OPTIMAL CURRENCY SHARES IN

CURRENCY RESERVES OF CENTRAL BANKS

by

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Abstract

Central banks are always be concerned about minimization of the risk of loss in the value of their currency reserves portfolio, i.e. its variance, while risk-free yield remains a secondary goal. Basically, interest and exchange rates affect portfolio variance and yield. This paper analyses historical data on interest and exchange rates and uses a mean-variance framework in order to find optimal allocations of currencies in the currency reserves subject to the special needs of central banks. Making assumptions on special needs of central banks, i.e. minimum requirement to keep part of reserves in dollars and euros, setting target yield and others, I find optimal currency shares in the currency reserves for a particular level of expected return. Analysis shows that (i) setting a reference currency allows central banks to get a high yield in some years without bothering much about risk. In addition, (ii) central banks may adjust optimal currency shares when they expect exchange rates trends.

Keywords: currency reserves, reference currency, special needs of central banks

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Introduction

Effectiveness of foreign exchange reserves management has long been a subject investigated by many researchers. The studies range according to the principal issues research in financial economics seeks to address: social cost (Rodrik, 2006), diversification (Dooley et al, 1989), profitability of foreign exchange reserves, the impact of reserves on financial markets (Higgins and Klitgaard, 2004) and so on.[1] The growth of reserves in recent years has been dramatic. Figure 1 illustrates this growth. Evidence suggests that at the end of 2007 the stock of foreign exchange reserves was 6.4 trillion dollars, having doubled in four years.

The demand for foreign exchange reserves comes from several sources. Emerging and developing economies accumulate foreign assets as protection against financial crises and as a consequence of foreign exchange market intervention to prevent appreciation of their own currencies (Edwards, 1983; Frenkel, 1983; Papaioannou et al., 2006). Additionally, countries treat an accumulation of foreign exchange reserves as a key factor for self-protection from sudden stops of capital flows. A higher (net) level of liquid foreign exchange reserves make them better withstand panics in financial markets and sudden reversals in capital flows (Feldstein, 1999). There is convincing evidence that being liquid reduces the probability of suffering a financial crisis (Rodrik, 2006). Whatever reasons for accumulation exist, reducing the risk of loss in the value of foreign exchange reserves due to market conditions and maintenance of liquidity of foreign exchange reserves are among the key issues monetary authorities, especially central banks, need to consider in an ongoing manner.

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[1] Also there are some studies about the extent to which management of reserves may be driven by non-economic considerations. See Aizenman, Joshua and Reuven Glick, *"Sovereign Wealth Funds: Stylized Facts about their Determinants and Governance,"* 2008, NBER Working Paper No. 14562.

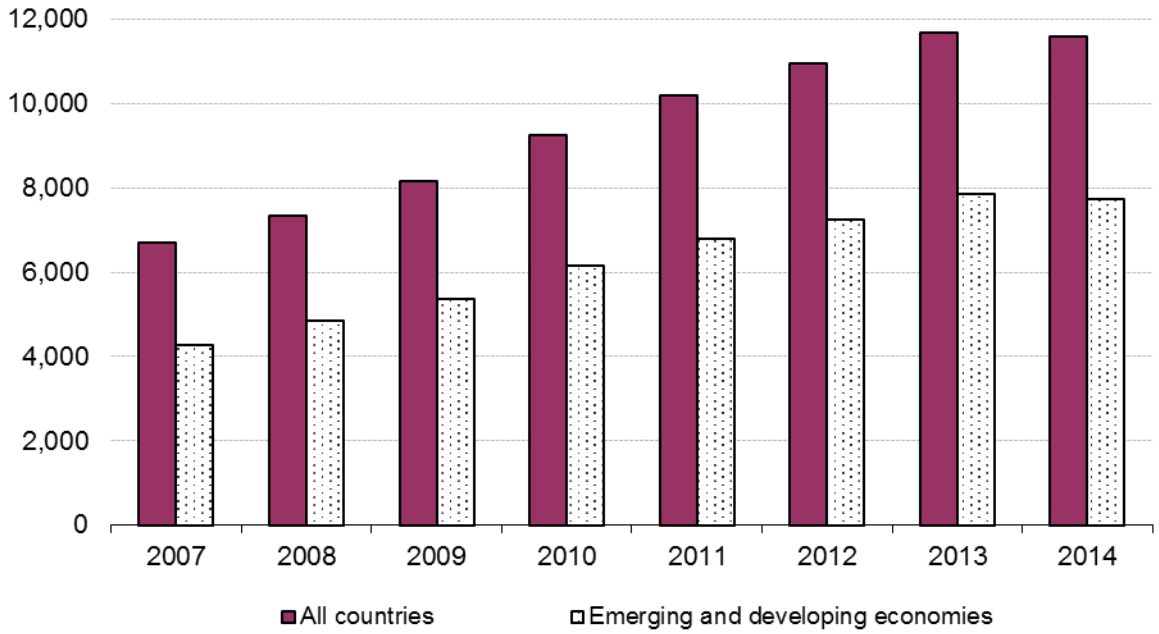

Figure 1. Global Foreign Exchange Reserves (billion USD year-end)

Notes: The Figure reports the accumulation of foreign exchange reserves during 2007-2014. All values are expressed in equivalent of US dollars.
Source: calculated using IMF Statistics Department COFER database and International Financial Statistics.

Generally, monetary authorities aim to reduce the impact of negative market forces on the deterioration of the value of foreign exchange reserves. In many countries the central banks represent the monetary authority which is in charge of the management of foreign exchange reserves. These reserves mainly consist of precious metals (generally, gold), securities with a risk-free rate of return and currency reserves. Currency reserves are the most liquid part of foreign exchange reserves. For many emerging and developing countries these liquid reserves come from export of commodities (precious metals, oil, natural gas, and etc.). Once those commodities are converted into financial assets, they are invested as a part of liquidity portfolio.[2] In general, liquidity comes at the cost of risk taking on the management of the currency reserves.

