of these histories respectively:

$$\mu_0^2, \mu_0(1-\mu_0)\mu_0^2, \mu_0^2(1-\mu_0)^2\mu_0^2, ...$$

Therefore, the probability of herding on In is the sum of the probabilities of these histories:

$$Pr(HerdIn|\mu_0) \approx \frac{\mu_0^2}{1-\mu_0+\mu_0^2}$$

This approximation is very close to the exact value of $Pr(HerdIn|\mu_0)$ even for small n as the elements of the sum are decreasing quickly. Moreover, this is also the expected fraction of consumers

[6]Bikhchandani et al. (1992) defines *information cascade* the same way as I refer to *herd behavior*. Because in this model information cascades is the cause of herd behavior this defintions are interchangeable.

buying the product. ■

The probability of herding on In and the probability that the number of consumers exceed the certain threshold are not the same; nevertheless these probabilities are very close to each other especially for large N. Proposition 3.17 provides some intuition about the market behavior of consumers. It shows that due to strategic complementarity in actions consumers are reluctant to enter the market. When the expected product quality of the new product equals status quo for $\mu_0 = \frac{1}{2}$ the probability that consumers switch is

$$Pr(HerdIn|\mu_0 = \frac{1}{2}) = \frac{1}{3}$$

This result provides an explanation for why it is hard for companies to enter markets with strategic complementarities. This model suggest that if consumers are prearranged in an order this hesitation to switch might be easily explained by a failure to convince early movers to enter the market. Therefore, companies should do everything to convince early movers because due to herding effects and complementarity it is critical for their success.

Corollary 3.18. *The probability of herding on not purchasing the product if its quality is good is:*

$$Pr(HerdOut|\theta = 1) \approx \frac{1-q}{1-(1-q)q}$$

Proof. The proof is similar to Proposition 3.17. Histories that lead to herding on In are the following:

$$(Out), (In, Out, Out), (In, Out, In, Out, Out)...$$

and the probabilities of these histories given $\theta = 1$:

$$1-q, q(1-q)^2, q^2(1-q)^3...$$

Therefore, the probability of herding on Out given $\theta = 1$ is the sum of the probabilities of these histories:

$$Pr(HerdOut|\theta = 1) \approx \frac{1-q}{1-(1-q)q}$$

■

For $q = \frac{3}{4}$ this probability becomes $\frac{4}{13}$ suggesting that even if the product is of good quality and the signal is quite precise the company fails to convince enough consumers to buy the product: *excess inertia* occurs. This result is a direct consequence of herding and strategic complementarities. Even if there would be enough information for consumers to realize that the company produces good product this information can not spread due to herding. Therefore, early signals make more influence on the final behavior of consumers. Because signals are not totally precise, this leads to herding on not purchasing the product with some probability. This result gives an alternative explanation to for *excess inertia* in markets with network externalities: herding blocks information revelation.

This section shows that companies face a difficult challenge by introducing a new product to a market where consumers enjoy network externalities only if enough of them buys the new product. A coordination problem arises: early players tend to hesitate to enter the market because they can not be certain that others will follow them. The next section focuses on how companies can overcome this challenge and how advertisements alter the decision of consumers.

4 Advertisement as a public signal of product quality

In reality firms use marketing efforts, especially advertisements in order to alter the decision of consumers. One usual way to model advertising is to assume that it serves as a signal of product quality. In this section I introduce public advertisments into the model in Section 3.

4.1 Setup

In order to model advertisement I introduce public signal θ_p which is observed by every player in the beginning of the game[7].

This assumption refers to non targeted advertisement efforts, for instance television ads or billboards[8]. In the model consumers observe a public signal and have some belief about its precision. This public signal represents advertisement and similarly to the private signal depends on the state of the world θ. In the beginning of the game players observe a public signal θ_p and then update their beliefs about θ accordingly. During the analysis I only focus on the case when this public signal equals 1. In the basic interpretation of the model this is realistic as a firm thinking of market entry would not use advertisement against the product or alternatively the -1 signal represents the case when the firm does not advertise. Moreover, this assumption only restrict the game to a given subset of information and does not influence the behavior of players in equilibrium, as all players observe the public signal of advertisement and it is common knowledge. The beliefs of players about the precision of the public signal are given by Table 3,

	$\theta_p = 1$	$\theta_p = -1$
$\theta = 1$	p	$1-p$
$\theta = -1$	$1-p$	p

Table 3: The distribution of the public advertisement signal

where $p = \frac{1}{2}$ means that players believe advertisement can not provide valuable additional infor-

[7]The most important assumption about the public signal is that it is common knowledge. Therefore, it also makes sense to assume that every player observes it just before her decision, the two are equivalent.

[8]In a strict sense there does not exist non targeted advertisement, as the probability of observing the advertisment is in every case higher for specific subgroups of the population. So, when I use the expression *non targeted advertisement* I refer to these widely available forms of marketing.

mation. Through the model I assume $p \geq \frac{1}{2}$, so that consumers believe that advertisements are not controversial as a signal of product quality θ. One possible interpretation is that the firm is certain that its product is of high quality ($\theta = 1$); however consumers do not fully believe marketing efforts due to several reasons. Firms with low product quality would also want to send the same advertising signal, so in order to fully account for advertisement the strategic motives of the firm should also be endogenous and accounted for in equilibrium. Nevertheless, parameter p incorporates some characteristics of the underlying game between the firm and its consumers. Another interpretation is that consumers observe a marketing competition between the status quo product and the new one and believe that the winner of the competition has the better product with probability p. The equilibrium concept and the solution procedure remains the same as in the basic model, the only difference is in the beliefs of players about θ which has several consequences.

