Tender value and the extent and form of corruption in public procurements: A game theory approach

by

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Abstract

The main focus of this paper is to analyze the relationship between tender size and corruption in public procurements. Based on a slightly modified model developed by Mizoguchi and Van Quyen (2014), this paper establishes a theoretical framework for the analysis of the corrupt behaviour of competing firms under uncertainty in the information available to the opponent firm. Utilizing the developed framework, this paper aims to answer why high value, specific tenders are more exposed to corruption than average or low value tenders. A main result of the analysis is that clear competition requires a high probability of repetition and high discount factor, with the former being rather rare in case of large tenders. An additional goal of the paper is to determine why bid rigging is a more prevalent form of corruption than kickbacks. The results suggest that bid rigging is characteristic to procurements with higher probability of repetition, thus they can be observed more often.

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Contents

1 Introduction

In developed economies, public procurements account for a major part in government spendings (OECD, 2015). These spendings can be severely affected by corruption, resulting in inefficiencies and a great cost to society (Transparency International, 2010; Hafner et al., 2016). Economic analysis of corruption in public procurements is therefore essential.

Due to the complexity of the procurement process, corruption may take many forms. The most common is bid rigging, when bidding firms form a horizontal collusion (PwC and Ecorys, 2013). The exact outcome of such a collusion may vary, they may pre-decide the winner and submit conforming bids, split the market based on consumers or geography or decide on submitting bids in a series of procurements such that they win the procurements taking turns (OECD, 2009). Bid rigging is followed by kickbacks in frequency, when a firm ”kicks back” an amount of money to the procurement official in exchange for winning the procurement (PwC and Ecorys, 2013).

Based on PwC and Ecorys (2013), most cases either include bid rigging or kickbacks. They also find that both of these forms of corruption positively correlate with the tender size, that is, large procurements are more exposed to both bid rigging and kickbacks.

This paper is devoted to examine two questions: Why is bid rigging more prevalent than offering kickbacks and how the tender size matters, that is, why is corruption more characteristic in case of large procurements.

After a review of relevant literature in Section 2, the first part of Section 3 develops a model of a procurement framework based on Mizoguchi and Van Quyen (2014) in order to analyze the questions. In this model, two firms compete in either a single procurement or a series of procurements. At the beginning of a game, nature determines the initial type of the firms. Firms have two distinct types: informed or uninformed. An informed firm is aware of the channels of corruption and can offer a kickback to the procurement official. An uninformed firm, in contrary, can not offer a kickback. In a dynamic setting, informed firms remain informed for all periods, while an uninformed firm becomes informed (and remains informed for all subsequent periods) by observing the opponent offering a kickback. That is, if an uninformed firm's informed opponent offers a kickback, the uninformed firm becomes informed as well.

In each stage of the game, firms submit a bid consisting of a price, a quality and - if informed - a kickback. The price stated in the bid can not exceed the commonly known reservation price of the government, normalized to one. The quality stated in the bid can be twofold: a firm either offers a low quality or a high quality. A low quality product or service can be provided by both firms facing a cost normalized to zero. A high quality is assumed to require specific knowledge, and thus the cost of a high quality product may be different for the two firms (a procurement may fit a firm better than the other). To model this difference, the cost of providing a high quality is determined by nature at the beginning of each period based on a commonly known uniform prior distribution.

The bids are evaluated by subtracting the price from the quality stated in the offer. If a kickback is offered, the procurement official is willing to participate in fraudulent bid evaluation, and modify the value of the bid of the firm offering the highest kickback in a way, that it wins the procurement. A firm's payoff in case of losing the procurement is zero, while the winning firm's payoff is the price decreased by the cost and the kickback.

The second part of Section 3 develops equilibria for both a stage game and an infinitely repeated game. In the stage game, one equilibrium is identified. In this equilibrium, an informed firm always offers a kickback. This behavior is rather intuitive: if a procurement is unique, that is, the game is only played once, firms have only one chance to win and realize a positive payoff. Therefore if they are informed and have a chance to increase their probability of winning by offering a kickback, they will do so. This has a very important implication. Fair competition arises if and only if both firms are uninformed. This means that the general characteristics of the procurement framework have a severe effect on the outcome in case of unique procurements. The only way to decrease the extent of corruption is to introduce such improvements to the procurement framework that decrease the probability of being informed.

In the dynamic game, three different equilibria are identified that describe different forms of corruption (additional equilibria may exist). Firms either (1) participate in a fair competition without offering a kickback in any period, (2) enter a bid rigging scheme by the second period or (3) offer a kickback in all periods starting with the second period. A main finding of this paper is that the equilibrium characterizing the game mainly depends on the probability of repetition.

The fair competition equilibrium is characterized by a trigger strategy. If any firm offers a kickback in any period, both firms offer the highest affordable kickback in all subsequent periods. Considering this strategy, fair competition may arise two ways: Either both firms are uninformed and therefore corruption is impossible or there is at least one informed firm, but the punishment defined for a corrupt behavior deters any informed firm from offering a kickback. The latter case not only requires a severe punishment, but also a high probability of repetition. Even with the most severe punishment defined in the equilibrium strategy (zero payoffs for all periods after offering a kickback), an informed firm would offer a kickback if the game was expected to end soon. Therefore the high probability of repetition is key to sustain such an equilibrium, as it is a major determinant of the perceived value of possible future payoffs.

An other equilibrium identified may result in bid rigging. In this equilibrium, both firms play according to the stage game equilibrium strategies in the first stage. This way the initial types of the firms become a common knowledge by the second stage, as an informed firm offers a kickback in the stage game, while an uninformed firm does not. If any firm is informed, both firms become informed by the second period. Once both firms are informed, they submit bids in a way, that one of the firms wins in every odd period, while the other wins in every even period. If any firm deviates in any period, both firms offer the highest affordable kickback in all subsequent periods, resulting in a zero payoff for both firms. This trigger strategy, with a probability of repetition high enough, makes this equilibrium sustainable, as none of the firms will want to deviate once both of them are informed.

To make sure that none of the firms wants to deviate in the first period, the probability of repetition has to be low enough. If the probability of repetition is too high, an informed firm may take the risk and offer no kickback in the first period. If its opponent is uninformed, the informed firm can reveal this information by the second period, and offer a kickback maximizing its payoff in the second period, thus increasing the cumulative payoff by delaying the bid rigging scheme. Therefore bid rigging requires the probability of repetition to fall into a specific interval.

The third type of equilibrium is characterized by a series of kickbacks starting in period two (if any of the firms is informed). The strategies define the stage game equilibrium strategy to be played in the first period. Having an informed firm, both firms become informed by the second period. Once both firms are informed, both of them offer the highest kickback in all subsequent periods, resulting in a zero payoff. From the second period, there is no incentive to deviate for any firm, as considering the strategy of the opponent, any bid results in a payoff of zero. In the first period, no deviation of an uninformed firm affects the future gameplay and the strategy is a best response to the opponent's strategy in the first stage, thus there is no profitable deviation. By taking the risk of not offering a kickback in the first period, the informed firm may reveal that its opponent is uninformed by the second period. If its opponent is uninformed, the informed firm may either (1) sustain a fair competition by not offering a kickback in any subsequent period or (2) offer a kickback resulting in the highest payoff in the second period before entering the kickback scheme. If the probability of repetition is low enough, an informed firm will have no incentive to deviate to trying to sustain a fair competition or delaying the kickback scheme.

Section 4 analyzes the questions of the paper using these equilibria derived in Section 3. The relationship between the prevalence of corruption and the tender size may originate in the probability of repetition. Assuming that large tenders are rather specific (as they require carrying out a complex process or a process to a large extent), there is a negative relationship between the tender size and the probability of repetition. Since a high probability of repetition is key in sustaining fair competition, large procurements associated with lower probability of repetition are, indeed, more likely to end up in a corrupt state.

