An Optimal Debt Management Strategy for Hungary

by

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Abstract

This thesis analyzes the expected costs and risks associated with Hungarian government securities with the purpose of constructing an optimal debt management strategy over a five-year horizon. In order to accomplish this objective, two intermediate issues are resolved. First, different empirical yield curve models are compared based on in-sample fitting. I find that more complex models fit the yield curve better, but additional restrictions are necessary for them to be interpretable as factor models. Second, the yield curve models are compared based on pseudo out-of-sample forecasting performance. Using various autoregressive, vector-autoregressive and state-space models for prediction, I conclude that the Nelson-Siegel model in a state-space framework outperforms the benchmark random walk forecast on longer maturities and horizons. This model is then used to carry out an *ex ante* prediction for the 2016-2020 horizon. Ten thousand scenarios are generated using a Monte-Carlo simulation describing the evolution of the term structure of interest rates. The scenarios are used to calculate the expected debt charges associated with bonds. Finally, the mean-variance and the mean-conditional cost-at-risk efficient frontiers are established. The policy conclusion is that reducing the share of very long and very short maturity government bonds in favor of medium maturity papers would be a Pareto-improvement to the current Hungarian debt portfolio.

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Chapter 1

Introduction

An important fact in economic life is that most governments in the world run a budget deficit. The immediate consequence of this phenomenon is that governments must finance their extra expenditures by borrowing. The issuance of government securities is the primary way of obtaining extra revenue. Government bonds in developed countries offer a low-risk investment alternative for institutional, corporate and individual investors alike. With the government having the largest debt portfolio in a country, it can also play an important role in controlling or influencing the financial markets. Government securities also serve as benchmark for other sectors as well.

Literature refers to severals country level statistics as debt. It can mean the debt of households, enterprises, regional governments or the central government. The focus of this thesis is central government debt and throughout this paper, I refer to that particular concept when writing about debt.

The actor responsible for issuing government debt is the sovereign debt manager. Practically, in most cases, it is the government acting through the Ministry of Finance or a specialized organization. The challenge of a debt manager is to create a stable financing

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strategy within certain constraints that meets the required level of borrowing set by the government's economic policy. Different financings strategies lead to different size, structure and characteristics of the government debt. At its core, this is a portfolio management exercise to find an optimal trade-off between costs and risks inherent to the debt portfolio. Structuring the maturity of debt, issuing zero-coupon securities and bonds that pay fixed or variable interest as well as issuing bonds in sovereign and foreign currency are all decisions the debt manager has to make.

In Hungary, the role of the debt manager falls to the Government Debt Management Agency Pte. Ltd., abbreviated ÁKK in Hungarian. Its mission "is to finance the government debt and the central government deficit at the lowest costs in the long run taking account of risks, at a high professional level and by using sophisticated methods" (ÁKK Zrt, 2016). According to official government debt data by ÁKK and a GDP estimate by the Hungarian Government (Magyarország Kormánya, 2016), the central government debt ratio of Hungary on December 31th, 2015 was 73.3% of the GDP, or in nominal terms, HUF 24 699.72 billion. To highlight the importance of debt management, gross debt service charges according to ÁKK data were close to 4% of the Hungarian GDP. This is only about one percentage point less compared to the country's health or education expenditures based on World Bank data. With the low interest rate regime in contemporary Europe, we will certainly see the debt service charges fall, but the amount nonetheless depends very much on debt management strategy.

The research question of the thesis is the following: what proportion of different HUF-denominated bonds[1] should Hungary issue to minimize its debt service charges and exposure to risk, in conform with its mission stated above? The question has a clear policy

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[1] For the sake of simplicity, often the term 'bonds' is used throughout this thesis, referring to all government securities including Treasury bills

relevance. In an economic crisis, a country with a highly risky debt portfolio might have to refinance a large share of its maturing debt at very high interest rates. On the other hand, a low-risk portfolio of long maturity bonds usually has higher debt charges in the long run. A large debt-to-GDP ratio combined with high interest rates might lead to a debt spiral. Moreover, the debt portfolio of a country is not necessary efficient in a mathematical sense. It can be possible to reduce both the expected costs as well as risks at the same time.

To answer the research question, I develop a debt management model for Hungary that is able to calculate an efficient cost-risk frontier of debt portfolios and can accommodate a wide variety of policy objectives and constraints. However, to accomplish this, it is first necessary to model and forecast the term structure of interest rates. The term structure of interest rates - also known as the yield curve - is one of the most important topics in finance. Knowledge of the yield curve is required for calculating the present value of securities and thus used in pricing options, bonds or derivatives. For a sovereign debt manager, bond pricing is absolutely vital to calculate interest expenditure and eventually the entirety of the debt service charges.

Term structure modeling is necessary for two main reasons. First, at any given moment, we can only observe the prices and yields of a limited amount of debt securities (bonds and bills) which cover those select maturities. However, for most calculations, it is necessary to know other points of the yield curve as well. Furthermore, the observed bonds usually include both zero-coupon bonds and bonds that pay interest periodically, which imposes additional complexities to the estimation of the zero-coupon yield curve. In Hungary, there are no HUF-denominated zero-coupon government securities with a maturity of over one year. Furthermore, Hungarian bonds use different interest compounding[2], which is

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[2]The used methodology is linear compounding for zero-coupon T-bills and effective compounding for interest bearing bonds.

problematic as the goal is to estimate a single yield curve.

Second, term structure models cannot only be used for estimating the yield curve in a static, cross-sectional way, but are also useful for taking the dynamic time-series aspects of yields into account and forecast the term structure of interest rates. With an adequate forecasting model at hand, it is possible to generate scenarios for the future and determine the optimal debt management strategy based upon it.

There is a lack of consensus in literature as well as among practitioners regarding term structure estimation and forecasting models. This thesis contributes to existing literature by comparing several empirical yield curve models using Hungarian data. There is also an ongoing discussion about the exact estimation procedure and algorithm to use for yield curve fitting. I show that simple local optimization algorithms are adequate for this task if reasonable starting values and constraints are given. This paper also demonstrates that restricting the optimization space using dynamic constraints can facilitate the interpretation of empirical term structure models as factor models. Another novel result of the thesis is the comparison of Nelson-Siegel-Svensson and Legendre yield curve models based on forecasting performance. I find that the four-factor Legendre model offers a viable alternative to the popular Nelson-Siegel model.

Estimating and forecasting the Hungarian term structure of interest rates are necessary steps to calculate the costs and risks associated with HUF-denominated government securities. With the estimated costs and risks, it is possible to create an efficient frontier of debt portfolios. All results of the thesis from yield curve estimation to portfolio optimization serve as building blocks to answer the research question.

The outline of this thesis is as follows: Chapter 2 introduces the definitions and basic bond mathematics necessary to understand the rest of the thesis. Chapter 3 gives an account of the existing models used for term structure estimation and forecasting, as well as models concerning debt management strategies. A cross-country overview is then

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given showing the wide array of models used in practice. Chapter 4 deals with the practical estimation of the Hungarian yield curve. Several models introduced in Chapter 3 are characterized and compared based on curve fitting performance and other econometric characteristics. In Chapter 5, several forecasting methodologies are introduced and then used to predict the future yields through a number of different term structure models. The models and methodologies are then evaluated and compared based on pseudo out-of-sample forecasting accuracy and precision. The best model is then selected to generate a large number of forecast scenarios. Chapter 6 is about building a practical debt management model for HUF-denominated government bonds based on the scenarios calculated in Chapter 5. An optimal debt portfolio is then estimated and explicit policy recommendations are given based on the results. Chapter 7 concludes. All calculations and figures throughout the paper were made using MATLAB. For calculations in Chapters 4 and 6, I used parts of ÁKK's debt management software (currently under development) with several extensions and modifications.

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Chapter 2

Bond Pricing and the Term Structure of Interest Rates

The purpose of this chapter is to provide an understanding of the fundamental concepts of bond mathematics and the term structure of interest rates. The definitions and formulas in this chapter are used extensively throughout the thesis.

Bonds are debt instruments. When a bond is purchased, the buyer lends money to the issuer (the government in this case). The issuer is obliged to pay the principal value of the bond plus interest if applicable. We can group bonds into two major categories: interest bearing and zero-coupon bonds. Several different types of interest bearing bonds exist. Fixed interest rate bonds periodically pay a constant coupon. Step-up bonds are similar in a sense that the coupon is predetermined when the bond is issued, but the coupon payments are not constant but (usually) increasing. Floating rate bonds have a time-varying coupon that is determined periodically based on a reference interest rate. There are also inflation-linked bonds. One type is similar to floating rate bonds, meaning that the inflation is the reference rate that determines the coupon. For the second type, the consumer price

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index has a 'capital uplift' effect, meaning that inflation directly increases the capital of the bond. Zero-coupon bonds on the other hand pay no interest. They are issued for a price under their face value. The face value is then repaid when the zero-coupon bond matures.

Pricing zero-coupon bonds can be done in three different ways depending on interest rate compounding. The price of a zero-coupon bond prevailing at time t for the maturity T ($t \leq T$) can be calculated the following way using simple linear compounding:

$$P(t,T) = \frac{1}{1 + \tau(t,T) R_{lin}(t,T)} \tag{2.1}$$

where:

P denotes the price,

τ denotes the time,

and R denotes the zero-coupon (or spot) yield.

We can also use effective compounding, when interest is paid and compounded k times per year. The pricing formula for effective compounding is:

$$P(t,T) = \frac{1}{\left(1 + \frac{R_{eff}(t,T)}{k}\right)^{\tau(t,T)}} \tag{2.2}$$

The third way is continuous compounding, which can be calculated as follows:

$$P(t,T) = e^{R_{log}(t,T)\tau(t,T)} \tag{2.3}$$

The price of a fixed coupon bond can be calculated by summing the prices of zero-coupon bonds, where all coupon payments as well as the principal payment are considered to be zero-coupon bonds.

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$$p(t) = \sum_{i=1}^{n} P(t, T_i)c_i + P(t, T_n)M = \sum_{i=1}^{n} P(t, T_i)CF_i \tag{2.4}$$

where:

c_i denotes the i^{th} coupon payment,

M denotes the principal payment (face value) paid at maturity,

and CF_i denotes the i^{th} cash-flow of the bond.

Either of these formulas can be used to construct the term structure of interest rates, also referred to as zero-coupon or spot yield curve. The term structure is a function:

$$T \to R(t, T) \tag{2.5}$$

The yield curve is typically upward-sloping, meaning that longer maturity bonds pay more interest. Inverted (downward sloping) curves usually indicate an expectation for the short rates to fall. It can also be associated with an expected economic downturn as central banks usually react to a slowing economy by easing monetary policy. The conventional tool for loosening the monetary policy is cutting the short rate. Therefore, expecting an economic slowdown is in line with expecting the short-term interest rates to fall and thus, one possible cause of inverted yield curves. Flat yield curves can be seen when economic expectations for the short rate are mixed and interpreted differently by investors. Humped yield curves usually occur when the macroeconomic expectations are uncertain. When the yield curve is humped, medium maturity bonds have a higher or lower interest compared to both long and short maturity bonds.

The forward rate for a period $[T, S]$ can be calculated as:

$$F(t, T, S) = -\frac{\ln P(t, S) - \ln P(t, T)}{\tau(T, S)} \tag{2.6}$$

On the bond market, the market maker sets a two-way price quotation. The bid price

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is the maximum price the buyers are willing to pay while the ask price is the minimum price the sellers are willing to sell the bond for. The following inequality holds for the bid and ask prices:

$$P_{bid} \leq P \leq P_{ask} \tag{2.7}$$

The true P price of a bond is only theoretical. In this thesis, the market price (or mid price) will be used as the price of a bond which can be calculated as:

$$P = P_{mid} = \frac{P_{bid} + P_{ask}}{2} \tag{2.8}$$

The bid-ask spread for prices is the following:

$$s = P_{ask} - P_{bid} \tag{2.9}$$

The yield-to-maturity (YTM) is an internal rate of return measure for a bond that can be calculated from the price. The continuously compounded YTM $y(t)$ of a interest bearing bond is the unique solution to the following equation:

$$p(t) = \sum_{i=1}^{n} CF_i e^{-y(t)\tau(t,T_i)} \tag{2.10}$$

An exact inverse formula does not exist and it is impossible to calculate the YTM algebraically in most cases, therefore a numerical estimation method has to be used. A notable exception is when there is only a single cash-flow remaining (for example, in case of zero-coupon bonds).

There is an inverse relationship between the price and the YTM of a bond. We can discern between bid and ask YTMs as well. However, in case of YTMs, the inverse relationship to prices means that the bid YTM is larger than the ask YTM. Therefore, the

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bid-ask inequality is reversed:

$$y_{ask} \leq y \leq y_{bid} \tag{2.11}$$

The final concept to be introduced is duration. Duration is a measure of price sensitivity. The Macaulay-duration is a weighted average of time until all cash-flows are acquired. The modified duration measures the change of price if the yield changes by an infinitesimal amount. The Macaulay-duration of a bond with continuous compounding can be calculated from the YTM:

$$MacD(t) = \frac{\sum_{i=1}^{n} \tau(t, T_i) CF_i e^{y(t)\tau(t,T_i)}}{p(t)} \tag{2.12}$$

The modified duration can be calculated by taking the derivative of the price with respect to the YTM. When continuous compounding is used, the two duration measures are equal in absolute value.

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Chapter 3

Literature Review

This chapter examines the vast literature surrounding term structure modeling along with the papers regarding optimal debt strategy models. First, the most popular yield curve models are introduced along with corresponding literature regarding the making of forecasts using said models. The next part of this chapter deals with the comparison of the real-world applications of a wide variety of yield curve models across different countries. Finally, a comprehensive overview is given regarding the different debt management models used worldwide. Here, theory is very closely related to practice as some of the practitioners of debt management publish their theory and experiences regarding debt management frameworks themselves.

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3.1 The Theory of Term Structure Modeling and Forecasting

Yield curve modeling has almost 40 years of literature. Despite advances in theoretical modeling, there is a lack of consensus on which types of models to use in practice. The two main classes of theoretical yield curve models include the no-arbitrage approach and the equilibrium approach, also known as affine models. Affine models focus on the time series aspects of the yield curve. The dynamics of an affine term structure model are driven by the instantaneous rate, also called the short rate. However, these models have the underlying assumption that longer maturity interest rates are a linear function of the short rate. This is contradictory to empirical evidence as yield curves are often nonlinear in practice. Notable contributing articles concerning the equilibrium approach of yield curve modeling include the often used Cox-Ingersoll-Ross (1985) model, other one-factor models by Vasicek (1977), Ho and Lee (1986), Duffie and Kan (1996) and Dai and Singleton (2000) as well as the two-factor model of Longstaff and Schwartz (1992). However, from a practical point of view, the drawback of these models is the poor out-of-sample forecasting performance, as noted by Duffee (2002). The other class of theoretical models are the no-arbitrage models. These models attempt to find a perfect cross-sectional fit of the term structure of interest rates so that no arbitrage opportunities exist. On the other hand, these models take no account of the time series aspects of yields, which makes them inadequate for forecasting. Prominent contributions to the no-arbitrage family include the Hull and White (1990) and the Heath-Jarrow-Morton (1992) model.

As both classes of theoretical models display inherent weaknesses in the practical field of interest rate forecasting, it comes as no surprise that empirical models started appearing as well. The common point of empirical term structure models is that while they have weaker or absolutely no theoretical foundation, they exhibit a strong in-sample fitting of

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the yield curve and some of them are also adequate for out-of-sample forecasting.

The first class of empirical models are the so-called spline models. Splines are piecewise polynomial functions and can be used for yield curve fitting. Splines can be given great flexibility with the introduction of extra parameters that can be done by increasing the number segments and knots. A large enough number of parameters result in a function almost perfectly fitting the observable points of the term structure but can result in a function with a lot more local minima and maxima than a yield curve should have. There is also a criterion in yield curve fitting that spline functions should remain reasonably "smooth" in a mathematical sense (twice continuously differentiable) to avoid breaks in the term structure of interest rates. The first usage of splines in yield curve estimation was by McCulloch (1975) with a cubic spline interpolation. The problem with constructing discount curves using cubic splines is, however, that a polynomial function diverges at longer maturities while the long end of the term structure tends to flatten out in reality. Therefore, several other variations of the spline model have been introduced. Noteworthy examples include the exponential spline (Vasicek and Fong, 1982), the B-spline (Steeley, 1991; Bolder and Stréliski, 1999), the smoothing spline (Fisher et al., 1995), the variable roughness penalty (VRP) (Waggoner, 1997) and the Merrill Lynch spline (Li et al., 2001). The main argument against spline models is their complete detachment from theoretical foundations. They are a mathematical tool that can be used to interpolate any curve without taking economic considerations or the specific characteristics of the term structure into account. Nevertheless, the exponential spline model exhibits reasonably good forecasting performance, outperforming all theoretical models according to Bolder (2006).

The second class of empirical models is usually called the Nelson-Siegel-Svensson (NSS) family. The original model is credited to Nelson and Siegel (1987), who created a static model to fit a cross-section of the term structure. Their model contains four parameters. The first three are related to the level, slope and curvature of the yield curve, while the

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fourth parameter is a decay constant which determines how fast the factor loadings on the second and third parameters tend to 0. The model is reinforced by the analysis of Litterman and Scheinkman (1991), proving that a large percentage of the variation in the yield curve can indeed be explained by three factors through a principal component analysis. They also confirmed that the principal components really correlate to the level, slope and curvature of the yield curve.

Over time, several extensions have been made to the basic Nelson-Siegel model. For example, Björk and Christensen (1999) introduced a second slope component, Bliss (1996) used a separate decay factor for the slope and the curvature, while the model of Svensson (1994) has an additional curvature component with its own decay factor, resulting in a model with six coefficients. A detailed comparison of these models has been carried out by De Pooter (2007) who introduced an Adjusted Svensson model to correct the shortcomings of the Svensson model, reducing the collinearity between the two curvature components.

Out of all term structure models, the Nelson-Siegel-Svensson class has been so far most extensively analyzed in terms of out-of-sample forecasting performance. The original static model of Nelson and Siegel (1987) has been converted to a dynamic framework by Diebold and Li (2003), who interpret the first three coefficients as latent factors while fixing the decay parameter to linearize the model. They show that their forecasts can beat the random walk (no-change) results when forecasting more than a month ahead. Beating the random walk is an important benchmark in light of the fact that affine models are incapable of doing so (Duffee, 2002). More evidence about the forecasting performance of NSS models has been provided by Bolder (2006) who showed that the Nelson-Siegel model provides a more consistent forecast compared to the exponential spline model, although the two models did provide a similar performance. Further results by De Pooter (2007) show that the Björk-Christensen four-factor model outperforms the three-factor Nelson-Siegel model (along with other benchmark models like the random walk and the unrestricted vector

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autoregression) in out-of-sample and does not suffer from collinearity problems akin to the Svensson model. He suggests an AR(1) specification to forecast the factors. There are also efforts to bridge the gap between the Nelson-Siegel approach and the theoretical models. Noteworthy examples include the affine arbitrage-free Nelson-Siegel model by Christensen et al. (2011) and its bilateral extension by Mouabbi (2014).

A third class of empirical models include the usage of orthogonal Legendre-polynomials to estimate the term structure of interest rates. This model is used for example by Hubig (2013) and Almeida (2005) and was constructed as an alternative to the Nelson-Siegel-Svensson family of models. There is, however, no literature concerning the forecasting capability of the Legendre model. Exact mathematical characterization of some empirical models mentioned here are part of Chapter 4.

There are several notable papers concerning the Hungarian yield curve. Kopányi (2010) uses mainly theoretical (affine) models, Makara (1998) analyzes spline models, while Fegyveres (2014) and Reguly (2015) compare spline and different Nelson-Siegel-Svensson models. However, neither of these articles deal with out-of-sample forecast evaluation.