[2] This research focus on liquidity portfolio of central banks only and does not cover allocation of financial assets to specific funds created for designated purposes (Infrastructure Investment Funds, Stabilization Funds and etc.).

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Thus, as one of their main issues, central banks deal with systematic risk in order to minimize losses in currency reserves. Since currency reserves are under risk due to the volatility of interest rates and exchange rates, the main issue for central banks is to allocate foreign currencies in their currency reserves with minimized risk from changes in associated interest rates and exchange rates.

This paper estimates the ability of central banks to reduce the risk of loss in the value of currency reserves, subject to changes in interest rates and exchange rates. The priority is to protect the value of a country's currency reserves through safe investment in assets or claims of first-class institutions.[3] First, using a mean variance framework (and making various assumptions about currency returns), I obtain the optimal currency shares in the currency reserves of central banks from 2008 to 2015. Next, the framework incorporates some special liquidity needs of central banks.

Comparing the allocations I obtain from optimization give the results as follows. First, since interest rates and exchange rates are very volatile within the considered period, the mean-variance optimizer yields very unstable results. Also significant shifts in shares of currencies do not significantly increase portfolio variance in the framework. Second, setting a reference currency (US dollar) allows central banks to avoid risk of loss just by increasing the share of dollars in the currency reserves.[4] That is, variance of the portfolio has an inverse relationship with the share of dollars in the portfolio. Central banks are risk averse and yield a risk-free rate

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[3] Basically, first-class institutions are commercial banks or financial groups with solid credit ratings.
[4] The reference currency is the currency in which country values its reserve holdings. I assume that US dollar is a primary reference currency for many countries. Meanwhile, countries, such as EU member states, might set Euro as reference currency. Then, a general approach for the present analysis will be the same, while reserve holding policy of central banks night be different.

of return for the currency reserves when their reserves are pegged in dollars.[5] Therefore, this inverse relationship may partly explain the inclination of central banks to hold a sizable fraction of the currency reserves in dollars. Third, volatility of exchange rates reveals how this volatility is associated with yield in the reference currency. The empirical analysis suggests that volatility of exchange rates does contribute to increase in yield and does not significantly affect portfolio variance in most cases. This phenomenon is obvious in years when the Euro, Pound or Japanese yen depreciated against the reference currency. Therefore, in such a case, central banks can afford a higher yield for their currency reserves rather than risk-free rate of return.

The organization of the rest part of the paper is as follows. Next section explains the methodology I use for the analysis. The third section describes the data used and composition of variance-covariance matrix, while the fourth section shows the results of analysis. I set the fifth section as a discussion part and the final section summarizes this paper.

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[5] I specify a risk-free rate of return for the currency portfolio in the Results of Analysis part of this paper.

Methodology

I follow evidence that central banks use (explicitly or implicitly) risk diversification strategies (RBS, 2003, 2005; ECB, 2005).[6] Basically, central banks prefer to hold a sizable fraction of the currency reserves in the currencies of the country's main trading partners and in the currencies of international liabilities (Papaioannou et al., 2006).[7] In such a case, an issue central banks confront is to find the set of portfolio components (currencies) which minimizes the variance of the portfolio (currency reserves) for the expected return of each feasible portfolio (Merton, 1972).[8]

Evidence suggests that central banks invest part of their currency reserves in money market time deposits. Share of time deposits in currency reserves placed abroad differs among central banks (see Table A.1 in Appendices). Still, those time deposits represent the handiest part of currency reserves for central banks because, given the predetermined short term outflow and inflow of foreign exchange, central banks also use time deposits to finance such flows and avoid liquidity mismatch. Thus, I assume that every central bank would like to find an optimal allocation of currencies in its currency reserves in order to pursue at least two goals, i.e. (i) provide liquidity in a particular currency and (ii) earn some interest which compensates for risks associated with holding time deposit in that particular currency.

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[6] Collections of articles by the Royal Bank of Scotland and the European Central Bank suggest that central banks do follow a portfolio optimization strategy, while taking into account some unique features of monetary authorities.
[7] Usually countries do not release the currency composition of their reserves. Truman et. al (2006) found out that only three of top 21 holders of foreign exchange reserves disclosed the currency composition of their foreign exchange reserves during 2000-2004 occasionally. Only in 2015 some countries agreed to disclose asset classes of their reserves only (see Data Template on International Reserves and Foreign Currency Liquidity of IMF).
[8] As Merton states, the frontier of all feasible portfolios which can be constructed from several securities is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected returns.

I use London Interbank Offered Rate (LIBOR) as a good approximation of monthly money market deposit rate. Since central banks tend to place funds with top rated foreign banks, LIBOR in each currency serves as a benchmark to define level of interest of major banks in funds. This approach assumes that central banks have the same risk profile as banks which participate in LIBOR setting. Although both are different type of institutions, major foreign banks basically favor to deal with central banks because of low risk profile of the latter and preferential attachment of central banks in favor of those institutions.

Following an assumption that central banks are risk averse, for each period (year), central banks minimize the currency part of the reserves as follows:

$$\min \frac{1}{2} w'Vw + \lambda[\overline{R}_p - w'\overline{R}] + \gamma[1 - w'e] + \mu w \qquad (1)$$

subject to:

$w_i \geq 0,\ i = 1,...,n$

in which $\overline{R} = (\overline{R}_1, \overline{R}_2, ..., \overline{R}_n)'$ is a $n \times 1$ vector of expected returns of the currency reserves, V is $n \times n$ full rank covariance matrix of currency returns and $w' = (w_1, w_2, ..., w_n)'$ is a $n \times 1$ vector of portfolio shares. $\overline{R}_p$ is an expected (weighted average) return of the portfolio which consist of the sum of appropriate returns on currency deposits multiplied by the appropriate shares in the portfolio.[9] As a central bank does not engage in short selling, each currency share in optimal allocation w_i is equal to zero or positive. The locus is to minimize the portfolio variance

[9] In (1), the problem actually minimizes one-half the portfolio variance to avoid carrying extra "2" in the first-order condition (2). The solution is the same as minimizing the total variance and only changes the scale of the Lagrange multipliers.

