Learning and Spillovers under Imperfect Competition

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Abstract

I analyze a model with sequential markets under the presence of learning-by-doing and learning spillovers. I focus on the role that product differentiation plays in firm competition under the presence of these joint economies, and how it impacts optimal quantities, prices and profits. Furthermore, I analyze the welfare implications of both learning-by-doing and learning spillovers. I obtain the following main results. (1) A low level of learning efficiency enhances both profits and consumer surplus, while high levels of learning efficiency may lead to a decline in profits, or even market centralization, which can explain some rather puzzling persisting market structures. (2) The presence of spillovers increases both profits and consumer surplus and in some cases has the potential to prevent market centralization. (3) The fraction of the inefficiency originating from learning-by-doing decreases as competition becomes fiercer, while the fraction of the inefficiency originating from spillovers remains a substantial part of the total inefficiency regardless of product differentiation.

Keywords: Learning-by-Doing, Learning Spillovers, Market Centralization, Market Inefficiencies

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[1]The thesis explained herein represent the own ideas of the author, and do not necessarily reflect the opinion of *Central European University Budapest Foundation CEUBPF*.

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1 Introduction

For a long time now, economists have been aware and widely utilized the concept of learning-by-doing. Early empirical work such as that of Wright (1936) clearly depict how costs of producing an aircraft decrease as the cumulative number of aircrafts produced increases. Moreover, realizing the fact that firms within the same industry generally employ a similar technology, has led to the introduction and development of the idea of learning spillovers. Taking into account these factors has greatly helped in both understanding and shaping optimal firm behavior. The main body of research, be it theoretical or empirical, has greatly focused on the implications that these factors have for market performance. With a few exceptions, prevailing assumptions of this body of research have been the existence of a pre-defined market structure and the production of a perfectly homogeneous good. Although these studies have enriched our collective understanding of the impact that learning-by-doing and market spillovers have, the strong assumption of a homogeneous good and the usual assumption of a fixed market structure greatly limit the applicability of this knowledge, and consequently, leave us still in doubt in regards to the effect that these concepts have in more general cases, in more realistic cases, where all the firms do not produce the exact same good.

As a result, the aim of this work is to unveil the potential that learning-by-doing and learning spillovers have not only on determining market efficiency, but also in shaping the market structure. To the best of my knowledge this work presents the first analysis of the impact of learning-by-doing and learning spillovers in a market with a varying degree of product differentiation. This allows for the robustness examination of previous findings, and for a better comprehending of the role and importance that product differentiation has regarding these concepts. Furthermore, recognizing the structure shaping potential of these concepts allows for the explanation of puzzling market structures such as the perseverance of a highly profitable monopoly regardless of the lack of any concrete entry barriers. Finally, the structure of my model allows me to decompose the market inefficiencies and by analyzing them separately I identify the impact that product differentiation has on the magnitude of these inefficiencies.

I set up a non-cooperative, two stage duopoly model in which firms produce a differentiated good. Firms choose optimal quantities to produce given a linear demand curve originating from consumers quadratic utility function in line with Vives and Singh (1984). Linear demands have also been employed by Fudenberg and Tirole (1983), Dasgupta and Stiglitz (1988), and so on, while Spence (1981) utilizes a constant elasticity demand curve, which results in an equilibrium described by a system of nonlinear equations, for which only numerical solutions are possible. I focus on a two stage duopoly as it allows me to identify all the effects that guide optimal behavior [2]. I incorporate learning-by-doing in the first stage, thus, the higher the first stage production, the lower the second stage production costs will be. I employ a linear learning curve, which calls for some limitations but which when combined with a linear demand curve allows for explicit solutions of the optimal quantities produced, prices, profits, consumer and total surplus. Moreover, I introduce learning spillovers, implying that not only does cumulative production decrease firms own costs but to some extent the costs of the rivals as well. For the perfectly homogeneous goods case, considerable work regarding learning spillovers has been done by Ghamawat and Spence (1985), which proves helpful for my analysis.

The inclusion of learning-by-doing and learning spillovers leads to a change in the behavior of firms regarding the selection of the level of optimal quantities to produce. The strategic effects that impact this decision have been analyzed in detail by Qiu (1997) for a sequential model under the presence of $R\&D$, these effects are similar with the ones identify in this work. As will be seen in detail, firms will no longer rely on marginal cost pricing (equating marginal revenues to marginal costs), rather, to choose first period quantities, beyond initial marginal costs firms will take into account the second period gained efficiency and the strategic effects. As has been found in empirical works, such as Argote and Epple (1990), learning efficiency varies between different industries. Furthermore, comparing results obtained from Irwin and Klenow (1994), Gruber (1998), Thornton and Thompson (2001), Bloom et al. (2013), and so on, leads to the conclusion that the extend to which knowledge spillovers are significant is also industry specific. Consequently, to approximate different industries, in my analysis I have specific parameters, such as μ which controls the learning-by-doing efficiency, η controls the level to which knowl-

[2]While the inclusion of additional firms would change the weight that each effect has, it does not introduce any new, or reduce any existing strategic effects, thus I focus on a duopoly rather than an oligopolistic market. The same claim holds for the addition of subsequent time periods.

edge spills over, while γ controls the degree of product differentiation. Varying these parameters allows me to approximate different industries and see the implications of learning-by-doing and learning spillovers far beyond the basic homogeneous goods case with a fixed learning-by-doing efficiency and spillover magnitude. The analysis results on the following main findings.

Firstly, a low level of learning efficiency enhances both profits and consumer surplus regardless of the level of product differentiation, while high levels of learning efficiency may lead to a decline in profits, or even market centralization if the products greatly substitute one another. When the goods are highly differentiated, the strategic effects are quite low, thus the introduction of learning-by-doing, by increasing firm efficiency leads to higher profits. However, when the goods highly substitute one another, the firms pay great attention to the decisions of their competitors. Consequently, in an attempt to push them out in the second period, in equilibrium firms end up producing high first period quantities, which leads to low, or even negative first period profits. If sales will be low in the second period, which occurs if the goods highly substitute one another, it is possible that second period profits can not make up for the first period losses, thus, firms may choose to exit the market.

The fact that learning-by-doing may lead to market centralization has the potential to explain puzzling market structures, such as the persistence of a highly profitable oligopolistic, or monopolistic structures regardless of the lack of any obvious entry barriers. Such a case could be represented by an emerging market, similar to that of the market for smartphones, in which although there are great initial fixed costs, the expected profit level (when we ignore learning-by-doing) of the entering firm would justify these costs and ensure large profits[3]. However, by adding learning-by-doing in the picture, it becomes clear that the strategic effects that this concept gives rise to would make both the existing firm and the entrant much more aggressive, which would surely lead to negative profits. As a result, firms outside the market decide not to enter regardless of the large profits that existing firms may be receiving.

[3]Learning-by-Doing has the potential to explain such puzzling market structures under a high level of product substitutability and a high learning efficiency, conditions that I believe are met by the market for smartphones.

Secondly, the presence of spillovers increases both profits and consumer surplus and in some cases has the potential to prevent market centralization. By introducing learning spillovers competition becomes less aggressive in the first period and more efficient in the second, thus more profitable for the existing firms. Regardless of the lowering of the competitiveness factor, the additional efficiency that these firms gain from the spillovers, by ensuring that they can produce less costly in the subsequent period, results in a higher consumer surplus as well. Bloom et al. (2013) have decomposed the effects of spillovers in that of technology spillovers and the business stealing effect. While the first effect, by making firms more efficient increases market performance, the latter, by demotivating firms to produce as much (since rivals also benefit from the cumulative production) may lead to a decrease in the optimal quantities produced. In their work Bloom et al. (2013) find that technology spillovers dominate and thus spillovers improve market performance. This is in line with the findings in this work, where, although I identify cases in which the business stealing effect might dominate in the first period, I conclude that the gained efficiency in the second period more than compensates for this fall and thus leads to a more efficient market.

Finally, by identifying the optimal quantities chosen by the social planner and by decomposing the inefficiencies present in this model, I find that the fraction of the inefficiency originating from learning-by-doing decreases as competition becomes fiercer. On the other hand, the fraction of the inefficiency originating from learning spillovers remains a substantial part of the total inefficiency regardless of product differentiation. This seems somewhat intuitive, as learning-by-doing is internalized by the firms, the fiercer the competition is, the more they will rely on the utilization of this factor. While in the other hand, the spillover effect, by benefiting not only the firm but also its competitor, generates largely different motives for the firms compared to those of the social planner. Consequently, regardless of the degree of product differentiation, the inefficiency originating from spillovers remains largely unchanged.

This paper is organized as follows. I review the theoretical literature while continuing by briefly presenting key findings from the body of empirical work in Section 2. I introduce the model and findings for both the case with and without learning spillovers in Section 3. Section 4 focuses on the source and variation of market inefficiencies that are present in this setting. Finally, in Section 5 I conclude this study with a few policy implications.

2 Overview of Previous Research

2.1 Theoretical Research

Among the seminal works in the literature of learning-by-doing is the work of Spence (1981), who studies a continuous time model under the presence of learning-by-doing, which is taken into account by modeling costs as a decreasing function of cumulative production. The author derives both the precommitted and the subgame perfect equilibrium and finds a minimal difference between the equilibriums originating from the different solution concepts. Spence utilizes a constant elasticity demand, which results in an equilibrium described by a system of nonlinear equations. Given the impossibility to solve the model analytically Spence reports different numerical solutions. The paper furthermore analyzes the entrance deterrence effect that learning-by-doing has, and the author finds that a moderate level of learning efficiency create the greatest barriers to entry. This may seem rather intuitive, as if learning efficiency is very low, it is almost negligible, while if it is very high, the entering firms can quickly catch up with the existing firms.

Spence contributes to the literature further with his work Spence (1984) on which he analyzes market performance under varying levels of concentration, spillovers and learning efficiency. He also notes that $R\&D$ spillovers have a positive effect via cost reduction and a negative effect via incentive reduction, effects that will be present in the learning-by-doing case as well. Among the main findings of this work is that market performance is significantly better when spillovers are large. Ghamawat and Spence (1985) also conclude that spillovers in the learning-by-doing case improve market performance by finding that the increased efficiency outweighs the decrease in firms incentive to expand initial production. The authors state that while it is often the case that in $R\&D$ the disincentive effect of spillovers usually dominates the efficiency effect, this is not the case for spillovers in learning-by-doing, since here profits directly depend on the production level, while in the $R\&D$ case the disincentive motive can easily lead firms to conduct no $R\&D$ investment. Thus, although the spillover effects are comparable, the trade-off between efficiency and disincentives fundamentally differs between models with $R\&D$ and learning-by-doing.

Fudenberg and Tirole (1983) also study the market performance implications of the presence of learning-by-doing. They focus both on the precommitted and the subgame perfect equilibrium. In the former case they find that the presence of learning leads to a sequentially increasing output, while in the latter, under the presence of strategic considerations firms can be led to choose decreasing output paths. Moreover, they find that a monopolist firm learns slower than a social planner, and that welfare can be improved by transferring production incentives to the latter, more mature phase.

Bulow et al. (1985) study the implications that joint economies have on the firm's optimal decisions. The relevance of this work is further emphasized by its critique of both the work of Spence (1981) and Fudenberg and Tirole (1983), the former being criticized for not finding a significant difference between the precommitted and the subgame perfect equilibrium, as a result of the artificially chosen constant elasticity near (-1), while the latter is criticized for its conclusion that first period output is higher in a sequential game. The authors mention that this latter finding results from the linear demand and quantity competition, which, under the assumption of a homogeneous good, always gives rise to strategic substitutes. While had the goods been strategic compliments, these results would have reversed. Dasgupta and Stiglitz (1988) investigate how learning possibilities affect the structure of the industry. They emphasize the importance of the non-symmetric case, in which, even a minimal asymmetry has the potential to lead an oligopolistic market to a fully centralized market.

In the aforementioned studies, a prevailing assumption is that of homogeneous goods. Thus it is not straightforward how robust these findings are to the more general case where the goods are not necessarily identical. The only exception is the work of Bulow et al. (1985) who although analyzes a case of sequential market with perfect substitutes, emphasizes the role of the goods being strategic compliments or strategic substitutes. Thus, it becomes clear that it is of interest to check the robustness of the previous findings, and the further implications of learning-by-doing and learning spillovers under different levels of product differentiation, which is what this study aims to do.

The work of Qiu (1997) represents the work closest to the one conducted in this paper. While Qiu (1997) focuses on comparing the efficiency differences between cournot and bertrand competition its relevance arises from its sequential model with $R\&D$ in a differentiated duopoly, where in the first period firms choose the cost reducing $R\&D$ levels while in the second period they choose their production levels. As I state below, some of the effects identified in this paper are similar to the effect presented under the model I analyze.

Regardless of the foundations and seemingly rational assumptions characterizing the theoretical work, it is of great importance to verify whether the aforementioned factors in fact play a significant role in the practical sense. This motivates the next subsection in which I analyze the empirical work covering the concept of learning-by-doing and learning spillovers.

2.2 Empirical Research and Evidence

I start the unfolding of the empirical work with one of the earliest empirical treatments of the matter, that of Wright (1936) who identifies a negative relationship between the labor costs of producing an aircraft and the cumulative number of produced aircrafts. After the publication of this work numerous other researchers were interested to find if this relationship holds in manufacturing and in general in other industries as well. These studies, to a large extend, confirmed the findings of Wright . Among the early proponents of learning-by-doing was The Boston Consulting Group, who in their work BCG (1972) recognized the wide applicability of the learning curve.

