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THREE ESSAYS IN FINANCIAL ECONOMICS

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DISCLOSURE OF CO-AUTHORS CONTRIBUTION

Title of the work: Individual Investors Exposed (Chapter 3)
Co-author: Kata Váradi

The nature of the cooperation and the roles of the individual co-authors and the approximate share of each co-author in the joint work are the following: The paper was developed in cooperation with Kata Váradi. My contribution was the development of the theoretical model, the analysis of the data and writing. Kata Váradi was responsible for obtaining the data.

10.14754/CEU.2017.09

Abstract

My thesis contributes to understanding how innovative financial assets affect the behavior of financial market participants. The first two chapters focus on why credit rating agencies failed to correctly assess the riskiness of innovative structured products, like those of collateralized debt obligations. The third chapter investigates how the introduction of retail structured products may lead to systematic patterns in aggregate retail investor behavior.

The subprime crisis began to unfold when markets realized that structured products designed to be safe are, in fact, toxic. Credit rating agencies prolonged this misperception by granting triple-A credit ratings to many of these assets. In an applied game theoretic setting, I derive the conditions in Chapter 1 under which credit rating agencies operating in a duopoly, similarly to S&P and Moody's, are likely to provide overly optimistic assessments of risk. The main innovation of Chapter 1 is that I allow agencies to learn about each other's assessments during the rating process. Importantly, learning enables agencies to cater credit ratings, that is, offer a higher rating to a given issuer based on the other agency's more favorable assessment. Catering is harmful for social welfare as it reduces the informativeness of ratings. I show that the negative welfare implications of catering are most severe when the skewness of the rated assets' payoff is large, similarly to the payoffs of collateralized debt obligations. Chapter 2 builds on the framework of Chapter 1 and investigates how a rating agency calibrates its information technology as a response to changes in its business environment and also whether it has sufficient incentives to invest into information acquisition. I show that the agency's business environment has a strong effect on calibration and, in turn, on rating standards. Additionally, while the agency's ability to calibrate may have the benefit of alleviating conflicts of interests in the industry, when these conflicts are extreme, the agency chooses to ignore additional information about rated assets' quality. This helps to reconcile the empirical evidence documented in the literature on structured ratings, according to which agencies ignored valuable information that was available to them at the time they issued their ratings.

The third chapter is joint work with Kata Váradi and it focuses on retail structured products that are derivatives designed by banks for individual investors. Retail structured products became increasingly popular in the last decade as they enabled individual investors to trade with complex assets, that were previously not

available to them. We analyze both empirically and theoretically a subset of these products, called knock-out warrants. Individuals can trade with knock-out warrants through stock exchanges and they allow individuals to place leveraged speculative bets in the market of their chosen underlying, like a stock index or a commodity. We show theoretically that in these markets individuals, on average, are likely to bet on price reversals, even if at the individual level investors randomly choose the direction of their respective bet. Using proprietary data from a bank, we provide supportive evidence for our prediction. We speculate that the setup of these markets may be beneficial for the banks if they need to hedge their own exposure to the underlying asset.

The results of my thesis suggest that the presence of innovative financial assets often affect the behavior of market participants. In particular, assets with highly skewed payoffs may change market outcomes in unforeseen ways. The skewed payoffs of collateralized debt obligations seem to have an adverse effect on rating agencies' incentives to exercise due diligence. On the other hand, the skewed payoffs of knock-out warrants results in unintentional but predictable aggregate behavior of individual investors.

Chapter 1: Credit Rating Catering

I analyze how competition between credit rating agencies affects market efficiency. As a main innovation, I introduce information flows between rating agencies that take place during the rating process. In the model, rating agencies cannot commit to truthfully reveal their information, but they have to pay a penalty whenever a project carrying their high rating defaults. The key insight is that competing agencies in a duopoly are tempted to cater ratings, that is, agencies selectively offer better ratings to issuers based on the more favorable assessment of the other agency. As a main result, I show that catering in a duopoly may lead to lower welfare than achieved with a monopolist agency even though agencies in a duopoly have more information. Two conditions are key to this result. First, agencies frequently need to disagree about fundamentals, which creates opportunities to cater ratings. Second, the rated asset's payoff needs to be sufficiently skewed to the left, which makes catering in a duopoly relatively cheap. These features seem to match the characteristics of complex structured products, which are difficult to precisely evaluate and have skewed payoffs by design.

Chapter 2: The Benefits of Loose Rating Standards

I study the information technology choice of a credit rating agency. The information technology classifies projects as good or bad and may commit two types of errors: it may classify a project as good that later defaults or it may classify

10.14754/CEU.2017.09

a project as bad that would otherwise succeed. After its private signal about the project's quality is realized, the rating agency cannot commit to truthfully reveal it. However, the agency faces penalties if a project with a high rating defaults. When the penalties are relatively high the agency cannot afford to misclassify bad projects. However, when the penalties are relatively low the agency does not want to misclassify good ones. In the latter case the agency's conflict of interests is alleviated as projects classified as bad will, in fact, be correct, which maximizes the expected penalty for misreporting their signals. When the agency cannot commit to truthfully reveal its signals, it will not want to invest in a more precise technology because the additional information is ignored. I connect the results to stylized facts on fluctuations of rating standards.

Chapter 3: Individual Investors Exposed (joint with Kata Váradi)

We show in a simple model that investors' aggregate position is influenced by the menu of available products. Our focus is on exchange traded call and put knock-out warrants, because they are popular among individual investors who seek directional bets. If investors allocate their funds randomly between calls and puts, then their aggregate position will depend on the relative leverage of the offered call and put warrants. By construction, the leverage of calls will be higher than the leverage of puts after recent declines in the underlying. Hence, investors will take a long position after declines in the underlying and a short position after increases in the underlying, on average. This behavior is equivalent to betting on price reversals. We present supporting empirical evidence for our predictions. Using a unique, proprietary data set obtained from a bank, we are able to compute the aggregate position of retail investors who hold knock-out warrants. We speculate that this might be beneficial for banks' liquidity management if banks act as market makers on the underlying asset's market.

10.14754/CEU.2017.09

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10.14754/CEU.2017.09

Contents

10.14754/CEU.2017.09

10.14754/CEU.2017.09

Tables

10.14754/CEU.2017.09

Illustrations

10.14754/CEU.2017.09

10.14754/CEU.2017.09

1

Credit Rating Catering

> ...a Chase investment banker complained that a transaction would receive a significantly lower rating than the same product was slated to receive from another rating agency: ”There’s going to be a three notch difference when we print the deal if it goes out as is. I’m already having agita about the investor calls I’m going to get.” Upon conferring with a colleague, the Moody’s manager informed the banker that Moody’s was able to make some changes after all: ”I spoke to Osmin earlier and confirmed that Jason is looking into some adjustments to his [Moody’s] methodology that should be a benefit to you folks.” Wall Street and the Financial Crisis: Anatomy of a Financial Collapse. U.S. Senate (2011)

1.1 Introduction

The rise and fall of mortgage-backed securities shook the credit ratings industry. Rating innovative structured products was a lucrative business prior to the subprime crisis: just within a decade it became the principal source of profit for the big rating agencies.[1] However, the collapse of housing and the widespread downgrades of structured products evaporated profits from rating such securities.[2] Furthermore, as the standard business practice is that issuers pay for ratings, agencies were accused of issuing biased ratings. Thus, the performance of credit rating agencies and the role of the ratings industry became a widely debated topic among academics, regulators and the general investing public, leading to U.S. Senate hearings and in-

[1] Structured ratings revenue of Moody’s tripled between 2000 and 2005, becoming the largest source of revenue. Also, a former S&P employee stated that ”revenues grew tenfold between 1995 and 2005 and rating volumes grew five or six fold” in the residential mortgage segment (Raiter, 2010).

[2] E.g. in 2015 less than 20% of Moody’s revenue originated from rating structured products.

10.14754/CEU.2017.09

vestigations by the Department of Justice.[3] The results of these inquiries led many to question agencies' integrity.

I analyze how competition between credit rating agencies affects market efficiency. As a main innovation, I introduce information flows between rating agencies that take place during the rating process. In the model, rating agencies cannot commit to truthfully reveal their information, but they have to pay a penalty whenever a project carrying their high rating defaults. The key insight is that competing agencies in a duopoly are tempted to cater ratings, that is, agencies selectively offer better ratings to issuers based on the more favorable assessment of the other agency. As a main result, I show that catering in a duopoly may lead to lower welfare than achieved with a monopolist agency even though agencies in a duopoly have more information. Two conditions are key to this result. First, agencies frequently need to disagree about fundamentals, which creates opportunities to cater ratings. Second, the rated asset's payoff needs to be sufficiently skewed to the left, which makes catering in a duopoly relatively cheap. These features seem to match the characteristics of complex structured products, which are difficult to precisely evaluate and have skewed payoffs by design.

In my model, rating agencies assign ratings to issuers seeking favorable ratings for their respective projects. The rating process consists of two stages. It begins with agencies assigning preliminary ratings to issuers based on their own information technology, that generates a noisy signal for each project. Importantly, agencies learn the preliminary ratings assigned by the other agency. Then, in the second stage of the rating process, agencies assign offered ratings to issuers, which may be better than the respective preliminary ratings initially assigned. At this point – consistently with industry standards – issuers can decide whether to pay a rating fee for the agencies to disclose their offered ratings. However, issuers are only willing to pay for high ratings, which gives an incentive for the agency to manipulate. Given the two stages of the rating process, agencies have two opportunities to manipulate ratings. First, they can manipulate preliminary ratings by misreporting their signals to issuers. This can be done by assigning better than justified preliminary ratings. I label this kind of manipulation *inflating.* Second, as agencies learn about issuers' preliminary ratings assigned by the other agency they can also selectively assign better offered ratings. I refer to this method of manipulation as rating *catering.* [4] While a monopolist agency is only able to inflate ratings, as it cannot condition manipulation on another agency's preliminary ratings, agencies in a duopoly may both inflate and cater ratings.

[3] In 2015 S&P reached a settlement agreement of $1.4 billion with the U.S. Justice Department, several states and a pension fund over dispute of inflating its subprime-mortgage ratings. Also, its executives admitted that business relationships affected modeling updates.

[4] I follow Griffin et al. (2013) as labeling selective manipulation of rating catering.

10.14754/CEU.2017.09

The first insight is that catering may be more tempting for agencies in a duopoly than inflating ratings for a monopolist. Agencies find it optimal to manipulate the ratings of those projects first that are less likely to default in order to minimize the expected penalty. Since catering conditions manipulation on a second signal that could suggest a high rating, catered projects are less likely to default than inflated ones. Hence, agencies in a duopoly find catering harder to resist.

In equilibrium inflating and catering ratings always reduce welfare. Therefore, from a social perspective, agencies should never enable the financing of projects that they believe to be bad, even if the other agency has a favorable assessment. Catering enables the financing of projects with contradicting signals (at best), which implies – together with the symmetric signal structure – that catering has the same negative effect on welfare as financing a randomly chosen project.

As the main contribution, I show that a duopoly may lead to lower welfare than a monopoly despite the additional information brought by the second agency. If (i) the penalty is not sufficiently high to deter manipulation, (ii) ex ante project payoffs are sufficiently skewed to the left – small gains occur with high probabilities and large losses occur with low probabilities – and (iii) there are frequent disagreements between agencies then a monopoly will lead to higher welfare. Frequent disagreements are a result of relatively low signal precision and uncorrelated signal errors, which creates opportunities to cater ratings. Skewed payoffs imply that the expected penalty for catering in a duopoly is lower than the monopolist's expected penalty for inflating. Together, these conditions make catering relatively attractive and harmful.

My model contributes to the discussion on why conflicts of interest may be especially pronounced for structured ratings.[5] The conditions that lead to the main result match stylized facts on structured products. First, structured products are complex, which makes their risk difficult to correctly assess, leading to frequent disagreements.[6] Second, the goal of issuers in the structured segment was to design assets that receive AAA ratings from agencies, which certifies that their credit risk is low.[7] This led to the design of AAA tranches that had skewed payoffs.[8] The opening quote above illustrates the mechanism of rating catering that took place in the structured segment.

Equilibrium characteristics of the duopoly are consistent with stylized facts documented by the empirical literature. First, issuers find it optimal to purchase ratings

[5] The widespread downgrades that took place in the structured segment during the subprime crisis was unprecedented (Griffin and Tang, 2012).

[6] Griffin et al. (2013) provide evidence of large disagreements between S&P's and Moody's model implied ratings. See the discussion in Section 1.4.3.

[7] Coval et al. (2009b) report that about 60 percent of all structured products were AAA-rated globally. To the contrary, in the corporate segment only 1 percent of the issues had AAA ratings.

[8] Coval et al. (2009a) labeled these assets "economic catastrophe bonds", since they were designed to only fail in the worst economic state.

from both agencies in equilibrium. This holds for the corporate segment, where both S&P and Moody's cover virtually the whole sector. Also, in the structured segment Griffin et al. (2013) report that about 85% of AAA rated CDO capital was rated by the two largest agencies. Investors are only willing to finance projects that disclose high ratings from both agencies, preventing issuers from *shopping ratings,* that is, only disclosing favorable ratings and hiding bad ones from investors. Second, the model predicts that as conflicts of interests arise, agencies in a duopoly start rating manipulation by catering ratings. This is in line with the evidence provided by Griffin et al. (2013), who analyze the structured segment and find that agencies cater for selected issuers by improving their model implied ratings when issuers obtained better ratings from the competing agency. Also, catered projects default with higher probability ex post in the model, which is consistent with catered ratings experiencing larger subsequent downgrades (Griffin and Tang, 2012). Third, as agencies increase manipulation with competition, rating standards may deteriorate. This is in line with the evidence presented by Becker and Milbourn (2011), who show that as the market share of Fitch increased between 1995 and 2006 in the corporate segment, ratings became less informative.

1.1.1 Related literature

To the best of my knowledge, this is the first paper to model information flows between credit rating agencies during the rating process. I show that as incentive problems arise, agencies cater ratings by selectively improving the ratings of issuers who managed to obtain better assessments from the other agency. As a main contribution I show that adding another agency with conditionally independent information may lead to lower efficiency.

A group of papers argue that competition might lead to rating shopping. The presence of naive investors and asset complexity (Skreta and Veldkamp, 2009) and the inability of investors to observe undisclosed contacts between issuers and agencies (Farhi et al., 2013; Faure-Grimaud et al., 2009; Sangiorgi and Spatt, 2016) allow issuers to shop ratings by only disclosing favorable ones to investors. Instead of shopping, I show how competition can lead to lower welfare because of the catering done by agencies.

Attracting business by manipulating ratings can be an equilibrium outcome. Mathis et al. (2009) show that agencies only reveal their information honestly as long as their income is sufficiently diversified. Opp et al. (2013) analyze how the regulatory advantages associated with high ratings leads a rating agency to inflate its ratings. Bolton et al. (2012) demonstrate that conflicts of interest in the ratings industry together with the trusting nature of institutional investors create incentives to manipulate ratings. Compared to this literature I emphasize that catering can be very tempting to agencies because the other agency's more optimistic assessment might actually be correct. However, catering is still socially undesirable.

The theoretical literature provides mixed predictions on the effects of competition on rating agencies' concerns for their reputation. Bouvard and Levy (2013) and Camanho et al. (2012) argue that competition between agencies may be welfare reducing due to the decreased value of reputation. On the other hand, competition may also have a disciplining effect on agencies, because of the threat of entry (Frenkel, 2015). However, if markets trust the incumbent agency, competent potential entrants may fail to enter in the first place (Jeon and Lovo, 2011). Bar-Isaac and Shapiro (2013) find that if investors do not punish an agency if another agency also gave a high rating for a bad asset then the cost of being incorrect is lower with multiple agencies. Compared to this literature I demonstrate that competition may lead to lower welfare because it enables agencies to cater ratings, which is a more sophisticated method of manipulation.

Ratings may serve multiple purposes. It has been pointed out by recent contributions that if ratings are widespread referred to in regulations, ratings may be used by investors for regulatory arbitrage (Opp et al., 2013).[9] Additionally, credit ratings could provide a coordination mechanism. Multiple equilibria may exist if issuers can choose the riskiness of their projects (Boot et al., 2006) or when to default (Manso, 2013) and ratings may help in equilibrium selection. Compared to these, here the only purpose of ratings is to convey information between issuers and investors in order to alleviate trade.

Taking a broader perspective, the model belongs to the literature analyzing certification intermediaries. The seminal model introduced by Lizzeri (1999) allows certifiers to choose and commit among general disclosure rules. However, Strausz (2005) shows that sufficient rents are needed to prevent the certifier from signing side-contracts with sellers. Compared to these studies I focus on agencies' incentive to free ride on each other's information in order to minimize the costs associated with rating manipulation.[10]

The remainder of the paper is organized as follows. The next section will present the benchmark model with a monopolist agency. Section 3 provides the equilibrium for a duopoly of agencies. Section 4 gives the main result on welfare ranking a monopoly and a duopoly. The final section concludes.

[9] Among others, Bongaerts et al. (2012) and Kisgen and Strahan (2010) provide empirical evidence that the regulatory role of ratings affects market outcomes.

[10] For a more extensive review of the recent theoretical literature see Jeon and Lovo (2013).

1.2 Benchmark model

Consider a game with three types of players: issuers, a rating agency and investors. All of them are assumed to be risk neutral. There is a unit mass of issuers indexed by j and each of them has a project with uncertain quality. Projects can be either of good type $(\theta_j = g)$ or of bad type $(\theta_j = b)$. Good projects never default, while bad projects always default. The net present value (NPV) of a good (bad) project is $V_g = R - 1$ $(V_b = -1)$, where projects return $R > 1$ in case of no default and each project requires 1 unit of capital. The share of issuers with good projects is denoted by π_g, implying that the average project has value of $\bar{V} = \pi_g V_g + (1-\pi_g)V_b = \pi_g R - 1$.

Assumption 1 (Average project has negative NPV)**.**

$$\bar{V} < 0 \iff \pi_g < 1/R \tag{1.1}$$

It is assumed that the average NPV of projects is negative, implying that without rating agencies the market would break down.[11]

Issuers do not know their projects' type and have an outside option of 0.[12] This implies that there is no ex ante informational asymmetries between issuers and investors.[13] Though issuers may learn about their project during the rating process, they cannot credibly convey what they learn to investors. Hence, learning during the rating process does not affect their outside option.[14]

The rating agency has access to a rating technology that produces signals about projects, $s_j \in \{a, b\}$ with properties

$$Pr(s_j = a|\theta_j = g) = Pr(s_j = b|\theta_j = b) = 1 - \alpha, \quad \alpha \in \left(0, \frac{1}{2}\right) \tag{1.2}$$

where $\alpha = 0$ implies a perfectly informative technology and $\alpha = 1/2$ means that the signal is uninformative. The signals are produced at zero costs.[15]

In modeling the contract between issuers and the agency I follow the industry standard, which is the issuer-pays business model. That is, the agency sets the rating fee, f, which only has to be paid by issuers if they choose to disclose their respective rating.[16] I also assume that investors cannot observe rating fees[17] which

[11] This assumption simplifies the exposition as it rules out equilibria in which all projects would be financed.

[12] Having 0 outside option greatly simplifies derivations. However, it reduces the bargaining power of those issuers who learn that their project is good.

[13] I discuss this assumption in more detail below.

[14] This will no longer be the case with two rating agencies, where purchasing the other agency's rating is, in principal, an outside option for issuers. Furthermore, this option's value may be affected by what issuers learn during the rating process.

[15] All results go through if there are some fixed costs associated with setting up the rating technology. However, introducing variable costs may undermine agencies' incentives to generate information.

[16] Bizzotto (2016) analyzes the fee structure decision of a rating agency and finds that in equilibrium an agency only asks for a fee if the rated project is sold and does not ask for an upfront fee.

[17] While in the corporate segment the fee schedules are published by the major rating agencies, in the

10.14754/CEU.2017.09

are contracted at the beginning of the rating process.[18] Importantly, the rating fee is a flat fee, implying that it may not be contingent on signals or ratings.[19]

After the rating fee is set, the rating agency transforms its signals into ratings, $r_j \in \{A,B\}$. The agency may *inflate* ratings with inflating probability ε by offering an A rating to a project with a b signal, but always conveys a good signal honestly, by offering an A rating to issuers with a signals:[20]

$$\varepsilon = Pr(r_j = A|s_j = b), \ \ Pr(r_j = A|s_j = a) = 1 \tag{1.3}$$

Given the offered rating and the rating fee, issuers choose whether they want to pay the rating fee in exchange for the agency disclosing it to investors.

Investors observe ratings and bid for the projects, given its rating(s). Investors only condition their bids on project ratings, implying that they cannot make inference from the distribution of ratings.[21]

Giving A ratings to bad projects is costly for the agency. Whenever a sold project with an A rating defaults, all players learn that the respective project was bad (as only bad projects default) and the respective agency made a potentially intentional error. Hence, the respective agency endures a *penalty* of c. I assume that the penalty is a direct monetary cost imposed by a regulator.[22]

Figure 1.1 illustrates the timeline of the game when there is a monopolist agency. The timeline makes it clear that issuers enter the rating process with knowing the

structured segment they are not. As it is noted in Langohr and Langohr (2010) "There are no public fee schedules for the rating of structured finance instruments, only vague guidelines. The time and complexity involved in the rating of these instruments varies much more widely." (Langohr and Langohr, 2010, p. 185). Assuming that investors cannot observe fees simplifies the derivations as it prevents investors from conditioning their beliefs on rating fees. I.e. when investors form beliefs about the value of a project, they can only condition their beliefs on ratings but not on rating fees. This is in line with the general modeling approach of this paper, emphasizing that from the investors' point of view the rating process is a black box.

[18] This latter assumption is not binding in the current setup. Since there are a unit mass of issuers, agencies do not face uncertainties about the distribution of realized signals. Therefore, setting fees before or after signal realizations makes no difference.

[19] In the current setup this is not restrictive as agencies and issuers would never find it optimal to renegotiate the rating fee during the rating process. In Appendix D I discuss the positive average NPV case, where this assumption may become binding.

[20] In this respect, the setup is similar to Bolton et al. (2012) and Opp et al. (2013). Requiring the agency to give high ratings to issuers with high signals is not restrictive in the setup, since it does not influence the agency's bargaining power. In settings where the agency may issue unsolicited credit ratings, like in Fulghieri et al. (2014), threatening to issue bad ratings may be optimal for the agency. However, when average NPV is positive, this assumption may become binding for a duopoly of agencies. See Appendix D for the discussion of the positive average NPV case.

[21] In the current setup there is no aggregate uncertainty, as all players know the prior share of good projects. If investors could learn from the distribution of ratings then they could perfectly infer the amount of inflating, which is unrealistic. Learning from the distribution would be a reasonable assumption if combined with aggregate uncertainty. However, these would unnecessarily complicate the model.

[22] See footnote [3]. The penalty may also be thought of as a (reduced form) reputation loss for the agency, though that would have nontrivial implications for the welfare measure and the welfare result. See Section 1.4.2 for further discussion.

10.14754/CEU.2017.09

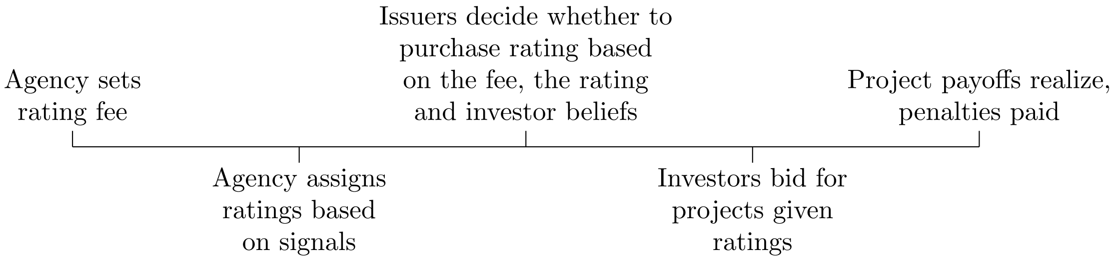

Figure 1.1 Timeline of the game with a monopolist agency.

rating fee and only make the purchasing decision once they have also learned the rating they are offered.[23] For a definition of an equilibrium see Section 1.3.

1.2.1 Solution with commitment

In order to demonstrate the basic commitment problem of the agency, I first provide the solution of the game assuming that investors can observe the amount of rating inflating. This way the agency can commit to any inflating level it prefers.

Issuers are only willing to pay the rating fee for an A rating, because purchasing a B rating only reveals the worst signal at a cost. Given issuers' strategy, one can formulate the problem of the rating agency as

$$\max_{\varepsilon,f} \Pi = \max_{\varepsilon,f} (\mu_a + \varepsilon \mu_b)[f - c(1 - p_A(\varepsilon))], \quad \text{s.t. } f \leq p_A(\varepsilon)R - 1, \tag{1.4}$$

where μ_a denotes the total mass of projects that receive a signals and, similarly, μ_b denotes the total mass of projects that receive b signals.[24] The posterior success probability of A-rated projects, when the agency is inflating with probability ε is denoted by $p_A(\varepsilon)$. The agency's choice variables are the rating fee (f) and the amount of inflating (ε).

Assumption 2.

$$p_A(0)R - 1 > (1 - p_A(0))c. \tag{1.5}$$

[23] The timeline also helps to illustrate why it does not matter within the current setup whether issuers know their own project's type. Since there are no upfront fees or costs associated with asking for an offered rating, issuers would still always ask all agencies for offered ratings, regardless whether they have a good or a bad project (i.e. issuers with bad projects can freely ask for an offered rating even if they correctly anticipate that they will be offered a bad rating). Also, since issuers' outside option does not depend on their projects' true type, knowing the true type does not affect their outside option, which is normalized to zero in the paper. Finally, issuers' payoffs are not affected by their project's realized return (i.e. they do not hold claims on the projects' payoff after selling them as they do not have anything to pledge), that is, they only care about selling their respective project.

[24] Observe, that there is no uncertainty in the sizes of μ_a and μ_b, which implies that the agency will choose the same disclosure rule before and after the realization of the signals.

Assumption 2 guarantees that the agency wants to participate in providing ratings. It says that the maximum equilibrium rating fee with zero inflating ($p_A(0)R - 1$) must exceed the expected penalty of A-rated projects with zero inflating. This is likely to hold when the rating technology is sufficiently precise ($p_A(0)$ is close to 1) and the penalty (c) is sufficiently low. In principal, with a more precise rating technology the agency can afford to operate with higher penalties, as its signal will commit fewer errors.

Consider the agency's constraint. Issuers are only willing to pay the rating fee if they can recover it by selling the project to investors. In turn, investors' valuation depends on the amount of inflating. Hence, by lowering the amount of inflating, the agency increases the value of an A rating which makes room for setting higher fees.

Now consider the objective. The first term in parentheses captures the total mass of issuers who are offered an A rating. Clearly, the total mass increases with inflating. The term in brackets is the difference between the rating fee and the expected penalty the agency has to pay after each A rating it discloses. This term is decreasing in inflating because the average default probability, $1 - p_A(\varepsilon)$, is increasing in inflating, leading to higher expected penalties.

In order to solve (1.4) one needs to assume that the constraint binds, use it to substitute out the rating fee (f) and find optimal inflating (ε). The first-order condition will satisfy

$$\frac{\partial \Pi}{\partial \varepsilon} \propto p_B R - 1 - c(1 - p_B) < 0, \tag{1.6}$$

where p_B is the success probability of a project that is offered a B rating. Since all B-rated projects had b signals, p_B also equals to the success probability of projects with b signals. The derivative is proportional to the difference between the NPV of an inflated project ($p_B R - 1$) and the expected marginal penalty for inflating ($c(1 - p_B)$).

The agency perfectly internalizes the adverse effects of inflating. First, its profits will decrease by the NPV of the inflated project, and second, it is also required to pay a penalty in expectation. Hence, if the agency can commit to a given inflating level, it would always set it to zero.

Observe that agency profit is linear in inflating. This follows from the fact, that (i) inflating the rating of an additional project always adds a project to the financing pool with the same expected quality, i.e. a project with a b signal and (ii) there are no convexities in the penalty schedule, i.e. the unit penalty does not depend on the total mass of defaults. This linearity will imply that the agency either prefers a corner solution (inflate all ratings or none) or it is indifferent regarding the inflating level in equilibrium.

Why is zero inflating optimal? Because investors are competitive, issuers secure all gains from trade. However, the monopolist agency extracts all the surplus from issuers. This gives the agency an incentive to maximize the gains from trade, which is increasing in the amount of information revealed by the agency. Hence, the agency wants to reveal all of its information, which is implemented by zero inflating.

1.2.2 *Solution without commitment*

Now I relax the assumption that investors can observe the amount of inflating, which implies that the agency can no longer commit to any given inflating level. The agency's problem becomes

$$\max_{\varepsilon,f} \Pi = \max_{\varepsilon,f} (\mu_a + \varepsilon\mu_b)[f - c(1 - p_A(\varepsilon))], \text{ s.t. } f \leq \hat{p}_A R - 1, \tag{1.7}$$

where the only difference from (1.4) is in the constraint: the agency takes investor beliefs about the success probability of A-rated projects ($\hat{p}_A$) as given.[25] Hence, changing the amount of inflating does not affect investor valuations.

Note that mass sizes μ_a (projects with a signals) and μ_b (projects with b signals) are deterministic, since there are a unit mass of issuers. Hence, the agency knows the distribution of realized signals before the beginning of the rating process. This makes it possible for the agency to set the rating fee optimally even before learning signal values. Therefore, the problem in (1.7) can be thought of as the agency maximizing its payoff conditional on signal realizations, but can also be interpreted as selecting the optimal fee and disclosure rule before signals realize.

In equilibrium investor beliefs must be consistent, implying

$$\hat{p}_A = p_A(\varepsilon^*), \tag{1.8}$$

where ε^* is the agency's optimal inflating level. It is instructive to decompose the profit function into issuers with different signals. Assuming the constraint in (1.7) binds, the problem may be written as

$$\max_{\varepsilon} \Pi = \max_{\varepsilon} \mu_a[\hat{p}_A R - 1 - c(1 - p_A(0))] + \varepsilon\mu_b[\hat{p}_A R - 1 - c(1 - p_B)], \tag{1.9}$$

where the first term is the profit from providing A ratings to issuers with a signals. It

[25] Formulating the problem this way makes the comparison to the case with commitment straightforward.

is clear that this does not depend on inflating. The second term captures the profit from offering A ratings to issuers with b signals. The objective in (1.9) makes it easy to see that agency profit is linear in inflating.

The first order condition of the agency follows immediately from (1.9),

$$\frac{\partial \Pi}{\partial \varepsilon} \propto \hat{p}_A R - 1 - c(1 - p_B), \tag{1.10}$$

where inflating an additional project has two effects: first, it increases profits by the rating fee paid by the inflated project and, similarly to the case with commitment, it decreases profits by the expected marginal penalty of inflating. It follows from (1.10) that if investors are optimistic by believing that A-rated projects succeed with a high probability and the penalty is sufficiently low, the agency will want to inflate all ratings. Hence, in such a case zero inflating cannot be an equilibrium outcome.

The following lemma summarizes the equilibrium.

Lemma 1 (Equilibrium with a monopolist agency)**.** *Under Assumptions 1 and 2*

(i) Issuers always purchase A ratings and never purchase B ratings.

(ii) Agency sets

$$\varepsilon^* = \begin{cases} 0, & \text{if } c(1-p_B) \geq p_A(0)R - 1 \\ \dfrac{\mu_a[p_A(0)R - 1 - c(1-p_B)]}{\mu_b[c(1-p_B) - (p_B R - 1)]}, & \text{if } c(1-p_B) < p_A(0)R - 1 \end{cases} \tag{1.11}$$

$$f^* = p_A(\varepsilon^*)R - 1. \tag{1.12}$$

(iii) Investor beliefs satisfy $\hat{p}_A = p_A(\varepsilon^*)$, $\hat{p}_B < p_A(\varepsilon^*)$, $\hat{p}_\emptyset < 1/R$.

The proof is straightforward. Given that the agency is not inflating ratings, its marginal benefit from inflating is the expected NPV of an A-rated project ($p_A(0)R - 1$). Similarly, its marginal cost of inflating is the expected marginal penalty of the inflated project ($c(1 - p_B)$). When the marginal cost exceeds the marginal benefit, the only equilibrium is zero inflating.

However, when the marginal cost is lower than the marginal benefit without inflating, the agency cannot commit to zero rating inflation. Therefore, investors will decrease their valuations to the point, where the agency is indifferent regarding the amount of inflating (so the first order condition (1.10) equals to zero). Thus, $\hat{p}_A$ will be implied by

$$\hat{p}_A R - 1 = c(1 - p_B), \tag{1.13}$$

which means that in an equilibrium with some inflating the marginal cost equals the marginal benefit of inflating. Combining (1.8) and (1.13), one finds the equilibrium success rate of financed projects,

$$p_A(\varepsilon^*) = \frac{1+c(1-p_B)}{R} \;\; for \;\; \varepsilon^* > 0. \tag{1.14}$$

Finally, solving (1.14) for equilibrium inflating, ε^*, gives (1.11). Observe, that the equilibrium success probability, $p_A(\varepsilon^*)$, is increasing in the penalty and in signal precision. A higher penalty increases the expected marginal penalty for inflating, alleviating conflicts of interests. Similarly, improved signal precision (lower signal noise, α) also increases the expected marginal penalty because inflated projects are now more likely to default. This implies that the net effect of a better technology (taking into account the agency's potential behavioral response of increased inflating) is increased equilibrium success rate.

As the penalty approaches zero, it is clear from (1.13) that the expected NPV of financed projects also goes to zero. In the special case, when the penalty is zero ($c = 0$), there is no cost of inflating. However, equilibrium condition (1.8) still needs to be satisfied, implying that the agency will pick an inflating level that will result in A-rated projects carrying zero expected NPV, $p_A(\varepsilon^*) = 1/R$.

