# ESSAYS ON SOCIAL NETWORKS AND ECONOMIC BEHAVIOR

by

Vladimir Mikhailov

Submitted in partial fulfillment of the requirement for the degree of Doctor of Philosophy at Central European University

Advisor: Adam Szeidl

Budapest, Hungary

CENTRAL EUROPEAN UNIVERSITY
DEPARTMENT OF ECONOMICS

The undersigned hereby certify that they have read and recommend to the Department of Economics for acceptance a thesis entitled **"Essays on Social Networks and Economic"** by **Vladimir Mikhailov**.
Dated: June 7, 2017

I certify that I have read this dissertation and in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.
Chair of the Thesis Committee:

_____
Botond Kőszegi

I certify that I have read this dissertation and in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.
Advisor:

_____
Ádám Szeidl

I certify that I have read this dissertation and in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.
Internal Examiner:

_____
Ádám Zawadowski

I certify that I have read this dissertation and in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.
External Examiner:

_____
Yann Bramoullé

I certify that I have read this dissertation and in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.
External Member:

_____
János Kiss Hubert

i

CENTRAL EUROPEAN UNIVERSITY
DEPARTMENT OF ECONOMICS

Author: **Vladimir Mikhailov**
Title: **Essays on Social Networks and Economic Behavior**
Degree: **Ph.D.**
Date: **June 07, 2017**

Hereby, I testify that this thesis contains no material accepted for any other degree at any other institution and that it contains no material previously written and/or published by another person except when the appropriate acknowledgement is made.

Signature of the author: …………………..

CEU eTD Collection

## Abstract

This thesis consists of 3 unconnected single-authored chapters. Each chapter investigates a particular aspect of social networks' influence on the behavior of economic agents.

In Chapter 1 analyze a model in which criminal networks are viewed as embedded in the social network. Agents in the society are assumed to have social preference for or against illegal activity and, accordingly, can help or harm the criminals without actively partaking in crime. I derive predictions for crime participation as a function of centrality in a given network, as well as the effect of network structure on aggregate crime. The equilibrium number of criminals exhibits an inverse-U pattern with respect to public support for crime. If crime is strongly disliked by the society, it is only committed by the most peripheral agents. If, on the other hand, there is social sympathy for the crime, then it is only the most central individuals who become criminals. In terms of network structure, I find that social antipathy towards crime can mean that denser networks exhibit less crime than sparser ones, which reverses the result of Ballester et al. (2006) that denser networks produce more crime on aggregate. I also find that, depending on the society's attitude, an increase in sanctions might fail to deter or even increase aggregate crime. The results reconcile several apparent conflicts between existing models and empirical evidence.

Chapter 2 is my job market paper. I use a unique data set which maps out the complete social network within a community of Indian student migrants at a large university in central Kazakhstan to identify endogenous peer effects in assimilation among the community's members. Upon arrival, students are randomly assigned into small academic groups, consisting only of fellow Indians. I use the resulting exogenous variation in social ties to implement a quasi-experimental empirical strategy. Positive peer effects are identified in ability to speak the local language and to acquire local friends. At least a part of the effect is explained by complementarity between assimilation efforts of friends, implying that the peer effects 'snowball ' into a social multiplier of 1.4. Finally, assimilation is shown to increase overall GPA, conditional on hours of study. The results suggest that taking advantage of the social multiplier within existing migrant clusters might be a viable alternative to policies, such as settlement quotas, designed to prevent clustering.

In chapter 3 I study the problem of a monopolist, who relies on word of mouth in order to diffuse the information about the product through a social network of consumers. The product can be of a certain quality, which is proportional to the probability that the consumer has a positive experience with the good. If the quality is low, the consumer might have a bad experience and choose to give a negative review to friends, discouraging them from purchasing. Discouraged consumers create bottlenecks in the information passage process. I first take quality as exogenously given and show that in highly connected networks, negative WOM makes demand less elastic than the fully-informed case, so the monopolist charges a higher price. Raising the price in this case is a 'vaccine' against negative reviews. Later, I endogenize the quality choice and show that if the quality-boosting technology is expensive, then price and quality are substitutes, and the optimal quality goes down with network connectivity, while price goes up.

# Chapter 1
## "Love Thy Criminal Neighbor: Patterns of Crime in Social Networks"

Do criminals come from the core or the periphery of the social networks? Does the density of social networks deter or boost criminal activity? Social sciences deliver contradictory answers. In this chapter I develop a model in which criminal networks are embedded in the fabric of society. Members of a criminal network form an illicit underbelly to the social network. Embeddedness leaves criminals vulnerable to actions by their non-criminal peers. I also allow for society as a whole to feel sympathy or antipathy for the particular type of crime. These features of the model introduce a mechanism by which social sympathy can influence the network location of criminals, as well as the overall level of crime. The possibility of action by non-criminal agents forces better connected people to make different criminal choices than the less connected ones. The mechanism helps explain the puzzles, delivers new comparative statics and gives predictions on the effect of sanctions on aggregate crime.

There are 3 central contributions of this paper. The first is to show that socially unpopular crimes are committed by people on the periphery of the social network, while the popular crimes are committed by the central people. The first part of the statement arises from the fact that legitimate agents want to hurt the criminals when crime is undesirable. Being on the fringe of society means being least exposed to such wrath of the crowd and being able to hide. The second part of the statement is due to the fact that in cases of social support for the criminal "cause" the legitimate agents always help the delinquents. Members of the interconnected central component benefit the most from such help and commit a lot of socially desirable crime. Their actions leave the lesser-supported agents with criminal opportunities which are insufficient to cover the costs of crime.

The second main contribution is to show that denser networks decrease aggregate crime only if the crime is sufficiently disliked by society. If people are close to indifferent or support the crime, then adding extra ties to an existing social network increases the aggregate levels of delinquency. This happens because legitimate agents always help offenders carry out the crime if they consider it socially beneficial. Therefore, an extra link in the network necessarily means an extra bit of help for the criminal, which raises his effort. For cases when crime is socially disliked, on the other hand, an extra link exposes the criminal to an additional bit of harm, thus reducing his criminal effort.

The third main contribution is to investigate the effect of an increase in expected punishment on aggregate crime. In my model an increase in punishment also increases the intensity of peer effects in crime, because criminals are assumed to learn their "craft" behind bars. People who commit the socially disliked offenses are located on the periphery and do not get to benefit from the boost in peer effects, so their criminal activity drops, creating a deterrence effect. On the other hand, crimes which people support are committed by the tight core of the network. For them increased peer effects win over a rise in the expected cost, bringing the aggregate crime up.

# Chapter 2
## "When Sandeep Met Sergey: Peer Effects in Social Assimilation of Foreigners"

Countries often want to integrate and assimilate their migrant populations. In particular, social assimilation, defined as knowledge of the language and creation of social ties with locals, is considered an important goal. Yet, economists know little about this kind of assimilation. What are the mechanisms behind it? What are its effects on economic outcomes at the destination country? What types of policies could be implemented in order to foster it? A lot of the debate has focused on the challenges posed by immigrants' desire to settle in groups, forming persistent ethnic clusters. Policymakers have to find an optimal way to make use of social networks within these clustered communities of foreigners in their assimilation strategies.

In this paper I hypothesize the peer effects in social assimilation to be an active channel of co-nationals' influence on each other's assimilation outcomes. The paper has three main contributions. The first contribution is to use a unique and uniquely suited data set in order to identify positive endogenous peer effects within a community of foreigners in acquisition of language skills and friendships with locals. The data set covers a community of Indian educational migrants at Karaganda State Medical University in central Kazakhstan. The community is ideal for this investigation due to its homogeneity and complete racial, religious, linguistic and cultural separation from the rest of the city. Upon arrival, students are randomly placed into small academic groups of 7 to 15 fellow Indians for administrative purposes, providing a source of exogenous variation in social ties among them. This variation allows me to devise instrumental variable strategies in order to tackle the myriad of endogeneity issues that plague peer effect estimations.

The second contribution is to investigate the mechanisms behind the peer effects. There are two competing mechanisms - conformity and complementarity. Complementarity implies that foreigners actively help each other learn the local language, while conformity implies that foreigners simply mimic each other's language skills attainment. Determining the mechanism if of fundamental importance to designing optimal assimilation policies. I show that the peer effects are at least partially driven by complementarities between assimilation efforts of students in my sample. As a result, a social multiplier arises, which can potentially be exploited in order to extract large cumulative gains from targeted assimilation-related interventions.

The paper's third contribution is to use the exogenous variation in social ties to show that assimilation causes the GPA of Indian students to go up, controlling for study hours. Consequently, co-ethnic networks may have a lasting positive effect of helping foreigners be more productive throughout their spell at the destination.

# Chapter 3
## "Zero Stars! Price, Quality and Negative Word of Mouth"

Negative word of mouth (WOM), the act of telling others one's unpleasant experiences with a good or a service, is ubiquitous and an important determinant of demand. Yet, few formal economic models of NWOM exist. Empirical marketing research on NWOM has shown that it is more powerful than positive WOM and that in the age of social media firms have not quite figured out how to deal with it. In particular, it has been shown that lowering the price of the good is not an effective strategy, as it does not lead to an increase in sales. A natural question, then, arises regarding the firm's optimal pricing strategy in face of NWOM.

In this chapter I add negative WOM to the theoretical framework of Campbell (2013). In my model a monopolist wants to diffuse the information about the good to an initially uninformed network of consumers. To do so, the monopolist has to pick the price and the quality that would stimulate WOM communication. Quality is costly. However, if the quality is imperfect, the consumers may share negative reviews with each other, thus reducing demand. There are three sets of results in the chapter. First, I make use of the so-called *cavity method* (Newman and Ferrario (2013)) to calculate the expected the demand for the product in an arbitrary social network of consumers who engage in both positive and negative WOM. I contribute to the literature on demand formation under WOM by showing that for any degree distribution, demand always falls in the intensity of negative WOM, but increases in network density.

The second set of results covers monopolist's pricing behavior under negative WOM in several settings. I show that in dense networks negative WOM reduces the price elasticity of demand, allowing the monopolist to charge a higher price compared to the situation where consumers are fully informed. So, the ability to share negative information ends up reducing consumer welfare. The intuition is that raising the price can serve as a 'vaccine' by causing a greater reduction in negative WOM than in the positive WOM. In addition, I show that whenever the monopolist can reduce the probability of bad reviews directly by selecting higher product quality, price and quality may be either compliments or substitutes, depending on the cost of quality-improving technology. This set of results has implications for antitrust and regulatory policies.

The third set of model's predictions characterizes the negative WOM's relationship with formal advertising. I show that NWOM may induce a negative relationship between product quality and the level of informative advertising. A negative correlation of this nature is, puzzlingly, sometimes observed in markets where WOM is likely to be strong (Kwotka (1995)). For the targeted advertising, I prove that it is suboptimal to target the individuals with the highest degree, and that the optimal degree increases with the intensity of negative WOM.

# Acknowledgments

I would like to thank my advisor Adam Szeidl for three years of intellectual inspiration, prompt replies, and steadfast guidance. Personally or professionally, one could hardly wish for a better mentor. After a decade of running regressions and solving optimization problem, it is thanks to him that I think I finally understand some economics.

I would like to thank the entire economics department for creating a stimulating atmosphere of continuous involvement in active high level research. More broadly, CEU as a unique university has provided me with six wonderful years that have completely changed the course of my life and made me a better person. I truly hope that the burning fires of history do not consume other young people's chances of having similar experiences.

As a part of my PhD studies I was lucky enough to spend three months at Cambridge University and to interact with an incredible group of social and economic networks researchers. For that I must thank Sanjeev Goyal, who not only made the visit possible, but also devoted a lot of his own time towards overseeing my development as an economist.

I believe the thesis is much improved by the astute comments and suggestions of examiners Adam Zawadowski and Yann Bramoullé. I thank them for their time and efforts.

My family far far away... My gratitude, respect and love for you neither belongs nor can fit in these lines. Suffice to say that for 11 years you've somehow managed to maintain a very authentic, affectionate and animated connection with what was increasingly becoming a pixelized Skype image.

To my wife (and the whole Tatai clan). You gave a sense of home and purpose to the life of a nomad. You dragged me along when I didn't particularly feel like moving forward. You reminded me that enjoying today and planning for tomorrow were not mutually exclusive. A "thank you" is in order, even though it can never be enough.

# Contents

# Chapter 1

# Love Thy Criminal Neighbor: Patterns of Crime in Social Networks

## 1.1   Introduction

Is a typical delinquent more likely to hail from the heart or the fringe of the friendship network? Social sciences deliver contradictory answers. Some existing models predict that it should be the people with most friends who commit the most crime (Ballester et al. (2006)), and that criminal networks hinge on a few well-connected individuals. Yet, "loner" behavior or, in network terms, low degree centrality is often found in empirical work to be the most important correlate of delinquency (Akers (1998), Andrews and Bonta (2014)). Another question with often contradictory answers is whether social network density encourages or hinders illegal activity. Models predict tighter societies to exhibit more crime through network effects (Calvo-Armengol and Zenou (2004),, Ballester et al. (2006)), while a number of empirical papers, such as Yamamura (2011) or Land et al. (1991), show that denser and more coherent neighborhoods produce less crime. These two apparent puzzles suggest that the networks' effect on crime is imperfectly understood.

In order to better understand these puzzles, I develop a model in which criminal networks are embedded in the fabric of society, rather than being independent entities. Members of a criminal network typically maintain their normal occupations and, crucially, social contacts in an attempt to hide their transgressions from the authorities. Such networks, therefore, tend to form an illicit underbelly to the social network. Embeddedness leaves criminals vulnerable to actions by their non-criminal peers. As a reflection of such vulnerability, agents in my model can directly influence the lawbreakers' actions without taking part in the crime. I also allow for the society as a whole to feel sympathy or antipathy for any particular type

of crime. Taken together these features of the model introduce a mechanism by which social sympathy can influence the network location of criminals, as well as how much crime they commit. The intuition is that a promise of an action by friends forces better connected people to make different criminal choices than the less connected and, therefore, less vulnerable ones. The mechanism helps explain the puzzles, delivers new comparative statics and gives predictions on the effect of sanctions on aggregate crime.

The model is described in Section 2. It is the linear-quadratic peer effects model used by Calvo-Armengol and Zenou (2004) and Ballester et al. (2006) with the added possibility for legal agents to help or harm the criminals. Decision to help or harm depends on two motive. The first motive is for the agents to act in accordance with how they feel about the crime - what I refer to as "social sympathy". Not all acts legally defined as crimes are equally condemned by the society. Most people would unsympathetic towards a bank robbery and hurt the perpetrating criminals by reporting them to the police. However, the same people might feel sympathy for the plot to illegally overthrow their country's malevolent dictator and, therefore, help the criminals by keeping quiet. The second motive is doing right by one's friends. A person might feel more reluctant to report an impending bank robbery to the police if the perpetrators are his close friends who could face prison due to his actions. I study Nash equilibria of a model which includes these motives.

The first central result of this paper, presented in Section 3.2, is that unpopular crimes are committed by people on the periphery of the social network, while the popular crimes are committed by the central people. The first part of the statement arises from the fact that legitimate agents want to hurt the criminals when crime is undesirable. Being on the fringe of society means being least exposed to such wrath of the crowd and being able to hide. The second part of the statement is due to the fact that in cases of social support for the criminal "cause" the legitimate agents always help the delinquents. Members of the interconnected central component benefit the most from such help and commit a lot of socially desirable crime. Their actions leave the lesser-supported agents with criminal opportunities which are insufficient to cover the costs of crime.

There is an abundance of empirical support for the first half of the result by both psychologists and economists, who find violent crime to be connected to antisocial or "loner" behavior. According to Loeber and Hay (1997), antisocial personality disorder has been found to promote chronic forms of violent behavior in up to 75% of cases. In a structural test of the Ballester et al. model Liu et al. (2012) find using the Adolescent Health survey data that delinquents tend to have lower values of the social inclusion index than their non-criminal peers. As far as the theoretical work on criminal networks goes, Baccara and Bar-Isaac (2008) find that an isolated binary cell might be the optimal network structure

for terrorism. Since terrorism is unlikely to provoke sympathy, my model would also predict terrorist to be extremely isolated. However, my result gives a prediction about network location of any criminal, not just the terrorists.

Empirical veracity of the second part of the statement is harder to ascertain, with little previous work done in social sciences. According to Taki and Coretti (2013),, there are reasons to believe that the so-called "Arab Spring" was instigated by efforts of a few activists who were extremely popular on the social media. Additionally, anecdotal evidence supports the notion that revolutionary leaders are people who are central to their community, with the likes of Vladimir Lenin and Fidel Castro both born to well-off well-connected families. My model suggests that such people are selected to be leaders not because they care more about society but because they are pushed towards becoming figureheads for the social movement by help and encouragement from peers. This result highlights the importance of leading cliques in a context of social upheaval, suggested in theoretical work by Chwe M. (2000).

The theoretically derived result above suggests that people who commit serious crimes interact with each other less than people committing petty crime. I use simulations to show that the amount of active links between criminals in a given network, indeed, goes down as punishment increases and social sympathy falls. Pickpockets interact with each other more than larcenists, who interact with each other more than murderers. This observation was made empirically in a seminal paper by Glaeser et al. (1996).. Their work, however, did not focus on investigating the difference between different types of crime. This paper, therefore, puts forward a potential explanation for that important discovery.

In Section 3.3 I discuss what kind of social networks facilitate crime. The section reconciles two apparently contradicting theories regarding the effect of tighter social networks on crime. Calvo-Armengol and Zenou (2004) argue that denser networks multiply crime through increased interactions. Another school of thought, assembled under the label of social disorganization theory (Shaw and McKay (1942)), suggests that denser and more cohesive social networks should reduce aggregate crime, because they provide better conditions for informal social control. The implication is that less densely populated, more ethnically fragmented and migration-prone neighborhoods, i.e. ones with sparser networks, should exhibit more crime. The two ideas are not only contradicting in theory but are also supported by contrasting empirical evidence. Using panel data on violent crimes in Chicago Browning et al. (2004) show that higher levels of network interaction increase violent crime. At the same time Yamamura (2011) and Land et al. (1991) show that, respectively, cigarette consumption and violent crime decrease in areas where social networks are likely to be tighter.

The second main contribution of this paper is to show that denser networks decrease aggregate crime only if the crime is sufficiently disliked by society. If people are close to

indifferent or support the crime, then adding extra ties to an existing social network increases the aggregate levels of delinquency. The intuition is that legitimate agents always help offenders carry out the crime if they consider it socially beneficial. Therefore, an extra link in the network necessarily means an extra bit of help for the criminal, which raises his effort. For cases when crime is socially disliked, on the other hand, an extra link exposes the criminal to an additional bit of harm, thus reducing his criminal effort. Violent crime in Land et al. and cigarette consumption go down in network density because these activities are disliked by society, and the researchers are not explicitly controlling for social attitudes. Browning et al. discover a positive effect of network density on violent crime, because they control for what is referred to as "collective efficacy." Collective efficacy is measured as a composite index of people's wishes for their neighborhood and answers to crime-related hypothetical questions. The measure arguably picks up the negative attitudes towards violent crime. Therefore, through inclusion of collective efficacy, Browning et al. are able to separate the two effects which allows their measure of social interactions to only pick up the multiplier effect of networks on crime. Such effect is positive, in line with Calvo-Armengol and Zenou. This discussion suggest that failure to properly account for prevailing social attitudes towards crime might introduce considerable bias into any estimation of peer effects in delinquency. A further implication of the mechanism is that tighter social networks might be partially responsible for higher levels of corruption in developing countries.

The final main contribution, presented in Section 3.4, is to investigate the effect of increase in expected punishment on aggregate crime. Standard theory predicts that higher punishment should decrease people's involvement in crime through deterrence. However, multiple studies have found either no deterrence (Cover and Thistle (1988) or Cornwell and Turmbull (1994)) or even a positive effect (Gneezy and Rustichini (2000)) of punishment on undesired activity. In my model an increase in punishment also brings about a boost in criminal activity. The reason is that it increases the intensity of peer effects in crime, because criminals are assumed to learn their "craft" behind bars. People who commit the socially disliked offenses are located on the periphery. They have no friends, so they do not get to benefit from increased peer effects. Their criminal activity drops, creating a deterrence effect. On the other hand, crimes which people support are committed by the tight core of the network. For them increased peer effects win over a rise in the expected cost, bringing the aggregate crime up.

Collectively, the results presented in this paper have three main implication for designing cost-effective anti crime and intervention policies. The first is that it is important to understand the public's attitude towards the type of crime before changing criminals sanctions. The best time to toughen up on crime might be after an intense media campaign convincing

people that that particular crime is bad for society. Also, in case of social support, easing up on crime might help reduce its aggregate amount. The second has to do with targeted attacks to remove few vital nodes from the illegal network, which are becoming a popular method of combating crime (Morselli and Giguere (2006)). If a certain criminal activity is heavily disliked by society, then such interventions might by systematically futile. Criminals in such cases are hovering on the periphery and removing a few of them would not cause a large reduction in crime. On the other hand, a targeted intervention might be very effective for socially supported crimes, like internet piracy. Conversely, any attempts to organize a socially-desirable revolution should be focused on engaging the most prominent individuals in the social network. Thirdly, in areas, where social networks are traditionally very dense for cultural or historic reasons, campaigning against crime and promoting anti criminal attitudes might be an alternative to a stricter judicial system or an increase in spending on public safety.

The paper relates to the public economics literature on social interaction in crime and patterns of criminal activity in the society. Sah (1991) studies crime participation rates in an economy consisting of several groups. Consistent with my results, he finds that different segments of society can have different participation rates even if faced with the same socio-economic conditions. He allows agents to have subjective view of the punishment probability but does not factor in public sympathy towards the crime into his model or derive a clear pattern of crime entry decisions. In one of the few theoretical papers which also let criminals participate in the labor market and care about other people Calvo-Armengol et al. (2007) use search and matching framework to examine the effect of strong ties in the social network on the interplay between criminal and labor outcomes. They do not account for social sympathy towards crime either but find that in economies with intensive peer interactions judicial punishment is not sufficient to deter crime, and the policies instead need to be based on interaction patterns.

## 1.2   The Model

There is a finite set $\mathcal{N} = \{1, 2 ... N\}$ of agents in the economy. The agents belong to a network of social connections, denoted by $g$. In this paper, $g$ is given exogenously.[1] If a social connection between agents $i$ and $j$ is present, then $g_{ij} = 1$, while it is 0 otherwise. Consequently, the social network is represented by its graph adjacency matrix $G$. For simplicity, I use reciprocal undirected links, meaning that $G$ is symmetric. Allowing the network to be

---

[1]For an example of a study where the criminal network structure is determined endogenously, see Calvo-Armengol and Zenou (2004) or Liu et al. (2012).

directed does not substantively alter the results.

Each agent $i$ has to choose two actions. His criminal effort level $e_i \geq 0$ and $a_i$ which I refer to as contribution to the criminal "cause". Any action that an agent takes without actively participating in the crime and that has payoff-relevant consequences for his criminal friends can be interpreted as such contribution. The contribution can be positive or negative, corresponding to assisting or harming the criminals. A positive contribution might, for example, mean hiding the criminal friends from justice and a negative contribution might mean tipping off the police. In order to assure equilibrium existence, it is sometimes necessary to introduce maximum crime effort $\bar{e}$ which can correspond, for example, to 24 hours in the day. Apart from caring about the expected payoff to their own illicit activity, agents also have social preferences. They affect that part of the payoff through $a_i$. Every agent's utility is the sum of two parts:

$$
u_i \quad = \quad \underbrace{k_i(e, a_{-i}, g)}_{expected\ criminal\ payoff} \quad + \quad \underbrace{s_i(a, e_{-i}, g)}_{societal\ payoff}
$$

More specifically, the criminal payoff can be broken down into three components:

$$
k_i(e, a_{-i}, g) = \underbrace{y_i(e_i)}_{gain} \quad - \quad \underbrace{p f_i(e_i, g)}_{expected\ punishment} \quad + \quad \underbrace{h_i(e_i, a_{-i}, g)}_{contribution\ from\ others}
$$

where $p$ is the probability of getting caught and punished for one's criminal actions. Expected payoff increases in contribution from friends, so $\partial h_i / \partial a_j > 0$ if $g_{ij} = 1$, and falls in aggregate crime, corresponding to congestion costs, i.e. $\partial y_i / \partial e_j < 0 \ \forall \ j \neq i$. Being locked up in jail has been empirically shown by Bayer et al. (2009) to increase delinquents' criminal capital. Intuitively, because the $f_i$ term captures 'lifetime' criminal punishment, it can be reduced by friends' crime in one of two ways. First, if one spends his jail time surrounded by more experienced and skilled criminals, he will learn from them and become less likely to get caught next time. Second, if the agent is part of a group of experienced skillful criminals when free, he might be less likely to get caught in the first place, compared to an agent who has no one from whom to 'learn' the craft. For that reason the punishment term incorporates peer effects from having criminal connections, so $\partial f_i / \partial e_j < 0$ if $g_{ij} = 1$. For tractability I pick linear-quadratic functional form, analogous to Calvo-Armengol and Zenou (2004):

$$
k_i(e, a_{-i}, g) = \underbrace{e_i(1 - \delta e_i - \beta \sum_{j \in \mathcal{N}} e_j)}_{gain} \quad - \quad \underbrace{p f_0 e_i (1 - \sum_{j \in \mathcal{N}} g_{ij} e_j)}_{expected\ punishment} \quad + \quad \underbrace{e_i \sum_{j \in \mathcal{N}} g_{ij} a_j}_{contribution\ from\ others}
$$

6

where $f_0$ is the punishment per unit of crime for an isolated agent, $\beta \in (0;1]$ is the degree of congestion, $\delta \in [0;1]$ determines the moral cost of crime. I define $\phi = pf_0 > 0$ as the expected punishment cost per unit of crime for an isolated agent. The agent needs to choose positive criminal effort in order to receive help from or be sanctioned by his friends.

The societal payoff component of the utility function can also be broken into three components:

$$s_i(a, e_{-i}, g) = \underbrace{v_i(a_i, e_{-i}, \gamma)}_{concern\,for\,society} + \underbrace{l_i(a_i, e_{-i}, g)}_{concern\ for\ friends} - \underbrace{c_i(a_i)}_{cost\ of\ action}$$

This part of utility depends on agent's own contribution, crime efforts of others and, crucially, the parameter $\gamma \in \Re$ which I refer to as *social sympathy*. For example, $\gamma \longrightarrow -\infty$ might be appropriate when the criminal activity is a drug cartel that is purely destructive from the society's point of view. On the other hand, $\gamma \longrightarrow \infty$ might represent the case of agents in the economy illegally plotting to overthrow an oppressive dictator. Adopting the linear-quadratic functional form, the societal payoff becomes:

$$s_i(a, e_{-i}, g) = \underbrace{\gamma a_i \sum_{j \neq i} e_j}_{concern\,for\,society} + \underbrace{\lambda \sum_{j \in \mathcal{N}} g_{ij} a_i e_j}_{concern\ for\ friends} - \underbrace{\frac{1}{2}a_i^2}_{cost\ of\ action}$$

The first two terms reflects the agents' two incentives to contribute. First, they get positive utility from aligning contribution decisions with the social sympathy, that is by assisting $(a_i > 0)$ criminals if $\gamma > 0$ and harming them $(a_i < 0)$ otherwise. Second, by trying to do the socially optimal thing the agent imposes an externality on his criminal friends. The size of $a_i$ related externality is equal to $a_i \sum_{x \in \mathcal{N}} g_{ij} e_j$. Being concerned for their well-being, the agent wants to internalize such side effect of his contribution. Parameter $\lambda \in (0;1]$ measures the degree of concern and can also be thought of as strength of social ties. The presence of such altruism is motivated by the empirical observation that people often care about the way their contribution directly affects their friends. For example, they might choose to not report to the police a crime in which their friends are involved, even if they are opposed to that type of crime. Examples of such behavior are discussed in detail in Section 1.2.1. Implicitly, I also assume that criminals do no internalize the externality that their *criminal* action imposes on their friends. This assumption simply relies on the idea that, unlike the non-criminal contributions, illegal acts themselves are almost by definition selfish. If $\gamma < 0$ the two incentives contradict each other, potentially forcing the agent to assist the

7

crime that he dislikes. Assembling all the parts I obtain the following utility function:

$$u_i = e_i(1 - \delta e_i - \beta \sum_{j \in N} e_j + \sum_{j \in N} g_{ij}a_j) - \phi e_i(1 - \sum_{j \in N} g_{ij}e_j) + \gamma a_i \sum_{j \neq i} e_j + \lambda a_i \sum_{j \in N} g_{ij}e_j - \frac{1}{2}a_i^2 \quad (1.1)$$

The utility function has a standard cost-benefit structure used in Ballester et al. (2006), Calvo-Armengol and Zenou (2004), Liu et al. (2012). The institutional punishment term in equation 1.1 implicitly sets parameter $\eta = 1$ in $\phi e_i(1 - \eta \sum_{j \in \mathcal{N}} g_{ij}e_j)$. This is a standard assumption made in the literature to simplify analysis. Setting $\eta = 0$ would not qualitatively change any of the main results of this paper. The results survive because all of them are predicated on social sympathy $\gamma$ belonging to certain intervals. Setting $\eta = 0$ only changes the intervals' values. A positive $\eta$, however, is crucial for reducing equilibrium multiplicity. Intuitively, when peer effects in crime matter, agents become much less 'interchangeable' from the network centrality point of view. Such heterogeneity makes equilibrium conditions harder to satisfy.

If $\lambda$ and $\gamma$ are both equal to 0, then the model reduces to a version of the model studied in Calvo-Armengol and Zenou (2004). The main results of this paper concern equilibrium behavior for various ranges of social sympathy $\gamma$. So, all of the subsequent propositions would hold even if altruism parameter $\lambda$ were set to 0. There are two reason why altruism is nevertheless important in this model. First, a positive $\lambda$ introduces another reason why heterogeneity in terms of the network position is important, thus making agents even less 'interchangeable' in equilibrium and further reducing multiplicity issues. Second, positive altruism allows for positive non-criminal contributions even when $\gamma < 0$. In the next section I argue that people often help their criminal friends despite being opposed to the crime (negative social sympathy). This situation can only occur with $\lambda > 0$. In turn, as I show in Section 1.3.2, positive contributions are necessary for emergence of equilibria in which criminals are connected to each other. Consequently, positive altruism creates the mechanism in my model through which gangs may form, i.e. subnetworks of connected agents who are perpetrating a socially disliked crime.

## 1.2.1 Discussion of the Model

In this section I provide evidence and give justification for various assumptions made throughout this paper.

## Modeling Choices

*Linear-quadratic utility.* While, clearly, a special choice, linear-quadratic utility is used in analyzing a wide variety of economic models with externalities, such as models of conformism (Bernheim (1994), Akerlof (1997)) or research and development (Goyal and Moraga (2001)). In particular, it is widely used in studying games on networks due to the elegance and convenience of dealing with linear best response systems and the possibility of analyzing the outcomes explicitly as a function of the network structure.

*Congestion in crime.* This may be justified in three different ways. First, the supply of criminal opportunities might be limited. A narcotics dealer might lose a lot of his business if a lot of competition enters the market due to a supply shock. Alternatively one can think of very specific crime, like stealing a certain work of art. If one person steals it, the others don't get to. Second, certain illegal acts might only be valuable to practitioners as an act of rebellion. If adolescents are consuming marijuana only to get back at society, then their payoff to doing so is likely to decrease in the number of marijuana smokers. Third, although not explicitly modeled in this paper, there might be dynamic concern about the effect of crime on future legal enforcement (Sah (1991)). A rise in armed robbery might increase the police presence in the area, thus decreasing expected payoff to armed robbery.

*No victims in the social network.* Criminal activity in the model only has an impact on society as a whole but no direct adverse impact on the neutral non-criminal agents. Such approach means that victims of crimes are people outside of the social network $g$. The network, therefore, should be thought of as a medium-sized community, such as a town or a neighborhood. People there would mostly know each other, and the crime or any form of illicit activity would be directed outwards, at outsiders to the neighborhood or at an external entity, such as the government.

*No peer effects in non-criminal contributions.* There is no skill to be learned and applied in contributing to delinquents, and the agents do not expect to suffer any institutional punishment because they do not actively participate in the crime itself. It is possible that in certain cases, such as interpreting assistance as raising awareness about the issue on the social media[2], the network effects might exist in contribution towards or against the crime as well. However, these cases are specific and that type of peer effects is not the focus of this study.

---

[2]Multiple studies have shown that moods and attitudes exhibit contagion-like spread on social media. See, for example, Coviello et al. (2014).

**Evidence for Assumptions**

*Embedded criminals and possibility of non-criminal contribution.* There is plenty of anecdotal evidence supporting such claim. For example, Morselli and Giguere (2006) conclude after examining the "Caviar" network involved in a drug-trafficking operation in Quebec, that while the criminal subnetwork hinged on three important traffickers, several legitimate actors were crucial to maintaining the law abiding appearance of the setup. The 2014 article in *The Guardian* wrote of the status of Hamas fighters in Gaza: "Hamas and other militants are embedded in the (Gaza) population. Their fighters are not quartered in military barracks, but sleep at night in their family homes[3]." The military wing of Hamas is designated as a terrorist group by the US, Canada and the UK, among others.

Social antipathy can make people want to contribute to the fight against crime. It is not unheard of for individuals to take matters in their own hands by complementing or even replacing the formal judicial arrangement. For example, a recent surge of violent crimes in Mexico has strengthened the informal vigilante system. According to *InSight Crime* foundation "lynchings of suspected thieves, rapists and murderers are [...] common in rural villages and fringe urban neighborhoods" in the country.[4]

*Varying social sympathy and publicly supported crimes.* The assumption that people reserve different amount of such sympathy for different crimes is hardly controversial. Dealing illicit drugs is probably less outrageous than murder. In countries where it is outlawed, homosexuality arguably worries citizens less than rape. Moreover, anecdotal evidence suggests that certain illegal acts, such as opposing an oppressive government by any means necessary, might be viewed as socially beneficial by the people. The same *Guardian* report goes on to state that "most people defend (the militants's) "right to resist" - and support for Hamas rises." Another example of largely publicly backed crime is internet piracy. On October 1st, 2012 the headquarters of a Swedish hosting company were raided by the authorities. As a result, in the next two weeks membership of the Swedish Pirate Party had increased from 7,600 to 14,000. I must remark here that socially approved criminal activity can but does not have to be welfare-improving. It must merely be perceived as such by the people. I also view social sympathy as independent from social norms. Social norms represent an implicit agreement between the society members on what is considered acceptable. Social sympathy reflects the extent to which a typical member of the society rationalizes certain illegal acts as damaging or helpful to his social group.

*Concern for friends matters and can overcome social antipathy.* Agents' desire to inter-

---

[3]http://www.theguardian.com/world/2014/jul/24/gaza-hamas-fighters-military-bases-guerrilla-war-civilians-israel-idf

[4]http://www.insightcrime.org/news-analysis/vigilantes-fight-criminal-acapulco-mexico

nalize the externality that their actions impose on criminal friends is central to the model and appears to be reflected by anecdotal evidence. Hamas fighters in Gaza enjoy so much support not only due to perceived justness of their cause but also due to having strong social ties with the civil population. The sign of $\gamma$ for the case of Gaza strip unrest is debatable, but regardless of the sign, ordinary Palestinians would still be reluctant to hand their brothers, sons or friends over to the Israeli authorities. Therefore, social antipathy on its own does not prevent people from choosing positive contributions. Perhaps, the most striking bit of anecdotal evidence in support of this assumption is presented by the case of Boston Marathon bombing on 15th of April 2013. That day, after his name was released to the public, Dzhokhar Tsornaev, the younger of the two perpetrators and a student at the time at University of Massachusetts Dartmouth, texted "If yu want yu can go to my room and take what's there" to his college friend Dias Kadyrbaev[5]. The text prompted Kadyrbaev and two more of their classmates to enter Dzokhar's dormitory room and smuggle out his laptop, a flash drive and a box of fireworks. None of the three had anything to do with the bombing or any of the other crimes, and it is entirely possible that all three abhor terrorist and extremist activity. Theirs was likely a gesture of help towards a friend, who happened to be a violent criminal.

## 1.3 Patterns of Crime

In this section I first derive the equilibrium and then proceed to introduce the three key results of the paper.

### 1.3.1 Nash Equilibrium

In the game all people simultaneously decide how much criminal effort to exert and how much to contribute to the criminal cause. The equilibrium concept is Nash equilibrium. Formally, each agent $i$ is solving the following constrained maximization problem:

$$\max_{a_i,e_i} e_i(1-\delta e_i-\beta\sum_{j\in\mathcal{N}}e_j+\sum_{j\in\mathcal{N}}g_{ij}a_j)-\phi e_i(1-\sum_{j\in\mathcal{N}}g_{ij}e_j)+\gamma a_i\sum_{j\neq i}e_j+\lambda a_i\sum_{j\in\mathcal{N}}g_{ij}e_j-\frac{1}{2}a_i^2 \quad (1.2)$$

$$s.t.\ e_i\geq 0$$

---

[5]http://www.newyorker.com/news/news-desk/why-are-dzhokhar-tsarnaev-friends-going-to-prison

The utility function is strictly concave in own actions for $\beta$, $\delta > 0$ [6]. Therefore, the best responses of agents are functions given by the first order conditions.

The game is a mixture of strategic substitutes and strategic compliments. This form of strategic interaction means that the game cannot be analyzed via a standard set of tools applied to games of strategic compliments that take advantage of supermodularity (Vives (2005)) to make general statements about existence and nature of the Nash equilibria.

The first order condition of problem 1.2 with respect to the contribution gives the following best response of agent $i$:

$$BR_{a_i}(e_{-i}) = \sum_{j \neq i} (\gamma + \lambda \phi g_{ij}) e_j \tag{1.3}$$

Observe that this best response does not directly depend on contributions of other agents in the society. It depends only on the overall level of crime in the social network. The optimal contribution goes up with social sympathy and agents' concern for their friends.

Criminal effort is constrained to be non-negative. To handle corner solutions, following the approach in Bramoulle and Kranton (2007) and Bramoulle et al. (2014), the best response function of individual $i$ on the crime effort margin can be written as:

$$BR_{e_i}(e_i, a_i) = \max\left\{ 0, \frac{1}{2(\delta + \beta)} \left[ (1 - \phi) - \sum_{j \neq i}(\beta - \phi g_{ij})e_j + \sum_{j \in \mathcal{N}} g_{ij} a_j \right] \right\} \tag{1.4}$$

From Equation 1.4 it is clear that an agent can be kept away from crime by both, high congestion costs and harming contributions from network peers. By definition, any Nash equilibrium of the game must satisfy equations 1.3 and 1.4 simultaneously for all agents. In order to assure equilibrium existence for all parameter combinations , one needs to introduce $\bar{e}$ as the upper limit on the criminal effort[7].

**Definition 1.** *Any Nash equilibrium in which at least one agent selects the maximum level of crime $\bar{e}$ is called 'Maximum Crime' equilibrium.*

For $\phi < 1$ maximum crime equilibria always exist when social sympathy $\gamma$ is high. Specifically, as $\gamma$ goes to $\infty$ all agents choosing $\bar{e}$ becomes the unique equilibrium for any parameter combination. In the subsequent discussion I do not focus on maximum crime equilibria. The main reason is that selecting a numerical value for $\bar{e}$ is not straightforward. While no crime is a natural lower bound on criminal activity, setting $\bar{e}$ to 0.1 or 100 has

---

[6]In that case the Hessian matrix $H = \begin{pmatrix} -1/2 & 0 \\ 0 & -2(\beta + \delta) \end{pmatrix}$ is always negative-definite.

[7]See the Appendix for formal proof of equilibrium existence.

no real life counterpart. Moreover, equilibria sets may be different for different values of $\bar{e}$. For these reasons I only focus on non maximum crime equilibria in this paper. Appendix 2 gives formal conditions that any maximum crime equilibrium must satisfy and provides a numerical example.

Since the utility function 1.1 is strictly concave in own actions, I only study pure strategy Nash equilibria. In any possible non maximum crime equilibrium the set $\mathcal{N}$ is split into people who choose 0 crime and people who choose positive crime. Throughout this paper I refer to the former as "workers" and the latter as "criminals". Both categories, however, choose non-zero contributions.

**Definition 2.** *Suppose an equilibrium involves a set $\mathcal{C} \subset \mathcal{N}$ of criminals and $\mathcal{W} = \mathcal{N} \backslash \mathcal{C}$ of workers. Partition the adjacency matrix $G$ into $G_\mathcal{C}$, $G_\mathcal{W}$, $G_{\mathcal{W}\mathcal{C}}$, where $G_\mathcal{C}$ includes all links $g_{ij}$ s.t. $i, j \in \mathcal{C}$, $G_\mathcal{W}$ includes includes all links $g_{ij}$ s.t. $i, j \in \mathcal{W}$ and $G_{\mathcal{W}\mathcal{C}}$ includes all links $g_{ij}$ s.t. $i \in \mathcal{W}$, $j \in \mathcal{C}$. I refer to such partition as a "network split" induced on set $\mathcal{N}$ by the equilibrium crime entry decisions and the equilibrium itself as a "split" equilibrium.*

Adjacency matrix $G$, along with any other square matrix, can always be partitioned according to the definition of split equilibrium. For example, $A_{\mathcal{W}\mathcal{C}}$ is the same matrix as $A$, except the rows corresponding to agents in $\mathcal{C}$ and the columns corresponding to agents in $\mathcal{W}$ are taken out. In this paper I apply such matrix notation to the identity matrix, denoted by $I$, and a matrix of ones, denoted by $U$. I also apply the notation to matrices $F$ and $CF$, where $F_{\mathcal{W}\mathcal{C}} = G_{\mathcal{W}\mathcal{N}} U_{\mathcal{N}\mathcal{C}}$ is the $|\mathcal{W}|$ by $|\mathcal{C}|$ matrix in which $ij$'th entry is the number of friends of $i$, and $CF_{\mathcal{W}\mathcal{C}} = G_{\mathcal{W}\mathcal{N}} G_{\mathcal{N}\mathcal{C}}$ is a matrix whose $ij$'th entry is the number of common friends worker $i$ has with criminal $j$. Using this notation and the best-response functions 1.3 and 1.4, I obtain the following convenient characterization of any non-$\bar{e}$ equilibrium:

**Proposition 1.** *(Characterization) Contributions $\boldsymbol{a}$ and criminal efforts $\boldsymbol{e}$, along with a network split into criminals $\mathcal{C}$ and workers $\mathcal{W}$ form an equilibrium if and only if:*

*i)* $\boldsymbol{a} = [\gamma U_{\mathcal{N}\mathcal{C}} - \gamma I_{\mathcal{N}\mathcal{C}} + \lambda\phi G_{\mathcal{N}\mathcal{C}}] \boldsymbol{e}_\mathcal{C}$

*ii)* $\underbrace{(2\delta + \beta)I_\mathcal{C} + \beta U_\mathcal{C} - \phi G_\mathcal{C} - \gamma(F_\mathcal{C} - G_\mathcal{C}) - \lambda CF_\mathcal{C}}_{B} \Big] \boldsymbol{e}_\mathcal{C} = (1 - \phi)\boldsymbol{1}$

*iii)* $[\beta U_{\mathcal{W}\mathcal{C}} - \phi G_{\mathcal{W}\mathcal{C}} - \gamma(F_{\mathcal{W}\mathcal{C}} - G_{\mathcal{W}\mathcal{C}}) - \lambda CF_{\mathcal{W}\mathcal{C}}] \boldsymbol{e}_C \geq (1 - \phi)\boldsymbol{1}$

The three conditions arise from assembling best responses in Equations 3 and 4 into matrix form. The first condition is just a matrix version of Equation 3. The second condition recognizes that for all agents in $\mathcal{C}$ criminal best response is given by the first order condition in Equation 4. The third condition precludes deviation on the extensive margin of crime by

requiring that the non-negative constraint on criminal effort bind for all agents in $\mathcal{W}$.

Matrix $B$ contains all the criminal subnetwork interactions and shows that they are carried out in four ways. First, there is a direct utility loss or gain that criminals experience from their friends' contributions, expressed by the term $\gamma(F_{\mathcal{C}} - G_{\mathcal{C}})$. Second, there is an indirect local neighborhood effect through the common friends term $\lambda C F_{\mathcal{C}}$. That term indicates that clustering plays an important role in crime decisions. For example, suppose agents $i$ and $j$ have a friend $z$ in common. In such case, any crime committed by agent $i$ alters $z$'s contribution decision, which in turn influences the crime decision of $j$. Third, there is the peer effects in learning term $\phi G_{\mathcal{C}}$ term. Fourth, criminals affect each other through congestion costs $\beta U_{\mathcal{C}}$. Finally, the term $(2\delta + \beta)I_{\mathcal{C}}$ represents moral cost of crime and contains no interactions. Observe that congestion and peer effects are the only network interactions that would remain if one were to exclude the societal preference part of the utility function.

Proposition 1 does not imply uniqueness. Multiplicity is possible, stemming from interchangeability of agents with similar network centrality. This is standard in the literature and in line with observed differences in crime for neighborhoods with similar fundamentals (Glaeser et al. (1996)).

## 1.3.2 Who Participates in Crime?

### Tendency from Periphery to Core

The nature of the equilibrium network split depends on the values of expected punishment $\phi$ and social sympathy $\gamma$. I this section I study the equilibrium location of criminals in the social network as a function of the crime's nature, determined by these two parameters.

I begin by introducing the interplay between $\gamma$ and $\phi$ using the stylized environment of a *generalized star*. Such network only has two types of agents in terms of network position: core agents and peripheral agents. The $N_1$ core agents are all connected to each other. Each core agent is connected to $K$ peripheral agents for whom that is the only social tie (Figure 1). Figure 2 shows the equilibrium network split in a generalized star in Figure 1 for any possible combination of $\gamma$ and $\phi$ and highlights the first main result of the paper. For $\phi < 1$ as $\gamma$ grows, the equilibrium crime subnetwork shifts from periphery to subsuming the entire network, to the core. This is an example of what I refer to as *fringe-all-core* profile. Showing that such profile is the basic comparative static of the equilibrium criminal set $\mathcal{C}$ with respect to $\gamma$ is the first central result of this paper. The general result is given by Proposition 2 with the intuition for it to follow.

**Definition 3.** *(Fringe-all-core profile). The equilibrium crime entry decisions exhibit a fringe-all-core profile with respect to social sympathy if there exists a sequence of $\gamma$ values,*
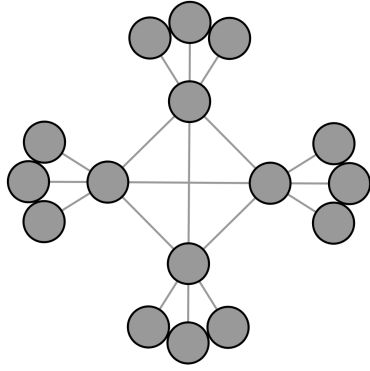
**Figure 1.1:** Generalized Star with $N_1 = 4$ core and $N_1 K = 12$ peripheral agents.
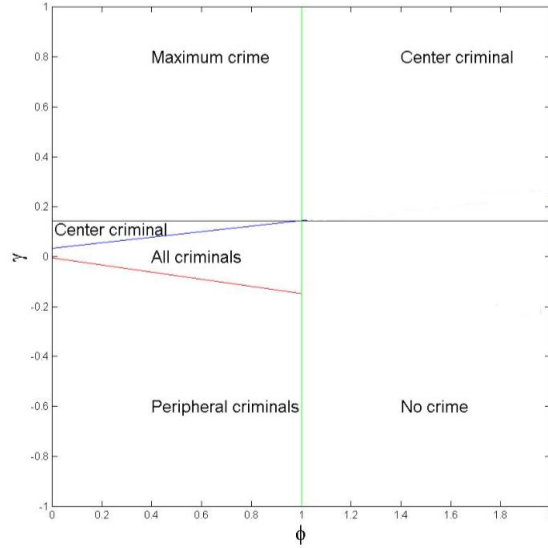
**Figure 1.2:** Equilibrium network split in $\phi$, $\gamma$ space ($\lambda = 0.1$, $\beta = 1$, $\delta = 0.1$).

*satisfying $\gamma_1 \leq \gamma_2 < 0 \leq \gamma_3 \leq \gamma_4 < \gamma_5$ such that:*

1. *If $\gamma \in (-\infty; \gamma_1]$, then the unique equilibrium involves the agents with the smallest number of friends being the only criminals.*

2. *If $\gamma \in [\gamma_2; \gamma_3]$, then the unique equilibrium is for all agent to become criminals.*

3. *If $\gamma \in [\gamma_4; \gamma_5]$, then there exists an agent with $F_c$ friends such that the equilibrium network split is given by $\mathcal{W} = \{i | F_i < F_c\}$ and $\mathcal{C} = \{i | F_i > F_c\}$. Agents with exactly $F_c$ friends can become either workers or criminals.*

The following proposition establishes that such pattern is always possible for any network structure.

**Proposition 2.** *There exist a $\tilde{\phi} < 1$ such that for any network structure and any $\phi < \tilde{\phi}$, except for a 0-measure set [8], the equilibrium crime entry decisions exhibit a fringe-all-core profile.*

The pattern implies that the number of equilibrium criminals should change in an inverse-U shape with respect to social sympathy[9]. As $\gamma$ becomes large and negative, only the least

---

[8]The solution doesn't exist when the best response system is a singular matrix. However, given that $\gamma, \delta, \lambda, \beta$ and $\phi$ are continuous parameters, the set of all possible combinations of them that would give singularity has a Lebesgue measure of 0 on $\Re^5$

[9]The pattern is not as stark for many networks for $\phi \longrightarrow 1$ case. But the general periphery-core transition remains. Case of $\phi \longrightarrow 1$ is discussed in greater detail in Section 4.3.

connected agent commits crime. This happens for three reasons. First, the least connected agent is the one who is least affected directly by the negative contribution from his friends. Second, if all highly connected agents become workers, there is no congestion. Third, if no crime is being committed elsewhere, then there is no one influencing the least connected agent's friends, so their negative contributions are not too large. Intuitively, that agent has the least to lose from angering his social network peers. This result is in line empirical findings by psychologists and sociologists that antisocial behavior is an important correlate of violent crime (Akers (1998), Andrews and Bonta (2014)).

On the other extreme as $\gamma \longrightarrow \infty$ the only equilibrium is maximum crime as mentioned in the previous section. However, there necessarily exists a range of positive $\gamma$ values for which only the best connected agents become criminal. Such *core only* equilibrium is due to congestion costs. For small positive $\gamma$ every would-be criminal receives a positive contribution. The core, however, receives a larger share of the overall contribution and their subsequent high criminal efforts generate prohibitively high congestion costs for the lesser supported fringe[10]. This phase corresponds to most socially connected people carrying out socially supported but punishable activities. An example could be starting an illegal internet file-sharing platform to facilitate piracy of copyrighted material. In order to start such service one needs contacts, equipment and even brand recognition. If a lot of people contribute to several existing services by, say, sharing news about them online, then the internet piracy sector becomes an oligopoly, driving potential new platforms out through congestion costs which in this case can be though of as entry barriers.

Existence of the all-in equilibrium for $\gamma$ around zero is due to lack of punishment. If social sympathy is nearly absent and there is almost no price to pay, then everyone finds it optimal to exert some small crime effort. This situation corresponds to petty crimes.

Proof of Proposition 2 rests on the fact that $\gamma$ has two effects on crime entry decisions, as expressed by conditions 2 and 3 in Proposition 1. In both conditions $\gamma$ only affects the $\gamma(F_\mathcal{C} - G_\mathcal{C})$ term, so as $\gamma$ grows the number of friends starts to dominate crime entry decisions[11]. On one hand, rising $\gamma$ makes crime even more attractive to agents, who are already criminals. They want to exert even more effort, thus also increasing congestion costs. Congestion costs drive criminals with less friends out. On the other hand, rising $\gamma$ also makes

---

[10]Without congestion costs, the criminal subnetwork still spreads from periphery to all agents as $\gamma$ grows. However, for a further increase $\gamma$, the equilibrium goes straight to maximum crime, so the entry pattern is no longer inverse-U shaped. My view is that complete absence of congestion costs in crime is unlikely. At best they might be discontinuous in a sense of being almost non-existent for low levels of aggregate crime and being large for very high levels and would necessarily kick in when everybody is a criminal.

[11]This dominance why it is most convenient to use the number of friends as a measure of social importance in Proposition 2. Simulations show that a similar core to periphery transition can be generated using the Katz-Bonacich centrality .

crime seem comparatively more attractive to workers with a lot of friends, enticing them to deviate, thus violating condition 3. Therefore, as $\gamma$ grows, any equilibrium split is only possible on a range of $\gamma$ values before lesser connected criminals drop out of or more connected workers come into the criminal subnetwork. Formally, for large negative $\gamma$ I show that these forces mean that condition 2 is violated ($\mathbf{e}_i < 0$ for some $i \in \mathcal{C}$) for any set $\mathcal{C}$, except for the set of agents with the smallest number of friends. For $\gamma > 0$ I show that at least one agent must always choose to be a worker and that such agent cannot have more friends than the least connected criminal, otherwise condition 3 would be violated for him.