3.2 Term Structure Modeling in Practice: A Cross-Country Overview

The most popular theoretical and empirical yield curve models have been introduced and categorized. In this section, I switch from theory to practice and show which term structure models are actually used by sovereign debt managers. The most comprehensive overview yet was conducted by the Bank for International Settlements (2005). It is worth mentioning that some procedures may have changed since the conduction of this study.

Table 3.1 shows the yield curve estimation methodology from a number of developed countries. Clearly the most popular term structure estimation method is the Svensson

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Table 3.1: Methods of term structure estimation (based on Bank for International Settlements (2005) and ECB (2016))

Debt Manager	Method	Error Minimization	Maturity Spectrum
Belgium	Nelson-Siegel or Svensson	Weighted prices	0 to 16 years
Canada	Merril Lynch spline	Weighted prices	3 months to 30 years
European Central Bank	Svensson	Yields	0 to 15 years
Finland	Nelson-Siegel	Weighted prices	1 to 12 years
France	Nelson-Siegel or Svensson	Weighted prices	0 to 10 years
Germany	Svensson	Yields	1 to 10 years
Hungary	Cubic spline	Weighted prices	3 months to 15 years
Italy	Nelson-Siegel	Weighted prices	0 to 30 years
Japan	Smoothing spline	Prices	1 to 10 years
Norway	Svensson	Yields	0 to 10 years
Spain	Svensson	Weighted Prices	0 to 10 years
Sweden	Smoothing Spline and Svensson	Yields	0 to 10 years
Switzerland	Svensson	Yields	1 to 30 years
United Kingdom	VRP	Yields	0 to 30 years
United States	Smoothing spline	Weighted prices	0 to 10 years

model. The Nelson-Siegel is somewhat less popular. There are also some countries using spline models with very different methodologies ranging from the cubic spline to the VRP. It is noteworthy that no single country is using any of the theoretical models shown before, only the empirical models. There are two countries (Belgium and France) that use the Nelson-Siegel and the Svensson model conditionally. That is, the Svensson model can be reduced to the Nelson-Siegel by taking away the two extra parameters it has. This way, if the estimation proves those last two coefficients to be statistically insignificant, the Nelson-Siegel formula is used, otherwise if the model fitting improves, the full Svensson parametrization is estimated. Sweden is another special case: they use smoothing splines to estimate the forward curve and fit the Svensson model to the forward curve instead of the zero-coupon curve.

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There is a duality in error minimization practices. There is no clear preference in the usage of weighted price error and zero-coupon yield error minimization. From a computational point of view, yield error minimization is faster, as term structure models calculate the yields and not directly the price. Calculating the prices from the yields is another step that requires extra computation. However, as there are usually no long maturity zero-coupon bonds, yields have to be obtained either by stripping, which method entails the calculation of coupon yields and principal yields separately or using yield data from swaps. Weighted price error minimization can be mathematically equivalent to yield error minimization if weighting is done using the inverse squared modified duration (Berenguer et al., 2013). They give a mathematical proof that this weighting scheme can be used to account for price error heteroscedasticity given that the yield errors are homoscedastic and thus yield error minimization is an efficient estimator. The assumption that yield errors are homoscedastic is based on Vasicek and Fong (1982). Should that assumption prove to be false, both estimators are inefficient. According to the Bank for International Settlements (2005), most estimation methods using weighted price error minimization use the inverse of the duration or modified duration as weights. The cubic spline model used by ÁKK (based on Makara (1998)) makes use of an entirely different weighting scheme: the squared inverse of the bid-ask spread. This reflects the liquidity of a given bond to a certain extent, as more liquid bonds usually have a lower spread and are thus given a higher weight. The results of the two weighting schemes are somewhat similar as bond duration and the width of the spread are positively correlated. Hladíková and Radová (2012) give a comprehensive overview of different bond weighting schemes (including inverse spread and inverse duration) when using price error minimization and the Nelson-Siegel model. They rank the different methods based on error metrics, smoothness and stability. They rank the duration-weighted models among the best and the spread-weighted ones among the worst. Nevertheless, for spline models, weighting using the squared inverse bid-ask spread

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might still be a valid method.

In case of the maturity spectrum there are no fundamental differences. Nine out of sixteen countries have the instantaneous rate as a lower bound, the rest of the countries choose not to include observations with a maturity of under 3 months or 1 year. The latter seems quite extreme as the lower end of the term structure tends to be quite volatile and bonds with a maturity under 1 year can be an important part of the financing strategy. The upper bounds range from 10 to 30 years. It is worthwhile that the high end of the maturity spectrum is not always reached, as the longest maturity outstanding bond might have a significantly lower remaining maturity than original maturity.

3.3 Debt Management Models

This section gives a general overview of debt management strategies and international guidelines, gives an account of exemplary quantitative debt management models from Turkey, Italy and Canada and introduces the current Hungarian debt strategy. The latter is especially important as it contains a set of guidelines for the Hungarian debt portfolio which are used in Chapter 6 as constraints for portfolio optimization.

A large number of countries have a publicly available and well-documented debt management strategy. Debt strategies usually contain the projected financing needs of the government, the proposed composition of bonds to be issued next year as well as underlying guidelines and constraints. Restrictions can be put on the share of foreign currency denominated bonds, the duration of the debt portfolio, and the composition of zero-coupon bonds, fixed interest bonds, floating bonds and inflation-linked bonds. However, actual government debt management practices are diverse. Quite a few countries have their own quantitative debt management models based on sound forecasting practices and portfolio optimization. These mathematical models may or may not be available to the public.

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These governments base their actual debt strategy (at least partly) on the results of these calculations. Countries without a quantitative model base their debt strategy on foreign best practice, experience from previous years and other qualitative guidelines.

The International Monetary Fund and the World Bank have published a set of qualitative guidelines for public debt management. The key idea of the document regarding the objective of public debt management is the following: "The main objective of public debt management is to ensure that the government's financing needs and its payment obligations are met at the lowest possible cost over the medium to long run, consistent with a prudent degree of risk" (IMF & World Bank (2015)). The paper advises against too much exposure to risks associated with foreign currency, short-term maturity debt and floating rate debt, warning against the implications of a crisis, when the inverted yield curve would mean that these obligations become the most expensive.

The Turkish model for debt management by Balibek and Köksalan (2010) is based on the country's Medium Term Fiscal Plan and provides a three-year strategy. They use a simplistic vector autoregression (VAR) forecasting approach utilizing macroeconomic variables as well as short and medium maturity interest rates without using any term structure models. The forecasts are used to generate a large number of possible scenarios. A three-dimensional efficient frontier is then calculated based on costs, risk (measured by conditional cost-at-risk) and liquidity risk.

The Italian debt management model is designed by Consiglio and Staino (2012). They use a stochastic programming approach to generate a set of scenarios for the starting year. An iterative procedure is then used to generate scenario trees for 2 and 3 year forecasts as well. Goodness-of-fit for the forecasting is not measured. The actual forecasting procedure is a multivariate mean-reverting model. Nine different bonds are used in the portfolio optimization exercise with maturities ranging from 3 months to 30 years. The two-dimensional efficient frontier is constructed by minimizing cost with respect to conditional

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cost-at-risk. At the lower bounds of the conditional cost-at-risk, medium maturity bonds are used. When allowing for a riskier portfolio, they are replaced gradually by short and long bonds. However, they assume that all bonds are issued at the beginning of the simulation and if short maturity bonds are issued they simply expire without the need for re-issuing them. This way, unsurprisingly, the 3-year bond is deemed the lowest risk alternative for a 3-year strategy.

By far the most complex of the quantitative debt management models is the Canadian one by Bolder and Deeley (2011). They use several different term structure models (including affine, exponential spline and Nelson-Siegel) for forecasting and combine them with macroeconomic forecast. A consistent methodology is used throughout the paper to determine optimal portfolio weights. Using different models, the cost is minimized subject to certain risk measures. Multiple risk measures are used, including variance, cost-at-risk, conditional cost-at-risk, rollover risk and conditional volatility. The simulation spans ten years. The models can account for several possible constraints like issuance structure or average time to maturity. Optimal portfolio weights are calculated for several sets of proposed scenarios ranging from average to extreme budgetary and debt-service charge risks as well as normal, flatter and steeper yield curves.

The current Hungarian debt strategy (ÁKK Zrt, 2015) is partly based on a quantitative model. The mathematical model is not available to the public. The guidelines set three strategic goals. First, reducing the 73.3% debt-to-GDP ratio every year until it reaches 50% according to constitutional requirements. Second, restructuring financing by increasing domestic financing. The primary tool to achieve this purpose is focusing on retail bond purchases, offering favorable retail sector-exclusive investment opportunities. Third, reducing the share of foreign currency denominated debt. The latter is now confined to the range of 25-35%. The large majority of this debt is denominated in Euro and USD, and exposure to non-Euro currencies is reduced to zero by hedging with swaps. The share of

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fixed interest bonds in the portfolio, hereinafter the fix-to-float ratio is constrained to the range of 61-83% for domestic bonds and to 61-71% for foreign currency denominated debt. The share of bonds with a maturity under one year and floating bonds has to be in the 17-39% range for the domestic bond portfolio and in the 29-39% range for foreign currency denominated debt. These constraints are designed to protect against sudden fluctuations in the short end of the yield curve. The duration of the portfolio is also restricted to 2.5 to 3.5 years. With these constraints in mind, the planned 2016 issuance structure includes 91% domestic bonds with 9% foreign, and total net financing of HUF -762 billion to reduce the debt-to-GDP ratio and the share of foreign bonds in particular.

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Chapter 4

An Empirical Model for the Hungarian Yield Curve

In this chapter several term structure models introduced in Chapter 3 are built, fitted to estimate the Hungarian yield curve and compared based on multiple goodness-of-fit statistics. The analysis includes Nelson-Siegel, Svensson and Legendre-type term-structure models. First, the models are given an exact mathematical characterization. Second, problems in the exact model calibration are resolved, such as in which manner prices should be computed or which numerical optimization method should be used. Third, the results are compared and analyzed.

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4.1 Model Framework

In this section, several term structure models from the Nelson-Siegel-Svensson family as well as the Legendre model are given an exact mathematical characterization. The basic model by Nelson and Siegel (1987) is a static three factor model. They assume that the forward curve can be approximated using the sum of a constant and a Laguerre function:

$$F_t(\tau) = \beta_{t,0} + \beta_{t,1} e^{-\frac{\tau}{\lambda_t}} + \beta_{t,2} \left(\frac{\tau}{\lambda_t}\right) e^{-\frac{\tau}{\lambda_t}} \tag{4.1}$$

where:

τ is the maturity,

λ is an exponential decay constant,

β_0 is the long-term factor,

β_1 is the short-term factor,

and β_2 is the medium-term factor.

The derived spot rate curve is:

$$y_t(\tau) = \beta_{t,0} + \beta_{t,1} \left(\frac{1 - e^{-\frac{\tau}{\lambda_t}}}{\frac{\tau}{\lambda_t}}\right) + \beta_{t,2} \left(\frac{1 - e^{-\frac{\tau}{\lambda_t}}}{\frac{\tau}{\lambda_t}} - e^{-\frac{\tau}{\lambda_t}}\right) \tag{4.2}$$

It is worth mentioning that the parametrization by Diebold and Li (2003) is somewhat different, using $\lambda\tau$ instead of τ/λ. It is easy to compute both the instantaneous short rate and the long rate using the Nelson-Siegel model:

$$\lim_{\tau \downarrow 0} y_t(\tau) = \beta_{t,0} + \beta_{t,1} \; ; \qquad \lim_{\tau \to \infty} y_t(\tau) = \beta_{t,0} \tag{4.3}$$

The economic interpretation for the three factors is straightforward. The loading on β_0 is 1 for every maturity, thus it determines the level of the yield curve and is related to long-term interest rates. The loading for second factor, β_1 starts at 1 and decays exponentially to 0, therefore it is related to the slope of the curve. The speed of the decay is determined by λ. Increasing β_1 increases the short interest rates but has no effect on the long rates. Lastly, the loading of β_2 starts out at 0, reaches its maximum of approximately 0.2984 at a maturity determined by λ and decays to zero. This factor is related to the curvature of the

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yield curve, has no effect on short and long rates, only medium-term interest. A common estimation procedure in literature (Diebold and Li, 2003) is fixing λ so that the loading on the third factor reaches its maximum at the 30-month maturity. This thesis measures the maturity τ in years and the corresponding λ value (for 2.5 years) is approximately 1.39. Figure 4.1 shows the factor loadings for the Nelson-Siegel model with λ set to 1.39.

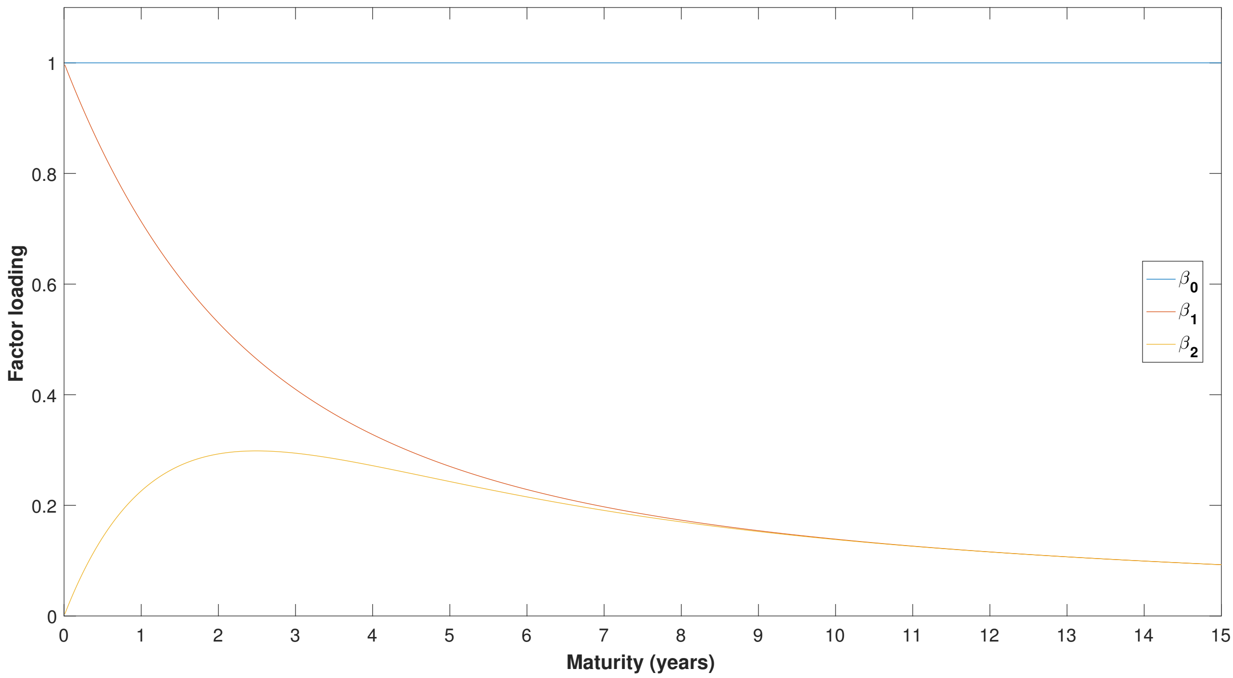

Figure 4.1: Factor loadings for the Nelson-Siegel model

A possible extension for the Nelson-Siegel model is one by Björk and Christensen (1999). Adding an extra slope component the equation becomes:

$$y_t(\tau) = \beta_{t,0} + \beta_{t,1}\left(\frac{1-e^{-\frac{\tau}{\lambda_t}}}{\frac{\tau}{\lambda_t}}\right) + \beta_{t,2}\left(\frac{1-e^{-\frac{\tau}{\lambda_t}}}{\frac{\tau}{\lambda_t}} - e^{-\frac{\tau}{\lambda_t}}\right) + \beta_{t,3}\left(\frac{1-e^{-\frac{2\tau}{\lambda_t}}}{\frac{2\tau}{\lambda_t}}\right) \quad (4.4)$$

The extra slope component which decays at a faster rate adds some flexibility to the short end of the yield curve. A crucial difference between this four-factor model and the original Nelson-Siegel model is that the short rate can now be calculated as $\beta_{t,0} + \beta_{t,1} + \beta_{t,3}$.

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The Adjusted Svensson model (De Pooter (2007) based on Svensson (1994)) adds an extra curvature and decay component and can be parametrized as:

$$y_t(\tau) = \beta_{t,0} + \beta_{t,1}\left(\frac{1-e^{-\frac{\tau}{\lambda_{1,t}}}}{\frac{\tau}{\lambda_{1,t}}}\right) + \beta_{t,2}\left(\frac{1-e^{-\frac{\tau}{\lambda_{1,t}}}}{\frac{\tau}{\lambda_{1,t}}} - e^{-\frac{\tau}{\lambda_{1,t}}}\right) + \beta_{t,3}\left(\frac{1-e^{-\frac{\tau}{\lambda_{2,t}}}}{\frac{\tau}{\lambda_{2,t}}} - e^{-\frac{2\tau}{\lambda_{2,t}}}\right) \tag{4.5}$$

The maximum loading on the second curvature factor is approximately 0.4968. It increases and decays faster compared to β_2. Also, $\arg\max_{\beta_3(\tau)} \approx \lambda_2$. The long and short rates can be calculated exactly as in the Nelson-Siegel model. Figure 4.2 shows the factor loadings for both extended NSS models.

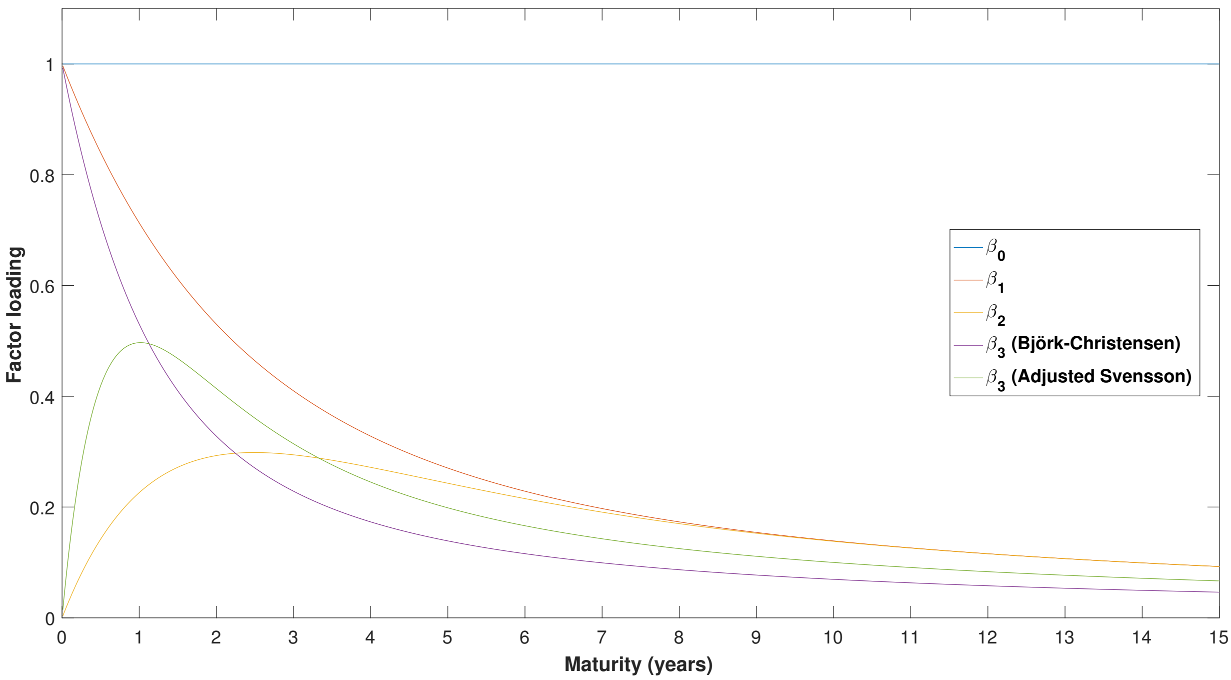

Figure 4.2: Factor loadings for the extended NSS models

Finally, the Legendre model using n factors and no decay components can be written based on Almeida (2005) as:

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Let

$$x(\tau_t) = \frac{2\tau_t}{\max(\tau_t)} - 1 \tag{4.6}$$

We can realize that the value of x is -1 for a bond maturing at this time instant and 1 for the longest maturity bond.