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σ^2 which is equal to $w'Vw$, subject to the constraints of portfolio expected return and portfolio shares:[10]

$$\frac{\partial f}{\partial w} = Vw - \lambda\overline{R} - \gamma e + \mu = 0 \quad (2)$$

$$\frac{\partial f}{\partial \lambda} = \overline{R}_p - w'\overline{R} = 0 \quad (3)$$

$$\frac{\partial f}{\partial \gamma} = 1 - w'e = 0 \quad (4)$$

$$\frac{\partial f}{\partial \mu} = w = 0 \quad (5)$$

I start with the formation of variance-covariance matrices of currency returns which needs to represent risks associated with changes in interest rates and volatility in exchange rates. Since changes in exchange rates follow a random walk, variance-covariance matrix incorporates exchange rate risks.[11] The next is to minimize variance-covariance matrices subject to constraints (2), (3), (4) and (5) at target yield level which central banks desire to have from the management of their currency reserves. The further step which is undertaken in the following chapter is to incorporate new constraints which represent special needs of central banks into the mean-variance framework described here.

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[10] The equation (3) is a first-order condition of (1) with respect to Lagrange multiplier λ. The equation (4) represents a condition when portfolio's weights sum to one. The equation (5) indicates that central banks are not allowed to take advantage from short selling.

[11] Papaioannou et al. (2006) assumes that when the exchange rate follows a random walk, the central bank's expected currency appreciation/depreciation is equal zero, so that the central bank just realizes the one year interest rate on the particular currency deposits. However, data from 2000 to 2007 is not consistent with this assumption. As exchange rates have an important implication in this analysis, I incorporate exchange rates into the variance-covariance matrix.

Data Description

I start our estimation of optimal currency shares by analysis of data on interest rates and exchange rates from 2008 to 2015. This period is particularly interesting for the analysis due to the expansionary monetary policy of major central banks around the world during and in the aftermath of the global financial crisis of 2008-2009 and European Sovereign Debt Crisis of 2012. Further, I follow the assumption that central banks tend to hold a huge fraction of their reserves in the currency of the nation's external debt and in the trade invoicing currency (Papaionnou et al, 2006), as well as that central banks favor currencies which are part of the Special Drawing Rights of IMF. Thus, I comprise data on monthly deposit rates and exchange rates of four international currencies, namely the US dollar (dollar, hereafter), the Euro, the British pound sterling (pound, hereafter) and the Japanese yen (yen, hereafter) to assess how central banks could allocate these currencies with a minimum risk for prescribed expected return.[12] In order to find optimal allocations of currencies at the desired yield level, I build variance-covariance matrices of currency returns and minimize them subject to constraints from (2) to (5).

I propose that central banks express returns from currency reserves in dollar terms. This allows central banks to set deposits in dollars as risk-free investment with respect to the exchange rate. Thus, deposits in the other three currencies – euro, pound and yen, become risky investment due to volatility of exchange rates. In order to incorporate exchange rate risk, annual yields on deposit rates of euro, pound and yen deposits are expressed in terms of dollars. As a result, return on deposits in all three currencies, R_p, can be represented in percentage terms.

[12] Data on interest and exchange rates extracted from the web page of the Research Division of the Federal Reserve Bank of St. Louis: https://research.stlouisfed.org, accessed May 25, 2016.

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Empirical evidence suggests that central banks set a target profit level – yield, from the management of their reserves. In fact, central banks, at least, prefer to get a rate of profit which complies with low risk taking. So, the purpose of central banks is to obtain a targeted yield at an identified low variation of the portfolio. In addition, central banks do not bother very much about transaction costs because these costs tend to become very low (Niehans, 1971; Portes and Rey, 1998) and central banks do not face them very often.

It is important to note that central banks do not tend to change the composition of their currency portfolio often due to policy constraints and rational expectation of near term predetermined drain of particular assets. With respect to such behavior, I assume that central banks prefer to invest currency reserves (or at least part of it) to monthly deposits in order to provide better liquidity and earn corresponding rate of return.

Table 1 presents the results from the data set under the above-stated assumptions. This table illustrates a sensitivity of optimal allocations to the yield and risk tolerances of central banks. The optimizer yields unstable results due to very volatile exchange rates. Significant drop in interest rates in 2008 and 2009 by major central banks led to very low return on time deposits, while due to swings in exchange rates became a major factor that influenced the analysis. For example, the allocation of yen increases drastically in 2010-2011 under any yield level due to the appreciation of the yen against the dollar.[13]

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[13] The yen started appreciating against the dollar since the end of 2007; in 2012 it reached a record low level of 75.72 yen per dollar. Due to such a trend, yield from deposits in yen became very profitable in dollar terms in 2010 and 2011.

Table 1. Mean-variance optimal currency allocations assuming random of exchange rates and no transaction costs (%)