Afterwards, the availability of highly detailed microlevel data, and the expected presence of both learning-by-doing and learning spillovers influenced numerous researchers to study the market of the semiconductor industry (usually computer electronic data storage devices). Within this industry, Irwin and Klenow (1994) provides one of the first systematic empirical analysis and finds that both learning-by-doing and learning spillovers are highly significant. Moreover, Irwin and Klenow emphasize that an additional unit produced by a firm increases its efficiency about three times more than an additional unit of another firms production. Implying that although spillovers are present they are far from perfect, a possibility that I surely allow for in my analysis. Similar conclusions have been reached by Gruber (1998), who beyond confirming the presence of both effects finds that spillovers are quite persistent even beyond state borders, implying thus the existence of spillovers in a global level.

In their work Argote and Epple (1990) emphasize the role that learning-by-doing has on productivity growth and focus on the variation of learning curves (the efficiency of learning-by-doing). They identify reasons for this variation, such as organizational forgetting, employee turnovers, transfer of knowledge across products and across organizations, and so on. Necessary to emphasize is also the work of Lieberman (1984) who beyond finding a strong and significant learning effect finds that both the time trend and current output loose their significance once cumulative output is introduced in the regression, implying that learning is a function of cumulative output rather than current output or time. Similar conclusions have also been reached by Rapping (1965) and Sheshinski (1967) thus giving great validity to this functional form. These findings are highly important as one could suspect that the firm's experience regardless of the amount of output is responsible for the gained efficiency, which is shown to not be the case. Thornton and Thompson (2001) find that learning-by-doing and learning spillovers were highly responsible for the productivity growth in World War II shipbuilding. Nonetheless, their estimates of knowledge spillovers are considerably lower than the ones estimated in the semiconductor industry. Consequently, the authors imply that learning spillovers may depend more on the technology utilized by the industry than the nature of the competition.

More recently, and most relevantly the study of Bloom et al. (2013) focuses on spillovers originating from $R\&D$ investment, which happen to have a very similar nature with the spillovers originating from learning-by-doing. This study emphasizes that the presence of spillovers gives rise to two effects that determine equilibrium behavior. The first is knowledge spillovers (a positive effect), which can increase the productivity of all firms operating in a similar technological realm. The second effect is the so called market rivalry effect, or business stealing effect (a negative effect), which demotivates firm production as it makes competitors more efficient. Using a panel of U.S firms during 1981-2001 the study finds that the knowledge spillover effect dominates the business stealing effect, implying that spillovers are in fact desirable, and consequently that there will be under investment in spillovers, which as will be shown, is a result consistent with the theoretical model developed in this work.

The body of empirical research examined above, greatly supports the idea that incorporating learning-by-doing and learning spillovers in the industrial optimal decision making process is not important only in a theoretical framework but is highly supported by data as well. The variation of the findings emphasizes that the implications change from industry to industry, implying that for a proper approximation of the different industries, there is a crucial need for variations among the theoretical learning efficiency, spillover magnitude and product differentiation, which I carry out in this work.

3 Model Setup and Results

3.1 The Utility Function and The Demands

The utility function is assumed to be quadratic and strictly concave in line with Vives and Singh (1984):

$$U_t = \alpha q_{1t} + \alpha q_{2t} - \frac{\left(\beta q_{1t}^2 + \beta q_{2t}^2 + 2\gamma q_{1t} q_{2t}\right)}{2}$$

Where q_{it} represents the amount of good i consumed in period t[4]. In addition, the utility function is assumed to be separable and linear in a numeraire good, thus eliminating income effects on the monopolistic sector, making partial equilibrium analysis possible. The representative consumer maximizes each period:

$$\max_{q_{1t}, q_{2t}} U_t - p_{1t} q_{1t} - p_{2t} q_{2t} \qquad t = 1, 2$$

While discounting future periods by a δ factor. Here p_{it} represents the price of good i at time t. This gives rise to the following linear demand curves:

$$p_{1t} = \alpha - \beta q_{1t} - \gamma q_{2t}$$
$$p_{2t} = \alpha - \beta q_{2t} - \gamma q_{1t}$$

Holding the parameter values constant gives rise to the prevailing assumption that the demand in both time periods remains unchanged. The value of γ determines whether the goods are independent, compliments or substitutes, where in particular[5]:

Compliments	Independent	Substitutes
$\gamma < 0$	$\gamma = 0$	$\gamma > 0$

Furthermore, the fraction γ/β captures the degree of product differentiation. Thus when $\gamma \to 0$ the products are independent, while when $\gamma \to \beta$ the products are homogeneous.

[4] Also in line with Vives and Singh (1984) the following restrictions have to be satisfied to ensure a well defined function: $\beta_1\beta_2 - \gamma^2 > 0$, $\alpha_i\beta_j - \alpha_j\gamma > 0$, where the subscripts become relevant in the non-symmetric case.

[5] In my analysis I only consider the case where $\gamma \in [0, \beta]$. I apply the following restriction since in the Vives and Singh (1984) model the γ parameter controls both the product differentiation, and the complimentability/substitutability of the products. This is natural and desirable when the goods are substitutes, however, as $\gamma \to -\beta$, the goods become better compliments of one another while at the same time becoming more homogeneous, which is a confusing and questionable feature.

3.2 The General Learning Curve

In line with the concept of learning-by-doing, firm costs are assumed to decrease as the total quantity of production increases. In general then, costs in time t can be expressed as:

$$c_{it} = f(c_{i1}, Q_{it}, Q_{jt})$$

Where c_{i1} represents the initial costs of firm i, Q_{it} represents the total production until time period t of firm i while Q_{jt} represents the total production until time period t of firm j, the competing firm. The function $f(\cdot)$ is a decreasing function of total output of firm i and j, $\partial f/\partial Q_{it} < 0$ and $\partial f/\partial Q_{jt} < 0$. In general it is a desirable feature to assume that the second derivatives are positive, implying that the marginal gain in efficiency from learning-by-doing decreases. However, in my analysis I assume a linear learning curve, thus:

$$c_{it} = c_{i1} - \mu \sum_{s=1}^{s=t-1} q_{is} - \eta\mu \sum_{s=1}^{s=t-1} q_{js}$$

And since I analyze the model in two time periods[6]:

$$c_{12} = c_{11} - \mu q_{11} - \eta\mu q_{21}$$

$$c_{22} = c_{21} - \mu q_{21} - \eta\mu q_{11}$$

Here μ represents the efficiency of learning-by-doing, or in other words, μ determines how much costs reduce with each quantity produced. While η represents the amount of spillovers. Naturally $\eta \in [0, 1]$, with $\eta = 0$ representing no spillovers and $\eta = 1$ representing the case with perfect spillovers. The perfect spillovers case implies that the cost reduction from competitors production is as effective as the cost reduction originating from firm's own production. While the no spillovers case implies that no level of production from the competitor can affect firm's costs. Thus, it seems natural to assume, as is supported by empirical evidence, that the degree of spillovers is between these extreme cases.

[6]Ideally: $\lim_{Q_{it}, Q_{jt} \to \infty} f(c_{i1}, Q_{it}, Q_{jt}) \to \underline{c} \geq 0$, where $\underline{c}$ represents the lowest possible level to which costs can be reduced. However, since this is not attainable with a linear function the following assumption is made: $c_{i1} \geq \mu q_{i1} + \eta\mu q_{j1} + \underline{c}$.

3.3 Firms General Maximization Problem

Since there are two time periods, firm i maximizes the following profit function:

$$\Pi_i = \pi_{i1} + \delta\pi_{i2} = q_{i1}\left(p_{i1} - c_{i1}\right) + \delta q_{i2}\left(p_{i2} - c_{i2}\right)$$

Where δ represents the discount factor. To find the subgame perfect equilibrium I first solve for the second period problem, in which:

$$\frac{d\pi_{i2}}{dq_{i2}} = MR - MC = 0$$

Hence for the second period, marginal cost pricing holds. This leads to:

$$q_{i2} = \frac{2\alpha\beta - \alpha\gamma - 2\beta c_{i2} + \gamma c_{j2}}{4\beta^2 - \gamma^2}$$

Thus q_{i2} by becoming a function of c_{i2} and c_{j2}, also becomes a function of q_{i1} and q_{j1}. The first period maximization results in:

$$\frac{d\Pi_i}{dq_{i1}} = \underbrace{\frac{\partial\pi_{i1}}{\partial q_{i1}}}_{MR-MC} + \delta\left(\underbrace{\frac{\partial\pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}}_{\text{Savings Motive}} + \underbrace{\frac{\partial\pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}}_{\mu\text{ Strategic Effect}} + \underbrace{\frac{\partial\pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i1}}}_{\eta\text{ Spillover Effect}}\right) = 0$$

For more details refer to **Appendix 1.1**[7]. Simultaneously these effects determine the equilibrium level of q_{i1}. The first part, $(MR - MC)$, originates from first period profit maximization, and in the absence of learning-by-doing it would be the effect that would solely determine first period production. However, the fact that a higher level of production in the first period reduces costs in the second period introduces the *Savings Motive*, thus motivating firms to produce more in the first period. The third effect, the μ *Strategic Effect*, originates from the fact that the goods are strategic substitutes, thus, the more one firm produces, the less will the other firm produce. This competition pushing effect motivates firms to produce even further beyond the $(MR-MC)$ level. While in a model with say $R\&D$ there would be explicit costs that the firms would have to pay to achieve lower subsequent period costs, in the case of learning-by-doing the cost that

[7] Although Qiu (1997) analyzes a model with $R\&D$ rather than learning-by-doing, he identifies similar effects that determine optimal firm behavior.

the firms pay is the departure from marginal cost pricing, thus, firms pay by receiving lower first period profits. The last effect is the η *Spillover Effect*, which implies that the firms take into account that by producing more in the first period they will also make their competitors more efficient via knowledge spillovers. Although for goods that are substitutes this effect is negative, as will be seen in **Learning-by-Doing with Spillovers**, in equilibrium, this effect does not always decrease first period production.

In the case of a precommitment equilibrium, where firm i does not account for the effect that its own production has on q_{j2}, the last two terms would be absent, this would also be the case for a monopoly and the social planner. Regardless of the equilibrium concept it is clear that there is a departure from marginal cost pricing, where the optimal level of q_{i1} is no longer set to equate marginal costs with marginal revenues, rather, it also takes into account the effect it has on the second period optimization.

3.4 Learning-by-Doing with no Spillovers

I begin this analysis with the case in which there are no spillovers, particularly $\eta = 0$. Having explored the effects that the introduction of learning-by-doing has on market efficiency and structure, in the next chapter I introduce spillovers as well. If there are no spillovers the first order conditions that have to be satisfied for a subgame perfect equilibrium reduce to the following:

$$\frac{d\pi_{i2}}{dq_{i2}} = MR - MC = 0$$

$$\frac{d\Pi_i}{dq_{i1}} = \underbrace{\frac{\partial \pi_{i1}}{\partial q_{i1}}}_{MR-MC} + \delta \left(\underbrace{\frac{\partial \pi_{i2}}{\partial c_{i2}} \frac{\partial c_{i2}}{\partial q_{i1}}}_{\text{Savings Motive}} + \underbrace{\frac{\partial \pi_{i2}}{\partial q_{j2}} \frac{\partial q_{j2}}{\partial c_{i2}} \frac{\partial c_{i2}}{\partial q_{i1}}}_{\mu \text{ Strategic Effect}} \right) = 0$$

The only difference from the previous general case is that the last term in the second first order condition, the *η Spillover Effect*, no longer appears. This leads to the following optimal quantities:

$$q_{i1} = \frac{(\alpha - c)\left(8\beta^3 + 4\beta^2(\gamma + \delta\mu) - 2\beta\gamma^2 - \gamma^3\right)}{(2\beta - \gamma)(2\beta + \gamma)^3 - 4\beta^2\delta\mu^2}$$

$$q_{i2} = \frac{(\alpha - c)\left(4\beta^2 - \gamma^2\right)(2\beta + \gamma + \mu)}{(2\beta - \gamma)(2\beta + \gamma)^3 - 4\beta^2\delta\mu^2}$$

Proposition 1.

An increase in the efficiency of learning μ increases first and second period production regardless of the degree of product differentiation.

See **Proof 1** in the **Appendix**. The intuition behind **Proposition 1**, is the following. Since in the first period there is an opportunity to decrease second period costs, and also to push competition away via the *μ Strategic Effect*, firms find it optimal to produce beyond $MR = MC$, consequently increasing first period production. Furthermore, as can be seen by the initial first order condition, there are no strategic elements in determining second period production, and since the costs are now lower, both firms by equating $MR = MC$ in the second period find it profitable to produce more[8].

For products that are perfectly homogeneous Fudenberg and Tirole (1983) reach the same conclusions, however, their finding by not allowing for variation in product differentiation, can be considered as a specific case of the above proposition. This result largely depends on the linearity of the model. Bulow et al. (1985) have pointed out that the reaction of firms, whether they increase or decrease their production under the presence of interrelated economies, depends on whether the goods are strategic compliments or strategic substitutes. With linear demand functions, if goods are substitutes they will be by default strategic substitutes as well. Thus, it is not straightforward whether these implications would hold in a nonlinear model.