Also let us denote the k^{th} Legendre-polynomial at x by $P_k(x)$. Then the yield can be calculated using the Legendre-model as:

$$y_t(x_t) = \sum_{k=0}^{n-1} \beta_{t,k} P_k(x_t) \tag{4.7}$$

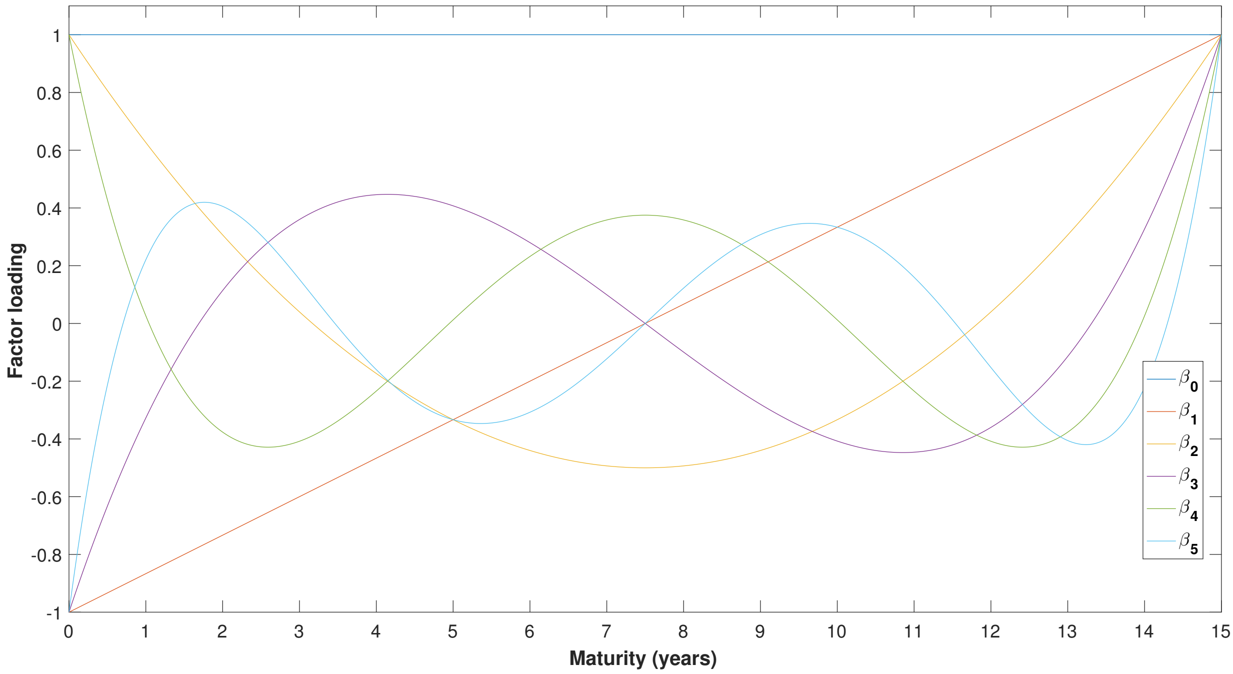

Figure 4.3: Factor loadings for the Legendre model

Figure 4.3 shows the factor loadings for a six-factor Legendre model with a maximum maturity of 15 years. Calculating the long and short yields is somewhat more complex compared to the NSS models. The long rate is the sum of all factors while the short rate

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is an alternating sum of the factors:

$$y_t(0) = \sum_{k=0}^{n-1} (-1)^k \beta_{t,k} \ ; \qquad y_t(\max \tau_t) = \sum_{k=0}^{n-1} \beta_{t,k} \tag{4.8}$$

It is important to notice that the Legendre model is defined at maturity 0 and the long rate is defined at the maximum maturity in the fitting sample instead of infinity.

4.2 Data and Model Calibration

Even though the Nelson-Siegel-Svensson models are frequently used in term structure modeling, there is a lack of consensus in the literature about the calibration of the model. This is mainly caused by the fact that the optimization problem at the core of the estimation has several numerical difficulties due to the non-convexity of the problem and the abundance of local optima (Gilli et al., 2010). They suggest using differential evolution, a stochastic optimization algorithm for estimating the parameters.

The estimation of the Hungarian yield curve is carried out using the ÁKK dataset of daily price quotes as well as cash-flow, YTM and duration data for Hungarian bonds[1]. The time window of the estimation ranges from January 2002 to December 2015. I also use ÁKK's yield data created using the Fama and Bliss (1987) smoothed bootstrap. Bootstrapping draws a random sample with replacement from an observed discrete distribution. A smoothed bootstrap adds a small random noise to each sampled element in order to better represent the theoretical continuous underlying distribution.

The first obstacle to overcome in calibrating the models is that Hungarian government bonds, as mentioned before, use different pricing schemes: linear for zero-coupon T-bills

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[1]Hungarian bonds in the estimation include zero-coupon Diszkontkincstárjegy (DKJ) T-bills and Magyar Államkötvény (MÁK) interest bearing bonds, all denominated in HUF.

and effective for interest bearing bonds. For the estimation, I recalculate every bond price using continuous compounding to get a consistent yield curve. The next issue is determining the objective function. The common practices include the minimization of yield errors or weighted price errors. I use both methods. For minimizing weighted price errors, the methodology of Berenguer et al. (2013) is used: the inverse of the squared modified duration for weights. The objective function itself is the following:

$$\min \sum \left((P^{est} - P^{obs})^2 \cdot \frac{1}{ModD^2} \right) \tag{4.9}$$

where:

P^{est} denotes the estimated price

and P^{obs} denotes the observed price.

The estimated price is calculated as:

$$P^{est} = \sum_{i=1}^{n} CF_i e^{-y(t)\tau(T_i,t)} \tag{4.10}$$

And y is calculated using either of the term structure models.

The objective function for the zero-coupon yield error minimization problem is simply:

$$\min \sum (y^{est} - y^{obs})^2 \tag{4.11}$$

where observed yields come from ÁKK's Fama-Bliss dataset. The greatest advantage of using yield error minimization is revealed in case of Legendre or fixed λ Nelson-Siegel-Svensson models. In these cases, the optimization problem becomes entirely linear and can be solved with a simple Ordinary Least Squared (OLS) regression. In any other case, a numerical optimization algorithm must be used.

Contrary to the observations made by Gilli et al. (2010), I found that using stochastic global optimization algorithms yield almost no improvement over the results of determin-

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istic local solvers. Stochastic solvers also have the drawback of relying on random numbers and being orders of magnitude slower[2]. The reduced speed is due to the fact that global solvers usually search in a much larger parameter space compared to local solvers. On the other hand, local optimization algorithms search in a reduced space close to a given starting value. However, this also means that local solver algorithms are dependent on being provided with a good starting value, else it is prone to converging to a local optimum far from the global optimum.

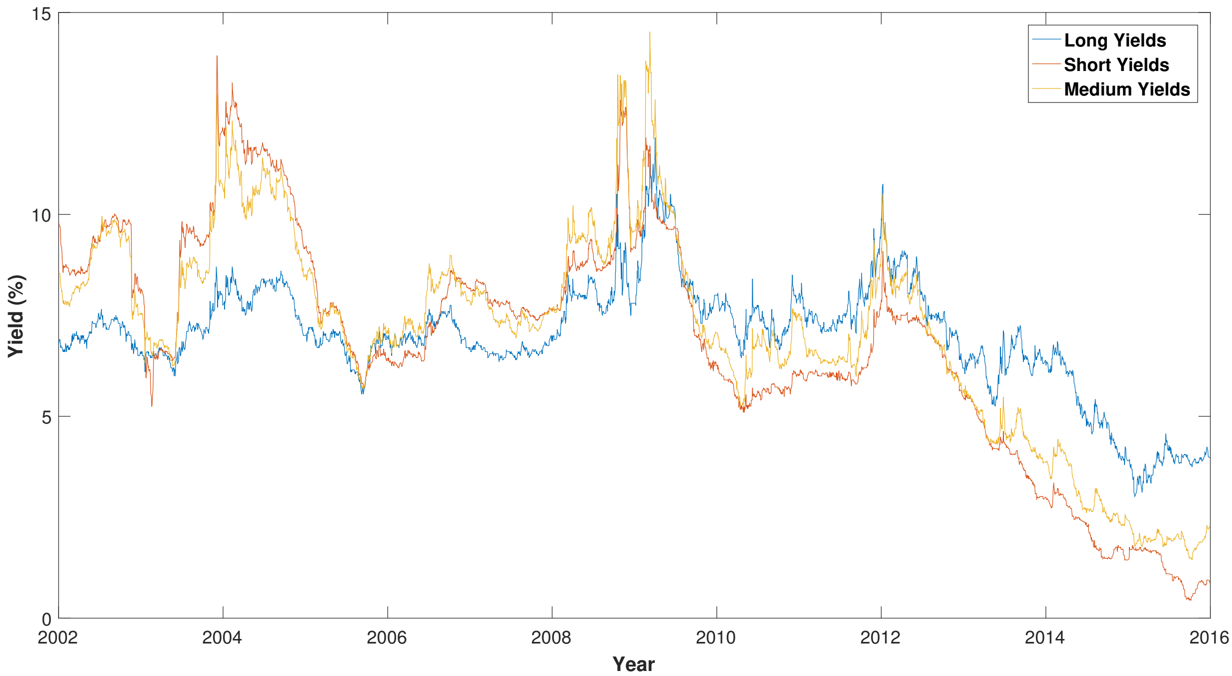

Figure 4.4: Hungarian yields 2002-2015

Let us denote the average yield of observed bonds over 10 years of maturity by y_{long}, the average yield of bonds with a maturity under 6 months with y_{short} and the average yield of bonds between 1.5 and 3.5 years of maturity with y_{med}. The temporal evolution

[2]Based on estimation results with the deterministic interior-point ($fmincon$) versus the stochastic genetic algorithm (ga), simulated annealing ($simulannealbnd$) and particle swarm optimization ($particleswarm$) solvers of MATLAB.

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of these yields is shown in Figure 4.4. I set the starting values of the factors to:

$$\beta_0 = y_{long}; \quad \beta_1 = y_{short} - y_{long}; \quad \beta_2 = \frac{2y_{med} - y_{long} - y_{short}}{0.2984}; \quad \beta_3 = 0 \tag{4.12}$$

This builds on the assumption that the models can indeed be interpreted as factor models and the theoretical equations for calculating the long and short rate hold. This is the reason for using these "residual" yields in the starting values of β_1 and β_2 The third factor's starting value contains a division by 0.2984, its maximal factor loading for the purpose of rescaling. The fourth factor (used by the extended models) is initially set to 0 as a null hypothesis that using three factors is enough. If the economic assumptions for the factors hold, we can expect the estimated parameters to be highly correlated with the residual yields shown in Figure 4.5.

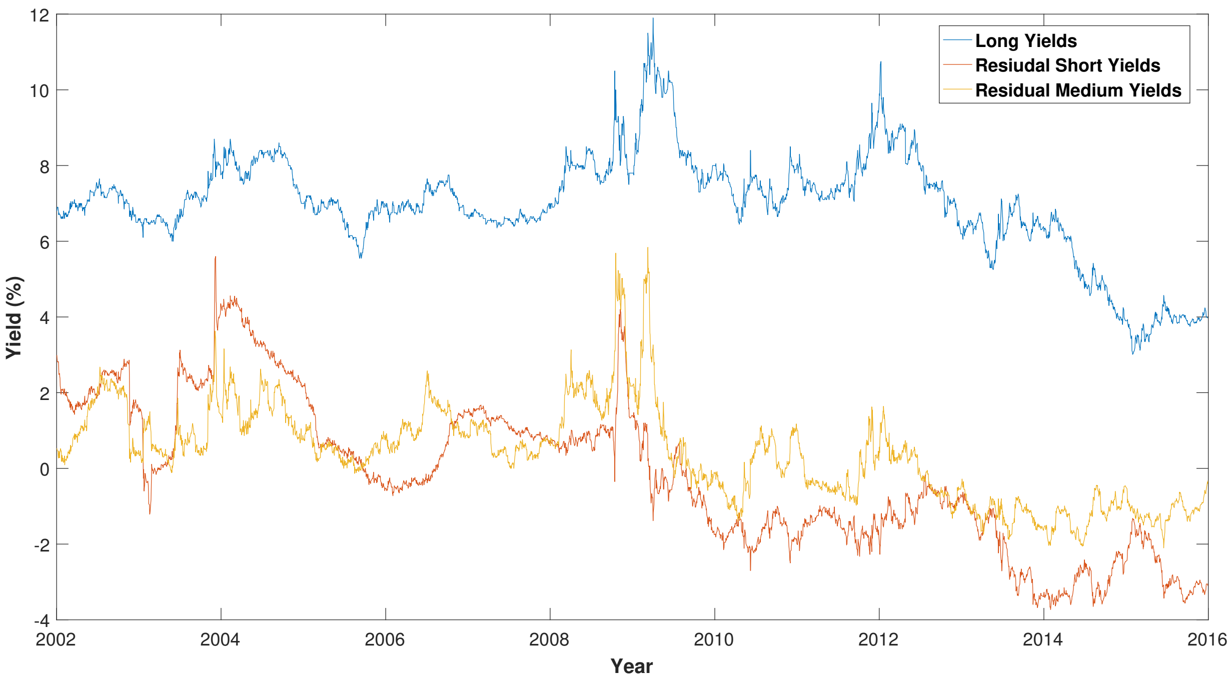

Figure 4.5: Residual yields 2002-2015

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4.3 Results and Evaluation

In this section, models are evaluated based on goodness-of-fit and time series criteria. The first measure of fit is mean absolute error (MAE). We can calculate the MAE for a single day with n observed bond prices as follows:

$$\text{MAE} = \frac{\sum_{i=1}^{n} |P_i^{est} - P_i^{obs}|}{n} \tag{4.13}$$

The next measure used for evaluation is root mean squared error (RMSE).

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (P_i^{est} - P_i^{obs})^2}{n}} \tag{4.14}$$

These two widely used measures can be calculated in a similar manner for YTMs as well. The third fitting criterion is the hit ratio which can be calculated as:

$$\text{Hit ratio} = \frac{\sum_{i=1}^{n} h_i}{n}, \quad \text{where } h_i = \begin{cases} 1 & \text{if } P_i^{est} \in [P_i^{bid}; P_i^{ask}] \\ 0 & \text{if } P_i^{est} \notin [P_i^{bid}; P_i^{ask}] \end{cases} \tag{4.15}$$

The hit ratio for prices and YTMs is exactly the same. Lower MAE and RMSE but higher hit ratio values characterize a better fitting model.

Table 4.1 shows the goodness-of-fit statistics (MAE and RMSE for prices and YTMs as well as the hit ratio, averaged out over time) for three different models with two versions each. All these models are fitted on weighted prices and use variable λ values. Some constraints are added to reduce the feasible region and make the task computationally easier. The box constraints for the first factor is $[0; 30]$ and $[-30; 30]$ for the other β factors. A non-negativity constraint for the adequate short rate for each model is also imposed. The λ decay factors are constrained so the curvature factors reach their maximum maturity between 1 and 15 years. For the Adjusted Svensson models a $\lambda_2 \geq \lambda_1$ relation is

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also introduced.

Table 4.1: Goodness-of-fit statistics for variable λ models

Model	MAE_p	MAE_y	$RMSE_p$	$RMSE_y$	Hit Ratio
Restricted Nelson-Siegel	0.1991	0.0782	0.3171	0.0985	0.8832
Unrestricted Nelson-Siegel	0.1575	0.0669	0.2352	0.0859	0.9044
Restricted Björk-Christensen	0.2084	0.0819	0.3292	0.1040	0.8596
Unrestricted Björk-Christensen	0.1494	0.0611	0.2220	0.0778	0.9143
Restricted Adjusted Svensson	0.1407	0.0630	0.2059	0.0814	0.9142
Unrestricted Adjusted Svensson	0.1279	0.0592	0.1854	0.0770	0.9259

Additionally, the restricted models also use moving box constraints for the first three factors in order to facilitate correlation between the factors and the residual yields of Figure 4.5. The first and second factors are restricted to their respective starting values +/- 1, while the third factor has a +/- 3 allowed range compared to its own starting value. The stricter of the static and moving box constraints are applied for each instance of optimization.

As a general rule, unrestricted models perform better than restricted ones and the Adjusted Svensson model has a better fit than the Nelson-Siegel or the Björk-Christensen as observed from Table 4.1. Unsurprisingly, more factors or less restrictions offer a better in-sample fit. There is a higher discrepancy in price errors compared to YTM errors as observed prices are usually close to 100, whereas observed YTMs are usually below 10. All models offer a decent in-sample fit overall.

Table 4.2 shows in-sample fit statistics for four more models. The Nelson-Siegel and Björk-Christensen ones use a fixed λ of 1.39. All models are estimated using an OLS regression for yield error minimization (bootstrapped yields) without any constraints for the β factors.

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Table 4.2: Goodness-of-fit statistics for OLS estimated models

Model	MAE_p	MAE_y	$RMSE_p$	$RMSE_y$	Hit Ratio
Nelson-Siegel Fixed λ	0.1922	0.0808	0.2895	0.1034	0.8601
Björk-Christensen Fixed λ	0.1589	0.0653	0.2390	0.0827	0.9063
4-Factor Legendre	0.1832	0.0874	0.2584	0.1111	0.8347
6-Factor Legendre	0.1314	0.0632	0.1855	0.0827	0.9116

Constant decay factors do not hurt the goodness-of-fit statistics significantly. The Björk-Christensen model fits better than its restricted, variable λ counterpart. The four factor Legendre model has a worse fit compared to the similar Björk-Christensen, but the six factor model's statistics are comparable to the Adjusted Svensson models of Table 4.1. Based on fitting criteria alone no clear recommendation can be made for forecasting.

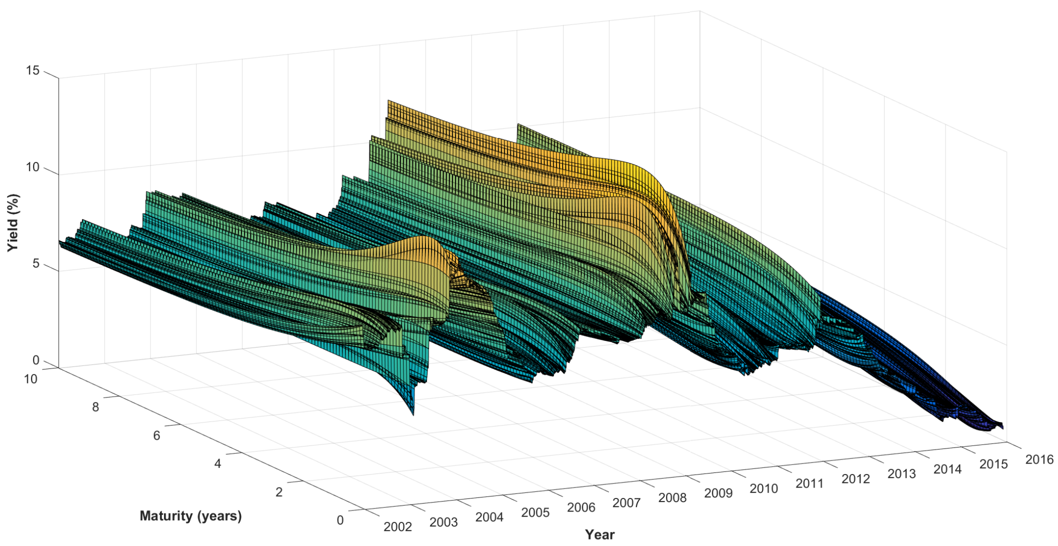

Figure 4.6: Hungarian term structure of interest rates 2002-2015

The entire Hungarian term structure of yields for the entire time interval and maturities up to 10 years can be seen in Figure 4.6. It is calculated using the restricted Nelson-Siegel model. We can see many different yield curve shapes including flat, upward sloping, humped and inverted. The short end of the term structure is particularly volatile. We can

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also notice that by now, the short yields are close to 0.

