From One to one, to many:
the *Parmenides* 142b-144b

Florin George Călian

Department of Philosophy

Central European University

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Supervisor: Professor Gábor Betegh

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Declaration

This dissertation is the result of my own work and includes nothing, which is the outcome of work done in collaboration except where specifically indicated in the text. It has not been previously submitted, in part or whole, to any university of institution for any degree, diploma, or other qualification.

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Abstract

This dissertation is a contribution to the reassessment of Platonic ontology and mathematical conceptions as found in the beginning of the second hypothesis of the second part of the *Parmenides* (142b-144b), in what I argue to be Plato's ontological argument and the argument for the generation of numbers. I support the reading of these two specific arguments as a coherent continuum: how the *being* of *one* was conceived by Plato as necessarily different from *one*, differentiating thus himself from Eleatic philosophy and setting the grounds for multiplicity, which leads Plato into a discussion on the *primordia* of numbers. There are numbers (the mathematical argument) only because *one* and *being* can be separated (the ontological argument). My analysis of the argument for the generation of numbers is substantiated by an evaluation of Aristotle's testimony in *Metaphysics* A6, a testimony which, I argue, is built upon this passage of the *Parmenides*. The dissertation provides an analytical commentary on the stages of the arguments, in an attempt to place the arguments on the map of Plato's philosophy. I demonstrate that the ontological and mathematical arguments are actual Platonic arguments, and not merely dialectic exercises, which have traceable conclusions in Plato's philosophy of the so-called late dialogues, especially the *Theaetetus*, the *Sophist*, and the *Timaeus*.

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Special thanks must go to my two elective affinities, to my wife Antoaneta, who sensibly put up with my writing blocks and other caprices all these years, and to our daughter, Sophie.

This thesis is dedicated to Ciprian Aristotel (1977-2000)

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"*If someone were to reduce Plato to a system, he would render a great service to mankind…*" Leibniz (*Philosophical Papers and Letters*, 1989, 659)

"*There is an argument, suggested by a passage in Plato's* Parmenides*, to the effect that, if there is such a number as 1, then 1 has being; but 1 is not identical with being, and therefore 1 and being are two, and therefore there is such a number as 2, and 2 together with 1 and being gives a class of three terms, and so on. This argument is fallacious, partly because "being" is not a term having any definite meaning, and still more because, if a definite meaning were invented for it, it would be found that numbers do not have being – they are, in fact, what are called logical fictions'.*" Russell (*Introduction to Mathematical Philosophy*, 1993, 138)

"*Anybody who sets out to report Plato's opinions can properly be asked to tell us on what principles he interprets the evidence at his disposal*" Crombie (*An Examination of Plato's Doctrines*, 1962, 14)

1. The Parmenides Question

The *Parmenides* is a key Platonic dialogue, which raises controversial and contradictory interpretations. The first part of the dialogue contains both the most coherent and detailed depiction of one central version of the theory of forms as it is to be found in the middle dialogues and, at the same time, the most upsetting counter-arguments to the theory.[1] Each counter-argument leaves us with the impression of undermining the edifice of the theory of forms as it is displayed in the *Phaedo* and the *Republic*. The first part of the *Parmenides* does not reach any theoretical philosophical conclusion regarding the ontological status of the forms, except that they are necessary for thought and speech. The second part of the dialogue is introduced by Plato as a necessary dialectical exercise for a better training in philosophy. How to understand these complicated structures and Plato's critique of his own theory of forms is not certain at all.

One might think that referring to previous scholarship would offer some guidance in reading the *Parmenides*; however, even from within the Academy, through the Neo-Platonists, and up to contemporary commentators, there is no scholarly agreement on a generally accepted modus of interpretation: the *Parmenides* remains a puzzling dialogue[2] and there is no real consensus in respect to its

[1] I am aware that it is only a textual artifice to constantly mention *a* theory of forms; Plato's dialogues propose no definitive theory of forms that remains unchanged in the entire corpus. In several dialogues versions of the theory of forms are presented, and in different parts of Plato's work different facets of the theory are displayed with subsequent emendations, testifying thus to a continuous re-thinking process on the theory of forms. The discussions in the *Parmenides* of aspects pertaining to the theory of forms are representative for the development of the theory in what has been identified as the middle period.

[2] Mary Louise Gill, and Paul Ryan, Plato *Parmenides* (Hackett Publishing, 1996), 1.

purpose[3], topic, and techniques[4]. Similar to ancient readings of the dialogue, with the complex exception of the Neo-Platonists[5] - as even in their case the overall interpretation was not fully unanimous – contemporary studies demonstrate that the dialogue's structure and arguments are far from clear.

The second part of the dialogue has been even more puzzling and contentious for scholars. Although it formally makes up a whole with the first part of the dialogue, the articulation of these two components is highly problematic; moreover, at first glance the second part seems difficult to link not only with the first part, but with ideas expressed in Plato's other dialogues as well. This part of the dialogue remains a piece of curiosity (both in the Platonic corpus and in the history of philosophy), and it has often been left aside as not providing any reliable philosophical implications that might prove relevant for reconstructing Plato's philosophy[6].

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[3] G. Vlastos thinks that the dialogue is an example of 'honest perplexities'; see "The Third Man Argument in the Parmenides." In *Studies in Plato's Metaphysics*, ed. R. E. Allen, (London: Routledge & Kegan Paul, 1965), 231–63.

[4] G. E. L. Owen considers the dialogue as an example of sophistry. See Owen, G. E. L. "Notes on Ryle's Plato." In *Logic, Science and Dialectic*, (London: Duckworth, 1986), 85–103. The contemporary advocate of the "mining of sophistry" is Kelsey Wood. See Kelsey Wood, *Troubling Play: Meaning and Entity in Plato's Parmenides* (SUNY Press, 2012). On the other hand, for Cornford, the arguments in the *Parmenides* are not sophistry, but valid arguments. Francis Macdonald Cornford, *Plato and Parmenides: Parmenides' Way of Truth and Plato's Parmenides* (Trubner & Company Limited, 1939).

[5] For the atypical way of reading the *Parmenides* by the Neo-Platonists, especially by Proclus, see, for example, my study, which focuses mainly on the intricate prologue: "*Clarifications* of Obscurity: Conditions for Proclus's Allegorical Reading of Plato's *Parmenides*" in *Obscurity in Medieval Texts*, ed. by L. Dolezalova, J. Rider and A. Zironi (Medium Aevum Quotidianum, Sonderband 30), (Krems: Institut für Realienkunde des Mittelalters und der frühen Neuzeit, 2013), 15-31.

[6] As for example, Allen takes the dialogue as being aporetic, "akin to Beta of Aristotle's *Metaphysics*", (VII), "it presents metaphysical perplexities, not positive doctrine" (289), Reginald E. Allen, *Plato's Parmenides* (Yale University Press, 1997).

If there is a line of thought in the second part that could be linked with other conceptions from Plato's philosophy that is, I argue, the second hypothesis, with its first arguments. This hypothesis, the longest of the eight hypotheses that make up the second part of the *Parmenides*, attracts particular attention [7] through its bewildering transition from a discussion about being and oneness to an argument that seems to be discussing the generation of numbers. From this second hypothesis, my research focuses mainly on the argumentation series from 142b1 to 144a4 that consists of two core arguments, which I have called the *ontological* and *mathematical* arguments. In order to avoid any confusion, I use the term hypothesis (the Greek term for each proposal is ἡ ὑπόθεσις) for the inferences that follow from *if one is* and, respectively, *if one is not*, and the term argument for the main arguments within the hypothesis. [8] Taking the beginning of the second hypothesis as essential for a reconstruction of some key problems of Plato's philosophy, I discuss the conceptual map of the first two arguments and their possible ontological and mathematical implications for Plato's philosophy of mathematics and ontology. My dissertation proposes an interpretation of the ontological argument in its connection with the argument for the generation of numbers, arguing for the idea that these two

[7] See, for example, Kahn's comment on it: "Within this system of eight deductions, we must recognize the exceptional position of Deduction 2. Taken together with its appendix on the instant of change, Deduction 2 is as long (15 Stephanus pages) as the seven other deductions combined. Although all the deductions make some positive contribution, Deduction 2 presents philosophical thought on an entirely different scale, as an outline theory of the conceptual properties required for spatio-temporal being and becoming." Kahn, *Plato and the Post-Socratic Dialogue*, 21.

[8] These inferences are usually referred to as deductions (Kahn, Ryle, Owen, Allen, Rickless), arguments (Scolnicov), suppositions (Turnbull) or hypotheses (Conford). I refer thus to the first argument (142b5-143a2), and the second argument (143a4-144a5) for what is within the second hypothesis (142b-155e). For a rejection of the term 'hypothesis' see Scolnicov, *Plato's* Parmenides, 3.

arguments are not two separate philosophical exercises, but form a coherent whole, the main outcome of which is the conception that *one is multiple*.

The analytical reconstruction of the arguments, with a focus on the structure of the ontological argument and of the argument for the generation of numbers, is presented in Chapter Two. There are two constituents of the argument that I highlight in this outline: the ontological premises, and the mathematical premises, bound together through the commonality of the concepts they are built upon: *one*, *being* and *difference*.

Chapter Three presents an analysis of the ontological presuppositions of the argument, and tries to answer the question of why Plato conceives *one* in terms of something made up of parts. I analyse the use of the verb "to be" in the proposition *one is*, and I propose a reading based on the understanding of "to be" a complete meaning of "to be" in the first hypothesis, and for an incomplete meaning of "to be" in the second hypothesis. One further argument for confirming that the second hypothesis becomes a building block for Plato's later philosophy is the fact that it can validly be read as a refutation of the first hypothesis – which brings into discussion ideas reminiscent of Parmenidean Monism – and main conceptions from this second hypothesis are later found in a more articulated form in the *Sophist*.

Chapter Four makes the transition towards the second argument, the argument for the generation of numbers. It sketches the background for a discussion concerning Plato's philosophy of mathematics from the perspective of the history of Greek mathematics. Different topics related to ancient Greek mathematics are discussed, such as numerical notation, and ordinality versus cardinality. The chapter

discusses also Plato's relation to mathematics, especially arithmetic, the role of mathematics in philosophical training, and the connection between mathematics and dialectics.

Chapter Five is dedicated to a thorough analysis of the argument for the generation of numbers and to how Plato uses specific ontological kinds (*one*, *being* and *difference*) to reach the concept of *multiple* and, through a controversial inference, the concept of *numerosity*. *Twoness* and *threeness* are analysed as first numbers, but also as paradigmatic for *even* and *odd*. Another focus is the use of multiplication as compared to that of addition. In this context, I also discuss the problem of the missing prime numbers in the generation process. As supporting discussion, I provide a reading of the argument in the light of Aristotle's *Metaphysics* A6, where the idea that Plato understood numbers as generated, with the exception of prime numbers, is advanced.

Chapter Six underlines potential corollaries between the second hypothesis and late dialogues such as the *Theaetetus*. The chapter is an attempt to link the separate domain argument from the *Parmenides* (133a-134e) with the *Theaetetus* (185c9-d4), where sets of common determinations (in particular *being*, *difference*, *odd* and *even*) could be used as a possible way out of the epistemic and ontological dilemmas left open by the first part of the *Parmenides*.

Chapter Seven looks at the nature of generation – whether it should be understood as a theoretical account, as a logical analysis, or as a generation *per se*. I argue that Plato understood the process of generation not as a chronological process, but as an ontological axiomatization of numbers. I advance the idea that the

argument for the generation of numbers is heralding Plato's conception of the nature of one (*one is many*) and the generation of the world, as interplay between one and multiple, between ontology and arithmetic.

2. The Parmenides – Second Part, the First Two Arguments of the Second Hypothesis (142b-144b)

The second hypothesis could have significant impact on how we understand Plato's philosophy of mathematics in particular, and the ontology of the late dialogues in general. Also, it can give some hints on how to interpret the full second part of the *Parmenides*.[9] The whole hypothesis is dense with philosophical ideas.[10] Parmenides rhetorically asks (142b):

> "If one is, we are saying, aren't we, that we must agree on the consequences for it, whatever they happen to be?"[11]

The question stays as another starting-pont about the *one*, which has other consequences than those presented in the first hypothesis.[12] In the following lines I

[9] Charles Kahn points that the second hypothesis "is the centerpiece of Part Two" of the *Parmenides*", Charles H. Kahn, *Plato and the Post-Socratic Dialogue: The Return to the Philosophy of Nature* (Cambridge University Press, 2013), 38. Robert Turnbull argues for that too: "The supposition two is indeed the key to understand the second part of the *Parmenides*". Robert G. Turnbull, *The Parmenides and Plato's Late Philosophy* (University of Toronto Press, 1998), 70.

[10] As Kahn very plastically put it, in the second hypothesis, the "stream of arguments good, bad, and ingenious, is a rich flotsam of philosophical insights." Kahn, *Plato and the Post-Socratic Dialogue*, 39.

[11] For the *Parmenides*, when the text does not raise important questions, pertaining to the translation itself I quote the translation of Mary Louise Gill and Paul Ryan. I also consulted other English translations for comparison. Part of the English translations of the *Parmenides* that I inventoried are: T. Taylor (1793), G. Burges (1854), B. Jowett (1892), H.N. Fowler (1926), A.E. Taylor (1934), F.M. Cornford (1939), J. Warrington (1961), R.E. Allen (1984), K. Sayre (1996), M.L. Gill and P. Ryan (1996), A. K. Whitaker (1996), T. Turnbull (1998), A.E. Coxon (1999), S. Scolnicov (2003), A. Hermann and S. Chrysakopoulou (2010). Constantly checked were the translations of Cornford, Allen, Gill-Ryan, Turnbull and Scolnicov. With some few exceptions for the *Parmenides*, all translations from other works of Plato are quoted from the John M. Cooper *Plato: Complete Works*, unless otherwise specified.

set out analytically the first two arguments of the second hypothesis and I provide an outline by reconstructing step by step the progression of thought as found at 142b5-143a2 and 143a4-144a5. The first argument, in turn, is composed of two other demonstrations 142b-c and 142d-143a that illustrate that *one* and *being* must be conceived independently. They are two different expressions of the same ontological argument which states and restates that being is different from one (see Figure 1).

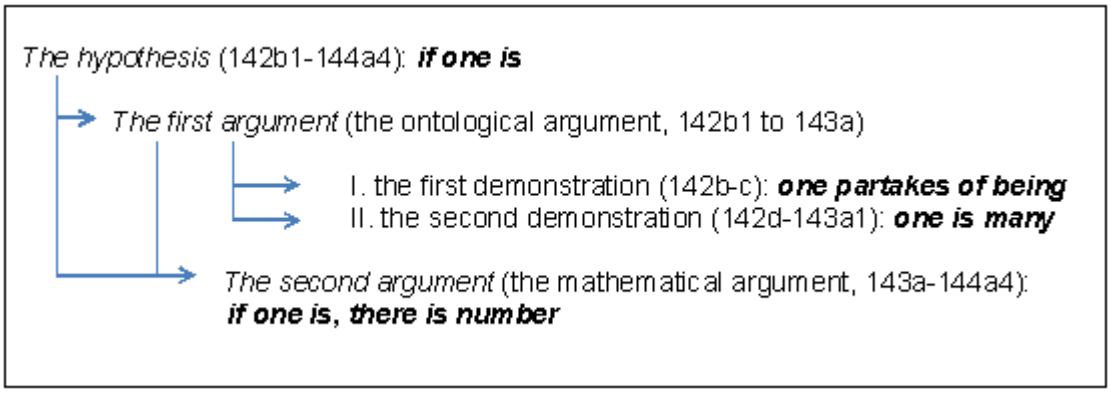

Figure 1. The structure of the arguments (142b1-144a1)

[12] The first hypothesis, "if one is" leads to negative consequences about the one: one has no parts and is not a whole (137c5–d3), has no extremities or middle (137d4–6), one does not have limits (137d6–8), no shape (137d8–138a1), is not in itself and not in another, has no contact (138a2–b6), is neither in motion nor at rest (138b7–8), not in motion (138b8–c1), not altered (138c1–4), not carried (138c4–6), does not revolve (138c6–d2), does not undergo translation (138d2-e7), is not in motion (138e7–139a3), is not at rest (139a3–b2), is neither the same as nor different from either itself or another (139b4–5), the one is not different from anything (139c3–d1), is neither different from a different nor the same as itself (139d1–e4), the one is neither like nor unlike anything (139e7–8), is neither equal nor unequal to anything (140b6–c4), one does not come to be, nor is it older or younger than or of the same age as anything (140e1–141a4), the one is not anything (141e7–10), the one is not one (141e10–142a1), the one has no relation and no name, and there is about it no discourse, knowledge, perception, or opinion (142a1–6). I have followed here the analytical differentiations of Samuel Scolnicov, *Plato's Parmenides* (University of California Press, 2003), 80-94.

The whole hypothesis, divided into two parts (142b5-143a2, and 143a4-144a5), aims to prove that the *one is multiple*, and the manner in which Plato constructs this demonstration makes use of an argument for the *generation of numbers*.

2.1. Outline for 142b-144b

The first argument (the ontological argument) 142b1 to 143a:

(142b1-5) Parmenides returns to the hypothesis from the beginning (ἐξ ἀρχῆς):

(142b) "if one is, can it *be*, but not partake of being?"

I. (142b-c) [if one is (ἓν εἰ ἔστιν), there is both one and being]

1. If the/a/ one *is*,

2. one is not the same as being (as the being of the one),

2.1. otherwise 'one is' would be the same as 'one (is) one'.

3. „is" signifies something other than "one,"

4. (therefore) ∴ one partakes (μετέχειν) of being (οὐσίας).

II. (142d-143a1)

1. one is a whole, being and one are its parts (μόρια),

2. (142e1) oneness is not absent (ἀπολείπεσθον) from the being(-ness) part, and being(-ness) is not absent from the oneness part;

3. each of the two parts possesses oneness and being, the part is composed of at least two parts, endlessly, since oneness always possesses being and being always possesses oneness.

4. (142e7-143a1) by necessity, it always comes to be two, it is never one (ἀνάγκη δὺ᾽ ἀεὶ γιγνόμενον μηδέποτε ἓν εἶναι),

5. ∴ (143a1) *the one is infinitely many* [unlimited (ἄπειρον) and multitude (πλῆθοσ)].

The second argument (the mathematical argument) 143a-144a4:

III. (143a-b) [the introduction of difference]

1. *one* is not *being*.

2. *one* has a share in *being* (ἓν οὐσίας μετέσχεν).

3. therefore one and *its* being are *different*.

4. one is not different from being by virtue of its oneness.

5. being is not different from one by virtue of being itself.

6. ∴ therefore the difference of one and being is due to difference.

6. ∴ therefore there is difference and it is distinct from one and being.

IV. (143c-144b) [Through the argument from one admitting all numbers, from "member of a pair" and "two," Plato constructs a theory for the whole number system]

7. (143c3) if we have three distinct entities we can pick out pairs (τινε)[13] (say being and difference, or being and one, or one and difference).

8. (143c4) a pair is rightly called 'both'[14] (ἀμφοτέρω) ["x", "y" = "both (x,y)"].

9. (143d2) what is called both is two (δύο) ["δύο" is identified as a set with two members corresponding to the cardinal number two].

10. (143d2-3, 4-5) each of the two is one (δύο ἦτον > ἓν εἶναι).

11. (143d7) one added to any sort of pair is three (τρία γίγνεται) [a set off three members corresponding to the cardinal number three].

(if two & three, then all the numbers)

12. (143d7-8) three is odd (Τρία... περιττὰ), and two even (δύο ἄρτια),

13. (143d9-e2) if there are two (δυοῖν), there must be twice (δὶς), since two is twice one (τῷ τε δύο τὸ δὶς ἓν),

[13] It is not clear if there is an exact correspondent in Greek for "pair". Perhaps it should be taken in the following manner: (a,b), (b,c), (a,c).

[14] Couple.

14. (143e1-2) if three (τριῶν), also thrice (τρίς), since three is thrice one (τῷ τρία τὸ τρὶς ἕν).

15. (143e3) there must be "twice two" (δύο δὶς).

16. (143e3) there must be "thrice three" (τρία τρὶς).

17. (143e5) there must be twice three (τρία δὶς) and thrice two (δύο τρίς)

18. ∴ (143e7) there *will* be even times even (ἄρτιά ἀρτιάκις), odd times odd (περιττὰ περιττάκις)[15], odd times even (ἄρτια περιττάκις), and even times odd (περιττὰ ἀρτιάκις).

19. ∴ (144a3) there is no number that does not necessarily exist.

20. (144a4) if one is, *there must also be number* (Εἰ ἄρα ἔστιν ἕν, ἀνάγκη καὶ ἀριθμὸν εἶναι).

2.2. Reading through the argument

These two arguments (142b5-143a2 and 143a4-144a5) are part of the longest hypothesis of the second part of the *Parmenides*. Both of Plato's arguments that display that *one* is *multiple* are at first glance almost unintelligible, and they seem to contradict Plato's philosophy in other dialogues, and any kind of philosophy of mathematics. Plato's intention – namely to show that one is not only one, that one is

[15] Plato uses the Attic form περιττός , ή, όν of περισσός.

a multitude – is apparently logically incongruent.[16] For the purpose of this dissertation the question of the validity of the arguments is of secondary importance since I don't aim to discuss whether Plato's argumentation is valid or not, or if he intentionally entertained a paralogism. The discussion and subsequent conclusion that the argument is fallacious do not make it less important for Plato's philosophy. In this dissertation I argue precisely that Plato uses the premises of these arguments in other dialogues and that for Plato these arguments are philosophically grounded. Also it is equally important to look at the arguments' intentions, philosophical suppositions, and inferences that can be drawn therefrom.

Demonstrating that *one* is not only *one*, both arguments have the same purpose, namely to prove that one is many (through a regress *ad infinitum* argument) and to approach the *one* from two different angles: its relation with two independent concepts, *being* and *difference*, and its relation with numbers. For some commentators we are dealing with *independent* arguments[17]. My own view is that the arguments are related and are *dependent* on each other, not only through their purpose, but also

[16] Cornford comments in this regard: "The reasoning appears to be fallacious and has been condemned as such, owing to the ambiguity of the term 'part'. Infinite divisibility is commonly applied to magnitudes. If our 'One Being' were a magnitude, we could imagine it endlessly divided into parts (smaller magnitudes) each of which would *be* and be *one*. But if […] the 'One Being' is simply 'one entity' of whatever kind, it seems illegitimate to regard its being and its unity as parts resulting from division and capable of subdivision." Francis Macdonald Cornford, *Plato and Parmenides: Parmenides' Way of Truth and Plato's Parmenides* (K. Paul, Trench, Trubner & Company Limited, 1939), 139. Or Schofield: "Through confounding the truth and the reference of the statement 'one is' in Parmenidean fashion, Plato treats 'one' and 'is' as belonging to *the one that is*, and so by an easy step takes *one* and *being* to be its parts." Malcolm Schofield, "A Neglected Regress Argument in the *Parmenides*", The Classical Quarterly, 23 (1973), 44. A similar conclusion is made by Wood also: "A difficulty for interpretation is that Parmenides in the second beginning mimes sophistry by juxtaposing different linguistic intentions in a provocative and suggestive way" Kelsey Wood, *Troubling Play: Meaning and Entity in Plato's Parmenides* (SUNY Press, 2012), 94.

[17] See, for example, Kenneth Sayre, *Plato's Late Ontology A Riddle Resolved* (Parmenides Pub., 2005), 54-58.

because of an internal relation: there are numbers (the second argument) only and only if *one* and *being* (the first argument) are separated in the first place.

The main lines along which Plato differentiates one and being and institutes numbers are:

I. although *one* participates in *being*, *one* is not *being*; one and being are two distinct entities;

II. *one* is not different from *being* because of its *oneness*, but because of *difference*. From these three entities one can generate numbers.

2.2.1. One and being

The ontological argument (142b5-143a2) has two demonstrations in which the same idea is restated, that one has one and being as its parts. As I stated earlier, I take the two demonstrations of the ontological argument as two ways of arguing the same thing, namely that *one* and *being* must be conceived independently, that one is not only one, but always comes to be two and never one. The two demonstrations are two expressions of the same argument. Here the character Parmenides argues, and his opponent, Aristotle[18], admits without further inquiries, that, although *one* participates in *being*, it is not *being*. The same language is used in the first part of the dialogue (129a-b):

[18] It is customarily assumed that the partner of the dialogue with Parmenides is not Aristotle, the philosopher, but one of the Thirty Tyrants who ruled Athens after it had been defeated by Sparta in the Peloponnesian War.

ἔστι τῷ μετέχειν ἀμφοῖν ὅμοιά τε καὶ ἀνόμοια αὐτὰ αὑτοῖς.

A legitimate question is whether the agglutination of one and being represents another facet of participation. The language of participation, which is specific to the relation of the participated things with that of forms, is used here in a natural way, without any further qualifications. Plato uses thus a vocabulary which he had already used in other dialogues, but for the relationship between forms and physical particulars (the *Phaedo*, the first part of the *Parmenides*). It is not so obvious how to interpret the language of participation in the demonstration, which could drive the demonstration into an unexpected direction. One could consider the following possibilities: 1. one is part of the sensible world and being is part of the intelligible; 2) one and being are both part of the intelligibles, the participation relation is in the realm of the intelligibles; 3) there is no separation anymore between intelligible and sensibles (since any kind of separation was negated in the first part of the *Parmenides*); 4) even if the language of participation is used, one should not refer to the intelligible-sensible distinction. The argument points towards a particular relation between one and being. The option 2) would fit the arguments of the scholars who take the discussion to be about one, as being a discussion about the form one. One of the most immediate consequences that can be drawn is that there is participation (μετέχειν) in the realm of the intelligibles, participation between intelligible entities. This participation, if one and being would be conceived as forms,

would question the principle of purity of forms (*Phaedo*, 74c-d, *Symposium*, 211e), and will make some forms to be dependent of other forms.[19]

Going further, if one *is*, it means that *one has being*, therefore *one* and *being* are *two* distinct entities; and the *is*-ness could be conceived separately from *one*. Each part (τὸ μόριον) of one - i.e. *one* and *being* – turns out to be *one* and *being* as well, and so on, *ad infinitum*. The division by two entities could be schematized (Figure. 2) in the following manner (by multiples of two):

One Being

One Being; One Being

One Being; One Being; One Being; One Being;

δύ' ἀεὶ γιγνόμενον μηδέποτε ἓν εἶναι

(always becomes two and never one, 143.a.1)

O B - 2

O B + O B - 4

[19] For a defense of the participation see Kahn, *Plato and the Post-Socratic Dialogue*, 24: Plato introduces us with the second hypothesis to a "new notion of participation (*metechein*) as a connection between Forms or concepts."

O B; O B + O B; O B - 8

O B; O B; O B; O B + O B; O B; O B; O B – 16

O B; O B; O B; O B; O B; O B; O B; O B + O B; O B; O B; O B; O B; O B; O B; O B – 32

Figure 2. The division of One and Being

It is unclear whether Plato took the division of *one* into two entities as a progressive multiplication, or as proper division. Perhaps it is not without reason that the Greek word used by Plato for part is τὸ μόριον, whose meaning is part, piece or member, but also, in arithmetic, fraction.[20] The process implies either or both:

a) External multiplicity: the same as number series of multiples of two: 2, 4, 8, 16, 32, etc. The concept of one, whatever it stands for, does not have in it the concept of being; being is external, and it is added to it.

b) Internal division: progression within one, as fractions by two: ½, ¼, 1⁄8, 1/16, etc. One and being are indistinct entities, part of the entity one-being. Being is internal to one, and it is not added, but it is differentiated from within it, by division.

[20] Diophantus uses τὸ μόριον as "fraction with 1 for numerator", as "fraction in general", "denominator of a fraction", and another use that would incline the balance towards division, rather than multiplication is in the expressions: μορίου or ἐν μορίῳ "divided by". See Liddell-Scott, the entry for τὸ μόριον.

Whether it is a process of division[21] or of multiplication[22], one can see that the ontological power of *one* turning into *one* and *being* produces only powers of duality. The line on which Plato develops his texts seems to move imperceptibly from ontology towards arithmetic and back again, as there seems to be a continuum from ontological differentiation towards number generation, which further operates on the first differentiation of being from one. Once the differentiation is made, ontological and mathematical inferences follow.

Later in the hypothesis, a positive integer is obtained (for example, 3 is 2+1). Here Plato could also be speaking about the counting of divisions (1/3=1/2+1/1) and this would imply that what Plato has in mind is the generation of ratios. But a ratio or fraction cannot be even or odd, thus Plato must be speaking about whole numbers. Another reason for favouring multiplicity would be that one, since is the unit for calculation, cannot be divided.

At this point, it is worth noticing that for Aristotle "the infinite by addition is the same thing as the infinite by division" (*Physics*, 206b3). Accordingly, for Aristotle

[21] Cornford reads the argument as using division. Later, where the numbers are brought into discussion, Cornford interprets addition and multiplication as an alternative to division. Francis Macdonald Cornford, *Plato and Parmenides: Parmenides' Way of Truth and Plato's Parmenides* (K. Paul, Trench, Trubner & Company Limited, 1939), 138-140. "The sort of division here intended can only be the mental act of distinguishing the two elements in ' One Entity'" (Ibid., 139).

[22] In the *Phaedo* (96e6-97b3), *two* is formed either by addition or by division. Examining the natural cause of things, Socrates shows his perplexity, giving the example of the *becoming* (generation) of two: "I will not even allow myself to say that where one is added to one either the one to which it is added or the one that is added becomes two, or *that the one added and the one to which it is added become two* because of the *addition* of the one to the other. […] Nor can I any longer be persuaded that when *one thing is divided*, this *division* is *the cause of its becoming two*, for just now the cause of becoming two was the opposite. At that time it was their coming close together and one was added to the other, but now it is because one is taken and separated from the other" (translation by G.M.A. Grube, my emphasis). Socrates' perplexity is given by the fact that one cannot have two opposite causes – addition and division – for having the same effect, which is *two*. What is important in this passage, it is to see that the origin of two can be thought in both ways.

there should be no significant distinction between external progressive infinite (a) and internal division (b). In chapters 4-8 of book 3 of the *Physics*, dedicated to the problems of the infinite, Aristotle mentions several times how Plato conceives infinity: "the Pythagoreans identify the infinite with the even… But Plato has two infinites, the Great and the Small" (203a10-15).[23] Further on, Aristotle comments that Plato

> "made the infinites two in number, because it is supposed to be possible to exceed all limits and to proceed *ad infinitum* in the direction both of increase and of reduction. Yet though he makes the infinites two, he does not use them. For in the numbers is not present the infinite in the direction of reduction, as the monad is the smallest; nor is the infinite in the direction of increase, for he makes numbers only up to the decad" (Aristotle, *Physics*, 206b30-33).[24]

In our hypothesis, Plato seems to use at least one type of infinite. The two infinities could be progressions of numbers from one, with *one* as axis:[25]

…. 1/4, 1/3, 1/2, 1, 2, 3, 4, ….

Compared to our mathematics, which takes the infinites as gravitating around zero:

{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}

[23] It is not clear if "the Small" infinite (203a15) must stop at unity or stays for the division of one. On the other hand for some adepts of the unwritten doctrines, the whole expression "the Great and the Small" it a synonym for indefinite dyad. See, for example, Kenneth M. Sayre, *Plato's Late Ontology: A Riddle Resolved : With a New Introduction and the Essay, "Excess and Deficiency at Statesman 283C-285C"* (Parmenides Pub., 2005).

[24] Translation by R. P. Hardie and R. K. Gaye.

[25] This would imply that is possible to go beyond unity in the direction of decrease, and therefore Zeno's paradox of motion could be a problem concerning the division of unity.

From Aristotle's comments one can understand that the transition from one to multiple could be performed both by increase and reduction, and Plato's argument show that *one* is in both directions *many*.

2.2.2. One, being, and difference and numbers

The first argument (142b5-143a2) presents the generation of entities by *two*, in pairs. As Plato sets off from only two terms and applies to them strictly multiplication or division, the results are restricted to multiples of two. Thus, in order to obtain other variables, which are not bound by twoness, the introduction of a third element seems to be inevitable. The next argument does precisely that.

The second argument (143a4-144a5) restates the same division of *one*, this time introducing, alongside one and being, *difference* – as *the* principle of differentiation between one and being.[26] One is not different from being because of its oneness (of being *one*), nor because of its being (of being *being*), but the differentiation between them is by virtue of difference and otherness (τῷ ἑτέρῳ τε καὶ ἄλλῳ ἕτερα ἀλλήλων, 143b7). It seems to me that it is this *triadic differentiation* – one, being, difference – that imperceptibly brings about numerical capitalisation, whose presence in the discussion does not immediately switch the attention directly to numbers per se.

[26] Kahn sees in the whole second hypothesis "the *a priori* conditions for natural science, recognizing alternative, incompatible possibilities, with no basis for deciding between them." Kahn, *Plato and the Post-Socratic Dialogue*, 2013, 42.That means that there are different alternatives that give the same result, in our case the division of *one* through *one* and *being* by necessity, and the division of *one* into *one* and *being* with the help of *difference*.

Nevertheless, the switch does occur. The argument continues with picking up pairs (being and difference, or being and unity, or unity and difference), and with the proposition that one added to any sort of pair is three. And from here Plato gives us the recipe to the complete multiplication chart: (143e7) there will be even times even (ἄρτιὰ ἀρτιάκις), odd times odd (περιττὰ περιττάκις), odd times even (ἄρτια περιττάκις), and even times odd (περιττὰ ἀρτιάκις); thus all the numbers (144a4). The conclusion of the argument is that if there is such a thing as one, there must also be number (εἰ ἄρα ἔστιν ἕν, ἀνάγκη καὶ ἀριθμὸν εἶναι, 144a).

In this argument Plato uses the concepts of *one*, *being* and *difference* in mathematical operations. Their surprising counting creates the argumentative possibility of structuring metaphysical entities according to numbers, or, in other words, finding numerical patterns in the intelligible realm; up to number three. After number two and three, the reader finds himself or herself plunged in what seem to be strictly arithmetical operations and the concepts that opened the argument seem to vanish and to leave the floor to numerical values alone: numbers are the products of multiplications between two and three. Even if their products are not called numbers, it is obvious that it is about numbers as results of these products. The first occurrence of the term number is in the conclusion, where there is also a return of the ontological and underlying concept of *one*, a fact that demonstrates that the ontological basis which started the whole argumentation also grounded the subsequent mathematical discussion and has never actually left the process of the generation of numbers: "Then if that is so, do you think there is any number that

need not be?" - "In no way at all." - "Therefore, if one is, there must also be number" (144a4-5).

3. The Ontological Argument: Being and One, and Difference (142b1-143b)

3.1. One as predication, one as subject: the use of "is"

Why did Plato develop an argument which insists on showing that one *partakes* of being? This chapter tries to explore the philosophical reasons for differentiating *one* from *being*. Both concepts, one and being, are in need of clarification.

First of all, one might ask what kind of entity ἕν is. In the *Phaedo* (97b8-9) Socrates, taking one as unity, admits that he does not know its origin: "I do not any longer persuade myself that I know why a unit or anything else comes to be".

The concept of ἕν could stand for: the unity of Form, of any Form[27] (in *Phaidon* 78d and the *Republic* 476a the Forms are indivisible and unitary); the Form of the one, of something unitary, which keeps together any entity; a metaphysical one (similar to the way Plotinus conceived it); a quantitative unity which is the beginning of numerosity; the unitary being from Parmenides' poem etc. Any unduly definite solution would risk either a too comprehensive or a too limited definition.[28]

[27] In *Philebus* (15a-b), the forms are presented as monads: "the one is not taken from the things that come to be or perish, as we have just done in our example. […] But when someone tries to posit man as one, or ox as one, or the beautiful as one, and the good as one, zealous concern with divisions of these unities and the like gives rise to controversy", translation by Dorothea Frede. Here the passage should be read together with the Dilemma of participation (130e-131e), especially 131b: "being one and the same, it will be at the same time, as a whole, in things that are many and separate; and thus it would be separate from itself".

[28] Charles Kahn resumes at least three possibilities for understanding τὸ ἕν in the *Parmenides*: "(1) the One itself as a Platonic Form; (2) the property of unity or being one; and (3) any entity whatsoever, anything that is one. There is also a special case of (3) envisaged in Deduction 2, where the One takes on the properties of extension, time, and change, and hence can apparently be identified with the natural world." Kahn, *Plato and the Post-Socratic Dialogue*, 19.

I have chosen to translate ἓν as *one*, although it can rightly be translated as *the one*[29] or by *unity*[30] as well, but any of these other two variants would strain the meaning to a more restricted sense. Moreover, *one* as entails semantic neutrality could make this concept easily liable to be understood with any of the meanings listed above.

Secondly, the concept of being as it figures in the one-being pair is closely linked with the use of the verb εἶναι. The second hypothesis starts with:

> ἓν εἰ ἔστιν, ἆρα οἷόν τε αὐτὸ εἶναι μέν, οὐσίας δὲ μὴ μετέχειν; (142b5-6)

Here Plato thinks that if "X is", we have "X" and "is", as separated entities. "Is" is different from "one", that is to say, *being* and *one* are *different* entities, and by the virtue of this *difference* one can formulate an ontological statement such as "one is". If they are not *different*, one cannot assert such claims as "one is". Thus *one is* if and only if "*is*" is something else than *one*.[31]

[29] Owen prefers for ἓν *one* and *the one* and uses them indifferently. G. E. L. Owen "Notes On Ryle's Plato" in Gail Fine, *Plato: Metaphysics and Epistemology. 1* (Oxford University Press, 2000), 299. Scolnicov translates ἓν as one. Scolnicov, *Plato's Parmenides*

[30] Allen translates as unity. Reginald E. Allen, *Plato's Parmenides* (Yale University Press, 1997). Ryle also translates like that. RYLE, Gilbert, "Plato's *Parmenides*" (1939), in *Studies in Plato's* Metaphysics, ed. Reginald E. Allen (London: Routledge & Kegan Paul, 1965).