Lemma 1.
For any given α, β, $c \in [0, \alpha)$, $\delta \in [0, 1]$ and $\gamma \in [0, \beta]$:

$$\mu \leq \frac{2c\beta}{\alpha}$$

is sufficient but not necessary to ensure a non-negative second period cost $c_{i2} \geq 0$. While in the extreme case, as $c \to \alpha$, the following condition is sufficient:

$$\mu \leq 2\beta$$

See **Proof 2** in the **Appendix**. The need for a restriction on μ comes from the fact that I utilize a linear learning function. Thus, without any restrictions on learning efficiency it could be the case that the second period costs become negative. Having stated **Lemma 1** I move on to establish **Theorem 1**, which is among the main points presented in this paper.

[8]Of course, the fact that the costs of the rival are also lower push firms to produce less, however, since firm's own costs have a higher significance, in the second period firms end up producing higher quantities.

Theorem 1.

When learning efficiency is low, an increase in learning efficiency increases profits regardless of the degree of product differentiation. While when learning efficiency is above a certain threshold, a further increase in learning efficiency decreases profits if the goods highly substitute one another.

$$\frac{\partial\,\Pi_i}{\partial\,\mu} > 0 \quad \textit{if} \quad \begin{cases} \mu < \mu^* \\ \textit{for any } \gamma \\ \textit{for any } \frac{c}{\alpha} \end{cases} \qquad\qquad \frac{\partial\,\Pi_i}{\partial\,\mu} \leq 0 \quad \textit{if} \quad \begin{cases} \mu^* \leq \mu < \mu^{**} \\ \gamma^* \leq \gamma \\ \left(\frac{c}{\alpha}\right)^* \leq \frac{c}{\alpha} \end{cases}$$

If the $^c/_\alpha$ ratio is higher than a certain threshold, a high level of learning efficiency can lead to market centralization if the goods highly substitute one another.

$$\left.\begin{matrix} \frac{\partial\,\Pi_1}{\partial\,\mu} > 0 \quad \textit{in fact} \quad \Pi_1 \to \Pi^M \\ \\ \frac{\partial\,\Pi_2}{\partial\,\mu} < 0 \quad \textit{in fact} \quad \Pi_2 \to 0 \end{matrix}\right\} \quad \textit{if} \quad \begin{cases} \mu^{**} \leq \mu \\ \gamma^* \leq \gamma \\ \left(\frac{c}{\alpha}\right)^{**} \leq \frac{c}{\alpha} \end{cases}$$

Regardless of all the changes in profits, even regardless of market centralization, an increase in learning efficiency always increases Consumer Surplus and Total Surplus.

$$\frac{\partial\,CS}{\partial\,\mu} \geq 0 \qquad \frac{\partial\,TS}{\partial\,\mu} \geq 0 \qquad \textit{for all permitted parameter values.}$$

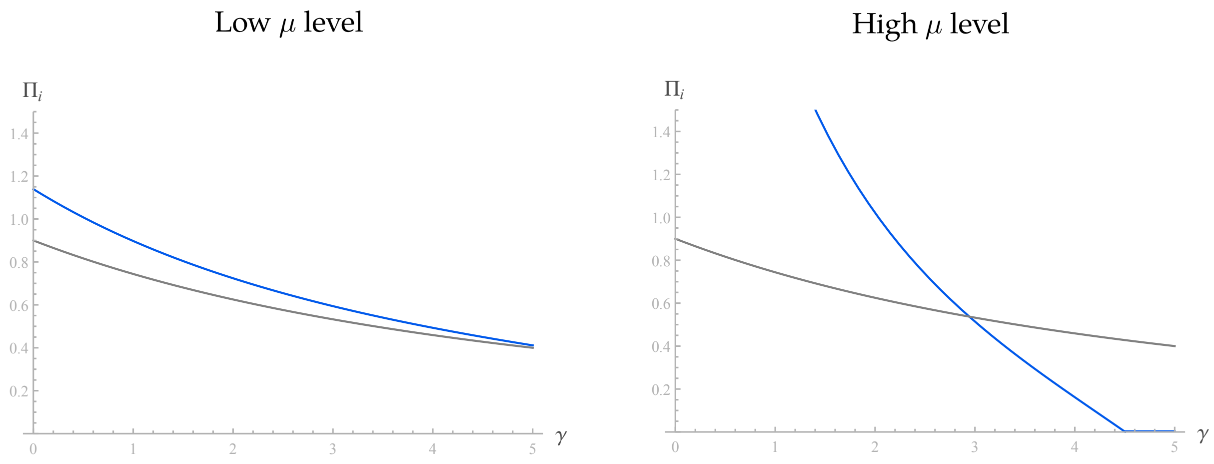
See **Proof 3** in the **Appendix**, and **Figure 1**. The fraction $^c/_\alpha$ can have two interpretations. It can primarily be perceived as the initial potential for profit. When c is quite close to α, which happens to be the initial value of the marginal utility of the consumers, there is not much scope for profit, as with the slightest decline in marginal utility the consumer will not be interested to pay even the costs of production, let alone an additional profit markup. On the other hand, if c is significantly lower than α, there is a large potential for profit. The marginal utility can decline significantly as the consumers consume larger quantities while still being interested to pay the costs c, potentially with a good markup on top. Thus the first interpretation of $^c/_\alpha$ is the *Initial Potential for Profit*.

A second interpretation of the fraction can be the following. Keeping in mind that costs can not decrease below 0, the higher the initial cost is, the higher is the potential for learning. If the initial costs are quite high, then learning-by-doing is very attractive as a mechanism to decrease these costs. However, if the initial costs are already near 0, learning-by-doing does not have a large potential impact in cost reduction. Thus, the second interpretation of c/α is the *Learning Potential*. The last part of **Theorem 1** allows for the possibility of profits decreasing below 0. In this case the two firms would find themselves in the following situation:

		Firm 2	
		Stay	*Exit*
Firm 1	*Stay*	$\Pi_1 < 0, \Pi_2 < 0$	$\Pi^M > 0$, 0
	Exit	0, $\Pi^M > 0$	0, 0

In which case, for reasons not captured in this model I assume that one of the firms chooses to leave thus leading the other firm to enjoy Π^M monopoly profits. Without loss of generality, I assume that the firm that leaves is *Firm 2*[9]. **Figure 1** graphically represent the effect that a low level of μ and a high level of μ have on profits, for an additional plot of profits on μ space see **Figure 4**.

Figure 1: The Impact of Learning-by-Doing on Firm Profits

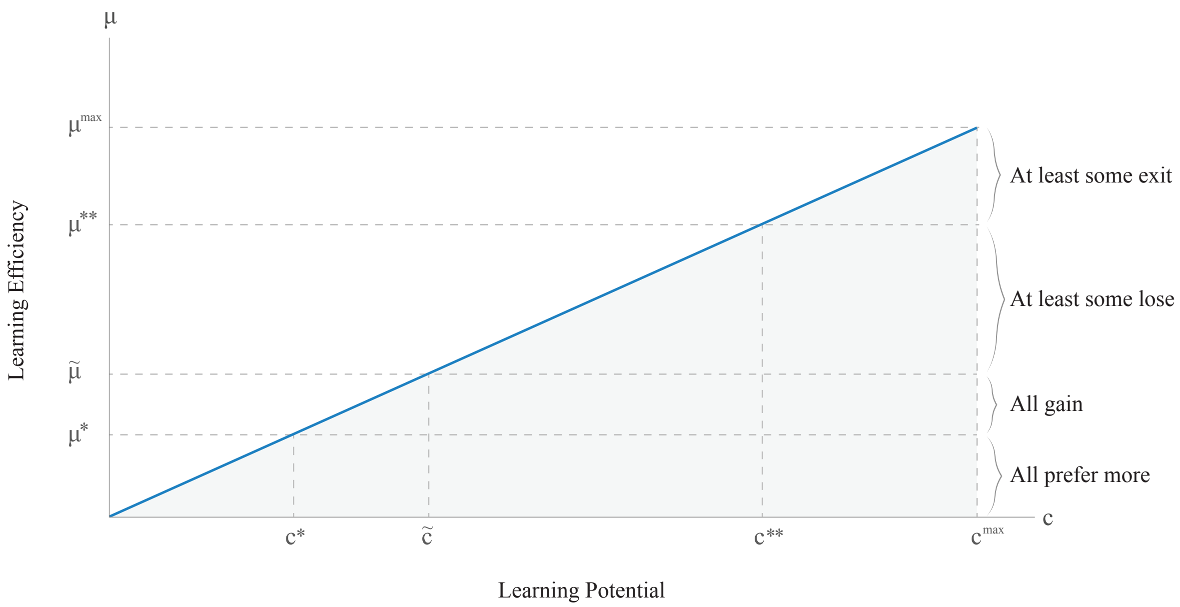


The Gray line represents profits with no learning-by-doing, while the Blue line represents profits under the presence of learning-by-doing. The graph has been plotted using the following parameter values: $\alpha = 15$, $\beta = 5$, $c = 12$, $\delta = 1$, on the left side $\mu = \mu^* = 0.42\beta = 2.1$, and on the right side μ is equated to the highest constrained value $\mu = \frac{2c\beta}{\alpha} = 8$.

[9]Of course this is the case if exiting is costless. Otherwise, if the negative profit is in fact lower than the cost of exiting firms would find it optimal to withstand the losses. Furthermore, here I am also assuming that once a firm leaves it no longer has the option to re-enter the market in the next period.

As can be seen, a low level of μ increases profits regardless of the degree of product differentiation γ, however, for large values of μ, under a high degree of substitutability (high γ), firms profits decrease, in fact in the plot, in the region where γ is close to β (which happens to be 5), market centralization will occur. Of course, μ can reach these high levels only if the fraction c/α is sufficiently high. The conclusion that a high learning efficiency may lead to market centralization has also been pointed out by Fudenberg and Tirole (1983) for the perfectly homogeneous goods case, which as can be seen in the graph, is the case with the highest potential for market centralization. However, from this broader analysis it becomes clear that learning-by-doing can lead to market centralization only when the learning efficiency and the c/α fraction is significantly high, and goods are good substitutes of one another (a special case of which is of course the perfectly homogeneous goods case).

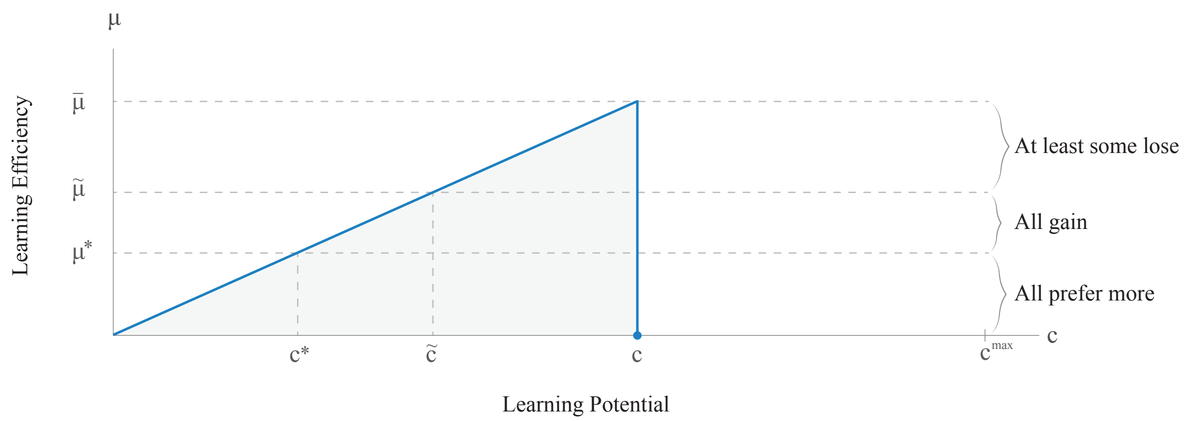
Figure 2: Learning Efficiency Restrictions and Thresholds



The Blue line represents the highest possible value of μ for a given c, while the gray area represents all other lower values that μ may take. The maximal value of μ restriction originates from **Lemma 1**, while the thresholds are explicitly expressed in **Proof 3: Part 2** in the **Appendix**.

To further clarify the potential effect that μ may have on profits, **Figure 2** presents all possible values that μ can take. As can be seen, when μ is relatively low, all prefer a higher learning efficiency. However, as $\mu > \mu^*$ increases, as shown in **Proof 3: Part 1** in the **Appendix**, if the goods highly substitute one another, firm's profits start to decrease, nonetheless firms still gain compared to the outcome with no learning. This is the case until $\mu \to \tilde{\mu}$, beyond which point, if the products highly substitute one another, firm's profits will in fact be lower in the presence of learning-by-doing. This comes as a result of producing beyond the level where $MC = MR$ in the first period. Regardless of the gained efficiency in the second period, the loss of profits from the first period now takes over. Moreover, as $\mu > \mu^{**}$, which is possible only if $c > c^{**}$ (leading to a high value of c/α), if the products are highly substitutable, profits fall below 0, thus leading to market centralization. The reason for such a drastic fall in profits is the high level of c. When c is low, departing from marginal cost pricing results only in the reduction of first period profits. However, as c approaches α, departing from marginal cost pricing causes firms to incur losses in the first period. If competition is not to high, the first period losses can be justified by the second period gains. However, when competition is high, which in this model means that the products are highly substitutable, second period production is quite low, as such, it fails to yield a profit large enough to justify the first period losses. Of course, all of this is possible only given a high level of c.