Looking at (1.11) it is clear that inflating increases with the value of a honest rating ($p_A(0)R - 1$) and decreases with the expected marginal penalty ($c(1 - p_B)$). The cutoff condition for no inflating is more likely to be satisfied when the penalty is high and when the rating technology is noisy (high α). This captures the basic incentive conflict of a rating agency: the better (i.e. more valuable) its information the less he can resist the temptation to inflate.

Both fee revenue and penalties linearly increase with the mass of financed bad projects. This implies that when there is inflating in equilibrium the expected marginal cost and revenue of inflating is equal for all levels of inflating, leading to the profit neutrality of rating inflation. Hence, if investors mistakenly choose an off equilibrium $\hat{p}_A$ that is marginally higher than the respective equilibrium belief (and the agency and issuers know this) then the rating agency would find it optimal to respond by inflating *all* ratings. Observe, that the profit neutrality of inflating does not mean that the rating agency is earning zero profit, as it is always profitable to sell A ratings to projects that received a signals. The expected penalty for providing ratings to these projects is lower than the rating fee.

In equilibrium only A-rated projects will be financed. This implies that off equilibrium beliefs on B-rated and unrated projects have to be sufficiently pessimistic. In particular, investors need to be at least marginally more pessimistic about B-rated projects, than A-rated projects, otherwise issuers would find it optimal to purchase

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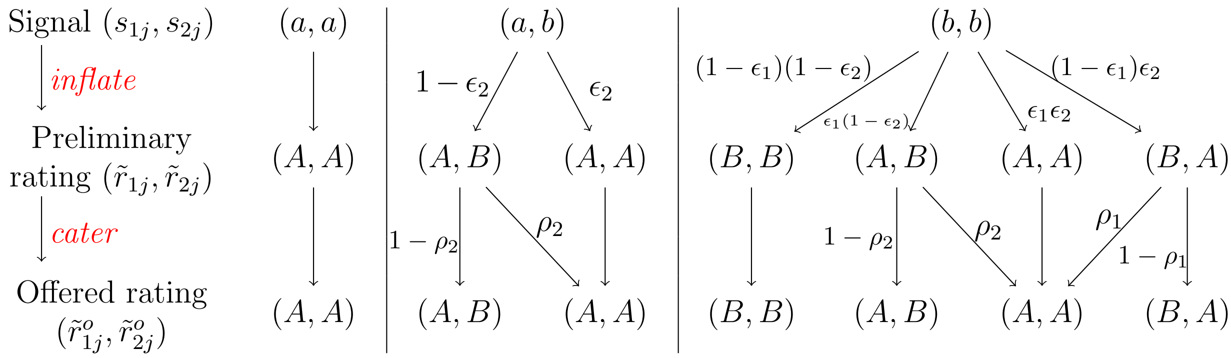

Figure 1.2 The rating process: from signals to offered ratings. The figure illustrates how signals are transformed into offered ratings during the rating process when agencies inflate preliminary ratings with ε_i and cater offered ratings with ρ_i. Note that receiving (b,a) signals is not shown as it is symmetric to the (a,b) case.

B-ratings. This is not restrictive, as investors understand that *B* ratings are a result of *b* signals, implying negative expected NPV. Also, investors need to believe that purchasing a project without a rating would lead to a loss, on average. This is, in fact, guaranteed by Assumption 1 and provides the sufficient incentive for issuers to purchase *A* ratings.

1.3 Duopoly

Introducing a second agency and allowing information to flow between the agencies requires a more detailed modeling of the rating process. Agencies are indexed by i. Both agencies have access to a rating technology, that produces signals ($s_{ij} \in \{a,b\}$) about each project, as described in (1.2). Rating errors are independent between the rating technologies, implying that the combined information of agencies in a duopoly is superior compared to a monopolist's information.[26]

Signals produced by the rating technology are transformed into *preliminary* ratings, $\tilde{r}_{ij} \in \{A,B\}$. Agencies are allowed to *inflate* preliminary ratings with probability ε_i, just as in the benchmark case:

$$\varepsilon_i = Pr(\tilde{r}_{ij} = A | s_{ij} = b), \;\; Pr(\tilde{r}_{ij} = A | s_{ij} = a) = 1 \tag{1.15}$$

Importantly, after preliminary ratings are assigned, agencies learn the preliminary ratings assigned by the other agency. If the other agency assigned an *A* pre-

[26] In Appendix B I analyze the effects of market structure through the merger of two agencies, which keeps the amount of information constant.

10.14754/CEU.2017.09

liminary rating then an agency may *cater* to these issuers by improving their *offered* ratings, $r^o_{ij} \in \{A,B\}$. One can think of catering as a conditional manipulation method, which conditions on the other agency's preliminary rating. As before, agencies cannot assign a worse offered rating than their assigned preliminary rating:

$$\rho_i = Pr(r^o_{ij} = A|\tilde{r}_{ij} = B, \tilde{r}_{-ij} = A), \;\; Pr(r^o_{ij} = A|\tilde{r}_{ij} = A) = 1. \tag{1.16}$$

otherwise $r^o_{ij} = \tilde{r}_{ij}$.

Figure 1.2 summarizes the rating process. Note that once projects receive an a signal or an A preliminary rating they will always be assigned an A offered rating from the respective agency. This follows from the fact that agencies cannot deflate ratings during the rating process.[27] Second, a project with two bad signals can only be offered an A rating if at least one agency chooses to inflate its preliminary rating. Hence, without inflating, catering only affects projects with mixed signals.[28]

The rating process in Figure 1.2 is a reduced form for modeling how issuers go back and forth between rating agencies, seeking a favorable rating. I show in Appendix C that this corresponds to a game in which issuers ask agencies sequentially (and possibly multiple times) for ratings before deciding which ones to purchase.

Given offered ratings and fees, issuers choose the ones (if any) they want to purchase and disclose to the public. Issuers have four pure strategies (over which they may implement mixed strategies), namely, purchase agency 1's rating, purchase agency 2's rating, purchase both and purchase none. Formally, the strategy of issuers is represented with a function $D(r^o_1, r^o_2, f_1, f_2) \to [0,1]^4$, where the 4 outputs are the probabilities assigned to the respective pure strategies.[29]

Ratings observed by investors are $r_{ij} \in \{A, B, \emptyset\}$, where $\emptyset$ stands for undisclosed, implying that the given agency-issuer offered rating was not disclosed.[30] Investors observe ratings and bid for the projects, given its rating(s).

Figure 1.3 gives the timeline for the game with the duopoly. The main difference compared to the monopolist case is that agencies in a duopoly may revise the rating that they are offering to issuers during the rating process based on each other's preliminary ratings.

Below I state the Perfect Bayesian Equilibrium of the game.

[27] In the current setup this is not restrictive as agencies have all the bargaining power (they set the rating fee by making a take it or leave it offer). Hence, by deflating ratings, agencies could not corner issuers any further.

[28] Agencies cannot improve the offered rating of a project if it received a B preliminary rating from the other agency. However, this is not restrictive in equilibrium.

[29] E.g. $D() = \{0,0,1,0\}$ corresponds to "purchase both", while $D() = \{0.5, 0.5, 0, 0\}$ means "with 50% purchase agency 1's rating and with 50% purchase agency 2's rating"

[30] Note that in the benchmark case undisclosed ratings played no role, but here, in principal, it could happen that an issuer only discloses a single A rating, as this could be sufficient information for investors to purchase the project.

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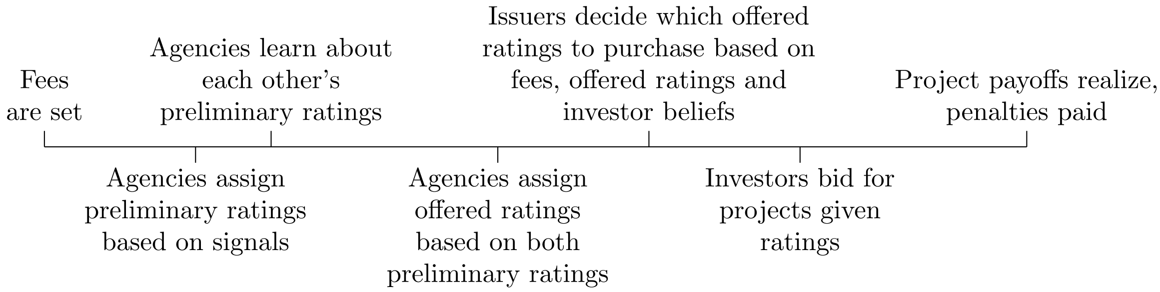


Figure 1.3 Timeline of the game with a duopoly of agencies.

Definition 1 (Equilibrium). *1. Issuer $j \in [0,1]$ optimally chooses which offered ratings to purchase (if any), given fees (f_i), offered ratings r^o_{ij} for $i \in \{1,2\}$ and investor beliefs.*

2. Rating agencies optimally set fees and manipulation levels ($f_i, \varepsilon_i, \rho_i$), given issuers' ratings purchase strategies, fees and manipulation levels set by the other rating agency and investor beliefs.

3. Investor beliefs about success probabilities are correct for all rating combinations. Hence, by bidding competitively for projects, they break even in expectation.

1.3.1 Equilibrium in duopoly

Let $\hat{p}_{r_{1j}r_{2j}}$ be investor beliefs about the conditional success probability of project j with $r_{1j}, r_{2j} \in \{A, B, \emptyset\}$ ratings. Since purchasing a B rating is never optimal for issuers, as it reveals the worst possible information at a cost,[31] the relevant beliefs are $\hat{p}_{AA}, \hat{p}_{A\emptyset}, \hat{p}_{\emptyset A}$, as projects could potentially be financed with these rating combinations.

Similarly to the notation introduced above, define $p_{r_{1j}r_{2j}}(\varepsilon_1, \varepsilon_2, \rho_1, \rho_2)$ as the posterior probability that a project is good, given that it received $r^o_{1j}, r^o_{2j} \in \{A, B, \emptyset\}$ offered ratings, when agencies are inflating and catering ratings with $\{\varepsilon_1, \varepsilon_2, \rho_1, \rho_2\}$. E.g.

$$p_{AA}(\varepsilon_1, \varepsilon_2, \rho_1, \rho_2) = Pr(\theta_j = g | r^o_{1j} = A, r^o_{2j} = A, \varepsilon_1, \varepsilon_2, \rho_1, \rho_2) \quad (1.17)$$

In order to solve the model, one has to make a guess about the equilibrium rating purchase strategy of issuers (and later verify) in order to formulate the profit

[31] This follows from restricting agencies' strategy space by not allowing them to manipulate a signals (A preliminary ratings) into B preliminary ratings (B offered ratings).

10.14754/CEU.2017.09

function of agencies. Suppose issuers who are offered A ratings from both agencies find it optimal to purchase both and no other issuer finds it optimal to purchase ratings.

If issuers have to purchase both ratings in order to sell their projects, the sum of the rating fees proposed by the two agencies cannot exceed the price investors are willing to pay for a project with two A ratings:

$$f_1 + f_2 \leq \hat{p}_{AA}R - 1, \tag{1.18}$$

which implies that agencies will always set the highest possible fee, otherwise they would be leaving money on the table[32]:

$$f_i = \hat{p}_{AA}R - 1 - f_{-i}. \tag{1.19}$$

Given the guess on issuer behavior and the optimal fee the problem of agency 1 may be formulated as

$$\begin{aligned} \max_{\varepsilon_1,\rho_1} \Pi_i = \max_{\varepsilon_1,\rho_1} & \mu_{aa}[f_1 - c(1 - p_{AA}(\mathbf{0}))] + \\ & + [\mu_{ba}(\varepsilon_1 + (1-\varepsilon_1)\rho_1) + \mu_{ab}(\varepsilon_2 + (1-\varepsilon_2)\rho_2)][f_1 - c(1-\pi_g)] + \\ & + \mu_{bb}[\varepsilon_1\varepsilon_2 + \varepsilon_1(1-\varepsilon_2)\rho_2 + \varepsilon_2(1-\varepsilon_1)\rho_1][f_1 - c(1-p_{BB})], \end{aligned} \tag{1.20}$$

where the first line captures the profit from issuers with two good signals (their total mass being μ_{aa}). The second line is the profit from providing A ratings to issuers with mixed signals (their total mass being $\mu_{ab} + \mu_{ba}$), where the first bracket is the mass of issuers with mixed signals that are either inflated or catered, while the second bracket consists of the difference between the rating fee and the expected penalty for allowing the sales of a project with mixed signals. The third line is the profit from providing A ratings to issuers that received b signals from both rating technologies (their total mass being μ_{bb}). The first term in brackets gives the probability of these issuers being offered A ratings from both agencies. In particular, such an issuer's rating may be inflated by both agencies ($\varepsilon_1\varepsilon_2$) or may be inflated by one agency and catered by the other ($\varepsilon_i(1-\varepsilon_{-i})\rho_{-i}$). The final bracket is the difference between the rating fee and the expected penalty for enabling the sales of a project with two b signals.

The marginal benefit from selling a rating is always the rating fee, however, the marginal cost depends on the signals of the issuer purchasing the rating. The marginal cost will be the highest for issuers with two bad signals, $c(1-p_{BB})$, since

[32] Since issuers only purchase ratings if they are offered an A rating from both agencies, reducing fees does not generate additional demand for ratings.

these projects are very likely to default. Hence, agencies will be reluctant to inflate ratings. Instead, they will be more likely to cater ratings first, as the expected penalty for providing A ratings to issuers with mixed signals equals $c(1-\pi_g)$. This follows from the fact that signals are unbiased, so if a project receives contradicting signals, then the posterior success probability will be equal to the prior:

$$p_{AB}(\mathbf{0}) = \frac{\pi_g \alpha(1-\alpha)}{\mu_{ab}} = \pi_g. \tag{1.21}$$

Since I will only focus on symmetric equilibria, equilibrium fees proposed by the agencies will coincide, $f_1 = f_2 = (\hat{p}_{AA}R-1)/2$. The equilibrium fee also has to be at least as high, as the expected penalty without inflating and catering, which is stated in the following assumption.

Assumption 3.

$$\frac{p_{AA}(\mathbf{0})R-1}{2} > (1-p_{AA}(\mathbf{0}))c \tag{1.22}$$

The interpretation of Assumption 3 is the same as the interpretation of Assumption 2. Assumption 3 guarantees that when agencies are not manipulating ratings, the rating fee has to be larger than the expected penalty per project, otherwise, agencies would not want to participate in the market.

Finally, investor beliefs have to be consistent in equilibrium. This will be satisfied if

$$\hat{p}_{AA} = p_{AA}(\varepsilon_1^*, \varepsilon_2^*, \rho_1^*, \rho_2^*). \tag{1.23}$$

Since only those issuers will purchase ratings who are offered A ratings from both agencies, the equilibrium can be pinned down by this condition together with the first order conditions of (1.20), while other beliefs will only appear on the off-equilibrium path.

The following lemma states the symmetric Perfect Bayesian Equilibrium for the game with two agencies.[33]

Lemma 2 (Equilibrium with two agencies)**.** *Under Assumptions 1 and 3*

(i) Issuers with $\{A,A\}$ offered ratings purchase both, otherwise they purchase none.

[33] In Section 1.3.3 I discuss other equilibria and equilibrium refinements.

(ii) *Equilibrium inflating and catering satisfy*

$$\begin{cases} \varepsilon^* = \rho^* = 0 & \textit{if } \frac{p_{AA}(\mathbf{0})R-1}{2(1-\pi_g)} \leq c \\ \varepsilon^* = 0, 0 < \rho^* \leq 1 & \textit{if } \frac{p_{AA}(0,0,1,1)R-1}{2(1-p_{BB})} \leq c < \frac{p_{AA}(\mathbf{0})R-1}{2(1-\pi_g)} \\ 0 < \varepsilon^* < 1, \rho^* = 1 & \textit{if } c < \frac{p_{AA}(0,0,1,1)R-1}{2(1-p_{BB})} \end{cases}$$

(iv) *Equilibrium beliefs satisfy:* $\hat{p}_{AA} = p_{AA}(\varepsilon^*, \rho^*)$, $\hat{p}_{A\emptyset}, \hat{p}_{\emptyset A} \leq \frac{\hat{p}_{AA}R+1}{2R}$, $\hat{p}_{\emptyset\emptyset} \leq 1/R$

The proof of Lemma 2 can be found in the Appendix, together with formulae for equilibrium inflating and catering.

The first result is that only those projects are going to be financed, which are offered *A* ratings from both agencies. The intuition is as follows. If issuers offered *A* ratings from both agencies would randomly purchase a single rating then this would lead to agencies competing in fees in order to attract business. However, if fees are sufficiently low, issuers offered *A* ratings from both agencies will find it optimal to purchase the second rating so they could distinguish themselves from issuers who are only offered an *A* rating from a single agency. This mechanism rules out pure strategy equilibrium candidates in which projects with a single public rating obtain financing.[34]

Given that issuers purchase both *A* ratings when offered, issuers with contradicting offered ratings would not want to purchase the single *A* rating because investors will correctly infer that the undisclosed rating of these issuers has to be *B*. This implies that they will value such projects accordingly: their valuation will not exceed the value of an average project, which is negative by Assumption 1.[35]

The second finding is the "pecking order" nature of rating manipulation methods. When the penalty is sufficiently high, agencies will not manipulate ratings. However, when the penalty is too low to prevent manipulation, agencies will first start to cater ratings and once they have catered all ratings, they will also start inflating ratings. The reason they start with catering is simple: catered projects default with lower probability than inflated projects, which makes them cheaper to manipulate as the associated expected penalty will be lower.

Figure 1.4 illustrates the pecking order nature of manipulation methods. If the penalty (c) is high, then agencies in a duopoly do not cater (solid line) nor inflate (dotted line) ratings, as shown in region A. In regions B+C+D they cater but do not inflate. Finally, they cater for everyone and also inflate in region E.

[34] The structure of the equilibrium is different with equilibria reached in models of rating shopping, where some issuers find it optimal to only disclose a single rating in equilibrium (Bolton et al., 2012; Skreta and Veldkamp, 2009; Sangiorgi and Spatt, 2016). In Appendix C I discuss the case when issuers sequentially ask agencies for ratings and show that the same equilibrium is reached.

[35] This follows from the fact, that in the best case scenario these issuers obtained contradicting signals and conditional on obtaining contradicting signals, a project has average NPV, which is negative.

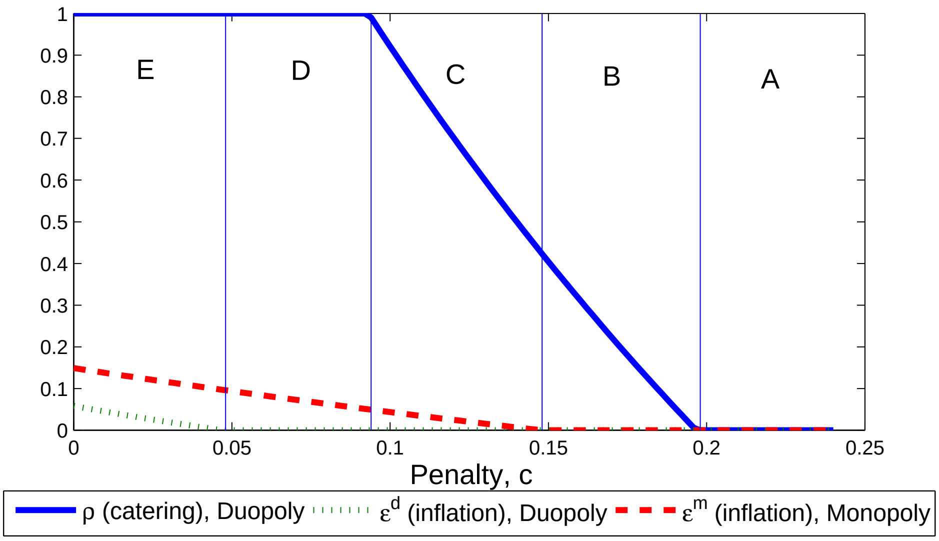


Figure 1.4 Equilibrium manipulation levels as a function of the penalty. Vertical lines indicate the ranges of the penalty that correspond to the parameter regions in Lemma 1 and 2. In regions A+B a monopolist does not inflate ratings, while in regions C+D+E it does. Agencies in a duopoly do not cater nor inflate ratings in region A, they cater in regions B+C+D and they also inflate in region E. Parameters are: $R = 1.2$, $\alpha = 0.05$, $\pi_g = 0.5$

As agencies propose take-it-or leave it fees and issuers with two A offered ratings purchase both, agencies are able to increase fees up to the point where issuers are indifferent between their outside option and purchasing both ratings, hence $f_i = (\hat{p}_{AA}R - 1)/2$.[36]

Given the other agency's strategy, agencies do not have an incentive to deviate from these fees. If one agency were to increase its fee, his revenue (together with other agency's) would drop to zero as issuers would not find it optimal to purchase any ratings. Interestingly, decreasing fees also decrease profits. There are two cases to consider. First, suppose agencies do not find it optimal to cater ratings. That implies that selling A ratings to issuers with mixed preliminary ratings would generate losses for the agencies (the rating fee is smaller than the expected marginal penalty for catering). For the same reason, agencies do not want to attract these issuers with lower rating fees. Additionally, since the demand for A ratings is perfectly inelastic

[36] One could easily introduce Nash bargaining to make this part of the model more realistic. Note that while Nash bargaining clearly affects allocation it would also affect efficiency, as if issuers are able to bargain lower fees then this would decrease the temptation to cater and inflate ratings.

from issuers who are offered A ratings from both agencies, lower fees lead to lower overall fee revenue from these issuers.

Now consider the case when agencies cater in equilibrium. In this case the expected marginal penalty of catered projects equals the rating fee. This implies that agencies earn zero profits by selling ratings to catered projects. It follows that if an agency were to marginally decrease its rating fee, then selling A ratings to issuers with mixed offered ratings would lead to a loss, since now the rating fee is smaller compared to the expected penalty for catering.

It also follows that agencies would not want to deviate by setting contingent fees. Since they are collecting rating fees from a homogeneous group of issuers (i.e. issuers who manage to sell projects face the same constraint) agencies have no reason to set contingent fees. However, this result is somewhat specific to the setup here, as I show in Appendix D that if the average NPV of projects is positive (and additional conditions are satisfied), then agencies may have an incentive to set contingent fees, if possible.

Since only projects with two A ratings will be financed, investor beliefs about the default probabilities of these projects must be correct in equilibrium. Following the structure of the equilibrium, beliefs about issuers with different rating combinations will be on the off-equilibrium path. For projects with one disclosed A rating, investors believe that these are worth at most the rating fee, otherwise all issuers who can, would only want to purchase one rating. Also, issuers without any disclosed ratings are believed to be at most average, which prevents their financing.

1.3.2 Relation to stylized facts

The equilibrium derived above provides two key predictions. First, issuers always find it optimal to purchase both offered ratings in equilibrium. Second, if conflicts of interest arise, agencies will have a strong temptation to seek out each other's information and cater ratings.

The two largest agencies, S&P and Moody's both cover virtually all corporate bonds in the US, which is consistent with the predictions of the model.[37] However, Bongaerts et al. (2012) reject the hypothesis that this behavior is due to the informativeness of the second and third rating, while in the model the only motive to purchase a rating is to convey information as I abstract from regulatory advantages. Also, my model stays silent on the role of unsolicited ratings. In equilibrium, revealing unsolicited ratings does not increase the bargaining power of agencies.[38]

[37] Doherty et al. (2012) investigate the entrance of S&P in rating insurance companies, which was previously dominated by A.M. Best. They report that among those insurers who already had a rating from A.M. Best, 83% of them requested a new rating from S&P during their sample.

[38] Fulghieri et al. (2014) build a model to analyze the role of unsolicited ratings while Bannier et al. (2010) investigate empirically why unsolicited ratings are lower on average.

10.14754/CEU.2017.09

On the other hand, Griffin et al. (2013) report that about 85% of AAA rated capital in the CDO market was rated by both Moody's and S&P. Furthermore, they also find that having an additional AAA rating paid off to issuers on average, as it increased the funds they could raise by more than the cost of the additional rating, which suggests that investors valued the second rating. Hence, in this respect the model is closer to the structured segment.

I show that agencies in a duopoly always start manipulating ratings by catering and only if they have catered to everyone will they consider assigning inflated preliminary ratings. By using the other agency's information agencies minimize their liability. This supports the empirical findings of Griffin et al. (2013) who manage to reconstruct agency model implied ratings and conclude that an agency was more likely to make upward adjustments to model implied ratings when the other agency's model was more optimistic. Moreover, these adjustments were surprisingly large. They report that when Moody's (S&P's) model allowed for 10% larger AAA tranche size, S&P (Moody's) adjusted its model implied AAA tranche size by over 7% (3.5%). Also, upward adjustments were significantly larger than downward adjustments and the latter were inconsequential, since they did not result in worse ratings.[39] Kraft (2015) provides evidence that agencies cater to those issuers who have ratings-based performance-priced loan contracts.

1.3.3 Other equilibria and robustness

Here I briefly discuss the uniqueness and the robustness of the equilibrium. I will show that the equilibrium presented above is the only symmetric pure strategy equilibrium in which transactions take place between players.

There are other Perfect Bayesian Equilibria. First, there are a set of trivial equilibria, in which investors believe that regardless of the disclosed ratings the project has negative NPV. Hence, issuers do not purchase ratings at all. Second, there is a large set of asymmetric equilibria, in which investors believe that a rating from one agency does not reveal sufficient amount of information, but the other agency's does. This leads us back to equilibria that are similar to the ones derived for a monopolist agency, since issuers are only going to purchase ratings from one agency. However, since learning could take place between agencies even in an asymmetric equilibrium, the asymmetric equilibria are not always going to be identical to the monopolist case. The above illustrate that perfectness leaves us with multiple equilibria.

Now consider robustness. Would the equilibrium of Lemma 2 survive trembling

[39] Griffin et al. (2013) argue that downward adjustments are a mechanical result of issuers targeting the rating models of one agency. In cases when the targeted agency's model is stricter, the other agency makes downward adjustments.

hand type refinements? Suppose issuers with $\{A,A\}$ offered ratings forget to purchase one of their ratings with a total mass of $2\eta \to 0$ (so that a mass of η forgets to purchase from each agency). Then it will no longer be optimal for issuers with $\{A,B\}$ and $\{B,A\}$ offered ratings not to purchase their A ratings because now they have the opportunity to pool with $\{A,A\}$ issuers who made a mistake. However, always purchasing their A rating cannot be an equilibrium outcome as it will imply that the expected NPV of projects with a single A rating is negative, since η is close to zero. This leaves us with mixed strategies, which imply that in equilibrium issuers with $\{A,B\}$ and $\{B,A\}$ offered ratings must be indifferent between purchasing and not purchasing their A rating. Their indifference condition may be written as:

$$\frac{\hat{p}_{AA}R-1}{2} = \frac{\eta(\hat{p}_{AA}R-1)+\kappa(\hat{p}_{AB}R-1)}{\eta+\kappa} = \hat{p}_{A\emptyset}R-1, \tag{1.24}$$

where a κ mass of issuers with $\{A,B\}$ offered ratings purchase their A ratings. The condition in (1.24) says that the equilibrium rating fee is equal to the value of a project that only reveals a single A ratings. This implies that the value of a project with a single A rating is exactly the half of the value of a project with two A ratings. Hence, when off-equilibrium beliefs satisfy $\hat{p}_{A\emptyset} = \hat{p}_{\emptyset A} = \frac{\hat{p}_{AA}R+1}{2R}$ then the equilibrium will be robust to trembling hand refinements. It is clear that the solution to the first equality in (1.24) gives the well defined equilibrium mass of issuers with mixed offered ratings, κ, who choose to purchase their A rating. The results are symmetric for issuers with $\{B,A\}$ offered ratings.

The result that issuers purchase both A ratings is in line with the unraveling principle (Grossman, 1981), also found by Sangiorgi and Spatt (2016) in the context of credit rating agencies. According to the unraveling principal, if a seller is able to disclose verifiable information about its product's quality, it will want to do so, otherwise investors would believe that the product is of the worst possible quality. In my setting, investors expect that issuers will learn about project quality from both agencies, because issuers receive offered ratings for free. Hence, in an equilibrium in which issuers only disclose a single A rating, there will be issuers who did not disclose the second A rating. But this would be profitable for them (and the agency) as it would allow them to differentiate themselves from issuers with mixed offered ratings.

1.4 Comparing monopoly and duopoly welfare

This section compares outcomes between market structures. Importantly, comparisons are made holding the exogenous penalty, c, fixed across market structures.

This corresponds to a thought experiment in which a regulator collects the penalty whenever a highly rated project defaults, irrespective of market structure and rating mistakes do not bare other consequences for the agency.

However, if one interprets the penalty as a reputation cost, there are theoretical arguments suggesting that they will endogenously vary with market structure. Whether it will be larger in a monopoly or duopoly largely depends on how investors punish agencies for being wrong.[40, 41] For example, if investors do not punish agencies for errors that are made by both of them, then moving from a monopoly to a duopoly would imply smaller penalties in a duopoly. This would also strengthen the welfare results of the paper.

1.4.1 Manipulation methods and rating standards of a monopoly and a duopoly.

How do the parameter regions of Lemma 1 and 2 relate? Figure 1.4 shows an example, highlighting the amount of inflating and catering in a monopoly and duopoly. Vertical lines divide the penalty (c) into five regions. A monopoly does not inflate (dashed line) in regions A+B, while it does in regions C+D+E. Agencies in a duopoly do not manipulate in region A, cater (solid line) but do not inflate (dotted line) in regions B+C+D and both cater and inflate in region E.

In region B of Figure 1.4 agencies in a duopoly cater ratings while a monopolist reports honestly, which implies that agencies in a duopoly may start manipulating ratings at higher penalty levels. Below I state a sufficient condition for this result.

Corollary 1.

$$p_A(0)R - 1 < (p_{AA}(\mathbf{0}) - p_A(0))R \text{ is sufficient for } \frac{p_A(0)R-1}{1-p_B} < \frac{p_{AA}(\mathbf{0})R-1}{2(1-\pi_g)}$$

See the proof in the Appendix. Corollary 1 says that when the marginal value of the second honest rating exceeds the marginal value of the first honest rating then there will be a range of penalty levels, such that issuers in a duopoly will cater and a monopolist does not inflate ratings in equilibrium (region B in Figure 1.4). Intuitively, when a second honest opinion is highly valuable, agencies in a duopoly will start catering ratings at higher penalty levels. Also, catering is less harmful than inflating ratings in a monopoly since catered projects default with

[40] For example, Bar-Isaac and Shapiro (2013) consider two grim-trigger-strategies in an infinite horizon game with a duopoly of agencies: the "independent punishment" strategy, according to which if an agency is caught misreporting, investors will ignore its subsequent ratings and the "linked punishment" strategy, in which investors do not punish agencies if both of them make the same mistake. Investor's strategy affects the continuation value of the agencies through their survival probabilities: with linked punishment, they are more likely to survive mistakes, as if the other agency also committed the same mistake, they will not be punished.

[41] See footnote 24 in Bolton et al. (2012) for further discussion on this.

lower probability. The condition in Corollary 1 is likely to be satisfied when signal noise α is neither too low and neither too high, as the too extremes $\alpha = 0$ ($\alpha = 0.5$) imply $p_{AA}(\mathbf{0}) = p_A(0) = 1$ ($p_{AA}(\mathbf{0}) = p_A(0) = \pi_g$). With moderate signal noise investors are willing to pay a significant premium for a second *a* signal as the probability that a bad project receives a single *a* signal is α, but receiving two *a* signals is only α^2.

The next corollary relates inflating levels across market structures.

Corollary 2. *Let ε^{mon} (ε^{duo}) denote equilibrium inflating in a monopoly (duopoly). Then $\varepsilon^{duo} \leq \varepsilon^{mon}$.*

Corollary 2 implies that there will always be a nonempty set of parameter values where a monopolist inflates ratings but agencies in a duopoly do not inflate. In terms of Figure 1.4, this implies that the region D+C has positive measure. As agencies in a duopoly only start inflating ratings once they cater for everyone, it is easy to show that inflating levels in a duopoly will always be lower than in a monopoly. Catering for everyone implies that all issuers who are inflated by at least one agency will automatically obtain a second *A* rating through catering. This makes inflating expensive for agencies as projects with two bad signals are likely to default.

Equilibrium rating standards are the result of the precision of the rating technology and equilibrium manipulation levels. I define rating standard as the the NPV of a financed project:

$$RS^{mon}(\varepsilon^*) = p_A(\varepsilon^*)R - 1 \quad \text{and} \quad RS^{duo}(\varepsilon^*, \rho^*) = p_{AA}(\varepsilon^*, \rho^*)R - 1, \tag{1.25}$$

where $RS^{mon}(\varepsilon^*)$ and RS^{duo} stand for rating standard in a monopoly and a duopoly, respectively. It is important to emphasize that rating standards in (1.25) capture the quality of financed projects, not the total value of financed projects. This implies that efficiency may be decreasing in rating standards, if higher rating standards prevent the financing of not only many bad projects, but also many good projects.[42]

Corollary 3. *Rating standards $RS^{mon}(\varepsilon^*)$ and $RS^{duo}(\varepsilon^*, \rho^*)$ are at least weakly*

(i) increasing in the penalty, c.
(ii) increasing in rating precision, $1 - \alpha$.
(iii) decreasing in the share of good projects, π_g, if there is manipulation in equilibrium, otherwise increasing in the share of good projects.

[42] For example, a technology that only assigns an *A* rating to good issuers with a total mass of zero will constitute the highest rating standard, $R - 1$, but will also result in zero efficiency as only issuers with zero mass secure financing.

When agencies find it optimal to manipulate ratings, rating standards are increasing with the penalty, as a higher penalty helps overcome conflicts of interests. Of course, when agencies find it optimal to report ratings truthfully, increasing the penalty does not affect agency behavior (unless if it leads to the violation of Assumption 2 or 3, in which case the agencies shut down operation).