Proposition 2 requires punishment parameter $\phi \leq \tilde{\phi} < 1$. Evidently from Figure 2, $\phi = 1$ is an important threshold. Crossing it allows for no crime equilibrium. The threshold is natural, because if $\phi$ is exactly equal to 1, then the marginal cost for an isolated criminal of an extra bit of criminal effort exactly equals the benefit. A formal discussion of what happens for $\phi > 1$ is presented in Section 4.1.

The fringe-all-core pattern is an extreme result. Numerical computations show that for $\phi < 1$ the transition from fringe to core is more gradual, as shown by the following example.

**Core-Periphery in an Erdos-Renyi Random Network.**

Figure 3 demonstrates the full gradual inverse-U equilibrium participation profile for a ten node Erdos-Renyi random network in Figure 4. Equilibria are mostly unique, but sometimes there is multiplicity due to interchangeability of agents with similar network positions. The figure gives one possible evolution trajectory for the equilibrium set of criminals with respect to $\gamma$. Picking any other equilibrium in cases when there are multiple equilibria would yield a similar pattern. For any social sympathy value below $-0.567$, the equilibrium set of criminals includes only the marginal agents 3 and 7, each with two friends. The average network degree of criminals and the size of the the equilibrium set grow steadily until $\gamma$ reaches $-0.124$. Beyond that point, congestion costs drive the fringe out, and the criminal subnetwork decreases in size, while average degree keeps growing. The only equilibrium possible with positive $\gamma$ is the one in which the highest-degree agents 1, 2, 6, 8 and 10 form the criminal underbelly. In terms of Proposition 2, the cutoff amount of friends $F_c$ equals to five. These agents are highly inter-connected, almost forming a five-clique. Simulations confirm that it is always the most tightly-knit community that accommodates the criminal subnetwork in any equilibrium when crime is supported ($\gamma > 0$).

Example 1 demonstrates a general gradual transition pattern. Least connected agents become criminals when $\gamma$ is far below 0. As $\gamma$ grows, more socially important individuals start joining the criminal subnetwork, because crime is attractive, and they are not hampered too much by their peers' negative contribution anymore. Eventually the criminal subnetwork
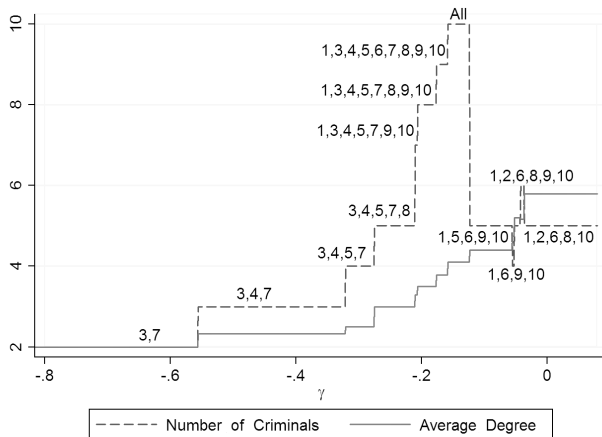
**Figure 1.3:** Number, identities and average degree of equilibrium criminals as a function of $\gamma$ ($\phi = 0.5$, $\lambda = 0.5$, $\beta = 1$, $\delta = 0.1$).
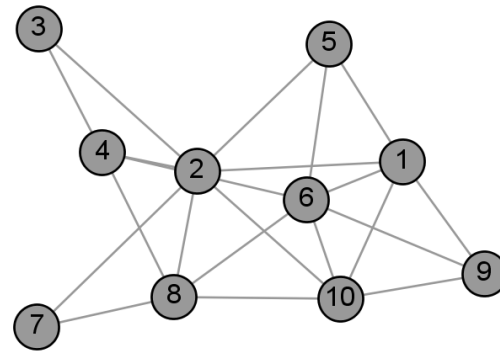
**Figure 1.4:** 10 node ER random network.

reaches its maximum size, and the marginal agents start dropping out because there aren't enough crime opportunities left to them.

### Evidence on Entry Decisions

In this section I link the implications of Proposition 2 and the frindge-all-core profile to several bits of empirical evidence.

*Fringe-all-core profile.* The theoretic framework in, among others, Ballester et al. (2006) and Calvo-Armengol and Zenou (2004) predicts that it is the best connected individuals who benefit the most from committing crimes. The fringe-all-core profile with respect to social sympathy goes further and connects that prediction with several seemingly conflicting pieces of evidence. On one hand, an empirical correlation between delinquency and anti-social behavior is persistent in the psychology literature (Loeber and Hay (1997)). Using an economic model Liu et al. (2012) have found schoolchildren most prone to violent delinquency to be more socially isolated. This evidence is consistent with the fringe-all-core profile, according to which agents with least friends commit the most violent crimes. Social sympathy and a threat of action by the legal agents in the network create conditions for such people to become offenders instead of the better connected ones, who the theory predicts to benefit more from crime.

On the other hand, it appears that many acts, which are punishable by law and often involve violence, are carried out by people who have a significant social presence. Upheavals, like the "Arab Spring," are typically orchestrated by several extremely well-connected and well-supported activists (Coviello et al. (2014)). Historically, the great revolutionaries, like

18

**(a)** $\gamma = -1$, $\phi = 0.95$     **(b)** $\gamma = -0.5$, $\phi = 0.5$     **(c)** $\gamma = -0.1$, $\phi = 0.1$



**(d)** $\gamma = 0.1$, $\phi = 0.1$     **(e)** $\gamma = 0.8$, $\phi = 0.1$
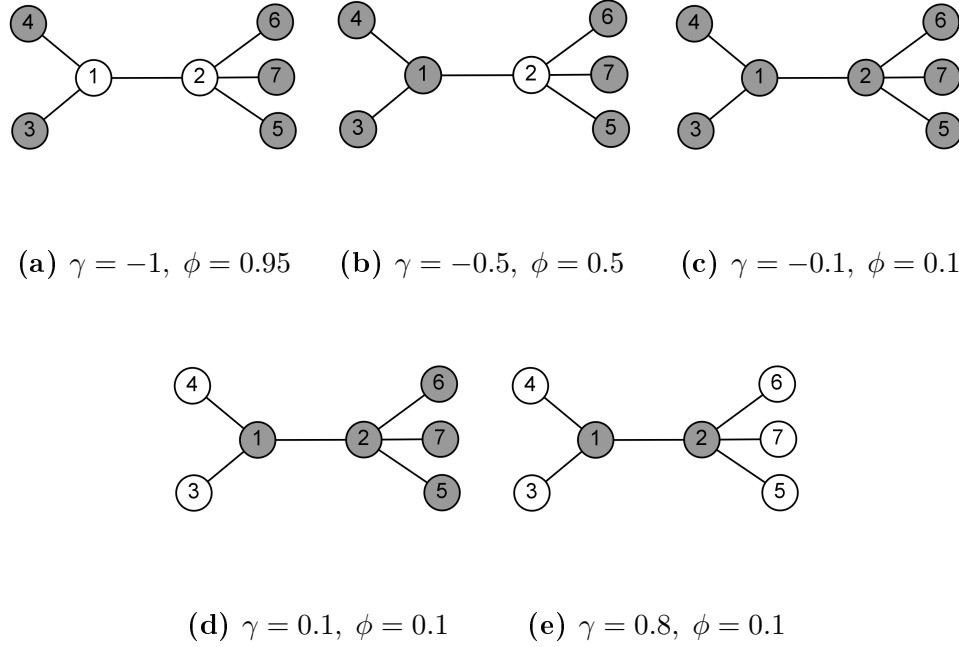
**Figure 1.5:** Equilibrium crime interaction as a function of $\gamma$ and $\phi$ ($\lambda = 0.1$, $\delta = 1$, $\beta = 1$).

Fidel Castro or Vladimir Lenin came from well-off well-connected families. According to "TPB AFK: The Pirate Bay Away from Keyboard", a recent documentary about the illegal file-sharing service "The Pirate Bay," Fredrik Neij, one of the founders, is a married father of three and enjoys an evening at a pub with his many friends. Social upheaval, revolutions and internet piracy, therefore, are illegal acts which, based on anecdotal evidence at least, are committed by people who do not appear to belong to periphery of the social network. This anecdotal evidence is in support of the fringe all core profile. Social unrest and internet piracy draw sympathy from the general public, and the profile predicts that offenses like that should be carried by people with many friends.

*Amount of social interactions varies by crime type.* The fringe-all-core profile suggests that the more heinous the crime (the lower the $\gamma$), the less friends criminals have on average. Therefore, one would expect the amount of friendships between two criminals to decrease in $\gamma$, meaning that active criminals interact less with each other if the society really dislikes their actions. This reasoning provides an explanation for Glaeser et al. (1996) empirical observation that the expected size of an interacting clique of agents varies by the type of crime, being smallest for murder and rape and largest for petty crime. In the context of my model one would expect the seriousness of crime to be reflected by higher punishment $\phi$ and lower social sympathy $\gamma$.

19
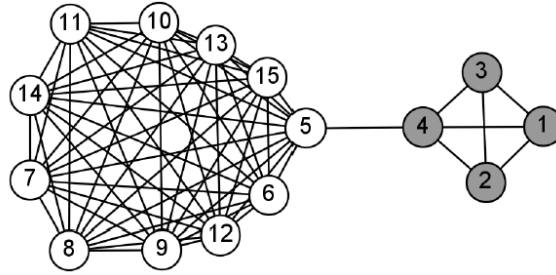
**Figure 1.6:** Minority is pushed to violent crime ($\gamma = -0.6$, $\phi = 0.95$, $\beta = 1$, $\delta = 1$, $\lambda = 0.5$).

To that end Figure 5 present the evolution of the set of criminals in the unique equilibrium with respect to the two parameters in a bridge network. The empty nodes are workers and the filled-out nodes are criminals. For high punishment and antipathy (Panel a) there are no active links between criminals. Crime is strongly disliked and its practitioners need to be able to hide. This situation corresponds to truly heinous crime, like rape and murder. Moderate punishment and antipathy (Panel b), corresponding to serious crimes, like larceny, allow for two out of 6 links to be active. Finally, all links are engaged in case of petty crime (small $\phi$ and $\gamma$) in Panel c. The model, therefore, suggests that non-criminal contributions might constitute the mechanism behind the Glaeser et al. (1996) finding. Figure 5 also shows the model's prediction regarding socially supported and mildly punished crime. The amount of interaction should fall, but the surviving links connect the most influential people.

*Severe segregation pushes minorities to violent crime.* An extensive literature documents that neighborhoods populated by highly-segregated minority exhibit more violent crime [12]. This is also a prediction of the model. Marginalizing one small community creates an artificially large fringe in the social network, because minority groups tend to form less friendships on average (Currarini et al. (2009)). That is where the violent crime is predicted to reside. Figure 6 shows an example of that in society with an extremely segregated crime-infested minority neighborhood. Ex ante the agents are identical. The threat of negative contribution from their large community keeps the majority away from crime, because they have nowhere to hide. The segregated minority does not interact enough with the majority to benefit from such informal policing. All else the same, if there are more social links between minority and majority, then violent crime does not infest the minority neighborhood to the same extent. A degree-preserving rewiring of the links to include more inter-group ties reduces the aggregate crime level.

*Amount of criminals should vary in an inverse-U pattern with social support.* It is difficult to identify an episode of public sympathy changing rapidly for a certain crime. Consumption of marijuana and growing support for its legalization arguably provides one such instance.

---

[12]See, for example, a study by Patacchini and Zenou (2012) on predominantly black boroughs in London.
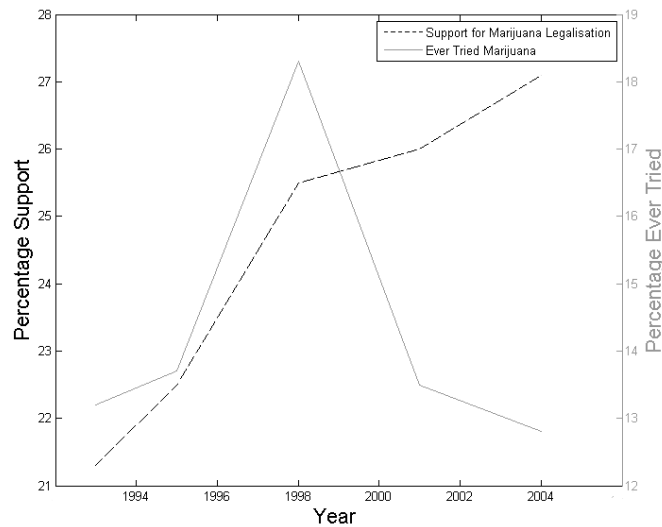
**Figure 1.7:** Co-evolution of support for legalization and usage of marijuana in Australia.
Source: NDSHS.

Figure 7 shows time series of support for legalization of marijuana and percentage of people
who have tried the drug according to Australia's National Drug Safety Household Survey,
described in detail by Clements and Zhao (2014). An inverse-U evolution pattern of per-
centage of marijuana consumers coincides with a steady rise in support for legalization of
the substance. Interpreting consumption as crime and calls for legalization as evidence of
sympathy, the figure can be explained in the following way. In the early 90s the usage was
considered unacceptable and became an attribute of social pariahs, who found it easier to
hide their habits from society. With time marijuana became more tolerated, and people were
no longer afraid of their social network peers "ratting them out" to the police. Everybody
who was curious tried the stuff, resulting in the spike in consumption. As the social atti-
tudes towards marijuana grew even more accepting, its consumption became trendy. The
demand on the black market shifted up further, driving up the price, making marijuana a
luxury good available to the wealthy and well connected. The original consumers were, thus,
crowded out.

Of course, this explanation is not bulletproof. The ceteris paribus assumption almost
certainly does not hold, and the pattern might be generated spuriously. Notably, the proba-
bility of being prosecuted might have been varying at the same time. Additionally, no data
is available on average connectedness of marijuana users in each year. Nevertheless, the
inverse-U pattern and an accompanying increase in public support is encouraging as far as
the model's predictions go.

### 1.3.3 When Do Networks Facilitate Crime?

**Interaction of Tightness and Social Sympathy**

The notion of tightness or close-knittedness of the social network is often brought up in both theoretical and empirical research on crime. And yet there is no consensus on whether tighter network reduce or, in fact, boost crime. Calvo-Armengol and Zenou (2004) and Ballester et al. (2006) show that networks amplify crime, meaning that uniformly denser networks should exhibit more of it than sparser ones. At the same time, a large empirical literature testing the predictions of social disorganization theory, seems to indicate (Yamamura (2011), Land et al. (1991)) that undesired activity falls in proxies for network tightness. Such proxies include religious and ethnic uniformity, lack of migration and population density.

The result that tighter networks should exhibit more crime is usually attributed to risk-sharing or an increase in peer effects in learning-by-observing brought upon by extra links. Such mechanism is still at work in my model. However, it is now only a part of the story, because social preferences can make an extra link work in the opposite direction. Two people who share a social bond do not have to help each other commit crime. A new link simply means that the criminal is exposed to influence from an extra person, which can mean extra negative contribution. So, an additional link does not have to increase crime in the economy. This is the second key result of the paper and is formalized by the following proposition.

**Proposition 3.** *(Tighter Networks) Suppose two networks $g$ and $g'$ are such that $g \subset g'$ and denote by $e^*(g, \phi, \gamma)$ an equilibrium criminal profile under $g$. Then, all equilibria where the network split is the same under both $g$ and $g'$ have the following properties:*

*i) If $\gamma \geq 0$, then $\sum_i^N e_i'^*(g', \phi, \gamma) \geq \sum_i^N e_i^*(g, \phi, \gamma)$.*

*ii) If $\gamma$ is sufficiently negative, then $\sum_i^N e_i'^*(g', \phi, \gamma) < \sum_i^N e_i^*(g, \phi, \gamma)$.*

Proposition 3 applies to situations when equilibrium network network split remains the same under both networks. The Proposition shows that social preferences and sympathy offer an explanation to seemingly contradicting predictions of Calvo-Armengol and Zenou (2004) and the social disorganization theory. The Proposition shows that it is possible for a tightly-knit network to exhibit more crime on aggregate than a sparse one of the same size, but only if the social sympathy is not overly-negative. For sufficiently large negative value of $\gamma$ denser social networks yield less crime. Intuitively, if $\gamma$ is large and negative, then an extra link means that the criminal is exposed to more negative contribution, making him choose less crime. That in turn makes his friends's negative contribution even larger, making the second degree friends also commit less crime. The logic for positive $\gamma$ is identical. The

Proposition, therefore, shows that social attitudes can act as a complement to the formal justice system, particularly in tightly-knit societies. The proof is an iteration on the fact that aggregate crime in a network with one added link equals to aggregate crime in the original network plus a term whose sign depends on the sign and magnitude of $\gamma$.

Two issues need to be clarified. First, Proposition 3 does apply to the rare instances of equilibria multiplicity. For example, if agents $\{1, 2, 3\}$ being criminal and agents $\{4, 5, 6\}$ being criminal are both permissible equilibria under $g$ and $g'$ for $\gamma > 0$, then in both cases the aggregate crime is going to be larger under $g'$. Second, it is possible for the equilibrium set of criminals to change as links are added to the network. Suppose social sympathy $\gamma$ is large and negative. Then in accordance with Proposition 2 the agent with fewest friends is the only criminal. However, one can typically provide such agent with enough new links to make some other agent the one with fewest friends, thus changing the equilibrium split. Nevertheless, the intuition behind Proposition 3 appears to be preserved even when equilibrium splits change. Specifically, while I was not able to come up with a formal proof, the following property seems to emerge in simulations when $g \subset g'$:

1) If $\gamma \geq 0$, then every equilibrium under $g'$ has higher aggregate crime than every equilibrium under $g$.

2) If $\gamma$ is sufficiently negative, then every equilibrium under $g'$ has lower aggregate crime than every equilibrium under $g$.

Appendix 3 provides some evidence for this statement. In it, I demonstrate for a concrete set of parameters how the equilibrium split and the aggregate crime change as links are gradually added to an empty 7-node network, making it complete.

### Evidence for $\gamma$ Directing the Effect of Tightness

A reduction in aggregate undesired activity for denser networks if $\gamma < 0$ is in accordance with multiple empirical studies. Yamamura (2011) studies panel data on cigarette consumption in various Japanese prefectures. Smoking, of course, is not a crime, so in terms of the model the punishment $\phi$ is set to 0. However, smoking is typically discouraged in modern societies and it is arguable that $\gamma$ is negative. Yamamura finds that an increase in population density, a fall in migration or a fall in population turnover reduce the cigarette consumption. Socially coherent and tighter-knit prefectures, therefore, consume less cigarettes. Land et al. (1991) perform a similar analysis on a panel data of crime rates in US cities for 1960, 1970 and 1980. They find that rates of all violent crimes are positively correlated with city's size, percentage of minorities and percentage of divorces. All of these variables reflect lack of social network

tightness and, therefore, confirm that sparser networks should produce more violent crime. The reason that Calvo-Armengol and Zenou contradict those findings by predicting a strictly positive effect of network tightness on crime is that they do not focus on social sympathy and only capture an increase in peer effects. I am not aware of research on whether crimes that arguably correspond to positive $\gamma$, such as internet piracy, increase in network tightness. Carrying out such work would be an interesting empirical test of Proposition 3.

Finally, Proposition 3 gives a prediction with respect to corruption. One would expect there to be social antipathy towards corruption due to its detrimental effects on the economy's efficiency. However, such antipathy is arguably not as strong as antipathy towards violent crime. Moreover, Liu (1985) argues that the more efficient firms should be able to pay bigger bribes and get the best government contracts. In such a way corruption might actually improve efficiency, and if people are aware of this channel, then one might argue a positive $\gamma$ for some types of corruption. Therefore, it seems reasonable to suggest that social antipathy towards corruption is not negative enough to overcome people's desire to help their friends. Then, according to Proposition 3, corruption should be more rampant in societies where social networks are tighter. This prediction can rephrased as "there should be more corruption in developing countries," because social networks there are tighter due to population density, low urbanization, emphasis on kinship and importance of informal risk sharing. Attitudes towards corruption are typically less negative in the developing countries as well, so one can justify assuming a $\gamma$ that is not too negative. In particular, 84.6% of the respondents of the World Values Survey in developed countries consider accepting a bribe as never justifiable, while the percentage falls to 73.3% in less developed countries (Gatti et al. (2003)). This prediction seems to be upheld empirically, with developing countries mostly found to be more corrupt (Svensson (2005)). Standard explanations of the link between low development and corruption include inherent paucity of institutions and lack of accountability by politicians due to insufficient levels of human capital among the citizens. My model suggests that an alternative explanation could come form people providing assistance to escape justice to or turning a blind eye on (i.e. contributing positively) a corrupt politician who belongs to their own tightly-knit caste, clan or ethnic or religious group.

### 1.3.4   When Does Punishment Deter Crime?

Going beyond network-related predictions, the model has the capacity to deliver insights on more general issues, such as the effect of sanctions on crime. Standard economic theory predicts that sanctions should reduce crime. A consensus in the literature is that the reduction happens through either deterrence or incapacitation. My model is not dynamic, and, there-

fore, any effect of an increase in punishment $\phi$ on crime must be attributed to deterrence. However, the very existence of a deterrence effect of criminal sanctions is a debated issue. For every economic study that argues existence (Corman and Mocan (2000), Gonzalez-Navarro (2013)) there is one that finds the effect to be negligible or non-existent (Cover and Thistle (1988), Cornwell and Turmbull (1994)). Moreover, some argue that heavier punishment might increase the undesired behavior. For example, Gneezy and Rustichini (2000) found that introducing late arrival fees at a daycare center actually encouraged tardiness. The possibility of contributions by non-criminals in my model allows for the effect of an increase in criminal sanctions on crime to go in either direction. This third main contribution of the paper is formalized by the following proposition:

**Proposition 4.** *(Deterrence) Suppose the network has an single agent with the lowest degree $\underline{F}$. Suppose also that $\phi < 1$ and that the equilibrium exists and is denoted by $e^*(g, \phi, \gamma)$. Then, the following two things are true:*

1) *There always exists $\underline{\gamma} < 0$ such that $\frac{\partial \sum_i^N e_i^*(g,\phi,\gamma)}{\partial \phi} < 0 \ \forall \ \gamma < \underline{\gamma}$.*

2) *If $\gamma$ is high enough, then $\frac{\partial \sum_i^N e_i^*(g,\phi,\gamma)}{\partial \phi} > 0$.*

An increase in $\phi$ can raise or lower the aggregate crime, depending on social sympathy. Only relatively large negative $\gamma$ necessitates existence of a deterrence effect. For positive $\gamma$ and even small negative $\gamma$ an increase in punishment might bring about an increase in the aggregate crime. Figure 8 demonstrates this point by providing responses of equilibrium crime to punishment at different levels of social sympathy in a given network.

The mechanism through which social sympathy affects deterrence relies on the networked nature of society. Specifically, the results rely on variation of centralities among agents. An increase in $\phi$ has two separate effects on the equilibrium crime in the model. First, it causes a scaling down of criminal effort for all agents due to lower expected payoff. Second, it raises the peer effects in crime. The latter effects counteracts the first one. If $\gamma$ is low, then, following Proposition 2, it is mostly the peripheral people who are committing the crime. Criminal networks are not formed, and there are no links among active criminals. Consequently, these agents do no benefit from strengthened peer effects. On average for them the scaling down effect wins over, bringing the aggregate crime down. On the other hand, for high $\gamma$ the central people join the criminal subnetwork in equilibrium. There are a lot of links between active criminals, so it is possible for the strengthened peer effects to overwhelm the scale-down effect. The existence of the deterrence effect, therefore, relies entirely on where the society lies on the 'fringe-all-core' pattern. The existence of a unique agent with $\underline{F}$ friends simply assures that there is one agent who is more fringe that anyone
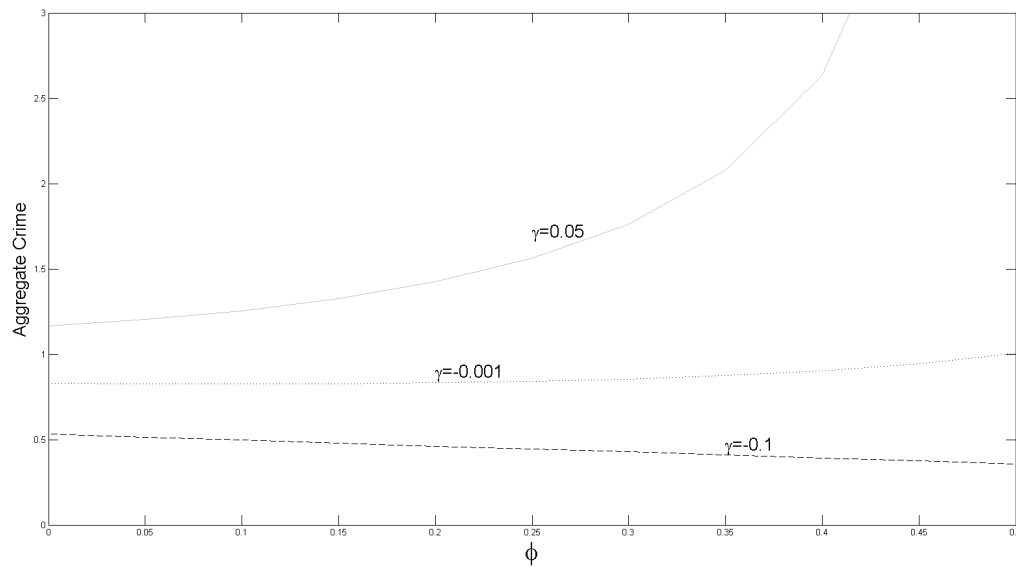
**Figure 1.8:** Effect of punishment on aggregate crime (15 node ER network, $\beta = 1$, $\delta = 0.1$, $\lambda = 0.1$).

else. This is a sufficient but not necessary condition to preclude situations in which there is a clique of agents who all have $\underline{F}$ friends (see, for example, Figure 1.6). In such situation there would be an active clique of criminals even if $\gamma \longrightarrow -\infty$, and the strengthening of peer effects might still overcome the scaling down effect, violating part 1 of Proposition 4[13].

The model offers one explanation for the contradicting findings in the literature on deterrence. For example, in the Gneezy and Rustichini case, late arrival to the daycare center likely corresponds to $\gamma$ close to zero. Introduction of fines might have caused an outrage. Parents might have encouraged each other to arrive late on purpose as an act of defiance against the perceived draconian measures. It also explains why Israel controversial policy of demolishing homes of restive Palestinians seems to have no deterrence effect[14]. Acts countering the perceived Israeli oppression likely garner social sympathy among regular Palestinians, resulting in positive $\gamma$. On the other hand, $\gamma$ corresponding to murder and burglary is likely large and negative, hence the strong deterrence effect of increased policing in New York City found by Corman and Mocan (2000).

Proposition 4 suggests that it is crucial to adequately gauge present social attitudes towards a certain type of crime before introducing any change in criminal sanctions. If the illegal activity in question does not anger the people sufficiently, then a change in legislation

---

[13]Notice that this discussion implies that Proposition 4 does not apply, for example, to complete networks.

[14]http://www.economist.com/news/middle-east-and-africa/21676991-policy-doesnt-seem-be-achieving-its-objectives-punishment-demolitions-east

might be a pure waste of public money or even instigate a crime wave. The model suggests that when a country's oppressive government "turns the screw" on opposition, it might, in fact, facilitate further public unrest. Conversely, looser enforcement might sometimes lower the aggregate amount of crime and could be considered.

Even though, for any given $\gamma$, the sign of the derivative of aggregate crime with respect to punishment is fixed, Figure 8 shows that a U-shaped pattern can sometimes be generated (the dotted line) for a given set of parameters. Another prediction of the model, therefore, is that a change in sanctions not only shifts the level of crime, but also the composition of the equilibrium set of criminals. There are reasons to believe that such prediction is empirically relevant. Sherman (1993), for example, suggests that defiance might be an important motif. If people are heterogeneous in their degree of defiance, then heavier-punished crimes might attract more defiant individuals. Another motif is that heavier-punished crimes might attract more skillful criminals and scare away the "part-timers". Person's degree in the social network might be correlated with criminal skill or defiance. There is precious little research done in social sciences on the composition effect of a change in criminal sanctions. My model suggests that filling in this gap could be important for anti crime policies.

There is one more subtle issue that the model fails to capture. A deterrence effect of punishment on crime might come from an increase in either severity or certainty of sanctions. In the model $\phi$ is the expected punishment cost. Therefore, it is impossible to disentangle the two, even though the distinction has been found empirically relevant by Entorf and Spengler (2008).

## 1.4  Crime and Network Structure

In this section I present three results which allow for a better understanding of the network structure's effect on crime and the forces that affect equilibrium criminal behavior.

### 1.4.1  Extreme Punishment and "Lone Hero" Equilibrium

All results so far dealt with the case of $\phi < 1$. In this section I talk about equilibrium criminal set for $\phi > 1$, i.e. for extreme punishment cases. In such cases, there always exists a no-crime equilibrium[15]. However, unlike in Calvo-Armengol and Zenou (2004), while a zero crime equilibrium is always possible for $\phi > 1$, there can also be equilibria with positive crime. Consequently, the fact that criminals are embedded in the social network implies that

---

[15]To see this, observe that an agent gets payoff of exactly 0 in a proposed no-crime equilibrium. If he deviates unilaterally to a positive crime level $\tilde{e}$, he would receive a payoff of $(1 - \phi)\tilde{e} - (\beta + \delta)\tilde{e}^2 < 0$. The contribution $a_i$ always merely reacts to aggregate crime and doesn't allow for deviation either.

making punishment so high as to render crime unattractive to an isolated individual might not be enough to eliminate it completely. This observation might seem surprising, but it arises arises from the fact that an increase in $\phi$ also increases peer effects and the helping motive, making crime more attractive to agents with a lot of friends. Therefore, beyond $\phi = 1$, agents with a lot of friends might still find it optimal to make friends carry out the crime, particularly if $\gamma$ is high. The following is the counterpart of Proposition 2 for $\phi > 1$ cases:

**Proposition 5.** *Suppose $\phi > 1$, and denote the identity and the degree of the highest-degree node in g by $\overline{F}$ and those of the lowest-degree node by $\underline{F} > 1$. Then, regardless of the overall network structure, two things are true:*

    *i) $\gamma < -\frac{\phi - \beta + \lambda \underline{F}}{\underline{F} - 1}$ is sufficient to ensure that in the unique equilibrium $e_i^* = a_i^* = 0 \ \forall i$.*

    *ii) There exists $\gamma^*$ such that for any $\gamma \geq \gamma^*$ the unique positive crime equilibrium is characterized by a network split into $\mathcal{C} = \left\{ \overline{F} \right\}$ and $\mathcal{W} = \left\{ i \vert \ i \neq \overline{F} \right\}$.*

Extreme punishment alone is not enough to completely deter crime and needs to be combined with social antipathy. Moreover, Proposition 5, combined with numerical simulations, show that the basic comparative static of the equilibrium criminal set with respect to $\gamma$ remains the same even $\phi$ is fixed to be larger than 1. For large negative $\gamma$ no crime is ever committed. As $\gamma$ grows more central agents join the criminal subnetwork until eventually only the person with most friends remains.

For $\gamma > 0$ and $\phi > 1$ all non-criminal contributions are positive and large relative to crime effort. These contributions are what allows agents to commit the crime which is prohibitively costly for isolated individuals. Therefore, for high sympathy and extreme punishment one can think of criminals in the equilibrium as "lonely heroes" or the spearheads of revolution. For instance, a citizen in a buttoned-up autocratic regime might give a revealing interview to a foreign news channel, even though such act might entail a life in prison sentence for treason. These people carry out the costly illicit acts because of being pushed to do so by peers. The coexistence of lone hero and no-crime equilibria in this case seems historically accurate. A well-supported person needs to decide whether to go through with the socially desirable but very risky crime or take the safe option of doing nothing. History, particularly the 19'th century, is full of doomed rebellions that were born out of public anger but sparked into life by actions of a few prominent and, presumably, well-connected individuals, such as the 1825 revolution in Russia or the 1848 revolution in Hungary. Settling on a crime versus no-crime equilibrium for $\gamma$ high and $\phi > 1$ is, therefore, a matter of central agents being brave enough to provide such spark.

## 1.4.2    Sympathy Adjusted Centrality

Proposition 1 suggests that equilibrium criminal effort depends on how many friends the criminal has and the degree of clustering in his social neighborhood. This is in contrast with Ballester et al (2006) conclusion that criminal effort is proportional to Katz-Bonacich centrality of agents. In my model the optimal effort of each equilibrium criminal is also proportional to a particular network characteristic, which is formalized by the following corollary to Proposition 1.

**Corollary 1.** *If the equilibrium is characterized by the set $\mathcal{C}$ of criminals, then their efforts are given by:*

$$\boldsymbol{e}_{\mathcal{C}}^* = \frac{1-\phi}{2\delta + \beta + \beta \sum_{i\in\mathcal{C}} d_i(g,\gamma,\phi)} \boldsymbol{d}(g,\gamma,\phi)$$

*where*

$$\boldsymbol{d}(g,\gamma,\phi) = [I - \frac{1}{2\delta + \beta} \underbrace{(\phi G_{\mathcal{C}} + \gamma(F_{\mathcal{C}} - G_{\mathcal{C}}) + \lambda C F_{\mathcal{C}})}_{D}]^{-1}\boldsymbol{1} \qquad (1.5)$$

The matrix $D$ contains all social interactions between criminals. $D_{ij} = 0$ can only happen if $i$ has no friends. Therefore, $D_{ij}$ measures the importance of link $ij$ to the social network weighted by $\phi$ and $\gamma$. $D$ can be thought of as an adjacency matrix of a weighted graph.

According to Debreu and Herstein (1953), whenever the parameters are such that $\rho(D)$,
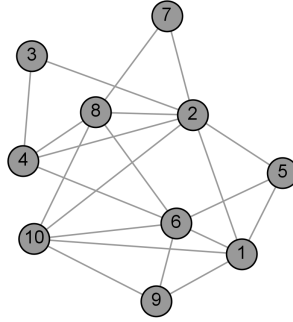


**Figure 1.9:** *Random 10-node Erdos-Renyi network.*

the spectral radius of the matrix, is less than $2\delta + \beta$ (which happens for cases when crime has a very high cost compared to punishment) the inverse $[I - (1/(2\delta + \beta))D]^{-1}$ can be represented as an infinite matrix sum. In that case $\mathbf{d}(g,\gamma,\phi) = [I - (1/(2\delta + \beta))D]^{-1}\mathbf{1} = \sum_{k=0}^{\infty}(1/(2\delta + \beta))^k D^k \mathbf{1}$ and $\mathbf{d}_i(g,\gamma,\phi)$ is the sum of weights of all possible weighted walks originating from $i$ on the criminal subnetwork. Consequently, for such parameter combinations $\mathbf{d}(g,\gamma,\phi)$ can be thought of as a vector of centrality measures. I refer to this centrality measure as *sympathy adjusted* centrality. It is a counterpart in my model to the Katz-Bonacich centrality, which

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mathbf{b}(g,\phi)$ | 1.689 | 1.889 | 1.287 | 1.555 | 1.448 | 1.793 | 1.298 | 1.687 | 1.432 | 1.708 |
| $\mathbf{d}(g,\gamma,\phi)$ | 1.604 | 2.159 | 0.021 | 0.971 | 0.718 | 1.995 | 0.11 | 1.507 | 0.742 | 1.673 |

**Table 1.1:** Comparison of sympathy adjusted centrality and Katz-Bonacich centrality for an Erdos-Renyi random graph ($\phi = 0.1$, $\gamma = 0.015$, $\lambda = 0.5$, $\delta = 0.1$, $\beta = 1$).

using the notation of my model would be given by $\mathbf{b}(g,\phi) = [I - (1/(2\delta + \beta))G_{\mathcal{C}}]^{-1}\mathbf{1}$. The two centrality measures are equal if $\gamma = \lambda = 0$.

The sympathy adjusted centrality subsumes the Katz-Bonacich centrality. Table 1 shows the difference between the two measures in a 10-node Erdos-Renyi random network shown in Figure 9. Introduction of social preferences makes the differences in crime efforts more pronounced. Nodes 2 and 3 commit, respectively, most and least amount of crime in both cases. However, their equilibrium efforts given by Katz-Bonacich centrality are of the same magnitude, while adjusting for sympathy makes node 2 commit about 100 times as much crime as node 3. Positive $\gamma$ here acts towards amplifying agent 2's illicit activity more so than anyone else's, almost pushing the peripheral agent 3 out of crime altogether. Greater divergence between the least and the most active criminals for positive social sympathy appears to be a general feature of the model. The non-criminal contributions drive a wedge between efforts of most and least prominent criminals, creating a sort of specialization and a natural hierarchy in the criminal subnetwork. For this reason an empirical study that does not in any way control for social sympathy might severely overestimate peer effects in crime.

The only difference in terms of ranking is that agents 5 and 9 are switched. Agent 9 commits more crime than agent 5 when social sympathy is included because all of his friends are also friends with each other, forming a 4-clique. Agent 5's local neighborhood is not as interconnected. Belonging to a less tightly-knit group means that his social contacts do not push each other to contribute as much towards the crime and ultimately give him less help. The tightest community commits the socially beneficial crime.

### 1.4.3 Equilibria in Generalized Star

The stylized environment of generalized star allows to derive the exact cutoffs in $\gamma$ for the fringe-all-core profile. This is handy for studying the system's behavior as $\phi \longrightarrow 1$.

**Proposition 6.** *(Generalized Star) Suppose $\phi < 1$. Then for a generalized star of arbitrary size and structure there always exist $\gamma_1 < 0$, and $\gamma_2 < \gamma_3$ such that:*

   *i) If $\gamma \in (-\infty; \gamma_1]$, then there exists an equilibrium network split in which all peripheral agents become criminals and all core agents become workers.*

**(a)** $N_1 = 3,\ K = 4$ **(b)** $N_1 = 3,\ K = 10$ **(c)** $N_1 = 3,\ K = 25$ **(d)** $N_1 = 1,\ K = 4$

**(e)** $N_1 = 10,\ K = 4$ **(f)** $N_1 = 25,\ K = 4$ **(g)** $N_1 = 1000,\ K = 150$
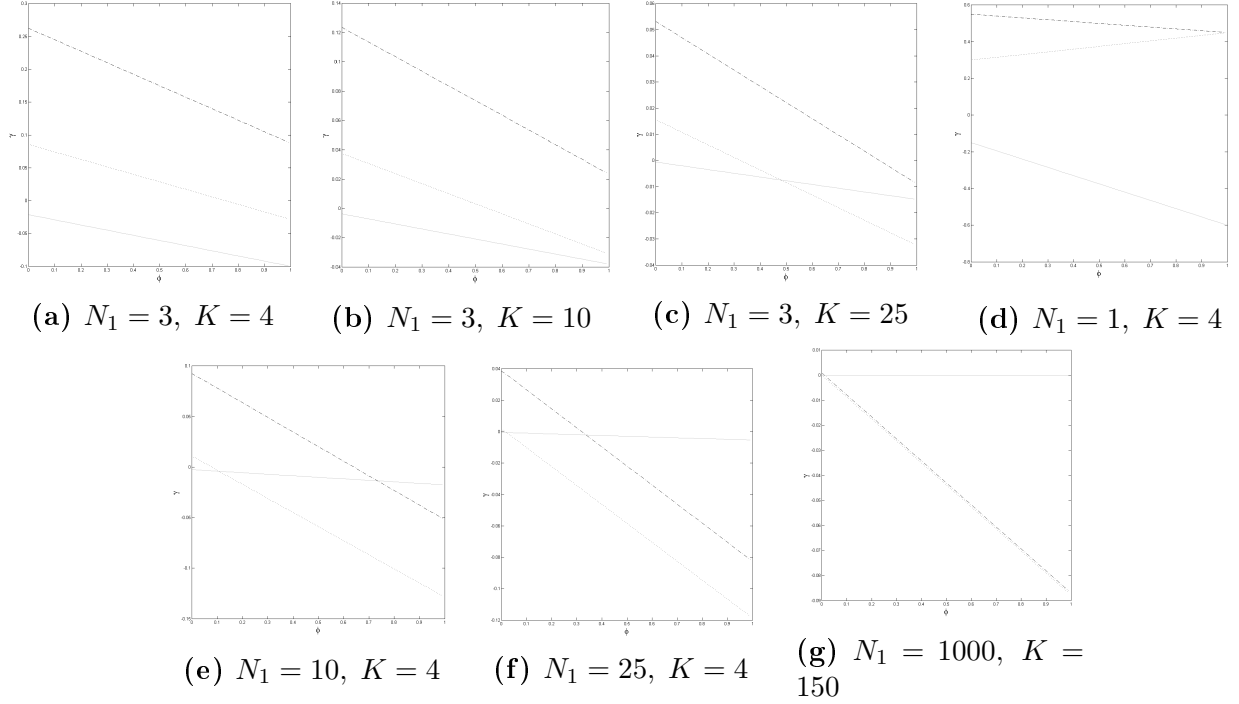
**Figure 1.10:** Equilibria in a generalized star. Periphery-only under solid line. All-in between solid and dotted lines. Core-only between dotted and dash-dot lines ($\lambda = 0.1,\ \beta = 1, \delta = 0.1$).

*ii) If $\gamma$ is between[16] $\gamma_1$ and $\gamma_2$, then there exists an equilibrium where all agents are criminals.*

*iii) If $\gamma \in (\gamma_2; \gamma_3]$, then there exists an equilibrium network split in which all peripheral agents become workers and all core agents becoming criminals.*

The exact cutoff values are derived in the Appendix. In accordance with Proposition 2, the generalized star network of any size and structure always adheres to fringe-all-core profile for small enough $\phi$. Figure 10, however, demonstrates that whether the pattern is preserved for $\phi \longrightarrow 1$ depends on the size of the star and the relative thickness of the peripheral layer $K$. Evidently there are many stars for which the fringe-all-core profile holds exactly for any combination of $\phi$ and $\gamma$, not just for small enough $\phi$.

For $N_1 > 1$ all three cutoffs are represented by downward sloping lines in $\phi$, $\gamma$ space. As the network grows $\lim_{N_1 K \longrightarrow \infty} \gamma_1 = 0$, while $\lim_{N_1 K \longrightarrow \infty} \gamma_2 = \lim_{N_1 K \longrightarrow \infty} \gamma_3 = -\lambda$. The lines defined by $\gamma = 0$ and $\gamma = -\lambda$, therefore, become important thresholds. If $\gamma \in [-\lambda; 0]$, then the periphery-only equilibrium coexists with an interior one. If, on the other hand, $\gamma > 0$, then the equilibrium is maximum crime. There is never a core-only equilibrium. The convergence happens because as $N_1 K \longrightarrow \infty$ the periphery starts to dominate the core in

[16]which of these two values is bigger depends on exact parameter values

size. Yet, the degree of each core agent goes to infinity, while the degree of periphery agents remains fixed at 1. This divergence makes core infinitely more sensitive to social antipathy. The periphery-only and all-criminal equilibria coexist because the core contributes positively towards crime due to concern for their numerous friends, while the much larger periphery contributes negatively. The core agents, therefore, can only commit crime if they all help each other which becomes an equilibrium selection issue. If $\gamma > 0$, then the periphery also contributes positively, and the infinite contribution from the core leads the system into maximum crime equilibrium. If $\gamma < -\lambda$, then even the core dislikes crime enough to always hurt the criminals, thus forcing each other out.

## 1.5    Concluding Remarks

The model in its current form is designed to represent crime. It is hard to think of many other activities which are institutionally punishable, subject to moral and congestion costs, exhibit learning-behind-bars peer effects and are able to attract harm or help from the non-engaged population. However, the framework can potentially be adapted to other scenarios of costly actions where both peer effects and opinion of the crowd matter. Such scenarios could include coming out as homosexual in a conservative society or choosing to obtain education in an environment which discourages high achievement. To do so, one would need to change the nature of peer effects, alter the way in which agents internalize their contribution's externality on friends and remove formal punishment. Studying the way in which social sympathy affects participation patterns in those situations might also be of interest for policy makers.

One possible extension of the current model is to formally add agent heterogeneity and investigate the resulting homophily's effect on entry into crime. It could be that people reserve different level of social sympathy for the same crime based on whether it is committed by a member of their group or not. In such case the model can give predictions with regards to spread of crime in a society characterized by inter-ethnic group interactions.

Finally, traditional models of peer-effect in crime ignore the fact that the criminal network is embedded in the wider social network and clearly influenced by it. This omission means that when such models are taken to the data in attempt to estimate peer effects, the estimator might be biased upwards or downwards, depending on whether the society exhibits sympathy or antipathy towards the type of illicit activity. An important direction for applied work would be to structurally estimate the model in order to obtain results on peer-effects which take such bias into account. Finding a way to estimate the social sympathy parameter $\gamma$ for a given crime in a given society is also of interest.

# Appendix 1: Proofs

**Equilibrium Existence**

Best responses on the contribution margin are always given by the FOC of Problem 1.2 for all players and pose no obstacles to equilibrium existence. The fact that crime is constrained to be non-negative, however, leads to a possibility (for high $\gamma$ and low $\phi$) of non-existence. In such situations, agents always want to unilaterally deviate to committing more crime to take advantage of positive contributions by their friends. Introduction of $\bar{e}$ remedies this problem. Best response given by the Equation 1.4 in this can be re-written as:

$$\min \left\{ \bar{e}, \max \left\{ 0, \frac{1}{2\delta + \beta} \left[ (1 - \phi) - \sum_{j \neq i} (\beta - \phi g_{ij}) e_j + \sum_{j \in N} g_{ij} a_j \right] \right\} \right\} \qquad (1.6)$$

The best response mapping $\mathbf{BR}_e = (BR_{e_1}, ..., BR_{e_n})$ is continuous from $[0, \bar{e}]^{|\mathcal{N}|}$ and has fixed point by the Brouwer fixed-point theorem. Therefore, there necessarily exists a Nash equilibrium.

**Proof of Proposition 1**

For any vector $\mathbf{a}$ of contributions and $\mathbf{e}$ of criminal efforts to form a Nash equilibrium characterized by a network split into sets $\mathcal{C}$ and $\mathcal{W}$, $\mathbf{a}$ must be given by Equation 1.3. Using the matrix notation gives the i). If agent $i \in \mathcal{C}$, then by definition $e_i > 0$. Using the interior FOC of Problem 2 with respect to criminal effort and the fact that workers exert no criminal effort, yields $(2\delta + \beta) e_i + \sum_{j \in \mathcal{C}} (\beta - \phi g_{ij}) e_j - \sum_{j \in \mathcal{N}} g_{ij} a_j = 1 - \phi, \ \forall i \in \mathcal{C}$. In the matrix form this gives condition $[(2\delta + \beta) I_\mathcal{C} + \beta U_\mathcal{C} - \phi G_\mathcal{C} - G_{\mathcal{C}\mathcal{N}} \mathbf{a}] \mathbf{e}_\mathcal{C} = (1 - \phi) \mathbf{1}$. If agent $i \in \mathcal{W}$, then by definition $e_i = 0$, and Equation 1.4 requires a series of inequalities $\sum_{j \in \mathcal{C}} (\beta - \phi g_{ij}) e_j - \sum_{j \in \mathcal{W}} g_{ij} a_j \geq 1 - \phi$ to be satisfied simultaneously. Assembling these inequalities into one matrix inequality gives condition $[\beta U_{\mathcal{W}\mathcal{C}} - \phi G_{\mathcal{W}\mathcal{C}} - G_{\mathcal{W}\mathcal{N}} \mathbf{a}] \mathbf{e}_C \geq (1 - \phi) \mathbf{1}$. Because optimal $\mathbf{a}$ is always given by the first order condition, plug $\mathbf{a} = [\gamma U_{\mathcal{N}\mathcal{C}} - \gamma I_{\mathcal{N}\mathcal{C}} - \lambda \phi G_{\mathcal{N}\mathcal{C}}] \mathbf{e}_\mathcal{C}$ into the other two conditions and perform matrix multiplication to obtain the result. $\boxdot$

**Proof of Proposition 2**

The equilibrium crime entry decisions exhibit a fringe-all-core profile if there exists a sequence $\gamma_1 \leq \gamma_2 < 0 \leq \gamma_3 \leq \gamma_4 < \gamma_5$ satisfying each of the three statements that constitute Definition 2. I prove each of the three statements that constitute the fringe-all-core profile separately:

**Statement 1)** Without loss of generality denote the agent with the lowest amount of friends by 1 and his number of friends by $\underline{F}$. I break the proof of this statement into 3 claims:

**Claim 1** *For $\gamma \longrightarrow -\infty$ agent 1 becoming the only criminal is always an equilibrium.*

Using condition ii) in Proposition 1 and the fact that in the proposed equilibrium agent 1 is the only criminal, I obtain his optimal strategy to be $e_1 = (1-\phi)/(2(\delta+\beta)) - \gamma \underline{F} - \lambda \underline{F}$ which is positive whenever $\gamma < (2(\delta+\beta) - \lambda \underline{F})/\underline{F}$. For the proposed profile to be an equilibrium, none of the other agents must find it optimal to unilaterally deviate to crime. According to condition iii) in Proposition 1, this means that matrix inequality $[\beta U_{\mathcal{W}1} - (\phi - \gamma)G_{\mathcal{W}1} - \gamma F_{\mathcal{W}1} - \lambda CF_{\mathcal{W}1}]e_1 \geq (1-\phi)\mathbf{1}$ must hold. Here the matrix pre-multiplying $e_1$ is just a column vector of length $N-1$. Plugging $e_1$ into the matrix obtain that each of the series of inequalities becomes: $(\beta - (\phi-\gamma)g_{i1} - \gamma F_i - \lambda\phi CF_{i1}) > 2(\delta+\beta) - \gamma\underline{F} - \lambda\phi\underline{F} > 0$. By definition $F_i > \underline{F} \; \forall i \neq 1$, so these inequalities are all satisfied simultaneously for $\gamma$ low enough. Therefore, there exists a $\tilde{\gamma}$ such that any $\gamma < \tilde{\gamma}$ allows for an equilibrium where agent 1 is the only criminal.

**Claim 2** *For $\gamma \longrightarrow -\infty$ there can not be an equilibrium in which an agent with more then $\underline{F}$ friends becomes the only criminal.*

Suppose it were possible for agent with $\underline{F}+k$ friends to be the only criminal in equilibrium for some $k$. Denote that agent by $k$. Then, the condition for the agent 1 with $\underline{F}$ friends to remain a worker and not deviate to crime in equilibrium is given by $\beta - (\phi-\delta)g_{1k} - \gamma\underline{F} - \lambda\phi CF_{1k} > 2(\delta+\beta) - \gamma(\underline{F}+k) - \lambda(\underline{F}+k)$. But there necessarily exists $\hat{\gamma}$ such that for any $\gamma < \hat{\gamma}$ the inequality fails for any $k$.

**Claim 3** *For $\gamma \longrightarrow -\infty$ there is no equilibrium in which multiple agents with different amount of friends become criminal.*

Any other set $\mathcal{C}$ of criminal agents can only arise in equilibrium if the two conditions

$$e_{\mathcal{C}} = (1-\phi)\underbrace{(2\delta+\beta)I_{\mathcal{C}} + \beta U_{\mathcal{C}} - \phi G_{\mathcal{C}} - \gamma(F_{\mathcal{C}} - G_{\mathcal{C}}) - \lambda CF_{\mathcal{C}}]}_{B_{\mathcal{C}}}{}^{-1}\mathbf{1} > \mathbf{0}$$

and

$$\underbrace{(\beta U_{\mathcal{WC}} - \phi G_{\mathcal{WC}} - \gamma(F_{\mathcal{WC}} - G_{\mathcal{WC}}) - \lambda CF_{\mathcal{WC}}))}_{P_{\mathcal{WC}}}e_C \geq (1-\phi)\mathbf{1}$$

hold simultaneously. When $\gamma$ become large the first condition becomes approximately $e_{\mathcal{C}} \approx \frac{1-\phi}{-\gamma}(F_{\mathcal{C}} - G_C)^{-1}\mathbf{1} > \mathbf{0}$. This can be expressed as:

$$e_{\mathcal{C}} \approx \frac{1-\phi}{-\gamma}(G_C(U_{\mathcal{C}} - I_{\mathcal{C}}))^{-1}\mathbf{1} = \frac{1-\phi}{-\gamma}(U_{\mathcal{C}} - I_{\mathcal{C}})^{-1}G_C^{-1}\mathbf{1}$$

Which, solving for the first inverse, becomes:

$$e_{\mathcal{C}} \approx \frac{1-\phi}{-\gamma(|\mathcal{C}|-1)} \begin{pmatrix} (2-|\mathcal{C}| & 1 & . & 1 \\ & 1 & . & . & . \\ . & & . & . & . \\ . & & . & . & . \\ 1 & & . & . & (2-|\mathcal{C}|) \end{pmatrix} G_{\mathcal{C}}^{-1}\mathbf{1} = \frac{1-\phi}{-\gamma(|\mathcal{C}|-1)} \begin{pmatrix} (2-|\mathcal{C}|) & 1 & . & 1 \\ & 1 & . & . & . \\ . & & . & . & . \\ . & & . & . & . \\ 1 & & . & . & (2-|\mathcal{C}|) \end{pmatrix} \begin{pmatrix} \sum_j g_{1j}^{-1} \\ . \\ . \\ . \\ \sum_j g_{|\mathcal{C}|j}^{-1} \end{pmatrix}$$

Where I denote $\sum_j G_{\mathcal{C},ij}^{-1}$ by $\sum_j g_{ij}^{-1}$ $\forall i$. Since the only cases considered are when $|\mathcal{C}| \geq 2$, $2 - |\mathcal{C}| \leq 0$. The fact that $G_{\mathcal{C}} G_{\mathcal{C}}^{-1} = I_{\mathcal{C}}$, implies the following two expressions:

$$\sum_i \sum_j g_{ij}^{-1} F_i = |\mathcal{C}|$$

$$\sum_i \sum_j \sum_{z \neq i} g_{ij}^{-1} g_{zj} = 0$$

which can only both be true if $\sum_j g_{ij}^{-1}$ is positive for some $i$ and negative for others, because $G_{\mathcal{C}}$ is not generically a positive monomial matrix. It follows that at least 1 element of $e_{\mathcal{C}}$ is always negative as $\gamma \longrightarrow -\infty$, violating the equilibrium condition. Hence, as $\gamma \longrightarrow -\infty$ there can be no equilibrium in which agents with more than $\underline{F}$ friends form part of the criminal subnetwork.