Figure 3: Learning Efficiency Restrictions and Thresholds for a specific c

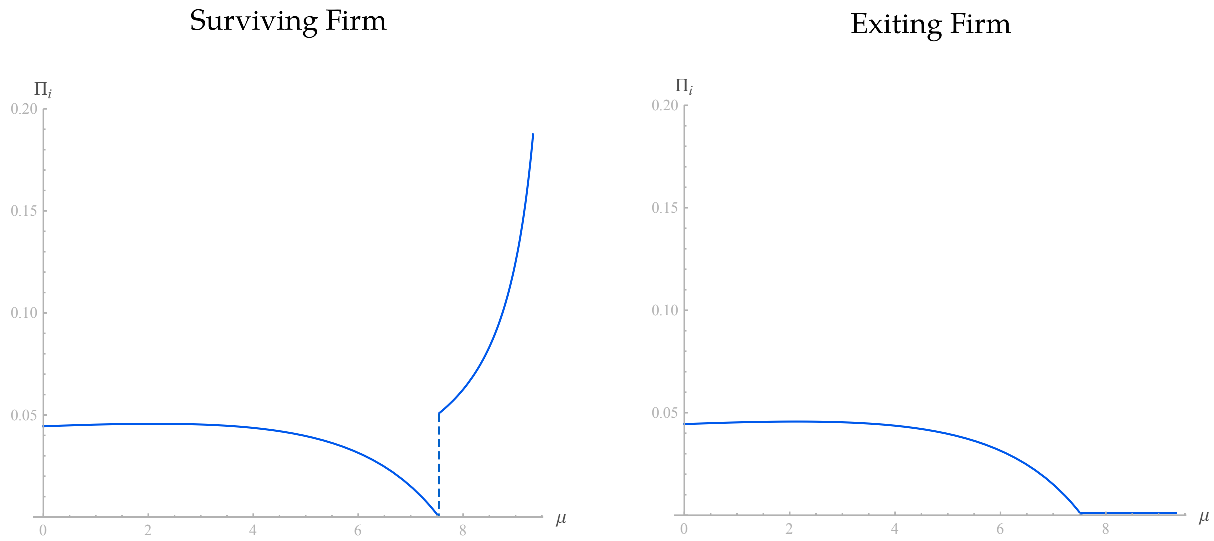


The Blue line represents the highest possible value of μ for a given c, while the gray area represents all other lower values that μ may have. The maximal value of μ restriction originates from **Lemma 1**, while the thresholds are explicitly expressed in **Proof 3: Part 2** in the **Appendix**..

As can be seen in **Figure 3**, for a low level of c, there is no possible value of μ that can lead to market centralization. In fact, if c was even lower, it could be the case that firms would gain from learning-by-doing, regardless of the degree of product differentiation. Thus it becomes clear that learning-by-doing can damage firm profits under high level's of learning efficiency and a high learning potential (alternatively a low initial potential for profit).

Furthermore, **Theorem 1** states that even if centralization occurs consumer surplus and total surplus will still increase further. This represents no typical case, especially regarding consumer surplus. After market centralization it is usually expected that consumer surplus will decline. However, since market centralization can only occur when the learning efficiency is significantly high, a monopoly, by producing more in the lack of competition, benefits consumers in the first period with high production levels, and in the second period with both a high production level and high production efficiency[10]. Since total surplus also increases from this centralization, it becomes clear that learning-by-doing can lead to cases of a natural monopoly.

Figure 4: Learning-by-Doing as an Entrance Barrier



The Blue line represents profits. The graphs have been plotted using the following parameter values: $\alpha = 15$, $\gamma = \beta = 5$, $c = 14$ and $\delta = 1$. Market centralization occurs when μ reaches $\mu^{**} = 1.5\beta = 7.5$, a threshold defined at **Proof 3: Part 2** in the **Appendix**. For the plot on the left, after the discontinuity, thus for $\mu > 7.5$ the profit level has been divided by 8 to allow for a better fitting plot, thus monopoly profits are in fact 8 times larger than they appear in this plot.

It is worth presenting one more plot regarding the effect of learning-by-doing on profits. As can be seen in **Figure 4**, when market centralization occurs, the surviving firm enjoys large monopoly profits, while the firm that exits does not find it profitable to remain in the market. Ex-post, from a purely empirical analysis this would seem as a rather puzzling situation, in which, there is a monopoly that enjoys high profits while there are no evident entry barriers, yet the monopoly structure of the market persists. However, by adding learning-by-doing in the picture, it becomes clear that the strategic effects that this concept gives rise to would make

[10]With linear demands, the fact that the products are substitutes implies that they are strategic substitutes as well. And as is the case with strategic substitutes, the higher the production level of the competing firm, the less the firm produces. Following this logic, and as was shown in **Lemma Prerequisites: Part 2** in the **Appendix**, when the goods are independent ($\gamma = 0$), or when there is a monopoly, firm's production is maximal.

both the firm that exited (remained outside of the market) and the monopolist much more aggressive, which would surely lead to negative profits. As a result, the firm outside the market decides not to enter regardless of the large profits that the monopolist may be receiving. Thus, beyond the other effects discussed in this work, it becomes clear that learning-by-doing also has the potential to play the role of an entry barrier [11].

The main implications of this section is that while both consumers and firms may desire a bit of learning to none, high levels of learning efficiency has the potential to decrease firm profits, in fact it has the potential to lead to market centralization. Consequently, this structure shaping possibility of learning-by-doing has the potential to explain some persisting puzzling market structures. While Dasgupta and Stiglitz (1988) point out that in the perfectly homogeneous goods case, if there is asymmetry between the competing firms in the form of a initial cost advantage, then learning-by-doing may lead to market centralization, here, I have shown that not only is this the case for non homogeneous goods as well, but that market centralization can arise even with perfectly symmetric firms. Regardless of all these effects, a higher level of learning efficiency will always increase both consumer and total surplus.

[11] For a more in depth analysis on the deterrence potential of learning-by-doing for the perfectly homogenous case see Dasgupta and Stiglitz (1988). Moreover, Spence (1981) pays a significant amount of attention to the entry barrier nature of learning-by-doing, however, in Spences model firms outside of the market decide not to enter as a result of the cost disadvantage that exists between them and the early entrants (this disadvantage originates from learning-by-doing). On the other hand, in the case I analyze, firms decide to exit/not to enter, as a result of the high aggressiveness level originating from learning-by-doing. Thus, in the case I analyze, it is the negative expected profits, rather than the cost disadvantage that ensures the persistence of the market structure.

3.5 Learning-by-Doing with Spillovers

While the no spillovers case marks the beginning of this analysis, it is clear that the no knowledge spillovers assumption is rather strong. Though the magnitude may differ, as was shown in **Empirical Research and Evidence**, firms tend to benefit from the gained knowledge of other producers within the same industry. Thus I continue by analyzing the case where spillovers are present, $\eta > 0$, which as was previously stated, leads to the following second period costs:

$$c_{i2} = c_{11} - \mu q_{i1} - \eta \mu q_{j1}$$

The restriction on η is the following, $\eta \in [0, 1]$. Hence η determines what portion of learning leaks from one firm to the other, with $\eta = 1$ representing the case with perfect spillovers, while $\eta \leq 1$, represents cases with imperfect spillovers. Following **Firms General Maximization Problem** leads to the following subgame perfect equilibrium quantities:

$$q_{i1} = \frac{(\alpha - c)\left(8\beta^3 + 4\beta^2(\gamma + \delta\mu) - 2\beta\gamma(\gamma + \delta\eta\mu) - \gamma^3\right)}{2\beta\delta(\eta + 1)\mu^2(\gamma\eta - 2\beta) + (2\beta - \gamma)(2\beta + \gamma)^3}$$

$$q_{i2} = \frac{(\alpha - c)\left(4\beta^2 - \gamma^2\right)(2\beta + \gamma + \eta\mu + \mu)}{2\beta\delta(\eta + 1)\mu^2(\gamma\eta - 2\beta) + (2\beta - \gamma)(2\beta + \gamma)^3}$$

If the goods are independent, and if there is no spillovers or learning, the joint economies no longer exist and the maximization problem reduces to the trivial monopoly maximization problem with the somewhat familiar result:

$$\lim_{\gamma,\mu,\eta \to 0} q_{it} \to \frac{\alpha - c}{2\beta}$$

Proposition 2.

An increase in the level of spillovers η increases first period production if γ is low, decreases first period production when products are close substitutes (high γ), while always increasing second period production.

See **Proof 4** in the **Appendix**, and **Figure 5**. The intuition of **Proposition 2** is the following. When goods are almost independent (low levels of γ), the firms are not much concerned with their competitors (both the μ and the η strategic elements are week when γ is low). However, the fact that now knowledge spillovers are present, implies that production in the second period will be more efficient, which leads to higher second period quantities. This strengthens the savings motive, consequently leading to a higher first period production. However, when the goods highly substitute one another (high γ), the firms are greatly concerned with the choices made by their competitors (both the μ and the η strategic elements are strong when γ is high). Internalizing the fact that with each unit produced firms also increase the efficiency of their competitor has a greater impact than does the cost saving effect, thus first period quantities decrease. In line with Bloom et al. (2013) terminology, the business stealing effect dominates when goods are high substitutes of one another, thus leading to a decrease in first period production, while the learning effect dominates when goods are highly differentiated, increasing first period production. While Fudenberg and Tirole (1983) find that the introduction of spillovers leads to a decrease in the first period output, I have shown that this is a characteristic of their perfectly homogeneous goods case, and does not necessarily carry out in the more general setting analyzed in this paper.

I expect the behavior regarding the choice for the first period quantity to hold even in a more general model. However, the same can not be said for the second period quantity, which, in this model, always increases with spillovers. Until some implicitly defined γ^*, where both firms produce more in the first period, it is straightforward that second period production will be higher as well, as cost would have decreased further. However, the fact that second period production increases with spillovers even beyond γ^* implies that the efficiency fall from producing lower first period quantities is more than offset from the knowledge spillovers, or the additional efficiency gained from the competitors production. This, I believe to be a characteristic of the special model that I analyze in this work.

Lemma 2.

For any given α, β, $c \in [0, \alpha)$, $\delta \in [0, 1]$, $\gamma \in [0, \beta]$ and $\eta \in [0, 1]$:

$$\mu \leq \frac{\beta\left(c - \alpha + \sqrt{\alpha^2 + c^2}\right)}{\alpha}$$

is sufficient but not necessary to ensure a non-negative second period cost $c_{i2} \geq 0$. While in the extreme case, as $c \to \alpha$, the following condition is sufficient:

$$\mu \leq \sqrt{2}\beta$$

See **Proof 5** in the **Appendix**. After introducing spillovers, **Lemma 1** no longer represents the adequate restriction on μ to ensure positive second period costs. As can be seen, because of spillovers, a more restrictive constraint must be imposed on the learning efficiency. Having stated **Lemma 2**, I move on to establish **Theorem 2**.

Theorem 2.

For a relatively moderate level of learning efficiency:

$$\frac{\partial \Pi_i}{\partial \eta} > 0 \qquad \frac{\partial CS}{\partial \eta} > 0 \qquad \textit{and consequently} \qquad \frac{\partial TS}{\partial \eta} > 0$$

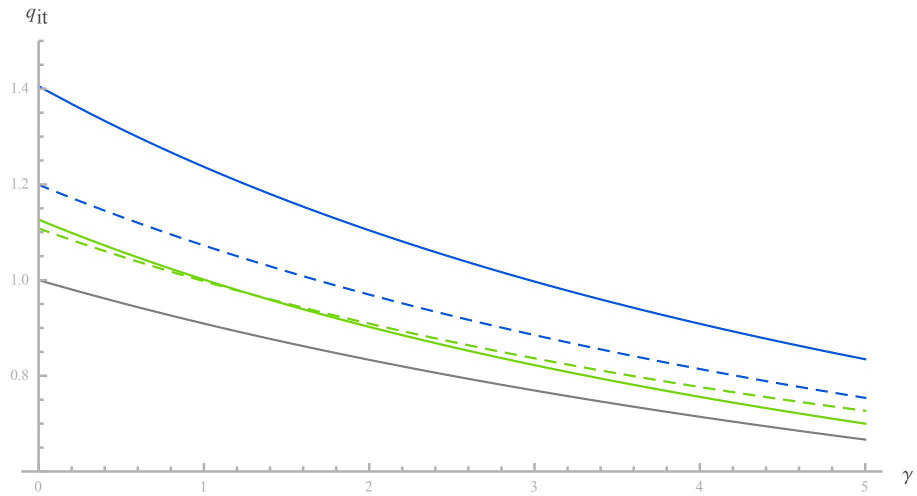
When going from a state with no spillovers to a state with spillovers.

See **Proof 6**[12] in the **Appendix**. Thus, introducing spillovers in a state with no spillovers is welfare enhancing. To see why this happens I start from analyzing the effect that spillovers have on profits with the help of **Figure 5**.

Figure 5 represents the produced quantities, in line with **Proposition 2**. When the goods are highly substitutable (high γ), the introduction of spillovers demotivates firms to produce as much as they were planning to in the absence of spillovers, thus first period production reduces towards the $(MC = MR)$ level of production, consequently increasing first period profits. Furthermore, the loss in efficiency from this fall in production is more than offset from the efficiency gained from spillovers, which ensure that second period production is both higher and more efficient, leading to higher second period profits. When the goods are almost independent (low γ), the introduction of spillovers increases first period production even further, consequently decreasing first period profits, however, this is more than offset from the drastic increase in profits in the second period. On the other hand, the consumers surplus, regardless of the cases

[12] While countless numerical observations convince me that when there is no market centralization, profit, consumer surplus and total surplus always increase in η, regardless of the level of learning efficiency and the initial level of η, because of the complicated structure of the functions it is not possible to algebraically prove this, thus I present a specific case.