Assumption 4.1. *θ_p and its distribution is common knowledge. Players observe it and know that others also observe it and that others also know that others observe it and so on.*

Proposition 4.2. *Under Assumption 4.1 introducing a public signal of advertisement θ_p is equivalent to a change in the common prior μ_0.*

$$Pr(\theta = 1|\mu_0, \theta_p = 1) = \frac{p * \mu_0}{p * \mu_0 + (1-p) * (1-\mu_0)}$$

Proof. Because θ_p becomes common knowledge it changes the common prior μ_0 for every player and this becomes also common knowledge.

$$\mu_0' = Pr(\theta = 1|\mu_0, \theta_p = 1) = \frac{Pr(\theta_p = 1|\theta = 1) * Pr(\theta = 1)}{Pr(\theta_p = 1)} = \frac{p * \mu_0}{p * \mu_0 + (1-p) * (1-\mu_0)}$$

■

4.2 Equilibrium of the game with advertisement as a public signal

Similarly to the notations in the basic model and in order to make analysis easier to follow. Lets introduce the following notations:

Notation 4.3. *Let denote $\bar{p}'$ the belief of Player 1 that a herd will emerge on state In if she ob-*

served positive private and public signals: $\bar{p}' = Pr(In > 0.5N|\theta_1 = 1, \theta_p = 1)$*. Similarly:* $\underline{p}' = Pr(In > 0.5N|\theta_1 = -1, \theta_p = 1)$.

Notation 4.4. *Let denote* $\bar{q}'$ *the belief of Player 1 that the product is of good quality if she observed positive private and public signals:* $\bar{q}' = Pr(\theta = 1|\theta_1 = 1, \theta_p = 1)$*. Similarly:* $\underline{q}' = Pr(\theta = 1|\theta_1 = -1, \theta_p = 1)$.

4.2.1 The signal of advertisement is more precise than private signals

Proposition 4.5. *If the public signal is more precise than the private signal,* $\theta_p = 1$ *and* $\mu_0 = \frac{1}{2}$ *every consumer plays* In *regardless of her signal leading to perfect herding on purchasing the product.*

Proof. Every consumer enters the market if the first player does for $\theta_1 = -1$. Player 2 can not infer the signal of Player 1, so she has to make the same decision if $\theta_2 = -1$. Hence, Player 2 will also choose In and a perfect herd emerge. Player 1 always enters the market if she knows that everyone will follow her which belief is correct in equilibrium. Therefore, it is enough to prove that the first consumer always enters the market with the belief that it generates a herd.

According to Proposition 3.7 the first consumer does not enter the market even for $\theta_1 = -1$ and commor prior μ_0 if (6) is satisfied. Similarly, Player 1 always enters if:

$$\underline{p}' \geq 3 - 4\underline{q}'$$

Player 1 knows that by purchasing the product for $\theta_1 = -1$ she generates a herd, therefore $\underline{p} = 1$ and the previous equation becomes:

$$\underline{q} \geq \frac{1}{2}$$

In order to prove the proposition it is enough to show that this inequality is satisfied. Due to Proposition 4.2 a public signal $\theta_p = 1$ changes the common prior to:

$$\mu_0' = \frac{p * \mu_0}{p * \mu_0 + (1-p) * (1-\mu_0)} = p$$

then using Bayesian updating given μ_0':

$$\underline{q}' = \frac{(1-q)*\mu_0'}{(1-q)*\mu_0' + q*(1-\mu_0')}$$

Substituing μ_0' and some algebra gives that $\underline{q} \geq \frac{1}{2}$ if

$$\frac{(1-q)p}{(1-q)p + q(1-p)} \geq \frac{1}{2}$$
$$p \geq q$$

which is the condition needed to be proven. ■

Corollary 4.6. *If the public signal is more precise than the private signal of players herding on purchasing the product occurs with probability 1.*

Proof. This statement is a direct consequence of Proposition 4.5. ■

Corollary 4.7. *A very precise advertisement signal prevents social learning.*

Proof. A very precise advertisement signal overwhelms a negative private signal for every player; therefore, everyone enters the market. Even if there is enough information about the true state of the world consumers do not have the chance to socially learn about it as private signals can not be infered by the actions of players. ■

The firm clearly benefits from marketing efforts that generate strong belief in advertisement. A very precise public signal overwhelms any information carried in the private signals leading to perfect herding on buying the product. Consumers correctly infer from the information available that the product is probably of high quality and do not hesitate to enter the market. As a result, consumers always switch from status quo and *inertia* does not emerge.

4.2.2 Advertisement is only a weak signal of product quality

The more interesting and realistic case is when the precision of the advertisement signal is weak and hence, consumers do not herd with probability 1 on entering the market. The behavior of consumers can be analyzed and the equilibrium of the game can be found in the same way as of the basic game in Section 3 which allows to use the same argumentation intensively. As I will show only the behavior of Player j's when j is an even number differs. Nevertheless, advertisements make significant differences in the behavior of consumers in favour of the firm: consumers are much more willing to purchase the product.

Theorem 4.8. *The following strategies constitute a Perfect Bayesian Equilibrium under Assumptions 3.1, 3.5 and 3.8 if $p > \frac{1}{2}$. The best response strategy of Player i if i is an odd number in equilibrium is the following:*

$$a_i = \begin{cases} In & \textit{if Player i observed } P_i(2 \leq) \textit{ regardless} \theta_i \\ In & \textit{if Player i observed } P_i(0) \textit{ and} \theta_i = 1 \\ Out & \textit{if Player i observed } P_i(0) \textit{ and} \theta_i = -1 \\ Out & \textit{if Player i observed } P_i(\geq -2) \textit{ regardless} \theta_i \end{cases}$$

and the best response strategy of Player j if j is even odd number in equilibrium is the following:

$$a_j = \begin{cases} In & \textit{if Player j observed } P_j(1 \leq) \textit{ regardless} \theta_j \\ In & \textit{if Player j observed } P_j(-1) \textit{ and} \theta_j = 1 \\ Out & \textit{if Player j observed } P_j(-1) \textit{ and} \theta_j = -1 \\ Out & \textit{if Player j observed } P_j(\geq -3) \textit{ regardless} \theta_j \end{cases}$$

Proof. In equilibrium Player 1 generates a herd by entering the market for $\theta_1 = 1$. Therefore, $\bar{p}' = 1$ and Player 1 enters the market for if $\bar{p}' \geq 3 - 4\bar{q}$ which is always satisfied because $\bar{q}' \geq \frac{1}{2}$. Proposition 3.7 shows that for $\theta_1 = -1$ Player 1 does not purchase the new product if $\underline{p}' \leq 3 - 4\underline{q}'$. The proof of Proposition 4.5 shows that $\underline{q}' \leq \frac{1}{2}$ if $p \geq q$. Thus, $3 - 4\underline{q}' \geq 1$ and $\underline{p}'$ is always smaller or equal 1 as it denotes a probability. Every odd player after Player 1 with $P_i(0)$ faces the same situation and makes the same decision. In equilibrium herding emerges on In even after one action In; therefore Players observing $P_i(2 \leq)$ follow the herd and purchase the product.