The three claims together prove that the proposed strategy is an equilibrium for $\gamma \longrightarrow -\infty$ and rule out any other equilibrium, proving the statement.

**Statement 2)** If all agents are criminal, then $\mathcal{N}$ and $\mathcal{C}$ are equivalent. Evident from Proposition 1, in any such equilibrium the contribution levels are given by $[(2\delta + \beta)I_{\mathcal{N}} + \beta U_{\mathcal{N}} - \phi G_{\mathcal{N}} - \gamma(F_{\mathcal{N}} - G_{\mathcal{N}}) - \lambda CF_{\mathcal{N}}]e_{\mathcal{N}} = (1-\phi)\mathbf{1}$. Any such equilibrium is unique because it is given by the solution to the $|\mathcal{N}|$ by $|\mathcal{N}|$ system of equations. Such equilibrium exists if $\gamma = \phi = 0$ and is given by $e_i^* = 1/(2\delta + (N+1)\beta) > 0$ $\forall i$. By continuity, if $\phi = 0$, there exists a range of both positive and negative $\gamma$ values which give interior equilibrium. Therefore, by continuity there must also exist a $\tilde{\phi} > 0$ such that for any $\phi < \tilde{\phi}$ there exists a range of $\gamma$ values $[\gamma_2; \gamma_3]$ which gives interior equilibrium.

To see that no other non-interior equilibria are possible in that range, observe that when $\phi = \gamma = 0$, any possible network split equilibrium has $|\mathcal{C}|$ people become criminals and $|\mathcal{W}|$ people become workers. The criminals all choose the same level of crime:

$$e_c^* = \frac{1-\phi}{2\delta + \beta + (|\mathcal{C}|+1)\beta}$$

For this to be an equilibrium, for the workers it must be true that:

$$\beta \mathcal{C} e_c^* \geq (1 - \phi)$$

$$\Leftrightarrow$$

$$\frac{|\mathcal{C}|\beta}{2\delta + \beta + (|\mathcal{C}| + 1)\beta} \geq 1$$

which is never possible. So, the unique Nash equilibrium when $\phi = \gamma = 0$ is when all agents become criminals and choose $e_i^* = 1/(2\delta + (N+1)\beta)$. By continuity, there must exist $\tilde{\phi}$ and a range $[\gamma_2; \gamma_3]$ such that this equilibrium remains unique. That $[\gamma_2; \gamma_3]$, therefore, conforms to Definition 2.

**Statement 3)** Any non-maximum crime equilibrium must satisfy Proposition 1. The equilibria unravel if $\gamma$ very large. Statement 2 shows that there must be a range of combinations $\gamma > 0$, $\phi > 0$ which deliver an interior Nash equilibrium before it unravels. I split the proof of the statement into 2 lemmata.

**Lemma 1.** *For small enough $\phi$ there always exists a range of positive $\gamma$ values in which at least one agents chooses to become a worker.*

*Proof.* I prove the lemma by showing that there always exists a range of positive $\gamma$ values for small enough $\phi$ such that agent with $\underline{F}$ friends is the only worker. The proof is made up of 2 claims.

**Claim 1** *If $\gamma = \gamma_3 + \varepsilon$, then for small $\varepsilon > 0$ the non-negativity constraint binds for the agent with $\underline{F}$ friends only.*

Without loss of generality denote agent with $\underline{F}$ friends by 1. Following Statement 2, if $\gamma \in [\gamma_2; \gamma_3]$, the all agents become criminals in the equilibrium. This means that for $\gamma = \gamma_3$ the first order condition with respect to criminal effort for one or more agents gives exactly 0 as the interior solution. Plugging the best response along the contribution margin into the FOC for criminal effort, obtain that the interior criminal effort by any agent $i$ is given by $e_i = (1 - \phi - \sum_{j \neq i} (\beta - \phi g_{ij} - \gamma F_i + \gamma g_{ij} - \lambda \phi C F_{ij}) e_j) / (2(\delta + \beta) - (\lambda + \gamma) F_i)$. The numerator is always smallest and the denominator largest for agent 1 if $\gamma > 0$, because by definition he has the least friends and, therefore, friends in common with others. It must, therefore, be agent 1 for whom the $e_i \geq 0$ constraint starts to bind first. So, for $\gamma = \gamma_3 + \varepsilon$ the non-negative constraint for agent 1 binds, making him choose 0 crime effort.

**Claim 2** *There always exists $\varepsilon$ such that for $\gamma = \gamma_3 + \varepsilon$ all other agents choose positive crime levels.*

For $\gamma \in [\gamma_2; \gamma_3]$ all agents are criminal, so $[I_{\mathcal{N}} - (1/(2\delta + \beta)) \underbrace{(\phi G_{\mathcal{N}} + \gamma(F_{\mathcal{N}} - G_{\mathcal{N}}) + \lambda G^2)}_{D}]^{-1} \mathbf{1} \geq$

**0**. For $\gamma = \gamma_3$, $D$ is a positive matrix. By Debreu and Herstein (1953) this means that the spectral radius $\rho(D) < (2\delta + \beta)$, i.e. $\gamma$ and $\phi$ are small relative to $\delta$ and $\beta$. By definition of $\gamma_3$ for $\gamma = \gamma_3 + \varepsilon$ the inequalities $[I - (1/2\delta + \beta)D]^{-1}\mathbf{1} \geq \mathbf{0}$ no longer hold at the same time, so $\rho(D) > (2\delta + \beta)$. Suppose now that column 1 and row 1, corresponding to agent 1, are taken out of matrix $D$. The resulting matrix $D_{-1}$ is a principal submatrix of $D$ which by Theorem 1.6 in Berman and Plemmons (1979) implies $\rho(D_{-1}) < \rho(D)$. Hence, there necessarily exists a small positive $\varepsilon$, such that if $\gamma = \gamma_3 + \varepsilon$, then $[I - (1/2\delta + \beta)D_{-1}]^{-1}\mathbf{1} > \mathbf{0}$ holds, and $[I - (1/2\delta + \beta)D]^{-1}\mathbf{1} > \mathbf{0}$ does not hold. Therefore, all agents, other than 1, choose interior crime levels.

The two claims put together mean that there exists such $\varepsilon > 0$ that agent 1 chooses to be a worker for $\gamma = \gamma_3 + \varepsilon$ and everybody else chooses to be criminal. Thus, for small enough $\phi$ there must exist a range of positive $\gamma$ values which deliver an "1 out, others in" equilibrium network split. The logic described above can potentially be iterated several times. Therefore, in $\gamma > 0$ and small $\phi$ cases for different network structures there might exist various non-interior equilibria characterized by network splits. $\square$

**Lemma 2.** *Suppose $\phi$ is sufficiently small and $\gamma \in [\gamma_4; \gamma_5]$. Then for any pair of agents $\{ij\}$ such that $i$ is has less friends than $j$, it can not be the case that in equilibrium $i$ is a criminal and $j$ is not.*

*Proof.* Suppose this were possible. In that case condition iii) in Proposition 1 must be violated for agent $i$ and held for agent $j$, so that only $i$ wants to unilaterally deviate to crime. But if $\gamma > 0$, $\phi$ is small and $j$ has more friends, then this is impossible because whenever the inequality $\sum_z (\beta - \phi g_{iz} - \gamma(F_i - g_{iz}) - \lambda CF_{iz})\mathbf{e}_{\mathcal{C}} > 0$ holds for $i$, the inequality $\sum_z (\beta - \phi g_{jz} - \gamma(F_j - g_{jz}) - \lambda CF_{jz})\mathbf{e}_{\mathcal{C}} > 0$ automatically holds for $j$. So, whenever $i$ has an incentive to unilaterally deviate to crime, so does $j$. $\square$

Lemmata 1 and 2 together imply the existence of a cutoff number of friends $F_c$ in the $[\gamma_4; \gamma_5]$ interval. It follows from Lemma 1 that for small enough $\phi$ the highest interval $[\gamma_4; \gamma_5]$ always exists and is greater than 0. Moreover the interval is such that the equilibrium is a network split inside it but maximum crime for $\gamma > \gamma_5$. Therefore, by Lemma 1 if $\gamma \in [\gamma_4; \gamma_5]$, then there exists and agent with smallest amount $F_c$ of friends among all agents in the set $\mathcal{C}$. By Lemma 2 all the agents with less than $F_c$ friends must belong to the set $\mathcal{W}$ in equilibrium, concluding the proof of Statement 3). $\boxdot$

### Proof of Proposition 3

Two networks $g$ and $g'$ are called adjacent if $g' = g \cup \{ij\}$. Without loss of generality assume

that $g' = g \cup \{12\}$. Use the previously established fact that:

$$B_{\mathcal{C}}(g, \phi, \gamma)e^*_{\mathcal{C}}(g, \phi, \gamma) = (1 - \phi)\mathbf{1} = B_{\mathcal{C}}(g', \phi, \gamma)e'^*_{\mathcal{C}}(g', \phi, \gamma)$$

To simplify notation call $e'^*_i(g', \phi, \gamma) = e'_i$ and $e^*_i(g, \phi, \gamma) = e_i$ $\forall i \in \mathcal{C}$. Also, call $B_{\mathcal{C}}(g, \phi, \gamma) = B$ and $B_{\mathcal{C}}(g', \phi, \gamma) = B'$ Then, in matrix notation obtain:

$$B'e' = (1 - \phi)\mathbf{1} = Be' - \begin{pmatrix} \lambda e'_1 + \phi e'_2 - \gamma e'_2 + \gamma \sum_i e'_i + \lambda \sum_i g'_{i2}e'_i \\ \lambda e'_2 + \phi e'_1 - \gamma e'_1 + \gamma \sum_i e'_i + \lambda \sum_i g'_{i1}e'_i \\ \lambda g'_{31}g'_{32}(e'_1 + e'_2) \\ . \\ . \\ . \\ \lambda g'_{N1}g'_{N2}(e'_1 + e'_2) \end{pmatrix}$$

Next, pre-multiply both sides of the equation by $e^T$ to obtain:

$$\underbrace{e^T(1 - \phi)\mathbf{1}}_{\sum_i e_i} = \underbrace{e^T B}_{\approx (1-\phi)\mathbf{1}^T} e' - e^T \begin{pmatrix} \lambda e'_1 + \phi e'_2 - \gamma e'_2 + \gamma \sum_i e'_i + \lambda \sum_i g'_{i2}e'_i \\ \lambda e'_2 + \phi e'_1 - \gamma e'_1 + \gamma \sum_i e'_i + \lambda \sum_i g'_{i1}e'_i \\ \lambda g'_{31}g'_{32}(e'_1 + e'_2) \\ . \\ . \\ . \\ \lambda g'_{N1}g'_{N2}(e'_1 + e'_2) \end{pmatrix}$$

which becomes:

$$(1 - \phi)\sum_i e_i \approx (1 - \phi)\sum_i e'_i - M$$

where

$$M = (\gamma + \lambda)(e'^2_1 + e'^2_2) + 2\phi e'_1 e'_2 + \gamma(e'_1 + e'_2)\sum_{i=3}^N e'_i + \lambda(e'_1 + e'_2)\sum_{i=3}^N g_{i1}g_{i2}e'_1 + \lambda(e'_1 \sum_{i=3}^N g_{2i}e'_i + e'_2 \sum_{i=3}^N g_{1i}e'_i)$$

If $\gamma \geq 0$, then $M > 0$, which means that $\sum_i^N e'^*_i(g', \phi, \gamma) > \sum_i^N e^*_i(g, \phi, \gamma)$. Because $M$ is monotonically growing in $\gamma$, there always exists a threshold negative value of $\gamma$ below which $M < 0$. For such range of $\gamma$ values $\sum_i^N e'^*_i(g', \phi, \gamma) < \sum_i^N e^*_i(g, \phi, \gamma)$.

If $g \subset g'$, then there always exists a sequence of adjacent networks $\{g_0, ..., g_K\}$, such that $\forall 0 \leq k \leq K - 1$, $g_k \subset g_{k+1}$ and $g_{k+1} = g_k \cup ij$ for some $ij$, $g_0 = g$ and $g_K = g'$. Iterating the inequality across pairs of adjacent networks gives the result. $\quad\boxdot$

**Proof of Proposition 4**

By definition of equilibrium, dropping the $\mathcal{C}$ subscript for simplification, the vector of crime efforts is equal to:

$$e^*(g, \phi, \gamma) = (1 - \phi) \underbrace{[(2\delta + \beta)I + \beta U - \phi G - \lambda CF - \gamma(F - G)]}_{B}^{-1} \mathbf{1}$$

The aggregate crime in equilibrium, therefore, can be written as:

$$\sum_i^N e_i^*(g, \phi, \gamma) = \mathbf{1}^T \left( \frac{B}{1 - \phi} \right)^{-1} \mathbf{1}$$

Using the chain rule and the matrix-by-scalar derivative rule obtain:

$$\frac{\partial \sum_i^N e_i^*(g, \phi, \gamma)}{\partial \phi} = \frac{\partial \left( \mathbf{1}^T \left( \frac{B}{1-\phi} \right)^{-1} \mathbf{1} \right)}{\partial \phi} = -\mathbf{1}^T \left( \frac{B}{1 - \phi} \right)^{-1} \frac{\partial \left( \frac{B}{1-\phi} \right)}{\partial \phi} \left( \frac{B}{1 - \phi} \right)^{-1} \mathbf{1}$$

where each element of $\mathbf{1}^T \left( \frac{B}{1-\phi} \right)^{-1}$ and $\left( \frac{B}{1-\phi} \right)^{-1} \mathbf{1}$ is non-negative by definition of equilibrium. The overall sign of the derivative, therefore, depends on the signs of the elements in $\frac{\partial \left( \frac{B}{1-\phi} \right)}{\partial \phi}$. Specifically, the derivative is guaranteed to be positive if $\frac{\partial \left( \frac{B}{1-\phi} \right)}{\partial \phi}$ is a negative matrix and to be negative if it is a positive matrix.

$$\frac{\partial \left( \frac{B}{1-\phi} \right)}{\partial \phi} = \frac{1}{(1 - \phi)^2} [(2\delta + \beta)I + \beta U - G - \lambda CF - \gamma(F - G)]$$

Given that all parameters, except for $\gamma$ are positive, the matrix necessarily becomes positive in the $\gamma \longrightarrow -\infty$ limit, proving the first claim of the Proposition.

The matrix $\frac{\partial \left( \frac{B}{1-\phi} \right)}{\partial \phi}$ would similarly become negative in the $\gamma \longrightarrow \infty$ limit, but the non-existence of equilibrium in that limit precludes from talking about comparative statics. However, a combination of $\gamma$ and $\lambda$ being large enough relative to $\delta$ and $\beta$ is sufficient to make the matrix negative. The equilibrium often exists for such cases and, if it does, then the derivative is positive, proving the second claim of the Proposition. $\boxdot$

**Proof of Proposition 5**

**Statement i)** The proposed strategy is an equilibrium whenever $\phi \geq 1$, because the agent

receives a payoff of 0. A unilateral deviation from such strategy profile along the contribution margin brings no extra utility but is costly, and the deviation along the criminal margin from 0 to $\hat{e} > 0$ changes the extra payoff to $\hat{e}(1 - \phi - (\delta + \beta)\hat{e}) < 0$. Clearly, playing $a_i = 0 \; \forall i$ is best response in this case. So, no profitable unilateral deviation is possible.

To see that such equilibrium is unique for the proposed parameter range, suppose that there exists a different equilibrium profile $\tilde{e}$ given by a network split into sets $\mathcal{C} \neq \varnothing$ and $\mathcal{W}$ such that $\tilde{e}_i > 0 \; \forall i \in \mathcal{C}$ and $\tilde{e}_i = 0 \; \forall i \in \mathcal{W}$. Agent $i$'s direct payoff to criminal activity is $(1 - \phi)\tilde{e}_i - \delta\tilde{e}_i^2 - \beta\tilde{e}_i \sum_{j \in \mathcal{C}} \tilde{e}_j + \phi\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij}\tilde{e}_j + \tilde{e}_i \sum_{j \in \mathcal{N}} g_{ij}a_j$. The profile can only be an equilibrium if this expressions is non-negative for all $i \in \mathcal{C}$, otherwise there is an incentive to deviate to exerting 0 crime effort. Plug the best response of agents on the contribution margin, given by Equation 1.3 to obtain the following expression for proposed equilibrium criminal payoff:

$$(1 - \phi)\tilde{e}_i - \delta\tilde{e}_i^2 - \beta\tilde{e}_i \sum_{j \in \mathcal{C}} \tilde{e}_j + \phi\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij}\tilde{e}_j + \tilde{e}_i \sum_{j \in \mathcal{N}} g_{ij} \sum_{z \neq j}(\gamma + \lambda g_{jz})\tilde{e}_z$$

Expand the last term to get:

$$(1 - \phi)\tilde{e}_i - \delta\tilde{e}_i^2 - \beta\tilde{e}_i \sum_{j \in \mathcal{C}} \tilde{e}_j + \phi\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij}\tilde{e}_j + \gamma\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij} \sum_{z \neq j}\tilde{e}_z + \lambda\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij} \sum_{z \in \mathcal{C}} g_{jz}\tilde{e}_z$$

Use the facts that $g_{ij} = g_{ji}$, and that $g_{ij} = 1$ if and only if $\{ij\} \in g$ to simplify the sums:

$$(1 - \phi)\tilde{e}_i - \delta\tilde{e}_i^2 - \beta\tilde{e}_i \sum_{j \in \mathcal{C}} \tilde{e}_j + \phi\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij}\tilde{e}_j + \gamma\tilde{e}_i F_i \sum_{j \in \mathcal{C}} \tilde{e}_j - \gamma\tilde{e}_i \sum_{j \in \mathcal{C}} g_{ij}\tilde{e}_j + \lambda\tilde{e}_i \sum_{j \in \mathcal{C}} CF_{ij}\tilde{e}_j$$

$$\underbrace{(1 - \phi)\tilde{e}_i - \delta\tilde{e}_i^2}_{<0} + \underbrace{\tilde{e}_i}_{>0} \left[ \sum_{j \in \mathcal{C}}(\phi g_{ij} - \beta + \gamma F_i - \gamma g_{ij} + \lambda CF_{ij})\tilde{e}_j \right]$$

Profile $\tilde{e}$ can not be an equilibrium if for all $i \in \mathcal{C}$ it is true that:

$$\sum_{j \in \mathcal{C}}(\phi g_{ij} - \beta + \gamma F_i - \gamma g_{ij} + \lambda CF_{ij})\tilde{e}_j < 0$$

A sufficient condition is:

$$(\phi g_{ij} - \beta + \gamma F_i - \gamma g_{ij} + \lambda CF_{ij}) < 0 \; \forall ij$$

which is necessarily satisfied if $\gamma < -\frac{\phi g_{ij} - \beta + \lambda CF_{ij}}{F_i - g_{ij}} \; \forall ij$. Clearly, the inequality is satisfied for all pairs as $\gamma \longrightarrow -\infty$. Take the worst case scenario and obtain the sufficient condition on

$\gamma$ to be $\gamma < -\frac{\phi - \beta + \lambda \underline{F}}{\underline{F} - 1}$.

Hence, for this combination of parameters there can not be a nonempty set $\mathcal{C}$ such that $\tilde{e}_i > 0 \ \forall \ i \in \mathcal{C}$, so, the no-crime equilibrium is unique.

**Statement ii)** Similar to proof of Statement 1 of Proposition 2. $\quad\boxdot$

### Proof of Corollary 1

Use condition *ii)* from Proposition 1. Bring the $\beta U_{\mathcal{C}} \mathbf{e}_{\mathcal{C}}^*$ term to the right-hand side and use the fact that $U\mathbf{e}^* = \sum_i e_i^* \mathbf{1}$. On the left-hand side re-arrange the terms to obtain the following expression:

$$(\delta + \beta)[I_{\mathcal{C}} - \frac{1}{\delta + \beta} \underbrace{(\phi G_{\mathcal{C}} + \gamma(F_{\mathcal{C}} - G_{\mathcal{C}}) + \lambda C F_{\mathcal{C}})}_{D}]\mathbf{e}^* = (1 - \phi + \beta \sum_{i \in \mathcal{C}} e^*)\mathbf{1}$$

Pre-multiply both sides by $(I - \frac{1}{2\delta + \beta}D)^{-1}$ to get $(2\delta + \beta)\mathbf{e}^* = (1 - \phi + \beta \sum_{i \in \mathcal{C}} e_i^*)\mathbf{d}(g, \gamma, \delta)$. Pre-multiply both sides by $\mathbf{1}^T$ to obtain:

$$(2\delta + \beta) \sum_{i \in \mathcal{C}} e_i^* = (1 - \phi + \beta \sum_{i \in \mathcal{C}} e_i^*) \sum_{i \in \mathcal{C}} d_i(g, \gamma, \phi)$$

Rearrange the terms to obtain the result. $\quad\boxdot$

### Proof of Proposition 6

I'll prove each of the three statements separately:

**Statement i).** There are only 2 types of agents defined by their network position, a core type and a peripheral type. In the proposed equilibria all agents within each type are identical and always choose the same strategy. Due to stylized nature of the generalized star, it is possible to know everything about each agent's connections. Specifically, each core member has $N_1 + K - 1$ friends, $N_1 - 2$ friends in common with other members of the core, no friends in common with the $K$ periphery agents that are attached to him and 1 friend in common with each of $(N_1 - 1)K$ peripheral agents that are not attached to him. Similarly, each periphery member has 1 friend, which is in common with $K - 1$ other peripherals , and no friends in common with the other $(N_1 - 1)K$ periphery members. Also he has no friends in common with the central agent that he is attached to and 1 friend in common with all other core agents.

Plugging that information into Proposition 1, obtain that the following two conditions must be satisfied: $e_p = (1 - \phi)/(2\delta + (N_1 K + 1)\beta - \gamma N_1 K - \lambda \phi K) > 0$ for peripheral agents to become criminals and $(\beta N_1 K - \phi K - \gamma(K(N_1(N_1 + K - 1) - 1)) - \lambda(N_1 - 1)K)e_p > 1 - \phi$ for core members to stay out. Plugging $e_p$ into the second condition and simplifying gives

2

The equilibrium conditions become:

1) $[(2\delta+\beta)I_C+\beta U_C-(G_C+\gamma(F_C-G_C)+\lambda CF_C)]e_C+\bar{e}[\beta U_{CS}-(G_{CS}+\gamma(F_{CS}-G_{CS})+\lambda CF_{CS})]1_S = (1-\phi)1_C$

2) $[\beta U_{WC}-(G_{WC}+\gamma(F_{WC}-G_{WC})+\lambda CF_{WC})]e_C+\bar{e}[\beta U_{WS}-(G_{WS}+\gamma(F_{WS}-G_{WS})+\lambda CF_{WS})]1_S \geq (1-\phi)1_W$

3) $\bar{e}[(2\delta+\beta)I_S+\beta U_S-(G_S+\gamma(F_S-G_S)+\lambda CF_S)]1_S+[\beta U_{SC}-(G_{SC}+\gamma(F_{SC}-G_{SC})+\lambda CF_{SC})]e_C \leq (1-\phi)1_S$

Any Nash equilibrium needs to satisfy all three. I refer as 'Maximum Crime' equilibrium to any situations when at least one agent is a super criminal.

The most harmless way to implement a $\bar{e}$ threshold without altering the results in this paper is to choose an arbitrary high value of $\bar{e}$ and then to study the equilibria as $\gamma$ changes. I do this for the uneven bridge network (Figure 1.5) by picking $\bar{e}=2$. I also set $\delta=0.1$, $\beta=1$, $\lambda=0.1$, $\phi=0.1$:

The fringe-all-core pattern is preserved under conditions of Proposition 2. However, as $\gamma \longrightarrow \infty$, the equilibria change in the following manner:

- For $\gamma \in (0.2; 0.5)$ the unique equilibrium has 1 and 2 as criminals and the rest as workers.

- For $\gamma \in (0.5; 0.73)$, the unique equilibrium has 1 and 2 as super criminals and the rest as workers.

- For $\gamma \in (0.73; 0.79)$, there are are multiple equilibria when 1 and 2 are super criminals and some subset of 6, 7, 9 are criminals, and 4, 3 are workers.

- For $\gamma \in (0.79; 1.22)$, there multiple equilibria when 1 and 2 are super criminals, and the rest are split in various ways into workers and criminals.

- For $\gamma \in (1.22; 1.24)$, there is a unique equilibrium when 1 and 2 are super criminals and the rest are criminals.

- For $\gamma > 1.24$ there is a unique equilibrium where all agents are super criminals.

This pattern is preserved when I simulate different large networks. First, the fringe-all-core pattern emerges. Then, the core agents become super criminals. Then others progressively lesser and lesser - connected agents join them as super criminals, until the entire network reaches $\bar{e}$.

| Link Added | # of Equilibria | Agg. Crime | # Criminals | Split Survives? |
|:---:|:---:|:---:|:---:|:---:|
| none | 1 | 0.7683 | 7 | - |
| 1-2 | 1 | 0.8486 | 7 | Yes |
| 3-4 | 1 | 1.0286 | 4 | No |
| 5-6 | 1 | 1.2000 | 6 | No |
| 1-7 | 1 | 1.4143 | 5 | No |
| 2-6 | 1 | 2.0172 | 3 | No |
| 3-5 | 1 | 3.1420 | 5 | No |
| 2-7 | 1 | 3.9541 | 5 | Yes |
| 1-3 | 1 | 6.9127 | 6 | No |
| 2-4 | 1 | 8.5816 | 7 | No |
| 2-5 | 1 | 9.3750 | 7 | Yes |
| 6-7 | 1 | 9.5 | 6 | No |
| 1-4 | 1 | 10.5 | 7 | Yes |
| 2-3 | 1 | 10.5 | 7 | Yes |
| 5-6 | 1 | 10.5 | 7 | Yes |
| 4-7 | 1 | 10.5 | 7 | Yes |
| 4-5 | 1 | 10.5 | 7 | Yes |
| 1-6 | 1 | 10.5 | 7 | Yes |
| 1-5 | 1 | 10.5 | 7 | Yes |
| 5-7 | 1 | 10.5 | 7 | Yes |
| 3-6 | 1 | 10.5 | 7 | Yes |
| 3-7 | 1 | 10.5 | 7 | Yes |

**Table 1.2:** Adding links to an empty 7 node network. $\lambda = 0.1$ $\beta = 1$ $\delta = 0.1$ $\phi = 0.1$, $\bar{e} = 1.5$, $\gamma = 0.5$

# Appendix 3: Changes in Equilibrium Split with Links Added

Table 1.2 shows how for $\gamma = 0.5$ the equilibrium changes as links are added to an initially empty 7 node network. Links are added consecutively until the network becomes complete. The equilibrium is always unique, but the split changes. Despite the set of criminals changing, adding never leads to reduction in aggregate crime. For the first 13 links crime strictly increases. After 13 out of the possible 21 links are added to the network, everyone becomes a 'super criminal,' i.e. reaches $\bar{e}$. Making the network complete does not disturb that equilibrium.

Table 1.3 shows the same transition for $\gamma = 0.5$. In that case adding links to the network strictly decreases the aggregate crime, regardless of whether there more or less criminals in equilibrium.

| Link Added | # of Equilibria | Agg. Crime | # Criminals | Split Survives? |
|:---:|:---:|:---:|:---:|:---:|
| none | 1 | 0.7683 | 7 | - |
| 1-2 | 1 | 0.7258 | 5 | No |
| 3-4 | 1 | 0.6429 | 3 | No |
| 5-6 | 1 | 0.5824 | 1 | No |
| 1-7 | 1 | 0.5625 | 6 | No |
| 2-6 | 1 | 0.5383 | 4 | No |
| 3-5 | 1 | 0.4814 | 4 | Yes |
| 2-7 | 1 | 0.4746 | 4 | Yes |
| 1-3 | 1 | 0.4401 | 5 | No |
| 2-4 | 1 | 0.4276 | 4 | No |
| 2-5 | 1 | 0.4059 | 6 | No |
| 6-7 | 1 | 0.3899 | 4 | No |
| 1-4 | 1 | 0.3776 | 5 | No |
| 2-3 | 1 | 0.3709 | 4 | No |
| 5-6 | 1 | 0.3703 | 4 | Yes |
| 4-7 | 1 | 0.3701 | 4 | Yes |
| 4-5 | 1 | 0.3491 | 4 | Yes |
| 1-6 | 1 | 0.3312 | 5 | No |
| 1-5 | 1 | 0.3248 | 4 | No |
| 5-7 | 1 | 0.3186 | 4 | Yes |
| 3-6 | 1 | 0.2998 | 4 | Yes |
| 3-7 | 1 | 0.2896 | 7 | No |

**Table 1.3:** Adding links to an empty 7 node network. $\lambda = 0.1$ $\beta = 1$ $\delta = 0.1$ $\phi = 0.1$, $\bar{e} = 1.5$, $\gamma = -0.5$

# Bibliography

[1] Akerlof, G.A. (1997): "Social Social Distance and Social Decisions," *Econometrica*, 65, 1005-1027.

[2] Akers, R.L. (1998): "Social Learning and Social Structure. A General Theory of Crime and Deviance," *Boston: Northeastern University Press*

[3] Andrews D.A., and J. Bonta (2014): "The Psychology of Criminal Conduct," *Routledge*.

[4] Baccara, M., and H. Bar-Isaac (2008): "How to Organize Crime," *Review of Economic Studies*, 75, 1039-1067.

[5] Ballester, C., Calvo-Armengol A., and Y. Zenou (2006): "Who's Who In Networks? Wanted: Key Player," *Econometrica*, 74, 1403-1417.

[6] Bayer B., Hjalmarsson R., and D. Pozen (2009): "Building Criminal Capital Behind Bars: Peer Effects In Juvenile Corrections," *Quarterly Journal of Economics*, 124, 105-147.

[7] Berman, A. and R. J. Plemmons (1979): "Nonnegative Matrices in the Mathematical Sciences," *Academic Press.*

[8] Bernheim, B.D. (1994): "The Theory of Conformity," *Journal of Political Economy*, 102, 841-877.

[9] Browning C.R., Feinberg S. L. and R.D. Dietz (2004): "Paradox of Social Organization: Networks, Collective Efficacy, and Violent Crime in Urban Neighborhoods," *Social Forces*, 83(2), 503-534.

[10] Bramoulle, Y, and R. Kranton (2007): "Public Goods in Networks," *Journal of Economic Theory*, 135, 478-494.

[11] Bramoulle, Y., Kranton, R., and M. D'Amours (2014): "Strategic Interaction and Networks," *American Economic Review*, 104, 898-930.

[12] Calvo-Armengol A., and Y. Zenou (2004): "Social Networks And Crime Decisions: The Role Of Social Structure In Facilitating Delinquent Behavior," *International Economic Review*, 45, 958-939.

[13] Calvo-Armengol A., Verdier T., and Y. Zenou (2007): "Strong and Weak Ties in Employment and Crime," *Journal of Public Economics*, 91, 203-233.

[14] Chwe M. (2000): "Communication and Coordination in Social Networks," *Review of Economic Studies*, 67, 1-16.

[15] Clements K., and X. Zhao (2014): "Economics and Marijuana: Consumption, Pricing and Legalisation," *Cambridge University Press.*

[16] Corman H., and N. Mocan (2000): "A Time-Series Analysis of Crime, Deterrence and Drug Abuse in New York," *American Economic Review*, 90(3), 584-604.

[17] Cornwell C., and W.N. Turmbull (1994): "Estimating the Economic Model of Crime with Panel Data," *Review of Economics and Statistics*, 76(2), 360-366.

[18] Coviello L., Sohn Y., Kramer A.D.I., Marlow C., Franceschetti M., Christakis N.A., and J.H. Fowler (2014): "Detecting Emotional Contagion in Massive Social Networks," *PLoS ONE*, 9(3), 1-6.

[19] Cover P.D., and J.P. Thistle (1988): "Time Series, Homicide, and the Deterrent Effect of Capital Punishment," *Southern Economic Journal*, 54(3), 615-522.

[20] Currarini S., Jackson M.O., and P. Pin (2009): "Time Series, Homicide, and the Deterrent Effect of Capital Punishment," *Econometrica*, 77(4), 1003-1045.

[21] Debreu, G., and I.N. Herstein (1953): "Nonnegative Square Matrices," *Econometrica*, 21, 597-607.

[22] Entorf, J., and E. Spengler (2008): "Is Being 'Soft on Crime' the Solution to Rising Crime Rates? Evidence from Germany," IZA DP No. 3710.

[23] Gatti, R., Paternostro S., and J. Rigolini (2003): "Individual Attitudes Toward Corruption: Do Social Effects Matter?* ,"World Bank Policy Research Working Paper 3122.

[24] Glaeser, E.L., Sacerdore, B., and J. Scheinkman (1996): "Crime and Social Interactions," *Quarterly Journal of Economics*, 111, 508-548.

[25] Gneezy,U., and A. Rustichini (2000): "A Fine is a Price," *The Journal of Legal Studies*, 29, 1-17.

[26] Gonzalez-Navarro, M. (2013): "Deterrence and Geographical Externalities in Auto Theft," *American Economic Journal:Applied Economics*, 5(4), 92-110.

[27] Goyal, G.A., and J.-L. Moraga (2001): "R&D Networks," *RAND Journal of Economics*, 32, 686-707.

[28] Land, K.C., McCall, P.L., and L.E. Cohen (1991): "Characteristics of U.S. Cities with Extreme (High or Low) Crime Rates: Results of Discriminant Analyses of 1960, 1970, and 1980 Data," *Social Indicators Research*, 24, 209-231.

[29] Liu, F.T. (1985): "An Equilibrium Queuing Model of Bribery," *Journal of Political Economy*, 93, 760-781.

[30] Liu, X., Patacchini, E., Zenou, Y., and L-F. Lee (2012): "Criminal Networks: Who Is The Key Player," Discussion Paper 8772, CEPR.

[31] Loeber, R., and D.F. Hay (1997): "Key Issues in the Development of Aggression and Violence From Childhood to Early Adulthood." *Annual Review of Psychology*, 48, 371-410.

[32] Morselli, C., and L-F. Giguere (2006): "Legitimate Strength in Criminal Networks," *Crime, Law & Social Change*, 45, 185-200.

[33] Patacchini E., and Y. Zenou (2012): "Urban Crime and Ethnicity," *Review of Network Economics*, 11, 1-30.

[34] Sah, R. (1991): "Social Osmosis and Patterns of Crime," *Journal of Political Economy*, 99, 1272-1295.

[35] Shaw, C., and H. McKay (1942): "Juvenile Delinquency and Urban Areas," *Chicago: University of Chicago Press*.

[36] Sherman, L.W. (1993): "Defiance, Deterrence, and Irrelevance: A Theory of the Criminal Sanction," *Journal of Research in Crime and Delinquency*, 30, 445-473.

[37] Svensson, J. (2005): "Eight Questions about Corruption," *Journal of Economic Perspectives*, 19, 19-42.

[38] Taki, M. and L. Coretti (2013): 'The Role of Social Media in Arab Uprisings - Past and Present," *Westminster Papers in Communication and Culture*, 9(2), 1-122.

[39] Vives, X.(2005): "Complementarities and Games: New Developments," *Journal of Economic Literature*, 43, 437-479.

[40] Yamamura E. (2011): "The effects of the social norm on cigarette consumption: Evidence from Japan using panel data," *Japan and the World Economy*, 23, 6-12.

# Chapter 2

# When Sandeep Met Sergey: Peer Effects in Social Assimilation of Foreigners

## 2.1 Introduction

Economists know little about social assimilation of foreigners. What are the mechanisms behind it? What are its effects on economic outcomes at the destination country? What types of policies could be implemented in order to foster it? In particular, a lot of the debate has focused on the challenges posed by immigrants' desire to settle in groups, forming persistent ethnic clusters[1]. Lazear (1999), Bauer et al. (2005), Chiswick and Miller (2002), Danzer and Yaman (2013) all argue that migrants in a larger cluster have lower language ability and interact less with the locals. However, this line of research does not deal with identifying precise channels of immigrants' influence on each other's social assimilation. The assumption is that the mere presence of co-nationals puts the breaks on one's assimilation by lowering incentives to integrate (Algan et al. (2013)). If this assumption is true, then a complete prevention of interactions between foreigners should be the best assimilation policy. Such strategy is, of course, not feasible. Therefore, policymakers have to find an optimal way to employ existing social networks within the immigrant communities in their approach. Isolating *active* channels of co-nationals' influence and the mechanisms behind them is the first step towards determining alternative, easier to implement policies.

This paper's first contribution is to identify endogenous peer effects in acquisition of language skills and friendships with locals as one such active mechanism. I collectively refer to

---

[1]70% of immigrant children in Norway, Denmark and Sweden go to schools where at least half of the pupils are of foreign background.
http://www.economist.com/news/europe/21684830-integrating-migrants-schools-will-not-be-easy-learning-hard-way

the two outcomes as *social assimilation.* The second contribution is to show the peer effects to be at least partially driven by complementarities between assimilation efforts of a foreigner and those of her friends. As a result, a social multiplier arises, which can potentially be exploited in order to extract large cumulative gains from targeted assimilation-related interventions. I use a detailed data set on a community of Indian educational migrants at Karaganda State Medical University in central Kazakhstan. Upon arrival, students are randomly placed into small academic groups of 7 to 15 fellow Indians for administrative purposes, providing a source of exogenous variation in social ties among them. This variation allows me to address the myriad of endogeneity issues that plague peer effect estimations. The paper's third contribution is to use that exogenous variation to show that social assimilation causes the GPA of Indian students to go up, controlling for study hours. Consequently, co-ethnic networks may have a lasting positive effect of helping foreigners be more productive throughout their spell at the destination.

Section 2 provides background on the empirical setting and describe the data set. The community that I am studying is one of students, rather than more conventional migrants, i.e. people who arrive in the new country in hope of obtaining better economic outcomes. I, however, argue that the community's social circumstances are closer to those of 'guest workers' who arrive in the country for an indefinite term, rather than international students in the western sense. First, the students initially arrive for the 5-year BA degree, but many choose to pursue a graduate degree, and some eventually choose to stay. As a result, many of them do not have a fixed return date in mind and may even view the move as permanent. Second, all students have to complete an internship at a local hospital in order to graduate, and many choose to work in order to support themselves. Therefore, there is an economic or workplace component to their presence at the destination country. Third, Indians study in English and barely take any joint classes with the locals. This academic segregation takes away most of the advantages pertaining to social assimilation that international students typically enjoy in comparison with economic migrants. Consequently, the unique data on Indian community that I use in this paper allow me to obtain results that may inform policies on migration at large. Structurally, the community is extremely cohesive, and isolated culturally, linguistically, ethnically but not residentially from the rest of the city. Observing the community's social network is crucial because existing assimilation literature invariably relies on neighborhood data, which likely results in misspecification of true social space (Zenou and Topa (2014)). My preferred measure of local language ability is a score on the comprehensive Russian language test. The number of local friends is measured by the number of local students that migrants nominate as friends.

Section 4 is devoted to checking whether academic groups truly are randomized. Ran-

domization is the key feature of the setting. Being assigned to the same academic group turns out to be the single strongest predictor of a friendship between two Indians, with 31% of nominated friendships occurring between group mates. This exogenous variation in friendships allows me to address the numerous confounding factors (Manski (1993), Jackson (2013)) in peer effect estimation by deriving relevant instruments. I check for validity of randomization by demonstrating that none of the pre-university characteristics influence the probability of being placed in the same group.

In section 5 I propose and implement two alternative identification strategies. Both of them make use of the fact that I observe two candidate reference peer groups - nominated friends and the university-assigned group, - one of which is exogenously created. My empirical methodology is a hybrid of two successful strategies: exploiting the random assignment (Sacerdote (2001), Zimmerman (2003), Bayer et al. (2009), Chetty et al. (2011), Thiemann (2016)), and exploiting the overlap between different peer groups (De Giorgi et al. (2016), Laschever (2005)). Personally nominated friendships are the more natural candidate for peer influence transmission. Therefore, I define 'peer effects' as the causal impact on social assimilation of a student of an increase in average social assimilation of her Indian friends in the nominated network. An OLS regression of such nature is not identified because the estimated effects are likely to be driven by correlated shocks, sorting of friendships on unobserved assimilation types, or simultaneity in decisions. One needs an instrument which is independent of both the friendship sorting process and the correlated shocks, such as taking language classes from a better teacher or living in a friendlier neighborhood.

Academic group averages of variables predetermined at the time of arrival in Kazakhstan are likely to make good instruments. The randomness means that group mates' average pre-university qualities are completely exogenous from the point of view of an individual migrant. The main identifying assumption, therefore, is that pre-university characteristics of group mates have no effect on migrant's assimilation beyond their impact on the assimilation of nominated friends. My proposed instrument for the Russian language regression is the fraction of the academic group that has taken a Russian language course before coming to Karaganda. In my most preferred specification, I instrument the *friends' average* observed Russian ability with the fraction of *group mates* who had pre-university Russian training, conditional on cohort, street of residence and Russian language teacher fixed effects, as well as a large number of controls. The identification comes from incompleteness of the overlap between the set of friends and the set of group mates for each migrants. The point estimate of the causal impact of migrant's friends Russian ability on her own is 0.66. The effect is economically large - one standard deviation increase in Russian test scores among migrant's friends boosts her own score by 1/3 of a standard deviation.

I implement an identical strategy in order to identify causal peer effects in ability to pick up local friends. Pre-university Russian training is not a valid instrument, because of its interaction with observed language ability. Therefore, the instrument for this outcome is the group's average answer (recorded at the time of matriculation) to the qualitative question of 'how important was an opportunity to experience new culture in your decision to come to Kazakhstan.' These answers provide a noisy measure of pre-university interest in interacting with locals. The point estimate of endogenous peer effects in a fully specified local friends regression is 0.33.

The main problem of the identification strategy described above is that group mates' characteristics might directly influence an Indian student's assimilation outcome. For example, a group that is on average more willing to embrace foreign cultures might attend more university-sponsored cultural events, resulting in more interactions with locals. For this reason, the second identification strategy is the preferred approach. I augment the method of Bramoulle et al. (2009) of instrumenting friends' outcome with characteristics of distant-degree friends. Specifically, I introduce the exogeneity of the group assignment into the model by subtracting the leave-out-self group means from all variables. Such transformation eliminates group correlated effects and the within-group component of network endogeneity. The endogenous peer effect in such case is expressed by the impact of the difference between *my friends and my group mates' friends* on the difference between *me and my group mates*. I then follow Bramoulle et al. in order to show that under mild conditions on the network topology, the difference in assimilation between my distant-degree friends and the distant-degree friends of my group mates can serve as a valid instrument for the difference between my friends and my group mate's friends. The intuition is that the characteristics of my distant degree friends only affect me through their effect on my first-degree friends' outcomes, i.e. the endogenous peer effect channel. Using this research design, the point estimates of endogenous peer effects are 0.59 in Russian ability and 0.49 in the number of local friends. The two identification strategies, therefore, give qualitatively similar and statistically indistinguishable point estimates, which is further evidence of peer effects' existence.

In section 6 I provide evidence for existence of a social multiplier assimilation. Endogenous peer effects might be driven by one or a combination of conformity and complementarity. With conformity, students are trying to minimize assimilation distance between themselves and their peers. With complementarity, the cost of assimilation effort falls with the effort of peers. The two mechanisms both predict a positive causal effect of an educational migrant's assimilation on that of her friends, but only complementarity results in a social multiplier (Boucher and Fortin (2016)). I provide three pieces of evidence for presence of complementarities. First, I present the answers to a series of qualitative hypothetical questions designed

to elicit Indians' own perception of the peer effect channels. The students appear to view both mechanisms as apt, but assign complementarity a larger weight. Second, I perform a version of the direct J test of social multiplier by Liu et al. (2013). Liu et al. suggest that if complementarity is part of the mechanism, then it is friends' *aggregate* assimilation that should matter and not their *average* assimilation. They propose a formal test to separate the two models. The results of the test confirm the presence of social multiplier in my data. Finally, I employ Boucher and Fortin (2016) approach of using isolated individuals in order to separate the mechanisms. For isolated individuals the estimated effect of exogenous characteristics on assimilation should be larger than for connected ones in presence of conformity. This difference arises because for connected individuals strong pressure to conform partially overrides own characteristics. Once the strength of conformity is determined, it can be used to back out the strength of complementarity from the endogenous peer effect coefficient that combines the two. This approach allows me to back out a social multiplier of approximately 1.4. To out the number into perspective, the effect of a policy that directly boosts the aggregate assimilation of the Indian community by 1% would snowball into overall of 1.4% increase through peer effects.

In section 7 I discuss the effect of social assimilation on productivity. It has been suggested in migration literature that assimilation is not only a desirable outcome in itself from the viewpoint of peaceful cohabitation of ethnicities, but also crucial for economic success. Wages and labor market outcomes have been shown to depend positively on having a local spouse (Baker and Benjamin (1997), Meng and Gregory (2005)), speaking the local language (Chiswick and Miller (1995), Dustmann and Fabri (2003)), and having local social capital (Adda et al. (2014), Mui Teng et al. (2015)). However, the research on both language skills and local friends acquisition is hampered by data limitations. All of the studies mentioned above work exclusively with survey data, which are prone to large and potentially malicious measurement errors. It is, for example, entirely possible that migrants who speak the local language at a near native level also have a better understanding of the limitations of their ability and, therefore, are less likely to report speaking it 'Very Well,' than migrants with only basic conversational skills. I use more precise measures of the two assimilation outcomes to show that social assimilation has a positive effect on productivity. The community's sole goal is to receive the degree and the skills needed to pass a qualification exam in India. Therefore, overall GPA can be though of as a measure of output. I use the group's average pre-university interest in foreign cultures as an instrument for student's 'assimilation index,' which is a combined measure of the two outcomes of interest. Controlling for hours of study, as well as a large array of personal characteristics, I estimate that one standard deviation increase in assimilation index causes a roughly .2 standard deviation increase in overall GPA.

Consequently, assimilation boosts foreigners' productivity.

At the first glance, the findings in this paper appear to contradict the documented lower assimilation for migrants in larger ethnic clusters (Danzer and Yaman (2013), Bauer et al. (2005)). However, all of those studies define migrants' social space as neighborhood and work with survey data, resulting in measurement errors both in outcomes and peer group definitions. More importantly, those results and mine do not necessarily negate each other. My data are not suited for identification of network *size* effects. Peer effects might be weaker if the network or the ethnic cluster is larger (perhaps, because in such case any given social tie receives much less weight), resulting in the observed negative correlation between community size and assimilation, documented in those studies. Another explanation could be that greater size of the community might move the nature of peer effects away from complementarity and towards conformity. Finally, the mere presence of social multiplier does not mean that ethnic cluster results in higher assimilation. The power of the multiplier needs to be harnessed through effective public policy.

## 2.2   Empirical Setting and Background on KSMU

The empirical setting for this paper is Karaganda State Medical University, a large public university of medicine in Karaganda, capital of the eponymous region in central Kazakhstan. From here onward I refer to the university by its acronym KSMU.

Karaganda is a coal-mining and industrial community in the center of Kazakhstan. Despite the seismic changes in the ethnic landscape of the population following Kazakhstan's independence from Soviet Union on 1991, people in Karaganda are not especially used to foreigners. According to the 2009 census, only 8,195 of the region's 1.3 mln. residents had arrived between 1999 and 2009 from outside the former Soviet countries. Almost all of those people were ethnic Kazakh repatriates from places like China or Mongolia. In particular, there were no arrivals from India. The World Bank bilateral migration flow matrix for 2013 also reports no permanent migration between India and Kazakhstan. The only other Indian citizens living in and around Karaganda are the few management staff of the Arcelor Mittal steel manufacturing facility in the nearby town of Temirtau. According to the census, only 14% of the region's urban population claim to be able to understand some English. These facts highlight the ethnic, linguistic, and cultural isolation of Indian students when they first arrive at KSMU, and suggest that the student body is likely to form a coherent community with strong bonds between members. The tightness of the community is only reinforced by abundance of familial relationships among the students. Out of 757 Indian students enrolled at KSMU at the time of data collection, 223 reported to have at least one relative
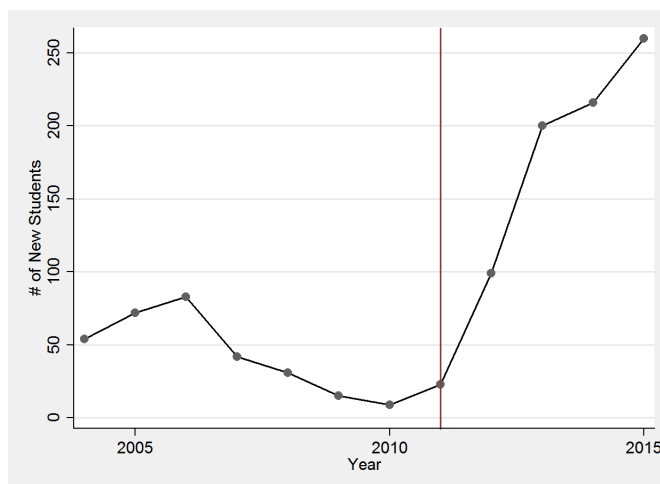
**Figure 2.1:** Indian students' cohort sizes throughout the years.

who attended or had graduated from the university. All students report no knowledge of non KSMU-affiliated people of Indian background in Karaganda. The Vice Dean of General Medicine is the only Indian contact person for them, who is not a fellow student. He is also the official recruiter of students in India and the main coordinator of their day-to-day activities. Most of the information on the history and evolution of this particular migration episode comes from his records.

Indian students started coming to KSMU as early as 1994, although the the numbers were low, and the stream was unsteady, with only a couple dozen every year throughout the 90s. The cohorts started to get larger in early 2000s. Figure 1 tracks cohort sizes throughout the years. The lull in late-2000s is explained by absence of direct recruiting efforts in India. The line separates all of the cohorts that are covered by my data. Lack of new arrivals in late-2000s is handy. Since the program is 5 years long, each student has an opportunity to interact with members of 9 different cohorts. I do not observe the post-2010 cohorts. However, the late-2000 lull means that for each student enrolled in 2016 I observe almost all of the Indian students with whom they have ever interacted at KSMU.

Although the university offers a large variety of undergraduate and graduate degrees, all of the Indians are enrolled in the 5-year Bachelor of General Medicine program. Advantage of this uniformity is that students are not able to choose what courses to take but instead progress along a predetermined path. As a result, all of them are exposed to very similar academic environments in terms of local peers, teachers and materials. The rate of attrition of enrolled Indian students is rather small. Out 798 who started the program since 2011, 757 were still enrolled at the time of the data collection in March 2016 - an attrition rate of just above 5%. According to the Vice Dean, almost all those 5% either never arrived in

Karaganda after being admitted and enrolled or withdrew from the university due to health or financial problems.

Along with official recruitment trips, word of mouth appears to play an important role in attracting Indian students to KSMU. As of 2015, 847 Indian students have graduated from the university during the tenure of the current Vice Dean (the period for which I have the information), majority of whom went on to become practicing medical professionals either in Rajasthan or in Delhi. In my data, 61.43% of all students report to have consulted with at least one alumnus before deciding to come to Karaganda.

The reverence that many in India hold of the medical profession and difficulty of obtaining a degree at home mean that students India medical schools around the world. Apart from established universities in the developed countries, KSMU is competing with universities from as China, Russia, Georgia, The Philippines, Ukraine, Kyrgyzstan, and Mauritius. According to the recruiter, students find KSMU attractive because they view it as the best value for money. The competition at a public medical school in India is about 20 people per place, while tuition at a private school can be as much as $12,000 per year. In comparison, yearly tuition at KSMU is less than $2,000 and the selection process is far less strict than at Indian public medical schools. An applicant must have gotten at least 50% on Rajasthan Board of Secondary Education Senior Secondary 12th Grade Exam (or equivalent exam for their state of residence), which I henceforth refer to as BSER, be least 17 years old, and pass subject tests in chemistry, mathematics, biology, and physics. The subject tests do not appear to be particularly stringent, with 93% of applicants succeeding in getting minimum amount of point required for admission.