Let us now look at the temporal correlation between the factors and the observed residual yields. Table 4.3 shows the correlation statistics. For the Legendre models, no factor correlates to medium term yields in a way comparable to NSS models and were marked with N/A. We can observe that short factors of all models except the unrestricted Adjusted Svensson have an above 90% correlation to the short residual yields, with NSS models having a higher correlation compared to Legendre models. However, only restricted or fixed λ NSS models have an acceptable correlation to the long yields, but still fall short of Legendre models. Medium term yields are similar in a sense that unrestricted models have very poor correlation.

Table 4.3: Correlation of factors and yields

Model	Long	Short	Medium
Restricted Nelson-Siegel	0.8866	0.9856	0.9497
Unrestricted Nelson-Siegel	0.5733	0.9592	0.8686
Restricted Björk-Christensen	0.8750	0.9747	0.9184
Unrestricted Björk-Christensen	0.5589	0.9102	0.3777
Restricted Adjusted Svensson	0.8760	0.9413	0.9018
Unrestricted Adjusted Svensson	0.0529	0.5950	0.3804
Nelson-Siegel Fixed λ	0.8310	0.9958	0.9887
Björk-Christensen Fixed λ	0.8103	0.9880	0.7422
4-Factor Legendre	0.9864	0.9017	N/A
6-Factor Legendre	0.9859	0.9143	N/A

The conclusion of this chapter is that a number of different models provide an adequate fitting of the Hungarian term structure of yields. More factors and less constraints provide a better fit at the expense of being less interpretable as factor models. This points to a possible over-fitting issue: perhaps more factors were used in certain cases than necessary. Over-fitting in theory hinders forecasting performance. This question can be best resolved by out-of-sample forecasting.

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Chapter 5

Forecasting the Term Structure of Interest Rates

This chapter deals with term structure forecasting. First, the methodology for different forecasting models, including random walk, autoregressive, vector autoregressive and state-space models is described. Second, term structure models from Chapter 4 are forecast using a number of methods. The forecasts are evaluated and compared based on an error measure. Third, the best model is selected to generate a number of random scenarios to be used for portfolio optimization in Chapter 6.

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5.1 Methods for Forecasting

The methodology in this section focuses on yields-only forecasts for the Hungarian, HUF-denominated term structure of interest rates. Including foreign currency denominated Hungarian bonds would require a more complex macro-financial model with multiple yield curves (for the Euro area, for example) and macro variables. That is beyond the scope of this thesis. For the purpose of forecasting, I switch from a daily to a monthly frequency using end-of-month yields and factor estimates from Chapter 4. Therefore, 168 in-sample periods are used. The objective is to forecast up to 60 months ahead so that a five-year debt management strategy can be established. To better judge prediction accuracy, shorter forecasts are used as well[1]. I use expanding time window pseudo out-of-sample forecasts starting from January 2004, the 24^{th} period. This means that first, forecasts up to five years are made using two years of observations. In the next step, 25 months of observations are used. The window expands until the forecast can no longer be evaluated using observations within the sample. For example, 1-month-ahead forecasts are computed until November 2015 while 5-year-ahead predictions are only made until December 2010 for the purpose of forecast evaluation. The used models can be sorted into three categories: direct forecast of yields, autoregressive forecasts using factors and state-space models. The methodology introduced here is largely based on Diebold and Li (2003) and De Pooter (2007).

The benchmark model for forecasting the yield curve is the random walk. In this model, at time t, a forecast for a yield of maturity τ, h months ahead is the following:

$$y^{est}_{t+h|t}(\tau) = y^{obs}_t(\tau) \tag{5.1}$$

This means that our best estimate for the future is the current yield, if we believe that yields follow a random walk process. Simple as it may seem, it is a surprisingly strong benchmark if we recall that most theoretical models cannot beat the random walk (Duffee, 2002).

The next competing model is a first-order autoregressive process on the yields, where:

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[1]Predictions for 1, 3, 6, 12, 24, 30, 48 and 60 months ahead are computed and evaluated.

$$y^{est}_{t+h|t}(\tau) = c(\tau) + \gamma y^{obs}_t(\tau) \tag{5.2}$$

We can carry out the autoregressive forecast for factors as well in a similar manner:

$$\beta^{est}_{i,t+h|t}(\tau) = c_i(\tau) + \gamma_i \beta^{obs}_{i,t}(\tau) \tag{5.3}$$

Each factor is forecast separately. The λ value(s) for NSS models are fixed to be equal to the in-sample median. This way all 10 models estimated before can be given an AR (diagonal VAR) forecast. The yields are then computed using the estimated future factors.

The final autoregressive method for forecasting factors is an unrestricted vector autoregression:

$$\boldsymbol{\beta}^{est}_{t+h|t}(\tau) = \mathbf{c}(\tau) + \boldsymbol{\Gamma}\boldsymbol{\beta}^{obs}_t(\tau) \tag{5.4}$$

The difference here is that past values of every factor are used for a simultaneous estimation of all future factors, allowing for cross-factor dynamics. However, as noted by Diebold and Li (2003), VAR models often produce poor forecasts, especially in cases where factors are supposedly independent. However, the VAR estimates are also required to initialize the state-space models, which is the third and final forecasting method to be introduced.

The fundamental assumption behind state-space models is that the factors driving the evolution of endogenous variables are not observable. In our case, this means that the observed data (yields) are driven by unobserved stochastic latent factors ($\beta_{1..n}$). State-space models consist of two equations. The measurement (or observation) equation describes the way of how yields are calculated from the latent factors. In general, the measurement equation of a state-space model can be described as:

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$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \mathbf{D}\mathbf{e}_t \tag{5.5}$$

where:

$\mathbf{y}_t$ is the observation vector (yields),

$\mathbf{C}$ is the measurement sensitivity matrix,

$\mathbf{x}_t$ is the state vector (latent factors),

$\mathbf{D}$ is the observation innovation matrix,

and $\mathbf{e}_t$ is a Gaussian white noise process.

The corresponding transition (or state) equation that describes the temporal evolution of the latent factors is:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\mathbf{u}_t \tag{5.6}$$

where:

$\mathbf{A}$ is the state transition matrix,

$\mathbf{B}$, state disturbance loading matrix,

and $\mathbf{u}_t$ is a Gaussian white noise process.

The term structure models described in Chapter 4 can all be given a state-space representation. I use a generalized version of the approach used by Diebold, Rudebusch, and Aruoba (2006) for the Diebold and Li (2003) model. The measurement equation for a term structure model with $n+1$ factors and k maturities can be written as:

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$$
\begin{pmatrix} Y_t(\tau_1) \\ Y_t(\tau_2) \\ \vdots \\ Y_t(\tau_k) \end{pmatrix} = \begin{pmatrix} 1 & L_1(\tau_1,\lambda) & L_2(\tau_1,\lambda) & \cdots & L_n(\tau_1,\lambda) \\ 1 & L_1(\tau_2,\lambda) & L_2(\tau_2,\lambda) & \cdots & L_n(\tau_2,\lambda) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & L_1(\tau_k,\lambda) & L_2(\tau_k,\lambda) & \cdots & L_n(\tau_k,\lambda) \end{pmatrix} \begin{pmatrix} \beta_{t,0} \\ \beta_{t,1} \\ \beta_{t,2} \\ \vdots \\ \beta_{t,n} \end{pmatrix} + \begin{pmatrix} \epsilon_t(\tau_1) \\ \epsilon_t(\tau_2) \\ \vdots \\ \epsilon_t(\tau_k) \end{pmatrix} \tag{5.7}
$$

The loadings for the corresponding $\beta 1..n$ factors are denoted by $L_{1..n}$. They are a function of the appropriate maturity ($\tau_{1..k}$) and the λ factors(s) where applicable. The loading of β_0 is 1 in every used term structure model.

We can write corresponding observation equation as:

$$
\begin{pmatrix} \beta_{t,0}-\mu_0 \\ \beta_{t,1}-\mu_1 \\ \beta_{t,2}-\mu_2 \\ \vdots \\ \beta_{t,n}-\mu_n \end{pmatrix} = \begin{pmatrix} A_{0,0} & A_{0,1} & A_{0,2} & \cdots & A_{0,n} \\ A_{1,0} & A_{1,1} & A_{1,2} & \cdots & A_{1,n} \\ A_{2,0} & A_{2,1} & A_{2,2} & \cdots & A_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{n,0} & A_{n,1} & A_{n,2} & \cdots & A_{n,n} \end{pmatrix} \begin{pmatrix} \beta_{t,0}-\mu_0 \\ \beta_{t,1}-\mu_1 \\ \beta_{t,2}-\mu_2 \\ \vdots \\ \beta_{t,n}-\mu_n \end{pmatrix} + \begin{pmatrix} \nu_t(\beta_0) \\ \nu_t(\beta_1) \\ \nu_t(\beta_2) \\ \vdots \\ \nu_t(\beta_n) \end{pmatrix} \tag{5.8}
$$

Mean-adjusted factors are used in the observation equation. The factor means are denoted by $\mu_{0..n}$. Using a vector notation, the two equations can be written as:

$$
\begin{aligned} \mathbf{Y}_t &= \mathbf{L}\boldsymbol{\beta}_t + \boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t \sim N(\mathbf{0},\mathbf{H}) \\ \boldsymbol{\beta}_t - \boldsymbol{\mu} &= \mathbf{A}(\boldsymbol{\beta}_t - \boldsymbol{\mu}) + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim N(\mathbf{0},\mathbf{Q}) \end{aligned} \tag{5.9}
$$

This is somewhat different compared to the general state-space formulation. Let us denote the mean-adjusted factors by $\mathbf{x}_t$ and the deflated yields by $\mathbf{y}_t$. They can be calculated

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as:

$$\mathbf{x}_t = \boldsymbol{\beta}_t - \boldsymbol{\mu} \qquad\qquad \mathbf{y}_t = \mathbf{Y}_t - \mathbf{L}\boldsymbol{\mu} \tag{5.10}$$

Substituting $\mathbf{x}_t$ and $\mathbf{y}_t$ into equation system 5.9, we get:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{L}\mathbf{x}_t + \boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \mathbf{H}) \\ \mathbf{x}_t &= \mathbf{A}\mathbf{x}_{t-1} + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim N(\mathbf{0}, \mathbf{Q}) \end{aligned} \tag{5.11}$$

The two Gaussian white noise processes $\boldsymbol{\epsilon}_t$ and $\boldsymbol{\mu}_t$ are assumed to be orthogonal. Matrices $\mathbf{A}$ and $\mathbf{L}$ in equation system 5.11 are analogous to matrices $\mathbf{A}$ and $\mathbf{C}$ of the general state space model described in equations 5.5 and 5.6. The respective noise processes have the following relationship:

$$\begin{aligned} \boldsymbol{\epsilon}_t &= \mathbf{B}\mathbf{u}_t \\ \boldsymbol{\nu}_t &= \mathbf{D}\mathbf{e}_t \end{aligned} \tag{5.12}$$

To reduce the number of parameters to be estimated and to ensure that $\mathbf{Q}$ is a proper covariance matrix and $\mathbf{H}$ is diagonal (so that changes in the different maturity deflated yields are uncorrelated) the following formulation is used:

$$\begin{aligned} \mathbf{H} &= \mathbf{D}\mathbf{D}' \\ \mathbf{Q} &= \mathbf{B}\mathbf{B}' \end{aligned} \tag{5.13}$$

All parameters of the state-space model are estimated simultaneously using the Kalman filter and by maximizing the likelihood function. It is worth mentioning that the state-

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space approach with the Kalman filter can also be used to estimate the in-sample factors, similar to Chapter 4, using starting values based on the observed yields instead of the static model estimation results. Even though this procedure does not require a separate set of in-sample results, it might lead to an inaccurate optimization as the starting values would be less precise.

The likelihood estimation of state-space models in general is a very challenging task in a mathematical sense and the optimization algorithms used are highly sensitive to starting values. Therefore, the following initial values are given: in-sample β factor means for $\boldsymbol{\mu}$, λ medians for themselves (for NSS models), the square root of the diagonal of the VAR(1) estimated covariance matrix of residuals for $\mathbf{B}$, the VAR(1) transition parameters for $\mathbf{A}$ and the lower triangle Cholesky-decomposition of the VAR(1) innovations covariance matrix for $\mathbf{D}$.

5.2 Evaluation and Comparison of Forecast Models

Forecasts are evaluated separately for model, maturity and horizon based on root mean squared error. This is computed in a similar manner as in the in-sample case:

$$\text{RMSE}_{\tau_i} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y^{est}_{\tau_i,t+h|t} - y^{obs}_{\tau_i,t})^2} \tag{5.14}$$

I also introduce the Trace RMSE based on Christoffersen and Diebold (1998) as a measure of averaging the errors between N maturities:

$$\text{TRMSE} = \sqrt{\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}(y^{est}_{\tau_i,t+h|t} - y^{obs}_{\tau_i,t})^2} \tag{5.15}$$

It is important to clarify that all forecasts are done using *ex post* forecasting procedures. The forecasts to determine the "future" values of the model are made using only past

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measurements but those "future" values are already available for evaluation. On the other hand, the ultimate goal of this chapter is to produce an accurate *ex ante* forecast up until 2020. Therefore, it is important to look at the stability of forecast errors throughout different maturities and forecast horizons. I also presume that unless there is very strong supporting evidence to use more complex models, the simplest one should be utilized as good *ex post* forecasting performance does not automatically imply that *ex ante* predictions will be accurate as well.

Tables A.1, A.2 and A.3, found in Appendix A, indicate that on short (sub 1-year) horizons almost no models manage to outperform the random walk. The direct AR(1) forecast of yields can only outperform the random walk on the longest horizon of 5 years based on Trace RMSE.

Among the AR(1) factor forecasts, the restricted Nelson-Siegel model has the best overall performance. This forecast beats the random walk on horizons greater than 1 year. It is interesting to observe that the generally less volatile longer maturity yields can be forecast better using term structure models (compared to the random walk), while the short yields tend to behave unpredictably based on the forecast errors. The AR(1) models, however tend to perform increasingly worse on forecast horizons below 1 year. Among the different model specifications, restricted AR(1) models tend to perform better compared to unrestricted or fixed λ ones contrary to their in-sample performance. Adding more factors does not improve forecasting performance.

The VAR(1) models provide different conclusions. First of all, the fixed λ and the six-factor Legendre models produced a few extreme outliers, estimated yields over 100% that increased their error statistics. This is a drawback of using unrestricted VAR models. On the other hand, models that did produce feasible results, in particular the unrestricted Nelson-Siegel and the restricted Adjusted Svensson, outperformed their AR(1) counterparts, especially on shorter horizons. However, on horizons shorter than 1 year, the

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VAR(1) models cannot beat the random walk either. The long/short maturity disparity is true for the VAR(1) models as well.

The greatest advantage of state-space models is their stability. They do not perform particularly well, but some specifications beat the random walk on longer forecast horizons and usually predict better on shorter horizons compared to their VAR(1) and AR(1) counterparts. No state-space model produces the unfeasible results of the fixed λ VAR(1) models. Further adding to the sense of stability is the fact that the all three basic Nelson-Siegel specifications yield the exact same result. This is the least complicated functional form and despite the different starting values, a similar optimum is found. Adding more factors does not improve the forecasting performance. It seems that the Hungarian interest rate dynamics are chaotic enough that a more complex functional form results in overfitting.

One of the novel contribution of this thesis is that Legendre models are indeed potent competitors to the NSS models even in pseudo out-of-sample forecasting performance. The four-factor Legendre model in a state-space framework matches the predictive accuracy of the basic Nelson-Siegel specification. A different functional form with adequate predictive capabilities can be useful in further studies as several papers (De Pooter et al., 2010; Bolder and Romanyuk, 2010; de Araújo and Cajueiro, 2013) show that combining multiple interest rate forecasts is capable of outperforming any single model. Such an approach, however, is not useful for this paper as it contradicts the dynamics necessary for simulating forecast scenarios.

Overall, the state-space models proved to be the most stable across forecast horizons. Some VAR models offer a better forecasting performance but others are extremely unstable. As the objective is to obtain an accurate true out-of-sample forecast, the Nelson-Siegel state-space model is the safest choice. It is stable enough that each in-sample model specification yields the same pseudo out-of-sample results. The Nelson-Siegel state-space model

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outperforms the random walk on longer horizons and maturities, and provides reasonable forecasts for shorter horizons as well. The reason for the decision towards the Nelson-Siegel model as opposed to the four-factor Legendre is that the factors of the former model are more convenient to interpret and have no problem of giving a valid approximation for yields above their maximum in-sample maturity. Figure 5.1 shows the in-sample factors of the restricted Nelson-Siegel model as well as their respective 5-year out-of-sample forecasts with a 95% prediction interval. The estimated decay parameter λ is 1.45, not much different from the 1.39 used by the fixed λ models in this thesis.

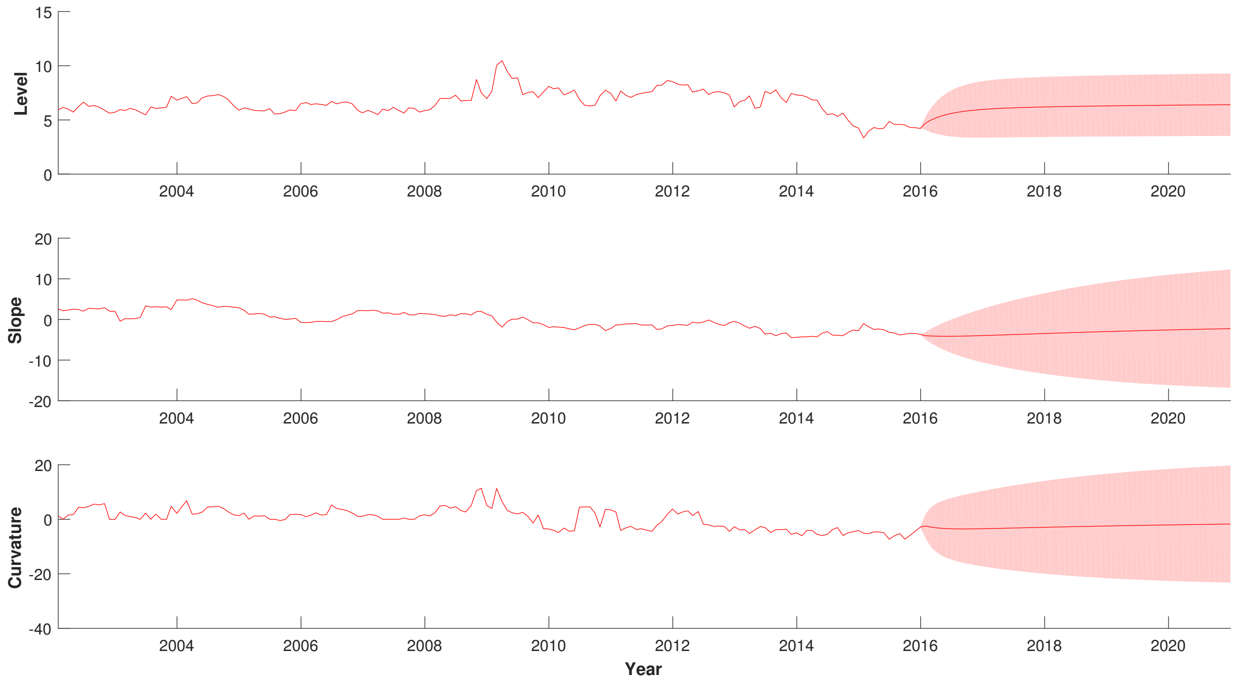

Figure 5.1: Nelson-Siegel factors in 2002-2020 with a 95% prediction interval

It is a natural consequence of using a mean-reverting model that the first factor influencing yield levels for all maturities is predicted to rise over the 5-year forecast horizon, starting from 2016. The created state-space model also allows for manually setting the return levels instead of using the historical means. It is possible to set a lower return level for β_0 if we assume that the process is not mean-reverting and the current low interest rate

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regime will prevail. This approach is worth investigating in future studies.