Player 2 clearly enters the market for $P_2(1)$ and $\theta_2 = 1$. If $P_2(1)$ and $\theta_2 = -1$ or $P_2(-1)$

and $\theta_2 = 1$ her belief about the product being good equals the precision of the public signal p. The easier way to see this is by imagining that Player 2 first observed her private signal and then updated her belief because of the public signal (the timing of signals does not matter if consumers use Bayesian updating). In equilibrium one additional positive signal initiates a herd on entering the market and everyone knows this, hence Player 2 purchases the product if it has better expected quality because it is certain that the treshold of consumers will be reached. Thus, for $P_2(1)$ and $\theta_2 = -1$ Player 2 enters the market if $p > \frac{1}{2}$.

For $P_2(-1)$ and $\theta_2 = 1$ Player 2 enters the market if $E[In|P_2(-1), \theta_{2,p} = 1] \geq 0$:

$$Pr(HerdIn|P_2(-1), \theta_{2,p} = 1) * [Pr(\theta = 1|P_2(-1), \theta_{2,p} = 1) * 1 + Pr(\theta = -1|P_2(-1), \theta_{2,p} = 1) * (-1)] +$$
$$+(1 - Pr(HerdIn|P_2(-1), \theta_{2,p} = 1)) * \left[Pr(\theta = 1|P_2(-1), \theta_{2,p} = 1) * \frac{1}{2} + Pr(\theta = -1|P_2(-1), \theta_{2,p} = 1) * (-\frac{3}{2})\right] \geq 0$$

or equivalently:

$$Pr(HerdIn|P_2(-1), \theta_{2,p} = 1)(4p - \frac{5}{2}) + \frac{3}{2} - 2p \geq 0$$

which is always satisfied in equilibrium if $p \geq \frac{1}{2}$ and if the probability of herding conditional on Player 2's information set exceeds $\frac{1}{3}$ (Appendix 8.1.3). This probability is approximately the ex-ante probability of herding on In and Proposition 3.17 proves that the condition is satisfied.

An easier way to see this is that the probability of herding is approximately $\frac{1}{3}$ without advertisement, and advertisement certainly increases this probability. Therefore, Player 2 purchases the product for $P_2(-1)$ and $\theta_2 = 1$. For $P_2(-1)$ and $\theta_2 = -1$ Player 2 plays Out. Player 2's belief about product quality is less than Player 1's after P1 observed a positive public and a negative private signal. Hence, Player 2 does not purchase the product neither. Every even player after Player 2 faces the same situation and makes the same decision. Players observing $P_j(\geq -3)$ follow the herd and play Out. ■

4.3 Herding with advertisement

Proposition 4.5 and 4.8 shows that herding on purchasing the product is more probable in the model with advertisement. Moreover, the effect of advertisement on herding is stronger for products with strategic complementarities.

Proposition 4.9. *Assume that the number of players is large, the common prior is μ_0 and there is public advertisement. Then if the public signal is more precise than the private the probability of herding on purchasing the product is 1. If the private signal is more precise (i.e. $p \leq q$)*

$$Pr(HerdIn|\mu_0 = \frac{1}{2}, \theta_p = 1) \approx \frac{pq + (1-p)(1-q)}{1 - pq + (1-p)(1-q) + (pq + (1-p)(1-q))^2}$$

provides a close approximation of the probability of herding on action In.

Proof. Corollaly 4.6 states that herding on entering the market is certain if the public signal is more precise. Histories that lead to herding on In are the following:

$$(In), (Out, In, In), (Out, In, Out, In, In)...$$

and the probabilities of these histories respectively:

$$pq + (1-p)(1-q), (p(1-q) + (1-p)q)(pq + (1-p)(1-q))^2, ...$$

Therefore, the probability of herding on In is the sum of the probabilities of these histories:

$$Pr(HerdIn|\mu_0, \theta_p = 1) \approx \frac{pq + (1-p)(1-q)}{1 - pq + (1-p)(1-q) + (pq + (1-p)(1-q))^2}$$

■

This approximation is very close to the exact value of $Pr(HerdIn|\mu_0, \theta_p = 1)$ even for small N as the elements of the sum are decreasing quickly. The probability of herding on In and the probability that the number of consumers exceed the certain threshold are not the same, nevertheless these probabilities are very close to each other especially for large N. As the probability of purchasing the product is increasing in the precision of the public signal, a lower bound for the probability of herding on In might be calculated by substituing $p = \frac{1}{2}$ to the inequality. For $p = \frac{1}{2}$ this expression becomes:

$$Pr(HerdIn|\mu_0 = \frac{1}{2}, \theta_p = 1) = \frac{q}{1 - q + q^2} \geq \frac{2}{3}$$

and the ex-ante probability of herding depends on the precision of advertisement signals. The lower bound is calculated by fixing the precision of the private signal q at $\frac{1}{2}$. Introducing advertisements increases the probability of herding due to two reasons, it changes the equilibrium behavior of consumers and it raises the common prior. The second effect depends on whether advertisements actually provide information or just consumers believe it does. Weakly precise advertisement signals change the probability of herding on purchasing the product from $\frac{1}{3}$ to at least $\frac{2}{3}$. Intuitively, advertisement changes the behavior of consumers who are reluctant to enter the market due to the risks of belonging to the minority.

Corollary 4.10. *The probability of herding on Out if $\theta = 1$ and θ_p=1:*

$$Pr(HerdOut|\theta = 1, \theta_p = 1) = \frac{(1-q)^2}{1-(1-q)q}$$

Proof. The proof is similar to Proposition 4.9 and can be found in Appendix 8.1.2. ■

For $q = \frac{3}{4}$ this probability becomes $\frac{1}{13}$, whereas it was $\frac{4}{13}$ without advertisement. A good firm is able to increase the probability of success significantly by using advertisement.