[31] The argument of the *Parmenides* resembles the *Sophist* (252a), where the Visitor claims that one cannot speak about motion and rest without *applying* being: "It seems that agreeing to that destroys everything right away, both for the people who make everything change, for the ones who make everything an unchanging unit, and for the ones who say that beings are forms that always stay the same and in the same state. All of these people apply being. Some do it when they say that things really are changing, and others do it when they say that things really are at rest."

This idea reminds us of the *Phaedo* and the first part of the *Parmenides*, where the assumption is made that a particular thing is not what it is by its own nature, but by virtue of another entity, which, in the case of the *Phaedo*,[32] is the form of precisely that particular thing or property. Plato seems to work with a specific principle of self-predication in which A=a by virtue of b. This particular understanding of the principle of self-predication must be linked with the outcome of the first hypothesis, in which the principle of self-predication A=a collapses. The outstanding conclusion at the end of the first hypothesis is that one is not even one (141e12): τὸ ἓν οὔτε ἕν ἐστιν. The second hypothesis is an attempt to fight against this very conclusion, by restating what seems to be the same premise *if one is*. The outcome in the second hypothesis is that one is one (A=A), but in order to maintain such position one should accept that one participates in being.

The two hypotheses state as initial premise the same proposition, but as the arguments unravel the reader senses a struggle between the two hypotheses in the manner of how *to be* is used and how the relation between *to be* and *one* should be understood. The examination of *one* in the second part of the *Parmenides*, is triggered by the initial phrase:[33]

εἰ ἕν ἐστιν, ἄλλο τι οὐκ ἂν εἴη πολλὰ τὸ ἕν; (137c4-5)

[32] Conf. the *Phaedo* (101c5): "no other reason for their coming to be two, save participation in twoness: things that are going to be two must participate in that, and whatever is going to be one must participate in oneness".

[33] "Finding a plausible interpretation of this passage is crucial for those who want to hold that all the conclusions of Part II are acceptable." Mary Louise Gill, *Introduction*, 64, n. 108.

Whatever ἕν stands for, first of all we must get a clear grasp of the syntactical structure of the first part of the phrase. We should point out the alternative possibilities for the syntactic position of ἕν, in the structure of εἰ ἕν ἐστιν; (137c)[34], and the two options, i.e. predication or subject, are equally likely, since ἕν does not have an article which would indicate that it is the subject. In turn, the choice of a syntactical role for ἕν in the proposition necessarily requires an analysis on how ἐστι functions: predicative or existential; incomplete or complete.

The "complete use" presupposes that no additional specifications are required; therefore one would have a valid and complete proposition in "X is", in which there is no need for additions. In a proposition like "Socrates is" the meaning is complete: Socrates exists. "Socrates is a philosopher" uses an incomplete meaning of "to be". Both examples have Socrates as subject; „is" and „is a philosopher" are predications about Socrates. The incomplete usage of "to be" requires formally something further for completion, "X is F." [35]

[34] The main hypotheses on which the argumentation is set up are: *if the one is* (I) and *if the one is not* (II). In their turn, each of the main hypotheses are subdivided in *consequences for the one* (I.1&2), and *for the many* (I. 3&4); and *consequences for the one* (II.1&2), and *for the many* (II.3&4). There is also an appendix for I.1&2 which does not comply with this symmetric development of the eight hypotheses: *if one is and is not* (I. App).

[35] Kahn, following Owen, points out that "from a linguistic point of view" the copulative instance (predicative or incomplete) has the primary role of the verb to be in Greek. Charles H. Kahn, *Essays on Being* (OUP Oxford, 2009), 2.

Since, in the first hypothesis, there is no article accompanying ἕν (137c4), the syntax of εἰ ἕν ἐστι makes possible two different understandings, one with ἕν as predication,[36] and another one with ἕν as subject:[37]

- If we take ἕν as subject, then εἶναι has a complete meaning, and εἰ ἕν ἐστι could be translated by "if one is", with an existential "is".

- If we interpret ἕν as a predicative noun, then εἶναι has an incomplete meaning, and εἰ ἕν ἐστιν could be translated by "if it is one", in which one is the predicative noun of to be. This variant would also be supported by the second part of the phrase, where πολλὰ seems to mirror in opposition the predicative noun ἕν and πολλὰ itself is a predicative noun.

- A third variable, which would be a special case of the second interpretation, in which *one* is tautologically predicative for the subject that is *one* as well, or, as Gill puts it, a statement of identity: "if it (one) is one."[38]

[36] Gill translates like: "*if it is one or if it is not one*?" (Mary Louise Gill, Plato's *Parmenides*, (Indianapolis: Hackett Publishing Company, 1996), 141.

[37] For Scolnicov "the presence or absence of the article has no significance." Samuel Scolnicov, *Plato's Parmenides.* (Berkeley: University of California Press, 2003), 79, n. 1. Therefore his translation is "*If the one is and if it is not?*," 78. Allen translates like „ *if one is or one is not?*," which implies also not a predicative statement. (Reginald Allen, Plato's *Parmenides* (Minneapolis: University of Minnesota Press, 1983, 17).

[38] A variation that is eventually rejected in Gill's translation (Gill, 68-69, 141).

To my understanding εἰ ἕν ἐστιν could be translated by "if one is", with an existential "is". In the second hypothesis ἓν εἰ ἔστιν, "is" is normally understood as complete since there is no other predication added to one. But in order to understand what Plato wants to say when he says ἓν εἰ ἔστιν, ἆρα οἷόν τε αὐτὸ εἶναι μέν, οὐσίας δὲ μὴ μετέχειν; (142b5-6),[39] we should follow Plato and force the use of εἶναι into a mixture of complete and incomplete uses. Or to accept that it is a complete use of "is", which permits or necessitates an additional completion.

I think that with this beginning of the second hypothesis Plato overwrites an incomplete meaning on top of the already extant complete meaning in order to display that "is" means something added to one. It is counterintuitive, if not totally absurd, but this is what Plato does. He says that one *is* if and only if it partakes to being. That means that Plato, in this special case understood "X is" as "X is is", the first *is* is copulative for X and the second "is" is the predication of being. Here, the "is" of one is distinct from one and equally important as one (at the same ontological level), unless we go back to the first hypothesis where "one is" means only "one is one". Plato explicitly shortly after the beginning of the second hypothesis says that:

[39] "if one is, were it possible for it itself to be, but without partaking to being?" (My translation, which I will further indicate when the case and which I provide due to the slight different understandings of the second hypothesis. As I specified in the introduction, whenever I quote without indicating the source, one must imply that I am using the translations gathered in John M. Cooper, ed., *Plato: Complete Works*.)

ἀλλ᾽ ὅμοιον ἂν ἦν λέγειν ἕν τε εἶναι καὶ ἓν ἕν. νῦν δὲ οὐχ αὕτη ἐστὶν ἡ ὑπόθεσις, εἰ ἓν ἕν, τί χρὴ συμβαίνειν, ἀλλ᾽ εἰ ἓν ἔστιν. (142c1-3)[40]

This is a tautology from which Plato wants to escape, as he stated a little bit earlier that "one" and "is" are different entities (ἡ οὐσία τοῦ ἑνὸς εἴη ἂν οὐ ταὐτὸν οὖσα τῷ ἑνί, 142b6-7),[41] namely that the "is" from "one is" denotes not a casual indication of one, as one simply is, but this "is" marks the full ontological value of "to be". Plato insists again that

ἆρα οὖν ἄλλο ἢ ὅτι οὐσίας μετέχει τὸ ἕν, τοῦτ᾽ ἂν εἴη τὸ λεγόμενον, ἐπειδάν τις συλλήβδην εἴπῃ ὅτι ἓν ἔστιν; (142c5-7)[42]

Bearing this in mind one could venture to understand, in the second hypothesis, Plato's lack of distinction between complete and incomplete uses of "to be" in the following manner: "X is" means "X is F", in which F is "being" distinct from "one". In order for something "to be", the arguments point to the fact that it's necessary to have *being* as a predication which is not a simple formula "X is", since "X" and "is"

[40] My translation: "But to say that one is and one is one would be the same thing. Now, in our hypothesis, we don't ask ourselves what needs to follow if one [is] one, but if one is."

[41] "The being of one must be/exist without being the same/identical as one". (my translation)

[42] "Then, when somebody briefly said that one is, this would be to say nothing else than one partakes in being". (my translation)

are brought together by an another "is", in the middle, which, in this instance, seems to function as incomplete. [43]

The overlapping of complete and incomplete uses of "to be" could mean that Plato did not envisage a rigorous distinction between these two, as much as modern readers would like to think. Scholars argue for contrary views: either that Plato marked out different uses of "is", or that he failed to make such a distinction.[44] Several studies, among which the most prominent are those of Charles Kahn, suggest that the distinction existential/predicative is a modern one, and this leads to the conclusion that it is improper to apply it to the ancient Greek language.[45] And, indeed, it is very difficult to find a clear distinction of this sort in other Platonic dialogues. On the other hand, if in the second hypothesis the priority of an incomplete use of "is" is difficult to perceive at first glance, in other dialogues, where Plato explicitly infers being from incomplete "is", it is more evident. [46] For Plato, in

[43] Michael Frede goes further and argues that Plato understood "is" only as *incomplete* predicate (Frede Michael, "Plato's Sophist on False Statements", in The Cambridge Companion to Plato, R. Kraut (ed.), (Cambridge: Cambridge University Press, 1992), 397–424.

[44] The representative studies which seem to support the position of a non-distinction between existence and predication in Plato's writings are those of Lesley Brown and Charles Kahn.

[45] Kahn finds a problem in considering "is" as "existence" in the Greek language. If a subject is thought of as 'real', and this is what is understood by existence, "then I [Kahn] would be inclined to deny that such a notion can be taken for granted as a basis for understanding the meaning of the Greek verb" (Charles Kahn, *Essays on Being* (Oxford: Oxford University Press US, 2009), 20.) He argues that a copula account like "X is F" should be taken as a veridical use of the type "X is truly F," especially in dialogues like *Phaedo* (65c-d) or *Lysis* (219c). Charles Kahn, "Some Philosophical Uses of "to be" in Plato", *Phronesis*, 26 (1981), 105–34.

[46] For example, in the *Republic* (477a) there is an overlap between existence and predication: "Tell us, does the person who knows know something or nothing? You answer for him. He knows something. Something that is or something that is not? Something that is, for how could something that is not be known?" In this fragment, "Socrates could be asking any or all of the following questions: (1) "Something that exists or something that does not exist?" (existential "is"); (2) "Something that is beautiful (say) or something that is not beautiful?" (predicative "is"); (3) "Something that is true or something that is not true?" (veridical "is")." John Cooper, note 17, 1103,

some other cases, the incomplete use of "is" implies the complete meaning of "is".[47] "X is F" implies "X is" and "X is not F" implies "X is not"[48] and, accordingly, this is, as Kahn put it, "a law of Platonic ontology."[49] The "illicit inference" from "X is not F" to "X is not" could be understood as "the triangle is not round, therefore the triangle is not." Plato's "logical rule" is that with "is" as incomplete one can infer even existence.[50] It is the reverse of *omnis determinatio negatio est*, through *omnis determinatio affirmatio est*.

While the *Sophist* has received concentrated attention concerning the use of "to be," the *Parmenides* has been relatively rarely examined. The *Sophist*'s priority in discussing the use of 'is' is due to the fact that in this dialogue Plato explicitly

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in *Plato, Complete Works*, Edited by John M. Cooper. Indianapolis: Hackett, 1997. Lesley Brown argues for a complete use of *esti* here. Brown, "The verb *to be*," 221-222.

[47] "For the philosophic analysis of predication, of course, other notions must come into play—not only existence but also instantiation and truth. These concepts are essentially interconnected. We recall that the notion of predication introduced by Aristotle's term *kategoreisthai* is not merely syntactical: *kategoreitai* means 'is truly predicated'", where truly refers to truth as "the correct ascription (or denial) of an attribute". Kahn, *Essays on Being*, 3-4 .

[48] See Brown, "The verb *to be*," 228. John Malcolm, "Some Cautionary Remarks on the 'is'/'teaches' Analogy," in *Oxford Studies in Ancient Philosophy*, 31 (2006) 290. Leigh, "The Copula and Semantic," 107. Gill, Plato's *Parmenides*, 71. Brown is talking about "traditional questions about whether Plato illicitly infers an existential from predicative 'is'," proposing to resolve the dilemma by arguing that "complete being is intimately related to, and derived from, incomplete being."

[49] One might try to establish a hierarchy between Plato's "X is F" and "X is". It is necessary to receive any predication in order to be (a), or it is necessary to be in order to be able to receive any predication (b)? In order to have a proposition as "X is", one must have "X is F". The possibility that we might have a proposition such as "X is" entails that actually "X is" is "*X is* is", as the second hypothesis stipulates. *X is* is" is not yet affirmation of identity, but a statement of the attributes of X. Even Kahn thinks that (a) is "a law of Platonic ontology that *X is F* implies *X is* "(Charles H. Kahn, *Plato and the Post-Socratic Dialogue: The Return to the Philosophy of Nature* (Cambridge University Press, 2013), 22.), but he claims the reverse as well, that (b) "for Plato a subject must have the attribute of being in order to have any attributes at all" (Ibid, 24.).

[50] The existential feature of "to be" should not be taken as implying physical existence, namely, that if something exists, it means that something exists physically. It is a "complete" usage of, but not in the sense of physical existence. If we translate the second hypothesis through "if the one exists" we do not imply the involvement of a physical existence of the one, but rather we attempt to underline a usage of "to be." "If the one exists" must be taken as "if the triangle exists" or "if the language exists". These entities do not involve a physical existence.

discussed the verb "to be." The *Parmenides*, as I hope to have proven above, produces at least equally interesting instances of "is", in the second hypothesis, where "is" is thought of as a distinct entity.

One of the most problematic illustrations of this backward type of inference is to be found in the *Sophist* at 238e:

> "I was the one who made the statement that *that which is not* should not share either in *one* or in *plurality*. But even so I've continued after all that to speak of it as *one*, since I say *that which is not*."

The puzzle is that that *which is not* cannot be thought, but still that *which is not* is something. In other words: "Not being is unthinkable, therefore not being is."[51] Brown nuanced the above statement by saying that "X is" should be understood as complete, keeping in mind that this is a complete use which allows for a further completion. Her assumption, according to Fiona Leigh, is that "there is a semantic continuity between complete and incomplete uses of εἶναι."[52] For Brown, the sentences that takes the form "X is" and "X is F" should be of the form *Jane teaches* and *Jane teaches French,*[53] where the verb "teaches" allows for an additional

[51] Malcolm, "Some Cautionary Remarks on the *is*," 293.

[52] Fiona Leigh, "The Copula and Semantic Continuity in Plato's Sophist," *Oxford Studies in Ancient Philosophy*, 34 (2008), 106.

[53] Lesley Brown, "Being in the *Sophist*: A Syntactical Enquiry", *Oxford Studies in Ancient Philosophy*, 4 (1986), 55. It seems that Kahn reinforced the analogy, considering the comparison as valid: "the relation between the verb *einai* in sentences of the form X is and X is Y is like that between the verb teaches in *Jane teaches* and *Jane teaches French*" (Charles Kahn, "A Return to the Theory of the Verb be and the Concept of Being," *Ancient Philosophy*, 24 (2004), 381–405.

completion. Thus, just "like "to teach" in English, it exhibits a certain continuity of meaning across uses."[54] Brown concludes that in fact there is no sharp distinction between the two ways of using "to be," like "X is F" and "X is."

In the second hypothesis if we read "one is" as complete, we should consider (similar to Brown's example with *Jane teaches*) that this "is" allows for further completion (that of being), which in Plato's words is: *if one is, one is participating in being*.

Thus, as I have tried to show above, following Brown and Kahn, there is no sharp distinction between complete and incomplete usages of "to be" in the second hypothesis; the apparent complete use of "is" intersects with an incomplete use, and Plato plays with this overlapping.

3.2. The *Parmenides* evaluates Parmenides

The complete use of "to be" in the first hypothesis dominates, to my knowledge, the choices of most interpretations. Taking ἓν as subject and "is" as complete, the first hypothesis argues in the fashion of the historical Parmenides that "if one is", *one* cannot have any predications, not even that of *to be*. The conclusion of the first

[54] Leigh, "The Copula and Semantic," 106. However, there is a critic on this "teaches" paradigm, (one of the most quoted): if one does not teach French, it does not follow that one does not teach. "It could fit if 'X is not F' is understood as universally quantified with respect to F in that if X does not teach any subject, then X does not teach. Indeed, if so taken, one could infer that, if X is not any F at all, X does not exist" (Malcolm, "Some Cautionary Remarks on the *is*," 290).

hypothesis is that ἓν is not even ἓν, (A = ~a),[55] even if the only predication allowed initially to *one* was that of being one, i.e. A is a.[56]

Considering the pivotal role that the principle of self-predication plays in conceptualizing the realm of intelligibles in Plato's philosophy, the altered restatement of the principle of self-predication moves fundamental concepts to a no man's land, where any epistemological attempt is debunked. Plato did go against such a conclusion, in order to maintain a principle of self-predication, which would correspond to the concept of *sameness* from the *Sophist* (255c), by tacitly switching from a complete 'is', in the first hypothesis, to an incomplete 'is', in the second hypothesis. One senses this change of meaning not in the opening question itself – which, at least in what the written Greek text demonstrates, is the same as the opening phrase of the second hypothesis – but in the subsequent answers to this opening question. After deciding to follow the consequences of the premise "if one is" no matter what they turn out to be, the first hypothesis deduces that ""If one is, one would not be many"[57] and continues with a list of negative propositions that concentrate on what *one is not*. On the other hand, the second hypothesis deduces from the same premise "if one is" a series of mostly affirmative sentences, beginning with "is it possible for it itself to be, but without partaking to being?" One must read through both deductions and up to their conclusion in order to retroactively

[55] Here 'a' stands for the property A stands for.

[56] "But if the one has any property apart from being one, it would be more than one; and that is impossible." ἀλλὰ μὴν εἴ τι πέπονθε χωρὶς τοῦ ἓν εἶναι τὸ ἕν, πλείω ἂν εἶναι πεπόνθοι ἢ ἕν, τοῦτο δὲ ἀδύνατον. (140a1–3)

[57] My translation.

understand that the premises of these two discussion lines are only formally identical.

The switch to an incomplete "to be" enables the *separation of being* from whatever is *one*, and establishes *being* and *oneness* as different. What Plato seems to suggest is that conceiving a distinct being would favour the possibility of *one* to be *one* (principle of self-predication), albeit partaking in being. By participation in being, *one* can have the predication of one. Plato's intention is to show how self-predication is made possible via being.

In the first hypothesis, from 'one is' (where the protagonists seems to use a complete "to be") one of the paradoxical conclusions is *that there is no one*; this use of "to be" proves not to be sufficient for maintaining that *one is*. In the second hypothesis Plato is thus exploring a possibility in which the being of one is given not by its univocal relation to itself, A=a, but by acknowledging being as a principle which underlies self-predication (for one to be one, one should also be being: A=a+b). In order for one to be identical with oneself one should partake in being. Being bestows being, identity, and self-predication.

Plato struggles in this second hypothesis to show that one is not only one, and being is not only being. Beside that one is one, and being is being, there is also a mixture, and one is being, and being is one. The identity between one and being, and the difference between one and being, is a step beyond the ontology of the historical Parmenides. By accentuating the division of one, the second hypothesis appears to

develop an argument, in the first instance, straight against Parmenidean ontology, and thus it also remains as a reply to the first hypothesis.

In Fragment B8, Parmenides regards ἔστιν (*what [it] is*) as whole οὖλον, one (μουνογενές), unmoved (ἀτρεμές) and perfect (ἀτέλεστον) (Fr. B8.4 DK). These are attributes that can be conceived conceptually independent of *being*, but not ontologically independent, as Plato does with *being* and *difference* when he introduces them as being ontologically distinct from *one*.[58] In Parmenides' poem, *being* is only *being*, and is *one* only by way of attribute. For Plato the idea that one is being entails that one partakes in being and each of them is a distinct intelligible ontological entity. Since the attributes of being in Parmenides' poem are not conceived as independent ontological entities, they do not allow any room for theorizing how being would develop into something different. Judging from a Parmenidean point of view, plurality would never be the case.

Parmenides' *being* does not partake in these attributes, in the way in which *one* partakes in *being* in the second hypothesis. Similar to what the first hypothesis records, namely that there is no development of one into something else, Parmenides underlines that the only development of being would be only being:

[58] Cornford takes one, being, and difference as three elements of unity: "Thus ' Unity itself ' is a whole or complex with at least three parts or elements, and so is many." Cornford, *Plato and Parmenides*, 143. I suppose that Cornford takes difference as part of one because if we conceive difference as originating in something totally independent of one, we would have two independent entities one and difference at the time when we already interrogate "if one is".

> "Nor is it divisible, since it is all alike and not any more in degree in some respect, which might keep it from uniting, or any inferior, but it is all full of Being. (25) Therefore it is all united, for Being draws near to Being (22-25)."[59]

Parmenides emphasizes here that being is indivisible and same with itself at all times. Being is everywhere being, and there is nothing in it that is something else. Being must be entire (οὖλον), or not at all (Fr. 8. 11). There is no place for a difference in it. Being is one (μουνογενές), but this is an attribute and not a part of it, while in the *Parmenides* one is being, and the being is part of it. In the poem being ends with being (Fr. 8.25). For the second hypothesis (142e), being is composed of at least two parts (oneness and being).

The second hypothesis goes in several respects substantially counter to Eleatic ontology, at least as it was perceived by Plato. Trying to refute the results of the first hypothesis, Plato "directly engages" Parmenides.[60] Plato expresses his dissatisfaction with the ontological consequences of Parmenides' poem, through a display, in the first hypothesis, of how he understood that the features of Parmenides' ontology

[59] οὐδὲ διαιρετόν ἐστιν, ἐπεὶ πᾶν ἐστιν ὁμοῖον,
οὐδέ τι τῇ μᾶλλον, τό κεν εἴργοι μιν συνέχεσθαι,
οὐδέ τι χειρότερον, πᾶν δ᾽ ἔμπλεόν ἐστιν ἐόντος·
τῷ ξυνεχὲς πᾶν ἐστιν · ἐὸν γὰρ ἐόντι πελάζει.

The translation follows that of Coxon. Also I used mainly Coxon's Greek text: A. H. Coxon, and Richard D. McKirahan, *The Fragments of Parmenides: A Critical Text with Introduction and Translation, the Ancient Testimonia and a Commentary* (Parmenides Pub., 2009), 70.

[60] Anscombe is convinced that the conclusion of the first hypothesis is a "truly Parmenidean conclusion". G. E. M. Anscombe, *From Parmenides to Wittgenstein: Volume 1: Collected Philosophical Papers* (Wiley, 1991), 25. Scolnicov follows a similar idea: "In this dialogue, Plato directly engages Parmenides, the most serious challenge to his own philosophy." Samuel Scolnicov, *Plato's Parmenides* (University of California Press, 2003), 2.

come together.[61] In my view, the second hypothesis is the Platonic response to the first hypothesis that concluded with the claim that nothing can be said about, and predicated of, the *one*. By separating being from one, the second hypothesis brings thus a correction to the first hypothesis, which I think is also a correction to the foundation of Eleatic ontology.[62]

The two hypotheses are mirroring each other (the negations of the first hypothesis become affirmations in the second)[63] and they are more meaningful if we consider them as part of a larger philosophical agenda, in which the *Parmenides* is not an empty exercise that leads to no philosophical conclusions. As Ross put it, treating the 'second part' as "primarily a gymnastic exercise does not exclude the possibility that in the course of it Plato may hit on positive ideas which will fructify in his later thought."[64]

[61] Scolnicov argues that: "As the *Parmenides* will make clear, Parmenidean ascription of being is "transparent." As Plato shows in Argument I, nothing is added to the Parmenidean one when it is said to be. To say 'the one' and to say 'the one is' is to say the same thing." See Samuel Scolnicov, *Plato's Parmenides* (University of California Press, 2003). Or, "Argument II is, together with the related Arguments III, V, and VII, an explication of *μέθεξις*, as opposed to Parmenidean being," Ibidem.

[62] For Plato, Parmenides' philosophy is a challenge and, at the same time, a philosophy that should be exceeded. At *Theatetus* 180e, criticizing Melissus and Parmenides, for claiming that "*Unmoved is the Universe*", Socrates says: "These philosophers insist that all things are One, and that this One stands still, itself within itself, having no place in which to move." The Visitor, at *Sophist* 242c, referring to Melissus and Parmenides says: "our Eleatic tribe, starting from Xenophanes and even people before him, tells us their myth on the assumption that what they call 'all things' are just one". Plato has his proper understanding of Eleaticism, in which the being of Parmenides is "one-being".

[63] Kahn stresses as well that "all of the attributes denied of the One in Deduction 1 are presented here [in the second one] in positive form." Kahn, *Plato and the Post-Socratic Dialogue*, 2013, 38-39.

[64] William David Ross, *Plato's Theory of Ideas* (Oxford: Clarendon Press, 1951), 100.

Most scholars, building on the ideas of Ryle[65] and Runciman,[66] think that it is the form of unity which is under examination in the second part of the *Parmenides*, and, consequently also in the first hypothesis. However, as I have argued, one should consider that what Plato examines in the first hypothesis, and the whole second part, refers to Parmenides' monism, an examination which becomes more developed in the *Sophist*. This is in line with the interpretation of Cornford,[67] Cherniss,[68] Guthrie,[69] Turnbull,[70] or Palmer[71] who argue that one should keep in

[65] He claims that "there is no internal evidence whatsoever" for the presupposition that the dialogue has actually as subject matter Parmenides' Monistic theory. Gilbert Ryle, „Plato's `Paramenides'", *Mind*, New Series, 48, nr. 190 (1939): 129-51.

[66] Rejecting the idea that the subject matter of the *Parmenides* could refer to Parmenidean monism, Runciman argues that "although certain arguments of the second part could be construed as referring to Parmenidean monism, it is clearly impossible so to interpret them all; and if Plato wished to discuss Parmenidean monism, he would not have done it in this intermittent way." And he thinks that "the ambiguities of the second part do not invalidate the contention that it is nevertheless the form of unity which is under discussion throughout." Walter Garrison Runciman, "Plato's Parmenides," *Harvard Studies in Classical Philology 64* (1959): 101. Runciman's critical observation that Plato wouldn't have discussed in such an intermittent way Parmenides' Monism can nevertheless be equally argued if Plato was discussing his form of unity. Why would an intermittent way go better with a discussion on the form of unity? I do go along Runciman's view that, indeed, certain arguments of the second part could refer to Parmenides' philosophy, especially in the first hypothesis.

[67] In the second hypothesis "the picture of a One Being [is] regarded as an all-inclusive whole and, as such, one and limited, and also as possessing continuous 'being'. So far it resembles Parmenides' One Being. The difference, however, is that our whole is divisible", Francis Macdonald Cornford, *Plato and Parmenides: Parmenides' Way of Truth and Plato's Parmenides* (K. Paul, Trench, Trubner & Company Limited, 1939), 144.

[68] "The quotations from the poem and the references to it are so frequent in Plato's writings that we may be sure when Plato was writing the *Parmenides* he had nothing more vividly before his mind than the poem which he mentions whenever he talks about the paradoxes of being." Harold Cherniss, "Parmenides and the *Parmenides* of Plato", *American Journal of Philology* (1932): 130.

[69] Plato "wanted to clear up the relationship between his own doctrine and the Eleatic thesis of One Being." W. K. C. Guthrie, *A History of Greek Philosophy: Volume 5, The Later Plato and the Academy* (Cambridge University Press, 1986), 57.

[70] Robert G. Turnbull, *The Parmenides and Plato's Late Philosophy: Translation of and Commentary on the Parmenides with Interpretative Chapters on the Timaeus, the Theaetetus, the Sophist, and the Philebus* (University of Toronto Press, 1998).

[71] John A. Palmer, *Plato's Reception of Parmenides* (Clarendon Press, 1999), especially 221-254. For Palmer, the second part of the *Parmenides* presents openly Parmenidean insights.

mind Parmenides' poem and that the subject matter of the second part of the *Parmenides* is the Parmenidean concept of "one-being".[72] One must be aware that it is not certain if Parmenides defended a numerical monism, and that it could be Melissus who developed an Eleatic monism.[73] However, according to John Palmer, Plato – contrary to our reading of the philosophy of Parmenides through the lengths of Plato's dialogues, to the effect that Parmenides was an adept of numerical monism – interprets and perceives Parmenides as an adept of numerical pluralism. The main problem with Palmers' interpretation is that at the beginning of the *Parmenides* (128c-d), Plato presents Zeno as arguing in his book against numerical pluralism.

Palmer dedicates special attention to the second hypothesis (second deduction according to Palmer).[74] For Palmer, some attributes of one from the second hypothesis reflect Parmenides' fragment 8, i.e. one is in itself, it is like itself, it is in contact with itself, while the contradictory attributes are in relation with one "in other aspects", aspects which are not in virtue of its own nature.

In my view, Plato, in the ontological argument, wants to show that it is necessary to conceive being and one as distinct entities which can give

[72] Cornford read it as a normal outcome of the Parmenidean one-being frame: "The reasoning is also valid *ad hominem*, in that Parmenides spoke of his One Being as an extended continuous magnitude with spherical shape. If it has these properties, it must be infinitely divisible in the ordinary sense," 139.

[73] See for example P. Curd, *The Legacy of Parmenides* (Princeton: Princeton University Press, 1998), 65, 240. Also J. Barnes, "Parmenides and the Eleatic One," *Archiv für Geschichte der Philosophie* 61 (1979), 1-12.

[74] Palmer, *Plato's Reception of Parmenides*. And also John Palmer, *Parmenides and Presocratic Philosophy* (OUP Oxford, 2009).

determinations to each other. Thus one partakes of being, one similarly being partakes of one. This is a mutual influence from both directions, while for Parmenides, Plato would say, being cannot take part in anything but itself. As far as I am concerned, while Plato takes as his starting point this Eleatic exclusivity that being is being only through itself, he nevertheless does not hesitate to emend this ontological uniformity, by directors some of those which were main attributes in Parmenides view towards an ontological status. If in the poem there is only one being, in the second hypothesis Plato emphasizes that we need at least two more entities (with the *Sophist* we will find out that we actually need four more entities, besides being) that have the same ontological status as being.

The relation of the *Sophist* with the *Parmenides* is stressed by scholars, but how exactly this relation came to be is still a matter of controversy.[75] With the second hypothesis, this relation should be more evident. Runciman noted in this regard:

> "I am myself unable to form any confident opinion as to how far the *Parmenides* should be regarded as the deliberate and conscious precursor of the *Sophist*. But that the *Sophist* does offer some solution to the problems raised in the *Parmenides* is, I think, beyond question."[76]

[75] The *Parmenides* and the *Sophist* have in common, at least formally, an excessive yearning to divide exhaustively the subject matter. In the *Parmenides*, the *one* is under a minute and a thoroughly divided scrutiny under its eight/nine hypotheses, about its existence and its relation with the *many*, and in the *Sophist*, the definition of the *sophist* is explored under seven conditions. In both of the dialogues, at the end of these very rigorous analyses, which give the impression rather of Aristotelian writings through their meticulous categorizations, a definite answer is not apparent. Besides formal circumstances, the dialogues are more internally related.

[76] Runciman, W. G. "Plato's Parmenides" Harvard Studies in Classical Philology 64 (1959): 119.

With few exceptions, the relation of the second hypothesis with the *Sophist* is less emphatic. Cornford was the first to underline the relation of the second hypothesis with the greatest kinds of the *Sophist*:

> "I do not suggest that the whole deduction [the second hypothesis] as stated in the *Parmenides* is intended to be valid, but the first steps seem to be guaranteed by the *Sophist*, which again explains that any proposition such as 'a One exists' involves the recognition of three terms: 'One', 'existence', and 'otherness'. This is used to convict Parmenides of contradiction when he asserts the existence of a One and yet denies plurality."[77]

The *Theaetetus* (183e) and the *Sophist* (217c) refer to the opening of the *Parmenides*, and not only dramatically. The *Parmenides* is a dialogue, as its title says, a dialogue related to Eleaticism and Parmenides' philosophy.[78] In the *Parmenides*, on the one hand Plato projects himself in the shadow of Parmenides' philosophy, and on the other hand he is trying to escape from the Eleatic paradigm. Parmenides criticized the theory of forms; Plato criticizes the Eleatic theory of *one being*.

[77] F. M. Cornford, "Mathematics and Dialectic in the Republic VI.-VII. (II.)," *Mind*, New Series, 41, no. 162 (April 1, 1932): 173–90. 179.

[78] Plato did not nominate Parmenides or Eleatic philosophy in the first hypotheses or through the whole second part of the dialogue because he builds his argumentation as an exercise. But he gives the title of the dialogue the *Parmenides*.

3.3. From different to *Difference*

A key premise of the second hypothesis is that Plato inferred the *existence* of *one* from the differentiation between one and its being, bringing thus in the dialogue what initially seems to be an innocent adjective (*different*), describing a relation between intelligibles; this adjective gets to be conceptualized in its corresponding noun and, by the end of the ontological argument, it turns out to be one of the cornerstone principles that Plato uses later in the *Sophist*'s ontology: "If one is" is possible through *difference* and as *difference*. In Plato's words:

> "So if being is something and the one is something different, it is not by its being one that the one is different from being, nor by its being being that being is other than the one. On the contrary, they are different from each other by difference and otherness."[79]

Being is both different from one and part of one. It is part of one as something different. *Difference* shows that *one* encloses two entities (*one* and *being*), and *being* is possible only by the *difference* between *being* and *one*. But the introduction of *difference* into the argument is made at a later point and after the distinction of *one* and *being* was already made, and argued that one is always two and never one. Since

[79] Translation by Gill. οὐκοῦν εἰ ἕτερον μὲν ἡ οὐσία, ἕτερον δὲ τὸ ἕν, οὔτε τῷ ἓν τὸ ἓν τῆς οὐσίας ἕτερον οὔτε τῷ οὐσία εἶναι ἡ οὐσία τοῦ ἑνὸς ἄλλο, ἀλλὰ τῷ ἑτέρῳ τε καὶ ἄλλῳ ἕτερα ἀλλήλων (143b2-6)

it was possible to make the difference between one and being already, one must suppose that *difference* was already in the background of the argument. If being is part of one, it is unclear how *difference* how to be understood: as part of *one* or as totally different from *one*. Difference is mainly a relational concept; it is the name that Plato uses to explain why one is not being and being is not one. Plato does something in the following manner. Let's say that a is one, b is being, and c difference. Since a≠b, Plato understands the non-identity relation as c. The introduction of difference, as a relational principle, could push us to decide if the ontological argument is not a mere conceptual argument, i.e a conceptual distinction, in which difference acts as a logical principle. Plato himself addresses the problem in this manner in *Sophist* 254d-e, where he asks whether sameness and difference are merely words (ὀνόματα) or kinds (γένη). In my analysis I take the discussion as being an ontological one, and maintaining that Plato does not make a sharp distinction between conceptual and ontological argumentation. For Plato, in the second hypothesis, a conceptual clarification means an ontological clarification.[80]

Both the *Parmenides* and the *Sophist* introduce *difference* as an operator between beings of the same genus or of different genera. *One* is not different from *being* by virtue of its oneness, and *being* is not different from *one* by virtue of being itself, the difference of *one* and *being* is due to *difference* (143a-b). For everything to be

[80] This aspect is more evident in respect to the *Sophist*. One may object here to the use of ontological thinking in the *Sophist*, and argue that the *Sophist* is an elaboration into the realm of philosophy of language. But in spite of the example of Theaetetus, that might lead us to language-and-logic related issues, we must not forget that the backdrop of the example is an ontological discussion generated in the dialogue by Parmenides' refusal to speak about non-being. At the same time it comes as a perfect complement to the *Parmenides*' second hypothesis, which seems essentially to be an argument on ontological grounds.

we need *being* (for *one* to be we need *being*, as external to one). But being is not enough; we also need *difference* (for one and being to be we need *difference*, as external to both of them). *Difference acts in a similar way as being*. Without *difference* any being would be without any determination, thus without a form. Difference informs a being into its different nature.[81] In the *Sophist*, one and being are different, but not in absolute isolation; the predication can be given if the entities can be blended.[82]

Two entities are different not because of their proper nature (which would be done by their proper Form, in dialogues as *Phaedo* and *Republic*), but, quite unexpectedly, by *difference*, which is a fully fledged concept on its own. Thus the Plato of the middle dialogues would say that the difference between a bed and a dog is due to their respective participation in the form of bed and in that of dog. This would be the *difference* pertaining to different *genera*. When referring to the same *genus*, the difference between an old yellow bed and a red one is due to the participation in the form of redness and yellowness, as well as in other forms, while partaking in the same form of *bedness*. By pointing towards *difference* for marking the difference between intelligibles, the *Parmenides* points to a switch in the way Plato understood intelligible objects.

[81] Perhaps there are some roots of this conceptual upgrade within the Academy in the depiction of Aristotle's type of *intensional definition*: i.e. genus–differentia definition.