Another key selling point of KSMU in the eyes of Indian applicants is its success in preparing students for Medical Council of India Screening Test (MCIST). India recognizes medical degrees from only five countries - Australia, Canada, the UK, the US, and New Zealand. Doctors from these countries are exempted from taking the test and are allowed to practice in India. Graduates from all other countries, including KSMU graduates have to pass the MCIST in order to prove their qualifications. According to the recruiter's data, 87% of the graduates pass the test on the first attempt, compared to the overall reported passing rate of only 21% for all foreign-trained doctors.

Typical class size for Indian students at KSMU is 13-15 people, although some key lectures are taught to an auditorium full of students. Unlike the Western-style system, students have little freedom in choosing what classes they take and have to move along a predetermined path. Lectures vary in length, but students spend much more time in classrooms than at a typical university in a western country. It is not uncommon for the entire period from 8 am to 5:30 pm to be occupied by lectures with lunch break at 2 pm. Students in fourth

and fifth year spend a bit less time in classes, but still often have lectures continuously from 8 am to 2 pm.

Defining local language in Kazakhstan is not straightforward. The two major ethnic groups are Kazakhs and Russians. Kazakh is the official language of government and public institutions. However, Russian is spoken more widely, particularly outside of the southern regions which border China. It is preferred in business and scientific circles and serves as lingua franca. According to the 2009 census in the city of Karaganda Russians made up 46.14% and Kazakhs made up 37.9% of the overall population of 460,039. Out of the region's entire urban population, 98.1% of people claim to understand spoken Russian, compared to mere 50.2% who claim to understand spoken Kazakh. No census figures are available for the city-by-city linguistic breakdown, but the trend should be even more pronounced in Karaganda itself and within the community of medical professionals in particular.

All of the curriculum for Indians at KSMU is taught it English. However, being able to speak Russian is likely to be advantageous in the process of studying, because of sometimes low standard of English among the faculty and existence of demonstration-based classes. Demonstration-based classes, such as the anatomy classes held in the university's morgue, are frequently taught to a joined group of local and Indian students and the instructor's commentary of the process is typically in Russian. Overall, half of the 757 Indian students answer 'yes' to the question of whether they feel like speaking Russian makes studying at KSMU easier. Only 5 people answer 'yes' to a comparable question about Kazakh language. For these reasons I focus on Russian language ability as my preferred measure of linguistic assimilation. Greater importance of Russian in day-to-day communication is also recognized by the university itself. It mandates two semesters of Russian and only one semester of Kazakh as part of the general education curriculum for new Indian students. As of the 2015/2016 academic year (the final cohort in my data), the language courses have been outsourced by the university to a local linguistic institute (LINGVO), making it difficult to compare the final grades of language courses across years.

Upon graduation, most of the students return to India, although, according to the recruiter's records the university's Indian alumni are currently working in Singapore, UAE, Russia, USA and UK. When asked what country they would prefer to practice medicine, 89.43% of my sample answered 'India', while only 2.25% answered 'Kazakhstan', which allows me to conclude that desire to remain in the country after graduation is unlikely to be the main driver of observed assimilation outcomes.

## 2.3   Data Description

The data were collected in early 2016 by a combination of supervised computer based surveys carried out in KSMU computer labs[2], and digitizing university records. In order to capture each migrant's social space as cleanly and thoroughly as possible, I presented them with alphabetized drop-down menus, containing all of the names of current Indian students at KSMU. In case of identical names, additional distinguishing features were provided, such as year of study or academic group number. Respondents were asked to select all of their closest friends, with a requirement of providing at least 2 names in order to avoid having isolated nodes in the network. Figure 2.2 displays the the network between migrants, color coded by year of study. Unsurprisingly, vast majority of links are between students in the same cohort. Otherwise, the network is very much a classical social network. It is dense, made up of a single connected component with average degree of 9.5. It is characterized by a high clustering coefficient of .157. The diameter is 7, and the average shortest path length is 3.5.

Being able to accurately identify each and every member of the migrant community and record the ties between them is the unique advantage of these data. The absence of other foreigners, Indian or otherwise, in Karaganda means that I am observing very precisely the relevant assimilation peer group. In fact the literature on ethnic neighborhoods typically takes the word 'neighborhood' quite literally. However, as Topa and Zenou (2014) point out, the overlap between geographical and social space of a person might be far from complete. In today's world of cheap electronics and fast internet connections, people living on the opposite ends of town, or in altogether different towns might exert more influence on each other than next door neighbors used to do 20 years ago. The regrettable success of terror organizations in long-distance online radicalization of Western youths is a testament to that. For that reason, going more granular and directly studying the actual interaction patterns between migrants is desirable. The freshness and the lack of historical context of this migration episode is another feature that makes it a viable target for research. Much of the debate on pros and cons of ethnic neighborhoods has so far been informed by the experiences of refugees from war-torn counties, black ghettos or Mexican immigrants in the United States. The questions of assimilation in those communities have either been heavily politicized or are fraught with painful history. Learning about interactions between co-nationals outside of any entrenched prejudice and before any social norms have formed is, therefore, of interest.

In the subsequent analysis, whenever I say assimilation, I mean two types of outcomes

---

[2]The surveys were carried out in English, rather than Hindi, the native language of all migrants in my sample.
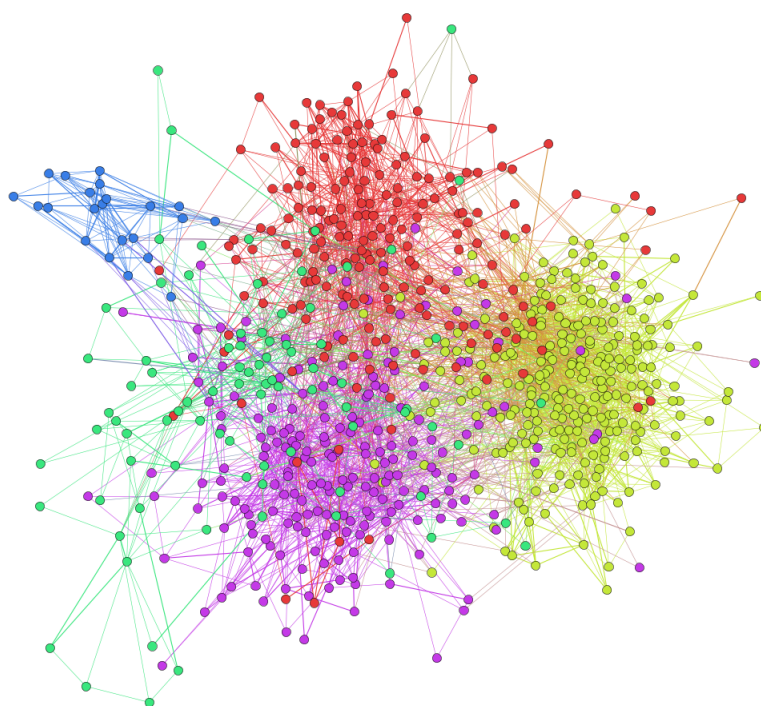
**Figure 2.2:** Network of Indian students, colored by year of study. Blue-5th year, green - 4th year, red - 3rd year, purple - 2nd year, yellow - 1st year.

in Karaganda. The first is the Russian ability, which I measure in two different ways. The primary one is the outcome of the standardized language test, taken by all migrants at the same time in early 2016. The results of the test are on 100 scale. The other measure is the self-assessment of listening comprehension, speaking, and reading ability, all on 1-10 scale from non existent to perfect. The second assimilation outcome is the amount of social capital in Kazakhstan, measured by the number of local friends. In order to measure the local social capital as accurately as possible, I provided respondents with a searchable digitized list of all local KSMU students to choose from, structured by year of study and academic program. Therefore, if an Indian student wanted to report a social tie with a friends, she had to select that person's name from the database. The friend was then identified in my data by an assigned number. The advantage of such approach was that it disciplineed respondents into providing a truer picture of their local social capital, then if they were directly asked to estimate their number of local friends. A direct question would have resulted in subjective and inaccurate reporting. In my subsequent analysis I use the number of local students who are nominated by migrants as friends as my preferred measure of local social capital.

In order to receive a fuller picture, I also ask the a non identity-based question about

the number of local friends *outside* KSMU. 64.2% report having none, with almost all others reporting very small numbers. Even though I expect these numbers to be extremely noisy, they still suggest that for vast majority of migrants interactions with fellow students are the only meaningful form of local social capital. Moreover, the two measures are strongly positively correlated, so I view the reported number of non-university friends simply as noise around the more precise identity-based measure from within KSMU.

Table 2.1 provides descriptive statistics of the main variables used in this study. In terms of socioeconomic background KSMU Indians form a rather homogeneous group. Seventy percent are male Hindus from the north-western state of Rajasthan. Rajasthan's capital Jaipur alone contributes 18% of the student body. Note that the state is the only one where KSMU holds regular recruiting events. Therefore, one might expect students from outside of Rajasthan to form a more selected group. Only 14% of the sample are female, but as many as 38.6% come from castes which are defined by the Indian government as disadvantaged[3].

Only 40 people report having lived abroad before coming to Kazakhstan. Consequently, for many the adjustment to the new country is hard - 43% report to have felt unsafe or been physically or verbally abused at least once in Kazakhstan due to their race. Nevertheless, they seem to be able to overcome these issues and form more than 4 close friendships with local students on average. Additionally, one can see by comparing mean test score and mean of self-assessed Russian ability that migrants tend to overestimate the quality of their linguistic knowledge (both variables were collected in early 2016, so there is no other reason for them to mismatch). Notice that, while many students would like to eventually practice medicine outside of India, as a group they have little desire to stay in Kazakhstan for any number of years after graduation. In my view, lack of desire to stay brings down the pressure to assimilate.

Table 2.1 includes the pre-KSMU outcomes which will be used as instruments in subsequent analysis. Forty three percent of the sample report to have had some formal training in Russian before coming to Karaganda for the first time. Having taken the course is a strong predictor of ultimate Russian ability and most likely is a signal of hidden underlying assimilation type. Before matriculation, students were asked to fill out a questionnaire with basic data about them. One of the questions that they are asked is 'how important was the desire to experience a new culture in your decision to come to KSMU?'. The answers are given on a 1-10 scale from 'not at all' to 'extremely.' Such answers can also be viewed to signals of underlying assimilation type.

BSER, which stands for Board of Secondary Education Rajasthan, refers to a high-stakes

---

[3]Formally, this means that they are designated as belonging to a 'Scheduled Tribe,' a 'Scheduled Caste,' or 'Other Backward Class.' I combine all of these people together into a single 'backward caste' category.
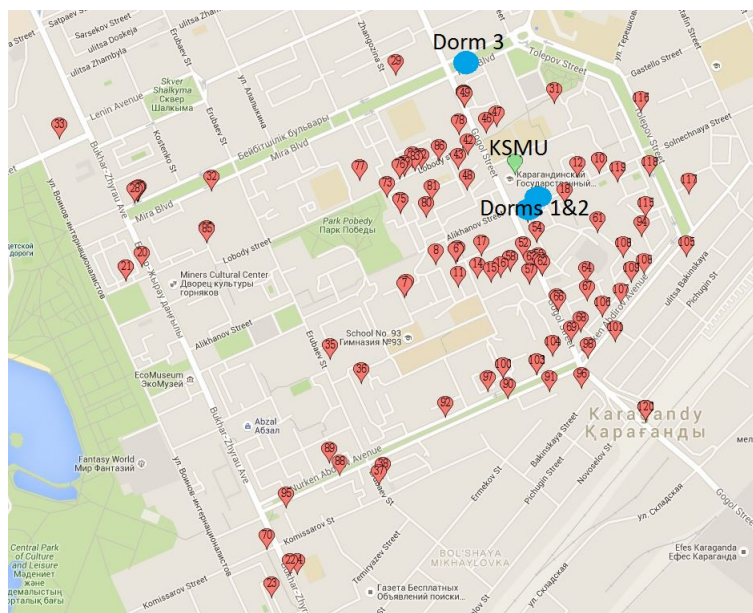
**Figure 2.3:** Map of Karaganda. Marked houses are home to at least one migrant.

exam that Rajasthan highschoolers have to take when they are 16. It is a standardized test in Hindi, English, Natural and Social sciences, mathematics and Sanskrit. Maximum possible score is 100, while, according to the official data, the highest ever score on the exam was 98.67. The only official average results that are available to public are the average number of students in Rajasthan getting the so-called '3rd division honors' (less than 40% score), '2nd division honors' (40-60%) and '1st division honors' (above 60%). In 2014 only 16.54% of all Rajasthan students got first division honors. Out of students from Rajasthan[4] enrolled at KSMU in 2016, the comparable figure is 82.82%, with average score of about 69 and maximum score of 97. Hence, the students as a group appear to be well above average as far as their observed pre-university ability is concerned. The very best Indian students prefer to either stay in India or attend medical schools in western countries, but KSMU Indians still represent a selected group of high-achieving individuals.

Majority of the students (54.82%) prefer to live off campus. The most typical living arrangement is for 4 students to share a 2-bedroom apartment. Living conditions are superior to the dorms, where one typically shares a small room with at least 3 other people, at roughly the same price on average. Figure 2.3 displays the map of central Karganda with indicators over all houses where migrants live. Indians typically sign long-term contracts with the landlord once they move into a new apartment, and, according to the recruiter, there is little mobility across town after that. Consequently, one might expect there to be

---

[4]For the 10% of the students that come from outside of Rajasthan, the score on BSER-like tests for their home state are converted to a 1-100 scale.

| Variable | Obs. | Mean | St. Dev. | Min | Max |
|---|---|---|---|---|---|
| Overall GPA | 757 | 2.712 | 0.522 | 1.53 | 4 |
| Russian Command (test) | 757 | 36.303 | 20.174 | 5 | 100 |
| Rus. Coprehension (1-10 self assessed) | 757 | 5.540 | 2.382 | 1 | 10 |
| Rus. Speaking (1-10 self assessed) | 757 | 5.382 | 2.274 | 1 | 10 |
| Rus. Reading (1-10 self assessed) | 757 | 5.664 | 2.501 | 1 | 10 |
| Had Rus. course pre KSMU | 757 | 0.435 | 0.496 | 0 | 1 |
| #Local friends | 757 | 4.264 | 2.455 | 2 | 10 |
| #Indian friends in Kaz. | 757 | 9.527 | 5.009 | 2 | 30 |
| Female | 757 | 0.139 | 0.346 | 0 | 1 |
| BSER | 757 | 68.885 | 9.554 | 44 | 100 |
| From Rajasthan | 757 | 0.900 | 0.301 | 0 | 1 |
| From Jaipur | 757 | 0.181 | 0.385 | 0 | 1 |
| Backward cast | 757 | 0.386 | 0.487 | 0 | 1 |
| Muslim | 757 | 0.108 | 0.311 | 0 | 1 |
| Age at year 1 | 757 | 18.094 | 1.728 | 15 | 32 |
| Lived abroad before | 757 | 0.050 | 0.218 | 0 | 1 |
| Married | 757 | 0.017 | 0.130 | 0 | 1 |
| Siblings | 757 | 1.996 | 1.684 | 0 | 15 |
| Relatives at KSMU | 757 | 0.703 | 1.696 | 0 | 10 |
| Lives in dorms | 757 | 0.452 | 0.498 | 0 | 1 |
| # Roommates | 757 | 2.413 | 1.229 | 0 | 8 |
| Shares house with local | 757 | 0.055 | 0.229 | 0 | 1 |
| Avg. hours study per week | 757 | 19.483 | 9.176 | 0 | 50 |
| Avg. hours party per week | 757 | 4.905 | 5.119 | 0 | 27 |
| Avg. hours study Russian per week | 757 | 5.367 | 6.021 | 0 | 52 |
| Avg. hours of sport per week | 757 | 4.342 | 3.908 | 0 | 14 |
| Avg. hours research per week | 757 | 1.150 | 1.938 | 0 | 15 |
| Avg. hours local media per week | 757 | 2.176 | 3.719 | 0 | 23 |
| Gets a scholarship | 757 | 0.148 | 0.355 | 0 | 1 |
| Does sports w/ locals | 757 | .211 | .409 | 0 | 1 |
| # Languages learned before KSMU | 757 | 2.099 | 1.371 | 0 | 11 |
| #Close friends in India | 757 | 10.300 | 45.519 | 0 | 1000 |
| Total monthly expenditure (1000 KZT) | 757 | 55.752 | 70.895 | 2.5 | 153.428 |
| Wants to work outside India | 757 | 0.144 | 0.351 | 0 | 1 |
| #Years intends to stay in Kaz. | 757 | 0.458 | 1.820 | 0 | 30 |
| Racially abused in Kaz. | 757 | 0.433 | 0.496 | 0 | 1 |
| Tries hard to make local friends | 757 | 4.229 | 2.609 | 1 | 10 |
| New culture is important | 757 | 5.790 | 2.212 | 1 | 10 |
| Local friends as valueable as Indian | 757 | 4.382 | 2.622 | 1 | 10 |
| Russian language is difficult | 757 | 5.036 | 2.356 | 1 | 10 |
| Unhappy if had to settle in Kaz. | 757 | 4.464 | 2.824 | 1 | 10 |
| University friends are temporary | 757 | 3.699 | 2.505 | 1 | 10 |
| Grades are important | 757 | 6.083 | 2.917 | 1 | 10 |

*Notes.* The last 6 variables are qualitative survey questions coded on a 1 to 10 scale from 'Strongly Disagree' to 'Strongly Agree'.

**Table 2.1:** Descriptive Statistics
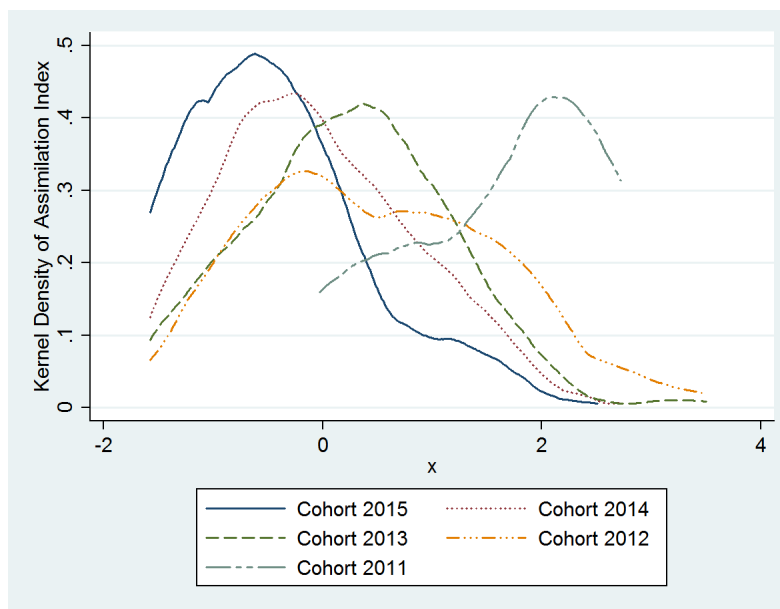
**Figure 2.4:** Assimilation Index by time spent in Kazakhstan

correlated assimilation shocks at the residence level. For example, certain neighborhoods in Karganda might be more ethnically mixed and, thus, more friendly to migrants, increasing assimilation of all of those who live there.

Since most students arrive in Karganda with rudimentary or absent knowledge of Russian and no social capital, one would expect each passing year to bring about a large marginal increase in assimilation if migrants do put in the effort. On the other hand, having local friends and speaking Russian is not mandatory, and most students have no desire to stay in Kazakhstan, so there is little explicit pressure to assimilate. In order to document a basic trend in assimilation, I create a standardized assimilation index. To do so, I first standardize the two assimilation outcomes of interest - Russian command and the number of local friends to be mean 0 and variance 1. I then add up the resulting variation, and divide it by $\sqrt{2}$ in order to get a mean 0, variance 1 variable. Figure 2.4 demonstrates the kernel density estimates for the distribution of assimilation index within each cohort. Additionally, an OLS regression of the index $(AI)$ on peer's averages and year fixed effects yields the following output (standard errors in square brackets):

$$AI_i = \underset{[.22]}{0.91} + \underset{[.08]}{0.49\overline{AI}_i} - \underset{[.24]}{1.2\,Year_{1i}} - \underset{[.23]}{.91\,Year_{2i}} - \underset{[.23]}{.7\,\,Year_{3i}} - \underset{[.23]}{0.61\,Year_{4i}}$$

Each earlier cohort is better assimilated than the one that follows it. Of course, these numbers muddle the time at destination effects together with cohort size and year of study effects. Nevertheless, the pattern towards increased assimilation year by year is strong and

robust to various estimation methods and inclusion of a large number of further controls. Therefore, the Indians do appear to assimilate during their time in Kazakhstan, and it is important to study the mechanisms behind that process.

## 2.4 Group Assignment

### 2.4.1 Assignment Procedure

The university administration simplifies the management of a large amount of foreign students by splitting them into groups of around 13 people on average[5]. In what follows, I refer to resulting groups as 'academic groups' (AG) and to students belonging to the same AG as 'group mates'. All group mates within an AG are Indians and take their classes and lectures together. Typically, a class is taught to not more than two AG's at the same time. Therefore, it is not unreasonable to expect group mates to impose various peer effect-type externalities onto each other.

The AG's are assigned either before or shortly after the students arrive in Karaganda. The assignment is done in the following way. First, group leaders are selected out of students who volunteer for such a role. The leader is in charge of maintaining in-class discipline, relaying complaints and wishes to the university administration and disseminating information among his group mates. After leaders are selected, their AG's are randomly drawn from the pool of all remaining students in their cohort. The administration tries to keep AG sizes as equal as possible, while keeping them down to 15 people or less. This way, with 200 new students there will be 10 AG's of 14 people and 4 AG's of 15 people. Such strategy, together with modest early attrition guarantees random variation in AG sizes. Therefore, the assignment process generates exogenous variation in both composition and size of the peer group.

Being in the same AG is a strong predictor of a friendship between two migrants. Of 3,606 reported friendships, 1,127 are between group mates. Figure 2.5 showcases this fact by highlighting all such ties within the migrant network. I exploit the exogenous variation in the friendship formation process in order to identify endogenous peer effects in assimilation.

### 2.4.2 Balance Checks

In order to proceed with my identification strategy I first provide evidence that the the AGs are, indeed, assigned exogenously. In my subsequent analysis by 'peer effect' I always mean

---

[5]Due to changing cohort sizes, the average size of a group was 14.2 in 2015, 13.5 in 2014, 12.83 in 2013, 9.58 in 2012, and 7.36 in 2011.
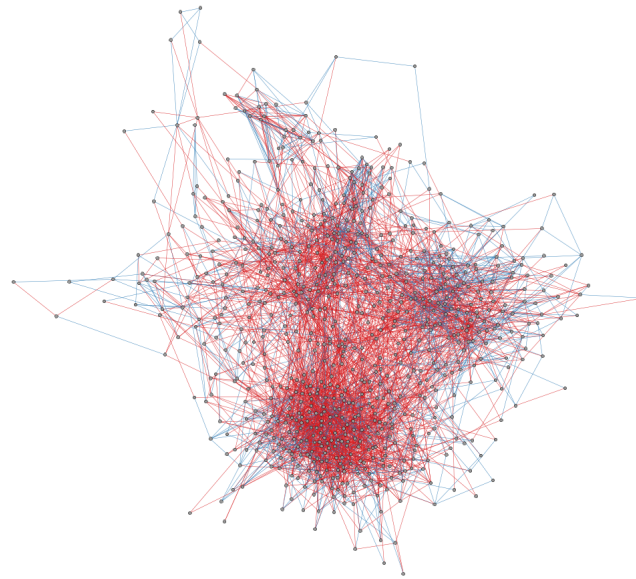
**Figure 2.5:** Group affiliation within the friendship network. All edges represent friendship nominations. The friendship links between AG mates are colored in blue.

the positive effect on migrant's assimilation of average assimilation of her peers. This means that for every migrant I construct 'leave-out-self' means of assimilation outcomes across the relevant peer group. Regressing the outcome on its leave-out-self group mean is a popular randomization check in the peer effects literature. As noted by Caeyers and Fafchamps (2016), among others, the leave-out-self nature of constructed group averages results in the so called "exlusion bias." The OLS estimator in a regression of a pre-university characteristic of a student on her AG's average of that characteristic is downward biased. For example, the regression of BSER score of a migrant on her group mates' average score results in an estimate of -.37 with a standard error of .14. This mechanical bias arises because excluding a high-BSER student from the computation of her AG mean is going to bring the mean down, while excluding a low-BSER student is going to push the mean up. The bias has potential to be particularly strong in my data, because group sizes are relatively small, and students tend to be quite diverse in their predetermined outcomes. Indeed, many other personal characteristics which are predetermined at the time of AG assignment appear to be strongly negatively correlated with the leave-out-self group means in OLS regressions. Therefore, this is not a suitable benchmark in my setting.

Another type of randomization check that is sometimes carried out in such circumstances is to regress the predetermined outcome on AG dummies (Chetty et al. (2011)). I believe that such a check is not suitable to my data either for three reasons. First, the sample size is

rather small. There are only 60 groups, and many variables, such as gender or religion, have little variation. Therefore, randomization would *appear* to be compromised in a (likely) event that several females or several Muslims are by chance assigned to the same group. Second, this method basically compares means across AG's. However, imagine a continuous variable, like height. It might be that in one group all people are 175 centimeters tall, whereas in a different group half is 150 and the other half is 200 centimeters tall. In such case, there is clear evidence of sorting but a regression of height on group dummies would not pick up on that. Third, it needs to be taken into account that certain group combinations are not possible. Two AG leaders can not possibly be assigned to the same group, nor can people from different cohorts.

For these reasons, I carry out the following randomization check. Since there are 757 migrants in my sample, there are 286,146 potential unique friendship ties (links) between them. I convert all of my data into link-level variables. This way, I obtain a data set with 286,146 observations, where all variables are characteristics of *potential links*, such as castes and ages of *both* migrants on either side of that link. For each link I also create a dummy variable, indicating whether the two migrants belong to the same AG or not. A test of balancedness of AG assignment, therefore, which reduces the complications mentioned above, is to regress the indicator for belonging to the same AG on predetermined link level outcomes, such as absolute difference in age and BSER scores, or caste and religion match between the two migrants. Under the null hypothesis of exogenous assignment, such a regression should only yield insignificant coefficient estimates.

In my data set there are 14 variables which can be viewed as predetermined as of the start of the program at KSMU. Combined, they should fail to predict a group match between two students. Table 2.2 contains the results of such balance tests separately for each cohort. In order to have a truly random baseline and to capture the natural tendency of certain regressors to produce spurious correlations, I pair the real regression output for each cohort (columns (1), (3), (5), (7) and (9)) with the output that arises when AG's are reshuffled in a manner that reproduces original assignment. Specifically and separately for each cohort, I keep group leaders in their places, and populate the groups around them with random students by sampling without replacements from the same cohort. I then regress the indicator for whether two students are 'matched' into the same reshuffled AG onto the predetermined attributes of each reshuffled 'link'. Those estimates are presented in columns (2), (4), (6), (8), and (10). Additionally, column (11) lists the output from the regression of the nominated friendship indicator on link level characteristics. Such regression picks out all of the personal characteristics which are important in friendship formation. This is of some interest, because the way friendship networks form between migrants could in itself

CEU eTD Collection

| Dep. Variable: indicator for whether i and j are: | Cohort 2015 | | Cohort 2014 | | Group Mates Cohort 2013 | | Cohort 2012 | | Cohort 2011 | | Friends All Cohorts |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Real (1) | Shuffled (2) | Real (3) | Shuffled (4) | Real (5) | Shuffled (6) | Real (7) | Shuffled (8) | Real (9) | Shuffled (10) | Real (11) |
| Caste match | .001 (.004) | .001 (.004) | .01 (.007) | .004 (.007) | .012 (.008) | .07 (.008) | .019 (.02) | .009 (.02) | .058 (.09) | -.07 (.09) | .006*** (.0008) |
| City match | .009 (.007) | -.002 (.007) | .007 (.007) | -.002 (.007) | -.0007 (.01) | -.07 (.01) | -.008 (.02) | -.01 (.02) | -.04 (.18) | -.15 (.17) | .004*** (.001) |
| State match | -.0003 (.004) | .006* (.007) | -.04 (.007) | -.002 (.007) | -.0007 (.01) | -.007 (.01) | -.008 (.02) | -.01 (.02) | -.04 (.18) | -.1 (.17) | .0008 (.001) |
| Age difference | -.0003 (.0007) | .0004 (.0007) | -.0009 (.0011) | -.0009 (.001) | -.0001 (.001) | .0005 (.0009) | -.001 (.004) | .002 (.004) | -.01 (.05) | .03 (.04) | -.002*** (.0001) |
| Gender match | .005* (.003) | .03 (.003) | .0066 (.0041) | .0066* (.0038) | .01 (.004) | .03 (.004) | .055*** (.011) | .018* (.011) | .003 (.08) | .06 (.09) | .004*** (.0005) |
| BSER difference | .0001 (.0001) | .0003 (.0002) | .0001 (.0002) | -.0005 (.0002) | .0001 (.0002) | -.0001 (.0002) | .0004 (.0006) | .0008 (.0006) | .001 (.004) | -.002 (.004) | .00003 (.00003) |
| Religion match | .0008 (.003) | .04 (.003) | .0004 (.004) | -.05 (.004) | .04 (.004) | .05 (.004) | .013 (.011) | .01 (.01) | -.07 (.07) | -.05 (.04) | .03*** (.0005) |
| Lived abroad match | .03 (.004) | -.005 (.004) | -.06 (.007) | -.003 (.007) | -.0005 (.005) | -.06 (.005) | .007 (.02) | .03 (.02) | - | - | -.001 (.0007) |
| Height difference | -.00016 (.00012) | -.0001 (.0001) | .00004 (.002) | -.0003 (.0002) | -.0001 (.0002) | .0007 (.0002) | -.0002 (.0007) | -.0005 (.007) | .006 (.004) | -.006 (.005) | -.0002 (.00002) |
| Scholarship match | -.00002 (.002) | -.002 (.003) | -.003 (.0041) | -.004 (.004) | .01 (.004) | -.003 (.004) | .005 (.01) | .004 (.015) | - | -.02 (.14) | -.001** (.0004) |
| #Languages diff. | -.0018* (.0011) | -.0012 (.001) | -.0018 (.0011) | -.0005 (.001) | -.0009 (.001) | -.0008 (.001) | -.007 (.005) | .003 (.005) | .007 (.03) | -.008 (.03) | -.0004** (.0002) |
| #Friends in India diff. | -.00005 (.00008) | .0002 (.0001) | -.0005 (.0005) | -.00003 (.00005) | 0 (.00002) | -.0003* (.00002) | -.0004 (.001) | .0005 (.001) | .01 (.03) | .025 (.24) | 0 (.00003) |
| Rus. pre KSMU match | -.001 (.002) | -.001 (.002) | -.003 (.004) | -.05 (.004) | -.004 (.004) | .0006 (.004) | -.006 (.01) | -.016 (.01) | -.05 (.06) | .027 (.062) | .0008** (.0004) |
| New culture important match | .002 (.004) | .009** (.0037) | .002 (.005) | -.06 (.005) | -.001 (.006) | -.07 (.006) | -.018 (.014) | -.04 (.014) | .01 (.03) | -.2*** (.07) | .005 (.006) |
| F-test's p-value | .4094 | .1154 | .2701 | .2593 | .9423 | .6788 | .0005 | .4984 | .9987 | .1435 | .0000 |
| $R^2$ | 0.00005 | 0.0006 | 0.0008 | 0.0008 | 0.0004 | 0.0006 | 0.0105 | 0.0034 | 0.0106 | 0.0804 | 0.0016 |
| N | 32232 | 32232 | 20196 | 20196 | 18231 | 18231 | 3619 | 3619 | 228 | 228 | 286146 |

**Table 2.2:** Balance tests.

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. All columns contain linear probability models. F-test for joint significance of pre-university variables. The sample size equals all possible pairings of 2 people in the relevant cohort. Two variables are omitted for 2011 cohort due to absence of variation.

67

have assimilation implications.

Overall, the data seem to pass the randomization test. In all cohorts but 2012 the F-test for joint significance comes nowhere near rejecting the null of no relationship. Moreover, the coefficients produced by real data are very similar in size, sign and statistical significance to the ones in reshuffled data. The one coefficient that falls out of the general pattern is the coefficient on gender match in cohort of 2012. There is clear evidence of sorting on gender among groups in that cohort. Indeed, according to the dean of foreign students, in that year some female students have explicitly stated a preference for a predominantly female group. As a result, 12 out of 18 females in that cohort were placed in only two groups, with the rest of the assignment protocol proceeding as usual. Such sorting is potentially problematic for my identification strategy, particularly if gender is strongly correlated with the underlying assimilation type. However, even for that cohort the data seem to be properly balanced on every other dimension. The exogenous variation, therefore, can be separated even for the 2012 cohort by controlling for gender composition of the peer group. I my subsequent analysis I always control for the gender composition, however the results remain qualitatively unchanged even if I exclude the 'female' AG's or the entire 2012 cohort, and are available upon request.

## 2.5   Peer Effects in Assimilation

Social scientists have long recognized that living in an ethnic cluster creates interaction patterns among migrants that are crucial in determining their eventual socioeconomic success and integration into the local society. Borjas (1995), for instance, has coined the term 'ethnic capital' to capture the idea that socioeconomic performances within an ethnic group are persistent across generations. More generally, the theory of ethnic neighborhoods has arisen (Edin et al. (2003)) in order to formally analyze the effects of ethnic clustering on migrants and minorities. Most of that the literature focuses on the role that networks play in the labor markets and finds a positive relationship between either the size or the density of the network and economic outcomes. Specifically, Calvo-Armengol and Jackson (2004) show in a theoretical model that job information transmission along social networks can reduce unemployment, more so the denser the networks. Notable empirical studies that establish positive causal effect of the size of the network of co-nationals on labor market outcomes include Munshi (2003) for U.S., Edin et al. (2003) for Sweden, and Damm (2009a) for Denmark. Potential mechanisms for positive effects include reduction in information asymmetries between firms and workers, as well as creation of jobs specific to the migrant community. On the other hand, Battisti et al. (2016) demonstrate that refugees in Germany

who are allocated to a larger cluster of co-nationals are initially more likely to be employed, but suffer from lower wages in the long run as they are less likely to invest in human capital. Beaman (2012) shows in a dynamic model that larger migrant networks can both reduce unemployment through information transmission and increase it through larger competition for jobs. She verifies these predictions using exogenous resettlement patterns of refugees in the United States.

Conventionally, economists define assimilation as the catch up in earnings and socioeconomic status of migrants compared to the natives (Borjas (1998), Chiswick et al. (1997)). However, there is a small literature focusing on migrants' assimilation in terms of identity, cultural and social norms. Baker and Benjamin (1997), Meng and Gregory (2005) focus on economic outcomes of assimilation through intermarriage. Specifically, Meng and Gregory use Australian data to show that having a local spouse results in an earning premium for migrants. Lundborg (2013) shows using data on refugees in Sweden that culturally more distant refugees from the Middle East and Africa spend more time in unemployment than ones from Europe and Latin America. Constant et al. (2013) exploit exogenous variation in migrant placements in Germany to show that ethnic clustering strengthens the newcomers' retention of an affiliation with the country of origin and weakens identification with the German society. Finally, Abramitzky et al. (2016) very broadly study cultural assimilation in historical context during the age of mass migration into the U.S. They document gradual assimilation pattern for migrant families, including intermarrying, higher literacy status and name changing. They show that within households brothers with more foreign sounding names were less likely to obtain education, had lower earnings, and faced higher unemployment.

The literature on language acquisition among migrants has followed two different trajectories. First, Chiswick and Miller (1995), Dustmann and Fabri (2003), among others, show that migrants' labor market success is more probable if their local language ability is high. Second, multiple studies have connected ethnic neighborhood size with poor linguistic ability. Lazear (1999) shows in a model of migrant assimilation that people who settle in a bigger ethnic cluster are less likely to learn the local language. Chiswick and Miller (2002) use U.S. Census data on adult male immigrants from non-English speaking countries to demonstrate that ethnic enclaves reduce an immigrant's own English language skills. Bauer et al. (2005) argue that the causation works in the opposite direction, and migrants with better local language ability choose to settle in places with fewer co-nationals. There is less known about the process through which migrants obtain local friends and their usefulness in terms of economic outcomes. However, local social capital must be important. The International Rescue Committee, for example, assigns local volunteer 'mentors' to all freshly

resettled refugees. Adda et al. (2014) present some evidence that social capital is important for immigrants' wages in Germany, while Mui Teng et al. (2015) descriptively suggest that for foreign workers in Singapore having a local-born friend leads to superior labor market outcomes. The research on importance of intermarriage also suggests that having close ties with the locals is beneficial for the immigrant. Danzer and Yaman (2013) additionally argue using evidence from Germany that living in a larger ethnic cluster increases the migrants' costs of interacting with locals, making them less likely to report having a local friend.

I this section I postulate existence of positive peer effects in language learning and build up of local social capital as another mechanism through which migrants can influence each others' assimilation outcomes. I first document existence of such peer effects, i.e. causal effect on migrant's assimilation outcome of the average outcomes of her friends. I then discuss differences in implications, depending on whether the peer effects are of reinforcing or conforming nature and make an attempt the tell the them apart in my data.

### 2.5.1    Identification

Causal identification of endogenous peer effects is complicated by a host of issues, summarized in Jackson (2013) and Angrist (2014). Suppose, there are $N$ migrants, $Y$ is the outcome of interest, and $X$ is an $N \times K$ matrix of exogenous variables. All network interactions are stored in a row-normalized $N \times N$ adjacency matrix $G$ such that $G_{ij} = \frac{1}{\#friends\ of\ i}$ if $i$ and $j$ are linked in the network and 0 otherwise. In such case $GX$ is an $N \times K$ matrix whose $nk$'the entry is the leave-out-self average of exogenous variable $k$ across migrant $i$'s network neighbors. Consequently, identifying peer effects implies causally identifying the coefficients in the following regression.

$$Y = \alpha + X\beta + GX\delta + \gamma GY + \varepsilon \tag{2.1}$$

where $\delta$ represent *exogenous* peer effects, i.e. effects of friends' characteristics; and $\gamma$ represents *endogenous* effects, i.e. true causal relationship between an outcome of one person and that of her peers. Most of this paper is devoted to identifying the endogenous peer effects. Below, I briefly list the main threats to identification and describe my method of dealing with them.

*Reflection Problem.* Manski (1993) in a seminal paper points out that OLS regressions of person's outcome on the mean outcome of her group are meaningless for 2 main reasons. First, if there are endogenous peer effects, then simultaneity of making decisions means that $Y$ and $GY$ interact in an infinite positive feedback loop, resulting in a mechanical correlation and a positive bias. Second, solving equation 2.1 for $Y$, pre-multiplying by $G$, and taking

expectation with respect to $X$ yields $E[GY|X] = \sum_{k=0}^{\infty} \gamma^k G^{k+1}(\alpha + \beta X + \delta GX)$. So, the presence of peer effect implies that conditional expectation of $GY$ is just a linear combination of the elements of power series $X$, $GX$, $G^2X$, etc. Consequently, an OLS regression can not tell exogenous and endogenous effect apart. I deal with these problems by using a variation of Bramoulle et al. (2009) method of instrumenting $GY$ with $G^2X$ and $G^3X$, *i.e.* characteristics of 2nd and 3rd degree friends.

*Endogenous networks.* The network captured by $G$ is not formed randomly. Typically, real life social networks arise through assortative matching on characteristics, resulting in homophily (Currarini et al. (2009)). Suppose, each migrant has a specific level of assimilation ability (proclivity to learn languages, create new friendships, etc) $\theta_i$ which is unobserved and, therefore, included in the error term. If migrants sort on assimilation ability, then $G \not\perp \theta$. Therefore, $Cov(GY, \varepsilon) \neq 0$, leading to inconsistent (most likely upward biased) OLS estimates of $\gamma$. In much of research on peer effects (Sacerdote (2001), Zimmerman (2003), Thiemann (2016)) the authors either ignore this problem or make ad-hoc assumptions about non-existence of endogenous effects and estimate exogenous effects only using randomized group assignment in some form. The advantage of my data is that I observe both the network data and the assigned AG's, which allow for exogenous variation in friendship creation, thus solving the endogenous networks problem.

*Correlated shocks.* Migrants who spend a lot of time together are likely to be exposed to similar shocks which might be driving the positive correlation between $Y$ and $GY$, leading to inconsistency and a likely upward bias of the OLS estimator of $\gamma$. For example, friends might take a language course together or be subject to racial abuse. I take care of this problem through a combination of clustering standard errors at AG level, including teacher and street of residence fixed effects (most probable sources of correlated shocks), and de-meaning the data through subtraction of leave-out-self AG averages.

*Measurement error and misspecification of peer group.* Manski (1993) and Jackson (2013) highlight that proper specification of reference peer group is an important and rarely-addressed issue with identification of peer effects. From equation 2.1 it is clear that a systematic misspecification of the adjacency matrix $G$ can lead to inconsistency of OLS. For several reasons, such concern should be minimal in my setting. First of all, the combination of lack of other migrants in Karaganda and lack of Indian arrivals prior to 2011 means that I can credibly claim to observe the entire pool assimilation peers. No important influencers should be omitted. Second, the fact that I also observe AG's helps me verify that survey respondents are revealing their friends truthfully. Over 31% of reported friendship ties are between group mates. Third, at the time of the data collection, students were asked to nominate their Indian friends from the same well-specified drop-down menu assigning a unique

identifier to each person. Therefore, there were neither errors in matching chosen names to real people, nor errors due to imperfect recall of names by respondents. Fourth, survey respondents theoretically could pick an unlimited amount of friends, so no important ties should be omitted. Fifth, the data also include self-reported friendship intensities (measured in the average amount of hours spent together in a typical week). I re do all of the subsequent analysis by weighting each friendship inversely in proportion with reported intensities. Such adjustment does not qualitatively change the results.

**Baseline Estimates and Exogenous Effects**

I begin by presenting OLS estimates of equation 2.1, using both reported friendship network and AG's in place of $G$. Note that both of the assimilation outcomes of interest are discrete variables with limited support. The empirical strategies in these paper largely ignore this issue. Given the amount of endogeneity concerns involved in peer effect estimation, such approach is convenient and taken in most prominent peer effect studies (Sacerdote (2001), Zimmerman (2003), Gaviria and Raphael (2001)). Throughout this paper I cluster standard errors at AG level, which also takes care of heteroscedasticity inherent to models with imperfectly continuous dependent variables. The estimates, presented in the first row of Table 2.3, have no causal interpretation. Nevertheless, studying the correlations is useful in order to establish a baseline.

The table lists and contrasts the results when using first the nominated friendship network and then the AG's as the definition of migrants' social space. In both cases, the raw correlations between migrant's own outcomes and the mean outcomes of her peers are large and statistically significant. However, when a full set of controls, as well as dummies for most likely sources of correlated effects (Russian language teacher, cohort, and street of residence) are included, the correlations become much smaller. They disappear entirely when AG's are used as the peer group of reference. This lack of correlation seems to indicate that endogenous effects must operate at the level of network and not AG's. So, group mates who are not close friends do not influence one's assimilation. Alternatively, it could be explained by a combination of the exclusion bias discussed earlier and the fact that with only 60 groups and so many regressors fixed at the group level there is simply not enough variation left for precise estimation. The full set of controls and fixed effects seem to explain away much of the correlation between migrant and her peers in language ability, but not in the number of local friends.

Curiously, observed pre-university ability, measured by BSER and being a scholarship recipient do not seem to be correlated with either language or local friends acquisition. However, having lived abroad and speaking multiple languages before attending KSMU

| Dep. Variable | G: Nominated Friendships | | | | | | G: AG ties | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Russian Command | | | #Local Friends | | | Russian Command | | | #Local Friends | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| $GY$ | .69*** | .48*** | .19** | .72*** | .65*** | .44*** | .74*** | .51*** | -.14 | .67*** | .54*** | .174 |
| | (.085) | (.09) | (.09) | (.09) | (.09) | (.11) | (.063) | (.086) | (.15) | (.09) | (.08) | (.14) |
| Backward cast | | -2.32* | -.02 | | .19 | .19 | | -.96 | -.16 | | .15 | .16 |
| | | (1.33) | (1.36) | | (.17) | (.16) | | (1.35) | (1.31) | | (0.2) | (.18) |
| Female | | 1.46 (2.22) | .66 | | -.46 | -.46 | | -.82 | -.13 | | -.6** | -.6** |
| | | | (2.04) | | (.33) | (.33) | | (2.02) | (1.94) | | (.28) | (.27) |
| Muslim | | -5.99*** | -6*** | | .34 (.3) | .34 | | -3.7*** | -4*** | | .55* | .36 |
| | | (1.83) | (1.94) | | | (.32) | | (1.81) | (1.82) | | (.29) | (.28) |
| Age at year 1 | | -.28 (.34) | -.34 | | .09* | .1* | | -.118 | -.27 | | .11* | .1* |
| | | | (.35) | | (.05) | (.05) | | (.36) | (.37) | | (.06) | (.05) |
| Lived abroad before | | 7.89*** | 7.24** | | -.33 | -.33 | | 6.96*** | 7.6*** | | -.31 | -.45 |
| | | (2.95) | (2.79) | | (.37) | (.37) | | (2.56) | (2.6) | | (0.42) | (.39) |
| BSER | | .008 (.069) | -.04 | | -.007 | -.007 | | .009 | -.006 | | -.004 | -.007 |
| | | | (.07) | | (.009) | (.009) | | (.07) | (.07) | | (0.009) | (.009) |
| From Rajasthan | | -6.12*** | -5.7** | | -.014 | -.014 | | -7*** | -6*** | | .06 | -.047 |
| | | (2.26) | (2.15) | | (.27) | (.27) | | (2.3) | (2.16) | | (0.28) | (.29) |
| Hours study Russian | | 1.09*** | 1.06*** | | | | | 1.13*** | 1.13*** | | | |
| | | (.13) | (.14) | | | | | (.14) | (.15) | | | |
| Was racially abused in Kaz. | | 2.41 (1.47) | 1.24 | | -.36* | -.36* | | .92 | 1.04 | | -.4 (.2) | -.37* |
| | | | (1.27) | | (.21) | (.21) | | (1.44) | (1.33) | | | (.2) |
| Lives in dorms | | 1.843 (1.37) | 4.65** | | .2 (.21) | .2 (.21) | | 3.57*** | 4.41** | | .19 | .13 (.2) |
| | | | (1.85) | | | | | (1.27) | (1.88) | | (.16) | |
| Receives scholarship | | -2.12 (1.78) | .34 | | .2 (.21) | .28 | | -.07 | .48 | | .29 | .24 |
| | | | (1.88) | | | (.31) | | (1.95) | (1.97) | | (.32) | (.32) |
| # languages spoken | | 2.95*** | 2.9*** | | -.03 | -.027 | | 3.01*** | 2.85*** | | -.003 | 0 (0) |
| | | (.767) | (.72) | | (.08) | (.082) | | (.83) | (.75) | | (.07) | |
| $GX$ | | Yes | Yes | | Yes | Yes | | Yes | Yes | | Yes | Yes |
| Cohort Effects | | | Yes | | | Yes | | | Yes | | | Yes |
| Teacher Effects | | | Yes | | | Yes | | | Yes | | | Yes |
| Street Effects | | | Yes | | | Yes | | | Yes | | | Yes |
| $R^2$ | 0.13 | 0.32 | 0.42 | 0.12 | 0.15 | 0.22 | 0.15 | 0.33 | 0.42 | 0.1 | 0.14 | 0.2 |
| $N = 757$ | | | | | | | | | | | | |

**Table 2.3:** OLS estimates of endogenous peer effect $\gamma$.

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors clustered at AG level in are in parentheses. Regressions control for answers to a wide array of qualitative survey questions, such as whether the migrant tries hard to find new local friends or finds that learning Russian is important.

| Dep. Variable | (1) Russian Command | (2) #Local Friends | (3) Hours Study Russian | (4) Stay After KSMU | (5) Hours Local Media |
|---|---|---|---|---|---|
| *AG Average of*: | | | | | |
| Had Russian course pre KSMU | 10.4** (4.76) | -.697 (.899) | 4.616** (2.274) | -1.146 (1.355) | -3.885** (1.875) |
| Backward Cast | 5.51* (3) | -.455 (.853) | 1.007 (1.787) | .115 (.377) | -2.428* (1.265) |
| BSER | .11(.23) | -.014 (.042) | .074 (.111) | -.016 (.026) | -.028 (.074) |
| Muslim | 5.48 (4.86) | -1.295 (1.241) | 6.771** (2.989) | .775 (.918) | -1.263 (1.671) |
| From Rajasthan | 5.36 (6.5) | -2.505** (1.071) | .326 (2.722) | 2.423*** (.822) | -1.935 (2.176) |
| Female | 1 (4) | -.764 (.794) | -2.422 (1.938) | .337 (.71) | .629 (1.053) |
| # Languages spoken | -.53 (1.55) | -.354 (.291) | -.38 (5.999) | .045 (.145) | .422 (.455) |
| # Siblings | .11 (.94) | .299 (.201) | -.792 (.481) | .021 (.108) | .527* (.28) |
| # Close friends in India | .09*** (.03) | .008 (.008) | .022 (.019) | -.016 (.013) | -.036 (.013) |
| Lived abroad before | 2.04 (7.49) | -1.721 (1.576) | -7.263* (4.115) | .636 (1.332) | 1.584 (2.655) |
| Age at year 1 | 1.94* (1.12) | -.191 (.215) | -.301 (.499) | -.32* (1.888) | -.402 (.396) |
| $R^2$ $N = 757$ | 0.5727 | 0.2137 | 0.1161 | 0.1799 | 0.1385 |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors in parentheses are clustered at AG level. All regressions include background controls, as well cohort, teacher and street fixed effects.

**Table 2.4:** OLS estimates of exogneous peer effects.

correlate with ability to pick up Russian. Muslims perform more poorly on the language test but report having more local friends. Perhaps, the latter fact is due to Islam being the most popular religion in Kazakhstan (although not in Karaganda itself), reducing the cultural barriers between Indian Muslims and the locals. This finding is in line with results in the literature that culturally less distant ethnicities assimilate better in terms of economic outcomes (Damm (2009a), Lundborg (2013)). Students who report having been racially abused in Karaganda also report fewer local friends, as do female students. Dorm residents are better Russian speakers, perhaps, because they are by definition exposed to local students and staff outside of classrooms. Migrants from Rajasthan (almost 90% of the sample) have poor language ability but report the same amount of local friends on average. Those who move to Karaganda at an older age, report more local friends.

Because AG's are assigned by chance, the group averages of pre-university outcomes are completely exogenous to migrants' outcomes in Kazakhstan. Therefore, given the empirical framework, if $G$ represents AG ties, the OLS estimator of the $\delta$ coefficient in the following regression can be be given causal interpretation (Sacerdote (2001)):

$$Y = \alpha + Pre\ KSMU\ Outcomes\ \beta + G * Pre\ KSMU\ Outcomes\ \delta + \psi_{cohort} + \phi_{teacher} + \eta_{street} + \varepsilon \quad (2.2)$$

Table 2.4 lists the estimates of the $\delta$ vector in an OLS regression of the two assimilation outcomes of choice, as well as reported average weekly hours spent studying Russian, reported number of years after graduation that the student would like to spend in Kazakhstan, and reported average weekly hours spent consuming local media. Not many group averages are significant in any of the regressions. Lack of statistical significance could be a sign absence of exogenous effects or indication that exogenous effects do not operate at the group level. Alternatively, it could be the results of the exclusion bias or the fact that with only 60 AG's exogenous assignment implies lack of power to identify the coefficients on variables that are close to fixed at the AG level.

The most important result in Table 2.4 is that migrants who were assigned to a group with higher fraction of peers with pre-KSMU Russian language courses exhibit higher Russian ability. The coefficient is economically large. A person whose entire group has taken a Russian course is predicted to have half a standard deviation higher test score than a person whose group has no course-takers. Having taken Russian before coming to KSMU is a strong predictor of migrant's own linguistic ability. Therefore, the causal nature of this result suggests that there exist positive peer effects in language acquisition among migrants. However, the channel through which the effect operate is not obvious. If having taken a Russian course before KSMU is just a signal of better innate ability to learn Russian, then

75

the effect is truly exogenous, i.e. the effect of having better peers. If, on the other hand, having taken a course in Russian signals higher *willingness* to learn (and, thus, effort put into learning) the language, then the effect is more likely endogenous in nature, i.e. migrants directly help each other learn Russian. In fact, column (3) shows that being assigned to an AG with 100% of Russian course takers increases one's average weekly hours devoted to studying Russian by 2/3 of a standard deviation, compared to being assigned to an AG with 0%. Therefore, it seems likely that the estimated effect is driven by migrants increasing their efforts to learn the language in response to an effort increase by their peers, which is more of an endogenous effects mechanism.

