NETWORK ANALYSIS OF THE HUNGARIAN INTERBANK MARKET – A MULTIPLEX APPROACH

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Abstract

Studying financial systems from a network perspective became an essential part of the economic literature after the financial crisis. This thesis analyzes the Hungarian interbank network on the basis of a recently emerged concept, namely the multiplex approach of the financial networks. This idea emphasizes that banks are connected through multiple channels, which should be considered when assessing systemic risk. Using a dataset of the Hungarian uncovered interbank transactions between 2003 and 2013 this thesis shows that the overnight and the longer-term maturity layers of this market have different characteristics in both cross-sectional and historical comparison. The analysis also reveals that the relative importance of the banks in the system is strongly varying across the layers, which should be taken into consideration in the SIFI assessment methodologies.

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1. INTRODUCTION

The recent financial crisis made it clear that understanding the network structure of the financial sector is crucial to assess systemic risk adequately. As Smaga (2014) points out, the economic literature provides various definitions for the concept of systemic risk. In this thesis, I will use this expression to the risk that emerges due to the interconnectedness of the financial institutions. It is the risk that an idiosyncratic shock affecting only one or few institutions will be amplified in the network due to the interbank connections, which finally leads to system-wide disturbances that may also have a real economic effect.

Beside "systemic risk" another expression started to be widely used in the literature about the financial system: the "too connected to fail" institutions. This concept took over the place of the "too big to fail" expression, and it emphasizes that the importance of an institution in the financial network does not only depend on its size but rather its embeddedness in the system. One key purpose of the network analysis of the financial market is the detection of these banks, the so called systemically important financial institutions (SIFIs).

The identification of the SIFIs became a crucial part of the macroprudential regulation of the central banks and financial authorities because the default or some serious solvency problems of these banks can have far-reaching effects. First, if a key bank defaults in the system then it causes direct losses to its creditors. Second, this effect may induce further defaults among the creditors, which means losses to their counterparties as well. In this way, the original default can be both directly and indirectly contagious, which potentially leads to solvency problems of institutions that were not partners of the originally problematic bank. As a third mechanism, the increased counterparty risk may result in a partial or full freeze of the interbank market. As a consequence, banks are compelled to find alternative ways of financing, which usually means higher costs since the interbank market is generally the cheapest way to get additional short-term liquidity.

In the recent years, a new approach appeared in the literature about the financial networks. This concept puts emphasis on the fact that financial institutions are connected through multiple layers forming a multiplex network. These papers suggest that if we combine the information that is incorporated in the various networks, then we get a more precise picture of the network structure of the interbank market, which ultimately can be used for more effective regulation. For example, banks can be directly connected through bilateral exposures in the uncovered interbank market, the covered interbank market (repo market) and the swap market (for example FX-swap). These networks may be further divided by the maturity of the transactions. Developing methods to analyze the separate sub-networks, their similar and diverse characteristics, and to assess how they contribute to the overall systemic risk is an interesting and currently expanding part of the financial network literature.

In this thesis, I will analyze the Hungarian uncovered interbank market using multiplex network approach. Although some papers have already analyzed the Hungarian financial network, this new multiplex approach has not yet been applied to Hungarian data. Albeit the uncovered interbank market is only one part of the complex financial network (see Section 3.1.), it can be useful to get an insight into the system. Furthermore, since the transactions are uncovered in this market, a potential distress could cause severe losses for the banks. Thus, knowing the characteristics of the network and detecting the systemically important institutions may be crucial not only from theoretical but also practical point of view.

The analysis is based on an anonymized dataset containing the uncovered interbank transactions between January 2003 and April 2013 provided by the Hungarian National Bank (MNB). The dataset provides information about the transacted amount, interest rate and maturity of every interbank transaction, which makes it possible to create sub-networks based on the different maturity of the deals. Since the uncovered interbank market is mainly used to get short-term liquidity, I create two layers: the overnight network containing transaction with one-day maturity, and the longer-term network that incorporates all other transactions.[1]

My first hypothesis is that the Hungarian uncovered interbank network shows small world characteristics where some large hubs are connected to many other banks, while the periphery-nodes only have a few counterparties. This finding would be in line with many empirical papers analyzing various interbank networks. Regarding the multiplex comparison, I expect that banks show different behavior in the overnight (ON) and the longer-term (LT) layers. This hypothesis is based on the presumption that these sub-networks serve different purposes for the banks. While the overnight network is the market to overcome short-term liquidity shocks (from the borrowing side) and expose excess liquidity (from the lending

[1] For every calculation including the creation of the networks I used the R programming language. The networks were visualized with Gephi.

side), the longer-term market may be a substitute for refinancing longer-term obligations from the non-interbank market.

The thesis is structured as follows. Chapter 2 contains a selected review of the relevant literature focusing on the single and multilayer approaches of the financial network analysis, as well as the identification of SIFIs. Chapter 3 gives a short description of the current Hungarian interbank market, and then it turns to the analysis of the interbank transaction network. The last section summarizes the findings.

2. LITERATURE REVIEW

This chapter provides a selected literature review about the network analysis of financial systems. In the first section, I summarize the main concepts about the relationship of the network structure of the financial market and the systemic risk, as well as the relevant empirical analysis about the Hungarian interbank network. Then I show papers that use the recently emerged multiplex approach. The last section gives an overview about the SIFI identification from both theoretical and practical point of view.

2.1. Financial networks – traditional approach

Many theoretical and empirical papers have tried to evaluate how the interconnectedness of the financial institutions affects the stability of the system. Some authors argue that more interconnected institutions form a more resilient system since in a loosely connected network banks have large exposures towards a few counterparties that can be the source of instability. For example, Allen and Gale (2000) emphasizes that a densely (preferably fully) connected financial network enhances risk-sharing of the banks since the losses of a defaulted bank are shared across its creditors, which makes the system more robust to shocks.

In contrast to this view, many authors started to analyze the increased systemic risk that arises from a dense interbank network. These papers focus mainly on the interbank contagion, when a shock of one or few institutions spread across the network and causes disturbances for other institutions as well. Gai and Kapadia (2010) argue that although the interconnectedness of the banks may reduce the probability of a severe distress in the network, it will amplify its effect through the interbank contagion channel. They call this phenomenon the "robust-yet-fragile" characteristics of the financial networks.

Battistion et al. (2012) drew a similar conclusion based on their dynamic modelling framework. The authors found that the connectivity of the institutions in the network and the probability of their default have a U-shaped relationship: in the case of a loosely connected network the increase of the interbank exposures is beneficial, but in a densely connected system the negative effect of interbank contagion exceeds the gain of risk sharing.

Acemoglu et al. (2015) also showed that the financial network disposes of a "robust-yet-fragile" property, but in their framework this phenomenon depends on the size of the shock rather than the density of the network. The authors claim that as long as the shock that hits the system is below a certain threshold, a denser network is much more robust than a sparse one. However, if some large shock affects the system, then contagion is more severe and quicker in a highly connected interbank network. In this case a dense network becomes fragile, while a sparse network is more resilient to the shock.

2.1.1. Hungarian application

Beside the theoretical foundations of financial networks, there is an extensive literature about the empirical analysis of specific networks. In the case of the Hungarian financial system, Lublóy (2006) provided an early paper in this context. She analyzed the Hungarian large-value transfer system (VIBER) that is a settlement system for transactions of larger amounts operated by the MNB. The network analysis of transactions in 2005 discovered that the linkages and the importance of the banks in this payment system were quite stable over time. Those banks that were central in one day tended to be important in the following days as well (i.e. the centrality indicators were strongly autocorrelated). In addition, the author identified the key institutions in the system based on various centrality indicators. The results showed that surprisingly not the largest banks were the most important ones, but rather those that were active in the USD/HUF FX swap market. This finding also indicates that the centrality of a bank from a network point of view is not always in line with its relative size (its total assets compared to other banks).

Another relevant empirical analysis was carried out by Berlinger et al. (2011) who studied the Hungarian uncovered interbank market using a transactional dataset between 2002 and 2009. The main purpose of this paper was the comparison of the pre-crisis and post-crisis networks. The authors found that although the number, the interest and the magnitude of the transactions did not show any structural change before the collapse of Lehman Brothers in September 2008, some network indicators like the density and the average closeness of the nodes started to show different dynamics beginning from 2006. It indicates that the banks anticipated the increased risks in the market long before the outbreak of the crisis.

As Lublóy (2006) pointed out, the FX swap market is crucial in the Hungarian financial network. Banai et al. (2015) carried out the network analysis of this market and showed that severe disruptions were detectable during the crisis, which led to the fragmentation of the network and the vanishing of some group of nodes. Although the scope of this paper did not include a thorough multi-layer analysis, the authors differentiated and compared the FX sub-markets of different maturities. Their results indicated that the maturity-layers showed different dynamics and reacted distinctly to the shock**s** of the financial crisis. This finding also indicates that the segmentation of the interbank markets and the comparison of the sub-network**s** can add new aspects to the financial network analysis.

2.2. Financial networks – multiplex approach

As discussed in the introduction of this thesis, the literature about the interbank networks has turned to the multi-layer approach only during the recent years. One early contribution to this literature was written by Montagna and Kok (2013) who developed an agent-based multi-layer model to analyze the European interbank market. Their dataset included detailed balance sheet information of 50 large EU banks at the end of 2011. Since the interbank exposures data were not available, the authors used simulated interbank networks. The most important finding of this paper was that the disregard of the interconnectedness of banks across multiple layers leads to a serious underestimation of systemic risk. They found that a shock affecting the system can be significantly intensified if the banks are connected in more than one network.

Bargigli et al. (2015) analyzed the Italian interbank network from the multiplex point of view. Their dataset contained interbank exposures that made it possible to create secured and unsecured exposure layers along different maturities. The authors suggested differentiating between two aspects of similarity: the topological similarity that compares the network characteristics, and the point-wise similarity that relates the node-level indicators like the centrality metrics. The analysis detected strong topological differences among the layers meaning that a connection between two banks in one of the sub-networks did not imply their linkage in other layers. Thus, the authors concluded that using the overall interbank network or only one of its sub-layers to assess systemic risk in the market can be highly misleading.

A pioneering paper was published by Aldasoro and Alves (2016) about the multiplex approach of interbank networks. Firstly, the authors amended the input-output model of Aldasoro and Angeloni (2013) by creating a multi-layer theoretical framework that makes it possible to assess the multi-layer systemic importance of the nodes. Secondly, they applied this approach to a dataset containing bilateral exposures among 53 large European banks at the end of 2011. The authors created layers by decomposing the network based on the instrument type (assets, derivatives and off-balance-sheet) and the maturity (less than one year, more than one year and unspecified) of the exposures.

Following the distinction of layer and node similarities suggested by Bargigli et al. (2015), Aldasoro and Alves (2016) first compared the adjacency matrices of the created interbank layers. They found that the instrument layers of the same maturity are not necessarily overlapping: for example while 81% of all bilateral connections are observable in the asset network, this share is only 48% in the case of the off-balance-sheet layer. Considering the maturity decomposition, the authors found that generally the different maturity-layers of the same instrument type show similar characteristics, while different instrument-layers are rather diverse.

As a next step, the authors also assessed how the systemic importance of the nodes is related across the layers. Their analysis included the correlation of various centrality metrics and the core-periphery structure across the layers. The results suggested that node-level indicators are positively correlated, thus systemic importance is quite persistent across layers of the analyzed network, which an opposite finding as of Bargigli et al. (2015). However, the authors emphasize that even in this case the decomposition of the systemic importance of the nodes into layer-specific contribution is an important aspect that can deepen our knowledge about the structure of the interbank market.

The literature of the multiplex financial network analysis is steadily growing with both theoretical modelling frameworks (see for example Peralta and Crisóstomo, 2016), and empirical applications (see for example Poledna et al., 2015). My comparison of the overnight and the longer-term layers of the Hungarian interbank network in Section 3.3.4. is based on these presented theoretical and empirical papers.

2.3. SIFI identification

As I emphasized in the introduction of this thesis, shocks can easily spread across the financial system and cause disturbances in that part of the network that was directly unaffected by the original shock. One of the primary purposes of the network analysis of the interbank market is to identify those institutions whose turmoil or default is likely to be very contagious in the network. These so called SIFIs are key nodes in the network, and therefore the regulators have a strong incentive to prevent their potential insolvency.

The identification of the SIFIs can be done in many ways, and there is a wide range of theoretical literature and practical methodologies to find these key nodes in the system. From a network point of view, the importance of a node can be captured with different metrics of centrality, like degree, betweenness, closeness or eigenvector centralities. These indicators evaluate the importance of a node in the network from various aspects. Therefore they often lead to divergent conclusions regarding the key institutions in the network.

Alter et al. (2015) applied these network metrics to identify the key institutions and the potential effect of their default in the German banking system. The authors combined these indices with data about the lending activity of the banks, which made it possible to evaluate the effects of correlated portfolio losses and interbank contagion in a holistic framework. Their analysis based on a dataset containing more than 1700 German banks supported that network centrality measures can be efficiently used to mitigate systemic risk. They showed that a capital allocation mechanism that takes into account the network centrality indicators can improve financial stability compared to a baseline model that disregards the interbank network.

A recent example for the SIFI identification in the Hungarian interbank market is the paper of Fukker (2017). The author applied the harmonic distance suggested by Acemoglu et al. (2015) to assess the centrality of the nodes. Both his simulations and the application to real data showed that this way of SIFI assessment performs similarly to the "usual" centrality metrics. However, this approach can be used as an indicator of financial stress in the system as it peaked around the financial crisis but was quite stable in normal times.

A more complex approach for the SIFI identification is proposed by Battiston et al. (2012) who introduced a DebtRank value measuring the importance of a node in the network. This metrics is based on the idea of feedback centrality that evaluates the role of a node not only by its place in the network but also by the importance of its neighbors. The DebtRank measures the "fraction of the total economic value in the network that is potentially affected by the distress or the default" of the particular node. (Battistion et al., 2012, p. 3) The calculation is made through the following steps:

1. First, the authors calculate the direct loss that the default of a node would cause to its partners based on the outstanding amounts. These losses are weighted with the

economic value of the counterparties, measured by the share of their exposures and the total exposures in the network.

2. Then, the authors determine the weighted losses of the partners of the neighbors that were directly affected, so they evaluate the spillover contagion of the original default. And they continue with the neighbors of these nodes and so on.
3. The total DebtRank is a weighted sum of these implied losses, where a dampening factor is used to assign a lower weight for nodes that are farther from the original node.

The authors applied the introduced DebtRank metrics for a special network of the FED emergency loan program during the financial crisis. They combined these data with the equity investments among the institutions to have a directed network for the estimation. The analysis revealed that the interbank network was highly interconnected with some very large borrowers with high DebtRank. It means that even a small shock of these banks could have caused the collapse of the system with great losses for the periphery nodes as well.

Another application of the DebtRank is the contagion analysis of Fink et al. (2015). They used this indicator on an end-of-year 2013 data about the German interbank exposures, and established a framework that is appropriate for the estimation of the effect of both idiosyncratic and system-wide shocks taking into account the interbank contagion channel. Their algorithm estimates how an increase of the probability of default (PD) of a bank affects its creditors, which increases the creditors' PD as well, that has an effect on their creditors, etc. By combining this methodology with real interbank exposure data, the authors estimated the effect of a real estate shock as an example. Their results showed that the indirect interbank contagion is responsible for the half of the total realized losses in the network. This modeling framework can be easily adjusted and therefore used to identify SIFIs or answer various policy questions.

Bluhm et al. (2013) also used the interbank contagion chain for the SIFI assessment. In their model, the banks are connected through their correlated balance sheets (non-interbank exposures) and also in the interbank market. The importance of a bank in the network is calculated as its contribution to the overall systemic risk. The marginal contributions are defined using the Shapley-value. They found that this SIFI identification and contagion simulation can be used for more effective policy making since the overall systemic risk can be reduced by adequate incentives. One measure they suggest is the taxation of interbank returns that would charge interbank transactions and therefore decrease the interconnectedness of banks and mitigate contagion.

Since my dataset contains anonymized interbank transactions, I cannot apply the more complex SIFI assessing methodologies due to the lack of real balance sheet (most importantly capital) data. Thus, for the SIFI identification in section 3.3.3. I will rely on the network centrality metrics.

2.3.1. SIFI assessment in practice

The recognition of the critical role of the systemically important financial institutions led to the change of regulation as well. The Basel Committee on Banking Supervision (BCBS) suggested a SIFI assessment methodology and a calculation of additional capital buffers these institutions should be required to hold to increase their loss absorbency as a part of the Basel III regulatory framework. (BCBS, 2013) Although the Committee recommended a common quantitative framework for the SIFI (or as they call them, global systemically important bank, G-SIB) identification, they emphasize that banks show large differences in their activities and portfolio structure. Thus, the SIFI assessment may be subject to qualitative refinements.

The BCBS (2013) publication uses an indicator-based methodology for scoring the importance of the banks. The indicators cover five categories that equally have 20% weight in the final score: size (total exposures), interconnectedness (interbank exposures), substitutes or financial institution infrastructure, complexity and cross-jurisdictional activity. After assigning a score to each institution, the Committee ranks them and suggests pre-defined additional capital requirements for each bucket (see Appendix Table 1).

The BCBS methodology is incorporated into the Capital Requirement Directive[2] (CRD IV) that serves as the harmonized banking regulation within the European Union. The Article 131 of CRD IV distinguishes the global (G-SII) and other (O-SII) systemically important institutions, where the latter ones are not key banks on international level but they are highly influential within the local financial network. A guideline for the assessment of such O-SIIs was issued by the European Banking Authority (EBA) in 2014 that strongly relies on the BCBS recommendations (EBA, 2014).

[2] Full text of the Directive is available at:
http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2013:176:0338:0436:En:PDF

Since neither of the Hungarian banks is qualified as a global systemic important institution by the Basel methodology, the SIFI assessment of the MNB is based on the EBA guidelines for the O-SII identification (MNB, 2016). In line with the standard methodology of the EBA guidelines, the scoring evaluates the banks with respect to four categories that actually include all aspects of the original Basel recommendation. Furthermore, the standard approach is amended with a supplementary methodology that already incorporates a network analysis of the interbank market. For the full list of indicators and the corresponding weights used by the MNB see Appendix Table 2.

Based on the determined scores of systemic importance the MNB requires additional capital buffers for the most significant O-SIIs. These buffers vary between 0.5% and 2% and will be gradually introduced until 2020. The first deadline when systemically important Hungarian banks have to comply with the required systemic risk buffer is 1st July 2017. For the list of Hungarian SIFIs identified by the MNB and the planned introduction of their additional capital buffer requirements see Appendix Table 3.

Although the suggested methodology of the Basel Committee is widely applied by the national banking authorities, some critics also emerged concerning the calculation. Benoit et al. (2017) argue that the calculation of the scores is statistically biased, and some indicators get disproportionally high weights in the total score. Their observation is that the consequence of the pure weighted sum of the five indicators is that the scores are dominated by the categories that have the largest cross-sectional variation. Thus, the indicators should be standardized (for example by their standard deviation) to be in line with the original concept of equal weights across the categories. The empirical analysis of the authors showed that this correction leads to much more robust SIFI identification.

As we see, the problem of SIFI identification is an essential part of the post-crisis banking regulation that focuses on the maintenance of financial stability. As the regulation of the SIFIs through additional capital buffers becomes part of the everyday macroprudential toolkit of the banking authorities, the deep analysis of the interbank network and the interconnectedness of the financial institutions are getting more and more crucial. In the following chapter, I will analyze and evaluate the network characteristic of the Hungarian interbank network, which can add some aspects to these problems from the network theory point of view.

3. EMPIRICAL ANALYSIS

The first two sections of this chapter give an overview of the basic characteristics of the Hungarian interbank network and my analyzed transactional dataset. Then I analyze the most recent available interbank network along four aspects: undirected network, directed network, SIFI identification and the multiplex analysis of the system. However, to get a clear picture of this market, it is not enough to study only the "current" (the latest available) snapshot. Therefore, the fourth section presents how the networks and their characteristics changed over time.