[82] A principle/premise that is to be found in the *Sophist*: τελεωτάτη πάντων λόγων ἐστὶν ἀφάνισις τὸ διαλύειν ἕκαστον ἀπὸ πάντων: διὰ γὰρ τὴν ἀλλήλων τῶν εἰδῶν συμπλοκὴν ὁ λόγος γέγονεν ἡμῖν, "To dissociate each thing from everything else is to destroy totally everything there is to say. The weaving together of forms is what makes speech possible for us," 259e4-6).

We naturally tend to think that *difference* is constantly relative to something different (*Sophist*, 255d). Less naturally, *difference* can be understood itself as subject and this *difference* can relate to itself (through being itself by itself), since *difference* is different in virtue of itself (258b–c, 259a–b). Perhaps the least natural is that *difference* provides the possibility to instantiate non-being (those things that are not are, 262e-263d) as something different, and not as absolute symmetrical opposition to *being* (as is shown in the example of true and false statements about Theaetetus).

The *per se* and *per aliud* distinction[83] is used also by Plato in the *Philebus* (51d).[84] Such division seems to be exposed by the pupil of Plato, Hermodorus,[85] and confirmed by Diogenes Laertius.[86] Plato could assume that being is understood through itself by itself (κατὰ ταυτόν) and in relation to other things (πρὸς ἑαυτό), pointing hence to "to be" as existence (κατὰ ταυτόν) and "*to be*" as predication (πρὸς ἑαυτό).[87] For many scholars these distinctions are related to the different types of predication and different senses of "to be".[88]

Any difference implies two *differentialia*, since difference is between at least two entities. The most explicit description of *difference* in Plato's corpus is in the

[83] This distinction is defended especially by Kahn and Meinwald.

[84] A similar schema is to be found in the *Timaeus* (51e-52d) between self-sufficient forms and participated things.

[85] See John M. Dillon, *The Middle Platonists, 80 B.C. to A.D. 220* (Cornell University Press, 1996), 8.

[86] See Mary Louise Gill, *Philosophos: Plato's Missing Dialogue* (OUP Oxford, 2012). 154.

[87] Palmer argues for the same distinction in *Republic* (436b8-9).

[88] Mary Louise Gill, "Method and Metaphysics in Plato's *Sophist* and *Statesman*", *The Stanford Encyclopedia of Philosophy* (Spring 2016 Edition), Edward N. Zalta (ed.).

Sophist,[89] where it is conceived as a central ontological concept, one of the greatest kinds, a concept which encompasses non-being (for example, 257b and 258e–259a). While *difference*, in the *Parmenides*, is necessary for the distinction of *one* and *being*, in the *Sophist*, it is indispensable in order to differentiate *difference* and *being*:

> "…Some of the *things that are* are *themselves by themselves* (*auta kath hauta*), whereas others are always said in relation to other things (*pros alla*)… But difference is always in relation to something different (*pros heteron*)… And this would not be the case, *if being and difference were not distinct*. For if difference partook of both forms, as being does, then something even among the different things could be different without being different in relation to something different. But in fact it has turned out for us that necessarily whatever is different is the very thing that it is from something different." (255c–d)

Difference differs from *being*, because *difference* exists only by relation of things (*pros alla*), while *being* is equally relative to other things (*pros alla*) but it is also itself by itself (*auto kath hauto*). Instead difference, according to the *Sophist*, is not in relation with itself, it is incapable of self-predication.[90]

[89] According to the *Sophist* (253e), difference permeates (through participation, *metechein*) the rest of the greatest kinds: "And we're going to say that it pervades all of them, since each of them is different from the others, not because of its own nature but because of sharing in the type of the different."

[90] Duncombe argues that *pros allela* is a palaeographically plausible reading for *pros alla*. Matthew Duncombe, "Plato's Absolute and Relative Categories at *Sophist* 255c14," *Ancient Philosophy* 31 (2012): 77-86

3.4. Concluding remarks

The ontological argument of the second hypothesis is exceptional in several respects: it starts by arguing that *one* participates in *being*, and this participation has repercussions and ramifications at different levels, first of all by negating the first hypothesis, secondly by proposing a new ontological principle, namely the *non-monolithic one*, and, what I will discuss in Chapter Five, by advancing a conception of the *principles of numbers* as derived from this basic multiplicity of one.

The first hypothesis concludes with a series of negative propositions (no predications can be given to *one*), while the second hypothesis argues for positive results (several predications can be given to *one*). The tenth negation of the first hypothesis, the negation of being, gives the start to the second hypothesis, which simply moves the discussion around its most important premise, the *being of one*.

The full meaning of the first hypothesis, of all absolute negations, in the light of the structure and the outcomes of the second hypothesis, proves to be a reassessment of Eleatic principles, used as a catalyst for the clearly non-Eleatic developments that Plato proposes through the second hypothesis.[91] With the first hypothesis one cannot predicate anything about the one (such as being, part/whole, location, motion/rest, same/different, like/unlike, equal/unequal, older/younger), even numbers, a prospect which Plato stands against. There is a need for understanding being as a determination, in order for one to be one (or for something

[91] Kahn leaves the analysis of the second hypothesis at the end of the commentary of the remaining seven hypotheses. He also emphasizes that the second hypothesis is crucial for the second part of the *Parmenides* and late dialogues. Kahn, *Plato and the Post-Socratic Dialogue*.

to be something), and thus to be. *Being* given as predication to *one* brings, as consequence, the *existence* of the subject for predication. Even if it is totally counterintuitive, these are the ontological implications of the ontological argument, and Plato appears to express here his own views.

If Plato's views are to be found in the ontological argument, as I have argued to be the case, in contrast to the views of the historical Parmenides, then what Plato does is not so different from the patricide in the *Sophist*. Plato does not name explicitly Eleatic ontology as his target, as he does in the *Sophist*; he does, however, propose a solution to the puzzle of the first hypothesis (namely to differentiate *being* from *one*, in order to state *one is*), a solution which would eventually lead him to, among other issues, the generation of numbers.

Another important feature of the ontological argument is the concept of *difference*. In order to differentiate *being* from *one*, and to have *being* as an independent entity, Plato uses difference as an ontological tool. Difference appears to be introduced as conceptualization of a trait (*different*) of one and being. One can accept that *one* and *being* are not identical with each other, and thus they are different.

4. Numbers, Mathematics and Plato

Before analysing the argument for the generation of numbers, I bring into the discussion ancient Greek mathematics in the time of Plato, followed by a synopsis of Plato's approach to Greek mathematics and his particular understanding of mathematical objects. The first section of this chapter insists on differences of paradigm between modern mathematics and Greek mathematics. The second section of the chapter presents specific tenets of Plato's philosophy of mathematical objects and numbers: an overview on how he understood the role of mathematics, the priority of dialectics compared to mathematical knowledge, the realist theory of mathematical objects.

Just as any discussion on natural philosophy or philosophy of science would require, any discussion on the philosophy of mathematics of Plato should also take into consideration any clues that the mathematical knowledge of his time would provide for a better grasp of his philosophy. At the same time, for a better grasp on the construction of the argument for the generation of numbers, it is important to draw attention to essential differences between ancient and modern understanding of mathematics. This will also highlight the limits of approaching Plato's view on mathematics through mathematics as we understand it today.

A fundamental topic in the modern philosophy of mathematics is the existence of numbers.[92] Even if we know numbers and operate with numbers, and

[92] Plato's dialogues do not raise problems regarding the existence of numbers. They do exist. In the *Republic*, their existence is not in question, and what is discussed is especially an epistemic feature, namely the faculty through which one can know numbers, kinds of numbers to

we develop an axiomatic system for them, this does not mean that numbers do exist. We think about each individual thing as being a unity – an entity which could be taken separately from other ones – and thus we think that this unity can be counted as being one (or the condition for counting entities that are more than one, as it was conceived in Greek mathematics), which together with another one, or other ones, forms a numerical class. This, however, does not tell us what number is, and where one should look for the explanation of the possibility of counting – whether in the order or in the distribution of things or in our epistemic structures, or perhaps even somewhere else. Number is not a predication of things, such as weight, or colour etc.,[93] and it is questionable if one can reduce number to numerical classes of things we put together.[94] These problems have been exercising the virtuosity of mathematicians and philosophers alike from antiquity up to present.[95]

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be known and where one should look for numbers. In the *Euthydemus* (290c1-6), after Plato draws a parallel between hunters and mathematicians, he specifies that mathematical objects do exist: "And again, geometers and astronomers and calculators (who are hunters too, in a way, for none of these make their diagrams; they simply discover those which already exist), since they themselves have no idea of how to use their prey but only how to hunt it, hand over the task of using their discoveries to the dialecticians – at least, those of them who do so are not completely senseless" [my emphasis].

[93] We cannot feel, see, hear, taste numbers. There is no sense perception that facilitates us to perceive numbers.

[94] This is a view, defended mainly by structuralism, which sees number as purely relational, which emerges from a structure of things. For a comprehensive defense of structuralism in the philosophy of mathematics see Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford University Press, 1997).

[95] An expression of the problems in the present philosophy of mathematics is formulated by *Benacerraf's Dilemma*, a paradigm for modern anti-Platonism. Paul Benacerraf points out that there is incongruence between the epistemology of mathematics and the metaphysics of mathematical objects. He uses a causal theory of knowledge to set out the difficulties raised by Platonic realism in which mathematical objects are not spatially and temporally localized, although the mathematicians who conceive them as mathematical objects are indeed spatially and temporally localized. Benacerraf thinks that the knowledge of mathematical objects should be achieved through causal interaction with mathematical objects, which poses conceptual and methodological difficulties. Therefore, mathematical propositions cannot relate to abstract objects.

4.1. Greeks on numbers

The modern historiography of mathematics is challenged by the fact that Ancient Greek mathematics and its philosophy appear to be both familiar and alien to our current debates on philosophy of mathematics. Scholars are divided in arguing either for difference[96] or for similitude[97] between what ancient numbers were in Greek mathematics and for Plato, and what the modern mathematical concept of numbers would be.

4.1.1. Greek mathematics, modern mathematics, and philosophy of mathematics

There is a correspondence between modern philosophy of mathematics and the Greek one. The difference consists chiefly in the complexity of modern philosophy of

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The dilemma was articulated by Paul Benacerraf in "Mathematical Truth," (*The Journal of Philosophy*, 1973, 70: 661-679).

[96] P. Pritchard, *Plato's Philosophy of Mathematics*, (International Plato Studies, 5.) Sankt Augustin: Academia Verlag, 1995. Pritchard rejects any comparison of mathematical Platonism with Frege's Platonism, and stresses that Plato's philosophy of mathematics is fundamentally different from the mathematical Platonism of contemporary debates in philosophy of mathematics.

[97] For an author as John Cleary, Plato's view on numbers is not to be surpassed by our modern views. Plato's theory on numbers is still functional (John J. Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers" In Gregor Damschen , Rainer Enskat and Alejandro G. Vigo (eds.), *Plato und Aristoteles - sub ratione veritatis. Festschrift für Wolfgang Wieland zum 70. Geburtstag*. Göttingen: Vandenhoeck & Ruprecht, 2003, 11). Douglas Blyth is more radical and tries to argue that the concept of number in Ancient Greece, especially for Plato in the argument for the generation of numbers, was similar to our modern understanding of number through set theory. Douglas Blyth, "Platonic Number in the *Parmenides* and *Metaphysics* XIII", *International Journal of Philosophical Studies* 8 (2000), 23-46. Set theory was developed by George Cantor around 1873 (published in 1874 in a paper: "On a Characteristic Property of All Real Algebraic Numbers"), and it is a field in mathematics which studies collections of objects, i.e sets. With Cantor set theory started to became a fundamental theory in mathematics. Blyth's claim that there is a resemblance between Greek mathematics and set theory is, I think, an exaggeration. Plato indeed pushed forward a concept of number which could be compared, only by its intention, with the axioms of set theory (Cantor, Zermelo) or mathematical logic (of Frege, Russell or Whitehead), but this comparison works only formally. Probably, we can go with Blyth and think that Plato's generation of numbers does not differ in its scope, but one should accept that it does differ in method and complexity. Both views aim to reduce the inner relations of numbers to essential categories.

mathematics, but paradigmatically the similarities are dominant. The contemporary general tendencies in the philosophy of mathematics overlap with the ancient Platonic-Aristotelian debate – with clear archetypal paradigms which are both complementary and contradictory. In essence, numbers are for Plato *a priori*, while for Aristotle they are *a posteriori*, and almost all that Aristotle thought about Plato's conception of mathematical objects is a redefinition of various Platonic views on mathematical objects, in the struggle to prove exactly the opposite. *Mutatis mutandis* the contemporary debate in the philosophy of mathematics takes place between Platonism and all variations of non-Platonism, with Aristotle as the first non-Platonist on numbers.[98] Our methods of investigation may be more sophisticated, but with regards to numbers, a mathematician can ultimately be a Platonist or anti-Platonist (which does not primarily imply Aristotelianism), and from these two extreme loci one may navigate to other possible nuances. A majority of developments in modern philosophy of mathematics are basically a reaction to Platonism in mathematics.

For the historian of mathematics the reason for approaching the Greek conception of numbers as familiar is the fact that we assume that Euclid's mathematics is universal. The language of mathematics is universal, as a paradigm for sciences and, in some cases, even for philosophy (e.g. Spinoza). It is also thought that everything which is discovered, proven by mathematicians in the field of

[98] It is of course the case that that pre-Platonic thinkers, for instance the early Pythagoreans, were also non-Platonists.

mathematics, is unchangeable.[99] Mathematicians assume that mathematics is absolute and transcultural, and thus there should be a necessary resemblance between Greek mathematics and our mathematics. It is generally believed that, especially from Euclid onwards, we share similar views on mathematics and numbers. For example, most scholars agree that for the ancient Greeks ἀριθμός is a collection of units[100] – "small collections of units make up larger collections; e.g. trio combined with duet makes quintet"[101]. Aristotle's notes that "in mathematical number no unit is in any way different from another" (*Metaphysics* 1080a22-3). This makes possible operations like addition, multiplication, subtraction, or division. Euclid (*Elements*, VII, def. 2) lays down a likely definition: Ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος (a number is a multitude composed of units), and this basic definition is part of what we work with today.

4.1.2. Discontinuities and incommensurabilities

On the other hand, the familiarity with Greek mathematics might prove illusory if we try to apply a Kuhnian paradigm in mathematics. Thomas Kuhn did not talk explicitly about mathematical paradigms themselves, but his theory does leave space for theorizing on the possibility of revolutions in mathematics as well. There are

[99] Bruce Pourciau, „Intuitionism as a (failed) Kuhnian revolution in mathematics", *Studies in History and Philosophy of Science* (2000): 297-329.

[100] See, for example, Anders Wedberg, *Plato's Philosophy of Mathematics* (Almqvist & Wiksell, 1955), 71. Thomas Heath, *A History of Greek Mathematics: From Aristarchus to Diophantus* (Courier Corporation, 1981), 69.

[101] John J. Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers", 9.

scholars who think that "Kuhnian revolutions in mathematics are logically possible, in the sense of not being inconsistent with the nature of mathematics."[102] And even more compelling in this sense is the idea that mathematics, "the one science where Kuhn apparently believed his ideas on incommensurability did not apply, is the science that reveals the deepest incommensurability of all."[103]

As opposed to natural sciences, previous discoveries in mathematics never seem liable to being questioned. Progress in mathematics seems to be linear and continuous, without paradigmatic upheavals and, even though there are examples in the history of mathematics which may fall under the heading of revolution,[104] the notion itself of revolution seems to contradict the very essence of the objects mathematics deals with.[105]

Some reject the possibility of revolutions in mathematics because such an assumption would entail that mathematics is not a science;[106] but whether or not mathematics itself is susceptible to revolutions, there are important differences between different ages and societies. There are reasons to think that, for example, the

[102] Pourciau, "Intuitionism as a (failed) Kuhnian revolution in mathematics", 297.

[103] Ibidem.328.

[104] Until the nineteenth century it was thought to be impossible to reject Euclid's geometry as being false.

[105] 'Revolutions never occur in mathematics' (Crowe, 1992, p. 19). Michael Crowe "Ten 'laws' concerning patterns of change in the history of mathematics" (1992)

[106] Mathematics may not be a science, since in mathematics one cannot record progressive steps which presuppose the rejection of old paradigms, as it is the case, for example, with Ptolemaic astronomy. Moreover, all sciences, with the notable exception of astronomy, are empirical sciences. A supporting argument against its status as science would be that, broadly speaking, science is based on induction, while mathematics on deduction. And the discussion may get even more complicated if we apply Popper's criteria, namely that a science is a science if it is falsifiable. But mathematics is not. If mathematics is not a science similar to natural science, than what it is?

discovery of incommensurable magnitudes[107] was a case of paradigm shift in Greek mathematics, which changed, at the time, the perception of numbers as integers. This point in the history of mathematics, even if one may question the extent to which it had triggered an actual revolution in mathematics, marked a conceptual shift and can help us understand the conceptual gap between our understanding and ancient Greek understanding of mathematics.

Studying Plato's philosophy of mathematics must thus take into consideration the framework of 4th century B.C. Greek mathematics and Greek numerical systems. Several variables, such as the meaning of number, the way of representing numbers or concepts about numbers are reasonably unlike our current notions on numerical systems. Pritchard seems to advocate a thorough discontinuity with regards to the understanding of what a number is.[108] Pritchard grounds his theory mainly on Klein's theory.[109]

Furthermore, I would add that a possible incommensurability between our mathematics and Plato's understanding of mathematics should also consider other more striking incommensurabilities such as those in the theory of mind (as I show in Chapter Six), religious beliefs, and different ontologies. To understand Plato's argument for the generation of numbers cannot be done exclusively through the

[107] There is no unit that makes it possible for a square to be commensurable with both its side and its diagonal. Plato (*Republic* VIII, 546c 4-5) calls the diagonal of the square ἀρρητον (irrational). Euclid called it ασύμμετρος (incommensurable). For a dense discussion see Heath 1921, pp. 90–91.

[108] Paul Pritchard, *Plato's Philosophy of Mathematics* (Academia Verlag, 1995), chapter 4.

[109] Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (Courier Corporation, 1992). Burnyeat seems to follow Klein and Pritchard, and to favour the discontinuity thesis. Cf. Burnyeat, "Plato on Why Mathematics is Good for the Soul," and also "Platonism and Mathematics: A Prelude to Discussion".

means of the history of mathematics and philosophy of mathematics. There is more in the argument; there is, to name some of the essential variables, a pre-Socratic frame of thinking that reduces multiplicity to basic principles, a mythology of generation (discussed in the chapter on arithmogony and cosmogony) and a non-numerical way of representing numbers.

4.1.3. Zero versus one; akrophonic versus alphabetic notation of numbers

There are several reasons to think that there are differences, if not incommensurabilities, between Greek and contemporary mathematics. To name at least two differences,[110] which are central when speaking about Plato's mathematics: 1) a different understanding of the number one and, as consequence, a different approach to the number series; 2) a different method of number notations which could have a notable influence on the mathematical and philosophical understanding of numbers.

[110] There are surely more differences, but I insist on those that are directly related to Plato's understanding of numbers. A specific divergence, which I touch upon, is pointed out by Jacob Klein: "Physics, as we know it today, is not conceivable without symbolic mathematics. We are used to this kind of symbolic expression to the extent that we have no difficulty in handling symbols and are not even aware of the fact that we are dealing with symbols. A school of thought which calls itself Logistic is trying to interpret this fact in its own way. I think, however, they do not understand it, because the existence of symbols appears to them to be self-evident. But symbols are in themselves a great problem. They didn't exist for the Greeks, at least not in the same way they exist for us." Jacob Klein, *Lectures and Essays* (St. John's College Press, 1985), 45.

1) Ancient Greek mathematics did not have the concept and a proper place for zero[111], and it did not conceive the number one as a proper number, since one was considered the source of all numbers and it itself could not be the result of anything. One was not a number, but was simply a unity and the condition of numbers.[112] This diverges profoundly from modern mathematics, in which all numbers gravitate around zero. By contrast, in Greek mathematics all numbers rise from one (unity), and have one as its centre. These clarifications alone set the floor for what is a major difference between the two ways of understanding numerosity.[113] (As I underline, for Plato one, as numerical unit, is obtained from duality – see Chapter Five).

2) Another difference, which is rarely discussed in specialized literature, in the case of Plato, is the various ways of representing numbers in ancient Greece. This itself could be a case of discontinuity in Greek mathematical culture in Plato's time. The Greeks, like all the ancients, didn't have proper numerals: they used the alphabet or system of numeric notation, similar to the Roman, to represent numbers. At a certain moment these two systems of denoting numbers were competing:

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[111] A Greek correspondence to zero could be μηδέν or oudέν, which could have stood for nothing. However, it looks that there was no numerical symbol for emptiness.

[112] Jacob Klein considers that there are three types of numbers in Plato's corpus, according to how the unit was understood: 1. sensible numbers – units are sensible things; 2. mathematical numbers – units are "pure" monads, independent of time; 3. eidetic numbers – units are an "assemblage of ideas, are nothing but a conjunction of *eide* which belong together". They belong together to a *genos*. Klein, Jacob (1968), *Greek Mathematical Thought and the Origin of Algebra*, Eva Brann (trans.) (MIT Press, Cambridge, MA, 1968). Reprinted, Dover Publications, Mineola, NY, 1992., 46-99, 90.

[113] In *The Foundations of Arithmeti*c, Frege makes a synopsis of the historical conception of number and he openly refutes the ancient understanding. *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl*, 1884, Breslau: W. Koebner (Austin's translation: 1974. *The Foundations of Arithmetic: A logico-mathematical enquiry into the concept of number*, 2nd ed. Blackwell, section 45).

i) The old Athenian system which, similar to Roman numerals, apparently emphasized the cardinality of number;[114] This way of representing numbers, called also the Attic notation,[115] develops a system in which the first letters of the numeral signifies the number (the only exception is unity, which is represented by a stroke "I"): Π – πέντε 5, Δ – δέκα 10 , Η - ἑκατόν 100, Χ – χίλιοι 1000, Μ – μύριον 10000. The structure is attested (and probably used much earlier) from 454 to about 95 B.C.,[116] and resembles the Roman numeral system: VI is ΠΙ, XIII is ΔΙΙΙ, CVI is ΗΠΙ. Numbers are understood as finite and composed entities.[117] The use of the stroke together with the number represented by the letter was "used almost exclusively to represent cardinal numbers."[118]

ii) The new Ionian system based on the alphabetical order of the letters of the Greek alphabet, was used to represent both ordinal and cardinal number. The Ionian system[119] consists in overlapping numerals with the Greek alphabet, a practice which was quite wide spread in the Mediterranean: α-1, β-2, γ-3, δ-4, ε-5, etc. This denoting system eventually became the

[114] John J. Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers" In Damschen, Gregor, Rainer Enskat and Alejandro G. Vigo (eds.), *Plato und Aristoteles - sub ratione veritatis. Festschrift für Wolfgang Wieland zum 70. Geburtstag* (Göttingen: Vandenhoeck & Ruprecht, 2003), 4.

[115] Called also Herodianic, after a passage attributed to the grammarian Herodian.

[116] Heath, *A Manual of Greek Mathematics*, 15.

[117] The system recorded only cardinal numbers, and it was used in metrology. Georges Ifrah, *The Universal History of Numbers: From Prehistory to the Invention of the Computer* (Wiley, 2000), 182.

[118] Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers", 25.

[119] Called also Milesian.

standard numerical system. The Ionian system was used for ordering (e.g. calendars).[120]

4.1.4. Ordinal and cardinal numbers

There is more than a historical detail with regards to the way in which the ancient Greeks made their calculations and represented numerosity. Plato was surely not unaware of Greek numerical notation systems in developing his philosophy of numbers – an issue which is seldom explored by scholars. Our language of arithmetic uses the Arabic numerical system. In Plato's time the language of arithmetic was different.

Going back as far as textual evidence makes it possible, one can see that, for example, in *Odyssey* (IV.411, 450-1, XVI. 245) "*arithmos* meant primarily a denumerable group, rather than a number propriety."[121] For John Cleary this suggests that "the older akrophonic system of Attic numerals favoured the concept of numbered group or cardinal whereas the introduction of the alphabetical system of numbering around 400 BCE made it at least possible to conceive of numbers as ordinals."[122] A cardinal meaning of *arithmos* should have had priority over ordinal distribution, at least for Homer. Plato is probably challenged by the introduction of

[120] Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers", 26.

[121] John J. Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers", 4.

[122] Ibidem

the alphabetical order, which "itself is internally ordered", being "suitable for representing ordinal numbers".[123]

The alphabetical system which encourages the ordinal meaning might support the number form theory in the following manner. In the *Phaedo*, Plato conceives 2 and 3 as not addible entities. These numbers are forms in themselves, and typify what means to be a form. Aristotle explicitly claims that "Plato used to say: that is, there is a first two and three, the numbers being non-combinable with one another" (*Met*. M 1083a 33-35, Annas).

For Douglas Blyth, an advocate of the resemblance of ancient and modern mathematics, Platonic numbers are distributed by arrangement in sequence, being basically ordinal.[124] Blyth considers that for Plato there is a difference between mathematical numbers, corresponding to intermediates, which are cardinals, and form numbers, which are ordinals. Analysing the model of generation of numbers from Parmenides, Blyth insists on its basic ordinal feature.

On the same lines, John Cleary ventures to say that Plato's original conception could be explained through the introduction of alphabetical numerals, which used to classify both cardinal and ordinal numbers (the old system being reserved only for cardinal numbers). "Plato seems to have been the first to see that ordinal number is logically prior to cardinal number, despite being temporally posterior."[125] On the

[123] Ibidem, 5.

[124] Blyth, "Platonic Number in the Parmenides and Metaphysics XIII".

[125] John J. Cleary, 26.

other hand, for philosophers such as Frege and Russell there is no priority[126] between ordinality and cardinality. The priority of cardinality, following Frege and Russell, seems to be accepted without reservations by mathematicians and the philosophers of mathematics alike. There are a few exceptions, as that of Dummett,[127] who argue that - following Cantor - "the ordinal number is more fundamental than that of cardinal number". He gives the example of counting the strokes of a clock, "we are assigning an ordinal number rather than a cardinal"[128]. A more neutral position in this problem is that of Shapiro, a defender of structuralism in philosophy of mathematics, who considers that "In a sense, the system of finite cardinal patterns, the system of finite ordinal patterns, and the system of strings have the same structure, namely, the natural-number structure."[129] It is very difficult to see ordinal numbers at work in the *Parmenides*, since from the very beginning we are faced with what seems a cardinal way of counting one, being and difference. If indeed Plato favored the ordinal feature of number in developing the form-number theory, for the generation of numbers the ordinal feature could be applied only to 1, 2, and 3.

[126] "The conclusion that ordinals are prior to cardinals seems to have resulted from confusion. Ordinals and cardinals alike form a progression, and have exactly the same ordinal properties. […] In order to prove that ordinals are prior to cardinals, it would be necessary to show that the cardinals can only be defined in terms of the ordinals. But this is false, for the logical definition of the cardinals is wholly independent of the ordinals", Bertrand Russell, *Principles of Mathematics* (Routledge, 2010), 243-1.

[127] Michael A. E. Dummett, *Frege: Philosophy of Mathematics* (Harvard University Press, 1991), 293.

[128] Ibid. Dummett complains that Frege did not pay more attention to Cantor's work in order to understand the priority of the ordinals. For Dummett, Frege "was well aware that Cantor was concerned with ordinal rather than cardinal numbers", but Frege rejected the distinction "as a mere divergence of interest, and never perceived its significance".

[129] Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford University Press, 1997), 116.

According to Cleary, the tension between the two ways of denoting numbers influences, seemingly, the view of Plato and Aristotle. Given these variables, one must be aware that discussing Plato purely on the basis of the dialogues, ignoring the circumstances, we might be missing any historical explanation on Plato's philosophical development regarding numbers. Also one must be aware that our understanding of the ancient conception of mathematics by referring only to numbers and their relations occurs with our deliberate overlooking of the philosophical and mythological backdrop it originally had (these features are more developed in the chapter on Arithmogony and Cosmogony).

4.2. Plato and mathematics

If there is any science, in the modern sense of the word, which totally seduced Plato, that is mathematics.[130] In the following lines I make a survey of Plato's relation to mathematics. Plato is not a mathematician, but in his dialogues mathematics and philosophy are interrelated. Compared to Descartes or Leibniz, where their interest and research in mathematics, and the connection they established between mathematics and philosophy, were evident and firm, the place of mathematics in Plato's philosophy is difficult to define (despite the obvious appreciation that he had for it). I think it is not a forced parallel if we think of Aristotle's philosophy as a

[130] One must emphasize that the special relation of philosophy with mathematics is traceable from the beginning of philosophy (starting with Thales), and almost all philosophers have an opinion about the ontological features of numbers, and had an interest in mathematics per se. Plato's interest in mathematics comes thus as a natural enterprise of philosophy.

compound of philosophy and biology, and, in a similar way, almost everything in Plato's philosophy is connected with mathematics: from ethics through armed strategy to physics and cosmogony. In several dialogues, Plato showed an intense interest in the elements and the nature of mathematics; he dealt with numbers, arithmetic, and geometry, and paid a vivid attention to mathematical methods.

Several studies argue that Plato's interest in mathematics had a great impact on the development of the discipline in the Academy and elsewhere.[131] A passage from Proclus' commentary on the first book of Euclid's *Elements*, entitled "Catalogue of Geometers", written probably by Aristotle's student, Eudemus, records that Plato was very advanced in mathematics.[132] Proclus testimony on Plato is not singular. In antiquity he was credited even with the discovery of geometrical analysis.[133]

On the other hand, for modern scholars Plato's contribution to mathematics does not immediately spring into attention as a main component of his thinking.[134] It has been argued that: "Plato's role has been widely exaggerated. His own direct

[131] Many scholarly works pay attention to Plato's mathematical preoccupation; for example, in the 50's, from 620 pages of annotated bibliography on Plato, from 1950 to 1957, 15 pages (396-412) record academic works dedicated to Plato's mathematics. See H. Cherniss, "Plato 1950-1957", *Lustrum* 4 (1959) 5-308 & 5 (1960) 321-656.

[132] "[Plato was] greatly advanced in mathematics in general and geometry in particular because of his zeal for these studies. It is well known that his writings are thickly sprinkled with mathematical terms and that he everywhere tries to arouse admiration for mathematics among students of philosophy". Proclus, *Commentary on the First Book of Euclid's* Elements, transl. Glenn R. Morrow (Princeton: University Press, 1970), 54.

[133] Diogenes Laertius, Lives (3.24) attributed to Plato the discovery of geometrical analysis. Three centuries later, *Anonymous Prolegomena Philosophiae Platonicae* (5.32-35) claimed the same thing. However, these testimonies are doubtful for the modern scholar.

[134] "The main problem in discussing Plato as a mathematician… is that most of the statements dealing with mathematics are, to the modern reader, at least couched in vague language," Roger Herz-Fischler, A mathematical history of division in extreme and mean ratio (Waterloo: Wilfrid Laurier Univ. Press, 1987), 79. See also Harold Cherniss, "Plato as Mathematician," Review of Metaphysics 4, no. 3 (1951): 418–419.

contributions to mathematical knowledge were obviously nil," or "Plato 'directed' research is fortunately not borne out of the facts."[135] Indeed, there is no explicit passage in Plato's corpus that might lead us to think that he perceived mathematics through the lens of a mathematician, and that he worked in a mathematical way on mathematical problems.

One must notice that the methods of mathematical research were not so well delineated in Plato's time[136], but he did assume a difference between mathematical research and philosophical research.[137] The disciplines of mathematics are very different from what we nowadays recognize under this heading. It is in the *Republic*[138], where he talks about four or five branches[139] of mathematics: (1) calculation (522c-526c)[140], (2) plane (526c-527c) and solid geometry (528a-e)[141], (3)

[135] Otto Neugebauer, *The Exact Sciences in Antiquity* (New York: Dover, 1969), 152. A similar scepticism is shared by Ian Muller: "it is very unlikely that Plato made substantive contributions to mathematics; indeed, many of the more specifically mathematical passages in his works have no clear and correct interpretation, and many of them can be read as the half-understandings of an enthusiastic spectator", Ian Mueller, "Mathematics and the Divine in Plato", in Teun Koetsier and Luc Bergmans, (eds.), *Mathematics and the Divine* (Amsterdam: Elsevier Science, 2005), 99-121, 101. See also Harold Cherniss, „Plato as Mathematician", *Review of Metaphysics* 4, nr. 3 (1951): 395-425.

[136] Mathematics as an independent, theoretical, discipline was not so finely circumscribed in Plato's time, and for Pythagoreans, mathematics (or what was understood as such) was intermingled with (or part of) religion.

[137] According to the divided line, there are at least different cognitive faculties involved.

[138] Mathematics starts to be part of the Guardians' education at their maturity, after they have been previously schooled in gymnastics and arts. The mathematical curriculum had the intention to train the mind and develop abstract thinking.

[139] These four disciplines turned into the quadrivium in the medieval period. Together with the trivium (grammar, logic and rhetoric) they formed the so-called seven liberal arts. There are scholars who take plane, on the one hand, and solid, on the other hand, as different disciplines, and they count five mathematical studies, e.g. Mitchell Miller, "Figure, Ratio, Form: Plato's «Five Mathematical Studies»", *Apeiron: A Journal for Ancient Philosophy and Science* 32, nr. 4 (1999): 73-88.

[140] "Well, if we can't find anything apart from these, let's consider one of the subjects that touches all of them. What sort of thing? For example, that common thing that every craft, every type of thought, and every science uses and that is among the first compulsory subjects for everyone. What's that? That inconsequential matter of distinguishing the one, the two, and the

astronomy (529a-530c)[142] and (4) harmonics (530d-531c)[143]. From these mathematical disciplines,[144] only calculation, astronomy and geometry are properly related to mathematics for modern science.[145]

4.2.1. On the value of mathematics

three. In short, I mean number and calculation, for isn't it true that every craft and science must have a share in that? They certainly must." (522c3-5) Here number and calculations are understood mainly as a techne. See Tom Angier, *Techne in Aristotle's Ethics: Crafting the Moral Life* (A&C Black, 2010), 22.

[141] "if geometry compels the soul to study being, it's appropriate, but if it compels it to study becoming, it's inappropriate."(522e4-5) "That's easy to agree to, for geometry is knowledge of what always is. Then it draws the soul towards truth and produces philosophic thought by directing upwards what we now wrongly direct downwards." (527b6-8) "Then as far as we possibly can, we must require those in your fine city not to neglect geometry in any way" (527c1-2). And "there is a world of difference between someone who has grasped geometry and someone who hasn't" (527c6-8).

[142] "And what about astronomy? … That's fine with me, for a better awareness of the seasons, months, and years is no less appropriate for a general than for a farmer or navigator" (527c9-528a3). "And since you reproached me before for praising astronomy in a vulgar manner, I'll now praise it your way, for I think it's clear to everyone that astronomy compels the soul to look upward and leads it from things here to things there" (528e4-7). It should not be a surprise that Plato does not pay attention to astronomical observation as such, since for him any science of perceptible things is a *doxa*.

[143] "It's likely that, as the eyes fasten on astronomical motions, so the ears fasten on harmonic ones, and that the sciences of astronomy and harmonics are closely akin, This is what the Pythagoreans say, Glaucon, and we agree, don't we? We do." (530d6-9). Both astronomy and harmonics are nowadays part of applied mathematics. However, it is not surprising to see them as part of non-applied mathematics. The Pythagoreans used to associate harmonics and mathematics until their identity: doing harmonics was equivalent with doing mathematics and vice-versa. Moreover, the guardians must understand only the mathematical character of harmonics, and not issues which belong to melody, rhythm, dance etc. Burnyet drew attention that one should deal with harmonics and astronomy in such a way that "lifts the mind out of and away from the sensible world". See Burnyeat, M. F., "Plato on Why Mathematics is Good for the Soul," 14-15.

[144] Since there was not available a proper number notation, algebra was not yet well developed. For Greek mathematicians it was difficult to elaborate and write equations or number problems. This could be the reason why they developed more problems in geometry, and used methods proper to geometry instead of algebra.

[145] Concerning astronomy, modern science still preserves a Pythagorean and Platonic point of view: even if astronomical knowledge is different, the assumption that the material world is based on number and numerical structure is similar to what Plato and Pythagoras affirmed.

The dialogues themselves do not provide a coherent and unitary view on Plato's ontology of mathematical objects; but they do provide rich references to mathematics and endorse Aristotle's claims that Plato was immersed in the problematic ontology of mathematical objects. The dialogues show more the interest that Plato had in mathematics as a tool, a model of rigorous thinking, and perfection. All the regular and perfect forms of geometry and the harmony of numbers appear as elements which are independent from alienability, they are pure and *per se*. Their importance consists in the fact that their contemplation can elevate their subject to the *real* truth. Mathematics for Plato pertains thus to a philosophical pedagogy and mathematical ignorance would be thus an error. For example, in *Gorgias*, Plato stresses the fact that geometry plays a social and moral role.[146] Neglecting geometry does not imply that Callicles doesn't know all the proofs from geometry, but that he is not *aware* of its basic elements (elements which we later find at Euclid as part of a mathematical thinking which had initially been developed in the Academy).