Rating standards are directly related to the precision of the rating technology, $(1-\alpha)$. When agencies report truthfully, a more precise signal leads to ratings that are more strongly correlated with project types, leading to increased standards. Interestingly, when agencies find it optimal to manipulate ratings, a more precise technology still leads to higher rating standards, even though agencies react to higher precision by increasing manipulation levels, as A ratings become more valuable. The intuition is clear from (1.14). The expected penalty associated with manipulated projects increases because agencies are more confident that projects obtaining bad signals will default. The only exception to this is when agencies in a duopoly find it optimal to cater but not inflate ratings. In this case, the marginal project being financed is the one with contradictory signals, which implies that their expected NPV does not depend on signal precision. Hence, in this case the effect of increased precision is exactly offset by the behavioral response of increased catering.

Surprisingly, rating standards strongly depreciate when the share of good projects increases and agencies are already manipulating ratings. Agencies respond to an increased share of good projects by increasing manipulation because if the share of good projects is higher, the probability that a manipulated project will succeed is also higher, reducing expected penalties. Thus, while the NPV of a randomly chosen project increases, rating standards decrease. On the other hand, when agencies do not manipulate in equilibrium, the higher share of good projects increases rating standards, as the success probability of A-rated projects increases.

The following corollary relates rating standards across market structures.

Corollary 4. *If*

$$2\pi_g - p_B - 1 > 0, \tag{1.26}$$

there always exists an interval for the penalty, c, such that $RS^{mon}(\varepsilon^*) > RS^{duo}(\varepsilon^*, \rho^*)$.

When agencies are not manipulating ratings, having an additional good signal always increases the probability that the given project is good. Hence, rating standards of the duopoly will always be higher in this case. However, when agencies manipulate ratings in equilibrium, surprisingly, the reverse could hold. For example, suppose that the monopoly inflates ratings while agencies in the duopoly cater (but do not inflate) ratings. Then

10.14754/CEU.2017.09

$$RS^{mon}(\varepsilon^*) - RS^{duo}(\varepsilon^*, \rho^*) = 1 + c(1-p_B) - [1 + 2c(1-\pi_g)] \propto 2\pi_g - p_B - 1,$$

implying that if the share of good projects (π_g) is sufficiently high, rating standards in a monopoly will be higher than rating standards in a duopoly. The condition (1.26) has a straightforward interpretation: if the success probability of a catered project in a duopoly (π_g) is large relative to the success probability of an inflated project in a monopoly (p_B) then agencies in a duopoly will be more aggressive in catering ratings than a monopoly will be in inflating ratings. Note that a necessary condition for (1.26) to hold is that the majority of the projects must be good ($\pi_g > 1/2$). Otherwise, agencies in a duopoly always produce higher rating standards.

In the current setup having a large ex ante success probability implies that project returns are skewed to the left: with large probability projects deliver small gains, but with small probability they lead to large losses. Hence, the above result suggests that a duopoly may lead to lower rating standards if asset returns are skewed to the left.

Rating standards are often in the focus of empirical work, as they capture the quality of ratings. However, it must be emphasized, that they do not tell us everything about efficiency, since they stay silent about the quantity of financed projects.

1.4.2 Welfare analysis

Does the additional information available in a duopoly lead to improved welfare? To measure welfare I use the total NPV of financed projects, normalized by the total NPV of successful projects ($\pi_g V_g$). I assume that the penalty paid by agencies is not considered waste, but it is collected by a regulator that distributes it among risk neutral consumers, who have constant, unit, marginal utility for money.[43] Welfare measures are directly related to rating standards introduced above, as they are the product of the quantity of financed projects times their average quality, the latter being equal to rating standards.

Below I state the main welfare result of the paper.

[43] At the other extreme, one may assume that the penalty is complete waste, which might be appealing if the penalty were interpreted as a reputation loss. In that case welfare equals to the agency's profit, since other players earn zero profit in expectation. Furthermore, if the penalty paid by agencies would reduce welfare, that would only strengthen the result that a monopoly may lead to higher welfare. To illustrate this, consider a case when the same projects are financed irrespective of market structure. Then agencies in a duopoly would have to pay a total penalty individually that is equal to the penalty paid by the monopolist. Hence, if paying the penalty is welfare decreasing, then a monopoly would produce higher welfare, even if the exact same projects are financed.

10.14754/CEU.2017.09

Proposition 1 (Comparing monopoly and duopoly welfare)**.** *If Assumptions 1-3 hold together with*

(i) $\mu_{aa}(p_{AA}(\mathbf{0})R-1)+2\mu_{ab}\bar{V}<0$ *(meaningful disagreement)*
(ii) $c<\bar{c}(\alpha,\pi_g,V_g)$ *(low penalty)*
(iii) $\pi_g>\bar{\pi}_g(\alpha,V_g,c)$ *(high ex ante success probability)*

then $W^{mon}(\varepsilon^*)>W^{duo}(\varepsilon^*,\rho^*)$*, where* $\bar{\pi}_g(\alpha,V_g,c)$ *and* $\bar{c}(\alpha,\pi_g,V_g)$ *are functions given in the Appendix and* $W^{mon}(\varepsilon^*)$ *is welfare in a monopoly while* $W^{duo}(\varepsilon^*,\rho^*)$ *is welfare in a duopoly.*

Proposition 1 provides sufficient conditions for the monopoly to produce higher welfare than a duopoly.

The first requirement is that agencies should meaningfully disagree about project fundamentals. Meaningful disagreement implies that if all projects with at least one *a* signal were to be financed then the market would break down as the average project would have negative NPV. Two factors drive this condition. First, if signals are noisy then agencies will frequently disagree which creates opportunities to cater ratings. Second, projects with mixed signals have to be sufficiently bad, so that catering is actually harmful.[44]

The second requirement is that the penalty has to be sufficiently low, making catering and inflating ratings cheap. This is straightforward. If catering and inflating would be expensive then agencies would always reveal their information honestly and the more information available in a duopoly would translate to higher welfare.

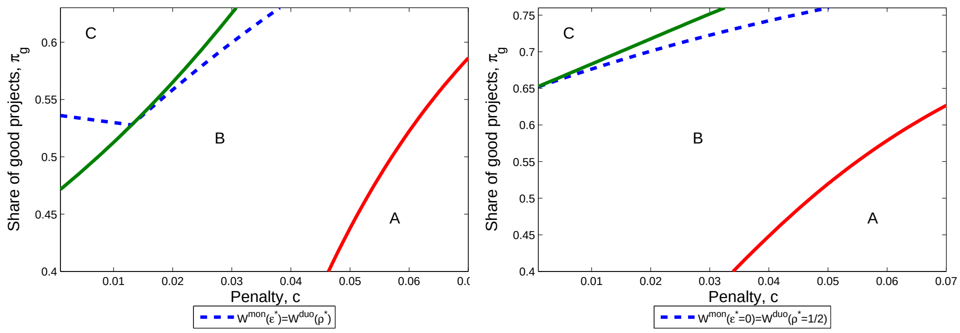
The third condition requires the ex ante success probability to be sufficiently high. This will imply that catering in a duopoly is much cheaper for agencies than inflating for a monopolist. The intuition is simple. If projects ex ante succeed with high probability than projects with mixed signals will also succeed with identically high probability. Hence, catering is relatively cheap because catered projects default with a low probability. On the other hand, inflating for a monopolist is relatively expensive as projects with a single bad signal are likely to default.

Proposition 1 clarifies why assets with skewed payoffs are likely to be catered. Ex ante payoffs are said to be skewed when a randomly chosen project yields a small gain with a high probability and a large loss with a small probability. The high success rate of such projects implies that agencies will only be caught with a low probability for catering. Hence, projects with higher skewness will be on the top of agencies' list when it comes to catering.[45]

What are the important mechanisms behind Proposition 1? First, in a duopoly there is more information available, since signal errors are independent across

[44] The condition of frequent disagreements has been also identified by the previous literature (Skreta and

10.14754/CEU.2017.09



(a) Low signal noise, $\alpha = 0.05$

(b) High signal noise, $\alpha = 0.1$

Figure 1.5 Welfare ranking of market structures. The solid lines divide the parameter space into regions A,B and C, which correspond to the regions of Figure 1.4. In regions A+B the monopolist does not inflate ratings ($\varepsilon^* = 0$), while in region C it does ($\varepsilon^* > 0$), as stated in Lemma 1. In region A agencies in a duopoly do not cater ratings ($\rho^* = 0$), while in regions B+C they do cater ($\rho^* > 0$), as stated in Lemma 2. Along the dashed line welfare is equal in a monopoly and duopoly, above the dashed line a monopoly leads to higher welfare, while below the dashed line duopoly leads to higher welfare. In region B the dashed line corresponds to the inverse of function $\bar{c}(\cdot)$ and in region C it corresponds to the function $\bar{\pi}_g(\cdot)$, both introduced in Proposition 1. Other parameters are: $R = 1.06$.

agencies. This increases the maximum potential welfare. Financing only those issuers who obtain good signals from both agencies limits the mass of financed projects, however, it leads to higher welfare through increased rating standards, as the projects that are excluded by the second signal carry negative expected NPV.

Second, increasing the number of agencies has a clear positive effect, as it decreases fees relative to the penalty, which helps agencies overcome conflicts of interests. Intuitively, lower rating fees decrease the temptation to manipulate ratings. In order to illustrate this result, consider the no noise limit, when $\alpha \to 0$. This is an interesting limit, because it keeps the available information constant across market structures (perfect information) and also holds constant the method of manipulation, as manipulated projects will certainly be bad projects. In this limit a monopolist would inflate ratings if $c < V_g$, as its rating fee converges to the NPV of a good project (V_g) and inflated projects default with probability $(1 - p_B) \to 1$, implying that the penalty (c) will be paid with certainty. On the other hand, agencies in a duopoly divide the fee revenue, which leads to a rating fee of $V_g/2$. Hence,

Veldkamp, 2009), because it creates opportunities to shop and disclose only the most favorable ratings. However, I emphasize that disagreements open the door for catering.

45 Note that conditioning the penalty on the size of the loss does not necessarily help, as the size of the loss may actually be smaller for assets with skewed payoffs.

they will only manipulate if $c < V_g/2$. Thus, a monopolist will always start inflating at higher penalty levels and will lead to (weakly) lower welfare in the no noise limit.[46]

Third, moving to a duopoly also changes the optimal method of rating manipulation. Since agencies in a duopoly can selectively manipulate ratings by cherry picking the relatively better ones through catering, they are more aggressive in manipulating ratings. This effect may be so strong that it outweighs the positive effects of added information and market structure. In particular, when incentive problems arise due to low penalties and the share of good projects is high catering ratings is very attractive to agencies.

Figure 1.5 illustrates the results of Proposition 1. The solid lines divide the parameter space into regions A,B and C, which correspond to the respective regions of Figure 1.4. In each panel, welfare in a monopoly and duopoly coincide along the dashed line. Welfare in a monopoly is higher if parameters fall above the dashed line and higher in a duopoly if parameters fall below the dashed line. In region A (when the penalty is high and the share of good projects is low) agencies do not manipulate ratings, regardless of market structure. In region B agencies in a duopoly find it optimal to cater ratings but a monopolist does not inflate ratings. This implies that the dashed line in region B corresponds to the set of parameters, which induces agencies in a duopoly to cater every second project ($\rho^* = 1/2$), as this will lead to equal welfare if a monopolist does not inflate.

In region C, when incentive problems are the worst due to low penalties and the high share of good projects, agencies in a duopoly cater ratings and a monopolist inflates ratings. On the left panel of Figure 1.5 the dashed line in region C highlights the parameter values, where welfare measures of the monopoly and duopoly coincide when a monopolist inflates with $\varepsilon^* > 0$ and agencies in a duopoly cater with $\rho^* > 1/2$.

The dashed lines in Figure 1.5 correspond to the functions $\bar{\pi}_g(\alpha, V_g, c)$ and $\bar{c}(\alpha, \pi_g, V_g)$ introduced in Proposition 1. The function $\bar{c}(\alpha, \pi_g, V_g)$ is the dashed line in region B (inverted) while $\bar{\pi}_g(\alpha, V_g, c)$ is the dashed line in region C, holding signal noise and the payoff of good projects fixed. Observe, that in the right panel, when signal noise is relatively high, the welfare ranking is always monotone in the penalty. This follows from the fact, that catering is more harmful in such a setting, since high signal noise leads to frequent disagreements among agencies, which provides ample opportunity to cater ratings.

[46] Alternatively, one can isolate the effect of market structure by analyzing the welfare implications of the merger of two agencies. This also keeps both the available information and manipulation methods constant as the merged agency can condition its manipulation strategy on both signals. See Appendix B, where I derive the welfare implications of a merger.

10.14754/CEU.2017.09

1.4.3 Relation to stylized facts

Does the structured segment qualify for the conditions stated in Proposition 1? We have already seen evidence for catering, which indicates that agencies expected low penalties.

Were disagreements meaningful enough? Griffin et al. (2013) argue that there were large differences between S&P's and Moody's model implied AAA tranche sizes. They report the average absolute difference to be 6% points, which is large, considering the sizes of junior tranches varied between 2-5%. Thus, there were significant differences, which demonstrate the complexity of rating structured assets.[47]

Additionally, AAA-rated structured products were supposed to be very safe, giving the impression to investors and agencies that their riskiness is similar to U.S. Treasuries. However, their large systematic risk was not publicized. As Coval et al. (2009a) demonstrate, tranching large portfolios of assets creates senior tranches that default with a low probability but they default in the worst economic state, which earns them the "economic catastrophe bond" title. Importantly, because they carry systematic risk, most of them will suffer losses if an economic catastrophe, like the recent crisis, occurs.

These empirical findings suggest that the circumstances preceding the crisis made catering of structured product ratings highly tempting for agencies. In turn, based on the model, a case could be made that a monopolist agency would have led to better outcomes, or, that expected retaliation by regulators were too small to deter the catering of seemingly safe assets.

1.5 Conclusion

This paper analyzes how market structure and the ability to calibrate rating models affects agencies' incentives to collect and reveal information. To this end, two new features are introduced. First, the rating process is modeled in detail, allowing agencies to learn about each other's assessments before providing a final rating. Second, agencies may flexibly set the tightness of their rating models. The analytically tractable framework makes it possible to analyze the efficiency implications of market structure, recognizing that the optimal method of rating manipulation varies with market structure.

I show that a monopoly may lead to higher efficiency, even if the combined in-

[47] To the best of my knowledge, similar measures from the corporate segment have not been documented. The reason for this could be that rating corporates is not a pure quantitative exercise, but requires the incorporation of qualitative information. However, Bongaerts et al. (2012) show that there is some disagreement between agencies in terms of the average level of ratings, which could indicate that agencies have different rating standards.

10.14754/CEU.2017.09

formation of agencies in a duopoly is superior. When issuers seeking finance offer assets similar to senior structured bonds agencies in a duopoly will have a strong incentive to cater ratings whenever the other agency has a more optimistic assessment. Hence, agencies fail to reveal their information. Additionally, I show that the incentives to collect additional information may disappear if agencies anticipate that they will not be able to reveal it in equilibrium.

The model presented here makes many assumptions in order to keep it tractable, some of which deserve further investigation. First, the welfare analysis abstracts from the cost of information production. In a duopoly if both agencies invest into information acquisition it will lead to an efficiency loss, since most of the time they will agree. However, the incentives to maintain the overall precision of the rating technology may be much lower in a duopoly due to learning, which mitigates the redundancies in information acquisition. Second, I only compare a monopoly with a duopoly, while the prevailing market structure consists of three larger agencies and a competitive fringe. Finally, allowing investors to learn from the cross section of ratings could serve as an important discipling device for agencies. These are left for future work.

10.14754/CEU.2017.09

2

The Benefits of Loose Rating Standards

2.1 Introduction

Information intermediaries can control how they produce and disclose information. Credit rating agencies are a leading example. They estimate financial ratios that investors care about, like the probability of default or the loss given default. But they choose to transform these measures into a scale which only gives a subjective ranking of riskiness. This makes it easier for business considerations to influence what kind of information is collected and how it is interpreted. For example, if an agency perceives that regulators are likely to retaliate if it attracts business by offering unjustifiably high ratings to issuers, then it may prefer a technology that only assigns top notch ratings to the safest of all assets. Similarly, if it wants to attract business, it may prefer a technology that is never unnecessarily strict with issuers.[1]

I study the information technology choice of a credit rating agency. The information technology classifies projects as good or bad and may commit two types of errors: it may classify a project as good that later defaults or it may classify a project as bad that would otherwise succeed. After its private signal about the project's quality is realized, the rating agency cannot commit to truthfully reveal it. However, the agency faces penalties if a project with a high rating defaults. When the penalties are relatively high the agency cannot afford to misclassify bad projects. However, when the penalties are relatively low the agency does not want to misclassify good ones. In the latter case the agency's commitment problem is alleviated as projects classified as bad will, in fact, be correct, which maximizes the expected penalty for misreporting their signals. When the agency cannot commit to truthfully reveal its signals, it will not want to invest in a more precise technology

[1] In 2015 as part of a settlement agreement S&P's executives admitted that business relationships affected modeling updates.

10.14754/CEU.2017.09

because the additional information is ignored. I connect the results to stylized facts on fluctuations of rating standards.

I develop a model in which a credit rating agency simultaneously chooses an information technology and a disclosure rule. However, it has no means by which it could commit to its choices. In the setup, the information technology is a binary signal that can commit false negative and false positive errors. While the unconditional probability that the technology misclassifies a project is fixed, the agency is free to set the composition of errors. At the extremes, it may choose a technology that only commits false positive errors or one that only commits false negatives. I will refer to the former as the 'tight' technology, since it never misclassifies bad projects. On the other hand, a technology only committing false negatives is referred to as 'loose', because it never misclassifies good projects. Since only those issuers pay the rating fee who are offered good ratings, the agency may find it optimal to loosen its signal or to inflate ratings by assigning high ratings to projects with low signals. Finally, investors fully aware of the incentive structure, bid for rated projects.

The first observation is that by choosing the composition of errors, the rating agency faces a quality-quantity trade-off: if it tightens its technology then good signals will imply higher success rates but the quantity of good signals will decline. In the extremes, projects classified as good by the tight technology will always succeed, but their mass will be low. On the other hand, when the agency chooses the loose technology projects classified as bad will always default and the mass of projects with good signals will be maximized.

Reporting truthfully is in the interest of the agency, since its profits are proportional to the value of information it can credibly convey. When the agency's commitment power is limited due to relatively low levels of the penalty, it has to find the optimal mix of signal errors and a disclosure rule (i.e. level of rating inflating). I show that under some conditions, the agency is generally better off by loosening its signal compared to increasing inflating. Both loosening and inflating increases the quantity and decreases the average quality of financed projects. However, if the signal is sufficiently precise to begin with, then loosening always adds bad and good projects proportionately to the financing pool, while inflating adds relatively more bad projects. In addition, loosening the technology also decreases the temptation to inflate, since the quality of projects classified as bad deteriorate with the looseness of the chosen signal. As a result, at very low levels of the penalty, the loose signal helps the agency honestly disclose its information.

As a main contribution, I show that the technology choice of the agency will be largely driven by its ability to minimize its payable penalties, which helps explain the well documented fluctuation in rating standards. Cheng and Neamtiu (2009) show that after the scandals of Enron and WorldCom rating accuracy and timeli-

10.14754/CEU.2017.09

ness improve among corporate ratings, presumably due to the increased threat of regulatory intervention. Ashcraft et al. (2010) and Griffin and Tang (2012) demonstrate that during the credit boom preceding the subprime crisis, agencies wanted to secure the lucrative rating fees associated with structured deals, which - together with unchanged perceived penalties - led to declining rating standards in the structured segment.

The model predicts that when the agency cannot commit to fully reveal its signals it will not be willing to invest at all into improving its overall signal precision. Generally, improving signal precision by reducing both false negatives and false positives affects agency profits through two channels. First, projects with good signals are more likely to succeed due to increased precision, which provides incentives to loosen the signal/increase inflating, resulting in reduced profits. Second, projects with bad signals are more likely to default due to increased precision, which increases the expected penalty for inflating. In turn, the agency inflates less, which enables it to provide more valuable information. However, when the agency anticipates that it will inflate ratings in equilibrium it chooses the loose technology, which already maximizes the expected penalty for inflating (projects with bad signals will always default). Hence, the second channel is shut down as a result of the agency's decision. Therefore, any improvement in signal precision will exactly be offset by increased inflating, which leads to the result that the agency is not willing to pay for improving signal precision. This is consistent with anecdotal evidence, which suggests that agencies were reluctant to update their credit risk models of structured products during the boom by utilizing new loan data. As the former head of S&P's Residential Mortgage Rating Group testified, "At the same time [early 2003], a data set of approximately 2.8 million loans was collected for use in developing the next version of the model. [...] Analysis [of the new data] suggested that the model in use was underestimating the risk of some Alt-A and subprime products. In spite of this research, the development of this model was postponed due to a lack of staff and IT resources. Adjustments to the model used in 2004, with the identified problems, were not made until March, 2005. To my knowledge a version of the model based on the 2.8 million loan data set was never implemented." (Raiter, 2010).

I also analyze the welfare implications of signal calibration. When the agency wants to avoid the costs associated with observed defaults of highly rated projects it will choose the tight technology. However, this may be suboptimal from a social standpoint if the forgone gain of successful projects is relatively large compared to the losses on bad ones. Interestingly, in such cases policies that try to alleviate the agency's commitment problem by increasing the penalty may be welfare decreasing, as the agency becomes overly tight.

10.14754/CEU.2017.09

2.1.1 An example

Consider the following simple example that illustrates the main intuition. There are four issuers who want to sell their projects. Two issuers have good projects (which always succeed) and two issuers have bad projects (which always fail). Only those projects can be sold, which obtain an A rating from the rating agency who has access to a rating technology. The rating technology is imperfect and it always misclassifies exactly one project. It can misclassify a good project by assigning a bad signal to it (tight technology), or misclassify a bad project by assigning a good signal to it (loose technology). Whenever an A-rated project fails, all players will learn that the agency committed an error, which results in a penalty of c for the agency. The same cost has to be paid by the agency if it mistakenly assigns an A rating to a bad project or if it deliberately inflates the rating. The agency can choose between the tight and the loose technology and can also choose whether to reveal its signals honestly. However, it cannot commit to any of its choices (i.e. investors cannot observe its chosen technology and disclosure rule). My focus is on the following question: how does the agency's choice of technology affect its incentives to reveal its signals honestly?

Suppose the rating fee is f which is paid by issuers who are offered an A rating for their respective projects. By assumption, $f \leq c$, which guarantees that the agency does not want to inflate the rating of a project that fails for sure (but does not rule out inflation if the project to be inflated succeeds with a sufficiently high probability). The tight technology classifies exactly one good project correctly and all others will receive bad signals. If the agency does not inflate ratings, it will give an A rating to one successful project, resulting in a payoff of f. The condition for the agency to report truthfully is that the expected cost of inflating is larger than the rating fee. The expected cost of inflating a single project is $2c/3$ since with probability $2/3$ the inflated project will default. Hence, the condition for truthful reporting is

$$\frac{3}{2}f < c. \qquad \text{(Incentive compatibility with tight technology)} \tag{2.1}$$

Now consider the loose technology. The loose technology classifies both good projects and also one bad project as good. If the agency finds it optimal not to inflate ratings then its payoff is $3f - c$, since all three A rated projects secure ratings, but one of them fails for sure. The incentive compatibility condition for truthful reporting in this case satisfies

$$f < c, \qquad \text{(Incentive compatibility with loose technology)} \tag{2.2}$$

since now the project to be inflated will default for sure. Hence, when the agency'a choice of rating technology influences its incentives to inflate ratings. With the tight technology incentive compatibility is a stronger requirement, since now there is a $1/3$ probability that an inflated project will succeed. On the other hand, with the loose technology the inflated project always fails, which increases the expected cost of inflating.

When the penalty is large compared to the rating fee $2f \leq c$, then the tight technology is always preferred, as it prevents the agency from paying a penalty. If the penalty is lower, $3f/2 < c < 2f$, then the agency still does not want to inflate ratings, no matter the technology it chooses. Since the payoff of the loose technology is larger in this range, it will choose it.

The interesting region is where the incentive compatibility constraint holds for the loose technology but does not hold for the tight technology, $f < c \leq 3f/2$. With the tight technology the agency cannot resist inflating all ratings, leading to a payoff of $4f - 2c$. This is smaller than the $3f - c$, which it earns with the loose technology. Hence, if the equilibrium rating technology satisfies the agency's incentive compatibility, the agency is able to earn higher profits.

Now suppose the agency could purchase an upgrade to its technology that does not misclassify projects, for a cost of I. What is the maximum I that it is willing to pay for the upgrade? With the upgrade the agency's profit changes to $2f - I$ if it does not inflate ratings, which is the case if $f < c$. If the agency previously found it optimal to purchase the loose technology then it will be willing to pay $I < c - f$, where $c - f$ is the revenue gain resulting from the upgrade.

Now consider the interesting case when $c = f$. Now the agency is indifferent between inflating and not inflating the rating of a (certainly) bad project. Also, the agency will be indifferent between obtaining the perfect technology for free and not obtaining it, since $c - f = 0$. Hence, when the agency cannot credibly commit to reveal its information it is not willing to pay any positive amount for an improvement in its information technology.

The model presented below sets the same problem in a richer context. Importantly, the rating fee will be endogenously set in a game where investors form beliefs about the payoff of financed projects, and issuers only pay the rating fee if they are convinced that they will be able to recover it from selling their respective projects. Also, by building on the adverse selection of issuers, the agency will never find it optimal to misreport all ratings, as this would lead to the breakdown of the market.

2.1.2 Related literature

To the best of my knowledge, this is the first paper to analyze a rating agency's trade-off between a tight and a loose technology. I show that when agencies do not face commitment problems, they are likely to choose the tight technology, as it leads to the highest rating standards. On the other hand, when agencies anticipate that in the future they will inflate ratings then they are more likely to choose the loose technology, as this alleviates the commitment problem, should they arise.

I build on the setup proposed by Farkas (2016). In that model, issuers seek favorable ratings from rating agencies in order to market their projects. Rating agencies have access to a rating technology that misclassifies a project with a probability that is independent of the project's true type. Here I relax this assumption in a setup where rating agencies calibrate the rating technology by setting the proportion of false negative and false positive errors, holding the total mass of errors fixed.

The theoretical literature uses various assumptions when it comes to introducing a rating technology. Table 2.1 gives an overview of typical assumptions. In all of these papers the rating technology is not the main focus, implying that the assumptions used to describe the rating technology in these papers are unrelated to the key findings. First, observe that there are two modeling strategies (broadly speaking). The first strategy is the CARA-normal framework, which usually stays tractable. However, in the CARA-normal framework it is very convenient to work with symmetric errors and one would have to sacrifice tractability if errors were not normally distributed. On the other hand, the second strategy is to assume a binary distribution: projects are either good or bad. Here variations include perfect information, independent classification errors and a technology that only commits false negative errors. The latter is usually justified by arguing that an information intermediary cannot convince the issuer of a good project that its project is bad, while it may make the mistake of offering a good rating to a bad project. My contribution here is to investigate the agency's aversion to false positive and false negative errors and understand its efficiency implications.

The closest to this paper is Cohn et al. (2016), who analyze the screening intensity choice of a credit rating agency. In their model, the focus is on the agency's equilibrium (costly) screening intensity in a setting where the technology only commits false negative errors (good projects are never misclassified), while I focus on how the equilibrium composition of errors affects the agency's incentive to reveal its information honestly.

The paper is related to research on persuasion with symmetric information (Brocas and Carrillo, 2007; Kamenica and Gentzkow, 2011), which derive the optimal distribution of signal errors of a sender who wants to persuade a receiver of a certain action. Compared to these papers, the payoff structure here is driven by the credit rating game, and information may be symmetric or asymmetric depending on the agency's technology choice. While I keep the unconditional probability of signal errors constant, I focus on the trade-off between false positive and false neg-

Table 2.1 *Groups of papers by the characteristics of the used information technology.*

Agencies can perfectly identify the project either at no cost or at some fixed cost.	Camanho et al. (2012), Frenkel (2015), Lizzeri (1999), Mathis et al. (2009)[2], Strausz (2005)
Agencies misclassify projects with a probability independent of project type.	Bolton et al. (2012), Farkas (2016), Jeon and Lovo (2011), Opp et al. (2013)
In a CARA-normal framework agencies observe an unbiased signal with a normally distributed error.	Sangiorgi and Spatt (2016), Skreta and Veldkamp (2009)
Technology only produces false negative errors, by misclassifying some bad projects as good.	Bar-Isaac and Shapiro (2013), Cohn et al. (2016), Bouvard and Levy (2013), Fulghieri et al. (2014)

ative errors. Importantly, I do not assume that the agency is able to credibly commit to any given rating technology.

From a broader perspective, the model is related to the literature on optimal prediction models when the outcome is binary (typically crisis/no crisis) and the decision maker's loss function may take a general form over false positives and false negatives. Demirguc and Detragiache (2000) focus on a policy maker, who wants to prepare in time if a banking crisis is to occur, while Alessi and Detken (2011) evaluate indicators for costly asset price booms with various loss functions. Sarlin (2013) uses a generalized loss function to evaluate early warning systems for financial crises. While I also focus on the trade-off between false positives and false negatives with binary outcomes, my main concern is how the choice of the information technology affects an agency's commitment power to honestly reveal its information.

The remainder of the paper is organized as follows. The next section introduces the setup and derives the equilibrium composition of signal errors. Section 3 investigates the agency's willingness to invest into signal precision. Section 4 derives the main welfare implications of signal calibration. Section 5 relates the results to the empirical literature. The final section concludes.

[2] Mathis et al. (2009) have an extension in which the share of false positive errors depends on the distribution of product characteristics.

10.14754/CEU.2017.09

2.2 The Model

2.2.1 Setup

Below I provide a brief summary of the model in Farkas (2016) when there is only a monopolist rating agency. The main novelty here is that at the beginning of the game the agency sets the composition of false positive and negative errors committed by its rating technology.

The players of the game are investors, issuers and a credit rating agency, all risk neutral. There are a unit mass of issuers indexed by $j \in [0,1]$ and each of them has one project, which is either good ($\theta_j = g$) or bad ($\theta_j = b$), where the prior probability of the project being good is π_g. All projects require one unit of investment, and good projects always return $R > 1$ while bad projects always default and return nothing. Thus, a good project has an NPV of $V_g = R - 1$, while a bad project has an NPV of $V_b = -1$. As the following assumption states, a randomly chosen project has negative NPV:

Assumption 1 (Average project has negative NPV)**.**

$$\pi_g V_g + (1-\pi_g)V_b < 0 \iff \pi_g < 1/R. \tag{2.3}$$

Only the rating agency has access to a rating technology that generates noisy signals about project quality. The rating technology works as a hypothesis test, where the null hypothesis is that the project is good and the alternative is that the project is bad. For each project j, the rating agency observes a signal $s_j \in \{a,b\}$. The probability of committing a type I error is denoted by α^I, which means that conditional on a true null (the project is good), we reject it (assign a b signal to it). Similarly, the probability of committing a type II error is denoted by α^{II}, which gives the probability of assigning an a signal, conditional on the projects being bad:[3]

$$\alpha^I = Pr[s_j = b|\theta_j = g] \text{ (type I error, false positives)}$$
$$\alpha^{II} = Pr[s_j = a|\theta_j = b] \text{ (type II error, false negatives)}$$

The agency may set the composition of type I and II errors with the constraint that the total mass of misclassified projects is bounded by below,

$$\alpha \leq \pi_g \alpha^I + (1-\pi_g)\alpha^{II}, \tag{2.4}$$

where α is the lower bound of the total mass of misclassified projects, $\pi_g \alpha^I$ is the

[3] Type I errors are referred to as false positives, since the test mistakingly alarms, while type II errors are referred to as false negatives, as the test fails to find the evidence proving that the project will default.

mass of misclassified good projects and $(1-\pi_g)\alpha^{II}$ is the mass of misclassified bad projects. It immediately follows that when the probability of misclassification is independent of the true type, then errors are symmetric, implying $\alpha = \alpha^I = \alpha^{II}$, which is imposed in Farkas (2016). The condition in (2.4) implies that the rating agency is free to set the ratio of false positive and false negative errors, with the constraint that the overall precision of its signal is limited.[4]

Figure 2.1 illustrates the basic idea of signal calibration. The agency observes noisy signals, hence, it cannot perfectly classify projects as good or bad. However, by calibrating its technology, it can influence the composition of projects that are categorized good and bad. When the agency chooses a loose signal, like on the left panel, then all good projects are correctly classified. However, a large fraction of bad projects are also classified as good. On the other hand, with the tight signal, like on the right panel, no bad projects are misclassified as good. By increasing signal tightness the agency is trading off quality for quantity, as a tighter signal increases the average quality but decreases the total mass of issuers receiving high (a) signals.

Assumption 2.

$$\alpha \leq \min\{\pi_g, 1-\pi_g\} \tag{2.5}$$

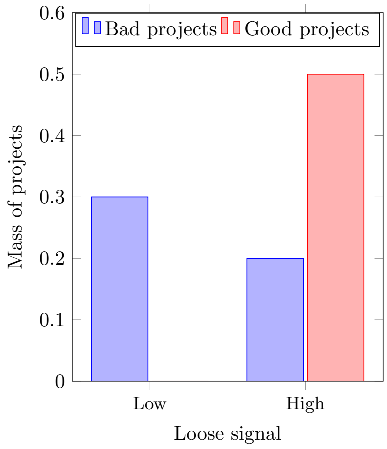
Assumption 2 says that the total mass of misclassified projects is lower than the mass of good projects and also lower than the mass of bad projects. This provides an upper bound for overall signal noise. It will greatly simplify the exposition, as if the agency chooses a corner solution, it guarantees that one of the errors (α^I or α^{II}) is equal to zero. In particular, I will call a technology *tight* when it does not commit false negative errors ($\alpha^{II} = 0$) and *loose* when it does not commit false positive errors ($\alpha^I = 0$).

Upon observing signals generated by its rating technology, the agency offers ratings, $r_j \in \{A, B\}$, based on its disclosure rule. The disclosure rule is characterized by ε, which is the probability of offering an A rating to a project that only has a b signal, while the agency always offers A ratings to projects with a signals.