year	2008		2009		2010		2011		2012		2013		2014		2015	
expected return	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights
0.5	0.00001	91.3	0.00000	98.7	0.00000	98.2	0.00000	95.8	0.00000	94.0	0.00000	91.7	0.000000	100.0	0.00005	64.5
		0.0		0.0		0.0		0.3		0.0		8.3	at 0.16%	0.0		0.0
		8.7		1.3		0.0		0.0		6.0		0.0		0.0		0.0
		0.0		0.0		1.8		3.9		0.0		0.0		0.0		35.5
1.0	0.00001	93.2	0.00000	94.5	0.00000	94.2	0.00001	88.0	0.00001	82.6	0.00002	78.3			0.00036	7.5
		0.0		0.0		0.0		0.8		0.0		21.7				0.0
		6.8		5.5		0.0		0.0		17.4		0.0				0.0
		0.0		0.0		5.8		11.3		0.0		0.0				92.5
1.5	0.00000	95.0	0.00001	90.3	0.00001	90.2	0.00001	80.1	0.00003	71.1	0.00006	64.9			0.00042	0.0
		0.0		0.0		0.0		1.3		0.0		35.1			at 1.07%	0.0
		5.0		9.7		0.0		0.0		28.9		0.0				0.0
		0.0		0.0		9.8		18.6		0.0		0.0				100.0
2.0	0.00000	96.8	0.00003	86.0	0.00001	86.2	0.00003	72.2	0.00005	59.6	0.00012	51.4				
		0.0		0.0		0.0		1.8		0.0		48.6				
		3.2		14.0		0.0		0.0		40.4		0.0				
		0.0		0.0		13.8		25.9		0.0		0.0				
2.5	0.00000	98.7	0.00005	81.8	0.00002	82.2	0.00005	64.4	0.00009	48.2	0.00019	38.0				
		0.0		0.0		0.0		2.4		0.0		62.0				
		1.3		18.2		0.0		0.0		51.8		0.0				
		0.0		0.0		17.8		33.2		0.0		0.0				
3.0	0.00000	98.7	0.00007	77.6	0.00004	78.3	0.00007	56.5	0.00013	36.7	0.00028	24.6				
		0.4		0.0		0.0		2.9		0.0		75.4				
		0.0		22.4		0.0		0.0		63.3		0.0				
		0.9		0.0		21.7		40.6		0.0		0.0				
3.5	0.00000	95.7	0.00010	73.4	0.00005	74.3	0.00009	48.7	0.00019	25.2	0.00039	11.2				
		0.5		0.0		0.0		3.4		0.0		88.8				
		0.0		26.6		0.0		0.0		74.8		0.0				
		3.8		0.0		25.7		47.9		0.0		0.0				

Notes: The table reports optimal currency weights of dollars, pounds, euros and Japanese yens at a specified rate of return which central banks require from their currency reserves. For example, at 3.5% yield in 2008 an optimal allocation of dollars, pounds, euros and Japanese yens was 95.7%, 0.5%, 0% and 3.8%, respectively. The associated portfolio variance was 0.0002%.

Full Table 1 is available in Appendices.

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In addition, Table 1 shows that a higher share of dollar in reserves results in a smaller variance of portfolios. As empirical evidence suggests (Henriksen et al, 2009), interest rates of these four currencies have a positive correlation. Thus, again the reason for this phenomenon (reverse relationship between share of dollar and portfolio variance) is the volatility of exchange rates rather than changes in interest rates. Exchange rate volatility accounts for a higher variance of portfolio. Therefore, in such a case, central banks have to be more concerned about changes in exchange rates rather than interest rates. Results in Table 1 explicitly show that whatever level of yields central banks choose, variance of portfolios in most cases is low. Further calculation shows that variance of these portfolios is less than 0.001 which is a pretty low risk level (and, also, can be seen as annual portfolio volatility). The standard deviation of the portfolios increases only in 2013 when the share of dollars drops less than 50% at a yield level of 3% and higher.

Introduction of new constraints

Since central banks pursue a reliable degree of liquidity in their reserves, they often require minimum shares of particular currencies in their currency reserve holdings. Because central banks are inclined to keep these shares in currencies of the main trading partners and external debt, I introduce new additional constraints reflecting this proposition. A minimum level of a particular currency is required to be held in the currency reserves at the cost of increased variance. In such a case,

$$w_{dollar} \geq 0.5 \quad (6)$$

$$w_{euro} \geq 0.1 \quad (7)$$

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Table 2 illustrates the results of mean-variance framework under constraints (6) and (7).[14] This table shows that constraints in shares of particular currencies do narrow the latitude for the yield. In particular, the highest yield central banks might earn in 2011 is 2.93%.[15] However, results from the current calculation do not differ much in terms of variance. The figures in Table 2 suggest that variances at different risk tolerances do increase moderately (see Table A.2 in Appendices). In 2014 and 2015 central banks would be unable to earn interest subject to my scenario: return on feasible optimal portfolio would be negative 1.1% and 0.5%, respectively. Since the share of dollar is fixed at its minimum requirement level, annual portfolio volatility (portfolio variance) at different risk tolerance again does not exceed 0.01%, except for the 2008-2009 years, when central banks might require higher yields.

Sharpe ratio for central banks

As an alternative approach to mean-variance framework, one may argue that central banks may target maximization of Sharpe ratio. Results are presented in Tables A.3.1 and A.3.2 in the Appendices. Sharpe ratio is stable across different portfolio allocations (except for 2008) in scenarios without constraints, while in case of more realistic scenarios of constraints introduced, it shows that higher returns are well compensated corresponding risks involved. Meanwhile, when annualized Sharpe ratio is computed from monthly data, it may be significantly inflated due to serial correlation of returns (Lo, 2002). Thus, I suggest that central banks should not target maximization of Sharpe ratio.

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[14] European Union (EU) member countries are exemptions from this template. It is reasonable to assume that EU member countries tend to hold a substantial share of euros in the currency reserves. The euro is a domestic currency in many EU countries. In this paper, I do not consider the case in which central banks keep a part of the currency reserves in the domestic currency due to space limitations.

[15] To compare, without such constraints, as Table 1 shows, central banks could earn 5% yield on its currency reserves.