In the *Republic*, Plato envisages what would be the perfect human education as comprising ten years dedicated to the study of mathematics and we might ask ourselves what so much mathematics for the philosopher kings would lead to. The requirement for studying mathematics for the duration of 10 years (537bd) was

[146] "Yes, Callicles, wise men claim that partnership and friendship, orderliness, self-control, and justice hold together heaven and earth, and gods and men, and that is why they call this universe a *world order*, my friend, and not an undisciplined world-disorder. I believe that you don't pay attention to these facts, even though you're a wise man in these matters. You've failed to notice that proportionate equality has great power among both gods and men, and you suppose that you ought to practice getting the greater share. That's because you neglect geometry" (*Gorgias* 507e6-508a7). For the relation between the social cosmos and natural cosmos, between political thinking and geometrical thinking see Jean-Pierre Vernant, *The Origins of Greek Thought* (Cornell University Press, 1984), 127-129.

complemented by the requirement to study dialectic for 5 years. This could be a natural demand as Plato was so seduced by mathematics, and held it as a model of rigorous thinking.[147] The instrumental understanding of mathematics is defended by Annas.[148] The absolute truth of mathematics was taken as a model for the absolute reality, complementary to the relative reality. Another view, which gains increasing approval, is that for Plato mathematics is constitutive of the Good.[149] According to Burnyeat, the concept of "unit" is the ground to study mathematics. "Unit" is "the highest value"[150], it is accessible "only by thought, not sight"[151] (524d-526b) and the Good is characterized as a unity, and it is "the unhypothetical first principle of everything"[152].

4.2.2. Dialectic and Mathematics

Plato talks about dialectic as a method that has precedence over the science of mathematics and that is the only means of achieving knowledge of the intelligibles. In the *Republic* (534b3–c5), dialectic comes as a necessary addition to the anamnesis

[147] The training in mathematics stays as condition to develop the ability of theoretical philosophy. It has a transitional place (*Republic* 531c-d) between *gymnastics* and *music* (the basic education) and *dialectics* (the definitive level).

[148] Annas Julia, "Philosophy and Mathematics," *An Introduction to Plato's Republic* (Oxford University Press, New York, 1981), 272-293. Mathematics is for Plato an example to follow because of its hypothetical-deductive method. Ibid. 289-90.

[149] M. F. Burnyeat, "Plato on Why Mathematics is Good for the Soul," in Timothy John Smiley, *Mathematics and Necessity: Essays in the History of Philosophy* (British Academy, 2000), 1-81.

[150] Ibid, 74.

[151] Ibid, 75.

[152] Ibid, 45.

theory. As the anamnesis theory is less and less used and present as a viable theory in late dialogues, the method of dialectic seems to have remained for Plato the exclusive way to access metaphysical entities. The end of the first part of the *Parmenides* explicitly concludes that there is a need for a logical exercise, a need for dialectic.[153] The need for exercise could be an important clue on how to corroborate the two parts of the *Parmenides*. Plato links the two parts by introducing the idea of a necessary training (ἡ γυμναοία) for answering the challenges of the theory of forms. There is a need for an exercise, for a proper training (135d), otherwise one will miss the truth (εἰ δὲ μή, σὲ διαφεύξεται ἡ ἀλήθεια).

Cornford tries to make sense of the relation between dialectic and mathematics.[154] He finds four elements of the contrast between mathematics and dialectic: (a) Objects;[155] (b) Methods of procedure;[156] (c) Movements of thought,

[153] In *Phaidros* (265c-d) it is assumed that dialectic is the way to grasp "what is".

[154] Cornford, "Mathematics and Dialectic in the *Republic* VI.-VII. (I.)", *Mind*, 1932, 37-52. The article lists the most important tendencies that were later developed in Platonic exegesis.

[155] For Cornford the division made by the divided line clearly points to some distinctive objects, but nothing points to intermediaries. The only Ideas (forms) that figure in the whole organization of education are moral (607b) and mathematical (510d) Ideas. For Cornford, moral Ideas are not a higher class, and the mathematical a lower one, since the only distinction lies in the natures of the two classes of Ideas: mathematics can use 'visible images, while of moral Ideas there are no 'visible images'.

[156] The contrast between mathematics and dialectic does not correspond to the difference between mathematical and moral Ideas. Dialectic applies to both fields of objects. In mathematics the method is deductive, a downward movement from premises to conclusion. Dialecticians have an upward movement in an opposite direction which is "is free from hypotheses" (510b7). What Plato means by hypothesis is a matter of controversy, and what exactly is 'hypothesis' in mathematics is also disputed. The usual example is taken from geometry which works with a set of unverified notions: the definitions of magnitude and its central attributes (such as straight, triangular), and the existence of magnitude, points, lines. Nevertheless definitions are not hypotheses; hypotheses are assumptions of the existence of things defined. Mathematics assumes definition of odd and even, triangular etc., but their existence as such should be demonstrated. On this point, Cornford thinks that Plato restricts hypotheses to existence and not to definition, and a definition is an account (logos). I would add that definitions are not ontological arguments (to prove in any way the existence of mathematical entities), they are, at most, epistemic arguments for correct descriptions of defined mathematical entities.

deductive and intuitive;[157] (d) States of mind, characteristic of the mathematician and of the accomplished dialectician.[158]

[157] Two ways of thinking are contrasted: deductive – downwards; intuitive – upwards. Noesis (in one of its senses) refers to the upward movement of intuition, while dianoia, (in one of its senses) refers to the downward movement of reasoning in deductive argument. A priori truth cannot be deduced or proven from the conclusion; it must be grasped by an act of analytical penetration. 'An act of analytical penetration' could correspond to the *elenctic* method: 'What is *F*?' e.g. *Laches* 'What is courage?', 'What is odd and even?' These issues have caused constant debate both for theoreticians of mathematics and for metaphysicians alike. A very appealing interpretation of what Plato might have had in mind points to concrete and more sophisticated mathematical methods, which, at that time, were innovative. For example, Proclus associates Plato's dialectic with the method of analysis in geometry. And Cornford quotes Proclus insisting that "by means of analysis carries the thing sought up to an acknowledged principle" (Eucl., I). From here, Cornford goes further in an attempt to enforce Proclus' position, refuting, unconvincingly, Thomas Heath's criticism: "Proclus's language suggests that what he had in mind was the philosophical method described in the passage of the *Republic* (511b), which of course does not refer to mathematical analysis at all" (p. 43-44). The method of analysis has indeed a very close connection to dialectic, even if Plato cannot take full credit for discovering it. "No doubt Plato did not invent the method of analysis, but the connection with dialectical method is closer than is here (Heath) suggested" (Ibidem. 44). "Plato may well have been the first to recognize as distinct the movement of thought involved in what Aristotle calls the analysis of a mathematical diagram." In his support, Cornford adduces several ancient testimonies, such as that of Aristotle: "Diagrammata are discovered by an activity. For it is by dividing (drawing lines in the given figure) that people discover them. If they had already been divided, they would have been obvious; as it is, they are present potentially. Why are the angles of the triangle equal to two right angles? 'Because the angles about one point are equal to two right angles. So if the line parallel to the side had already been drawn, the reason would have been immediately plain to inspection… the potentially existing (diagrammata) are discovered by being brought into actuality" (Met., 1051 a, 21) (Ibidem. 44). Another testimony brought by Cornford is that of Pappus who speaks about two directions of analysis: upwards - "analysis is the procedure which starts from the desired conclusion"; downwards – reversing the process to frame the theorem or demonstrate the construction in the logical order. Thus, for Cornford, "It is quite possible to accept the statement that Plato 'discovered' the method of Analysis, in the same sense as Aristotle discovered the syllogism." The geometer is "dispensing with… and contemplating the Idea of the triangle", which contains "in itself all 'essential properties' that can be drawn out" in theorems. The theorem is the fruit of contemplation, which penetrates by intuition to the latent properties 'contained by' the essence. Thus, "when dialectical method is applied to the definition of an Idea, the ascent is made by the 'synoptic' act of divining by intuition", which, according to Cornford, corresponds to the following schema of Proclus: a unity is the genus which is divided into species.

[158] For Cornford the state of mind of the mathematician is not *nous* due to at least these two main reasons: 1) "the reason they have reflected upon is an assumption that is not either demonstrated or seen to be indemonstrable"; 2) they don't have an intuitive apprehension of the indemonstrable principle of their whole science. On the other hand, *nous* has no argumentative power; it is "the perfectly clear vision, or unshakable grasp, of the completed structure of mathematical truth… by the light of the ultimate premise, intuitively seen and such that it cannot be questioned". In conclusion, Cornford states that *noesis* (as opposed to *dianoia*) implies: (1) the intuitive act of apprehending, by an upward leap, an Idea or a prior truth implicit in a conclusion, and (2) the state of mind of one who sees with perfect clearness a completed structure of truth illuminated by the unquestionable principle. But then again *dianoia* (as opposed to *noesis*) implies: (1) the downward movement of understanding following a deductive argument from premise to

According to G.E.L. Owen it is this conception of dialectic that was especially unpalatable[159] for Aristotle and differentiates[160] Aristotle from Plato. Dialectic has universal epistemological application and illustrates well one of Plato's principles: the principle of 'one over many'. Dialectic guides us towards "an unhypothetical principle," which is exactly what mathematics does not.[161] But how exactly dialectic gets rid of hypothesis (533) going to the *arche* of numbers is not unequivocally developed by Plato in the *Republic*.[162] As far as I see, for Plato *hypothesis* and *arche* are in mutual exclusion: once you get to an *arche*, there is no discussion of hypotheses anymore.

Plato contrasts dialectic with mathematics. For some scholars, this contrast is precisely between dialectic and the axiomatic and hypothetical method developed by mathematics.[163] The way of "saving" mathematics from being only mathematics is by practicing dialectic. Mathematical knowledge alone is not sufficient; it must be

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conclusion, and (2) isolated chains of reasoning depending on an assumption either not demonstrated or not seen to be indemonstrable.

[159] G. E. L Owen, *The Platonism of Aristotle*, (Oxford University Press, 1965). "it is his [Aristotle] criticism of the Platonic notion of the unity of science which perhaps more than anything else shapes the character of his philosophical method."

[160] Dialectic is "particularly important if we wish to understand the nature of the contrast between Plato and Aristotle" John David Gemmill Evans, *Aristotle's Concept of Dialectic* (Cambridge University Press, 1977), 7-8, 8.

[161] The priority over mathematics is noticed also in the *Philebus* (56d-58e). See also Norman Gulley, *Plato's Theory of Knowledge*, 173.

[162] It is assumed by Plato that dialectics and the good are intimate related, but why is like that is not argued, it is only illustrated by analogy of the sun.

[163] The contrast would be between axiomatic method and analytic method. "Mathematics developed by the analytic method gives justifications for its hypotheses pursuing this process until it reaches the unhypothetical principle of everything. What Plato criticizes is not mathematics, as such, but only the practice of certain mathematicians." Carlo Cellucci, *Rethinking Logic: Logic in Relation to Mathematics, Evolution, and Method* (Springer Science & Business Media, 2013), 44.

augmented by dialectical thinking.[164] As Burnyeat put it, one can speak here about a "meta-mathematical dialectic." Without it, mathematical knowledge is incomplete: "grasping what is, for we saw that, while they do dream about what is, they are unable to command a waking view of it as long as they make use of hypotheses that they leave untouched and that they cannot give any account of." (VII, 533b-c) Thus, for Plato, "dialectic is the only inquiry that travels this road, doing away with hypotheses and proceeding to the first principle itself, so as to be secure" (533d). In my own view the generation of numbers could be taken as an example of a dialectical exercise into the nature of numbers and the relation between ontological entities and numbers.

4.2.3. Form-numbers

Plato is usually charged by mathematicians and scholars with the allegation of mathematical realism. For Plato there are forms and things that participate in the forms, but he does not explicitly affirm that numbers are forms. It is thus assumed that Plato pushes forward a concept of number which had important repercussions on the history of mathematics and its philosophy. And indeed, for the Greeks numbers were perceived as instantiations of magnitudes (continuous and discontinuous), which means that numbers were understood spatially and

[164] A. H. Coxon, *The Philosophy of Forms: An Analytical and Historical Commentary on Plato's Parmenides, with a New English Translation* (Uitgeverij Van Gorcum, 1999), 119.

geometrically; thus numbers were *perceivable* numbers. Plato's philosophy of form-numbers goes against this frame. For him numbers are *abstracta*.

A theory of form numbers would be a natural result of Plato's theory of forms. In the *Phaedo*, the possibility to put together things in order to have numerosity is only because of forms of numbers, which, in their turn, belong to forms such as beauty or courage. In a manner that might seem counterintuitive for his contemporaries[165], in the *Phaedo* (101b9-c9),[166] Plato claimed that two things participate in the form of twoness, and one in the form of oneness, and perhaps one should think accordingly regarding all the things that form numerosities which participate in their respective forms. Here Plato refers to the fact that there are many sets of things which have two elements (two books, two people, a pair of eyes etc.). If so, the mathematical number two itself cannot be a form, only just one of the many things that participate in the form of twoness. It does not mean that the form of two[167] (the twoness) is composed by two (form) entities (each one being the form of unity), but that there is only one unique and uncompounded form for every two things. It is not a particular quality (or an adjective) of two, or three, or four, etc. Each number (as a form) is a unit and each numerosity, what is counted (not as a form), is composed of units. The form-number is a simple unity and only one entity;

[165] Except for the Pythagoreans, most probably that for the common Greek man a number was the product of adding, subtracting, dividing etc.

[166] This passage seems to be the more elaborated passage from the dialogues on the realism of natural numbers. See for a record of the rest of the passages Wedberg, *Plato's Philosophy of Mathematics*, 131, 154.

[167] For John Cleary "it remains unclear whether the Form of Two, for instance, is a collection of two ideal units or whether it is simply Twoness" p. 6.

the concrete numeral is a plurality, composed by entities.[168] That might entail that the numeral 2 consists of 1+1, but twoness, as a form number, is not composed of oneness and oneness. In other words, a numerosity which has two entities does not become two by addition, nor by division, but by participating in the form of two. Hence form-numbers are not generated through agglutination, but they are abstract entities, an infinite number of abstract and eternal entities. For group quantified entities there is a form number, and this ad infinitum.

One of the problems which Plato might have incurred, in conceiving numbers as forms, would have been how to fit the non-composed feature of forms (or self-predication, or separation, purity etc.) with the composed feature of numbers. The non-composed number-forms could not be part of mathematics since they cannot be divided into parts. His solution seems to be to abandon the composed feature of numbers. Two things may be two because of addition, but they form a unity that remains one, and, by advancing such a notion of numbers, Plato denies precisely the composed nature of such sets. But what about bigger numbers, such as 7543 for example? Is it a composed or non-composed number? According to the generation process, the only non-composed number is two, which is generated from duality. One is un-generated, and it is not a number. Three is the first number, which is a unification of one and two, indirectly, of unity and duality. It may be that through the generation process, Plato was developing a strategy in which there is room for

[168] This could be the one, as a number, which is discussed in the whole second part of the *Parmenides*, a one which is both a unity and plurality. Since two could be a unity and a plurality, and so on. By reading *to hen* in the *Parmenides* through to *arithmos* one should derive similar hypotheses.

both non-composed and composed numbers. Having only a non-composed number, which is two, may be enough for the rest of composed numbers as well. Two is seemingly composed, but not from another number, but from any of the two of the three kinds, a difference that points to the ontological foundations of numbers. Besides the tension between the composed and non-composed features of numbers, the plain number-forms would leave out the possibility of addition and multiplication, since these processes cannot be applied to form numbers, which are not composed of units.

One of the questions for Plato is "How many forms are there?", or, in the words of Mary McCabe: "How many forms are there for each set of particulars?"[169] If numbers were forms, then there would be an infinite number of form-numbers, and, thus, of forms. This would raise serious difficulties in building a consistent theory of forms, as it opens the complex problematic that reminds McCabe of the issues expressed in the Third Man Argument (TMA), namely "the question of counting forms"[170]. How many clone forms for largeness do we need to explain large things? One way out of this dilemma would be to keep the number of forms as a secondary and irrelevant question[171]. For sure the number of forms could not be infinite. Instead, the sensible world is infinite: the world *per se* has infinitely many determinations, and all the determinations are impossible to be fully known and fully counted. The infinite could not be part of the intelligible world, especially

[169] Mary Margaret McCabe, *Plato's Individuals* (Princeton University Press, 1999), 86.

[170] Ibid.

[171] "Yet this question might, on some views, be thought to be marginal", Ibid.

quantitatively. The intelligible world, with its precise and knowable determinations, must be finite in Plato's conception.

The matter of the precise number of forms was never addressed explicitly by Plato and no suggestion seems to be made in this direction. On the other hand he acknowledges in the *Parmenides* (141a) the infinite character of the series of integer numbers, which might be an argument in favour of the thesis that numbers were not thought of as forms proper, but they could be reduced to limited *primordia* of numbers (cf. Chapter seven, especially section 7.2).

A philosophy of numbers should narrow the discussion to a finite number of principles which generate numbers. In the *Timaeus*, 53a, Plato speaks about a limited number of the elementary constituents of the world: two types of triangles. This is contrasted by Aristotle in *De generatione,* 325b7, with the atomists who had an unlimited number of atomic shapes.[172] An analogy could be made with the limited number of forms. It may be that in order to keep away from infinity, Plato would feel compelled to reconsider numbers through basic entities, and, thereafter, to consider ways of combining them. The argument for generation of numbers could be conceived as such an attempt.

The simplest way of avoiding the infinite problem would have been to affirm only one form for all numbers, just as there is only one form of largeness for all large things. But Aristotle informs us that the Platonists "did not posit Ideas of classes

[172] See the commentary of the translator: Christopher John Fards Williams, *Aristotle's De Generatione Et Corruptione* (Clarendon Press, 1982), 131. The atomists in discussion are Leucippus or Democritus.

within which they recognized priority and posteriority (which is the reason why they did not maintain the existence of an Idea embracing all numbers)" (*Nicomachean Ethics* 1096a17-19).

A reduction of number to a few basic principles, comparable to that of the Pythagoreans,[173] would be the solution for an infinite number of number forms, and also a solution to the problems raised by the composed features of form-numbers. Thus the generation of numbers may come as an answer to these questions related to the theory of form-numbers.

At this point we should mention also that the relation between form number and participated things would fall under the Platonic principle of 'One over Many'. In the *Philebus* (14e) Plato says in so many words that one is many: "one is many and indefinitely many, and again that the many are only one thing." We find a similar account in the *Republic* (525e). The "one-many" problem (and the use of a peculiar principle of identity) recurs in several places in Plato's dialogues and was a constant philosophical thought. This principle, which is, through other philosophical issues, related with the problem of the relation of the form with the many participated things, is also directly related with the ontological status of numbers. Is the number 6 or 273 a unity or a plurality? As numbers, 6 and 273 are pluralities, as forms they are unities.

[173] In which the numbers are reduced to the tetractys, which is the base for numerosity.

4.2.4. Aristotle on Plato's philosophy of mathematics

Plato's dialogues do not present themselves as a coherent corpus. Each dialogue has its own problematic and each dialogue has distinguishable philosophical traits. It is Aristotle who perceives and presents Plato as a systematic philosopher with a coherent system. Nonetheless, Aristotle himself is inconsistent and presents different views on Plato's philosophical system in general, and on mathematics, in particular. Several of Aristotle's testimonies on the topic of Plato's philosophy of mathematics are in many regards conflicting and confusing, and complicate substantially any attempt at making sense of how Plato understood the ontology of mathematical objects. Aristotle attributed at least seven partly contradictory views to Plato:

a) numbers are forms (*Met*. 1073a17-22, 1090a16-17),

b) numbers (mathematical objects –τὰ μαθηματικά) are intermediary objects between forms and physical particulars (*Met*. 987b14-17, 1028b19-21, 1059b5-14, etc.),

c) individual instances exist by participation in numbers (*Met*. 987b12),

d) numbers are the products of the one and the dyad (*Met*. 987b22-35, 1092a23-24),

e) numbers are generated out of the dyad, except those which are prime (*Met*. 987b23-988a1),

f) form numbers are only up to the decad (*Phys.* 206b32-33, *Met*. 1084a10, 25),

g) forms are numbers (*Met*. 991b9, 1081a12, 1083a-1084a, *De Anima* 404b24-25).

As one can see, several of Aristotle's remarks are mutually exclusive, and Aristotle does not point that out himself. All these partially conflicting and competing testimonies do suggest that Plato's philosophy of mathematics was from the very beginning a controversial issue. Plato's dialogues give direct support for some of the Aristotelian claims, especially for (a), (b), and (c). The number-form theory (a) could fit the views in the *Phaedo* (101b9-c9, 103-106), while the assessment that numbers are intermediaries between forms and things (b) could find some grounds in the *Republic* (509d-511a),[174] depending on how one interprets the analogy of the divided line (epistemologically or/and ontologically)[175], and in the *Philebus* (56c-59d). The claim that things exist by participation to numbers (c) could be traced in the *Timaeus*, where, unlike any of the Aristotelian conceptions, physics and mathematics are intimately related. The *Timaeus* goes along with (c), taking mathematics as an essential feature of the physical world, although it is not evident how the

[174] In the *Republic*, the realm of mathematics is separated from the realm of forms through the divided line. It is a matter of controversy why Plato operates such a difference between the territory of forms and that of mathematics given the fact that he did not operate such a differentiation in other circumstances – on the contrary, he used mathematics as the example par excellence to prove that forms are real.

[175] Upon an epistemological reading, intelligible numbers are at the same ontological level with the forms, and only epistemic faculties are hierarchically differentiated; upon an ontological reading epistemic faculties overlap with an ontological hierarchy, and intelligible numbers are between the *sensibilia* and the realm of forms.

mathematical objects from the *Timaeus* can be linked with (a) and (b). However, in the *Timaeus*, Plato does not construct physical particulars through numbers, but through geometrical objects. Physical bodies are composed of particular geometrical entities. At its turn, the structure of these entities is determined by two types of right-angled triangles: isosceles (45°/45°/90°) or scalene, (30°/60°/90°). These triangles are the ultimate "atoms" of matter.

The supposition that Plato reduced numbers to one and the indefinite dyad (d) is excessively – and almost exclusively – defended by the Tübingen School as the real system of Plato, and it relies minimally on Platonic texts, and mainly on Aristotle's and post-Aristotelian testimonies. That Plato had thought of form-numbers only up to ten (f) apparently does not resemble anything in Plato's dialogues,[176] and this testimony is most unexpected and puzzling for what we usually assume about Plato's understanding of numbers. The report could be read also through the argument of the generation of numbers. It is important to underline that Aristotle supposed that Plato come to a halt with his generation of numbers at some point[177], and, if we rely completely on this claim, [178] Plato had a position closer to that of the Pythagoreans. That Plato thought that the forms are numbers (g) seems

[176] In *Physics* 206b30-33, Aristotle says: "For in the numbers the infinite in the direction of reduction is not present, as the monad is the smallest; nor is the infinite in the direction of increase, for he makes numbers only up to the decad." In *Metaphysics* XII, 1073a20, Aristotle reconfirms the restraint to the decad, and also the fact that this restriction is not present all the time: "for those who believe in Ideas say the Ideas are numbers, and they speak of numbers now as unlimited, now as limited by the number 10."

[177] See also Ross, *Plato's Theory of Ideas*, 179.

[178] This is an aspect which must not be ignored. John Dillon lays stress on this issue, in the beginning of his discussion of the middle Platonists: "a special importance is attached by him [Plato], as it was by the Pythagoreans, to the 'primal numbers', one, two, three and four, and their sum-total, ten (the Decad)." Dillon, 1977, 4.

to be a peculiarity of Aristotle's interpretation, and it lacks any direct reference in Platonic dialogues.[179]

Despite all these possibilities, the main scholarly controversy in the field is almost exclusively on a) versus b) – whether, according to Aristotle, Plato understood mathematical objects as forms[180] (P. Shorey[181] and H. Cherniss,[182] and, more recently, P. Pritchard,[183] and W. Tait[184]) or as intermediaries between forms and participated things (A. Wedberg,[185] and M. Burnyeat[186]). The grounds for these

[179] However, Ross points that sometimes Plato links specific forms with numbers. W.D. Ross, *Plato's Theory of Ideas*, p. 218. In this regard Plato could follow the Pythagoreans. According to Aristotle (*Magna Moralia* 1182a14, 1194a28), the Pythagoreans understood justice as a square number (also *Ross*, 218). Also Plato, as Ross points out, "regarded some Ideas as monadic, others as dyadic, and so on" (Ross, 218).

[180] Even if Plato, in most of his dialogues, is not explicit concerning the form feature of numbers, and he does not use forms as the most economical explanation for the "existence" of numbers, he is usually charged with mathematical realism by scholars and mathematicians alike. As I have pointed out, it is Aristotle who, in his struggle to reject Plato's assumptions on numbers (or what he thought that Plato assumed about numbers), states that for Plato numbers are forms, criticizing Plato for having separated numbers from things. Aristotle's rejection of Plato's philosophy of mathematics is part of his overall refusal to accept any kind of theory of forms, coming thus as a natural objection to Plato's conception of numbers. At the same time, Aristotle extrapolates many features of the forms to numbers, assuming that Plato applied the same conceptual frame to numbers. As in the case of Aristotle's rejection of the theory of forms, he argues that numbers should not be separated from things, but rather that they are the product of counting things: one cannot have numbers without things which are counted.

[181] P. Shorey, *The Unity of Plato's Thought* (Chicago: University of Chicago Press, 1968).

[182] H. Cherniss, *The Riddle of the Early Academy* (Berkeley: University of California Press, 1945).

[183] P. Pritchard, *Plato's Philosophy of Mathematics* (Sankt Augustin: Academia Verlag, 1995). For example, Pritchard thinks that there is "no indication that [Plato] had even noticed that he was talking about two distinct kinds of objects of thought" (160).

[184] W. Tait, "Noesis: Plato on exact science", *Reading Natural Philosophy: Essays to Honor Howard Stein*, ed. D. Malament, (Illinois: Open Court, 2002), 11–30.

[185] A. Wedberg, *Plato's Philosophy of Mathematics* (Westport, CT: Greenwood Press, 1977). One of the earliest defences is that of J. Adam, *The* Repu*blic of Plato,* (Cambridge: Cambridge University Press, 1902). For a critique of Adam, see P. Shorey which argues for number-form theory, "Ideas and Numbers Again," *Classical Philology* 22, no. 2 (April 1, 1927): 213–218.

[186] "For a Platonist the Forms are yet more real and still more fundamental to explaining the scheme of things than the objects of mathematics" M. Burnyeat, "Plato on why mathematics is good for the soul", in T. Smiley, ed., *Mathematics and Necessity* (Oxford: Oxford University Press, 2000), 1–82. See also M. Burnyeat, "Platonism and Mathematics: A Prelude to Discussion", in

two main conflicting views on Plato's understanding of mathematical objects rely heavily on Aristotle's testimonies, which favoured most the intermediary position of numbers. The two views seem to be irreconcilable, and scholars argue for one or the other position; one must add that scholars who support a) or b) assume that Plato had a stable theory, of the intermediary or of the number-forms, which basically is unchanged from the *Phaedo* and the *Republic* and *passim*.

If there is a place where Plato apparently made a difference between numbers and forms, it is in the *Republic*. Here, the epistemic or the ontological realm of mathematics is separated from the realm of forms. Plato uses a mathematical ratio[187] in order to divide different layers of reality. The realm of mathematics is separated from the realm of forms and *dianoia* – the discursive faculty – is epistemologically responsible for the understanding/perception of mathematical entities. The epistemic aspect of the divided line points out the fact that different cognitive faculties are required for the knowledge of forms, on the one hand, and the knowledge of mathematical objects, on the other. With respect to how one understands the divided line, there are two irreconcilable positions: 1). Plato understood mathematical objects as forms (or similar to them), and only their epistemic aspect is different; 2). Mathematical objects are not forms but, nevertheless, they are part of the intelligible realm.

Mathematics and Metaphysics in Aristotle, ed. A. Graeser (Bern & Stuttgart: Haupt, 1987), 212-40. Burnyeat thinks that "none of those who are sceptical of Aristotle's repeated and unambiguous ascription to Plato of a doctrine of intermediates has ever told us how mathematics *could* be about Forms instead." (Burnyeat, *Op.Cit*., 229).

[187] As an impossible division in mathematics, the topic of the divided line is intensively studied. The ratio of the divided line could be tributary to a Pythagorean conception on the mathematical relation between body and soul, matter and numbers, or, on the contrary, there could be no divided line at all based on any mathematical ratio.

There are several statements which could be taken as illustrative for a), for the number-form theory. Plato speaks about "the one itself" (524e6), "numbers themselves" (525d6), "the square itself and the diagonal itself" (510d5-8), and so on, in a manner reminiscent of how he refers to forms. Judging from these passages, it could be that Plato thought of mathematical objects as forms.[188] An illustrative example for this reading may be found in the article of Cornford[189]. Cornford argues that there is a difference between moral forms (the objects of *noesis*) and mathematical forms (the objects of *dianoia*), a difference, nonetheless that does not hinder them to be at the same ontological level. Epistemological clarity with regards to forms is for Plato an indicator that a specific concept is a form. Since one can perfectly define truth, beauty, love, and at the same time what the triangle, the square, the number, etc. is in a precise way, these must be forms; the perfect definition is a condition and a clue into the directions of forms. As we understand easily what the number 4 refers to, according to Plato's theory, the number 4 could therefore be a *form*.

For b), in the *Republic*, at 511d4-5, Plato insists that the epistemic faculty of *dianoia* is "something intermediate between opinion and reason" (ὡς μεταξὺ τι δόξης τε καὶ νοῦ τὴν διάνοιαν οὖσαν). For several scholars this epistemic distinction from the divided line advocates an ontological difference: numbers are not forms, but intermediates between forms and participated things. Why would Plato operate

[188] For others, nevertheless, these claims do not necessarily point. See Burnyeat, "Platonism and Mathematics," 219-20, note 19.

[189] Cornford, F. M., "Mathematics and Dialectic in the Republic VI-VII," *Mind* 41 (1932), (161):37-52, (162):173-90. Reprinted in Allen, R. E., ed. Studies in Plato's Metaphysics. London: Routledge and Kegan Paul, 1965.

such a difference between the domain of forms and that of mathematical objects[190] is unclear. He did not use such a differentiation in other circumstances – on the contrary, he used mathematics as the example *par excellence* to prove that *forms* are real. Does the distinction in the *Republic* imply that numbers are not forms? If they are not forms, what are they? Plato seems to be telling us that the domain of mathematics is different from that of the forms, but it isn't obvious how a class of mathematical objects should be taken separately from the forms. Even if they are intermediates, it still doesn't illuminate more the nature of numbers. If mathematical objects are different from the forms they still remain part of the intelligibles; thus there are two types of intelligibles, conceivable through different ways of cognitive access. The prospect of intermediate entities complicates the picture drawn by the theory of forms and mathematical objects even more, and what it actually does transmit to us is that numbers are, mainly, a lower (form) category. The need for intermediates would be justified by the need to be consistent with the principle of one over many. If numbers were forms, when one performed mathematical operations the principle of one over many would fail. Aristotle's testimony supports the view that Plato considered numbers as intermediates for this reason. Addition of two numbers of the same type (say 3+3) poses difficulties if one has to summon two forms of threeness. There could be a form of threeness but the mathematical operation of adding two threenesses remains in the realm of intermediates, and not in that of the forms.

[190] The divided line appears surprisingly as a sample of epistemology and ontology within a rigorous discussion regarding politics.

The tendency of scholarly reserch to choose between only one of these two possibilities – mathematical objects are forms or intermediaries – eclipses the rest of Aristotle's rich testimony on Plato's mathematical philosophy. The statement e), in which Aristotle criticizes Plato for leaving the generation of prime numbers aside, is very specific and seems to be alien of the content of any of the dialogues, with the exception of the *Parmenides* (142b-144b), and to the conventional way of seeing Plato as a Platonist regarding numbers. The following chapter examines in detail the argument for the generation of numbers and complements it with an analysis of Aristotle's testimony on prime numbers.

5. The Mathematical Argument. From One to Numbers (143c-144a4)

This chapter focuses on the second argument of the second hypothesis (143a4-144a5), namely on the process of the generation of numbers. It presents and discusses the ontological and mathematical implications of the argument. One might get the impression that Plato's aim is to theorize how we can obtain the first even and the first odd number, i. e. two and three, assuming thus evenness and oddness as sufficient conditions for the generation of all numbers. The following discussion tries to give a clearer picture of these suppositions by trying to read the argument in the wider context of Platonic dialogues, the framework of Greek mathematics and Aristotle's testimonies.[191]

From a strict ontological argumentation Plato switches abruptly towards what seems to be a mere counting procedure of intelligible entities, which stands as a basis on which a generation of numbers is constructed. Starting with three entities – one, being, and difference – Plato lays the foundation for all numbers. Having been given these three ingredients, the numbers two and three come to be, in a very complicated manner, and quite unexpectedly: multiplicity evolves from two and three.

[191] This chapter is a more detailed elaboration of what I have already published in a more condensed way: "One, Two, Three… A Discussion on the Generation of Numbers in Plato's *Parmenides*", in *New Europe College Stefan Odobleja Program*, Vainovski-Mihai, I. (ed.), 49-78.

It is a question if one, being and difference are the entities for reaching numerosity, or whther there could be any other entities than these three. There could be more entities, as for example, motion and rest, but only three entities are, for the moment, the necessary constituents to obtain numbers.[192] I advance the idea that these constituents have an ontological priority comparable to that of the composition and the generation of the world soul (*Timaeus* 34c-35b) where sameness, being, and difference are at the core of formation of the souls. We could speculate how ontological concepts of one, being and difference could underpin arithmetical structures: one gives to each number unity, being causes each number to be, and difference differentiates the numbers from each other.[193] Starting from other kinds, such as rest and motion, it would be difficult to think how they would contribute anything to numerosity. The relation between one, being, difference and numbers would be similar to the blending, combination, *koinonia*, of the forms towards participated things (*Republic*, 476a7-8). There is also the *koinonia* between the forms (*Sophist*, 253d). The model of the *Republic* would be closer. Numbers could partake in the one, being, and difference in the way I speculated above.

[192] Dougal Blyth argues that what actually Plato does is to count linguistic items, and numerosity arises from our ability of counting, Blyth, Dougal, "*Platonic Number* in the *Parmenides* and Metaphysics XIII". Blyth's interpretation, nevertheless, does not justify why Plato uses one, being and difference as starters for counting.

[193] Scholars do not explicate the bewildering link between kinds and the generation of numbers. To my knowledge, Anscombe is one of the few who tries to give an explanation for the role that these kinds play in the ontology of numbers. She says that "one itself is infinitely divided, each of the numbers being one." Anscombe, "The New Theory of Forms," in *From Parmenides to Wittgenstein*, 26.She does not, however, venture on to provide equally illuminating explanations on how being or difference would work for number.

5.1. The indistinctness of duality

The division or multiplication of one into one and being which in their turn multiply themselves into one and being, and so on, resembles the phases of cell division, where a cell is divided into two cells, which in their turn are divided into two cells themselves, and so on. In addition to the division of the one- being pair into identical "cells" of the same type (142d-143a), several nuances and gradations of duality are used in the second stage of the argument.

The argument builds on the following types of dualities:

- One is always two, it is never one
- From three distinct entities we can pick up pairs (τινε) (143c3);
- A pair is called both (ἀμφοτέρω) (143c4);
- What is called both is two (δύο) (143d2).

In the second hypothesis, there is an intensification of different facets of duality. In the following lines I will discuss particular problems raised by this inventory of lexical items denoting duality, offering possible interpretations.

5.1.1. The priority of pair relations. The internal relation of one-being

There is a conceptual distinctiveness of duality in our perceptions of items that are more than one. The priority of duality is obvious especially when we perceive the elements of the pairs which are not countable by way of a chronological order, as we

count items which are more than four. We perceive two items as a pair (when the items are of the same type) or as a group of two without counting them (i.e. there is one, there is the two, therefore there are three items). It goes the same with three items; one can perceive a group of three items without necessarily counting them. Perhaps it is the same with four, but surely five items necessarily make us count.[194]

The unity of duality is especially visible when dealing with pairs: the pair of items remain as a unity in themselves. The eyes are a pair, since one cannot decide which of the eyes comes first when counting. Since our body is mainly divided in two (two eyes, arms, legs, etc.), wq are inclined towards perception of dualities and pairs without counting them. It is not the same with our hand with five fingers. We know that there are five, and we check if other mammals have five or four fingers, thus we count them, and we don't perceive their number spontaneously.

There are several more examples which illustrate that it is difficult to define which member of the two items is the first in an ordinal way, as in the instance of the arms, the legs, the ears, a pair twins, etc. – the most common case of dual numeral in the ancient Greek language.[195] Simply, since it does not matter what way you start

[194] Concerning the acquisition of numbers in cognitive science, there is considerable evidence to support the idea that concepts such as 1, 2, 3, and possibly 4, are a priori, and the rest of the numbers are derivatively deduced. See, for example, Susan Carey, *The Origin of Concepts* (Oxford University Press, 2009), Ch. 4 *Core Cognition: Number*.

[195] The dual (*δυικός*), together with the plural (*πληθυντικός*), are grammatical categories that define number in Homeric Greek, for expressing things that are more than one, while single items are defined by the singular (*ἑνικός ἀριθμὸς*). The Attic dialect preserves the dual form, which otherwise is largely lost in other Greek dialects. The dual form is usually used in depictions of bodily parts ("two eyes", "two ears", "two arms", etc.) or of two persons that are thought together (Castor & Pollux, Achilles & Patroclus, Demeter & Kore, etc). The use of the dual form illustrates that Plato thought about *one* and *being* as being a joint pair. In English there are pairs for which we don't use the numeral two. We say, for example, a pair of slippers, not two slippers, a pair of gloves, not two gloves, a pair of earrings, and so on. More evident perhaps it is the unity of duality in 'a pair of scissors'. The language mirrors the thought that the two objects are in an

counting, the pair of items come together, while the pair relation is internal. Nobody would actually count the two eyes; their quantification is almost a priori. The parity of the two eyes is a given; it is a duality, and counting them individually is merely a process of division; its number, as a pair, is prior to the counting of two units.