We can see the predicted yields resulting from the Nelson-Siegel state-space forecast in Figure 5.2. The overall rising trend in yields is expected, knowing that the level factor is predicted to increase. Also, the long yields are predicted to be higher than the short yields for the whole forecast interval as the slope forecast stays above zero in Figure 5.1. The strong point of such a deterministic forecast is that it has allowed to distinguish between the forecasting accuracy of different model specifications and methodologies. Its inherent weakness is that it does not provide a complete distribution of yields. Using the deterministic forecast, the optimal strategy would be to issue only short maturity bonds as they are predicted to have a lower cost. To properly account for risks, a possible solution is a stochastic forecast that is able to generate a full distribution of predicted yields.

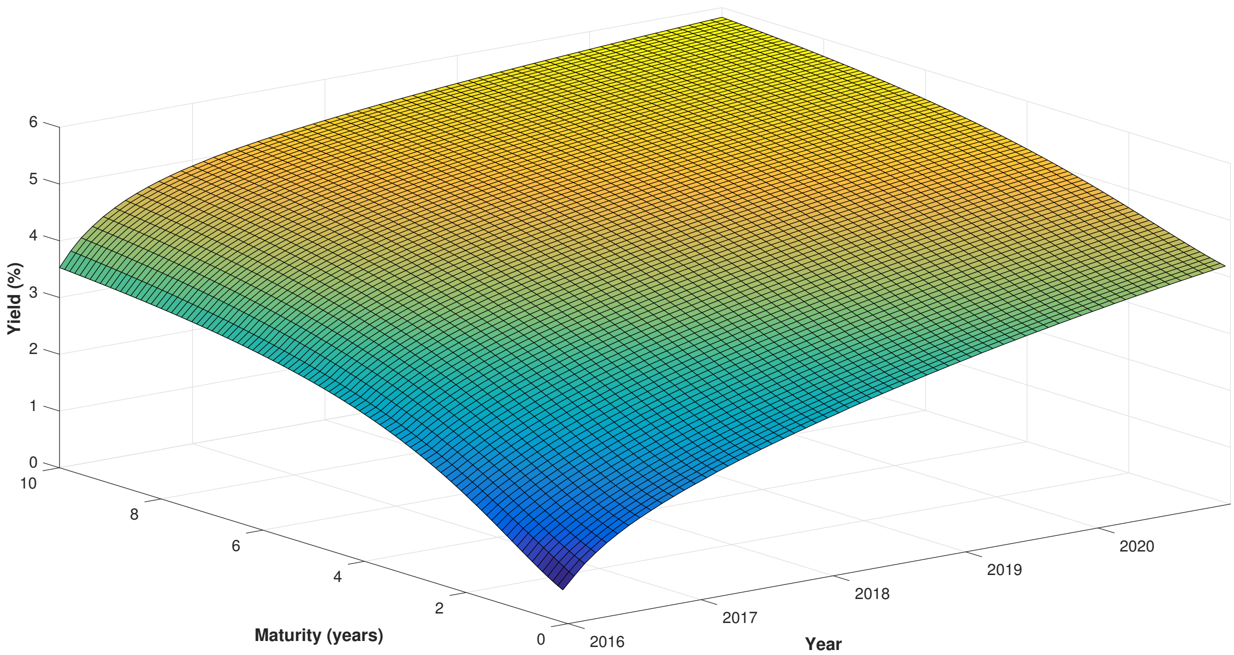

Figure 5.2: Term structure forecast 2016-2020

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5.3 Scenario Generation

The method for generating random forecast scenarios is Monte Carlo simulation. Instead of calculating the minimum mean squared error for each step as with the deterministic forecast, random numbers are drawn from the normal distribution for each measurement and transition equation. Deflated yields and mean adjusted factors are simulated and the unconditional means are added after the simulation. Ten thousand such paths are generated.

Figure 5.3 shows the distribution of short (3 month) and long (10 year) yields for a 1-month-ahead and 5-years-ahead forecast. We can see that on average, similar to the deterministic forecast, yield curves are upward sloping, inferred from the long yields having a higher mean. It is easy to observe that short yields are much more volatile in the long run compared to long yields. Negative short yields can plausibly happen in the future while negative long yields are very unlikely in the case of Hungary.

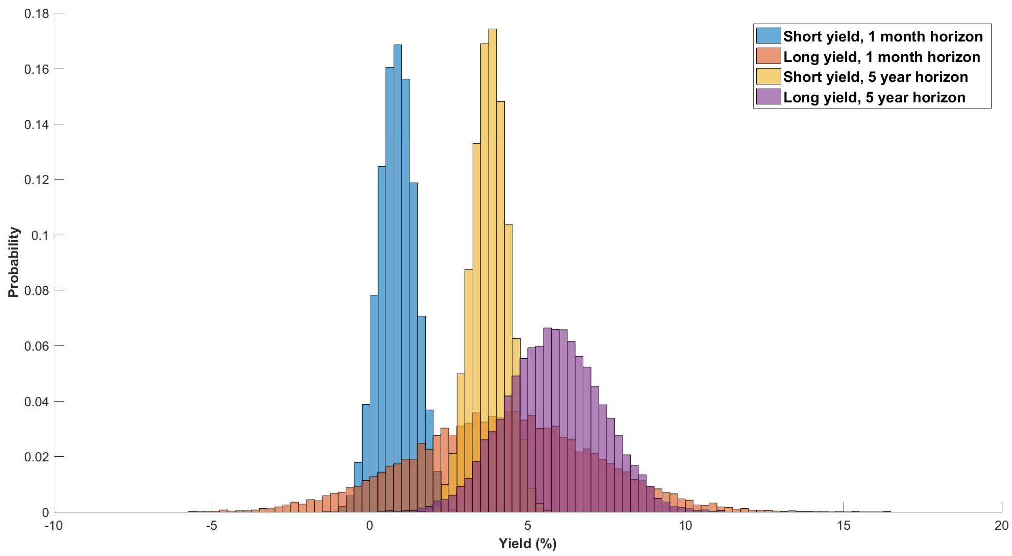

Figure 5.3: Distribution of yields

Descriptive statistics about the factors at the end of the forecasting period (December

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2020) can be seen in Table 5.1. The standard deviations mirror reality as the slope factor influencing the short yields is particularly volatile. The variance of the third factor is even higher, but it is important to remember that the maximum loading of the curvature is less than a third of the other factors. As the simulation draws from the normal distribution, the skewness and kurtosis values are as expected. It is particularly interesting that the mean of the curvature is negative. This means that on average, the yield curve has a slight downward hump. Therefore, the yield curve might often have a local minimum at 2.5-3 years of maturity. Furthermore, the last column shows that inverted yield curves occur with a 20% chance. This means that we can expect shorter maturity bonds to be cheaper than longer maturity bonds in most scenarios. In about 8.5% of the scenarios, the sum of the level and slope coefficient are below zero, meaning that the short rates are negative despite the general increasing trend of the predicted yields.

Table 5.1: Descriptive statistics for the 5-year horizon

Factor	Mean	StDev	Skew	Kurt	Min	Max	$P(\beta_i < 0)$
Level (β_0)	6.41	1.20	0.02	3.05	1.82	11.29	0.00
Slope (β_1)	-2.25	2.68	-0.00	3.04	-12.59	7.96	0.80
Curvature (β_2)	-1.76	3.28	-0.04	3.10	-14.30	11.68	0.70

From a cost-risk point of view, these results mean that longer maturity bonds are expected to have higher overall costs compared to lower maturity bonds for the forecast period. However, the high standard deviation of the slope means that lower maturity bonds will always be always carry higher risks for the debt manager with themselves.

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Chapter 6

Optimal Debt Management in a Cost-Risk Setting

This chapter deals with the construction of an optimal Hungarian debt portfolio of HUF-denominated government bonds. Using data from ten thousand randomly generated forecasting scenarios, the costs and risks associated with issuing several different types of government securities are estimated. From these estimates, an optimal portfolio of bonds is constructed. Questions and issues regarding the cost and risk preferences of the debt manager are given an overview as well. The final part of the chapter gives recommendations on the applications of the debt management model in policy.

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6.1 Bond Issuance Framework

The framework used for the purpose of this thesis contains several assumptions that are necessary for the portfolio optimization exercise to remain feasibly solvable with high precision and reasonably low computational expense. This is important as the goal is to construct the efficient frontier of a debt portfolio structure under different constraints. This task requires solving thousands of individual optimization problems. Therefore, the bond issuance framework is constructed in a temporally static manner so that the costs of the total portfolio remains a linear function of the initial portfolio weights. To accomplish this, new bond issuances always match the characteristics (like maturity and coupon type) of the matured paper. Another assumption of the model is that issuances in the model are scheduled to happen once every quarter. The interest rate for interest bearing bonds is set according to the predicted yield curve (differently for every scenario), rounded to 25 basis points. Interest-bearing bonds are issued at a price close to their face value (with some correction due to interest rate rounding), while zero-coupon T-bills are issued at a discount price, also according to the predicted term structure of interest rates. Demand does not play a role in the model framework. The assumption of the model is that there is always enough demand for bonds issued at market prices.

Furthermore, I assume that the government deficit path is and exogenously given and independent of the predicted yield curve paths and the debt management strategy. Although in reality, the performance of the debt manager can have some effect on the government deficit as debt service charges are a significant amount in the budget, modeling different deficit scenarios would require a more complex macro-financial forecast. The official estimates by the Hungarian government (Magyarország Kormánya, 2016) are used for the deficit path forecast.

Next, the papers used for the debt strategy are defined. Table 6.1 gives an overview of the HUF-denominated government securities issued in practice. I use eight different papers to model them. Currently, zero-coupon T-bills are issued with two different maturities in the Hungarian market. The shorter T-bill has a maturity of 3 months and the longer 12 months. In the past, there used to be 6-month zero-coupon T-bills as well. I model all interest-bearing T-bills using a 12-month zero-coupon security, as the share of 6-month

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and 24-month papers in the debt portfolio is very low in practice.

Table 6.1: Modeling government securities

Security	Type	Retail security	Maturity	Modeled by
DKJ	Zero-coupon T-bill	No	3 months 12 months	3-month ZC 12-month ZC
KKJ	Fixed interest T-bill	Yes	12 months	12-month ZC
FKJ	Fixed interest T-bill	Yes	6 months	12-month ZC
KTJ	Step-up T-bill	Yes	12 months 24 months	12-month ZC 12-month ZC
KTJP	Step-up T-bill	Yes	12 months	12-month ZC
MÁK	Fixed interest bond	No	3 years 5 years 10 years 15 years	3-year fix 5-year fix 10-year fix 15-year fix
	Floating bond		3 years 5 years	BUBOR float BUBOR float
BMÁK	Floating bond	Yes	4 years 6 years 10 years	12-month float 12-month float 12-month float
PMÁK	Inflation-linked bond	Yes	3 years 5 years	50% 12-month float 50% BUBOR float
Other	Floating bond	Varies	Varies	50% 12-month float 50% BUBOR float

Next on the list are fixed interest bonds. ÁKK issues 3, 5, 10 and 15-year fixed rate bonds. Even though the yield curve forecast only includes maturities up to 10 years, the Nelson-Siegel model allows extrapolation to longer maturity yields as well. All fixed interest bonds issued in practice are modeled by a similar fixed interest bond with the appropriate maturity. All fixed bonds pay interest once every year. Floating bonds are somewhat more problematic as the used model does not forecast inflation and there are some CPI-indexed bonds on the market. Two other types of indexing, both related to the term structure, have been used as well for recently issued floating rate bonds. The first type is used for the MÁK floating bond seen in Table 6.1 and is mainly targeted towards institutional investors. It pays interest quarterly and the reference curve is the 3-month

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Budapest Interbank Offered Rate (BUBOR). This is approximated in my model (denoted *BUBOR float*) using the 3-month interest rate and adding a 25 basis points spread, the historical mean difference between the two time series. Otherwise, the two data series have an over 99% correlation which is extremely high even when accounting for spurious correlation. The second reference rate, used by bonds targeting mainly retail investors (BMÁK) uses the weighted average of the last four 12-month zero-coupon T-bill auction results for setting the coupon. I benchmark this using the 12-month yields of the last two months as the corresponding zero-coupon auction is held every two weeks and denote this bond *12-month float*. Even though these bonds carry an additional interest premium, I do not elevate the coupon in the estimation. Only this bond is modeled after a security targeting retail investors. Not adding a premium is necessary in order to be comparable to securities available to the institutional sector that pay no premium. Floating bonds have a wide variety of maturities. However, only a single one with a maturity of 5 years is used for both floating bonds in the model as the coupons change every year (or quarter) anyways. Using the full maturity spectrum would only matter if average time to maturity constraints or a dynamic strategy was used. The share of CPI-indexed bonds are split equally between the two floating bond types in the model. The same applies to the small minority of floating bonds with a reference rate different to the ones already mentioned.

Figure 6.1 shows the modeled composition of Hungarian government bonds just before the start of the forecasting period based on ÁKK auction data. Overall, 20% of the securities have a maturity of maximum 1 year, 17% are floating bonds and the other 63% are fixed interest bonds. The dominant maturity of fixed interest bonds is 10 years, followed by 5-years and a roughly equal amount of 3 and 15-year bonds.

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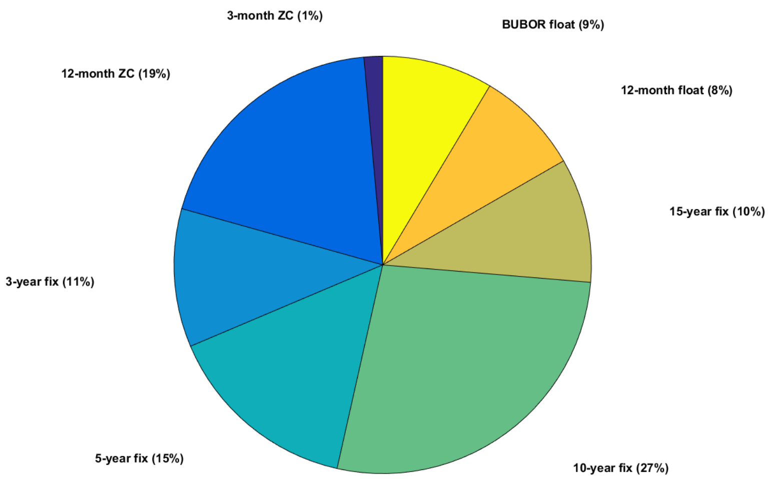

Figure 6.1: HUF-denominated bond portfolio, December 2015

It has to be mentioned, however, that this is only a rough approximation of the current outstanding series. In reality, bonds do not always have exact round maturities and some reopened bonds have a maturity somewhere between the displayed ones. Furthermore, 15-year bonds are sometimes reopened as 10-year bonds and 10-year bonds as 5-year bonds. Those bonds are split evenly between the modeled categories. It is not possible to give a more precise estimate due to the numerous exchange and repurchase auctions. Also, this is not the entire Hungarian government debt portfolio as loans and foreign currency denominated bonds are omitted.

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6.2 Portfolio Optimization with Constraints

In this section, I introduce two notions of risk and solve the corresponding optimization problems under different constraints. The cost part of cost-risk optimization is relatively straightforward. The cost of each bond can be calculated by summing capital gains and losses as well as interest expenditures falling into the forecast period (2016-2020) and averaging them out between the ten thousand scenarios. Also, the cost associated with a portfolio is just the weighted average of the individual costs. Therefore, only the costs associated with the eight individual bonds have to be calculated.

The first notion of risk is the standard deviation of the ten thousand cost realizations. This is of course the classic mean-variance approach by Markowitz (1952). This is a quadratic programming problem with linear constraints. Let us denote the (column) vector of portfolio weights by $\mathbf{w}$, the vector of individual bond costs by $\mathbf{c}$, the covariance matrix (calculated using the end-period realizations) by $\mathbf{H}$ and the total cost of a portfolio by $\bar{c}$. The basic quadratic programming problem with no additional constraints can be written using matrix notation as:

$$\begin{aligned} &\min_{\mathbf{w}} \mathbf{w}^T\mathbf{H}\mathbf{w} \\ \text{subject to: } &\mathbf{c}^T\mathbf{w} = \bar{c}_i \\ &\mathbf{0} \leq \mathbf{w} \leq \mathbf{1} \\ &\mathbf{1}^T\mathbf{w} = 1 \end{aligned} \tag{6.1}$$

The estimation procedure starts by determining the minimum and maximum total portfolio cost by linear programming. Then the interval between the two values is divided into a thousand equal parts giving $\bar{c}$. The quadratic programming exercise is then run for all one thousand different $\bar{c}_i$ cost constraints. The two other constraints here are trivial.

Next, a second and a third quadratic programming optimization is carried out similar to the first one, but with more constraints, for a total of three different models. The second one uses the duration and fix-to-float ratio constraints from the actual financing strategy (ÁKK Zrt, 2015). The duration of a portfolio is a weighted average of the $\mathbf{d}$ durations of each individual bond. It is important to mention that for example the duration of a

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portfolio consisting only of 3-year bonds is far below 3 (about 1.9). Furthermore, let $\mathbf{x}$ denote a binary vector, with zeros for floating rate bonds and zero-coupon T-bills and ones for fixed interest bonds. This vector is required to calculate the fix-to-float constraints. The corresponding quadratic programming exercise can be written as:

$$\begin{aligned} \min_{\mathbf{w}} \ & \mathbf{w}^T\mathbf{H}\mathbf{w} \\ \text{subject to: } & \mathbf{c}^T\mathbf{w} = \bar{c}_i \\ & \mathbf{0} \leq \mathbf{w} \leq \mathbf{1} \\ & \mathbf{1}^T\mathbf{w} = 1 \\ & 2.5 \leq \mathbf{d}^T\mathbf{w} \leq 3.5 \\ & 0.61 \leq \mathbf{x}^T\mathbf{w} \leq 0.83 \end{aligned} \tag{6.2}$$

For the third model I impose another set of constraints, hereinafter additional deviation constraints, that restricts the debt manager from not issuing certain bonds at all or switching too much from the current portfolio. This constraint is quite simple mathematically. Let us denote the vector of current portfolio weights from Figure 6.1 by $\mathbf{y}$. The constraint lets the portfolio weights differ by +/- 50%. Therefore the third model can be written as:

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$$
\begin{aligned}
\min_{\mathbf{w}} \ & \mathbf{w}^T\mathbf{H}\mathbf{w} \\
\text{subject to: } & \mathbf{c}^T\mathbf{w} = \bar{c}_i \\
& \mathbf{0} \leq \mathbf{w} \leq \mathbf{1} \\
& \mathbf{1}^T\mathbf{w} = 1 \qquad (6.3) \\
& 2.5 \leq \mathbf{d}^T\mathbf{w} \leq 3.5 \\
& 0.61 \leq \mathbf{x}^T\mathbf{w} \leq 0.83 \\
& 0.5\mathbf{y} \leq \mathbf{w} \leq 1.5\mathbf{y}
\end{aligned}
$$

Figure 6.2 shows the result of the three quadratic programming models. The costs are calculated as a percentage of the initial value of the debt portfolio (not the GDP) and a fifth root is used to calculate the yearly costs from the 5-year total. In the top left of the figure are the efficient frontiers for the different models along with the individual bonds and the current portfolio. It has to be mentioned, that contrary to the original Markowitz-problem with assets, the efficient frontier has a negative slope whereas the other end of the curve with a positive slope is suboptimal. It is easy to notice that most individual bonds are on (or at least very close to) the efficient frontier. This means that the correlation between the bond costs are really high and it is quite difficult to gain a Pareto-improvement by diversifying the portfolio. This is not surprising as the level of the yield curve has a congruent effect on all bond costs. The relation of the bonds in terms of cost and standard deviation is not surprising. Higher maturity bonds have a lower standard deviation and a higher cost. Furthermore, the two floating bonds are close to the zero-coupon T-bills corresponding to their reference yields. The current portfolio does not touch either of the efficient frontiers. Evidently, there are possible Pareto-improvements to that portfolio in a mean-variance setting.

