Is Relative Grading an Incentive towards Segregation?

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Abstract

In this paper I analyse students' course choice under the relative and absolute grading systems. I build a strategic group formation model where heterogeneous students simultaneously choose between two courses. Students differ in their types; they are either good or bad. While all the bad students are modelled as grade motivated, there is a share among good type students that exhibit course specific preferences. Results show that under relative grading, the model has two types of equilibria, interior where the share of good and bad students is equal in both classes, and corner where students of each type are segregated in separate classes. In the special case, when only bad students are grade motivated, only corner equilibria turn out to be stable. This result implies that the relative grading system induces students to segregate by type. No such incentive towards segregation is found when absolute grading is used.

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Contents

1 Introduction

Economists have long been intrigued by the signalling and the incentive power imbedded in grades. In today's schooling and business environment grades are, to a large extent, the sole determinant of who obtains scholarships, graduate school acceptance and job positions. Thus, it is quite natural for students and faculty to be driven by a grade oriented framework. The grading relationship between professors and students is considered an instance of the "principal student" model. While in the latter, workers choose their effort in order to maximize their earnings, in the former students decide how much effort to dedicate to studying in order to maximize their grades.

Consequences and effectiveness of different grading systems have become an important research topic in the literature of economics of education. The two most widely used grading schemes in schools and universities are absolute (criteria-referenced) and relative (norm-referenced) grading. A system is considered absolute when students' grades are determined by a fixed standard which is ex ante known. In this system, grades reflect the level at which the course requirements are mastered. On the other hand, a system is relative (also referred to as grading on the curve), when the standard, by which the grades are set, relies on the performance of other students in class. With relative grading, students' status is assessed, but not their learning attainment.

These two, substantially different, evaluation methods differ in their intended purposes, as well as on the incentives they trigger. Researchers have recently focused on understanding how grading methods affect students' learning outcomes and effort levels. Cherry and Ellis [2005] use an experimental design approach and show that the performance of students improves under the relative grading system. They argue that their result holds in class settings with a high number of enrolled students, where cooperation is not explicit.

Regarding effort, some studies have found that absolute grading in general induced students to exert more effort compared to grading on the curve (Dubey and Geanakoplos [2010]). In addition, in her theoretical model, Paredes [2012] also predict that for low levels of uncertainty, total effort levels are higher with absolute grading. Still, the empirical part of her work does not confirm the models predictions and does not show any significant difference in effort level when relative and absolute grading are compared.

While schools and especially universities offer students the possibility to choose between courses, theoretical and empirical studies have taken the grouping of student in classes as given. Thus, the focus has been on analysing the incentives that grading systems trigger once everyone is already grouped into a specific class, ignoring in this way the possibility that the class choice may have been initially influenced by the grading system. However, it is natural to assume that students choose courses strategically, especially when their individual value is weighted in relative terms. Motivated by this fact, in this paper I focus on analysing how course choice is influenced by different grading systems.

I develop a strategic group formation model where students simultaneously choose which course to enroll in, while considering the evaluation system under which they will be assessed. Students are modelled as heterogeneous in their types; they are either good or bad. For simplicity of analysis the number of available courses from which students can choose is limited to two. The first assumption of the model is that, excluding a share of good students that exhibit course preferences, all students are "grade-motivated", indicating that they simply care about maximizing their grades. In the paper I refer to these students as normal. The second assumption which is used, that attempts to capture the idea of how relative grading works in practice, is that students will obtain higher utility if they share a class containing a high percentage of bad students and a low percentage of good students. The paper's first result characterizes the equilibrium behaviour under relative grading and shows that the model has two types of equilibria, interior and corner ones. The interior equilibrium is such that the share of good and bad students is equal in both classes; on the other hand in corner equilibria all of the normal students are grouped in one class. The idea behind the interior equilibrium is straightforward, given that both classes have the same number good and bad students, individual deviations are not profitable, therefore no student has an incentive to change his action.