Of the background characteristics, surprisingly, average gender, prior linguistic knowledge and overall ability (measured by BSER) of the group appear to have no effects on assimilation. Higher fraction of Muslims in one's AG causes one to devote more hours to studying Russian, which seems surprising, since being a Muslim is negatively correlated with one's own Russian ability. Having older-than-average peers causes one to speak better Russian. Being assigned to a group with higher fraction of people who have lived abroad before reduces one's hours of Russian study. Presumably, the reduction occurs because more 'experienced' people themselves need to exert much lower efforts to learn the language and, thus, do not impose a positive externatlity on group mates. Finally, a higher fraction of peers from Rajasthan causes migrants to have both fewer local friends and greater desire to stay in Kazakhstan after graduation. These two results seem inconsistent. However, a higher fraction of people from the most prominently-featured state might imply a tighter within-group community, leading to both lower amount of local friends and perceived higher utility of stay in Kazakhstan.

**Endogenous Effects: Identification Strategy 1**

In this section I present the first of my two empirical strategies aimed at identifying the endogenous peer effects coefficient $\gamma$ in the following model of assimilation outcome $Y$:

$$Y_i = \alpha + \gamma \frac{1}{|N_i|} \sum_{j:G_{ij}>0} G_{ij}Y_j + X_i'\beta + \frac{1}{|N_i|} \sum_{k,\,j:G_{ij}>0} \delta_k G_{ij} X_{kj}'\delta + \psi_{cohort} + \phi_{teacher} + \eta_{street} + \varepsilon_i \quad (2.3)$$

where $|N_i|$ is the degree of migrant $i$ in the friendship network, and $X$ is a $N \times K$ matrix of exogenous variables. The identification strategy rests on the randomized nature of AG's. Pre-university characteristics are not affected by either subsequent assimilation or exposure to KSMU peers. Therefore AG averages of such characteristics may exogenous to assimilation decision and make good candidates for instruments of network averages of assimilation

outcomes.

I propose two candidate instruments, one for each assimilation outcome of interest. The fraction of student's AG mates who have taken a Russian course before coming to KSMU is a natural candidate instrument for the average Russian ability of her friends. Similarly, the AG average of reported importance of learning a new culture is a candidate instrument for the average number of local friends among a migrant's Indian friends. Define the adjacency matrix containing all the AG ties by $\tilde{G}$. Then, two conditions need to be satisfied for an AG average of a pre-university characteristic $X$ to be a valid instrument of friend's average outcome $GY$. The first condition is for $\tilde{G}X$ to be partially correlated with $GY$ after controlling for the observables. In both cases the proposed instrument is, indeed, strongly correlated with the friends' average assimilation outcomes, even with the full set of controls.

The second condition is for the exclusion restrictions to be satisfied. In this context the restrictions imply that $\tilde{G}X$ must only affect $GY$ through friends' average assimilation, i.e. through the endogenous peer effect channel. Broadly, there are two reasons why the restrictions may be violated. The first reasons is that the AGs are so important to the academic and social lives of the students that group-level characteristics may have a direct impact on assimilation. For example, it is conceivable that group average interest in foreign cultures is a proxy for AGs average unobserved ability which could have a direct effect on one's number of local friends beyond its influence on fellow Indians' behavior. Another possible violation would occur if group-level characteristics drove unobserved correlation shocks. For instance, groups with larger fraction of Russian course takers could attract greater attention and enthusiasm from language teachers. Such relationships could introduce a bias into IV estimates. The second reason is that due to the overlap between the sets of friends and the sets of groupmates, group covariates are likely to be correlated with friends mean friends covariates. For example, there turns out to be a positive correlation between the proportion of females among one's Indian friends and the fraction of Russian course takers among one's groupmates. Friends' mean covariates are also endogenous in the peer effect regressions, so the correlation may also introduce a bias into the IV estimation.

These important endogeneity concerns mean that the strategy can not claim to identify the endogenous effect of nominated friends. Instead, the $\gamma$ estimate is likely to be mash-up of endogenous and contextual effects coming from both group mates and nominated friends. I nevertheless present the strategy because a positive $\gamma$ estimate implies that peers do a have some form positive influence on each other. Table 2.5 contains the two stage least squares estimates. The estimates in the top row are large and statistically significant. When the full set of controls is included (Column (2)), the result suggests that a student's Russian test score goes up by 0.66 points (about 3% of a standard deviation) as a result of a 1 point rise

| Dep. Variable $Y$ | Russian Command | | # Local Friends | |
| --- | --- | --- | --- | --- |
| | (1) | (2) | (3) | (4) |
| $GY$ | .344 (.449) | .664* (.376) | 1.912*** (.265) | .332* (.178) |
| # Siblings | | -.293 (.245) | | .117** (.047) |
| # Languages Spoken | | 3.082*** (.499) | | -.151* (.08) |
| Age at year 1 | | -.284 (.32) | | 149*** (.047) |
| Lived abroad before | | 1.06 (2.233) | | -.219 .378 |
| BSER | | -.051 (.05) | | -.005 (.007) |
| Female | | 1.668 (1.732) | | -.513* (.29) |
| Muslim | | -6.236*** (1.547) | | .104 (.331) |
| Backward Cast | | 1.307 (0.91) | | .182 (.155) |
| From Rajasthan | | -2.825 (2.146) | | .021 (.369) |
| Group Leader | | 3.092* (1.853) | | -.044 (.283) |
| Lives in dorms | | 4.46** (1.781) | | .177 (.196) |
| Was racially abused | | 1.494 (1.03) | | -.313 (.218) |
| Rus. pre KSMU | | 14.312*** (.996) | | -.049 (.187) |
| Russian Command | | | | .013** (0.006) |
| # Local Friends | | .57*** (.201) | | |
| 1st stage p-val. for $H_0 : \beta_{instr} = 0$ | .000 | .001 | .000 | .000 |
| $R^2$ | 0.3904 | 0.6128 | 0.1161 | 0.3462 |
| $N = 757$ | | | | |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors in parentheses clustered at AG level. All regressions include cohort, teacher and street of residence effects. Regressions (2) and (4) additionally control for answers to a wide array of qualitative survey questions, such as whether the migrant tries hard to find new local friends or finds that learning Russian is important. Network averages of all controls are also included in columns (2) and (4).

**Table 2.5:** Endogenous peer effect estimates using identification strategy 1.

in average scores among her network friends. Column (4) shows that a student's number of local friends goes up by about 12% of a standard deviation if her Indian friends have 1 local friend more an average. Assuming that the exlusion restrictions are satisfied, these numbers can be viewed as evidence for existence of endogenous peer effects.

It seems peculiar that the estimates in Columns (1) and (3) of Table 2.5, which only include cohort, street, and teacher effects, are very different from Columns (2) and (4) which additionally include the full set of controls. The estimates in Columns (1) and (3) are likely biased due to the correlation between the instruments and the network averages of exogenous variables (the exogenous peer effects), which are not included in these regressions. After the exogenous variables and their networks averages are included, the bias disappears.

Note that the OLS versions of regressions in columns (2) and (4) with the same set of controls result in smaller estimates (Table 2.6). The OLS estimate of the endogenous effect in Russian command is .219 with a standard error .072, wheres the corresponding estimate for the number of local friends is .275 with a standard error of .11. Neither of the differences is statistically significant at 5%, but it is somewhat unusual for the the OLS to be downward biased in peer effects estimation, given that network endogeneity, correlated shocks and the reflection problem are all likely to be driving it upwards. There are a few reasons why a downward bias of OLS is plausible in my setting. First, the exclusion bias is likely to be quantitatively large. The reason for this is that there are a lot of friendships between students from different cohorts. Each year spent in Karaganda adds a lot to students' assimilation. Therefore, the leave-out-self mean level of assimilation is likely to be comparatively low for (a well assimilated) upperclassman and comparatively high for (poorly assimilated) freshman. Second, given a relatively small pool of potential friends to chose from, I do not think that migrants in my sample sort into friendships based mainly on an unobserved assimilation type. Results from section 4 show that friendship formation is mostly driven by AG assignment, caste, and religion. It does not appear to be correlated with many of the observed characteristics, which are directly related to assimilation type. Third, in all regressions I already control for most likely correlated effects, such as cohort, street of residence, and Russian language teacher effects, as well as network averages of many other Karaganda outcomes that are likely to affect assimilation, such as supporting local football team, having access to local media, playing sports with the locals or being racially abused by the locals. These controls likely pick up majority of correlated shocks, so an IV strategy adds little on that front. Therefore, it is entirely possible that the exclusion bias alone is enough to attenuate the OLS.

Results in Table 2.5 confirm the simple intuition that the number of local friends and Russian ability are part of the same assimilation process, and, therefore, are strongly corre-

| Dep. Variable $Y$ | Russian Command | | # Local Friends | |
| --- | --- | --- | --- | --- |
| | No Controls | Controls | No Controls | Controls |
| $GY$ | .84*** (.1) | .72*** (.15) | .87*** (.1) | .52** (.23) |
| $R^2$ | 0.145 | 0.548 | 0.143 | .325 |
| $N = 757$ | | | | |

*Notes.* \*, \*\*, and \*\*\* refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors in parentheses clustered at AG level. Column 2 and 4 include a full set of own and friends' characteristics, as well as cohort, street and teacher effects.

**Table 2.6:** Endogenous peer effect estimates using Bramoulle et al. strategy.

lated with each other. As a robustness check that takes this endogenous relationship into account, while still using my preferred instruments for each of the two variables, I also implement a GMM 3 stage least squares estimation (Wooldridge (2002)) for a 2×2 system of Russian command and the number of local friends. The results do not change qualitatively and are available upon request.

Another potential confounding factor in my estimation could be selective attrition. Suppose, for example, that only the poorly assimilated students drop out from each friendship cluster. Then, the fact that only well assimilated ones remain would create and illusion of peer effects. Unfortunately, I do not have access to students who began the university but did not finish for some reason. Overall, however, there were only 41 such dropouts in the last five years (5.4%). University does not keep their records. According to the dean of foreign students, majority of those cases were due to external events, such as family issues or financial problems, rather than failure to settle. Therefore, I do not think that ignoring them poses a strong identification threat.

**Endogenous Effects: Identification Strategy 2**

The exclusion restrictions imposed by IS1 are rather stringent, and there is a high chance that they may be violated. For these reasons in this section I proposed an alternative identification strategy for endogenous peer effects, as well as for telling the endogenous and contextual effects apart. The strategy is a variation of the method proposed by Bramoulle et al. (2009) who suggest to use $G^2 X$ and $G^3 X$ as instruments in equation (2.3). Vectors $G^2 X$ and $G^3 X$ contain characteristics of 2nd and 3rd degree friends. Table 2.6 provides the 2SLS[6] estimates of $\gamma$ using the Bramoulle et. al approach. Intuitively, the endogenous and contextual effects are separated because characteristics of migrant's distant-degree friends

---

[6]The results do not change qualitatively when a spatial 2SLS procedure is used (Lee (2003))
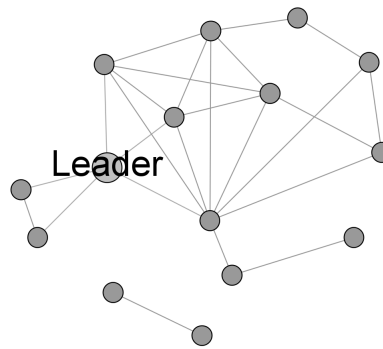
**Figure 2.6:** Friendship network within a typical academic group.

can only influence her own outcomes through their influence on the *outcomes* of her direct friends, which isolates the endogenous channel. The strategy, however, is unlikely to yield causal estimates because it does not address either the correlated effects or the network endogeneity issues. For this reason reason I propose a second identification strategy, which is a variation of Bramoulle et al. that makes use of group assignment and takes a more structural approach to correlated shocks.

Note first that the friendship network is far from complete. It is also incomplete *within* each AG. Figure 2.6 showcases the nominated friendship subnetwork within a typical group. In fact, 25.86% of all possible within-group friendship links are present in the nominated friendship network, so the within-group sub networks are quite dense. Consequently, correlated group shocks should arise. Moreover, of all possible correlated shocks, the group level ones are likely the most prominent for several reasons. First, students who are assigned to the same AG have the same Russian language teacher. Second, AG members participate in various cultural events, organized by the university as a single unit. Third, they take all of their classes together, and, as I mentioned before, the schedule is so intense that students spend half of their waking time in lecture halls. Fourth, after the third year of the program, students have to obtain hands-on experience in local hospitals, and AG members are typically assigned to the same one. Fifth, in case of racial abuse or other problems, students typically turn to the group leader for help and council, so an AG also forms a kind of support group, meaning that negative shocks are likely to be shared among group mates. For these reasons, I believe that most of assimilation-relevant correlated shocks, such as opinion-altering interactions with locals, or changes in teaching quality, are likely to take

place at an AG level[7]. Consequently, the underlying true model is given by:

$$Y = \mu_g + \gamma GY + \beta X + \delta GX + \eta \tag{2.4}$$

Where $\mu_g$ is a column-vector containing group specific correlated shocks. Assuming away the network endogeneity and the reflection problem, the model still can not be correctly identified by OLS, because group assignment largely determines friendships formation, so even if $E[\eta|GY] = 0$ holds, $E[\mu_g, |GY] \neq 0$. Therefore, the econometrician needs to find a way to difference the data and remove $\mu_g$, while leaving enough variation to identify the peer effects. I propose combining the information on nominated friendship and assigned AG ties in particular way. The entries of the vector $\mu_g$ are identical for all members of the same AG. Therefore, the vector would be wiped away if one were to subtract from each migrant's outcome the *the mean value of that outcome across all members of her AG*. This differencing can be performed by pre-multiplying equation 2.4 by the $(I - \tilde{G})$, where $I$ denotes the identity matrix and $\tilde{G}$ denotes the row-normalized adjacency matrix of the AG network. The transformed model becomes:

$$(I - \tilde{G})Y = \gamma(I - \tilde{G})GY + \beta(I - \tilde{G})X + \delta(I - \tilde{G})GX + (I - \tilde{G})\eta \tag{2.5}$$

Several things are noteworthy about the model in equation (2.5). First, the correlated effects disappear because $\tilde{G}\mu_g = \mu_g$. Second, the new error term $(I - \tilde{G})\eta$ is white noise due to randomness of $\tilde{G}$. Third, the differencing also wipes away everything else that is fixed at the group level. Crucially, this not only means removing the group level *observables*, such as cohort effects and teacher effects, or characteristics of the group leader, but also the group-level component of network endogeneity. Consequently the severity of the problem caused by potential sorting on unobserved assimilation types is reduced.

In order to better understand the transformation and the variation that remains, it is useful to spell out equation 2.5 for an individual migrant $i$. Denoting by $|AG_i|$ the size of migrant's AG and by $|N_i|$ the number of friends she has in the nominated network, obtain:

$$Y_i - \frac{1}{|AG_i|-1}\sum_{j:\tilde{G}_{ij}>0}Y_j = \gamma\left(\frac{1}{|N_i|}\sum_{j:G_{ij}>0}Y_j - \frac{1}{|AG_i|-1}\sum_{j:\tilde{G}_{ij}>0}\frac{1}{|N_j|}\sum_{z:G_{jz}>0}Y_z\right) +$$

$$\beta\left(X_i - \frac{1}{|AG_i|-1}\sum_{j:G_{ij}>0}X_j\right) + \delta\left(\frac{1}{|N_i|}\sum_{j:G_{ij}>0}X_j - \frac{1}{|AG_i|-1}\sum_{j:\tilde{G}_{ij}>0}\frac{1}{|N_j|}\sum_{z:G_{jz}>0}X_z\right) + \eta_i \tag{2.6}$$

---

[7]Identification strategy 1 might not solve the issue of group level shocks either, because the instruments (which themselves are group-level) might be systematically driving such shocks. For example, groups with a higher fraction of Russian course takers might spur the language teacher to perform better, which would not be captured by teacher dummies. Similarly, more culturally open AG's might be different at dealing with racial abuse

After the transformation the $\gamma$ coefficient identifies the effect on the difference between *me and my (assigned) friends* of the difference between *my (selected) friends and my (assigned) group mates' friends*. Note that ultimately it is the variation in group sizes $|AG|$ that helps identify the coefficient. The variation arises from varying cohort sizes, their indivisibility by the desired number of groups, and some early attrition. Hence, it should be orthogonal to assimilation differences between groups in a given cohort. Intuitively, the reason why the group size variation matters for peer effect identification is the following. As group size increases, the importance of influence of any group mate $j$'s friends on $i$'s value of assimilation outcome $Y$ starts to go to 0. Each migrant in a bigger group has smaller effect on her friends than a migrant in a smaller group does on hers. This heterogeneity in the endogenous effect's strength between different groups allows the econometrician to recover structural parameters from the reduced form.

The discussion above suggests that the OLS estimator of equation 2.5 is unlikely to be biased by endogeneity of the friendship network. For network's endogeneity to matter, each migrant would need to systematically pick friends whose hidden assimilation type is both lower then her own and higher than that of the friends of her assigned group mates. Such sophistication seems unlikely, particularly given the fact that vast majority of friendships between migrants have reportedly been initiated very soon upon arrival in Kazakhstan. However, one still can not give causal interpretation to the OLS estimate of equation (2.5) due to the bias introduced by reflection problem. The following result is an adaptation of the Bramoulle et al. (2009) Proposition 4, characterizing the possibility of identification of structural parameters from the reduced form in my setting.

**Proposition 7.** *Identification. Assume that no within group subnetworks are complete. Assume further that the $\beta\gamma + \delta \neq 0$. Social effects in the model provided by equation (2.5) are identified if and only if matrices $(I - \tilde{G})$, $(I - \tilde{G})G$, $(I - \tilde{G})G^2$ are linearly independent.*

Notice that the linear independence condition in this case is not very stringent. The network diameter here is just above 6, meaning that $G$, $G^2$ and $G^3$ are all independent from each other. Additionally, $\tilde{G}$ is almost certain to be linearly independent of both $G$ and $G^2$, because one obtains $\tilde{G}$ from $G$ through non-trivial reassignment of weights. Consequently, according to Proposition 1, it is nearly certain that the structural parameters $\beta$, $\gamma$, $\delta$ can be recovered if one is to estimate the reduced form of the model in equation (2.5).

What remains in order to causally identify the endogenous effects is to derive instruments for my setting which are analogous to characteristics of distant degree friends. Because $\gamma < 1$ by assumption and both $G$ and $\tilde{G}$ are row-normalized, I pre-multiply equation 2.5 by the

inverse of $(I - \gamma G)(I - \tilde{G})$ to obtain the reduced form:

$$Y = [(I - \tilde{G})(I - \gamma G)]^{-1}[\beta(I - \tilde{G})X + \delta(I - \tilde{G})GX + (I - \tilde{G})\eta]$$

Canceling out the inverses and using the series expansion of $(I - \gamma G)^{-1}$, the equation can be re-written as:

$$Y = \beta \sum_{k=0}^{\infty} \gamma^k G^k X + \delta \sum_{k=0}^{\infty} \gamma^k G^k G X + \sum_{k=0}^{\infty} \gamma^k G^k \eta \tag{2.7}$$

Finally, I pre-multiply the above equation by $(I - \tilde{G})G$ and take conditional expectations:

$$E((I - \tilde{G})GY|X) = \beta \sum_{k=0}^{\infty} \gamma^k (I - \tilde{G})G^{k+1}X + \delta \sum_{k=0}^{\infty} \gamma^k (I - \tilde{G})G^{k+2}X$$

Which means that $(I - \tilde{G})G^2 X$, $(I - \tilde{G})G^3 X$, etc. can be used as valid instruments for $(I - \tilde{G})GY$ in equation (2.5) in order to solve the reflection problem and identify $\gamma, \beta, \delta$. Intuitively, the instruments contain the difference for each migrant between the average characteristics of her second (third) degree friends and average characteristics of the second (third) degree friends of her AG mates.

Table 2.7 contains 2SLS estimates of equation 2.5 using the instruments described above. Note that in order to use the methodology, at least one explanatory variable is required. When only one explanatory variable is used (columns (1) and (3)), the estimates of the endogenous peer effect $\gamma$ are large and not statistically significant, most likely because the reduction in variation due to variable transformation is not properly overcome by including just one relatively week instrument. When the entire set of controls (and, therefore, instruments) is included, the estimates are reduced and become statistically significant. The estimated coefficients are very similar in size and statistically indistinguishable from the ones obtained using identification strategy 1. The similarity of the results, given that the two strategies tackle a different set of identification challenges, is a further evidence for existence of endogenous effects.

## 2.5.2 Placebo Tests and Robustness Checks

In this section I provide additional evidence that the two identification strategies solve the underlying issues and the results are not simply due to mechanical correlations arising from my methodology.

The first robustness check is a placebo test for the proneness of each identification strategy to result in spurious correlations. Specifically, Table 2.8 presents the estimates of

| Dep. Variable | $(I-\tilde{G})$*Russian command | | $(I-\tilde{G})$*#Local friends | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $(I-\tilde{G})G*Y$ | .631 (.595) | .625*** (.097) | .749 (1.18) | .385*** (.078) |
| $(I-\tilde{G})*$ | | | | |
| Lived abroad before | | 2.243 (2.052) | -.079 (.38) | -.226 (.36) |
| Rus. pre KSMU | 13.123*** (1.26) | 13.037*** (1.047) | | .066 (.174) |
| Age at year 1 | | -.588* (.327) | | .124*** (.046) |
| Shares house w/ local | | 6.08** (2.43) | | .575 (.372) |
| BSER | | -.041 (.054) | | -.0004 (.008) |
| Female | | .202 (1.756) | | -.34 (.266) |
| Muslim | | -6.712*** (1.501) | | .381** (.121) |
| Backward cast | | .479 (1.147) | | .318* (.171) |
| From Rajasthan | | -6.09*** (1.83) | | .223 (.277) |
| Group Leader | | 4.3** (1.87) | | -.139 (.18) |
| Lives in dorms | | 6.723*** (1.654) | | .025 (.25) |
| Was racially abused | | .962 (1.126) | | -.427*** (.111) |
| # Indian friends | | -.095 (.126) | | -.022 (.015) |
| Russian command | | | | .013** (.005) |
| # Local friends | | .764*** (.241) | | |
| $N = 757$ | | | | |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0: \beta = 0$. Standard errors in parentheses. Regressions (2) and (4) also include street of residence effects and additional controls for answers to a wide array of qualitative survey questions, such as whether the migrant tries hard to find new local friends or finds that learning Russian is important. For each control variable $(I-\tilde{G})X$, the corresponding exogenous effect $(I-\tilde{G})GX$ is also included in all regressions. $(I-\tilde{G})G^2X$ are used as instruments.

**Table 2.7:** Endogenous peer effects estimates using identification strategy 2.

| Characteristic: | IS1 (1st Stage) | Bramoulle et al. | IS2 |
|---|---|---|---|
| Height | .89 (.35) | .54** (.24) | .04 (.29) |
| Female | .02 (.04) | .95*** (.17) | .19 (.35) |
| New Culture | .41 (.3) | .66*** (.18) | .09 (.17) |
| Backward | .05 (.05) | .52*** (.18) | .04 (.27) |
| Jaipur | -.07 (.06) | -.006 (.24) | -.15 (.21) |
| Siblings | .25 (.2) | .47** (.19) | .15 (.24) |
| BSER | .02 (1) | -.09 (.2) | -.07 (.22) |
| Islam | .01 (.02) | 1.2*** (.13) | .09 (.77) |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. All regressions control for own and friends' characteristics, as well as cohort, street and teacher effects.

**Table 2.8:** "Endogenous effect" in pre-KSMU characteristics by different stretegies.

"endogenous" effects in 8 characteristics which are determined before the arrival in Kazakhstan. The first column contains the coefficient estimates from the regression of friend's average of a particular characteristic on the fraction of pre-university Russian course takers in the AG. None of the estimates are statistically significant, so the first stage partial correlation used in identification strategy 1 is not mechanical. The strategy of Bramoulle et al. (column 2) yields large significant estimates for 6 out of 8 characteristics, which is merely a confirmation that the strategy does not address network endogeneity. Column 3 shows that using the same set of covariates, identification strategy 2 yields estimates that are insignificant and close to 0 for all characteristics. So, the $(I - \tilde{G})$ transformation appears to solve the network endogeneity problem.

The second robustness check involves investigating whether it is the topology of the friendship network that is responsible for my results. Generating a new life-like friendship network on the same set of nodes is not a straightforward task. Therefore, I simply re-wire existing links in the following way. I first randomly reshuffle within each cohort the identification numbers that are assigned to migrants, while keeping all of the other data intact. Then, I use the the *original* provided identification numbers of Indian friends of each student, and re-create the adjacency matrix. After reshuffling, those ID's belong to randomly picked students instead of the real friends. In such a way I obtain a network with identical topology to the original one, except all the nodes have shifted identities, so there is no economic reason for peer effects to exist. Notice that, because I reshuffle identities separately within each year, the links that originally connected students from different cohorts continue to do so. Therefore, this methodology also tests for whether my results are explained mechanically by some younger students simply having more older friends.

The procedure also reshuffles the AG's. The number of groups and their sizes in each co-

| Method | OLS | | 2SLS | |
| --- | --- | --- | --- | --- |
| | (1) | (2) | (3) | (4) |
| *Estimate of $\gamma$ in*: | | | | |
| Russian command | .513*** | .044 | 2.95 | 2.49 |
| | (.096) | (.219) | (4.08) | (26.9) |
| # Local friends | 0.923** | -.0008 | 7.089 | -7.33 |
| | (.412) | (.115) | (765.3) | (98.07) |
| Controls | | Yes | | Yes |
| $N = 757$ | | | | |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. All output represents averages from 50 repetitions of the network re-wiring process. Standard errors in parentheses clustered at AG level. 2SLS estimates correspond to ones in Table 2.5, while OLS estimates correspond to ones in columns (1), (3), (5), and (6) of Table 2.3

**Table 2.9:** Robustness check: link re-wiring.

hort are preserved, but they are now populated by randomly picked migrants from the same cohort. After re-wiring the network, I set up all of the friends' averages and group mates' averages in the same way as before. I then re-estimate my preferred specification (Table 2.5) using these re-wired averages. I carry out the same operation 50 times, with average results listed in Table 2.9. Notice that the raw correlations between one's assimilation outcomes and the outcomes of ones 'friends' are positive, large and statistically significant (Column (1)). This mechanical correlation is driven by that fact that most friendships are between migrants in the same cohort. Since I preserve the network structure, the correlations remain in raw data but disappear completely once I control for the year effects (Column (2)). Columns (3) and (4) contain the averages of 50 IV estimates of $\gamma$ using (re-wired) instruments, as in Table 2.5. The estimates are wild, and the standard errors are enormous both with and without controls. This is an outcome of the weak instrument problem, because link re-wiring removes any systematic overlap between identities of a migrant's friends and those of her group mates. Overall, the estimates in Table 2.9 allow me to conclude that my endogenous effect results are not driven by the network topology or the particular structure of friendship ties between cohorts.

### 2.5.3   Peer Effects Heterogeneity

In this section I discuss the heterogeneity in the estimated endogenous peer effects in assimilation, in order to get a better understanding of the channels through which migrants influence each other.

**Russian knowledge**. Apart from the test scores, I also have data on migrants' self-assessment of their Russian ability. They rate their ability to speak, read and comprehend spoken Russian on a 1-10 scale. Because these are self assessment data, one might not necessarily expect to see peer effect in them. In fact, self-assessment index, measured as the sum of the three variables on a 3-30 scale explains about 40 % of variation in test scores. So, any peer effects that might be uncovered in these data are peer effects in own perception of Russian ability, rather than actual ability. Yet, this is not different from existing studies of language acquisition among migrants (Chiswick and Miller (1995, 2002), Dustmann and Fabri (2003)). My assumption is that the self-assessments are accurate at least relative to one another, thus carrying the information about the relative susceptibility of different components of the language skill to peer influence. The fraction of AG mates who have taken Russian before coming to Kazakhstan turns out to not be a valid instrument for network averages of these variables, because the first stage partial correlation is weak. Therefore, for each self-assessment category (reading, listening comprehension and speaking) I apply identification strategy 2 . The estimates are presented in Table 2.10. The estimated $\gamma$ in the regression of speaking ability is twice larger than the ones in regressions of reading and comprehension, and it is also the only statistically significant estimate out of the three. Therefore, there likely exist endogenous peer effects in speaking ability, but not comprehension or reading abilities. Given the existence of peer effects in overall Russian ability, these results suggest that migrants likely affect each other by practicing the language between themselves or otherwise boosting each others' speaking ability [8]. A policy implication is that migrants' assimilation should increase more rapidly if they are encouraged to talk to each other in the language of the destination country, rather simply take language courses together.

**Peer effects in social capital**. For each social tie with a local reported by migrants in my sample, I also record the primary language of communication with that person.

The three options are Russian, English and Kazakh. Of the 3,228 reported friendships with locals, 61% are English-based, 37% are Russian-based, and 2% are Kazakh-based. These friendships can be of fundamentally different nature. For example, friendships in which English is the primary language do not require additional language-learning effort

---

[8]43.33% of migrants report having practiced Russian with their Indian friends at least once during their time in Kazakhstan.

| Dep. Variable | Comprehension (1-10) | Reading (1-10) | Speaking (1-10) |
|---|---|---|---|
| | (1) | (2) | (3) |
| $GY$ ($\gamma$ estimate) | .304 (.374) | .298 (.368) | .553** (.229) |
| # Hours study Rus. | .059*** (.013) | .04** (.017) | .044*** (.013) |
| # Rus. pre KSMU | .927*** (.183) | .934*** (.195) | .794*** (.168) |
| # Languages spoken | .143* (.078) | .18** (.08) | .167*** (.064) |
| Age at year 1 | -.099* (.051) | -.109** (.049) | -.147*** (.045) |
| Lives in dorms | .027 (.269) | -.039 (.242) | .023 (.257) |
| BSER | .014* (.008) | .008 (.01) | .007 (.007) |
| Female | -.046 (.271) | .135 (.273) | -.244 (.248) |
| Backward cast | -.195 (.195) | -.262 (.197) | -.303* (.179) |
| Was racially abused | .09 (.158) | -.084 (.176) | .04 (.159) |
| Muslim | .023 (.3) | -.191 (.344) | -.09 (.305) |
| # Local Friends | .094*** (.035) | .077** (.035) | .091*** (.032) |
| 1st stage p-val. for $H_0$ : $\beta_{instr} = 0$ | .000 | .000 | .000 |
| $R^2$ | 0.2652 | 0.1829 | 0.2654 |
| $N = 757$ | | | |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors in parentheses clustered at AG level. All regressions include cohort, teacher and street of residence effects, as well exogenous effects $GX$ corresponding to every control variable $X$.

**Table 2.10:** 2SLS estimates of endogenous peer effects in self-assessed language ability by type.

| Dep. Variable | #Kazakh Friendships (1) | #Russian Friendship (2) | #English Friendship (3) |
|---|---|---|---|
| $GY$ ($\gamma$ estimate) | .884 (.696) | .704** (.327) | .487* (.277) |
| # Siblings | .003 (.006) | .033 (.027) | .085** (.041) |
| # Languages spoken | -.007 (.009) | -.051 (.042) | -.079 (.067) |
| Age at year 1 | -.004 (.005) | .064** (.028) | .084** (.039) |
| Lives in dorms | .052 (.037) | .086 (.142) | .122 (.173) |
| BSER | .001 (.001) | -.003 (.004) | -.003 (.005) |
| Female | .025 (.039) | -.49*** (.153) | -.041 (.217) |
| Backward cast | .006 (.026) | -.104 (.102) | .267** (.124) |
| Was racially abused | -.0003 (.026) | -.079 (.135) | -.179 (.135) |
| Muslim | -.007 (.043) | -.061 (.132) | .151 (.256) |
| Russian command | -.002** (.001) | .006 .004 | .008* (.004) |
| 1st stage p-val. for $H_0$ : $\beta_{instr} = 0$ | .005 | .000 | .000 |
| $R^2$ | 0.1073 | 0.1882 | 0.2880 |
| $N = 757$ | | | |

Notes. *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors in parentheses clustered at AG level. All regressions include cohort, teacher and street of residence effects, as well exogenous effects $GX$ corresponding to every control variable $X$.

**Table 2.11:** 2SLS estimates of endogenous peer effects in social capital by primary language.

from migrants. Therefore, they are less beneficial for linguistic skills acquisition. It is of interest to examine whether it is English or local language friendships that are mostly responsible for the endogenous peer effects in social capital acquisition. Because the number of friendships in each language is a fraction of the overall number of local friends, I can use identification strategy 1 and re-estimate the IV regression in column (4) of Table 2.5 separately for each language. Table 2.11 lists these estimates. The largest estimated effect is for Kazakh language friendships, but it is not statistically significant. The estimates for Russian and English friendships are both statistically significant. Under my assumptions, the IV estimates can be interpreted causally. So, there are endogenous peer effects in acquisition of both types of friendships.

The estimate for Russian language is much larger, which suggests that learning to communicate with locals in the local language is an activity in which migrants strongly depend on each other. An explanation could be that migrants perceive speaking to locals in Russian to be too daunting of a task until they see other Indians do it successfully. These kinds of friendships are likely to also be the main vehicle of linguistic improvement, resulting in additional spread of positive assimilation effects across the network. A policy implication is that if there exists a social multiplier, then providing fresh migrants with local partners in order to practice the language is likely to benefit the whole community, and not just that particular migrant.

## 2.6 Mechanisms and Social Multiplier

There is no consensus in migration literature about whether ethnic neighborhoods or enclaves are good or bad for migrants and local minorities. Some studies have found that migrants' costs of assimilation increase when they are settled together with larger group of co-nationals, making them less likely to assimilate (Danzer and Yaman (2013), Battisti et al (2016)), while other have found that larger ethnic networks provide additional job, business and risk-sharing opportunities (Munshi (2003), Edin et al. (2003)). Consequently,the question of optimal migrant assimilation policies, such as distribution of migrants across the host society remains open.

Countries with large migrant populations attempt to manipulate social networks among migrants by varying their dispersal policies, as well as entry quotas by skill and region of origin. Often these policies are driven by the prevailing political climate, rather than genuine attempts to help the newcomers in assimilation. Existence of endogenous peer effects in assimilation implies that the decision to assimilate is taken jointly by all migrants in the network. The way policymakers should account for the presence of such effects depends in

large part on whether the effects create a social multiplier (Glaeser et al. 2002).

There are three broad ways to spread migrants and refugees around the host country. They can be allowed to live where they want and to form and ethnic enclave of any size, which is the policy used in Germany and in Sweden for most of its modern history. They can be made live in close to isolation from co-nationals in places where there is available housing, which is the policy used in the UK. Finally, they can be spread randomly around the country in medium-sized groups, which is the policy of Denmark, among others. In this section I discuss complementarity and conformity as the two potential mechanisms behind the endogenous peer effects in assimilation. Depending on which of the two is salient, living in an ethnic enclave can either reduce or increase assimilation of a migrant relative to the counterfactual of being isolated at the destination. I then attempt to distinguish the mechanisms in my data.

### 2.6.1   Complementary vs. Conforming

In this section I apply to migration context the discussion in Boucher and Fortin (2016) of two competing microfoundations behind peer effects - complementarity and conformity.

*Mechanism 1. Complementarity.* This mechanism implies that migrants help each other assimilate through shouldering some of the implicit assimilation costs. For example, they can make it easier for each other to learn the language by studying together or practicing with each other. They can also make it easier to build local-specific social capital by sharing their local social contacts. Migrants, therefore, end up pushing each other toward higher assimilation levels. More specifically, one can imagine the following simple model. As before, denote the row-normalized adjacency matrix for the network between migrant by $G$. Suppose further, that each migrant has an assimilation type $\theta_i$, with higher types finding assimilation more beneficial. Migrants simultaneously choose assimilation effort $a_i$ by solving:

$$\max_{a_i} \ \theta_i a_i + \gamma_1 a_i G_i \mathbf{a} - \frac{1}{2}a_i^2 \tag{2.8}$$

$$s.t. a_i \geq 0$$

Parameter $\gamma_1$ measures the strength of peer effects. In this game of strategic complimentarities, the Nash Equilibrium always exists and is uniquely given by $\mathbf{a} = \sum_{k=0}^{\infty} \gamma_1^k G^k \underline{\theta}$. A migrant in an empty network would obtain $a_i^{iso} = \theta_i$. Therefore, in a model like this belonging to a network (i.e. living side by side with co-nationals)) is always preferable to being isolated in the new country, regardless of type distribution or network structure. This happens because of the existence of social multiplier. Suppose a policy reduces the cost of

assimilation, making each migrant's effective type $\theta_i + \epsilon$. Then, the associated increase in assimilation is given by $\mathbf{a}(\underline{\theta} + \epsilon) - \mathbf{a}(\underline{\theta}) = \epsilon \sum_{k=0}^{\infty} \gamma_1^k G^k \mathbf{1} = \frac{1}{1-\gamma_1} \epsilon \mathbf{1} > \epsilon \mathbf{1}$. The multiplier $SM = \frac{1}{1-\gamma_1}$ comes about because a migrant not only receives an $\epsilon$ positive shock but also benefits from her friends increasing their effort in response. Of course, a migrant who doesn't have access to a network of co-nationals might end up assimilating faster because she has no other choice but to form contacts with locals. Nevertheless, if $\gamma_1$ is large, isolationist policies are likely to be misguided, and governments should instead concentrate on harnessing the power of the social multiplier.

*Mechanism 2. Conformism.* Migrants might incur disutility from having assimilation levels different from those of their peers. Depending on the network structure and type distribution, such mechanism implies that migrant community might increase assimilation costs relative to the empty network counterfactual. Keeping all of the previous notation intact, one can imagine the following simple model that includes this motif:

$$\max_{a_i} \; \theta_i a_i - \frac{\gamma_2}{2} \left( G_i \mathbf{a} - a_i \right)^2 - \frac{1}{2} a_i^2 \tag{2.9}$$

$$s.t. a_i \geq 0$$

Parameter $\gamma$ now measures the strength of the mimicking motif. The unique Nash Equilibrium of this game is given by $\mathbf{a} = (1/(1+\gamma_2) \sum_{k=0}^{\infty} (\gamma_2/(1+\gamma_2))^k G^k \underline{\theta}$. Here, the response to an $\epsilon$ positive shock is given by $\mathbf{a}(\underline{\theta} + \epsilon) - \mathbf{a}(\underline{\theta}) = \frac{\epsilon}{1+\gamma_2} \sum_{k=0}^{\infty} \left( \frac{\gamma_2}{1+\gamma_2} \right)^k G^k \mathbf{1} = \epsilon \mathbf{1}$. So, the social multiplier is equal to 1. The intuition is that under pure conformity the positive shock influences everyone equally, so the relative differences between each migrant and her friends remain the same, and no indirect network effects are induced. Under this mechanism it is no longer true that migrants are necessarily better off in the network relative to being in an empty network. Suppose, migrants sort on assimilation types $\theta$, so the network has low-type and high-type communities. In such case, low types are mostly mimicking low types, whereas high types are mimicking mostly high types. For many such networks there exists a range of $\gamma_2$ values such that the rare connections between low and high types bring the high types' assimilation down by more than they boost the low types' assimilation. As a result, there is an overall drop compared to the assimilation of an isolated migrant $a_i^{iso} = \theta_i$.

Since the social multiplier $SM$ is necessarily equal to 1, isolationist policies do not necessarily imply forgone network benefits and might lead to higher assimilation than policies that allow migrants to settle in clusters. Clearly, the presence and the size of social multiplier in assimilation should should be taken into account when devising assimilation policies. However, telling the mechanisms apart is not straightforward, because their reduced form implications are observationally equivalent. The reaction function in case of complementarity

is:

$$a_i = \theta_i + \gamma_1 G_i \mathbf{a} \tag{2.10}$$
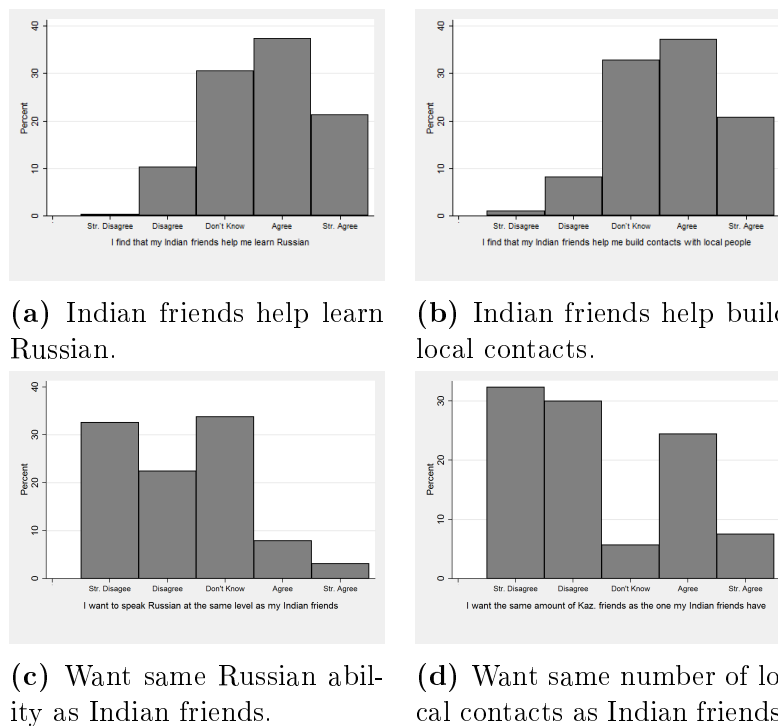
Whereas in case of conformism it is:

$$a_i = \frac{1}{1 + \gamma_2}\theta_i + \frac{\gamma_2}{1 + \gamma_2}G_i\mathbf{a} \tag{2.11}$$

Existing research into mechanisms behind peer effects typically relies on either testing the reduced form predictions from a more involved structural model (De Giorgi et al. (2016)) or experimental evidence (Bursztyn et al. (2016)). Conclusively distinguishing between the two mechanisms in my case based on non-experimental data is virtually impossible. Even a randomized control trial would be difficult in this context, since one would need to be able to create treatment conditions under which only one motif out of complementarity and conformity is active. So, conclusively identifying the microfoundation behind the peer effects from observed data is virtually impossible. Nevertheless, I lay out several pieces of evidence which, taken together, point towards presence of complementarity and, therefore, towards existence of a social multiplier.

### 2.6.2 Survey Evidence

In this section I lay out some qualitative survey evidence that points towards a combination of the two mechanisms being behind the estimated peer effects. Figure 2.7 presents the distribution of answers to four questions pertaining to how KSMU Indian students view their co-nationals' influence on them. Of all students, 58.78% (panel (a)) claim to believe that their Indian friends help them master Russian language, as opposed to only 10.7% of those who disagree. Similarly, 47.86% of migrants (panel (b)) believe that their Indian friends help them build social contacts among the locals, as opposed to 9.38% who disagree. These numbers indicate that student themselves feel that complementarities exist in assimilation. The bottom panels reveal something about the migrants' perception of the strength of the conforming motif. Roughly 55% of them (panel (c)) disagree that they necessarily want to have the same Russian language ability as their friends, while merely 11.1% agree, suggesting that migrants themselves do not view mimicking as an important peer effect channel in linguistic ability. Panel (d) shows that opinions regarding the amount of local friends are both more pronounced and more split; 62.35% of migrants disagree that they want the same amount of local friends as their Indian friends have on average, while 31.97% agree, with only a few people not expressing an opinion. Taken together, these raw numbers seem to suggest that migrants find each other useful in increasing assimilation, and majority of them

**(a)** Indian friends help learn Russian.

**(b)** Indian friends help build local contacts.

**(c)** Want same Russian ability as Indian friends.

**(d)** Want same number of local contacts as Indian friends.

**Figure 2.7:** Distribution of answers to four peer influence questions.

does not consciously mimic assimilation of their peers.

There is a variety of ways in which migrants can help each other assimilate. Table 2.12 summarizes students' answers to several questions aimed at identifying specific channels of influence. Studying Russian together appears to be an important one. However, because all students are required to take language courses, studying together might be a vehicle for excelling at exams, rather than for assimilation. Actually practicing the language together is a more valid indicator, and around a half of students report to engage in such activity. Almost 63% of the sample report to have been introduced to at least one local person by their Indian friends. This piece of evidence is backed up by the fact that on average two Indian friends have a 39% overlap in identities of reported local friends, suggesting that students might, indeed, be jointly accumulating social capital in Karaganda. Finally, students are reporting peer effects in media consumption. A regression of the assimilation index on dummy variables representing answers to the four questions yields an F-statistic of 17.85 and an $R^2$ of .087, suggesting a strong correlation between assimilation outcomes and reported peer influence channels.

An important feature of the conformism mechanisms is that a poorly assimilated migrant causes her peers do decrease assimilation. Two hypothetical survey questions were aimed at uncovering the perceived strength of such negative mimicking imperative. Specifically,

| Question | (1)<br>YES -1 | (2)<br>NO - 0 |
|---|---|---|
| *Have you ever been introduced to a local person by one of your Indian friends?* | 475 | 282 |
| *Have you ever practiced Russian with one of your Indian friends, outside of the first year language course?* | 328 | 429 |
| *Have you ever studied Russian together with one of your Indian friends?* | 478 | 279 |
| *Do you often watch local TV or consume other local media because your Indian friends are doing so?* | 586 | 171 |

*Notes.* *** implies p-value $< 0.01$ for a two-tailed t-test of mean=0.5.

**Table 2.12:** Peer influence channels.

19.16% of the students claim that they would be discouraged from taking a Russian language course if all their Indian friends had a negative perception of it. 34.21% feel that they would be discouraged from dating a local person if their Indian friends didn't approve. It is probable that answers to these questions are heavily influenced by implicit cultural norms[9]. Yet, taken together they suggest that migrants' perception is that the negative mimicking motif exists and is stronger in case of acquisition of local friends, rather than language skills.

Finally, students were asked whether they thought that their Russian language ability was the same as average ability of their Indian friends, and whether they had as many local social ties on average. In both cases approximately 79% of the sample responded positively, suggesting that majority of the students consider themselves to be as assimilated as their friends. Those people were then asked to select a reasons for the similarity. 66% picked that 'It is easier or more rewarding to learn language when your Indian friends are doing so as well,' (complementarity in language). 11% picked that 'You feel that your Indian friends might resent you if your Russian ability differed from theirs by a lot' (conformism in language). Similarly, 66% picked that 'You find it easier or more rewarding to interact with Kazakhstani people as a part of a group of Indians and/or you sometimes make friends with Kazakhstani acquaintances of your Indian friends,' (complimentarity in friendships). Around

---

[9]Indeed, females are much less likely to report willingness to proceed without peers' approval, even controlling for assimilation levels and other observables.

| Dependent Variable | Assimilation Index (1) | Russian Command (2) | #Local Friends (3) |
|---|---|---|---|
| Wants same Russian ability as Indian friends. | -.123*** (.03) | 4.91*** (.66) | .12 (.08) |
| Wants same local social capital as Indian friends. | -.09*** (.02) | .76 (.53) | -.47*** (.06) |
| Unlikely to take language course in spite of Indian friends. | -.008 (.08) | 1.83 (1.41) | -0.25 (.23) |
| Unlikely to date a local in spite of Indian friends. | -.34*** (.07) | -1.8 (1.44) | -1.1*** (.18) |
| N=757 | | | |

*Notes.* *, **, and *** refer to p-values less than .1, .05, and 0.01 respectively for a two-tailed test of $H_0 : \beta = 0$. Standard errors in brackets are clustered at AG level. All regressions control for cohort fixed effects, street of residence and Russian teacher effects.

**Table 2.13:** Peer influence and outcomes.

12% picked that 'You feel that your Indian friends might resent you if you had far more or far fewer Kazakhstani friends than them' (conformism in friendships). These data indicate that migrants themselves view reinforcing mechanism as more important than conformism.

Clearly, such qualitative survey data are not based on revealed preference arguments and, thus, are only suggestive. Nevertheless, students' perception of peer influence channels is likely to play a role in shaping outcomes. Morover, the survey seems to line up well with the observed outcomes and, thus, provides useful evidence for telling the mechanisms apart. For instance, there is a strong positive correlation between a migrant's claim to be similar to her friends in one of the assimilation outcomes and the true observed squared distance. Therefore, students seem to accurately perceive whether their assimilation levels are similar to those of their friends. Regressions in Table 2.13 summarize some relationships between assimilation and survey questions. Of particular interest is the fact that people with higher stated desire to conform have lower assimilation. Such relationship can be viewed as evidence that conformism is particularly damaging to the low types.

To conclude, the survey points to both mechanisms being salient in this particular migration episode. This evidence, therefore suggest that there exists a social multiplier.

### 2.6.3 Outcome-Based Evidence

In order to say something about the existence and the size of social multiplier, I employ two additional econometric approaches recently developed in the peer effect literature. I first use the method of Liu et al. (2013) that is designed to identify the *presence* of the complementarity motif (and therefore, the social multiplier) under certain assumptions. I then proceed with the method of Boucher and Fortin (2016) that estimates the *size* of the social multiplier by making use of isolated individuals.

**J test of Liu et al. (2013)**. Liu et al. propose a "statistical model selection test to detect which behavioral mechanism (conformity vs. complementarity) better represents the data." Their main argument is that the conformity mechanism is the microfoundation for the linear-in-means model that I've using in this paper, while complementarity is the microfoundation for the linear-in-aggregates model. In a linear-in-aggregates model, a migrant who has 10 Indian friends with average Russian test score of 75 should speak the language better than a migrant with three friends who have the same average score. This additional information on size of each individual's peer group can be used to tell the models apart.

Formally, Liu et al. design a specific J-test to distinguish between the two non-nested models. The model in which friends' aggregate assimilation matters can be specified as follows:

$$AI = \mu_g + X\beta_1 + GX\delta_1 + \gamma_1 G_{01} AI + \varepsilon_1 \tag{2.12}$$

Where $AI$ is the assimilation index, and $G_{01}$ is the non-normalized symmetric adjacency matrix of the undirected migrant network. As before, the model where the mean assimilation matters is specified as:

$$AI = \mu_g + X\beta_2 + GX\delta_2 + \gamma_2 GAI + \varepsilon_2 \tag{2.13}$$

The following is a version of test that Liu et al. propose in order to tell models in (2.12) and (2.13) apart:

1. Difference out the vector $\mu_g$. I perform the differencing by pre-multiplying the model by $(I - \tilde{G})$.

2. Estimate the "average effect" model in equation (2.13) using the quasi MLE method developed by Lee et al. (2010).

3. Obtain the predicted values $\hat{AI}_{av}$.

4. Plug the fitted values into the differenced "aggregate effect" model to obtain:

$$(I - \tilde{G})AI = \alpha(I - \tilde{G})\hat{AI}_{av} + (I - \tilde{G})X\beta_1 + (I - \tilde{G})GX\delta_1 + \gamma_1(I - \tilde{G})G_{01}AI + (I - \tilde{G})\varepsilon_1$$

5. Estimate the model by 2SLS using $G_{01}X$, $G^2X$ as instruments.

6. If $\hat{\alpha}_{2SLS}$ is equal to 0, then the data points to existence of social multiplier.

I apply the test to my data by estimating the fully-specified models of the assimilation index. I obtain $\hat{\alpha}_{2SLS} = -0.033$ (0.263). The J test, therefore, suggests that my data are better described by the linear-in-aggregates and not the linear-in-means model. By arguments of Liu et al. this result implies the existence of social multiplier due to endogenous peer effects.

The main concern with the J-test is that ultimately what it does is tell the linear-in-means model apart from the linear-in-aggregates model. The test's importance for my goals, therefore, relies on the assumption that the microfoundations for these two econometric models are conformity and complementarity respectively. This assumption is not testable. Indeed, Boucher and Fortin (2016) show that the linear-in-means model itself can be generated by either one of conformity and complementary and may or may not give rise to the social multiplier. For this reason I implement another method of assessing the existence and the size of the social multiplier in presence of endogenous peer effect that has been proposed by Boucher and Fortin.

**Identification based on isolated individuals by Boucher and Fortin**. The authors suggest making use of the fact that the individuals who are isolated in the network are not subject to peer effects. I do not have any isolated individuals in my data, so the have make an additional restriction. I assume that two categories of students are isolated in the assimilation network. The first category is the students in their first year of study at KSMU (cohort of 2015). The second category is the upperclassmen who only report to have social ties with freshmen. I make the assumption because formation of friendships and acquisition of linguistic skills is a lengthy process. So, it is unlikely that the freshmen have had enough time to truly impose any externalities on other in the few months from the start of the program till the date of the survey.