Table 2. Mean-variance optimal currency allocations assuming minimum dollar and euro weight of 50% and 10%, respectively (%)

year	2008		2009		2010		2011		2012		2013		2014		2015	
expected return	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights	variance	weights
0.5	0.00005	82.3	0.00025	50.3	0.00002	82.8	0.00001	50.0	0.00005	87.3	0.00005	89.7	0.000000	90.0	0.00019	50.0
		10.0		10.0		10.0		44.0		10.0		10.0	at -1.11%	10.0	at -0.5%	10.0
		7.7		0.0		0.0		0.0		2.7		0.0		0.0		0.0
		0.0		39.7		7.2		6.0		0.0		0.3		0.0		40.0
1.0	0.00053	50.0	0.00002	88.6	0.00003	78.8	0.00003	50.0	0.00008	75.8	0.00011	78.3				
		47.8		10.0		10.0		37.0		10.0		21.7				
		2.2		1.4		0.0		0.0		14.2		0.0				
		0.0		0.0		11.2		13.0		0.0		0.0				
1.5	0.00003	86.0	0.00003	84.4	0.00004	74.8	0.00005	62.6	0.00012	64.3	0.00017	64.9				
		10.0		10.0		10.0		18.0		10.0		35.1				
		4.0		5.6		0.0		0.0		25.7		0.0				
		0.0		0.0		15.2		19.4		0.0		0.0				
2.0	0.00003	87.8	0.00005	80.2	0.00005	70.8	0.00007	63.7	0.00016	52.8	0.00024	51.4				
		10.0		10.0		10.0		10.0		10.0		48.6				
		2.2		9.8		0.0		0.0		37.2		0.0				
		0.0		0.0		19.2		26.3		0.0		0.0				
2.5	0.00002	89.6	0.00008	75.9	0.00006	66.8	0.00009	56.4	0.00017	50.0	0.00025	50.0				
		10.0		10.0		10.0		10.0	at 2.12%	10.0	at 2.05%	50.0				
		0.4		14.1		0.0		0.0		40.0		0.0				
		0.0		0.0		23.2		33.6		0.0		0.0				
3.0	0.00002	87.6	0.00011	71.7	0.00008	62.8	0.00010	50.0								
		10.0		10.0		10.0	at 2.93%	10.0								
		0.0		18.3		0.0		0.0								
		2.4		0.0		27.2		40.0								
3.5	0.00002	84.7	0.00014	67.5	0.00009	58.9										
		10.0		10.0		10.0										
		0.0		22.5		0.0										
		5.3		0.0		31.1										

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Notes: The table reports optimal currency weights of dollars, pounds, euros and Japanese yens at a specified rate of return which central banks require from their currency reserves. For example, at 3.5% yield in 2008 an optimal allocation of dollars, pounds, euros and Japanese yens was 84.7%, 10%, 0% and 5.3%, respectively. The associated portfolio variance was 0.0022%.
Full Table 2 is available in Appendices.

In addition, this ratio relies on normal distribution assumption. Even though one can assume that exchange rate changes follow random walk, interest rate changes do not. Further, every central bank set the target yield based on their own goals and circumstances, and thus Sharpe ratio may be biased towards certain strategy of the central bank. Also Figure 2 shows that yields in dollar terms of euro, pound and yen deposits are very volatile over analyzed period of time. Sharpe ratio is inflated when volatility is high, since central banks, in order to keep the same Sharpe ratio for investments of currency reserves, should receive two percentage points more return for every unit increase in standard deviation of the portfolio under each scenario. It is difficult for central banks to balance between setting target yield (in advance) and maximization of excess return. Moreover, as table A.3.2 shows, maximization of Sharpe ratio is biased toward higher return on reserves. Once there is an opportunity to receive excess return which compensates for risk involved, dilemma between two strategies is subject to individual treatment case by case. However, in general, central banks state that the main goal of foreign exchange reserve management is to preserve its value and not to maximize excess return.

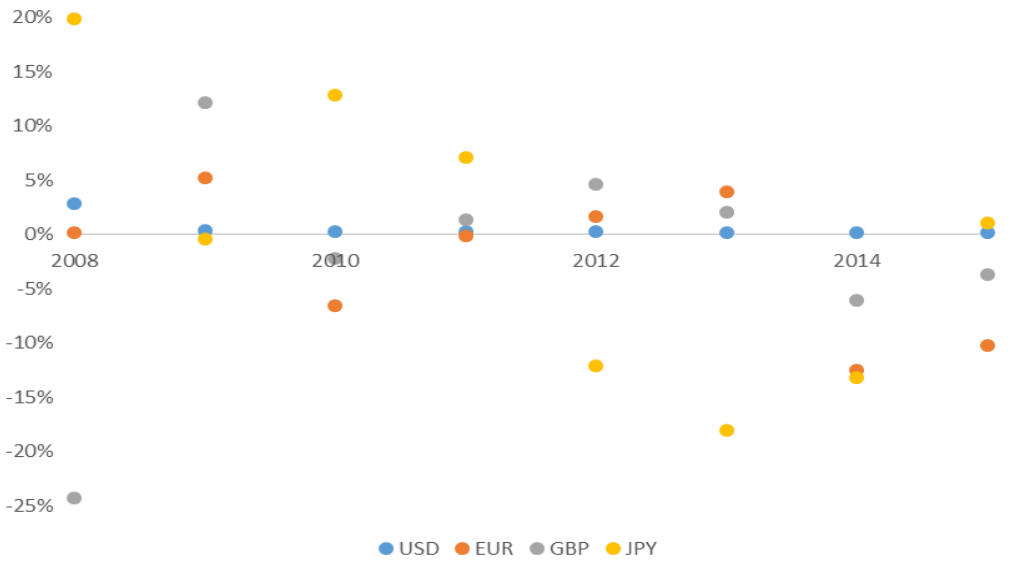

Figure 2. Return on the dollar, euro, pound and yen deposits in terms of US dollars.

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Results

The results from Table 1 and Table 2 show that it is crucial for central banks to set a reference currency based upon the optimal allocation of shares in their currency reserves. In the simple case, when central banks may desire to yield interest which reflects the Federal Funds Rate, there is no need for diversification of the currency reserves. In other words, central banks may prefer to be totally risk averse. As a result, an optimal allocation of currencies only consists of dollars and, therefore, central banks treat deposits as risk-free.[16] However, an optimal allocation of currency shares at different yield levels shows that portfolio variances are still low when they do include other currencies. That is, central banks may choose a desirable profit level without bothering very much about the associated risk.

Since central banks also have to ensure a liquidity of the currency reserves, they are inclined to hold a substantial proportion of the reserves in the currencies of the main trading partners and external debt. In this case, as Table 2 presents, the cost of liquidity is associated with a higher variance of portfolios at different yield levels. Even though, as in the case of no restriction on the minimum share of a particular currency, the variances of portfolios are still relatively low.