The corner equilibrium case presents a more intriguing distribution of students. In this situation, while one of the classes consists only of preferential good students that exhibit preferences towards that class, the other class has all the normal good and bad students grouped together. This is because of the fact that neither a good or bad type student would prefer being in a class with only good students. If a bad type student deviates and ends up in a class with only good student, he will be the weakest student there, and thus he will certainly obtain the lowest grade in class. In addition, a good type student prefers not to switch to the class consisting of all good students, given that the relative grading method will assign to some of the student high grades and to others low grades or even failing marks.

Given that there are multiple equilibria in the model, their stability is also checked. In the special case where all of the good students are preferential and only the set of bad students are motivated by grades, only corner equilibria prove to be stable. This result is striking since it shows that there will be segregation between good and bad students. Furthermore, the final result dealing with stability shows that the corner equilibrium in which all bad students will choose the class that has fewer preferential good students is more stable. This means that given the possibility between two classes, it is more plausible that the bad students group in the class which has fewer preferential good students.

The equilibrium and stability results show that relative grading serves as an incentive towards student segregation. On the other hand, according to the paper's findings there is no force pushing towards segregation when absolute grading is used. Several drawbacks of the relative grading method stand out from these results. First, it implies that teachers will not be aware that they are dealing with ability-segregated classes, since students willingly choose to segregate and are not systematized by schooling policies. Secondly, it shows that peer-effect between good and bad students will not be possible, given that students of different types are apart. And finally, the assumption that students ability is normally distributed, needed for relative grading to evaluate fairly, does not hold.

This paper presents, to the best of my knowledge, the first study that gives insight on how grading systems affect students' course selection. It contributes to the literature of economics of education, especially to the subcategory that focuses on analysing the incentive effects of grading systems. In addition, it also adds to the literature of strategic group and network formation, a rich survey of which is provided by Dutta and Jackson [2003], by adding an extra example where the tools of this field can be applied. While most of the work done in the literature of group and network formation assumes that agents exhibit homophily, the tendency to group with similar people (e.g., Currarini, Jackson, and Pin [2009]; Golub and Jackson [2012]; Alger and Weibull [2013]; Baccara and Yariv [2013]), the model presented here differs since in this case, while bad student want to group with other bad students, good type students have preferences towards grouping with the opposite type.

The remainder of this paper is organized as follows. In the next section, I present the model and its specifications under relative and absolute grading. In Section 3, the results on Nash equilibria and stability, together with their intuition are presented. I address the broader implications of the model's results in Section 4 and Section 5 concludes.

2 Model Setup

There is a continuum of student represented by the unit interval. Students are heterogeneous in their types; they are either good or bad. The type characteristic is denoted by a type parameter $t \in \{g, b\}$. The model developed here defines a game of complete information between students in the population, thus, it is assumed that all the students know each-others type. We let G and B denote the set of good and bad students respectively. Students must simultaneously choose to enroll in one of the two existing courses, X or Y. Therefore, the action space for each of them contains two elements $A = \{X, Y\}$. The classes are not necessarily required to have the same size and they may consist of students of different types.

All the bad type students are modelled as "grade-motivated", implying that they simply care about maximizing their grades. In this paper for simplicity I refer to the grade motivated students as normal students. On the other hand, good type students can be normal or preferential. Preferential students exhibit course specific preferences and gain utility only if their preferences are satisfied. The assumption in having preferential good students in the model was useful in tackling technical difficulties when analysing corner equilibria, however it also capture the fact that good students might not be simply incentivised by grades, but that they may have more developed interests regarding courses. Furthermore, it is natural to assume that because of their high ability they are not frightened in following their preferences. In this model, there exist a share φ_x of good students that prefer class X over Y, and a share φ_y of good students which prefer class Y over X. In the basic model that we consider, there will always be a positive share of preferential students, thus $\varphi_x, \varphi_y > 0$ must hold. However, preferential students always coexist with normal students and thus $\varphi_x + \varphi_y < G$. We let b_c and g_c denote the total number of bad and good students in class c respectively, where $c \in \{X, Y\}$.