The probability of repetition may serve as an answer to why bid rigging is more prevalent than offering kickbacks as well. Since out of the games that end up in a corrupt state bid rigging is characteristic to those that have a higher probability of repetition, games with bid rigging will on average last longer than games with kickbacks. This fact itself, however, does not explain the majority of bid rigging schemes, as it has to be accompanied by a proper distribution of probabilities of repetition as well to result in the observed structure.

Section 5 contains concluding remarks and some further research questions. First of all, the model does not provide a rigorous answer to why bid rigging is more prevalent than offering kickbacks. The results suggest that it depends on the distribution of probabilities of repetition as well. Empirical study is required to verify whether the true distribution supports the results. This paper focuses on the behavior of firms, and assigns a very simplified behavior to the procurement official. It would worth analyzing how the equilibria change if the procurement official is risk averse.

2 Background

Public procurements have a major role in the provision of public goods in modern economies. In 2013, OECD countries spent on average 13% of their GDP and 29.1% of their government expenditures on public procurements (OECD, 2015). As many authors point out, acquiring goods and services via public procurements not only accounts for a large share in economic activity, but also provide a fertile ground for several forms of corruption (see for example Strombom (1998)).

Even though quantitative measurement of the exact effects is difficult and uncertain, most studies agree that corruption in public procurements is a staggering burden on the society. Transparency International (2010) estimates a cost of 10-25% of the procurement value on average, while a study by RAND Europe prepared for the European Parliament has an aggregated estimate on the annual costs of corruption between €179bn and €990bn (Hafner et al., 2016). Analyzing procurement frameworks and understanding the motivations of economic agents is therefore an important public policy issue.

Public procurement is a complex process consisting of several subsequent procedures. A useful generalization of the procurement process was outlined by the OECD (2007, pp. 20-22), which breaks it up the to the following steps: First, the exact subject of the procurement (i.e. a specific good or service) is determined, and a tender is designed. Then depending on the exact type of the tender, firms are invited to participate in the procurement. This call for offers is followed by the bidding procedure and bid evaluation. A contract is awarded to the winner based on this evaluation. Afterwards, the contract is executed.

All stages of this process can be affected by some form of corruption. Based on OECD (2007, pp. 21), some typical forms of corruption in the preparatory phase of the procurement are limiting the time to submit the bids, specifying the tender to fit a selected firm, inviting firms that are „unlikely to submit competitive bids" or including possibilities to increase the value of the contract in later phases. The bidding procedure may be exposed to bribery and fraudulent bid evaluation, while the execution of the contract may lack control.

Even though there are many possible types of corruption in public procuremet-nts, only some of them occur with a high frequency. In an analysis of PwC and Ecorys (2013) prepared for the European Commission, corrupt practices are grouped into four main categories. These are bid rigging, kickbacks, conflict of interest, and other (PwC and Ecorys, 2013, pp. 145-146).

Bid rigging stands for a collusion among bidding firms. This horizontal collusion can take the form of: (1) cover bidding, when firms agree on submitting bids with higher prices than the pre-decided winner, (2) bid suppression, when firms decide not to bid except for the designated winner, (3) bid rotation, in which case firms agree on submitting bids in separate procurements resulting in winning procurements in turns or (4) market allocation, meaning that they split the market on the basis of consumers, geography or other separating factors (OECD, 2009, pp. 2).

Bid rigging, being the most common realization of corruption in public procurements (OECD, 2007; PwC and Ecorys, 2013), has been in the focus of several research papers. An early study by McAfee and McMillan (1992) examines bid rigging in auctions with and without the possibility of transfers among bidders in a single stage auction game. They find that from the seller's point of view there is no difference between the two settings, the payments equal the reservation price of the seller. In a public procurement framework this means that the procurer faces low value bids, that is, high prices and low quality. A more recent paper by Lengwiler and Wolfstetter (2010) finds that if a corrupt auctioneer orchestrating bid rigging and allowing bidders to modify the submitted prices acts as an agent of the seller, then it not only reallocates surplus from the seller to the agent and the buyer, but also makes all parties interested in maintaining the corruption. A wide selection of papers examined specific cases. For example price fixing in the Japanese construction industry was studied by McMillan (1991) and Ishii (2009), and cover bidding in highway construction was examined by Porter and Zona (1993).

According to PwC and Ecorys (2013, pp. 146.), bid rigging is followed by kickback as the second most frequent type of corruption in public procurements. Kickback stands for a vertical collusion in procurements, in which the winning firm "kicks back" money or pays a bribe to the corrupt procurement official in exchange for winning the procurement (PwC and Ecorys, 2013). Kickbacks have been analyzed by Auriol (2006), finding that their cost to the society is a multiple of the bribe paid. As Dastidar and Mukherjee (2014) point out, kickbacks result in a low quality, and depending on the bargaining power of the corrupt procurement official, a higher price. Therefore kickbacks have a similar effect as bid rigging in the society's point of view.

There is a conflict of interest if a procurement official's public and private interests differ. This may happen if the public official, their family members or other relatives have position in a firm that wins a procurement, evaluated by the official (PwC and Ecorys, 2013). Conflict of interest is similar to kickbacks in a sense, that it may result in private gains to the procurement official. However, the exact realization of the abuse of power can take many forms, which may be the reason for the lack of a general model describing this type of corruption in public procurements up to this day.

As there are many forms of corruption affecting public procurements, and it is the interest of the involved parties to hide their actions, corrupt procurements are hard to detect. Most guidelines on identifying corrupt procurements therefore use red flags to find procurements exposed to corruption (OECD, 2007). Even though different red flags may indicate different corrupt practices, PwC and Ecorys (2013, pp. 158.) find that one of the most important red flags, the tender size, correlates with the presence of bid rigging and kickbacks as well. This correlation suggests that large tenders are more exposed to corruption than average or low value tenders.

3 Model

This section develops the model of a procurement framework that allows for the analysis of the main questions of the paper. The setup is presented in Section 3.1, followed by the derivation of equilibria in Section 3.2.

3.1 Setup

Consider the model developed by Mizoguchi and Van Quyen (2014) with minor modifications as a starting point. The model of Mizoguchi and Van Quyen (2014) provides a simple framework, without any uncertainty in the type of the opponent. This model is presented in Section 3.1.1 in detail. Section 3.1.2 introduces uncertainty in order to establish a more realistic framework.

3.1.1 Basic model

Suppose that two firms are competing to win a public procurement. They can offer a bid which consists of a price, a quality and an amount of kickback. Following the notation of Mizoguchi and Van Quyen (2014), the bid of firm i can therefore be described by a price-quality-kickback triplet, (p_i, q_i, b_i). The procurement official decides the winner based on two criteria:

1. The price can not exceed the reservation price of the government ($\overline{p}$).
2. The bid offering the highest non-negative value (v_i) must be chosen, where

$$v_i \equiv \hat{q}_i - p_i$$

The corrupt procurement official is willing to participate in fraudulent bid evaluation, and thus alter the ranking by setting $\hat{q}_i > q_i$ such that $v_i > v_j$ if firm i offers the highest kickback to the official. Still sticking to the model of Mizoguchi and Van Quyen (2014), firms may choose of two different quality levels when making a bid: a low quality ($\underline{q}$) and a high quality ($\overline{q}$), where $0 \leq \underline{q} < \overline{q}$. Both firms can provide the low quality goods or services determined in the procurement facing a cost normalized to zero. The cost of a high quality good or service (c) may be different for the two firms, but it has the same, uniform distribution over the interval $[0, 1]$.

$$c \sim U[0, 1]$$

The payoff of the winning firm is given by the price decreased by the cost of providing the goods or services and the kickback, if offered. The firm losing the procurement realizes a payoff of zero.