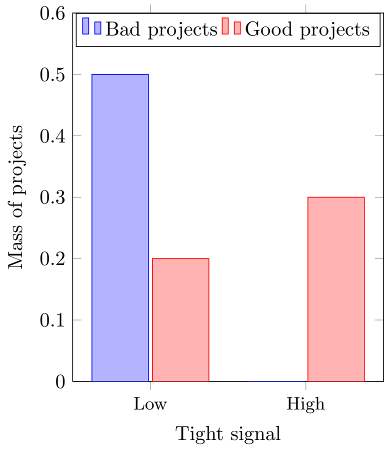
The chosen rating technology has a large influence on the distribution of signals and on the success rates of projects with given signals. Figure 2.2 illustrates these effects. On the left panel the solid (dashed) line shows the mass of issuers obtaining an a (b) signal, as a function of the share of good projects that receive a b signal, α^I, holding the total mass of misclassified projects constant.[5] When $\alpha^I = 0$ the

[4] The model presented here relates to models of rational inattention (Sims, 2003; Yang, 2015). However, instead of analyzing the general information acquisition problem of a rating agency, I focus on the trade-off between false positive and false negative errors, keeping the total mass of errors fixed. While this approach greatly increases tractability (without loss of intuition), it also comes at a cost, as the overall precision of the signal is fixed. In Section 2.3 I analyze the overall precision of the signal as a comparative static exercise.

[5] This implies that (2.4) holds with an equality.



(a) Low signal noise, $\alpha = 0.05$



(b) High signal noise, $\alpha = 0.1$

Figure 2.1 Signal calibration. This figure illustrates how calibrating the technology changes the composition of projects with given signal. For each project the agency observes a noisy signal. In order to classify projects, the agency sets a cutoff value. The left panel shows the extreme example of a loose signal, where every good project receives an a signal, but a large mass of bad issuers also receive a signals. On the right panel the other extreme is depicted. The tight signal only misclassifies some good projects as bad.

rating technology only commits false negative errors (loose technology), implying that every successful project receives an a signal in addition to some bad projects that also receive an a signal. As a result, the mass of issuers with a signals will be the largest at this point. At the other extreme, when the rating technology only commits false positive errors (tight technology), all bad projects receive b signals in addition to some successful projects, that also receive b signals. Hence, in this case the mass of issuers with a signals will be the lowest.

The right panel on Figure 2.2 show how conditional success probabilities, $p_A(\alpha^I, \varepsilon)$ and $p_B(\alpha^I)$, vary with the chosen rating technology. Observe, that the success probabilities depend on the technology choice and on the disclosure rule. The loose technology only produces false negatives ($\alpha^I = 0$), so all projects with b signals are definitely bad, implying that they always default ($p_B(0) = 0$). However, since some bad projects are classified as good, not all projects with a signals succeed. At the other extreme, when the technology is tight, projects with a signals always succeed ($p_A(\alpha/\pi_g, 0) = 1$), since no bad project is misclassified as good.

The quality-quantity trade-off is clear from Figure 2.2. Choosing the loose technology maximizes the quantity of projects with a signals, but leads to the lowest success rate of these projects. To the contrary, the tight technology guarantees that projects with a signals, will, in fact, succeed, but their total mass is the lowest, since the only error the technology makes is assigning b signals to successful projects.

Below I state the Perfect Bayesian Equilibrium of the game.

Definition 1 (Equilibrium)**.** *1. Issuers optimally choose whether to purchase the offered rating, given the rating fee (f) and investor beliefs.*
2. Rating agency optimally sets the fee, signal errors and manipulation levels ($f, \alpha^I, \alpha^{II}, \varepsilon$), given issuers' ratings purchase strategies and investor beliefs.
3. Investor beliefs about success probabilities are correct for all rating combinations.

It is important to emphasize that the agency cannot use any of its choices for signaling, i.e. investors do not observe the rating fee, the chosen rating technology (α^I, α^{II}) and the disclosure rule (ε). While in reality investors do, in fact, possess publicly available information about rating methodologies, they cannot be certain how those methodologies are implemented.[6] Additionally, rating methods may change unexpectedly.[7]

In order to formulate the agency's problem one needs to make a guess (and later verify) issuers' equilibrium strategy. Suppose issuers only consider purchasing A ratings but they never purchase B ratings. Then the agency's problem may be written as

$$\max_{\alpha^I,\varepsilon,f} \left[\mu_a(\alpha^I) + \varepsilon\mu_b(\alpha^I)\right]\left[f - c(1 - p_A(\alpha^I, \varepsilon))\right], \text{ s.t. } f \leq \hat{p}_A R - 1, \tag{2.6}$$

where I have assumed that (2.4) binds, which reduces the technology choice to setting false positives, α^I. The masses of projects with given signals ($\mu_a(\alpha^I), \mu_b(\alpha^I)$), together with the success probability of A-rated projects ($p_A(\alpha^I, \varepsilon)$) all depend on the technology choice, α^I, which captures the tightness of the technology. Investors' belief about the success probability of A-rated projects, $\hat{p}_A$, is formed independently of the agency's strategy, implying that the agency cannot commit to any given technology, α^I, or disclosure rule, ε.

The first-order conditions immediately follow

6 Benmelech and Dlugosz (2009) documents that S&P distributed its CDO Evaluator software that could be used by originators help design their products. However, its output was non binding for S&P which could also consider unmodeled features of a deal. Also, as Langohr and Langohr (2010, p. 191-192) note, "although these programs allow to anticipate some ratings and scenarios, many of the key assumptions and parameters underlying the quantitative models are not made explicit". Griffin and Tang (2012) analyze a large set of CDOs and find that "prior to April 2007, 91.2% of AAA-rated CDOs only comply with the credit rating agency's own AA default rate standard", which also suggests that agencies' are able to deviate from their published methodologies.

7 Langohr and Langohr (2010, p. 191-192) describe a case from 2003 when S&P unexpectedly revised some of its corporate ratings, claiming that it updated its methodologies related to pension liabilities.

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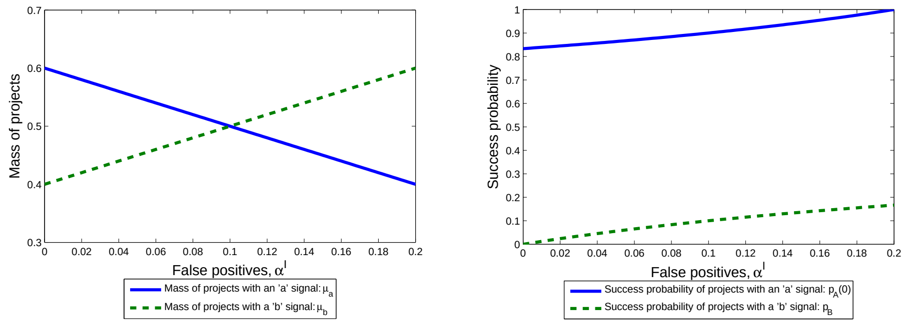

(a) Low signal noise, $\alpha = 0.05$

(b) High signal noise, $\alpha = 0.1$

Figure 2.2 Rating technology and signal characteristics. The left panel shows the mass of issuers obtaining a and b signals as a function of false positives (assuming (2.4) holds with an equality). The right panel shows the success probabilities of projects with a and b signals as a function of false positives (assuming (2.4) holds with an equality). The loose technology corresponds to the case when the share of false positives, α^I, is the lowest, $\alpha^I = 0$, while the tight technology corresponds to the case when it is the highest, $\alpha^I = \alpha/\pi_g$. Other parameters are: share of good projects: $\pi_g = 0.5$; total mass of misclassified projects: $\alpha = 0.1$.

$$\frac{\partial \Pi}{\partial \alpha^I} = \pi_g \varepsilon (c - 2f) \quad \text{and} \quad \frac{\partial \Pi}{\partial \varepsilon} = \mu_b(\alpha^I)[f - c(1 - p_B(\alpha^I))] \tag{2.7}$$

The two first-order conditions are very similar: both of them are linear functions of the rating fee and the penalty. The intuition for this is that inflating has similar effects as loosening the technology: additional projects obtain A ratings. Trading off a false positive error for a false negative error has the following effects. First, due to the additional false positive error the agency loses the rating fee collected from a good project that now receives a b signal. Second, the elimination of a false negative error saves the agency from paying the penalty for providing an A rating to a bad project, but it also reduces profits by the rating fee paid by the bad project's issuer.

Finally, the model is closed by requiring investors to have consistent beliefs about the success probability of A-rated projects:

$$\hat{p}_A = p_A(\alpha^{I*}, \varepsilon^*). \tag{2.8}$$

The condition in (2.8) makes it clear that investors cannot necessarily infer the amount of inflating, since the same rating standard can be achieved by various tightness-inflating combinations.

The following lemma gives the equilibrium.

Lemma 1 (Equilibrium)**.** *Under Assumptions 1 and 2*

(i) Issuers always purchase A ratings and never purchase B ratings.

(ii) Equilibrium technology and inflating satisfy

$$\begin{cases} \alpha^{I*} = \alpha/\pi_g, \;\; \varepsilon^* = 0 & \text{if} \;\; c \geq 2(R-1) \\ 0 \leq \alpha^{I*} < \alpha/\pi_g, \;\; \varepsilon^* = 0 & \text{if} \;\; 2[p_A(0,0)R-1] \leq c < 2(R-1) \\ \alpha^{I*} = 0, \;\; \varepsilon^* = 0 & \text{if} \;\; p_A(0,0)R-1 \leq c < 2[p_A(0,0)R-1] \\ \alpha^{I*} = 0, \;\; \varepsilon^* > 0 & \text{if} \;\; c < p_A(0,0)R-1 \end{cases}$$

(iii) Investor beliefs satisfy $\hat{p}_A = p_A(\alpha^{I*}, \varepsilon^*)$, $\hat{p}_B < p_A(\alpha^{I*}, \varepsilon^*)$, $\hat{p}_\emptyset < 1/R$.

The proof and the exact formulae for inflating and tightness are provided in the Appendix.

It is easy to see that when the penalty is very high, the agency will not inflate ($\frac{\partial \Pi}{\partial \varepsilon} < 0$) and choose the tight technology ($\frac{\partial \Pi}{\partial \alpha^I} > 0$) by setting $\alpha^I = \alpha/\pi_g$. Since the tight technology will never classify bad projects as good, the agency's total penalty is zero. This also implies that the agency can operate with any penalty level.

From Lemma 1 it is clear that as the penalty decreases the agency will respond by first choosing a looser technology and will only inflate if it is already using the loose technology and this still does not lead to sufficient commitment power.

Why does the agency prefer to loosen its technology instead of inflating signals drawn from a tight technology? From Assumption 2 it follows that the signal cannot misclassify more good projects than the total mass of bad projects: $\alpha \leq 1 - \pi_g$. Hence, projects classified as bad will always succeed with a probability smaller than $1/2$. This implies that as the penalty decreases the sign of $\frac{\partial \Pi}{\partial \alpha^I}$ will flip first among the two first-order conditions. At this point loosening leads to higher profits because the additional fee revenue is larger than the increased expected penalty, while inflating would still reduce profits as the expected penalty for inflating is larger than the rating fee. As a result, choosing the tight technology can no longer be an equilibrium outcome because if investors believe that the agency is choosing the tight technology, the agency has an incentive to choose the loose one.

Loosening the technology increases the agency's commitment power to report truthfully because a looser technology implies that projects with b signals are more likely to fail. In the extreme case, when the technology is loose, only bad projects are misclassified as good. Hence, projects classified as bad will always fail, maximizing the expected penalty for inflating.

Figure 2.3 illustrates the results of Lemma 1. The solid line shows the equilibrium value of false positive errors, α^I, as a function of the penalty, while the dashed line shows false positives when technology is restricted to be symmetric (i.e. $\alpha^I = \alpha^{II} = \alpha$), like in Farkas (2016). The solid vertical lines divide the pa-

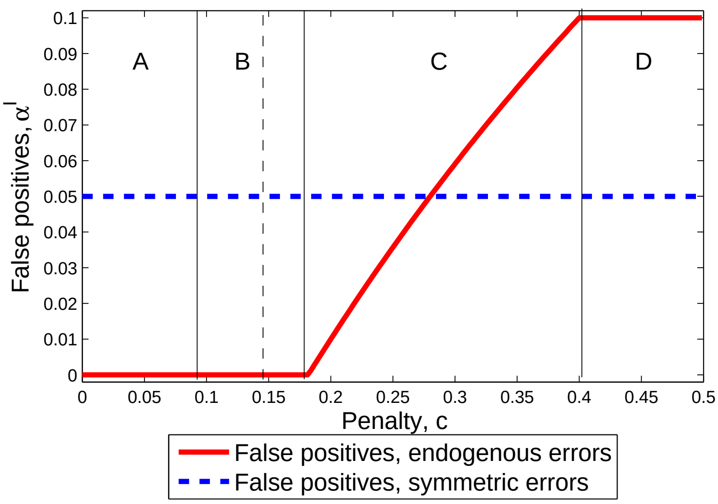

Figure 2.3 Equilibrium technology choice. This figure shows the equilibrium tightness of the technology (solid line) as a function of the penalty, together with the benchmark technology (dashed line). The solid vertical lines divide the parameter space to regions A,B,C and D, which correspond to the cases in Lemma 1. The dashed vertical line indicates the lowest penalty where the agency does not inflate with the restricted technology. Other parameters are: $\pi_g = 0.5$, $R = 1.2$, $\alpha = 0.05$.

rameter space into regions A,B,C and D, which correspond to the four cases of Lemma 1, while the dashed vertical line indicates the lowest penalty where the agency does not inflate with the restricted technology.[8]

The agency only inflates ratings in region A, while with restricted technology inflating starts at higher penalty levels. However, even though the agency does not inflate in region B, it cannot commit to tighter standards, because if investors would believe that the agency is tighter than the loose technology, the agency would find it optimal to choose the loose technology. In region C equilibrium requires $c = 2f$, implying that rating standards will be pinned down by the level of the penalty. Finally, in region D the agency chooses the tight technology, implying that it will not pay any penalties in equilibrium.

[8] See Section 2 of Farkas (2016) for a detailed analysis of the restricted technology case.

10.14754/CEU.2017.09

2.3 The incentives to invest into information

So far I have established how the rating agency trades off false positive and false negative errors. Now I turn to the question how the agency values an improvement in its information technology. Since the efficiency of this market largely depends on the information's quality, it is important to understand the incentives to acquire better information.[9]

An agency's willingness to pay for a more precise signal is captured by the partial effect of the total mass of correctly classified projects on agency profits $\partial\Pi/\partial(1-\alpha)$. When this derivative is close to zero the agency only has a small incentive to increase precision while if it is large then the agency will have a strong incentive to improve its technology.

I first present the results for the restricted technology and then turn to the more general case described above, where the share of false positive and negative errors is endogenized. Usually, increasing signal precision increases agency profits because it increases the information rent of the agency. However, there are exceptions and as I will show below, there could be substantial differences in the willingness to pay for a more precise signal.

2.3.1 Restricted technology

With restricted technology I show that decreasing signal errors, α, always increases profits.

Let Π^r denote equilibrium profits with the restricted technology. Then Π^r satisfies

$$\Pi^r = \begin{cases} \pi_g(1-\alpha)V_g-(1-\pi_g)\alpha(1+c), & \text{if } c(1-p_B)\geq p_A(0)R-1 \qquad (2.9) \\ \mu_a c[p_A(0)-p_B], & \text{if } c(1-p_B)< p_A(0)R-1 \qquad (2.10) \end{cases}$$

There are two regimes in this case to consider, either the agency inflates ratings or it reports its information honestly. In the honest regime the agency's profit equals to the difference between the total NPV of financed projects, $\pi_g(1-\alpha)V_g-(1-\pi_g)\alpha$, and total penalties, $(1-\pi_g)\alpha c$. While in the inflating regime profits will be equal to the difference between the rating fees collected from a-signal issuers, $\mu_a c(1-p_B)$, and their expected total penalty, $\mu_a c(1-p_A(0))$.

In the honest regime the partial derivative of the profit in (2.9) with respect to the total mass of correctly classified projects is

[9] Cohn et al. (2016) analyze this question by explicitly modeling costly information acquisition for a credit rating agency who may misreport its information. However, they restrict the agency's technology to only commit false negative errors.

$$\frac{\partial \Pi^r}{\partial(1-\alpha)} = \pi_g V_g + (1-\pi_g)(1+c), \tag{2.11}$$

which is always positive as $V_g, c > 0$ and $0 < \pi_g < 1$. Thus, in the honest regime increasing precision always results in higher profits for the agency. In this sense, the agency is always willing to pay some positive amount for reducing signal noise.

In the regime with inflating the agency will have a lower willingness to pay for a marginally more precise signal, since it knows that it will be tempted to inflate more if it has more valuable information. Formally, differentiating the profit in (2.10) with respect to signal precision yields

$$\frac{\partial \Pi^r}{\partial(1-\alpha)} = c\pi_g \left[1 + \frac{\mu_a}{1-\mu_a} + \alpha \frac{1-2\pi_g}{(1-\mu_a)^2}\right], \tag{2.12}$$

which is positive as long as $c > 0$ and zero only if $c = 0$. The positive sign is due to increased commitment power: the expected penalty for inflating is higher, because inflated projects default with higher probability if the signal is more precise. Thus, with the restricted technology agencies always strictly prefer a rating technology with lower signal noise.

2.3.2 Calibrated errors

When the agency is able to set the share of false positive and negative errors, it still holds that when the agency does not inflate ratings, decreasing the total mass of misclassified projects (α) increases profits. However, this will no longer hold in the worst state, when the agency inflates in equilibrium.

Let Π^u denote the equilibrium profit of the agency with calibrated errors (where u stands for unrestricted). Then Π^u satisfies

$$\Pi^u = \begin{cases} (\pi_g - \alpha)V_g & \text{if } \ c \geq 2(R-1) \\ (\pi_g - \alpha)c/2 & \text{if } \ 2[p_A(0,0)R-1] \leq c < 2(R-1) \\ \pi_g V_g - \alpha(1+c) & \text{if } \ p_A(0,0)R-1 \leq c < 2[p_A(0,0)R-1] \\ \pi_g c & \text{if } \ c < p_A(0,0)R-1 \end{cases} \tag{2.13}$$

It is easy to see that the partial derivatives of Π^u with respect to $(1-\alpha)$ will be positive for the regions where the agency does not inflate and equal zero when the agency does inflate in (2.13). When the agency inflates in equilibrium improving its technology will only induce it to inflate even more. In particular, since the loose technology is chosen in such a case, the agency already maximized the expected penalty for inflating. Hence, improving the technology cannot alleviate the commitment problem.[10]

I state these findings in the following proposition.

Proposition 1. *In the equilibrium described in Lemma 1, increasing signal precision* $(1-\alpha)$ *leaves agency profits unchanged if*

$$c < \frac{\pi_g}{\pi_g + \alpha} R - 1 \tag{2.14}$$

Proposition 1 implies that when the agency cannot avoid rating inflation, it will have no reason to invest into information acquisition. This is plausible, as it suggests that when the agency has a strong incentive to inflate ratings, it does not make sense to spend on information that will be ignored. On the other hand, it is also surprising, because the agency's only source of profit is its information rent.

Figure 2.4 illustrates this result. Panel (a) shows the distribution of project types over signals. Since the agency chooses the loose signal in region A, only some bad projects are misclassified as good. Panel (b) shows the distribution of project types over ratings. The shaded mass of bad projects captures rating inflating. Panels (c) and (d) illustrate the effects of increased signal precision. The agency still chooses the loose technology, which now commits fewer errors due to increased precision. However, increased precision is exactly offset by increased inflating as the distribution of project types over ratings is unchanged.

Observe that when the loose signal is chosen, inflating has the same effects as further loosening the technology (increasing α^{II} while keeping α^{I} constant). Both results in assigning an *a* signal to an additional project that is bad with certainty. In turn, loosening and inflating are equivalent tools for adjusting the value of an *A* rating. This also implies that the equilibrium derived in Lemma 1 can also be rewritten without inflating, as the agency only chooses to inflate when its equivalent to loosening. Hence, Lemma 1 can be thought of the case when (2.4) is a binding assumption, i.e. if in the short run the agency has to keep its overall signal precision constant.

2.3.3 Comparing results

Figure 2.5 illustrates the results. Similarly to Figure 2.3 the solid vertical lines divide the parameter space into regions based on Lemma 1. Additionally, the dashed vertical line indicates the lowest penalty level, where an agency with the restricted

[10] Note that in this region further loosening the technology has the exact same effect as increasing inflating. Both result in giving an *A* rating to a bad project. This also implies that the constraint (2.4) will not be binding: marginally increasing the mass of misclassified projects is exactly offset by lower inflating.

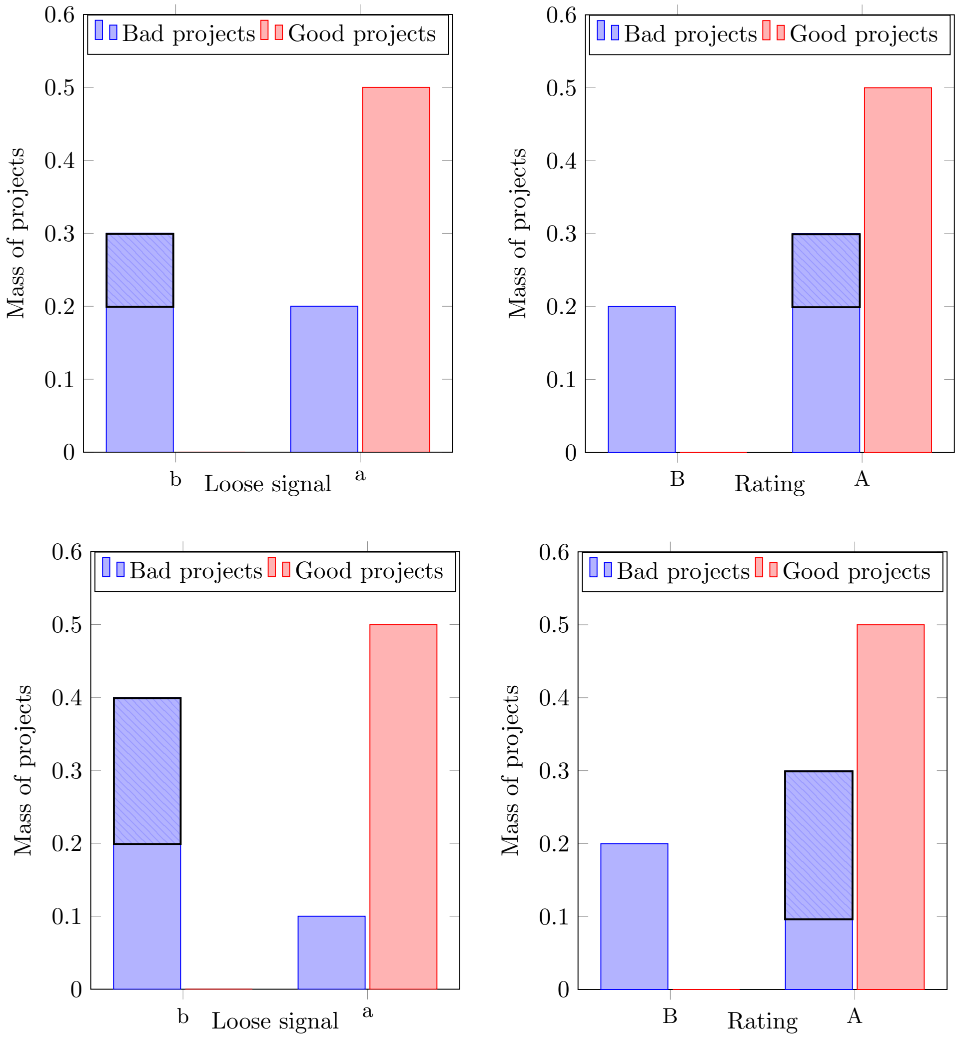

Figure 2.4 Improving signal precision in equilibrium region A. Panels (a) and (b) show how signals are transformed into ratings if the agency chooses the loose signal and also inflates in equilibrium. The shaded part of the bars represent the mass of inflated projects. Panels (c) and (d) show the implications of increasing signal precision.

technology does not inflate ratings. The solid (dashed) line shows the partial derivative of equilibrium agency profits with respect to the total mass of correctly classified projects, when technology is unrestricted (restricted) as a function of the penalty. A higher derivative indicates that the agency is willing to pay more for a marginal reduction in its signal's noise.

10.14754/CEU.2017.09

Depending on the penalty and whether technology calibration is possible, there could be substantial differences between the willingness of an agency to improve its precision. As a general tendency, a lower penalty leads to decreases in the returns to increased precision through two channels. First, an agency saves less on the penalty associated with false negative errors. Second, a lower penalty gives an incentive for increasing inflating which makes additional information less valuable for the agency. An exception to this rule happens when the agency chooses the loose technology but does not inflate (region B): in such a case the willingness to invest may be much larger, which reflects the result, that marginally improving signal precision in this region does not lead to a behavioral response by the agency (i.e. loosening or increased inflating).

The discontinuities in the slopes correspond to regime changes. With the restricted technology there is only one discontinuity, which corresponds to the point where the agency starts/stops inflating ratings. The slope jumps because the effect of increased signal precision is partially offset by increased inflating to the left of the vertical dashed line. With calibrated errors there are two jumps. First in region A the slope is zero since with the loose technology the agency can substitute one-to-one every misclassified bad project with an inflated one. In region B the agency still cannot commit to tightening standards but increasing precision does not lead to a behavioral response. In region C rating standards are pinned down by investor beliefs which implies that increasing precision forces the agency to loosen its technology. Finally, in region D the agency's profit increases by the NPV of a good project, since in this region reducing noise is identical to reducing false positives as the agency chooses the tight technology.

2.4 The welfare implications of signal calibration

Is it beneficial for welfare if an agency can freely set the composition of its errors (compared to the restricted technology that commits errors symmetrically)? Welfare is measured by the total NPV of financed projects normalized by the total NPV of successful projects. I assume that penalties paid by the agency is not considered waste, but it is collected by a regulator that distributes it among risk neutral consumers, who have constant, unit, marginal utility for money.[11] Let W^u (W^r) denote the welfare with calibrated (restricted) errors. The following proposition summarizes the relation between W^u and W^r.

[11] At the other extreme, one may assume that the penalty is complete waste. In that case welfare equals to the agency's profit, since other players earn zero profit in expectation.

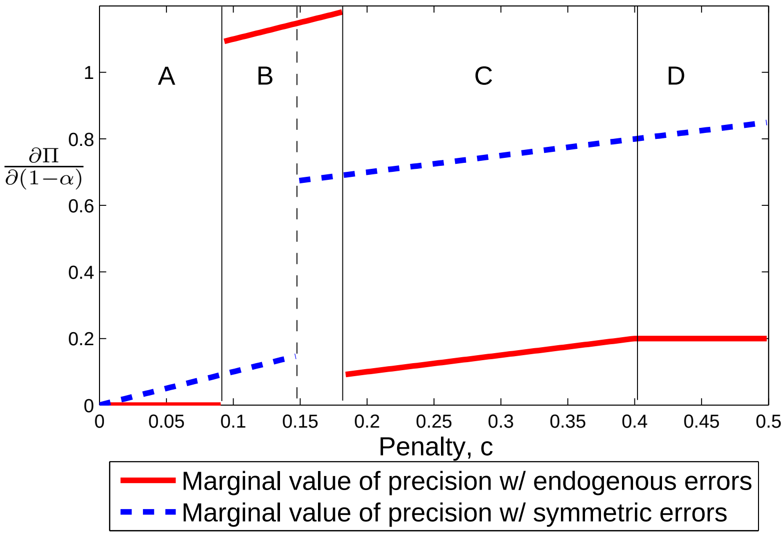

Figure 2.5 Incentives to improve signal precision. This figure shows how agency profits respond to a marginal increase in the mass of correctly classified projects: $\partial\Pi/\partial(1-\alpha)$. The dashed line shows the derivative for symmetric (restricted) technology while the solid line shows the derivative for the endogenous (calibrated) technology. The solid vertical lines divide the parameter space to regions A,B,C and D, which correspond to the cases in Lemma 1. The dashed vertical line indicates the lowest penalty where the agency does not inflate with the restricted technology. Other parameters are: $\pi_g = 0.5$, $R = 1.2$, $\alpha = 0.05$.

Proposition 2. *Suppose the agency with symmetric errors chooses to provide ratings.*

(i) *If* $c < p_A(0,0)R - 1$, *then* $W^u \geq W^r$
(ii) *If* $\alpha^{I*} \geq \alpha$ *and* $V_g < 1$ *then* $W^u > W^r$
(iii) *If* $\alpha^{I*} \geq \alpha$ *and* $V_g \geq 1$ *then* $W^u \leq W^r$

The proofs are straightforward. Generally, loosening the signal has two effects on welfare: first, it increases the agency's commitment power to honestly reveal its signal, as projects classified as bad with a looser signal are more likely to fail. Second, an additional bad *and* good project is financed. While the first effect is generally beneficial for welfare, the second effect may be harmful if losses on bad projects (which equals -1) are large compared to the gains on good projects (which equals $R-1$). The overall effect of signal calibration depends on two parameters: the level of the penalty, c, and the relative gain of good projects compared to the losses of bad projects, R.

If the penalty is sufficiently low, allowing an agency to calibrate its signal is always beneficial, as it increases commitment power when it is most needed. However, if the penalty is higher, such that the agency does not find it optimal to inflate, then allowing for calibration may lead to lower welfare. High penalties provide an incentive for the agency to choose the tight signal, however, if the gains of good projects are relatively large ($R > 2$), this is inefficient from a social standpoint, as committing false negative errors is less costly than committing false positives.

Figure 2.6 illustrates the results. Like above, solid vertical lines divide the parameter space into regions A, B, C and D, which correspond to the regions of Lemma 1 while the vertical dashed line notes the lowest penalty level at which the agency with symmetric errors does not inflate ratings. The solid (dashed) lines show equilibrium welfare as a function of the penalty when technology is endogenous (symmetric). The left panel illustrates an example when gains on good projects are smaller than losses on bad ones ($R < 2$) while the right panel shows an example for the complementary case, when $R > 2$.

Signal calibration may have important implications for welfare. First, in region A, allowing the agency to calibrate always increases welfare as the agency opts for the loose signal which maximizes its commitment power, leading to more informative ratings. In region B the agency chooses the loose signal and does not inflate. This may be an inferior outcome compared to symmetric errors if gains on good projects are small relative to losses on bad ones (left panel of Figure 2.6) as the loose signal only commits the relatively (and socially) more expensive error. However, if gains are large compared to losses, the loose signal is, in fact, welfare maximizing (right panel of Figure 2.6).

Observe, that welfare measures always cross in region C. The crossing point corresponds to the agency choosing symmetric errors in equilibrium, implying $\alpha^I = \alpha^{II} = \alpha$. At this point the agency finds it optimal to choose the symmetric technology, and because there is no inflating, welfare levels coincide.

The non-monotonicity of welfare suggests that policies aiming at enforcing the agency's commitment could be welfare decreasing. This is a surprising result given that with symmetric errors increasing the penalty is never welfare decreasing, as long as the agency is able to reach a nonnegative payoff. However, with calibration, the agency does not only respond by decreasing inflating but also by tightening its technology. But a tight technology is not socially desirable when gains on good projects are large compared to losses on bad ones (e.g. the right panel of Figure 2.6). Nevertheless, allowing for calibration enables the agency to operate even at very high penalty levels. The model demonstrates why the task for policy makers is complex. In particular, they need to design an environment in which a rating agency internalizes both types of errors. While false negative errors are observed,

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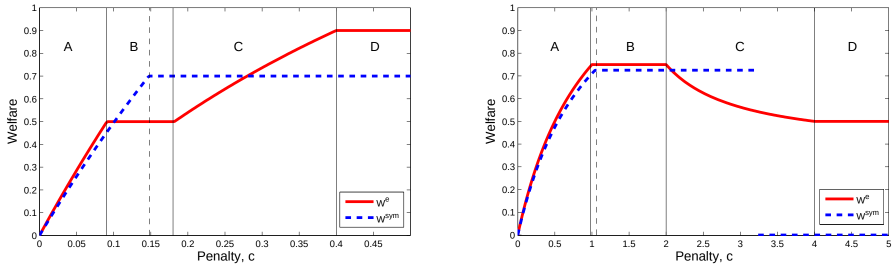

(a)
Low gains compared to losses:
$R = 1.2$, $\pi_g = 0.5$

(b)
High gains compared to losses:
$R = 3$, $\pi_g = 0.1$

Figure 2.6 Signal calibration and welfare. This figure shows how welfare changes with a monopolist agency if the agency endogenously sets the composition of errors (W^e) and when the agency is restricted to use a technology with symmetric errors (W^{sym}). Welfare is measured as the total NPV of financed projects, normalized by the total NPV of successful projects. On panel (a) the gains from good projects is low compared to losses on bad ones while on panel (b) the gains are relatively large compared to losses. Solid vertical lines divide the parameter space into regions A,B, C and D, while the dashed vertical line corresponds to the lowest penalty that is consistent with no inflating for the agency using the symmetric technology. Other parameters are: $\alpha = 0.05$.

false positive errors often stay unobserved, as issuers may decide not to undertake good projects if they are only offered low ratings.

2.5 Discussion

The strongest prediction of the model is that a rating agency can easily adjust its rating standards as a response to changes in its business environment. Moreover, changes in rating standards need not imply changes in the disclosure rule (i.e. inflating), but may reflect changes in how the agency calibrates its rating models.

It has repeatedly been argued that conflicts if interests in the structured segment were the most severe. The increasing demand for safe assets (Bernanke et al., 2011), the housing boom and the boom in structured finance (Coval et al., 2009b) all pressured rating agencies into assigning AAA ratings to an ever higher fraction of the deals. Hence, the model of this paper predicts that in such an environment agencies would prefer to have a technology that only commits errors of assigning AAA ratings to a higher than deserved share of a deal, thereby securing the relatively large rating fees.