2.1 Relative Grading

When relative grading is used, there are no fixed scores for certain grades. The grade is simply an expression of how well any student performed compared to the other students in class. To capture the idea behind relative grading we model the utility of a student $U(t, b_c, g_c)$ of type t, to depend on his own type parameter $t \in \{g, b\}$, and on the share of good and bad students with whom he shares a class. To simplify analysis, we suppose that preferential students enjoy a payoff of zero if they are not in their preferred class and a payoff of one otherwise. The first assumption of the model is that, under relative grading, the utility of both good and bad students is increasing in the share of bad students and decreasing in the share of good students in the same class. This implies that both types of students prefer sharing a class with students of bad type and avoid sharing a class with good type students. Given that numerous utility functions satisfy the mentioned assumption, in the upcoming analysis we focus on a specific utility for the basic model. The utility of student i of type t that chooses class c, where $c \in \{X, Y\}$, will be given by:

$$U_i(\alpha_t, b_c, g_c) = \frac{b_c}{b_c + g_c}\alpha_t,$$

where $\alpha_t \in [0, 1]$ and $\alpha_g > \alpha_b$.

This specified utility captures the setting that we are trying to model by satisfying the mentioned assumption. In this model, no spillovers are allowed between classes. This implies that the utility of a student in class X is not in any way affected by the share of bad and good students in class Y, same reasoning holds for the utility of students in class Y.

2.2 Absolute Grading

Under absolute evaluation, ex ante specified thresholds determine which grade students will gain. The utility of a student does not depend on the performance of other students in the class, but simply on his own type. If all students work well, they all have the possibility to obtain good grades. On the other hand, all of them may fail if they do not score above the pass/fail threshold. Since there is heterogeneity in students types, it is natural to assume that under absolute grading, good type students get higher grades because of their level of knowledge and bad type students obtain lower grades. To capture this fact, the utilities of normal students are specified as follows: A good type student will earn a utility of one and a bad type student will earn a utility of zero. Furthermore, the assumption regarding preferential good students is the same as with relative grading, meaning that if a preferential good student is in his preferred class he will gain a utility of one and if not, then he will get no utility.

3 Results

In this section I develop the paper's result. The focus will be on analysing the equilibrium and stability of endogenously formed classes with no spillovers.

3.1 Equilibria

I first characterize the equilibrium behaviour under relative grading. At the end of this section the result for absolute grading is provided as well. Given that the model is non-cooperative, Nash equilibrium is used as a solution concept and I focus only on pure strategies. A pure-strategy Nash equilibrium is an action profile with the property that no single student can obtain a higher payoff by unilaterally deviating. The characterization of pure-strategy Nash Equilibria is summarized in Proposition 3.1

Proposition 3.1. *Under relative grading, the model has two types of Nash equilibria, interior, where the share of bad and good students is equal in both classes, and corner, where all the normal students are grouped in one class.*

Proof. Suppose that a normal student i of type $t \in \{g, b\}$ chooses action $a_i^* = X$, then his utility will be:

$$U_i(t \mid X) = \frac{b_x^*}{b_x^* + g_x^*}\alpha_t.$$

If he switches and chooses class Y, he will have utility:

$$U_i(t \mid Y) = \frac{b_y^*}{b_y^* + g_y^*}\alpha_t.$$

If student i of type t chooses $a_i^* = X$ in equilibrium, then the following inequality must hold:

$$U_i(t \mid X) = \frac{b_x^*}{b_x^* + g_x^*}\alpha_t \geq \frac{b_y^*}{b_y^* + g_y^*}\alpha_t = U_i(t \mid Y).$$

Now, suppose student j of type t chooses action $a_j^* = Y$, then his utility will be:

$$U_j(t \mid Y) = \frac{b_y^*}{b_y^* + g_y^*}\alpha_t.$$

If he switches and chooses class X, he will have utility:

$$U_j(t \mid X) = \frac{b_x^*}{b_x^* + g_x^*}\alpha_t.$$

If student j chooses action $a_j^* = Y$ in equilibrium then the following inequality must hold:

$$U_j(t \mid Y) = \frac{b_y^*}{b_y^* + g_y^*}\alpha_t \geq \frac{b_x^*}{b_x^* + g_x^*}\alpha_t = U_j(t \mid X).$$

Therefore, if some students choose X and some choose Y, in equilibrium we will have:

$$\frac{b_x^*}{b_x^* + g_x^*} = \frac{b_y^*}{b_y^* + g_y^*} = \frac{B}{B + G}.$$

This shows that in equilibrium the share of bad students should be the same in both classes[1].