Similarly with items which do not naturally fall into a pair relation, such as two books, or two trees, we perceive a structure that is dual, without necessarily counting them. The twos are uncountable. With regards to items of different kinds it is not so evident that we perceive them directly as a pair, and distinguishing afterwards that the respective pair is made up of two countable but different items; we might also have another possibility: we conceive first that there is one item, then that there is also another item, and, thus, that there are two items. In the latter instance we conceive them separately, and in the first as forming a duo.

At the first view, the one-being pair would be rather more like a set of two different items, since it does not bring with it any of the immediacy of the usual pairs of two eyes, ears, etc., and, most stringently, since we don't have a pair of identical things, but a pair of two expressly different entities. Nevertheless, this ontological pair relation is more powerful than the pair relation of similar and identical things, which can be, if needed, conceived independently of each other. Before being two distinct entities, the one-being pair remains as an inseparable pair. Two items can exist independently of each other (a book from two books, a tree from

interdependent relation, a relation in which it is insufficient to speak about one member of the pair alone.

a pair of trees, even an arm from two arms), but being and one are understood by Plato as an internal parity.

5.1.2. From duality to two (δύο)

We would expect that the number two to be derived directly from the duality of one and being. On the contrary, the number two is derived from couples that can be made from a trinity (one, being, and difference). Plato explicitly states that since we have three distinct entities we can pick out three types of pairs (τινε): being and difference, or being and one, or one and difference. Plato specifies that a pair is called 'both' (ἀμφοτέρω) ["x", "y" = "both (x, y)"], and what is called both is two (δύο). Plato takes thus δύο (which matches the set with two members and the cardinal number two) as a consequence and derivation of ἄμφω.

Δύο refers to two things and not exactly to the abstract number two. In Greek mathematics, number refers not to abstract entities, but to numerosities. Should we conceive δύο as a proper abstract number (closer to a more modern understanding of number) or, in a tradition established by Greek mathematics, to a set of units? It seems that Plato goes against Greek mathematics[196] and conceived δύο as an abstract entity, as an abstract derivation of ἀμφοτέρω. The duality of ἄμφω resists a reduction to the ordinal or cardinal feature of the number two, while the cardinality of δύο is

[196] Wedberg thinks also that Plato's views on numbers could be understood against the background of his contemporary Greek mathematics, Anders Wedberg, *Plato's Philosophy of Mathematics* (Almqvist & Wiksell, 1955), 71.

posterior and a consequence of the pair condition of ἀμφοτέρω. If it is the other way around and the pair relation is posterior to cardinality, that there is prior numerosity of two elements, from which one can constitute a pair, then the whole construction would have no reason. The question as to why these pairs are picked out of threeness prior to their being two, could be answered by the fact that this way of proceeding enables a transition from duality (which is neither an ordinal series or a cardinal quantity) to cardinality. Duality remains an ontological relation which precedes any operation of counting. Plato pursues explicitly and insistently the following direction of thinking about pair, duality and two: first there is a pair (τινε) (143c3), since the pair is called both (ἀμφοτέρω) (143c4), we have two (δύο) (143d2), and each of the two (δύο ἦτον) is one (ἓν εἶναι) (143d4-5).

One of the reasons why the argument tries to start the number series with δύο could be part of the framework of Greek mathematics which conceived the first number of the number series as being the number two.[197] There could be a corollary in modern philosophical research in the work of Luitzen Egbertus Jan Brouwer (an opponent of the formalism of David Hilbert) who pioneered intuitionism and neo-intuitionism in philosophy of mathematics. For Brouwer, number generation starts from the intuition of pure twoness:

"From the intuition of the pure two-oneness, mathematics is generated by successive mental constructions, and these are always governed by inner intuition;

[197] For the understanding of the number two as the first number see Ross, Aristotle, *Physics*, (Oxford, 1936), 604. Wedberg, *Plato's Philosophy of Mathematics* (Almqvist and Wiksell,Stockholm, 1955), 23.

there is thus no place in it for the assumption of infinite totalities whose elements are not supposed to be generated step by step through an appropriate procedure."[198]

In Brower's words:

> "This intuition of two-oneness, the basal intuition of mathematics, creates not only the numbers one and two, but also all finite ordinal numbers, inasmuch as one of the elements of the two-oneness may be thought of as a new two-oneness, which process may be repeated indefinitely"[199]

Similar to Plato's view on the generation of number which starts from twoness (143c-d5), from pair to two, and from two to one, Brower's generation postulates the structure of duality as basis. The priority of duality was rejected by Aristotle in relation to Plato and by several modern philosophers of mathematics in relation to Brower.[200]

5.2. The origin of threeness

[198] This is the synthesis offered by Marco Panza and Andrea Sereni for Brower's mathematical philosophy. See M. Panza and A. Sereni, *Plato's Problem: An Introduction to Mathematical Platonism* (Springer, 2013), 88.

[199] L. E. J. Brouwer, *Intuitionism and Formalism* in Paul Benacerraf and Hilary Putnam, *Philosophy of Mathematics: Selected Readings* (Cambridge University Press, 1984), 77-89, 80. For a critique of Brower from Wittgenstein's perspective see Peter Michael Stephan Hacker, *Insight and Illusion: Themes in the Philosophy of Wittgenstein* (Clarendon Press, 1986), subchapter "The Brouwer Lecture", especially 125-127.

[200] Brower is not a Platonist in mathematics, but an intuitionist. What both the arguments of Plato and Brower have in common is that they are both non-Platonist arguments regarding the nature of numbers. Brower's intuitionism in regard to duality is not based on Plato's argument, but on Kant. See James Robert Brown, *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures* (Psychology Press, 1999), 115. Julia Annas also draws a parallel with Brower, but in the context of Aristotle's testimony that Plato derived numbers from "one" and "indefinite two". Cf. Julia Annas, *Aristotle's Metaphysics: Books M and N* (Clarendon Press, 1976), 43.

The mathematical argument from the *Parmenides* starts with δύο and, instead of going directly further, to three, Plato makes a loop to get to one. Surprisingly, the mathematical unity (ἓν εἶναι) for counting is obtained via δύο, and not as a given from the very beginning, from the initial ἕν. One is obtained from δύο, which should now stand as a unity for calculations, but Plato refers to it explicitly only once (143d7) when presenting the generation of a set with three members (τρία γίγνεται).

The ἕν was apparently already mentioned and used at the opening of the argument (142b-c). I think that the ἕν that we have at the beginning of the argument cannot be a mathematical ἕν, since its oneness refers rather to its ontological value than to its numerical property. Moreover, for this initial ἕν Plato's emphasis lies on its ontological plurality and not on its value as a unity – that we find, for example, in calculation – "since [one] it always proves to be two, it must never be one" (142e7).

The new ἕν (to which we get after and through number two!), which performs the role of mathematical unity, by the ordinary formula 1+1+1…. and so on, would be enough for expressing the generation of numbers, and, therefore, to have number 3 as the sum that is obtained from 1+1+1. Another possibility would be to have number 3 from the initial counting of the ontological trinity: one, being, and difference. Ross opts for the second version and hastily says that since we already have one, being, and difference, we have thus three countable things, which is the first odd number.[201] Ross assumes, in a way, what one would expect as the normal transition from the ontological argument to the argument for the generation of numbers, namely the counting of all three entities, since Plato had already counted

[201] William David Ross, *Plato's Theory of Ideas* (Clarendon Press, 1966), 187.

being and oneness as two. The text, however, does not follow this path. Plato does not follow what would be the obvious way for Ross and probably for most of us: multiplying one or counting the three entities; instead, he offers something unexpected and which was overlooked by Ross: the cardinal three is obtained by the addition of a unity to any pair (143d7).

How to determine what kind of one is added to two? Is it a left over one, after we pick up a pair, from the three entities? As for example: we pick a pair, let's say one and difference, from this pair we infer that there is two, and we add the remaining entity – being (which would be the remaining ontological entity). Is this the procedure Plato was thinking of, or, since we already have two, from the pair, and each member of the pair is now signposted as a mathematical one (as a mathematical entity), can one therefore use it also as a numerical one? I think that the last option is the case, and that Plato uses now, for the first time, the mathematical one.

One should question whether there was the necessity, in order to obtain three, of such a complex procedure (highlighting the one which is added to any pair). It is also questionable why the initial three entities of the argument - "difference is not the same as oneness or being" (143b6-7) - are not counted, and to refer directly to them as three entities. Are these entities (one, being, and difference) not appropriate for obtaining the number three or the set of threeness, as Plato does when he gets to sets of two? Is the way of obtaining three from 2+1 and not from 1+1+1 more relevant? In the following discussion I try to formulate possible answers.

One of the possibilities that I am considering is that obtaining the number three only from counting one, being and difference would not emphasize oddness as it does when formulating threeness as 2+1 (as example of the formula 2k+1 for odd number). The formula 2k+1 is illustrated in the figure 3.[202] What the argument seems to be doing is to display that 3 is not 1+1+1, but it is essentially 1 added to 2.

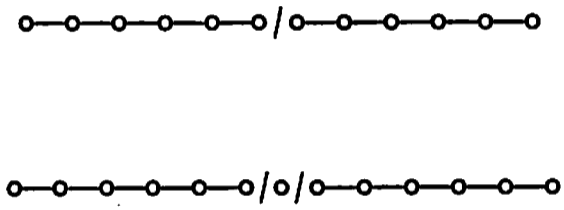

Figure 3. Visual representation of formula 2k+1

Going further with the emphasis on the formula 2k+1, we can see that this formula is not a peculiarity of this argument, and can be found in other Platonic dialogues as well. At *Phaedo*, 105c, Plato explicitly maintains that oneness is the sine qua non condition for an odd number to be odd:

> "if asked the presence of what in a number makes it odd (περιττὸς), I will not say oddness (περιττότης) but oneness (μονάς)."

[202] Apud W. R. Knorr, *The Evolution of the Euclidean Elements: A Study of the Theory of Incommensurable Magnitudes and Its Significance for Early Greek Geometry* (Springer Science & Business Media, 2012), 140.

For some scholars μονάς can stand here both for the form of one and also for unit.[203] As I understand it, in the light of the *Parmenides*, in the *Phaedo* Plato talks about odd as what stands out as having an extra one, a one which disables the evenness of the number. Plato is more explicit in this regard in the *Parmenides*, where the addition of one to an even results in odd. [204]

Plato's formula for the generation of numbers could be understood in the following manner. We have number 2 and number 2+1, and by their multiplication, 2x2, 2x(2+1), (2+1)x(2+1) and so on, the rest of numbers. This formula has the advantage of showing that there is need only for 2 and 1 to generate numbers. And since 1 is derived from 2, there is need only for 2 for generating numbers (as I mentioned in the previous subchapter, having 2 as the origin of numbers would be consistent with Greek mathematics). Similar to the origin of three, there is also an ambiguity regarding the origin of numerical one: it is not from the previous triad (one, being, and difference) that the numerical one is deduced, but from the memberships of the pair: "If there are two things, is there any way for each member of the pair not to be one? […]Therefore, since in fact each pair taken together turns out to be two, each member would be one" (143d2-5).[205] Thus, if there is one, there is 1, 2, 3, but not in this order. If one is, there are numbers are too, in the following order: 2, 1, and 2+1 (3).

[203] See, for example, David Gallop, *Phaedo* (Clarendon Press, 1975), 210.

[204] Liddell-Scott for περισσός also the form περισσότεροι which means "*more in number, extra*". This could refer to 1 which is added extra.

[205] Translation by Gill and Ryan: ὦ δ᾽ ἂν δύο ἦτον, ἔστι τις μηχανὴ μὴ οὐχ ἑκάτερον αὐτοῖν ἓν εἶναι; […] τούτων ἄρα ἐπείπερ σύνδυο ἕκαστα συμβαίνει εἶναι, καὶ ἓν ἂν εἴη ἕκαστον.

One must notice that the number three is the second number after the number two, and, as Julius Moravcsik suggests, it is a basic number.[206] The rest of numbers after two and three (which are the first two numbers) are secondary numbers.

A geometrical view on numbers that comes from a Pythagorean background may offer an alternative for the choice of the formula 2+1 for 3, in spite of the repetitive addition 1+1+. Number 3 might be conceptualized here as a triangular number.[207] The first triangular number, t1=1, forms the next triangular number t2, which is t1+2, which equals 3. The third triangular number, t3, would be t2+3, which equals 6, t4 would be t3+4, which equals 10, and so on (see figure 4). For 6 respectively 10 and the rest of number, Plato has already another formula (as even times odd) and does not use anymore a gnomon.[208]

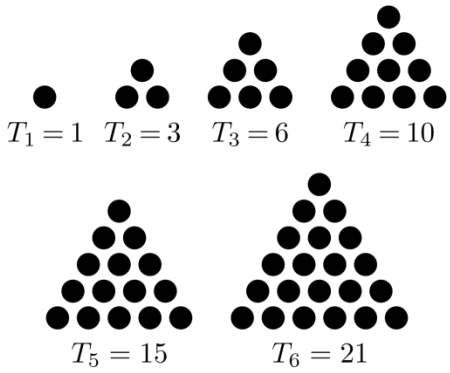

Figure 4. Triangular numbers

[206] Moravcsik mentions in passing this feature of number three: "(Plato might add the number 3 as basic if 1 is not acknowledged as a number)", Julius M. Moravcsik 'Forms and dialectic in the second half of the *Parmenides*" in Malcolm Schofield and Martha Craven Nussbaum, *Language and Logos: Studies in Ancient Greek Philosophy Presented to G. E. L. Owen* (Cambridge University Press, 2006), 135-154. 144.

[207] Most probably a discovery by Pythagoras. Heath, *A History of Greek Mathematics*, Vol. 1, 76.

[208] Gnomon is the number that is added to the previous triangular number. Knorr, *The Evolution of the Euclidean Elements,* 143.

5.3. Oddness and evenness

5.3.1. The pre-eminence of oddness and evenness

As I have shown above, Plato's formula for the generation of numbers follows the products of multiplication of 2 and of 2+1, reaching thus a differentiation of numbers into even and odd, whose preeminence in understanding numbers is evident all throughout the argument. The subsequent phrase after getting to 3 and 2 is that the first is odd, while the latter is even: τρία […] περιττὰ καὶ δύο ἄρτια (143d8-9). With this Plato's intention to assert a pattern of numbers becomes transparent and his eagerness to express a general mathematical classification of numbers justifies later readings of this passage as a passage about how numbers are generated.

This is not the only place where Plato understands numbers by what seems, for us, to be attributes of numbers: odd and even. In the argument for the generation of numbers, even and odd stay as natural in the process of multiplication. However, the process of generation of numbers could have been done without interfering with even and odd. The result would have been the same multiplying 2 and 3. The fact that Plato does not proceed directly to multiplication (which seems to be the terminal point of his argument) and insists on oddness and evenness demonstrates his perception of numbers through these two as identifications. The multiplication of evenness and oddness is for a modern eye gratuitous. The relation between 3 and

odd, and 2 and even is seen in the *Phaedo* as analogous to that between fire and hot.[209]

The argument is ostensibly redundant. Even times even (ἄρτιά ἀρτιάκις), odd times odd (περιττὰ περιττάκις), odd times even (ἄρτια περιττάκις), and even times odd (περιττὰ ἀρτιάκις) overlap in some respect. Odd times even and even times odd are identical, and even times even intersects with even times odd.[210] For example: 6x6 (even times even) is 12x3 (even times odd). Through these four operations Plato seems to exhaust all possible numbers as products of even and/or odd. Perhaps the overlapping that is inevitable given the above operation gives us a hint that for Plato an ordinary number could be understood in different ways, as we can see from the example of 36 above, which can be expressed in different ways, both as even times even or as even times odd. If we could use the terminology that pertains to the relation between sensibles for the intelligibles, then a number such as 36, being a result of even times even and even times odd, participates both in 'even times even' and 'even times odd', in the sense that their being is given by these principles of numbers.

[209] See *Plato's Phaedo* edited with introduction and notes by John Burnet (Oxford: Clarendon Press, 1963), 103. The analogy is seen by David Sedley in the following manner: "When heat approaches snow, the snow must either retreat, i.e. get out of its way, or perish, i.e. melt. Something equivalent must apply to fire in relation to heat, and to the number three in relation to oddness, even though in this last case the meanings of 'retreat' and 'perish' will have to be reinterpreted appropriately. (A guess: my three pairs of shoes are numerically odd; when I count them as six shoes, their oddness retreats; when I burn one pair, it perishes.) David Sedley, "Introduction to *Meno* and *Phaedo*", *Plato: Meno and Phaedo* (Cambridge University Press, 2010), xxxii.

[210] Heath, *A History of Greek Mathematics*, vol I, 72.

For Plato, it looks as though even and odd are not properties of numbers, but rather numbers are properties and derivations of even and odd.[211] And it seems that such classification was taken further in Greek mathematics, as Euclid goes along with it.[212] For some scholars, even, odd, and odd-even behave as species.[213] And what comes to be as important in the understanding of numbers is not the natural order of numbers, but how the universe is organised according to those species of numbers.[214] One could understand the combination of even/two and odd/three similar to the blending of the kinds/forms (the *Sophist*, 252e–253a). Compared with the *Phaedo*, where the number 6 would be the result of the form of hexad, from the argument in the *Parmenides* one would get to the conclusion that the number 6 is a combination of even/two and odd/three.

No meters what definition they may be given, one can see that through the identification or classification of the odd and even, one can subsequently identify all the numbers.[215] The classification of numbers into odd and even is so natural for

[211] "It is then not right to say "where there is fear there is also shame," but that where there is shame there is also fear, for fear covers a larger area than shame. Shame is a part of fear just as odd is a part of number, with the result that it is not true that *where there is number there is also oddness, but that where there is oddness there is also number.*" [my emphasis] *Euthyphro* (12d-e) and continues by specifying the same about evenness: "See what comes next: if the pious is a part of the just, we must, it seems, find out what part of the just it is. Now if you asked me something of what we mentioned just now, such as what part of number is the even, and what number that is, I would say it is the number that is divisible into two equal, not unequal, parts."

[212] As Thomas Heath noticed, "Euclid's classification does not go much beyond this [Plato's classification]". Thomas Little Heath, *A History of Greek Mathematics* (Oxford, The Clarendon Press, 1921), 72.

[213] See Jacob Klein, "The Concept of Number in Greek Mathematics and Philosophy," in Klein, *Lectures and Essays*, 43-52, 47.

[214] Ibid.

[215] For Brumbaugh, since "three is odd, and two even" (143d8), "the proof moves on from logistic or set theory to arithmetic, the theory of numbers treated as classes." R. S. Brumbaugh, *Plato on the One: the Hypotheses in the* Parmenides (Yale University Press, 1961), 97. Nevertheless,

Plato, as understood in this argument, that it gives the impression that oddness and evenness are principles of numbers. One of the most important aims within the argument for the generation of numbers is to arrive at the first odd and even numbers, and thus to proceed to the generation, being thus consistent to his commitments elsewhere (as in the *Republic* VII 524d, *Theaetetus* 198a, *Gorgias* 453e, or *Charmides* 166a) that the knowledge of numbers is the knowledge of the odd and even.[216]

5.3.2. Odd and even, limit and unlimited

Considering numbers through odd and even reminds us of Philolaus' affirmation (Fr. 5): "Number, indeed, has two proper kinds, odd and even, and a third mixed-together from both".[217] For some Pythagoreans, odd and even are part of what is known to us by the table of opposites (*Metaphysics* 986a22).[218] The odd-even pair follows immediately after the pair of limit and unlimited,[219] a pair which is part of the ontology of the *Philebus*. Aristotle confirms that odd and even are themselves

Brumbaugh does not insist more on the issue, and he takes the "definition of "twice" and "thrice" as relations between defined numbers." (Ibidem).

[216] Arithmetic is understood as the science of even and odd. Leonid Zhmud, *The Origin of the History of Science in Classical Antiquity* (Walter de Gruyter, 2006), 223.

[217] Translation by Huffman. Carl A. Huffman, *Philolaus of Croton: Pythagorean and Presocratic: A Commentary on the Fragments and Testimonia with Interpretive Essays* (Cambridge University Press, 1993), 178.

[218] According to Aristotle, the ten pairs of opposites are the following: limit-unlimited, odd-even, unity-plurality, right-left, male-female, rest-motion, straight-crooked, light-darkness, good-bad, square-oblong.

[219] For a discussion of the connection of odd with limit, and even with unlimited see also Jonathan Barnes, *The Presocratic Philosophers* (Routledge, 2013), 389-341.

specifications of limit and unlimited.[220] In the *Philebus*, the method of analysing sensible things is through limit and unlimited, and Plato strongly emulates the Pythagoreans,[221] especially Philolaus[222], and I am inclined to think that what is the ontological analysis through limit and unlimited in the *Philebus* is the configuration of numbers into even and odd in the *Parmenides*. In both cases one can see a Plato who explores the possibility of description through opposing principles. The huge importance of such principles is quantifiable if we consider that the scheme of division through one-many and limit-unlimited is presented as a divine gift. The divine origin conveys to us the epistemological power it has had on Plato and, consequently, the value of oddness and evenness in the ontology of numbers.[223]

[220] "the elements of number are the even and the odd, and of these the former is unlimited, and the latter limited" (*Met. 98622*). See also L. Sweeney, *Infinity in the Presocratics: A Bibliographical and Philosophical Study* (Springer Science, 2012), 79.

[221] Hackforth, in his commentary on *Philebus*, emphasizes the Pythagorean thinking as something obvious: "inasmuch as that classification is a counterpart of the real world of Forms, the logical problem is merged in the ontological, and Plato means us to understand that the Pythagoreans' endeavour to penetrate to the principle of Limit, which orders and 'informs' the unintelligible 'chaos' of the Unlimited, is essentially one with his own endeavour to trace the formal structure of the world that underlies, and gives its reality and meaning to, the world of sense experience." Hackforth, R., 1945, *Plato's Examination of Pleasure*, Cambridge: Cambridge University Press. 21.

[222] How much Plato borrowed or continues the Pythagoreans is a matter of endless debate. Even if Constance Meinwald considers that "Pythagorean scholarship is too diverse and contentious to be a starting-point for reading Plato" she finds in Plato's approach a way of improving some Philolaic tenets. Constance Meinwald (2002). *Plato's Pythagoreanism* Ancient Philosophy 22 (1):87-101., 87.

[223] "It is a gift of the gods to men, or so it seems to me, hurled down from heaven by some Prometheus along with a most dazzling fire. And the people of old, superior to us and living in closer proximity to the gods, have bequeathed us this tale, that whatever is said to be consists of one and many, having in its nature limit and unlimitedness" (*Philebus* 16c-d).

5.3.3. Hypothesizing odd and even

In the *Republic* Book VI, mathematicians are harshly criticized by Plato for not knowing what they are talking about.

> "Students of geometry, calculation, and the like hypothesize the odd and the even, the various figures, the three kinds of angles, and other things akin to these in each of their investigations, as if they knew them. They make these their hypotheses and don't think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone" (VI, 510c2-6).

In this passage Plato insists that there is more to mathematical entities than they are understood and used by mathematicians. Beyond mathematical entities there are principles or a principle which are or is to be discovered by dialectics (*Republic* 534b3–c5). What the process of dialectics consists in and what going beyond the hypotheses of mathematicians means is not thereupon exemplified by Plato.

It is not clear at all what Plato wanted to say by the phrase to "hypothesize the odd and the even", and why he considered that these should be taken as hypotheses for a mathematician.[224] Annas states that the hypotheses are like propositions, even if the text does not go in this direction.[225] For other scholars,

[224] Guthrie takes the proper meaning of *hupothesis* (to place under, to lay down), and interprets that what is laid down is a thing, W. K. C. Guthrie, *A History of Greek Philosophy IV Plato, the Man and his Diaogues: Earlier Period*. Cambridge: Cambridge University Press, 1975, 509. Other suggestions are that hypothesis can be taken as a concept, Crombie, I. M. *An Examination of Plato's Doctrines*. 2 Vols. London: Routledge &Kegan Paul, 1962, vol I, 113. A more elaborate study on of the *hupothesis* can be found in Robinson, Richard, *Plato's Earlier Dialectic.* 2d ed. Oxford: Clarendon Press, 1953, 93-100.

[225] Annas, Julia, *An Introduction to Plato's Republic*. Oxford: Clarendon Press, 1981, 287-90. Robinson has a similar interpretation as well, Robinson, *Plato's Earlier Dialectic*… 99–101, 152.

Plato's intention is to show that mathematicians are wrong because the hypothesis that all numbers are odd and even is not true, if one takes into account that there are irrational numbers as well.[226] This position, I believe, saddle Plato's idea with unnecessary complications and Plato's examination of the procedures of mathematicians does not have as object their falseness or validity, but rather a functional ignorance similar to that which is portrayed in Socratic dialogues; pious men who are proven not to know what is piety or just men who are proven not to actually know what is justice and so on provide explanatory parallels for mathematicians who are unable to prove their grounding hypotheses. If the pious man actually cannot be pious without knowing what piety is, then the mathematician does not actually do mathematics, even if he is working with numbers, since he does not know what numbers are. Mathematicians simply don't know philosophically what they are talking about: "we described them as to some extent grasping what is, for we saw that, while they do dream about what is, they are unable to command a waking view of hypotheses that they leave untouched and that they cannot give any account of" (533b8–c2).

Mathematicians do not "know" their hypotheses; they do not know what they are talking about, even if their "knowledge" is accurate and precise. They operate with concepts in ignorance of their ontological source. Thus the only difference between a mathematician and a philosopher mathematician is not that the latter knows more mathematics (most often the contrary), but that a philosopher is able to

[226] See, for example, A. E. Taylor, "A Note on Plato's Republic VI 510c2–5." *Mind* 43 (1934): 81–4.

question what the mathematician is working with; he does not operate mechanically with his concepts. For Plato this is precisely true knowledge – to be able to give an account about the first principles on which the hypotheses are built. What the mathematician does is not artificial or without value, it merely means that he is ignorant of the object matter of his discipline, since the objects of mathematics have a foundation which is not mathematical. Plato points towards the difference of epistemological objects for the philosopher and the mathematician. A mathematician who cannot philosophically explain his hypotheses has nevertheless a working knowledge which we could label as quantitative: numbers get their existence from mathematical operations which are performed in relation to sensibles. Even if the mathematician's knowledge is not based on sensibles, and thus he is doing an abstract mathematics, his knowledge is still not a real knowledge if he does not know the real origin of numbers. For the philosopher, when dealing with numbers, it is important to understand them by themselves, in their relation to intelligibles: "to discuss the numbers themselves, never permitting anyone to propose for discussion numbers attached to visible or tangible bodies (526d4-6)." Their origin is internal; their existence is based not on counting things, but on metaphysical principles.

If there is a place in the Platonic corpus that could be read as a possible 'account' on one of the entities underlying arithmetical investigation, i.e. the odd and the even, that would be the argument for the generation of numbers. This is the only place where the first odd and the first even are deduced from intelligibles. The generation of numbers points to the foundation of numbers in the following manner. Even and odd are a result of, or similar to, two and three. Two and three are a result

of structuring ontological entities. Going beyond odd and even, and three and two, we eliminate odd and even as hypotheses, they constitute now a mathematical knowledge based on the quantification of the basic ontological entities.

5.4. The prime numbers and the multiplication process

The use of mathematical operations by Plato is coming as a surprise,[227] and the generation process which has at its basis a mathematical operation goes against the usual frame of Plato's mathematical discussions elsewhere in the dialogues. In this hypothesis, already in the first argument, by the division or multiplication of *hen* into one and being a mathematical process by dividing or multiplying by 2 was used. If there are two, there must be twice/double, since two is twice one, if three, also thrice/triple, since three is thrice one (143d9). After showing how two is obtained from duality, and three from 2+1, Plato emphasizes that there is a multiplication process as well. The multiplication operation is inherent to the generation of two and three also, since Plato considers that if there is two, there must be twice, since two is twice one, and if three, also thrice, since three is thrice one. Two is thus also 2x1, and three is also 3x1. One can note here that Plato takes 3 now as being 3x1, and not 2+1 (as at 143d7).

[227] As Moravicsik put it: "There is nothing in Plato's ontology that corresponds to mathematical operations; the ontology reflects only mathematical truths," Julius M. Moravcsik 'Forms and dialectic in the second half of the *Parmenides*" in Malcolm Schofield and Martha Craven Nussbaum, *Language and Logos: Studies in Ancient Greek Philosophy Presented to G. E. L. Owen* (Cambridge University Press, 2006), 135-154. 144.

Contrary to what one would expect, namely to derive numbers through addition, Plato used multiplication instead. The rest of the numbers after two and three are products of multiplication: 2x2 (δὺο δὶς), 3x3 (τρία τρὶς), 2x3(δὺο τρίς), and 3x2 (τρία δὶς). Hence, there is even times even (ἄρτιά ἀρτιάκις), odd times odd (περιττὰ περιττάκις), odd times even (ἄρτια περιττάκις), and even times odd (περιττὰ ἀρτιάκις). It is argued that all numbers are generated from these two mathematical entities through a process of multiplication. The shortcoming of obtaining numbers only through multiplication is that primes, after two and three, remain unreached, since they are not multiplies of two or of three. Through multiplication alone one cannot obtain the complete number series; nevertheless, at the end of the argument, the opposite is claimed: there is no number left that does not necessarily exist (144a3).

Should we then consider the operation of addition as part of the argument as well? Otherwise the process of multiplication alone leaves prime numbers out (144a4). Prime numbers are integers greater than one divisible only by one and themselves, and they do not know of any pattern according to which they can be generated.[228] I would say that the main possible answers to these questions would be eventually restricted to the following options:

a) Addition is used together with the multiplication process, and thus a prime number such as 5 could be the result of 4+1, in the same way in which 3 was obtained from 2+1;

[228] Starting with the Pythagoreans, Greek mathematicians called prime numbers *linear* numbers or *rectilinear* numbers. The product of two linear numbers was a *plane* number (the sides are the numbers which have multiplied one another, cf. also *Theaetetus* 148a4). *Solid* number is the product of three prime numbers.

b) or, if we could take 1 as an odd number, primes could be a case of odd times odd, and thus 5 would be 5x1, and the operation of addition is not necessary anymore;

c) or primes could be thought as a subgroup of oddness;

d) another solution is to say that the argument is not complete, and Plato missed the fact that prime numbers are not generated. To this opinion Aristotle seems to yield in *Metaphysics* A6 (987b29-988a1): primes remain ungenerated. (See more in the following section on Aristotle).

e) Plato thought of primes as being ungenerated since one cannot reduce them to a factorial procedure, and primes would be similar to real form-numbers.

All these hypotheses have their limitations. In what regards a), b), and c), as they all have one type of generation or another as basis, one doesn't even get to the question of primes. And thus Aristotle's testimony would be actually a "mistaken gloss"[229].

It would be plausible to consider a) if we think that one usually defines the number series through addition, as n+1. For us, it is rather counterintuitive to conceive the generation of numbers through multiplication, and modern theories of numbers and philosophy of mathematics provide little analogous evidence. If Plato does not get rid of addition in the process of generation, as some scholars think is the

[229] Due to the problematic raised by the clause 'other than primes' this ended up being treated as a 'mistaken gloss'. See A.E. Taylor, *Aristotle on His Predecessors*, 104.

case (e.g. Cornford[230]), then prime numbers are quite easily obtained through the process of addition (5 it will be the result of 4+1 or 1+1+1+1+1). But if this is the case, why is 6, for example, not conceived through addition? And, implicitly, all the remaining numbers? Plato insists on the multiplication process instead, and if he also had in mind addition, then the multiplication process would be gratuitous.[231] Or multiplication could be considered, I would suggest, as a subspecies of addition, a particular kind of addition in which equal quantities are added for repetitive times. Numbers are a *repetitio* not of one, but of two and three.

A ground for leaving addition aside may be that multiplication provides a better understanding of numerosity, since each number can be reduced to (prime) factors, and it is easier to reduce numbers to basic factors of 2 and 3, than to 1 (e.g. 6=1+1+1+1+1+1 versus, *simplicius*, 6=3x2 or 2x3).[232] Only in this way the primary even and primary odd (which are twoness and threeness in the argument for the generation of numbers) are proven necessary for the number series. Each number, except the primes, is reduced to factorial operation, of the first even and first odd number.

[230] Cornford thinks that "Plato evidently includes addition and starts with that when he adds one term to another to make two, and two to one to make three." Francis Macdonald Cornford, *Plato and Parmenides* (Routledge, 2000). 141.

[231] As Moravcsik underlines "If we admit also addition, as Cornford does, then the whole section on multiplication becomes superfluous." See Julius M. Moravcsik 'Forms and dialectic in the second half of the *Parmenides*" in Malcolm Schofield and Martha Craven Nussbaum, *Language and Logos: Studies in Ancient Greek Philosophy Presented to G. E. L. Owen* (Cambridge University Press, 2006), 135-154, 144.

[232] On the contrary, for Aristotle "each number is said to be many because it consists of ones and because each number is measurable by one" (*Met*. I 6, 1056b23).

In what concerns b), Theon of Smyrna says that primes are called also odd-times odd, [233] referring thus to multiplication as well as core mathematical operation for the generation of numbers. It nevertheless leaves unsolved the issue of how one gets to the first primes, for example 5. Various other speculations have been made on the nature of primes, amongst which one defines them as a subdivision of oddness (c). Nicomachus took prime numbers as a subdivision of odd numbers.[234] Again, as in the previous case, although it might offer some insight into the nature of primes, this idea does not explain how we come to primes in the first place. Moreover, two is prime and even.

Regarding d), I would object to Aristotle's observation by saying that actually the first two prime numbers are generated: 2 is generated from the pair relation, and 3 from 2+1, so it would be difficult to convincingly maintain that Plato missed out the primes. I would instead say that what Plato did in this argument concerning numbers is to go for samples of number generation (an odd and an even, the unity of calculation moreover, and the first primes as well) whose aim was not to provide a law for the numeric generation of all numbers, but rather a tool for a principial understanding of numbers.

Primes escape any principial definition and therefore Plato consciously leaves them out (see next section). To this intentional leaving aside of primes in the generational pattern refers the e) option above. Any generated number (i.e one that

[233] cf. Heath, "Notes on definition 11", in Euclid, *The Thirteen Books of Elements*, Volume 2 (CUP Archive, 1926), 285. This would work also with Euclid's definition that "Πρῶτος ἀριθμός ἐστιν ὁ μονάδι μόνῃ μετρούμενος." ("A prime number is that which is measured by a unit alone" (Euclid, *Elements*, VII, def. 11). See also Heath, *A History of Greek Mathematics*, 278.

[234] Thomas Heath, *Mathematics in Aristotle* (Routledge, 2015), 84.

is composed) follows a pattern; prime numbers, due to their proper nature, are not liable to be generated by multiplication and hence they do not follow a pattern (at least a pattern construed by multiplication). Their absence (from 3 upwards) from the number sequence that is generated by Plato's argument might even drive us to the extreme assumption that perhaps primes are not numbers (analogous to the position of 1, which was not a number proper for Greek mathematics), but might be elements of numbers – one may use them to build numbers. [235] Thus we have in this process of generation: generated and ungenerated numbers. A number such as 10, for example, would be the product of a generated and an ungenerated number. It is factorial, it is even times odd (2x5), but since 5 is a prime number, i.e. ungenerated, then 10 would be the result of even times prime. If we accept e) then we imply that the process of generation does not aim at an exhaustive generation formula for all numbers (and we thus get rid of the issues related to primes), but rather describes the very structure of numbers under different categories (of even, odd, units and primes).

Any solution given to the prime problem must take into account that Plato's emphasis on multiplication[236] is justified by the fact that he aims at presenting what orders numbers (the ordinals of numbers), the building blocks of numbers, which

[235] Aristotle (*Posterior Analytics*, 96a37-38) says about primes that they are "not being measured by number" and "not being compounded from numbers"

[236] What Plato does resembles some archaic series of numbers similar to that of primitive populations: there is a story according to which Australian Aborigines limit their numbers system to *one* and *two* (i.e. a binary system), and out of them they can deduce composed numbers up to six. As, for example, three is made by *two* and *one*, while six is made by *two* and *two* and *two* (Dantzig, Tobias *Number and the Language of Science* (New York: Macmillan Company, 1954), 14). In this testimony six is obtained by multiplication, and not through addition. The aborigines count in pairs, i.e by multiplication.

are 2 and 3, while for the rest of the prime numbers they themselves are their proper building blocks. Prime numbers are random, and it is a question if Plato was aware that prime numbers can't be determined systematically.[237] Even for present-day mathematicians it is difficult to find algorithms that generate primes, for limited series. In *Metaphysics* A6 (987b29-988a1) Aristotle emphasizes that for Plato prime numbers remained ungenerated (see the next section for a detailed account).

The precise reference of Aristotle to the absence of prime numbers could show that there were discussions inside the Academy as to whether there is a way of generating prime numbers or not. Such a question would additionally imply that the rest of the numbers do follow a pattern, and that in the Academy a normal discussion around the composition of numbers entailed discussing also the generation of numbers.

Eratosthenes (276-195/194), who studied in the Platonic Academy under Arcesilaus of Pitane, wrote a work called *Platonikos*, in which he studied, among other aspects of mathematics (in Plato's philosophy as well), progressions and proportions. He developed an algorithm for finding primes, known as the sieve of Eratosthenes,[238] by finding first of all numbers which are composed of prime numbers. In a given limit, one should mark all multiples of 2, after that all multiples of 3, we skip 4 since it is already marked as multiple of 2, we continue with all

[237] New researches in the problem of prime numbers propose algorithms for finding prime numbers. For example: Richard Crandall and Carl Pomerance, *Prime Numbers: A Computational Perspective* (Springer Science & Business Media, 2006).