The empirical literature has demonstrated that rating agencies failed to incorporate useful information into their credit ratings of structured products. Consistent with the proposed model, in each case the omitted information would have resulted in more false positives and less false negatives (i.e. tighter standards). Ashcraft et al. (2010) show that the detail of loan documentation and borrowers' FICO scores helps predict defaults of mortgage-backed securities, even after controlling for credit ratings. In terms of the model presented above, this can be interpreted as deteriorating rating standards since as time passed, the average FICO score of loans declined and the share of no/low documentation loans increased (Demyanyk and Van Hemert, 2011). As agencies chose to ignore these developments, rating standards declined. Coval et al. (2009a) show that AAA tranches of CDOs have a payoff similar to those of bonds that only default in the worst economic state (labelled "economic catastrophe bonds"). Nevertheless, their prices and ratings did not signal their systematic risk. Again, adjusting ratings by systematic risk would have lowered the size of the AAA tranches, leading to more false positive errors. Similarly, as Coval et al. (2009b) points out, parameter uncertainty was not given sufficient consideration. They demonstrate that ratings are very sensitive to modeling assumptions, especially for CDOs that have other CDOs in their collateral pool. Building parameter uncertainty into the models would have resulted in tighter standards, consistently with the prediction that agencies wanted to avoid false positives.

The model suggests that rating standards may continuously vary with the commitment power of the agency through model calibration. There is empirical evidence suggesting that rating standards significantly vary over time and in the cross section. Among others, Alp (2013) and Baghai et al. (2014) find large variation of rating standards over time in the corporate segment. Cheng and Neamtiu (2009) show that after the scandals of Enron and WorldCom rating accuracy and timeliness improve after 2002 among corporate ratings, presumably due to the increased threat of regulatory intervention. Ashcraft et al. (2010) and Griffin and Tang (2012) demonstrate how rating standards declined in the structured segment during the recent credit boom preceding the crisis, during a period in which the commitment of agencies was undermined due to lucrative rating fees.

The model also predicts that when agencies anticipate commitment problems, they do not have an incentive to invest into signal precision. This is in line with anecdotal evidence, which suggests that agencies in the structured segment were reluctant to invest into their credit rating models during the boom. In particular, agencies were reluctant to update their modeling assumptions by analyzing new loan data. It is safe to say that this inaction also decreased standards as the new data, coming from the boom period would have suggested higher default rates (i.e. boom in subprime lending) and higher default correlations (i.e. house prices appreciated nationwide) than used by rating agencies.[12,13] Importantly, this result requires agencies to freely set the composition of their errors.

While not modeled here explicitly, the model can reconcile why a rating agency with tighter standards might be more inclined to cater ratings in a duopoly, i.e. offer a higher rating if the other agency's assessment is more favorable. The incentive to cater is driven by the technology choice: with tight technologies catering is very attractive as it is certain that the other agency's more optimistic assessment is correct, while with loose technologies catering results in the financing of only bad projects. Griffin et al. (2013) show that in the structured segment preceding the subprime crisis S&P had somewhat tighter standards than Moody's and it also carried out larger upward adjustments when Moody's model implied ratings were more optimistic. In the model, taking rating technologies given, the amount of catering across agencies is influenced by two effects. First, the agency with tighter standards is likely to wrongly assign a bad signal to a good project, which makes it likely that the more optimistic agency is right if they were to disagree. On the other hand, having tighter standards convinces the other agency to cater ratings if they disagree on fundamentals. Until the agency's technology with the tighter standards is not too tight, the former effect will be stronger and the tighter agency has a stronger incentive to cater.

2.6 Concluding remarks

This paper analyzes the trade-off a rating agency faces when it calibrates the share of false positive and false negative errors committed by its information technology. The tractable framework makes it possible to investigate the drivers of the rating technology choice.

A monopolist rating agency may flexibly respond to changes in its business environment by calibrating its rating technology. While calibration does not necessarily

[12] As the former head of S&P's Residential Mortgage Rating Group testified, "Adequate staffing was not the only challenge faced in trying to maintain the quality of the rating process. The accuracy of the predictive models used to evaluate risk was also critical to the quality of the ratings. The version of LEVELS model developed in 1996 was based on a data set of approximately 250,000 loans. It was, I believe, the best model then used by a rating agency. As new models were programmed and tested, analysts continued to collect larger data sets for the next versions of the model. In late 2002 or early 2003, another version of the model was introduced based on approximately 650,000 loans. At the same time, a data set of approximately 2.8 million loans was collected for use in developing the next version of the model. By early 2004 preliminary analysis of this more inclusive data set and the resulting econometric equation was completed. That analysis suggested that the model in use was underestimating the risk of some Alt-A and subprime products. In spite of this research, the development of this model was postponed due to a lack of staff and IT resources. Adjustments to the model used in 2004, with the identified problems, were not made until March, 2005. To my knowledge a version of the model based on the 2.8 million loan data set was never implemented." Raiter (2010)

[13] "Moody's advised the Subcommittee that, in fact, it generally did not obtain any new loan data for its RMBS model development for four years, from 2002 until 2006, although it continued to improve its RMBS model in other ways." U.S. Senate (2011)

10.14754/CEU.2017.09

have a large influence on the overall informativeness of its information technology, it does affect the agency's payoff, rating standards and social welfare. In addition, the model is able to explain why agencies who do not report truthfully in equilibrium will be reluctant to improve their information technology.

It would be a fruitful exercise to investigate how increasing competition between rating agencies affects their choice of technology. In particular, agencies can only extract rents if they can avoid competing in rating fees. Choosing a looser technology could be optimal for agencies as a response to increased competition, as it maximizes investors' valuation for an additional high rating. However, this is left for future work.

10.14754/CEU.2017.09

3

Individual Investors Exposed

3.1 Introduction

The popularity of bank issued retail structured products increased significantly in the recent decade. In Germany, where these products are most popular, the total outstanding value of the exchange traded products (in Stuttgart and Frankfurt) was over EUR 68 billion in March 2016.[1][2] The purpose of structured products is to liberate the retail investor by providing access to a wide range of derivative products, which was only available for institutional investors previously. As it is often argued by banks, individual investors have limited access to derivatives markets because they cannot afford the costs of maintaining a margin account. Therefore, banks designed structured products such that investors cannot lose more than the money they invest, which makes it very convenient for them.

We show in a simple model that investors' aggregate position is influenced by the menu of available products. Our focus is on exchange traded call and put knock-out warrants, because they are popular among individual investors who seek directional bets. If investors allocate their funds randomly between calls and puts then their aggregate position will depend on the relative leverage of the offered call and put warrants. By construction, the leverage of calls will be higher than the leverage of puts after recent declines in the underlying. Hence, investors will take a long position after declines in the underlying and a short position after increases in the underlying, on average. This behavior is equivalent to betting on price reversals. We present supporting empirical evidence for our predictions. Using a unique, proprietary data set obtained from a bank, we are able to compute the aggregate position of retail investors who hold knock-out warrants. We speculate that this might be

[1] Deutsche Derivative Verband (2016)
[2] In order to put this number in perspective, consider the fact that the total market capitalization of the Deutsche Brse is in the range of EUR 1000 billion. Also, Célérier and Vallée (2017) show that the total amount of sold structured products (including non-exchange traded) since 2000 is over 2 trillion euros.

beneficial for banks' liquidity management if banks acz as market makers on the underlying asset's market.

A three period model is presented in which all players are non-strategic. Individual investors act as noise traders who randomly choose between taking a long or a short position. Also, their position is held for a random duration. A single call and a single put knock-out warrant is offered by a bank, who sets fair prices for these derivatives. Since introducing new warrants is time consuming, we take product characteristics to be exogenous and focus on the short run. Our main concern is how the characteristics (e.g. leverage) of the available warrants affect investors' aggregate position.

We focus on a narrow subset of exchange traded structured products, called knock-out warrants (also known as leverage/turbo certificates). Knock-out warrants are barrier options, that expire before their expiry date if the underlying's price reaches a prespecified barrier. Since barrier options have some characteristics that resemble futures contracts, banks often advertise knock-out warrants as futures contracts and investors use these to carry out speculative bets. A distinctive feature of knock-out warrants is that their leverage sharply varies with the underlying's value. When the underlying appreciates the leverage of a call (put) warrant decreases (increases).

Our model suggests that investors will take aggregate positions like they are betting on price reversals, despite the fact that they allocate funds randomly. When the call has a higher leverage than the put, randomly allocated funds result in a long position, because investors will gain more on the call than lose on the put if the underlying appreciates. Since the leverage of the call will be larger after recent declines in the underlying's value, investors will tend to have a long position after recent contractions. Importantly, if investors could access more standardized markets (i.e. futures), their position would not systematically vary with the underlying's price, which is consistent with how the literature usually thinks about noise traders.[3]

We provide supporting empirical evidence for the model's predictions. Using transaction level data obtained from a bank[4], we are able to track individual investors' aggregate holdings of knock-out call and put warrants that have the German DAX index as their underlying. We show that investors tend to increase their long exposure when the calls have higher leverage than the puts. In turn, investors tend to have a net long (short) position after recent declines (increases) in the DAX.

Figure 3.1 in the Appendix illustrates how investors' aggregate position changes together with the asymmetry in the leverage of offered calls and puts during our

[3] E.g. see Kyle (1985) or Stambaugh (2014) for a more recent example.
[4] Our bank wishes to remain anonymous, therefore we will not disclose its name or the respective stock exchange where the transactions took place.

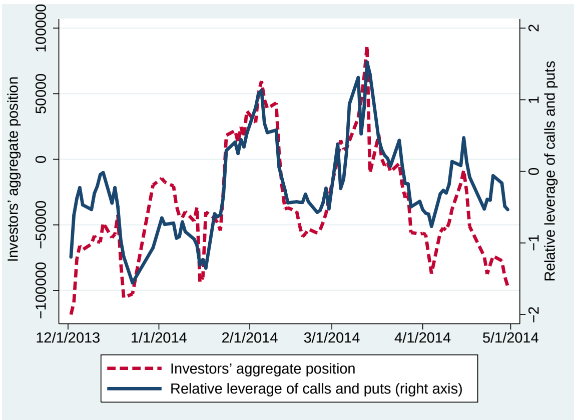

Figure 3.1 Relative leverage of offered call and put warrants and investors' aggregate position. Relative leverage (solid line, right axis) is defined as the difference between the Log leverage of the highest levered call and the Log leverage of the highest levered put, where leverage is the absolute percent change in the warrants value if the underlying changes by one percent. Investors' aggregate position (dashed line) is the difference between the total number of call contracts and put contracts investors hold at the end of the day.

sample. The correlation is surprisingly strong. When the leverage of the offered call warrant is large relative to the leverage of the offered put warrant, investors are likely to hold more call contracts than put contracts, which implies that they take an aggregate long position.

The above mechanism would not be relevant if there were only minor differences between the leverage ratios of offered knock-out calls and puts. However, the leverage of the knock-out warrants are very sensitive to the value of the underlying. In particular, as the underlying approaches a warrant's strike price, its leverage explodes.

Similarly to vanilla options, an issuing bank could theoretically sustain a symmetric menu of knock-out warrants but because issuing a new product is costly and takes some time this will not be the case in practice. Intuitively, after a change in the DAX index, the available vanilla options are roughly symmetric: there will be both calls and puts that are either in-the-money, out-of-the-money and near at-the-money. On the other hand, barrier options studied here can only be in-the-money, otherwise they would be knocked-out. Hence, after an increase in the DAX index a bank would want to issue a new knock-out call in order to offer a cheap warrant for investors who would like to bet on further increases in the DAX. However, since issuing a new product can be costly and definitely requires time[5], the bank will

[5] Informal discussions with industry experts revealed that administrative costs associated with new product

10.14754/CEU.2017.09

wait until the increase of the DAX is meaningful enough to justify the issuance of a new call. These delays in the issuing process often lead to a state where there is a significant difference between the leverages of knock-out calls and puts, as Figure 3.1 illustrates.

As a first order approximation, the profitability of issuing knock-out warrants depends on the margins banks charge (the difference between a warrant's ask price and its theoretically fair value) and banks hedging costs.[6] However, banks may participate in the underlying's market through multiple venues, which may affect their hedging costs. For example, if banks act as market makers on the underlying's market, then selling knock-out warrants to individual investors would decrease the banks' overall hedging costs. To illustrate, suppose banks mitigate selling pressure on the underlying's market by purchasing from the underlying during price decreases. Simultaneously, if investors increase their long position during price decreases then this provides a natural hedge for the banks' market making activities. Hence, by transmitting the order flow from the underlying's market to the warrant exchange, the bank would be able to save on its hedging costs.

3.1.1 Related literature

To the best of our knowledge, this is the first paper to analyze how the menu of available products may influence the aggregate position of individual investors. Banks offer knock-out warrants that enable individual investors to carry out directional bets, both long and short, in a convenient way. We show in a model and provide empirical evidence that if investors allocate their funds randomly across long and short positions, the asymmetric leverage of offered products will lead investors to speculate on price reversals.

The theoretical literature has used various assumptions to describe the behavior of individual investors. In seminal papers, noise traders introduced into rational expectation equilibrium models are often associated with individual investors who trade randomly due to exogenous reasons, like liquidity shocks (Kyle, 1985; Glosten and Milgrom, 1985). While these papers' main concern is market liquidity, we focus on individual investors' aggregate position when they behave as noise traders, who randomly allocate their funds.

A group of theoretical papers have emphasized the role of investors' heterogeneous beliefs in financial markets. See Xiong (2013) for a recent survey. This research agenda's main aim is to understand the asset price implications of heterogeneous beliefs. While we only focus on investors' aggregate position, hetero-

issuance could be significant at smaller exchanges. Similarly, the time required to introduce a new product varies significantly between exchanges.

[6] Wilkens and Stoimenov (2007) reports that knock-out certificates, on average, sell at premium of 2-6% relative to upper boundaries computed from option prices.

geneous beliefs arise naturally, since while some investors hold long speculative positions, others hold short positions simultaneously. Hence, belief heterogeneity offers a plausible microfoundation for investor behavior in our setting.

Proposed theories of the disposition effect provide complementary mechanisms that are consistent with our findings. According to the disposition effect, investors have a larger propensity to sell stocks from their portfolio that were recent winners compared to stocks that were recent losers (Odean, 1998). This is consistent with investors betting on reversals: after recent appreciations of the underlying the value of calls increases, while the value of puts decreases. Hence, if investors have a larger propensity to close positions that secure gains, they will sell the calls, which is equivalent to betting on a price reversal. For example, the theory of realization utility proposed in Barberis and Xiong (2012) assumes that investors gain an extra utility from selling assets that have appreciated during the holding period. This naturally leads to the disposition effect and is also consistent with our findings.

Theoretical papers analyzing the strategic behavior of banks who issue innovative financial assets for individual investors is scarce. Kondor and Kőszegi (2017) build a model in which investors do not take into account that a bank has private information when it wants to sell an innovative asset to them. Compared to their model we only focus on knock-out warrants and analyze how the menu of warrants affects investor exposures. By focusing of short run effects we abstract from banks' strategic behavior, which would be a natural (though not trivial) next step.

The closest to our paper in terms of the type of data used is Baule (2011) who merges quote data with transactions data in order to recover the order flow of investors. However, Baule (2011) focuses on discount certificates and investigates how banks use anticipated order flow in setting their quotes. The German tax code is exploited for identification, which favors investments held over a year and find that investor demand is high for products that mature just over a year. In contrast, we focus on highly leveraged products, which are typically held for much shorter periods (often within day) and presumably for the reason to speculate on directional movements. Importantly, investors can only open long positions using discount certificates, while in our setting investors may just as easily open short positions. Thus, our design enables pessimists to easily participate which is better suited to analyze investors' aggregate speculative position.

In a broader sense, our interest in investor behavior and product design relates to the literature on how individual investors affect asset prices. Ben-David and Hirshleifer (2012) documents that investors are more likely to sell assets that gained or lost a significant amount since purchase, compared to ones that hardly changed in value. An (2016) builds on this pattern and shows that stocks with both large unrealized gains and losses produce abnormal returns. It is argued that this is a result of individual investors exerting selling pressure on these stocks, due to their large movements. We also find some indirect evidence in line with these findings. In particular, trading activity concentrates to the most levered warrants, which have the most volatile returns. While we do not address possible asset pricing implications of our findings, if banks hedge their open positions originating from the warrants market on the underlying's market, investor behavior could be transmitted.

There is a growing literature that analyzes the pricing of structured products. The dominating idea in this literature is the "life-cycle hypothesis". According to the life-cycle hypothesis issuers quote prices that contain higher margins at the beginning of the product's life (when investors are more likely to purchase the products) and decrease the margin during the product's life. Since many investors resell the products to banks before maturity, the banks profit from decreasing the margins over time. The reason banks are able to do this is that short selling of structured products is not allowed. The idea of a life cycle effect was put forward by Wilkens et al. (2003). Wilkens and Stoimenov (2007) study the pricing of leverage products similar to the ones in the current paper. They find that quoted prices significantly exceed upper hedging boundaries and also theoretical values. Henderson and Pearson (2011) provides evidence suggesting significant overpricing for retail structured products in the U.S. Compared to these papers we assume that warrants are priced fairly, and since we focus on the asymmetries of offered products in a static framework, the dynamics of margins should not play a significant role here.

Banks have been very active in designing innovative products in order to attract individual investors' attention. This started a literature that investigates the complexity and performance of these products. Célérier and Vallée (2017) show that as yields decreased during recent years banks offered more complex products in order to attract investors with lucrative headline returns. Compared to them we focus on a standardized segment of retail structured products and focus on how individuals use these to place speculative bets.[7]

The rest of the paper is organized as follows. The next section introduces the model for individual investors' aggregate position and derives our testable hypotheses. Section 3 presents our empirical evidence. In section 4 we discuss alternative mechanisms that could lead to similar observations. The final section concludes.

3.2 A model of individual investor positions

Consider a three period binomial tree, where $t \in \{0,1,2\}$. There is a single risky asset, which has a value of S_t at t. Between periods the risky asset either increases with a factor of $u > 1$ or decreases with a factor of $d < 1$. For the sake of exposition,

[7] While knock-out warrants are standardized, they are also difficult to be correctly appraised by individual investors. However, we do not address pricing in this paper.

10.14754/CEU.2017.09

we are going to work with an equal probability tree, implying that the probability of moving up and down the tree between periods is always $1/2$. These probabilities also coincide with the risk-neutral measure, implying that $1 = (d+u)/2$. We normalize $S_0 = 1$. As a result, $S_1 \in \{u,d\}$ and $S_2 \in \{u^2, ud, d^2\}$.

There are individual investors who cannot trade directly with the underlying but can purchase warrants from a bank, who prices the warrants fairly. Trading between the bank and investors takes place in periods $t = 0$ and $t = 1$, while in period $t = 2$ the warrants expire. Those purchasing warrants at $t = 0$ may resell their warrants to the bank at $t = 1$ or hold their warrants until maturity (until $t = 2$). There are two warrants offered by the bank: one knock-out call, and one knock-out put.

Investors randomly choose between taking a long or a short position, and pick assets accordingly. Similarly, they hold on to their chosen assets for a random duration. At the beginning of trading periods $(t = 0, 1)$ n investors arrive, each endowed with m money to invest. Each investor independently decides whether to take a long position or to take a short position, where the probability of going long is 1/2. If an investor chooses to take a long position, she will spend her money on knock-out calls, otherwise she will purchase knock-out puts. Additionally, a fraction δ of investors who bought warrants during $t = 0$, resell their warrants to the bank during $t = 1$.

Here is the timing of events:

$t = 0$

1. S_0 is revealed.
2. Trading takes place: n investors arrive, who individually randomize between taking a long or a short position using their funds, m.

$t = 1$

1. S_1 is revealed.
2. Trading takes place: those who purchased securities in period 0 resell them with probability δ. Additionally, n investors arrive, who individually randomize between taking a long or a short position using their funds, m.

$t = 2$

1. S_2 is revealed.
2. Securities mature and are settled.

In the remainder of this section we will first analyze investors' trading activity and aggregate position if they would have access to the futures market. This will serve as a useful benchmark. Second, we turn to the case where investors are faced with the two products offered by the bank.

10.14754/CEU.2017.09

3.2.1 The futures market as a benchmark

Consider a futures contract that expires at $t = 2$. The futures contract will serve as a benchmark, which allows us to analyze how investors would trade with futures if they had access to futures markets. Importantly, we assume that a risk neutral market maker is able to take the other side of investors' net order flow, implying that individual investors' aggregate demand does not affect prices.

The futures market offers investors the opportunity to take long or short positions in the underlying risky asset. The value of a futures contract at t is denoted by F_t. Since we abstract from discounting, the value of the futures will always be equal to the value of the underlying, that is $F_t = S_t$.

It is standard practice that trading with futures requires a margin account. That is, if an investor wants to open a futures position (buy or sell futures contracts), she needs to transfer the initial margin into the margin account as collateral. In reality the initial margin is typically set between 5% to 10% of the contract's value and its amount depends on the underlying asset's volatility. Let the initial margin be denoted by *margin*. We abstract from margin calls in our setting by assuming that

$$\max\{1-d^2, u^2-1\} < \mathit{margin}, \tag{3.1}$$

that is, the initial margin is large enough to cover all potential losses that may occur before maturity. For example, if an investor purchases one contract (takes a long position) at $t = 0$, she has to post $\mathit{margin} \times S_0$ as collateral. In the worst case (from the investor's perspective), when the underlying depreciates to $S_2 = d^2$, she loses $(1-d^2)S_0$, which is smaller than the collateral posted by inequality (3.1). Thus, the initial margin is large enough to cover even the largest losses.

Let H_0^{FL} (H_0^{FS}) denote the total number of bought (sold) futures contracts by investors in period 0. If investors individually randomize between buying and selling futures contracts, then the expected number of bought and sold contracts will satisfy (using the expected value of the binomial distribution)

$$E[H_0^{FL}] = E[H_0^{FS}] = \frac{\text{Total expected funds invested}}{\text{Initial margin} \times F_0} = \frac{nm/2}{\mathit{margin}}, \tag{3.2}$$

since the total expected money flowing to buys and sells is $nm/2$ and the initial margin requirement is *margin*. Investors buy and sell more contracts if (i) there is a higher number of them (higher n) (ii) investors have more money to invest (higher m) and if the initial margin required is smaller.

Importantly, investors buy and sell the same number of futures contracts in expectation. This result is consistent with how it is often thought about noise traders, namely, that they do not systematically buy or sell assets. As a result, on average, investors do not have an aggregate exposure to the underlying. Volumes of buys and sells will also be symmetric.

The same argument applies for period $t = 1$. In turn, no matter how the underlying evolves, in expectation investors will have zero exposure in both trading sessions, which also implies that their aggregate position will not systematically vary with the underlying.[8]

To sum up, we emphasize the conditions required for the above result: (i) margin requirements for long and short contracts are not correlated with past returns, (ii) investors' preferences for opening new long (instead of short) positions are not correlated with past returns and (iii) investors' decision to close a position is not correlated with past gains/losses. We continue with the discussion of the knock-out warrant market, where due to different product characteristics, condition (i) will no longer hold, leading to systematic patterns in aggregate investor positions.

3.2.2 Investors in the warrants market

Consider knock-out warrants. A knock-out call offers long exposure and the knock-out put offers short exposure. Similarly to the futures discussed above, they expire at $t = 2$. Importantly, we assume that the strikes and barriers satisfy

$$K^{call} = B^{call} < d^2 \leq u^2 < B^{put} = K^{put}, \tag{3.3}$$

where K^{call} (K^{put}) is the strike price of the call (put) and B^{call} (B^{put}) is the barrier of the call (put). Setting the barriers equal to the strikes helps our exposition, while setting the barriers outside of the underlying's range simplifies derivations, since it guarantees that no knock-out events occur before maturity (this is similar to inequality (3.1), which eliminates margin calls from the setup). As a consequence, the payoff of the warrants will coincide with the payoff of vanilla options. While this makes the model less realistic, the model is still sufficiently rich to illustrate the main intuition.

The call's payoff at $t = 2$ is

$$C_2 = S_2 - K^{call} \tag{3.4}$$

while the put's payoff is

$$P_2 = K^{put} - S_2. \tag{3.5}$$

Following the simple structure, it is straightforward to determine the fair prices

[8] Note that this last statement would also hold if margin requirements would be different for long and short positions (but constant over time).

of the warrants for $t = 0,1$. In particular, the solutions are $C_t = S_t - K^{call}$ and $P_t = K^{put} - S_t$.[9]

E.g. if $S_1 = u$, then under the risk-neutral measure

$$C_1 = E[C_2|S_1 = u] = E[S_2 - K^{call}|S_1 = u] = (u^2 + ud)/2 - K^{call} = u - K^{call}. \quad (3.6)$$

One of the features that separates futures markets from bank issued knock-out warrants is that in the latter investors are not required to post margins as (3.3) guarantees that investors cannot lose more than the invested amount (i.e. $C_2, P_2 > 0$).

Now consider the distribution of trading activity. Let H_t^C (H_t^P) denote the *gross* number of calls (puts) that investors purchase from the bank at t. During $t = 0$, the expected number of calls and puts bought by investors is (where the expectation is over individual investors' randomization over going long or short)

$$E[H_0^C] = \frac{nm}{2C_0} = \frac{nm}{2(1-K^{call})}, \ E[H_0^P] = \frac{nm}{2P_0} = \frac{nm}{2(K^{put}-1)}, \quad (3.7)$$

that is, all else equal, investors will purchase higher number of contracts if (i) there are a higher number of investors (higher n), (ii) investors have more money to spend (higher m) and (iii) if the given product is cheaper (lower C_0 or lower P_0). Note that we have used the normalization $S_0 = 1$. Since all warrants are priced fairly, the expected profit of purchasing any calls or puts is zero. However, their payoff, in general, depends on the evolution of the risky asset (S_t), implying that they will have exposure to the underlying. Interestingly, the exposure a contract offers is independent of its price, since option payoffs are linear in the underlying. Formally, let V be the value of investors' warrants at the end of period $t = 0$,

$$V_0 = H_0^C(S_0 - B^{call}) + H_0^P(B^{put} - S_0), \quad (3.8)$$

where we have used that the barriers are equal to the strikes.

We define EXP_t as investors' aggregate exposure (which we will also refer to as investors' aggregate position) to the underlying at the end of period t, which, at $t = 0$, is equal to

$$EXP_0 = \frac{\Delta V_0}{\Delta S_0} = H_0^C - H_0^P. \quad (3.9)$$

Equation (3.9) can be interpreted as follows. Suppose investors hold H_0^C calls and H_0^P puts at the end of period $t = 0$, and the price of the underlying, S_0, increases by one unit. Then the aggregate value of investors' portfolio will increase by $H_0^C - H_0^P$.

[9] In this setup the warrants are similar to real life knock-out warrants, since their value also falls close to their intrinsic value.

10.14754/CEU.2017.09

Thus, an additional call (put) contract increases (decreases) investors' exposure by one unit, irrespective of the contract's properties (i.e. barriers and strikes). This implies that buying a cheap contract has the same effect on investors' exposure as buying an expensive one. The expectation of exposure is equal to

$$E[EXP_0] = E[H_0^C] - E[H_0^P] = \frac{nm}{2} \frac{B^{call} + B^{put} - 2S_0}{(S_0 - B^{call})(B^{put} - S_0)} \tag{3.10}$$

Since the denominator in (3.10) is always positive, the sign of the aggregate exposure depends on the level of the barriers, B^{call} and B^{put}, relative to S_0. When B^{call} and/or B^{put} is relatively large (but still satisfying the inequalities in (3.3)) then the exposure will be positive: if the underlying increases the combined value of the warrants increases. The intuition is simple. When B^{call} and/or B^{put} is relatively large then the price of a call, C_0 will be small compared to the price of a put, P_0. This results in an investor portfolio consisting of more call contracts than put contracts. Hence, investors, on average, will profit if the underlying increases and lose if the underlying decreases. Investors' expected exposure will be zero if S_0 is the arithmetic mean of the barriers, as in that case the calls and puts trade at the same price.

Now consider the trading session at $t = 1$. The new traders take positions similarly as before, implying that the expected *change* in investors' aggregate position from $t = 0$ to $t = 1$ will be

$$E[EXP_1 - EXP_0|S_1] = \frac{nm}{2} \left[\frac{B^{call} + B^{put} - 2S_1}{(S_1 - B^{call})(B^{put} - S_1)} - \delta \frac{B^{call} + B^{put} - 2S_0}{(S_0 - B^{call})(B^{put} - S_0)} \right] \tag{3.11}$$

where $E[EXP_1|S_1]$ is the expected exposure of investors at the end of $t = 1$, conditional on the value of the underlying and δ is the fraction of investors who choose to resell their warrants to the bank before they expire.[10] The first term in brackets corresponds to the total exposure of those investors who arrived in period 1, while the second term captures resells of investors who arrived in period 0. It is clear from (3.11) that when $S_1 = d$ is realized, then the price of the call depreciates, and, simultaneously, the price of the put appreciates. Thus, investors who arrive in period 1 will take a longer position compared to period 0 investors because calls are now relatively cheaper than before. Similarly, when $S_1 = u$ is realized, investors will decrease their aggregate position. Thus, investors will be betting on price reversals.

The following proposition summarizes the main predictions from the model:

[10] Note that expectations are taken over individual randomizations on whether to take a long or a short position.

Proposition 1. *1. Share of transactions involving calls does not depend on past returns.*

2. Share of call contract trading value is positively correlated with past returns,

$$Cov\left[\frac{[\delta H_0^C + H_1^C]C_1}{nm + \delta[H_0^C C_1 + H_0^P P_1]}, S_1 - S_0\right] > 0. \tag{3.12}$$

3. Share of call contract volumes is negatively correlated with past returns,

$$Cov\left[\frac{\delta H_0^C + H_1^C}{H_1^C + H_1^P + \delta[H_0^C + H_0^P]}, S_1 - S_0\right] < 0. \tag{3.13}$$

4. Investors bet on price reversals, implying $Cov[EXP_1 - EXP_0, S_1 - S_0] < 0$

The proofs are straightforward, and shown in Appendix F. Since investors individually randomize between taking long or short positions and the probability of reselling warrants to the bank is independent of past returns, the share of transactions involving calls will not depend on past returns.

The second statement is that the share of call contract value (value of exchanged call contracts normalized by the total value of exchanged call and put contracts) is increasing with past returns. The intuition for this result is that if the underlying appreciates, the value of calls held by investors increases relative to puts. In turn, since the selling decision is independent of past returns, the calls resold by investors will have higher value than the puts.

Third, the share of call contract volume is decreasing with past returns. The idea is that after appreciations of the underlying call contracts become relatively more expensive, implying that for the same amount of investment, investors can only purchase a lower number of call contracts.

Finally, investors take positions as if they were betting on price reversals. This is the most surprising prediction of the model, given that investors allocate their funds randomly between assets. Since the price of calls (puts) is monotone increasing (decreasing) in the underlying, if the underlying increases puts are going to be cheaper than calls and this will lead new investors to purchase a higher number of puts, effectively betting on a reversal.

It is important to emphasize that these results are conditional on not hitting any of the barriers, as we have assumed in (3.3). The moment a barrier is hit, investors' position will be closed automatically, implying that in that moment they are reinforcing market movements. However, since we have also assumed away margin calls in our benchmark case, we believe that introducing a more realistic setup in both cases would not affect our main qualitative prediction.

To conclude, while the exposure individual investors may take with calls and puts is very similar to the exposure provided by futures, the contract features of

10.14754/CEU.2017.09

knock-out warrants are fixed in the short-run, which leads to an asymmetry of product characteristics between offered calls and puts. In turn, if individual investors allocate their funds randomly, their aggregate exposure will systematically vary with the asymmetries of the available menu. While the asymmetries in this simplified setup are only driven by the evolution of the underlying, in reality banks have the option to strategically time the introduction of new warrants, which gives them the (potentially valuable) power to influence individual investors' aggregate position.

3.3 Empirical evidence

We have obtained a unique, transactions level data set from a bank operating in Europe.[11] The knock-out warrants in our data have the German DAX index as their underlying asset and the data covers 100 trading days. The main advantage of our data is that it enables us to track investors' aggregate holdings of the various warrants. Also, since the data is at the transactions level, we can construct measures of trading activity that can directly be related to the ones introduced in our model. This makes it feasible to test the predictions of our model (the statements in Proposition 1).

3.3.1 The market for exchange traded retail structured products

The German Derivatives Association provides an overview of exchange traded structured products.[12] The two largest exchanges where these products are traded are Euwax (European Warrant Exchange) in Stuttgart and Frankfurt Smart Trading, which is a specialized segment of Deutsche Brse in Frankfurt. Trading is concentrated to these exchanges, but most exchanges in Europe offer the opportunity for banks to issue structured products (and our data also comes from a smaller exchange).

There are over twenty banks actively issuing structured products. The issuing bank has to determine product characteristics and is required to continuously quote bid and ask prices, guaranteeing a liquid secondary market for its assets.

Importantly, short selling is prohibited in these markets. This implies that banks may quote ask and bid prices above the fair value of the derivatives, as its not possible to achieve arbitrage profit through short selling.

When the product is introduced, the outstanding number of contracts is zero. The

[11] Since the bank who provided us with the data wishes to remain anonymous, we do not disclose its identity, nor the identity of the stock exchange in which its products are issued.
[12] See Deutsche Derivative Verband (2014).

number of outstanding contracts increases if investors purchase products from the bank and decreases if investors resell the assets to the issuing bank. As investors may submit limit orders, it is possible investors' orders cross, which does not affect the outstanding number of contracts. However, in practice the bank is involved in most transactions.

A major difference between purchasing a structured product from a bank via a stock exchange and buying options in an options market is that the stock exchange does not require the bank to hold a margin account. In other words, by purchasing a structured product from a bank, investors hold the credit risk of the issuing bank. As Baule et al. (2008) show, the issuer's credit risk is in fact reflected in its quotes, i.e. issuers with lower credit risk quote higher prices.