Now, suppose that all normal students choose class X, then the utility of a normal student i of type t is:

$$U_i(t \mid X) = \frac{B}{B + g_x^*}\alpha_t.$$

If some student deviates and chooses class Y, he will gain utility:

$$U_i(t \mid Y) = \frac{0}{0 + \varphi_y^*}\alpha_t = 0.$$

As $\frac{B}{B+g_x^*}\alpha_t > 0$ holds according to our assumptions for any given B, g_x^* and α_t, no student will have an incentive to deviate from class X. Using the same reasoning it can be shown that there is no incentive to deviate if all normal students choose class Y. Thus,

[1]Since $\frac{g_x^*}{b_x^*+g_x^*} = 1 - \frac{b_x^*}{b_x^*+g_x^*}$, then in equilibrium: $\frac{g_x^*}{b_x^*+g_x^*} = \frac{g_y^*}{b_y^*+g_y^*} = \frac{G}{B+G}$, the share of good students is equal in both classes as well.

we have two corner Nash equilibria. First, one where all normal students are in class X and class Y contains only the share φ_y of preferential students, and the second one where all normal students pick class Y and class X consists only of the share φ_x of preferential students. □

In our case, an equilibrium is considered corner if all of the normal students choose the same class, and interior when some normal students choose one class and others choose the other class. Therefore, in interior equilibria, neither share of normal students equals to zero ($b_c,\ g_c \neq 0$, where $c \in \{X, Y\}$).

The intuition behind Proposition 3.1 is as follows. A student will decide to stay in a class if and only if he cannot gain a higher utility by unilaterally deviating. Given that the analysis focuses on a continuum of students, a single students deviation has an insignificant role in increasing or decreasing ones utility. Let us first consider the case of the unique interior equilibrium, where the share of good and bad students is equal in both classes. In this case, by deviating from the equilibrium action to another class, any given student would gain the same utility as by remaining in his own class. Therefore, since no student can earn a higher utility from deviation, none of them will have an incentive to change their behaviour.

In the case of corner equilibria, all of the normal students choose the same class, either class X or class Y. Hence, here one of the two existing classes contains only preferential good students, which cannot gain from deviation. The other class consists of a mixture of good and bad students grouped together. If a good student deviates and chooses to attend the class with only preferential good students, according to the utility we have specified, they would gain a payoff of zero, since the share of bad students in that class equal to zero. The same holds if the deviation is made by a bad type student. Therefore, since no student can benefit from deviation, they do not have an incentive to depart from their equilibrium action.

Simply put, the results for corner equilibria show that under relative grading, a good type student prefers being in a class that contains good and bad students, rather than sharing a class with only good students. This result is intuitive given that when students are graded on the curve, not all of them obtain good grades. As journalist and author Malcolm Gladwell states, “In a class full of Einsteins, relative grading would amount to some of the Einsteins getting 10, some of them getting 6, and some of the Einsteins failing!”. Furthermore, the result also implies that a bad student prefers sharing a class with other good and bad students, rather than being the only bad student in a class full of good students. This is intuitive as well, since if you are the only bad student in a class where all of your peers are good, you will for sure obtain a bad grade. Thus, a bad student prefers being in a class with other bad students, so that he can have a chance to get an average grade.

3.2 Stability

When characterizing the equilibrium behaviour we only checked if students have an incentive to individually deviate, however it can be that even if unilateral deviations are unprofitable, deviation of coalitions of students may give rise to higher gains from deviating. A Nash Equilibrium may be considered unstable if a small change in the students actions could lead to a different action vector, once every student best responds. Given that there are corner and interior equilibria, it is reasonable to consider their stability. When analysing stability, I focus on the special case where all of the good students are preferential, so when $\varphi_x + \varphi_y = G$.

Definition 3.2. *A Nash equilibrium is stable if there exists an $\varepsilon > 0$ such that when a smaller than ε share of students change their actions relative to the Nash equilibrium:*

1. *Each student who did not change prefers to keep their current action and*

2. *Each student who changed prefer to revert to her equilibrium action*

By using the Stability concept defined above, we obtain the following result in Proposition 3.3.