Corollary 4.11. *Advertisement signals have a stronger influence on the behavior of consumers in markets with strategic complementarities, in the sense that it changes the probability of herding by a larger extent.*

Proof. In order to prove this statement, first the benchmark model has to be specified[9]. The two models differ only in their payoffs, in the benchmark model consumers get θ if they enter the market, whereas they get 0 if they do not. This benchmark model is equivalent to the basic setup in Bikhchandani et al. (1992), therefore the probability of herding without advertisement is the same on both actions: $\frac{1}{2}$. Table 4 presents the probabilities of herding on purchasing the product under different conditions, with the basic assumption that $\mu_0 = \frac{1}{2}$. The last two rows of Table 4 show that the effect of advertisement on herding is larger in the model with strategic complementarities. ■

[9]A more detailed analysis of the benchmark model is presented in Appendix 8.1.5.

$Pr(HerdIn)$	Benchmark model	Strategic complementarities
Without advertisement	$\frac{1}{2}$	$\frac{1}{3}$
With weak advertisement	higher than $\frac{2}{3}$	higher than $\frac{2}{3}$
With strong advertisement	1	1
Effect of weak advertisement	higher than $\frac{1}{6}$	higher than $\frac{1}{3}$
Effect of strong advertisement	$\frac{1}{2}$	$\frac{2}{3}$

Table 4: The effect of advertisement signals on herding

The main difference between the two models is that consumers who are indifferent behave differently, with strategic complementarities they do not enter the market as they are afraid of the punishment if there is not enough consumers entering the market, whereas without strategic complementarities they use mixed strategies with equal probability on purchasing and not the product. Advertisement breaks this indifference and convinces consumers to initiate a herd. It also increases the belief of consumers about the probability of herding which only matters with strategic complementarities. The result is a significant growth in the probability of purchasing the product.

4.4 The social welfare effects of advertisement

Intuitively, providing more precise information to every consumer should increase their welfare. Indeed, every player welcomes more precise information as it increases their payoffs in expectation, however, the welfare effects might be controversial. If an information cascade occurs it blocks information revelation and in the end of the game the available information is less for the community of players. Therefore, more precise information in the beginning of the game might hurt subsequent players by preventing them to learn from the behavior of their predecessors.

In order to examine social welfare effects of advertisement I compare the welfare of consumers for different precision of advertisement. I will focus on the case when $\theta_p = 1$ and calculate the ex-ante expected payoff of consumers. The equilibrium behavior depends on the relation of p and q which is taken into consideration.

Assumption 4.12. *Consumers belief about the precision of the public advertisement signal matches reality.*

Proposition 4.13. *The expected payoff given $\theta_p = 1$ for $p > q$ and $p < q$ is*

$$2p - 1, \frac{p+q-1}{1-q+q^2}$$

and for $p = q$ the left hand side is smaller if $q \in (\frac{1}{2}, 1)$.

Proof. The proofs are presented in Appendix 8.1.4. ■

Corollary 4.14. *More precise public signal might lead to social welfare loss.*

Proof. Proposition 4.13 indicates that around $p = q$ social welfare is decreasing in p. Figure 1 presents the expected payoff of players for $q = \frac{3}{4}$ and different values of p. The figure shows

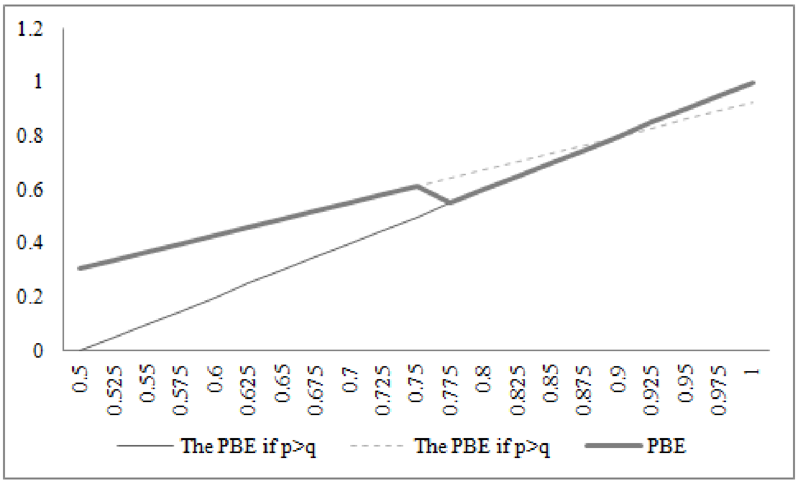

Figure 1: Expected payoffs

a significant decrease in expected payoffs around $p = \frac{3}{4}$. The payoff of the equilibrium when every player purchases the product exceeds the one for $p < q$ only for large p (for $q = \frac{3}{4}$ around $p = 0.9$). ■

Due to a very precise positive advertisement signal early consumers purchase the product by overwhelming their possibly negative private signal. This behavior prevents subsequent consumers from learning the signal of their predecessors and completely blocks social learning. In contrast, a weak advertisemnt signal also leads to herding with high probability, however, it offers an opportunity to at least some social learning which increases social welfare if the precision of the advertisement signal does not exceeds the precision of the private signal significantly.

5 Autonomous consumers

During the analyis in the basic model and the extension with advertisement I assumed that each consumer is sophisticated: they predict the behavior of subsequent players given their beliefs, infer signals from actions and use Bayesian updating intensively and correctly. However, in reality there might be heterogeneity among consumers in the extent to which they care about the behavior of others or behave rationally. Some consumers might decide based on their own signal without making steps to observe the action of other consumers or to try to infer their behavior. In this section I introduce a departure from strict rationality and examine the consequences on the behavior of players and on the final outcome of the game. Bikhchandani et al. (1992) shows that after an information cascade is established an agent with more precise information might be able to break it. As I will show autonomous consumers are able to break herd behavior as well.

5.1 Autonomous players in the basic model

Definition 5.1. *Autonomous player.*
Autonomous players do not make inferences from the action of predecessors and think that everyone else behaves autonomously.

Definition 5.2. *Sophisticated player.*
Sophisticated players observe the behavior of others and make inference based on these actions according to Bayes' rule.

Autonomous players maximize their payoffs according to the belief based on only their own signals and information. The belief of autonomous players about how others behave might be defined in several ways. In the definition I assumed that autonomous players believe that everyone else is autonomous as well. Another potential belief is that everyone else chooses the same action as they do. As I will show, these two type of beliefs result in the same behavior. Sophisticated players use the information they can get by inference to update their beliefs and to predict the behavior of subsequent players.