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[16] That is because the variance of a portfolio which only consists of dollars is lowest and only reflects changes in interest rates; exchange rates do not affect the variance at all.

Discussion

I suggest that central banks may apply the outcomes from the conducted analysis in order to response to exchange rate movements. It is important for central banks to expect such movements in exchange rates. An ability of central banks to come up with expected volatility of exchange rates, or at least have a substantive view on case, allows them to take a low risk and maximize the yield from the currency reserves by optimal allocation of currency shares.

The data set on exchange rates from 2008 to 2015 has an important feature. The yield of central banks from the currency reserves decreases in years when the pound and euro depreciated against the dollar, and vice versa. Depreciation of the pound and euro put a downward pressure on the yield from deposits in these currencies. In such a case, since deposits in euros and pounds are risky, central banks would prefer to keep their currency reserves in the reference currency and, therefore, get a risk-free rate.

Meanwhile, the exchange rate of pound and interest paid for deposits in this currency are less volatile compared to the euro and yen. In this analysis I assume, for simplicity, that central banks do not hedge exchange rate risks of their currency reserves. In fact, they do hedge exchange rate risks, but only to the extent at which they think it may deteriorate dollar value of reserves in that particular currency. That is, central banks usually do not apply exchange rate risk management tools to part of the currency reserves which are dedicated to provide extensive liquidity. Thus, their assumption about non-hedging of currency portfolio under analysis may be well grounded to some extent.

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Volatility of exchange rates may occur due to different factors. For example, the Federal Reserve may decide to apply further "quantitative easing" in order to encourage commercial banks to supply credit the real sector of an economy. This policy may further boost the US economy and/or cause inflation. Boosting the US economy may lead to the appreciation of the dollar. In such a case, central banks may consider keeping their currency reserves in the reference currency, i.e. in dollars. However, quantitative easing also puts an upward pressure on inflation and, therefore, the dollar may depreciate against the pound, euro and yen. In this case, central banks may prefer to increase the shares of those currencies in their currency reserves. As evidence shows, the Bank of England, the European Central Bank and Bank of Japan did apply the same policy. Their policies may lead to the same outcomes in future. However, value of currencies will depend on the willingness of monetary authorities to the extent of pushing money into their national economies.

Central banks that seek to respond to such expansionary policies and reallocate their currency reserves should be able to evaluate medium term outcomes that affect exchange rates. For example, at present the Federal Reserve states its readiness to increase its benchmark interest rate in the summer, and probably once more at the end of the current year. Meanwhile, the BOJ, as well as the ECB and BOE do not have any room for such increase. Moreover, both interest rates for the euro and yen deposits are in negative territory nowadays. Thus, central banks may assume that without a solid ground for economic growth, currencies of those advanced economies may depreciate against the dollar. In my view, the results of analysis in Table 2 partially reflect such expectations. Negative yield on currency reserves invested in time deposits may suggest that in short term holding euro and pound may be reduced to minimum level possible.

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On the whole, central banks should always be concerned about exchange rate volatility. If central banks expect any trend in exchange rates, then they can pursue one of two approaches in the allocation of shares in their currency reserves. In particular, foreseen depreciation of the dollar against other currencies may lead central banks to increase shares of the pound and euro in their currency reserves, and vice versa.[17]

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[17] In many countries central banks do not set its policy independently. Sometimes there is a pressure from other government authorities (for example, Ministry of Finance) to get a higher rate of return from the reserves when there is a feasible opportunities to earn money.

Conclusion and Policy Implications

Foreign exchange reserves have grown rapidly in recent years. Currently, about two thirds of international reserves in the world are held in US dollars.[18] Even though reserve holdings in dollars are considered to be risk-free, monetary authorities, especially central banks, prefer to diversify their currency reserves at the cost of taking additional risk. In such cases, central banks keep timing to find optimal allocation of currencies in their reserves.

This paper has considered an optimal allocation of currencies in the currency reserves when central banks assess the profitability of their currency reserves in a reference currency – the dollar. I have used a mean-variance framework to estimate optimal currency shares by introducing constraints which reflect some special needs of central banks. First, I have assumed that central banks cannot take advantage from short selling of its assets. Then, I have assessed changes in portfolio variances by further introducing another two constraints, which specify a requirement of holding minimum shares of dollars and euros in the currency reserves.

There are two findings from the conducted analysis. First, I emphasize the importance of setting a reference currency for central banks. Central banks are able to set investments in the reference currency, i.e. time deposits in dollars, as risk-free. Then, central banks should only be concerned about changes in the Federal Funds Rate. As the dollar reserves are risk-free, a higher share of dollars in the reserves leads to less variance of optimal allocation portfolios, and vice versa. This kind of inverse relationship between the share of dollars and the variance of portfolios may partly explain why central banks prefer to keep a sizable fraction of their reserves in dollars.

[18] See IMF Statistics Department COFER database and International Financial Statistics.

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Further analysis (Table 2) shows that the special needs of central banks do not lead to substantial increase of portfolio variance. The analysis shows that volatility of exchange rates reveals how volatility can be associated with profitability of reserves in dollar terms. Empirical analysis suggests that appreciation of currencies against the dollar does contribute to an increase in yield, but central banks in fact may not care much about increase in portfolio variance. That is, central banks may afford a higher yield without bothering much about risk. On the other hand, depreciation of the pound, euro and yen against the dollar narrows the yield. In this case, central banks may switch holding a relatively higher share of dollars in the currency reserves in order to avoid losses from their deposits in pounds and euros.

I believe that central banks can use the conducted empirical analysis in particular scenarios. Evidence suggests that an increasing number of central banks pursue similar optimization strategies (Papaioannou et al., 2006). If central banks to develop scenarios that depend on movements in the future, they will be able to allocate optimal shares of particular currencies in the currency reserves with low risk using a mean-variance framework. Of course, there might be other "special needs" central banks have to consider in each case. Given, for example, the present environment of low interest rates, the expectation of future changes in interest rates by the Federal Reserve and appreciation of the dollar against its major peers may induce central banks to increase the share of dollars in their currency reserves.