Proposition 3.3. *In the special case when $\varphi_x + \varphi_y = G$, only corner equilibria are stable in the model.*

Proof. We first show that the interior equilibrium is not stable. At the interior equilibrium, when $\varphi_x + \varphi_y = G$, each of the bad students in class X will gain utility: $\frac{b_x^*}{b_x^*+\varphi_x^*}\alpha_b$, and each of the bad students in class Y will gain utility: $\frac{b_y^*}{b_y^*+\varphi_y^*}\alpha_b$, where: $\frac{b_x^*}{b_x^*+\varphi_x^*}\alpha_b = \frac{b_y^*}{b_y^*+\varphi_y^*}\alpha_b$, since the share of bad and good students is the same in both classes.

Let $\varepsilon > 0$ and let a mass of at most ε bad students in class X changes their behaviour relative to the equilibrium[2]. Thus, when considering their best-responses, the bad students in class X that did not change their response have an incentive to deviate and not to keep their current action, which violates the first requirement for an equilibrium to be judged stable. Furthermore, the bad students that altered their actions and chose class Y have no incentive to revert their decision to the prior equilibrium given that they would earn a lower utility. Therefore, since the resulting strategy profile, once everyone has best responded, is not the starting interior equilibrium, it cannot be considered stable. Same reasoning holds if we let a share of bad students from class Y deviate to class X.

[2]In this setting, a bad student in class X that did not change his action gains utility: $\frac{b_x^*-\varepsilon}{b_x^*-\varepsilon+\varphi_x^*}\alpha_b$, and a bad student in class Y has utility: $\frac{b_y^*+\varepsilon}{b_y^*+\varepsilon+\varphi_y^*}\alpha_b$. Given that in equilibrium $b_x^* = b_y^* \geq 0$ and $\varphi_x^* = \varphi_y^* \geq 0$, and we specified that $\varepsilon > 0$, the following inequality holds: $\frac{b_x^*-\varepsilon}{b_x^*-\varepsilon+\varphi_x^*}\alpha_b < \frac{b_y^*+\varepsilon}{b_y^*+\varepsilon+\varphi_y^*}\alpha_b$.

Now, we show that corner equilibria are stable. Consider first the corner equilibrium where all bad students choose class X. Let i be a bad type student. If student i is in class X he gains utility: $\frac{B}{B+\varphi_x^*}\alpha_b$, and if he is in class Y he gains a utility of zero. Let $\varepsilon > 0$, if a mass of at most ε of bad students changes their behaviour relative to the equilibrium, and choose class Y and not X, then the payoff to a bad student that did not change his action and remained in class X will be equal to $\frac{B-\varepsilon}{B-\varepsilon+\varphi_x^*}\alpha_b$, and the payoff to a bad student that deviated and chose class Y will be now equal to: $\frac{\varepsilon}{\varepsilon+\varphi_y^*}\alpha_b$. In order for the corner equilibrium to be stable, we need to check if the two stability criteria hold. First we check to see what conditions need to be met for a student who did not change his behaviour to prefer to keep his current action. In this case, the utility to a non-deviating bad student equals $\frac{B-\varepsilon}{B-\varepsilon+\varphi_x^*}\alpha_b$. On the other hand, if he deviates he gains $\frac{\varepsilon}{\varepsilon+\varphi_y^*}\alpha_b$. He will not deviate if and only if: $\frac{B-\varepsilon}{B-\varepsilon+\varphi_x^*}\alpha_b \geq \frac{\varepsilon}{\varepsilon+\varphi_y^*}\alpha_b$, which implies that $\frac{B\varphi_y^*}{\varphi_y^*+\varphi_x^*} \geq \varepsilon$. Now, we check when a bad student that changed his action would like to revert back to the equilibrium. A deviant gains utility of $\frac{\varepsilon}{\varepsilon+\varphi_y^*}\alpha_b$, and he will only revert his action if $\frac{\varepsilon}{\varepsilon+\varphi_y^*}\alpha_b < \frac{B-\varepsilon}{B-\varepsilon+\varphi_y^*}\alpha_b$, which implies that $\frac{B\varphi_y^*}{\varphi_y^*+\varphi_x^*} > \varepsilon$. Therefore, in order for the stability criteria to be met $\varepsilon < \frac{B\varphi_y^*}{\varphi_y^*+\varphi_x^*}$ must hold. Given that ε is specified to be a very small share this inequality always holds. The same reasoning holds if we focus at the equilibrium where all bad students choose class Y and a share of them deviate to class X. Hence, corner equilibria are stable. $\square$