Assumption 5.3. *The fraction of autonomous players is γ. Sophisticated consumers do not know* *whether a specific player is sophisticated or autonomous.*

Corollary 5.4. *Autonomous players always follow their own signal if $q > \frac{1}{2}$: for a positive signal they purchase the product, whereas for a negative signal they do not.*

Proof. Autonomous players believe that the probability that the product is good after they observed a positive signal is $q > \frac{1}{2}$. First, start with the belief in the definition: they assume that everyone else after them behaves autonomously. The expected fraction of subsequent consumers getting $\theta_i = 1$ is q, therefore if the number of players is large and $q > \frac{1}{2}$ the probability that there will be enough consumers to enjoy network externalities equals 1. Hence, the expected value of purchasing the product is:

$$1 * [q * 1 - (1 - q)(-1)] = 2q - 1 > 0$$

and autonomous consumers with $\theta_i = 1$ always enter the market. For $\theta_i = -1$ the expected value of playing In is always smaller than 0 and they choose Out. With the belief that everyone else chooses the same action as they do it is easy to see that autonomous players follow their own signal: just their belief about product quality matters. ■

This concept of autonomous players shares some characteristics with inferential naivite introduced by Eyster and Rabin (2010). BRTNI players best respond to the belief that their precedessors followed their own signal. BRTNI players assume that other players follow their own signal, just as autonomous players do. Corollary 5.4 shows that autonomous players follow their own signal in this setup.

Assumption 5.5. *A sophisticated Player 1 follows her signal: purchases the product if and only if $\theta_1 = 1$.*

This assumption is similar in spirit to Assumption 3.6 and it also has similar consequences. The most important reason behind this assumption is to make sure that perfect herding can not occur in the basic setup. First, I find the equilibrium of the game given this assumption, then I show what conditions have to be satisfied in order Assumption 5.5 to hold. Because this condition might be easily satisfied, with this procedure I prove that the assumption holds under equilibrium, hence contradictions does not arise.

Proposition 5.6. *In the model with autonomous players if Assumption 5.5 is satisfied sophisticated consumers are able to find out the true value of the state θ. Herding by sophisticated players on the wrong state occurs with probability 0. Sophisticated consumers almost always choose to purchase the product for $\theta = 1$ in the long run.*

Proof. A usual result of herding models is that the only case when players do not to learn the true value of θ is when an information cascade occurs on the wrong state and blocks information revelation. I show that every false information cascade is broken, therefore players learn the quality of the product with probability 1. I prove the proposition for $\theta = 1$. For $\theta = -1$ the proof would be similar.

Due to Assumption 5.5 the first sophisticated player follows her signal which is a satisfactory condition to prove the statement. According to Corollary 5.4 autonomous players always follow their signals. The optimal behavior of sophisticated players are complicated, however can be described by trigger strategies: for high enough beliefs they always purchase the product. The assumption that Player 1 enters the market ensures that if players before observing their signal have $\mu_i = \mu_0$ they enter the market for a positive signal. The consequence of the assumption is that players follow their own signal if μ_i is close to μ_0.

Theorem 3.14 shows the equilibrium of the basic model; sophisticated players follow their own signal until a herd is initiated. Because autonomous consumers do the same, the behavior of consumers is the same before a herd is established and sophisticated players can not recognize autonomous players. After a herd is initiated on Out, i.e. after a sophisticated player did not follow her own signal $\theta_i = 1$, the behavior of naive and sophisticated consumers differ for $\theta_i = 1$.

A sophisticated consumer observing a herd on Out has more information than someone who initiated the herd because in this case an action Out carries some information about naives: with some probability Out happened because a autonomous player observed a negative signal. However, this information is much weaker than the information content of a private signal. In contrast, an action $a_i = In$ reveals that Player i is naive and that $\theta_i = 1$. Naive players always provide new information. Sophisticated players break the herd if their belief about θ reaches a threshold which happens with probability 1. If it did not happen after observing a single In action they still follow the herd. An action In again reveals a positive private signal and after a while sophisticated players start to follow their own signals. Because the true state of the world is $\theta = 1$ consumers observe good signals with higher probability, hence sophisticated agents from the overwhelming number of action In infer that the signal is of good quality and herd on In.

Herds on the true state break if enough consecutive negative signals are inferred. This happens with probability 1 due to the properties of binomial distribution. Any number of consecutive negative signals happen with positive probability and a herd is always broken if this number reaches a treshold. Nevertheless, in the long run the law of large numbers ensures that the fraction of positive signals is close to its expected value q and sophisticated consumers almost always choose to purchase the product. The bigger the fraction of naive players is the faster consumers learn the true product quality. ■

Corollary 5.7. *A sophisticated Player 1 enters the market for a positive signal if* $q \leq \frac{3}{5}$*. In equilibrium Assumption 5.5 is equivalent with this statement.*

Proof. Proposition 5.5 shows that sophisticated consumers always herd on the true state of the world. If $\theta = 1$ sophisticated consumers herd on purchasing the product. Because the expected fraction of naives playing In equals Player 1 belief about $\theta = 1$: q[10], the expected number of consumers certainly exceeds the threshold needed to benefit from network externalitities. Therefore, Player 1 correctly infers that

$$\bar{p} = Pr[In > 0.5N|\theta_1 = 1] = Pr[\theta = 1|\theta_1 = 1] = q$$

i.e. the probability that Player 1 benefits from network externalities given a positive signal equals her belief about the probability that $\theta = 1$ given a positive signal. According to Proposition 3.6 Player 1 enters the market for $\mu_0 = \frac{1}{2}$ if:

$$\bar{p} \geq 3 - 4q$$
$$q \geq 3 - 4q$$
$$q \geq \frac{3}{5}$$

[10]See Appendix 8.2.2 for the properties of binomial distribution.

■

With autonomous players the private signal has to be more precise for Player 1 to enter the market. Because there are strategic complementarities Player 1 takes into consideration the probability that she benefits these network externalities. As Player 1 wants to avoid entering the market if subsequent players do not follow her, the belief μ_1 about the probability of a good product have to be strong enough. Player 1 has exactly the same μ_1 with autonomous players. If Player 1 purchases the product subsequent sophisticated consumers are much more likely to purchase it, however, Player 1 does not have any influence on naive players. Without naive players Player 1's influence on the final outcome is stronger. Therefore, Player 1 benefits network externalities with smaller probability and she is more reluctant to enter the market.