As a minor modification to the original model, suppose that both the reservation price of the government and the high quality equals one. This restriction simplifies a number of calculations without any major effect on the main results.

$$\overline{p} = \overline{q} = 1$$

Therefore a stage game in the original model can be described by the following steps: First, nature chooses c_i and c_j, a private information of each firm. Firms then make their bids. The procurement official chooses the bid with the highest kickback as winner (assume, that if kickbacks offered are the same, the official randomly chooses the winner). If no kickbacks are offered, the procurement official chooses the bid offering the highest value (assume, that if values offered are the same, the official chooses the one offering a higher quality, and even the quality being the same the official randomly chooses the winner). The winner pays the kickback, if offered, and provides the goods or services at the quality and price stated in the bid. In the rest of this paper, such a gameplay will be referred to as a stage game in the fully informed state, where fully informed refers to the assumption that both firms are aware of the channels of bribery, and therefore they can both offer a kickback when submitting a bid.

This basic model will be evaluated in different informational frameworks as well, as defining equilibria in different environments significantly contributes to the equilibria in the uncertain setting. Suppose that the two firms play the game with a lack of information on whom to offer a kickback, that is, eliminate b_i and b_j from the bids. In this case, the true values of the offers (v_i and v_j) are evaluated, and the bid with the higher value wins the procurement. Such a gameplay will be referred to as a stage game in the uninformed state, where uninformed state means that no firms have information on the channels of bribery.

Finally, this model will be analyzed in a setting where only one of the two firms has the information on whom to offer a kickback. That is, one of the firms submits a bid consisting of a price, a quality and a kickback, while the other firm submits a price and a quality only. This version of the basic model will be referred to as a stage game in the asymmetrically informed state.

3.1.2 Introducing uncertainty

Assuming that firms have information on the type of the other player (informed or uninformed) is a great simplification, and results in unrealistic equilibria. This section therefore develops such a framework, in which the type is initially a private information of each firm, and therefore they face uncertainty in the state of the world.

Suppose that firms have a common prior probability of being uninformed, denoted by ψ. In a dynamic setting, suppose that an uninformed firm can reveal the channels of corruption, and thus become informed if its informed opponent pays a kickback.

Therefore the game can be described by the following steps. First, nature determines whether a firm is initially informed or uninformed with commonly known prior probabilities $1-\psi$ and ψ respectively. Then the first stage of the game begins. Nature chooses c_i and c_j, a private information of each firm. Firms then make their bids. An informed firm's bid consists of a price, a quality and a kickback, if offered. An uninformed firm's bid only consists of a price and a quality. The procurement official chooses the bid with the highest kickback as winner (assume, that if kickbacks offered are the same, the official randomly chooses the winner). If no kickbacks are offered, the procurement official chooses the bid offering the highest value (assume, that if values offered are the same, the official chooses the one offering a higher quality, and even the quality being the same the official randomly chooses the winner). The winner pays the kickback, if offered, and provides the goods or services at the quality and price stated in the bid. For the next stage of the game, an informed firm remains informed, while an uninformed firm remains uninformed unless its informed opponent pays a kickback in the current stage.

3.2 Equilibria

This section presents the equilibria of the games defined in Section 3.1. First the equilibria for the basic model are developed, as they contribute to the equilibria in the uncertain setting as well.

3.2.1 Equilibrium of the stage game in fully informed state

A pure strategy equilibrium in the fully informed state has been identified by Mizoguchi and Van Quyen (2014). They find that the bid $(\overline{p}, \underline{q}, \overline{p})$ always constitutes a perfect bayesian equilibrium. A detailed proof is to be found in Appendix 1, but the intuition behind this equilibrium strategy is clear: both firms having the information on whom to offer a kickback actually participate in a Bertrand competition in kickbacks, as the amount of kickback is the only determinant of the winner. Therefore the equilibrium strategy defines a bid consisting of the reservation price of the government, a low quality and a kickback that equals the price, resulting in a zero payoff. Considering that the opponent's equilibrium strategy includes offering a kickback, the requirement for winning the procurement reduces to offering the highest kickback. Deviating to a lower kickback would automatically result in losing the procurement and a payoff of zero. Considering that the price in the equilibrium bid is the reservation price of the government, deviating to a higher kickback results in a negative payoff, as the expenditures from paying the kickback can not be covered by the income from price.

Therefore there is no profitable deviation in kickback. The only profitable deviation may be in price and quality. Since the probability of winning is determined based on the kickback offered, it is not affected by the price and the quality. Only the payoff in case of winning is affected by them. The payoff in case of winning obtains its maximum where the price equals the reservation price of the government, and the quality is low. Therefore there is, indeed, no profitable deviation from the equilibrium strategy.

3.2.2 Equilibrium of the stage game in uninformed state

Since in the uninformed state the bid with the higher value wins the procurement, firms participate in a first price auction in values. It also holds, that firms have an identically distributed reservation value ($\overline{v}$), that depends on the realization of the cost of providing high quality goods or services. Since this first price auction is revenue equivalent to a second price auction in values, the equilibrium strategies of the firms can be characterized by a bidding function, derived by calculating the expected reservation value of the losing firm conditional on the reservation value of the winning firm. Therefore for each firm, the value of their bids is determined by a well defined bidding function:

$$b(\overline{v_i}) = \frac{\overline{v_i}^2 + \underline{q}^2}{2\overline{v_i}}$$

The exact price and quality of the bid is uniquely determined by the value of the bid with the restriction that if a value can be provided by a low and a high quality as well, the high quality is to be chosen. This strategy will henceforth be referred to as the uninformed state bidding function. A detailed proof of the equilibrium strategy is presented in Appendix 2.

This equilibrium results in a strictly positive expected payoff ($E(\pi_i)$), as the bidding function defines a lower value to bid than the reservation value of the firms. The expected payoff only depends on the low quality, and is given by

$$E(\pi_i) = \frac{1}{6} - \frac{\underline{q}^2}{2} + \frac{\underline{q}^3}{3}$$

A detailed calculation of the expected payoff is to be found in Appendix 2 as well.

3.2.3 Equilibrium of the stage game in asymmetrically informed state

The equilibrium in the asymmetrically informed state is easy to determine. Suppose that the uninformed firm plays according to the uninformed state bidding function. Then in the asymmetric state, the informed firm's best response is to bid $(\overline{p}, \underline{q}, \epsilon)$ where ϵ is an infinitesimal kickback. Regardless the strategy of the uninformed firm, due to the kickback offered, the informed firm wins the procurement with a profit of $1 - \epsilon$. Deviating to any other strategy with a non-zero kickback results in a strictly lower payoff, as

$$p_i - c_i - b_i < 1 - \epsilon \quad \forall p_i \neq 1 \text{ or } b_i \neq \epsilon \text{ or } q_i \neq \underline{q}$$

Deviating to any other strategy with a zero bribe can not result in a higher payoff. If the firm decides not to pay a bribe, the game reduces to the uninformed state, in which the expected profit of the firm (if it plays according to the best response, uninformed state bidding function) is

$$E(\pi_i) = \frac{1}{6} - \frac{\underline{q}^2}{2} + \frac{\underline{q}^3}{3} < 1 - \epsilon \quad \forall \underline{q}$$

Therefore the informed firm has no incentive to deviate from bidding $(\overline{p}, \underline{q}, \epsilon)$. Considering the strategy of the informed firm, the uninformed firm has no incentive to deviate from the uninformed state bidding function, as regardless the bid submitted, the uninformed firm loses the procurement and has a zero payoff.

3.2.4 Equilibrium in the stage game with uncertainty

In the stage game, the informed firm does not have to consider the possibility that its opponent is uninformed and observes the channels of corruption, as the game is only played once. Therefore the informed firm will always offer a kickback. By contradiction, suppose that an informed firm's candidate equilibrium strategy allocates a positive probability to any bid $(p_i, q_i, 0)$. But reallocating the same probability to the bid (p_i, q_i, ϵ), where ϵ is an infinitesimal bribe, results in a higher expected payoff. Note that in limits, it does not modify the payoff of the firm in case of winning the procurement. Played against an uninformed firm or an informed firm playing according to the candidate equilibrium strategy, the bid with probabilities reallocated to (p_i, q_i, ϵ) results in an increased probability of winning the procurement. Therefore an informed firm always has incentive to deviate from a strategy allocating positive probability to a bid with zero kickback, and therefore no such strategy can be be sustained in equilibrium.