Figure 5: The Impact of Spillovers on First and Second Period Quantities

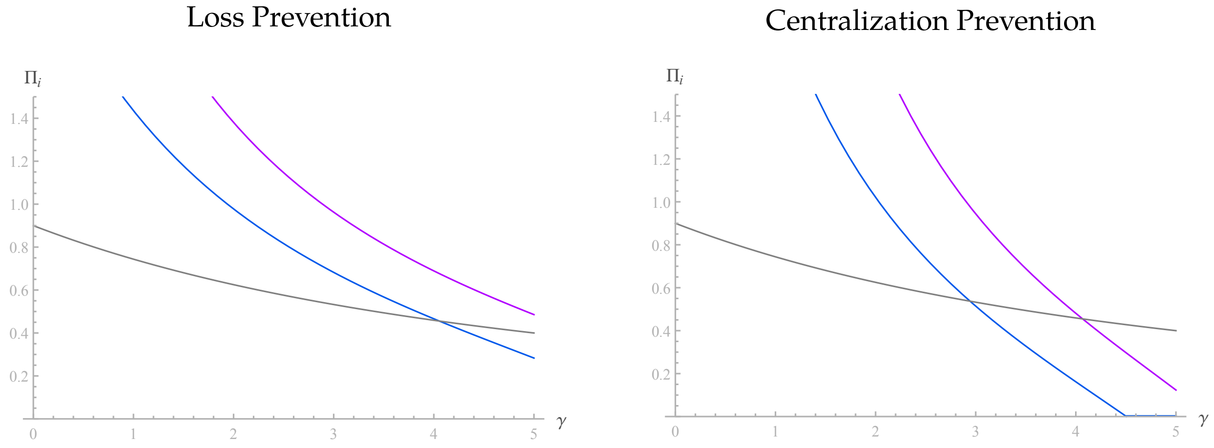


The Gray line represents quantities in the absence of learning and spillovers, The Green lines represent first period production, while the Blue lines represent second period production. The Dashed lines represent production in the presence of learning-by-doing, but in the absence of spillovers, while the Full lines represent quantities when spillovers are present. The graphs have been plotted using the following parameter values: $\alpha = 15$, $\beta = 5$, $c = 5$, $\delta = 0.5$, $\mu = 1.8$ and $\eta = 1$. The nature of the function remains unchanged even for lower levels of η, however, the Full lines begin to converge towards the Dashed lines, thus making it more challenging to distinguish the effect of η.

where first period production may decrease, always increases as a result of the dominating effect of the increase in second period production, which also implies even lower second period prices. These results, while seemingly intuitive, may be a characteristic of the model at hand and there is no reason to believe that they hold for a more general model. Nonetheless, the effects at play, which is what this analysis focuses on, surely persist regardless of generality.

Beyond profits and consumer surplus, spillovers play an important role in preventing market centralization. From **Lemma 2**, it becomes evident that with perfect spillovers there can be no market centralization (as μ^{**} is no longer feasible). However, even with imperfect spillovers, as shown in **Theorem 2**, profits increase. This could possibly prevent profits from dropping below 0, and thus prevent market centralization. I illustrate two cases in **Figure 6**. In the first case, if there would be no spillovers and if the goods would be high substitutes of one another (high γ), firms would have lower profits compared to the case with no learning. However, with a level of spillovers of 30%, not only is this loss prevented, but profits are now higher than the no learning case for any possible γ. On the right side, spillover levels of 20% by ensuring that profits remain positive, ensures that there is no market centralization. Of course these are very specific cases but nonetheless they explicitly portray the impact that spillovers have on profits.

Figure 6: The Impact of Spillovers on Firm Profits



The Gray line represents profits when there is no learning-by-doing or learning spillovers, while the Blue line represents profits under the presence of learning-by-doing only and the Violet line represents profits under learning-by-doing and knowledge spillovers. The graphs have been plotted using the following parameter values: $\alpha = 15$, $\beta = 5$, $c = 12$, $\delta = 1$, on the left side $\mu = 6$ and $\eta = 0.3$, while on the right side $\mu = 8$ and $\eta = 0.2$.

In their work Ghamawat and Spence (1985) note that while it is often the case in settings with $R\&D$ investment for the disincentive effect of spillovers to dominate the efficiency effect, this is not the case for spillovers in learning-by-doing. With learning-by-doing profits depend both indirectly (via cost reduction) and directly on the production level (the control variable of interest), while in the $R\&D$ case, in which profits only indirectly depend on $R\&D$ investment (the control variable of interest), the disincentive motive can easily lead firms to conduct no $R\&D$ investment. Thus, although the spillover effects are comparable, the trade-off between efficiency and disincentives fundamentally differs between models with $R\&D$ and models with learning-by-doing. This claim clearly prevails in my analysis, in which, as was shown, spillovers increase both profits and consumer surplus.

The purpose of this section is to emphasize that regardless of the fact that spillovers make competitors more efficient as well, and thus may reduce production incentives, spillovers increase both firm profits and consumer surplus. Moreover, as was shown, learning spillovers have the potential to prevent market centralization. Initially this may be perceived as a positive feature and one that would advocate for even higher levels of spillovers. However, as was demonstrated in **Learning-by-Doing with no Spillovers**, market centralization, since it occurs in the form of a natural monopoly is not welfare decreasing. Thus the centralization prevention effect of spillovers is not by default desirable.

4 Market Inefficiencies

4.1 The Social Planner

To start the analysis of market inefficiencies it is necessary to identify the output levels that would maximize social welfare. To do so I solve for the so called social planner who maximizes total surplus:

$$\max_{q_{11},q_{12},q_{21},q_{22}} TS$$
$$= \max_{q_{11},q_{12},q_{21},q_{22}} U_1(q_{11},q_{21}) - q_{11}c_{11} - q_{21}c_{21} + \delta\left(U_2(q_{12},q_{22}) - q_{12}c_{12} - q_{22}c_{22}\right)$$

If both $\mu > 0$ and $\eta > 0$, thus if both learning-by-doing and learning spillovers are present, the social planners equilibrium quantities are:

$$q_{i1}^s = \frac{(\alpha - c)(\beta + \gamma + \delta(\eta + 1)\mu)}{(\beta + \gamma)^2 - \delta(\eta + 1)^2\mu^2}$$

$$q_{i2}^s = \frac{(\alpha - c)(\beta + \gamma + \eta\mu + \mu)}{(\beta + \gamma)^2 - \delta(\eta + 1)^2\mu^2}$$

Since there are no strategic effects, the social planner's precommitted equilibrium is equivalent to the subgame perfect equilibrium, as would be the case for a monopoly. These equilibrium quantities also reduce to the somewhat familiar social planner's output when there are no joint economies and the products are independent:

$$\lim_{\gamma,\mu,\eta\to 0} q_{it}^s \to \frac{\alpha - c}{\beta}$$

Before continuing, since the quantities produced by the social planner differ from the decentralized solution, **Lemma 2** is no longer sufficient to ensure positive second period costs. Thus a new restriction is needed.

Lemma 3.

For any given α*,* β*,* $c \in [0, \alpha)$*,* $\delta \in [0, 1]$*,* $\gamma \in [0, \beta]$ *and* $\eta \in [0, 1]$*:*

$$\mu \leq \frac{c\beta}{2\alpha}$$

is sufficient but not necessary to ensure a non-negative second period cost $c_{i2} \geq 0$*. While in the extreme case, as* $c \to \alpha$*, the following condition is sufficient:*

$$\mu \leq \frac{\beta}{2}$$

See **Proof 7** in the **Appendix**. It is clear that **Lemma 3** is the most restrictive lemma. This comes as a result of the higher production quantities that the social planner finds optimal to choose.

Proposition 3.

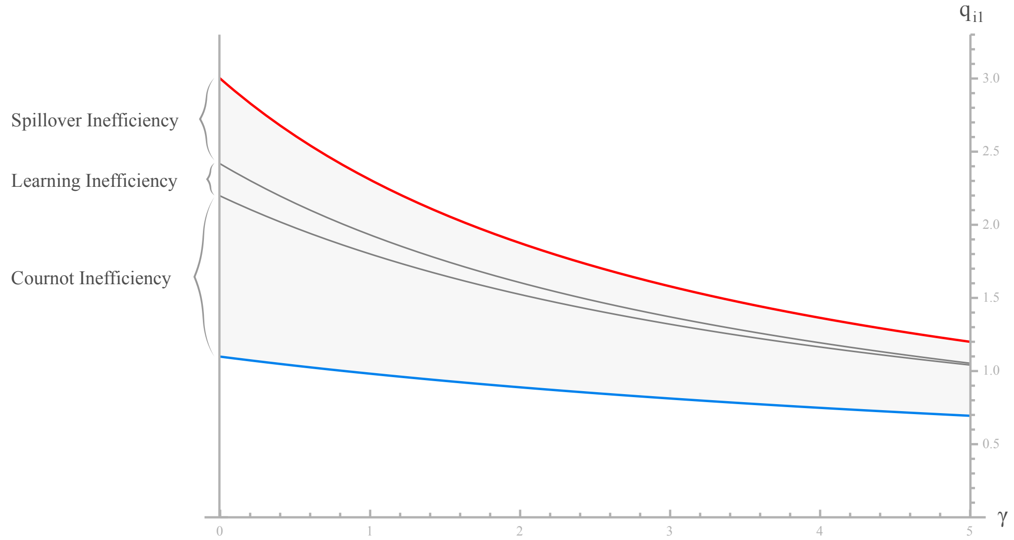
The Social Planner produces more than the Decentralized Economy for any given level of product differentiation.

See **Proof 8** in the **Appendix**. Similar conclusions have been reached by Fudenberg and Tirole (1983) and Dasgupta and Stiglitz (1988) for the perfectly homogeneous case with learning-by-doing, and by Qiu (1997) for the Vives and Singh (1984) model with $R\&D$. And as noted by Spence (1984) given the discrepancy between the profit maximizing firm and the welfare maximizing social planner there is no a priori reason to assume that the decentralized equilibrium will match with the social planner's equilibrium choices. In fact, since marginal revenue is steeper than the demand curve, in general it is expected that the decentralized solution will be lower, as suggested in **Proposition 3**.

4.2 Decomposing Quantity Inefficiency

I utilize **Figure 7** to analyze the decomposed inefficiencies.

Figure 7: Quantity Produced Inefficiency Decomposition



The Blue line represents the optimal quantities from the decentralized equilibrium, while the Red line represent the social planner's optimal quantities. The space between the Blue line and the lower Gray line represents the cournot inefficiency, the space between the two Gray lines represents the inefficiency arising from learning-by-doing, and finally the space between the upper Gray line and the Red line represents the inefficiency arising from spillovers. The graphs have been plotted using the following parameter values: $\alpha = 15$, $\beta = 5$, $c = 5$, $\delta = 1$, $\mu = 0.83$ and $\eta = 1$.

See **Appendix 1.11** for the proper decomposition. By *Cournot Inefficiency* I refer to the discrepancy arising from the fact that the social planner maximizes total surplus while the firms maximize profits for a given level of costs. Introducing learning incentivizes both firms and the social planner to increase their optimal quantities. However, since the social planner maximizes total surplus while the firms maximize their profits, the optimal quantity increase of the social planner originating from the presence of learning-by-doing will be higher than the optimal quantity increase of the firms, thus the discrepancy between the optimal values grows further, this additional inefficiency is referred to as the *Learning Inefficiency*. Finally the *Spillover Inefficiency* refers to the additional inefficiency arising from the different impact that the presence of spillovers has on the optimal choice of the social planner and the decentralized economy.

Proposition 4.

The fraction of the inefficiency arising from learning-by-doing has a tendency to decrease as the goods become perfect substitutes.

See **Proof 9** in the **Appendix**. The intuition of **Proposition 4** is the following. The higher the degree of substitutability between the goods the more incentivised the firms are to increase their production as the μ *Strategic Effect* becomes stronger. Thus the strengthening of competition pushes the firms to further utilize their learning capacity, and since their initial level is lower than the social planner's level, the discrepancy originating from learning decreases as $\gamma \to \beta$. In the concrete example presented in **Figure 7**, the *Learning Inefficiency* drops from 11.5% when the goods are independent to 2.3% when the goods are homogeneous. The same can not be said regarding the spillover inefficiency. As can be seen in the concrete example in **Figure 7**, regardless of the degree of product differentiation the *Spillover Inefficiency* remains a significant proportion of the total inefficiency. In the concrete example presented in **Figure 7**, the *Spillover Inefficiency* starts at 30.5% when the goods are independent and drops to 28.9% when the goods are homogeneous, marking a drop of only 1.6%. Although **Figure 7** represents a specific case, the fall of the *Learning Inefficiency* as $\gamma \to \beta$ and the persistence of the *Spillover Inefficiency*, are in fact general features. The only difference from this specific case and other possible cases is that as $c \to \alpha$ allowing for $\mu \to \beta/2$, and as $\eta \to 1$, the *Learning Inefficiency* and *Spillover Inefficiency* becomes more and more significant.