The predictions of the model with autonomous players is able to match reality better. Some players buy the product independently of the behavior of other players, while some of them stick to the status quo. Consumers in the end find out product quality while it may take significant time. The model also captures the intuition that early consumers in markets with strategic externalities might be reluctant to enter the market even if their private information indicates so if they do not trust enough this information.

5.2 Advertisement with autonomous players

The definition of autonomous players can be applied to the model with advertisement as well. Naive players observe the public signal; therefore their decision is based on the common prior, their private signal and the advertisement signal. Proposition 5.8 characterizes their behavior.

Proposition 5.8. *Autonomous players always purchase the product if $\theta_p = 1$ and the advertisement signal is more precise than their private signal: $p > q$. Otherwise, they follow their own signal.*

Proof. Just as previously I only focus on the case when $\theta_p = 1$. Due to Corollary 5.4 autonomous players purchase the product if their belief exceeds $\frac{1}{2}$. This is always the case for $p > q$ even if their private signal is negative, therefore they purchase the product if everyone else does. They believe that every player thinks the same way and enter the market. If $p < q$ and their private signal is also positive they clearly purchase the product. If their private signal is negative their belief of θ is smaller than $\frac{1}{2}$ and they do not purchase the product. ■

Corollary 5.9. *In the case when the advertisement signal is more precise than the private signal and $\theta_p = 1$ every consumer purchases the product.*

Proof. The statement is a direct consequence of Propositions 4.5 and 5.8. The behavior of sophisticated consumers does not change as they correctly infer that autonomous consumers always buy the product as well. ■

Corollary 5.10. *Assume that a sophisticated Player 1 purchases the product for a positive private signal. Then, if the private signal is more precise than the advertisement signal sophisticated consumers are able to find out the true value of the state θ. Herding by sophisticated players on the wrong state occurs with probability 0. Sophisticated consumers almost always choose to purchase the product for $\theta = 1$ in the long run.*

Proof. The proof is similar to the proof Proposition 5.6. Because autonomous players always follow their own signal they provide new information which is enough for sophisticated consumers to learn the true product quality. ■

Corollary 5.11. *Player 1 purchases the product for a positive private signal if her belief*

$$\bar{q}' \geq \frac{3}{5}$$

where $\bar{q}' = Pr[\theta = 1|\theta_{1,p} = 1]$.

Proof. This statement is a consequence of Corollary 5.7 and Proposition 3.6. The belief of Player 1 that the number of consumers exceeds the treshold in order to benefit network externalities equals her belief about θ because consumers learn the true product quality. According to Proposition 3.6 Player 1 enters the market if:

$$\begin{aligned} Pr[In > 0.5N|\theta_{1,p} = 1 &\geq 3 - 4\bar{q} \\ \bar{q}' &\geq 3 - 4\bar{q}' \\ \bar{q}' &\geq \frac{3}{5} \end{aligned}$$

■

Therefore, for Player 1 to enter the market for $\theta_1 = 1$ it is enough if the private and the public signal provide enough information, i.e. if the sum of their precision is high enough. Because all sophisticated players know that social learning will lead to herding on the true state of the world: a weakly precise advertisement does not change significantly the behavior of sophisticated consumers: it makes a difference only if the private signal is very unprecise. Due to social learning a firm with a good product quality always succeeds in the long run.

6 Conclusions, considerations and possible extensions

The results of this thesis indicate that introducing payoff externalities influence the behavior of consumers: it decreases significantly the probability of herding on purchasing the product. Early players are reluctant to enter the market because they are afraid of being the only consumers of the new product. The signal of the first player has to be precise enough; otherwise she does not take this risk. Even if the product is of good quality the failure of social learning might lead to excess inertia; consumers herd on not purchasing the product. Therefore, the model provides an alternative, herding based explanation why even firms with good product quality face a significant entry barrier in markets with network externalities.

Informative advertising is a very efficient tool in overcoming the excess inertia of consumers, a positive informative advertisement changes the belief of players that the product is of good quality and also increases the belief of early movers that others will follow their entry decision. However, a very precise advertising signal causes every consumer herding on purchasing the product totally preventing social learning. Hence, increasing the precision of the advertisement signal might hurt players.

The assumption that consumers are heterogenous to the extent to which they take into consideration the behavior of others makes the results of the model more realistic. Autonomous consumers care only about their own signals, therefore they never follow the herd and always break information cascades. Early players have less influence on the final outcome and thus early sophisticated players hesitate even more to enter the market. Even a weak advertisement signal might help to oversome this hesitation, if the sum of the precision of the private and public signal is high enough sophisticated agents follow their own signal and learn the true state of the world in the long run. Thus, in this case firms with good product quality always succeed.

The modeling setup I used is able to capture the characteristics of markets with network externalities, several motives of consumers and their consequences on behavior. However, several assumptions might seem too strict and loosening them could lead to significant improvements. First of all, the form of payoff externality is specific, and was introduced with the movitation to reflect certain characteristics of markets with network externalities where a consumer base have to be installed in order to enjoy network externalities. This assumption limits the possible range of markets for which my results can be applied.

The binary signal structure might seem too restrictive: consumers might care about the extent to which positive signals differ suggesting richer signal space. For instance, customers usually evaluate products in a 1-to-5 rating scale. In rational herding models like Bikhchandani et al. (1992) richer signal spaces usually prevent herding; however Dasgupta (2000) shows that with strong payoff externalities herding might occur. Probably the same would be true in the setup I used: the assumption that at least half of the consumers have to choose the new product to benefit positive payoff externalities provides a strong incentive for herding. In this paper the impact of advertising is not continous; only when it succeds to lift beliefs above a certain threshold does it have significant effects. Due to richer signal spaces this result might be changed and the effect of advertising on behavior may be weaker.

Similarly to standard herding models, I assumed that the order of consumers is exogenous. Chari and Kehoe (2003) assume that the order of signals is exogenous, i.e. in every round one of the consumers gets a private signal, and every consumer in every round decides whether to enter the market or not (in their case invest or not). In every period the aggregate amount of investments is observed and investors might invest or wait for more information. This endogeneous order of investors makes analysis more complicated and also results in herding behavior. The endogeneous timing assumption is more realistic; however, the results on herding do not change dramatically. Advertisemens probably would have the same effect: early players would not hesitate that much to enter the market, therefore the probability of herding on purchasing the product would increase significantly.