Central banks may require a higher yield even though the pound, euro and yen depreciate against the dollar and interest rates stay low. In such a case, despite rebalancing the shares in the currency reserves, currently there is increasing pressure on central banks to invest in higher return assets. Going forward, it would be thus interesting to extend this mean-variance

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framework allowing for currency returns from derivative instruments, such as dual currency deposits, which is the combination of money market deposit and currency option. Allocation of part of the currency portfolio to such an instrument may give central banks more flexibility at the cost of higher risk taking.

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Appendices

Table 1. Mean-variance optimal currency allocations assuming random of exchange rates and no transaction costs (%)

year	2008		2009		2010		2011		2012		2013		2014		2015	
expected return	var	weights	var	weights	var	weights	var	weights	var	weights	var	weights	var	weights	var	weights
0.5	0.00001	91.3	0.00000	98.7	0.00000	98.2	0.00000	95.8	0.00000	94.0	0.00000	91.7	0.000000	100.0	0.00005	64.5
		0.0		0.0		0.0		0.3		0.0		8.3	at 0.16%	0.0		0.0
		8.7		1.3		0.0		0.0		6.0		0.0		0.0		0.0
		0.0		0.0		1.8		3.9		0.0		0.0		0.0		35.5
1.0	0.00001	93.2	0.00000	94.5	0.00000	94.2	0.00001	88.0	0.00001	82.6	0.00002	78.3			0.00036	7.5
		0.0		0.0		0.0		0.8		0.0		21.7				0.0
		6.8		5.5		0.0		0.0		17.4		0.0				0.0
		0.0		0.0		5.8		11.3		0.0		0.0				92.5
1.5	0.00000	95.0	0.00001	90.3	0.00001	90.2	0.00001	80.1	0.00003	71.1	0.00006	64.9			0.00042	0.0
		0.0		0.0		0.0		1.3		0.0		35.1			at 1.07%	0.0
		5.0		9.7		0.0		0.0		28.9		0.0				0.0
		0.0		0.0		9.8		18.6		0.0		0.0				100.0
2.0	0.00000	96.8	0.00003	86.0	0.00001	86.2	0.00003	72.2	0.00005	59.6	0.00012	51.4				
		0.0		0.0		0.0		1.8		0.0		48.6				
		3.2		14.0		0.0		0.0		40.4		0.0				
		0.0		0.0		13.8		25.9		0.0		0.0				
2.5	0.00000	98.7	0.00005	81.8	0.00002	82.2	0.00005	64.4	0.00009	48.2	0.00019	38.0				
		0.0		0.0		0.0		2.4		0.0		62.0				
		1.3		18.2		0.0		0.0		51.8		0.0				
		0.0		0.0		17.8		33.2		0.0		0.0				
3.0	0.00000	98.7	0.00007	77.6	0.00004	78.3	0.00007	56.5	0.00013	36.7	0.00028	24.6				
		0.4		0.0		0.0		2.9		0.0		75.4				
		0.0		22.4		0.0		0.0		63.3		0.0				
		0.9		0.0		21.7		40.6		0.0		0.0				
3.5	0.00000	95.7	0.00010	73.4	0.00005	74.3	0.00009	48.7	0.00019	25.2	0.00039	11.2				
		0.5		0.0		0.0		3.4		0.0		88.8				
		0.0		26.6		0.0		0.0		74.8		0.0				
		3.8		0.0		25.7		47.9		0.0		0.0				
4.0	0.00000	92.7	0.00014	69.2	0.00007	70.3	0.00013	40.8	0.00025	13.7	0.00050	0.0				
		0.5		0.0		0.0		3.9		0.0	at 3.92%	100.0				
		0.0		30.8		0.0		0.0		86.3		0.0				
		6.7		0.0		29.7		55.2		0.0		0.0				
4.5	0.00001	89.7	0.00018	65.0	0.00009	66.3	0.00016	33.0	0.00032	2.3						
		0.6		0.0		0.0		4.5		0.0						
		0.0		35.0		0.0		0.0		97.7						
		9.7		0.0		33.7		62.6		0.0						
5.0	0.00001	86.8	0.00023	60.7	0.00011	62.3	0.00020	25.1	0.00033	0.0						
		0.6		0.0		0.0		5.0	at 4.6%	0.0						
		0.0		39.3		0.0		0.0		100.0						
		12.6		0.0		37.7		69.9		0.0						

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Notes: The table reports optimal currency weights of dollars, pounds, euros and Japanese yens at a specified rate of return which central banks require from their currency reserves. For example, at 3.5% yield in 2008 an optimal allocation of dollars, pounds, euros and Japanese yens was 95.7%, 0.5%, 0% and 3.8%, respectively. The associated portfolio variance was 0.0002%.

Table 2. Mean-variance optimal currency allocations assuming minimum dollar and euro weight of 50% and 10%, respectively (%)