[238] The work of Eratosthenes did not survive. The sieve is credited to Eratosthenes in the work of Nicomachus *Introduction to Arithmetic*, 13. 2–4.

composite numbers of 5, and so on.[239] By marking and finding composite numbers, we leave out primes.

In my view, Eratosthenes' way of showing the composite nature of numbers resembles Plato's generation of numbers, in which the main emphasis is not on anticipating and identifying sequences of numbers, but on the composite feature of numbers. An Eratosthenes would read Plato's recipe for the generation of numbers as a demonstration for the existence of two types of numbers: generated (multiples of odd and even and their subsequent combinations)[240] and ungenerated (primes), and since there is no pattern for primes, Plato leaves them out, as Eratosthenes does later.

5.5. Aristotle and the generation of numbers in the *Parmenides*

There are reasons to think that it could be this argument Aristotle had in mind in Metaphysics A6, when he points out that Plato assumed a production of numbers, with the exception of prime numbers: "except those which were prime, could be neatly produced out of the dyad as out of some plastic material". Whether

[239] For a discussion of the sieve of Eratosthenes in modern number theory concerning the distribution of primes see: Alina Carmen Cojocaru and M. Ram Murty, *An Introduction to Sieve Methods and Their Applications* (Cambridge University Press, 2005), 63-74; and Benjamin Fine and Gerhard Rosenberger, *Number Theory: An Introduction via the Distribution of Primes* (Springer Science & Business Media, 2007), 198-201.

[240] Multiplication preserves also a geometrical representation, and thus Plato would be tributary to the way of Greek mathematics, which, before Euclid, used to represent numbers diagrammatically, and not symbolically. See Paul Pritchard, *Plato's Philosophy of Mathematics* (Academia Verlag, 1995), 23-24. Also Wedberg, *Plato's Philosophy of Mathematics*, 25. Blyth, "*Platonic Number* in the *Parmenides* and Metaphysics XIII", 29, 40.

Aristotle's reading of the *Parmenides* is accurate or not, this could be a proof that the argument for the generation of numbers was considered by Aristotle as part of Plato's philosophy in a consistent way, and not as an empty dialectical exercise, without any weight. In the first section I discuss possible links between Aristotle's *Metaphysics* A6 (987b33-988a1) and the *Parmenides*, through an exploration of current scholarship, while the second section points to Aristotle's critique (A9) of the relation between the duality and the cardinality of two.

5.5.1. Metaphysics A6 *(987b29-988a1) on generation of numbers and prime numbers*

One of the issues with such testimonies is that in general Aristotle's Plato does not fully correspond with what we find in the dialogues. Several scholars argue, in a tradition established by Cherniss,[241] that Aristotle is inaccurate and did not succeed in properly understanding Plato.[242] Aristotle's critique would be inappropriate, since Plato did not in fact assume what Aristotle criticized (especially in what concerns the critique of form numbers). On the opposite side, Paul Pritchard, author of a monograph on Plato's philosophy of mathematics, insists that Aristotle did

[241] *Aristotle's Criticism of Plato and the Academy* (Baltimore: Johns Hopkins Press, 1944); *The Riddle of the Early Academy* (Berkeley: University of California Press, 1945). In fact, the whole rejection of Aristotle started earlier than Cherniss with an article of Cook Wilson (*On the Platonist Doctrine of the Asymbletoi Arithmoi*, 1904). Burnyeat notices that "a surprisingly large number of scholars defer to Cook Wilson's judgment that Aristotle is wrong" (Burnyeat, *Platonism and Mathematics*, 234.)

[242] Cherniss takes Aristotle's testimony on intermediates as a typical misunderstanding of Plato's philosophy (Cherniss, *op.cit.*, 75-78).

understand Plato, but he didn't agree with his theory.[243] Other scholars arrive at the very important conclusion that Aristotle argues "against every variation,"[244] of Platonic forms, nevertheless "he never appears to feel that he has refuted it once and for all."[245] His rejection of forms comes in some points together with his rejection of Plato's theory of numbers. In *Metaphysics* M, Aristotle gives a historical and theoretical account of conceptions about numbers of his precursors. He claims that Plato "says the first kind of number, that of the Forms, alone exists, and some say mathematical number is identical with this" (1080b23-24). And further: "some think that those which are the objects of mathematics are different from those which come after the Ideas; and of those who express themselves otherwise, some speak of the objects of mathematics and in a mathematical way — viz. those who do not make the Ideas numbers nor say that Ideas exist; and others speak of the objects of mathematics, but not mathematically; for they say that neither is every spatial magnitude divisible into magnitudes, nor do any two units make 2" (1080b36-1708). This would prove an improbable view for Aristotle who considers Plato's conception of numbers as a doctrine of ἀσύμβλητοι ἀριθμοί. However, Aristotle's anti-Platonist position is not maintained throughout his work. For example, in *Posterior Analytics* I. 13, Aristotle surprisingly claims that "mathematics is concerned with forms; its objects do not exist according to some substrate" (79a7-8).[246]

[243] Paul Pritchard, *Plato's Philosophy of Mathematics* (Academia Verlag, 1995).

[244] Philip, J.A, *Pythagoras and Early Pythagoreanism*, 77.

[245] Philip, J.A, *Pythagoras and Early Pythagoreanism*. Canada: University of Toronto Press, 1966, 67.

[246] Even if Aristotle refers here to his forms (not the Platonic ones), there is still an opening towards a Platonism of mathematical objects. A similar remark is to be found in

In my view, Aristotle's various and contradictory opinions about Plato's philosophy in general, and about philosophy of mathematics could be justified if we admit that Plato himself did not maintain a constant philosophical position in his dialogues, and thus the dialogues leave room for a variety of possible interpretations. Aristotle is an interpreter of the dialogues, but also of the first Platonists of the Academy – who had advanced their own interpretations of Plato – and a proper identification in Aristotle's critique between these Platonists' and Plato's own ideas is not clearly textually marked. To make the difference between when exactly Aristotle is interpreting Plato and when he is setting out an interpretation of the first successors of Plato is thus a very difficult task.

In Metaphysics A6, Aristotle quite naturally associates Plato's philosophy with that of the Pythagoreans, showing also where Plato's philosophy differs from that of the Pythagoreans.[247] What is particularly remarkable in Aristotle's confident testimony is his assertion that, as with the Pythagoreans, a central point in Plato's philosophy was to give an account of numbers.

Aristotle claims on several occasions that for Plato numbers are generated, but it is only in one place that he explicitly insists that Plato's process of generation leaves out prime numbers. It is the passages in *Parmenides* 142b-144b which seem to

Metaphysics M 1-3, where he maintains that mathematical objects are prior to sensible things in definition. Aristotle preserves, as it seems, in his matter-form theory some of the Platonist assumptions which he is struggling to reject.

[247] For some scholars this association is artificial. See, for example, Harold Fredrik Cherniss, *Aristotle's Criticism of Plato and the Academy* (Johns Hopkins University Press, 1944), 194-195. For a critique of Cherniss see Steel, *Plato as seen by Aristotle* in Steel ed., *Aristotle's Metaphysics Alpha*, 189-190.

be precisely the place where one should look for something that could substantiate Aristotle's testimonies. In A6 (987b29-988a1), Aristotle affirms:

> "His divergence from the Pythagoreans in making the One and the numbers separate from things, and his introduction of the Forms, were due to his inquiries in the region of definitory formulae (for the earlier thinkers had no tincture of dialectic), and his making the other entity besides the One a dyad was due to the belief that the numbers, except those which were prime, could be neatly produced out of the dyad as out of a plastic material"[248]

This passage on the generation of numbers, together with the whole chapter A6, is notably problematic. At first glance, the generation of numbers from the *Parmenides* seems not to be the argument Aristotle had in mind in A6,[249] since, one may argue, "the generation of numbers does not seem to have been a concern of Plato."[250] Furthermore, Aristotle claims that for Plato numbers "could be neatly produced out

[248] (Ross' translation). τὸ μὲν οὖν τὸ ἓν καὶ τοὺς [30]ἀριθμοὺς παρὰ τὰ πράγματα ποιῆσαι, καὶ μὴ ὥσπερ οἱ [31] Πυθαγόρειοι, καὶ ἡ τῶν εἰδῶν εἰσαγωγὴ διὰ τὴν ἐν τοῖς λό-[32]γοις ἐγένετο σκέψιν (οἱ γὰρ πρότεροι διαλεκτικῆς οὐ μετεῖ-[33]χον), τὸ δὲ δυάδα ποιῆσαι τὴν ἑτέραν φύσιν διὰ τὸ τοὺς [34] ἀριθμοὺς ἔξω τῶν πρώτων εὐφυῶς ἐξ αὐτῆς γεννᾶσθαι ὥσ-[988a1]περ ἔκ τινος ἐκμαγείου. Primavesi's new revised edition of the Greek text does not differ at this passage from that of Ross. See Carlos Steel ed., *Aristotle's Metaphysics Alpha: Symposium Aristotelicum* (Oxford: Oxford University Press, 2012).

[249] In general it is thought that Aristotle does not refer at all to the *Parmenides*. For a discussion of the pro and cons arguments see: Donald J. Allan, "Aristotle and the *Parmenides*" in Ingemar Düring and Gwilym Ellis Lane Owen, *Aristotle and Plato in the Mid-Fourth Century: Papers of the Symposium Aristotelicum Held at Oxford in August, 1957* (Elanders Boktryckeri Aktiebolag, 1960), 133-144. Also A. E. Taylor, *The Parmenides of Plato* (Oxford: Clarendon Press, 1934), especially Appendix C: Aristotle and the *Parmenides*, 128-134.

[250] David Amirthanayagam, "Plato and the Measure of the Incommensurable. Part II. The Mathematical Meaning of the Indeterminate Dyad" in *The St. John's Review Volume* XLVI (2002), 25-62, 43. This is one of the common positions of modern scholarship, namely that Plato never thought about numbers as being generated.

of the dyad as out of some plastic material (*ekmageion*)[251]", and the dyad is apparently missing from the *Parmenides*: most scholars argue for the absence of the use of such concept as the indefinite dyad in the dialogues, and that also includes the *Parmenides*.

David Ross thinks that "the *Parmenides* does not help us, for there is no question there of the indefinite dyad."[252] Also, in a note to his translation of the *Metaphysics*, he claims that in the *Parmenides* "primes are not there excepted" and "nothing in the works of Plato corresponds exactly to what Aristotle says here."[253] However Ross does not say more about why he thinks that primes are not missing from the *Parmenides'* generative process. The claim is quite surprising, and Ross does not provide any further indications.

A possible clue is given in another place, and that is in Ross' edition of the Greek text: "the numbers, including 2, are produced by the ordinary processes of addition and multiplication from 1."[254] Nevertheless, for Ross, Aristotle's account in

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[251] *Ekmageion* raises problems of understanding. John Dillon translates it as "mould", A.E. Taylor as "matrix". It can be also translated as "impress", "model" etc. The same word is used in the *Timaeus* (50c) for the receptacle, and in the *Theaetetus* 191c-d (as a wax tablet – *kerinon ekmageion* –, an image used by Homer, as Plato indicates, and which is perceived as "the gift of Memory, mother of the muses"), and 196a. In the *Laws*, 800b, it can be understood as "model". According to Ross, the meaning from the *Timaeus* is that which Aristotle had most certainly in mind. (Cf. Ross, *Arist' Metaph.*, 176). Turnbull takes Aristotle's matrix as referring exactly to the pair one-being from the *Parmenides*, which "appears to be the special sort of two that such a dyad would be." Turnbull, 74.

[252] W. D. Ross, *Aristotle's Metaphysics* (Oxford: Clarendon Press, 1924), 174.

[253] W. D. Ross, Volume VIII *Metaphysica*, The Works of Aristotle Translated into English, (Oxford: Clarendon Press, 1928), 987b, footnote 1.

[254] W. D. Ross, *Aristotle's Metaphysics*, 174. The remark is insufficient, since, as I showed, numbers, the number two included, are not conceived as being generated from the number one, but from one added to twoness and, from this, further on, from twoness and threeness. In *Plato's Theory of Ideas*, Ross is more accurate: "this proof is rather perfunctory, since it makes no provision for prime numbers other than 2 and 3." See William David Ross, *Plato's Theory of Ideas* (Clarendon Press, 1951), 187.

A 6 "is not quite accurate" since in N3 1091a9-12 "it is only 2 and its powers that could be neatly produced out of the 1 and the indefinite dyad."[255] Indeed, despite of what Aristotle claims in A6, at N3 1091a10 he remarks that Platonists "cannot in any way generate numbers other than those got from 1 by doubling"[256], consequently the dyad generates multiples of two[257] (powers of two), a series of 2, 4, 8, 16, 32, etc. Ross' observation that in the *Parmenides* "there is no question… of the indefinite dyad" is too easily assumed since the first part of the argument generates a series of dualities,[258] even if numbers are not explicitly mentioned.

On the other hand, Julia Annas argues for the contrary. Aristotle's testimony on one and the indefinite dyad seems to Annas to have an embryonic appearance in the argument for the generation of numbers.[259] The first argument (142b5-143a2) of

[255] W. D. Ross, *Volume VIII Metaphysica*, Book A6, 987b, footnote 1.

[256] This statement contrasts with that from *Met.* 1081a, 14, in which it is stated that: "*Number* comes from the 1 and the indefinite dyad, and the principles and the elements are said to be principles and elements of number". Scholars do not agree if this statement is ascribed by Aristotle to Plato or to some of his followers. Its meaning is problematic and addresses a complicated challenge for historians of philosophy. Instead of clarifying Plato's understanding of numbers, these type of statements create ambiguities: e.g. "but what this apparently simple statement means has remained a mystery until modern times" Ivor Bulmer-Thomas, "Plato's Theory of Number," *Classical Quarterly* 33, no. 02 (1983): 375.

[257] Even if Ross rejects an appropriation between A6 and the *Parmenides*, he thinks that the indefinite dyad ascribed by Aristotle "might be assigned to 2… such as [it] is expressed in *Parmenides*." Ross, Aristotle's *Metaphysics, 175*.

[258] A similar idea is expressed by Cornford, in his attempt to reject Alexander and Simplicius (*Phys*., 454, 22 ff.), regarding the *Lectures on the Good*: "Plato in his latest phase derived numbers from the One and the 'Indefinite Dyad.' The second factor, so far as numbers are concerned, is the principle of plurality: 'Each of the numbers, in so far as it is a particular number and one and definite, shares in the One; in so far as it is divided and is a plurality, in the Indefinite Dyad If plurality can somehow be deduced from ' the existence of a One ', we can dispense with the existence of the Indefinite Dyad as a second primitive hypothesis." Since "the Parmenides (143) contains an argument which does in fact deduce a plurality of numbers from 'the existence of a One". Cornford, "Mathematics and Dialectic in the Republic VI.-VII. (II.)". 178-179.

[259] *Aristotle's Metaphysics M and N*, 48.

the *Parmenides* "resembles the working of the indefinite two",[260] while the second argument (143a4-144a5) "indicates a process which is reminiscent of the way one works as a principle."[261] Thus Annas considers that one could read and appropriate Aristotle's remark on Plato's dyad (not only the instance from A6, but also those from M and N) in the light of *Parmenides* 142b-144b.[262]

John Dillon is more restrained and argues that an ontological reading of the *Parmenides* starts later, with Speusippus, which, in the case of the second hypothesis, implies "an account of how One, when combined with the indefinite Dyad (under the guise of 'Being') produces, first the whole set of natural numbers, and then progressively, the various lower levels of reality".[263]

According to A. E. Taylor, it is this very argument from the *Parmenides* that Aristotle had in mind in his "perversion of Plato's theory of numbers" with the one and 'indeterminate Duality'.[264] Moreover Taylor says that Aristotle does not refer to the *Parmenides* only in A6, but also at N, 1091a11, where Aristotle states that "number, according to him [Plato], cannot be generated except from one and the

[260] Annas must have in mind this proposition: "it always comes to be two, it is never one". Annas already sees here a correspondence to "the way that numbers of the form 2^n are produce from 2." Annas, *Aristotle's Metaphysics: Books M and N* (Clarendon Press, 1988), 48-49.

[261] Julia Annas, *Aristotle's Metaphysics: Books M and N*, 48.

[262] Other positions such as that of Sayre support the position that "there is no evidence that Aristotle was thinking of this particular derivation" of numbers from the *Parmenides*. See Kenneth M. Sayre, *Metaphysics and Method in Plato's Statesman* (Cambridge University Press, 2006), 198.

[263] John Dillon, Syrianus's *Exegesis of the Second Hypothesis of the Parmenides: The Architecture of the Intelligible Universe Revealed*, in John Douglas Turner, Kevin Corrigan (ed.) "Plato's Parmenides and Its Heritage: Volume II: Reception in Patristic, Gnostic, and Christian Neoplatonic Texts" (Society of Biblical Literature Writings from the Greco-Roman World Supplement), 133.

[264] Alfred Edward Taylor, *Aristotle on His Predecessors. Being the First Book of His Metaphysics, Translated from the Text Edition of W. Christ, with Introduction and Notes by A.E. Taylor* (Chicago: Open Court Publishing Company, 1910), 104.

indefinite dyad." For Taylor this claim "unmistakably refers to the same passage of the *Parmenides*."[265]

I find Taylor's and Annas' views that Aristotle in *Metaphysics* A6 referred to the *Parmenides* more convincing especially because there is an explicit reference to the problem of prime numbers in the *Parmenides*. If Aristotle had referred to the *Parmenides* this would prove mainly that the argument on the generation of numbers was an issue which Plato considered as an actual possibility, and we should pay it equal consideration as we do to the theory of form numbers.

At the same time, the beginning of the second hypothesis (142b-144b) could be essential to advance the view, as stressed by the Tübingen School, that Plato had "unwritten doctrines" which were not fully developed in the written dialogues. Paralleling the dyad with the division of one into one and being at 142b5-143a2 is a vast enterprise, but what I have insisted upon so far is the duality of one and the consequences for numbers thereupon. Moreover, it is beyond the scope of my present research to assess the implications of a possible oral teaching of Plato, since it is contradictory to reconstruct an allegedly exclusive oral teaching from a written one. I try thus to avoid as much as possible the methodology of the Tübingen School, when interpreting the arguments from the *Parmenides*. What is important in the economy of my study – that Plato's own philosophical position can be recognized in the second argument of the second hypothesis of the *Parmenides* – is to emphasize

[265] A. E. Taylor, *Aristotle on His Predecessors: Being the First Book of His Metaphysics* (Open Court, 1910), 104.

that Aristotle was aware of such perpetual division of one into the duality of one and being, and that he overlaps this division with a function attached to the dyad.[266]

The difference between a possible Tübingen interpretation of the passage and that which is expressed in this dissertation is that a Tübingen reading makes difficult to follow the development of the greatest kinds introduced in the *Sophist* as a development of the second hypothesis of the *Parmenides*. My reading gives to the *Sophist* a more coherent appraisal: the greatest kinds of the *Sophist* are an elaboration of an ontological project which is only partially elaborated in the *Parmenides*' second hypothesis.[267]

The relation between the *Parmenides* and Aristotle's *Metaphysics* A6 should be satisfactorily explained without the exclusive lens of the "unwritten doctrines". One can have a more nuanced reading of 142b-144b (the division of one into one and being), especially 143a-b (the establishing of the three entities: one, being, difference), and 143c-143e2 (the transition from pair-duality to cardinality), in which several divisions and dualities are at work in order to point to a generative process. These dualities used by Plato do not refer necessarily directly to a possible concept of indefinite dyad, that Plato had it in mind to avoid formulating it like that in the *Parmenides*. Instead one can speculate that Aristotle read the *Parmenides*, and had this argument for the generation of numbers in sight when he is criticizing the limits of

[266] Here are some basic claims of Aristotle in regard to the unwritten doctrines: one and the indefinite dyad are the causes of everything (*Metaphysics* 987b20-22, 988a10-15); one and the indefinite dyad create everything else (Aristotle, *Metaphysics* 988a10-15); one overlaps apparently with the Good, *Metaphysics* 988a14-15 together with 1091b13-14).

[267] In the *Sophist* 255e, Plato gives to *difference* the same ontological power as in the *Parmenides* 143a-b.

the indefinite dyad. He could conflate this argument with his understanding of the indefinite dyad.

In my view the duality of one goes straight against the unwritten doctrines in which the One and the indefinite dyad are the two independent, opposing principles.[268] As I have shown, the duality of one, or being form one is, is opposed the monolithic feature of one from the first hypothesis (137c–142a). In contrast to the unwritten doctrines, where the dyad is a principle which is independent of one, the duality of one (i.e. the dyad) comes from the relation between the one and (its) being, and restates the philosophy of the Parmenidean poem with some particular and new distinctions.

According to Ross, after the *Parmenides*, the closest Platonic dialogue which could be taken as an important clue to what Aristotle asserts about the indefinite dyad, or the great and the small,[269] is the *Philebus*,[270] a dialogue which develops a theory around limited and unlimited, quite a special theory in the whole of the Platonic corpus. In the *Philebus* (23c-26d), Plato differentiates the limited from the unlimited. In the *Philebus*, another distinction is made, between un-generated and generated entities. One – the indivisible unit – is unborn and un-generated and it overlaps extensively with the form (15a-c). In the *Philebus*, one resembles the limit, while the great and the small overlap with the unlimited. The limited and unlimited,

[268] More, the vocabulary on the indefinite dyad is not fixed. The adepts of the unwritten doctrines speak about indefinite two, or the Unlimited, or the Great and the Small. This unfixed vocabulary marked also the ancient commentators of Plato, and not all scholars accept that they speak about the principle of indefinite dyad.

[269] Plato uses a similar concept also in *Statesman* 284a1-e8.

[270] Allen, in the commentary of his translation of the *Parmenides*, is an advocate of the link between the second part of *Parmenides* and *Philebus*. Allen, PP, [316-317].

as they are conceptualized in the *Philebus*, are for Ross key concepts for the understanding of the generation "of the ideal numbers", for which he finds a strong support in this dialog, since "the One answers exactly to the 'limit' of the *Philebus*, and the great and small to the 'unlimited' of the *Philebus*."[271] His solution to the question of generation is that "successive numbers were the result of successive applications of limit or definiteness to unlimited plurality."[272] Ross accepts the limit and unlimited as mirroring one and the great and small, taking them as criteria for scrutinizing numbers. In other words, numbers can be reduced to these two principles. Ross' reading seems to be in conflict with Plato's statement that the ideal numbers are not composite, i.e. irreducible. But, even if Ross accepts that the *Philebus*' limited and unlimited are the ingredients for numbers and each number can be reduced to these two concepts, in the generation in the *Parmenides*, one cannot find "principles answering to the One and the great and small".

5.5.2. Aristotle on the number two: "the relative is prior to the absolute"

Picking out pairs may hint at a particular mathematical operation. But how can one understand what Plato does when he picks out pairs (τινε, and thus to have ἄμφω) from a group of three ontological entities – one, being and difference? Is it a

[271] Ross, *Plato's Theory of Ideas*, 204.

[272] Ross, Ibid., 205.

conceptual operation? Is it just a manner of speaking about how metaphysical principles are thought about in their coming together?

In the following pages I draw a parallel between Plato's aim to emphasize the priority of ἄμφω and τινε in order to obtain δύο and Aristotle's critique in *Metaphysics* A9, of what he sees as Plato's conception of the priority of the dyad to number two. The *Parmenides* provides again, just as in the case of A6, a very probable reference to what A9 might be referring to. In A9, Aristotle develops an argument in which he emphasizes that it is actually impossible to assume that the cardinality of two is posterior to that of the dyad, as Plato seems to think in Aristotle's view. My interest is to show that, at least in the *Parmenides*, Plato does not do what Aristotle claimed when he said that Plato placed the relative in the absolute, without identifying the indefinite dyad with Plato's concept of pair-duality of the *Parmenides*.

According to Aristotle:

> "in general the arguments for the Forms destroy the things for whose existence we are more anxious than for the existence of the Ideas; for it follows that not the dyad but number is first, i.e. that the relative is prior to the absolute"[273] (990b18-20) [my emphasis].

The tension is between the dyad and the form numbers. Aristotle's argument is as follows:

i) the dyad is prior to the forms (of numbers);

ii) the form number 2 is posterior to the dyad;

[273] Ross' translation.

iii) but the dyad already contains the cardinality of two;

iiii) therefore a contradiction.

In spite of the coherence of the argument, it is difficult to think that Plato would have accepted proposition iii) – namely, to have the indefinite dyad as an instance of δύο. First of all, in a tradition established by the Pythagoreans, Plato would think about the indefinite dyad as a metaphysical principle, while duo would be a set of two members which would correspond to the number two. Moreover, the one of the "one is" (of the pair relation one-is) should not be understood as the number one, as I have already shown in the sections on the origin of twoness and threeness (5.1., 5.2.).

If we accept that Aristotle's *Metaphysics* A6 was aimed at the *Parmenides* (as Taylor and Annas seem to assume), then the argument from A9 would not work for the *Parmenides*, since in the *Parmenides* the cardinality of two is obtained with difficulty at a later stage of the argument from duality, which, in its turn, is obtained from pair. In the following lines I want to enforce the idea that what Plato does in the *Parmenides* is an attempt to answer the kind of critique, probably already formulated in the Academy by the time Plato was writing the *Parmenides*, which was later displayed by Aristotle in A9.

When Plato speaks about the strong connection between one and being (one is a whole, being and one are its parts (μόρια), 142d), and also their difference (the difference of one and being is due to difference, 143b), he does not immediately jump to justifying a countable series of entities, but to the fact that these three entities make up pairs.

Plato was often accused of illicit inferences in this argument and, as far as I can see, Plato's sliding from an ontological discourse into a mathematical one while still working with ontological terms (one, being and difference) is such an illicit inference. Picking up the pair of one and being from the threefold one, being and difference, makes perfect sense, but when we are confronted with pairs of one and difference, or difference and being alone, the ontological grounding disappears and the impression we get is that Plato indulges in a gratuitous pair-making exercise. An explanation for this exercise could be that the pair relation is central for Greek philosophy and mentality (see, for example, the table of pairs of opposites attributed to some Pythagoreans) and Plato takes the idea of pairness even further than the ontological pair (one-being, a justified pair) applying pair-relations even where such connection seems more difficult to swallow.[274]

As far as I understand, and as I showed in 5.1 (The indistinctness of duality), for Plato what is τινε comes first in the argument for its pairness; that is, for its unity and not for its numerical value. It is only in the subsequent lines of the argument that τινε (143c3) is translated into ἀμφοτέρω (143c4) and the concept of twoness is suggested and thus the number δύο is deduced (143d2). In the early stages of the ontological argument, one is shown to be two, and never one (τούτω τὼ μορίω ἀεὶ ἴσχει, 142e), but this inference has only the role of indicating one's plurality, that one

[274] Perhaps one underlying reason for why Plato gets carried away in his pair-picking process is his preference for the method of division (*diairesis*) which we often meet in late dialogues. In the *Sophist* 253d, the Stranger rhetorically asks "Aren't we going to say that it takes expertise in dialectic to divide things by kinds and not to think that the same form is a different one or that a different form is the same?"

is more than one while it still remains one, a statement that contradicts the position of the philosopher Parmenides (as I have argued in chapters 3.1, 3.2).

In a presumably genus-species reading, there are two possibilities: a) two (as a species) is derived from ἄμφω (the genus) Plato, b) ἄμφω (as a species) presupposes already two (as genera) Aristotle. In the case of a) it is taken for granted that cardinality is a derivation of *ampho*, while for b) the cardinality is already there before analyzing ἄμφω. Plato operates a distinction between them, and he grants priority to ἄμφω as opposed to the counted two. Even if the pair seems to be a species of the cardinality of two, in this instance Plato conveys a different conception (143d2): ὧ δ᾽ ἂν ἄμφω ὀρθῶς προσαγορεύησθον, ἆρα οἷόν τε ἄμφω μὲν αὐτὼ εἶναι, δύο δὲ μή; οὐχ οἷόν τε. ὧ δ᾽ ἂν δύο ἦτον, ἔστι τις μηχανὴ μὴ οὐχ ἑκάτερον αὐτοῖν ἓν εἶναι; οὐδεμία. τούτων ἄρα ἐπείπερ σύνδυο ἕκαστα συμβαίνει εἶναι, καὶ ἓν ἂν εἴη ἕκαστον. (“Can things that are correctly called 'both' be both, but not two? “They cannot.”—“If there are two things, is there any way for each member of the pair not to be one?”—“Not at all.”—“Therefore, since in fact each pair taken together turns out to be two, each member would be one.”).

Plato’s insistence on picking out a certain pair (τινε) which “is correctly called ‘both’” (ὀρθῶς ἔχει καλεῖσθαι ἀμφοτέρω), before saying that we are speaking about two, demonstrates that he didn’t consider obtaining δύο through a simple and univocal procedure (ex. 1+1) directly from counting two items. The procedure of obtaining two is the following: if we take a τινε, from a threefold (or other sets), we must point out that τινε is also an ἀμφοτέρω. For Plato, only after realising the pairness of a unity, can one conceive the individuality of its respective composing

units, and thus ἀμφοτέρω is also δύο. After having moved from pairness to duality, the process that leads from twoness to one seems to be, once again, a process of division (as we have initially been confronted with the division of one into one and being, see Figure 1, The division of One and Being, section 2.2.1. One and Being): we divide into individual members (e.g. ἑκάτερος, which will be the opposite of ἀμφότερος), and thus we have independent members. (One cannot help noticing the synonymous series Plato is delving into here and help wondering whether Plato simply gets carried away by words, with an almost rhetorical speech. And such a reading would cancel any ontological allegations, leaving us at the mercy of the Greek language!)[275]

The initial effort of picturing one as being actually a pair and part of a pair is reasonable if we take the ontological argument as an argument about how one is multiple and how this basic multiplicity (pairness of one and being) is the sine qua non condition of any unit or unity, forcing us to conceive that any being must be a unity and any unity must be a being. Plato phrases this pair as the ontological principle (cf. chapter 3 The Ontological Argument: Being and One, and Difference (142b1-143b)).

Coming back to Aristotle's critique in A9, namely that "the relative is prior to the absolute", I have tried to emphasise so far that for Plato the relative (i.e. number

[275] R. E. Allen thinks that the argument lacks in English the force that it has in Greek: "The exact force of his argument cannot be reproduced in English. Greek possesses, as English does not, a dual as well as a singular and plural; when Parmenides argues that since it is possible to mention Unity and to mention Being, each of two has been mentioned, the English "two" is more explicit than the text, which contains only the genitive dual *αὐτοῖν*. It is from this feature in the syntax of his language that Parmenides goes on to infer that both have been mentioned, and that since both have been mentioned, two have been mentioned." Plato, *The Dialogues of Plato, Volume 4: Plato's Parmenides, Revised Edition*, tr. R. E. Allen, (New Haven: Yale University Press, 1998), 262.

2) is posterior to pairness (which, for Plato, is not directly the number 2, thus a relative, but rather a principle). The pair relation comes first, and afterwards its numerosity, namely that there are two things. To my understanding, for Plato, ἀμφοτέρω (duality) is an instance of τινε, and not of the cardinality of δύο (number two), in the sense that duality refers to pairs, while two refers to any two units. Aristotle is conscious that for Plato and Platonists "not two units make two" (1082b19, tr. Annas). Two is not obtained by the addition of one to one; rather, each member of two is one.

As I have emphasized above, Plato's elaborated process for obtaining two is quite complicated and, to some extent, it might seem gratuitous. It is not obvious, from the *Parmenides* alone, why Plato used such an excess of concepts around duality. But it could be obvious if we assume that Plato could have been aware of such critique from the Academy at the time of elaborating the *Parmenides*. Plato might have encountered a similar critique that later was formulated by Aristotle; so he constructed such a detour argument.[276] The derivation of two from pairs is a counterexample to what Aristotle says in A9. [277] As I see it, the cardinality of the dyad is simply an Aristotelian reading, and unsubstantiated by the Platonic text.[278]

[276] Maybe not just accidentally the discussion partner of Parmenides is named Aristotle. As Anthony Kenny elegantly put it "by a flattering anachronism" Plato makes Aristotle contemporary with Parmenides, Anthony Kenny, *Ancient Philosophy* (Clarendon Press, 2004), 65.

[277] I would agree with Ross here, when he says that "Aristotle is not quite fair in assuming that the indefinite dyad is an ordinary member of the class of 2's", Ross in *Aristotle's Metaphysics*, Oxford, 196.Even if the pair-duality concept would overlap with the indefinite dyad, it will be not so evident that such a meta-principle as the indefinite dyad would already contain the relative number two.

[278] See Taylor, *Aristotle on His Predecessors*, 120.

5.6. Concluding remarks on the argument for the generation of numbers

A manner in which one could understand these three entities – *one*, *being* and *difference* – and no other, as the fundamentals of numbers for Plato, is by conceiving *one* as the principle that gives to each number unity, *being* – what makes each number be, and *difference* – what differentiates numbers. The introduction of *difference* makes possible averting the collapse of the first hypothesis, by differentiating *one into many*, (Cf. chapter three, especially the section 3.2. The *Parmenides* evaluates Parmenides), but it can play the same role in what concerns numbers, namely *to differentiate multitude into numbers*.

The first integer that is deduced and used is two (δύο). The manner in which two is obtained makes up most of the argument. Two is not obtained by the addition of one to one; it is deduced from pair relations and only after the identification of two is one deduced as member of two. The result of the argument is consistent with Greek mathematics in which the series of integers is conceived as starting with two as the first number of the series.

Three is the first odd number, if we exclude one as being odd (a debatable issue for ancient Greek mathematicians). The first remark that Plato makes, after generating three, is to say that "three is odd." In the economy of the multiplicative generation, compared with the rest of the numbers, which are generated by multiplication, threeness is the result of addition (2+1). Since only for this number the process of adding 1 is used explicitly, the reason behind this method of obtaining the number three could be in order to emphasize its oddness. It would have been

natural to use addition for *two*, but Plato probably intentionally avoids that in order to preserve addition for oddness, as a method of highlighting oddness (as 2k+1), and to conceive two in its factorial aspect, as even.

The ingredients of numbers are: two, one, three (or one, even, and odd), while the structure of a mathematical unit is always a one-being pair. We don't have for each number a form-number (as the theory of forms as described in the *Phaedo* and the *Republic* would imply); instead we have the product of each number from one, two, and three by a process of multiplication, similar to the blending of the forms. The argument suggests that this procedure is complete and all numbers, primes included, we are led to believe, are in one way or another generated.

The odd and even are inferred from the complicated numbering of one, being and difference. It is an account in which Plato establishes that the odd and the even there are a result of pairs and trinities. The pairs and trinities are, at their turn, derived from counting pure logical and metaphysical principles. Plato's insistence on two and three, and therewith, even and odd, demonstrates the idea that he was looking for *principles* of numbers. Contrary to our understanding of numbers, for Plato even and odd were necessary features of numbers and the philosophy of numbers was the philosophy of even and odd.

The mathematical argument arises as a natural consequence of the ontological argument. One of the questions is whether this argument should be taken in a strictly mathematical sense or whether, very plainly, it refers to something else. My interpretation is that Plato brings number into discussion making use of the analogy most directly relevant (i.e. a mathematical analogy) to speak about the progression

of one into many when he speaks about 1 into 2, 3, 4, etc. Further mathematical considerations – how to get primes, how to get odd - may get into an eventual actual mathematical discussion, but the basis that triggered the argument is ontological. However the argument shows at work, in a condensed way, a lot of purely mathematical ideas which obviously fuelled discussion within the Academy (since Aristotle took this as a reference point when discussing philosophy of mathematics).

6. From Ontology through Numbers to Epistemology

As I have argued above, and especially in Chapters Four and Five, Plato's argument for the generation of numbers is significant for the history of philosophy of mathematics. The ontological argumentation remains a possible ground for the mathematical argument, and through the ontological principles it proposes it spreads clear ramifications beyond its argued mathematical purpose. The aim of this chapter is to show that the upshots of both arguments permeate at different ontological levels: in the intelligible realm, in our epistemic faculties, and also the sensibilia. Other potential connections between *Parmenides* 142b-144b and arguments from Plato's late dialogues can be found, to show primarily that the ontological and mathematical arguments in the *Parmenides* condensed philosophical conceptions that were later put to use. The most obvious permeation of the ontological argument is the discussion of the greatest kinds in the *Sophist*, and in the generation of the soul from the *Timaeus*. My purpose is to underline possible ramifications of the ontological and mathematical arguments in passages where the linkage with the *Parmenides* (142b-144b) is not so evident, as for example, the *Theaetetus* (185c9-d4). I will argue that the ontological argument could actually provide some answers to the problems raised by the first part of the *Parmenides*, not in a direct manner, but empowered by other dialogues, such as the *Theaetetus*.