According to Deutsche Derivative Verband (2016) the most popular structured products in Germany measured by the value investors hold are capital protection products. However, measured by the number of offered products, leverage products constitute the majority, which might indicate the higher profit margin on these products.

3.3.2 Product characteristics of knock-out warrants

It is useful to fix ideas about the particular derivatives that are analyzed, as it is often difficult to navigate within the variety of retail structured products.

We have two types of options, knock-out calls (KO call) and knock-out puts (KO put). The KO call is a call option with a barrier and is known as a down-and-out call, as the barrier is set below ('down') the price of the underlying and the right to exercise disappears once the price hits the barrier ('out'). However, there is a twist. The option has a residual value if the barrier is hit, and the upper bound of the residual value is the difference between the barrier and the strike. Formally, the payoff function of a KO call is

$$\text{Payoff of KO call} = \begin{cases} S(T) - K & \text{if } S(t) > B \;\; \forall\, t,\; t_0 < t < T \\ 0 \leq Residual(\leq B - K) & \text{if } \exists t\text{: } S(t) \leq B, t_0 < t < T \end{cases}$$

where $S(t)$ is the price of the underlying at time t, t_0 is the date when the product was issued, B is the barrier price and K is the strike price and they satisfy $B > K$ for the KO call options.

The upper bound of the residual is $B - K$. Once the price hits the barrier the bank has to close its hedging positions (i.e. sell the underlying), and if the price jumps below the barrier then the bank cannot close the position at the barrier. Hence, the residual value will depend on the price at which the bank manages to close

10.14754/CEU.2017.09

its position. This essentially shifts the trade execution risk to the holders of the option.[13] This is similar to a stop-loss order, where execution is not guaranteed.[14]

KO put contracts are put options and can be approximated by up-and-out puts. The barrier is set above the price of the underlying ('up') and the right to exercise is lost once the barrier is crossed ('out'). Similarly to the KO call, there is a residual value if the barrier is hit. Formally, the payoff of a KO put is

$$\text{Payoff of KO put} = \begin{cases} K - S(T) & \text{if } S(t) < B \;\; \forall\, t,\; t_0 < t < T \\ 0 \leq Residual(\leq K - B) & \text{if } \exists t\text{: } S(t) \geq B, t_0 < t < T \end{cases}$$

where the barrier is always below the strike, $B < K$.[15]

In Appendix G we compare the Delta and the Vega of a KO call with those of a vanilla call and a futures contract, in order to illustrate how KO warrants are priced relative to these derivatives.[16] Here we only focus on leverage, which makes KO warrants unique.

Since the Delta of the KO call is always close to 1, a first-order approximation for the value of a KO call is its intrinsic value, $S(t) - K^{call}$ and for a KO put it is $K^{put} - S(t)$. Exactly for this reason, banks often advertise such warrants as futures contracts, with an automatic stop-loss (instead of getting a margin call). Note that our model generates exactly these prices, as $C_t = S_t - K^{call}$ and $P_t = K^{put} - S_t$, due to the fact that we abstracted from knock-out events occurring before maturity.

Using these approximations to express the leverage of a KO call, one finds that

$$\text{Leverage of KO call} \approx \frac{S(t)}{S(t) - K^{call}} \tag{3.14}$$

It is clear from (3.14) that leverage explodes as the underlying's price approaches the strike price.

Figure 3.2 highlights how the leverage of the KO call is related to the leverage of the vanilla call and that of a futures.[17] As expected, the futures always has a leverage equal to the inverse of the required margin. The leverage of the vanilla call is close to flat, which highlights the fact that most of its value is the time value.

[13] This argument assumes that the banks always hedge their exposure. However, if banks do not completely hedge their exposure then they do not have to close their hedging positions if a KO event occurs. Hence, banks might be able to profit from not giving the maximum residual value to investors.

[14] Note that if the option reaches the expiry date without a knock-out event then the payoff of $S(T) - K$ is received at the expiry date, T, but if the barrier B is hit then the residual value will be transferred to the investors' account a few business days after the knock-out event.

[15] In practice, there is also a multiplier that scales the KO products. Since this multiplier does not affect the analysis, we chose not to complicate the exposition with it.

[16] Delta measures the unit change in the derivative's price if the underlying's price increases by 1 unit. Vega measures the percent change in the proce of the derivative when the underlying's volatility increases by 1% point.

[17] Note that the parameters used for this figure are given in Table G.1 in Appendix G.

10.14754/CEU.2017.09

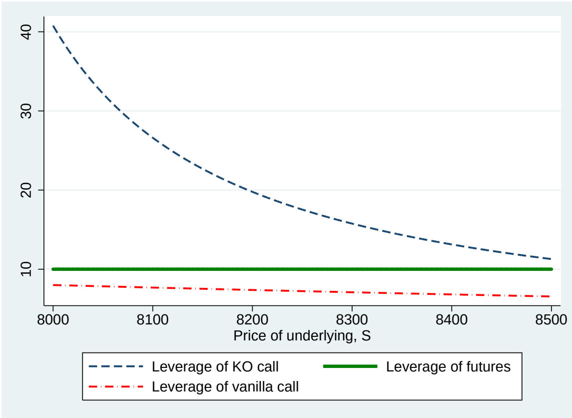

Figure 3.2 Leverage of a KO call, a vanilla call and a futures contract as a function of the underlying's price. Leverage is defined as an elasticity of the derivative's value with respect to the underlying's price. Note that option leverages were numerically approximated based on the Black-Scholes model. Parameters used are provided in Table G.1. For the leverage of the futures, we have assumed a margin of 10%.

To the contrary, the value of a KO call will be very close to the intrinsic value, which will imply that when its intrinsic value is low, its leverage will be high.

The mechanical relation of KO call and put prices, and also their leverage is illustrated in Figure 3.3. Suppose a bank wants to issue a new KO call. Then it has to determine the call's terms, e.g. its strike price, barrier and maturity. Figure 3.3 illustrates that the chosen strike price for the call has an important effect on the initial value and leverage of the new product. Importantly, the new call's strike may result in a call contract that is either much cheaper and has higher leverage compared to the already available put product, or much more expensive with a lower leverage.

To sum up, the KO call does seem to be closer to a futures contract than to a vanilla call (stable Delta close to 1), but notable differences still remain. The crucial property that sharply differentiates the KO call from both a vanilla call and a futures is its leverage, which is highly sensitive to the underlying's price. Importantly, when a bank introduces a new KO product it may significantly alter the prevailing asymmetries in the leverages of offered KO calls and KO puts. Since the product with the highest leverage will turn out to be the most popular product in this market, changes in the relative leverage of KO calls and KO puts will influence investors' aggregate exposure.

10.14754/CEU.2017.09

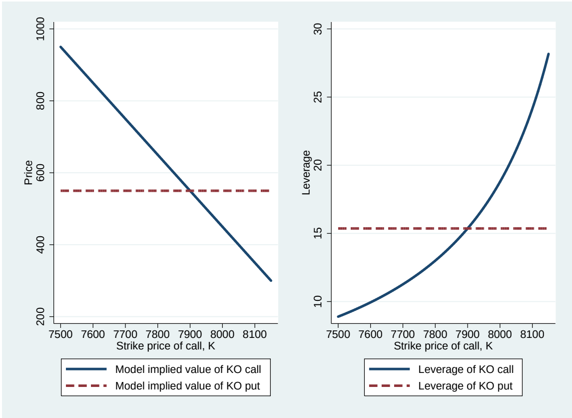

Figure 3.3 Value and leverage of a KO call and a KO put as a function of the strike price of the offered KO call. Parameters for the KO put are $K = 9000$, $B = 8750$, while for the KO call the barrier is always set to $B = K + 250$. Parameters common to both products are $S = 8450$, $r_f = 0$, $T = 1$, $\sigma = 0.2$. Model implied values are based on Black-Scholes assumptions.

3.3.3 Supply of products

The issuing bank guarantees that it will continuously provide liquidity for the issued products by quoting bid and ask prices. In particular, the bank quotes ask prices as long as investors have not bought all the contracts that a bank initially set out as the maximum number of contracts (this rarely happens in practice) and the bank starts quoting bid prices whenever the outstanding number of contracts is greater than zero.

Table 3.1 gives a summary of the products introduced since 2008 by the bank that provided us with the transactions data.[18] Commodities (oil, natural gas, gold, silver, copper) and stock indices are by far the most popular underlyings. More than half of the products were knocked-out before their respective expiry date and as of May 2016 there are about a hundred different products offered.

[18] It is interesting to compare the available KO options on our developing market exchange with those available on developed ones. We will use the German Euwax exchange as a comparison for this purpose. The most striking differences are the variety and number of products that are offered. Recall, that according to Table 3.1, there were only about a hundred of available KO products. At the Euwax exchange, the number of available KO products is in the 6 digit range and they are available for virtually any underlying product in the world. This suggests that costs associated with product issuance in a developing country's exchange may be higher. The second notable difference is in the type of structured products that are being offered. While the majority of products offered are also leveraged products at Euwax (Deutsche Derivative Verband, 2016), over 40% of the products are not leveraged. These non-levered products are typically either capital protection products or discount products, that have a cap on the maximum payoff. Capital protection products are the most popular ones at Euwax, measured by the total value of open contracts. The revealed choice of our bank to focus on KO products suggest that these are the most profitable products to issue at our exchange. This is also supported by the indirect evidence, that at Euwax while the majority of products are leveraged, they only constitute about 2-3% of open contracts, measured by their value.

10.14754/CEU.2017.09

Table 3.1 *Issuance history of KO warrants. This table shows the number of warrant issues issued by the bank, that provided us with the transactions data, by the type of the underlying product and the trading status of the warrants as of May 2016.*

Underlying category	Knocked-out	Expired	Listed as of 05/2016	Delisted
Commodities	154	141	21	4
FX	55	84	19	0
Government debt	11	11	2	0
Regional blue chip	67	3	41	0
Stock indices	127	80	14	0

In order for the bank to earn a profit on introducing KO products, it needs to generate order flow. Intuitively, it is easier to achieve large interest in a given product, if its underlying is volatile[19] and salient. Figure 3.4 shows the number of available

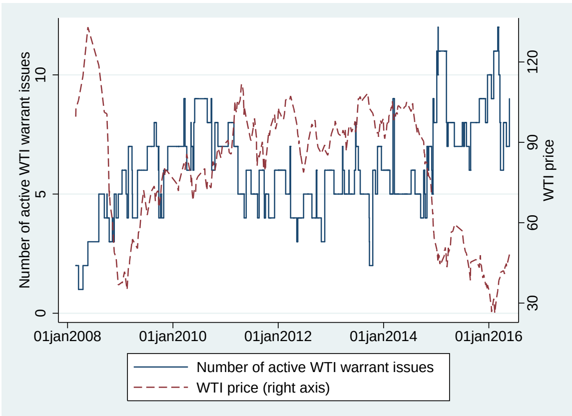

Figure 3.4 Example of product issuance. This figure shows the number of active warrant issues, that have the WTI Oil price as their underlying, together with the evolution of the underlying's price (right axis).

KO products during the sample that have the WTI Oil price as their underlying. It is easy to see that after the price collapse of 2008 and 2015 the number of available products sharply increased. This increase partly reflects the fact that the bank is constantly trying to offer products with high leverage, implying that after large price movements, it wants to replace old products (as the older products were either knocked out, or their leverage decreased). However, it is also reasonable to increase the menu of products in order to cater to investors who allocate their attention to headline news.

[19] Kumar (2009) demonstrates that individual investors are drawn to high volatility (lottery-type) assets in financial markets.

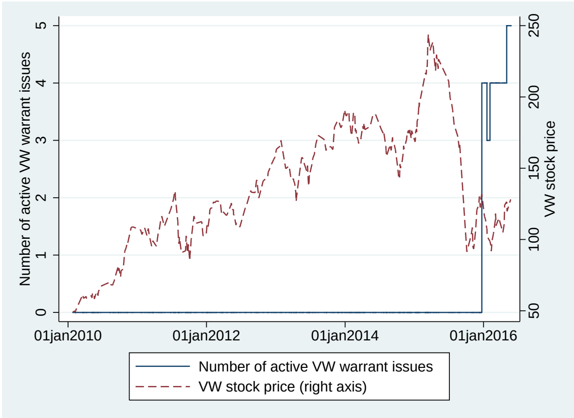

Figure 3.5 Example of product issuance II. This figure shows the number of active warrant issues, that have the Volkswagen stock price as their underlying, together with the evolution of the underlying's price (right axis).

To illustrate how the bank's decision is affected by salience of events, Figure 3.5 shows that the bank introduced KO products for Volkswagen, just after the break of the emissions scandal. It introduced two KO calls, and two KO puts at the end of December 2015, respectively.

Supply of warrants on the DAX

Figure 3.6 illustrates how the DAX index evolved during our sample period and also shows the offered knock-out warrants represented by their barriers. Barriers below the DAX represent the calls (solid horizontal lines) and barriers that are above the DAX represent the puts (dashed hotizontal lines). In the first half of our sample the barriers of two puts were hit leading to knock-out events. After these knock-out events, the bank introduced new puts with higher barriers and also introduced a new call with a higher barrier in order to offer a call contract with higher leverage.

It is clear from Figure 3.6 that as the underlying randomly fluctuates and the menu of offered products varies, the underlying will usually be closer to either a call's barrier or a put's barrier. The distance between the DAX index and the barrier is a good proxy for the respective warrant's value. This will imply that in any moment, the composition of supplied products will be asymmetric. In particular, most of the time there will be a sizable difference between the values of the cheapest call (the call with the highest barrier) and cheapest put (the put with the lowest barrier).

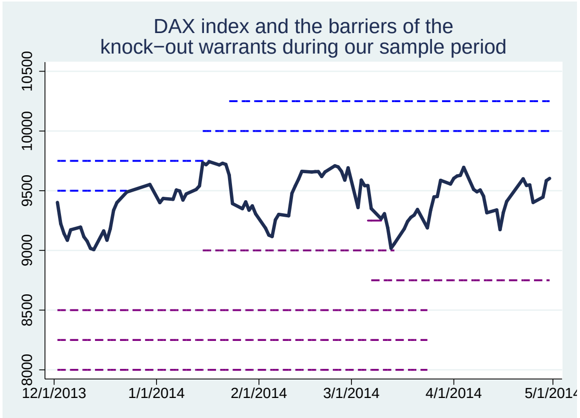

Figure 3.6 DAX index and offered knock-out warrants during our sample. Dashed horizontal segments indicate the lifespan of knock-out put contracts represented by their barriers. Solid horizontal segments indicate the lifespan of knock-out call contracts represented by their barriers. After introduction, warrants either get knocked-out (4 cases) or expire at maturity (3 cases). In three cases the products only expired after our sample period ended.

Table 3.2 *Frequency of transactions for all trading days and all products in our sample. Investors either buy contracts from the bank, or resell them to the bank. Settlement transactions (i.e. after a knock-out event or after the option expires) are omitted. Note that a transaction may involve any number of contracts from a given warrant.*

	KO put	KO call	Total
Investor buys	6537	5309	11846
Investor resells	4754	4016	8770
Total	11291	9325	20616

3.3.4 Descriptives of transactions data

We have obtained transactions data from a bank on KO calls and puts that have the German DAX index as their underlying. Our data consists of all transactions that took place between the bank and investors from December 2013 to April 2014 (100 trading days).[20] There were 6 KO calls and 4 KO puts actively traded in the given sample period.

Table 3.2 provides the frequency of transactions. KO puts seem to be slightly more popular among investors than calls, which is likely to reflect that it is more difficult to find adequate substitutes for puts. The significant number of investment

[20] I.e. investor-investor transactions are not recorded, as the bank did not participate in these. Investor-investor transactions happen when investors submit orders that cross within the bid and ask prices set by the bank.

resells also suggest that the holding period of these products is short and that the majority of investors does not wait for their purchased products to expire.

Table 3.3 *Descriptive statistics of daily data. This table shows the mean, the 10th, 50th and 90th percentiles of the respective variables' distribution when the transactions data is aggregated at the daily level, separately for KO puts and KO calls. Transactions is the total number of transactions per day (investor buys+investor resells), Volume (# of contracts) is the total number of contracts exchanged during the day, Investors' net buy (# of contracts) is the difference between the number of outstanding contracts at the end of the day and the number of outstanding contracts at the beginning of the day. Trading value is the total daily value of the contracts that were exchanged between the bank and investors. Net change in investors' position is the difference between Investors' net buy of calls and puts. Settlement transactions (i.e. after a knock-out event or after a warrant expires) are omitted.*

	Mean	**p10**	**p50**	**p90**
KO puts				
Transactions	113	56	106	166
Volume (# of contracts)	34682	10155	27852	70154
Investors' net buy (# of contracts)	1499	-11072	1590	14920
Trading value (euros)	240091	64767	199986	487363
Volume/|Investors' net buy|	8.08	1.64	4.24	15.75
KO calls				
Transactions	93	29	78	187
Volume (# of contracts)	20705	3524	15464	45609
Investors' net buy (# of contracts)	1059	-7551	636	9665
Trading value (euros)	167697	41535	147712	323279
Volume/|Investors' net buy|	8.48	1.67	4.69	16.48
Net change in investors' position	-440	-18812	53	19702

Table 3.3 shows investors' trading activity by various measures. On the median day, there are 106 transactions involving KO put products and 77 involving KO call products. The total value of daily volume is around 240 000 (170 000) euros for KO put (call) products. This amounts to an average volume of 35 thousand contracts for puts and 20 thousand for calls, respectively.[21] Importantly, there is large variation

[21] In practice the payoff is multiplied by the so-called multiplier, which scales the value of the contracts. In our case the multiplier is always 1/100, which implies that with a leverage of roughly 10 an investor needs to purchase 100 contracts if she wishes to have the same exposure as buying one unit of the underlying. For example, suppose the underlying is at S=10 000 euros, and the strike and barrier prices of a KO call are both 9000 euros. Then with a multiplier of 1/100 the KO call will roughly be worth (10000-9000)/100=10 euros. Purchasing a hundred of these costs 1000 euros. Using (3.14), one finds that its leverage is approximately 10. Hence, buying the underlying for 10 000 euros gives the same exposure as buying leveraged KO calls for 1000 euros, which have a leverage of 10. As evident from this example, one can obtain a given exposure cheaper by choosing a product with higher leverage.

in the daily change of investors' net position. This suggests that investors' net buy of calls and puts are negatively related (i.e. investors tend to buy calls and sell puts simultaneously). While these are not large magnitudes by any standard, recall that the data only covers the products for a single bank issued in a single exchange for a single (but undoubtedly popular) underlying. Also, the average leverage of the warrants is above 12, which implies that actual positions are an order of magnitude larger.

The main strength of our data is that it enables us to track investors' aggregate position. As Table 3.3 demonstrates, there is significant variation in investors' position (Net change in investors' position), which we will focus on below.

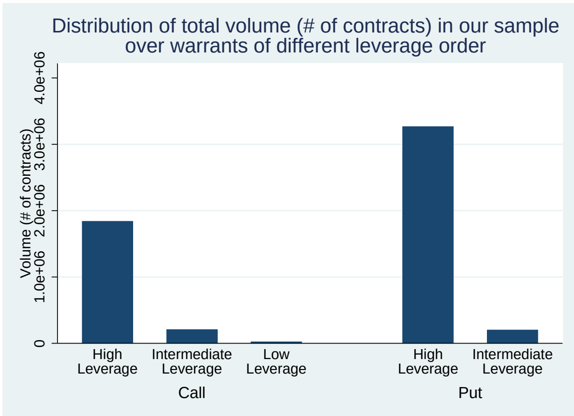

Figure 3.7 Total volume (# of contracts) by leverage order. This chart shows the total volume in our sample, separately for calls and puts, over the leverage order of available warrants. Note that only those trading days were considered when at least two different calls and puts were available.

To investigate the relationship between popularity and leverage we have calculated the total volume by the order of leverage, separately for calls and puts. Figure 3.7 shows the results. It is clear, that the highest leveraged product is the most popular one.[22] This implies that whenever a bank issues a new call or put that has higher leverage than the already available ones products, it is likely to attract most of the trading activity.

3.3.5 Results

Does investors' aggregate position systematically vary with the available menu of contracts? We have already shown that the most levered products will be the most

[22] Note that the result also holds if trading activity is measured by the number of transactions or the trading value.

10.14754/CEU.2017.09

Table 3.4 *Distribution of trading activity and past returns. Transaction share of calls is the daily number of transactions involving call contracts divided by the total number of transactions (calls+puts) during the day. Trading value share of calls is the total value of calls that were exchanged during the day divided by the total value of contracts that were exchanged during the day (calls+puts).* $100\log(\frac{DAX_t}{DAX_{t-1}})$ *is the overnight return of the DAX (from close on day* $t-1$ *until open on day* t*),* $100\log(\frac{DAX_t}{DAX_{t-2}})$ *is the return on the DAX from the close of day* $t-2$ *until the open of day* t *and* $100\log(\frac{DAX_t}{DAX_{t-5}})$ *is the return on the DAX from the close of day* $t-5$ *until the open of day* t*.*

	Transaction share of calls			Trading value share of calls		
	(1)	(2)	(3)	(4)	(5)	(6)
$100\log(\frac{DAX_t}{DAX_{t-1}})$	-0.00670			0.0726***		
	(0.0239)			(0.0245)		
$100\log(\frac{DAX_t}{DAX_{t-2}})$		-0.0264**			0.0283**	
		(0.0101)			(0.0125)	
$100\log(\frac{DAX_t}{DAX_{t-5}})$			-0.0263***			0.00582
			(0.00498)			(0.00688)
Constant	0.429***	0.430***	0.431***	0.425***	0.427***	0.428***
	(0.0130)	(0.0126)	(0.0117)	(0.0152)	(0.0153)	(0.0157)
Observations	100	100	100	100	100	100
R-squared	0.001	0.053	0.189	0.051	0.042	0.006

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

popular ones within KO puts and within KO calls (Figure 3.7). Now we turn to the more interesting question, whether asymmetries in the product characteristics of the offered calls and puts systematically affects the position investors take.

The regressions in Tables 3.4 and 3.5 directly test, whether the data is consistent with the predictions of Proposition 1. The explanatory variables are always the past returns of the DAX index for three different time horizons. Returns are always computed up to market open of trading day t from the market close of trading day $t-1$ (overnight), $t-2$ and $t-5$, respectively.

The first prediction of the model is that the share of transactions involving call contracts (over the total number of transactions) is unrelated to past returns. This is tested in columns (1)-(3) of Table 3.4. The negative and statistically significant coefficients suggest that past returns have predictive power. In particular, after appreciations of the underlying trade tends to concentrate in puts, which are now cheaper (and offer higher leverage).

The second hypothesis is that the exchanged value of call contracts (normalized

Table 3.5 *Distribution of trading activity and past returns II. Volume share of calls is the daily volume of call contracts divided by total daily volume (calls+puts). Net change in investors' position is the difference between investors' net position after the market closes on day t and investors' net position when the market closes on day $t-1$, where the net position is measured by the difference between the number of call and put contracts held by investors. Net change in investors' position corresponds to $EXP_1 - EXP_0$ in the model.* $100\log(\frac{DAX_t}{DAX_{t-1}})$ *is the overnight return of the DAX (from close on day $t-1$ until open on day t),* $100\log(\frac{DAX_t}{DAX_{t-2}})$ *is the return on the DAX from the close of day $t-2$ until the open of day t and* $100\log(\frac{DAX_t}{DAX_{t-5}})$ *is the return on the DAX from the close of day $t-5$ until the open of day t.*

	Volume share of calls			Net change in investors' position		
	(1)	(2)	(3)	(4)	(5)	(6)
$100\log(\frac{DAX_t}{DAX_{t-1}})$	0.0152			-12,438***		
	(0.0353)			(3,154)		
$100\log(\frac{DAX_t}{DAX_{t-2}})$		-0.00595			-6,574***	
		(0.0163)			(1,169)	
$100\log(\frac{DAX_t}{DAX_{t-5}})$			-0.0233**			-2,795***
			(0.00903)			(649.6)
Constant	0.373***	0.374***	0.375***	-31.39	-185.0	-334.9
	(0.0198)	(0.0197)	(0.0191)	(1,448)	(1,417)	(1,487)
Observations	100	100	100	100	100	100
R-squared	0.001	0.001	0.065	0.145	0.217	0.141

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

by the total value of exchanged calls and puts) is increasing with past returns. Columns (4)-(6) of Table 3.4 provide some supporting evidence for this prediction.

The apparently conflicting results of Table 3.4 suggest that there is considerable heterogeneity within investors, which our model does not capture. In particular, the results could be reconciled if there were a group of investors with short holding periods (i.e. day traders), who are drawn to leverage but only submit relatively small orders. This would explain why there are a lot of transactions involving higher levered products together with the finding that trading value is concentrated in recently appreciated warrants (i.e. in calls after positive past returns). The high values of gross volume over net volume in Table 3.3 does, in fact, suggest a high share of day trading.

The third prediction is that the volume share of calls is decreasing with past returns. We find some supporting evidence for this as shown in columns (1)-(3)

of Table 3.5. The estimated coefficient in column (3) suggests that a one standard deviation increase of the past 5-day return (of 2.1%) implies an over 13% decrease in the volume share of calls relative to its mean. This result is consistent with our mechanism suggesting that investors buy larger number of contracts if they are cheaper, simply because the value of their investment is assumed to be fixed. However, it may also be strengthened by the presence of day trading, as noted above.

The final and most interesting prediction is that investors systematically bet on price reversals. This is true if investors tend to increase their long positions after depreciations of the underlying and vice versa. We find strong supportive evidence for this prediction as shown in columns (4)-(6) of Table 3.5.

The significant and large coefficients suggest that the basic mechanism highlighted by our model drive the aggregate behavior of investors. Note that this prediction would stay intact even in the presence of day traders, who, by definition, do not leave positions open overnight. Hence, they would not affect investors' end of day aggregate position.

To sum up, the key prediction of our model seems to be supported by our data. However, additional assumptions on trading behavior are required if one would like to better capture the evolution of our trading activity measures. A fruitful extension that could possibly provide a better understanding of the data would involve investors being drawn to volatility (in our case this coincides with leverage), which is a documented characteristic of individual investor behavior (Kumar, 2009). However, we leave this for future research.

3.4 Alternative explanations

Our simple model is able to reconcile why investors bet on price reversals, on average. However, it is important to discuss alternatives, that might also be consistent with observed patterns.

3.4.1 Disposition effect

According to the disposition effect, investors have a larger propensity to sell assets that have gained value since purchase than to sell those that lost value since purchase. Among others, Odean (1998), Grinblatt and Keloharju (2001), Feng and Seasholes (2005) analyze the trading histories of individual investors in the U.S. and find supporting evidence for the disposition effect.

Unfortunately, our data does not enable us to link trades to individual investors, which prevents us from directly analyzing the behavior of individual investors.

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However, since a significant amount of trading volume comes from day trading[23], we can see how investors respond on the aggregate to within day developments.

Figure 3.8 shows the within day distribution of trading activity. As expected, most transactions occur right after the exchange opens and just before the exchange closes for the day. In particular, due to day trading, the propensity to buy is largest in the morning hours and the propensity to sell is largest before the exchange closes.

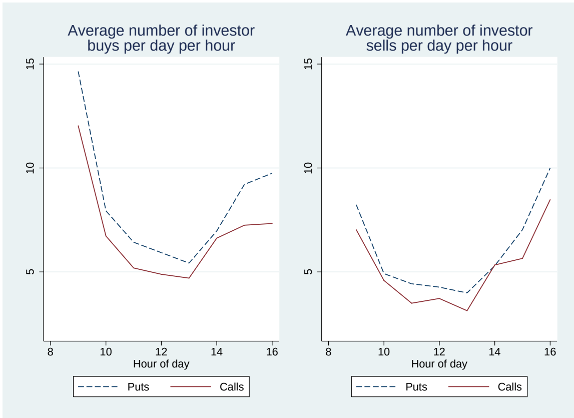

Figure 3.8 Investors' trading activity within trading hours. This graph shows the average number of buy transactions and resell transactions investors carry out with the bank.

Figure 3.9 breaks down trading intensities by the sign of the within day return of the underlying. Consistent with the disposition effect, investors are more likely to submit sell orders for calls during the afternoon if the underlying's price increased within the day. Similarly, they are more likely to sell puts during the afternoon if the price decreased during the day. This pattern suggests that the selling decision is not completely random.

Furthermore, looking at purchases, it seems that investors' decision to buy also depends on within day movements. In particular, investors seem to be following a "double down" strategy: investors are more likely to purchase calls during the afternoon, when the price decreased during the day and more likely to purchase puts when the price increased during the day (both within and between calls and puts). This is consistent with the findings of Ben-David and Hirshleifer (2012), who suggest that investor overconfidence has the following effects: when the price of the chosen product rises, overconfident investors realize the gains as everything went as planned. However, when the chosen product depreciates, overconfident

[23] See Table 3.3.

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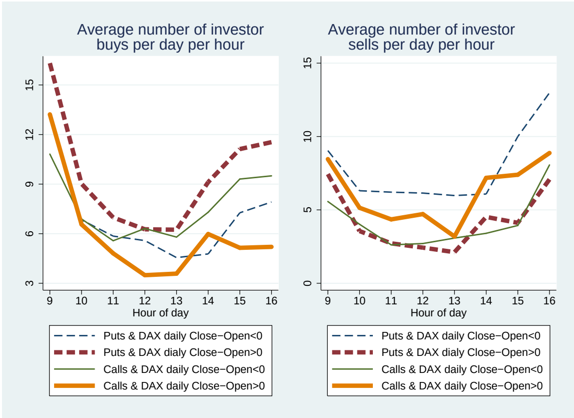

Figure 3.9 Investors' trading activity within trading hours by within day market movement. This graph shows the average number of buy and resell transactions by the hour of day, computed separately for days when the DAX index increased during the day (Close-Open¿0) and decreased during the day (Close-Open¡0). Solid (dashed) lines correspond to calls (puts) while thick (thin) lines correspond to days when the DAX increased (decreased) within the day.

investors now see an even bigger opportunity to gain, therefore they increase their holdings of the given product.

The above suggest that the disposition effect is in line with our findings. However, we have to emphasize that the disposition effect - while widely studied - is an empirical finding. Hence, the disposition effect cannot explain our results. We can only say that we have provided additional evidence which is in line with the disposition effect.

3.4.2 Belief in mean reversion, negative feedback trading

If a fraction of investors believe that the underlying's price exhibits mean reversion then betting on price reversals is a legitimate strategy, which would directly lead to our empirical results. There is evidence from lab experiments suggesting that people mistakingly extrapolate random series. Andreassen and Kraus (1990) find that if people familiar with basic economics are presented with a sample of a stock price series that has a visible trend, subjects are likely to extrapolate that trend. Also, if the presented series does not exhibit a trend, people extrapolated mean reversion. Weber and Camerer (1998) and more recently Jiao (2016) also use lab experiments to determine the possible mechanisms that lead to the disposition effect and present evidence in favor of biased beliefs toward mean reversion.

The two competing explanations in Jiao (2016) are the preference based explanation and the belief based explanation. While the belief in mean reversion falls in the latter, prospect theory is the most prominent candidate in explaining the disposition effect. In particular, when risk attitudes are different for gains and losses, investors might be risk seeking if they hold losers and therefore choose not to sell the loser. On the other hand, in the domain of gains, investors might be more risk-averse, which induces them to close their positions. Jiao (2016) provides evidence for the belief based explanation. The disposition effect was strongest in those subjects who predicted more mean reversion.

Identifying belief in mean reversion with the data at hand is, at best, challenging. The reason is simple: our model predicts that investors who allocate their funds randomly will systematically bet on price reversals. However, any trading strategy derived from the assumption that investors believe in mean reversion would provide the same prediction.

A possible identification strategy would be to simulate trading in a simple model where some traders behave like the ones described in our model, while others believe in mean reversion (and/or momentum). By matching moments of trading activity measures and investors' positions one could, in principle, identify the share of mean reversion traders. However, while it is clear that our small sample is not suitable for such an exercise, the principal source of identification would be how strongly investors bet on price reversals. Intuitively, if investors are betting on reversals more aggressively than investors in our model then that would be evidence for some investors believing in mean reversion.

To conclude, the regressions in Table 3.5 could be easily interpreted as evidence for both our model and any model where investors tend to believe in mean reversion, on average. What is clear, however, is that not all investors derive their strategies from the same belief of mean reversion, as large amounts of call and put contracts are held simultaneously by investors. Hence, we suggest that our model of randomly allocated funds may go a long way in capturing the heterogeneity of investor beliefs and behavior in the market for knock-out warrants.

3.5 Concluding remarks

We analyze both theoretically and empirically how the menu of available knock-out warrants may influence the aggregate position investors take. As a first order approximation, we model individual investors by simply assuming that they allocate their funds randomly between products offering long and short positions, similarly to noise traders.

As our main contribution, we demonstrate theoretically, and also provide suggestive empirical evidence from bank issued warrants that investors' aggregate

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position may systematically vary with the menu of offered products. If investors allocate their funds randomly between long and short positions, and the products on the long and short side have very different leverages then investors will take a position corresponding to the product's position that has the higher leverage. In turn, we show that large differences in the leverages of offered products may prevail, leading to a large aggregate position for investors. Also, since the leverage of calls will be larger after recent contractions of the underlying, investors will tend to bet on price reversals, even though they allocate their funds randomly. We speculate that the systematic variation in investors' position may be beneficial for banks' liquidity management if it operates as a market maker on the underlying's market.

While we have made the first steps in understanding investor behavior in the markets for knock-out warrants, we have ignored the potentially strategic behavior of the supply side. It is likely that the behavior of individual investors influences bank's strategies. E.g. banks may strategically time the introduction of new warrants and they may strategically quote prices. For purposes of risk and/or liquidity management banks might be inclined to design products that leave individual investors systematically exposed to the underlying, on average. Also, since the bank has the information advantage of knowing the number of outstanding contracts, it may find it optimal to use this information when setting its quotes, which also affects investor positions. Hence, a better understanding of the supply side would be a natural next step.