There are 267 individuals who satisfy this definition of isolation. Accounting for group effects, their assimilation can be modeled as:

$$(I - \tilde{G})AI = (I - \tilde{G})X\beta + (I - \tilde{G})\varepsilon_1 \tag{2.14}$$

The remaining 490 migrants are assumed to solve the following assimilation problem in which both the complementary ($\gamma_1$) and conforming ($\gamma_2$) motifs are present:

$$\max_{AI_i} (\mu_{gi} + X_i\beta + (GX)_i\delta + \varepsilon_{2i})AI_i + \gamma_1 AI_i G_i\mathbf{AI} + \frac{\gamma_2}{2}(G_i\mathbf{AI} - AI_i)^2 - \frac{1}{2}AI_i^2 \quad (2.15)$$

$$s.t. AI_i \geq 0$$

Resulting in the following reduced form relationship:

$$AI = \frac{\mu_g}{1+\gamma_2} + \frac{1}{1+\gamma_2}X\beta + \frac{1}{1+\gamma_2}GX\delta + \frac{\gamma_1 + \gamma_2}{1+\gamma_2}GAI + \frac{1}{1+\gamma_2}\varepsilon_2 \quad (2.16)$$

Under the assumption and if the model is correct, the estimated coefficient on the exogenous variable $X$ should be smaller for upperclassmen than for the first years, because the true $\beta$ is suppressed by the the conformity term $1/(1-\gamma_2)$. Essentially, if the pressure to conform is very strong, then it overrides the migrant's individual characteristics, resulting in their diminished observed effect.
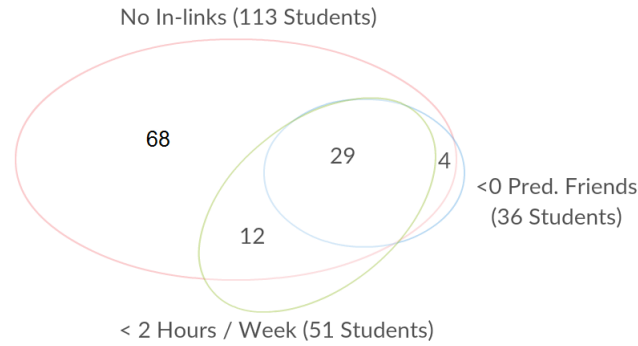
I estimate equation (2.14) by OLS and equation (2.16) using the identification strategy 2 in order to obtain the reduced-form coefficients. From the estimates of $\hat{\beta}_{OLS}$ and $\left(\frac{\hat{\beta}}{1+\gamma_2}\right)_{IS2}$, I back out the strength of the conformism motif $\hat{\gamma_2} \approx 0.42$. The combined endogenous peer effect is estimated as $\left(\frac{\hat{\gamma_1+\gamma_2}}{1+\gamma_2}\right)_{IS2} = 0.509$. Substituting for $\hat{\gamma_2}$ and solving yields the strength of the complementarity motif $\hat{\gamma_1} \approx 0.3$.

The social multiplier implied by my endogenous peer effect estimates, therefore, can be calculated as $\hat{SM} = 1/(1-\hat{\gamma_1}) = 1.43$. A policy that exogenously reduces assimilation cost by $\epsilon$ would boost each migrant's outcome by $1.43\epsilon$ due to the increase in productivity of assimilation efforts through network ripple

My implementation of Boucher and Fortin's methodology relies on two key assumptions. The first assumption, which is fundamental to their approach, is that the $\beta$ coefficients from equations 2.16 and 2.14 are the same. This assumption is not testable and would be violated if, for example, own religion affects isolated and connected students in systematically different ways.

The second assumption is that freshmen are isolated in the network. This is a strong assumption which I make because all of the students in my sample were asked to nominate at least 2 friends, leaving no one formally isolated. The request was made to ensure that the row-normalized adjacency matrix could be created for the linear-in-means model. No isolated individuals can be present in the network if one is to estimate and identify such a model. In order to ease the concern about defining isolation in a particular ad hoc way, I propose three alternative definitions of isolation:

1. Students with no in-degrees in the directed network. There are 113 of them.

No In-links (113 Students)

68    29    4

12    <0 Pred. Friends
(36 Students)

< 2 Hours / Week (51 Students)

**Figure 2.8:** The overlaps between differently defined set of 'isolated' students.

2. Students who report to spend less than 2 hours in total with their friends during the average week. There are 51 of them.

3. Students for whom the predicted number of friends (fitted value from an OLS regression of the # of friends on individual characteristics) is less than 0.

The three sets of students defined by the three alternative measures of isolation exhibit a significant amount of overlap. Consequently, one would expect the social multiplier estimates based on these definitions to be similar. Figure 2.8 documents the overlap precisely. Using these alternative definitions yields estimated social multiplier of 1.26, 1.04 and 1.11 respectively. So, the size of the multiplier depends substantially on how isolated individuals are defined.

## 2.7    Assimilation and Productivity

Much of the migration assimilation literature (Borjas (1998) Chiswick et al. (1997), Meng and Gregory (2005)) focuses on whether or not migrants are able to catch up with locals in terms of wages overtime. The basic argument is that because migrants are positively selected all they need in order to erase the earnings gap between themselves and the locals is to accumulate some destination-specific human capital, which they do overtime. Since Indian migrants in my sample are students, their output can readily be observed in the form of GPA. Given that I also observe study hours, the data gives me an opportunity to explicitly investigate whether increased assimilation has any effect on migrants' productivity. In this section I explore such effects.

Clearly, earning good grades is not tantamount to future workplace productivity. Yet, countless studies (Loura and Garman(1993), James et al. (1989) to name a few) have documented positive correlations between university GPA and future wages. For migrants in
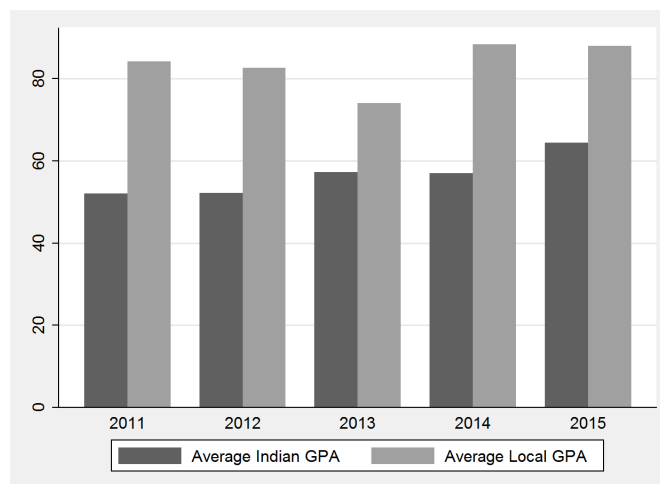
**Figure 2.9:** Observed cohort average Fall 2015 term GPA (as percentage of 4) of Indians and locals enrolled in the 'General Medicine' baccalaureate.

my sample, attending the university is the only reason for migration. Therefore, one would expect doing well in the program to be their ultimate goal. Additionally, 70.81% of people in my sample agree with the statement that grades at KSMU are important to their future career and they do all they can to maximize them. Hence, I view GPA as a valid measure of output in Kazakhstan.

Even though there is no direct need to speak Russian in order to do well, there are reasons to think that being better assimilated can make students more productive. Demonstration-based classes, such as anatomy, are often taken together with local students. They sometimes feature commentary in Russian, making some linguistic ability useful. There might also be out-of-classroom advantages to speaking the local language, such as ability to talk more informally with an instructor, or keeping in touch with freshest news and tendencies around the university. Having local friends is beneficial for better understanding of the educational system, customs in dealing with professors and for direct help in studying (locals are better students on average). Finally, being better assimilated might make a student more comfortable with and less distracted by the minutiae of living in an alien place. I believe that variations of such positive assimilation effects on migrants' productivity are likely to exists at any 'real' workplace.

Grades at KSMU are assigned on a standard letter scale, with an A fetching 4 grade points, B fetching 3, etc. Figure 2.9 depicts differences in Fall 2015 GPA between Indians and locals within the same cohort of the same program. Clearly, Indian students start off lagging behind and, while some catch up does occur, the locals as a group are still able to achieve better grades as upperclassmen. Importantly, grades at KGMU are not assigned on the curve, so the numbers are comparable across cohorts, and there are no spillovers from

| Method:<br>Dep. Variable: GPA | OLS<br>(1) | OLS<br>(2) | OLS<br>(3) | IV<br>(4) | IV<br>(5) |
|---|---|---|---|---|---|
| Assimilation Index | | .25*** (.04) | .17*** (0.4) | .26** (.12) | .21* (0.12) |
| Russian Command | .008*** (.002) | | | | |
| #Local Friends | .029* (.016) | | | | |
| Friends' GPA | | | | | .49***(.09) |
| Age at year 1 | .01 (.03) | .007 (.025) | .009 (.03) | .001 (.015) | .003 (.014) |
| BSER | 0.008* (.004) | 0.008** (.004) | .008* (.004) | 0.005** (.002) | .004** (.002) |
| Hours study | .007* (.004) | .012*** (.004) | .007* (.004) | .0042 (.002) | .002 (.002) |
| Female | -.27** (.11) | -.33*** (.11) | -.26** (.11) | -.11** (.06) | -.1* (.05) |
| Islam | -.1 (.13) | -.05 (.12) | -.12 (.13) | -.06 (.06) | -0.4 (0.6) |
| Backward | -.02 (.08) | -.08 (.07) | -.02 (.08) | -.02 (.05) | -.004 (.04) |
| Shares house w/ local | .45*** (.13) | .49*** (.12) | .45*** (.14) | .15* (.09) | .176** (.086) |
| AG leader | .16 (.13) | .21 (.13) | .17 (.13) | .06 (.07) | .076 (.071) |
| Married | .35 (.27) | .33 (.23) | .38 (.27) | .22* (.13) | .2* (.12) |
| Lived abroad before | -.01 (.15) | -.003 (.15) | .008 (.15) | -0.02 (.09) | -.007 (.08) |
| # Languages spoken | -.05 (.03) | -.05 (.03) | -0.4 (.03) | -.03** (.02) | -.027 (.018) |
| Wants to work abroad | .19* (.1) | .2** (.09) | .02** (.01) | .1* (.06) | .09 (.06) |
| Grades are important | .01 (.01) | .018 (.013) | 0.1 (0.1) | .001 (.015) | .002 (.008) |
| GPA disaster | -.19* (.11) | -.19 (.1) | -.19* (.11) | -.11* (.06) | -.1 (.05) |
| From Jaipur | .02 (.09) | 0 (.09) | .02 (.09) | .01 (.05) | .01 (.04) |
| Friends' BSER | .02* (.01) | .015 (.009) | .017* (.008) | 0.013*** (.005) | .009* (.005) |
| Friends' hours of study | -.001 (.01) | 0.008 (.009) | 0 (.01) | .002 (0.005) | -.004 (.005) |
| Street Effects | Yes | | Yes | Yes | Yes |
| Group Effect | Yes | | Yes | Yes | Yes |
| 1st stage p-val for $H_0$ :<br>$\beta_{instr} = 0$ | | | | .000 | .000 |
| $R^2$ | .2851 | .1716 | .2830 | .2349 | .2865 |

*Notes.* N=757. Standard errors in parentheses are clustered at AG level.

**Table 2.14:** Effects of assimilation on GPA.

one person increasing their studying efforts compared to the rest of the group. There is only one student in my sample with perfect overall GPA, and no students with overall GPA of 1, so in what follows I do not worry about the truncation of data at those points.

The two assimilation outcomes of interest - the number of local friends and Russian language ability, - unsurprisingly, have similar variation. Therefore, since I am interested in the effect on output of assimilation as a whole, and in order to maximize power, I use the combined index to capture assimilation. I first estimate an OLS regression of the overall GPA on assimilation index and controls. Even though I include reported weekly studying hours, as well a large number of personal characteristics, such the observed pre-university ability measured by the BSER, the OLS only documents correlations. In order to say something about the causal effect of assimilation on GPA, I again use AG average answers to pre

university question about importance of new culture. It seems likely that being assigned to a group which is collectively more willing to assimilate is going to increase one's assimilation through peer effects, so the effect is identified on the assumption that the instrument has no direct effect on GPA.

Table 2.14 presents the estimation output. Dependent variable is the standardized (mean 0, variance 1) overall GPA in the program. Because AG mates take many of their classes together, I include group effects in the fully specified model in order to absorb all of the potential variation between learning environments. Additionally, I cluster standard errors at the group level, which also takes care of heteroscedasticity inherent in models with imperfectly continuous dependent variable. Column (1) demonstrates that both assimilation outcomes are positively correlated with the GPA, even after inclusion of a large set of controls and group effects. The rest the columns contain regressions with the assimilation index (top row) as the variable of interest. The most preferred OLS estimate (column (4)) implies that one standard deviation increase in assimilation index is associated with a 0.17 standard deviation increase in overall GPA. The IV estimate in column (5) is large and statistically significant, implying that that a one standard deviation increase in assimilation index causes a 0.21 standard deviation (or about 0.12 grade points) increase in overall GPA.

The IV estimate is slightly larger than the OLS. The main identification threat and a source of potential upward bias in IV estimates is the possibility that the instrument affects GPA through channels other than assimilation. Specifically, one's willingness to embrace new culture could be a proxy for unobserved components of ability. Therefore, migrants in an AG where such willingness is high might do better academically due to possible peer effects in focusing, productivity or ambition. I do not think that this is a severe problem for 2 reasons. First, the specification in column (5) includes friends' average GPA as an additional control. The estimated coefficient on that variable is large and significant, so it should pick up majority of ability-related peer effects. The fact that coefficient on assimilation index doesn't change much from its inclusion suggests that the problem is relatively small. Second, the IV specifications include group dummies. The instrument is a leave-out-self group average of a variable. Intuitively, in the first stage the migrant's difference between own and group's average (including self) assimilation index is regressed onto the difference between own and group's average (excluding self) opinion on importance of foreign culture, weighted by group sizes. So, the identifying variation comes from groups sizes, which are arguably random and shouldn't be compromised by ability-related peer effects.

A possible reason for why the OLS could be downward-biased is that better assimilated students might be invited to many social gatherings with locals and lose focus on academic achievements, which would not be picked up by controlling for hours of study. Alternatively,

better assimilated students might alienate their Indian classmates and, thus, miss out on positive spillovers in studying and practicing the material with each other. Overall, while I do not go into exploring the exact mechanism in detail, the results in Table 2.14 contribute to the migration literature by providing evidence of social assimilation's positive impact on migrants' productivity in the destination country.

## 2.8  Policy Implications and External Validity

Many western countries have acknowledged and made use of the fact that the manner in which migrants (and particularly refugees) are spaced geographically influences assimilation. Ultimately, migrant and refugee dispersal policies are nothing but attempts to manipulate the size and the structure of social network between new arrivals. Similarly, the tight control over skilled migration quotas that exists in many developed countries, often at a state or province level, allows for prevention of ethnic enclaves, making the migrant social space sparser. Sweden for much of its modern history had no pronounced dispersion policies (Bevelander et al. (2013)). In Denmark the refugee council aims at attainment of randomly created local ethnic clusters of 70-100 co-nationals (Damm, 2009a). In the UK dispersal means assigning people to available housing, which often results in refugees being grouped together in the most economically disadvantageous areas (Stewart 2009).

The nature of peer effects matters when deciding which dispersal policy is likely to be most effective. The complementary mechanism insures that migrants lower each other's costs and results in a social multiplier. Consequently, as long as they are not completely isolated from the locals, letting migrants form networks might be the best way to proceed. Instead of focusing on minimizing the exposure to co-nationals, governments should focus on harnessing the power of the social multiplier. If the mechanism is purely conforming, then there is no social multiplier, and potential sorting on hidden assimilation types could result in pockets of complete rejection of assimilation's very idea. In such case, a random dispersal that results in a small ethnic cluster (e.g. the Danish policy) might be the best way to go, because it reduces sorting opportunities.

By all appearances, the peer effects arise because the network of migrants is jointly determining eventual assimilation outcomes. Migrants share each others' local social ties, practice and study the language with each other. Crucially, this implies that for a fresh migrant merely being exposed to an already assimilated co-national might not be as effective as trying to navigate the new country side-by-side with other fresh migrants. Therefore, the most cost effective migrant dispersal policy might be to settle migrants with *similar* observed assimilation together in a cluster. Such policy would not only insure that migrants enjoy job

information spillovers between each other in the short run, but also help each other build productive linguistic skills and social capital that will benefit them long term. Consequently, providing language training and ability to interact with local volunteers is likely to not only improve assimilation of a particular migrant, but also indirectly the assimilation of her migrant friends through peer effects. The size of the estimated social multiplier suggests that relatively small interventions can have large cumulative effects if carried out properly.

This paper does not deal with implications of ethnic neighborhoods on migrants' own welfare. I do not attempt to come up with the counterfactual of an isolated migrant trying to assimilate. It is not inconceivable that a migrant left alone would make a bigger effort to assimilate in response to loss of support network. Consequently, I do not take a stand on what the optimal size or the structure of the migrant networks should be in order to maximize integration. My conjecture is that the optimal size should balance out the positive peer effect-type externalities that I document in this paper and the potential loss of intensity of integration effort due to increased access to fellow countrymen, documented by Danzer and Yaman (2013) or Chiswick and Miller (2002). Identifying such an optimal migrant group size seems like a fruitful area for future research.

*External Validity.* The importance of my results for policy depends largely on whether the conclusions carry over to a more general migration setting. I believe there are three main aspects of each migration episode that determine the pressure on migrants to assimilate themselves and to influence their co-nationals.

First, the nature of migration. All of the migrants in my sample are students. The circumstances of educational migration are rather different from those of a typical economic migrant or a refugee. On one hand, their stay of five years is relatively long, suggesting that they have time to reap the rewards of investing early into linguistic skills and social capital. On the other hand, almost none of them intend to stay in Kazakhstan upon graduation, and the usefulness of Russian language and Kazakh social ties while practicing medicine in India or first world countries is questionable. Overall, though, I think that migrating to study in English at a foreign university creates similar assimilation pressure to migrating for work at a foreign firm where most of the staff is local.

Second, the cultural distance between the two countries. The specifics of migrating to central Kazakhstan probably increase the pressure to assimilate, compared to migrating into Western countries. Few people are used to foreigners and even fewer speak English, creating a fertile ground for hostilities. Therefore, KSMU Indians might feel a higher need to assimilate as a kind of defense mechanism. On the other hand, Kazakhstan is a secular and religiously diverse country, with people of many faiths living together in peace, meaning that there is no threat of discrimination on religious grounds. Religion might play an important

role in assimilation (Lundborg (2013)). Indeed, with Islam being one of the main religions in Kazakhstan, students of Muslim background have somewhat different assimilation patterns. Overall, I do not think that KSMU Indians are any more likely to assimilate due to the particulars of the cultural difference between the two countries, than, for example, the Turkish guest workers in Germany.

Third, the specific structure of the migrant community. This is where migrants in my sample differ from a hypothetical 'average migrant' in several important ways. As evidenced by the BSER scores, they are strongly positively selected, suggesting that they might find it easier to learn the local language. More importantly, the community is extremely tight. There are many familial ties between members, and all of them have the same target - to finish the university in five years. Therefore, the community likely has collectively arrived at some kind of 'assimilation requirement' for successful graduation, with older cohorts helping younger ones in achieving it. Such alignment of ultimate goals probably also means that migrants are more willing to help and more efficient in helping each other, because they benefit equally from assimilation-boosting activities.

To sum up, I believe that migrants in my sample do not face a particularly high pressure to integrate, compared to an 'average migrant.' Therefore, it is unlikely that the *existence* of endogenous peer effects in assimilation is explained by the nature of the migration episode. However, a particularly tight nature of the community and the fact that all of the migrants are in Kazakhstan for the same purpose probably boost the effects' *strength*. Consequently, while peer effects in assimilation should exists in every migrant community, they are likely to usually be of lower intensity than the ones documented in this paper.

## 2.9    Concluding Remarks

In this paper I show that migrants in an ethnic cluster are not merely blocking each other's social assimilation simply by reducing incentives to exert assimilation effort. instead, they play a direct role in each other's assimilation by exerting peer effect-type externalities in acquisition of linguistic skills and local social capital. Therefore, assimilation decisions appear to be made jointly by the entire migrant community, which provides additional scope for assimilation policies. Moreover, the presence of the social multiplier means that properly managed ethnic clusters have the potential to result in higher assimilation outcomes than complete isolation of migrants at the destination.

My results are based on a cross section, making me rely on exogenous variation for identification. Repeating the data collecting exercise for several years in a row could result in a panel data set, in which each migrant cohort could be traced out through the entire stay

in Kazakhstan. That would permit a truly dynamic study of social assimilation, as well as investigation into the effects of cohort size in order to properly contrast my results with the literature that finds negative relationship between the size of ethnic cluster and assimilation.

Additionally, it would be desirable to carry out a field experiment with subsequent fresh cohorts in order to not only properly disentangle but also quantify the relative strength of competing peer effect mechanisms. Such information would allow better tailored assimilation policies.

Finally, testing whether my results are universal or an artifact of the specific migration episode is necessary. For that, a large scale replication of this paper involving a larger and less cohesive migrant community would be desirable. Such work, however, would invariably encounter difficulties in accurately mapping out the migrants' social space, measuring both language ability and local social capital, and finding a suitable identification strategy.

# Proof of Proposition

*Proof.* The proof follows the logic of Bramoulle et al. (2009) Proposition 4.

The AGs are not allowed to make up completely connected and separate components, because that would eliminate all of the variation in $(I - \tilde{G})GX$ for such group's members for any variable X.

Reduced form for $Y$ looks like:

$$Y = \beta(I - \gamma G)^{-1}X + \delta(I - \gamma G)^{-1}GX + (I - \gamma G)^{-1}\varepsilon$$

Therefore, reduced form for $(I - \tilde{G})Y$ looks like:

$$(I - \tilde{G})Y = (I - \tilde{G})[\beta(I - \gamma G)^{-1}X + \delta(I - \gamma G)^{-1}GX + (I - \gamma G)^{-1}\varepsilon]$$

Hence, two sets of structural parameters $(\beta, \gamma, \delta)$ and $(\beta', \gamma', \delta')$ lead to the same reduced form for $(I - \tilde{G})Y$ if and only if $(I - \tilde{G})(I - \gamma G)^{-1}(\beta I + \delta G) = (I - \tilde{G})(I - \gamma'G)^{-1}(\beta'I + \delta'G)$. Now, post-multiply both side of the equation by $(I - \gamma G)(I - \gamma'G)$ to obtain:

$$(I - \tilde{G})(I - \gamma G)^{-1}(\beta I + \delta G)(I - \gamma G)(I - \gamma'G) = (I - \tilde{G})(I - \gamma'G)^{-1}(\beta'I + \delta'G)(I - \gamma G)(I - \gamma'G)$$

Next, observe that due to the symmetry of $G$, matrices $(I - \gamma G)$, $(I - \gamma'G)$, $(\beta I + \delta G)$, and $(\beta'I + \delta'G)$ all commute:

$$(I - \tilde{G})(I - \gamma G)^{-1}(I - \gamma G)(I - \gamma'G)(\beta I + \delta G) = (I - \tilde{G})(I - \gamma'G)^{-1}(I - \gamma'G)(I - \gamma G)(\beta'I + \delta'G)$$

Canceling out the inverses and re-arranging the terms, obtain that the two sets of parameters lead to the same reduced form if and only if:

$$(\beta - \beta')(I - \tilde{G}) + (\delta - \delta' + \beta'\gamma - \beta\gamma')(I - \tilde{G})G + (\gamma'\delta - \gamma\delta')(I - \tilde{G})G^2 = 0 \qquad (2.17)$$

**If direction**.

Suppose, the matrices $(I - \tilde{G})$, $(I - \tilde{G})G$, $(I - \tilde{G})G^2$ are linearly independent. In that case for equation (2.17) to hold the following 3 equations must be satisfied:

$\beta = \beta'$

$\delta + \beta'\gamma = \delta' + \beta\gamma'$

$\gamma'\delta = \gamma\delta'$

Hence, the $\beta$'s must be equal for indeterminacy if the matrices are linearly independent. What remains is to prove that $\delta$'s and $\gamma$'s must also be equal. From the third equation it

is clear that there must exist $\lambda \neq 0$ such that $\gamma' = \lambda\gamma$ and $\delta' = \lambda\delta$. Substitute these back to the second equation to obtain: $\delta + \beta\gamma = \lambda(\delta + \beta\gamma)$. Hence, the equations hold only if $\lambda = 1$, which proves that if the matrices are linearly independent, then the reduced form is identified.

**Only if direction**.

Suppose matrices $(I - \tilde{G})$, $(I - \tilde{G})G$, $(I - \tilde{G})G^2$ are linearly dependent. In that case, $\lambda_1(I - \tilde{G}) + \lambda_2(I - \tilde{G})G = (I - \tilde{G})G^2$. But this means that there are only 2 equations to be satisfied by the three-parameter sets:

$\beta - \beta' + \lambda_1(\gamma\delta' - \gamma'\delta) = 0$
$\delta - \delta' + \beta'\gamma - \beta\gamma' + \lambda_2(\gamma\delta' - \gamma'\delta) = 0$

Hence, in case of linear dependence social effects are unidentified.

Extension of the proof to multiple control variables follows trivially from Bramoulle et al. (2009) and is omitted. $\square$

# Bibliography

[1] Abramitzky R., Boustam L.P., and K. Eriksson (2016):"Cultural Assimilation During the Age of Mass Migration," NBER working paper 22381.

[2] Adda J., Dustmann C., and J.-S. Gorlach (2014): "Migrant Wages, Human Capital Accumulation and Return Migration," working paper.

[3] Algan Y., Bisin A., Manning A., and T. Verdier (2013): "Cultural Integration of Immigrants in Europe," *Oxford Scholarship Online*.

[4] Angrist J. (2014): "Perils of Peer Effects." *Labor Economics*, 30, 98-108.

[5] Baker M., and D. Benjamin (1994): " The performance of immigrants in the Canadian labor market," *Journal of Labor Economics* 12(3), 369-405.

[6] Battisti M., Peri G., and A. Romiti (2016): "Dynamic Effects of Co-Ethnic Networks on Immigrants' Economic Success," NBER working paper 22389.

[7] Beaman L. (2012): "Social Networks and the Dynamics of Labour Market Outcomes: Evidence from Refugees Resettled in the U.S.," *Review of Economic Studies*, 79(1), 128-161.

[8] Bevelander P., Bilde R. H., Dahlstedt I., Eskelund M, Hansen L.M., Macura M., Pedersen K. G., and L. Ostby (2013): " Scandinavia's Population Groups Originating from Developing Countries: Change and Integration," Nordic Council of Ministers, TemaNord 2013:561.

[9] Borjas G. (1995):"Ethnicity, Neighborhood, and Human Capital Externalities," *American Economic Review*, 85(3), 365-390.

[10] Borjas G. (1998): "To Ghetto or Not to Ghetto: Ethnicity and Residential Segregation," *Journal of Urban Economics* 44, 228-253.

[11] Boucher V., and B. Fortin (2016): "Some Challenges in the Empirics of the Effects of Networks," *The Oxford Handbook on the Economics of Networks*, Chapter 12.

[12] Bramoulle Y., Djabbari H, and B. Fortin (2009): "Identification of peer effects through social networks," *Journal of Econometrics*, 150, 41-55.

[13] Bursztyn L., Ederer F., Ferman B., and N. Yuchtman (2014):"Understanding Mechanisms Underlying Peer Effects: Evidence From Field Experiment On Financial Decisions," *Econometrica*, 82, 1273-1301.

[14] Calvo-Armengol A., and M.O. Jakson (2004): "The Effects of Social Networks on Employment and Inequality," *American Economic Review*, 94(3), 426-254.

[15] Chetty R., Friedman J.N., Hilger N., Saez E., Whitmore Schanzenbach D., and D. Yagan (2011): "How Does Your Kindergarten Classroom Affect Your Earnings? Evidence from Project STAR," *The Quarterly Journal of Economics*, 126(4), 1593-1660.

[16] Chiswick B., Cohen Y., and T. Zach (1997): "The Labor Market Status of Immigrants: Effects of the Unemployment Rate at Arrival and Duration of Residence," *Industrial and Labor Relations Review*, 50 (2), 289-303.

[17] Chiswick B., and P. Miller (1995): "The Endogeneity between Language and Earnings: International Analyses,"Journal of Labor Economics, 13(2), 246-288.

[18] Chiswick B., and P. Miller (2002): "TImmigrant Earnings: Language Skills, Linguistic Concentrations and the Business Cycle,"Journal of Population Economics, 15(1), 31-57.

[19] Currarini, S., Jackson M. O., and P. Pin (2009): "An Economic Model of Friendhip: Homophily, Minorities, and Segregation" *Econometrica*, 77, 1003-1045.

[20] Cayers, B., and M. Fafchamps (2016): "Exclusion Bias in the Estimation of Peer Effects" working paper.

[21] Daam A.P. (2009a): "Ethnic Enclaves and Immigrant Labor Market Outcomes: Quasi-Experimental Evidence," *Journal of Labor Economics*, 27, 2, 281-314.

[22] Daam A.P. (2009b): "Determinants of Recent Immigrants' Location Choices: Quasi-Experimental Evidence," *Journal of Population Economics*, 22(1), 145-174.

[23] Danzer M.A., and F. Yaman (2013): "Do Ethnic Enclaves Impede Immigrants' Integration? Evidence from a Quasi-experimental Social-interaction Approach," Review of International Economics, 21(2), 311-325.

[24] De Giorgi G., Frederiksen A., and L. Pistaferri (2016): "Consumption Network Effect," NBER Working Paper 22357.

[25] Dustmann C., and F. Fabri (2003):"Language Proficiency and Labour Market Performance of Migrants in The UK," *The Economic Journal*, 113, 696-717.

[26] Dustmann, C., and J-S. Gorlach (2015): "The Economics of Temporary Migration," CReAM Discussion Paper CPD 03/15.

[27] Edin, P.-A., Fredriksson P., and O. Aslund: "Ethnic Enclaves and the Economic Success of Immigrants - Evidence from a Natural Experiment," *Quarterly Journal of Economics*, 118, 329-357.

[28] Gaviria, A., and S. Raphael (2001): "School-Based Peer Effect and Juvenile Behavior," *Review of Economics and Statistics*, 83, 257-268.

[29] Glaeser, E.L., Scheinkman J.A., and B.I. Sacerdote (2002): "The Social Multiplier," *Journal of the European Economic Association*, 1, 345-353.

[30] Jackson M. O. (2013): "Unraveling Peers and Peer Effects: Comments on Goldsmith-Pinkham and Imbens' "Social Networks and the Identification of Peer Effects"," *Journal of Business and Economic Statistics*, 31:3, 270-273.

[31] James E., Alsalam N., Conaty J.C, and D.-L. To (1989) "College Quality and Future Earnings: Where Should You Send Your Child to College?" *The American Economic Review*, 79(2), 247-252.

[32] Laschever, R. (2005):"The Doughboys Network: Social Interactions and Labor Market Outcomes of World War I Veterans," Mimeo, Northwestern University.

[33] Lazear E.P. (1999): "Culture and Language," *Journal of Political Economy* 107, 95-126.

[34] Lee L.-F.(2003): "Best Spatial Two-Stage Least Squares Estimators for a Spatial Autoregressive Model with Autoregressive Disturbances," *Econometric Reviews*, 22, 307-335.

[35] Lee L.-F., Liu, X., and X. Lin (2010): "Specification and estimation of social interaction models with network structures," *The Econometrics Journal*, 13 (2), 145-176.

[36] Liu X., Patacchini E., and Y. Zenou (2013), "Peer Effects: Social Multiplier or Social Norms?", *CEPR Discussion Paper* No. 9366.

[37] Loury L.D., and D. Garman (1993): "Affirmative Action in Higher Education," *The American Economic Review*, 83, 2, 99-103.

[38] Lundborg P. (2013): "Refugees' Employment Integration in Sweden: Cultural Distance and Labor Market Performance,"*Review of International Economics*, 21, 219-232

[39] Manski C. (1993): "Identification of Endogenous Social Effects: The Reflection Problem," *Review of Economic Studies*, 60, 531-542.

[40] Meng X., and R.G. Gregory (2005): "Intermarriage and the Economic Assimilation of Immigrants," *Journal of Labor Economics*, 23(1), 135-174

[41] Mui Teng Y., Koh G., and D. Soon (2015): "Migration and Integration in Singapore. Policies and Practice," *Routledge*.

[42] Munshi K. (2003): "Networks in The Modern Economy: Mexican Migrants in The U.S. Labor Market," *The Quarterly Journal of Economics*, 118(2) 549-599.

[43] Sacerdote B. (2001): "Peer Effects with Random Assignment: Results from Dartmouth Roommates," *The Quarterly Journal of Economic*, 116, 681-704.

[44] Stewart E. (2009): "The integration and onward migration of refugees in Scotland: a review of the evidence," UN Refugee Agency Policy Development and Evaluation Service research paper 174.

[45] Thiemann P. (2016): "The Persistent Effects of Short-Term Peer Groups in Higher Education," *Working Paper*.

[46] Topa G., and Y. Zenou (2014): "Neighborhood and Network Effects," CEPR Discussion Paper No. 10126.

[47] Zimmerman D.J. (2003): "Peer Effects in Academic Outcomes: Evidence from a Natural Experiment," *Review of Economics and Statistics*, 85(1), 9-23.

[48] Wooldridge J.M. (2002):"Econometric Analysis of Cross Section and Panel Data," *MIT Press*, p. 194-198.

# Chapter 3

# Zero Stars! Price, Quality, and Negative Word-of-Mouth

## 3.1 Introduction

Negative word of mouth (WOM), the act of telling others one's unpleasant experiences with a good or a service, is an important determinant of demand. One marketing study found that a single negative online review leads to a loss of about 30 customers on average[1]. The more severe the bad experience, the more likely the consumer is to engage in negative WOM instead of directly complaining to the manufacturer (Richins (1983)). A large body of research in marketing attempts to identify the best ways of dealing with bad reviews. In particular, it has been shown that lowering the price is not an effective strategy (Book et al. (2016)). In this paper I build on the work of Campbell (2013) to show that under negative WOM lowering the price often leads to a *reduction* in the consumer awareness about the monopolist's product. Instead, increased awareness can be achieved by a price hike. I also investigate how the intensity of negative WOM affects the optimal choices of quality and advertising.

There are three sets of results in the paper. First, I make use of the so-called *cavity method* (Newman and Ferrario (2013)) to find the demand for the product in an arbitrary social network of consumers who engage in both positive and negative WOM. I contribute to the literature on demand formation under WOM by showing that for any degree distribution, demand always falls in the intensity of negative WOM, but increases in network density. The second set of results covers monopolist's pricing behavior under negative WOM in several settings. I show that in dense networks negative WOM reduces the price elastic-

---

ity of demand, allowing the monopolist to charge a higher price compared to the situation where consumers are fully informed. So, the ability to share negative information ends up reducing consumer welfare. The intuition is that raising the price can serve as a 'vaccine' by reduces the negative WOM more than the positive WOM. I also show that whenever the monopolist can reduce the probability of bad reviews directly by selecting higher product quality, price and quality may be either compliments or substitutes, depending on the cost of quality-improving technology. This set of results has implications for antitrust and regulatory policies. The third group of model's predictions characterizes the negative WOM's relationship with formal advertising. I show that such WOM may induce a negative relationship between product quality and the level of informative advertising. A negative correlation of this nature is, puzzlingly, sometimes observed in markets where WOM is likely to be strong (Kwotka (1995)). For the targeted advertising, I prove that it is suboptimal to target the individual with the highest degree, and that the optimal degree increases with the intensity of negative WOM.

Few theoretical studies have explicitly considered negative WOM. Mahajan et al. (1984) extend the Bass model to allow for the possibility of both good and bad information flow in a diffusion setting. Their model only deals with the derivation of diffusion rates under different WOM regimes, and does not analyze firm's behavior or include a consumer network. In Section 2 I introduce a model that does have those ingredients. I derive the properties of demand in presence of negative WOM on a network of consumers characterized by an arbitrary degree distribution. Consumers are heterogeneous in their degrees and valuations for the good. They do not aggregate or actively search for information like they do in Galeotti (2010). Instead, following Campbell (2013), the WOM dynamics are governed by a statistical process whose parameters are controlled by the firm. Each consumer, who becomes aware of the good, decides whether to purchase it based on his valuation. The intensity of negative WOM is given by $\gamma$. Following a purchase, the consumer has either a positive experience with probability $\gamma$ or a negative experience. In case of a positive experience, consumer shares the product information with his social network friends, who may then choose to buy the product themselves (positive WOM). In case of negative experience, the consumer gives his friends a bad review. Any consumer who receives a bad review becomes discouraged and does not buy the product, regardless of the price or the number of positive reviews received. This assumption reflects the recurring empirical observation that consumers take negative reviews to be more trustworthy, making negative WOM more potent than positive WOM (Mizerski (1982), Skowronski and Carlston (1989), Herr et al. (1991), Bone (1995)). Discouraged consumers do not participate in WOM, creating bottlenecks and blocking transmission of positive information about the product.

I use the cavity method in order to compute the fraction of discouraged and the fraction of positively informed consumers upon completion of the WOM process. In a nutshell, the method involves removing a random node from the network (creating a cavity) and assuming existence of probabilities that the node's friends are either positively informed or discouraged by some *other* consumer. Doing so removes the need to worry about the direction of information flow between the consumer and his friends. Once the probabilities are assumed, they can be computed indirectly by deriving the so-called self-consistent conditions. The pair of interdependent self-consistent conditions, one each for the fraction of discouraged and the fraction of informed consumers, govern the WOM. The interdependence is the main friction in the model from the firm's point of view. WOM creates positively informed consumers if and only if it creates discouraged consumers. The price needs to be low enough to allow the positive information to spread, but lowering the price also stimulates negative WOM.

The above mechanism means that the fraction of positively informed consumers behaves in an unexpected manner with respect to price. If the price is low, then both types of WOM are very active, and the bottlenecks created by discouraged consumers are significant. Raising the price removes some of the bottlenecks and, paradoxically, *increases* the fraction of the population that is positively informed. Consequently, a price increase has three effects on demand. First, there is the regular reduction in quantity demanded due to fewer people willing to pay the higher price. Second, there an additional reduction in quantity demanded because positive WOM is suppressed. Third, the price increase means that fewer negative reviews are being shared and fewer people become discouraged, removing some of the bottlenecks in the information transmission process. In some networks the positive effect on demand of uncorking the bottlenecks overwhelms the reduced intensity of positive WOM, leading to an overall decrease in price elasticity compared to the fully informed case.

There are two ways to look at the probability of positive experience in my model. It may be viewed as the level of consumers' 'nastiness', i.e. their inherent proclivity to badmouth the product. Alternatively, it can be viewed as a measure of product's quality. For example, in 2016 a small number of reports about Samsung Galaxy Note 7 smartphone catching on fire spread virulently across the internet. The probability of the phone catching on fire is what I call quality. In Section 4 I investigate the monopolist's quality and price choices. The first result states that if the quality is exogenously given, and the network is dense, then the optimal price is higher under WOM than in case of fully informed population. This result is a reversal of Campbell (2013). Intuitively, when the social network is dense, each negative review can reach and discourage many consumers, making demand very inelastic in price. The monopolist uses the high price as a 'vaccine' against negative WOM. This result has implications for market regulation policies. The monopoly profit under negative WOM is

smaller than under positive WOM, but the price is higher. The firm's markup over marginal cost, therefore, is much smaller than what the high price and market share suggest. So, a competition authority may find it useful to estimate the level of negative WOM before imposing regulations or bringing up an antitrust case against the firm.

To the best of my knowledge there is no credible evidence of firms systematically increasing prices in response to negative WOM. Marketing literature, however, is replete with evidence that *lowering* the price in response to negative publicity is not an effective in a variety of contexts, because it fails in significantly increase quantity demanded (Book et al. (2016) in travel industry, Chatterjee (2001) in online retail, Ahluwalia et al. (2000) in sports apparel). This evidence is consistent with my finding that negative WOM makes demand very price-inelastic. One bit of anecdotal evidence in support of firms actually raising prices in response to NWOM concerns Samsung. After the PR disaster of its exploding Galaxy Note 7 phone, Samsung raised the prices on its newest flagship phone Galaxy S 8 by $100[2].

I also show that the relationship between price and the intensity of negative WOM depends on whether for each consumer the product valuation and the network degree are correlated. If they are independent, then the optimal price goes down in the intensity of negative WOM. On the other hand, when the most popular individuals also have the highest valuations, the optimal price goes up in the intensity of negative WOM. Intuitively, in case of independence, the more aggressive the negative WOM, the stronger the price hike 'vaccine' needs to be. However, when the most popular individuals also have the highest valuations, changing the price barely affects WOM. In such cases, the bulk of WOM is carried out by high degree consumers, whose high valuations make them immune to price changes. Consequently, the monopolist sets a high price when the negative WOM is intense in order to extract all of the surplus from high-valuation individuals, and a low price when the negative WOM is weak in order to attract the low-valuation consumers.

The final exercise in Section 4 is to analyze a joint price and quality choice. Continuing with the Galaxy Note 7 case, Samsung investing in further battery checks would be an example of choosing higher quality. The key result is that when such investment is cheap, price and quality are complements. So, whenever it is optimal to directly lower the intensity of negative WOM, it is also optimal to raise the price. However, if the technology is expensive, price and quality become substitutes. Intuitively, when the technology is cheap, the firm can afford to eliminate the negative WOM by choosing high quality. It can then pick a high price because the demand is very large at any reasonable price level. On the other hand, when the technology is costly, the firm can only afford to provide low quality. Negative WOM becomes very active, and the firm has to set a high price as a vaccine to salvage some profit.

---

[2]http://www.techradar.com/news/samsung-galaxy-s8-price

In Section 5 I investigate the impact of negative WOM on advertising. I view advertising as purely informative and, therefore, as a substitute for WOM. The results in that section contribute to the literature on firm's marketing strategies in presence of inter-consumer communication that includes both theoretical (Campbell (2013), Galeotti and Goyal (2009)), and empirical (Coulter et al. (2002), Van den Bulte and Joshi (2007)) work. I first consider fractional advertising, i.e. the firm providing the information directly to some portion of the population. I show that advertising and price are compliments, and both increase in the intensity of negative WOM. So, presence of advertising makes the optimal price rise in probability of bad reviews. The intuition is that when negative WOM is intense, there are very few positively informed consumers, and the firm needs to advertise heavily to have any demand at all. However, consumers who receive the information from the firm are still likely to give a bad review and discourage their friends, so the firm sets a high price as a vaccine. On the other hand, when negative WOM is weak, many consumers are positively informed, and the firm does not need to advertise extensively. Instead, it sets a low price to stimulate positive WOM. These results shed light onto the observation that advertising is sometimes negatively correlated with quality in markets where consumer reviews are likely to matter. For example, Kwotka (1984) finds the quality of eye examinations to be lower at practices that advertise.

The second application in Section 5 is to investigate whether the firm that has access to social network data should advertise to consumers with the highest degrees. The answer depends on the intensity of negative WOM. If the probability of bad reviews is high, then the firm should target the highest degree consumer, because he is likely to both be uninformed and to set off a WOM cascade should he like the good. If, however, the probability of bad reviews is low, then the high degree individual is likely to have received information through WOM, and the firm should instead target the low-degree consumers who are likely to be uninformed.

## 3.2   Negative WOM and Formation of Demand

In this section I introduce the model of negative WOM and derive the demand for the product. I also derive the expressions for the fractions of the population who participate in both negative and positive WOM as a function of price and quality.

### 3.2.1 The Setting

There is a population of consumers $N = 1, 2, ..., n$ with heterogeneous valuations of the good that the monopolist is trying to sell. The valuations $\theta_i$ are i.i.d. for each consumer and distributed uniformly on the $[0, 1]$ interval. Initially, all consumers are uninformed about the good's existence. Once they become informed, they buy it if and only if $\theta_i \geq P$. The benchmark demand for the fully informed population, therefore, is $(1 - P)$.

Consumers are arranged in a friendship network which is described by a graph consisting of $N$ nodes and $E$ edges, such that $E \subseteq \{(i, j) | i \neq j \in N\}$, where $(i, j)$ denotes a tie between two nodes. Each potential consumer, therefore, is described by both his valuation $\theta$ and his degree $k$. I denote by $\Phi(k, \theta)$ the joint distribution of these two dimensions of consumer heterogeneity. From the joint distribution, the marginal probability that a randomly picked person is of degree $k$ becomes:

$$p_k = \int_\theta \Phi(\theta | k) d\theta \tag{3.1}$$

Degrees and valuations are independent by assumption, but I study one important case of correlation in Section 4.3. The mechanics of WOM in this paper follow Campbell (2013). An individual engages in positive WOM, i.e. passes the information about the good to his friends with probability $v(\theta_i, P)$, which is a function of both the private valuation and the price set by the monopolist. Throughout this paper I maintain the simplifying assumption that the individual engages in WOM if and only if he buys the good.

$$v(\theta_i, P) = 1 - P \tag{3.2}$$

The main novelty of my model is to incorporate the possibility of negative reviews into the WOM process. I assume that the monopolist's product possesses a certain quality $\gamma < 1$ which the monopolist takes as given (I relax this assumption later by allowing for implementation of a quality-boosting technology). When a consumer buys the good of quality $\gamma$, he has a $(1 - \gamma)$ chance of having a negative experience with it. One can think of this as the probability of a new smartphone overheating, clothes coming apart at the seam, or a car being less fuel efficient than promised. The value of a 'bad' product to consumer is 0. By assumption, consumers are not aware of the possibility of bad product, and do not incorporate $\gamma$ into their expected utility of buying the good. If a consumer has a bad experience with the product, he gives a negative review to all of his friends. Receiving a negative review informs the friends of the product's poor quality, setting their valuation to 0 and discouraging them from purchase. As a result, the individuals who received a negative review do not participate in the WOM process (positive or negative) themselves. Implicitly, I assume that people only trust

those reviewers who have themselves used the product. The timing of the model is as follows:

(1) The monopolist picks price (and possibly quality $\gamma$) after forming the expectations of demand arising from the WOM process.

(2) Initially $\varepsilon \approx 0$ fraction of consumers become aware of the good.

(3) An informed individual $i$ buys the good if $\theta_i > P$. After buying, he either has a positive experience and gives positive information to his friends (probability $\gamma$) or has a negative experience and passes negative information to his friends (probability $1-\gamma$). The individuals who receive a positive news decide whether to buy the good based on their valuation. The individuals who receive bad news become 'discouraged,' do not buy the good and do not participate further in WOM.

(4) In case a consumer receives conflicting reports, he goes with the negative one and becomes discouraged.

(5) Steps 3 and 4 repeat until uniformed consumers no longer have a chance to become either informed or discouraged.

The setup and the timing so far is the same as in Campbell (2013), with the possibility of negative WOM being the only difference. My way of solving for the demand, while similar to Campbell's in that it relies on the generating function formalism, is somewhat methodologically different. I make use of the so-called 'cavity' method often used in statistical physics in order to solve graph theoretic problems (Newman and Ferrario (2013)). The method allows me to incorporate the negative WOM into the model in relatively straightforward way.

### 3.2.2 Demand under Negative WOM

In order to clarify the notation I begin by listing several properties of generating functions that I make use of extensively in this paper. This account draws heavily on Callaway et al. (2000), Newman (2005) and Newman et al. (2009). I re-direct to those papers any reader who wishes to obtain a more through understanding of how to apply the generating function formalism to solving various graph-theoretic problems.

Before proceeding, two issues need to be made explicit. First of all, the approach relies on assuming that the social network is represented by a random network with an arbitrary degree distribution. This is what is known in network science as the Configuration Model. The main idea is that $N$ nodes are created, each with a number of 'stubs.' i.e. initially empty

out-links. Each free stub of each node is then connected in a uniform random fashion with a free stub of another node until no free stubs remain. Such approach allows for unparalleled analytic tractability but comes at a cost of imposing restrictive structure on the network. In particular, it proves to be incompatible with certain properties of real life social networks, such as high level of clustering or low diameter. Intuitively, the reason is that when the network is large, the chances of any two connected nodes both randomly forming a link with the same third node go to zero. Clustering can be introduced into my model by using the methodology developed in Newman (2009), but this is not an exercise that I pursue.

The second issue is that the particular method that I am using in this paper relies not only on the assumption of random degree distribution, but also on the number of nodes $N$ going to infinity. This assumption results not merely in the absence of clustering, but also the probability of a cycle of any length going to zero. Consequently, the network exhibits a tree-like structure. The assumption reduces the problem's complexity by removing the need to account for the fact that some of the information that a consumer receives might have initially originated from that consumer. The advantage of the model is that all theoretical results on the expected spread of WOM are *exactly* right in the $N \longrightarrow \infty$ limit. Additionally, despite the restrictive assumptions, the configuration model has proven to perform well in predicting the size and the patterns of disease outbreaks (Newman and Ferrario (2013)).

A social network with degree distribution $\{p_k\}$ can be described by its probability generating function $g_0(.)$.

$$g_0(x) = \sum_{k=0}^{\infty} p_k x^k \tag{3.3}$$

The function is a polynomial in its argument, and its $k$'the coefficient is the probability that a randomly picked person has degree $k$ in the social network. In my subsequent calculations I will need to consider not only randomly chosen chosen nodes, but randomly chosen edges as well. The probability that a node at the end of a randomly chosen edge has degree $k$ is no equal to $p_k$. For example, there is no way to reach a node of degree 0 by following a random edge. Therefore, in order to express the distribution of degrees at the end of a randomly picked edge one needs to introduce a normalization. A properly normalized distribution distribution $\{q_k\}$ is known as the *excess degree* distribution and is derived as:

$$q_k = \frac{(k+1)p_{k+1}}{E(k)} \tag{3.4}$$

121

and is characterized by the probability generating function:

$$g_1(x) = \sum_{k=0}^{\infty} q_k x^k \qquad (3.5)$$

From equations 3.3 and 3.5 it is straightforward to obtain the following properties of generating functions which I will use extensively in subsequent analysis:

$$g_1(x) = \frac{g_0'(x)}{E(k)} \; ; \; g_0(1) = 1 \; ; \; g_0'(1) = E(k) \qquad (3.6)$$

The first task is to calculate the probability $G$ that a randomly picked consumer $i$ receives positive information about the good upon completion of the WOM process. The cavity method for carrying out such computation involves first considering the probabilities that any given neighbor of $i$, call him consumer $j$, receives the positive or negative information when $i$ is removed from the network (thus creating a cavity in network's structure). Formal definitions for these probabilities are:

**Definition 4.** *I call any neighbor $j$ of consumer $i$ "Externally Informed" if he receives positive and no negative information about the good when $i$ is removed from the network. I denote the probability of such event by u.*

**Definition 5.** *I call any neighbor $j$ of consumer $i$ "Externally Discouraged" if he receives negative information about the good when $i$ is removed from the network. I denote the probability of such event by v.*

Taking $u$ and $v$ as given, what is the probability that a randomly picked consumer is positively informed about the good? Being positively informed informed means receiving positive information from at least one friends *and* not receiving negative information from any of them. I compute this probability by backwards inductions. Suppose, $i$ has $I$ friends who are externally informed. Then

$$G_1 = \sum_{b=1}^{I} \binom{I}{b} ((1-P)\gamma)^b P^{I-b} = ((1-P)\gamma + P)^I - P^I \qquad (3.7)$$

is the probability that at least one of $i$'s $I$ externally informed friends buys the good (each with probability $(1-P)$) and has a positive experience with it (each with probability $\gamma$)), while all other externally informed friends do not buy the good (each with probability $P$).