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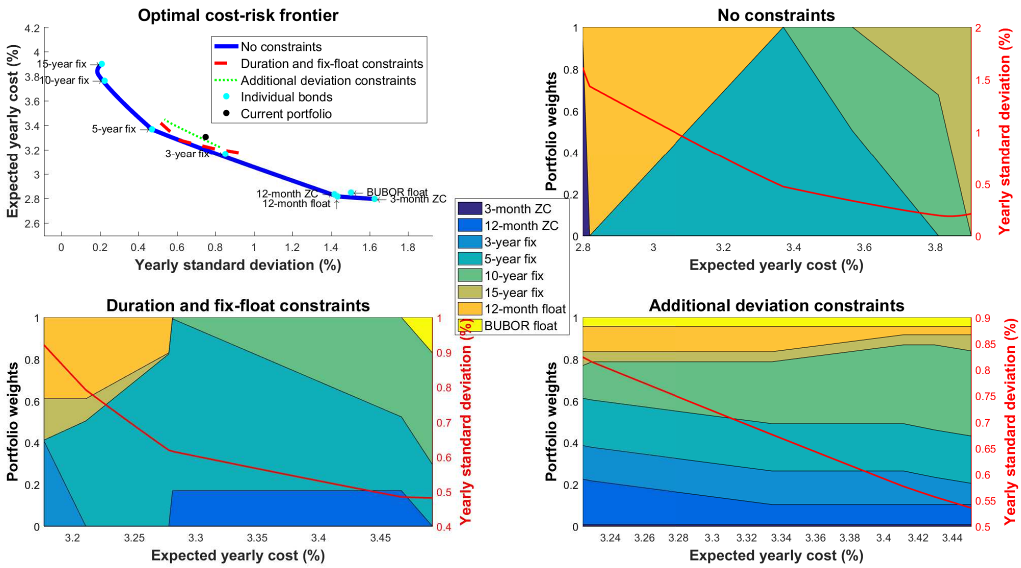

Figure 6.2: Mean-variance efficient frontier and portfolios

The other figures show the optimal portfolio weights and the corresponding standard deviation for different expected costs and constraints. There is a non-linear trade-off between cost and risk. The most efficient bond overall seems to be the 5-year fixed interest bond. The discount bonds and the BUBOR-indexed papers tend to be outclassed by the 12-month discount bond based floater. Clearly, the bottom right figure using the third model specification indicates that these suboptimal bonds are kept at their minimum allowed levels, replacing them with longer maturity fixed bonds or the 12-month indexed floater. Now, I move on to a more modern risk metric: the conditional cost-at-risk.

The fundamental problem with the mean-variance approach is that it is mathematically possible, that a bond has a higher cost compared to another in every scenario and yet its cost has a higher standard deviation. It can plausibly happen with two high maturity bonds where the standard deviation is relatively low compared to the cost. One more advanced and very commonly used risk metric for assets is value-at-risk (VaR). The complementary measure for liabilities which is relevant for the debt manager in this particular is cost-at-

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risk (CaR). The most frequently used cost-at-risk measure is the CaR_{95} which is (in this case with discrete scenarios) the 95^{th} percentile of expected costs falling into the forecast horizon. This risk measure has two shortfalls. First, it ignores the remainder of the tail distribution. Second, it is not convex and therefore mathematically very difficult to optimize.

Neither variance nor VaR (or CaR) are coherent risk measures according to the axioms defined by Artzner et al. (1999). However, the conditional value-at-risk (CVaR), or in the debt portfolio case, conditional cost-at-risk (CCaR) satisfies all said axioms and also accounts for costs above the CaR. The CCaR measure is the average cost, provided that the cost is larger than the corresponding CaR. This measure, introduced by Rockafellar and Uryasev (2000) can be optimized using linear programming for given scenarios, which fits the framework of this thesis.

Let z denote the α % cost-at-risk, $\mathbf{R}$ the matrix of costs associated with every portfolio and scenario and $\mathbf{u}$ a vector of auxiliary variables. The linear programming exercise corresponding to the first mean-variance model can be written as:

$$\begin{aligned}
&\min_{\mathbf{w},z} z + \left(\frac{1}{1-\alpha}\right)\mathbf{1}^T\mathbf{u} \\
\text{subject to: } &\mathbf{c}^T\mathbf{w} = \bar{c}_i \\
&\mathbf{0} \leq \mathbf{w} \leq \mathbf{1} \\
&\mathbf{1}^T\mathbf{w} = 1 \\
&\mathbf{R}\mathbf{w} - \mathbf{1}z - \mathbf{u} \leq \mathbf{0} \\
&\mathbf{u} \geq \mathbf{0}
\end{aligned} \tag{6.4}$$

The duration, fix-float and portfolio switching constraints can be added to this model similar to the standard deviation case.

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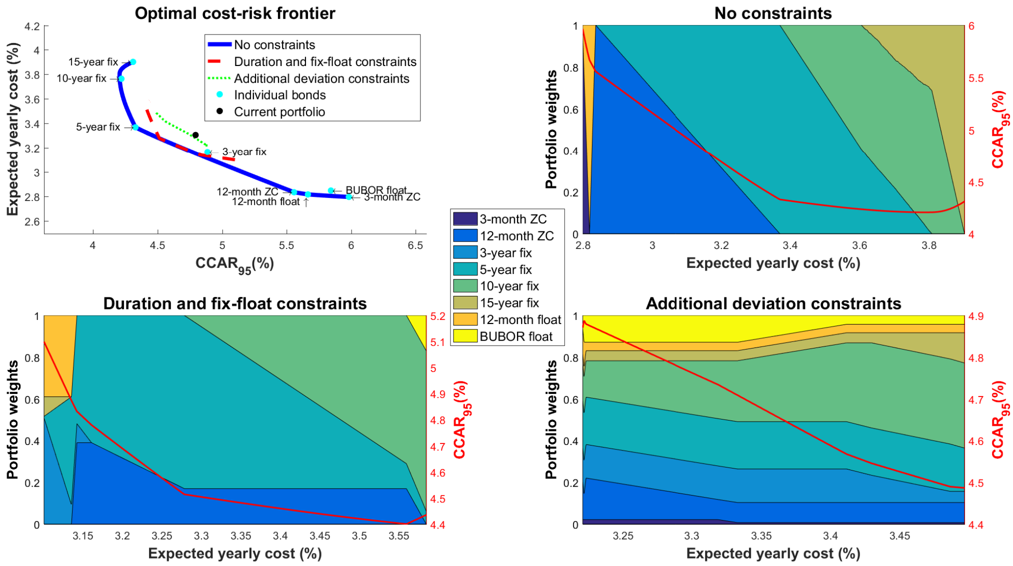

Figure 6.3: Mean-CCaR$_{95}$ efficient frontier and portfolios

Figure 6.3 shows the mean-CCaR efficient frontier with constraints similar to the mean-variance optimization. The main difference compared to Figure 6.2 is that the 15-year bond is entirely suboptimal compared to the 10-year bond. The results of the unconstrained optimization is that the optimal portfolio usually contains two bonds. As the allowed cost increases, the corresponding minimal CCaR portfolio switches from 3-month to 1-year then 5-year, 10-year and finally 15-year bonds. However, the last part actually reaches the inefficient, positively sloped end of the curve. Using duration and fix-float constraints, the 12-month indexed floater is the optimal choice for the lowest cost portfolio, combined with the 3-year fixed interest bond and the 15-year bond to increase the duration. Even though the 15-year bond is suboptimal, combining it with short maturity bonds enables a lower total cost than using the 10-year bond due to the duration constraints. The minimum cost portfolio, however, has a high conditional cost-at-risk even with duration and fix-float constraints. Allowing a higher cost, the five-year and finally the 10-year bond comes into the optimal portfolios, along with the 12-month zero-coupon bond to satisfy the fix-float

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constraint. However, switching to the 10-year bond offers noticeably diminishing returns when it comes to risk. When additional deviation constraints are taken into account as well, the feasible domain of portfolios becomes very limited as suboptimal bonds take up a large part of the portfolio.

Figure 6.4 shows that the results remain robust even when using a 99% CCaR. The average cost of a 3-month bond in the worst 1% of the scenarios is less than one percentage point higher than in the worst 5% of the yield paths.

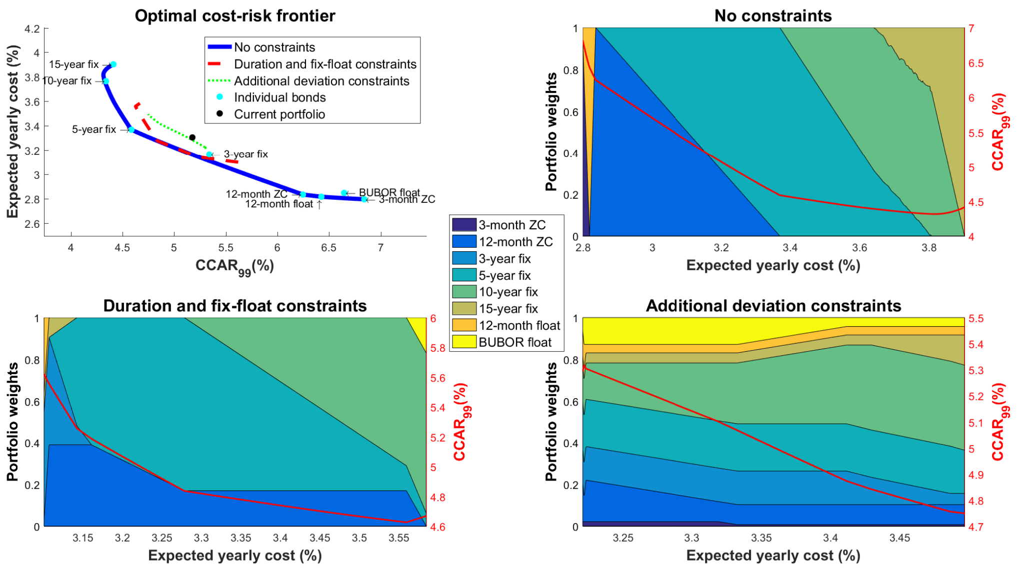

Figure 6.4: Mean-CCaR$_{99}$ efficient frontier and portfolios

Table 6.2 shows the cost discrepancy between the 3-month zero-coupon as well as the 5 and 15-year fixed interest bonds for different samples. The most important observation is that the cost difference between the 5 and 15-year bonds is much higher on average in favor of the former compared to the worst-case scenarios in favor of the latter. Whereas if we compare the 5-year bond to the 3-month bond we can see that the debt charge discrepancy in the worst case scenarios is more in favor of the 5-year bond than the average cost favors the 3-month bond. Therefore, the main conclusion of the CCaR-optimization is that bonds

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with a maturity over 5 years offer no real benefit even in the worst-case scenarios and have a higher cost on average. The other end of the maturity spectrum with the BUBOR-indexed floating bond and the 3-month zero-coupon T-bill has the drawback of being highly risky with only a limited compensation on the cost part.

Table 6.2: Average yearly debt charges for bonds in different scenarios

Bond	All scenarios	Worst 500 scenarios	Worst 100 scenarios
3-month ZC	2.80%	5.98%	6.85%
5-year fix	3.37%	4.33%	4.59%
15-year fix	3.90%	4.31%	4.41%

Table 6.3 illustrates the effects of reducing the share of the suboptimal portfolios. Two additional portfolios are constructed as an alternative to the current HUF-denominated debt portfolio modeled using eight bonds.

Table 6.3: Exemplary portfolios

Measure	Current portfolio	Optimal I.	Optimal II.	Plausible
3-month ZC (%)	1.44	1.31	0.00	2.00
12-month ZC (%)	19.21	16.79	17.00	15.00
3-year fix (%)	10.72	6.73	0.00	5.00
5-year fix (%)	15.12	25.17	80.05	40.00
10-year fix (%)	27.18	24.92	2.95	20.00
15-year fix (%)	9.64	8.29	0.00	4.00
12-month float (%)	8.11	12.35	0.00	14.00
BUBOR float (%)	8.59	4.43	0.00	0.00
Cost (%)	3.31	3.29	3.29	3.29
Standard Dev. (%)	0.75	0.73	0.61	0.70
$CCAR_{95}$(%)	4.79	4.76	4.51	4.69
$CCAR_{99}$(%)	5.18	5.14	4.83	5.05
Fix-float ratio (%)	62.66	68.41	83.00	69.00
Duration (years)	2.91	2.76	2.54	2.75

The *Optimal I.* portfolio is calculated using a mean-variance optimization, allowing a cost of 3.29%. It honors the additional deviation constraints as well. The same mean-

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variance approach is used for the *Optimal II.*, but without the additional deviation constraints. It still honors the duration and fix-float constraints. Of course, having 80% of the portfolio in one bond is not a realistic alternative in practice. Finally, the *Plausible* portfolio is manually constructed and also stays within the duration and fix-float bounds. The purpose of the *Plausible* portfolio is to demonstrate the effects of reducing the share of 15-year bonds, slightly reducing the share of floating bonds and switching the indexing of floating bonds to the 12-month zero-coupon yields. I consider these changes feasible in the current situation as a long-run goal. The shares of different securities are given as percentages of the portfolio. The other measures are defined as before.

As we can see, the *Optimal I.* portfolio is a Pareto-improvement to the current debt portfolio in all four cost and risk measures. Moreover, the *Optimal II.* portfolio is Pareto-improvement for all three risk measures even to the *Optimal II.* portfolio, while its costs stays the same. The *Plausible* portfolio is in the middle of the latter two. The expected benefits according to the model are quite small, especially when only smaller changes are made to the current portfolio. Nevertheless, it illustrates the improvements in both costs and risks when reducing the share of suboptimal securities in favor of medium maturity bonds.

Besides simplifications concerning the issuance structure and the lack of demand constraints, the results regarding the efficient frontiers require only two assumptions about the utility function of the debt manager, namely that the debt manager is risk averse and cost minimizing. Both are quite natural and require no further explanation. Finding an exact optimal portfolio could be possible in two ways. First, the well-known tangency or mean-variance efficient portfolio requires a risk-free rate. However, when it comes to the largest entity on a market, there is no way of determining an exogenous risk-free borrowing rate. Another possible way would be setting the debt manager's Arrow-Pratt risk-aversion coefficient or in general, the utility function. Both are usually measured either experimen-

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tally, using tools of behavioral finance or empirically, analyzing the past choices of actors. Neither approach is in the scope of this thesis, but offer interesting possibilities for further research. Therefore, I consider the efficient frontier a better representation of optimality as opposed to selecting just a single arbitrary point and labeling it the one optimal portfolio.

6.3 Policy Recommendations

Approximately 38% of the HUF-denominated outstanding Hungarian government securities have an original maturity of over 10 years. I have shown that in a medium-term debt strategy they offer no real benefit in terms of risk compared to 5-year bonds, while having a significantly higher average debt service charge. Reducing their overall share in favor of medium maturity bonds would reduce the costs associated without significantly increasing risks. Of course, in reality, this cannot be done immediately, only gradually, by reducing long-maturity bond issuance.

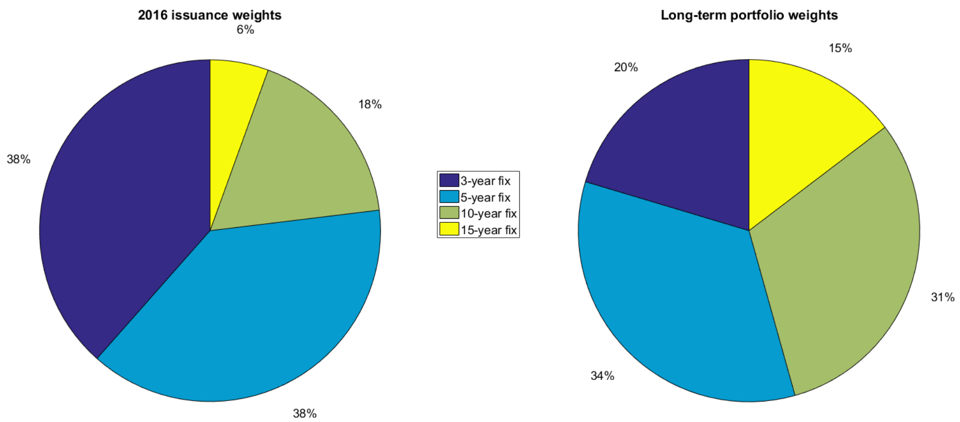

Figure 6.5: Fixed bond portfolio weights

Figure 6.5 shows the fixed bond portfolio weights in relation to each other. Both the

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targeted 2016 issuance weights (ÁKK Zrt, 2015) and their theoretical long-term[1] weights are shown. The long-term weights are calculated by multiplying the initial amounts with their respective maturities. For example, if the same amount of 3 and 15-year bonds are issued each year, there will be 5 times as much in circulation of the latter than the former in the long run.

Following the 2016 issuance strategy means that the collective share of the two highest maturity bonds would drop from 38% to 28% of the total HUF-denominated portfolio. This is a step in a direction consistent with the results of my model and perhaps a target below 25% could be achieved as well by reducing long-maturity bond issuances even more.

A second concern is the structure of floating bonds. There are three main types of indexing of current Hungarian floaters with an approximately equal share. Setting the reference rate to 12-month discount bond as opposed to the 3-month BUBOR incurs almost no additional costs while reducing risk. This can also partly offset the duration-reducing effect of issuing less long-maturity bonds. Assessing the benefits and drawbacks of CPI-indexed bonds requires further studies. Combining this change with the reduction of the share of 15-year bonds would still lower the duration of the debt portfolio by about two months. This is still inside the set bounds, but reducing it to 2.75 +/-0.5 years seems to be a reasonable change. The fix-to-float and duration constraints in the actual debt strategy greatly restrict the set of feasible portfolios, but the constrained efficient frontier still shares a section with the unconstrained one for all risk measures. Therefore an efficient portfolio can be reached even when taking the fix-to-float constraint into account. Also, both zero-coupon T-bills do play an important role in establishing the short end of the yield curve through auctions and maintaining the liquidity needs of the budget, therefore

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[1]The long term weights are reached if the issuance structure does not change over time and all currently outstanding bonds mature.

a certain amount of them should always be present on the market.

A third issue is the promotion of retail financing as opposed to institutional investors. Most bonds specifically targeting retail investors have a one to three percentage points interest rate premium. The vast majority of retail bonds are floaters or have a maturity no greater than 1 year. If we look back at the expected yearly cost of the different portfolios, we can see that they are in the 2.8-4% range. The premium payed above the market rate can be larger than the 1.2% difference between the expected cost of the cheapest (3-month T-bill) and the most expensive (15-year bond) government security in the model. Issuing retail bonds with a bonus coupon is suboptimal mathematically, but could be regarded as a sort of transfer and a tool of redistribution from a policy point of view. There is certainly some ambiguity in judging this question, but having two separate markets as financing sources is certainly beneficial and may justify an increased interest expenditure.