The aim of this section is to initially demonstrate that regardless of the degree of product differentiation, the decentralized market underproduces compared to the optimal quantity that would maximize social welfare. Furthermore, this section aims to demonstrate that since learning-by-doing is internalized by the firm it does not add much to the already existing discrepancy between the decentralized and optimal output, especially as competition becomes more aggressive, while on the other hand spillovers remain a substantial share of the inefficiency regardless of the degree of product differentiation.

5 Conclusions

Firstly, in this analysis I have extended the work of Fudenberg and Tirole (1983) by demonstrating that knowledge spillovers need not lead to a decrease of first period production, and by identifying conditions under which learning-by-doing can lead to market centralization. I have further extended the work of Dasgupta and Stiglitz (1988) by showing that asymmetry in the form of a cost advantage is not necessary for learning-by-doing to lead to market centralization, and that this can occur with perfectly symmetric firms as well.

Beyond these extensions, and more importantly, the main finding of this work is that low levels of learning-by-doing efficiency are beneficial for both firms and consumers, while high levels of learning-by-doing efficiency may lead to a decrease of firm profits, even centralization of the market, which can potentially explain some puzzling market structures. Moreover, I have shown that the presence of spillovers benefits both firms and consumers and may help avoid profit losses and may even prevent market centralization. Finally I have shown that since learning-by-doing is internalized by the firms, it does not greatly increase the discrepancy between the decentralized and the optimal output quantities, while on the other hand spillovers greatly contribute to the increase of this inefficiency. From these findings I derive the following policy implications:

- Even though there may be numerous factors that justify policies aiming to prevent market centralization, in this work I have attempted to show that the market centralizing forces driven from learning-by-doing, in general, are not socially harmful, since this centralization will not benefit only the surviving firm but consumers as well (the monopoly will be a natural one). Thus, all else being equal, if market centralization is driven by increased efficiency, it is advised to let the evolution of the market structure run its course.
- Furthermore, I have endeavored to show that the only case in which it may be justifiable to control/reduce the magnitude of spillovers is if the aim is to centralize the market, or to allow for market centralization to occur. In any other case, based on this analysis, the reduction of spillovers is expected to be socially harmful. By emphasizing the fact that profits depend directly on quantities produced, I aim to point out that policymakers need not greatly worry about firm incentives as much as in the $R\&D$ case.

- Finally, the fact that learning-by-doing is well internalized from the firms implies that in general the added inefficiency does not have much potential to justify market intervention. However, if it can be argued that there is a high potential for market spillovers, then market intervention, via different tools such as subsidies or taxes, may in fact prove to be socially improving.

6 Appendix

1.1

Taking the total derivative of Π_i:

$$d\Pi_i = \frac{\partial \Pi_i}{\partial \pi_{i1}}\left(\frac{\partial \pi_{i1}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i1}}{\partial q_{i2}}dq_{i2} + \frac{\partial \pi_{i1}}{\partial c_{i1}}dc_{i1}\right) + \frac{\partial \Pi_i}{\partial \pi_{i2}}\left(\frac{\partial \pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i2}}dq_{i2}\right)$$

$$+\frac{\partial \Pi_i}{\partial \pi_{i2}}\left(\frac{\partial \pi_{i2}}{\partial q_{i2}}\frac{\partial q_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial q_{i2}}\frac{\partial q_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i2}}dq_{i2} + \frac{\partial \pi_{i2}}{\partial q_{i2}}\frac{\partial q_{i2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial q_{i2}}\frac{\partial q_{i2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i2}}dq_{i2}\right)$$

$$+\frac{\partial \Pi_i}{\partial \pi_{i2}}\left(\frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i2}}dq_{i2} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i2}}dq_{i2}\right)$$

Realizing that $\frac{\partial \pi_{i2}}{\partial q_{i2}} = 0$, since second period quantities are already maximized, the above total derivative reduces to:

$$d\Pi_i = \frac{\partial \Pi_i}{\partial \pi_{i1}}\left(\frac{\partial \pi_{i1}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i1}}{\partial q_{i2}}dq_{i2} + \frac{\partial \pi_{i1}}{\partial c_{i1}}dc_{i1}\right) + \frac{\partial \Pi_i}{\partial \pi_{i2}}\left(\frac{\partial \pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i2}}dq_{i2}\right)$$

$$+\frac{\partial \Pi_i}{\partial \pi_{i2}}\left(\frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i2}}dq_{i2} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i1}}dq_{i1} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i2}}dq_{i2}\right)$$

Furthermore, dividing by dq_{i1}, and realizing that $\frac{dq_{i2}}{dq_{i1}} = 0$, and that because of constant marginal costs $\frac{dc_{i1}}{dq_{i1}} = 0$:

$$\frac{d\Pi_i}{dq_{i1}} = \frac{\partial \Pi_i}{\partial \pi_{i1}}\frac{\partial \pi_{i1}}{\partial q_{i1}} + \frac{\partial \Pi_i}{\partial \pi_{i2}}\left(\frac{\partial \pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i1}}\right)$$

Which reduces to:

$$\frac{d\Pi_i}{dq_{i1}} = \frac{\partial \pi_{i1}}{\partial q_{i1}} + \delta\left(\frac{\partial \pi_{i2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{i2}}\frac{\partial c_{i2}}{\partial q_{i1}} + \frac{\partial \pi_{i2}}{\partial q_{j2}}\frac{\partial q_{j2}}{\partial c_{j2}}\frac{\partial c_{j2}}{\partial q_{i1}}\right)$$

If the goods are substitutes:

$$\frac{d\Pi_i}{dq_{i1}} = \frac{\partial \pi_{i1}}{\partial q_{i1}} + \delta\left(\underbrace{\underbrace{\frac{\partial \pi_{i2}}{\partial c_{i2}}}_{(-)}\underbrace{\frac{\partial c_{i2}}{\partial q_{i1}}}_{(-)}}_{(+)} + \underbrace{\underbrace{\frac{\partial \pi_{i2}}{\partial q_{j2}}}_{(-)}\underbrace{\frac{\partial q_{j2}}{\partial c_{i2}}}_{(+)}\underbrace{\frac{\partial c_{i2}}{\partial q_{i1}}}_{(-)}}_{(+)} + \underbrace{\underbrace{\frac{\partial \pi_{i2}}{\partial q_{j2}}}_{(-)}\underbrace{\frac{\partial q_{j2}}{\partial c_{j2}}}_{(-)}\underbrace{\frac{\partial c_{j2}}{\partial q_{i1}}}_{(-)}}_{(-)}\right)$$

If the goods are compliments:

$$\frac{d\Pi_i}{dq_{i1}} = \frac{\partial \pi_{i1}}{\partial q_{i1}} + \delta \left(\underbrace{\underbrace{\frac{\partial \pi_{i2}}{\partial c_{i2}}}_{(-)} \underbrace{\frac{\partial c_{i2}}{\partial q_{i1}}}_{(-)}}_{(+)} + \underbrace{\underbrace{\frac{\partial \pi_{i2}}{\partial q_{j2}}}_{(+)} \underbrace{\frac{\partial q_{j2}}{\partial c_{i2}}}_{(-)} \underbrace{\frac{\partial c_{i2}}{\partial q_{i1}}}_{(-)}}_{(+)} + \underbrace{\underbrace{\frac{\partial \pi_{i2}}{\partial q_{j2}}}_{(+)} \underbrace{\frac{\partial q_{j2}}{\partial c_{j2}}}_{(-)} \underbrace{\frac{\partial c_{j2}}{\partial q_{i1}}}_{(-)}}_{(+)} \right)$$

If, however, the goods are independent, in particular, if $\gamma = 0$, the last two arguments no longer appear as $\frac{\partial \pi_{i2}}{\partial q_{j2}} = 0$, and the only motive for the firm to deviate from marginal cost pricing in the first period remains the savings motive.

1.2

Proof 1.
When $\eta = 0$ first period quantity is equal to:

$$q_{i1} = \frac{(\alpha - c)\left(8\beta^3 + 4\beta^2(\gamma + \delta\mu) - 2\beta\gamma^2 - \gamma^3\right)}{(2\beta - \gamma)(2\beta + \gamma)^3 - 4\beta^2\delta\mu^2}$$

$$\frac{\partial q_{i1}}{\partial \mu} = \frac{(\alpha - c)4\beta^2\delta\left(4\beta^2\delta\mu^2 + 2\mu(2\beta - \gamma)(2\beta + \gamma)^2 + (2\beta - \gamma)(2\beta + \gamma)^3\right)}{\left((2\beta - \gamma)(2\beta + \gamma)^3 - 4\beta^2\delta\mu^2\right)^2}$$

The sign of this derivative depends on the value of

$$4\beta^2\delta\mu^2 + 2\mu(2\beta - \gamma)(2\beta + \gamma)^2 + (2\beta - \gamma)(2\beta + \gamma)^3$$

which under the given initial assumptions is positive for any permitted value of the parameters, thus

$$\frac{\partial q_{i1}}{\partial \mu} > 0$$

regardless of the value of γ.

When $\eta = 0$ second period quantity is equal to:

$$q_{i2} = \frac{(\alpha - c)\left(4\beta^2 - \gamma^2\right)(2\beta + \gamma + \mu)}{(2\beta - \gamma)(2\beta + \gamma)^3 - 4\beta^2\delta\mu^2}$$

$$\frac{\partial q_{i2}}{\partial \mu} = \frac{(\alpha - c)\left(4\beta^2 - \gamma^2\right)\left(8\beta^2\delta\mu(2\beta + \gamma) + 4\beta^2\delta\mu^2 + (2\beta - \gamma)(2\beta + \gamma)^3\right)}{\left((2\beta - \gamma)(2\beta + \gamma)^3 - 4\beta^2\delta\mu^2\right)^2}$$

The sign of this derivative depends on the value of

$$8\beta^2\delta\mu(2\beta + \gamma) + 4\beta^2\delta\mu^2 + (2\beta - \gamma)(2\beta + \gamma)^3$$

which under the given initial assumptions is positive for any permitted value of the parameters, thus

$$\frac{\partial q_{i2}}{\partial \mu} > 0$$

regardless of the value of γ. ■

1.3 Lemma Prerequisites

The motive of **Lemma Prerequisites** is to show that first period production is maximal when $\delta \to 1$, $\gamma \to 0$ and $\eta \to 1$. This will prove useful for proving both **Lemma 1** and **Lemma 2**.

Part 1: $\frac{\partial q_{i1}}{\partial \delta} > 0$

$$\frac{\partial q_{i1}}{\partial \delta} = \frac{(\alpha - c)2\beta\mu(2\beta-\gamma)(2\beta+\gamma)^2(2\beta-\gamma\eta)(2\beta+\gamma+\eta\mu+\mu)}{(2\beta\delta(\eta+1)\mu^2(\gamma\eta-2\beta)+(2\beta-\gamma)(2\beta+\gamma)^3)^2}$$

Given the usual restrictions, this derivative is clearly positive, implying that as δ increases from 0 to 1, first period production increases[13].

Part 2: $\frac{\partial q_{i1}}{\partial \gamma} < 0$

$$\frac{\partial q_{i1}}{\partial \gamma} =$$

$$\frac{(2\beta+\gamma)(c-\alpha)}{(2\beta\delta(\eta+1)\mu^2(\gamma\eta-2\beta)+(2\beta-\gamma)(2\beta+\gamma)^3)^2} \cdot \begin{pmatrix} (\gamma-2\beta)^2(2\beta+\gamma)^3 \\ +2\beta\delta\mu(2\beta+\gamma)\left(4\beta^2(\eta+2)-4\beta\gamma(\eta+2)+3\gamma^2\eta\right) \\ +4\beta\delta(\eta+1)\mu^2\left(2\beta^2(\eta+1)-\beta\gamma(\eta+3)+\gamma^2\eta\right) \end{pmatrix}$$

It is clear that the sign of the derivative depends inversely on the sign of the within bracket last part, which can be shown to be monotonically increasing in η, thus it is vital to check its sign when $\eta = 0$, which reduces the function to:

$$4\beta^2\delta\mu^2(2\beta-3\gamma)+16\beta^2\delta\mu(\beta-\gamma)(2\beta+\gamma)+(\gamma-2\beta)^2(2\beta+\gamma)^3$$

For low values of μ this function is positive when $\gamma = 0$, concave, and positive when $\gamma = \beta$ (thus positive for any γ), while for large values of μ, it monotonically decreases in γ. Furthermore, the level of μ needed for the function to become negative is :

$$\mu > \frac{3\sqrt{3}\beta}{2}$$

Which I only assume to be unfeasible, while I formally prove this in **Lemma 2**. Under this assumption, it is clear that the part upon which the sign of the derivative is inversely related is positive, thus the whole derivative is negative. This implies that as goods become better compliments of one another, the first period production reduces regardless of all the other parameter values[14].

[13]This is rather intuitive, as the more future oriented firms are ($\delta \to 1$) the more they will produce in the first period to ensure a more efficient and competitive position in the second period. On the other hand if the firms are completely myopic ($\delta = 0$), they do not diverge first period production from the $(MR - MC)$ level at all.

[14]This follows from the fact that the goods are strategic compliments, thus the more the production of the other firm plays a role, the less the firm produces.

Part 3: $\gamma \to 0\,,\ \frac{\partial q_{\text{i1}}}{\partial \eta} > 0$

$$\text{for } \gamma = 0 \qquad \frac{\partial q_{\text{i1}}}{\partial \eta} = \frac{\delta\mu^2(\alpha - c)(2\beta + \delta\mu)}{\left(\delta(\eta+1)\mu^2 - 4\beta^2\right)^2}$$

Which is clearly positive.