Given $G_1$ it is easy to see that

$$G_2 = \sum_{I=1}^{k-d} \binom{k-d}{I} (uG_1)^I (1-u)^{k-d-I} = (u((1-P)\gamma+P)+1-u)^{k-d} - (uP+1-u)^{k-d} \quad (3.8)$$

is the probability that at least 1 out of $i$'s $k-d$ non-discouraged friends becomes externally informed, buys the good and has a positive experience with it, while all others either don't get externally informed or don't buy. Here, $d$ is the number of $i$'s externally discouraged friends. Next, consider the probability

$$G_3 = \sum_{d=0}^{k} \binom{k}{d} ((1-v)G_2)^{k-d} v^d = (1-(1-P)(1-\gamma)(1-v)u)^k - (1-(1-P)(1-v)u)^k \quad (3.9)$$

This is the probability that a randomly picked consumer of degree $k$ receives positive information from at least one and negative information from none of his friends. Finally, the desired probability of a random consumer becoming positively informed upon completion of the WOM process is simply $G_3$ averaged across all possible degrees:

$$G = \sum_{k=0}^{\infty} p_k((1-(1-P)(1-\gamma)(1-v)u)^k - (1-(1-P)(1-v)u)^k) = \quad (3.10)$$

$$= g_0(1-(1-P)(1-\gamma)(1-v)u)^k - g_0(1-(1-P)(1-v)u)^k$$

Because the degrees and valuations are uncorrelated, the probability that a randomly picked informed consumer has high enough valuation to buy the good is simply $1-P$. Consequently, the demand for monopolist's product is equal to:

$$D(P,\gamma) = (1-P)G \quad (3.11)$$

If the quality $\gamma$ is set to 1, then equation 3.11 is subject to the same system dynamics as Campbell (2013), despite being derived with a different technique. What remains is to derive expressions for $u$ and $v$. In words, $u$ is the probability that the consumer at the end of a randomly picked friendship gets positive information from at least one of his $k-1$ remaining friends and no negative information. Such probability is:

$$u = g_1(1-u(1-P)(1-v)(1-\gamma)) - g_1(1-u(1-P)(1-v)) \quad (3.12)$$

I refer to equation 3.12 as a *self consistency* condition for $u$. Similarly, $v$ is the probability that a consumer at the end of a randomly picked friendship receives negative information

**(a)** Social Network          **(b)** WOM process
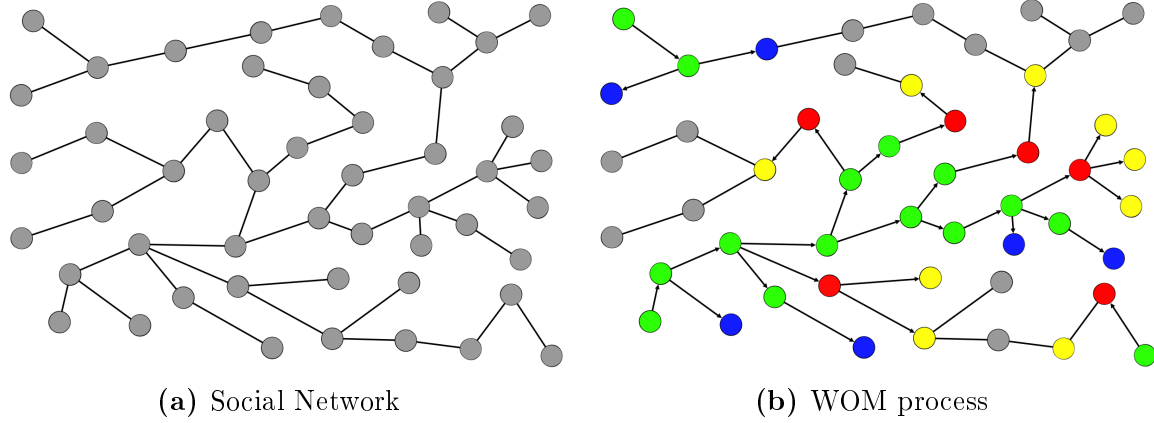
**Figure 3.1:** Realization of WOM. Green nodes - bought the good and liked. Red nodes - bought and disliked. Yellow nodes - discouraged. Blue - informed, did not buy. Grey - not informed.

from at least one of his remaining $k-1$ friends. Using similar logic as equation 3.10, the self consistency condition for $v$ can be derived as:

$$v = 1 - g_1(1 - u(1-P)(1-v)(1-\gamma)) \tag{3.13}$$

A sample outcome of the WOM process defined by equations 3.11-3.13 is depicted in Figure 3.1 for the same network as in Campbell (2013) Figure 1. Note that, as discussed before, the network in the figure exhibits a tree-like structure and has no cycles. Figure 3.1 panel (b) features an agglomeration of green dots. These dots form the so-called *giant component*. In this basic version of the model, a giant component needs to form in order for the monopolist to have a positive level of demand. If the giant component does not form, then demand is zero, because the pockets of informed people are going to be localized, while the size of the population goes to infinity. The monopolist can stimulate the WOM by lowering the price. Intuitively, the more people buy, the more people talk, and the more consumers become informed. Equations 3.12 and 3.13 together define a system in price $P$ and quality $\gamma$ which exhibits a non-linear phase transition in terms of the emergence of the giant component. Proposition 8 formalizes this point.

**Proposition 8.** *When $\theta$ and $k$ are uncorrelated, the critical price $P^c$ below which WOM creates positive fractions of informed and discouraged consumers is given by:*

$$P^c = \frac{g_1'(1)\gamma - 1}{g_1'(1)\gamma} \tag{3.14}$$

Equation 3.14 defines as a function of $\gamma$ the upper threshold on the price that the monopolist can charge while still maintaining non-zero demand. Above that critical price *both*

**(a)** $\gamma = 0.9$  **(b)** $\gamma = 1$ (Campbell)
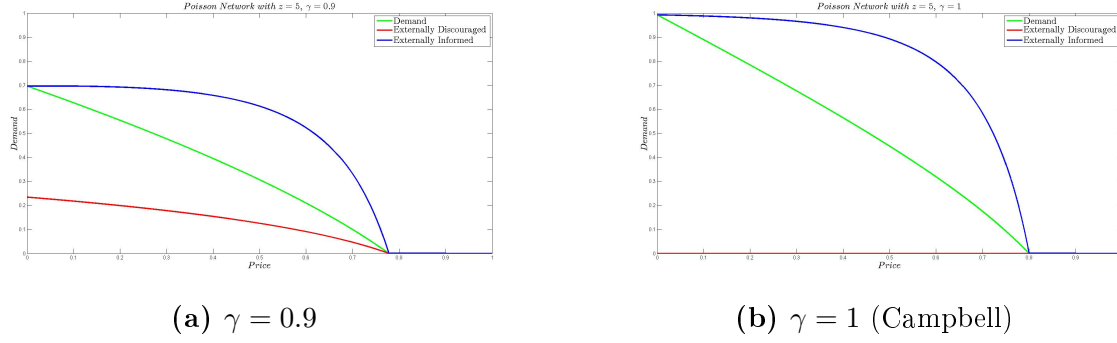
**Figure 3.2:** WOM dynamics comparison for different $\gamma$.

$v$ and $u$ are equal to 0. So is the demand, because if no one is informed, no one buys. If no one buys, no consumer has a negative experience, so the fraction of population that is discouraged also tends to 0. Therein lies the key friction in the model from the perspective of the monopolist. Low enough price has to be chosen in order to stimulate demand, but as soon as demand becomes positive, so does the fraction of discouraged population. Because discouraged people do not buy or participate in WOM, large chunks of the network are excluded from the market with no access to information.

The critical price grows in $\gamma$. As quality improves, the negative WOM becomes less intense, and it is possible to sustain non-zero demand at higher prices. Therefore, if the firm can choose both price and quality, lowering the price and raising the quality appear to be substitutes when it comes to stimulating demand. In Section 4.2 I expand on this intuition and show that whether it is correct depends on the cost of quality-boosting technology.

Figure 3.2 provides a comparison as a function of price between the outcome of the negative WOM process and the case of $\gamma = 1$ when consumers necessarily have a positive experience. Clearly, the fraction of externally discouraged people is zero when there is no possibility to share negative information. However, with $\gamma < 1$ the fraction monotonically falls in price, so the monopolist faces a trade-off between increasing demand and increasing the discouraged population at any price level. Panel (a) in Figure 3.2 has $\gamma = 0.9$, which is close to 1. Yet the maximum possible demand at zero prices is much smaller in that case than in the $\gamma = 1$ case. So, even a small probability of negative WOM can create large reductions in demand. Proposition 9 formalizes some patterns of demand's behavior.

**Proposition 9.** *When $\theta$ and $k$ are uncorrelated the demand $D(P, \gamma)$ has the following properties:*

*(i) $D(P, \gamma)$ is continuous in both arguments. Moreover, $D(P, \gamma) = 0$ whenever $(1 - P)\gamma g_1'(1) \leq 1$ and $D(P, \gamma) > 0$ otherwise.*

(ii) $\frac{\partial D}{\partial P} < 0$ *whenever* $(1-P)\gamma g_1'(1) > 1$

(iii) $\frac{\partial D}{\partial \gamma} > 0$ *whenever* $(1-P)\gamma g_1'(1) > 1$

(iv) *There exists $\underline{\gamma}$ such that the elasticity of $D(P, \gamma)$ is smaller than the perfectly informed elasticity $P/(1-P)$ whenever $\gamma \in (\underline{\gamma}, 1)$, $P \longrightarrow 0$, and greater otherwise.*

Unsurprisingly, demand is only non-zero when condition 3.14 for emergence of a giant component is satisfied. Demand falls in price despite the fact that so does the fraction of discouraged and uninformed population. So, negative WOM does not create a Giffen good. Demand necessarily increases in quality $\gamma$, because higher quality reduces the amount of discouraged people and, hence, the bottlenecks that prevent the spread of good information. For low price/high quality combination WOM makes demand *less* elastic than the case of complete information.

I am not able to derive an analytic result on the effect of an increase in average degree on demand. However, simulations with Poisson degree distribution show that in presence of negative WOM increasing the average degree boosts demand at high price level and reduces demand at low price level. The former happens because higher degree raises the critical price (Equation 3.14). Thus, positive demand is sustained where it would have been zero in less dense networks. The latter happens because at low price level higher average degree allows for negative WOM to spread fast, creating a large problem of uninformed or discouraged consumers.

## 3.3 Discussion of Model's Assumptions

Several assumptions are made in this paper in order to make the model tractable and to highlight the key frictions that negative WOM introduces into the firm's problem. I this section I attempt to briefly justify some of those assumptions.

*Monopoly.* Absence of competition is, of course, a very special environment for the firm to be in. However, most markets where WOM truly has bite, such as markets for experience or luxury goods and services, are characterized by some degree of market power. Extending the model to a Bertrand duopoly would reduce the firms' ability to manipulate WOM through price but wouldn't fundamentally change the process. My guess is that in such case most predictions would carry over but become less sharp.

*Nature of positive WOM.* Even though I am using a slightly different method of deriving demand, the positive information transmission process remains unperturbed compared to the Campbell (2013) model. In his work Campbell performs several extensions and generalizations of the WOM protocol, by making it more sparse or altering transmission probabilities.

None of those extensions significantly change his results, which leads me to conclude that the basic framework described in the previous section adequately captures the key elements of the WOM process. Consequently, I do not pursue any extensions of the positive WOM.

*Nature of negative WOM.* The key assumption in the model is that once a customer has a bad experience with the product, he immediately discourages all of his friends from purchasing it. This might seem extreme, but because the process is considered in the limit of a large network, $\gamma$ may also be viewed as a probability that the customer who bought the good discourages any one of his neighbors in separation. Additionally, the assumption in my model that negative WOM trumps positive WOM is supported by plenty of empirical evidence (Mizerski (1982), Skowronski and Carlston (1989), Herr et al. (1991)). No amount of good reviews are going to convince a person to buy the car that has been known to spontaneously run out of gas in the middle of a driveway.

*Exogenously given* $\gamma$. In many sections of this paper I take quality $\gamma$ as exogenously given. There are several ways to justify such approach. First, quality is typically considered to be a long-run choice that, unlike price, is difficult to alter in response to WOM. Second, the firm might know that the product is good by not perfect ($\gamma \longrightarrow 1$), but that fixing it would be prohibitively costly. Third, $\gamma$ can be viewed as the fraction of hard-to-please customers who always choose to badmouth the small flaws in the product. Even in this latter case the firm might still have a good idea of the value of $\gamma$. For example, by looking at the product's average rating on Amazon or similar platforms.

*Naive consumers.* Consumers in the model make decisions following rules-of-thumb. Allowing them to anticipate the bad experiences, or to search for and aggregate the information in a more sophisticated way would make the model immensely complicated. Introducing these elements is of interest for future research. There is also no strategic concern on behalf of the monopolist due to absence of repeated interactions. This assumption has one of two possible implication for the applicability of my results. Either the model describes a durable good monopolist, or it is an approximation of a repeated-interaction case where all of consumer's history with the firm is taken as one 'transaction.'

## 3.4   Pricing under Negative WOM

Pricing decisions by firms are among the most fundamental acts of economic decision making. Pricing in presence of negative WOM has not been theoretically studied before and is of interest in today's economy, where purchasing decisions are increasingly based on peer reviews. In this section I study monopolist's pricing choices in three different environments.

### 3.4.1 Pricing Given Quality

In this section I assume that degrees and valuations are uncorrelated, and that quality $\gamma < 1$ is set exogenously, so the monopolist has no option to implement a quality-improving technology. This assumption corresponds to a situation when some features of the product are going to disappoint some fraction of people no matter what the monopolist does.

Because quality is fixed, the only choice variable that the monopolist has to control both negative and positive WOM is the price. On top of the usual effects, increasing the price here affects the fractions of externally discouraged and externally informed people. The monopolist, therefore, has to choose the price that maximizes the profits given the expectations of the behavior of these two fractions. Using the expression for demand derived in Section 2, the monopolist solves the following problem:

$$\max_P (1 - P)(P - c)[g_0(1 - u(1 - v)(1 - \gamma)(1 - P)) - g_0(1 - u(1 - v)(1 - P)] \quad (3.15)$$

$$s.t. \ (1 - P)\gamma g_1'(1) > 1$$

In case of the full information benchmark, the negative WOM does not play any role, so demand is $1 - P$, and the optimal price is $P_{FI} = \frac{1+c}{2}$. Proposition 10 compares the optimal pricing outcome of this model against the full information and the Campbell ($\gamma = 1$) benchmarks.

**Proposition 10.** *Suppose valuations are uncorrelated with degrees and $(1 - P)\gamma g_1'(1) > 1$. Then, the following things are true:*

i) *For large $E(k)$ there exists $\underline{\gamma}$ such that $\forall \gamma \in [\underline{\gamma}, 1)$, the optimal price chosen by the monopolist $P^*_{NWOM}$ is greater than the fully informed choice $P_{FI}$. In all other cases, $P_{FI}$ is larger.*

ii) *For large $E(k)$ the optimal price $P^*_{NWOM}$ is an upside-down parabola in $\gamma$. In all other cases $\frac{\partial P^*_{NWOM}}{\partial \gamma} > 0$*

Part one of the proposition states that for high enough average network degree negative WOM might encourage the monopolist to charge a higher price than in the fully-informed (and, therefore, also Campbell) case. In this setting the quality is given exogenously, so this result can also be viewed as the possibility of negative WOM harming consumers. If they could stop sharing negative information, then more of them would buy the product and do so at lower prices.

The optimal price chosen by the monopolist may be higher than the fully informed case because in some circumstances negative WOM reduces the price elasticity of demand. In
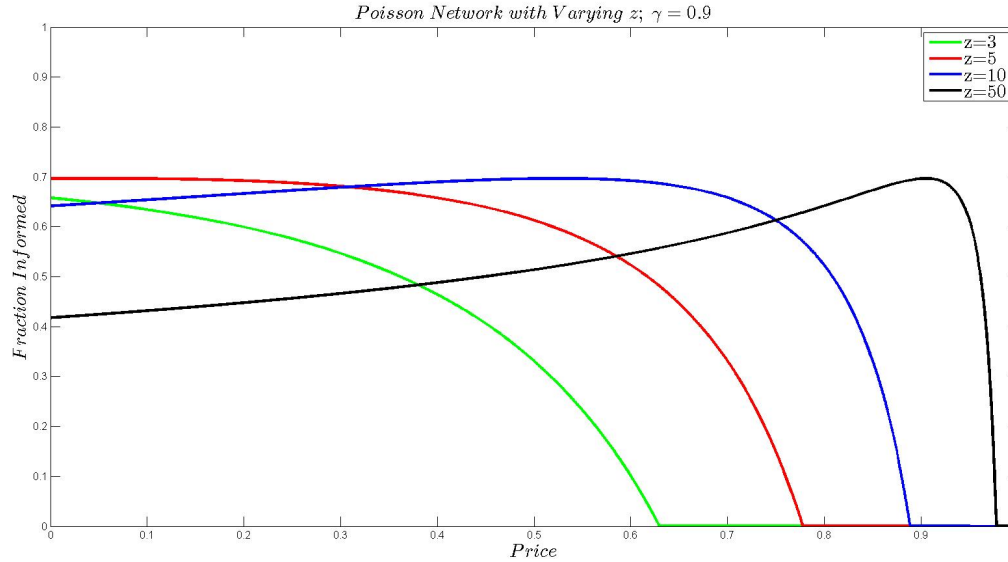
**Figure 3.3:** Fraction of externally informed as a function of price as network density varies.

order to understand the intuition behind this result one needs to consider price's effect on the fraction of externally informed consumers $u$. Without the negative WOM the effect is invariably negative - $u$ falls in price, because less people buying the good means less people talking about the good. In presence of negative WOM this is no longer true. As price goes up, fewer people buy the good, leading to fewer negative experiences, fewer discouraged consumers, and removal of bottlenecks in the information transmission process. This point is further illustrated by Figure 3.3. In order to maximize the fraction of informed consumers the monopolist should pick $P = 0$ if average degree is small, but $P \longrightarrow 1$ if it is large. Intuitively, under negative WOM the fraction of discouraged and uninformed individuals will be very large in dense networks because bad news spread fast. In such networks increasing the price acts as a 'vaccine' against the bad news. A price increase, then, does three things to the demand. It suppresses the fraction of discouraged consumers, increases the fraction of informed consumers and reduces the fraction of informed consumers who value the good enough to buy it. The net effect is to reduce demand but at a slower rate than in a fully informed case, reducing the elasticity and allowing monopolist to choose a higher price. This logic, of course, is only true when $\gamma$ is bounded away from 0. If the quality is really low, then $P_{FI}$ is above the critical price, and the monopolist is forced to pick something lower.

The second part of Proposition 10 gives a comparative static of the optimal price with respect to exogenous quality. This relationship also depends on the average network degree. If the average degree is small, price grows in quality. If the average degree is large, then the price grows for low quality but falls as quality approaches 1. Here, again, the 'vacci-

129
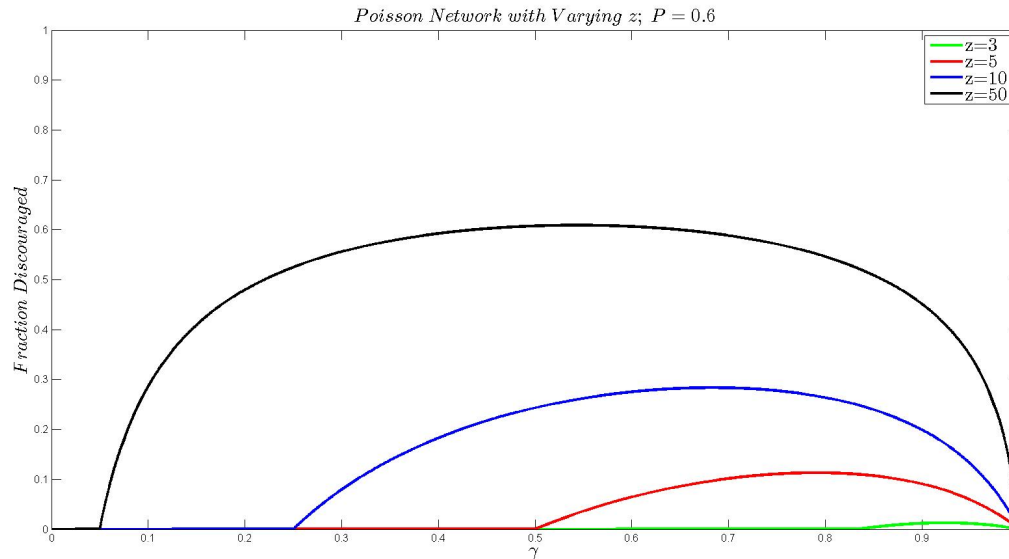
**Figure 3.4:** Fraction of discouraged as a function of $\gamma$ as network density varies.

nation' intuition rings true. However, it is now the fraction of discouraged consumers that exhibits an interesting non-linearity. Figure 3.4 show on an example of a Poisson network that fraction $v$ is inversely U-shaped in $\gamma$. The reason for why $v$ initially grows in quality is that in order to have discouraged consumers someone in the population must have bought the product in the first place, so the fraction grows together with the fraction of informed population. It is only at high $\gamma$ values that the fraction of discouraged consumers starts to fall, because there is hardly anyone having a negative experience after buying the good. Consequently, as $\gamma$ rises, it initially *reduces* the price elasticity of demand by reducing the 'vaccination' effect of a price increase. This reduction in elasticity is more pronounced the higher the average degree (Figure 3.4), so the monopolist's optimal price grows even faster with $\gamma$. However, $v$ vanishes when $\gamma \longrightarrow 1$, so at some point it starts falling in quality. In case of high average degree, the fall is rapid enough for the price elasticity of demand to start increasing in $\gamma$, inducing a lower optimal price.

Monopolist's profits are lower in case of $\gamma < 1$ than in Campbell's $\gamma = 1$ case, even though the price is higher. The upshot is that if a firm with market power is facing a moderate amount of negative WOM in a dense social network, it has to raise the price as a way of attenuating consumer discouragement. Yet, its economic profits are relatively small and the demand is inelastic, so it might not be wise to bring up an antitrust case against the firm or impose regulations on it. Such measures would only bring small efficiency gains.

Clearly, the average degree within the social network of consumers is an important determinant of the monopolist's pricing decisions. I am not able to derive a general comparative

static for the effect of average degree on price, but the following proposition gives a result for Poisson networks and appears to hold in simulations with other degree distributions.

**Proposition 11.** *Suppose valuations and degrees are not correlated, and the degree distribution is Poisson with mean $z$. Then $\frac{\partial P^*_{WOM}}{\partial z} \geq 0$.*

The monopolist's optimal price choice goes up in average network degree when negative WOM is present. The intuition behind this result can also be discovered in Figure 3.4. As long as some negative WOM is possible, higher average degree necessarily increases the fraction of discouraged consumers, introducing bottlenecks that reduce the proportion of informed population. This mechanism reduces price elasticity of demand, increasing the optimal price chosen by the monopolist.

Results in this section are particularly insightful in light of the recent explosion of social media and social networking sites. Since $\gamma$ is endogenously given, one might interpret it not as quality but simply as the propensity of consumers to share with each other the information on certain less-than-perfect features of the product. Each person now can maintain many more connections than in past decades, and the average degree of an individual in a typical social network keeps growing. Consequently, Proposition 11 means that consumers are hurt by the opportunity to vent frustrations about the product to all 5,000 of their Facebook friends. Negative WOM increases the prices and reduces information.

### 3.4.2   Choosing Price and Quality Menu

I this section I abandon the assumption that quality $\gamma$ is exogenously given. Instead, the monopolist can now choose the price and quality combination that maximizes profits in presence of negative WOM. It is, therefore, possible to choose $\gamma = 1$ and eliminate bad reviews altogether, bringing the model in line with Campbell (2013). I model the cost of quality improvement as a constant increase $c_q$ in the *marginal* cost of producing a unit of the good. The demand is derived from the WOM process in the same way as in Section 2. So, the monopolist's problem now becomes:

$$\max_{P, \gamma} (P - c - c_q\gamma)D(\gamma, P) \tag{3.16}$$

$$s.t. \ (1 - P)\gamma g_1'(1) > 1$$

Where $D(\gamma, P)$ is given by equation 3.11. One can think of $\gamma$ selection as a decision on how many costly steps to include in the firm's quality control mechanism. For example, a manufacturer of smartphones could implement an additional screen durability test.

131

Modeling the monopolist's behavior in such way introduces another trade-off into the problem. Choosing low $\gamma$ is cheap but results in a large number of discouraged people, increasing the monopolist's incentives to 'vaccinate' by raising the price. Setting $\gamma = 1$ means eliminating all bad reviews, and immediately boosting demand. However, the high marginal cost raises the price, suppressing positive WOM and significantly lowering demand. Choosing the right combination of price and quality in this setting, therefore, is a matter of picking the right strategy to combat negative WOM. A priori it is not clear whether price and quality are complements or substitutes. In other words, it is not clear whether negative WOM guarantees high product quality at high prices. The following proposition characterizes some properties of the monopolist's optimal choice of price and quality menu:

**Proposition 12.** *Suppose degrees and valuations are independent. Then the following things are true:*

(i) *If $c_q \longrightarrow 0$, then $\gamma^*_{NWOM} = 1$, $P^*_{NWOM} \longrightarrow P^*_{Campbell} < P_{FI}$.*

(ii) *There exists $\overline{c_q}$ such that price and quality are substitutes for $c_q \geq \overline{c_q}$.*

(iii) *If degree distribution is Poisson, and $c_q > \overline{c_q}$, then $\frac{\partial P^*_{NWOM}}{\partial E(k)} > 0$, $\frac{\partial \gamma^*_{NWOM}}{\partial E(k)} < 0$.*

The first part of the proposition simply establishes the baseline. As long as quality is relatively costless, the monopolist chooses to eliminate negative WOM and stimulate positive WOM through low prices. Demand always grows in $\gamma$, so by implementing such menu monopolist obtains the largest demand at the lowest cost.

The statement in part (i) of the Proposition is the basis for part (ii). With costless quality-boosting technology, consumers enjoy low prices and high quality. As the cost becomes large, price increases and quality drops. Figure 3.5 provides an illustration of how the optimal price and quality menu changes with respect to the cost of technology. The intuition is the following. Increasing $c_q$ puts downward pressure on the optimal choice of $\gamma$. However, because the choice is constraint at $\gamma = 1$, the firm initially still chooses to completely eliminate negative WOM. Increasing cost of such action makes the firm charge a higher price to make up for lost profits. Once the cost of technology becomes too large to maintain $\gamma = 1$, price and quality become compliments, because the monopolist now faces negative WOM and has to lower the price to stimulate the WOM's positive component. Finally, as $c_q$ becomes extremely large, price and quality become substitutes. The intuition is that it is now very costly to achieve quality, so the monopolist hardly provides any. Low quality creates huge bottlenecks in positive information transmission, so the monopolist has to resort to setting a high price as a 'vaccine'.

Of course, in absence of reputation concerns or repeated interactions, the monopolist
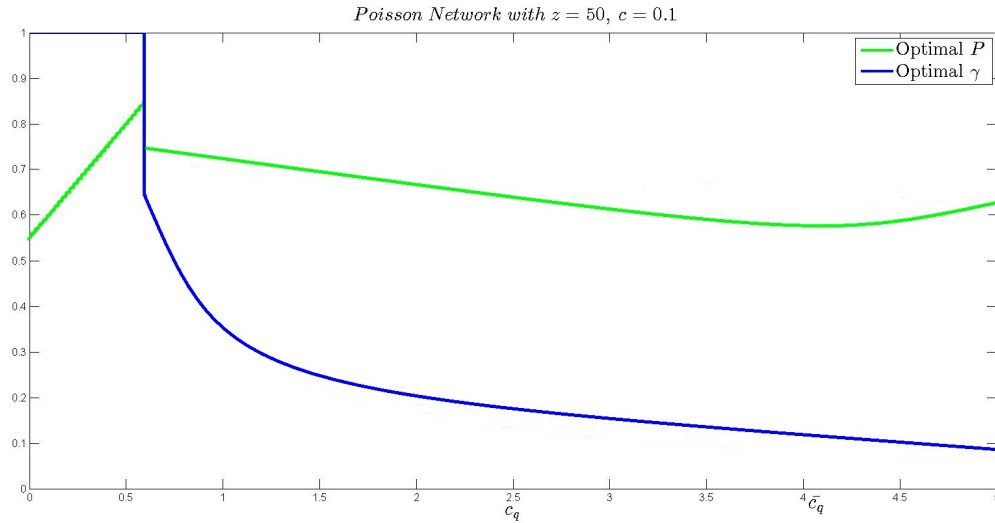
**Figure 3.5:** Monopolist's optimal choice of price and quality as a function of technology cost $c_q$.

would have no incentive to invest anything into improving the quality if the population were fully informed. In that sense, the possibility of engaging in the negative WOM allows the consumers to keep the monopolist honest and forces quality provision. Part (ii) of the Proposition also suggests that when quality is very costly, a partial cost subsidy from the authorities might be effective in raising consumer welfare, as it would both increase the quality and lower the price of the good.

Part (iii) of Proposition 12 says that the intuition from the previous section is preserved when the monopolist can choose both quality and price. As the average degree in the social network goes up, optimal price chosen by the monopolist always increases. When price and quality are substitutes, a price increase means a fall in quality. This divergence happens because in denser networks the problem of discouraged customers becomes more severe, so the monopolist has to raise the price in order to alleviate some of it. The quality improving technology is so expensive, that the optimal quality choice is already low, and lowering it a little further would not reduce the demand much, but would significantly decrease the cost. Consequently, in this setting negative WOM means that growing network density both lowers the quality and raises the price, imposing a double toll on consumer welfare. On the flip side, if the cost of improving the quality is low enough, then denser networks force the monopolist to both provide the quality and increase the price.

133

### 3.4.3 Pricing with Correlated Valuations and Degrees

In this section I revert to the assumption that quality $\gamma$ is given. Instead, I relax another assumption by imposing a correlation between consumers' valuations and their social network degrees. As Campbell (2013) points out, certain goods are more valuable to people with a lot of connections. Smartphones or video game consoles are more useful to people with high degree in the social network. Large part of the appeal of various luxury items is an opportunity to showcase them to others. In recent years, with the advent of celebrity bloggers and YouTube stars, firms have become more susceptible to the whims of the popular. A system in which stars with high valuations for luxury goods express their opinions to their armies of social network followers is now firmly in place. For example, Calvin Klein, a fashion designer once said that his company "booked (models) because of how many followers they have online.[3]"

Clearly, in case of products described above it is no longer true that valuations and degrees are independent. People with high network degrees are likely to value the product highly as well. Formally, there is joint distribution of degrees $k$ and valuations $\theta$ given by $\Phi(\theta, k)$. So, even though I still assume the marginal distribution of valuations to be uniform, it is no longer true that at price $P$ a randomly picked person of degree $k$ buys the product and engages in positive WOM with probability $(1 - P)\gamma$. Instead, the probability is given by

$$b_k = \gamma \int_P^1 \phi(\theta|k)d\theta \tag{3.17}$$

Similarly, the probability that a node at the end of a randomly picked link buys the product now becomes

$$B = \sum_{k=0}^{\infty} q_{k-1} \int_P^1 \phi(\theta|k)d\theta \tag{3.18}$$

Where, as before, $\{q_k\}$ is the excess degree distribution such that $q_k = \frac{(k+1)p_{k+1}}{<k>}$. Using equation 3.18, one can derive the probability that the consumer with excess degree $k$ receives at least one positive review and no negative reviews as:

$$G_c = (1 - u(1 - v)B(1 - \gamma))^k - (1 - u(1 - v)B)^k \tag{3.19}$$

---

[3]http://www.forbes.com/sites/declaneytan/2016/05/11/are-fashion-bloggers-able-to-convert-followers-into-buyers/#7ead1f895fdd

Combining the two equations above yields the demand for the firm's product in this more general setting:

$$D(P) = \sum_{k=0}^{\infty} p_k \int_P^1 \phi(\theta|k)d\theta \left[(1 - u(1-v)B(1-\gamma))^k - (1 - u(1-v)B)^k\right] \qquad (3.20)$$

The self consistent conditions become:

$$u = \sum_{k=0}^{\infty} q_k \left[(1 - u(1-v)B(1-\gamma))^k - (1 - u(1-v)B)^k\right] \qquad (3.21)$$

and

$$v = 1 - \sum_{k=0}^{\infty} q_k(1 - u(1-v)B(1-\gamma))^k \qquad (3.22)$$

In this framework negative WOM adds a further layer of complication to the monopolist's problem. If high valuation consumers also have high degrees, then the WOM process is extremely active, because the high degree nodes buy the product even when prices are high and then share reviews with their many connections. However, should one of the 'stars' dislike the good, the consequences of negative WOM become more severe, since the amount of discouraged people grows fast. Intuitively, positive correlation between degrees and valuations alters the monopolist's ability to use price as a 'vaccine' and changes the pricing strategy with respect to the negative WOM. The following Proposition makes this point more clear using a concrete $\Phi(\theta, k)$. Proving a more general result doesn't seem feasible, but the proposition appears to hold in simulations with other correlation structures.

**Proposition 13.** *Suppose the joint distribution of degrees and valuations $\Phi(\theta, k)$ is such that $\phi(1|\theta < \underline{\theta}) = 1$ and $\phi(k^*|\theta > \underline{\theta}) = 1$. Then, for high enough $k^*$, $\frac{\partial P^*}{\partial \gamma} < 0$.*

The distribution in the proposition describes a network in which there are only two types of consumers. There are stars with high degree whose valuation for the good is at least $\underline{\theta}$, and peripheral consumers of degree 1 with low valuation for the good is less than $\underline{\theta}$. In such star-follower setting the optimal price rises in the intensity of negative WOM, which is in contrast with the basic setting in Section 4.1.

In order to understand the intuition for this result one first needs to understand the information flow structure. Only 3 types of network segments are possible. First, there may be diads of customers which never become informed. Second, there may be segments made up of 1 star and $k^*$ followers. These segments never receive the information either. Almost all consumers, however, belong to the third type - a large component with many stars connected to each other. These large components are the only portion of the network where

**(a)** $\gamma = 0.8$                                            **(b)** $\gamma = 0.9$
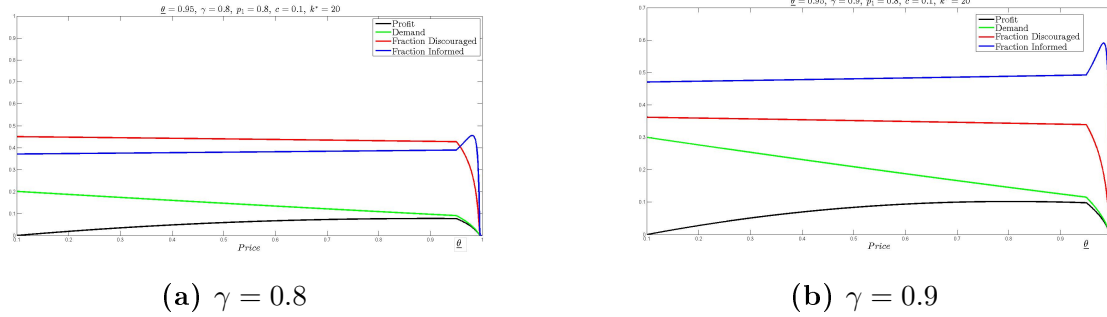
**Figure 3.6:** Demand, profits and information passage as a function of price.

WOM is active. Peripheral nodes play no role in the WOM because they have to receive the information from their stars and, once they do, they have no one left with whom to share the information. Consequently, *all* of the WOM in this setting is carried out by the stars.

Figure 3.6 displays the evolution of demand and fractions $v$ and $u$ with price in this setting for different $\gamma$. The WOM process evolves in the following manner. Up until the price reaches $\underline{\theta}$, the fraction of informed and discouraged individuals remains stable. The reason is that on that segment the stars always buy the good if informed. Since all of the meaningful WOM communication is carried out between them, the amount of WOM remains unchanged, and fractions $u$ and $v$ are stable for $P < \underline{\theta}$. The demand on that segment is simply a downward sloping line reflecting the drop-off in purchases by the peripheral consumers. For $P > \underline{\theta}$ Figure 3.6 shows an immediate spike in the fraction of informed consumers. The reason the spike occurs is that beyond that price some stars decide not to buy the good, so they no longer can discourage their friends in case of a bad experience.

From the discussion above it is clear that in this setting, unlike Section 4.1, the monopolist has very limited use for price as a 'vaccination' instrument. The only act of vaccination that would make a difference against negative WOM is to raise the price above the $\underline{\theta}$ threshold. Varying the price below the threshold does not achieve anything as far as manipulating consumer information goes. Because most of the consumers are peripheral, monopolist does not find it profitable to cut them off by setting the price too high, and always chooses $P < \underline{\theta}$, so the vaccination motif is removed entirely. Why does the optimal choice of price go down in $\gamma$? Setting $P < \underline{\theta}$ ensures that almost all stars buy the good. If $\gamma$ is low, many of them have a bad experience and discourage their armies of followers. Since those discouraged followers are not going to buy the good, the monopolist might as well extract most surplus from the stars by setting $P \longrightarrow \underline{\theta}$. On the other hand, if $\gamma$ is high, then almost all peripheral consumers receive glowing reviews from their stars, so it makes sense for the monopolist to attract this positively informed majority by lowering the price.

Proposition 6 implies that in the star-follower setting, which is typical of today's WOM

136

networks, consumers are punished with higher prices for engaging in negative WOM. Simulations show that the problem becomes even more severe if the fraction of stars in the population grows or stars obtain more followers. So, just like in Section 4.1, increasing the average network degree invariably leads to higher prices. An interesting extension of this model would be to allow the monopolist to price-discriminate between stars and followers. In reality firms often do that by offering stars free samples of the good.

## 3.5 Advertising under Negative WOM

One of the basic functions of advertising is to raise awareness about the product. In that sense, advertising can be thought of as an alternative to WOM. Many studies have looked at the interaction between WOM and firm's advertising practices both theoretically (Campbell (2013), Galeotti and Goyal (2009)), and empirically (Coulter et al. (2002), Van den Bulte and Joshi (2007)). In this section I examine how the firm's advertising strategy is affected by negative WOM. For that purpose I revert to exogenously given quality $\gamma < 1$, as well as uncorrelated product valuations and network degrees. The monopolist has to contend with a possibility that a customer who directly receives product information has a bad experience and discourages his friends.

### 3.5.1 Fractional Advertising

I this section I study the monopolist's choice of fractional advertising in face of negative WOM. Formally, the monopolist can choose the fraction $\omega$ of consumers who receive the information directly, at a constant unit marginal cost $\alpha$. This type of advertising is not targeted, so $\omega$ represents the probability that a consumer of random type hears about the product independently of WOM. I assume that whenever a consumer receives conflicting information, WOM trumps advertisement. In practice this assumption means that if a person receives both an ad and negative report from a friend, he becomes discouraged. There are numerous models out there that make predictions on the interaction of price and the level of advertising (Bagwell (2007)). The question that I ask in this section is how those two decisions are affected by negative WOM.

In the interest of clarity, I go through the derivation of the demand and the self-consistency conditions for this problem. As before, I derive the expressions step-by-step, starting from an individual who has $k$ friends, out of whom $D$ are discouraged, $I$ are informed and $B$ bought the product and liked it. The total probability of having at least 1 out of $I$ informed friends buy and like the product and not having any of them dislike the

product is unchanged from Section 4.1:

$$A_1 = \sum_{B=1}^{I} \binom{I}{B}((1-P)\gamma)^B P^{I-B} = ((1-P)\gamma + P)^I - P^I \qquad (3.23)$$

The total probability that at least 1 out of $K - D$ non-discouraged friends becomes informed of the product (either by WOM or through advertising), buys it and enjoys it, while all of the other non-discouraged friends either don't get the information or don't buy is:

$$A_2 = \sum_{I=1}^{k-D} \binom{k-D}{I}(\omega + (1-\omega)u)^I A_1^I ((1-\omega)(1-u))^{k-D-I} = \qquad (3.24)$$

$$= ((\omega + (1-\omega)u)((1-P)\gamma + P) + (1-\omega)(1-u))^{k-D} - ((\omega + (1-\omega)u)P + (1-\omega)(1-u))^{k-D}$$

Finally, the total probability that a person with $k$ friends has at least one friend who becomes informed, buys and likes the good, while all of the other friends either become discouraged, or fail to become informed, or don't value the good highly enough to buy, is:

$$A_3 = \sum_{D=1}^{k} \binom{k}{D} v^D ((1-v)A_2)^D = (1 - (1-\gamma)(1-P)(1-v)(u+\omega-u\omega))^k - \qquad (3.25)$$

$$-(1 - (1-P)(1-v)(u+\omega-u\omega))^k$$

$A_3$ is the probability that a consumer of degree $k$ becomes positively informed of the good as a result of the combination of WOM and advertising. Consequently, the probability that a person at the end of a randomly picked link is informed is simply $g_1(A_3)$. So, the self-consistent condition that defines such probability is:

$$u = g_1((1 - (1-\gamma)(1-P)(1-v)(u+\omega-u\omega)) - g_1(1 - (1-P)(1-v)(u+\omega-u\omega)) \quad (3.26)$$

Similarly, the self-consistent condition for the probability that a consumers at the end of a randomly-picked link becomes discouraged as a result of the WOM and advertising is:

$$v = 1 - g_1(1 - (1-\gamma)(1-P)(1-v)(u+\omega-u\omega)) \qquad (3.27)$$

The probability that a randomly-picked person is positively informed is equal to:

$$A_0 = g_0((1 - (1-\gamma)(1-P)(1-v)(u+\omega-u\omega)) - g_0(1 - (1-P)(1-v)(u+\omega-u\omega)) \quad (3.28)$$

The demand in this setting combines the two types of informed consumers. First, those among the non-discouraged who became informed through advertisement and purchased the good. Second, those who didn't receive the advertisement but became informed through WOM and bought the good:

$$D(P, \omega, \gamma) = (1 - P)(\omega(1 - v) + (1 - \omega)A_0) \tag{3.29}$$

The task of the monopolist is to pick the optimal price and advertising menu in order to maximize profits given negative WOM.

$$\max_{\omega, P} (P - c)D(P, \omega, \gamma) - \alpha\omega \tag{3.30}$$

Note that it is no longer necessary for the monopolist to kick-start the WOM and to generate a giant components in order to have non-zero demand. If the cost of advertising is low enough, the firm can do away with WOM entirely and simply deliver the information directly to the entire population. The following proposition characterizes the monopolist's optimal choices of price and advertising given quality $\gamma$.

**Proposition 14.** *Suppose the degree distribution is Poisson, and the advertising cost $\alpha$ is small. Then, $\exists \underline{\gamma}$ such that $\frac{\partial \omega^*}{\partial \gamma} \leq 0$, $\frac{\partial P^*}{\partial \gamma} \leq 0$ for $\gamma \in [0, \underline{\gamma}]$, while for $\gamma \in (\underline{\gamma}, 1)$, $\omega^* = 0$, $\frac{\partial P^*}{\partial \gamma}$ behaves in accordance with Proposition 3.*

The proposition states that price and advertising compliments from the point of view of the firm. If the level of advertising goes up, so does the optimal price. If the firm boosts demand with ads, then the optimal action is to reap the reward by setting a higher price.

Crucially, the main implication of Proposition 14 is that the monopolist's ability to reach consumers directly through advertising changes the relationship between optimal price and quality established in Section 4.1. As long as the level of advertising is positive, the monopolist's optimal choice of price goes down in quality. The intuition for this result is the following. When $\gamma$ is approaching zero, WOM on its own would fail to kick-start the demand. Therefore, in order for the demand to be non-zero, the firms needs to directly inform a large fraction of the social network. However, by assumption a negative personal review from a friends trumps the positive information received directly from the firm. Therefore, if the quality if low, much of the advertising is in vain, because a randomly picked consumer is likely to be discouraged and not receptive. This mechanism implies that the 'vaccination' effect of a price increase is the largest at low quality levels, giving the monopolist at incentive to keep the price high. When quality increases, there is less need to advertise, because the WOM already does a decent job of disseminating the product information. Advertising is

costly, so the firm cuts back and instead stimulates WOM by lowering the price. Finally, as $\gamma$ approaches 1, the firm abandons advertising completely, so the model reverts to the baseline studied in Section 4.1. If the average network degree is high, then the optimal price keeps falling in quality, otherwise it begins to rise.

In this setting, the more aggressive the negative WOM and the lower the product's quality, the higher the price and the more intense the advertising. Empirically, advertising intensity is usually found to be positively correlated with product's quality (Caves and Greene (1996), Tellis and Fornell (1988)). However, there is some evidence for a negative correlation as well. Moreover, most of such evidence is discovered in markets where negative WOM is likely to be a truly powerful force. For example, Kwoka (1984) finds the quality of eye examinations to go down with advertising intensity in the optometry industry. The result presented in this section may help inform further empirical studies on the interaction of price and quality.

### 3.5.2 Degree-Targeted Advertising

With the advent of platforms, such as Facebook, firms have obtained an unprecedented access to the data on consumers' patterns of usage of social networks. As a result, a school of thought has emerged in the marketing science literature stating that the most effective use of firms' advertising budget is to target those consumers that have the most friends (Coulter et al. (2002), Van den Bulte and Joshi (2007)). However, Watts and Dodds (2007) and Campbell (2013) argue that this may, in fact, not be the best approach. In this section I show that the presence of negative WOM implies that targeting the highest-degree consumer may not be optimal, and that the actual optimal degree to target depends on the intensity of negative WOM.

Specifically, I analyze the situation in which the monopolist gains access to the degree distribution data and, after initially setting the price and the quality, tries to complement the (N)WOM process from Section 4.1 by optimally targeting consumers of certain degree. The main assumption behind this analysis is that WOM contains a stronger signal for a consumer than advertising. Whenever a person receives both a targeted ad from the firm and a negative review from a friend that person becomes discouraged. In other words, the firm has nothing to gain from advertising to someone who has already been reached by WOM. The firm's goal, therefore, is to use targeted advertising to trigger WOM in the grey areas of the network that would not otherwise be reached. To solve this problem, the monopolist has to determine the expected marginal increase in demand from targeting a consumer of degree $k$. Such consumer has to be uninformed by WOM in the absence of targeted advertising. A consumer can only be uninformed if his $k$ neighbors are all discouraged, uninformed, or do

not value the good highly enough to buy it. For any given neighbor the probability of this happening is:

$$\zeta = \underbrace{v}_{discouraged} + (1-v)(\underbrace{1-u}_{uninformed} + \underbrace{uP}_{low\ valuation}) \tag{3.31}$$

Consequently, a consumer of degree $k$ is uninformed upon completion of the WOM process with probability $\zeta^k$. With probability $\gamma(1-P)$ an uninformed consumer buys the good and relays positive information to his $k$ neighbors. In turn, each of the neighbors is uninformed with average probability $g_1(\zeta)$. The neighbors then may relay the information to their own $c_1$ neighbors, where $c_1$ is the average excess degree in the network. A naive guess would be that the monopolist should target the highest degree consumers, particularly if the quality is high, because they can pass the information to many others at a low risk of discouraging them. However, the following proposition shows that such intuition is not correct.

**Proposition 15.** *Assume that the WOM process has occurred on a network after monopolist chose particular $P < P^c$ and $\gamma$. Then, two things are true:*

*(i) it is optimal for the monopolist to advertise to the person of degree $k^*$, where:*

$$k^* = \max\left\{0, -\left(\frac{1}{\ln(\zeta)} + \frac{c_1}{\sum_{j=0}^{\infty}(\gamma c_1(1-P)g_1(\zeta))^{j+1}}\right)\right\}$$

*(ii) $\frac{\partial k^*}{\partial \gamma} < 0$.*

The proposition states that optimal target degree in the social network is not only typically bounded below $\infty$, but also goes down with respect to quality $\gamma$. The basic intuition for why it is not optimal to target the highest degree individual is preserved from the Campbell (2013) setting. The WOM process is biased towards providing information to the high-degree individuals. Therefore, it is likely that the highest degree individual would already be in possession of the information, either positive or negative, upon completion of the WOM process. Consequently, targeting such a person would be a waste.

The intuition for why the optimal degree goes down with quality is as follows. If quality is very low, then WOM fails to kick-start the demand, and the monopolist maximizes the expected marginal increase in demand by targeting the highest degree individual. However, as soon as $\gamma$ is high enough for WOM to yield a giant component, the monopolist's task changes. The firm now needs to identify the highest degree consumer who is likely to belong to the area of the network that has been cut off from the information by the negative WOM-induced bottlenecks. If $\gamma$ is low, then such areas are going to be large and likely contain high degree individuals, who should then be targeted by the firm. If $\gamma$ is approaching 1, then

the high degree individuals are unlikely to belong to the cut-off areas of the network, so the monopolist should instead target more isolated consumers.

## 3.6 Concluding Remarks

In this paper I have studied pricing, advertising and quality-setting behavior of a monopolist, who has to manage both positive and negative word-of-mouth spread of the information about the product. The main discovery is that the monopolist has an option of using a price increase as a vaccine against the negative WOM. Several pricing and quality choice results emerge that are in contrast with existing models.

Empirically testing a number of the model's theoretic predictions would represent an interesting avenue for future research. Sections 4.3 and 5.1 both suggest that the price might increase with the intensity of negative WOM. This is a counter intuitive prediction in itself, because instead of keeping the firm honest, negative reviews end up reducing consumer surplus. Testing whether negative WOM increases prices in real life and singling out the precise mechanism seems important for practical application of my results. For example, an antitrust authority might want to invest in disrupting the star-follower nature of certain WOM networks in order to increase consumer well-being. In Section 5.2 I conclude that the network degree of the consumer who should be targeted by advertising goes up in the intensity of negative WOM. Verifying this hypothesis could have important implications from the marketing standpoint. If the prediction is true, then similar degree-targeting strategies may have opposite results depending on the nature of the product.

The model studied in this paper is stripped of strategic behavior by the firm. My suspicion is that if a firm were to be monopolisticly competitive, rather than outright monopolistic, most of the results would still go through. However, introducing repeated interactions and reputations concerns would likely alter the patterns of behavior.

## Appendix: Proofs

I begin with several Lemmas which will simplify the subsequent proofs.

**Lemma 1** *For $\gamma < 1$, $v$ is equal to 0 if and only of $u$ is also equal to 0.*

**Proof** For the if direction, plug $u = 0$ in the the self consistency condition 3.13 to obtain:

$$v = 1 - g_1(v + (1 - v)) = 1 - g_1(1) = 0$$

by the properties of generating functions.

For the only if direction, suppose $u > 0$, then for $v$ to be 0, the following must be true based on self consistent condition:

$$0 = 1 - g_1(u(1 - P)(\gamma - 1) + 1)$$

by assumption $\gamma < 1$, and $P = 0 \Leftrightarrow u = 0$. Therefore, the only way to satisfy the equation is to set $u = 0$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\boxdot$

**Lemma 2** *Suppose that $u(P, \gamma)$ and $v(P, \gamma)$ are given by the SCC's in equations 3.12 and 3.13. Then, the following hold:*

*(1) $v(P) = 0$, $u(P) = 0$, and $\frac{\partial u}{\partial P} = 0, \frac{\partial v}{\partial P} = 0$, $\frac{\partial u}{\partial \gamma} = 0, \frac{\partial v}{\partial \gamma} = 0$ whenever $P$, $\gamma$, and $\{p_k\}$ are such that $g'(1)(1 - P)\gamma < 1$.*

*(2) $u$, $v$ are both continuous in $P$ and $\gamma$.*

**Proof** By Lemma 1, $v > 0 \Leftrightarrow u > 0$. Therefore, the two SCC's can be considered separately by assuming that $v$ is SCC for $u$ and $u$ in SCC for $v$ are simply positive parameters. Such approach is necessary, because the two SCC's make a highly non-linear system that would otherwise be difficult to study analytically.

Consider two functions in $u$ and $v$ defined by the SCC's. The SCC for $u$ for any given $v$ becomes:

$$f(u|v) = g_1(1 - u(1 - P)(1 - v)(1 - \gamma)) - g_1(1 - u(1 - P)(1 - v)) \qquad (3.32)$$

The SCC for $v$ for any given $u$ becomes:

$$h(v|u) = 1 - g_1(1 - u(1 - P)(1 - v)(1 - \gamma)) \qquad (3.33)$$

The solution $u = v = 0$ necessarily satisfies both expressions. The functions are polynomials in their respective arguments, and, hence, continuous. Therefore, the task is to show that there also exists at least one non-zero solution if $g'(1)(1 - P)\gamma > 1$. Consider the SCC for $v$ separately. For any given $u > 0$, taking the first derivative obtains:

$$h'(v) = -u(1 - P)(1 - \gamma)g_1'(1 - u(1 - P)(1 - v)(1 - \gamma)) \qquad (3.34)$$

which is necessarily negative on $v \in [0, 1]$, because the derivative of the generating function is just a polynomial in a small positive number with positive coefficients. But $h(1) = 0$ and

$h(0) = 1 - g_1(1 - u(1 - p)(1 - \gamma) > 0$. Given that the function is decreasing everywhere, for any $u \in [0, 1]$ there necessarily exists exactly one solution to the SCC $h(v) = v$ on the $[0, 1]$ interval.

Consider now the function $f(u)$. Its first derivative is:

$$f'(u) = (1-v)(1-P)[g_1'(1-u(1-P)(1-v)) - (1-\gamma)g_1'(1-u(1-P)(1-v)(1-\gamma))] \quad (3.35)$$

Observe that $f(0) = 0$ and $f(1) \in (0, 1)$. Therefore, for each $v$ there necessarily exists a $u \in (0, 1)$ that satisfies the condition $f(u) = u$ as long as $f'(0) > 1$. This condition can be spelled out as:

$$(1 - v)(1 - P)\gamma g_1'(1) > 1 \quad (3.36)$$

which is precisely the condition the defines $P^c$ be Proposition 1. Consequently, as long as the condition is satisfied there exists a solution to the system of SCC's, such that $u$, $v$ both $\in (0, 1]$. Hence, if the condition fails, $u$, $v$ necessarily equal 0, and their slopes equal 0 as well in both arguments. $\qquad \boxdot$

**Lemma 3** *Suppose that $u(P, \gamma)$ and $v(P, \gamma)$ are given by the SCC's in equations 3.12 and 3.13 and $g'(1)(1 - P)\gamma > 1$. Then, the following hold:*

*(1)* $\frac{\partial v}{\partial P} < 0$.