Proposition 3.3 shows that if all the good students are preferential, only corner equilibria, where all bad students choose the same class, are stable. The intuition behind this result is as follows. In this setting, good students can only gain a positive utility if they are in their preferred class and a utility of zero otherwise. Hence, only the bad students are the ones that may gain from deviation. In this special case, the bad type students can finally achieve what they always wanted, form a class together with students of their own type and avoid good students as much as possible. This is more profitable for them, since they do not have high competition. Intuitively, being in a class with more bad students, increases the chances of receiving a higher grade compared to being in a class with fewer bad students.

Proposition 3.4. *The corner equilibrium in which all bad students choose the class with fewer preferential good students is more stable.*

Proof. The proof is similar with the proof of Proposition 3.3. Let us consider the case where there are more students that prefer class Y compared to class X and show that the equilibrium in which all bad student choose class X is more stable compared to the equilibrium in which all of them choose class Y. In order for the equilibrium in which all bad students choose class X to be stable, $\varepsilon < \frac{B\varphi_y^*}{\varphi_y^*+\varphi_x^*}$ must hold as it was shown in the proof of Proposition 3.3. On the other hand for the equilibrium where all bad students choose class Y to be stable $\varepsilon < \frac{B\varphi_x^*}{\varphi_x^*+\varphi_y^*}$, must hold. Given that $\varphi_x^* < \varphi_y^* \Longrightarrow \frac{B\varphi_y^*}{\varphi_y^*+\varphi_x^*} < \frac{B\varphi_x^*}{\varphi_x^*+\varphi_y^*}$. This implies that a larger share of students is needed to change their equilibrium behaviour in order for the equilibrium in which all bad students choose class X to lose its stability. □

Proposition 3.4 claims that given the possibility to choose between two classes with preferential good students, it is more plausible that the bad type students will choose the one that has a smaller share of preferential good students, since their utility will be higher in that class. This result enforces even more the segregation feature that was discovered when checking for stability of equilibria.

3.3 Equilibria under absolute grading

In this part of the results, I characterize the equilibria of the model under absolute grading and I summarize them in Proposition 3.5.

Proposition 3.5. *Under absolute grading, all the profiles in which preferential students choose their preferred class are Nash equilibria.*

Proof. Normal good and bad students are neutral between the two class choices since they gain the same utility by being in any of them. A bad student gains a utility of 0 and a good student a utility of 1. Therefore, once all the preferential students choose their preferred classes, no profitable deviations are possible and thus, no student wants to move. □

While quite intuitive, this result presents an interesting feature of absolute grading, which shows that there are no forces pushing towards student segregation by type when this evaluation method is used.

4 Broader Implications

The results presented in the previous section show that under relative grading, it is plausible that students will be segregated into different classes according to their types, while this will not be likely the case under absolute grading. As many educational institutions are transitioning to relative grading method and since in higher education this is common practice, it is of interest to examine the implications that this segregation brings.

It is crucial to note that, while heavily criticized and often opposed, in practice there are many institutional programs that intentionally separate students judging either by their ability or by their prior performance. Two of the mostly used systems for student segregation are tracking and ability grouping. The difference between the two rises in the way they organize the curriculum. With tracking, the program is structured to match students ability or performance; on the other hand, under ability grouping all students face a uniform curriculum, even though they are separated.