3.1. The Hungarian interbank network

Before analyzing the transactional dataset, it is worth to assess the importance of the interbank lending within the Hungarian financial network. As I mentioned in the introduction, the main purpose of the interbank network is the get additional short-time liquidity. Thus, banks probably do not finance their longer-term assets from the interbank network, but it serves as an important market to overcome liquidity shocks that affect the system in an asymmetric way.

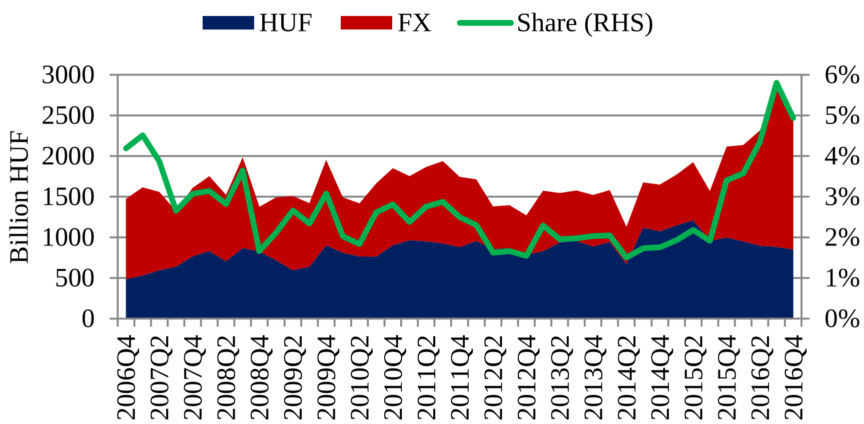


Figure 1 – Hungarian interbank exposures (end-of-quarter)

Source: MNB

In Figure 1 we can see that amount of the overall interbank exposures among the Hungarian credit institutions based on quarterly MNB data. The green line (right-hand side axis) shows the share of the interbank exposures compared to the total liabilities of the sector. Based on aggregated data the interbank exposures amounted around 2-3% of the total liabilities during the 2006-2016 period, while it peaked in 2016 reaching almost 6%. Regarding the denomination, only 1/3 of the latest available interbank exposure data was in HUF, but as we can see the share of FX exposures was quite varying over time.

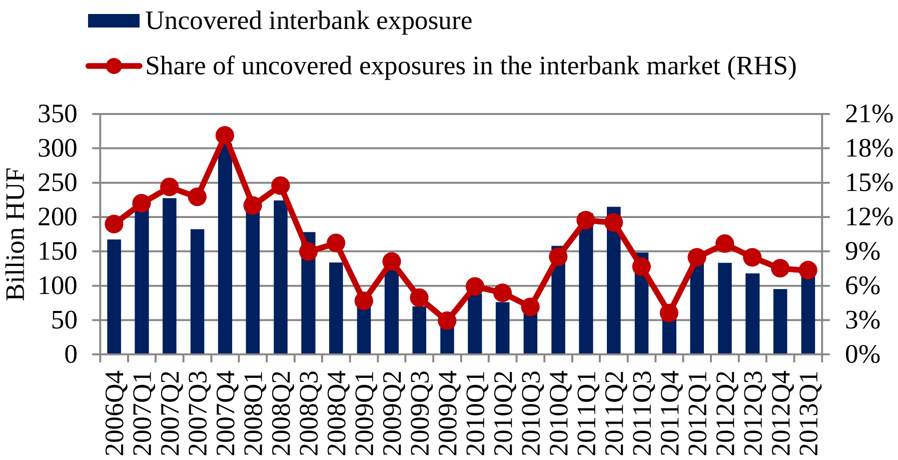


Figure 2 – Hungarian uncovered interbank exposures (end-of-quarter)

Source: MNB and own dataset (provided by the MNB)

Although banks are connected through the overall interbank market, from a systemic risk point of view the uncovered interbank lending is the most interesting sub-market since only marginal counterparty risk emerges if the exposures are covered with securities. Unfortunately, there is no openly accessible data about the decomposition of the formerly presented interbank exposures, so I used my transactional database to calculate end-of-quarter uncovered interbank exposures. In Figure 2 we can see these exposure values compared to the overall aggregated interbank exposures (red line). The presented time period is the intersection of my 2003-2013 transactional dataset and the 2006-2016 aggregated time series.

As we can see in Figure 2 the uncovered interbank lending summed up to 150 billion HUF in average over this period, which meant 9.5% of the total interbank market. However, this ratio was quite volatile, reaching almost 20% in 2007Q4 but falling to 3% in 2011Q4. Compared to the total liabilities over this period, we can conclude that the uncovered interbank market had a share of 0.5% in average. This value does not seem to be notable, but a default of an important borrower in this market can cause serious losses for its counterparties.

The previous statement can be supported with a simple calculation. Based on the data used for Figure 1 the total aggregated liabilities of the credit institutions were 31 158 billion HUF in average between 2006 and 2016. Using the 0.5% average share of the uncovered interbank market, such exposures summed up to 155.8 billion HUF. As I will present in Section 3.3.2., due to the high concentration on the borrowing side, the most important bank can be responsible for 1/3 of the total borrowed amount. If we consider a potential default of this key institution, then it causes 51.9 billion HUF instant loss for its counterparties. If we compare it to the 124 billion HUF average pre-tax profit of the credit institutions over this 11 years period (see Appendix Figure 1), then we can see that the potential systemic risk in this sub-network is not as negligible as it seemed to be based on its share among the total liabilities (or total assets). Of course, it is an extremely simplified calculation that does not take into account the cross-sectional and historical variation of the interbank lending activity. Unfortunately, due to the anonymity of my transactional dataset, I cannot individually compare the interbank lending behavior of the banks to their balance sheet. Such an analysis could reveal more appropriate conclusions.

3.2. Data description

The dataset provided by the Hungarian National Bank (MNB) contains uncovered interbank transactions between January 2003 and April 2013. The data is anonymized, so the banks are not identifiable. Therefore, I will use the assigned IDs as "names". Beside the source and target IDs the following information is available for every transaction: start date, finish date, transacted amount and interest rate. From the start and finish date I derived the maturity of the deals (adjusted with weekends and holidays). The maturity will serve as the basis of the segmentation of the market into overnight and longer-term sub-markets.

The data contains 102 941 transactions among 45 banks.[3] However, as one can expect, neither the number of the transaction nor the number of active banks was stable in the observed period. To get an insight into the historical changes of the market I created monthly one-year windows from the dataset. For example, the "April 2010" network refers to a network that is built based on the interbank transactions with start date between 1st April 2010 and 31st March 2011. These monthly one-year networks will be used during the whole analysis.

[3] Actually, the dataset contains 53 separate institutions but those that belong to the same banking group are merged together.

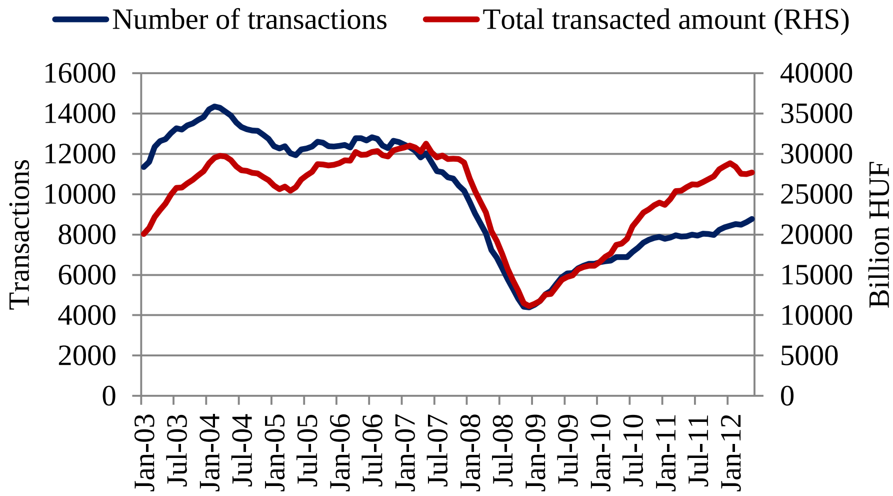


Figure 3 – Total transactions in the one-year networks

In Figure 3 we can see the total number and amount of the interbank transactions. Every observed point represents a one-year network beginning at the indicated month. Before the 2008-2009 financial crisis, the time series were quite stable: the banks transacted around 25-30 thousand billion HUF through 10-12 thousand transactions. However, as the crisis unfolded and reached the Hungarian banking sector the interbank lending activity dramatically decreased. In the November 2008 network, right after the collapse of Lehman Brothers, the transacted amount was around one-third of the pre-crisis level.

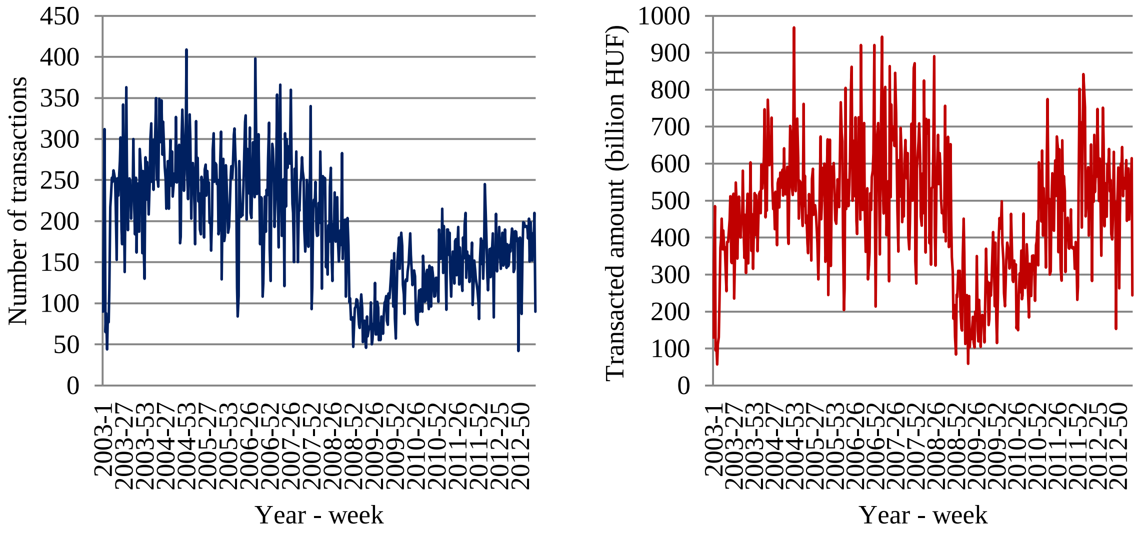


Figure 4 – Weekly uncovered transactions

The narrowing of the market is even more apparent if we consider the weekly aggregated transactions and transacted amounts in Figure 4. Right after the Lehman default, the average 234 weekly transactions (average of all weeks before 46th week of 2008) fell to 83 transactions (average of weeks in the following one year). The fallback was similarly strong if we consider the transacted amounts: it decreased from the average weekly 520 to 210 billion HUF. As Berlinger et al. (2011) points out this phenomenon reflects that the Hungarian banks reacted to the increased uncertainty in the market with pulling back their interbank activity rather than increasing the interest rates. After the crisis, the uncovered interbank lending activity started to increase, but it did not reach the pre-crisis level even in the latest observed network (but it probably did in the following years).

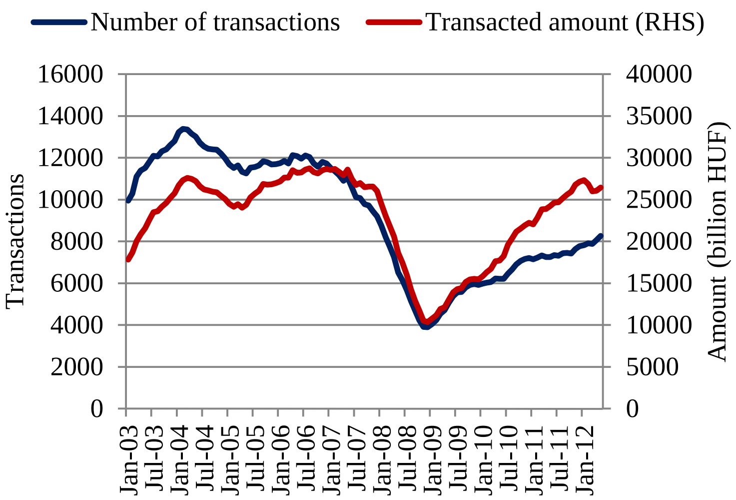


Figure 5 – Overnight transactions in the one-year networks

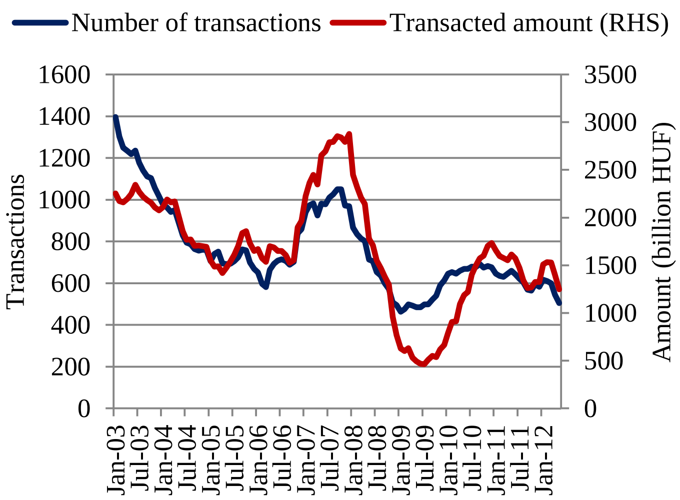


Figure 6 – Longer-term transactions in the one-year networks

Since the main focus of my thesis is to separate and compare the two identifiable layers of the interbank network, it is important to have an insight into the basic dynamics of these sub-markets. As Figure 5 and Figure 6 present, we can see different patterns for the two layers. In this case, it is also worth to divide the analyzed period into the pre-crisis, crisis and post-crisis years because the dynamics of the analyzed metrics usually show clearly detectable differences among these time periods.

Regarding the number of transactions and the total transacted amount in the case of the overnight sub-network, we can see similar dynamics as for the whole market, which is not surprising knowing that the majority of the interbank transactions are overnight. However, the longer-term sub-market shows different patterns. In the pre-crisis period it gradually decreased until 2006, then within one year, the total transacted amount almost doubled. The crisis also left its mark on this sub-market since the transactions fell below their pre-crisis level during 2008 and 2009. Although this sub-market consolidated to some extent, we cannot see signs of any increasing trend. It may indicate that banks introduced more severe interbank exposure limits towards each other during the crisis period and these limits have not been loosened since then.

After having an overall picture of the transactions in the market, in the following Sections I will analyze the latest available one-year network that begins on 1st May 2012. Then in Section 3.4., I present the historical analysis of the interbank lending data together with various network-level and node-level metrics.

3.3. Analysis of the latest network

The data in my transactional database make it possible to build different types of networks that help me to analyze the Hungarian interbank market from various aspects. One aspect is the type of connection among the nodes. If we build an undirected network, then we connect two banks if any transaction occurred between them during the observation period. In this way, we can get an insight into the interconnectedness of the banking sector, which is highly important for the assessment of systemic risk and potential contagion.

However, we can get additional information about the system if we also take into consideration the direction of the transactions by building a directed network. In this way, we can also analyze which banks appear mostly on the lending, on the borrowing or both sides of the transactions. This approach makes it possible to differentiate the SIFIs on the borrowing and the lending sides of the market.

In the next sub-section, I will identify the most important characteristics of the undirected network. Then I turn to the analysis of the directed system. Based on the directed network I identify the most important institutions in the third sub-section. The fourth part is a fundamental element of the multiplex comparison, where I evaluate both the layer and the node similarities of the overnight and the longer-term submarkets.

3.3.1. Undirected network

The latest one-year window in my dataset contains the interbank transactions between May 2012 and April 2013. To get an insight into the “current” state of the Hungarian interbank market, I will use the network built using this time period. This network contains 34 financial institutions and 8 777 transactions among them. In this part, I won’t differentiate between the overnight and the longer-term layers because this sub-section serves as a general overview of the interbank market.

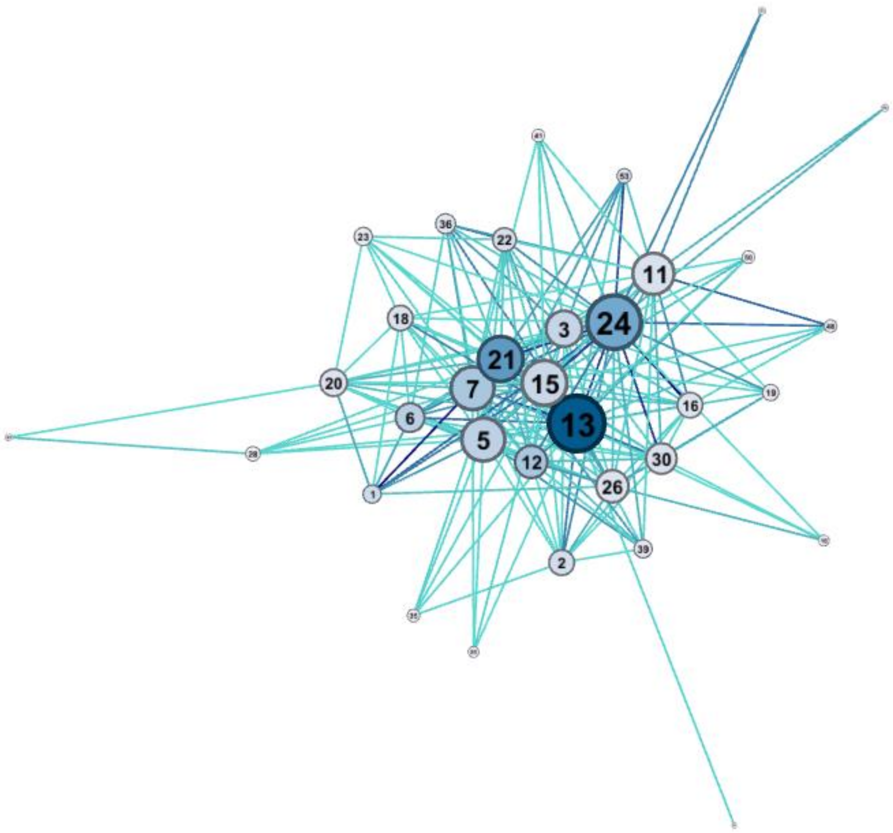


Figure 7 – Undirected one-year network (May 2012)

Figure 7 presents the latest available one-year network. It is an undirected graph where two nodes (banks) are connected if at least one transaction occurred between them during the observation period. The color of the links indicates the amount transacted between the two adjacent banks during this one year period: the darker is the line, the greater is the transacted amount. The basic metrics of the network are summarized in Table 1.

Table 1 – Basic metrics of the undirected network

Indicator	Value
Number of banks	34
Number of connections	197
Diameter	3
Average degree	11.588
Average path length	1.699
Density	0.351
Betweenness centrality	0.165
Global clustering coefficient	0.577
Average local clustering coefficient	0.733

The first thing one can observe is that the graph seems to be quite dense, which means that banks are connected with many other banks in the uncovered interbank market. The network consists of one giant connected component, so every node can be reached by any other node through some path that contains at most 3 nodes as the diameter of the network shows. Based on this we can conclude that the Hungarian interbank network has a small world characteristic, but it is not surprising given that it is a network with a quite small number of nodes.

The average degree in the network is around 11.6 meaning that a randomly selected bank is expected to be connected with 11-12 other institutions. This phenomenon has an ambiguous effect on financial stability from a theoretical perspective. First, if many banks are connected on the market, then it is easier to find new transaction partners for an institution if one of its partners is not capable of taking part in the network anymore. On the other hand, in a highly connected network financial distress can spread much faster if some key counterparties become insolvent. Therefore, it is crucial to identify the key banks in the network, which I will present in Section 3.3.3.

To quantify the density of the network we can use various metrics. First, we can calculate the density which is the ratio of the realized links and the potential maximum number of links. In this network, there are 34 nodes meaning that the maximum number of links can be 561. Since there are 197 observed links, the density of the interbank network is 0.351. Another meaningful indicator is the average local clustering coefficient which is 0.73 in this network. It can be interpreted as the probability that two randomly chosen neighbors of a node are connected to each other. The global clustering coefficient that measures the closed triangles in the network also gives us a hint about the connectedness of the nodes is 0.58.