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References

Alessi, Lucia, and Detken, Carsten. 2011. Quasi real time early warning indicators for costly asset price boom/bust cycles: A role for global liquidity. *European Journal of Political Economy*, **27**(3), 520–533.

Alp, Aysun. 2013. Structural Shifts in Credit Rating Standards. *The Journal of Finance*, **68**(6), 2435–2470.

An, Li. 2016. Asset Pricing When Traders Sell Extreme Winners and Losers. *Review of Financial Studies*, **29**(3), 823–861.

Andreassen, Paul B., and Kraus, Stephen J. 1990. Judgmental extrapolation and the salience of change. *Journal of Forecasting*, **9**(4), 347–372.

Ashcraft, Adam B., Goldsmith-Pinkham, Paul, and Vickery, James. 2010. *MBS ratings and the mortgage credit boom*. Staff Reports 449. Federal Reserve Bank of New York.

Baghai, Ramin P., Servaes, Henri, and Tamayo, Ane. 2014. Have Rating Agencies Become More Conservative? Implications for Capital Structure and Debt Pricing. *The Journal of Finance*, **69**(5), 1961–2005.

Bannier, Christina E., Behr, Patrick, and Güttler, Andre. 2010. Rating opaque borrowers: why are unsolicited ratings lower? *Review of Finance*, **14**(2), 263–294.

Bar-Isaac, Heski, and Shapiro, Joel. 2013. Ratings quality over the business cycle. *Journal of Financial Economics*, **108**(1), 62–78.

Barberis, Nicholas, and Xiong, Wei. 2012. Realization utility. *Journal of Financial Economics*, **104**(2), 251–271.

Baule, Rainer. 2011. The order flow of discount certificates and issuer pricing behavior. *Journal of Banking & Finance*, **35**(11), 3120 – 3133.

Baule, Rainer, Entrop, Oliver, and Wilkens, Marco. 2008. Credit risk and bank margins in structured financial products: Evidence from the German secondary market for discount certificates. *Journal of Futures Markets*, **28**(4), 376–397.

Becker, Bo, and Milbourn, Todd T. 2011. How did increased competition affect credit ratings? *Journal of Financial Economics*, **101**(3), 493–514.

Ben-David, Itzhak, and Hirshleifer, David. 2012. Are Investors Really Reluctant to Realize Their Losses? Trading Responses to Past Returns and the Disposition Effect. *Review of Financial Studies*, **25**(8), 2485–2532.

Benmelech, Efraim, and Dlugosz, Jennifer. 2009. The alchemy of CDO credit ratings. *Journal of Monetary Economics*, **56**(5), 617–634.

Bernanke, Ben S., Bertaut, Carol, DeMarco, Laurie Pounder, and Kamin, Steven. 2011. *International Capital Flows and the Returns to Safe Assets in the United States, 2003-2007*. International Finance Discussion Papers 1014. Board of Governors of the Federal Reserve System.

Bizzotto, Jacopo. 2016. *Fees, Reputation, and Rating Quality*. Mimeo.

Bolton, Patrick, Freixas, Xavier, and Shapiro, Joel. 2012. The Credit Ratings Game. *The Journal of Finance*, **67**(1), 85–112.

Bongaerts, Dion, Cremers, K. J. Martijn, and Goetzmann, William N. 2012. Tiebreaker: Certification and Multiple Credit Ratings. *Journal of Finance*, **67**(1), 113–152.

Boot, Arnoud W. A., Milbourn, Todd T., and Schmeits, Anjolein. 2006. Credit Ratings as Coordination Mechanisms. *Review of Financial Studies*, **19**(1), 81–118.

Bouvard, Matthieu, and Levy, Raphael. 2013. *Two-sided reputation in certification markets*. Carlo Alberto Notebooks 339. Collegio Carlo Alberto.

Brocas, Isabelle, and Carrillo, Juan D. 2007. Influence through ignorance. *RAND Journal of Economics*, **38**(4), 931–947.

Camanho, Nelson, Deb, Pragyan, and Liu, Zijun. 2012. *Credit Rating and Competition*. Mimeo.

Célérier, Claire, and Vallée, Boris. 2017. Catering to Investors through Security Design: Headline Rate and Complexity. *Quarterly Journal of Economics (forthcoming)*.

Cheng, Mei, and Neamtiu, Monica. 2009. An empirical analysis of changes in credit rating properties: Timeliness, accuracy and volatility. *Journal of Accounting and Economics*, **47**(1-2), 108–130.

Cohn, Jonathan, Rajan, Uday, and Strobl, Günter. 2016. *Credit Ratings: Strategic Issuer Disclosure and Optimal Screening*. Mimeo.

Coval, Joshua, Jurek, Jakub W., and Stafford, Erik. 2009a. Economic Catastrophe Bonds. *American Economic Review*, **99**(3), 628–66.

Coval, Joshua, Jurek, Jakub, and Stafford, Erik. 2009b. The economics of structured finance. *The Journal of Economic Perspectives*, **23**(1), 3–25.

Demirguc, Asli, and Detragiache, Enrica. 2000. Monitoring Banking Sector Fragility: A Multivariate Logit Approach. *World Bank Economic Review*, **14**(2), 287–307.

Demyanyk, Yuliya, and Van Hemert, Otto. 2011. Understanding the Subprime Mortgage Crisis. *Review of Financial Studies*, **24**(6), 1848–1880.

Deutsche Derivative Verband. 2014. *The structured products sector in figures*. Source: http://www.derivateverband.de.

Deutsche Derivative Verband. 2016. *Market Volume in Derivatives*. Source: http://www.derivateverband.de.

Doherty, Neil A, Kartasheva, Anastasia V, and Phillips, Richard D. 2012. Information effect of entry into credit ratings market: The case of insurers' ratings. *Journal of Financial Economics*, **106**(2), 308–330.

Farhi, Emmanuel, Lerner, Josh, and Tirole, Jean. 2013. Fear of rejection? Tiered certification and transparency. *RAND Journal of Economics*, **44**(4), 610–631.

Farkas, Miklós. 2016. *Credit rating catering*. Mimeo.

Faure-Grimaud, Antoine, Peyrache, Eloïc, and Quesada, Lucía. 2009. The ownership of ratings. *The RAND Journal of Economics*, **40**(2), 234–257.

Feng, Lei, and Seasholes, Mark S. 2005. Do Investor Sophistication and Trading Experience Eliminate Behavioral Biases in Financial Markets? *Review of Finance*, **9**(3), 305–351.

Frenkel, Sivan. 2015. Repeated Interaction and Rating Inflation: A Model of Double Reputation. *American Economic Journal: Microeconomics*, **7**(1), 250–80.

Fulghieri, Paolo, Strobl, Günter, and Xia, Han. 2014. The economics of solicited and unsolicited credit ratings. *Review of Financial Studies*, **27**(2), 484–518.

Glosten, Lawrence R., and Milgrom, Paul R. 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, **14**(1), 71–100.

Griffin, John M., and Tang, Dragon Y. 2012. Did subjectivity play a role in CDO credit ratings? *The Journal of Finance*, **67**(4), 1293–1328.

Griffin, John M., Nickerson, Jordan, and Tang, Dragon Y. 2013. Rating shopping or catering? An examination of the response to competitive pressure for CDO credit ratings. *Review of Financial Studies*, **26**(9), 2270–2310.

Grinblatt, Mark, and Keloharju, Matti. 2001. What Makes Investors Trade? *The Journal of Finance*, **56**(2), 589–616.

Grossman, Sanford J. 1981. The Informational Role of Warranties and Private Disclosure about Product Quality. *Journal of Law and Economics*, **24**(3), 461–83.

Henderson, Brian J., and Pearson, Neil D. 2011. The dark side of financial innovation: A case study of the pricing of a retail financial product. *Journal of Financial Economics*, **100**(2), 227 – 247.

Jeon, Doh-Shin, and Lovo, Stefano. 2011. Natural barrier to entry in the credit rating industry. In: *Paris December 2010 Finance Meeting EUROFIDAI-AFFI*.

Jeon, Doh-Shin, and Lovo, Stefano. 2013. Credit rating industry: A helicopter tour of stylized facts and recent theories. *International Journal of Industrial Organization*, **31**(5), 643–651.

Jiao, Peiran. 2016. Belief in Mean Reversion and the Disposition Effect: An Experimental Test. *Journal of Behavioral Finance (forthcoming)*.

Kamenica, Emir, and Gentzkow, Matthew. 2011. Bayesian Persuasion. *American Economic Review*, **101**, 2590–2615.

Kisgen, Darren J., and Strahan, Philip E. 2010. Do Regulations Based on Credit Ratings Affect a Firm's Cost of Capital? *Review of Financial Studies*, **23**(12), 4324–4347.

Kondor, Péter, and Kőszegi, Botond. 2017. *Financial Choice and Financial Information*. Mimeo.

Kraft, Pepa. 2015. Do rating agencies cater? Evidence from rating-based contracts. *Journal of Accounting and Economics*, **59**(2), 264–283.

Kumar, Alok. 2009. Who Gambles in the Stock Market? *The Journal of Finance*, **64**(4), 1889–1933.

Kyle, Albert S. 1985. Continuous Auctions and Insider Trading. *Econometrica*, **53**(6), 1315–35.

Langohr, Herwig, and Langohr, Patricia. 2010. *The rating agencies and their credit ratings: what they are, how they work, and why they are relevant*. Chichester: John Wiley & Sons.

Lizzeri, Alessandro. 1999. Information revelation and certification intermediaries. *The RAND Journal of Economics*, **30**(2), 214–231.

Manso, Gustavo. 2013. Feedback effects of credit ratings. *Journal of Financial Economics*, **109**(2), 535–548.

Mathis, Jerome, McAndrews, James, and Rochet, Jean-Charles. 2009. Rating the raters: are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics*, **56**(5), 657–674.

Odean, Terrance. 1998. Are Investors Reluctant to Realize Their Losses? *The Journal of Finance*, **53**(5), 1775–1798.

Opp, Christian C., Opp, Marcus M., and Harris, Milton. 2013. Rating agencies in the face of regulation. *Journal of Financial Economics*, **108**(1), 46–61.

Raiter, Frank L. 2010. *Written Statement to the Senate.* Written Statement. Permanent Subcommittee on Investigations. United States Senate.

Rubinstein, M., and Reiner, E. 1991. Breaking Down the Barriers. *RISK*, **4**(8), 28–35.

Sangiorgi, Francesco, and Spatt, Chester. 2016. Opacity, credit rating shopping and bias. *Management Science, forthcoming*.

Sarlin, Peter. 2013. On policymakers' loss functions and the evaluation of early warning systems. *Economics Letters*, **119**(1), 1–7.

Sims, Christopher A. 2003. Implications of rational inattention. *Journal of Monetary Economics*, **50**(3), 665 – 690. Swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.

Skreta, Vasiliki, and Veldkamp, Laura. 2009. Ratings shopping and asset complexity: A theory of ratings inflation. *Journal of Monetary Economics*, **56**(5), 678–695.

Stambaugh, Robert F. 2014. Presidential Address: Investment Noise and Trends. *The Journal of Finance*, **69**(4), 1415–1453.

10.14754/CEU.2017.09

Strausz, Roland. 2005. Honest certification and the threat of capture. *International Journal of Industrial Organization*, **23**(1), 45–62.

U.S. Senate. 2011. *Wall Street and the Financial Crisis: Anatomy of a Financial Collapse.* Majority and Minority Staff Report. Permanent Subcommittee on Investigations.

Weber, Martin, and Camerer, Colin F. 1998. The disposition effect in securities trading: an experimental analysis. *Journal of Economic Behavior & Organization*, **33**(2), 167–184.

Wilkens, Sascha, and Stoimenov, Pavel A. 2007. The pricing of leverage products: An empirical investigation of the German market for 'long' and 'short' stock index certificates. *Journal of Banking & Finance*, **31**(3), 735 – 750.

Wilkens, Sascha, Erner, Carsten, and Rder, Klaus. 2003. The Pricing of Structured Products in Germany. *The Journal of Derivatives*, **11**(1), 55 – 69.

Xiong, Wei. 2013 (Mar.). *Bubbles, Crises, and Heterogeneous Beliefs*. NBER Working Papers 18905. National Bureau of Economic Research, Inc.

Yang, Ming. 2015. Coordination with flexible information acquisition. *Journal of Economic Theory*, **158, Part B**, 721 – 738. Symposium on Information, Coordination, and Market Frictions.

10.14754/CEU.2017.09

4

Appendices

Appendix A Proofs of Chapter 1

Proof of Lemma 2. First, consider the symmetric equilibrium strategy of issuers who are offered A ratings from both agencies. In a symmetric equilibrium, where agencies can only use pure strategies ($f_1 = f_2$), the following must hold:

$$D(A,A,f_1,f_2) = [q,q,p,1-p-2q], \tag{A.1}$$

where function outputs correspond to action probabilities 'purchase agency 1's rating', 'purchase agency 2's rating', 'purchase both' and 'purchase none', respectively. Now consider the case when $p > 0$ and $q > 0$. This would imply that issuers with $\{A,A\}$ offered ratings are indifferent between purchasing both or purchasing only one A rating from one of the agencies. In turn, issuers with mixed offered ratings always want to purchase their A rating in order to pool with $\{A,A\}$ issuers who only purchase a single rating. Hence, $D(A,B,f_1,f_2) = [1,0,0,0]$ and $D(B,A,f_1,f_2) = [0,1,0,0]$. Also, the marginal value of a second A rating $((\hat{p}_{AA} - \hat{p}_{A,\emptyset})R)$ has to be equal to the equilibrium rating fee. With such strategies it is easy to show that agencies always have an incentive to marginally decrease their fees. If agency 1 decreases its fee marginally, there will be two effects. First, there will be a jump in agency 1's fee revenue, as issuers with $\{A,A\}$ offered ratings now find it optimal to always purchase agency 1's rating. Second, since agency 1 is selling ratings in a higher proportion to issuers with $\{A,A\}$ offered ratings, its expected penalty per issuer will decrease. Both effects increase agency profits. However, fees cannot be arbitrarily low, as this will prevent agencies from providing ratings. Thus, there is no symmetric equilibrium with $p > 0$ and $q > 0$, since agencies either want to decrease fees or they do not want to provide ratings at all. Second, consider the case when $p = 0$ and $q > 0$. In this case agencies have an incentive to compete in fees and the same argument holds as above. Third, consider the case when $0 < p < 1$ and $q = 0$. In this case agencies would want to marginally decrease fees so issuers respond by $p = 1$. Hence, the only candidate left is $p = 1$ and $q = 0$. This implies that issuers with contradictory offered ratings cannot pool with $\{A, A\}$ issuers, leading to $D() = [0, 0, 0, 1]$ for all issuers who are not offered A by both agencies. The equilibrium strategy of issuers gives the profit function (1.20). Consider its partial derivatives with respect to ρ_1 and ε_1: $\frac{\partial \Pi}{\partial \rho_1}$ $\frac{\partial \Pi}{\partial \varepsilon_1}$. Assume that in equilibrium $0 < \varepsilon_1^* < 1$ and $0 < \rho_1^* < 1$. This implies that both partial derivatives must equal zero. However, it is easy to show that this leads to a contradiction. Similarly, the candidates in which (i) $\varepsilon_1^* = 1$ and $0 < \rho_1^* < 1$ and (ii) $0 < \varepsilon_1^* < 1$ and $\rho_1^* = 0$ also lead to contradictions. Hence, the only candidates left are those in which

$$\frac{\partial \Pi}{\partial \varepsilon_1} < 0, \frac{\partial \Pi}{\partial \rho_1} < 0, \tag{A.2}$$

implying $\rho_1^* = \varepsilon^* = 0$ (region A in Figure 1.4),

$$\frac{\partial \Pi}{\partial \varepsilon_1} < 0, \frac{\partial \Pi}{\partial \rho_1} = 0, \tag{A.3}$$

implying $0 < \rho_1^* < 1$ and $\varepsilon^* = 0$ (region B+C in Figure 1.4),

$$\frac{\partial \Pi}{\partial \varepsilon_1} < 0, \frac{\partial \Pi}{\partial \rho_1} > 0, \tag{A.4}$$

implying $\rho_1^* = 1$ and $\varepsilon^* = 0$ (region D in Figure 1.4), and

$$\frac{\partial \Pi}{\partial \varepsilon_1} = 0, \frac{\partial \Pi}{\partial \rho_1} > 0, \tag{A.5}$$

implying $\rho_1^* = 1$ and $0 < \varepsilon^* < 1$ (region E in Figure 1.4). It is easy to show, that given agency 2's strategy and parameters, exactly one of the above condition pairs will hold. Hence, what is left to show is that given that one condition holds, there is only a unique symmetric equilibrium. The expected marginal cost of catering is $c(1 - \pi_g)$ if there is no inflating and $c(1 - p_{BB})(> c(1 - \pi_g))$ if there is inflating, while the expected marginal cost of inflating is always $c(1 - p_{BB})$. Since the cost of catering is smaller, $(p_{AA}(\mathbf{0})R - 1)/2 \leq c(1 - \pi_g)$ has to be satisfied for the no catering/no inflating equilibrium. If $(p_{AA}(\mathbf{0})R - 1)/2 > c(1 - \pi_g) > (p_{AA}(0, 0, 1, 1)R - 1)/2$ then the symmetric equilibrium catering level will be implied by

$$c(1 - \pi_g) = (p_{AA}(0, 0, \rho^*, \rho^*)R - 1)/2 \tag{A.6}$$

which has exactly one solution as $p_{AA}(\dot{)}$ is strictly decreasing in ρ_1 and ρ_2. The solution gives equilibrium ρ^* to be

$$\rho^* = \frac{\mu_{aa}[p_{AA}(\mathbf{0})R - 1 - 2c(1-\pi_g)]}{2\mu_{ab}[2c(1-\pi_g) - \bar{V}]} \tag{A.7}$$

If $\frac{p_{AA}(0,0,1,1)R-1}{2(1-\pi_g)} > c > \frac{p_{AA}(0,0,1,1)R-1}{2(1-p_{BB})}$ will imply that catering all ratings ($\rho^* = 1$) is optimal, and no inflating is also optimal as the expected marginal penalty of inflating $c(1-p_{BB})$ is larger than the equilibrium rating fee $\frac{p_{AA}(0,0,1,1)R-1}{2}$. Finally, if $c < \frac{p_{AA}(0,0,1,1)R-1}{2(1-p_{BB})}$ the agency will find it optimal to inflate ratings. The symmetric equilibrium level of inflating is implied by

$$c = \frac{p_{AA}(\varepsilon^*, \varepsilon^*, 1, 1)R - 1}{2(1-p_{BB})}, \tag{A.8}$$

which has a unique solution as the success probability is strictly decreasing in inflating. The solution for ε^* is implied by

$$2\varepsilon^* - \varepsilon^{*2} = \frac{(\mu_{aa} + 2\mu_{ab})[p_{AA}(0,0,1,1)R - 1 - 2c(1-p_{BB})]}{\mu_{bb}[2c(1-p_{BB}) - (p_{BB}R - 1)]} \tag{A.9}$$

Proof of Corollary 1.

$$p_A(0)R - 1 < \frac{p_{AA}(\mathbf{0})R - 1}{2} \underbrace{\rightarrow}_{1-\pi_g < 1-p_B} \frac{p_A(0)R - 1}{1-p_B} < \frac{p_{AA}(\mathbf{0})R - 1}{2(1-\pi_g)} \tag{A.10}$$

Proof of Corollary 2. It is easy to show that

$$\frac{p_A(0)R - 1}{(1-p_B)} > \frac{p_{AA}(0,0,1,1)R - 1}{2(1-p_{BB})}, \text{ and}$$
$$\varepsilon^{mon} > \varepsilon^{duo} \text{ for } c = 0$$

The first inequality implies that the monopolist will start to inflate at a higher penalties. The second inequality says that at $c = 0$ inflating in a monopoly will always be larger. Given these, the only way $\varepsilon^{mon} < \varepsilon^{duo}$ can occur, is if $\varepsilon^{mon} = \varepsilon^{duo}$ has two (or more) roots in c on the interval where $\varepsilon^{duo} > 0$ and $c \geq 0$. However, this can easily be ruled out as the equality $\varepsilon^{mon} = \varepsilon^{duo}$ has exactly two roots in c and one of them is always negative.

Proof of Corollary 3. Proof of (i): when agencies do not manipulate in equilibrium, rating standards are unaffected by marginal changes in c. However, when they do manipulate, rating standards will be increasing in c, because[1]

$$\left.\frac{\partial p_A(\varepsilon^*)}{\partial c}\right|_{\varepsilon^*>0} = \frac{1-p_B}{R} > 0 \tag{A.11}$$

$$\left.\frac{\partial p_{AA}(\varepsilon^*,\rho^*)}{\partial c}\right|_{\varepsilon^*=0,\rho^*>0} = \frac{2(1-\pi_g)}{R} > 0, \quad \left.\frac{\partial p_{AA}(\varepsilon^*,\rho^*)}{\partial c}\right|_{\varepsilon^*>0,\rho^*>0} = \frac{2(1-p_{BB})}{R} > 0 \tag{A.12}$$

Proof of (ii): when agencies do not manipulate ratings, it is clear that $p_A(0)$ and $p_A(\mathbf{0})$ are strictly increasing in $1-\alpha$, which gives the result. When agencies do manipulate in equilibrium,

$$\left.\frac{\partial p_A(\varepsilon^*)}{\partial(1-\alpha)}\right|_{\varepsilon^*>0} = -\frac{c}{R}\frac{\partial p_B}{\partial(1-\alpha)} > 0, \tag{A.13}$$

$$\left.\frac{\partial p_{AA}(\varepsilon^*,\rho^*)}{\partial(1-\alpha)}\right|_{\varepsilon^*=0,\rho^*>0} = 0 \tag{A.14}$$

$$\left.\frac{\partial p_{AA}(\varepsilon^*,\rho^*)}{\partial(1-\alpha)}\right|_{\varepsilon^*>0,\rho^*>0} = -\frac{2c}{R}\frac{\partial p_{BB}}{\partial(1-\alpha)} > 0, \tag{A.15}$$

which give the result. Proof of (iii): p_B and p_{BB} are increasing in π_g, which imply that $p_A(\varepsilon^*)$ and $p_{AA}(\varepsilon^*,\rho^*)$ (given that there is some manipulation) are decreasing in π_g. On the other hand, $p_A(0)$ and $p_{AA}(0,0)$ are increasing in π_g.

Proof of Corollary 4. First, observe that rating standards in a monopoly and duopoly will be equal if $\rho^* = 1/2$ in a duopoly and $\varepsilon^{*mon} = 0$. The c that implements $\rho^* = 1/2$ is $c = \frac{p_A(0)R-1}{2(1-\pi_g)}$. It is clear that if $2(1-\pi) < 1-p_B$ then a monopoly will find $\varepsilon^{*mon} = 0$ to be optimal at this point. Now if c marginally decreases then a monopolist will still not inflate, while agencies in a duopoly will choose $\rho^* > 1/2$, which imply that rating standards in the monopoly will be higher.

Proof of Proposition 1. Observe that $\varepsilon^* = 0$ in a duopoly because of the first condition. In order to rank welfare, one needs to solve for the parameters, where $W^{mon}(\varepsilon^*) = W^{duo}(\rho^*)$. There are two regions to consider. First, in region B it is clear that one needs to find a solution to $W^{mon}(0) = W^{duo}(1/2)$. The solution to this is

$$\bar{c}(\alpha,\pi_g,V_g) = \frac{\pi_g V_g(1-\alpha)-\alpha(1-\pi_g)}{2(1-\pi_g)\mu_a} \tag{A.16}$$

[1] With a slight abuse of notation, let $p_{AA}(\varepsilon^*,\rho^*) = p_{AA}(\varepsilon^*,\varepsilon^*,\rho^*,\rho^*)$.

Second, there may be a second solution in region C, where the monopoly inflates ratings in equilibrium, $W^{mon}(\varepsilon^* > 0) = W^{duo}(\rho^* > 1/2)$. The solution to this is

$$\underline{c}(\alpha, \pi_g, V_g) = \frac{(1-2\alpha)[1+\alpha(2\pi_g-1)(1-\pi_g+\pi_g V_g)+\pi_g(2\pi_g+V_g-3)]}{2(1-\alpha)(1-\pi_g)\mu_a} \tag{A.17}$$

Finally, at $c = 0$: $W^{mon}(\varepsilon^*) = W^{duo}(\rho^*) = 0$. It is easy to show that if $\bar{c}(\alpha, \pi_g, V_g)$ does not exist, then $\underline{c}(\alpha, \pi_g, V_g)$ also does not exist and $W^{duo}(\rho^*) > W^{mon}(\varepsilon^*)$ always. If $\bar{c}(\alpha, \pi_g, V_g)$ does exist then there are two cases. Either the solution to $\underline{c}(\alpha, \pi_g, V_g)$ does not exist (as on the right panel of Figure 1.5) and $W^{duo}(\rho^*) > W^{mon}(\varepsilon^*)$ for all $c < \bar{c}(\alpha, \pi_g, V_g)$ or the solution to $\underline{c}(\alpha, \pi_g, V_g)$ does exist, in which case $W^{duo}(\rho^*) < W^{mon}(\varepsilon^*)$ for all $\underline{c}(\alpha, \pi_g, V_g) < c < \bar{c}(\alpha, \pi_g, V_g)$ and $W^{duo}(\rho^*) > W^{mon}(\varepsilon^*)$ for $c < \underline{c}(\alpha, \pi_g, V_g)$ (this happens on the left panel of Figure 1.5). The function $\bar{\pi}_g(\alpha, V_g, c)$ is the inverse of $\underline{c}(\alpha, \pi_g, V_g)$ with respect to π_g, which is well defined as $\underline{c}(\alpha, \pi_g, V_g)$ is strictly decreasing in π_g.

Appendix B Merger of two agencies

In order to isolate the effect of market structure and information, I analyze the welfare implications of a merger of two rating agencies. Analyzing a merger isolates the effect of market structure when comparing the merged agency with a duopoly (since information is held constant), but also isolates the effect of an additional signal when comparing the merged agency with the monopolist (since both operate as monopolists).

The merged agency observes both signals, keeping the amount of information constant before and after the merger. Similarly, a merged agency may condition its manipulation method on both signals, which implies that manipulation methods are also held constant. In particular, if the penalty is sufficiently low, a merged agency finds it optimal to offer A ratings to issuers with mixed signals, just like agencies in a duopoly start by catering ratings.

I assume that the distribution of projects is such that if all projects with mixed signals were financed then the financed projects would have negative NPV.[2]. This simplifies the analysis without loss of intuition, as agencies in a duopoly and the merged agency will never want to enable the financing of a project with two b signals, but may find it optimal to enable the financing of some projects with mixed signals.

Formally, the merged agency's objective is

[2] This corresponds to the condition (i) in Proposition 1

$$\max_{\tilde{\rho},\tilde{f}} \mu_{aa}[\tilde{f} - c(1 - p_{AA}(\mathbf{0}))] + \tilde{\rho}(\mu_{ba} + \mu_{ab})[\tilde{f} - c(1 - \pi_g)], \quad \text{s.t.} \quad \tilde{f} \leq \hat{p}_A R - 1 \tag{B.1}$$

where $\tilde{\rho}$ is the probability of assigning an A rating to an issuer with mixed signals and $\tilde{f}$ is the rating fee of the merged agency.

Observe, that assigning a good rating to issuers with mixed signals ($\tilde{\rho}$) has the same impact on welfare as catering (ρ), since both methods guarantee financing of issuers with mixed signals. Hence, welfare in a merger may be defined using the welfare measure of the duopoly, as

$$W^{merger}(\tilde{\rho}^*) = W^{duo}(\tilde{\rho}^*) = \frac{\mu_{aa}(p_{AA}(\mathbf{0})R - 1) + \tilde{\rho}^* 2\mu_{ab}\bar{V}}{\pi_g V_g}. \tag{B.2}$$

The merged agency finds it optimal not to manipulate, if

$$c > \frac{p_{AA}(\mathbf{0})R - 1}{1 - \pi_g}, \tag{B.3}$$

which, together with the condition for no catering in a duopoly stated in Lemma 2 implies that the merged agency requires a penalty twice as large than agencies in a duopoly for truthful reporting. This directly follows from the fact that if there is no manipulation, the sum of the two rating fees in a duopoly is the same as the rating fee after the merger.

The following proposition states the results.

Proposition B.1 (Welfare implications of a merger/added signal)**.***(i) If the penalty is sufficiently high, then a merger does not affect welfare:*

$$W^{duo}(0) = W^{merger}(0) \;\; \textit{if} \;\; c \geq \frac{p_{AA}(\mathbf{0})R - 1}{1 - \pi_g} \tag{B.4}$$

(ii) If the penalty is sufficiently low, then a merger is welfare decreasing:

$$W^{duo}(\rho^*) > W^{merger}(\tilde{\rho}^*) \;\; \textit{if} \;\; c < \frac{p_{AA}(\mathbf{0})R - 1}{1 - \pi_g} \tag{B.5}$$

(iii) If $c \in [\underline{c}, \frac{p_A(0)R-1}{1-\pi_g})$ *then*

$$W^{merger}(\tilde{\rho}^*) < W^{mon}(\varepsilon^*), \;\; \textit{where} \;\; \underline{c} < \frac{p_A(0)R - 1}{1 - \pi_g} \tag{B.6}$$

Otherwise $W^{merger}(\tilde{\rho}^*) \geq W^{mon}(\varepsilon^*)$.

The proofs are straightforward. When the penalty is sufficiently high to deter the merged agency from manipulation then a merger does not affect welfare as only those projects obtain financing in both cases who have two *a* signals. However, if the merged agency finds it optimal to assign *A* ratings to some issuers with mixed signals then the merger is welfare decreasing, because either agencies in a duopoly do not cater ratings at all if

$$\frac{p_{AA}(\mathbf{0})R-1}{2(1-\pi_g)} \leq c < \frac{p_{AA}(\mathbf{0})R-1}{1-\pi_g} \tag{B.7}$$

or cater less, as rating standards are always higher if agencies in a duopoly cater ratings

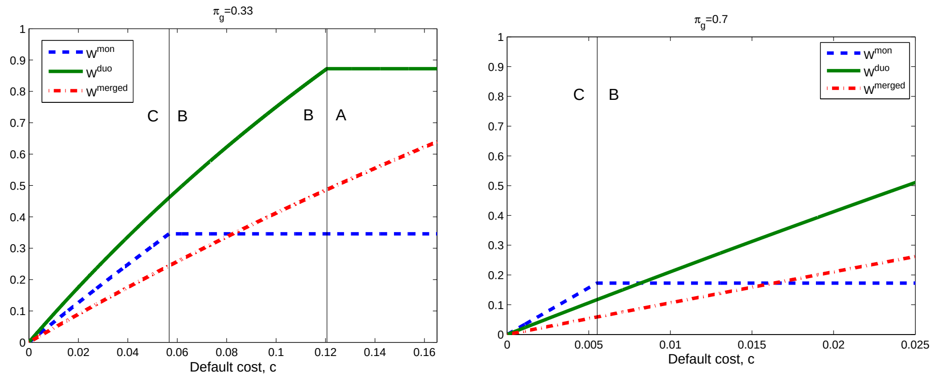
$$p_{AA}(\rho^*) = \frac{2c(1-\pi_g)+1}{R} > \frac{c(1-\pi_g)+1}{R} = p_{AA}(\tilde{\rho}^*) \tag{B.8}$$

The surprising result in Proposition B.1 is that moving from a (merged) monopoly to a duopoly may leave welfare unchanged if agencies do not manipulate in either market structures. Thus, the standard intuition, that a duopoly always leads to lower deadweight loss compared to a monopoly breaks down in the current setup. The fundamental source of inefficiency here is signal noise, which results in the financing of some bad projects and also prevents the financing of some good projects. Hence, welfare is only increasing with the quantity of financed good projects, while in the standard model welfare is generally increasing with quantity. If information is held fixed welfare cannot change beyond an upper bound by changing market structure, as both market structures use the information in the most efficient way (i.e. only allowing the financing of projects with two *a* signals).

Proposition B.1 also implies that a merger does not increase information rents. This is because agencies in a duopoly do not compete in fees, but extract all the gains from trade. However, if there were more than two agencies, there could be incentives to merge as more agencies could lead to equilibria where agencies are forced to compete in rating fees.[3] On the other hand, merging does have the benefit that penalties only have to paid once after the merger.

The additional signal of the merged agency is only welfare improving compared to a monopoly if the penalty is sufficiently high. However, the additional signal also undermines honest disclosure through two channels. First, it increases the value of its honest rating, which enables the agency to increase its fee. This makes rating

[3] This prediction is in line with the history of mergers in the ratings industry. Becker and Milbourn (2011) illustrates the multiple mergers that led to the appearance of Fitch. Also, S&P is a result of the 1941 merger between Standard Statistics and Poor's Publishing. On the other hand, the two dominant agencies constitutes a stable market structure.



(a) Low share of good projects

(b) High share of good projects

Figure B.1 Welfare implications of a merger. Vertical lines indicate parameter regions, corresponding to the regions introduced in Figure 1.4. Parameters are: $\alpha = 0.05$ and V_g is set such that $\mu_{aa} p_{aa} + 2\mu_{ab}\bar{V} = -0.1$. In panel (a) $V_g = 0.17$, in panel (b) $V_g = 0.03$.

manipulation more tempting. Second, the merged agency can now selectively manipulate the ratings of issuers with mixed signals, that are better, on average, than issuers who's ratings may be considered for inflating in a monopoly. This decreases the expected penalty for manipulation, due to the more sophisticated selection of manipulated projects.

Figure B.1 illustrates how welfare changes with the penalty in a monopoly (dashed line), a duopoly (solid line) and a merged agency (dash-dot line). As stated in Proposition B.1, welfare after a merger is never greater than welfare in a duopoly. The point where a merged agency caters for every second issuer with mixed signals ($\tilde{\rho}^* = 1/2$) corresponds to the intersection of W^{merger} (dash-dot line) and W^{mon} (dashed line). Since at this penalty level the monopolist always finds it optimal not to inflate ratings ($\varepsilon^* = 0$), the welfare measures always coincide at the corresponding penalty level. The value of $\underline{c}$ used in Proposition B.1 is zero on both panels of Figure B.1.[4]

[4] However, in some extreme cases, when the share of good projects (π_g) is very low, the welfare measures could intersect again for some penalty level close to (but still above) zero.