Since the informed firm offers a kickback in the stage game, the uninformed firm always loses the stage game played against an informed firm. Therefore in equilibrium, an uninformed firm will bid according to the uninformed state bidding function. Any bid of the uninformed firm yields a zero payoff if the opponent is informed, therefore there is no profitable deviation from the candidate equilibrium strategy if the opponent is informed. If the opponent is uninformed, the game reduces to the stage game in the uninformed state, and therefore the candidate equilibrium strategy, indeed, constitutes an equilibrium. The equilibrium strategy of the uninformed firm is therefore to bid according to the uninformed state bidding function.

In order to find the equilibrium strategy of an informed firm, first see that the informed firm's bid always consists of a price of $\overline{p}$, a quality $\underline{q}$ and a kickback. Any strategy allocating positive weight to (p_i, q_i, b_i), where $p_i \neq \overline{p}$ or $q_i \neq \underline{q}$ is strictly dominated by bidding according to a strategy that reallocates the same probability to the bid $(\overline{p}, \underline{q}, b_i)$. It has already been established that an informed firm always offers a kickback, therefore the winner is determined based on the amount of kickback offered. This means that p_i and q_i does not affect the probability of winning, only the payoff in case of winning. Since the payoff in case of winning is $p_i - c_i - b_i$, and it obtains its maximum if $p_i = \overline{p}$ and $q_i = \underline{q}$, any strategy allocating positive weight to the bid (p_i, q_i, b_i) is dominated by a strategy reallocating the same probability to bidding $(\overline{p}, \underline{q}, b_i)$. Therefore in equilibrium, an informed firm always bids the reservation price of the government, a low quality, and a positive bribe.

See that no pure strategy can be an equilibrium strategy. Consider any pure strategy $(\overline{p}, \underline{q}, b_i)$ as a candidate equilibrium strategy of the informed firm. The same strategy can not be the best response of an other informed firm j, as bidding $(\overline{p}, \underline{q}, b_i + \epsilon)$, where ϵ is an infinitesimal positive quantity, results in a strictly higher expected payoff, as

$$\lim_{\epsilon \to 0} \psi(1 - b_i - \epsilon) + (1 - \psi)(1 - b_i - \epsilon) > \psi(1 - b_i) + (1 - \psi)(1 - b_i)\frac{1}{2}$$

Therefore for any bid $(\overline{p}, \underline{q}, b_i)$, it is worth deviating to paying a higher bribe, and by doing so, winning the procurement against any type of opponent. The only pure strategy that can not be eliminated this way as a candidate equilibrium strategy is bidding $(\overline{p}, \underline{q}, \overline{p})$, as offering a higher bribe would result in a negative payoff. But considering $(\overline{p}, \underline{q}, \overline{p})$ as a candidate equilibrium strategy, it is worth deviating to bidding $(\overline{p}, \underline{q}, \epsilon)$, where ϵ is an infinitesimal bribe, as it results in a higher expected payoff (as playing against an uninformed firm still results in winning the procurement):

$$\lim_{\epsilon \to 0} \psi(1-\epsilon) + 0(1-\psi) = \psi > 0 \quad \forall \psi \neq 0$$

Therefore no pure strategy of the informed firm can be an equilibrium strategy. Consider now mixed strategies. It must hold for any equilibrium mixed strategy, that taking any two bids with a positive probability, the expected payoff from the two bids considering the equilibrium strategy of the opponent is equal. If this condition did not hold, the firm could increase its expected payoff by reallocating probability from the bid with lower expected payoff to the bid with higher expected payoff, and thus it would not be an equilibrium strategy. Therefore it must hold that $E(\pi_i|b_i)$, the expected profit if $(\overline{p}, \underline{q}, b_i)$ is played must be constant for all $(\overline{p}, \underline{q}, b_i)$ that has a positive probability. The expected profit considering the equilibrium strategy of the opponent given a bid with kickback b_i is

$$E(\pi_i|b_i) = \psi(1-b_i) + (1-\psi)F_j(b_i)(1-b_i)$$

where $\psi(1-b_i)$ is the probability of the opponent being uninformed multiplied by the profit conditional on the other player being uninformed, while $(1-\psi)F_j(b_i)(1-b_i)$ is the probability of the other player being informed multiplied by the expected profit conditional on the other player being informed and that its mixing can be characterized by the cumulative distribution function F_j. Note, that since there are no differences between two informed firms, $F_j = F_i$, therefore in equilibrium the expected profit conditional on the bribe can be captured by

$$E(\pi_i|b_i) = \psi(1-b_i) + (1-\psi)F_i(b_i)(1-b_i)$$

Consider a mixed strategy such that F_i is continuous and differentiable, and the corresponding probability density function is strictly positive on an interval beginning at the infinitesimal bribe ϵ and ending at $\overline{b_i}$. Then since $F_i(\epsilon) = 0$ we have

$$\lim_{\epsilon \to 0} E(\pi_i|\epsilon) = \lim_{\epsilon \to 0} \psi(1-\epsilon) + (1-\psi)F_i(\epsilon)(1-\epsilon) = \psi$$

Therefore a mixed equilibrium strategy with cumulative distribution function F_i must yield an expected profit of ψ conditional on any bid with positive probability. Therefore it must hold that

$$E(\pi_i|b_i) = \psi(1-b_i) + (1-\psi)F_i(b_i)(1-b_i) = \psi \quad \forall\, b_i \text{ where } f(b_i) \neq 0$$

It is straightforward by rearrangement that the only distribution satisfying the constraint above is

$$F_i(b_i) = \frac{\psi b_i}{(1-\psi)(1-b_i)}$$

The probability density function is given by

$$f_i(b_i) = \frac{\mathrm{d}\, F_i(b_i)}{\mathrm{d}\, b_i} = \frac{\psi}{(1-\psi)(1-b_i)^2}$$

Since F_i must be a proper cumulative distribution function, it must hold that:

$$1 = \frac{\psi \overline{b_i}}{(1-\psi)(1-\overline{b_i})}$$
$$\overline{b_i} = 1-\psi$$

The candidate equilibrium strategy of the informed firm is therefore to bid $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

This strategy is an equilibrium if and only if there is no profitable deviation. Suppose that an informed firm's opponent plays according to this candidate equilibrium strategy if it is informed, while according to the uninformed state bidding function if uninformed. Deviating to any mixed strategy that only allocates positive weight to bids with b_i in the interval $(0, 1-\psi]$ does not increase the expected payoff, as F_j is constructed in such a way, that any such bid results in the same expected profit ψ. For any bid including a bribe $b_i > 1-\psi$ the firm can win the procurement by probability one if the opponent plays according to the candidate equilibrium strategy, and thus receive a profit of $1-b_i$. But $\forall\, b_i > 1-\psi$ holds that $1-b_i < \psi$. Therefore any deviation from the candidate equilibrium strategy allocating positive weight to any bribe $b_i > 1-\psi$ results in a lower expected payoff than playing the candidate equilibrium strategy. Since there is no profitable deviation, playing the candidate strategy indeed constitutes an equilibrium.

Therefore in equilibrium an uninformed firm always submits a bid according to the uninformed state bidding function, while an informed firm bids $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

3.2.5 Equilibria in the infinitely repeated game with uncertainty

Consider now the case when two firms compete in a series of procurements, and the number of repetitions is uncertain or infinite. By setting the effective discount factor properly, any such game can be modeled by an infinitely repeated game. Suppose that the probability that the game enters a next stage is ξ, while the one period discount factor is ρ. Then the effective discount factor (δ) is

$$\delta = \xi\rho$$

The infinitely repeated game has multiple equilibria, out of which this paper identifies three. These equilibria require different effective discount factors in order to ensure that none of the firms deviate.