These metrics suggest that the Hungarian interbank network based on one-year transactions is quite dense and the banks are strongly connected to each other. However, I have to emphasize that these values reflect a network built using *transactions* over a one-year period. Thus, it enables me to study the interbank connections in a more robust way, where day-to-day variations do not influence the results. However, the findings are not comparable with such empirical analyses that use the interbank *exposures* as a basis. For example, Bargigli et al. (2015) analyzed the Italian interbank market, and they found that the density of the network was around 0.01 (1%) at the end of 2012. Similarly, the German interbank network analyzed by Alter et al. (2015) had a density of 0.007 (0.7%) at the end of March 2011. But these networks are based on the interbank outstanding amounts (which is a stock variable) rather than the transactions over a certain period (which a flow variable). Thus, concluding that the Hungarian interbank network is by magnitudes denser than in other countries would be misleading.

Degree distribution

One key indicator that helps us to understand the characteristics of our network is the degree distribution; that is how many connections a node has in the system. Based on Barabási (2016) real life networks tend to have power-law degree distribution, which can be described by the following equation:

$$p_k = k^{-\gamma}$$

, where p_k stands for the probability that a node has degree k, and γ is the degree component. In this case the degree distribution is approximately a straight line on a log-log scale graph. This means that in such networks there are a few nodes with extremely high degree, the so called hubs, while a lot of nodes have only a few links. This phenomenon, i.e. the existence of large hubs ensures that the network has a small world characteristic.

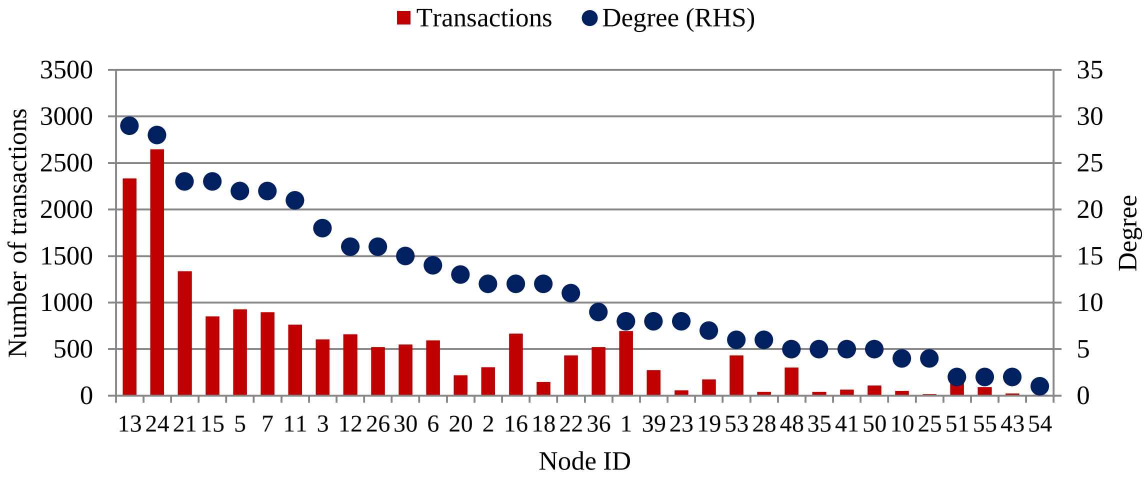


Figure 8 – Degrees and transactions in the undirected network

In Figure 8 you can see the degrees of the banks in descending order marked by blue circles. Contradicting the aforementioned empirical evidence of the observed degree distributions we cannot see nodes with extremely high degrees. The degrees of the nodes seem to be equally distributed meaning that this network cannot be regarded as scale-free based on the unweighted degree distribution. The scale of the degrees is quite wide compared to the total number of the banks in the network: the least active institution had transactions with only one partner, while the most active one was involved in transactions with 29 other banks (which is 88% of the total possible number of counterparties).

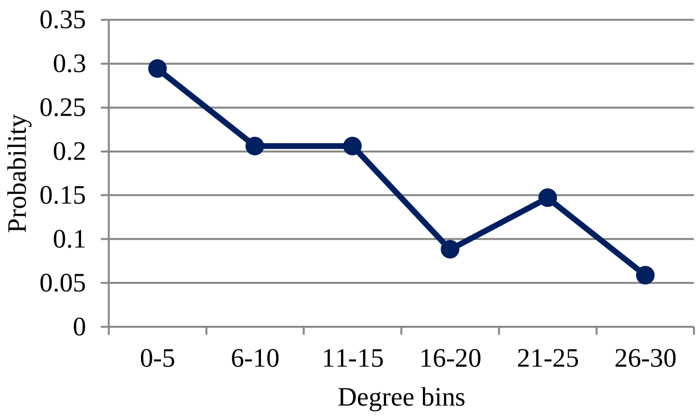


Figure 9 – Degree distribution in the undirected network

Since there are only 34 nodes in this network, it has no sense to calculate the empirical degree distribution because almost every node has a different degree. However, if we bin the degrees into larger categories then it already shows something about the functional form of the underlying degree distribution. In Figure 9 we can see this degree distribution can be regarded as closely linear on a linear-linear scale, so it is definitely not a scale-free network. This result is somewhat surprising because Aldasoro and Alves (2016) pointed out that the empirical analyses of the interbank markets usually find that the networks have scale-free property. Banai et al. (2013) also found scale-free characteristics for the Hungarian overnight FX-swap network.

However, the pure unweighted degree distribution is probably not the best indicator if we want to assess the structure of the network. Since the Hungarian interbank market is a small market with only 34 active institutions (in the May 2012 – May 2013 period), it is absolutely not surprising that we cannot see much difference in the number of total connections. What really determine the role a bank plays in this network are the number and the total amount of the transactions rather than the number of its counterparties. If we have a look at the red bars in Figure 8, then we can conclude that contrary to the unweighted degree we definitely can detect institutions with outlying number of transactions (which can be called as the weighted degree using the number of transactions as weights). Precisely, nodes 24 and 13 are the ones that are involved in almost 30% of all transactions.

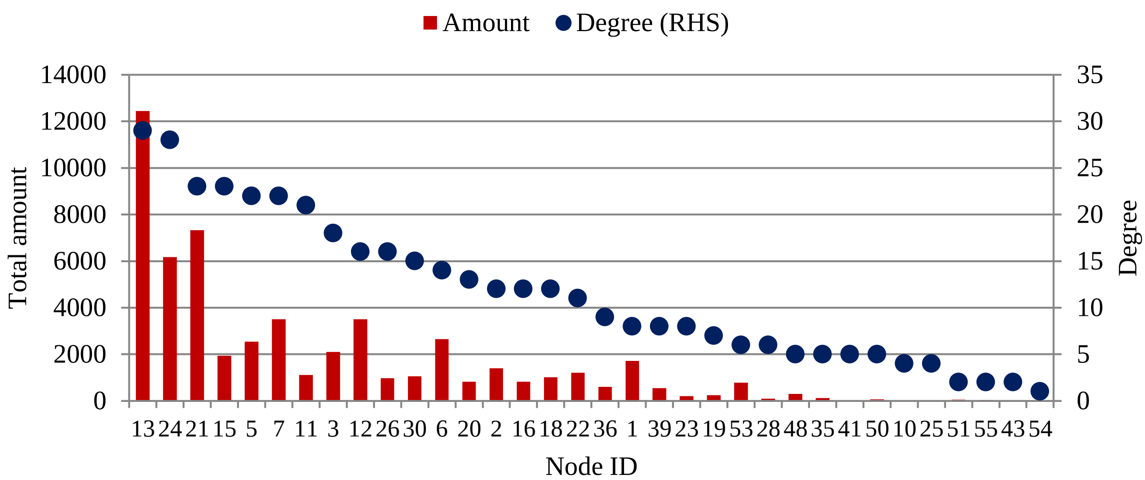


Figure 10 – Degrees and transacted amounts in the undirected network

The weighted degree distribution is even more unequal if we use the total transacted amounts as weights. As plotted in Figure 10, Bank 13 is extremely active based on the transacted amounts and banks 21 and 24 also transacted twice as much as the fourth Bank in the row. It infers that although the pure degree distribution covered it, there is some sign of scale-free property in the Hungarian interbank network with a few big hubs and many nodes with a smaller weighted degree.

Concentration

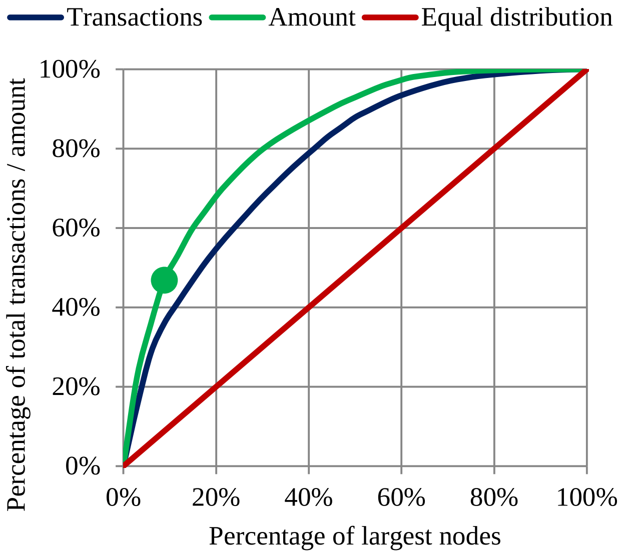


Figure 11 – Lorenz-type curves of the undirected network

Another way to assess how unequal is the distribution of the weighted degrees is by using the methodology of determining income inequality within a country, which is the Lorenz curve and GINI coefficient. For the interbank network in Figure 11 I plotted a Lorenz-type curve for both weighted degree distributions. We can see how unequal are the distributions weighted with the number and the amount of transactions if we compare them to the red 45° line indicating the completely equal distribution. The curves represent the cumulative share in the number or amount of transactions counting from the largest (most important) nodes towards the smallest ones. For example, the green point in the green curve indicates that the three largest banks (3/34 = 8.8%) are involved in 47% of the total transactions in the network based on the transacted amounts. Hence the far are the curves from the 45° red line the unequal are the weighted degree distributions. As we can see the inequality is larger if we consider the transacted amounts rather than the number of transactions, which is supported by the GINI coefficients that are 66% and 54%, respectively.

So far I have presented the main characteristics of the network and which role the banks play in it compared to each other. We saw that banks tend to have quite a lot counterparties, and a few banks are responsible for the majority of the transactions. However, it is also an important question how diversified is the "interbank portfolio" of one particular node. We don't know yet whether the banks' transactions are equally distributed among their counterparties or banks tend to transact with only a few key other banks irrespectively of the number of their total counterparties. For example, it can be possible that a less active institution with 6 counterparties accomplishes 90% of its transaction with only one counterparty and 10% with the other five. From a financial stability point of view, the diversified are the portfolios of the institutions the stable is the market since the collapse of one key counterparty has a smaller effect on the other institutions. To get an answer to this problem I applied the Herfindahl-Hirschmann Index that is generally used to assess the concentration of a market. In this case the "market" is the interbank portfolio of an institution, and the Index is calculated for Bank *i* as follows:

$$HHI_i = \sum_j \left(\frac{tr_{ij}}{TR_i}\right)^2$$

, where tr_{ij} stands for the total transacted amount between banks *i* and *j*, and TR_i indicates the total transacted amount of Bank *i*. The larger is the value of the Index, the more unequal is the distribution of the transacted amount across the counterparties of the particular bank.

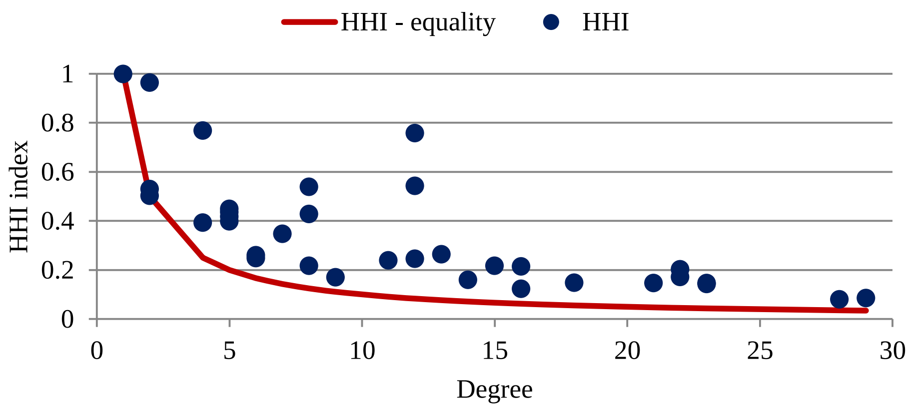


Figure 12 – HHI index by degree

In Figure 12 we can see the calculated HHI indices by the number of degrees (number of counterparties) for the analyzed financial institutions. The red line indicates the theoretical totally equal distribution, which is $\frac{1}{k_i}$ for Bank *i* (where k_i stands for the degree of Bank *i*). As we can see the observed HHI distribution approximately follows the shape of the theoretical totally equal distribution, which means that banks with more counterparties tend to have more diversified transaction portfolio as well. Banks with more than 15 counterparties are quite close to the red line indicating that their transactions are fairly balanced across their counterparties. However, the HHI indices are quite diverse for banks with fewer counterparties. For example, we can see that there are two banks with 12 counterparties but with HHI around 0.55 and 0.75 meaning that despite of the number of their partners they prefer to transact only with 1-2 other institutions. Concerning the potential contagious effect of the insolvency of a bank in the network, highly concentrated interbank portfolios like these two cases are quite dangerous.

Comparison with theoretical networks

As we could see, the Hungarian interbank network cannot be regarded as scale-free based on the pure degree distribution, but it seems to have this characteristic if we weight the degrees of the nodes. In this section, I compare the structure of the observed interbank network with theoretically grounded network models: the Erdős-Rényi random graph and the Barabási-Albert model.

Based on Barabási (2016) the Erdős-Rényi model is a fully random network based on two input parameters: the number of nodes (*N*) and the number of links (*L*) or the probability that two nodes are connected (*p*). These random networks have binomial degree distribution. For this comparison, I use the *G(N,p)* model where *N* is set to 34 just as in the interbank network, while *p* is defined as the density of my interbank network (the number of links compared to its theoretical maximum).

The other theoretical model I use for the comparison is the Barabási-Albert (BA) model. The algorithm models the dynamic evolution of a network on the basis of two main concepts: growth and preferential attachment (Barabási, 2016). Growth means that in every step new nodes are added to the network. The concept of preferential attachment defines how these new nodes connect to the already existing ones: the probability that a new node *B* "chooses" the old node *A* depends on the degree of *A*. It means that those nodes that are connected to many other nodes in the network are preferred counterparties of the new entrants. As Barabási (2016) shows, such networks have a power law degree distribution. For the comparison I simulated Barabási-Albert networks through the following steps: (1) In the first period there is only one node, and in every step a new node is added to the system. (2) In every period the number of created edges equals the current number of nodes in the network, where the probability of choosing a given node linearly depends on its degree, and multiple edges are enabled.

Table 2 – Metrics of the interbank and theoretical networks

Indicator	Interbank network	Erdős-Rényi random network	Barabási-Albert scale-free network
Number of banks	34	34	34
Number of connections	197	197	367
Diameter	3	3	2
Average degree	11.588	11.546	21.605
Average path length	1.699	1.660	1.345
Density	0.351	0.350	0.655
Global clustering coefficient	0.577	0.348	0.666
Average local clustering coefficient	0.733	0.350	0.669

The average network metrics of these random graph based on 50-50 simulations are summarized in Table 2. The results are interesting in the sense that the interbank network based on transactions over one year cannot be regarded as fully random since the clustering coefficients show that it is much more concentrated than the corresponding Erdős-Rényi graphs. On the other hand, the chosen BA model that simulated the evolution of the network resulted in a much denser network based on the density indicator, although the clustering coefficients are much closer to the interbank ones. Thus, the analyzed interbank network cannot be classified as fully random or scale-free network according to these examples, since it shows some characteristics of both types.

3.3.2. Directed network

So far I have presented the main characteristics of the undirected interbank network. However, this approach disregards one important aspect of the interbank lending, namely the directions of the transactions. If we add this information and build a directed network, then it already enables us to differentiate between the lending and the borrowing sides of the interbank market. In addition, I will separate the data along another dimension, namely the maturity of the transaction by creating an overnight and a longer-term network since the main focus of this thesis is the comparison of these network layers.

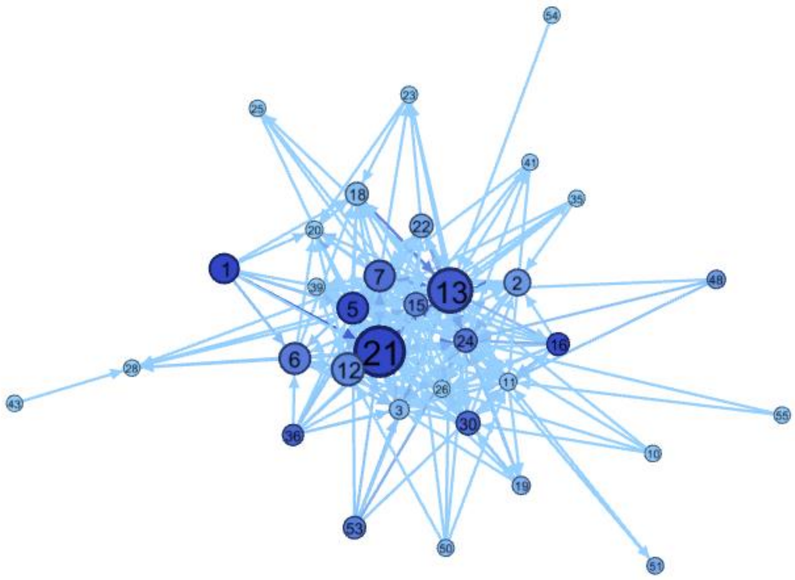


Figure 13 – Graph of the overnight network (colored by lending activity)

(Node color: out-transactions, node size: lent amount, edge color: transacted amount)

Figure 13 plots the overnight directed network where the transactions have a maturity of one working day (weekends and holidays are not counted). A directed link goes from a source node (lender) to a target node (borrower) if there was at least one transaction in this direction during the one-year period (May 2012 – April 2013). The darkness of the nodes represents the number of out-transactions (lending activity) while the size of the nodes indicates the total lent amount. There are many nodes with darker blue color indicating that a lot of banks are active lenders in the overnight interbank market.

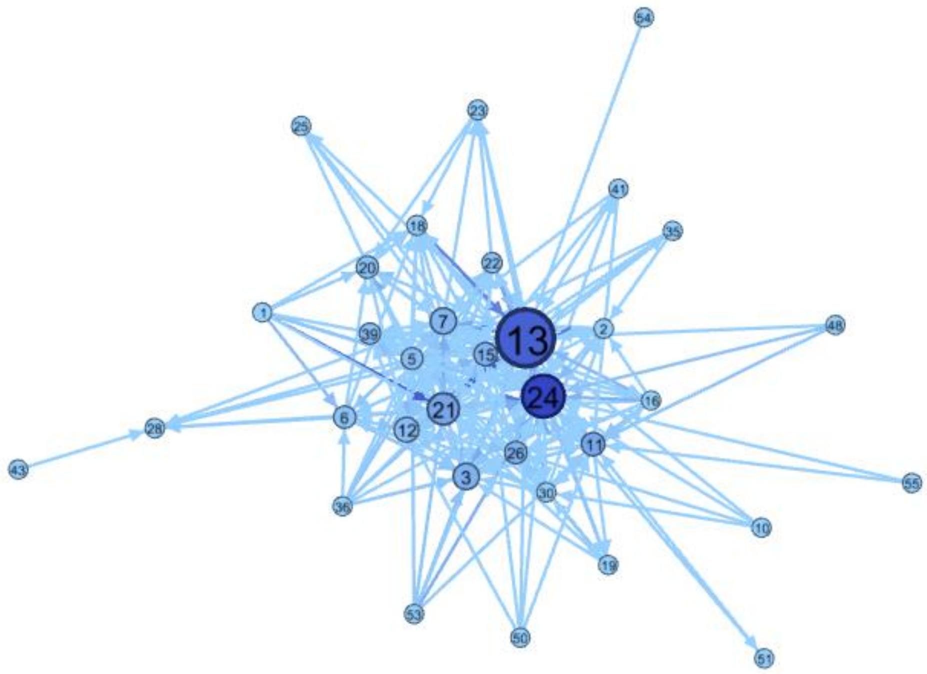


Figure 14 – Graph of the overnight network (colored by borrowing activity)

(Node color: in-transactions, node size: borrowed amount, edge color: transacted amount)

If we change the color and the size of the nodes in order to reflect the borrowing-side of the overnight market, we can see that banks 13 and 24 are extremely important institutions in the network. Both the number (color) and the total amount (size) of their borrowing activity are extremely high compared to the other banks.