The segregation that results from the relative grading system between good and bad students however, differs from the above mentioned practices, since it is not intentionally designed by institutions. While under ability programs students exogenously separated, under relative grading students will be incentivized to willingly separate on their own. In particular, bad students will segregate themselves. This shows that the grading system has the potential to cause the phenomenon of ability grouping. Therefore, the first implication is that, when students are intentionally segregated, teachers and schools are aware of this fact and know that they are dealing with different types of classes; however this is not the case when students willingly segregate on their own. The research on ability grouping is directly linked with the study of peer effects, and it is one of the most controversial issues in the literature of education. Evidence suggests that grouping students into classes of same ability and providing them with an identical curriculum, has no considerable effect on their achievement (Loveless [1999]). While it might be reasonable for bad students to segregate in order for them to maximize their grades, there are drawbacks regarding knowledge sharing. Bad students will not take into consideration that they may gain from the good students, thus peer effects between different types will, to a large extent, be lost. Furthermore, there may be negative effects on students that are grouped with only good students, when evaluated on the curve. They can be easily demoralized if their relative ranking drops. Research also suggests that good student suffer from lower self-worth and greater anxiety during exams in ability-segregated classes.

Relative grading is an assessment method that was originally based on the assumption that the ability of students, especially in large classes, is most likely normally distributed; however this may not always be the case. As we have seen from the model's results, it is plausible that one of the classes will contain only good students while the other class will be composed of a majority of bad students. In this case, students' abilities are likely the same within a class. When taking an exam in the class of good type students, more than half of them may achieve a near perfect score, which is not even closely a normal distribution, still only a small percentage of them will obtain high grades, since relative grading fits letter grades to the curve of a normal distribution. Kulick and Wright [2008], address this issue in their research and claim that with relative grading the best grade is not always assigned to the best students. They use mathematical models and simulations to identify a flaw in relative grading system, and reason that this method will correctly assign grades if students' abilities are actually normally distributed, implying that only a small share of students perform exceptionally, the majority of students achieves average scores and a minor part of them perform poorly. Therefore, while bad students may gain higher grades by segregation, the opposite holds true for good students, a considerable number of which will be poorly graded.

5 Conclusion

The model's results show that students' course choice is affected by the grading system. Under relative grading students are induced to segregate by type into different classes. The stability analyses show that when all good students are incentivised by course preferences and not by grades, the equilibrium that is more likely to arise is the one in which the bad students will choose to segregate themselves into a class with the smallest number of preferential good students. No such force pushing towards segregation by type is observed under absolute grading. The novel segregating incentive of the relative grading revealed in this paper calls for attention to the implications that this feature might have on the education system in general. Most crucially, the results show that given the similarity of students in the same class, the assumption that students abilities are normally distributed, which is needed in order for relative grading to provide a fair evaluation, does not hold.

References

Ingela Alger and Joergen W. Weibull. Homo Moralis-Preference Evolution Under Incomplete Information and Assortative Matching. *Econometrica*, 81(6):22692302, 2013.

Mariagiovanna Baccara and Leeat Yariv. Homophily in peer groups. *American Economic Journal: Microeconomics*, 5:69–96, 2013.

Todd L. Cherry and Larry V. Ellis. Rank-order grading improve student performance? Evidence from a Classroom Experiment. *International Review of Economics Education*, 4:1:9–19, 2005.

Sergio Currarini, Matthew O. Jackson, and Paolo Pin. An economic model of friendship: Homophily, minorities, and segregation. *Econometrica*, 77:1003–1045, 2009.

Pradeep Dubey and John Geanakoplos. Grading exams: 100, 99, . . . , 1 or A, B, C? Incentives in games of status. *Games and Economic Behavior 69*, 1:7294, 2010.

Bhaskar Dutta and Matthew O. Jackson. On the formation of networks and groups. *in: B.Dutta, M. O. Jackson (Eds), Models of the Strategic Formation of Networks and Groups, Springer, Berlin*, 2003.

Benjamin Golub and Matthew O. Jackson. How Homophily Affects the Speed of Learning and Best Response Dynamics. *Quarterly Journal of Economics*, 127(3):1287–1338, 2012.

George Kulick and Ronald Wright. The impact of grading on the curve: A Simulation Analysis. *International Journal for the Scholarship of Teaching and Learning*, 2(2), 2008.

Tom Loveless. The tracking wars: State Reform Meets School Policies. *Brookings Institution Press, Washington, DC*, 1999.

Valentina Paredes. Essays on the economics of education. *UC Berkeley Electronic Theses and Dissertations*, 2012. (Phd Thesis).