PROPOSITION 1. If the effective discount factor is at least $\frac{5}{6} + \frac{q^2}{2} - \frac{q^3}{3}$, there exists an equilibrium in which both firms submit bids according to the uninformed state bidding function in each period regardless their type, that is, fair competition is sustainable.

A classical example presenting the importance of repetition in sustaining fair competition is the procurement of generic pharmaceuticals in Mexico. Between 2003 and 2006, each region had its own procurements for all the pharmaceuticals, therefore the procurements were rather ad hoc and unique. In 2007, the social security agency of Mexico began to award national contracts annually, establishing a repeated procurement framework. This resulted in an average price decrease of 20% in case of the most important pharmaceuticals, indicating a decrease in corruption (OECD, 2011, pp. 11).

The intuition is that informed firms do not offer a kickback, as by doing so they would reveal that they are informed, and their potentially uninformed opponent would become informed as well. By setting the fully informed state equilibrium strategy to be played in the rest of the periods (which results in a zero payoff), an informed firm paying a kickback would realize a large payoff once, and suffer a loss in all subsequent periods.

Consider therefore the following strategies: An uninformed firm's bid in the first period is characterized by the uninformed state bidding function. For all subsequent periods, the firm bids according to the same bidding function, unless it becomes informed. If an uninformed firm becomes informed (that is, its opponent pays a kickback), it bids according to the equilibrium strategy of the fully informed state in all subsequent periods. An informed firm's first period bid is characterized by the uninformed state bidding function. For all subsequent periods, the informed firm bids according to the same bidding function, unless its opponent pays a kickback. If its opponent pays a kickback, it bids according to the equilibrium strategy of the fully informed state in all subsequent periods.

These strategies result in an equilibrium as long as the effective discount factor is high enough:

$$\delta = \xi\rho \geq \frac{5}{6} + \frac{q^2}{2} - \frac{q^3}{3}$$

A detailed proof of the equilibrium with the derivation of the required effective discount factor is presented in Appendix 3. This is, however, not the only equilibrium.

PROPOSITION 2. If the effective discount factor is in the interval $\frac{\sqrt{5}-1}{2} \leq \delta \leq \frac{5}{6} + \frac{q^2}{2} - \frac{q^3}{3}$, there exists an equilibrium in which firms play their stage game equilibrium strategies in the first period, and with at least one informed firm participating in the procurement, they enter a bid rigging scheme starting in the second period.

Repetition is a key factor in case of bid rigging as well. A study by Ishii (2009) analyzing bid rigging in Japan states that bid rigging is an exchange of favors among firms (submitting a bid such that the other firm can win), and being the predecided winner in a procurement requires former favors towards the other players. Therefore sustaining a bid rigging scheme requires repetition such that favors can be exchanged. These favors are characteristic of the equilibrium defined below as well.

By playing the strategy defined for the stage game in the first period, the game either enters the fully informed state or the uninformed state by the second period. If the strategies are well defined and at least one of the firms is informed, firms may end up in bid rigging, that is, winning the procurement taking turns. Consider the following strategies.

Firm i: In the first period, if uninformed, bid according to the uninformed state bidding function. If informed, bid $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

For the rest of the periods, bid according to the uninformed state bidding function as long as being uninformed. If becoming informed (or being initially informed) bid $(\overline{p} - \epsilon, \underline{q}, 0)$ in every even period, and bid $(\overline{p}, \underline{q}, 0)$ in each odd period as long as firm j bids $(\overline{p}, \underline{q}, 0)$ in each even period after becoming informed. If firm j bids anything else than $(\overline{p}, \underline{q}, 0)$ in an even period, bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods.

Firm j: In the first period, if uninformed, bid according to the uninformed state bidding function. If informed, bid $(\overline{p}, \underline{q}, b_j)$ where $b_j \in (0, 1-\psi]$ and the probabilities over b_j are characterized by the probability density function

$$f_j(b_j) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_j)^2} & \forall\, b_j \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

For the rest of the periods, bid according to the uninformed state bidding function as long as being uninformed. If becoming informed (or being initially informed) bid $(\overline{p} - \epsilon, \underline{q}, 0)$ in every odd period, and bid $(\overline{p}, \underline{q}, 0)$ in each even period as long as firm i bids $(\overline{p}, \underline{q}, 0)$ in each odd period after becoming informed. If firm i bids anything else than $(\overline{p}, \underline{q}, 0)$ in an odd period, bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods.

The result is a bid rigging scheme from the second period. This equilibrium is stable as long as

$$\frac{\sqrt{5}-1}{2} \leq \delta \leq \frac{5}{6} + \frac{\underline{q}^2}{2} - \frac{\underline{q}^3}{3}$$

It is interesting to observe that both a too high and a too low discount factor results in deviation. In case of a too low discount factor, it is worth deviating from the bid rigging, as a one period payoff from deviation is higher than the net present value of the bid rigging payoffs from all the subsequent periods. In case of a too high discount factor, it is worth deviating for an informed firm in the first period by bidding according to the uninformed state bidding function. This way, if the opponent is uninformed, the informed firm could offer an infinitesimal kickback in the second period, and thus obtain a higher payoff in the first two periods. The detailed derivation of the equilibrium and its requirements are presented in Appendix 4.

PROPOSITION 3. If the effective discount factor is at most $\frac{5}{6}+\frac{q^2}{2}-\frac{q^3}{3}$, there exists an equilibrium in which informed firms always offer a kickback.

There has been a number of procurements affected by kickbacks in recent years. In Hungary for example, there have been allegations concerning the procurement of gripen fighter jets (Index.hu, 2012) and impeachment in case of the procurement of Alstom metro trains (Index.hu, 2016). A common feature of these procurements is their specificity, that is, the probability of a future similar procurement is very low. This is captured in the model by the probability of repetition.

If the discount factor is low, that is, the probability of repetition is low or the present value of future payoffs is low, firms will try to win the first procurement by offering a kickback.

Consider the following strategy for both firms: If uninformed, bid according to the uninformed state bidding function in the first period. If informed, bid $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

For an informed firm, bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods. For an uninformed firm, bid according to the uninformed state bidding function as long as uninformed, and bid $(\overline{p}, \underline{q}, \overline{p})$ in all periods after becoming informed. This strategy results in an equilibrium as long as

$$\delta \leq \frac{5}{6} + \frac{\underline{q}^2}{2} - \frac{\underline{q}^3}{3}$$

Note, that both firms being uninformed, the equilibrium is a fair competition. At least one of the firms being informed, the game enters the fully informed state by the second period. Considering the equilibrium strategy of the opponent, there is no profitable deviation after the second period. In the first period, the bid of the uninformed firm does not have any effect on the future gameplay, thus there is no profitable deviation for an initially uninformed firm. An informed firm may deviate by not offering a kickback, and this way delaying the kickback scheme or sustaining a fair competition. However, the effective discount factor being low, the informed firm is impatient and does not take the risk of offering no kickback for possibly higher future payoffs. The detailed derivation of the equilibrium is presented in Appendix 5.

4 Analysis

This section presents an analysis of two questions based on the model presented in Section 3. The first part of the analysis seeks an explanation to the relationship between corruption and tender size, while the second part of the analysis aims to find a reason for the observation, that bid rigging is a more frequent form of corruption than kickbacks.

4.1 Tender size and the extent of corruption

High value procurements are more exposed to corruption than average or low value procurements (OECD, 2007, pp. 27.). The reason of this phenomenon may originate in repetition. It is a fair assumption that high value tenders are specific. Either they require carrying out a complex processes or performing some activities to a large extent. If this assumption holds, it means that large procurements are rather unique, and there is a low or zero chance of repetition.

With a zero chance of repetition, the outcome can be characterized with the stage game equilibrium. The stage game equilibrium strategy for an uninformed firm is to submit a bid according to the uninformed state bidding function, while in case of an informed firm it is to bid $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

It is clear from the equilibrium strategies, that an informed firm in a very specific tender, where there is no chance of repetition, will always offer a kickback and therefore the tender will be corrupt. The intuition is clear. Having only one chance to win a procurement and realize a positive payoff provides an incentive for any informed firm to offer a kickback, and thus increase the probability of winning.