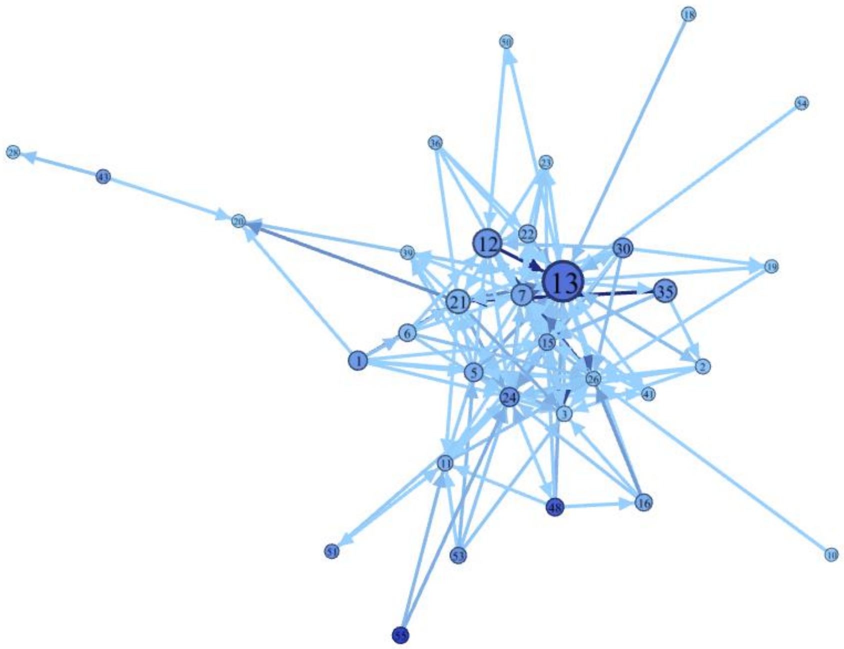


Figure 15 – Graph of the longer-term network (colored by lending activity)

(Node color: out-transactions, node size: lent amount, edge color: transacted amount)

In contrast to the short-term network, the longer-term network is much less dense with the density of 0.137 (Figure 15). Since I am analyzing the uncovered interbank market, it is anticipated that the longer-term sub-market contains fewer transactions because the uncovered lending for longer maturity is riskier for the lender institution than the overnight lending. The average interest rate is slightly higher in the longer-term market, which also refers to higher risk (see Table 3).

As we can see in the longer-term part of the market, bank 13 is the most important lender based on the total transacted amount, while there are some other institutions that transact a lot but only small amounts (e.g. bank 55, see Appendix Figure 2 and Appendix Figure 3).

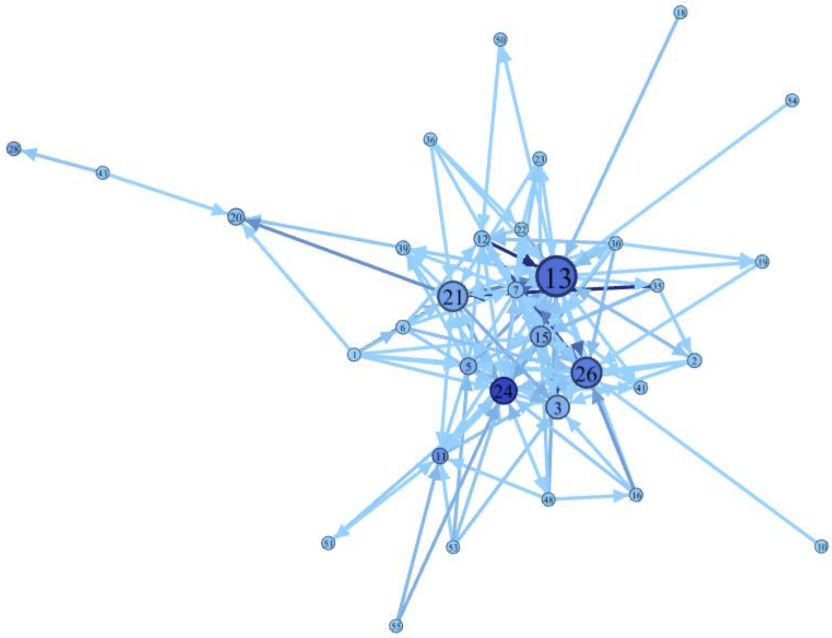


Figure 16 – Graph of the longer-term network (colored by borrowing activity)

(Node color: in-transactions, node size: borrowed amount, edge color: transacted amount)

In the borrowing part of the longer-term layer more banks appear as important institutions beside bank 13 (Figure 16). It also shows that in this sub-market the lender and borrower parts are quite different and there are a lot of banks that are key nodes only in one of the networks. This phenomenon suggests that in the assessment of the Hungarian interbank market the analysis of the directed network is preferable rather than the whole undirected one.

Table 3 – Metrics of the overnight and the longer-term layers

	Overnight layer	Longer-term layer
Number of banks	34	33
Number of transactions	8272	505
Average transaction per bank	243.294	15.303
Average interest rate (%)	5.715	5.833
Average maturity (day)	1	8.952
Average transacted amount (billion HUF)	3.197	2.472
Diameter	3	4
Average in-degree / out-degree	8.118	4.394
Average path length	1.680	2.025
Density	0.246	0.137
Betweenness centrality	0.192	0.256
Global clustering coefficient	0.578	0.433
Average local clustering coefficient	0.483	0.440

In Table 3 we can compare the basic metrics of the two directed sub-networks. The first observation is that almost every bank that is active in the overnight sub-market appears in the longer-term as well. However, both the average number and average amount of the transactions are significantly lower in the latter one, which indicates that the longer-term layer constitutes a sparser network. The densities, the average path lengths and the clustering coefficients also support this finding. While the global clustering coefficient of the overnight layer is almost the same as in the undirected network, the average local clustering coefficient is much lower indicating that the differentiation between the lending and borrowing side in a directed network leads to a less connected system. Since the diameter is 3 and 4 in the two layers, it is still true that every node can be reached with a small number of steps from every other node.

Degree distribution

For the comparison of the activity of the banks in the lending and borrowing sides of the two layers, I will use the degree distributions and the transacted amounts. For the assessment of the importance of a node in the network, I find the transacted amounts preferable against the number of transactions because the latter indicator can be misleading if a bank has many transactions with smaller quantities.

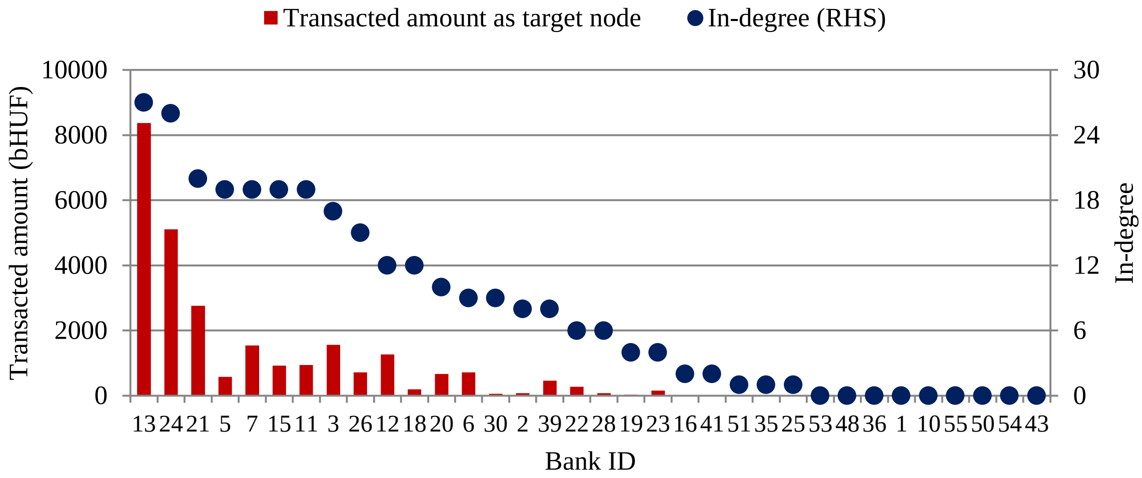


Figure 17 – In-degrees and borrowed amounts in the overnight layer

If we have a look at the relationship of the in-degree and borrowed amounts in the overnight network (Figure 17), we can see that 9 banks (26.5%) have zero in-degree meaning that they were active only on the lending side of the overnight interbank transactions. Similarly to the undirected network, we cannot see outlying hubs regarding the pure in-degree distribution, but there is clearly one large hub (bank 13) if we use the in-strength calculated with the borrowed amounts. Bank 24 also seems to be an important node in this sub-network; together with bank 13, they are responsible for 51% of the total borrowed amounts. While bank 21 also borrowed almost 2 800 billion HUF during this one-year period, the other banks are negligible players. Thus, it seems that on the overnight borrowing side of the market only a few banks are active.

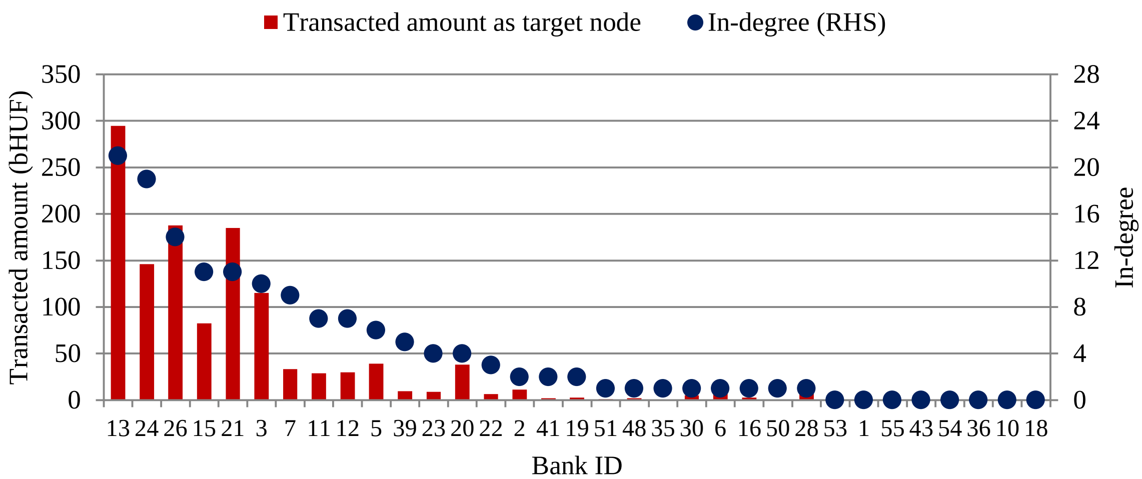


Figure 18 – In-degrees and borrowed amounts in the longer-term layer

In the longer-term borrowing sub-network, two main conclusions can be drawn (Figure 18). Firstly, half of the banks have zero or one in-degree, which indicates that much fewer institutions borrow for longer-term maturity than overnight. Secondly, while the in-degrees of banks 13 and 24 are also outstanding of the distribution, the in-strength is more balanced among the most active banks (the top five) than in the overnight network. It means that although fewer banks are active in this sub-network, they seem to be similarly important based on the borrowed amounts.

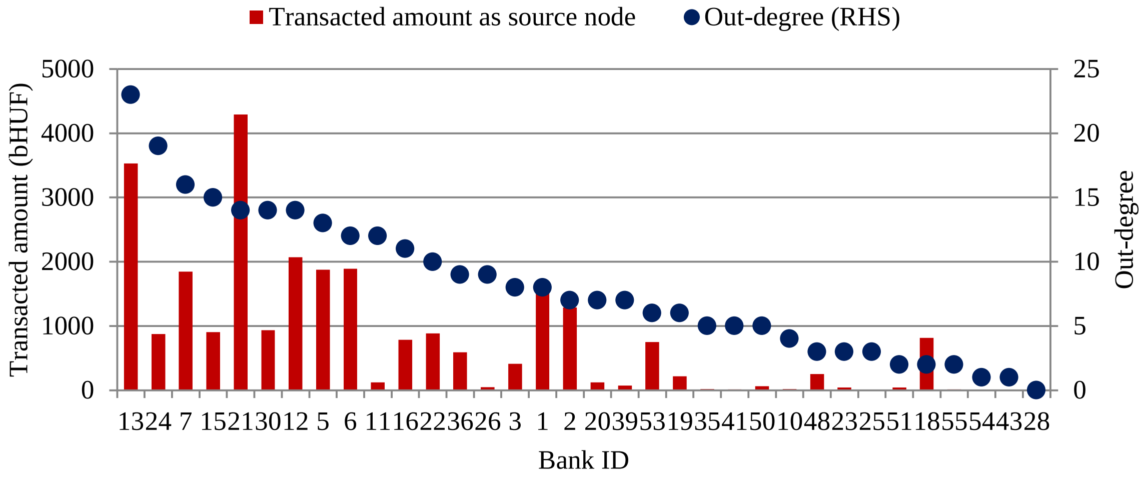


Figure 19 – Out-degrees and lent amounts in the overnight layer

Since there are only a few active players on the borrowing side in both layers, we can expect that the remaining institutions appear as lenders in the network. Indeed, looking at Figure 19, we can see that the distribution of the out-strength is much more balanced than in the borrowing side. There are 13 banks that transacted more than the average 777.8 billion HUF, while this number was only 8 in the overnight borrowing sub-network. Another important difference is that the former clear positive relationship between the number of counterparties (degree) and the transacted amount is not observable. It means that there are numerous banks that lend huge amounts only for a few partners (for example banks 1 and 2).

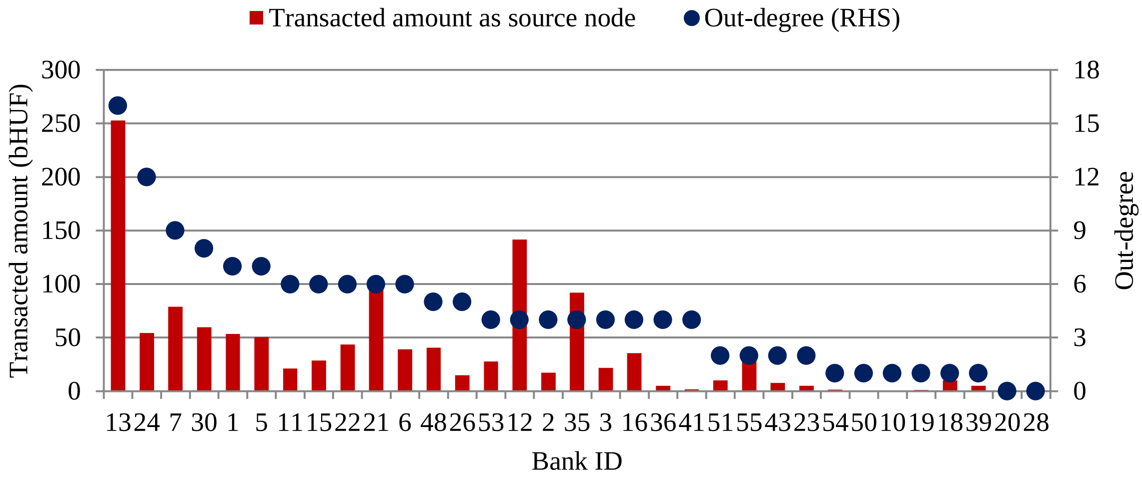


Figure 20 – Out-degrees and lent amounts in the longer-term layer

In contrast to the more balanced overnight lending side, we can see that bank 13 is definitely the most important lender in the longer-term layer being responsible for 20% of the total lent amount (Figure 20). Although many other institutions appear as a lender for longer maturity, the distribution of the out-strength is quite unequal with three larger nodes beside bank 13.

Concentration

Thus, based on these figures we can detect some differences between the overnight and the longer-term sub-networks. While in the former one the transacted amounts seem to be more equally distributed among the largest banks in the lending side compared to the borrowing side, the opposite can be observed for the longer-term layer. To evaluate in a more quantitative way how this observation affects the overall concentration of the two sides, I applied the Herfindahl-Hirschmann Index presented in the last sub-section. In this case, the index is calculated as follows:

$$HHI = \sum_i \left(\frac{TR_i}{TR}\right)^2$$

In the formula TR_i stands for the total transacted amount of bank i, while TR indicates the total transacted amount in the analyzed layer. The higher is the HHI index of the market the concentrated it is, which is potentially riskier concerning financial stability.

Table 4 – HHI index in the overnight and the longer-term layers

	Overnight layer		Longer-term layer	
	HHI	Theoretical minimum HHI	HHI	Theoretical minimum HHI
Borrowing side	0.163	0.029	0.131	0.030
Lending side	0.080	0.029	0.084	0.030

The results are summarized in Table 4. As expected from the previous graphs the HHI concentration index is much larger in the borrowing sides for both sub-markets. Comparing the sides across layers, we cannot see any difference in the case of the lending-side concentration, while the longer-term borrowing sub-market is less concentrated than the overnight one, which supports the observed differences between Figure 17 and Figure 18. The inequality of the participation in the borrowing and lending sides of the sub-markets can be assessed with the already presented Lorenz-type curves as well.

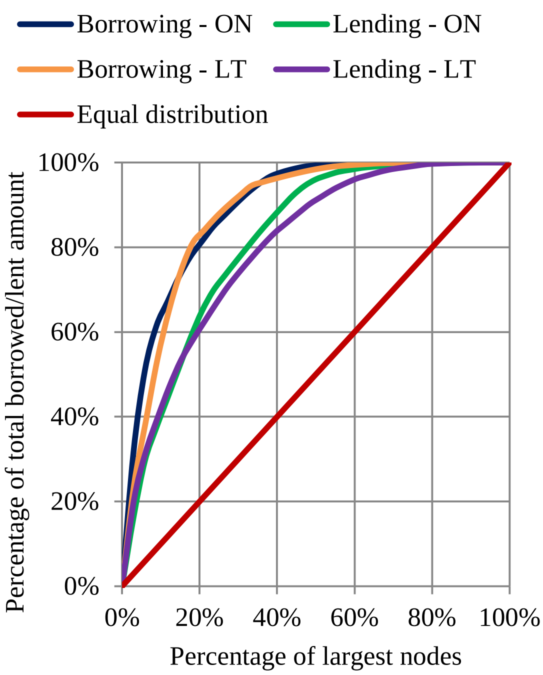


Figure 21 – Lorenz-type curves of the directed networks

Figure 21 also supports the finding, that the participation on the lender side of the markets is much more even than on the borrower side. Similarly to the undirected network, we should also have a look at the in- and out-portfolios of the banks to assess the potential risks of concentration.