The relation of the second part of the *Parmenides* to its first part is a matter of controversy. For some scholars, the second part provides solutions to the problems

raised in the first part,[279] while for others the two parts of the dialogue share no connection.[280] In my own view, the separation of *being* from *one* in the second part of the *Parmenides* could come as an essential answer not only to the issues raised by the Parmenidean ontology (as I have argued in Chapter Three), but also to Platonic ontology which shows some of its limitations in the first part of the dialogue. Assuming that *one*, *being*, and *difference* are independent ontological entities, Plato may use these entities to explain not only the origin of numbers, but also the origin of knowledge, of the soul, and even of the forms. These ontological kinds could have a similar function in the case of forms as they have in the case of numbers: one would give to each form unity, being makes them be, and difference differentiates forms. Plato does not speak openly, as in the case of numbers, about the role of these kinds with regards to forms (though the *Sophist* could be read as a development of the ontological argument with regards to the way in which forms are individuated). Plato does speak, however, about the role of being and difference in the generation of the soul in the *Timaeus*, and, as I will argue in the next sections, the ability of the mind to know is circumscribed by Plato through particular kinds such as *being*, *difference*, *odd* and *even*. According to the *Theaetetus* (185c9-d4), we arrived at these kinds not by sense perception, but by our proper thinking. From the perspective of the ontological argument, the development of the *Theaetetus* could be surprising, but,

[279] For recent arguments for a strong relation between the two parts see Samuel Scolnicov, *Plato's Parmenides* (University of California Press, 2003). See also Samuel C. Rickless, *Plato's Forms in Transition: A Reading of the Parmenides* (Cambridge University Press, 2006).

[280] Ryle is a typical advocate of the idea that the two parts of the dialogue are only lightly related, and that they were probably written at different moments. Cf. Gilbert Ryle, "Plato's *Parmenides*" (1939), in *Studies in Plato's* Metaphysics, ed. Reginald E. Allen, London: Routledge & Kegan Paul, 1965.

in the next sections, I argue that there is an octopus-type relation connecting the ontological argument, the *Theaetetus,* and the first part of the *Parmenides*.

The first part of the *Parmenides* ends in a forthright epistemic and ontological difficulty. The last refutation of the theory of forms, known as 'the separate domains argument', and referred to as 'the greatest difficulty' (133a-134e), stands as a natural, and easily foreseeable, outcome in the development of the criticism of the forms. According to Plato, one can reject this argument only if one "happened to be widely experienced and not ungifted", and if one "consented to pay attention while in your effort to show him you dealt with many distant considerations" (133b-c). These possible "distant considerations" could start, in my understanding, with the ontological argument and undergo a development with the *Theaetetus* (185c9-d4).

In the first section 6.1., I argue against the idea, defended by some scholars, that the first part of the *Parmenides* consists of unsound arguments, whose utility was intended for the student of philosophy, in order to uncover their fallacies. I insist that the "greatest difficulty" was actually a real challenge for Plato. Secondly, in section 6.2., I will try to show that the second hypothesis is the inauguration of the 'distant' answer, an answer which is more fully developed in the *Theaetetus* (185c9-d4). The ontological argument of the second hypothesis does not answer directly to the *Parmenides* 133a-134e, but it can give an indirect answer, through the *Theaetetus* (185c9-d4).

6.1 The Separate Domains Argument in the *Parmenides* (133a-134e)

The separate domain argument proposes an absolute separation between the intelligible and the sensible domains which has ontological and epistemic consequences: the two realms thus absolutely separated, the knowledge of the intelligibles appears impossible. If there are forms, they belong exclusively to the intelligible realm, a realm which is totally separated from our cognitive capacities, and from sensibilia. They belong to their own world, sharing no relation with our mind. Our knowledge is only of this world, and the world can be known only through our capacities, which are in no way linked with the world of forms. The argument concludes that the only *relation* that actually exists is among forms themselves, on the one hand, and among sensibles, on the other hand. That is to say that, if forms are (in the world of forms), we do not have any access to them, since our knowledge is limited to our ordinary life, in which things relate among themselves, but not with forms. That means that if forms do exist, they are necessarily unknown to us (135a). Plato also affirms the reverse: the knowledge of gods[281] cannot reach us.[282] In an analogous manner to human nature which does not have access to the intelligible realm, the divine nature itself would not have access to the physical world either.

[281] Plato's god here reminds us of Aristotle's god which thinks only about thinking.

[282] The same idea is to be found in book X of the *Laws*, where three views on misapprehensions of the one who commits an unholy act are presented: "it is because of one of three possible misapprehensions: either, as I said, he believes (1) the gods do not exist, or (2) that they exist but take no thought for the human race, or (3) that they are influenced by sacrifices and supplications and can easily be won over" (884b5-9). The existence of gods ignoring our world is a form of antique deism with which Plato was not unfamiliar, and that, doubtlessly, he struggled with.

The Separate Domains Argument sums up previous arguments into a predictable dilemma. In this argument Plato expresses plain reservations concerning the theory of forms. All previous arguments (130e-133a) aimed at ontological questions, while the last argument points also towards epistemological matters. The argument stands as the decisive crescendo to all the difficulties regarding the theory of forms, presenting one of the most radical critiques of the theory, a critique which, from Aristotle onwards, is hardly superseded by other subsequent critics. Parmenides himself specifies that the separation of the domains is the greatest difficulty that one may encounter about the theory of forms.

Parmenides does not immediately start the list of refutations with 'the greatest difficulty', but keeps it for the final argument (does this mean that the previous arguments, included the TMA, are less difficult compared with the greatest difficulty?), and gives the impression that he permanently bears it in mind as the leitmotiv of the refutations: if one can surpass the main ontological problems of the forms, an additional problem will always remain, pertaining to human and divine nature alone. That means that together with the TMA, each argument announces, gradually, 'the greatest (μέγιστον) difficulty' (133b4). Plato's greatest problem, which is that of Aristotle too, is the epistemological gap between two layers: the intelligible forms and the participated things.

If we accept the premise that forms are in fact separated from participated things, they must be unknowable since (1) our mind is related only with participated things, and (2) there is no relation between things and forms. The main statement of the greatest difficulty is:

> "Suppose someone were to say that if the forms are such as we claim they must be, they cannot even be known."

This surprising statement goes directly against the *Republic*, where the vertical ascension of the divided line moves our epistemic abilities towards intelligibles. Here we are left to figure out only a horizontal line in which all our epistemic abilities are reduced to the sensible world. The argument relies thus on an absolute ontological difference between forms and participated things.[283] The difference is both ontological and epistemological. At the same time the conclusion of the argument states an epistemological gap between forms and our possibility to know them. Thus the argument points to ontological and epistemic differences. What is a given is that here there are two realms which are *different* and that they *are*, and what is questioned is how these realms are related, and therefore how the realm of forms can be accessed by us.

The modern version of Plato's argument is to be found in the so-called *access problem* or Benacerraf's dilemma.[284] As I already mentioned in Chapter Four, Benacerraf uses a causal theory of knowledge in order to show the difficulties raised by Platonic realism (in which mathematical objects are not spatially and temporally localized, although the mathematicians who conceive them as mathematical objects

[283] Several commentators take this separation as an absolute principle of separation, see for example Mary Margaret McCabe, *Plato's Individuals* (Princeton University Press, 1999) 91. William Prior, *Unity and Development in Plato's Metaphysics (RLE: Plato)* (Routledge, 2012), 75-82.

[284] Benacerraf "Mathematical Truth," (*The Journal of Philosophy*, 1973): 661-679.

are themselves spatially and temporally localized). Our knowledge is in space and time, while mathematical objects are outside of space and time.[285]

For several scholars this argument, together with the previous arguments of the first part of the *Parmenides*, is meant not to be taken seriously.[286] This would imply that the man of 'wide experience' (who is selected by Plato as the person who can find the solution to the puzzle) can find the solution to the argument by showing the argument's unsoundness.[287] But I think that this might be a wrong track.[288] The tenacity of the philosophically experienced man consists not in showing what a poor argument is fabricated here, but in providing a solution to it. In my view the argument is logically valid.[289] The argument retains an unquestionable validity, and Plato was overwhelmed by its implications. The epistemic objections to the possibility of intelligible knowledge are worthy to be considered. Our cognitive

[285] See the beginning of Chapter Four for another reference to Benacerraf's views, especially to his anti-Platonism regarding mathematical objects.

[286] See for example Mueller, I., "*Parmenides* 133A–134E: Some Suggestions", *Ancient Philosophy*, 1983: 3–7. Bostock considers that especially the separate domain argument "is so riddled with fallacy that it is hard to believe that Plato could ever have taken it seriously" David Bostock, *Plato's Phaedo* (Clarendon Press, 1986), 206.

[287] F Lewis, "Parmenides on Separation and the Knowability of the Forms: Plato's Parmenides 133A ff.", *Philosophical Studies*, 35, 1979: 105–127.

[288] Several commentators think that Plato is not serious with the critique of the forms and that the questions raised are only for paideic sake: the reader is challenged to find a solution to the fallacious arguments. See, for example, Cornford, *Plato and Parmenides*. 96. On the contrary, I think that the problems raised by the *Parmenides* are honest and sincere perplexities in face of the theory of forms. For some modern philosophers, the problems of the *Parmenides* are still a philosophical issue, particularly 'the separate domain argument'. See, for example, Michael Esfeld și Christian Sachse, *Conservative Reductionism* (Routledge, 2012).69: "If one conceives universals as something that exists beyond the empirical world, the question remains unanswered what it means that the tokens in the empirical world take part in or instantiate the universals. The problem that this question highlights has already been pointed out by Plato himself in the *Parmenides* (130e–133a), and it still is an open issue."

[289] For a recent defence of the argument's validity see Matthew Duncombe, "The Greatest Difficulty at *Parmenides* 133 C–134 E and Plato's Relative Terms" in Brad Inwood, *Oxford Studies in Ancient Philosophy* (OUP Oxford, 2013), 43-61.

possibilities do not have access to the *inteligibilia*, the realm of the forms remain fully transcendent for the sensibles and for our epistemic faculties.[290]

It is thought that Plato never replied to the arguments of the first part of the *Parmenides*, and especially to the 'greatest difficulty'. One must be aware that the greatest difficulty is not simply an applied problem of how one can get access to the domain of the forms, and a possible epistemic response to this dilemma would be to invoke the anamnesis theory.[291]

At least on two notable occasions, in the *Meno* (76a6–7) and in the *Phaedo* (74b2–3), Plato assumes that by recollection one can have access to specific intelligible ideas. But Plato does not make use, in the *Parmenides*, of the precise anamnesis answer.[292] I would venture to say the at that moment of composing the *Parmenides*, Plato does not see the anamnesis theory[293] as a viable epistemic option, especially since here Plato is profoundly questioning basic features of the philosophy which is specific to the middle dialogues. His avoidance of *recollection* could be a side-effect of the possibility that he was rethinking his theory of forms,

[290] In this respect the argument resembles Kant's difference and distinction between 'transcendent' (opposed to immanent) and 'transcendental' (*grosso modo*, a way of cognition which is related with our cognitive domains). Our cognitive categories do not have access the things in themselves.

[291] In Plato's earlier dialogues, the theory of forms and the immortality of the soul, and thus anamnesis theories, are strongly connected.

[292] According to Bostock, Plato does not make any attempt to replay with the theory of recollection because "*perhaps* Plato was now feeling that this doctrine was altogether too extravagant a solution of the problems that led to it". Bostock, *Plato's* Phaedo, 206.

[293] Allen is also very circumspect and thinks that the consequences of the argument "cannot be avoided by the doctrine that learning is Recollection: for it is an essential part of that doctrine that sensibles remind us of Ideas by reason of their participation in or likeness to Ideas, and there is now no participation. Nor can souls be intermediate between sensibles and Ideas if there is no connection between Ideas and sensibles." Reginald E. Allen, *Plato's Parmenides* (Yale University Press, 1997), 197.

but also that he was reconsidering his views on soul and its epistemic faculties. When elaborating such criticism against the theory of forms, Plato was actually revising not only the theory of forms, but also his philosophy of mind, and, implicitly, his theory of knowledge.

One can notice a revision of the standard theory of knowledge (as it is displayed in the *Meno*) in the *Theaetetus*, where the anamnesis theory is not used again as the most economical answer for epistemic problems. The *Meno*, where recollection is the epistemic answer, and the *Theaetetus* mirror each other, as both are centred on the problem of knowledge, and both of them assert the fact that we should turn our attention from sense-perception to the mind that does the judgments. According to the *Theaetetus* (185e), looking at the instruments of the mind, can one figure out how knowledge is possible. One important difference between these two dialogues is that the conception about the soul regarding its cognitive faculties, as described in the *Theaetetus*, does not resemble the conception of cognitive faculties of the soul as conceived in the *Meno*. In both dialogues, Plato operates with different views on the nature of the soul and its cognitive faculties.

The consequences of the aporetic argument are that forms are inaccessible for us to be known, and, in turn, we and the participated things are unreachable by gods. This epistemological gap, grounded on the ontological gap, does not deny that *forms are*. Forms are separate, absolute, and they can be reached only if you are on the same level with them; only in this way one can have an epistemic relation with the forms. It is also impossible to know the divine realm; our knowledge is the knowledge of our world, and not of forms.

Possible (distant) solutions to the challenge of the separate domains would be: 1) ontologically, to show that there are some elements that the two realms share; 2) epistemologically, that the soul could perceive the common elements of both realms; since we cannot be on the same level with forms, a solution could be to share something that the forms share.

The prefiguration of the common elements of both realms is to be found in the ontological and mathematical argument, and in its fuller theoretical depiction of generation of the soul, in the *Timaeus*. In the next section I argue that there are good reasons to think that also in the *Theaetetus* a solution is schematically advanced. *Being* and *difference* of the ontological and mathematical arguments are proposed as instruments of the soul (*Theaetetus* 185c-e); instruments to perceive and understand *sensibilia*.

6.2 Knowing Kinds and Knowing Numbers - the *Theaetetus* (185c9-d4)

In the *Theaetetus*, Plato seems to continue his critique of the theory of forms in the *Parmenides*. There is no direct reference to the theory of forms in the *Theaetetus*, and Plato does not provide recollection (of the *Phaedo* or of the *Meno*) as a possible answer to the problems raised by the limited possibilities of knowledge. As I underlined in the previous section Plato does not take recollection as a reliable option when it comes to "the greatest difficulty" as well. I argue that one of the

reasons for which the anamnesis theory is not used by Plato in reply to the 'separate domain argument' could be that Plato is revising his overall theory concerning the epistemic power of the soul. In my understanding, Plato is looking for *a priori* cognitive concepts of the soul and mind (*Theaetetus*, 185c9-d4) and these concepts surprisingly prove to be the entities that underline the world of the intelligible realm (the *Sophist*).

What is under examination in the *Theaetetus* is what knowledge is.[294] In the refutation of Theaetetus' claim that 'knowledge is perception' (151d-186e), Plato develops a slightly modified view on the soul's epistemic abilities (compared with those from the *Meno*, *Phaedo*, or the divided line of the *Republic*), in which the ability of the mind to know is expressed by the use of such concepts as *being*, *difference*, *sameness*. These concepts are called the "commons" of everything that we perceive, and, more important, our mind comprehends these concepts not by sense perception, but by its proper thinking.

Socrates starts the inquiry into the 'commons' in the following manner (185a):

> "Now take a sound and a colour. First of all, don't you think this same thing about both of them, namely, that they both are?"

Through *both are* Plato understands that both sound and colour have a share in *being*. Plato insists on what looks to be the obvious: One cannot perceive what is not. For

[294] Guthrie points out that with the Theaetetus "for the first time Plato has chosen to make knowledge itself the main subject of enquiry, setting aside for the purpose all preconceived ideas such as appear unchallenged in the *Phaedo-Republic* group." W. K. C. Guthrie, *A History of Greek Philosophy: Volume 5, The Later Plato and the Academy* (Cambridge University Press, 1986), 65.

Plato knowledge presupposes a differentiation of *being* from the objects of knowledge.[295] And in order to differentiate between the objects of perception, one needs first to conceive their being. Socrates adds that at the same level with being, there is *difference* and *sameness* ("each of them is different from the other and the same as itself" 185a4-5). *Oneness* is added as well ("both together are two, and each of them is one"). What strikes us in this passage is that the prerequisites for the generation of numbers are to be found as the prerequisites for the possibility of perception as well.[296]

After these clarifications, Socrates has a very precise request for the mathematician Theaetetus:

> "Now through what does that power function which reveals to you what is common in the case of… all things — I mean that which you express by the words 'is' and 'is not' and the other terms used in our questions …? What kind of instruments will you assign for all these? Through what does that which is percipient in us perceive all of them?"[297]

[295] I mark here that this is the usual way in which Socrates introduces the definiendum in the early dialogues, namely that there is such a thing as the thing in question (e.g. justice/that justice is).

[296] Turnbull considers too that there is a relation between the generation of the soul, the second hypothesis and the 'commons'. See Robert G. Turnbull, *The Parmenides and Plato's Late Philosophy: Translation of and Commentary on the Parmenides with Interpretative Chapters on the Timaeus, the Theaetetus, the Sophist, and the Philebus* (University of Toronto Press, 1998), 159.

[297] Translation by M. J. Levett, revised by Myles Burnyeat.

To this question which aims at something else which is not perception, but makes perception possible, Theaetetus, a Socratic interlocutor very much active in the discussion compared with other interlocutors of Platonic dialogues, and, considering the absence of any conclusive remark on perception up to this moment in the dialogue, answers in a totally unexpected way (185c9-d4),

> "You mean being and not-being, likeness and unlikeness, same and different; also one, and any other number applied to them. And obviously too your question is about odd and even, and all that is involved with these attributes; and you want to know through what bodily instruments we perceive all these with the soul.
>
> Socrates: You follow me exceedingly well, Theaetetus. These are just the things I am asking about."

There is something in the soul that makes us able to perceive and to know, among other things, both basic kinds (*being* and *not-being*[298], *same* and *different*), and mathematical features (*odd* and *even*). The occurrence of these mathematical features alongside basic kinds comes unwarranted. But, for Theaetetus, it is natural to say that *odd* and *even* are at the same level with *being* and *difference* in their epistemic role. How can one perceive *odd* and *even* as entities that share something with *being*, *difference* and *one* is unclear. What we can notice is that there is an intimate relation between mathematical features and essential kinds of the world, and a

[298] The presence of not-being as part of the 'commons' is unsuitable since, according to the *Sophist*, *not-being* would be something, if one may say that, to the extent that it would be a derivation and consequence of the basic kind *difference*.

mathematician like Theaetetus insists on pointing that out. Plato, through the character Theaetetus, presents a view in which to know means to know also in a mathematical way. Thus, for Plato, our epistemic frame is mathematical, and the world too. Socrates marks out these *instrumentalia* through the concept of 'commons':[299]

> "All I can tell you is that it doesn't seem to me that for these things there is any special instrument at all, as there is for the others. It seems to me that in investigating the common features of everything the soul functions through itself."

Here, Plato indicates that there is a fissure between the *instrumental* features of sense-perception (through which we know), and the *non-instrumental* features of the soul (which makes possible knowledge). "The soul functions through itself" may suggests that there is something similar to what we would call a principle of identity, and a principle of being, if one may say so, inside of us.

There is an on-going discussion as to whether the 'commons' here refer to the theory of the forms or not.[300] Using basic kinds to give us an intimation of the epistemic abilities of the mind, Plato affirms that there are *primitive* principles which have logical priority in the world. For Cornford, one can speak here with certainty

[299] There is no special organ, similar to sense organs which are responsible for hearing or seeing to perceive identity, and to differentiate between sounds and visual stimuli.

[300] Revisionists like Owen, Burnyeat or Bostock assume that one cannot speak about forms in the *Theaetetus*, since Plato does not endorse the theory of ideas at this point anymore.

about forms. On the other hand, Robinson points out a tension in Cornford's reading of forms in the *Theaetetus*: "If Plato does mention Forms here, he cannot at the same time be trying to make them conspicuous by their absence,"[301] compared with "Plato here avoids using the word [form] because he is determined to say as little about the Forms as possible."[302]

For Robinson, this change is better seen in the lack of any overt appeal to the ontology of the two worlds.[303] For this specific discussion, in the *Theaetetus*, I think that the focus is not on forms[304], but on what I would call the *cognitive domains* of the soul. As Robinson puts it, "Plato has here [in the *Theaetetus*] turned his attention away from the world of Forms to the mind of man,"[305] and Robinson thinks one should take into account the kinds of the *Sophist*.[306]

If the 'commons' are forms, then there are only a few forms that can be perceived, and these should be the most essential of them. There is no need to say

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[301] Richard Robinson, "Forms and Error in Plato's *Theaetetus*", *The Philosophical Review*, Vol. 59, No. 1 (Jan., 1950), 3-30, 10.

[302] Francis Macdonald Cornford, *Plato's Theory of Knowledge: The Theaetetus and the Sophist* (Courier Dover Publications, 2003), 106.

[303] "There is little or no talk about two worlds. Socrates does not mention now any of those thrilling attributes, such as eternity and divinity, which in the middle dialogues had made the Forms objects of worship and love." Richard Robinson, "Forms and Error in Plato's Theaetetus", 3.

[304] Plato does not develop an answer which is depended exclusively on the theory of forms, since, as far as I can see, all the discussion in the *Theaetetus* is related to the 'separate domain argument' and, thus, there is an indirect questioning of the validity of the theory of forms, in the sense that the theory of forms must be emended.

[305] Robinson, "Forms and Error in Plato's Theaetetus", 18.

[306] "These things that are "common to all" are obviously Forms in Cornford's view. Whether they are Forms is not obvious to me, and to my mind this issue rests rather on the intricate question of the *megista gene* of the *Sophist*, whether they are Forms or not any issue about which I have not yet succeeded in reaching a confident opinion." Richard Robinson, "Forms and Error in Plato's Theaetetus", *The Philosophical Review*, Vol. 59, No. 1 (Jan., 1950), pp. 3-30, 10.

that we know the form of the bed or of the number five through one to one correspondence (for the object of knowing there is a cognitive form which corresponds to that object). With the cognitive kinds one can identify some features of the bed or of number five which give them *identity*, *differentiates* them from another chair or number six, gives them *being*. In other words, we don't perceive anymore a chair due to the form of chairness that acts on the participated thing, nor do we remember the form of the chairness, but we do perceive what is essential to a particular chair, that it *is*, that it is *one*, and that it is a specific being (chair) *different* from other beings.

For some scholars, when talking about 'commons', Plato speaks about *predicates*,[307] since "Socrates and Theaetetus speak in a way which takes for granted that 'good' and 'beautiful' are by now among the predicates that the Protagorean theory caters for"[308]. Sedley, an advocate for the predicates' position, affirms: "I use 'predicates' for these commons, because… in this part of the dialogue they are considered purely in a predicative capacity"[309]. It is the same with 'being', or 'number'. Sedley argues that "from 'being' (*ousia*), the class of these 'common' predicates is extended (185a11–186b1) to numbers."[310] Just as being is part of the commons, number is as well. Being part of the commons may give us an inkling as to what kind of number is enumerated here as one of the commons. The concept of

[307] D. N. Sedley, *The Midwife of Platonism: Text And Subtext in Plato's Theaetetus* (Clarendon Press, 2006), 106-107.

[308] Ibidem, 53.

[309] Ibidem, 106.

[310] Ibidem, 106.

number appears again in the vicinity of concepts, such as being, difference, etc., and the context is very similar to the unexpected development of the argument in the *Parmenides*, where from an apparently ontological premise one gets to a mathematical conclusion. As far as I can see, being, difference, odd, even are the same entities mentioned both in the *Theaetetus*, and the *Parmenides*.[311]

For Sedley, "all that Socrates has done is to isolate a set of predicates to which we have access independently of the use of our sense-organs,"[312] and "seems reasonably harmless for us to label these a priori." Still, Sedley recognizes that these entities are more than simple predicates:

> "I do not intend 'a priori' as a loaded term, informed by Kantian or any other presuppositions, but just as a handy way to categorize those entities, predicates or concepts, *whichever they may be*, that Socrates regards as capable of being understood by dialectical investigation."[313]

The dialectical process of collecting and dividing, as it is expressed in the *Sophist* (253d), is possible due to the fact that there are such 'commons' of the mind.[314] N.R. Murphy ponders that:

[311] Also, *being* and *difference*, from the ontological argument *build up* the soul in *Timaeus* 35a-b. Cf. Turnbull, *The* Parmenides *and Plato's Late Philosophy*.

[312] Sedley, *The Midwife of Platonism*, 106.

[313] Ibidem, 107.

[314] It seems that the method of dialectic, as styled in the *Republic*, makes an indirect introduction to how the mind works for grasping the intelligibles. In the *Theaetetus*, grasping the basic kinds is not through dialectic, but it is a precondition for dialectic.

> "dialectic explains the objects of mathematics by studying what the *Sophist* calls the κοινωνία εἰδῶν or what in the *Republic* is called 'going higher' (ἀνωτέρω)." [315]

Even without the argument about generation of numbers, one should enquire about the relation of kinds and numbers. Having an ontology based on greatest kinds, as in the *Sophist*, one could ask about the relation of the greatest kinds with numbers. Finding the *primordia* of numbers, Plato, in the *Republic* (531d-534e), thinks that an *arche* of mathematics can be found through dialectic. Also the crucial way through universal knowledge is through dialectic, which can unify the diversified knowledge.

The *commons* could be taken as an answer to the separate domain in the following manner: we perceive the main features of *sensibilia* with the soul; some of the main features of *sensibilia* are the main features of intelligibles and of numbers (the *Parmenides*) (i.e. *being* and *difference*). Thus, the essential features of things are the essential features of intelligibles too; the soul can perceive both the features of things and the features of the intelligibles. Thus separate domains, the realm of *sensibilia*, and that of the intelligibles, are not fully separated, because of the cognitive capacities of the soul which can distinguish in sensibilia and in the intelligibles the

[315] N.R. Murphy, Plato's *Republic*, Oxford, 1951, 195.

basic logical/ontological kinds, and *being* and *difference*.[316] Plato seems to think that one, being, and difference are ontologically present in both realms.

We perceive that things *are*, that each of them is *one*, and that they are *different*. The main basis for *identity* and *difference* in the world is placed in our mind (it does not mean that they are not features of the world too, independent of our mind). Our mind grasps the features of the world not solely by sense perception. Special concepts of the mind, such as sameness, difference, and being make possible the perception of important features of reality. These features *are in* the reality, and *are part* of our mind. Plato limits his application of the 'commons' of the mind to sensibilia, and does not go in the direction of the intelligible world. This is, I think, coherent with the overall rhetoric of the dialogue, which avoids any direct reference to the intelligible world or forms. But, according to the *Sophist*, these 'commons' refer to and are, first of all, part of the intelligible world. Sharing in these commons, which seem to play a role of logical forms (meta-forms?), one could have access to the intelligible forms in a manner that is similar to the way in which we have access to sensible things. We perceive the common features of sensible things by the common features of the mind. Analogically, if the forms are (and the 'greatest difficulty' argument accepts that the forms are), one could perceive them by the commons in our mind, because the main ingredients of the intelligible realm, of the soul, and of the sensible world are sameness, being, difference.

[316] The composition of the soul explicates the capacity of judgement about sameness and difference, on the principle that 'like is known by like' (at least this is what Aristotle thinks in *De Anima* 406b28–31). Plato's *dualism*, with the introduction of the world soul, in the *Timaeus*, is transformed into a *trialism*: the intelligibles, the world soul, the world as such. The soul is everywhere and animates everything, and thus informs matter.

6.3. The second hypothesis and the late dialogues

As I have tried to show in the previous section, the second hypothesis is linked to the first part of the *Parmenides* and the *Theaetetus*. Late dialogues also, especially through the *Sophist* and the *Timaeus*, develop the ideas of the second hypothesis into a consistent philosophical agenda. It is a question if there is such group as late dialogues. As I will argue later on, the developments in the second hypothesis might help in enforcing a certain chronology of Platonic dialogues.

With the exception of some dialogues which raise the question of the authorship of Plato, the exact number of Platonic dialogues is apparently definite. However, in order to show the role of the *Parmenides*' argument for the development of important tenets of the late dialogues, one should inquire if one can speak of such a group of dialogues, which are labelled as late dialogues, and in what resides their coherence. One must keep in mind that even if the body of the dialogues as a whole survived, it arrived without any pre-established order that might give us a clue about the sequence in which the dialogues were written – an issue which bears an importance greater perhaps than the chronology of the work of any other ancient or modern philosopher.[317]

[317] With some notable exceptions, when it is crucial to know which work comes first, as, for example, in the case of Wittgenstein: which was the first, the *Tractatus* or the *Investigations*? For

It is not the chronology itself that is significant for our purpose, but the existence of a *pattern of thinking* specific to a group of dialogues which also overlap in what regards stylistic particularities. The current standard periodization, as opposed to the ancient one, which was thematically conceived,[318] is based mainly on stylometric analysis.[319] There are some particularities in Plato's style of writing and the way of problematizing from middle dialogues to late ones. Independent of the classification made, judging from Plato's treatment or avoidance of hiatus, particles, specific expressions, there is also a recurrent use of some key concepts in the *Parmenides, Timaeus, Theaetetus* and *Sophist*, key concepts that oblige us to think of

a scholar who will work after 2000 years on Wittgensteinian philosophy to know the order, and the chronological distance between them, is essentially relevant.

[318] Accordingly each group would correspond to a specific theme (conf. Diogenes Laertius 3.56-62). In antiquity there were different orders for the dialogues. There were the nine tetralogies, as established by Thrasyllus, while Aristophanes grouped some of the dialogues in trilogies. This shows that there is certain type of magnetism between some dialogues (dictated by commonality of themes, methods, Socrates' presence or absence, internal cross-references, etc.) which imposes their grouping, and which makes the isolated reading of any one of them insubstantial. One might verify that not each dialogue has internal links with other ones, but some of them can be grouped, for example, posing narration as criterion: how many of them are narrated (9), how many from these are narrated by Socrates (6), how many are narrated for an indeterminate audience (5), and so on. Different groups can always be made, depending on which permutation function one applies.

[319] For modern scholarship, the most objective and accepted grouping is that based on stylometry. Inaugurated by Campbell (1867) and Ritter (1888) divides the thirty-six dialogues into three groups: 1): *Apology, Charmides, Crito, Cratylus, Euthydemus, Euthyphro, Gorgias, Hippias Minor, Ion, Laches, Lysis, Menexenus, Meno, Phaedo, Protagoras, Symposium*; 2): *Republic, Phaedrus, Parmenides, Theaetetus; 3): Sophist-Statesman, Philebus, Timaeus-Critias, Laws.* For the limits of studies on dialogues' stylometric analysis see Jacob Howland, "Re-Reading Plato: The Problem of the Platonic Chronology," *Phoenix* 45 (1991), 189-214

these dialogues as related.[320] The classical stylometric groups overlap to a certain degree with the thematic groups labelled as early, middle, and late.[321]

There is a substantial overlap between the third thematically-based group and the third stylometry-based group (*Theaetetus, Sophist, Statesman*, *Philebus*, *Timaeus, Critias*, *Laws*) and one particularity of the third group is the less frequent use of the theory of forms (with the exception of the *Timaeus*), to the point of its total absence. The late dialogues also have in common, as I want to show, a specific and explicit use, in key places, of concepts as *one*, *being*, and *difference* (especially in *Theaetetus*, *Sophist*, *Timaeus*).

The root of the conceptualization of these notions as related entities is the *Parmenides*. The main investigation of the thesis – if the ontological argument and the mathematical argument are part of Plato's philosophy – shows a line of thinking which presents the transition from middle dialogues to late ones as exhibiting in the argument for the generation of numbers abbreviated features of the late dialogues. For my research it is important to consider the dialogues as being connected mainly thematically[322], and not especially chronologically[323]. Thus I underline a tendency in

[320] See Charles H. Kahn, *Plato and the Post-Socratic Dialogue: The Return to the Philosophy of Nature* (Cambridge University Press, 2013). 157. See also Kahn, "On Platonic Chronology" in Annas and Rowe (2002) 93–128.

[321] Schleiermacher splits the dialogues into three groups: elementary, preparatory (or mediatory), and constructive.

[322] Thematically in general way, not that it is the same theme in the so-called late dialogues. I claim that there are some lines of thought that come and come again.

[323] For a critique of the problems raised by the chronological studies and by the inferences based on chronological assumptions see Debra Nails, *Agora, Academy, and the Conduct of Philosophy*, Kluwer, 1995, especially chapter 4, 5 and 6. She argues that the proposed chronologies do not overlap and that they are circular, and she emphasizes that if we base the order of composition on "affinity of style", we have "precious little independent evidence", 113.

Plato's philosophy in regard to specific dialogues which can be grouped together regardless of their precise chronological order (whether the *Theaetetus* is later than the *Parmenides* or not).

6.4. The Unitarian *versus* Revisionist theses and *Parmenides 142b-144b*

This section aims to show that the *Parmenides* directs us to an ontological frame in the late dialogues, which begins with the argument for the generation of numbers. One of the enquiries regarding late dialogues is whether the theory of forms is abandoned.[324] Does Plato build a genuine philosophical system around the theory of forms? Is he a systematic philosopher in his late dialogues? Or was he rather trying to raise suitable questions, than to give definitive answers, in an analogous way to a contemporary analytic philosopher? These problems have been dividing scholars without any decisive result. What is clear is that the dialogues, and especially late ones, hardly fit into a philosophical system. The theory of forms is not fully abandoned, I argue, but it undergoes considerable modification, a type of revision, in respect to the relation between forms and numbers, and the prominence of *being*, and *difference* in respect with numbers and forms. Particular modifications were

[324] Owen's efforts of persuasion for changing the dating of the *Timaeus* is part of the revisionist frame, and advances the idea that Plato fully rejected the theory of forms. Despite Owen's article, the *Timaeus* is nowadays accepted as a late dialogue, although the revisionist thesis still haunts scholars, especially for dialogues such as the *Theaetetus*. David Bostock considers that Plato "abandoned the whole notion of forms as standard examples". According to Bostock, Plato "does not stop talking about forms, but this way of thinking of them does not recur in any of his writings after the *Parmenides*." David Bostock, *Plato's Phaedo* (Clarendon Press, 1986), 207.

marked by scholars, other are still to be explored. Emendations of the theory of forms in relation to Plato's theory of numbers in late dialogues are little explored.

Plato seems to be looking for an alternative or improved understanding of the ontology of the theory of forms. The idea that Plato in his late dialogues goes beyond a "pure" theory of forms, as it is developed in the *Phaedo*, is discussed by scholars. Patricia Curd takes the *Parmenides* and the *Sophist* as starting points of Plato's awareness that the theory of forms - in reality of Parmenidean character - has its limitations.[325] Any attempt to understand his partially-revised theory of forms should take into consideration that Plato is not explicitly a systematic philosopher who develops and presents a constant philosophical system throughout the dialogues. One might build a system of his philosophy by putting together his dialogues (with more or less arbitrary harmonization), and this could be done depending on what the reader emphasizes, but the dialogues *per se* do not depict a monolithic philosophical system.

Contemporary scholarship on Plato is divided into two apparently irreconcilable positions. On the one hand, there are scholars (Ryle, Robinson, Owen, McDowell, etc.) who argue that one cannot systematize Plato around the theory of forms (1) since consistent references to the theory of forms are missing in late dialogues, thus Plato didn't endorse his theory after the *Parmenides*. On the other hand, there are scholars (Cornford, Ross, Sedley, Chappell, etc.) who think that (2) even if the theory of forms is not explicitly mentioned in late dialogues, the reader

[325] Patricia Curd, *The Legacy of Parmenides: Eleatic Monism and Later Presocratic Thought* (Princeton University Press, 1998).Ch. VI, 'Problems of Unity', and 'Unity is Many.

should always bear it in mind as the underlying reference system, since Plato did maintain a unitary philosophy throughout all his dialogues. According to the *Revisionist* position (1) Plato consistently revised his philosophical commitments and one cannot resort to the theory of forms as explanatory for late dialogues, whereas with the *Unitarian* view (2), a consistent effort of harmonizing late dialogues with early dialogues is needed, since even if there is no explicit reference to the theory of forms (with the exception of *Timaeus*), it remains the main background theory of the dialogues.[326]

The two interpretations exclude each other, especially when applied to dialogues such as the *Parmenides*, the *Theaetetus*, and the *Sophist*. A Plato who has at the core of his philosophy, in the late dialogues, a generation process of numbers, would be a revisionist since it will be difficult to accommodate a generative process with a static theory of forms. Or, a more Unitarian view, that the forms are not generated, but would not take the numbers to be forms. At the same time, a Plato who thinks that the generation process is not one in time (an interpretation similar to that of the non-literalist reading regarding creation of the world in the *Timaeus*) could accommodate a generative process with a more dynamic theory of forms. In the next chapter, in the section 7.2., I have advanced the possibility of reading the generation process not as a real chronological process, but as a process which expresses mainly ontological relations between numbers, and odd and even, and between ontological kinds and numbers.

[326] Of course, there is the case of scholars who think that there is a theory of forms in the background of the late dialogues, but they don't think necessarily that the forms are in the background already in the early dialogues.

Independent of how we understand the generation process, in time or timelessly, both the *Revisionist* and *Unitarian* positions could be more nuanced depending on the way we interpret the argument for the generation of numbers. It may actually lead us to a third possible scenario, a *Generative* position (3), in which the theory of forms undergoes changes, and is not entirely rejected. Together with numbers, and the soul, forms, at their turn, are to be accounted for and 'generated' through the basic ontological kinds (which would mean that the structure of the intelligible forms could be given, as in the case of numbers, through meta-ontological kinds such as oneness, sameness, being, difference, and possible other kinds).[327] The Generative scenario would attenuate the extreme *Revisionist* position, and will be a more nuanced reading of the *Unitarian* scenario.

In the *Phaedo*, in the reply of Socrates to Cebes, the difficulty of generation comes to the fore, and generation is there presented in the larger context of generation and destruction (95e – 96a). Socrates struggled to find the cause for generation and destruction, and he confesses that:

> "I do not […] know why a unit or anything else comes to be, or perishes or exists" (97b4-6).