The only case when a large, specific tender is clear is when both firms are uninformed. The probability of such state is ψ^2. Therefore the number of clear large procurements is predicted to decrease in a quadratic way in the prior probability of being uninformed. This means, that the extent of corruption is very sensitive to the general characteristics of the procurement framework.

Suppose now that repetition is possible, but uncertain. This situation is modeled by the infinite repetition framework. The results suggest that in this setting a clear competition can be sustained if and only if

$$\delta = \xi\rho \geq \frac{5}{6} + \frac{\underline{q}^2}{2} - \frac{\underline{q}^3}{3}$$

while bid rigging can arise if and only if

$$\frac{\sqrt{5}-1}{2} \leq \delta \leq \frac{5}{6} + \frac{\underline{q}^2}{2} - \frac{\underline{q}^3}{3}$$

That is, sustaining clear competition requires both a high discount factor and a high probability of repetition, while bid rigging requires an effective discount factor in a specific interval. This means that large tenders do not have to be unique to end up in a corrupt state, it is enough to have a low probability of repetition. If the probability of repetition is low, neither a fair competition nor bid rigging can be sustained.

The results suggest that low corruption requires either a state, in which both firms are uninformed or (1) high probability of repetition and (2) high discount factor. The first case is not related to the tender size. In the second case however, since the requirements are likely to be violated as tender size or tender specificity increases, large, specific tenders are indeed expected to end up in a corrupt state more often.

4.2 The form of corruption

It is clear, that if a game ends up in a state, where both firms are uninformed, it results in a clear competition unless firms cooperate. Such an outcome is way less probable if both or one of the firms is informed. It requires both (1) a high probability of repetition and (2) a high discount factor. If any of these conditions do not hold, firms will have an incentive to deviate from clear competition, and either enter a bid rigging scheme or a series of kickbacks.

Consider now the case where a game ends up in a corrupt state. That is, take a case where either one of the firms or both firms are initially informed, and the effective discount factor is too low to sustain a clear competition. The exact outcome of the game depends on the effective discount factor. Note, that for any positive effective discount factor, firms prefer the bid rigging scheme to the kickback scheme, as it results in the same expected payoff in the first period and either the same or strictly higher payoffs in the subsequent periods (as they win the procurements taking turns). Therefore it is fair to assume that if bid rigging is sustainable, the bid rigging equilibrium is the outcome of the game.

Now take a look at the main determinant of the equilibrium strategy, that is, the effective discount factor. The effective discount factor is a product of the discount factor and the probability of repetition. Assuming that the discount factor is constant for all the procurements, the only factor determining whether a game between two firms (with at least one of them informed) ends up in a clear competition, a bid rigging scheme or a kickback scheme is the probability of repetition.

The results suggest that given a discount factor (ρ), the game ends up in a fair competition if the probability of repetition exceeds a cutoff:

$$\xi \geq \frac{\frac{5}{6} + \frac{q^2}{2} - \frac{q^3}{3}}{\rho}$$

A bid rigging scheme arises in a lower region of the probability of repetition, namely where

$$\frac{\frac{\sqrt{5}-1}{2}}{\rho} \leq \xi \leq \frac{\frac{5}{6} + \frac{q^2}{2} - \frac{q^3}{3}}{\rho}$$

While the equilibrium is a kickback scheme if the probability of repetition is low, that is

$$\xi \leq \frac{\frac{\sqrt{5}-1}{2}}{\rho}$$

Notice that bid rigging requires a higher probability of repetition than kickbacks. Therefore among the games that end up in the corrupt state there are games with bid rigging associated with a higher probability of repetition, and games with kickbacks associated with a lower probability of repetition. As a kickback scheme arises when the probability of repetition is low, these games end sooner than the games with bid rigging (on average). Therefore even though the exact share of games with bid rigging and games with kickbacks depends on the distribution of the probability of repetition among games, it is likely that analyzing a given period, bid rigging will be more prevalent than kickbacks.

5 Concluding remarks

This paper examined two questions, namely how the tender size matters, that is, why is corruption more characteristic in case of large procurements and why is bid rigging more prevalent than offering kickbacks.

Based on a model of a procurement framework, the analysis found that a main determinant of the outcome is the probability of repetition. Fair competition requires a high probability of repetition. As large tenders are likely to be specific or even unique, fair competition in case of large tenders is hard to sustain.

The form of corruption is also related to the probability of repetition. Analyzing games where corruption arises showed that bid rigging is characteristic to games with higher probability of repetition, while kickbacks arise when this probability is lower. Since games with bid rigging are repeated longer, it is likely that bid rigging offers more frequently than kickbacks. This result however depends on the distribution of probabilities of repetition. An empirical study on the repetition of procurements between given firms can either support or confute this theory.

Further research is needed to model the behavior of the procurement official more correctly. Throughout this paper, the procurement official was assumed to be risk neutral. Introducing a risk averse official would result in a more realistic model and may alter the equilibria as well.

6 Appendices

This section contains extensive calculations and proofs for for several equilibria stated in Section 3.

Appendix 1: Establishing the pure strategy equilibrium for the stage game in the fully informed state

Consider the candidate equilibrium strategy, which is for each firm to bid $(\overline{p}, \underline{q}, \overline{p})$. It is clear that this strategy being played, the expected payoff of the firms is zero. Considering the candidate equilibrium strategy of the of firm j, any deviation with $b_i < \overline{p}$ results in losing the procurement and a payoff of zero. Still considering the equilibrium strategy of the of firm j, any deviation with $b_i > \overline{p}$ results in winning the procurement with a payoff of $p_i - b_i$. But the reservation price of the government implies the constraint $p_i \leq \overline{p}$, therefore the payoff of firm i would be strictly negative. Considering the candidate equilibrium strategy of the of firm j, any deviation with $b_i = \overline{p}$ results in the same probability of winning the procurement, as it is determined by the kickback offered. Since the payoff in case of losing the procurement is zero regardless the bid, and the probability of winning the procurement is unchanged if $b_i = \overline{p}$, there is incentive to deviate from the candidate equilibrium strategy if and only if by deviating the payoff is higher in case of winning. But

$$p_i - c_i - \overline{p} < \overline{p} - 0 - \overline{p} \quad \forall p_i \neq \overline{p} \text{ and } q_i \neq \underline{q}$$

therefore there is no bid resulting in a higher payoff than the bid specified in the candidate equilibrium strategy. Since there is no profitable deviation, bidding $(\overline{p}, \underline{q}, \overline{p})$ is indeed an equilibrium strategy.

Appendix 2: Establishing the pure strategy equilibrium for the stage game in the uninformed state

Since in the uninformed state the bid with the higher value wins the procurement, firms participate in a first price auction in values. The reservation values of the firms are independent and identically distributed on the interval $[\underline{q}, 1]$. It is clear, that the probability density function of the reservation value ($\overline{v}$)is

$$f(\overline{v}) = \begin{cases} \underline{q} & \text{for } \overline{v} = \underline{q} \\ 1 & \text{else} \end{cases}$$

and the cumulative distribution function is

$$F(\overline{v}) = \overline{v}$$

Using the revenue equivalence theorem (first applied by Vickrey (1961) and later generalized by Myerson (1979) and Riley and Samuelson (1981)) this first price auction in value is revenue equivalent to a second price auction in value, as it is a common knowledge that the reservation values are independently drown from the same, strictly increasing distribution. Therefore the bidding function can be derived by calculating the expected reservation value of the losing firm conditional on the reservation value of the winning firm. Consider that firm i has a reservation value $\overline{v_i}$ and wins the procurement. Then the expected reservation value of firm j characterizes the bidding function of firm i, therefore the bidding function of firm i is

$$b(\overline{v_i}) = \frac{\overline{v_i}^2 + \underline{q}^2}{2\overline{v_i}}$$

After determining the value to bid, the corresponding price and quality can be calculated. This method of submitting bids is henceforth referenced as the uninformed state bidding function.