1.4

Proof 2.
From:

$$c_{i2} = c_{i1} - \mu q_{i1} \geq 0$$

$$\implies \mu \leq \frac{c_{i1}}{q_{i1}} = c_{i1} \cdot \frac{(2\beta-\gamma)(2\beta+\gamma)^3 - 4\beta^2\delta\mu^2}{(c-\alpha)\left(-8\beta^3 - 4\beta^2(\gamma+\delta\mu) + 2\beta\gamma^2 + \gamma^3\right)}$$

*As shown in **Lemma Prerequisites** I can let $\gamma \to 0$ and $\delta \to 1$, and rewriting c_{i1} as c reduces the above inequality to:*

$$\mu \leq \frac{c(\mu - 2\beta)}{c - \alpha}$$
$$\implies \mu \leq \frac{2\beta c}{\alpha}$$

And since $\lim_{c\to\alpha} \frac{2\beta c}{\alpha} = 2\beta$, in the extreme case $\mu \leq 2\beta$ is sufficient.■

1.5

Proof 3.
Part 1: $\frac{\partial \Pi_i}{\partial \mu}$
Without loss of generality I assume that $\delta = 1$. Lower values of δ would certainly change the values of the thresholds and the values of the function, however, it would not change the general behavior of the functions, which I am interested in analyzing. The only exception is the case when $\delta = 0$ (fully myopic firms), in which case in both periods we are back to marginal cost pricing, however, this is not a case that I focus on in this work.

$$\frac{\partial \Pi_i}{\partial \mu} =$$

$$\frac{2\beta(c-\alpha)^2}{\left((2\beta-\gamma)(2\beta+\gamma)^3 - 4\beta^2\mu^2\right)^3} \cdot \left(\begin{array}{c} (\gamma-2\beta)^2(2\beta+\gamma)^5\left(4\beta^2 - 2\beta\gamma - \gamma^2\right) - 32\beta^5\mu^4 \\ +\mu(2\beta+\gamma)^3\left(64\beta^5 - 96\beta^4\gamma + 24\beta^2\gamma^3 - 2\beta\gamma^4 - \gamma^5\right) \\ -12\beta^2\gamma\mu^2(2\beta-\gamma)(2\beta+\gamma)^2(4\beta+\gamma) \\ +4\beta^2\mu^3\left(-32\beta^4 - 32\beta^3\gamma + 4\beta\gamma^3 + \gamma^4\right)) \end{array}\right)$$

*It is clear that the denominator decreases in μ. Allowing for the highest theoretically possible value of $\mu = 2\beta$ as shown in **Lemma 1**, the denominator reduces to:*

$$\left(16\beta^3\gamma - 4\beta\gamma^3 - \gamma^4\right)^3$$

Which, given the typical restrictions, is always positive. Thus the sign of the derivative depends on the sign of the last inner bracket part. For low values of μ (these values will be shortly explicitly defined), this function is always positive. However, for values of μ larger than a particular threshold, this function becomes negative for high γ. To illustrate this, lets assume $\mu = 0$, which would imply that we are checking the change from going from a state with no learning-by-doing, to a state with the presence of infinitesimal learning efficiency. In this case the inner bracket part reduces to:

$$(\gamma - 2\beta)^2(2\beta + \gamma)^5 \left(4\beta^2 - 2\beta\gamma - \gamma^2\right)$$

Which is positive regardless of the value of γ. On the other hand, letting μ reach its highest constrained value of $\mu = \frac{2c\beta}{\alpha}$, and considering the case when the goods are highly homogeneous (in fact considering the perfectly homogeneous case, $\gamma \to \beta$) the inner bracket part reduces to:

$$\frac{\beta^9 \left(243\alpha^4 - 512c^4 - 1888\alpha c^3 - 2160\alpha^2 c^2 - 594\alpha^3 c\right)}{\alpha^4}$$

Which is negative whenever c passes a certain threshold (which I will shortly define).

Part 2: Explicit Thresholds

To identify the maximal value of μ for which all firms, regardless of product differentiation, desire more learning efficiency I do the following. I take the $\frac{\partial \Pi_i}{\partial \mu}$ derivative, equate $\gamma = \beta$, and $\delta = 1$, equate the reduced derivative to 0, and solve for μ, which leads to the first explicitly defined threshold:

$$\mu^* = 0.42\beta$$

To identify the maximal value of μ for which all firms, regardless of product differentiation, gain from learning-by-doing I do the following. I take the profit function Π_i, equate $\gamma = \beta$, and $\delta = 1$, equate the reduced function to the profit function in the absence of learning ($\mu \to 0$), and solve for μ, which leads to the next explicitly defined threshold:

$$\tilde{\mu} = 0.73\beta$$

To identify the maximal value of μ for which no firms exit, regardless of product differentiation, I do the following. I take the profit function Π_i, equate $\gamma = \beta$, and $\delta = 1$, equate the reduced function to 0, and solve for μ, which leads to the final μ explicitly defined threshold:

$$\mu^{**} = 1.50\beta$$

*From the μ identified thresholds the c thresholds can easily be derived via the restriction originating from **Lemma 1**:*

$$\mu \leq \frac{2c\beta}{\alpha}$$

Which leads to the following c thresholds:

$$c^* = 0.212\alpha \qquad \tilde{c} = 0.36\alpha \qquad c^{**} = 0.75\alpha$$

The only threshold left undefined is γ^*, which can not be expressed explicitly.

Part 3: $\frac{\partial CS}{\partial \mu}$

Firstly I define CS to be:

$$CS = U_1(q_{11}, q_{21}) - q_{11}p_{11} - q_{21}p_{21} + \delta \left(U_2(q_{12}, q_{22}) - q_{12}p_{12} - q_{22}p_{22}\right)$$

Once more, without loss of much generality I assume that $\delta = 1$:

$$\frac{\partial CS}{\partial \mu} =$$

$$\frac{2(\beta+\gamma)(c-\alpha)^2}{\left((2\beta-\gamma)(2\beta+\gamma)^3-4\beta^2\mu^2\right)^3} \cdot \left(\begin{array}{c} (\gamma-2\beta)^2(2\beta+\gamma)^5\left(8\beta^2-\gamma^2\right) \\ +\mu(2\beta-\gamma)(2\beta+\gamma)^3\left(96\beta^4-24\beta^2\gamma^2+\gamma^4\right) \\ +12\beta^2\mu^2(2\beta-\gamma)(2\beta+\gamma)^2\left(8\beta^2-\gamma^2\right) \\ +4\beta^2\mu^3\left(32\beta^4-8\beta^2\gamma^2+\gamma^4\right) \end{array} \right)$$

*Once more, via a logic similar to **Proof 3: Part 1**, it becomes clear that the sign of the derivative depends on the inner bracket part, which given the typical restrictions is always positive.*

***Part 4:** $\frac{\partial TS}{\partial \mu}$*
Firstly I define TS to be:

$$TS = U_1(q_{11}, q_{21}) - q_{11}c_{11} - q_{21}c_{21} + \delta\left(U_2(q_{12}, q_{22}) - q_{12}c_{12} - q_{22}c_{22}\right) = CS + \Pi_1 + \Pi_2$$

Once more, without loss of much generality I assume that $\delta = 1$. Since CS always increases, while Π_i may decrease for high levels of γ only, it is sufficient to check if TS increases for the extreme case, $\gamma \to \beta$, and if so, this implies that TS increases for any value of γ. In this case:

$$\frac{\partial TS}{\partial \mu} = \frac{8(c-\alpha)^2(9\beta+4\mu)\left(108\beta^3+45\beta^2\mu-8\beta\mu^2-4\mu^3\right)}{\left(27\beta^2-4\mu^2\right)^3}$$

Which once more, via a logic applied to the above proofs, can be shown to always be positive.

Part 5: Market Centralization Case
The lowest values for which the market centralizes is μ^{**} which is possible only if $c \geq c^{**}$. It is straightforward to conclude that profits will increase for the surviving firm, as it will enjoy monopoly profits as opposed to 0 or negative profits, while the firm that leaves will have 0 profits. Furthermore, once the market is centralized the profit function becomes:

$$\Pi^M = \frac{(c-\alpha)^2(\beta\delta+\beta+\delta\mu)}{4\beta^2-\delta\mu^2} \qquad \frac{\partial \Pi^M}{\partial \mu} = \frac{\delta(c-\alpha)^2(2\beta+\mu)(2\beta+\delta\mu)}{\left(\delta\mu^2-4\beta^2\right)^2}$$

And since the derivative is clearly positive, monopoly profits increase in μ. To check what happens to CS as the market centralizes, I equate both $\gamma = \beta$ and without much loss of generality $\delta = 1$. I further subtract the monopoly level of CS_M, which leads to:

$$\frac{\beta(c-\alpha)^2\left(567\beta^4-288\beta^3\mu-268\beta^2\mu^2+52\beta\mu^3+34\mu^4\right)}{(\mu-2\beta)^2\left(27\beta^2-4\mu^2\right)^2}$$

Which is positive whenever $\mu < 1.19\beta$, 0 if $\mu = 1.19\beta$, and negative for $\mu > 1.19\beta$, implying that for for values of $\mu > 1.19\beta$, $CS_M > CS$. And since market centralization can only occur when $\mu \geq 1.5\beta$, I conclude that whenever market centralization occurs, CS increases to CS_M. Furthermore, after having jumped to CS_M, which is equal to:

$$CS_M = U_1(q_{11}) - q_{11}p_{11} + \delta\left(U_2(q_{12}) - q_{12}p_{12}\right) = \frac{\beta(c-\alpha)^2\left(4\beta^2(\delta+1)+8\beta\delta\mu+\delta(\delta+1)\mu^2\right)}{2\left(\delta\mu^2-4\beta^2\right)^2}$$

Taking the derivative with respect to μ and once more equating $\delta = 1$:

$$\frac{\partial CS_M}{\partial \mu} = \frac{2\beta(c-\alpha)^2}{(2\beta-\mu)^3}$$

Which is clearly positive.

Finally I follow the same procedure for TS, concluding that whenever $\mu > 0.56\beta$, $TS_M > TS$. And since market centralization can only occur when $\mu \geq 1.5\beta$, I conclude that whenever market centralization occurs, TS increases to TS_M. Alternatively, since both Profit and CS increase, it is straightforward to imply that TS also increases. Furthermore, after having jumped to TS_M, which is equal to:

$$TS_M = U_1(q_{11}) - q_{11}c_{11} + \delta\left(U_2(q_{12}) - q_{12}c_{12}\right)$$

$$= \frac{(c-\alpha)^2\left(12\beta^3(\delta+1) + 16\beta^2\delta\mu - \beta\delta(\delta+1)\mu^2 - 2\delta^2\mu^3\right)}{2\left(\delta\mu^2 - 4\beta^2\right)^2}$$

Taking the derivative with respect to μ and once more equating $\delta = 1$:

$$\frac{\partial TS_M}{\partial \mu} = \frac{(c-\alpha)^2(4\beta-\mu)}{(2\beta-\mu)^3}$$

*Which once more is positive. For a better comprehending of the impact that μ has on TS, CS and Π_i view **Figure 7** in the **Appendix** under **Additional Plots**.* ■

1.6

Proof 4.
Once more, without loss of much generality I assume that $\delta = 1$.

$$\frac{\partial q_{i1}}{\partial \eta} = \frac{2\beta\mu(\alpha-c)\left(2\beta\mu^2(\gamma\eta-2\beta)^2 + \mu(\gamma-2\beta)(2\beta+\gamma)^2(-2\beta+2\gamma\eta+\gamma) + \gamma(\gamma-2\beta)(2\beta+\gamma)^3\right)}{\left(2\beta(\eta+1)\mu^2(\gamma\eta-2\beta) + (2\beta-\gamma)(2\beta+\gamma)^3\right)^2}$$

It is clear that the sign of the derivative depends on the sign of:

$$2\beta\mu^2(\gamma\eta-2\beta)^2 + \mu(\gamma-2\beta)(2\beta+\gamma)^2(-2\beta+2\gamma\eta+\gamma) + \gamma(\gamma-2\beta)(2\beta+\gamma)^3$$

Which is positive for low values of γ, while for values $\gamma > \gamma^*$ (γ^* being an implicitly defined threshold), the derivative turns negative.[15]

$$\frac{\partial q_{i2}}{\partial \eta} =$$

$$\frac{\mu(\alpha-c)}{\left(2\beta(\eta+1)\mu^2(\gamma\eta-2\beta) + (2\beta-\gamma)(2\beta+\gamma)^3\right)^2} \cdot \begin{pmatrix} (\gamma-2\beta)^2(2\beta+\gamma)^4 \\ +2\beta\mu(2\beta-\gamma)(2\beta+\gamma)^2(2\beta-\gamma(2\eta+1)) \\ +2\beta\gamma(\eta+1)^2\mu^2\left(\gamma^2-4\beta^2\right) \end{pmatrix}$$

It is clear that the sign depends on the inner bracket par, which is positive for any permitted parameter values.■

[15] The only exception, a case that I ignore, is when $\mu \to \sqrt{2}\beta$, a threshold defined in **Lemma 2**, which is only possible as $(c \to \alpha)$, and if $\eta \to 0$, in this very particular case, the derivative would be positive for any γ. However, since the derivative itself depends on $\alpha - c$, not only is this case very unique, but also its impact is immensely small.