Consumers might be heterogenous in their beliefs about the precision of the advertisement signal, i.e. they might react differently to advertisements. For instance, some consumers may not think that advertisements carry relevant information (*nonbelievers*) whereas others might totally believe in it (*believers*). In this case, consumers can not know whether a specific player whose action they observed decided to buy the product due to her signal or due to advertisements. An interesting case would be when some *believers* always purchase the product due to advertisements. Then, *nonbelievers* observing an action In can not know whether it was made by a *nonbeliever* with a positive signal or a *believer*. Whereas and action Out would fully reveal the identity and signal of the player. Therefore, the information content of an action In is significantly smaller and a longer line of actions In is necessary to initiate a herd. Hence, *nonbelievers* herd in In with smaller probability. An interesting question to consider is whether this phenomena could make advertisements controversial, i.e. whether it could make the firm worse off.

In the models I developed information spreads only through actions, consumers are not allowed to use any other forms of communication. This assumption reflects the widely shared view in economics that interests make communication unreliable (cheap talk). This problem with direct communication certainly arises in this environment with strategic complementarities in payoffs; every consumer who purchased the product is interested in convincing others to enter the market. However, in reality, consumers communicate in many channels: they talk to each other, they evaluate products and they write customer reviews. This communication is not always motivated by interests, as in reality consumers migth be honest, especially if the incentives for dishonesty is weak. This communication could make information spreading much more efficient, especially if consumers who purchased the product share their experiences. Without biases this would lead to efficient social learning: agents would quickly learn the true product quality. However, there are several biases present altering consumer experience and communication which would be an interesting area for further analysis. For instance, expectations might influence experience with the product and alter evaluation due to several reasons. With reference dependent preferences the utility of consumers is influenced by expectations (Kőszegi and Rabin, 2006). Due to cognitive dissonance, when experience is below expectations consumers might overvalue their experience to the level of their expectations[11].

Customer reviews provide some information on product quality without perfect precision. Naive consumers might be assumed to suffer from several biases and communicate their experience honestly. Observers can not know the identity or interests of specific evaluators and sometimes only an aggregate score of evaluations is available. These constitute significant barriers to social learning and the implications for markets with strategic complementarities would be interesting to discover.

[11]Consumers experience cognitive dissonance when their expectations or beliefs and experience differs and try to eliminate this dissonance by modifying each of them. Bayesian updating may be considered as an extreme case when only beliefs are altered.

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8 Appendix

8.1 Proofs for several statements

8.1.1 The proof of Proposition 3.7

Proposition 3.7. *Under Assumption 3.5 Player 1 plays Out for* $\theta_1 = -1$.

Proof. Player 1 compares the expected value of playing *In* and 0, the expected value of playing *Out*. Let denote $\underline{q} = Pr(\theta = 1|\theta_1 = -1)$.

$$\begin{aligned} E[In|\theta_1 = -1] = Pr(In > 0.5N|\theta_1 = -1)[1 * Pr(\theta = 1|\theta_1 = -1) - 1 * Pr(\theta = -1|\theta_1 = -1)] + \\ [1 - Pr(In > 0.5N|\theta_1 = -1)][0.5 * Pr(\theta = -1|\theta_1 = -1) - 1.5 * Pr(\theta = -1|\theta_1 = -1)] \\ = \underline{p}[\underline{q} - (1 - \underline{q})] + (1 - \underline{p})[0.5\underline{q} - 1.5(1 - \underline{q})] \leq 0 \end{aligned}$$

$$\underline{p} \leq 3 - 4\underline{q} \tag{6}$$

For $\mu_0 = 0.5$ this expression simplifies as $\underline{q} = 1 - q$. Otherwise

$$\underline{q} = Pr(\theta = 1|\theta_1 = -1) = \frac{Pr\theta_1 = -1|\theta = 1) * Pr(\theta = 1)}{Pr(\theta_1 = -1)} = \frac{(1 - q) * \mu_0}{(1 - q) * \mu_0 + q * (1 - \mu_0)} \tag{7}$$

Substituing back (7) to (6) and after some algebra:

$$\frac{1 + \underline{p}}{3 - \underline{p}} \leq \frac{q}{1 - q}\frac{1 - \mu_0}{\mu_0}$$

Player 1 plays *Out* if Assumption 3.5 is satisfied. ■

8.1.2 Proof of Corollary 4.10

The probability of herding on not purchasing the product if its quality is good and there is positive and weak public advertisement:

$$Pr(HerdOut|\theta = 1, \theta_p = 1) \approx \frac{(1 - q)^2}{1 - (1 - q)q}$$

Proof. The proof is similar to Proposition 3.17. Histories that lead to herding on Out are the following:

$$(Out, Out), (In, Out, Out, Out), (In, Out, In, Out, Out, Out)...$$

and the probabilities of these histories given $\theta = 1$ and $\theta_p = 1$:

$$(1-q)^2, q(1-q)^3, q^2(1-q)^4, ...$$

Therefore, the probability of herding on Out given $\theta = 1$ and $\theta_p = 1$ is the sum of the probabilities of these histories:

$$Pr(HerdOut|\theta = 1, \theta_p = 1) \approx \frac{(1-q)^2}{1-(1-q)q}$$

■

8.1.3 Proof of Proposition 4.8

In Proposition 4.8 the equilibrium strategy of Player 2 is In for $P_2(-1)$, $\theta_{2,p} = 1$ if the following inequality is satisfied:

$$E[In|P_2(-1), \theta_{2,p} = 1] = Pr(HerdIn|P_2(-1), \theta_{2,p} = 1)(4p - \frac{5}{2}) + \frac{3}{2} - 2p \geq 0$$

1. If $4p - \frac{5}{2} > 0$, i.e. if $p > \frac{5}{8}$:

$$\begin{aligned} Pr(HerdIn|P_2(-1), \theta_{2,p} = 1) &\geq \frac{2p - \frac{3}{2}}{4p - \frac{5}{2}} \\ &\geq \frac{1}{2} - \frac{1}{16p - 10} \end{aligned}$$

   and for $p \geq \frac{5}{8}$: $\frac{1}{2} - \frac{1}{16p-10} \geq \frac{1}{3}$.