In the uninformed state, firms have a strictly positive expected payoff. A firm with reservation value $\overline{v_i}$ has a $\overline{v_i}$ probability of winning according to the cumulative distribution function of $\overline{v}$. A firm losing the procurement makes zero profit, and a firm winning makes a profit of $\overline{v_i}$ decreased by the bid $b(\overline{v_i})$. Therefore the expected profit is

$$E(\pi_i) = \int_{\underline{q}}^{1} \overline{v_i}\left(\overline{v_i} - b(\overline{v_i})\right) \mathrm{d}\overline{v_i} = \int_{\underline{q}}^{1} \overline{v_i}^2 - \frac{\overline{v_i}^2 + \underline{q}^2}{2} \mathrm{d}\overline{v_i} = \frac{1}{6} - \frac{\underline{q}^2}{2} + \frac{\underline{q}^3}{3}$$

Since the derivative

$$\frac{\mathrm{d}\, E(\pi_i)}{\mathrm{d}\, \underline{q}} = \underline{q}^2 - \underline{q}$$

is strictly negative on the whole domain of $\underline{q}$ and

$$\lim_{\underline{q} \to 1} E(\pi_i) = 0^+$$

the expected profit of the firms is strictly positive.

Appendix 3: Establishing the competitive equilibrium for the infinitely game with uncertainty

In an infinite repetition setting, there exists an equilibrium in which zero bribes can be sustained, if the effective discount factor δ is high enough. Consider the following strategies: An uninformed firm's bid in the first period is characterized by the uninformed state bidding function. For all subsequent periods, the firm bids according to the same bidding function, unless it becomes informed. If an uninformed firm becomes informed (that is, its opponent pays a bribe), it plays the equilibrium strategy derived for the informed state in Appendix 1 for all subsequent periods. An informed firm's first period bid is characterized by the uninformed state bidding function. For all subsequent periods, the informed firm bids according to the same bidding function, unless its opponent pays a bribe. If its opponent pays a bribe, it plays the equilibrium strategy derived for the informed state in Appendix 1 for all subsequent periods.

In case of an uninformed firm, it is clear that such a strategy is indeed a best response to the other player's strategy. First, note that the uninformed firm can never pay a bribe as long as it's opponent does not pay a bribe. Therefore regardless the deviation an uninformed firm may consider, its opponent playing according to the candidate equilibrium strategy will submit a bid defined by the uninformed state bidding function in all periods. Since the best response for such strategy of the opponent is to bid by the bidding function, any deviation in any period only affects that period's payoff, and it can not increase the payoff. Therefore for an uninformed firm there is no profitable deviation from the equilibrium strategy.

In case of an informed firm, considering any deviation that includes bids with zero bribe in all periods does not alter the behavior of the opponent, that is, it's opponent will bid according to the bidding function in all periods. Since the best response to that strategy of the opponent is to bid based on the same bidding function, no deviation incorporating zero bribe can increase the expected payoff of the firm. Consider now a deviation including a non-zero bribe in some periods. Until the first such period, the opponent will play according to the bidding function, therefore no deviation in the previous periods can increase the expected payoff for those periods. Therefore the most profitable possible deviation incorporating non-zero bribe in some periods still has to define playing with respect to the bidding function, until paying a bribe for the first time. After paying a bribe, however, the opponent will bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods, resulting in a zero profit in all subsequent periods regardless the bids defined for those periods by the deviation. Therefore it is worth deviating if and only if the payoff from paying a bribe exceeds the expected net present value of payoffs in case of paying according to the candidate equilibrium strategy. The highest payoff from paying a bribe is achieved, if the bid is $(\overline{p}, \underline{q}, \epsilon)$. Therefore it is worth deviating if and only if

$$\frac{1}{1-\delta}\left(\frac{1}{6} - \frac{\underline{q}^2}{2} + \frac{\underline{q}^3}{3}\right) < 1$$

That is, a clear competition without corruption can be sustained if and only if

$$\delta = \xi\rho \geq \frac{5}{6} + \frac{\underline{q}^2}{2} - \frac{\underline{q}^3}{3}$$

Appendix 4: Establishing the bid rigging equilibrium for the infinitely game with uncertainty

Consider the following strategies. Firm i: In the first period, if uninformed, bid according to the uninformed state bidding function. If informed, bid $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

For the rest of the periods, bid according to the uninformed state bidding function as long as being uninformed. If becoming informed (or being initially informed) bid $(\overline{p} - \epsilon, \underline{q}, 0)$ in every even period, and bid $(\overline{p}, \underline{q}, 0)$ in each odd period as long as the firm j bids $(\overline{p}, \underline{q}, 0)$ in each even period after becoming informed. If firm j bids anything else than $(\overline{p}, \underline{q}, 0)$ in an even period, bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods.

Firm j: In the first period, if uninformed, bid according to the uninformed state bidding function. If informed, bid $(\overline{p}, \underline{q}, b_j)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_j are characterized by the probability density function

$$f_j(b_j) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_j)^2} & \forall\, b_j \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

For the rest of the periods, bid according to the uninformed state bidding function as long as being uninformed. If becoming informed (or being initially informed) bid $(\overline{p} - \epsilon, \underline{q}, 0)$ in every odd period, and bid $(\overline{p}, \underline{q}, 0)$ in each even period as long as the firm i bids $(\overline{p}, \underline{q}, 0)$ in each odd period after becoming informed. If firm i bids anything else than $(\overline{p}, \underline{q}, 0)$ in an odd period, bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods.

For an initially uninformed firm, there is no profitable deviation as long as the effective discount factor is high enough. Considering the equilibrium strategy of the opponent, it is easy to see that there is no profitable deviation in the first period. Since the first period bid of the uninformed firm does not affect the rest of the game, it would worth deviating if and only if it resulted in a higher expected payoff in the first period. But playing against an informed opponent, the first period payoff of an uninformed firm is zero regardless the bid submitted. Playing against an other uninformed firm, bidding according to the uninformed state bidding function is a best response. Therefore there is no profitable deviation from the candidate equilibrium strategy for an initially uninformed firm in the first period. Considering the equilibrium strategy of the opponent, the state of the world is revealed by the second period. If remained uninformed, the opponent must be uninformed as well. Therefore bidding according to the uninformed state bidding function in each period is, indeed an equilibrium strategy. If became informed, the game enters a fully informed state. The candidate equilibrium strategy defines winning the procurement taking turns. In a period in which the firm wins the procurement there is no incentive to deviate as it would not affect the rest of the gameplay and already results in the highest possible payoff. In a period in which the firm loses the procurement there is no incentive to deviate as long as the effective discount factor is high enough. Any deviation results in a series of zero payoff for the rest of the game. The best deviation therefore maximizes the payoff in the period of the deviation. The maximal payoff is achieved by bidding $(\overline{p} - \gamma, \underline{q}, 0)$ where γ is infinitesimal and $\gamma > \epsilon$. This results in a payoff of one in limits. It does not worth deviating as long as

$$1 \leq \frac{\delta}{1-\delta^2}$$

This holds as long as

$$\delta = \xi\rho \geq \frac{\sqrt{5}-1}{2}$$

For an initially informed firm, the derivation from the second period is identical. In case of an informed firm, it does not worth deviating in the first period as long as the discount factor is low enough to not to try revealing the type of the opponent.