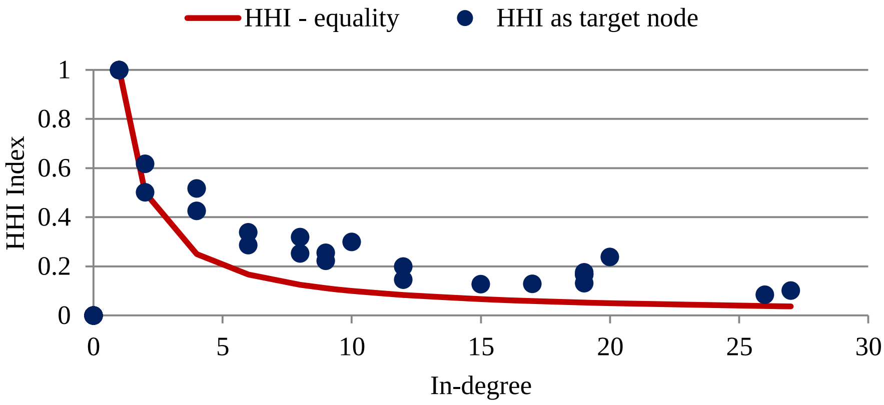


Figure 22 – HHI index by in-degree on the borrowing side of the overnight layer

In Figure 22 we can see the HHI index of the banks in the overnight layer, calculated using the borrowed amounts. The observed values follow quite closely the red line that indicates the totally equal distribution. It means that those banks that are active borrowers in the overnight interbank market tend to diversify their portfolios. The evaluation of this finding regarding systemic risk can be ambiguous. As Gai and Kapadia (2010) points out, diversification enhances risk-sharing and lowers the probability of a distress, but at the same time, it increases the contagion if a key node defaults in the network.

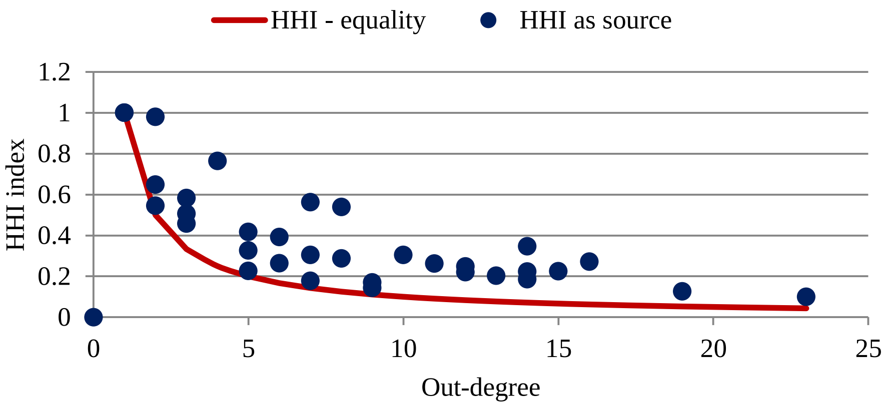


Figure 23 – HHI index by out-degree on the lending side of the overnight layer

On the lending side of the market banks tend to choose their counterparties in a more concentrated way (Figure 23). It may be the consequence of the counterparty limits. The HHI indices show similar pattern in the longer-term networks as well.

As a conclusion, we can state that few large institutions are responsible for the majority of the borrowed money in the overnight layer, while this distribution is more balanced in the longer-term sub-network. From a financial stability point of view, a highly concentrated overnight borrowing side is not a preferred state. If only a few banks are responsible for the majority of the interbank borrowing, then a potential default or distress of these key institutions can cause severe direct losses to their counterparties. However, it is true for both layers that much more banks appear as lenders in the network. Since it facilitates the liquidity of the market, this is a favorable condition.

3.3.3. SIFI identification

One main purpose of the network analysis of the interbank connections might be the identification of the banks that play a pivotal role in the system, the so called SIFIs. The literature review about the SIFI identification in Section 2.3 highlighted that various approaches could be found in the empirical analyses to assess the importance of the nodes in the network.

The SIFI identification can be based on simple and more complex network metrics as well. The most basic approach uses the pure degree centralities and strength of the nodes: the more links, transactions or transacted amounts a bank has the important it is the whole system. Of course, it is advantageous in this case to differentiate the lending and borrowing side of the market, so analyzing a directed network.

In the last section, we saw that based on these simple metrics banks 13 and 24 are the greatest players on the borrowing side. In this case, we analyze either the unweighted degrees, the transactions-weighted strength or the amounts-weighted strength, we get the same result regarding the potential SIFIs. In contrast, on the borrowing side of the market, the indicators lead to different results. While based on the pure out-degrees also banks 13 and 24 are the most important ones, the transactions-weighted strength already shows five similarly important banks, and the amounts-weighted strength clearly indicates that bank 13 and 21 is are the key nodes.

These examples show that various metrics can lead to entirely different conclusion concerning the SIFI identification since they capture different aspects of the network. For example, in the case of the default of a bank, the pure in- and out degrees show how many other institutions may be affected, while the strength metrics give us some information about how severe the contagious effect might be. Using only these simple metrics on their own is probably not the best way to find the key banks in the system.

Opsahl centrality

However, if we create a more complex indicator by combining the information these metrics carry, we can get a much more meaningful index. For this purpose, I will use the Opsahl centrality index proposed by Opsahl et al. (2010), which is calculated as follows.

$$C_{OPS,i}^{in} = \left(k_i^{in}\right)^{1-\alpha} * \left(w_i^{in}\right)^{\alpha}$$

$$C_{OPS,i}^{out} = (k_i^{out})^{1-\alpha} * (w_i^{out})^{\alpha}$$

In the equations k_i^{in} and k_i^{out} stand for the in- and out-degrees of node i, while w_i^{in} and w_i^{out} indicate the sum of the weights of the in- or out-edges corresponding to node i. The parameter α serves as a weight for the two metrics. If $\alpha = 0$ then we get the simple in- and out-degrees, while $\alpha = 1$ gives back the in- and out-strength of the node.

I calculated the Opsahl centrality indices using the transacted amounts as edge weights. Since both the number of counterparties and the strength of the nodes carry useful information, I decided to use $\alpha = 0.5$ as weight parameter. The results calculated using the latest available network (May 2012 to April 2013) for both the overnight and the longer-term layers are summarized in Figure 24 and Figure 25. The higher is the Opsahl centrality of a node the important it is in the network. The red and green lines on the graphs indicate the cutoffs of the top 3 key borrowers and lenders, respectively.

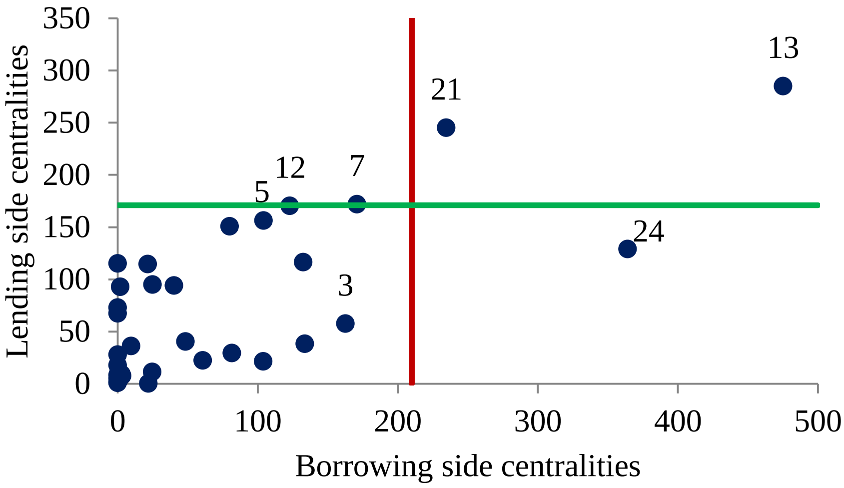


Figure 24 – Opsahl centralities in the overnight layer

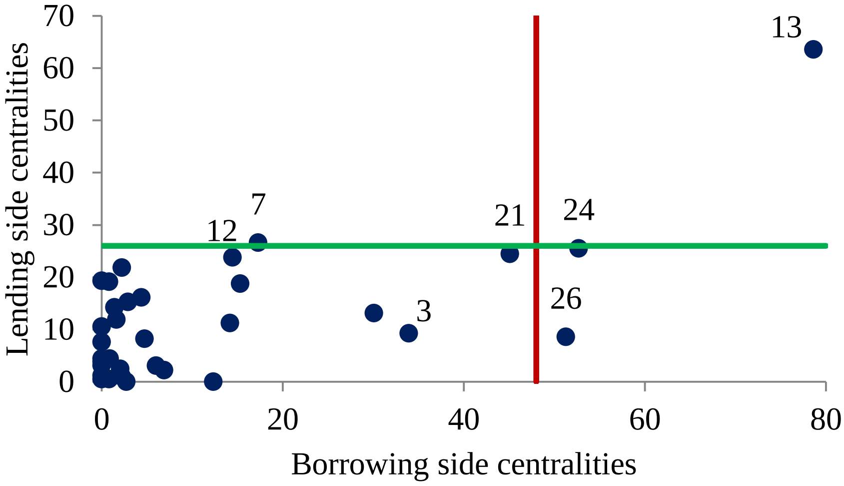


Figure 25 – Opsahl centralities in the longer-term layer

The first observation is that although the Opsahl centralities of the nodes in the borrowing and the lending market show some positive correlation (within the graphs), we can find several examples that appear only on one side of the market as an important node. For example bank 12 seems to be a crucial lender in both layers, but it is not a particularly active borrower. In contrast, bank 26 is the third most central node in the longer-term borrowing side, while it is a negligible lender in this layer.

Bank 13 is apparently the most important node in the network in both layers and on both sides. Its relative importance is exceptionally high in the longer-term network. As we can see, banks 24 and 21 also seem to be key nodes in the network. The main conclusion based on the Opsahl centralities is that the relative importance of the banks is greatly varying across the sides of the market. These results are in line with the findings of the last section, and support the importance of the market segmentation based on the direction of the transactions.

Betweenness centrality

So far I mainly focused on the direct relationships between the nodes for the SIFI identification. However, the importance of a node in a network does not only depend on its (weighted or unweighted) links but its position in the system as well. This aspect can be captured by the betweenness centrality that quantifies how central a node is in the network. This indicator is calculated as the number of the shortest paths between all possible pair of nodes that goes through the analyzed node compared to the theoretical maximum of such paths. Expressed mathematically following the notations of Jackson (2010):

$$C_{BETW,i} = \sum_{k \neq j: i \notin \{k,j\}} \frac{\frac{P_i(kj)}{P(kj)}}{(n-1) * (n-2)/2}$$

In the equation $P_i(kj)$ stands for the number of shortest paths between nodes k and j that go through node i, while $P(kj)$ is the number of all shortest paths between nodes k and j. The closer is the value of $C_{BETW,i}$ to one, the important a node is in the network.

In the interbank network the central placement of a bank can mean that it may bind together such banks that have zero limits towards each other. For example banks A and B are not allowed to transact with each other, but since bank C is an important counterparty of both banks, they are part of the same connected component of the network. Thus, centrally lying smaller banks can facilitate the flow of interbank funding and therefore make the market more liquid and less segmented.

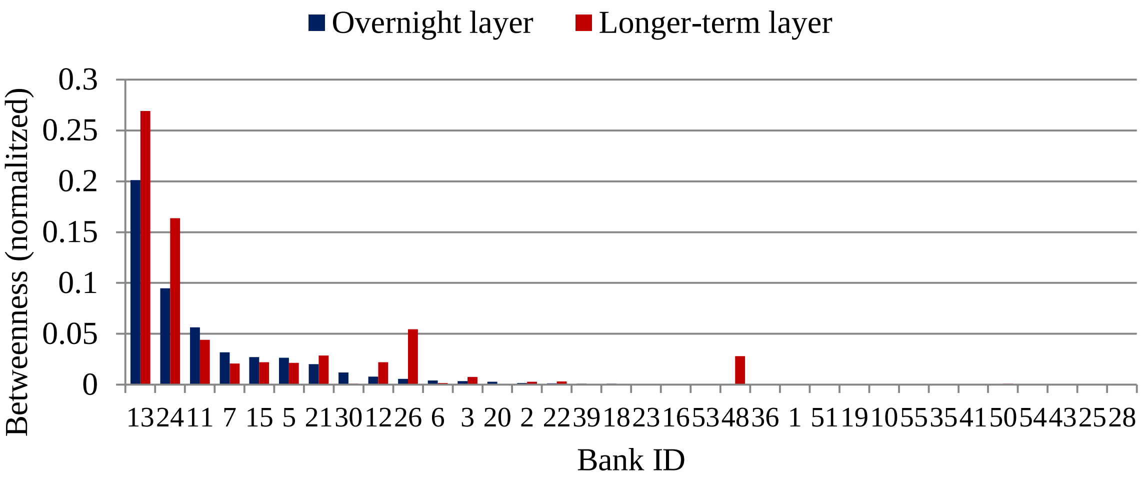


Figure 26 – Betweenness centrality in the latest directed network

As we can see in Figure 26, more than 47% of the nodes have zero betweenness centrality meaning that no shortest path goes through them. On the other end of the scale bank 13 is clearly the most central node in both layers since 20% and 27% of all directed shortest paths include this bank in the overnight and the longer-term layers, respectively. It is twice of the value of the second most important bank (node 24) in the case of the overnight sub-network. Since these banks were key nodes based on the degree and Opsahl centrality measures as well, we can conclude that this network has small world characteristics like other empirical networks, where large hubs with a high number of links serve as linking nodes for smaller, less important ones.

Closeness – harmonic centrality

Another usual way to assess the centrality of a node is by using the closeness centrality. This indicator is calculated as the reciprocal of the fairness, that is the sum of the pairwise shortest paths between the nodes.

$$C_{CLO,i} = \frac{1}{\sum_{j \neq i} d(i,j)}$$

In the equation $d(i,j)$ indicates the shortest path (distance) between nodes i and j. However, since I analyze a directed network, it can happen that there is no directed path between two nodes. For example bank 53 has zero in-degree meaning that it is not reachable from any other node. To overcome this problem I will use the normalized Harmonic centrality proposed by Rochat (2009).

$$C_{HARM,i}^{IN} = \frac{1}{n-1} * \sum_{j \neq i} \frac{1}{d(i,j)}$$

$$C_{HARM,i}^{OUT} = \frac{1}{n-1} * \sum_{j \neq i} \frac{1}{d(j,i)}$$

If there is no directed path between nodes i and j, then $\frac{1}{d(i,j)}$ and $\frac{1}{d(j,i)}$ are set to zero. In this way we can get comparable and meaningful centrality values for all nodes.

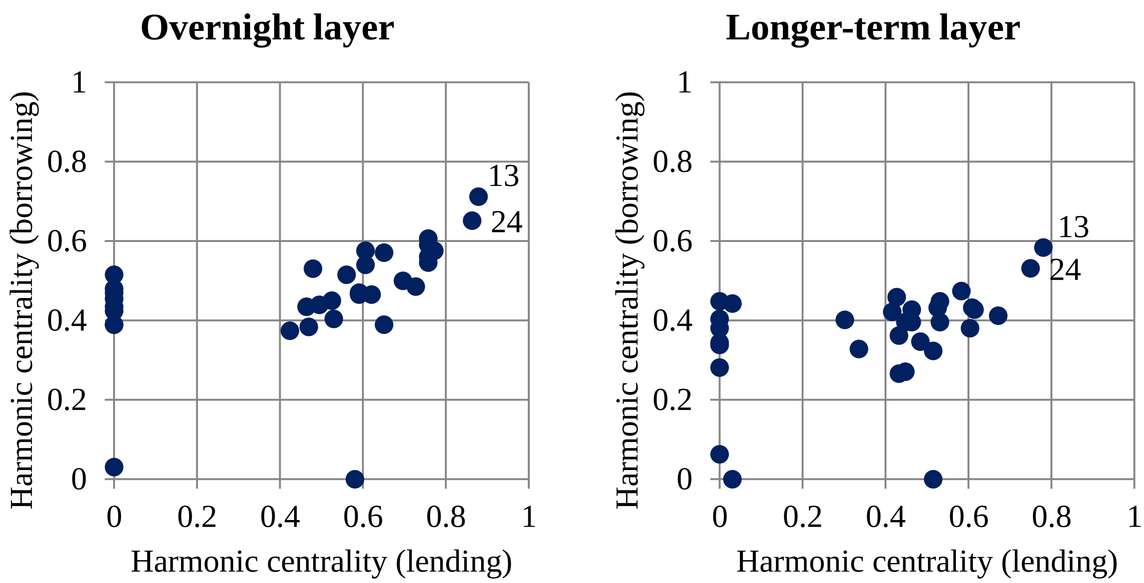


Figure 27 – Harmonic centralities in the latest directed network

Figure 27 presents the scatter plot of the calculated harmonic centralities of the borrowing and the lending sides of the market for the two layers. If we disregard the nodes with zero value for either of the centralities, then we can see a moderate positive relationship between the two metrics. It indicates that among those banks that are active on both sides of the market the more central ones on the one side tend to be important on the other side as well. This relationship seems to be much more robust than in the case of Opshal centralities, which indicates that different network metrics capture various aspects of being systemically important, thus using only one indicator can be misleading in some cases. Based on this way of SIFI assessment banks 13 and 24 are also the key nodes on both sides of the network.

Summary of the SIFI identification

As I presented in the previous sub-chapters, one can assess the relative importance of the banks in the interbank network with various metrics. Since they capture different aspects of the role a node plays in the network, one can make different conclusion concerning the SIFIs in the market. Thus, to assess systemic importance one should not rely on only one indicator because it can disregard crucial elements. In Table 5, I summarized the results of my SIFI analysis. The indicators are categorized into three groups based on their information content: they assess the importance of a node in the overall network or in the borrowing/lending side.

Since I discovered important differences between the two sides of the market, the undirected degree centrality is not a useful indicator. For the economically meaningful assessment of the overall importance of the nodes, the betweenness centrality may be used. It evaluates how important a bank is in the sense of connecting the institutions in the interbank market. If a bank that is a key player in this aspect collapses, then it may lead to the disjunction of the market with group of banks that are not connected anymore. It is particularly possible knowing that the key drivers of the interbank lending activity are the limits the banks have towards each other. Thus, a centrally lying bank may not be replaced due to these fix limits.

The indices that try to identify the key banks on the borrower side of the interbank market are extremely important since the default of a key borrower can be very contagious. First, a collapsed SIFI has a direct effect on its lenders in the form of immediate loss. The number of the potentially affected institutions and the severity of the direct contagion can be captured by the in-degree, in-strength and "In-Opsahl" centrality indicators. However, there is an indirect contagious effect of a potential default as well through the further links in the network. If the lenders of the defaulted banks will not be able to meet their liabilities either then it will cause losses for their lenders as well, and so on. The severity of this potential spillover contagion can be assessed by the analysis of the Harmonic centrality.

Putting together these aspects I can conclude that banks 13 and 24 are clearly the most important nodes on the borrowing side of the interbank market. Their collapse or even some smaller disturbance could have large direct and indirect contagion through the network. Unfortunately, I cannot analyze further these institutions for having a clearer picture of their activity since my database contains anonymous data.

On the lending side of the market, the same metrics can be used for the assessment of the direct and indirect contagion of a potential default. However, the insolvency of a key lender is likely less problematic from a financial stability point of view. The main consequence of such event is that the market becomes less liquid since a key liquidity provider is not active anymore. It can lead to higher funding costs for the institutions that cannot get enough liquidity from the interbank market.

Of course, if more lending banks draw back from the market, for example, due to the lack of trust towards other banks as we saw during the 2008-2009 financial crisis, the market may freeze, and potentially the central bank has to step in by providing liquidity for the banking system. Thus, knowing the SIFIs on the lending side of the market is also crucial, although in my opinion not as important as in the borrowing side.