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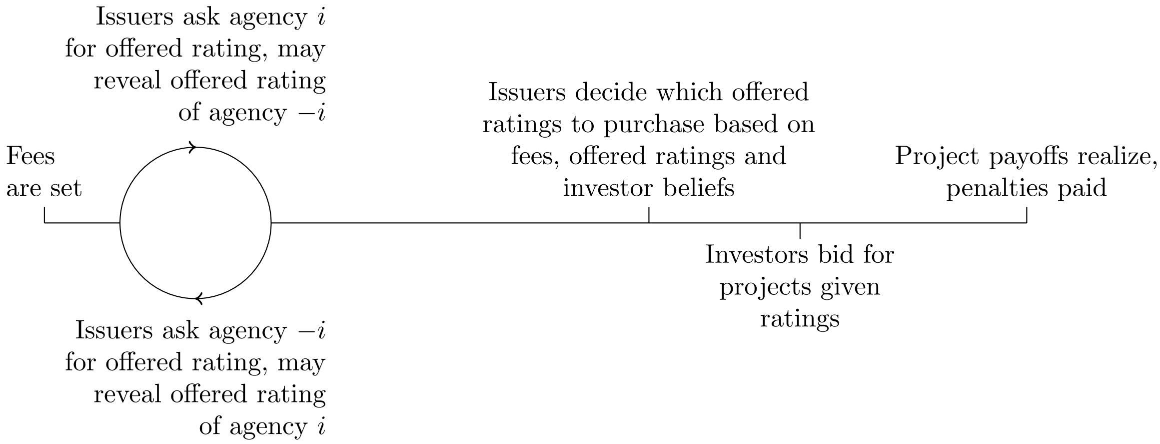

Figure C.1 Timeline of the sequential game with a duopoly of agencies.

Appendix C Sequential rating process

This section provides a sequential ratings game, in which the rating process is divided into multiple rounds. I show that the game in the main text is a reduced form of this game, as both of them lead to the qualitatively identical equilibrium.

Consider the timeline for the sequential rating game:

1. Agencies $i = 1,2$ set fees, f_i.
2. Given fees, issuers may ask one agency for an offered rating.
3. The agency observes a signal s_{ij} about each asking issuer and produces an offered rating $\tilde{r}_{ij}$.
4. Given fees and offered ratings, issuers may ask one agency for an offered rating.
    1. Issuers may reveal the offered rating of the other agency made in previous rounds.
    2. Given its signal and the other agency's offered rating (if revealed), the agency offers a rating for the asking issuer, $\tilde{r}_{ij}$.

    4. is repeated until issuers do not want to ask any of the agencies for an offered rating.
5. Issuers decide whether to pay the fees to the agencies for disclosing their offered ratings to investors.
6. Investors bid for the projects, given their respective ratings.
7. Project cash flows become realized and penalties are paid by agencies.

The timeline is also illustrated on Figure C.1. Agencies optimally choose how they transform signals[5] and revealed offered ratings of the other agency into offered ratings. Importantly, since issuers and agencies may interact multiple times, the agency is also able to identify issuers. Hence, agencies' rating assignment strategies may be represented by the following function:

$$\tilde{r}_{ij} = \begin{cases} A & \text{if} \quad s_{ij} = a \\ A & \text{if} \quad s_{ij} = b, \tilde{r}_{-ij} \neq A, j < \varepsilon_i \\ A & \text{if} \quad s_{ij} = b, \tilde{r}_{-ij} = A, j < \rho_i \\ B & \text{otherwise,} \end{cases} \tag{C.1}$$

where the agency sets variables ε_i and ρ_i. Similarly to the model in the main text, all issuers with a signals will be offered A ratings by (C.1). If the agency-issuer signal is b and the issuer does not reveal an A offered rating from the other agency (implying that it either reveals nothing or a B offered rating) then the agency inflates its rating to A if its index is smaller than ε_i. Similarly, if the agency-issuer signal is b and the issuer reveals an A offered rating from the other agency then the agency caters its rating if its index is smaller than ρ_i. If issuer types are randomly distributed on the unit segment then ε_i and ρ_i will correspond to inflating and catering probabilities introduced in the main text. Importantly, the offered rating assignment rule of the agency does not depend on which round the issuer asks for an offered rating. This captures the opacity of the market: if issuer-agency contacts were observable then issuers could commit to only ask a single agency for a rating. However, this seems unrealistic.

In the sequential game issuers not only have to decide which ratings to purchase but they also have to plan the sequence in which they ask agencies for offered ratings and they also have to decide whether they want to reveal offered ratings by the other agency. Formally, let $G(H)$ be a function that gives issuer j's potentially mixed strategy over the following action set: $\{$ ask agency 1; ask agency 2; ask none $\}$, where H contains the history of offered ratings made by the agencies. I will conjecture and later verify that issuers always reveal their offered rating proposed by the other agency.

Below I state the Perfect Bayesian Equilibrium of the sequential game.

Definition C.1 (Equilibrium)**.** *1. Issuers optimally choose functions $G(H)$ and their rating purchase decisions, given beliefs of investors and rating fees.*

2. Rating agencies optimally set fees and manipulation levels (ε_i, ρ_i), given issuers' strategies, investor beliefs and fees and manipulation levels set by the other rating agency.

[5] Note that agencies' information technology always generates the same errors. That is, applying the rating technology to a given issuer always produces the same realization of the signal.

3. *Investor beliefs about success probabilities are correct for all rating combinations.*
4. *There is some finite $\bar{k}$ for which if $k > \bar{k}$ then none of the issuers want to ask for an offered rating.*

It is easy to see that all equilibria in which $\bar{k} > 3$ are redundant, since three rounds is enough for issuers to convey all information between agencies.

Are there symmetric equilibria, in which $\bar{k} = 1$, that is, issuers only want to ask a single agency for an offered rating? Suppose there is such an equilibrium. Then in that equilibrium agencies must compete in rating fees in order to attract issuers. Agencies will break even if they set fees equal to $f_i = c(1 - p_A(0))$, as in this case the rating fee equals the expected penalty of a project with a single a signal. Since agencies propose identical fees, issuers find it optimal to ask agencies 1 and 2 with equal probabilities in the first round. However, issuers who are offered a B rating would also find it optimal to ask the other agency for a rating in round 2, in the hope that the other agency will receive an a signal about their project's quality. Hence, there are no symmetric equilibria in which all issuers find it optimal to only ask a single agency for a rating.[6]

Are there symmetric equilibria in which $\bar{k} = 2$? First, consider equilibria candidates in which all issuers only purchase a single A rating, if offered at least one.[7] If that is the case, agencies must compete in rating fees in order to attract business. Since issuers with at least one a signal will be financed, the rating fee equals to $f_i = c(1 - p_{AA}(0,0,1,1))$. Issuers randomly ask one of the agencies for an offered rating in the first round then they ask the other agency in the second round. However, agencies now have an incentive to marginally decrease the rating fee. By doing so, all the $\{A,A\}$ issuers would purchase the rating of the deviating agency, which would increase fee revenue and decrease the total penalty because the average quality of these projects are higher (their success rate increases to $p_{AA}(0,0,1/2,1/2)$). The same argument applies for equilibrium candidates in which issuers with $\{A,A\}$ offered ratings randomly purchase one or both. Thus, equilibrium candidates in which at least some issuers with $\{A,A\}$ offered ratings only purchase a singe A cannot be supported with non-random fees.

Now consider equilibria candidates in which issuers with $\{A,A\}$ offered ratings purchase both. This will be identical to the equilibrium of Lemma 2 as long as agencies find it optimal not to cater ratings ($p_{AA}(\mathbf{0})R - 1 \leq 2c(1 - \pi_g)$). However, if agencies do cater ratings ($\rho_i > 0$) then issuers who were offered a B rating from

[6] An equilibrium with a single round may be reached if one is willing to assume that agencies can credibly threat to offer B ratings to issuers with a signals and issuer-agency contacts are transparent. The latter implies that agencies can infer that in the second round (which is on the off-equilibrium path) only those issuers ask for an offered rating who were offered a B rating by the other agency in the first round.

[7] That is, issuers with $\{A,B\}$ and $\{B,A\}$ offered ratings will only purchase the A while issuers with $\{A,A\}$ offered rating will randomly purchase one.

the first agency they asked and an A rating from the second agency they asked have an incentive to go again to the first agency and reveal the second agency's rating in the hope that their offered rating will be adjusted. In such cases the equilibrium of Lemma 2 is reached with a minimum of $k = 3$ rounds.

For the arguments above to hold it needs to be shown that issuers would never find it optimal not to reveal the other agency's offered rating. Hiding a B rating could never help an issuer, since agencies treat issuers with B ratings the same way as they treat issuers with no revealed ratings. Also, hiding A ratings is never optimal due to the pecking order of manipulation methods, i.e. $\rho_i \geq \varepsilon_i$.

The following Lemma states the equilibrium of the sequential game:

Lemma C.1 (Equilibrium of the sequential rating game)**.** *1. Agencies set rating fees and manipulation levels as in Lemma 2.*

2. Equilibrium beliefs are as in Lemma 2.

3. Issuers ask

- *agency $i \in \{1,2\}$ for an offered rating with equal probabilities in round 1.*
- *agency $-i$ in round 2.*
- *agency i if agency i offered B while agency $-i$ offered A in the final, third round. Otherwise they do not ask for an offered rating in round 3.*

Also, issuers always reveal the offered rating of the other agency.

Appendix D Positive average NPV

In this appendix I briefly discuss the implications of violating Assumption 1. That is, I describe the case where the expected NPV of a randomly chosen project is positive. I focus on describing the equilibria in a monopoly and a duopoly. Insights from this exercise are as follows:

1. There is always an equilibrium in which all issuers sell their projects to investors without ratings.
2. With positive average NPV issuers may have a valuable outside option (i.e. selling a project without disclosing *all* of its ratings). This increases issuers' bargaining power, which limits rating agencies' ability to extract all the surplus.
3. If issuers do have a valuable outside option, agencies will decrease the rating fee to the point where issuers are indifferent between purchasing and not purchasing the rating.
4. In a duopoly, issuers may find it optimal to shop ratings in equilibrium. That is, if an issuer is offered an A and a B, he may find it optimal to purchase the A only. However, issuers who are offered A from both will purchase both in equilibrium.

5. Modeling assumptions that are non-binding in the main text may become binding when combined with positive average NPV. In particular, agencies may have an incentive to set contingent rating fees or to ”downward cater” ratings.

D.1 Equilibria with a monopolist agency

Trivial equilibria

First, with positive average NPV the market could function without rating agencies, as now investors are willing to purchase projects based on their priors. This leads to a set of PBE in which investor beliefs satisfy

$$\hat{p}_{\emptyset} = \pi_g \quad \hat{p}_A \leq \min\left\{1, \pi_g + \frac{c(1-p_A(0))}{R}\right\}. \tag{D.1}$$

That is, investors' posterior for unrated projects coincides with the prior and they are sufficiently pessimistic about rated projects. In particular, they have to be so pessimistic that deters the agency from providing ratings.

Note that off-equilibrium beliefs play a crucial role here and the equilibria may not be robust to trembling hand refinements. I.e. if an agency accidently offers an honest A rating to a tiny mass of issuers and the issuers accidently purchased it, their posterior success rate would be larger compared to investor beliefs. Of course, when the penalty is so high that the agency finds it optimal not to offer ratings even if investors correctly assess A-rated projects, the trivial equilibrium is unique and robust.

Robustness of the benchmark equilibrium

The equilibrium described in Lemma 1 will be robust to trembling hand errors with positive expected NPV as long as projects receiving b signals have non-positive expected NPV:

$$p_B R - 1 \leq 0. \tag{D.2}$$

The condition in (D.2) guarantees that even if some issuers tremble, and do not purchase their A ratings, they still cannot sell their unrated project, because they are now pooled with issuers who were only offered a B rating. However, there is a notable difference, since the probability of inflating now may reach $\varepsilon^* = 1$, in which case only a rating fee of zero is consistent with trembling hand refinements. If the agency always inflates, it cannot extract rents.

What happens if the inequality in (D.2) does not hold? Then projects receiving b

signals carry positive expected NPV. In turn, if investors accidentally purchase unrated projects, they will learn that they are valuable. This increases issuers' outside option to

$$p_B R - 1 > 0. \tag{D.3}$$

As a result, the equilibrium of Lemma 1 will no longer hold. The following lemma gives the equilibrium in this case:

Lemma D.1 (Equilibrium with a monopolist agency - positive expected NPV). *Assuming $[p_A(0) - p_B]R \geq c(1 - p_A(0))$ and that inequality (D.3) holds*

(i) Issuers always purchase A ratings and never purchase B ratings.

(ii) Agency sets

$$\varepsilon^* = \begin{cases} 0, & \text{if } [p_A(0) - p_B]R \leq c(1 - p_B) \\ \text{Implied by } p_A(\varepsilon^*) = \dfrac{(R-c)p_B + c}{R}, & \text{if } [p_A(0) - p_B]R > c(1 - p_B) \geq (\pi_g - p_B)R \\ 1, & \text{if } (\pi_g - p_B)R > c(1 - p_B) \end{cases}$$

$$f^* = I(\varepsilon^* < 1)[p_A(\varepsilon^*) - p_B]R. \tag{D.4}$$

(iii) Investor beliefs satisfy $\hat{p}_A = p_A(\varepsilon^*)$, $\hat{p}_\emptyset = p_B + I(\varepsilon^* < 1)(\pi_g - p_B)$, $\hat{p}_B \leq 1/R + (p_A(\varepsilon^*) - p_B)$.

The proof is straightforward and follows the logic of the proof of Lemma 1. Compared to Lemma 1, note that the rating fee is now only the difference between the NPV of an A-rated project and the NPV of a B-rated one, since issuers have the valuable outside option not to disclose any ratings and still sell the project for $p_B R - 1$. Also, if the penalty is sufficiently low, the agency may find it optimal to inflate all ratings, implying that A-rated projects will succeed with the prior probability, π_g. In this case, the agency can no longer extract rents ($f^* = 0$) as trembling errors would reveal that unrated projects are the same as A-rated ones. In this region welfare will coincide with the welfare achieved in the trivial equilibrium discussed above.

Issuers still find it optimal to always ask the agency for a rating. In principal, one could envision an equilibrium in which issuers randomize between asking and not asking for a rating, hoping that they can sell their projects even without a rating. However, it is easy to rule out such equilibrium candidates. In particular, if issuers were to randomize, then the agency would have an incentive to marginally decrease its rating fee, making issuers no longer indifferent between asking and not asking for a rating. The agency would have an incentive to do so, as it would experience a jump in its profits.

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D.2 Equilibrium with a duopoly of agencies

This section makes the following points:

1). Except for the trivial equilibria, there are no equilibria in which issuers who are offered A ratings from both agencies do not purchase both (unraveling principle, i.e. Grossman (1981)).
2). Issuers who are offered mixed ratings may find it optimal to purchase the A. Thus, it may happen in equilibrium that some issuers reveal only a single rating (shop ratings).
3). I formally show the symmetric equilibrium for one set of parameters.

Issuers who are offered A ratings from both agencies purchase both in a symmetric equilibrium (except in the trivial equilibria).

To see this, suppose there is an equilibrium in which such issuers randomize between purchasing one or both ratings, or prefer to only purchase a single rating over purchasing both. Given this strategy, agencies will compete in rating fees, as proposing a marginally lower rating fee than the competitor attracts business discontinuously (from the best issuers), leading to a jump in profits. This follows from the fact that the demand for a rating from $\{A,A\}$ issuers is now perfectly elastic (i.e. always purchase the cheaper rating).

Suppose there is an $\tilde{f}$ at which agencies break even. That is, the total penalty paid by agencies equals their total fee revenue. Note that in this hypothetical equilibrium issuers with mixed offered ratings always purchase their A ratings and there is a positive mass of issuers who are offered A ratings from both agencies pick one random (either because they are indifferent between purchasing both or one or they strictly prefer to have only one, but indifferent about which one).

It is clear that if agency 2 is playing $\tilde{f}$ then agency 1 does not want to charge a marginally higher fee. A marginally higher fee would only leave issuers with $\{A,B\}$ ratings purchasing agency 1's rating, leading to negative profits due to the fact that the expected penalty for selling ratings to them is higher.

However, agency 1 would be better off if it marginally decreased its rating fee. The reason is that now all issuers with $\{A,A\}$ offered ratings would purchase agency 1's rating (since its cheaper) implying that agency 1 has to pay a lower expected penalty per sold rating, leading to positive profits. Hence, $\tilde{f}$ is not an equilibrium.[8]

Now consider 2). Issuers with mixed offered ratings do not purchase their offered A rating under Assumption 1 (with negative average NPV). The intuition for this result is that if investors believe that their undisclosed rating is a B then they

[8] Throughout the paper I restrict agencies to pure strategies. If agencies could implement mixed strategies, there could possibly be additional equilibria.

are average in expectation at best. However, if the average NPV is positive, revealing a single A rating is likely to convince investors that the project is worth financing (since even if the undisclosed rating is B the project can still be valuable in expectation).

Issuers with mixed offered ratings will find it optimal to purchase the A rating if

$$\hat{p}_{A\emptyset}R - 1 - f \geq 0 \;\; and \;\; \hat{p}_{A\emptyset}R - 1 - f \geq \hat{p}_{\emptyset\emptyset}R - 1 \tag{D.5}$$

That is, the rating fee must be at most the value of a single A-rated project and issuers must be at least as well off with the purchase than selling the project without any rating. It is clear from these conditions that the rating fee and investor beliefs play a key role in this decision. Below I derive the equilibrium for a set of parameters where issuers may find it optimal to purchase the single rating.

Lemma D.2 (Equilibrium with a duopoly of agencies - positive expected NPV)**.** *Assume the following hold:*

$$(\mu_{aa} + \mu_{ab})[p_{AA}(\mathbf{0}) - \pi_g]R \geq c[\mu_{aa}(1 - p_{AA}(\mathbf{0})) + \mu_{ab}(1 - \pi_g)] \tag{D.6}$$

$$p_{BB}R - 1 < 0 < \frac{p_{AA}(\mathbf{0})R - 1}{2} < \pi_g R - 1 \tag{D.7}$$

Then

(i) Issuers always purchase A offered ratings and never purchase B offered ratings.

(ii) Equilibrium inflating and catering satisfy

$$\begin{cases} \varepsilon^* = \rho^* = 0 & \text{if } [p_{AA}(\mathbf{0}) - \pi_g]R \leq c(1 - \pi_g) \\ \varepsilon^* = 0, 0 < \rho^* \leq 1 & \text{if } \frac{[p_{AA}(0,0,1,1) - \pi_g]R}{1 - p_{BB}} \leq c < \frac{[p_{AA}(\mathbf{0}) - \pi_g]R}{1 - \pi_g} \\ 0 < \varepsilon^* \leq 1, \rho^* = 1 & \text{if } c < \frac{[p_{AA}(0,0,1,1) - \pi_g]R}{1 - p_{BB}} \end{cases}$$

(iii) Rating fees are given by:

$$f_i = [p_{AA}(\varepsilon^*, \varepsilon^*, \rho^*, \rho^*) - \pi_g]R \tag{D.8}$$

(iv) Equilibrium beliefs satisfy: $\hat{p}_{AA} = p_{AA}(\varepsilon^*, \rho^*)$, $\hat{p}_{A\emptyset}, \hat{p}_{\emptyset A} = \pi_g$, $\hat{p}_{BB}, \hat{p}_{\emptyset\emptyset} \leq 1/R$.

The proof follows the same logic as the previous ones. When there is an interior solution for catering it is implied by

$$[p_{AA}(0, 0, \rho^*, \rho^*) - \pi_g]R = c(1 - \pi_g) \tag{D.9}$$

and when there is an interior solution for inflating it is implied by

$$[p_{AA}(\varepsilon^*, \varepsilon^*, 1, 1) - \pi_g]R = c(1 - p_{BB}), \tag{D.10}$$

which guarantee that the agency will be indifferent between the amount of catering and inflating in equilibrium. In turn, they will be pinned down by investors having consistent beliefs.

Inequality (D.6) guarantees that the total revenue of the agency is larger than its total penalty, given that it does not manipulate ratings. Agency i's total revenue is the sum of the rating fees collected from issuers who are offered an A rating by agency i. Note that these issuers' other rating might be A or B.

The first inequality in (D.7) implies that issuers who are offered B ratings from both agencies carry projects that have negative expected NPVs. This assumption supports investor beliefs such that selling projects to investors without ratings will not be a valuable (outside) option for issuers.

However, the last inequality in (D.7) implies that if agencies were to set rating fees according to Lemma 2 and investors believed that projects with a single A rating succeed with the prior probability, then issuers would be better off purchasing only a single rating. Thus, the outside option of purchasing a single rating is valuable and effectively forces agencies to lower the rating fee to the point where issuers are indifferent between purchasing one or both ratings:

$$\hat{p}_{AA}R - 1 - 2f \geq \pi_g R - 1 - f \rightarrow (\hat{p}_{AA} - \pi_g)R \geq f, \tag{D.11}$$

implying that the rating fee will correspond to investors' marginal valuation for the second A rating.

Observe, that when the agency does not manipulate ratings, selling A ratings to issuers that were only offered a B by the other agency leads to an expected loss. The intuition is simple: in this region the rating fee is smaller than the expected penalty for selling a rating to an issuer with mixed signals (i.e. catering), that is why agencies do not want to cater ratings in the first place.

Agencies would be better off if they could prevent these loss generating rating purchases. Here, two modeling assumptions become binding. First, agencies could prevent the sales by making rating fees contingent on the other agency's preliminary rating (or delaying the proposal of the rating fee until they learn each other's preliminary ratings). Also, if they could "downward cater" offered ratings, that is, offer a B after learning that the other agency only assigned a B preliminary rating, then issuers could not shop ratings.

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Appendix E Proofs of Chapter 2

Proof of Lemma 1. Assume the following strategies for the agency:

$$\begin{cases} \alpha^{I*} = \alpha/\pi_g, \;\; \varepsilon^* = 0 & \text{if } c \geq 2(R-1) \\ \alpha^{I*} = \dfrac{2(\pi_g V_g - \alpha) - c(\pi_g + \alpha)}{2\pi_g(V_g - 1 - c)}, \;\; \varepsilon^* = 0 & \text{if } 2[p_A(0,0)R-1] \leq c < 2(R-1) \\ \alpha^{I*} = 0, \;\; \varepsilon^* = 0 & \text{if } p_A(0,0)R - 1 \leq c < 2[p_A(0,0)R-1] \\ \alpha^{I*} = 0, \;\; \varepsilon^* = \dfrac{\pi_g(V_g - c) - \alpha(1+c)}{(1+c)(1-\pi_g-\alpha)} & \text{if } c < p_A(0,0)R - 1 \end{cases}$$

Then, the first-order conditions will satisfy

$$\begin{cases} \left.\dfrac{\partial \Pi}{\partial \alpha^I}\right|_{\alpha^{I*},\varepsilon^*} > 0, \;\; \left.\dfrac{\partial \Pi}{\partial \varepsilon}\right|_{\alpha^{I*},\varepsilon^*} < 0 & \text{if } c \geq 2(R-1) \\ \left.\dfrac{\partial \Pi}{\partial \alpha^I}\right|_{\alpha^{I*},\varepsilon^*} = 0, \;\; \left.\dfrac{\partial \Pi}{\partial \varepsilon}\right|_{\alpha^{I*},\varepsilon^*} < 0 & \text{if } 2[p_A(0,0)R-1] \leq c < 2(R-1) \\ \left.\dfrac{\partial \Pi}{\partial \alpha^I}\right|_{\alpha^{I*},\varepsilon^*} < 0, \;\; \left.\dfrac{\partial \Pi}{\partial \varepsilon}\right|_{\alpha^{I*},\varepsilon^*} < 0 & \text{if } p_A(0,0)R-1 \leq c < 2[p_A(0,0)R-1] \\ \left.\dfrac{\partial \Pi}{\partial \alpha^I}\right|_{\alpha^{I*},\varepsilon^*} < 0, \;\; \left.\dfrac{\partial \Pi}{\partial \varepsilon}\right|_{\alpha^{I*},\varepsilon^*} = 0 & \text{if } c < p_A(0,0)R-1 \;. \end{cases}$$

Hence, the agency cannot increase its profit by deviating. From Assumption 2 it follows that there cannot be an equilibrium where the agency does not choose the loose technology but inflates. Therefore, besides the trivial equilibrium in which there is no trade, there are no other equilibria.

Appendix F Proofs of Chapter 3

Proof of Proposition 1.

1. By definition, the share of transactions involving calls is i.i.d. and the probability of resells, δ, is also independent of all variables.
2. Using that $E[S_1 - S_0] = 0$, the covariance may be written as

$$E\left[\frac{[\delta H_0^C + H_1^C]C_1}{nm + \delta[H_0^C C_1 + H_0^P P_1]}(S_1 - S_0)\right], \tag{F.1}$$

   Let the share of call contract trading value be denoted by $f(S_1, L_0, L_1)$, where L_t is the number of traders who arrive in period t and decide to open a long position (they are binomially distributed i.i.d. random variables). The expectation in (F.1) will always be positive because taking L_0 and L_1 fixed the expectation can be written as

$$\frac{1}{2}(u-1)[f(u,\bar{L_0},\bar{L_1})-f(d,\bar{L_0},\bar{L_1})] \tag{F.2}$$

which will always be positive as $f(\cdot)$ is increasing in S_1. Since this holds for all realized values of L_0 and L_1, it also holds in expectation.

3. The proof follows the same logic as the previous one.
4. The covariance may be written as

$$E\left[\left(H_1^C-H_1^P-\delta(H_0^C-H_0^P)\right)(S_1-S_0)\right]=E\left[\left(H_1^C-H_1^P\right)(S_1-S_0)\right] \tag{F.3}$$

where the equality follows from the fact that L_0 and S_1 are independent, that is $E[(H_0^C-H_0^P)(S_1-S_0)]=(H_0^C-H_0^P)E[S_1-S_0]=0$. Then for all realizations of L_1, the covariance simplifies to

$$\frac{1}{2}(u-1)\left[\frac{L_1 m}{u-K^{call}}-\frac{L_1 m}{d-K^{call}}+\frac{(n-L_1)m}{K^{put}-d}-\frac{(n-L_1)m}{K^{put}-u}\right], \tag{F.4}$$

which is always negative as $d<u$.

Appendix G The Delta and Vega of a KO call

In order to build some intuition for the pricing of KO products, it is useful to compare the behavior of their model implied prices to futures prices and to European vanilla option prices. These comparisons are useful because banks market these products by advertising that they have similar characteristics to futures, except that once the barrier is hit, their positions are automatically closed, so the investors do not have to hold a margin account. Also, by comparing to vanilla options, the effects of path dependence will become clear.

Consider a KO call. The following table summarizes the benchmark parameters for the contract used in this example. The underlying price is assumed to evolve consistent with Black-Scholes assumptions, implying that it can be characterized by the risk-free interest rate (r_f) and the volatility of returns (σ) under the risk-neutral measure:

Figures G.1-G.6 summarize the results. Using the Black-Scholes model, one can find a closed form solution for the price of the KO call, assuming that the residual

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Table G.1 *Benchmark product characteristics for a KO call. The parameter values above will be used to illustrate the characteristics of KO warrants.*

Parameter	Value
Strike price, K	7750
Barrier, B	8000
Underlying's price, $S(0)$	8250
Maturity in years, T	0.5
Risk free interest rate, r_f	0.05
Volatility of the underlying, σ	0.2

value in case of a knock-out event is the difference between the barrier and the strike price.[9,10]

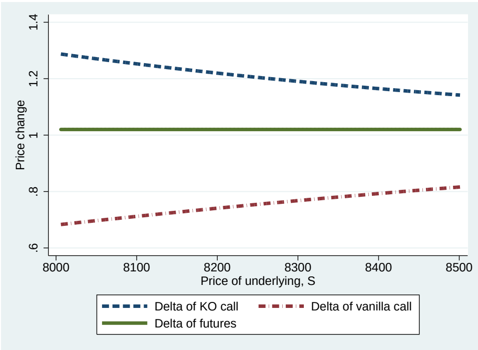

Figure G.1 This figure shows the Δ of a KO call, a vanilla call and a futures contract as a function of the underlying's price. The Δ is the absolute change in the price of the derivative when the underlying's price changes by one unit. Parameters used are provided in Table G.1.

Figures G.1-G.3 show the Δ (absolute price increase of given product if the price of the underlying increases by 1 unit) of the KO call, a vanilla call and a futures contract. The futures contract always has a $\Delta = (1+r_f)^T$ and it is well known that the vanilla call always has a $\Delta < 1$. Compared to these, the KO call has a larger Δ, implying that it is not identical to a futures contract. Observe on Figure G.1 that Δ converges to 1 for both options as they become heavily in the money. Figure G.3 highlights the effect of volatility on Δ. Contrary to a vanilla call, the KO call has a more stable Δ, implying that it is more useful for directional bets than a vanilla call.

9 Rubinstein and Reiner (1991) were the first to provide a closed form solution for the value of barrier options under the Black-Scholes assumptions.

10 With this assumption the residual value can me modelled as an American style binary put option, which has a closed form solution in the Black-Scholes world.

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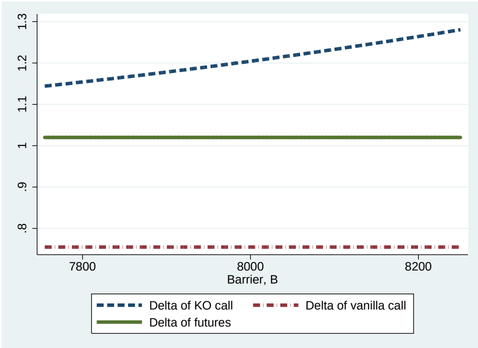

Figure G.2 This figure shows the Δ of a KO call, a vanilla call and a futures contract as a function of the barrier. The Δ is the absolute change in the price of the derivative when the underlying's price changes by one unit. Parameters used are provided in Table G.1.

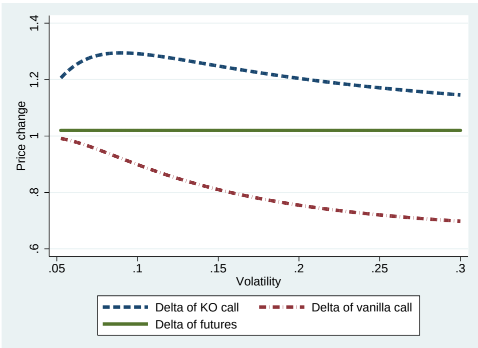

Figure G.3 This figure shows the Δ of a KO call, a vanilla call and a futures contract as a function of the underlying's volatility. The Δ is the absolute change in the price of the derivative when the underlying's price changes by one unit. Parameters used are provided in Table G.1.

Figures G.4-G.6 show the Vega (percent change in asset's value if volatility increases by 1% point) of the three products. Naturally, the futures always has a Vega equal to zero. Interestingly, the KO call has a negative Vega, implying that the KO call's value is decreasing in volatility. This is in sharp contrast with the properties of a vanilla call and highlight the important affect the barrier plays. Higher volatility increases the probability of a knock-out event resulting in the lowest possible payoff. Observe, that as volatility increases, the Vega of the KO call will converge to zero as the knock-out event becomes a certainty (Figure G.6).

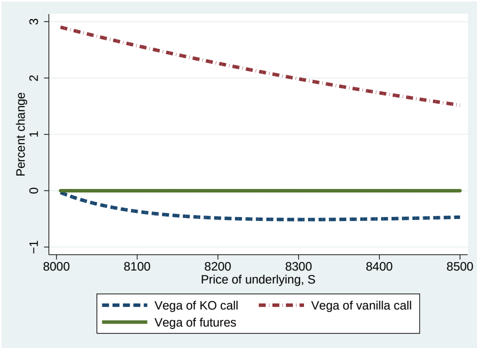

Figure G.4 This figure shows the Vega of a KO call, a vanilla call and a futures contract as a function of the underlying's price. The Vega is the percent change in the price of the derivative when the underlying's volatility increases by 1% point. Parameters used are provided in Table G.1.

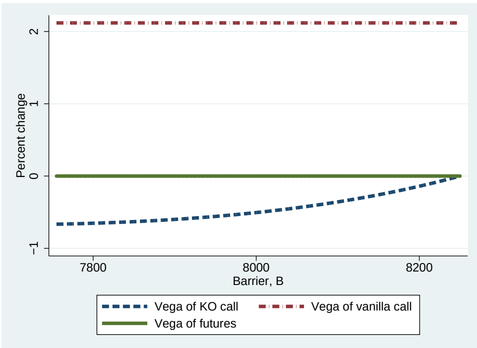

Figure G.5 This figure shows the Vega of a KO call, a vanilla call and a futures contract as a function of the barrier. The Vega is the percent change in the price of the derivative when the underlying's volatility increases by 1% point. Parameters used are provided in Table G.1.

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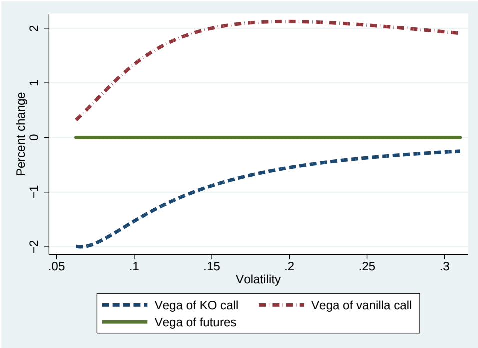

Figure G.6 This figure shows the Vega of a KO call, a vanilla call and a futures contract as a function of the underlying's volatility. The Vega is the percent change in the price of the derivative when the underlying's volatility increases by 1% point. Parameters used are provided in Table G.1.