First, note that the strategy defined for the first period is the equilibrium strategy of the stage game. Therefore there is no other strategy that would result in an increase in the expected payoff for the first period. A profitable deviation should therefore increase the net present value of the expected payoffs for the subsequent periods. Also note, that if the opponent is informed or a kickback is offered, the rest of the gameplay is not affected by any deviation. Therefore the rest of the gameplay is affected if and only if the opponent is uninformed and the deviation includes offering no kickback. Of all the bids offering no kickback, the best deviation is to bid according to the uninformed state bidding function (as it is the best response for the bid of an uninformed opponent). By probability $1-\psi$ the opponent is informed, and the deviation results in a payoff of zero in the first period, while not affecting the rest of the periods. By probability ψ the opponent is uninformed, and the rest of the gameplay is affected. In this case, the rest of the game can be threefold. Either the firm does not offer a kickback in any period and a fair competition is sustained, or the firm offers a kickoff in a later period and the bid rigging scheme arises or the firm offers a kickoff in a later period and exits the bid rigging scheme as well by deviating. It is worth deviating to trying to establish a fair competition for firm j

if and only if

$$\psi\left(\frac{\frac{1}{6}-\frac{\underline{q}^2}{2}+\frac{\underline{q}^3}{3}}{1-\delta}\right) > \psi + \frac{\delta^2}{1-\delta^2}$$

But there is no such setting that this inequality would hold. Let us denote the fair competition profit by x, then

$$x \equiv \frac{1}{6}-\frac{\underline{q}^2}{2}+\frac{\underline{q}^3}{3}$$

The condition on the deviation can be rewritten as

$$\psi\left(\frac{x}{1-\delta}\right) - \psi - \frac{\delta^2}{1-\delta^2} > 0$$

The left hand side is strictly increasing in x. The maximum value of x is $\frac{1}{6}$. By setting x to its maximum we have

$$\frac{\psi}{6(1-\delta)} - \psi - \frac{\delta^2}{1-\delta^2} > 0$$

Its derivative in ψ is $\frac{1}{6(1-\delta)} - 1$, which is positive for all $\delta > \frac{5}{6}$ and negative for all $\delta < \frac{5}{6}$. Consider any $\delta > \frac{5}{6}$. For any such δ, the left hand side obtains its maximum if $\psi = 1$. But even in that case

$$\frac{1}{6(1-\delta)} - 1 - \frac{\delta^2}{1-\delta^2} < 0 \quad \forall \delta > \frac{5}{6}$$

Consider any $\delta < \frac{5}{6}$. For any such δ, the left hand side obtains its maximum if $\psi = 0$. But even in that case

$$-\frac{\delta^2}{1-\delta^2} < 0 \quad \forall \delta < \frac{5}{6}$$

Therefore it is not worth deviating to trying to establish a fair competition for firm j. Since firm i's payoff from the candidate equilibrium strategy is even higher, it is not worth for firm i to deviate as well.

Now consider that the firm does not try to keep up the fair competition, only tries to find out whether the opponent is uninformed, and if it is, then make the first deviation by paying an infinitesimal bribe instead of mixing. Since the rest of the gameplay is the same, it is worth deviating if and only if the payoff in the expected payoff in the first two periods exceeds the expected payoff of the candidate equilibrium strategy. For firm j that is

$$\psi\left(\frac{1}{6}-\frac{q^2}{2}+\frac{q^3}{3}+\delta\right)>\psi$$

The inequality holds if and only if

$$\delta>\frac{5}{6}+\frac{q^2}{2}-\frac{q^3}{3}$$

therefore it is not worth deviating as long as

$$\delta\leq\frac{5}{6}+\frac{q^2}{2}-\frac{q^3}{3}$$

Since firm i's payoff from the candidate equilibrium strategy is even higher, it is not worth for firm i to deviate as well.

Consider now the strategy in which the firm tries to reveal the uninformed type of the other player, offer a kickoff in a later period and exit the bid rigging scheme as well by deviating. This is worth if and only if

$$\psi\left(\frac{1}{6}-\frac{q^2}{2}+\frac{q^3}{3}+\delta+\delta^2\right)>\psi+\delta$$

But note, that since $\delta>\delta^2$, this requires an even higher δ than the previous deviation. Therefore if it is not worth deviating by the previous strategy, it is not worth deviating by this strategy as well.

Therefore the bid rigging can be sustained as long as

$$\frac{\sqrt{5}-1}{2} \leq \delta \leq \frac{5}{6} + \frac{q^2}{2} - \frac{q^3}{3}$$

Appendix 5: Establishing the kickback equilibrium for the infinitely game with uncertainty

Consider the following strategy for both firms: If uninformed, bid according to the uninformed state bidding function in the first period. If informed, bid $(\overline{p}, \underline{q}, b_i)$ where $b_i \in (0, 1-\psi]$ and the probabilities over b_i are characterized by the probability density function

$$f_i(b_i) = \begin{cases} \frac{\psi}{(1-\psi)(1-b_i)^2} & \forall\, b_i \in (0, 1-\psi] \\ 0 & \text{else} \end{cases}$$

For an informed firm, bid $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods. For an informed firm, bid according to the uninformed state bidding function as long as uninformed, and bid $(\overline{p}, \underline{q}, \overline{p})$ in all periods after becoming informed.

For an initially uninformed firm, there is no profitable deviation. Considering the equilibrium strategy of the opponent, it is easy to see that there is no profitable deviation in the first period. Since the first period bid of the uninformed firm does not affect the rest of the game, it would worth deviating if and only if it resulted in a higher expected payoff in the first period. But playing against an informed opponent, the first period payoff of an uninformed firm is zero regardless the bid submitted. Playing against an other uninformed firm, bidding according to the uninformed state bidding function is a best response. Therefore there is no profitable deviation from the candidate equilibrium strategy for an initially uninformed firm in the first period. Considering the equilibrium strategy of the opponent, the state of the world is revealed by the second period. If remained uninformed, the opponent must be uninformed as well. Therefore bidding according to the uninformed state bidding function in each period is, indeed an equilibrium strategy. If became informed, the game enters a fully informed state. Since in this case the opponent bids $(\overline{p}, \underline{q}, \overline{p})$ in all subsequent periods, any bid in any period results in a zero payoff. Therefore there is, indeed, no profitable deviation.

Now consider an initially informed firm. First, note that the strategy defined for the first period is the equilibrium strategy of the stage game. Therefore there is no other strategy that would result in an increase in the expected payoff for the first period. A profitable deviation should therefore increase the net present value of the expected payoffs for the subsequent periods. Also note, that if the opponent is informed or a kickback is offered, the rest of the gameplay is not affected by any deviation. Therefore the rest of the gameplay is affected if and only if the opponent is uninformed and the deviation includes offering no kickback. Of all the bids offering no kickback, the best deviation is to bid according to the uninformed state bidding function (as it is the best response for the bid of an uninformed opponent). By probability $1 - \psi$ the opponent is informed, and the deviation results in a payoff of zero in the first period, while not affecting the rest of the periods. By probability ψ the opponent is uninformed, and the rest of the gameplay is affected. In this case, the rest of the game can be twofold. Either the firm does not offer a kickback in any period and a fair competition is sustained, or the firm offers a kickoff in a later period and a series of bidding $(\overline{p}, \underline{q}, \overline{p})$ by the opponent arises. It is worth trying to establish a fair competition if and only if

$$\frac{\psi}{1-\delta}\left(\frac{1}{6} - \frac{\underline{q}^2}{2} + \frac{\underline{q}^3}{3}\right) > \psi$$

By rearranging, the kickback equilibrium can be sustained as long as

$$\delta \leq \frac{5}{6} + \frac{\underline{q}^2}{2} - \frac{\underline{q}^3}{3}$$

It is worth trying to delay the kickback offering if and only if

$$\psi\left(\frac{1}{6}-\frac{q^2}{2}+\frac{q^3}{3}+\delta\right)>\psi$$

By rearranging, the kickback equilibrium can be sustained as long as

$$\delta\leq\frac{5}{6}+\frac{q^2}{2}-\frac{q^3}{3}$$

Therefore if the discount factor is not too high, there is no profitable deviation. If the discount factor is high enough, it is worth for an informed firm to try to establish a fair competition, as the net present value of the possible future payoffs from a fair competition is higher than the payoff from one single period.

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