In my network banks 13, 24 and 21 seem to be the most important banks in the lending side, while some other institutions can also be regarded as key nodes in some aspects. As a combined evaluation I can conclude that banks 13 and 24 are the systemically most important institutions in the analyzed Hungarian interbank network. Probably these institutions are among those 8 Hungarian banks that are required to hold additional capital buffer due to their key role in the banking system.[4]

[4] For the list of these institutions and the current capital buffers see the following MNB statement: https://www.mnb.hu/letoltes/az-egyeb-rendszerszinten-jelentos-intezmenyek-azonositasa-2016-en-honlap.pdf

Table 5 – Summary table of the SIFI identification

Centrality index	Network definition	Economic meaning	Information content about the contagion of the bank's default	Identified SIFIs, ON (in order)	Identified SIFIs, LT (in order)
Overall importance in the market					
Betweenness (with directed links)	Number of shortest paths through the node	Relative importance in connecting disjoint groups of banks	Potential effect on market liquidity: the default of a bank with high betweenness can lead to the fragmentation of the market into more connected components.	13, 24	13, 24
Borrowing side of the market					
In-degree	Number of edges to the node	Number of counterparties the bank borrows from	Number of directly affected institutions: immediate loss for the lender banks.	13, 24	13, 24
In-strength (in-transactions)	Sum of edge weights to the node	Number of borrowing	Severity of the direct contagion: immediate loss for the lender banks.	24, 13	24, 13, 26
In-strength (borrowed amount)	Sum of edge weights to the node	Amount of borrowing	Severity of the direct contagion: immediate loss for the lender banks.	13, 24	13, 26, 21
Opsahl (in-degree and in-transactions)	Weighted index of the in-degree and in-strength	Relative importance in the market based on the number of lending partners and the number of borrowing	Severity of the direct contagion: immediate loss for the lender banks.	24, 13	24, 13, 26
Opsahl (in-degree and borrowed amount)	Weighted index of the in-degree and in-strength	Relative importance in the market based on the number of lending partners and the amount of borrowing	Severity of the direct contagion: immediate loss for the lender banks.	13, 24, 21	13, 24, 26, 21
Harmonic (borrowing)	Reciprocal of the directed distances with other nodes (starting at the node)	Relative importance in the market based on the borrowing activity from other banks	Severity of the indirect contagion: the default of a more central bank spreads more quickly through the network.	13, 24	13, 24, 26

Centrality index	Network definition	Economic meaning	Information content about the contagion of the bank's default	Identified SIFIs, ON (in order)	Identified SIFIs, LT (in order)
		Lending side of the market			
Out-degree	Number of edges from the node	Number of counterparties the bank lends to	Number of directly affected institutions: less available, thus likely more expensive funding for the borrower banks.	13, 24	13, 24
Out-strength (out-transactions)	Sum of edge weights from the node	Number of lending	Severity of the direct effect: less available, therefore likely more expensive funding for the borrower banks.	1, 21, 5, 16, 13	55, 48, 13
Out-strength (lent amount)	Sum of edge weights from the node	Amount of lending	Severity of the direct effect: less available, therefore likely more expensive funding for the borrower banks.	21, 13	13, 12, 21
Opsahl (out-degree and out-transactions)	Weighted index of the out-degree and out-strength	Relative importance in the market based on the number of borrowing partners and the number of lending	Severity of the direct effect: less available, therefore likely more expensive funding for the borrower banks.	13, 21, 24, 5, 7	13, 24, 48
Opsahl (out-degree and lent amount)	Weighted index of the out-degree and out-strength	Relative importance in the market based on the number of borrowing partners and the amount of lending	Severity of the direct effect: less available, therefore likely more expensive funding for the borrower banks.	13, 21, 7, 12, 5	13
Harmonic (lending)	Reciprocal of the directed distances with other nodes (ending at the node)	Relative importance in the market based on the lending activity to other banks	Severity of the indirect effect: the default of a more central bank can cause more severe liquidity problems in the market.	13, 24	13, 24

3.3.4. Multiplex comparison

As we can saw in the previous two sections, there are some detectable differences between the overnight and longer-term layers based on the network metrics. To quantify this finding, we can use various indicators that assess the similarity (or the distance) of the two layers. For the analysis, I will distinguish two aspects of the similarity: the comparison of the sub-networks as a whole, and the assessment of the relative importance of the nodes in the two layers.

Layer similarity

The first question one could pose is whether the existence of a directed connection in one of the layers contains information about the existence of the connection in the other one. As Aldasoro and Alves (2016) suggests, the Jaccard Similarity Index is one possible indicator that can help to answer this question. The index is computed as follows:

$$J(\boldsymbol{x}, \boldsymbol{y}) = \frac{|\boldsymbol{x} \cap \boldsymbol{y}|}{|\boldsymbol{x} \cup \boldsymbol{y}|}$$

In the equation **x** and **y** stand for the vectors to compare. To apply the index in this framework I transformed the unweighted adjacency matrices of the sub-networks into row vectors. Thus, the nominator of the index contains the total number of cases when there is a directed connection between Banks *i* and *j* in both sub-networks for every (*i*,*j*) pair. The denominator is the sum of the cases where there was a directed connection between every pair of banks in either of the two networks (including the nominator, i.e. when there was a connection in both sub-networks). Therefore, the greater is the index, the similar are the compared sub-networks based on the existence of connections. The index in my framework is calculated as follows, where ON stands for the overnight layer and LT for the longer-term layer.

$$J(ON, LT) = \frac{number\ of\ links\ in\ both\ ON\ and\ LT\ layers}{number\ of\ links\ in\ either\ ON\ or\ LT\ layers\ (or\ in\ both)}$$

Although the Jaccard Similarity Index contains important information about the co-existence of links, to fully assess the similarity or difference of the layers, we should take into consideration those cases as well, when there is no connection between banks i and j in both sub-networks. Thus, I will also use the Simple Matching Coefficient (SMC) to compare the unweighted adjacency matrices.

$$SMC(ON, \mathrm{LT}) = \frac{number\ of\ matching\ links\ in\ ON\ and\ LT\ layers}{number\ of\ total\ possible\ links}$$

The nominator of SMC counts the cases where the existence or non-existence of a directed link is the same in the two sub-networks, while the denominator is simply the total number of possible directed connections (which is $n * (n-1)$ for n number of nodes).

Table 6 – Layer similarity indicators

Similarity indicator	Value
SMC	0.875
Jaccard Index	0.488

As we can see in Table 6, for the latest available overnight and longer-term sub-networks the SMC indicator is 0.875 showing a strong similarity between the layers. The value means that the "existence dummies" of the possible links are the same in 87.5% of the cases. However, the Jaccard Index is 0.488 that means that the similarity is much lower if we compare only the existing links. To evaluate the Jaccard Index we can consider that in the case of two totally random networks where the existence of a link is 50%, the index is expected to be $\frac{0.5*0.5}{3*0.5*0.5} = \frac{1}{3} = 33.3\%$, while two totally similar networks have an index of 1.

Based on these result I conclude that although the layers seem to be quite similar as a whole (based on SMC), I cannot infer that those banks that are connected in one layer are probably connected in the other one too (as the Jaccard index shows). It leads to the question whether the absence of this link similarity is symmetric: can either of the layers tell something about the other layer's links or neither of them is useful for "forecasting" the connections in the other one? To answer this question I slightly modified the Jaccard index by creating "conditional" indices in the following way:

$$CJ_{OV}(ON, LT) = \frac{number\ of\ links\ in\ both\ ON\ and\ LT}{number\ of\ links\ in\ ON}$$

$$CJ_{LT}(ON, LT) = \frac{number\ of\ links\ in\ both\ ON\ and\ LT}{number\ of\ links\ in\ LT}$$

The modified indices are summarized in Table 7.

Table 7 – Conditional Jaccard indices

Similarity indicator	Value
Conditional Jaccard index | link in ON	0.500
Conditional Jaccard index | link in LT	0.952

The Conditional Jaccard indices clearly show what the simple index covered: the lack of similarity is asymmetric. While only 50% of the links in the overnight network is present in the longer-term network, the share is 95.2% in the other direction. It means that those banks that transact for longer maturity also tend to be connected in the overnight network, but this is not true in the other way around. Of course, this result is not surprising in my dataset about the uncovered interbank lending, but it indicates an important methodological aspect, namely that the similarity of the layers shouldn't be assessed only by the overall comparison of the adjacency matrices since it can hide important characteristics of the overall network.

The SMC and Jaccard indices used the unweighted adjacency matrices of the sub-networks for their comparison. However, as we could see in section 3.3.1., to analyze the characteristics of the interbank network the weighted degree distributions are much more informative indicators. To compare the sub-networks without losing this information, I will use the Cosine index suggested by Aldasoro and Alves (2016). The index is calculated as follows:

$$C(\boldsymbol{ON}, \boldsymbol{LT}) = \frac{\boldsymbol{ON} * \boldsymbol{LT}}{||\boldsymbol{ON}|| * ||\boldsymbol{LT}||}$$

In the formula, $\boldsymbol{ON}$ and $\boldsymbol{LT}$ stand for the weighted adjacency matrices of the overnight and longer-term layers (transformed into row vectors). The numerator is the scalar product of the vectors, while the denominator contains the product of the length (norm) of the vectors. Thus, the index calculates the cosine of the angle between the two row vectors. As a consequence, the result is constrained between -1 and 1 irrespectively of the magnitude of the used weights, which makes it comparable across the used weights and over time. Since in my analyzed networks the used adjacency matrices cannot contain negative numbers, the index can take values between 0 and 1 (the latter one would mean perfect matching).

Table 8 – Cosine indices

Similarity indicator	Value
Cosine index (weights: number of transactions)	0.404
Cosine index (weights: transacted amount)	0.629

I applied the cosine index for the two sub-networks with both the number of transactions and the transacted amounts as weights (Table 8). Similarly to the unweighted similarity indices, these results also suggest that the two layers have some similar characteristics. The positive relationship is much stronger if we use the transacted amounts as weights rather than the number of transactions. As a comparison, Aldasoro and Alves (2016) found that that the Cosine similarity is 0.43 between the long-term (more than one year) and short-term (less than one-year) interbank market in their network consisting of 53 large European banks. Based on the amount-weighted index, the different maturity-layers of the Hungarian market seem to be more overlapping.

Nodes similarity

Beside the assessment of the similarity of the two sub-networks, it is also an important question whether banks that play a key role in one of the layers are key nodes in the other one as well. To quantify this relationship, I estimated the correlation between the metrics used for the SIFI identification between the two layers. Higher correlation means that key banks in one sub-network are expected to be important in the other one as well. The results are summarized in Table 9.

Table 9 – Correlations of node metrics

	Metrics	**Pearson Correlation Coefficient**
	Betweenness centrality	0.964
Borrowing side	In-degree	0.885
	In-strength (transaction)	0.925
	In-strength (amount)	0.863
	Opsahl centrality (borrowing, transaction)	0.952
	Opsahl centrality (borrowing, amount)	0.922
	Harmonic centrality (borrowing)	0.734
Lending side	Out-degree	0.862
	Out-strength (transaction)	0.275
	Out-strength (amount)	0.758
	Opsahl centrality (lending, transaction)	0.682
	Opsahl centrality (lending, amount)	0.849
	Harmonic centrality (lending)	0.810

To assess the importance of the banks from a network point of view, we can use the betweenness centrality is presented in the last section. Among the analyzed correlations this metrics has the highest value (0.964). It means that those banks that are central nodes in one of the networks tend to be central ones in the other one as well. However, if we divide the networks into borrowing and lending side, then the picture is more complex.

Comparing the borrowing and the lending sides we can find systematically lower correlation coefficients in the latter case (except for the harmonic centrality). The banks that are active borrowers in one of the sub-networks seem to be active in the other one as well, but this cannot be unanimously stated for the lending side. Although the pure out-degrees seem to be strongly correlated between the overnight and longer-term networks, the strength and Opsahl centrality indices are definitely lower, especially when calculated with the number of transactions. Thus, we can conclude that banks that provide much liquidity for other institutions in the overnight market are not necessarily key lenders in the longer-term sub-network (and vice versa).

As a conclusion, the two analyzed subnetworks of the Hungarian interbank market, namely the overnight and the longer-term layers are somewhat overlapping, but the similarity is definitely not symmetric. First, I found some asymmetry regarding the co-existence of the links among the banks: transactions in the longer-term network seem to imply transactions in the overnight one, but reversely it is not necessarily observable. Second, concerning the similarity of the roles the banks play in the network, I can conclude that the borrowing behavior of the banks is expected to be similar in the two sub-markets, but it is not certainly true for the lending sides.

3.4. Historical analysis

In the previous sections, I analyzed the latest available network in my dataset. However, it also important how the uncovered interbank market changed over time since a snapshot network is not necessarily able to provide a fully detailed description of the market. In this section, I will show the historical realizations of the calculated network metrics focusing on the network characteristics, SIFIs in the markets and the comparison of the overnight and longer-term network.

3.4.1. Network characteristics

Network metrics

As we could see in Figure 3 in Section 3.1, the liquidity in the interbank network strongly declined during the crisis period. From a network perspective, such a phenomenon may have various effects. First, it can happen that the former links remained among the banks, but the frequency and the amount of the transactions decreased. That would mean unchanged density and path lengths in the network. However, if banks ceased to transact with some of their former counterparties, then it would appear as a less dense and more fragmented network.

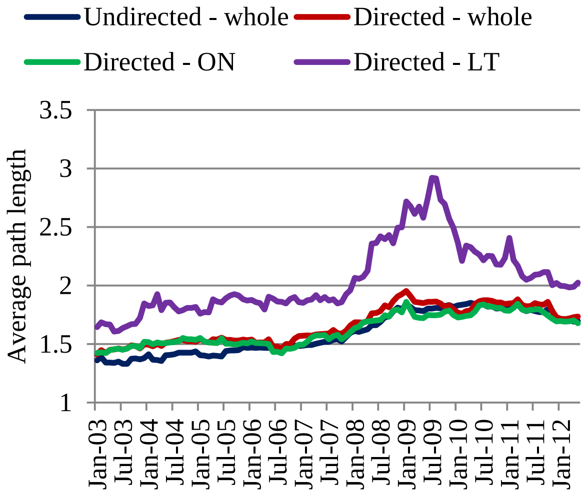


Figure 28 – Average path length of the networks

Regarding the average path lengths in the analyzed networks, three of them seem to be only slightly different in the crisis period (Figure 28). For the directed or undirected networks and the overnight directed network the slowly increasing trend of the average path lengths was already observable before the crisis. However, we can see much more robust changes in the long-term network. Interestingly, the strong increase of the path lengths preceded the Lehman collapse and began a few months earlier. It can indicate that there were some prior signs of the financial distress in the Hungarian interbank market as well (for example due to the spillover effects of the collapse of the US subprime mortgage market).

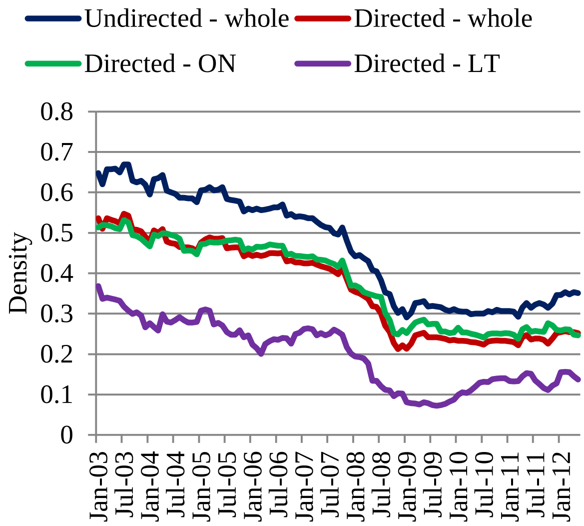


Figure 29 – Density of the networks

Comparing the density of the four network approaches the results are in line with the expectations: the undirected network has the highest while the longer-term directed network the lowest density parameter during the whole sample period (Figure 29). The dynamics of these metrics is more interesting and shows similar pattern across the networks: it gradually decreased until 2009 and then stabilized. It indicates that the Hungarian interbank market was much denser during the 2000s than in the first years of the 2010s.

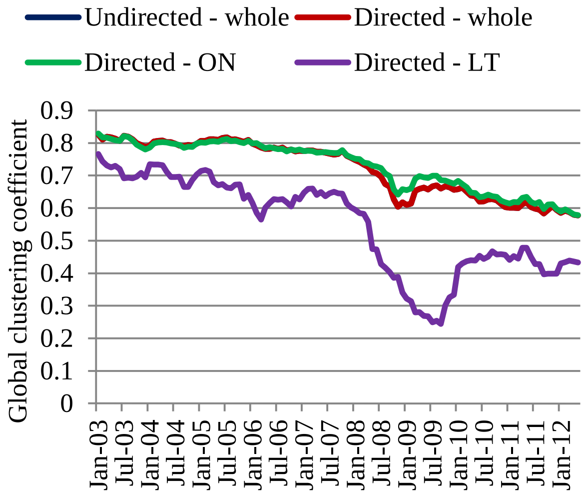


Figure 30 – Global clustering coefficient of the networks

We can see similar pattern concerning the global clustering coefficient (Figure 30). Since it is also an indicator of the connectedness of the networks it leads to the same conclusion as the density. However, it adds some information about the dynamics of the longer-term network since the decreasing trend strongly accelerated during the crisis. That was followed by a consolidation period, and it seems to be stabilized in the last two observed years. Based on these graphs we can conclude that the longer-term network is much more sensitive to the financial disturbances than the overnight network. It is a rational behavior since in a period of uncertainty the financial institutions obviously don't want to lend each other with higher maturity, especially in the uncovered market.

Concentration

As we saw in Section 3.3.2., the lending and borrowing sides of the market are quite different regarding the market concentration. In the latest network the lending side is much more balanced with some equally important institutions while the borrowing side is dominated by banks 13 and 24. However, it is only a snapshot of the interbank market, so this status does not necessarily reflect a long-run equilibrium.

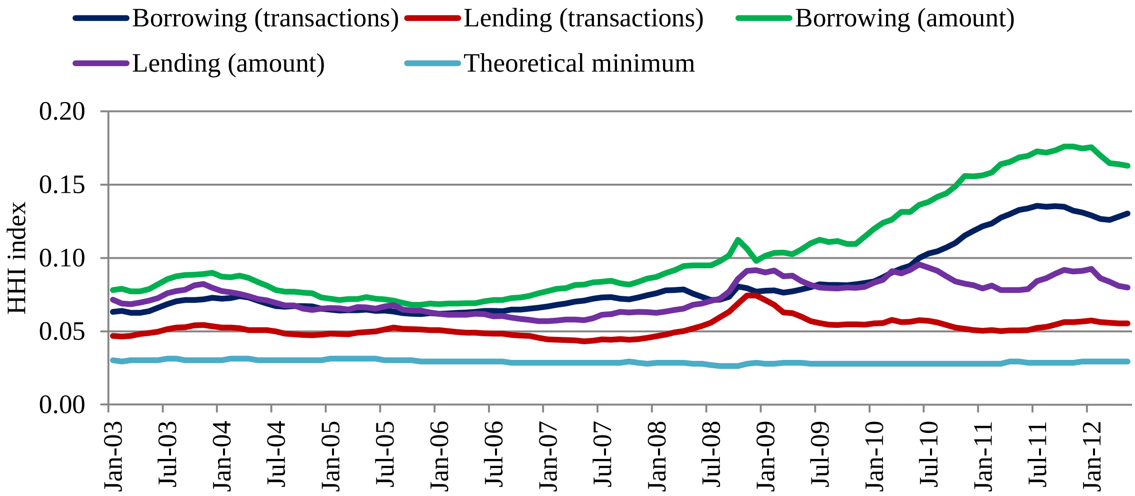


Figure 31 – HHI indices of the overnight layer

In Figure 31 we can see the historical values of the HHI indices for the borrowing and lending parts of the overnight market. The first astonishing observation is that the huge differences detected in the latest available network were absolutely not typical before the crisis. All HHI indices were between 0.04 and 0.08 in the pre-crisis years referring to a much more balanced uncovered overnight interbank market. However, in the post-crisis years the borrowing side started to transform and become less unbalanced. Such changes are observable in the lending side when the HHI is calculated with the transacted amounts, but the increase of the index is more moderate.