1.7

Proof 5.
From:

$$c_{i2} = c_{i1} - \mu q_{i1} - \eta\mu q_{j1} \geq 0$$

Realizing that in the symmetric case $q_{i1} = q_{j1}$*, and rewriting* c_{i1} *as* c*:*

$$\implies \mu \leq \frac{c}{q_{i1}(1+\eta)} = c \cdot \frac{2\beta\delta(\eta+1)\mu^2(\gamma\eta - 2\beta) + (2\beta-\gamma)(2\beta+\gamma)^3}{(\eta+1)(\alpha-c)\left(8\beta^3 + 4\beta^2(\gamma+\delta\mu) - 2\beta\gamma(\gamma+\delta\eta\mu) - \gamma^3\right)}$$

As shown in the ***Lemma Prerequisites*** *I can let* $\gamma \to 0$*,* $\eta \to 1$ *and* $\delta \to 1$*, which reduces the above inequality to:*

$$\mu \leq \frac{c\left(\mu^2 - 2\beta^2\right)}{(2\beta+\mu)(c-\alpha)}$$
$$\implies \mu \leq \frac{\beta\left(c - \alpha + \sqrt{\alpha^2 + c^2}\right)}{\alpha}$$

And since $\lim_{c\to\alpha} \frac{\beta\left(c-\alpha+\sqrt{\alpha^2+c^2}\right)}{\alpha} = \sqrt{2}\beta$*, in the extreme case* $\mu \leq \sqrt{2}\beta$ *is sufficient.* ■

1.8

Proof 6.
Once more, without much loss of generality I assume that $\delta = 1$*, and for a relatively moderate level of learning, given the constraint from* ***Lemma 2****,* $\mu \in [0, \sqrt{2}\beta]$*, I equate* $\mu = \beta$*, which requires neither a very high, nor a very low level of c. Furthermore, after taking the derivative, I equate* $\eta = 0$*, to see what is the effect of going from a state with no spillovers, to a state with spillovers. In this case:*

$$\frac{\partial \Pi_i}{\partial \eta} = \frac{4\beta^5(c-\alpha)^2\left(432\beta^6 + 432\beta^5\gamma - 128\beta^3\gamma^3 - 38\beta^2\gamma^4 + 3\beta\gamma^5 + 2\gamma^6\right)}{\left(12\beta^4 + 16\beta^3\gamma - 4\beta\gamma^3 - \gamma^4\right)^3}$$

The denominator is positive for any permitted parameter values, thus the sign of the derivative depends on the sign of:

$$432\beta^6 + 432\beta^5\gamma - 128\beta^3\gamma^3 - 38\beta^2\gamma^4 + 3\beta\gamma^5 + 2\gamma^6$$

Which is also positive for any $\gamma \in [0, \beta]$*. Following the same steps:*

$$\frac{\partial CS}{\partial \eta} =$$
$$\frac{2\beta(\beta+\gamma)(c-\alpha)^2}{\left(12\beta^4 + 16\beta^3\gamma - 4\beta\gamma^3 - \gamma^4\right)^3} \cdot \left(\begin{array}{c} 1728\beta^9 + 864\beta^8\gamma - 1248\beta^7\gamma^2 - 928\beta^6\gamma^3 - \gamma^9 \\ +144\beta^5\gamma^4 + 274\beta^4\gamma^5 + 52\beta^3\gamma^6 - 20\beta^2\gamma^7 - 9\beta\gamma^8 \end{array}\right)$$

The denominator is positive for any permitted parameter values, thus the sign of the derivative depends on the sign of the inner bracket last part, which is also positive for any $\gamma \in [0, \beta]$*. Consequently, since both* Π_i *and* CS *increase with* η*, it follows that:*

$$\frac{\partial TS}{\partial \eta} > 0 \quad ■$$

1.9

Proof 7.
*Similar to **Lemma Prerequisites** I initially show the following:*

$$\frac{\partial q_{i1}^s}{\partial \delta} = \frac{\mu^2(\alpha - c)(\eta+1)^2(\beta+\gamma+\eta\mu+\mu)}{\left((\beta+\gamma)^2 - \delta(\eta+1)^2\mu^2\right)^2} > 0$$

$$\frac{\partial q_{i1}^s}{\partial \gamma} = \frac{(c-\alpha)\left(2(\eta+1)\mu(\beta+\gamma)+(\beta+\gamma)^2+\delta(\eta+1)^2\mu^2\right)}{\left((\beta+\gamma)^2 - \delta(\eta+1)^2\mu^2\right)^2} < 0$$

$$\frac{\partial q_{i1}^s}{\partial \eta} = \frac{\mu(\alpha-c)\left(2\delta(\eta+1)\mu(\beta+\gamma)+(\beta+\gamma)^2+\delta(\eta+1)^2\mu^2\right)}{\left((\beta+\gamma)^2 - \delta(\eta+1)^2\mu^2\right)^2} > 0$$

Furthermore, from:

$$c_{i2} = c_{i1} - \mu q_{i1}^s - \eta\mu q_{j1}^s \geq 0$$

Realizing that in the symmetric case $q_{i1}^s = q_{j1}^s$, and rewriting c_{i1} as c:

$$\implies \mu \leq \frac{c}{q_{i1}^s(1+\eta)} = c \cdot \frac{(\beta+\gamma)^2+\delta(\eta+1)^2\mu^2}{(\eta+1)(\alpha-c)(\beta+\gamma+\delta(\eta+1)\mu)}$$

As shown above, I can let $\gamma \to 0$, $\eta \to 1$ and $\delta \to 1$, which reduces the above inequality to:

$$\mu \leq \frac{c\left(\mu^2 - 2\beta^2\right)}{(2\beta+\mu)(c-\alpha)}$$
$$\implies \mu \leq \frac{c\beta}{2\alpha}$$

And since $\lim_{c\to\alpha} \frac{c\beta}{2\alpha} = \frac{\beta}{2}$, in the extreme case $\mu \leq \frac{\beta}{2}$ is sufficient. ■

1.10

Proof 8.
*From **Lemma Prerequisites: Part 2** it is clear that the decentralized equilibrium quantity decreases in γ, consequently its highest values is reached when $\gamma = 0$. On the other hand, as shown in **Proof 7**, the social planner's optimal quantity also decreases in γ, reaching its lowest value when $\gamma = \beta$. Once more, without much loss of generality I assume that $\delta = 0$:*

$$\underbrace{q_{i1}^s}_{\gamma\to\beta} - \underbrace{q_{it}}_{\gamma\to 0} = \frac{2\beta\eta\mu(\alpha-c)}{(\eta\mu+\mu-2\beta)\left((\eta+1)\mu^2-4\beta^2\right)}$$

*The numerator is clearly positive, while the denominator for any value of $\eta \in [0,1]$ and as defined in **Lemma 3** for $\mu \in [0, \frac{\beta}{2}]$, is also always positive. As a result, since the lowest value of q_{i1}^s is larger than the highest value of q_{i1}, this implies that $q_{i1}^s \geq q_{i1}$ for any γ.*

To show that $q_{i2}^s \geq q_{i2}$ the exact same procedure must be followed, thus I omit it from the proof. ■

1.11 Quantity Inefficiency Decomposition

Initially, to correct for the standard *Cournot Inefficiency* I let $\mu = 0$ and $\eta = 0$, and calculate the fraction:

$$\frac{q_{i1}}{q_{i1}^s} = \frac{2\beta + \gamma}{\beta + \gamma}$$

Multiplying the decentralized equilibrium optimal quantity q_{i1} with this fraction leads to the social planners optimal quantity if there is no learning, which is represented in **Figure 7** by the lower gray line, this quantity is refereed to as q_{it}^{cc}, for cournot corrected. Furthermore, to correct for the inefficiency originating from learning-by-doing I only equate $\eta = 0$, while now $\mu \geq 0$, and calculate the fraction:

$$\frac{q_{it}^{cc}}{q_{it}^s} = \frac{\beta(\beta+\gamma+\delta\mu)\left(16\beta^4+16\beta^3\gamma-4\beta^2\delta\mu^2-4\beta\gamma^3-\gamma^4\right)}{(2\beta+\gamma)\left(-\beta^2-2\beta\gamma-\gamma^2+\delta\mu^2\right)\left(8\beta^3+4\beta^2(\gamma+\delta\mu)-2\beta\gamma^2-\gamma^3\right)}$$

Multiplying q_{it}^{cc} by this fraction leads to the social planners optimal choice under the presence of learning-by-doing, which is represented in **Figure 7** as the upper gray line, this quantity in refereed to as $q_{it}^{(c\&\mu)c}$, for cournot and learning corrected.

1.12

Proof 9.
*Using the quantities defined in **Proof 8**, the difference $q_{it}^{(c\&\mu)c} - q_{it}^{cc}$ represents the fraction of the inefficiency originating from learning (the region between the two gray lines in **Figure 7**). While $q_{it}^s - q_{it}$, represents the total inefficiency, thus, under the assumption of $\delta = 1$ and $\eta = 0$ (since now we are focused on the inefficiency originating from learning only):*

$$\frac{q_{it}^{(c\&\mu)c} - q_{it}^{cc}}{q_{it}^s - q_{it}} = \frac{\textit{Learning Inefficiency}}{\textit{Total Inefficiency}} = \frac{\mu\left(8\beta^4+4\beta^3(\gamma+\mu)-4\beta^2\gamma^2-4\beta\gamma^3-\gamma^4\right)}{(\beta+\gamma)\left(\mu\left(4\beta^3-2\beta\gamma^2-\gamma^3\right)+\beta(2\beta-\gamma)(2\beta+\gamma)^2\right)}$$

Represents the fraction of the inefficiency that originates from learning-by-doing. The derivative of this fraction with respect to γ is:

$$\frac{1}{(\beta+\gamma)^2\left(\beta(2\beta-\gamma)(2\beta+\gamma)^2-\mu\left(-4\beta^3+2\beta\gamma^2+\gamma^3\right)\right)^2}\cdot$$

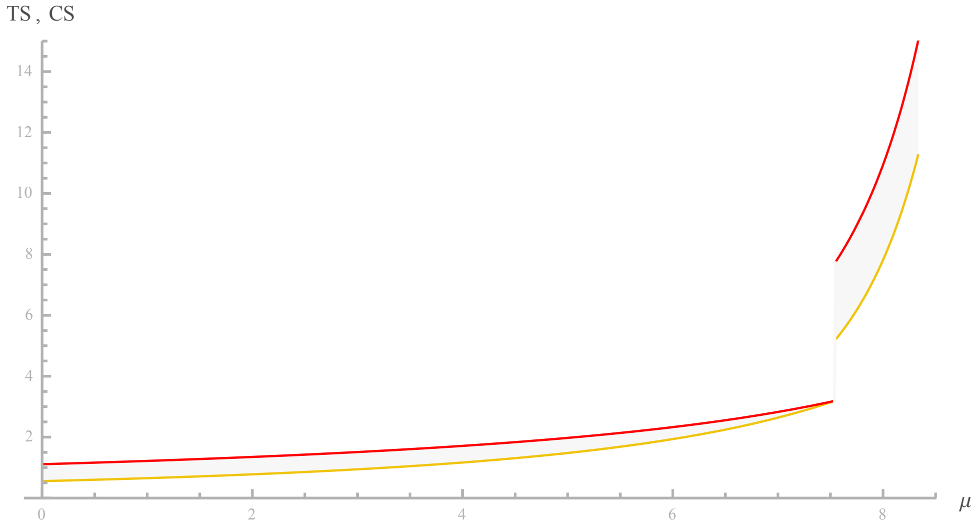
$$\begin{pmatrix} +4\beta^2\mu^2\left(-4\beta^3+4\beta^2\gamma+9\beta\gamma^2+4\gamma^3\right) \\ -\mu\left(64\beta^6+16\beta^5\gamma-52\beta^4\gamma^2-24\beta^3\gamma^3+4\beta^2\gamma^4+4\beta\gamma^5+\gamma^6\right) \\ -\beta(2\beta+\gamma)^2\left(16\beta^4+8\beta^3\gamma+8\beta^2\gamma^2+8\beta\gamma^3+\gamma^4\right) \end{pmatrix}$$

*The sign of the derivative depends on the latter inner bracket part, which is negative for any $\gamma \in [0, \beta]$ and $\mu \in [0, \frac{\beta}{2}]$, as defined in **Lemma 3**. While if $\eta \geq 0$, for large values of η, it can be shown that the derivative will initially be positive, nonetheless, once more, as $\gamma \to \beta$, the derivative turns negative and the fraction of the inefficiency originating from learning decreases.* ■

1.13 Additional Plots

For a better comprehending of the impact that μ has on CS and TS, I present the following graph.

Figure 8: The impact of μ on TS, CS and Π_i



The Red line represents the level of TS, while the Yellow line represents the level of CS. The graphs have been plotted using the following parameter values: $\alpha = 15$, $\gamma = \beta = 5$, $c = 12.5$ and $\delta = 1$.

In this particular case market centralization occurs when μ reaches $\mu^{**} = 1.5\beta = 7.5$, a threshold defined at **Proof 3: Part 2**. As can be seen, both before and after market centralization TS and CS increase as μ increases. The shaded region between TS and CS represents profits, which shrink to 0 as $\mu \to \mu^{**}$, and become positive once more after centralization occurs.

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