The policy recommendation can be summarized as follows: reducing the proportion of bonds with costs tied to the two ends of the term structure in favor of medium maturity bonds results in a Pareto-improvement regarding costs and risks. It is also feasible, as evidenced by some steps being already taken in that direction. Diversifying the debt portfolio of bonds does not have a particularly large positive effect due to the high correlation between bond costs throughout the different predicted scenarios. There is no need for drastic measures in changing the debt portfolio of Hungary. The current debt portfolio is not optimal in a mathematical sense, but reasonably close to different measures of efficiency. A gradual transition to a medium-maturity bond heavy portfolio and a small reduction in the overall portfolio duration is the suggested policy approach based on the results of my model.

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Chapter 7

Conclusions

In this thesis, I set out to answer a single research question: what is the optimal issuance structure of HUF-denominated bonds that offers possibly low debt service charges while minimizing risk levels as much as possible? The answer to this question, which is the main policy recommendation of this paper, is that it would be beneficial to reduce the proportion of bonds with a very long or very short maturity and increase the share of medium maturity bonds in the funding structure.

In pursuit of giving a sound, well-founded answer to the main research question, several interim tasks needed to be accomplished. First, I compared different empirical term structure models by in-sample fit and correlation of their factors with corresponding yields. The main conclusions were that more flexible and complicated models have a better in-sample fit, but cannot be interpreted very well as factor models. A novel result, going beyond existing literature, is that using dynamic constraints restricting the estimated parameters results in a much higher correlation with the appropriate time series of yields, in expense of a slightly worse fit.

Next, I used the in-sample estimates of ten term structure models to forecast the

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yield curve on a five-year horizon using three forecasting techniques. I found out using a pseudo out-of-sample expanding window forecast that the random walk benchmark can be beaten reliably by most models and methodologies when forecasting yields with a maturity exceeding 5 years on a horizon longer than one year. About a third of them outperform the random-walk on forecasting horizons over one year when all maturities are aggregated. The Nelson-Siegel model in a state-space formulation was selected as the model to generate scenarios with, but the four-factor Legendre model also showed some promise and is a viable candidate for further studies combining interest rate forecasts.

Finally, I calculated the ten thousand Monte-Carlo simulated forecast paths of Nelson-Siegel factors using a state-space model with the purpose of analyzing the costs and risks of government bonds. I calculated the efficient frontier using both a mean-variance approach and a conditional cost-at-risk approach. The two results were similar in some aspects, but the latter approach did a better depiction of why bonds on both ends of the maturity spectrum are inefficient. However, I also concluded that the cost difference between long and short maturity bonds is in fact smaller than the interest premium payed for retail bonds.

The most important policy-related conclusion is that the share of the 15-year bonds should be reduced in favor of medium maturity bonds and the reference rate of floating bonds should be tied to the 12-month interest rate rather than the 3-month BUBOR rate in order to limit the exposure to the very volatile short end of the yield curve. The duration of the Hungarian debt portfolio would be reduced as a result of lowering the share of 15-year bonds, but would still remain inside the constraints set by the guidelines in effect. According to the results of my model, these two changes would reduce the debt charge without increasing risks.

The model could be improved in many ways. For example, the methodology of forecasting can be extended to include macroeconomic factors as well. This would allow including

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foreign currency denominated and inflation-indexed bonds in the estimation. Using combined interest rate forecasts could further improve the predictive performance as well. It is also possible to modify the state-space approach by using dynamic covariance estimates, a Markov regime switching model or by using a different mean-reverting level for the factors, which may allow simulating the effects of different economic shocks including crisis scenarios. There are also several ways to improve the portfolio optimization by alleviating some of the imposed restrictions and assumptions. Most important of all would be including demand into the equation and treating it endogenously. Also, a dynamic issuance structure could be used as well, along with a more complicated optimization procedure with dynamic issuance or rollover constraints. Furthermore, more types of risks measures and constraints could be used, including conditional volatility, rollover risk as well as budgetary or refinancing constraints.

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Appendix A

Forecast Results

The tables in the following pages contain forecast error statistics for 32 competing models. The first model (Y-RW) is the yield random walk (no-change) benchmark forecast. Root mean squared errors (in yield percentage points) are displayed for that model. The error statistics of every other model are displayed as a fraction of the random walk error. Figures in **bold** indicate a better performance of a model compared to the random walk. In every model, bootstrapped yields of 22 maturities[1] are used (crucial for VAR and state-space models) and prediction errors for 9 maturities are displayed. The first numerical column contains the Trace RMSE computed using the error statistics of the displayed maturities.

Forecasts using term structure models are put into three categories based on the estimation procedure: autoregressive AR(1) models are denoted by AR, vector autoregressive VAR(1) models by VAR(1) and state-space models by SSM. The models are denoted the following way: prefixes stand for the model and suffixes for the estimation type. Used mod-

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[1]The following maturities are used: 3, 4, 5, 6, 8, 10, 12, 15, 18, 21, 24, 30, 36, 42, 48, 52, 60, 72, 84, 96, 108 and 120 months.

els are: direct yields forecast (Y), Nelson-Siegel (NS), Björk-Christensen (BC), Adjusted Svensson (AS), and Legendre (LE). The second part of the nomenclature determines the model specification. Restricted (R) and unrestricted (U) models refer to the ones in Table 4.1, numbers 4 and 6 refer to the number of Legendre factors, while fixed λ models come from Table 4.2. The suffixes refer to the estimation procedure. All ten term structure models are estimated using each procedure in addition to the yield-AR and yield-RW, thus giving the total of 32 models.

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Table A.1: Prediction RMSE for a 1-month forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	0.61	0.57	0.64	0.65	0.65	0.66	0.63	0.62	0.56	0.51
Y-AR	1.41	1.44	1.40	1.45	1.44	1.42	1.40	1.39	1.38	1.38
AR Models										
NS-Fix-AR	1.46	1.46	1.47	1.56	1.50	1.44	1.42	1.39	1.38	1.43
NS-R-AR	1.45	1.53	1.49	1.54	1.48	1.42	1.39	1.37	1.36	1.40
NS-U-AR	1.66	1.91	1.78	1.75	1.64	1.54	1.52	1.51	1.58	1.67
BC-Fix-AR	1.97	2.55	2.25	2.15	1.98	1.84	1.77	1.67	1.58	1.58
BC-R-AR	1.71	1.93	1.88	1.90	1.74	1.63	1.59	1.53	1.49	1.54
BC-U-AR	2.63	3.20	2.93	2.90	2.67	2.47	2.40	2.27	2.24	2.35
AS-R-AR	1.62	1.87	1.70	1.64	1.53	1.46	1.49	1.52	1.67	1.78
AS-U-AR	2.41	2.96	2.72	2.57	2.27	2.09	2.10	2.11	2.33	2.55
LE-4-AR	1.41	1.44	1.41	1.46	1.43	1.39	1.39	1.38	1.38	1.38
LE-6-AR	1.41	1.45	1.40	1.46	1.43	1.40	1.39	1.37	1.36	1.39
VAR Models										
NS-Fix-VAR	1.06	1.04	1.11	1.12	1.09	1.04	1.02	1.01	1.01	1.06
NS-R-VAR	1.11	1.10	1.16	1.15	1.14	1.09	1.08	1.08	1.08	1.12
NS-U-VAR	1.19	1.12	1.11	1.13	1.17	1.16	1.18	1.22	1.29	1.37
BC-Fix-VAR	1.10	1.15	1.12	1.13	1.14	1.09	1.06	1.05	1.04	1.07
BC-R-VAR	1.16	1.06	1.17	1.24	1.20	1.16	1.15	1.13	1.14	1.20
BC-U-VAR	1.60	1.26	1.39	1.57	1.58	1.57	1.64	1.65	1.78	2.00
AS-R-VAR	1.20	1.12	1.13	1.12	1.14	1.12	1.16	1.23	1.37	1.49
AS-U-VAR	1.63	1.42	1.58	1.69	1.66	1.57	1.56	1.59	1.73	1.88
LE-4-VAR	1.07	1.12	1.12	1.10	1.09	1.05	1.04	1.04	1.05	1.06
LE-6-VAR	1.12	1.20	1.17	1.15	1.14	1.10	1.08	1.07	1.06	1.08
State-Space Models										
NS-Fix-SSM	1.05	1.05	1.08	1.08	1.06	1.01	1.01	1.01	1.03	1.12
NS-R-SSM	1.05	1.05	1.08	1.08	1.06	1.01	1.01	1.01	1.03	1.12
NS-U-SSM	1.05	1.05	1.08	1.08	1.06	1.01	1.01	1.01	1.03	1.12
BC-Fix-SSM	1.14	1.16	1.23	1.21	1.17	1.11	1.09	1.07	1.07	1.14
BC-R-SSM	1.09	1.14	1.16	1.13	1.10	1.05	1.03	1.03	1.06	1.14
BC-U-SSM	1.19	1.24	1.23	1.23	1.20	1.15	1.14	1.14	1.17	1.25
AS-R-SSM	1.09	1.14	1.13	1.12	1.10	1.06	1.04	1.04	1.04	1.08
AS-U-SSM	1.11	1.21	1.18	1.14	1.10	1.05	1.03	1.04	1.07	1.16
LE-4-SSM	1.06	1.12	1.11	1.09	1.07	1.02	1.01	1.01	1.02	1.04
LE-6-SSM	1.08	1.16	1.14	1.11	1.09	1.05	1.04	1.03	1.03	1.05

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.2: Prediction RMSE for a 3-month forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	0.95	0.90	0.97	1.03	1.03	1.01	0.97	0.95	0.86	0.81
Y-AR	1.12	1.19	1.16	1.13	1.10	1.09	1.09	1.09	1.12	1.15
AR Models										
NS-Fix-AR	1.25	1.32	1.33	1.31	1.23	1.19	1.20	1.18	1.19	1.22
NS-R-AR	1.24	1.38	1.37	1.32	1.23	1.18	1.18	1.15	1.14	1.16
NS-U-AR	1.46	1.73	1.66	1.52	1.39	1.32	1.33	1.31	1.36	1.43
BC-Fix-AR	1.89	2.64	2.37	2.06	1.82	1.68	1.61	1.51	1.43	1.40
BC-R-AR	1.63	2.09	1.99	1.79	1.59	1.48	1.43	1.36	1.31	1.31
BC-U-AR	2.24	2.98	2.72	2.41	2.16	2.02	1.96	1.85	1.81	1.82
AS-R-AR	1.49	1.86	1.74	1.55	1.39	1.31	1.32	1.32	1.41	1.49
AS-U-AR	2.18	2.83	2.68	2.38	2.10	1.94	1.88	1.79	1.78	1.83
LE-4-AR	1.11	1.23	1.22	1.15	1.07	1.04	1.05	1.04	1.07	1.15
LE-6-AR	1.11	1.24	1.21	1.15	1.08	1.04	1.04	1.03	1.07	1.14
VAR Models										
NS-Fix-VAR	1.20	1.22	1.28	1.26	1.22	1.18	1.17	1.16	1.14	1.17
NS-R-VAR	1.17	1.20	1.26	1.22	1.17	1.14	1.13	1.12	1.10	1.11
NS-U-VAR	1.09	1.14	1.14	1.09	1.05	1.02	1.03	1.05	1.11	1.19
BC-Fix-VAR	1.28	1.40	1.37	1.33	1.29	1.25	1.23	1.20	1.17	1.18
BC-R-VAR	1.17	1.16	1.23	1.24	1.19	1.16	1.15	1.14	1.12	1.14
BC-U-VAR	1.32	1.24	1.29	1.30	1.27	1.27	1.31	1.33	1.42	1.54
AS-R-VAR	1.14	1.14	1.16	1.11	1.06	1.05	1.09	1.13	1.23	1.33
AS-U-VAR	1.17	1.21	1.22	1.17	1.12	1.10	1.11	1.13	1.21	1.35
LE-4-VAR	1.17	1.26	1.26	1.21	1.17	1.14	1.13	1.12	1.10	1.13
LE-6-VAR	1.17	1.24	1.25	1.21	1.18	1.15	1.13	1.11	1.11	1.11
State-Space Models										
NS-Fix-SSM	1.09	1.12	1.16	1.13	1.08	1.05	1.05	1.05	1.05	1.09
NS-R-SSM	1.09	1.12	1.16	1.13	1.08	1.05	1.05	1.05	1.05	1.09
NS-U-SSM	1.09	1.12	1.16	1.13	1.08	1.05	1.05	1.05	1.05	1.09
BC-Fix-SSM	1.26	1.37	1.43	1.37	1.28	1.22	1.19	1.15	1.10	1.11
BC-R-SSM	1.17	1.34	1.33	1.23	1.16	1.11	1.09	1.08	1.06	1.08
BC-U-SSM	1.33	1.51	1.52	1.43	1.32	1.25	1.22	1.20	1.20	1.25
AS-R-SSM	1.19	1.29	1.29	1.23	1.18	1.14	1.13	1.12	1.11	1.14
AS-U-SSM	1.11	1.25	1.22	1.14	1.08	1.04	1.04	1.04	1.07	1.13
LE-4-SSM	1.11	1.20	1.20	1.15	1.11	1.08	1.07	1.06	1.05	1.08
LE-6-SSM	1.14	1.21	1.22	1.18	1.14	1.11	1.10	1.09	1.08	1.10

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.3: Prediction RMSE for a 6-month forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	1.32	1.39	1.44	1.48	1.43	1.36	1.28	1.24	1.12	1.06
Y-AR	1.16	1.17	1.18	1.15	1.14	1.16	1.15	1.14	1.14	1.15
AR Models										
NS-Fix-AR	1.26	1.27	1.31	1.30	1.25	1.24	1.24	1.22	1.21	1.21
NS-R-AR	1.24	1.28	1.32	1.29	1.24	1.22	1.22	1.19	1.18	1.18
NS-U-AR	1.40	1.58	1.55	1.45	1.35	1.32	1.31	1.28	1.28	1.30
BC-Fix-AR	1.73	2.12	2.01	1.83	1.69	1.62	1.57	1.48	1.40	1.36
BC-R-AR	1.58	1.87	1.82	1.67	1.54	1.48	1.44	1.38	1.33	1.31
BC-U-AR	1.80	2.18	2.06	1.88	1.74	1.68	1.64	1.56	1.52	1.51
AS-R-AR	1.39	1.63	1.58	1.44	1.32	1.28	1.26	1.24	1.28	1.32
AS-U-AR	1.77	2.15	2.09	1.89	1.71	1.62	1.57	1.49	1.46	1.46
LE-4-AR	1.15	1.22	1.22	1.17	1.11	1.10	1.10	1.09	1.10	1.15
LE-6-AR	1.14	1.23	1.22	1.17	1.12	1.10	1.09	1.07	1.09	1.14
VAR Models										
NS-Fix-VAR	1.27	1.24	1.30	1.30	1.27	1.26	1.26	1.24	1.24	1.25
NS-R-VAR	1.16	1.18	1.22	1.18	1.15	1.14	1.14	1.13	1.13	1.14
NS-U-VAR	1.05	1.10	1.10	1.05	1.01	1.01	1.01	1.02	1.06	1.12
BC-Fix-VAR	1.31	1.37	1.35	1.32	1.31	1.30	1.29	1.26	1.24	1.24
BC-R-VAR	1.15	1.15	1.19	1.18	1.15	1.15	1.14	1.13	1.13	1.14
BC-U-VAR	1.20	1.18	1.20	1.19	1.16	1.17	1.19	1.20	1.26	1.34
AS-R-VAR	1.10	1.11	1.12	1.07	1.03	1.03	1.07	1.10	1.19	1.27
AS-U-VAR	1.07	1.15	1.14	1.06	1.01	**1.00**	1.01	1.02	1.09	1.19
LE-4-VAR	1.15	1.20	1.20	1.16	1.13	1.13	1.13	1.11	1.11	1.11
LE-6-VAR	1.20	1.23	1.24	1.21	1.20	1.20	1.19	1.17	1.17	1.16
State-Space Models										
NS-Fix-SSM	1.09	1.12	1.14	1.11	1.07	1.06	1.07	1.06	1.07	1.09
NS-R-SSM	1.09	1.12	1.14	1.11	1.07	1.06	1.07	1.06	1.07	1.09
NS-U-SSM	1.09	1.12	1.14	1.11	1.07	1.06	1.07	1.06	1.07	1.09
BC-Fix-SSM	1.26	1.33	1.37	1.33	1.27	1.23	1.20	1.17	1.12	1.11
BC-R-SSM	1.21	1.38	1.35	1.26	1.19	1.15	1.14	1.10	1.07	1.06
BC-U-SSM	1.32	1.43	1.47	1.40	1.32	1.28	1.24	1.20	1.17	1.18
AS-R-SSM	1.16	1.22	1.23	1.18	1.14	1.12	1.12	1.11	1.11	1.13
AS-U-SSM	1.08	1.16	1.14	1.08	1.04	1.04	1.04	1.04	1.07	1.11
LE-4-SSM	1.09	1.14	1.15	1.11	1.07	1.06	1.06	1.05	1.05	1.06
LE-6-SSM	1.14	1.18	1.19	1.16	1.13	1.12	1.13	1.11	1.11	1.11

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.4: Prediction RMSE for a 1-year forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	1.98	2.27	2.27	2.26	2.15	1.99	1.87	1.77	1.58	1.46
Y-AR	1.08	1.09	1.09	1.07	1.06	1.08	1.07	1.07	1.07	1.08
AR Models										
NS-Fix-AR	1.08	1.05	1.10	1.10	1.07	1.08	1.08	1.07	1.08	1.10
NS-R-AR	1.03	1.02	1.06	1.05	1.02	1.03	1.03	1.02	1.03	1.06
NS-U-AR	1.11	1.22	1.22	1.14	1.07	1.06	1.04	1.02	1.02	1.05
BC-Fix-AR	1.33	1.46	1.44	1.36	1.29	1.28	1.25	1.21	1.19	1.19
BC-R-AR	1.26	1.38	1.38	1.31	1.23	1.21	1.19	1.16	1.14	1.15
BC-U-AR	1.27	1.41	1.39	1.30	1.23	1.21	1.18	1.15	1.13	1.14
AS-R-AR	1.12	1.23	1.24	1.16	1.08	1.05	1.03	1.01	1.03	1.07
AS-U-AR	1.27	1.44	1.44	1.33	1.22	1.18	1.15	1.11	1.10	1.12
LE-4-AR	1.08	1.11	1.11	1.07	1.03	1.04	1.05	1.05	1.08	1.15
LE-6-AR	1.08	1.11	1.11	1.07	1.04	1.04	1.04	1.04	1.08	1.14
VAR Models										
NS-Fix-VAR	1.27	1.20	1.26	1.28	1.27	1.28	1.28	1.28	1.29	1.31
NS-R-VAR	1.09	1.12	1.14	1.10	1.07	1.07	1.06	1.06	1.07	1.10
NS-U-VAR	**0.96**	1.03	1.03	**0.97**	**0.92**	**0.91**	**0.91**	**0.91**	**0.94**	**0.99**
BC-Fix-VAR	1.29	1.27	1.28	1.28	1.28	1.30	1.30	1.29	1.29	1.29
BC-R-VAR	1.09	1.11	1.13	1.10	1.07	1.07	1.07	1.06	1.07	1.10
BC-U-VAR	1.05	1.09	1.09	1.05	1.02	1.02	1.03	1.02	1.05	1.11
AS-R-VAR	**0.98**	1.01	1.02	**0.96**	**0.92**	**0.92**	**0.94**	**0.95**	1.02	1.10
AS-U-VAR	**0.98**	1.06	1.06	**0.99**	**0.93**	**0.91**	**0.90**	**0.90**	**0.94**	1.00
LE-4-VAR	1.06	1.10	1.10	1.05	1.03	1.03	1.04	1.03	1.04	1.06
LE-6-VAR	1.16	1.15	1.16	1.15	1.15	1.17	1.17	1.16	1.18	1.18
State-Space Models										
NS-Fix-SSM	**1.00**	1.02	1.04	1.00	**0.97**	**0.97**	**0.98**	**0.97**	**0.99**	1.03
NS-R-SSM	**1.00**	1.02	1.04	1.00	**0.97**	**0.97**	**0.98**	**0.97**	**0.99**	1.03
NS-U-SSM	**1.00**	1.02	1.04	1.00	**0.97**	**0.97**	**0.98**	**0.97**	**0.99**	1.03
BC-Fix-SSM	1.10	1.13	1.17	1.14	1.10	1.08	1.06	1.04	1.02	1.03
BC-R-SSM	1.06	1.14	1.14	1.08	1.02	1.02	1.01	**0.99**	**0.99**	1.00
BC-U-SSM	1.21	1.26	1.31	1.26	1.20	1.17	1.14	1.11	1.08	1.10
AS-R-SSM	1.07	1.12	1.13	1.09	1.05	1.04	1.04	1.03	1.04	1.07
AS-U-SSM	1.00	1.05	1.05	1.00	**0.96**	**0.96**	**0.97**	**0.97**	**0.99**	1.03
LE-4-SSM	**0.99**	1.02	1.03	**0.99**	**0.95**	**0.96**	**0.96**	**0.96**	**0.97**	1.00
LE-6-SSM	1.03	1.05	1.06	1.03	1.00	1.01	1.01	1.00	1.02	1.04