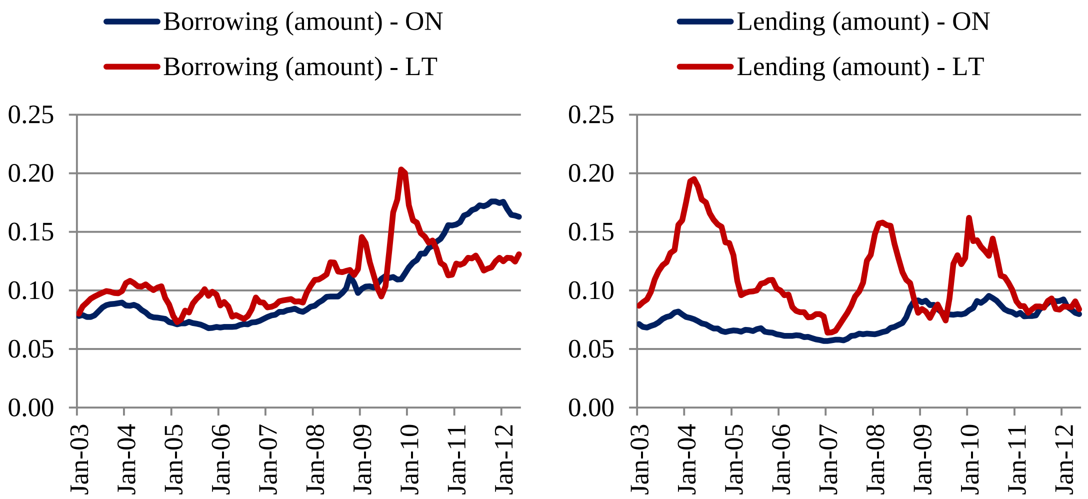


Figure 32 – HHI indices of the overnight and the longer-term layers

If we compare the HHI indices (calculated with transacted amount) of the overnight and the longer-term networks than we can state that the longer-term market concentration is highly volatile compared to the overnight concentration (Figure 32). It can be the consequence of the fewer number transactions in the market, where some large transactions can strongly influence the concentration indices.

To sum up, although the Hungarian banks were highly connected in the interbank market before the financial crisis, the network seems to have transformed during the post-crisis years into a denser and more concentrated market where a few key banks are responsible for large part of the transactions. It is especially true for the borrowing side of the market, while the lending activity is more balanced across the institutions.

At first sight, this result is somewhat surprising since in a period of uncertainty one could expect that banks draw back their lending rather than their borrowing activity due to the riskier counterparties, which would lead to more a concentrated lending side. However, this observation is explainable if we regard the counterparty limits that the banks have towards each other. In an unsecure period banks probably decide to lower their lending (exposure) limits towards smaller and therefore potentially riskier banks, and are willing to lend only to some few trustful institutions. It is particularly true for those banks that operate only as branches of large foreign banking groups because they usually have to apply the limits set in the headquarter of the bank. Since a large Western European bank probably doesn't let its Hungarian branch to have large exposures towards smaller and riskier Hungarian institutions, the branch will be able to have interbank exposures towards a few other institutions in the Hungarian interbank market.

3.4.2. SIFI identification

As we saw in Section 3.3.3., it can vary across both the sides of the market and the maturity layers, which banks prove to be systemically important. However, even if a bank seems to be SIFI in the current market, it is not necessarily true that it has been an important node in the network for the whole analyzed sample. In this section, I present how the systemic importance of the nodes changed over time both within and across the sub-networks. For the SIFI assessment, I rely on the Opsahl centrality index calculated with the transacted amounts because it does not only incorporate both the degree and the strength of the nodes, but it makes it possible to distinguish between the lending and borrowing sides of the market as well.

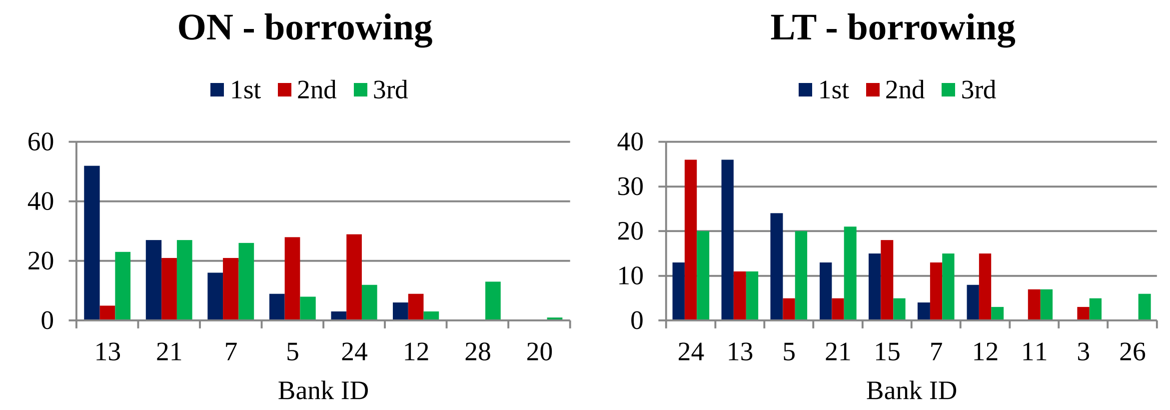


Figure 33 – Number of top three rankings of the key banks in the one-year networks, borrowing side

The first question one can pose is whether the systemic important nodes of the latest network reflect a longer-term equilibrium of the market. In Figure 33 we can see that in how many one-year networks (of the total 113) the banks were the first, second or third most important institutions based on their In-Opsahl centrality. Those banks appear in the graphs that were among the top three banks in any of the 113 networks.[5]

The left panel of Figure 33 shows that 8 banks shared the top 3 "places" in the SIFI ranking of the borrowing side in the overnight layer. As we can see bank 13 was the most important overnight borrower in 46% of the networks, while banks 21 and 7 also proved to be top SIFIs in one-third of the analyzed time period. The majority of the "second ranks" is shared quite equally among banks 24, 5, 7 and 21.

Compared to the overnight borrowing market the longer-term market shows a different picture (right panel of Figure 33). In this case, more banks appear among the top three nodes with more balanced rank distribution, which indicates that the SIFIs of the longer-term borrowing network changed more frequently over the 10 years. Bank 13 has clearly been the most influential bank in this network as well, but we can see that for example bank 24 has been much more important in this longer-term network than in the overnight one.

[5] The sum of the height of the same-color columns may exceed 113 because if more banks had the same centrality value then the same rank was assigned to all of them.

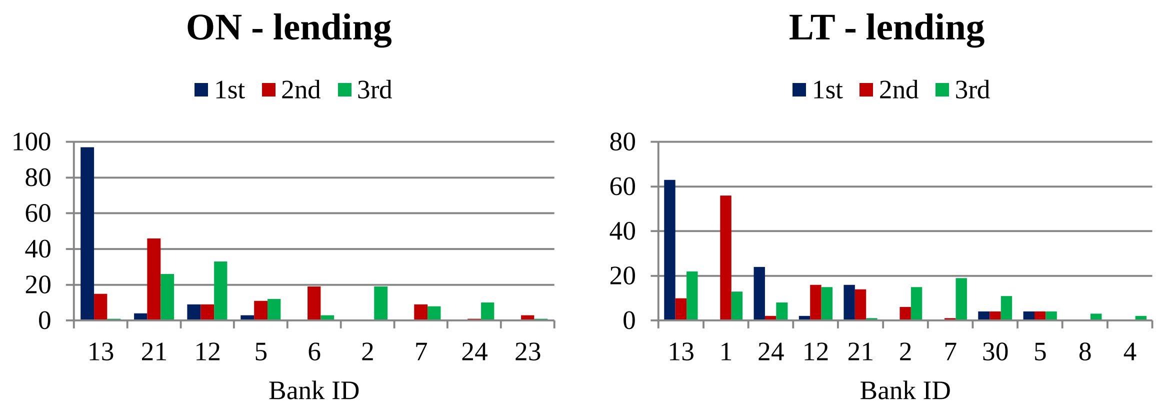


Figure 34 – Number of top three rankings of the key banks in the one-year networks, lending side

Looking at the lending sides of the overnight and the longer-term layers (Figure 34), we can conclude that concerning the first and second most important banks there has not been as much variation as in the borrowing side. Bank 13 was the most important SIFI in 86% and 56% of the ON and LT lending networks, while the distribution of the second places is also more concentrated, for example, bank 1 has been the second most important bank in 50% of the longer-term lending layer.

It may be surprising that in Section 3.3.2. I presented that the lending sides are much more balanced than the borrowing sides, while here we can see the opposite concerning the SIFI identification. However, Figure 33 and Figure 34 only present the relative ranking of the banks (ordinal scale), and they cannot evaluate their relative importance (cardinal scale). If we plot the standard deviations of the Opsahl centralities of the nodes over the analyzed period, then it supports that the relative importance of the nodes is more balanced on the lending side of the overnight layer than on its borrowing side (see Appendix Figure 4).

As the pool of the top three SIFIs varied over the analyzed time period, so did the role of the particular banks in the network. Clearly, bank 13 is the most important SIFI in the interbank network. However, if we have a closer look at its history, then we can see that this statement has not always been true in all four sub-networks.

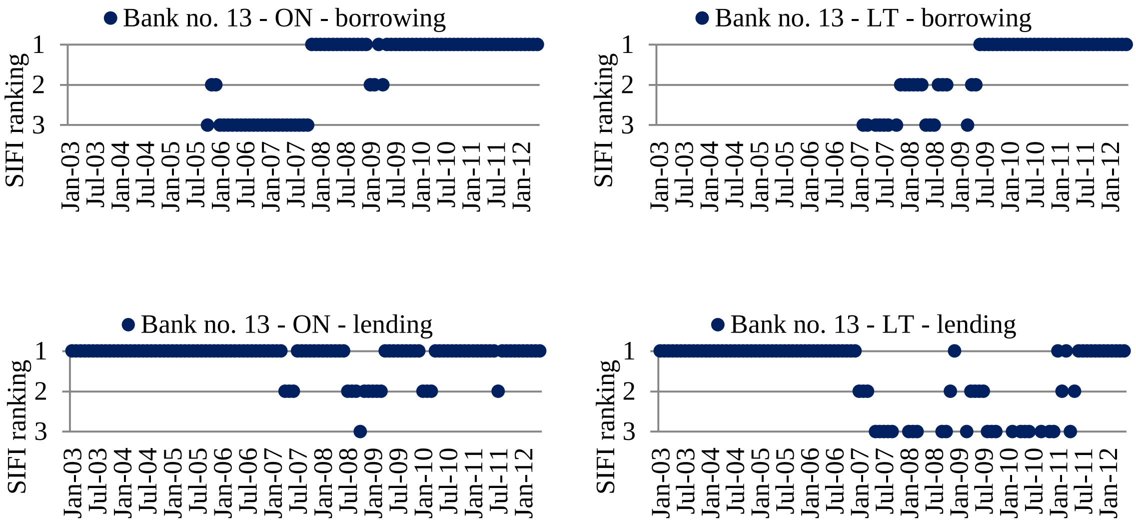


Figure 35 – Top three rankings of bank 13

In Figure 35 we can see the historical SIFI top 3 ranking of bank 13 in the four sub-networks: the borrowing and lending sides of the overnight and longer-term layers. Every dot indicates that bank 13 was the first/second/third important SIFI in the corresponding one-year network.[6] The upper-left graph shows that bank 13 was not among the top three overnight borrowers before October 2005. Moreover, it appeared in the top three most important borrowers in the longer-term layer only in the February 2007 network. Apparently, it has been an important SIFI in the lending part of the networks during the whole period (lower-left and lower-right graphs of Figure 35).

[6] For example the number of dots in the first row in the upper-left graph of Figure 35 is equal to the height of the blue (1st) column of bank 13 in the left graph of Figure 33.

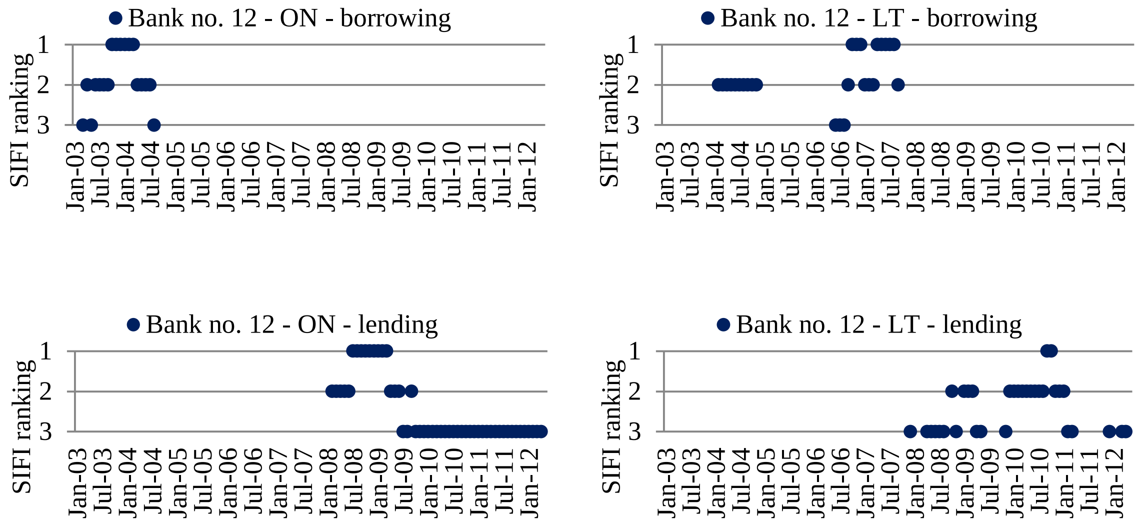


Figure 36 – Top three rankings of bank 12

Another interesting example is bank 12 that appeared in all graphs of Figure 33 and Figure 34 indicating that it has been a quite important institution in all parts of the interbank network. However, its role in the network changed significantly over time. In the first 2-3 years of the sample it was a core overnight borrower in the market as the upper-left graph of Figure 36 shows. During the pre-crisis years it also appeared as key borrower in the longer-term market as well (see the upper-right graph), but it was not among the top three lenders at all. But its behavior substantially changed after the crisis as it became a key lender in the market while it disappeared from the top three borrowers. We can state that in the post-crisis years bank 12 has been a key SIFI in the lending side of both the overnight and the longer-term sub-networks (lower-left and lower-right graphs).

Some more interesting example also show that the role the banks have played in the network strongly varied over time. For example bank 24 which is a key node in the latest network has always been a key longer-term borrower, but it only appeared as an important overnight borrower after the crisis (see Appendix Figure 5). Based on these observations I can conclude that in different parts of the sample different banks proved to be the systemically most important institutions. Unfortunately, I cannot identify the key factors that drove the changes of the behavior of some key banks due to the anonymity of my dataset, but a later analysis with not anonymized data could reveal interesting details about the interbank behavior of the Hungarian banks.

3.4.3. Multiplex network

Layer similarity

In Section 3.3.4 I presented that the overnight and the longer-term network have some common characteristics but the relationship is highly asymmetric and strongly depends on the chosen indicator of comparison. However, if we look at the historical realization of the analyzed parameters, then it turns out that the discovered similarities and differences have not always shown this picture.

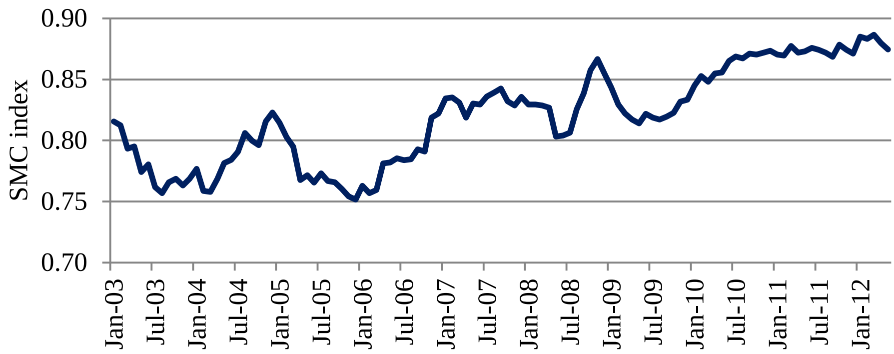


Figure 37 – Simple Matching Coefficient of the layers

The Simple Matching Coefficient was extremely high between the latest networks, but as we can see in Figure 37 it is the result of a positive trend that began around 2006. In the pre-crisis years this indicator varied around 0.77 indicating that ¾ of the directed unweighted connections was present in both the overnight and the longer-term one-year networks. Then this index started to slowly increase reached today's value of 0.88. Since I analyze yearly networks, these high numbers about the co-existence of the connections are not surprising.

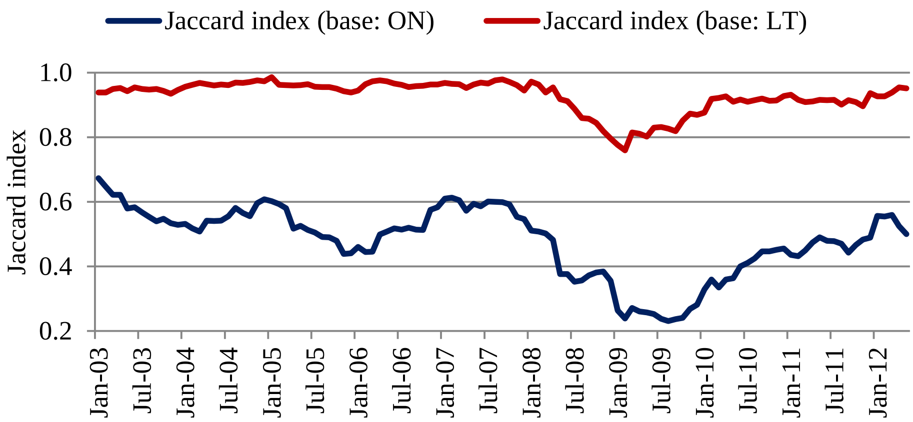


Figure 38 – Conditional Jaccard indices of the layers

The results are more interesting if we examine the asymmetry of the conditional Jaccard indices introduced in Section 3.3.4. As Figure 38 shows, if we take the longer-term network as the basis of the connections (red line) then the index is much more stable than in the case of the overnight network as a baseline (blue line). It indicates that the existence of a partnership in the longer-term interbank network has been a good proxy for the overnight connection over the observed period.

Apart from this observation, we can see that both indices fell during the crisis period. It signals that in times of uncertainty in the financial market the usually observable behaviors (for example the correlation of the transactions) change and interbank lending is not driven by the “everyday” processes anymore. As we moved away from the peak of the crisis the indices started to rise again, which supports the former inference.

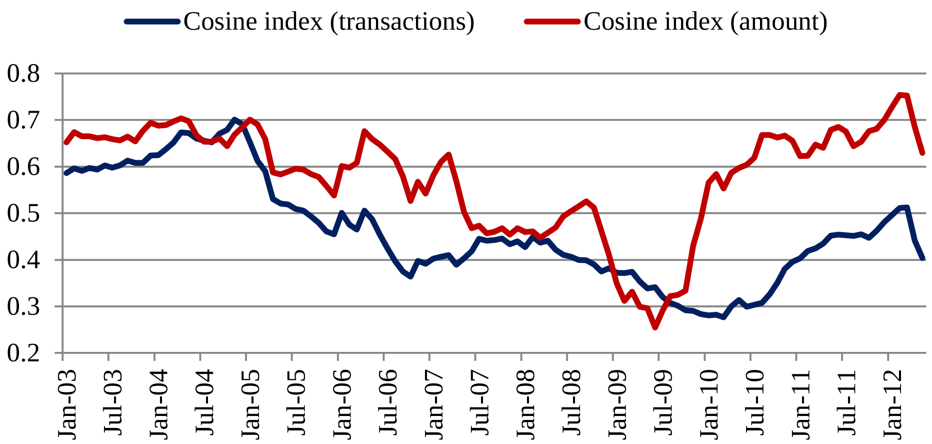


Figure 39 – Cosine indices of the layers

To compare not only the co-existence but the strength of the connections as well I applied the cosine index to the weighted adjacency matrices. In Figure 39 we can see how these indices (calculated with the number or the amount of the directed transactions) evolved over time. Concerning the dynamics we can see similar patterns for the two metrics: the similarity between the overnight and the longer-term networks was quite high until 2005, but it decreased to 0.3 around the peak of the financial crisis. So, this “weighted similarity” indicates that the sub-networks diverged during these years. It is an important observation since it infers that the overnight and the longer-term networks behave disparately during a crisis period.