year	2008		2009		2010		2011		2012		2013		2014		2015	
expected return	var	weights	var	weights	var	weights	var	weights	var	weights	var	weights	var	weights	var	weights
0.5	0.00005	82.3	0.00025	50.3	0.00002	82.8	0.00001	50.0	0.00005	87.3	0.00005	89.7	0.000000	90.0	0.00019	50.0
		10.0		10.0		10.0		44.0		10.0		10.0	at -1.11%	10.0	at -0.5%	10.0
		7.7		0.0		0.0		0.0		2.7		0.0		0.0		0.0
		0.0		39.7		7.2		6.0		0.0		0.3		0.0		40.0
1.0	0.00053	50.0	0.00002	88.6	0.00003	78.8	0.00003	50.0	0.00008	75.8	0.00011	78.3				
		47.8		10.0		10.0		37.0		10.0		21.7				
		2.2		1.4		0.0		0.0		14.2		0.0				
		0.0		0.0		11.2		13.0		0.0		0.0				
1.5	0.00003	86.0	0.00003	84.4	0.00004	74.8	0.00005	62.6	0.00012	64.3	0.00017	64.9				
		10.0		10.0		10.0		18.0		10.0		35.1				
		4.0		5.6		0.0		0.0		25.7		0.0				
		0.0		0.0		15.2		19.4		0.0		0.0				
2.0	0.00003	87.8	0.00005	80.2	0.00005	70.8	0.00007	63.7	0.00016	52.8	0.00024	51.4				
		10.0		10.0		10.0		10.0		10.0		48.6				
		2.2		9.8		0.0		0.0		37.2		0.0				
		0.0		0.0		19.2		26.3		0.0		0.0				
2.5	0.00002	89.6	0.00008	75.9	0.00006	66.8	0.00009	56.4	0.00017	50.0	0.00025	50.0				
		10.0		10.0		10.0		10.0	at 2.12%	10.0	at 2.05%	50.0				
		0.4		14.1		0.0		0.0		40.0		0.0				
		0.0		0.0		23.2		33.6		0.0		0.0				
3.0	0.00002	87.6	0.00011	71.7	0.00008	62.8	0.00010	50.0								
							at									
		10.0		10.0		10.0	2.93%	10.0								
		0.0		18.3		0.0		0.0								
		2.4		0.0		27.2		40.0								
3.5	0.00002	84.7	0.00014	67.5	0.00009	58.9										
		10.0		10.0		10.0										
		0.0		22.5		0.0										
		5.3		0.0		31.1										
4.0	0.00002	81.8	0.00018	63.3	0.00010	54.9										
		10.0		10.0		10.0										
		0.0		26.7		0.0										
		8.2		0.0		35.1										
4.5	0.00003	78.1	0.00023	59.1	0.00011	50.9										
		10.0		10.0		10.0										
		0.0		30.9		0.0										
		11.9		0.0		39.1										
5.0	0.00003	75.9	0.00028	54.8	0.00011	50.0										
		10.0		10.0		10.0										
		0.0		35.2		0.0										
		14.1		0.0		40.0										

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Notes: The table reports optimal currency weights of dollars, pounds, euros and Japanese yens at a specified rate of return which central banks require from their currency reserves. For example, at 3.5% yield in 2008 an optimal allocation of dollars, pounds, euros and Japanese yens was 84.7%, 10%, 0% and 5.3%, respectively. The associated portfolio variance was 0.0022%.

Table A.1.Summary of official reserve assets of several central banks (approximate market value)

(billion US dollars)

State	Official reserve assets	including total currency and deposits located abroad	of which with banks headquartered outside the reporting country	share of deposits with foreign banks in total currency reserves
1	2	3	5	6
Kazakhstan	27,875.6	10,473.8	899.2	8.6%
Kyrgystan	1,851.0	428.8	337.7	78.7%
Ukraine	12,721.5	2,976.8	1,978.7	66.5%
Armenia	1,680.7	493.6	459.6	93.1%
Belarus	4,111.0	1,591.8	1,069.3	67.2%
Georgia	2,456.8	2,252.2	455.0	20.2%
Saudi Arabia	587,084.1	188,110.7	187,698.9	99.8%

Source: Data Template on International Reserves and Foreign Currency Liquidity, IMF, as of June 5, 2016.

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Table A.2. Variance differences of optimal portfolios between two scenarios

expected return	2008	2009	2010	2011	2012	2013	2014	2015
0.5	0.00004	0.00025	0.00002	0.00001	0.00005	0.00005	0.00000	0.00013
1.0	0.00053	0.00001	0.00003	0.00002	0.00007	0.00008		
1.5	0.00003	0.00002	0.00004	0.00003	0.00010	0.00011		
2.0	0.00003	0.00002	0.00004	0.00004	0.00011	0.00012		
2.5	0.00002	0.00003	0.00004	0.00004	0.00008	0.00006		
3.0	0.00002	0.00003	0.00004	0.00003				
3.5	0.00002	0.00004	0.00003					
4.0	0.00002	0.00004	0.00003					
4.5	0.00002	0.00004	0.00002					
5.0	0.00002	0.00005						

Notes: The table shows portfolios variances from Table 2 less portfolios variance from Table 1.

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Table A.3.1. Sharpe Ratios for every scenario without constraints

year expected return (%)	2008	2009	2010	2011	2012	2013	2014	2015
0.5	-0.69	0.28	0.44	0.34	0.24	0.17	0.00	0.04
1.0	-0.69	0.30	0.45	0.34	0.24	0.17		0.04
1.5	-0.68	0.31	0.45	0.34	0.24	0.17		
2.0	-0.65	0.31	0.45	0.34	0.24	0.17		
2.5	-0.49	0.31	0.45	0.34	0.24	0.17		
3.0	0.20	0.31	0.45	0.34	0.24	0.17		
3.5	0.46	0.31	0.46	0.34	0.24	0.17		
4.0	0.52	0.31	0.46	0.34	0.24	0.17		
4.5	0.55	0.31	0.46	0.34	0.24			
5.0	0.56	0.31	0.46	0.34				

Table A.3.2. Sharpe Ratios for every scenario with constraints

year expected return (%)	2008	2009	2010	2011	2012	2013	2014	2015
0.5	-0.34	0.01	0.05	0.09	0.04	0.04	0.00	0.02
1.0	-0.08	0.15	0.12	0.14	0.08	0.08		
1.5	-0.24	0.20	0.18	0.18	0.11	0.10		
2.0	-0.17	0.23	0.23	0.22	0.14	0.12		
2.5	-0.08	0.25	0.28	0.25	0.17	0.15		
3.0	0.03	0.26	0.31	0.27				
3.5	0.14	0.27	0.35					
4.0	0.23	0.27	0.38					
4.5	0.31	0.28	0.41					
5.0	0.37	0.28	0.46					

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