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.5: Prediction RMSE for a 2-year forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	2.44	2.90	2.86	2.77	2.58	2.41	2.27	2.14	1.93	1.83
Y-AR	1.06	1.11	1.10	1.08	1.06	1.04	1.02	**1.00**	**0.98**	**0.97**
AR Models										
NS-Fix-AR	1.04	1.01	1.06	1.07	1.04	1.03	1.03	1.02	1.01	1.01
NS-R-AR	**0.98**	**0.99**	1.03	1.01	**0.98**	**0.97**	**0.96**	**0.95**	**0.93**	**0.94**
NS-U-AR	1.03	1.11	1.12	1.07	1.02	**0.98**	**0.96**	**0.94**	**0.92**	**0.92**
BC-Fix-AR	1.13	1.20	1.21	1.17	1.13	1.11	1.09	1.06	1.03	1.01
BC-R-AR	1.09	1.16	1.18	1.14	1.09	1.06	1.04	1.02	**0.99**	**0.97**
BC-U-AR	1.07	1.17	1.16	1.11	1.06	1.03	1.01	**0.98**	**0.94**	**0.93**
AS-R-AR	1.05	1.13	1.15	1.10	1.03	**0.99**	**0.97**	**0.95**	**0.94**	**0.95**
AS-U-AR	1.10	1.20	1.23	1.17	1.10	1.05	1.01	**0.98**	**0.94**	**0.93**
LE-4-AR	1.05	1.08	1.09	1.06	1.03	1.03	1.03	1.02	1.03	1.05
LE-6-AR	1.05	1.08	1.09	1.06	1.04	1.03	1.03	1.02	1.03	1.05
VAR Models										
NS-Fix-VAR	1.81	1.39	1.59	1.79	1.90	1.95	1.98	1.99	2.02	2.02
NS-R-VAR	1.10	1.15	1.16	1.13	1.09	1.07	1.06	1.05	1.04	1.04
NS-U-VAR	**0.99**	1.07	1.07	1.02	**0.97**	**0.95**	**0.93**	**0.92**	**0.91**	**0.91**
BC-Fix-VAR	1.71	1.43	1.53	1.67	1.79	1.84	1.87	1.87	1.86	1.82
BC-R-VAR	1.10	1.15	1.16	1.13	1.10	1.08	1.07	1.05	1.04	1.03
BC-U-VAR	1.02	1.11	1.10	1.05	1.01	**0.99**	**0.97**	**0.95**	**0.93**	**0.92**
AS-R-VAR	**0.99**	1.04	1.06	1.02	**0.97**	**0.95**	**0.94**	**0.93**	**0.95**	**0.97**
AS-U-VAR	1.04	1.12	1.14	1.09	1.03	**1.00**	**0.97**	**0.95**	**0.93**	**0.93**
LE-4-VAR	1.05	1.08	1.09	1.06	1.04	1.03	1.03	1.02	1.01	1.00
LE-6-VAR	1.36	1.21	1.27	1.34	1.40	1.43	1.45	1.46	1.47	1.43
State-Space Models										
NS-Fix-SSM	1.00	1.04	1.05	1.02	**0.99**	**0.97**	**0.97**	**0.96**	**0.95**	**0.95**
NS-R-SSM	1.00	1.04	1.05	1.02	**0.99**	**0.97**	**0.97**	**0.96**	**0.95**	**0.95**
NS-U-SSM	1.00	1.04	1.05	1.02	**0.99**	**0.97**	**0.97**	**0.96**	**0.95**	**0.95**
BC-Fix-SSM	1.08	1.09	1.13	1.12	1.09	1.07	1.05	1.03	1.00	**0.99**
BC-R-SSM	1.04	1.10	1.11	1.07	1.03	1.00	**0.99**	**0.98**	**0.96**	**0.95**
BC-U-SSM	1.10	1.16	1.20	1.17	1.12	1.07	1.03	**0.99**	**0.94**	**0.92**
AS-R-SSM	1.06	1.10	1.12	1.09	1.06	1.04	1.04	1.02	1.01	1.01
AS-U-SSM	1.05	1.12	1.12	1.08	1.04	1.01	1.00	**0.99**	**0.97**	**0.97**
LE-4-SSM	**1.00**	1.04	1.05	1.02	**0.99**	**0.97**	**0.97**	**0.96**	**0.95**	**0.95**
LE-6-SSM	1.02	1.05	1.06	1.04	1.01	**0.99**	**0.99**	**0.98**	**0.97**	**0.97**

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.6: Prediction RMSE for a 3-year forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	2.79	3.21	3.21	3.16	2.96	2.78	2.63	2.48	2.26	2.16
Y-AR	1.01	1.12	1.09	1.05	1.01	**0.97**	**0.94**	**0.92**	**0.89**	**0.87**
AR Models										
NS-Fix-AR	**0.99**	1.01	1.04	1.02	**0.99**	**0.97**	**0.95**	**0.94**	**0.92**	**0.92**
NS-R-AR	**0.95**	1.01	1.03	**0.99**	**0.94**	**0.92**	**0.90**	**0.88**	**0.86**	**0.85**
NS-U-AR	**0.97**	1.08	1.08	1.01	**0.95**	**0.92**	**0.89**	**0.87**	**0.84**	**0.83**
BC-Fix-AR	1.04	1.14	1.13	1.08	1.03	1.00	**0.98**	**0.95**	**0.92**	**0.90**
BC-R-AR	1.01	1.10	1.11	1.06	1.01	**0.98**	**0.95**	**0.92**	**0.89**	**0.87**
BC-U-AR	**0.99**	1.12	1.10	1.04	**0.98**	**0.95**	**0.92**	**0.89**	**0.84**	**0.83**
AS-R-AR	**0.98**	1.10	1.10	1.04	**0.97**	**0.92**	**0.89**	**0.87**	**0.85**	**0.86**
AS-U-AR	1.02	1.15	1.16	1.09	1.01	**0.96**	**0.93**	**0.90**	**0.86**	**0.85**
LE-4-AR	1.01	1.07	1.07	1.03	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**	**0.94**
LE-6-AR	1.01	1.07	1.07	1.03	**1.00**	**0.98**	**0.97**	**0.95**	**0.94**	**0.94**
VAR Models										
NS-Fix-VAR	3.52	2.18	2.78	3.41	3.75	3.93	4.00	4.07	4.14	4.11
NS-R-VAR	1.09	1.16	1.18	1.13	1.08	1.06	1.03	1.01	**0.99**	**0.97**
NS-U-VAR	**0.98**	1.08	1.07	1.02	**0.96**	**0.93**	**0.91**	**0.88**	**0.86**	**0.84**
BC-Fix-VAR	3.00	2.14	2.42	2.87	3.19	3.36	3.42	3.44	3.42	3.32
BC-R-VAR	1.10	1.18	1.18	1.14	1.10	1.07	1.04	1.02	**0.99**	**0.97**
BC-U-VAR	1.01	1.12	1.10	1.05	1.00	**0.97**	**0.94**	**0.91**	**0.87**	**0.85**
AS-R-VAR	**0.98**	1.08	1.08	1.03	**0.97**	**0.93**	**0.91**	**0.89**	**0.87**	**0.87**
AS-U-VAR	1.02	1.13	1.14	1.08	1.01	**0.97**	**0.94**	**0.91**	**0.88**	**0.86**
LE-4-VAR	1.01	1.06	1.07	1.03	**1.00**	**0.99**	**0.98**	**0.96**	**0.93**	**0.92**
LE-6-VAR	1.85	1.41	1.57	1.77	1.92	2.02	2.06	2.08	2.11	2.01
State-Space Models										
NS-Fix-SSM	**0.98**	1.06	1.06	1.01	**0.96**	**0.94**	**0.92**	**0.90**	**0.88**	**0.87**
NS-R-SSM	**0.98**	1.06	1.06	1.01	**0.96**	**0.94**	**0.92**	**0.90**	**0.88**	**0.87**
NS-U-SSM	**0.98**	1.06	1.06	1.01	**0.96**	**0.94**	**0.92**	**0.90**	**0.88**	**0.87**
BC-Fix-SSM	1.02	1.07	1.10	1.08	1.03	1.00	**0.97**	**0.94**	**0.91**	**0.89**
BC-R-SSM	1.01	1.11	1.11	1.05	**1.00**	**0.97**	**0.94**	**0.92**	**0.89**	**0.88**
BC-U-SSM	1.11	1.19	1.23	1.20	1.14	1.09	1.04	**0.99**	**0.92**	**0.88**
AS-R-SSM	1.02	1.09	1.10	1.06	1.01	**0.99**	**0.97**	**0.95**	**0.93**	**0.91**
AS-U-SSM	1.04	1.16	1.15	1.08	1.02	**0.99**	**0.96**	**0.94**	**0.91**	**0.88**
LE-4-SSM	**0.97**	1.06	1.06	1.01	**0.96**	**0.94**	**0.92**	**0.90**	**0.88**	**0.87**
LE-6-SSM	**0.96**	1.05	1.05	1.00	**0.95**	**0.93**	**0.91**	**0.89**	**0.87**	**0.85**

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.7: Prediction RMSE for a 4-year forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	2.90	3.18	3.21	3.26	3.08	2.94	2.81	2.69	2.45	2.35
Y-AR	1.01	1.19	1.14	1.07	1.00	**0.95**	**0.91**	**0.88**	**0.85**	**0.84**
AR Models										
NS-Fix-AR	1.05	1.13	1.15	1.09	1.04	1.01	**0.99**	**0.96**	**0.95**	**0.94**
NS-R-AR	**0.97**	1.10	1.10	1.02	**0.95**	**0.91**	**0.88**	**0.85**	**0.83**	**0.82**
NS-U-AR	**0.97**	1.16	1.14	1.03	**0.95**	**0.90**	**0.87**	**0.83**	**0.80**	**0.78**
BC-Fix-AR	1.07	1.25	1.22	1.12	1.06	1.01	**0.98**	**0.94**	**0.91**	**0.88**
BC-R-AR	1.03	1.20	1.19	1.09	1.02	**0.97**	**0.93**	**0.89**	**0.86**	**0.84**
BC-U-AR	1.01	1.21	1.18	1.07	**1.00**	**0.94**	**0.90**	**0.85**	**0.80**	**0.78**
AS-R-AR	1.00	1.19	1.18	1.06	**0.98**	**0.91**	**0.87**	**0.84**	**0.83**	**0.83**
AS-U-AR	1.04	1.25	1.24	1.12	1.02	**0.95**	**0.90**	**0.85**	**0.82**	**0.79**
LE-4-AR	1.00	1.15	1.13	1.04	**0.98**	**0.95**	**0.93**	**0.90**	**0.88**	**0.88**
LE-6-AR	1.00	1.15	1.13	1.04	**0.98**	**0.95**	**0.92**	**0.89**	**0.89**	**0.88**
VAR Models										
NS-Fix-VAR	8.99	5.31	7.08	8.69	9.62	9.98	10.12	10.17	10.42	10.27
NS-R-VAR	1.13	1.30	1.29	1.19	1.12	1.07	1.04	1.00	**0.98**	**0.96**
NS-U-VAR	**0.96**	1.13	1.11	1.01	**0.94**	**0.89**	**0.86**	**0.83**	**0.80**	**0.79**
BC-Fix-VAR	7.01	4.81	5.58	6.65	7.48	7.82	7.92	7.90	7.92	7.63
BC-R-VAR	1.15	1.33	1.31	1.21	1.14	1.09	1.05	1.01	**0.98**	**0.95**
BC-U-VAR	1.00	1.20	1.16	1.06	**0.99**	**0.94**	**0.90**	**0.85**	**0.81**	**0.79**
AS-R-VAR	**0.97**	1.14	1.13	1.03	**0.95**	**0.90**	**0.87**	**0.84**	**0.83**	**0.84**
AS-U-VAR	1.02	1.21	1.20	1.08	**1.00**	**0.94**	**0.89**	**0.85**	**0.82**	**0.80**
LE-4-VAR	1.01	1.13	1.12	1.04	1.00	**0.98**	**0.96**	**0.93**	**0.90**	**0.89**
LE-6-VAR	3.35	2.25	2.67	3.15	3.53	3.71	3.79	3.81	3.91	3.69
State-Space Models										
NS-Fix-SSM	**0.99**	1.15	1.14	1.04	**0.98**	**0.93**	**0.90**	**0.87**	**0.85**	**0.83**
NS-R-SSM	**0.99**	1.15	1.14	1.04	**0.98**	**0.93**	**0.90**	**0.87**	**0.85**	**0.83**
NS-U-SSM	**0.99**	1.15	1.14	1.04	**0.98**	**0.93**	**0.90**	**0.87**	**0.85**	**0.83**
BC-Fix-SSM	1.07	1.20	1.22	1.14	1.08	1.02	**0.98**	**0.93**	**0.89**	**0.86**
BC-R-SSM	1.12	1.34	1.30	1.18	1.09	1.04	**0.99**	**0.95**	**0.91**	**0.88**
BC-U-SSM	1.13	1.28	1.31	1.23	1.15	1.08	1.02	**0.95**	**0.88**	**0.83**
AS-R-SSM	1.06	1.19	1.19	1.10	1.05	1.01	**0.99**	**0.96**	**0.94**	**0.92**
AS-U-SSM	1.09	1.28	1.25	1.14	1.07	1.02	**0.99**	**0.95**	**0.92**	**0.89**
LE-4-SSM	**0.99**	1.15	1.13	1.03	**0.97**	**0.93**	**0.90**	**0.87**	**0.84**	**0.83**
LE-6-SSM	**1.00**	1.15	1.14	1.05	**0.98**	**0.94**	**0.91**	**0.88**	**0.86**	**0.84**

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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Table A.8: Prediction RMSE for a 5-year forecast horizon

	RMSE									
Maturity	**T**	6m	1y	2y	3y	4y	5y	6y	8y	10y
Yield Forecast										
Y-RW	3.19	3.68	3.69	3.66	3.40	3.19	3.00	2.83	2.56	2.45
Y-AR	**0.99**	1.14	1.10	1.04	**0.98**	**0.93**	**0.90**	**0.87**	**0.85**	**0.84**
AR Models										
NS-Fix-AR	1.07	1.15	1.16	1.10	1.05	1.02	1.01	**0.99**	**0.98**	**0.99**
NS-R-AR	**0.96**	1.09	1.08	1.01	**0.94**	**0.90**	**0.88**	**0.85**	**0.83**	**0.82**
NS-U-AR	**0.95**	1.11	1.09	**0.99**	**0.92**	**0.88**	**0.85**	**0.81**	**0.79**	**0.78**
BC-Fix-AR	1.05	1.20	1.17	1.09	1.04	**0.99**	**0.97**	**0.93**	**0.90**	**0.89**
BC-R-AR	1.01	1.14	1.13	1.06	**1.00**	**0.95**	**0.92**	**0.89**	**0.86**	**0.84**
BC-U-AR	1.00	1.17	1.13	1.05	**0.99**	**0.94**	**0.90**	**0.86**	**0.81**	**0.79**
AS-R-AR	**0.98**	1.14	1.13	1.03	**0.95**	**0.89**	**0.84**	**0.81**	**0.79**	**0.80**
AS-U-AR	1.01	1.20	1.18	1.08	**0.99**	**0.92**	**0.88**	**0.83**	**0.79**	**0.78**
LE-4-AR	**0.98**	1.12	1.10	1.01	**0.96**	**0.92**	**0.90**	**0.87**	**0.84**	**0.83**
LE-6-AR	**0.98**	1.12	1.10	1.01	**0.96**	**0.92**	**0.89**	**0.86**	**0.84**	**0.83**
VAR Models										
NS-Fix-VAR	22.14	12.15	16.52	20.92	23.66	24.97	25.71	26.26	27.13	26.93
NS-R-VAR	1.11	1.27	1.26	1.15	1.09	1.04	1.01	**0.98**	**0.95**	**0.94**
NS-U-VAR	**0.92**	1.08	1.06	**0.96**	**0.90**	**0.85**	**0.82**	**0.79**	**0.77**	**0.76**
BC-Fix-VAR	15.85	10.13	11.92	14.68	16.89	17.98	18.49	18.75	18.96	18.41
BC-R-VAR	1.16	1.34	1.32	1.20	1.14	1.08	1.05	1.01	**0.97**	**0.95**
BC-U-VAR	**0.96**	1.14	1.10	1.01	**0.94**	**0.89**	**0.86**	**0.82**	**0.77**	**0.76**
AS-R-VAR	**0.95**	1.11	1.09	1.00	**0.92**	**0.87**	**0.83**	**0.80**	**0.80**	**0.81**
AS-U-VAR	**0.98**	1.16	1.15	1.04	**0.96**	**0.89**	**0.85**	**0.81**	**0.78**	**0.77**
LE-4-VAR	1.01	1.12	1.10	1.03	**0.99**	**0.96**	**0.94**	**0.92**	**0.89**	**0.89**
LE-6-VAR	6.07	3.61	4.44	5.52	6.39	6.87	7.14	7.32	7.59	7.21
State-Space Models										
NS-Fix-SSM	**0.97**	1.11	1.10	1.01	**0.95**	**0.91**	**0.88**	**0.86**	**0.83**	**0.83**
NS-R-SSM	**0.97**	1.11	1.10	1.01	**0.95**	**0.91**	**0.88**	**0.86**	**0.83**	**0.83**
NS-U-SSM	**0.97**	1.11	1.10	1.01	**0.95**	**0.91**	**0.88**	**0.86**	**0.83**	**0.83**
BC-Fix-SSM	1.07	1.20	1.21	1.13	1.07	1.02	**0.99**	**0.94**	**0.90**	**0.87**
BC-R-SSM	1.20	1.42	1.38	1.25	1.17	1.11	1.06	1.02	**0.98**	**0.95**
BC-U-SSM	1.15	1.27	1.30	1.23	1.17	1.10	1.04	**0.98**	**0.90**	**0.86**
AS-R-SSM	1.09	1.23	1.22	1.13	1.06	1.02	**1.00**	**0.97**	**0.95**	**0.94**
AS-U-SSM	1.04	1.20	1.17	1.07	1.01	**0.97**	**0.94**	**0.91**	**0.89**	**0.87**
LE-4-SSM	**0.98**	1.12	1.10	1.01	**0.95**	**0.91**	**0.89**	**0.86**	**0.83**	**0.83**
LE-6-SSM	**0.99**	1.13	1.12	1.03	**0.97**	**0.93**	**0.90**	**0.88**	**0.85**	**0.84**

Figures in **bold** indicate outperformance of the random walk (Y-RW) model

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