Node similarity

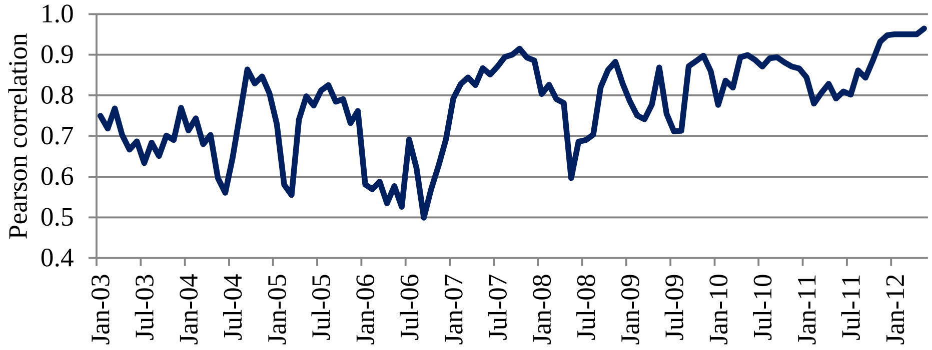


Figure 40 – Betweenness centrality correlation

Concerning the similarity of the role the nodes play in the two sub-networks, I found in Section 3.3.4. that the betweenness centrality indices are extremely correlated (0.964). However, based on Figure 40 this is absolutely not a long-time equilibrium since the current value is the highest in the analyzed period. The correlation of the betweenness centralities had an average of 0.78 with 0.11 standard deviation. Thus, the similarity of this metrics was quite varying, and we cannot see clear structural processes in the time series, only some slow positive trend after the crisis. This finding infers that the simultaneous importance of a bank in the two layers is not as unambiguous as the current state of the networks show; the positive connection is always true, but its strength is often changing.

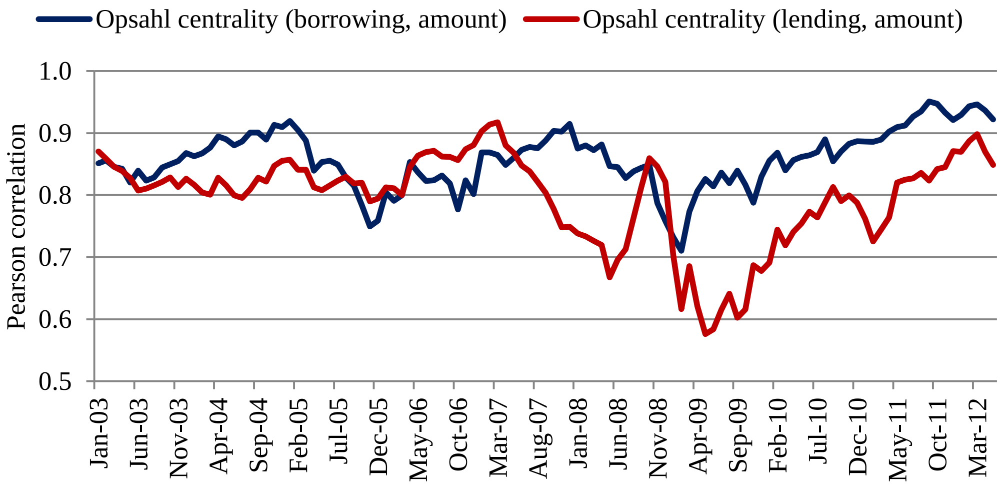


Figure 41 – Opsahl centrality correlations

If we compare the similarities in the borrowing and the lending part of the two networks, then we can see that the detected higher correlation of the borrowing side Opsahl similarities is rather a post-crisis phenomenon (Figure 41). Before the collapse of the Lehman Brothers the two sides of the market seemed to be roughly equally correlated across the layers. During 2008-2009 the two time series deviated, but their distance shrank in the last years of the sample. It infers that in “normal” times the importance of the banks in the two sides of the market is similarly correlated across the overnight and longer-term layers, but this connection becomes loose in times of turmoil.

As a summary, we can conclude that the multiplex comparison of the interbank sub-markets may be misleading if we focus on only one snapshot of the market. The connection between the layers and the relative importance of the nodes are not necessarily robust over time. Studying the dynamics of the similarities and understanding why the layers diverge in times of financial distress can deepen our knowledge about financial networks.

4. CONCLUDING REMARKS

This thesis intended to demonstrate that a financial network can be quite complex, and one has to apply various methodologies to get an insight into its structure. Following the recently emerged approach of the literature, I showed that the multiplex characteristic of the interbank network is an important aspect that has to be taken into consideration when assessing such systems.

The analysis of the Hungarian uncovered interbank network revealed that this market has a small world attribute, which supports my first hypothesis. However, this finding is probably the consequence of the low number of participants in this market. The relatively small size of the Hungarian banking sector makes it hard to compare it to other empirical papers that analyzed much larger networks with more thousand nodes. Nevertheless, I detected some signs of scale-free characteristic that is usually observed in the case of real financial networks.

From a systemic risk perspective, it is crucial how balanced the lending and the borrowing sides of the market are. My results show that a small number of key banks are responsible for the majority of the borrowed amounts, while on the lending side, banks appear to transact in a more equal way. This phenomenon is present in both the overnight and the longer-term layers. Regarding financial stability, it is a worrisome problem since the default or distress of such central institutions could cause severe losses and contagion in the whole system.

The multiplex comparison of the overnight and the longer-term layers confirmed that this approach is meaningful for financial networks. On the layer level, I found signs of asymmetry, namely that the connections among the banks in the overnight network do not imply that they lend to each other on longer maturity as well, while this implication was true in the other direction. The distance of the two layers significantly increased during the crisis period, which is the sign that the overnight and the longer-term sub-networks show different patterns in times of financial disturbances.

Regarding the node-level similarity, my results showed that in the latest available network the relative importance of the institutions on the borrowing side was strongly correlated across the layers, but it was just moderately accurate for the lending sides. However, the historical analysis revealed that the node-level similarity across layers has been strongly varying during the analyzed 10 years. In addition, as the historical assessment of the systemically important institutions discovered, banks tend to change the role they play in the system as well. For example, systemically important overnight lenders may become crucial longer-term borrowers. These findings support my hypothesis that banks show different behavior in the two layers, which indicates that the multiplex analysis of the system is crucial to understand the financial networks and to assess the SIFIs in both the sub-networks and the whole system.

My dataset made it possible to have an overview of the Hungarian uncovered interbank network. However, banks can be connected through other networks as well. To have a deeper understanding of the structure of this market, it would be beneficial to combine this dataset with information about the covered interbank lending, as well as the FX-swap transactions among the banks. A multiplex network analysis built on these layers could reveal essential details which may be used for improving SIFI assessment methodologies and ensuring a more resilient financial system.

5. APPENDIX

Appendix Table 1 – SIFI assessment weights of the BCBS methodology

Source: (2013, pp. 6, 12)

Category (and weighting)	Individual indicator	Indicator weighting
Cross-jurisdictional activity (20%)	Cross-jurisdictional claims	10%
	Cross-jurisdictional liabilities	10%
Size (20%)	Total exposures as defined for use in the Basel III leverage ratio	20%
Interconnectedness (20%)	Intra-financial system assets	6.67%
	Intra-financial system liabilities	6.67%
	Securities outstanding	6.67%
Substitutability/financial institution infrastructure (20%)	Assets under custody	6.67%
	Payments activity	6.67%
	Underwritten transactions in debt and equity markets	6.67%
Complexity (20%)	Notional amount of over-the-counter (OTC) derivatives	6.67%
	Level 3 assets	6.67%
	Trading and available-for-sale securities	6.67%

Bucket	Score range*	Higher loss absorbency requirement (common equity as a percentage of risk-weighted assets)
5	D–E	3.5%
4	C–D	2.5%
3	B–C	2.0%
2	A–B	1.5%
1	Cutoff point–A	1.0%

* All score ranges are equal in size. Scores equal to one of the boundaries are assigned to the higher bucket.

Appendix Table 2 – Indicators used for the SIFI assessment by the MNB

Source: (2016, p. 4)

	Criterion	Indicators	Weight
Standard methodology	Size	Total assets	20%
	Importance	Value of domestic payment transactions	20%
		Private sector deposits from depositors in the EU	
		Private sector loans to recipients in the EU	
	Complexity	Value of OTC derivatives	20%
		Cross-jurisdictional liabilities	
		Cross-jurisdictional claims	
	Interconnectedness	Intra financial system liabilities	20%
		Intra financial system assets	
		Debt securities outstanding	
Supplementary methodology	Supplementary indicators	Off-balance-sheet items (credit lines, guarantees)	20%
		Share in clearing and settlement system	
		Assets under custody	
		Interbank claims and/or liabilities (network analysis)	
		Market transaction volumes or values (network analysis)	

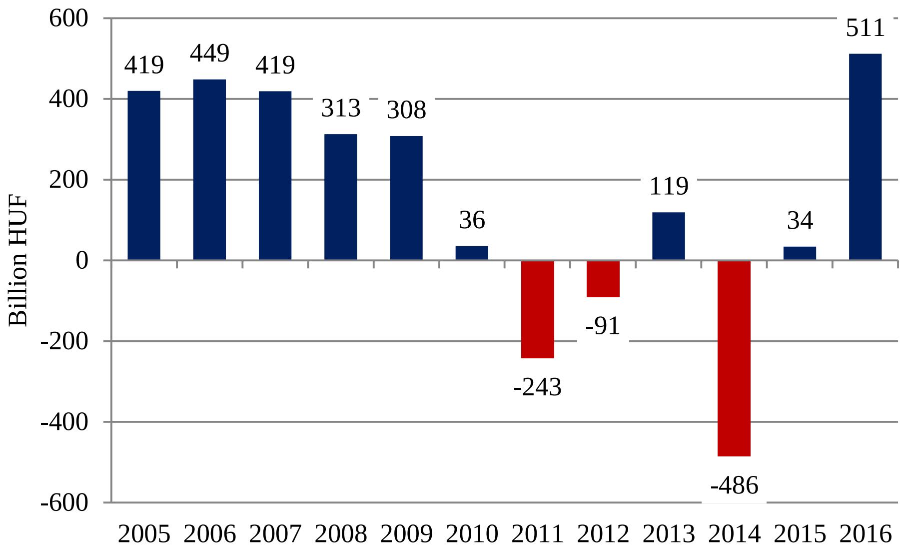
Appendix Table 3 – Hungarian SIFIs identified by the MNB and their capital buffer requirements

Source: MNB press release
Available at: http://www.mnb.hu/en/pressroom/press-releases/press-releases-2016/mnb-allows-more-time-for-banks-to-build-capital-buffers-in-order-to-support-lending

Institution	Capital buffer rate			
	For 2017	For 2018	For 2019	For 2020
OTP Bank Nyrt.	0.50%	1.00%	1.50%	2.00%
UniCredit Bank Hungary Zrt.	0.25%	0.50%	0.75%	1.00%
Kereskedelmi és Hitelbank Zrt.	0.25%	0.50%	0.75%	1.00%
Magyar Takarékszövetkezeti Bank Zrt.	0.125%	0.25%	0.375%	0.50%
Raiffeisen Bank Zrt.	0.125%	0.25%	0.375%	0.50%
Erste Bank Hungary Zrt.	0.125%	0.25%	0.375%	0.50%
CIB Bank Zrt.	0.125%	0.25%	0.375%	0.50%
MKB Bank Zrt.	0.125%	0.25%	0.375%	0.50%

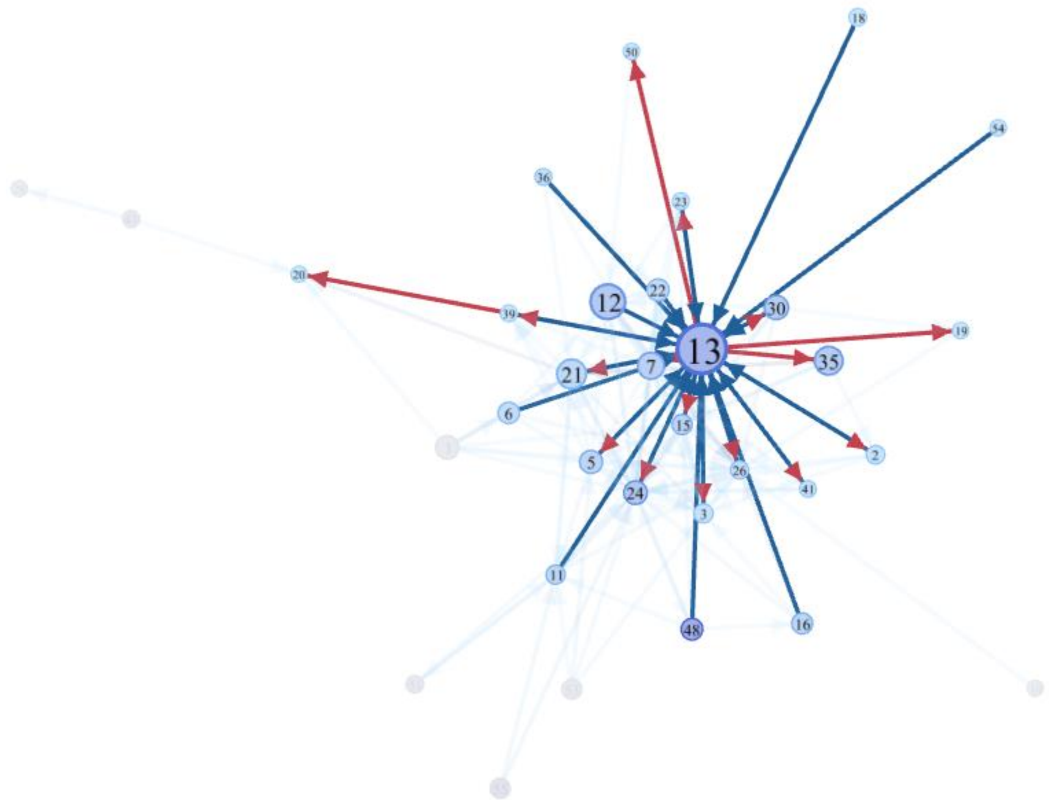
Appendix Figure 1 – Aggregated pre-tax profit of the Hungarian credit institutions

Source: MNB



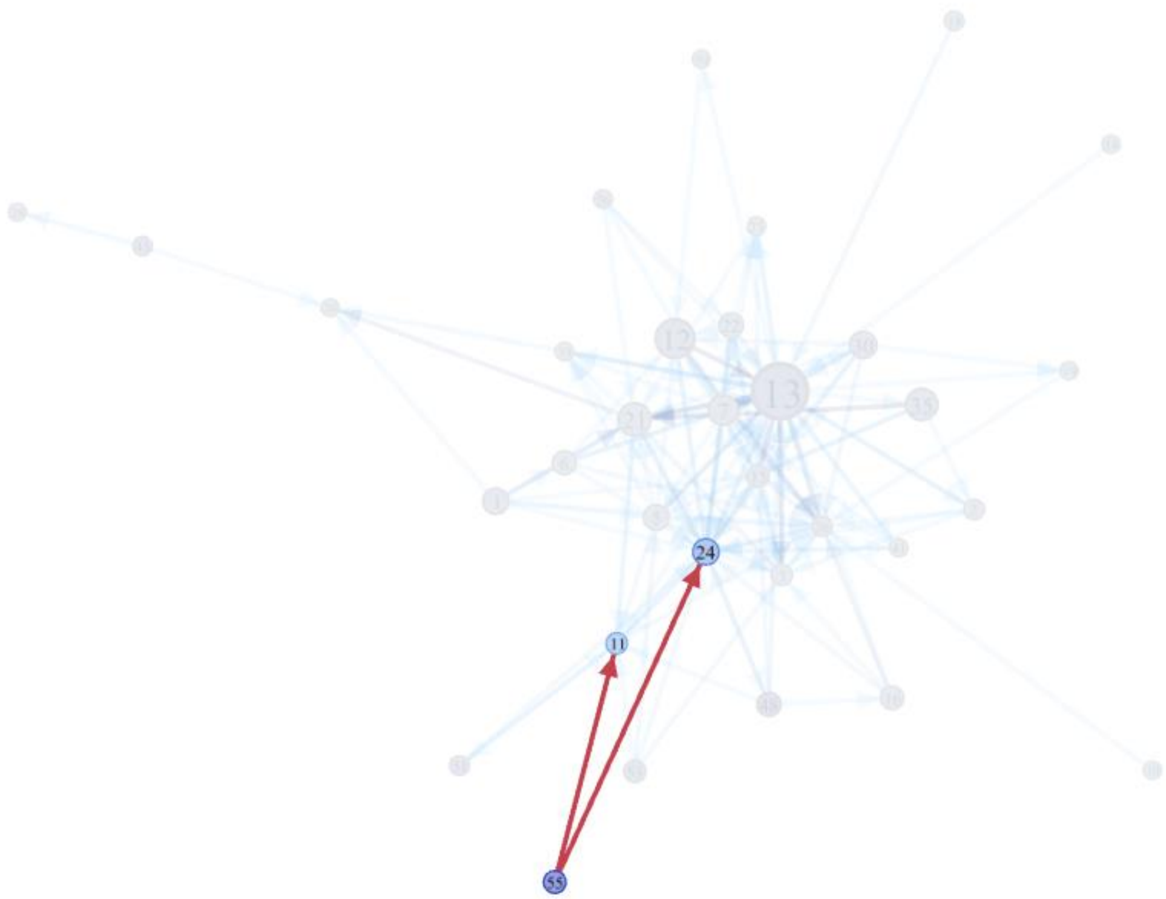
Appendix Figure 2 – Connections of Bank 13 in longer-term network

(Node color: out-transactions, node size: lent amount, edge color: transacted amount)

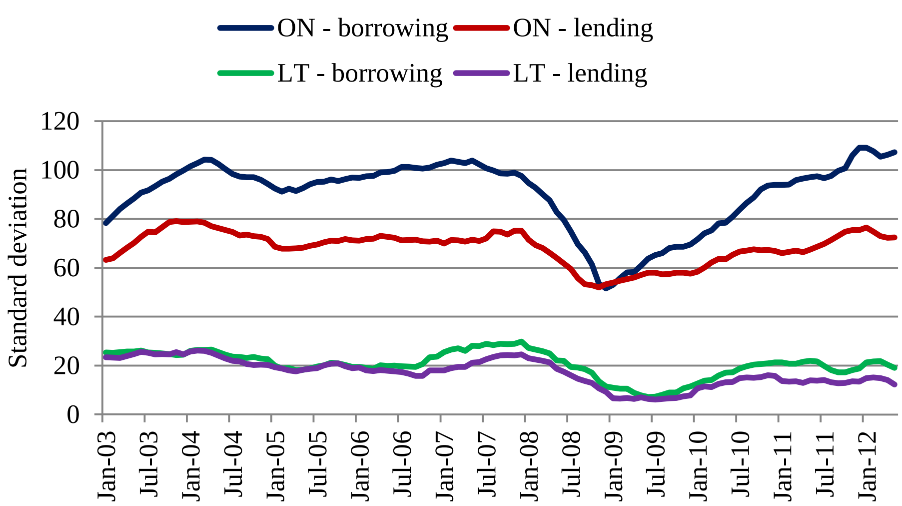


Appendix Figure 3 – Connections of Bank 55 in longer-term network

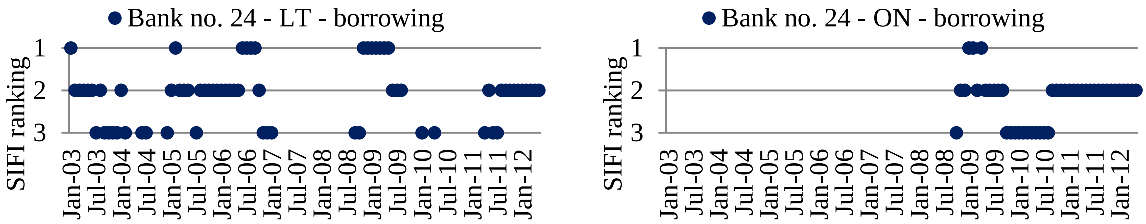
(Node color: out-transactions, node size: lent amount, edge color: transacted amount)



Appendix Figure 4 – Standard deviation of the Opsahl centralities



Appendix Figure 5 – Top three rankings of bank 24



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