2. if $4p - \frac{5}{2} = 0$, i.e if $p = \frac{5}{8}$: $\frac{3}{2} - \frac{5}{4} > 0$, so the inequality is satisfied.

3. If $4p - \frac{5}{2} < 0$, i.e. if $p < \frac{5}{8}$:

$$Pr(HerdIn|P_2(-1), \theta_{2,p} = 1) < \frac{2p - \frac{3}{2}}{(4p - \frac{5}{2}} \\ < \frac{1}{2} - \frac{1}{16p - 10}$$

and $\frac{1}{2} - \frac{1}{16p-10} > 1$ if $\frac{1}{2} < p < \frac{5}{8}$.

Therefore the inequality is always satisfied.

8.1.4 Proof of Proposition 4.13

Proof. Start with the case for $p > q$. Then Proposition 4.5 shows that players herd on In with probability 1, therefore, their expected payoff depends only on the common prior. For $\theta = 1$ the common prior equals the precision of the public signal: $\mu_0' = p$. Thus the expected payoff is:

$$p * 1 + (1 - p) * (-1) = 2p - 1$$

For $p < q$ the calculation is more complicated. The probability of herding in expectation is 1, and if consumers herd on Out they get payoff 0 independently of product quality. Hence, it is enough to calculate the probability of histories that lead to herding on In for $\theta = 1$ and $\theta = -1$ and then calculate their weighted average given μ_0'. Histories that lead to herding on In are the following:

$$(In), (Out, In, In), (Out, In, Out, In, In)...$$

and the probabilities of these histories:

$$\theta = 1 : q, (1 - q)q^2, (1 - q)^2 q^3, ...$$
$$\theta = -1 : 1 - q, q(1 - q)^2, q^2(1 - q)^3, ...$$

Fvery history yields 1 or -1 depending on the product quality for every player. Therefore, the expected payoff for $\theta = 1$ and $\theta = -1$ respectively:

$$\frac{q}{1-(1-q)q}, (-1)\frac{1-q}{1-(1-q)q}$$

The expected payoff is the weighted average of the two expressions above:

$$p * \frac{q}{1-(1-q)q} + (1-p)(-1)\frac{1-q}{1-(1-q)q} = \frac{p+q-1}{1-(1-q)q}$$

It is easy to show that around $p = q$ the switch to an equilibrium when every consumer purchases the product can not increase social welfare. In order to prove this lets compare the expected payoff of the two equilibria $2p - 1$ and $\frac{p+q-1}{1-(1-q)q}$ for $p = q$:

$$2q - 1 \geq \frac{2q-1}{1-(1-q)q}$$
$$q(2q^2 - 3q + 1) \geq 0$$

which is always satisfied if $q \in (\frac{1}{2}, 1)$. ■

8.1.5 The benchmark model without strategic complementarities

The benchmark model without strategic complementarities in actions is equivalent to Bikhchandani et al. (1992). I will examine th effects of advertisement in the basic BHW model in order to provide comparison. A strong advertising signal makes every consumer purchase the product; hence it leads to herding with probability 1. Introducing weakly precise ($\frac{1}{2} < p < q$) advertisement leads to herding with probability higher than $\frac{2}{3}$ on purchasing the product without payoff externalities. The only criteria for a consumer to purchase the product is her belief to exceed $\frac{1}{2}$. The histories that lead to herding on purchasing the product are the followings

$$(In), (Out, In, In), (Out, In, Out, In, In)...$$

which happens with probability $\frac{q}{1-q+q^2} \geq \frac{2}{3}$.

8.2 Some results in probability theory

8.2.1 Calculations of conditional probabilities

Bayes' rule:

$$Pr(A|B) = \frac{Pr(A \bigcap B)}{Pr(B)} = \frac{Pr(B|A) * Pr(A)}{Pr(B|A) * Pr(A) + Pr(B|A^c) * Pr(A^c)}$$

The conditional distribution of the underlying state θ given θ_i and other signals θ_j can be calculated using Bayes' rule. The probability that the another player gets a positive signal given a prior is:

$$Pr(\theta_i = 1|\mu_0) = Pr(\theta_i = 1|\theta = 1) * Pr(\theta = 1) + Pr(\theta_i = 1|\theta = -1)Pr(\theta = -1) = \mu_0 q + (1 - \mu_0)(1 - q)$$

The belief that $\theta = 1$ given a prior μ_i and a positive signal with precision q:

$$\begin{aligned} Pr(\theta = 1|\mu_i, \theta_i = 1) &= \frac{Pr(\theta = 1, \theta_i = 1|\mu_i)}{Pr(\theta_i = 1|\mu_i)} \\ &= \frac{Pr(\theta_i = 1|\theta = 1) * Pr(\theta = 1)}{Pr(\theta_i = 1|\theta = 1) * Pr(\theta = 1) + Pr(\theta_i = 1|\theta = -1) * Pr(\theta = -1)} \\ &= \frac{\mu_0 q}{\mu_0 q + (1 - \mu_0)(1 - q)} \end{aligned}$$

The belief of Player 2 after observing $P_2(1)$, a positive public and a negative private signal is:

$$Pr[\theta = 1|\theta_{1,p} = 1, \theta_2 = -1] = \frac{\frac{1}{2}qp(1-q)}{\frac{1}{2}qp(1-q) + \frac{1}{2}q(1-p)(1-q)} = p$$

8.2.2 Properties of Bernoulli and binomial distribution

During the paper I consider binary signals. The set of the possible states contains two elements $\theta \in \{-1, 1\}$ and the set of possible signals contains also two elements $\theta_i \in \{-1, 1\}$. If q denotes the precision of the signal, for a given θ one realization of the signal θ_i is a random variable with a *Bernoulli distribution* and parameter q. Whereas, the number of good signals given θ and N realizations follows a *binomial distribution* with parameter N and q.

The cumulative distribution function of the binomial distribution is very useful to determine the belief of players about subsequent signals.

$$F[K, N, q] = Pr[X \leq K] = \sum_{i=0}^{K} \binom{N}{i} q^i (1-q)^{N-i}$$

This expression shows that even if the state of the world is $\theta = -1$ the probability of several good signals is positive.

The expected value of the binomial distribution with $(-1, 1)$ is the following:

$$E[X] = N(1 * q + (1-q)(-1)) = N(2q - 1)$$