[327] Behind other forms (as the form of beauty, or the form of justice) there are logical forms (as the form of being, or the form of difference) which 'generate' them, in the sense that these logical forms assures the 'existence' of all forms, similar to the way in which these logical forms assures the 'existence' of all numbers.

Socrates admits his failure in finding a natural cause. But his interest in natural causes for coming to be and ceasing to be was, he insists, a passion of his youth. He is not satisfied with the explanations of natural sciences, and he acknowledges that he does not know, for example, the origin of *two* (96e6-97b3). As he often spoke about different types of generations in late dialogues - generation of the world, generation of the soul, generation of numbers - Plato was indeed immersed in several problems related with the problem of generation and could have been considering a scheme aimed at supporting the idea of the generation of numbers, comparable to the Pythagorean generation of numbers (cf. section 7.1.).

The reassessment of Plato's philosophy into a theory of the principles of numbers, in which generation is central, would be coherent with the Revisionist theory (1), and should not contradict the Unitarian theory either (2), since, as I think, for looking and constructing an axiomatic system for the theory of forms (and not only that), Plato goes beyond his early views on the forms (as they are presented and developed in the *Phaedo*, the *Republic*, the *Parmenides*), without necessarily rejecting them. The solution that I have advanced so far is not merely one of a compromise between (1) and (2), but an attempt to hint towards the reconstruction of a possible ontology of the late dialogues in which unclear, but fundamental passages – such as the generation of numbers, generation of the soul, and the commons of the soul – could be harmonized with both the theory of forms and that of the ontological kinds in Plato's late dialogues.

The theory of principles of numbers does not necessarily overlap with "the unwritten doctrine", which speaks about a "doctrine of principles" (*Prinzipienlehre*).

As I can see, there is a tendency in Plato's dialogues to look for ἀρχαί of numbers (to go beyond the hypotheses of mathematicians), which, at least for Aristotle, must have been the topic of oral discussions and indexed, therefore, as part of the unwritten Platonic philosophy. I am inclined to think that Plato is not committed dogmatically to a theory of ἀρχαί, based on one and indefinite dyad, but is looking for a theory of first principles. The second hypothesis may prove a testimony in this respect: numbers are conceived as generating from ἀρχαί, such as *one*, *being*, and *difference*.

It looks as though Plato switches from a "normative trio"[328] (Good, Beauty, and Justice) of the Socratic dialogues to possible other "normative trios" (Being, One, Difference, or Being, Sameness and Difference) of the late dialogues. With these trios, the discourse moves beyond ethical implications into a more technical discussion. The territory we are moving now into is what could be labelled logical-ontology or ontological-logic, because even if concepts such as being, one, etc. describe metaphysical entities, the argumentation in the *Parmenides* and the *Theaetetus* makes use of them as logical operators.

[328] Kahn, *Plato and the Post-Socratic Dialogue*, 181.

7. Arithmogony and cosmogony

7.1. Tetraktys *versus* triadic structure

In Greek mathematics *one* is the condition for and origin of numbers, whereas *one* itself is not a number. This view echoes a pre-Socratic manner of thinking, namely that everything must have a root. Finding an ultimate principle or the ultimate principles was a major program for the pre-Socratics; by deducing numbers from basic entities, Plato follows a similar path to pre-Socratic philosophers. A pre-Socratic framework, especially of the late pre-Socratics (i.e the Pythagoreans), applied to numbers, gives specific nuances to the generation process.

In the Pythagorean frame, the generation of numbers is reduced and translated into the language of the *tetraktys*. The sum of 1, 2, 3, 4 makes 10, and each numerical entity corresponds to *point, line, plane, solid* (*Met*. 1090b20-4), the basic geometrical entities which generate and create magnitude. The primal four entities lead to the decad, and each geometrical dimension presupposes the previous dimensions: the *linear* presupposes unity; the *surface* presupposes unity and linear dimension; *space* presupposes surface, linearity and unity. In short, the first four integers represent and concentrate everything, from number generation to psychology, and the constitution of the world. In *De Anima* (404b20-24), Aristotle gives an example of another hierarchical number theory: 1 corresponds to *mind* (nous), 2 to *science* (episteme), 3 to *opinion* (doxa), 4 to *sensation* (aisthesis). Here numbers are associated with psychological functions. The explicit fourfold numerological distinction from *De Anima* supports the idea that the divisions

concerning bodily or psychological functions were made, at least in the Early Academy, in function of some numerological virtues. According to Ross, the first four integers were an explanation both "for the formal structure of the world and for that of the mind." [329]

Aristotle perceives sometimes Plato's understanding of numbers as being very close to the *tetraktys* of the Pythagoreans (*Phys.* 206b32-33, *Met*. 1084a10, 25).[330] It can be that Aristotle's testimony on Plato, that "he makes numbers only up to the decad" (*Physics* 206b33), refers to Platonists of the Early Academy. The Early Academy is largely marked by an amalgamation of Platonic ideas with that of the Pythagoreans. In this regard we are dependent on Aristotle's indications. Similar to the Pythagoreans, Speusippus[331] as well seems to be an advocate of the metaphysical virtue of the decad.[332] In his treatise *On Pythagorean Numbers*, he lays emphasis on the *tetraktys*, and some scholars think that "he ascribed mystical properties to it"[333],

[329] Ross, *Theory of Ideas*, 179.

[330] Aristotle mentions more than once that Plato followed the Pythagoreans (*Met.* 987a 29), and, not accidentally, the discussion of Plato's forms is accompanied by discussion about the Pythagoreans. See, for a reassessment of the Pythagoreanism of Plato, Philip Sidney Horky, *Plato and Pythagoreanism* (Oxford University Press, 2013).

[331] It is quite difficult to rely on Aristotle's references on Early Academy and Speusippus, considering his hostility to the successor of the Academy. As Dillon remarks, for Speusippus' "philosophical views we are dependent very largely on references by his rival Aristotle, which are polemical, tendentious and, above all, allusive." See review of John Dillon of *Leonardo Taran: Speusippus of Athens. A Critical Study, with a Collection of the Related Texts and Commentary*. (The Classical Review, New Series, Vol. 33, No. 2 (1983), 225).

[332] His doctrine is similar to that of Hippasus. See Philip Sidney Horky, *Plato and Pythagoreanism* (Oxford University Press, 2013), 84.

[333] See Leonardo Tarán, *Speusippus of Athens: A Critical Study with a Collection of the Related Texts and Commentary*, Leiden: Brill, 1982, 20.

or, on the contrary, that for Speusippus "the perfection of the decad has to do with the decimal system, not with any alleged mystical properties".[334]

The textual evidence we have from Plato does not support the idea that Plato understood numbers in terms of the decad and contradicts Aristotle's report (Cf. also the section 4.3.4 Aristotle on Plato's philosophy of mathematics), but, I would argue that Aristotle's testimony suggests that the manner in which Plato understood numbers is somewhere in the middle between the decade of the Pythagoreans and that of Speusippus. The dialogues are silent about a potential bound of numbers at the decad, and the only passage from the dialogues where numbers go through a sort of reduction is the argument for the generation of numbers, where the scaling down is to one, two and three.[335]

Unlike the Pythagoreans and Speusippus, Plato develops a triadic structure of numbers. For Plato, *one*, *two*, and *three* are the sufficient principles for the generation of numbers. Taking into consideration that two overlaps with even (as 2k), and three with odd (as 2k+1), the ingredients for the generation of numbers seems for Plato to suffice as only three.[336] The Pythagoreans stopped at *four* (for the tetraktys), and

[334] Ibidem, 21.

[335] It is almost the only passage because in the *Laws* (818c4-5), Plato seems to make a similar remark, even if the discussion is not in regard to the nature of numbers, but in relation to the man's destiny: "if he can't recognize one, two and three, or odd and even numbers in general". However, the fugitive remark from the *Laws* could be taken as a clue that one, two, and three (and their corollaries odd and even) are to be understood as essential for knowing numbers.

[336] What Plato does resembles some archaic series of numbers similar to that of primitive populations: there is a story according to which Australian Aborigines limit their number system to *one* and *two* (i.e. a binary system), and out of them they can deduce composed numbers up to six. As, for example, three is made by *two* and *one*, while six is made by *two* and *two* and *two* (Dantzig, Tobias *Number and the Language of Science* (New York: Macmillan Company, 1954), 14). In this

Plato stops at *three*. The generation of numbers from a triadic structure is atypical considering the Pythagorean generation of numbers from a tetradic structure.[337] At the same time using only the first three numbers for generation could be interpreted as a Pythagorean variation. According to Aristotle, *De Caelo* 1.1, for the Pythagoreans, the number three is said to be the perfect, or complete, number.

My reading brings an additional nuance to that of scholars such as Cornford[338] or Sayre[339], who take Plato's generation of numbers as plain Pythagoreanism. The doctrine of the Platonic generation of numbers has indeed a Pythagorean background, especially Philolaic (as I emphasize in the next section), but the second hypothesis differs from 'plain Pythagoreanism', and from Speusippus, by emphasizing a triadic structure, both as regards the role of *one*, *two/even*, *three/odd*, and the role of the ontological principles *one*, *being*, and *difference*. As I see it, Plato may not have invented the process of generation of numbers – he adopted the process from the Pythagoreans – but he made some innovation around it, and founded the first three numerical entities (one, two, and three) on an ontological basis.

testimony six is obtained by multiplication, and not through addition. The aborigines count in pairs, i.e by multiplication.

[337] According to Sayre, "Plato had a mathematically more sophisticated generation of numbers based on then-current work of Eudoxus in the Academy." Kenneth M. Sayre, *Plato's Late Ontology: A Riddle Resolved : With a New Introduction and the Essay, "Excess and Deficiency at Statesman 283C-285C"* (Parmenides Pub., 2005), ch. 2, sec. 3.

[338] "In this revised form Plato restores the Pythagorean evolution of numbers from the One." Cornford, *Plato and Parmenides*, 138.

[339] Sayre, *Plato's Late Ontology: A Riddle Resolved*, p.53.

Plato's posterity, starting as early as with the first Platonists (especially Speusippus) could already have had an ontological interpretation of the second hypothesis. If Dodds[340] traced the origin of the ontological reading of the *Parmenides* back to Moderatus, Dillon[341] goes so far as to argue that Speusippus developed a similar reading, at least for the first two hypotheses.[342]

7.2 Eternal Generation *versus* Generation in Time

How should one properly conceive the *generation of numbers*? How should one understand such a process? What exactly does Plato do? Is it a real generation in action, firstly, from 2 obtaining 1, and from 2+1 obtaining 3, and secondly, from 2 and 3 by multiplication all the numbers? Could it mean that there was maybe a moment when numbers did not exist? Should one think about the generative process as being in time, in which each new number receives *existence* and appears from nowhere, as it would have not existed before, or is it rather a matter of reducing numbers to an axiomatic frame?

[340] E. R. Dodds, "*The Parmenides* of Plato and the Origins of the Neoplatonic One", Classical Quarterly 22 (1928), 129–142.

[341] J. Dillon, *The Heirs of Plato* (347-274BC), Clarendon Press, Oxford, 2003, 57–59.

[342] John Dillon considers that with Speussipus starts an ontological reading of *Parmenides*, which implies "an account of how One, when combined with indefinite Dyad (under the guise of 'Being') produces, first the whole set of natural numbers, and then progressively, the various lower levels of reality", John Dillon, Syrianus's *Exegesis of the Second Hypothesis of the Parmenides: The Architecture of the Intelligible Universe Revealed*, in John Douglas Turner, Kevin Corrigan (ed.) "Plato's Parmenides and Its Heritage: Volume II: Reception in Patristic, Gnostic, and Christian Neoplatonic Texts" (Society of Biblical Literature Writings from the Greco-Roman World Supplement), 133. However, I leave this possibility aside for the purpose of this discussion.

Aristotle says:

> "It is strange […] to attribute generation to eternal things, or rather this is one of the things that are impossible" (*Met*. 1091a22).

It is difficult not to ask ourselves, together with Aristotle, if the generation of numbers that Plato was contemplating – with the ontological terms he used as a springboard – was regarded as a chronological generation.[343] Or, bouncing back to the ontological foundation it was built upon, whether this generation was defined rather in terms of ontological priority and not succession in time. Does Plato attribute generation to eternal things? The generation process could be conceptualized closer to Plato's model, once we form ourselves a clearer idea whether the subjects of the generation are arithmetical numbers or form numbers. The uncertainty of the nature of generation resides not in the incomplete transmission of the concepts Plato used here, but "Plato may have been deliberately obscure on many points in his generation";[344] Plato's obscurity, I think, is not deliberate in order to mystify the terms he was working with, but it is determined rather by conceptual ambiguities that Plato was exploring. For example, Plato plays with different uses of "is", counts ontological entities, even if this is unusual,

[343] In the argument for the generation of numbers our epistemic ability as well is taken by Blyth into consideration, "the existence of numbers is proven from our capacity to count". Blyth, Dougal, "*Platonic Number* in the *Parmenides* and Metaphysics XIII", 23. But the texts provide little support to move the discussion towards proving generation by way of our epistemic capacities.

[344] John Niemeyer Findlay, *Plato: The Written and Unwritten Doctrines* (Routledge, 2012), 70.

deduces the series of numbers from evenness and oddness, etc. He explores and takes surprising paths that we usually avoid, considering them as aberrant, but the concepts themselves and their relation offer the ground for such a detour.

From the sequence of the argument one could advance the idea that there are grounds to consider a *chronological* progression between *numbers*, in the sense that multitude, and hierarchical multitude, involves a progression whose elements are ordered by their ontological position and/or their consequential position. As I emphasized in the previous section, the argument for the generation of numbers resonates with the Pythagorean understanding of the generation of numbers in some respects. Guthrie points out that the Pythagoreans didn't perform a sharp distinction between "logical and chronological priority."[345] Similar to Ross, Annas attributes to Aristotle's literalism the possibility that the Academy may have not distinguished "between a historical account and a logical analysis"[346]. Annas remarks that one should not think of an actual generation of numbers (in time), but of the existence of a number series, an existence proof and a classification of numbers.[347] Robert Turnbull ponders that, though *gignomai* is similar with the term used by Plato elsewhere, with the meaning of 'coming to be', involving a temporal meaning, "here

[345] W. K. C. Guthrie, *A History of Greek Philosophy: Volume 1, The Earlier Presocratics and the Pythagoreans* (Cambridge University Press, 1978).

[346] Annas, 211. According to Allen, in this case "numbers are simple essences incapable of analysis into ontologically prior and posterior elements." Plato's aim is not a *ratio essendi*, but a *ratio cognoscendi*. Allen, *Plato's* Parmenides, rev. ed., 266.

[347] Ibidem 265.

[in the *Parmenides*] is a standard one of Greek mathematicians for the generation of various mathematical series."[348]

Yet, Aristotle (*Met*. 1091a 12-29) took number generation as a process in time, even if the Academy had not endorsed such an interpretation.[349] There are scholars who recommend such a view, namely that "the generation of numbers was regarded by the early Pythagoreans as an actual physical operation occurring in space and time, and the basic cosmogonical process was identified with the generation of numbers from an initial unit, the Monad."[350] More recently, Schofield follows the same line of thought, and takes Aristotle's words *ad literam*, as a reliable testimony on number generation.[351] Aristotle's report on Philolaus is for Schofield accurate, which means that the Pythagoreans, and, particularly Philolaus, understood numbers as plainly part of the cosmos, and envisaged thus numbers and cosmology as closely related. Schofield argues in this regard against Huffman's interpretation of Philolaus, an interpretation which contradicts Aristotle in the idea that the Pythagoreans "identified creation of the material world with the generation of

[348]Turnbull, *The Parmenides and Plato's Late Philosophy*, 73.

[349] Criticizing Pythagoreans, Aristotle (*Met*. N. 1091a22-23) thinks that it is strange to attribute generation to eternal things. However, for Pythagoreans some numbers are generated, while others are not. For Aristotle, the claim that Plato had the numbers generated (*Met*. A. 987b22-35) doesn't seem to contradict a previous statement (*Met*. A, 987b16) that for Plato "the objects of mathematics are eternal and unchangeable". There are here two possibilities of interpretation that can be formulated as: a) either Aristotle did not realize the inconsistency, b) either Aristotle did not see the generation of numbers as contradictory to that of the eternity of numbers, the generation process being conceived as not being in time – therefore a technical description (but this would contradict his reading of the *Timaeus* – as a generation in time).

[350] Gerald James Whitrow, *Time in History* (Oxford: Oxford University Press, 1988), 40.

[351] Schofield, M. "Pythagoreanism: Emerging from the Presocratic Fog (Metaphysics A 5)", in *Aristotle Metaphysics Alpha*, Symposium Aristotelicum, ed. C. Steel (Oxford: 2012), 141—66.

numbers"[352]. The principles of numbers are the principles of reality, and the principle that the macrocosmos is essentially numerical: any number theory would thus entail a specific cosmological conception. Aristotle's testimony seems to follow this line of interpretation in *Metaphysics*, 987a 16, at least,[353] and for Philolaus,[354] the generation of numbers is clearly part of the cosmological process.[355] It is altogether an idea that demonstrates its Pythagorean origin, since for the Pythagoreans numbers are drawn from physics and are embodied in it: "it derives from their cosmology… *is* cosmology".[356]

For Schofield as well, the "apparently fantastic theory"[357] of Aristotle (985b23-986a21), that the generation of numbers is essentially a process in time, denotes the idea that number generation may be any kind of generation and generation in its primary sense: "the generation of the number series simply *is* the generation of the ordered system of 'the whole heaven.'"[358]

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[352] Huffman, *Philolaus of Croton*, 213.

[353] Huffman is skeptical on Aristotle's testimony: "There is no good support for Aristotle's assertion that the Pythagoreans identified the creation of the material world with the generation of numbers and thought of the first step in the generation of the cosmos as identical with the generation of the arithmetical unit." Huffman, *Philolaus of Croton*, 211. For a critique of Huffman see Schofield, M. "Pythagoreanism: Emerging from the Presocratic Fog (Metaphysics A 5)".

[354] Before Plato, Philolaus seems to be the only Pythagorean known to have composed a written cosmology. Conf. Kahn, *Plato and the Post-Socratic Dialogue*, 150. And Huffman, *Philolaus of Croton*.

[355] Huffman points that "the Pythagoreans identified the creation of the world with the generation of numbers, and thought of the first step in the generation of the cosmos [the central fire in Philolaus Fr.7] as identical with the generation of the arithmetical unit". Huffman, *Philolaus of Croton*, 211.

[356] Philip, J.A. *Pythagoras and Early Pythagoreanism*. Canada: University of Toronto Press, 1966, 76.

[357] Schofield, M. "Pythagoreanism: Emerging from the Presocratic Fog", 148.

[358] Schofield, Ibidem. 154-155.

For both Plato and Philolaus, who conceived one as "principle of all things"[359] (fr.8)[360], numbers have at their centre *one* (more exactly, for Plato, the *one that is*), an entity from which every number takes its *identity as unit*. It is again a Philolaic idea when Plato pictures numbers as originating in *one*; he adds, nonetheless, as conditions for the existence of numbers, the *being* of the *one*, and *difference*. For Plato, the *one* as encountered in the principial proposition of *one is*, is not a number, but an ontological entity, from which, together with other two ontological entities (being and difference), numbers are derived.

The *Parmenides* 153b-c may be interpreted as proof for Aristotle's understanding of the generation of numbers as a process in time. The terms in which Plato speaks here about number one bring apparent chronological implications - as the number one is older than the numbers which follow after it, which are younger:[361]

> "…of all the things that have number the one has come to be first. And the others, too, all have number, if in fact they are others and not another… But that which has come to be first, I take it, has come to be earlier, and the others later; and things that have come to be later are younger than what has come to be earlier. Thus the others would be younger than the one, and the one older than they."

[359] Tr. Huffman Carl A. Huffman, *Philolaus of Croton: Pythagorean and Presocratic: A Commentary on the Fragments and Testimonia with Interpretive Essays* (Cambridge University Press, 1993), 345,346. Or the "cause of all things" in the translation of Dillon and O'Meara, cf. Syrianus, *On Aristotle Metaphysics 13-14* (166,1) *Syrianus: On Aristotle Metaphysics 13-14* (A&C Black, 2014), 145. I am tempted to favor "cause" as a more appropriate translation since it makes more visible the accent on something that spurs generation.

[360] The fragment may be spurious (see Carl A. Huffman, *Philolaus of Croton*, 346.), but the idea cannot be eliminated from Philolaus' philosophy.

[361] The fact that one is understood here as a number, against Greek mathematics, shows that Plato overdraw *one* as a ontological principle.

From the quotation above, one may think that there are grounded reasons to take the process of generation of numbers as a process which is thought of as occurring primarily in time. But the understanding of number as something thought about as being in time is unusual if we picture it in the frame of the theory of form-numbers (where numbers are pure *abstracta*), and can hardly be accommodated even with conceptions of number that do not rely on the theory of forms and are more of an intuitive type of understanding of numbers (defended also by some of the modern philosophers) "entities discussed in mathematics [both geometrical figures and numbers] can properly be said to have a timeless existence".[362]

I would suppose that qualifications such as 'earlier' or 'later' which are attached to numbers of the series do not necessarily refer to their chronological organization (which could be taken metaphorically here), but to the order of their insertion in the numeric axis. Even if here the constituents of the number series seem to be arranging themselves in the numerical axis in a progression that implies also chronological progression, I wonder whether the argument for the generation of numbers must be understood as bringing also into discussion the chronological factor.

As I see it, the notion of generation itself is misleading, and in order to properly understand it, one must consider that Plato reflects upon numbers similarly

[362] William Kneale, "Time and Eternity in Theology", *Proceedings of the Aristotelian Society*, (1961), 98. For a critique of Kneale and of the timeless supposition see Quentin Smith, *Language and Time* (Oxford University Press, 2002), 208.

to a philosopher of mathematics who tries to formalize the number series. It may be that the attributes used to qualify numbers in *Parmenides* 153b-c – *older* and *younger* – hint rather at an ontological priority, than a necessary chronological succession. Moreover, to my understanding, there is a *hierarchical priority* of entities such as *twoness* and *threeness* on the one hand, and the *multiplicity* of numbers on the other hand. This logical priority is the ordering principle in the axiomatization of number series. One can speak about axiomatization in the sense that Plato identifies a limited number of primitive terms. These primitive terms are: *even*, *odd* and the *principle of multiplication*. From these primitive terms numbers are derived in a deductive way. According to this axiomatization any number should be explained though these primitive terms. It is a partial axiomatization if the primes remain out, but several solutions for the prime numbers can be given (cf. section 5.4: The prime numbers and the multiplication process). Thus with this axiomatization process of the numbers Plato is looking towards a foundation of mathematics.[363] Calling this argument an argument for the generation of numbers might mistakenly suggest to us that what we are looking at is a process in order to reconstruct the entire number series. Instead, what Plato does is to define what for him could be the *principia mathematica* – odd and even.

As I emphasized in Chapter Five, I think that what the argument for the generation of numbers provides is both an ontological and a mathematical

[363] For W. Tait, Plato is looking for a foundation of science by dialectic which is a deductive foundation of science. W. Tait, "Noesis: Plato on exact science". Even if the passages which are discussed by Tait are mainly the *Phaedo* (95a6–97b) and the *Republic* (509d-511d), the idea that Plato was interested in the foundation of sciences can be extrapolated to the argument for the generation of numbers.

understanding of numbers. The generation process has two coordinates: it delivers the principles and the ontological building blocks of numbers, and, at the same time, it draws a simple mathematical algorithm for identifying numbers. It is not only mere serialization, but a formalization of numbers - an attempt to understand the principles of numbers. I agree in this regard with Moravcsik, that the emphasis in the argument is not so much on the idea of a progression,[364] and we are dealing only with an analysis of the basic blocks of numbers[365]. The argument itself does not stipulate a chronological extension (only later does the second hypothesis develop in this regard), but mainly a conceptual one: from ontological concepts (one, being, and difference) through intermediate concepts between ontology and mathematics (even and odd) to numbers themselves.

At some key places, Plato speaks about the ultimate building blocks of numbers, of physical bodies, and of the soul. In the *Timaeus* the world-soul was created "chronologically" before the world itself; similarly the 'soul' of numbers (even and odd) are created before numbers themselves from the *being*, *difference* and *one* (which would play the role of *sameness*). Plato seems to think that not only the 'temporal' scale of the number series can be reduced to basic elements, but also the constitutive elements of the constitutive bodies of the cosmos. In the *Timaeus*, there is

[364] "Plato's task is presumably not to generate any series, but to ask what the basic concepts are whose interrelations underlie truths involving positive integers." Julius M. Moravcsik 'Forms and dialectic in the second half of the *Parmenides*" in Malcolm Schofield and Martha Craven Nussbaum, *Language and Logos: Studies in Ancient Greek Philosophy Presented to G. E. L. Owen* (Cambridge University Press, 2006), 135-154. 144.

[365] 'We can analyze any mathematical truths into a combination of odd and even numbers, multiplication, addition and their negatives, and the notions of unit, couple, and trio" Moravcsik, *Ibidem*.

a geometrical reduction of the physical bodies to right triangles which stand as basic 'atomic' entities. Setting face to face the regressive scheme in the *Timaeus* with the generation in the *Parmenides*, one can trace again a common effort of reducing physical dimension and arithmetic to a basic formula that aims to be pan-explanatory in its scope. Perhaps Plato's generation of numbers is not yet a cosmogonical account proper; but it is a principial discussion that provides the grounds for eventual cosmogonical accounts.

Conclusion

The dialogue *Parmenides* coinstitutes a defining moment in Plato's philosophy, and the ontological and mathematical arguments of the second hypothesis of the second part of the *Parmenides* make up the pivotal fragment of this significant moment. From the perspective offered by the deductions of the ontological argument, and from the spectacular use of the logical-ontological concepts in what I have argued to be an apparent mathematical argument, the reading of the ontological and mathematical arguments that I propose contributes to the reassessment of the role that the *Parmenides* plays within the group of the so-called late dialogues. The second hypothesis is the beginning of an ontological turn in Plato's philosophy, from a theory of forms (as it is to be found in dialogues of the *Phaedo-Republic* group) towards more logical-ontological concepts, such as *one*, *being*, and *difference*. The main logical concepts used in the ontological and mathematical argument – *one*, *being* and *difference* – will be part of the complex development of the greatest kinds in later dialogues.

This dissertation has advanced and substantiated the idea that one must be cautious when claiming that the second part of the *Parmenides* is "an exercise in dialectic rather than an exposition of doctrine."[366] As I argued (especially in Chapters Three and Five), there is more to be found in the second part of the *Parmenides*, and perhaps it may prove to be too disengaging to take these arguments

[366] William David Ross, *Plato's Theory of Ideas* (Clarendon Press, 1966), 187.

as empty dialectical exercises, void of any doctrinal and philosophical commitment. Formally, this involves indeed the display of an exercise (a 'mental gymnastics', as the dialogue insists), one of the most technical exercises of Plato's dialogues, but it turns out to be an exercise in which themes that would become recurrent in later dialogues are here used in decisive, albeit incipient, form.

For Plato there is a natural relation between ontology and numbers, a relation which for some scholars is expressed in the argument for the generation of numbers in terms of illicit inferences. Even if, judging from the logical coherence of the arguments, there might be an illicit inference, such as, for example, the counting of ontological kinds in plane arithmetic operations, towards number generation, I have showed that this does not rule out the possibility that the outcome of the inference be used by Plato as a building block for his later philosophy. One resulting assertion in my research is that Plato bases his arguments on previous considerations and builds upon these arguments and their subsequent conclusions in other epistemic and ontological discussions. As I showed, as important as it may be to point out possible paralogisms, it is equally important to underline the philosophical premises and presuppositions, and the arguments' intentions.

The *Parmenides'* second hypothesis, longer than the first hypothesis and actually the longest of all the hypotheses, is the opening set (especially through the ontological and mathematical arguments) of a long line of attempts to refute Parmenidean ontology. A main premise of the ontological argument is that Plato assumed the *existence* of *one* as the existence of its differentiations: "if one is" is possible through *difference* and as *difference*. Arguing for the division of *one* into *one*

and *being* (142b1-143a), and thus advancing the assumption that *one* partakes in *being*, Plato lays, in my understanding, the ground for the argument against Parmenides. Even if Plato does not openly assume his criticism against the philosopher Parmenides in the *ontological argument*, this critique is implicit all throughout. The fact that the dialogue is called the *Parmenides* is not only related with the critique of the theory of the forms, displayed by the character of the dialogue, Parmenides the philosopher, it is nevertheless related with the second part of the dialogue as well, where one can notice a deliberate use of arguments and claims about *being* in their direct relation to Parmenidean ontology. From the perspective of the ontological argument, the *Sophist* remains as the subsequent discussion contrasting Parmenidean ontology, where the deduction of the ontological arguments are extensively developed as proposals for conceiving *being* as a separate and independent, intelligible, substance, which, together with *difference*, would form an alternative ontological response to the philosopher Parmenides.

For Plato the result of the first hypothesis is a challenge, since one of the conclusions that the argument reaches is that one is not even one. Since the principle of self-predication plays an important role in conceptualizing the intelligibles in Plato's philosophy, the altered restatement of the principle of self-predication moves fundamental concepts to a no man's land, where any epistemological attempt is debunked. Plato rejects such a result, in order to preserve a principle of self-predication, by tacitly moving from a complete meaning of 'is', in the first hypothesis, to an incomplete 'is', in the second hypothesis. The purpose of this shift is to *separate* being from whatever is *one*, and to have *being* as an independent entity.

Through its participation in *being*, *one* can be predicated of *one*. In my argumentation I followed the thesis that Plato does not make the difference between a complete use and an incomplete use of 'to be', but he sometimes seems to prefer one of the two understandings to a higher degree. Plato apparently switches from a complete use of "to be" in the first hypothesis, to an incomplete use of "to be" in the second hypothesis, in the following manner: 'one is' means that 'one' is '*is*', and here 'is' appears to be used by Plato as predicative, and thus assumed this as a predication given to one ("'if one is', can it be, but not partake of being?", 142b6-7). The adjustment of meaning is not less evident in the opening question of the first two hypotheses (which, at least in what the written Greek text demonstrates, is the same: "if one is") but in the succeeding progressions from this opening hypothesis. The premises of the first and the second hypotheses are only formally identical. The second hypothesis deduces from the same premise "if one is" that the 'is' of one is a determination of one, and one partakes in this determination. Besides the interpretations we might have for the manner in which "to be" is used as a key to the understanding of the *one-being* pair, Plato introduces an unanticipated *ontological difference* that delineates *being* from *one*. Subsequently *oneness* and *being* are constituent parts of the *one* (142d6). As a result, *one* is *forced* through an infinite regression to be *multitude* (143a2), and, further on, to constitute numbers.

After dismantling the Parmenidean idea of a monolithic concept of *one*, by affirming that one is something more than *one*, Plato continues by exemplifying counting, and thus reaching the concept of number. The surprising counting of the ontological entities creates the argumentative possibility of structuring metaphysical

entities according to numbers, and up to the number three. After numbers two and three, the reader finds himself or herself plunged into what seem to be strictly arithmetical operations, and the concepts that opened the argument seem to vanish and leave the floor to numerical values alone: numbers are the products of multiplications between two and three.

At first glance, the argument's progression into a mathematical discourse comes too abruptly, but I have argued that one should bear in mind that there is a constant interplay between ontology and mathematics in Plato's philosophy. The imperceptible transition from ontology to mathematics all throughout the arguments might hint as the fact that, for Plato, there was an overlap between these two seemingly distinct territories. As I have argued, Plato does not develop a strictly mathematical procedure to generate numbers, but rather a metaphysical/logical one. Confusion arises when one is tricked into following mathematical operations, in their purely mathematical functions, for their application to ontological concepts.

Bearing in mind Plato's possible conception of mathematical objects (numbers are forms or intermediaries), one would expect that the argument for the generation of numbers has no relevance for building a theory of numerosity, and that the argument is more of a simple regress argument. The deduction that the reader should make is that Plato did not assume the ontological suppositions on which the argument is based, and thus the argument is only an artificial construction required by the rhetoric of the second part of the dialogue. But, as I have argued especially in Chapter Five, the argument, even in its mathematical assumptions, has more connections to, and implications for Plato's dialogues, than one would expect. The

conjectures around the derivation of numbers from very precise ontological kinds have various ramifications in Plato's philosophy. Ontological categories – *one*, *being*, and *difference* – perform a double role. First of all they surprisingly undergo mathematical processes and get counted. Secondly, by what seems to be their mere counting, they provide the principles of multiplicity and, adjacently, of numerosity. These ontological categories undergo the mathematical actions that they themselves ground and concentrate an entire science about the ontological foundation of mathematics.

I have also showed that one of the issues brought into discussion in what concerns Philolaus' generation of numbers is whether it is referred to as an actual process in time, and one can also think in a similar frame about Plato's generation of numbers, in the light of Aristotle's testimony as well. In my argumentation, Plato's generation of numbers is not directly thought of in terms of a chronological generation. Like Philolaus, Plato is interested in understanding the plurality of numbers by odd and even, and thus by using what seem to be the basic principles of numbers. The resemblance of this argument with Philolaus' fragments 6 and 7 could explain why Plato's argument leaves the impression that the reader should be familiar with such processes of generation. As I have showed, the process for the generation of numbers found in the *Parmenides* follows principally the argument of Philolaus, and presents a modified version of the Pythagorean generation of numbers which is not based on the first four numbers (1, 2, 3, 4 of the Pythagorean tetractys), but which is grounded on the first three numbers (in the order 2, 1, 3). What Plato's argument does is rather to propose an understanding of numbers not

in their accumulative progression, but through their constitutive principles, by reaching the first odd and the first even.

The beginning of the second hypothesis is a piece of Platonic philosophy which reassesses Eleatic monism proposing an ontological trio (*one*, *being*, and *difference*); and succeeds in demonstrating how this ontological trio underlies the possibility for the existence of many and, contiguously, of numbers. My claim for the legitimacy of the elaboration of the second hypothesis is even more reasonable if one considers that Plato's account here is part of a larger project which is developed in the *Sophist*, and partially in the *Timaeus*. In the *Timaeus*, the role of the greatest kinds is to construct the soul in such a way as to bridge the *intelligible realm* with the *material world*. The ontological argument could be a reaction not only to Parmenidean ontology, but also a reaction to the first part of the *Parmenides*, where Platonic ontology shows some of its limitations. There is an unexpected appropriation of the *Theaetetus*, and Chapter Six emphasizes that the logical-ontological concepts of the second hypothesis are called in the *Theaetetus* (185c9-d4) the 'commons', and they are understood as part of the cognitive domains of the soul (*being*, *difference*, *even*, *odd* etc.). The *Theaetetus* is especially important because the dialogue is dedicated expressly to the problem of knowledge, and could be read as a 'distant' answer, at least in its epistemic aspects, to a central problem that was illustrated in the first part of the *Parmenides*: 'the separate domains argument' (133a-134e). The key concepts for what empowers the linking of the two separate realms are announced by the ontological and mathematical argument. It is not a coincidence that a mathematician such as Theaetetus assumes that there is an intimate relation

between mathematical features and essential kinds of the world, and insists on pointing that out. What emerges is that the prerequisites for the generation of numbers are to be found as the prerequisites for the possibility of perception as well. In my view, an essential part of the answer to the 'greatest difficulty' of the *Parmenides* (133a-134e) should give a proper account of how it is possible to know both the sensible realm and the intelligible realm. One needs an epistemic bridge which epistemologically links the sensible things with the absolute realm and this bridge is given by the cognitive faculties of the soul in the *Theaetetus* (185c9-d4), and by the constitution of the soul in the *Timaeus* (135a).

To sum up, one of the contributions of the thesis is to expand the range of interpretative approaches to Plato's *Parmenides*, raising some not so obvious questions (such as, for example, why 3 is a derivation of 2+1 and not of 1+1+1) around the argument of the generation of numbers, showing that one may consider Plato's ontology of numbers in a different frame than that given by the current discussion - that Plato either had numbers as forms or as intermediates (which are conceived as ungenerated in both cases). The thesis advances that idea that conceiving numbers as form numbers would leave out the possibility of addition and multiplication of numbers, since these processes cannot be applied to form numbers, which are not composed of units.

The *Parmenides* offers probably one of the least allegorical cosmogonical accounts of Plato and further research should focus on how this logical narrative would fit with the other more vivid cosmogonical accounts, such as that in Plato's *Timaeus*. Further research could explore whether and to what extent the *Parmenides*,

with the complexity of the generative process, marks a turning point in Plato's philosophy and how later dialogues revised Plato's theory of forms into a *meta-theory of forms* and a *meta-theory of numbers*, in which more primitive logical forms are at the core of a new ontology, and new philosophical possibilities are explored: in the *Sophist* a new ontology is formulated, in the *Parmenides* a new theory for the generation of numbers, and in the *Timaeus* a new theory of the constitution of the soul. The reassessment of Plato's philosophy into a meta-theory, in which 'generation' could be central, as the reading key, would be coherent with the Revisionist theory, and should not contradict the Unitarian theory either (cf. section 7.2.), since, as I think, in looking for and constructing an axiomatic system for the theory of forms, numbers (and not only these), Plato goes beyond his early views on the forms (as it is presented and developed in the *Phaedo*, the *Republic*, the first part of the *Parmenides*), without necessarily refuting them. The solution that I have advanced so far is not merely one of a compromise between Revisionist and Unitarian theory, but an attempt to hint towards the reconstruction of a possible ontology of the late dialogues in which problematic, but fundamental passages – such as the generation of numbers or the generation of the soul – could be harmonized with both the theory of forms and that of the greatest kinds in Plato's late dialogues.

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