*(2)* *There exists $\underline{\gamma}$ such that for $\gamma \in (\frac{1}{(1-P)g_1'(1)}, \underline{\gamma}]$ $\frac{\partial u}{\partial P} < 0$, and for $\gamma \in (\underline{\gamma}, 1)$ $u(P, \gamma)$ is an upside-down parabola in $P$.*

**Proof** The first step is to differentiate the SCC's w.r.t. $P$.

$$\frac{\partial u}{\partial P} = [-g_1'(1 - u(1 - P)(1 - v)) + (1 - \gamma)g_1'(1 - u(1 - P)(1 - v)(1 - \gamma))]$$
$$[\frac{\partial u}{\partial P}(1 - v)(1 - P) - \frac{\partial v}{\partial P}u(1 - P) - u(1 - v)] \quad (3.37)$$

$$\frac{\partial v}{\partial P} = -(1-\gamma)g_1'(1-u(1-P)(1-v)(1-\gamma))[\frac{\partial u}{\partial P}(1-v)(1-P) - \frac{\partial v}{\partial P}u(1-P) - u(1-v)] \quad (3.38)$$

Assembling the two conditions in a matrix equation and solving it yields the expressions for the partial derivatives:

$$\begin{pmatrix} \frac{\partial u}{\partial P} \\ \frac{\partial v}{\partial P} \end{pmatrix} = A^{-1} \begin{pmatrix} \underbrace{(1 - \gamma)g_1'(S_1) - g_1'(S_2)}_{unknown\ sign} \\ \underbrace{-(1 - \gamma)g_1'(S_1)}_{<0} \end{pmatrix} u(1 - v) \quad (3.39)$$

Where:

$$A = \begin{pmatrix} [1 + (1-P)(1-v)((1-\gamma)g_1'(S_1) - g_1'(S_2))] & [-((1-\gamma)g_1'(S_1) - g_1'(S_2))u(1-P)] \\ [-(1-\gamma)g_1'(S_1)(1-P)(1-v)] & [1 + (1-\gamma)g_1'(S_1)(1-P)u] \end{pmatrix}$$ (3.40)

While $S_1 \equiv (1 - u(1-v)(1-P)(1-\gamma))$, and $S_2 \equiv (1 - u(1-v)(1-P))$.

In order to determine the sign of the partial derivatives, first determine the sign of the determinant.

$$|A| = 1 + (1-\gamma)(1-P)ug_1'(S_1) + (1-P)(1-v)[(1-\gamma)g_1'(S_1) - g_1'(S_2)]$$ (3.41)

By the properties of generating functions $g_1'(x) > 0 \forall x > 0$, and $g_1''(x) > 0$. The expression, therefore, is the smallest at the largest possible $\gamma$ such that $u(P, \gamma) = v(P, \gamma) = 0$. Such $\gamma$ is defined by threshold condition and is given by $\gamma = \frac{1}{(1-P)g_1'(1)}$. Plugging this values into equation 3.41 gives the value of 0. Therefore, the determinant can not be negative.

Consequently, the expression for $\frac{\partial v}{\partial P}$ becomes:

$$\frac{\partial v}{\partial P} = \frac{-u(1-v)(1-\gamma)g_1'(S_1)}{|A|}$$ (3.42)

The numerator is always negative, so the whole expression is negative as well. Hence, the fraction of the discouraged consumers necessarily falls with the price.

Now, consider the expression for $\frac{\partial u}{\partial P}$:

$$\frac{\partial u}{\partial P} = \frac{u(1-v)[(1-\gamma)g_1'(S1) - g_1'(S_2)]}{|A|}$$ (3.43)

The denominator is positive. However, the sign of the numerator varies and is determined by $[(1-\gamma)g_1'(S1) - g_1'(S_2)]$, which depends on $\gamma$. First, consider the case when when $\gamma$ is approaching $\frac{1}{(1-P)g'(1)}$ from above, so the $u, v \approx 0$. In this case the expression becomes:

$$(1-\gamma)g_1'(S1) - g_1'(S_2) \approx -\gamma g_1'(1) < 0, \forall P < P^c$$

So, for small $\gamma$ the fraction of externally informed individuals starts to fall with $P$. Consider now the case $\gamma \longrightarrow 1$ (but not equal to 1). If this is so, then $v \approx 0$, but $u > 0$. So, the expression becomes $\approx g_1'(1 - u(1-P)(1-\gamma))(1-\gamma) - g_1'(1 - u(1-P))$. Being polynomials both $g_1'(1 - u(1-P)(1-\gamma))$ and $g_1'(1 - u(1-P))$ are convex in $\gamma$. For $\gamma = \frac{1}{(1-P)g_1'(1)}$ they are both equal to $g'(1) > 1$ because the only solution to the SCC for $u$ is $u = 0$. At $\gamma = 1$, $g_1'(1 - u(1-P)(1-\gamma)) = g_1'(1)$, while $g_1'(1 - u(1-P)) < g_1'(1)$. If $P$ is ap-

145

proaching $P^c$ from below, then $u \approx 0$ and the sign of the numerator of the partial derivative is equal to the sign of $\approx -\gamma g_1'(1) < 0$. However, when $P \longrightarrow 0$ $u$ goes to 1. Therefore, in this limit $g_1'(S_2) = 0$. But for $\gamma$ just bellow $1 g_1'(S_2) < 1 - \gamma$ because of being convex. Additionally, at $\gamma$ just below 1 $g_1'(S_1) \approx g_1'(1) > 1$. Consequently, in this limit, $g_1'(S_1)(1 - \gamma) - g_1'(S_2) > (g_1'(S_1) - 1)(1 - \gamma) > 0$.

The derivative can change the sign at most once, because both $g_1'(S_2)$ and $g_1'(S_1)$ are monotonic in $P$. Therefore, for $\gamma \longrightarrow 1$ $u(P, \gamma)$ goes first up, then down with $P$ while for $\gamma \longrightarrow \frac{1}{(1-P)g_1'(1)}$ $u(P, \gamma)$ invariably declines in $P$. Hence, there exists a cutoff $\underline{\gamma}$ as defined in the statement of the lemma. ⊡

**Lemma 4** *Suppose that $u(P, \gamma)$ and $v(P, \gamma)$ are given by the SCC's in equations 3.12 and 3.13 and $g'(1)(1 - P)\gamma > 1$. Then, the following hold:*

*(1)* $\frac{\partial u}{\partial \gamma} > 0$.

*(2)* $v(P, \gamma)$ *is an upside-down parabola in $\gamma$.*

**Proof** In similar vein with Lemma 3, differentiate the SCC's w.r.t to the two fractions, and solve to matrix equation to obtain the partial derivatives:

$$\begin{pmatrix} \frac{\partial u}{\partial \gamma} \\ \frac{\partial v}{\partial \gamma} \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ -1 \end{pmatrix} u(1 - v)(1 - P)g_1'(S_1) \tag{3.44}$$

Where $A$, $S_1$, and $S_2$ are defined as before. Since the matrix is the same, its determinant is positive by previous argument. The expressions for $\frac{\partial u}{\partial P}$ becomes:

$$\frac{\partial u}{\partial \gamma} = \frac{[1 + u(1 - P)g_1'(S_2)]u(1 - P)(1 - v)g_1'(S_1)}{|A|} \tag{3.45}$$

Which is positive whenever the threshold condition is fulfilled because the derivative of the generating function is always positive when evaluated at a positive number. So, the fraction of externally informed necessarily increases in quality $\gamma$.

Next, consider the differentiated SCC for $v$.

$$\frac{\partial v}{\partial \gamma} = (1 - P)g_1'(1 - u(1 - P)(1 - v)(1 - \gamma))[\frac{\partial u}{\partial \gamma}(1 - v)(1 - \gamma) - \frac{\partial v}{\partial \gamma}u(1 - \gamma) - u(1 - v)] \tag{3.46}$$

Which can be re-written as:

$$\frac{\partial v}{\partial \gamma} = \underbrace{\frac{g_1'(S_1)(1 - P)}{1 + (1 - \gamma)ug_1'(S_1)(1 - P)}}_{>0} \left[(1 - v)(1 - \gamma)\frac{\partial u}{\partial \gamma} - u(1 - v)\right] \tag{3.47}$$

The sign of the expression is equal to the sign of the term in brackets.

$$\lim_{\gamma \longrightarrow 1} \left[ (1-v)(1-\gamma)\frac{\partial u}{\partial \gamma} - u(1-v) \right] = -u < 0 \tag{3.48}$$

$$\lim_{\gamma \longrightarrow (1/((1-P)g_1'(1)))} \left[ (1-v)(1-\gamma)\frac{\partial u}{\partial \gamma} - u(1-v) \right] = \frac{\partial u}{\partial \gamma} > 0 \tag{3.49}$$

So, the function $v(\gamma, P)$ is first increasing, then decreasing in $\gamma$. The derivative changes the sign only once because $u(\gamma, P$ is increasing and concave in $\gamma$. $\boxdot$

**Lemma 5** *Suppose that the distribution of degrees is Poisson with average degree $z$ and $u(z)$ and $v(z)$ are given by the SCC's in equations 3.12 and 3.13 and $z(1-P)\gamma > 1$, and . Then, the following hold:*

*(1) $\frac{\partial v}{\partial z} > 0$.*

*(2) $u(z)$ is an upside down parabola in $z$.*

**Proof** With Poisson degree distribution $g_1(x) = g_0(x) = e^{z(x-1)}$. Plugging this function into the definitions of $u$ and $v$ and differentiating w.r.t. $z$ yield the following matrix equation:

$$\begin{pmatrix} \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} \end{pmatrix} = \begin{pmatrix} [1 + (1-P)(1-v)((1-\gamma)e^{S_1} - e^{S_2})] & [-((1-\gamma)e^{S_1} - e^{S_2})u(1-P)] \\ [-(1-\gamma)e^{S_1}(1-P)(1-v)] & [1 + (1-\gamma)e^{S_1}(1-P)u] \end{pmatrix}^{-1}$$
$$\begin{pmatrix} -((1-\gamma)e^{S_1} - e^{S_2}) \\ (1-\gamma)e^{S_1} \end{pmatrix} u(1-v)(1-P) \tag{3.50}$$

Where $S_1 = -zu(1-P)(1-v)(1-\gamma)$, $S_2 = -zu(1-P)(1-v)$. The expression for the partial derivative of $v$ becomes:

$$\frac{\partial v}{\partial z} = \frac{u(1-v)(1-P)(1-\gamma)e^{S_1}}{|A|} \tag{3.51}$$

The determinant is never negative by the previous arguments, whereas the numerator is clearly strictly positive whenever the threshold condition is fulfilled. Therefore, the fraction of discouraged goes up in connectivity for Poisson degree distribution.

The expression for the partial derivative of $u$ is:

$$\frac{\partial u}{\partial z} = \frac{-((1-\gamma)e^{S_1} - e^{S_2})u(1-v)(1-p)}{|A|} \tag{3.52}$$

The sign of the expression is determined by the sign of $-((1-\gamma)e^{S_1} - e^{S_2})$. Both parts of the expression are convex and decreasing in $z$. For $z \longrightarrow 1/((1-P)\gamma))$ from above $u, v \approx 0$, so the expression becomes equal to $\gamma > 0$. For the expression to become negative in the limit of large $z$ the following inequality has to hold in that limit:

$$e^{-zu(1-v)(1-P)} < (1-\gamma)e^{-zu(1-v)(1-P)(1-\gamma)}$$

$$\frac{e^{zu(1-v)(1-P)(1-\gamma)}}{e^{zu(1-v)(1-P)}} < 1 - \gamma$$

Which in the limit becomes:

$$0 < 1 - \gamma$$

The function $u(z)$, therefore, first goes up, then down with connectivity. The derivative changes sign only once because both expressions in the numerator are monotonic in $z$. Therefore, $u$ is an upside-down parabola in $z$. ⊡

**Proof of Proposition 8**

By Lemma 1 $v = 0$ if and only if $u = 0$, so at the threshold critical price both $v$ and $u$ change discontinuously in the same direction. Therefore, in order to study the phase transition of the system given price, one only needs to study its behavior as $u$ approaches 0. To do this, take a Taylor expansion of equation 3.12 in powers of $u$ about the point when $u = 0$, obtain:

$$u = g_1(1) - g_1(1) + g_1'(1)u(1-v)((1-P)\gamma + P - 1) - g_1'(1)u(1-v)(P-1) + O(u^2) \quad (3.53)$$

But higher order terms can be ignored due to being vanishingly small and, since $v = 0$ as $u$ approaches 0, this can be simplified as:

$$1 = g_1'(1)\gamma(1-P) \quad (3.54)$$

which can be rearranged to give the result. ⊡

**Proof of Proposition 9**

(i) By Lemma 2, $u$ and $v$ are continuous in the relevant segments, so $D(P)$ is also continuous in those segments. Moreover, because both $u$ and $v$ are downward sloping in $P$, as $P$ approaches $P^c$ from below, these functions both approach 0. The demand $D(P)$ is 0 if and only if $u$ is 0, so it approaches 0 at $P = P^c$ from both above and below, thus being continuous on the entirety of the relevant domain.

(ii) In order to show that $\frac{dD}{dP} < 0$, consider the derivative.

$$\frac{\partial D}{\partial P} = \underbrace{-(g_0(S_1) - g_0(S_2))}_{<0} + (1-P)((1-\gamma)g_0'(S_1) - g_0'(S_2))$$

$$\left( \frac{\partial u}{\partial P}(1-P)(1-v) - \frac{\partial v}{\partial P}u(1-P) - u(1-v) \right) \quad (3.55)$$

Where by the properties of the generating functions, $g_0'(x) = zg_1(x)$. Consider now the term in big brackets. By Lemma 3 the following expression holds:

$$\frac{u(1-v)(1-P)}{|A|} = \left( \frac{\partial u}{\partial P}(1-P)(1-v) - \frac{\partial v}{\partial P}u(1-P) - u(1-v) \right) \quad (3.56)$$

The generating functions are increasing and convex in their arguments, so $g_0(S_1) - g_0(S_2) > g_1(S_1) - g_2(S_2)$. Combining these facts yields the following inequality:

$$\frac{\partial D}{\partial P} < -\left[ 1 + \frac{u(1-v)(1-P)zP}{|A|}(1-\gamma) \right] g_1(S_1) + \left[ 1 + \frac{u(1-v)(1-P)zP}{|A|} \right] g_1(S_2) \quad (3.57)$$

Notice that because all of the other terms necessarily are of a particular sign, the derivative can change sign at most once, just like the $(1-\gamma)g_0'(S_1) - g_0'(S_2)$ term. Therefore, it needs to be shown that derivative is negative for all possible values of $\gamma$.

Suppose, $\gamma \longrightarrow 1/((1-P)g_1'(1))$ from above. In such case for any $P < 1/g_1'(1)$, $u, v \approx 0$. Therefore, the expression 3.57 becomes approximately:

$$\frac{\partial D}{\partial P} < -g_1(1-\varepsilon_1) + g_1(1-\varepsilon_2) \quad (3.58)$$

Which is less than 0, because $\varepsilon_1 < \varepsilon_2$, even though they are both $\approx 0$. Similarly, if $\gamma \longrightarrow 1$, and $P \longrightarrow 1/g_1'(1)$ from below, then $u, v \approx 0$, so the expressions remains $\partial D/\partial P < -g_1(1 - \varepsilon_1) + g_1(1 - \varepsilon_2) < 0$.

Finally, if $\gamma \longrightarrow 1$, and $P \longrightarrow 0$, then by Lemma 3, $(1-\gamma)g_0'(S_1) - g_0'(S_2) > 0$, so the expression is negative again. Consequently, the demand falls in $P$.

(iii) The partial derivative of demand w.r.t $\gamma$ is:

$$\frac{\partial D}{\partial \gamma} = (1-P)\frac{\partial}{\partial \gamma}\left[ g_0(S_1) - g_0(S_2) \right] \quad (3.59)$$

Which is always greater than 0 above the threshold, by the arguments in Lemma 4.

(iv) Consider the expression:

$$\frac{\partial DP}{\partial PD} = -\underbrace{\frac{P}{1-P}}_{FI\ elasticity}[1 - \frac{1-P}{g_0(S_1)-g_0(S_2)}((1-\gamma)g_0'(S_1)-g_0'(S_2))\times$$

$$\left(\frac{\partial u}{\partial P}(1-P)(1-v) - \frac{\partial v}{\partial P}u(1-P) - u(1-v)\right)] \quad (3.60)$$

By arguments in Lemma 3, there always exists a $\gamma$ which satisfies the statement of the proposition such that the second term in the brackets is positive, meaning that the elasticity is smaller in absolute value. $\boxdot$

**Proof of Proposition 10**

(i) By Proposition 2, demand is continuous and differentiable in $P$. Therefore, the first order conditions are necessary.

$$\frac{P^*_{WOM}-c}{P^*_{WOM}} = \frac{1}{-\varepsilon_{WOM}}$$

$$\frac{P^*_{FI}-c}{P^*_{FI}} = \frac{1}{-\varepsilon_{FI}}$$

Therefore, if there exists $P' > P_{FI}$ such that $\frac{1}{-\varepsilon_{WOM}(P')} > \frac{1}{-\varepsilon_{FI}(P')}$, then by Lemma 3, the $WOM$ demand is less elastic than the fully informed demand for all $P < P'$ (a segment that subsumes $P_{FI}$), and $P^*_{WOM} > P_{FI}$. Therefore, one needs to show that such $P'$ exists.

$$1-\varepsilon_{WOM}(P') > \frac{1}{-\varepsilon_{FI}(P')} \Leftrightarrow \varepsilon_{WOM}(P') > \varepsilon_{FI}(P') \quad (3.61)$$

Take $P' = \frac{1+c+\sigma}{2}$, such that $\sigma << 1-c$.

$$\varepsilon_{WOM}(P') = -\frac{P}{1-P}[1 - \frac{1-P}{g_0(S_1)-g_0(S_2)}((1-\gamma)g_0'(S_1)-g_0'(S_2))$$

$$\left(\frac{\partial u}{\partial P}(1-P)(1-v) - \frac{\partial v}{\partial P}u(1-P) - u(1-v)\right)]|_{P=P'} \quad (3.62)$$

whether the expression is greater than $\varepsilon_{FI}(P')$ comes down to whether

$$((1-\gamma)g_0'(S_1)-g_0'(S_2))|_{P=P'}$$

is greater than 0.

$$(1-\gamma)g_0'\left(1-u(1-v)(1-\gamma)\left(\frac{1-c-\sigma}{2}\right)\right) - g_0'\left(1-u(1-v)\left(\frac{1-c-\sigma}{2}\right)\right) \quad (3.63)$$

Because generating function is polynomial in its argument and $u(1-v)$ goes up in $\gamma$, second term goes down with $\gamma$ in convex fashion from $g_0'(1) = E(k)$ when $\gamma = 0$ to $g_0'\left(1-u\left(\frac{1-c-\sigma}{2}\right)\right) < E(k)$, when $\gamma \longrightarrow 1$.

The term $g_0\left(1-u(1-v)(1-\gamma)\left(\frac{1-c-\sigma}{2}\right)\right)$ also equals $E(k)$ when $\gamma = 0$, however, by Lemma 3, the term is U-shaped in $\gamma$, and for $\gamma \longrightarrow 1$, the term equals $E(k)$. Therefore, in that limit, the following is always true:

$$\lim_{E(k)\longrightarrow\infty} = \frac{g_0\left(1-u(1-v)(1-\gamma)\left(\frac{1-c-\sigma}{2}\right)\right)}{g_0\left(1-u(1-v)\left(\frac{1-c-\sigma}{2}\right)\right)} = \frac{E(k)}{0} = \infty > (1-\gamma) \quad (3.64)$$

So, for any $c$, and $\sigma$ it is possible to find $E(k)$ high enough for the proposition to be true.
(ii) Again, at the optimal price the FOC must bind, therefore:

$$\frac{P_{WOM}^* - c}{P_{WOM}^*} = \frac{1}{-\varepsilon_{WOM}}$$

Consequently, one needs to show that $\frac{1}{-\varepsilon_{WOM}}$ is inverse U-shaped in $\gamma$. But

$$\frac{1}{-\varepsilon_{WOM}} = \frac{1}{\frac{P}{1-P}[1-B*C]} \quad (3.65)$$

where
$$B = (1-P)^2((1-\gamma)g_0'(S_1) - g_0'(S_2)) \quad (3.66)$$

and

$$C = \frac{u(1-v)}{(g_0(S_1)-g_0(S_2))(1+(1-\gamma)(1-v+u)(1-P)g_1'(S_1)-(1-v)(1-P)g_1'(S_2))} \quad (3.67)$$

So, it is enough to show that $B*C$ is inverse U-shaped in $\gamma$.
$\frac{\partial(B*C)}{\partial\gamma} = C\frac{\partial B}{\partial\gamma} + B\frac{\partial C}{\partial\gamma}$. For $\gamma \longrightarrow \frac{1}{(1-P)g_1'(1)}$ from above, by earlier arguments $B < 0$, $C > 0$, and $\frac{\partial B}{\partial\gamma} > 0$. Therefore, in order the sign the derivative in that limit, one needs to sign the derivative of $C$.
**Lemma 6** $\frac{\partial B}{\partial\gamma} < 0$ for $\gamma \longrightarrow \frac{1}{(1-P)g_1'(1)}$.

**Proof** using the SCC's for $u$ and $v$, C becomes:

$$C = \frac{g_1(S_1)(g_1(S_1) - g_1(S_2))}{(g_0(S_1) - g_0(S_2))(1 + (1-\gamma)(2g_1(S_1) - g_1(S_2))(1-P)g_1'(S_1) - g_1(S_1)(1-P)g_1'(S_2))} \tag{3.68}$$

Both the numerator and the denominator grow in $\gamma$. Therefore, it is enough to show that the numerator grows less rapidly.

Both $g_1(S_1)(g_1(S_1) - g_1(S_2))$ and $(g_0(S_1) - g_0(S_2))$ are convexly growing functions of $\gamma$, because generating functions are monotonic and convex in their argument. At $\gamma \longleftrightarrow \frac{1}{(1-P)g_1'(1)}$, both expressions are 0. At $\gamma = 1$, $g_1(S_1)(g_1(S_1) - g_1(S_2)) = 1 - g_1(1 - u(1-P) < 1 - g_0(1 - u(1-P) = (g_0(S_1) - g_0(S_2))$. Where the inequality comes from the fact that $Cov(k, (1 - u(1-p))^{k-1}) < 0$. Therefore, $(g_0(S_1) - g_0(S_2))$ is steeper than $g_1(S_1)(g_1(S_1) - g_1(S_2))$ for all permissible $\gamma$ values.

For the second term in the denominator:

$$\frac{\partial}{\partial\gamma}(1 + (1-\gamma)(2g_1(S_1) - g_1(S_2))(1-P)g_1'(S_1) - g_1(S_1)(1-P)g_1'(S_2)) =$$

$$\frac{\partial}{\partial\gamma}((1-\gamma)(2g_1(S_1) - g_1(S_2))(1-P)g_1'(S_1) - g_1(S_1)(1-P)g_1'(S_2)) >$$

$$(1-P)\frac{\partial}{\partial\gamma}((1-\gamma)(2g_1(S_1) - g_1(S_1))g_1'(S_1) - g_1(S_1)g_1'(S_2)) =$$

$$(1-P)\frac{\partial}{\partial\gamma}(g_1(S_1)((1-\gamma)g_1'(S_1) - g_1'(S_2)) \tag{3.69}$$

because $\frac{\partial g_1(S_1)}{\partial\gamma} > \frac{\partial g_1(S_2)}{\partial\gamma}$ by part (i) of the proposition. But when $\gamma \longrightarrow \frac{1}{(1-P)(g_1'(1)}$ the last expression is greater than $(1-P)\frac{\partial}{\partial\gamma}(g_1(S_1)(g_1'(S_1) - g_1'(S_2))$ which is greater than $\frac{\partial}{\partial\gamma}(g_1(S_1)(g_1(S_1) - g_1(S_2))$ for $\gamma > \frac{1}{(1-P)g_1'(1)}$. Consequently, both terms in the denominator grow more rapidly in $\gamma$ than the numerator, meaning that the derivative is negative. $\square$

By Lemma 6 $\frac{\partial(B*C)}{\partial\gamma} > 0$ when $\gamma$ approaches $\frac{1}{(1-P)g_1'(1)}$ from above. Consequently, in that limit $\frac{\partial P_{WOM}^*}{\partial\gamma} > 0$. By part (i) of this proposition, for sufficiently high $E(k)$ there exists $\underline{\gamma}$ such that $\forall \gamma \in [\underline{\gamma}, 1)$ $P_{WOM}^* > P_{FI}$. But Campbell (2013) studies the case when $\gamma = 1$ exactly and finds (Theorem 2) that $P_{WOM}^* \leq P_{FI}$. Therefore, there must exist a segment on the domain of $\gamma$ above $\underline{\gamma}$ such that $\frac{\partial P_{WOM}^*}{\partial\gamma} < 0$.

Therefore, for high enough $E(k)$, $P_{WOM}^*$ initially grows and then falls in $\gamma$. But the derivative can only change its sign once because the derivative of $C$ can only change its sign once (with the derivative of $|A|$), and its slope is always greater in absolute value than the slope of $B$ by properties of generating functions. So, the $P_{WOM}^*$ is inverse U-shaped. $\square$

**Proof of Proposition 11**

As with Proposition 10, it is enough to show that $-\varepsilon_{WOM}$ is falling in $z$. With Poisson degree distribution:

$$- \varepsilon_{WOM} = \frac{P}{1-P}[1 - \frac{1-P}{g_0(S_1) - g_0(S_2)}((1-\gamma)g_0'(S_1) - g_0'(S_2))$$

$$\left(\frac{\partial u}{\partial P}(1-P)(1-v) - \frac{\partial v}{\partial P}u(1-P) - u(1-v)\right)] \quad (3.70)$$

So, it is enough to show that:

$$\frac{\partial}{\partial z}\left[\frac{1-P}{g_0(S_1) - g_0(S_2)}((1-\gamma)g_0'(S_1) - g_0'(S_2))\left(\frac{\partial u}{\partial P}(1-P)(1-v) - \frac{\partial v}{\partial P}u(1-P) - u(1-v)\right)\right]$$

$$(3.71)$$

is larger than 0. Using the definitions of $u$ and $v$, the fact that $\{p_k\}$ is Poisson, and the expressions for partial derivatives from Lemma 3, the expression becomes:

$$(1-P)\frac{\partial}{\partial z}\left[\frac{(1-P)z(1-v)(u-\gamma(1-v))}{1+(1-\gamma)(1-P)u(1-v)z+(1-P)z(1-v)(u-\gamma(1-v))}\right] > 0 \quad (3.72)$$

Denote the numerator by $N$ and the denominator by $D$. D is positive for non-zero demand, so by quotient rule, the expression is true whenever $\frac{\partial N}{\partial z}D - \frac{\partial D}{\partial z}N > 0$. Observe that $D$ is equal to $N$ plus a positive term, so $D > N$ always.

I claim that both the numerator and the denominator grow in $z$. To show that this is true, I show that $(1-\gamma)(1-P)u(1-v)z$ grow in $z$.

$$\frac{\partial(zu(1-v))}{\partial z} = (1-v)u - zu\frac{\partial v}{\partial z} + z(1-v)\frac{\partial u}{\partial z} \quad (3.73)$$

using the expressions for the partial derivatives from Lemma 5, this equals:

$$(1-v)u - z\left(\frac{(1-P)u(1-v)^2\gamma(1-v-u)}{|A|}\right)$$

Using the fact that $|A| > 0$, the expression is positive whenever:

$$(1-v)u + z(1-v)^2(1-P)u^2(1-\gamma) + z(1-v)^2u(1-P)((1+\gamma)u + 2\gamma v - 2\gamma) > 0 \quad (3.74)$$

Which is always true, since $((1+\gamma)u + 2\gamma v - 2\gamma) > -u \ \forall\gamma$ such that $u > 0$. So, this term grows in $z$. The proof for $(1-P)z(1-v)(u-\gamma(1-v))$ is logically the same, and is omitted. Consequently, $D$ and $N$ both grow in $\gamma$.

If $N < 0$, then $\frac{\partial N}{\partial Z} D - \frac{\partial D}{\partial Z} N > 0$ is clearly satisfied. So, what remains to show that the inequality hold whenever $u - \gamma(1 - v) > 0$. Canceling out all of the repeated terms, the condition comes down to:

$$(1 + (1 - \gamma)(1 - P)z(1 - v)u)\frac{\partial}{\partial z}((1 - P)(1 - v)z(u - \gamma(1 - v))) -$$
$$(1 - P)(1 - v)z(u - \gamma(1 - v))\frac{\partial}{\partial z}((1 - \gamma)(1 - P)z(1 - v)u) > 0 \quad (3.75)$$

But $(1 - \gamma)(1 - P)z(1 - v)u > (1 - P)(1 - v)z(u - \gamma(1 - v))$, and the first partial derivative in the expression above is larger than the second because $v - 1$ grows in $z$ by Lemma 5. Hence, the inequality is always true. $\qquad \qquad \boxdot$

### Proof of Proposition 12
(i) By arguments in Lemma 4, $\frac{\partial D(\gamma, P)}{\partial \gamma} > 0$. Therefore, if $c_q \longrightarrow 0$, the profits are strictly increasing in $\gamma$, and the monopolist always picks the highest possible quality. If $\gamma = 1$, then the problem reduces to the one studied by Campbell with the same solution. And by Campbell Theorem 2, price is necessarily bellow $P_{FI}$.
(ii) The FOC for $\gamma$ is:
$$\gamma = \frac{P - c}{c_q} - \frac{D(P, \gamma)}{\partial D / \partial \gamma} \quad (3.76)$$

$P$ and $\gamma$ are substitutes if the right hand side goes down in $P$. The condition, therefore, is:
$$\frac{1}{c_q} - \frac{\partial D / \partial P \partial D / \partial \gamma - \partial^2 D / \partial P \partial \gamma}{(\partial D / \partial \gamma)^2} < 0 \quad (3.77)$$

Clearly, if the second term is positive, then for $c_q$ high enough the inequality is satisfied. Therefore, one needs to show that the second term is positive.

Using the the formulas from Lemma 4:
$$\frac{\partial D}{\partial P} = -(g_0(S_1) - g_0(S_2)) + (1 - P)((1 - \gamma)g_0'(S_1) - g_0'(S_2))\left(\frac{(1 - P)u(1 - v)}{|A|}\right) < 0 \quad (3.78)$$

$$\frac{\partial D}{\partial \gamma} = (1 - P)\frac{(1 - P)(\Lambda)}{|A|} + (1 - P)g_0'(S_1)u(1 - v) \quad (3.79)$$

Where:

$$\Lambda = g_0'(S_2) - g_0'(S_1))((1 - v)(1 + u(1 - P)g_1'(S_2))u(1 - P)(1 - v)g_1'(S_1) -$$
$$u^2(1 - P)(1 - v)g_1'(S_1)(1 - u(1 - P)(1 - v)g_1'(S_2)) \quad (3.80)$$

154

Using these equalities and again applying the definitions of derivative of $u$ and $v$ with respect to $\gamma$, the sufficient condition for $\partial D/\partial P \partial D/\partial \gamma - \partial^2 D/\partial P \partial \gamma > 0$ is:

$$- (g_0(S_1) - g_0(S_2)) + (1 - (1-P)^2 u(1-v))((1-\gamma)g_0'(S_1) - g_0'(S_2)) +$$
$$(1 - (1-P)u(1-v))((1-\gamma)g_1'(S_1) - g_1'(S_2)) + (1 + (1-v)u)g_1'(s_1) > 0 \quad (3.81)$$

If $(1-\gamma)g_0'(S_1) - g_0'(S_2) > 0$, then the condition automatically holds. So, assume it is less than 0:

$$- (g_0(S_1) - g_0(S_2)) + (1 - (1-P)^2 u(1-v))((1-\gamma)g_0'(S_1) - g_0'(S_2)) +$$
$$(1 - (1-P)u(1-v))((1-\gamma)g_1'(S_1) - g_1'(S_2)) + (1 + (1-v)u)g_1'(s_1) >$$
$$2(1 - (1-P)u(1-v))((1-\gamma)g_1'(S_1) - g_1'(S_2)) + g_1'(S_1) >$$
$$- 2(1 - (1-P)u(1-v))(1-\gamma) + g_1'(1 - u(1-v)(1-P)(1-\gamma)) >$$
$$- 2(1 - (1-P)u(1-v))(1-\gamma) + \frac{E(k-1)E(k)}{E(k)}(1 - u(1-v)(1-P)(1-\gamma)) \quad (3.82)$$

The final expression $> 0$ whenever the demand is non-zero. So, the original inequality is always satisfied.

(iii) If the two are substitutes, then $\left.\frac{\partial \gamma_{NWOM}^*(P)}{\partial P}\right|_{P=P_{NWOM}^*} < 0$ But from Proposition 11, optimal price is falling in expected degree for any $\gamma$. This doesn't change in the problem of joint choice of price and quality. Therefore, in case of substitutes, higher expected degree increases the prices and reduces quality. $\quad \boxdot$

**Proof of Proposition 13**

Suppose denote the fraction of consumers with degree 1 by $p_1$. Then, the excess degree distribution in this case is given by:

$$q_{k^*-1} = \frac{k^*(1-p_1)}{p_1 + (1-p_1)k^*}, \; q_0 = \frac{p_1}{p_1 + (1-p_1)k^*} \quad (3.83)$$

Similarly, the probability that the node of each degree buys the good if informed is:

$$b_{k^*} = \min\left(1, 1 - \frac{P-\underline{\theta}}{1-\underline{\theta}}\right), \; b_1 = \max\left(0, 1 - \frac{P}{\underline{\theta}}\right) \quad (3.84)$$

Consequently, the probability that the node at the end of a randomly picked link buys the good if informed is:

$$B = q_0 b_1 + q_{k^*-1} b_{k^*} \quad (3.85)$$

Then, from equations 3.21 and 3.22, the SCC for this problem become:

$$u = q_{k^*-1}\left(\underbrace{(1-uB(1-\gamma)(1-v))^{k^*-1}}_{S_1} - \underbrace{(1-uB(1-v))^{k^*-1}}_{S_2}\right) \tag{3.86}$$

$$v = 1 - q_0 - q_{k^*-1}\left((1-uB(1-\gamma)(1-v))^{k^*-1}\right) \tag{3.87}$$

And the demand becomes:

$$D(P,\gamma) = p_1 b_1(1-v)uB\gamma + (1-p_1)b_{k^*}((1-uB(1-\gamma)(1-v))^{k^*} - (1-uB(1-v))^{k^*}) \tag{3.88}$$

There two distinct portions of the demand curve. On $P \in [\underline{\theta}, 1)$ none of the low-degree types buys the good. Taking the derivative of the profit function wrt price on that segment yields:

$$\frac{\partial \pi}{\partial P} = \underbrace{\frac{(1-p_1)^2(1-2P+c)k^{*2}}{(1-\underline{\theta})^2(p_1+(1-p_1)k^*)}}_{>0} \times$$

$$\left[-S_1^{k^*-1}(1-\gamma) + S_2^{k^*-1}\right]\left(-u(1-v) + (1-P)(1-v)\frac{\partial u}{\partial P} - (1-P)u\frac{\partial v}{\partial P}\right) \tag{3.89}$$

The term $\left[-S_1^{k^*-1}(1-\gamma) + S_2^{k^*-1}\right]$ is always negative for high enough $k^*$, because $S_1 > S_2$ and both between 0 and. 1. The expression $\left(-u(1-v) + (1-P)(1-v)\frac{\partial u}{\partial P} - (1-P)u\frac{\partial v}{\partial P}\right)$ is negative by arguments presented in proof of Lemma 5. Consequently, profits are always falling on this segment for high enough $k^*$, so the monopolist never selects the price above the kink at $\underline{\theta}$.

What remains is to show that the optimal price falls in $\gamma$ when $P \in [0, \underline{\theta}]$. On that segment demand is continuous, so the FOC are necessary. Therefore it is enough to show that the price elasticity of demand goes up in $\gamma$. This derivative can be expressed as:

$$\frac{\partial}{\partial \gamma}\left(-\frac{\partial D(P,\gamma)}{\partial P}\frac{P}{D(P,\gamma)}\right) = -\frac{P}{D}\left(\frac{\partial^2 D}{\partial P \partial \gamma} - \frac{1}{D}\frac{\partial D}{\partial P}\frac{\partial D}{\partial \gamma}\right) \tag{3.90}$$

So, the expression in brackets must be negative. In order to sign the expression, first one needs to obtain the derivatives of $v$ and $u$ wrt to $P$ and $\gamma$. Performing the same exercises as in proofs of lemmas 3 and 4, obtain the following expressions:

$$\frac{\partial v}{\partial P} = \frac{p_1 T(1-v)u(1-\gamma)S_1^{k^*-2}}{|A|} \tag{3.91}$$

$$\frac{\partial u}{\partial P} = \frac{p_1 T u (1-v)(-(1-\gamma)S_1^{k^*-2} + S_2^{k^*-2})}{|A|} \tag{3.92}$$

$$\frac{\partial v}{\partial \gamma} = \frac{(TWu(1-v)S_1^{k^*-2})(1 + TWuS_1^{k^*-2})}{|A|} \tag{3.93}$$

$$\frac{\partial v}{\partial \gamma} = -\frac{-TWu(1-v)S_1^{k^*-2} + T^2W^2(1-v)^2uS_1^{k^*-2}S_2^{k^*-2}}{|A|} \tag{3.94}$$

Where $|A| = -1 - TW(1 - v - u)(1-\gamma)S_1^{k^*-2} + TW(1-v)S_2^{k^*-2}$, $T = \frac{(k^*-1)k^*(1-p_1)}{((1-p_1)k^*+p_1)^2\underline{\theta}}$, and $W = p_1(\underline{\theta} - P) + k^*(1-p_1)\underline{\theta}$.

Demand is given by equation 3.88. Taking the relevant derivatives and using the definitions of $b_1$, $b_{k^*}$, $B$, and the partials of $u$ and $v$ one can derive the exact expression for $\frac{\partial^2 D}{\partial P \partial \gamma} - \frac{1}{D}\frac{\partial D}{\partial P}\frac{\partial D}{\partial \gamma}$. After simplifying the large expression, the sufficient condition for it to be negative is:

$$-D(P,\gamma)(k^* - 2)T^2W^2S_1^{k^*}((1-\gamma)S_1^{k^*-3} - S_2^{k^*-3})u^3(1-v)^3 +$$
$$(k^* - 1)T^2W^2u^4(1-v)^4S_2^{k^*-1}((1-\gamma)S_1^{k^*-2} - S_2^{k^*-2})^2 < 0 \tag{3.95}$$

$$\Leftrightarrow$$

$$-D(P,\gamma)(k^* - 2)T^2W^2S_1^{k^*}((1-\gamma)S_1^{k^*-3} - S_2^{k^*-3})u^3(1-v)^3 +$$
$$(k^* - 1)T^2W^2u^4(1-v)^4S_2^{k^*-1}((1-\gamma)S_1^{k^*-2} - S_2^{k^*-2})^2 <$$
$$T^2W^2(k^*-1)(-D(P,\gamma)S_1^{k^*}((1-\gamma)S_1^{k^*-3}-S_2^{k^*-3})u^3(1-v)^3 + u^4(1-v)^4S_2^{k^*-1}((1-\gamma)S_1^{k^*-2}-S_2^{k^*-2})^2)$$
$$< \underbrace{T^2W^2(k^* - 1)D(P,\gamma)u^3(1-v)^3}_{>0} \times$$
$$\left[-S_1^{k^*}((1-\gamma)S_1^{k^*-3} - S_2^{k^*-3}) + S_2^{k^*-1}((1-\gamma)S_1^{k^*-2} - S_2^{k^*-2})^2\right] \tag{3.96}$$

Where the expression in square brackets is guaranteed to be negative for large enough $k^*$ because $S_1 > S_2$ and both are between 0 and 1. So, for large enough $k^*$, $\frac{\partial P_{NWOM}^*}{\partial \gamma} < 0$. ⊡

### Proof of Proposition 14
Since the degree is Poisson, the monopolist's objective function becomes:

$$(P - c)(1 - P)(\omega(1 - e^{S_1}) + (1 - \omega)(e^{S_1} - e^{S_2})) - \alpha\omega \tag{3.97}$$

where $S_1 = -z(1-\gamma)(1-P)(1-v)(u + \omega - u\omega)$ and $S_1 = -z(1-P)(1-v)(u + \omega - u\omega)$

The FOC with respect to $\omega$ gives:

$$\omega = \min\left\{1, \max\left\{0, -\frac{\alpha}{(1-P)(P-c)}\frac{1}{\frac{\partial u}{\partial \omega}+\frac{\partial v}{\partial \omega}} - \frac{1}{\frac{\partial u}{\partial \omega}+\frac{\partial v}{\partial \omega}}\left(-(1-u-v)+\frac{\partial u}{\partial \omega}\right)\right\}\right\}$$
(3.98)

Using the same methodology of partially differentiating the SCC's given by equations 3.26 and 3.27 and then solving the 2 by 2 system, I find:

$$\frac{\partial u}{\partial \omega} + \frac{\partial v}{\partial \omega} = \frac{e^{S_2}z(1-P)(1-v)(1-u)}{|A|}$$
(3.99)

where $|A| = 1 + z(1-\gamma)e^{S_1}(u + \omega - u\omega) - z((1-\gamma)e^{S_1} - e^{S_2})(1-P)(1-v)(1-\omega)$

I first show that $\exists \underline{\gamma}$ such that $\omega^* = 0$ for $\gamma \in [\underline{\gamma}, 1]$. Take the expression defined by equation 3.98 to the $\gamma \longrightarrow 1$ limit. In such limit $v \approx 0$, so $\lim_{\gamma \to 1}\omega^*$ becomes:

$$-\frac{\alpha(1 + e^{S_2}z(1-P)(1-\omega))}{(1-P)^2(P-c)z(1-u)e^{S_2}} - \frac{1 + e^{S_2}z(1-P)(1-\omega)}{(1-P)z(1-u)e^{S_2}}\left(u + \frac{e^{S_2}z(1-P)(1-u)}{1 + e^{S_2}z(1-P)(1-\omega)}\right)$$
(3.100)

This term is less than 0 for any permissible combination of parameter and variable values, so in this limit the interior solution would be negative. Consequently, the monopolist chooses $\omega^* = 0$.

Next, I show that for small $\alpha$ $\omega^*|_{\gamma=0} = 1$. To see this, first note that in such case $u = 0$, but $v > 0$ because by assumption negative WOM trumps advertisement. Plugging $\gamma = 0$ into equation 3.98 gives in the limit of small $\alpha$:

$$\omega^* \approx \frac{1 + ze^{-z(1-P)(1-v)\omega}}{z(1-P)e^{-z(1-P)(1-v)\omega}} = \frac{1}{z(1-P)(1-v)} + \frac{1}{1-P} > 1$$
(3.101)

So, $\omega^* = 1$ in this limit. The optimal advertising choice in such a way goes from 1 to 0 as $\gamma$ grows. But the function defined by interior solution to equation 3.98 may change the sign of its derivative with respect to $\gamma$ only once. Therefore, to prove that the function goes down in $\gamma$ on $[0, \underline{\gamma}]$ interval, it is enough to show that it goes down at $\gamma \longrightarrow 0$, which is true because the expression in equation 3.101 goes down in $\gamma$ by lemma 4.

To summarize, for small enough $\alpha$, the optimal advertising choice $\omega^*$ defined by equation 3.98 is equal to 1 for $\gamma \approx 0$, equal to 0 for $\gamma > \underline{\gamma}$ and is between is defined by a function that is downward-sloping in $\gamma$ which proves the result. The optimal price choice is also downward sloping in $\gamma$ because for Poisson degree distribution price and advertising are complements by Theorem 9 in Campbell (2013). $\boxdot$

**Proof of Proposition 15**

*(i)* A randomly picked agent with degree $k$ is uninformed with probability $\zeta^k$. After becoming informed, he buys the good with probability $(1 - P)$ and has a positive experience with probability $\gamma$. If he fails to buy or has a negative experience, then the information process stops right there. Each of his $k$ friends are uninformed with probability $g_1(\zeta)$ and also pass positive information to their $c_1$ friends with probability $(1-P)\gamma$. The process keeps repeating and can be expressed as:

$$\zeta^k(1-P)[(1-\gamma)+\gamma[1+k(1-P)g_1(\zeta)[(1-\gamma)+\gamma[1+\underbrace{c_1(1-P)g_1(\zeta)}_{\phi}[(1-\gamma+\gamma[...]]]]]] \quad (3.102)$$

Equation 3.102 can be re-written as:

$$\zeta^k(1-P)[(1-\gamma)+\gamma[1+\frac{k}{c_1}R]] \quad (3.103)$$

where

$$R = \phi[(1-\gamma)+\gamma[1+\phi[(1-\gamma)+\gamma[1+\phi[...]]]]] = \quad (3.104)$$

$$\phi(1-\gamma)+\phi\gamma+\phi^2\gamma(1-\gamma)+\phi^2\gamma^2+\phi^3\gamma^2(1-\gamma)+... =$$

$$\phi+\phi^2\gamma+\phi^3\gamma+... = \phi\sum_{j=0}^{\infty}(\phi\gamma)^j$$

So, in order to maximize the expected spread of the WOM process, the monopolist has to solve:

$$\max_{k} \zeta^k(1-P)\left[1+k\zeta^k\frac{1-P}{c_1}\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}\right] \quad (3.105)$$

$$s.t. \ k \geq 0$$

The FOC gives:

$$\zeta^k(1-P)\ln\zeta+\zeta^k\frac{(1-P)}{c_1}\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}+k\zeta^k\frac{(1-P)}{c_1}\ln\zeta\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}=0 \quad (3.106)$$

$$\ln\zeta+\frac{1}{c_1}\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}+k\frac{1}{c_1}\ln\zeta\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}=0$$

So, the solution to the problem is given by:

$$k^* = \max\left\{0,-\left(\frac{1}{\ln(\zeta)}+\frac{c_1}{\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}}\right)\right\} \quad (3.107)$$

*(ii)* In order to see that $k^*$ goes down with quality observe first that because $\frac{c_1}{\sum_{j=0}^{\infty}(\phi\gamma)^{j+1}}$ is finite, $\lim_{\gamma\longrightarrow 1/(g_1'(1)(1-P))} k^* = \infty$. Also, as $\lim_{\gamma\longrightarrow 1} k^*$ is finite because $\zeta = 1-u(1-P) \in [0,1)$ in such limit.

Taking the partial derivative of the interior solution of $k^*$ obtain:

$$\frac{\partial k^*}{\partial \gamma} = \frac{c_1^2(1-P)\left[\sum_{j=0}^{\infty}(j+1)(c_1(1-P)g_1(\zeta)\gamma)^j\right]}{\left(\sum_{j=0}^{\infty}(c_1(1-P)g_1(\zeta)\gamma)^{j+1}\right)^2}\left(g_1(\zeta)-\gamma g_1'(\zeta)\left((1-v)\frac{\partial u}{\partial \gamma}-u\frac{\partial v}{\partial \gamma}\right)\right)-$$

$$-\frac{(1-P)\left((1-v)\frac{\partial u}{\partial \gamma}-u\frac{\partial v}{\partial \gamma}\right)}{(\ln(\zeta))^2\zeta}$$

(3.108)

From lemmas 3 and 4 the expression $\left((1-v)\frac{\partial u}{\partial \gamma}-u\frac{\partial v}{\partial \gamma}\right)$ can be re-written as:

$$\left((1-v)\frac{\partial u}{\partial \gamma}-u\frac{\partial v}{\partial \gamma}\right) = \frac{u(1-v)(1-v+u)g_1'(S_1)}{|A|} > 0 \tag{3.109}$$

Since the expression is always positive, the partial derivative in equation 3.108 changes sign at most once. Therefore, to prove the statement it is enough to show that the derivative is negative in both limits $\lim_{\gamma\longrightarrow 1/(g_1'(1)(1-P))}$ and $\lim_{\gamma\longrightarrow 1}$.

Take the $\gamma \longrightarrow 1/(g_1'(1)(1-P))$ limit first. In such limit $\zeta = 1$ and the first term of the derivative is finite. Both the numerator and the denominator of the term

$$\frac{1-P}{(\ln(\zeta))^2\zeta}\frac{u(1-v)(1-v+u)g_1'(S_1)}{|A|}$$

go to 0. However, the numerator goes to 0 linearly, while the denominator goes to 0 exponentially, so the overall expression tends to $\infty$. Consequently, in this limit the derivative is negative.

Now, take the $\gamma \longrightarrow 1$ limit. A sufficient condition for the derivative to be negative in this limit is:

$$g_1(\zeta)-g_1'(\zeta)\left(\frac{\partial u}{\partial \gamma}-u\frac{\partial v}{\partial \gamma}\right) = g_1(1-u(1-P))-g_1'(1-u(1-P))\frac{u(1+u)c_1}{1-g_1'(1-u(1-P))} < 0 \tag{3.110}$$

By properties of generating functions $g_1(1-u(1-P)) < g_1'(1-u(1-P))$, so condition 3.110 is fulfilled whenever $\frac{u(1+u)c_1}{1-g_1'(1-u(1-P))} > 1$, which is always true because both the numerator and the denominator equal 0 in $P \longrightarrow P^c$ limit, but the numerator falls slower in $P$ and, thus, is larger for any value of $P$. $\boxdot$

# Bibliography

[1] Ahluwalia, R., Burnkrant, R.E., and H. Rao Unnava (2000): "Consumer Response to Negative Publicity: The Moderating Role of Commitment," *Journal of Marketing Research*, 37(2), 203-214.

[2] K. Bagwell, (2007): "The Economic Analysis of Advertising," in *Handbook of Industrial Organization*, Volume 3, edited by C. Mark Armstrong and Robert H. Porter, 1701-1844. Amsterdam: North - Holland.

[3] P.F. Bone (1995): "Word-of-Mouth Effects on Short-term and Long-term Product Judgments," *Journal of Business Research*, 32, 213-223.

[4] Book, L.A., Tanford, S., and Y. Chen (2016): "Understanding the Impact of Negative and Positive Traveler Reviews: Social Influence and Price Anchoring Effects," *Journal of Travel Research*, 55(8), 993-1007.

[5] Callaway, D. S., Newman, M E. J., Strogatz, S. H., and D. J. Watts. (2000): "Network Robustness and Fragility: Percolation on Random Graphs," *Physical Review Letters*, 85 (25), 5468-5471.

[6] A. Campbell (2013): "Word-of-Mouth Communication and Percolation in Social Networks," *The American Economic Review*, 103(6), 2466-2498.

[7] Caves, R.E., and D.P. Greene (1996): "Brands' Quality Levels, Prices and Advertising Outlays: Empirical Evidence on Signals and Information Costs," *International Journal of Industrial Organization*, 14, 29-52.

[8] P. Chatterjee (2001): "Online Reviews: Do Consumers Use Them?," *ACR 2001 Proceedings*, eds. M. C. Gilly and J. Myers-Levy, Provo, UT: Association for Consumer Research, 129-134.

[9] Coulter, R. A., Feick, L.F., and L. L. Price (2002): "Changing Faces: Cosmetics Opinion Leadership among Women in the New Hungary," *European Journal of Marketing*, 36 (11-12), 1287-1308.

[10] A. Galeotti (2010): "Talking, Searching, and Pricing," *International Economic Review*, 51(4), 1159-1174.

[11] Galeotti, A., and S. Goyal (2009): "Influencing the Influencers: A Theory of Strategic Diffusion," *RAND Journal of Economics*, 40 (3), 509-532.

[12] Herr, P.M., Kardes, F.R. and J. Kim (1991): "Effects of Word-of-Mouth and Product Attribute Information on Persuasion: an Accessibility-Diagnosticity Perspective," *Journal of Consumer Research*, 17(4), 454-462.

[13] Mahajan V., Muller E., and R.A. Kerin (1984): "Introduction Strategy for New Products with Positive and Negative Word-of-Mouth," *Management Science*, 30(12), 1389-1404.

[14] R.W. Mizerski: "An Attribution Explanation of the Disproportionate Influence of Favorable and Unfavorable Information," *Journal of Consumer Research*, 9(3), 301-310.

[15] M.E.J. Newman (2005): "Threshold Effects for Two Pathogens Spreading on a Network," *Physics Review Letters*, 95, 108701.

[16] M.E.J. Newman (2009): "Random Graphs with Clustering," *Physics Review Letters*, 103, 058701.

[17] Newman M.E.J., and C.R. Ferrario (2013): "Interacting Epidemics and Coinfection on Contact Networks," *PLoS ONE*, 8(8), e71321.

[18] Newman, M.E.J., Strogatz, S.H., and D.J. Watts (2005): "Random Graphs with Arbitrary Degree Distributions and Their Applications," *Physics Review E*, 64, 026118.

[19] J.E. Kwoka Jr. (1984): "Advertising and the Price and Quality of Optometric Services," *American Economic Review*, 74, 211-216.

[20] M. Richins (1983): "Negative Word-of-Mouth by Dissatisfied Consumers: A Pilot Study," *Journal of Marketing*, 47(1), 68-78.

[21] Skowronski, J.J., and D.E. Carlston (1989): "Negativity and Extremity Biases in Impression Formation: a Review of Explanations," *Psychological Bulletin*, 105(1), 131-142.

[22] Tellis, G.J., and C. Fornell (1988): "The Relationship Between Advertising and Product Quality Over the Product Life Cycle: A Contingency Theory," *Journal of Marketing Research*, 25, 64-71.

[23] Van den Bulte, C., and Y. V. Joshi (2007): "New Product Diffusion with Influentials and Imitators," *Marketing Science*, 26 (3), 400-421.