# THE EFFECT OF SECURED EMPLOYMENT ON THE PRECAUTIONARY MOTIVE

By

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# Abstract

The thesis aims to introduce a method for implementing endogenous employment choice in an intertemporal utility maximization macroeconomic setting. In the model the main source of agents' heterogeneity is the lack of enough wealth to safely choose a high-wage employment. Instead, individuals with low assets choose a riskless job until they can accumulate enough savings to switch to higher paying opportunity. Agents follow a pre-cautionary motive to evade the above-mentioned cycle and to mitigate the individual idiosyncratic risk that they face from working. This general equilibrium setup is analyzed in a Huggett economy (1993) where I produce a stationary equilibrium of savings distribution and interest rate. Compared to a benchmark model the inequality is relatively higher since the bottom ten percent of the agents have half of the assets. This resembles the empirical data as well.

## Acknowledgement

I wish to acknowledge the helpful comments and support of my supervisor, Tamás Briglevics. I am also indebted to my professors exceptionally to Andrzej Baniak and László Kóczy; I attended their curses gladly. They showed me how grate it is to be true economist, I hope I never have to work in any other field.

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# 1. Introduction

The goal of macroeconomics is to understand and predict the behavior of aggregate economic variables. Following the critique of Lucas (1976) and the revolutionary method of real business cycles (RBC) theory (Kydland & Prescott, 1977), the modern new Keynesian macroeconomic modelling has a microeconomic foundation. It is generally accepted that the statistical relationships observed in the historical data can be quite misleading and it can provide naïve prediction of the effects of economic policy changes. Parameters of the econometric models necessary change when the rule of the game was changed. Large shocks can divert the economy from previous equilibriums, and, although the aggregates seem to be unchanged, the detailed structure of the economy in two different time can be divergent.

Most of the dynamic stochastic general equilibrium (DSGE) models are based on a representative agent abstraction. It greatly reduces complexity of reality and assumes that the economy behaves as if one representative individual makes decision. This entity combines all the preferences and characteristics that the underlying households have. The consideration lies on the law of large numbers (LLN) that even if there were unexpected fluctuations in decisions, after aggregating them the positive and negative deviations will even out. (Friedman, 1953) The main drawback is that it can lose the predictive power if agents' decision rules are not differentiable functions of the state space and the model system moves further from the current equilibrium[1]. In contrast, heterogenous agent models provide a richer microeconomic foundation.

This paper starts with giving a sort summary of the heterogenous agent-based models developed for analyzing the macroeconomy by reviewing notable papers of the large literature beginning from the 1990s. These models are computationally burdensome and rely heavily on solving functional equations (called Bellman equation) which is one of the reasons why they started flourishing only in the last 90s. But with complexity comes great explanatory power; for example, to match and predict the distribution of wealth, income and other macroeconomic aggregates these models are especially useful. The thesis presents early articles such as the Aiyagari model (1994) or the Krusell-Smith model (1998). These models later became workhouse models and have been used to investigate many topics, including precautionary savings, the effect of liquidity constraints, the dynamics of inequality and the shape of the wealth distribution.

The early models tackled the problem of heterogeneity by introducing complete insurance markets, aka Arrow-Debreu market (Arrow & Debreu, 1954) where agents could buy state contingent claims on the future payoffs. It completely terminated the idiosyncratic risks for consumers. Although in financial economics this terminology is viable, general households do not have the option to insure themselves perfectly, for example against the possibility of unemployment. Due to this characteristic households smooth their consumption by using precautionary savings. As Aiyagari argues (1994):

[1] It can jump to another equilibrium such as in bank runs. (Diamond & Dybvig, 1983)

*Casual empiricism as well as formal evidence indicates that individual consumptions are much more variable than aggregate consumption [Deaton 1991]. Further, individual consumptions are not very highly correlated either with each other or with aggregate consumption as would be the case with complete frictionless Arrow-Debreu markets. This suggests that heterogeneity due to incomplete markets may be important. Heterogeneity is clearly necessary for studying the importance of borrowing constraints*.

As I see it, heterogenous agent models can be considered as a method of induction. Through predefined behavioral rules agents can generate distributions that can be matched with empirical data. It is particularly used when models employ

- binding constraints
- preferences that lead to different marginal utilities or propensity to save.
- individual idiosyncratic shocks

In contrast, econometric and representative agent models tend to use deduction. In the followings I would like to argue why heterogenous agent-based models should be part of the mainstream economics.

The main contribution of my thesis is a model of endogenous employment choice in an incomplete market setup. I analyze its effect on precautionary savings in a general economic model à la Huggett (1993) and find that it has a strong negative impact on the wealth inequality. Furthermore, agents can fall into cycles where they keep choosing a riskless, low-wage employment because they are liquidity constrained. This model could account for a wider left tail of the wealth distribution, although some alternation is still needed.

In Sections 2, I summarize some of the important papers in the literature of heterogenous macroeconomic models. Section 3 describes the endogenous employment choice model in a typical macroeconomic framework. At first, the agents' maximization problem, the computational strategy and the partial solution is presented. In the second part it is implemented in a general equilibrium set-up following Huggett (1993). That section also makes a comparison with a benchmark model where there is no riskless job opportunity. Section 4 concludes with some remarks.

## 2. Evolution of heterogenous macroeconomic models

It is generally accepted that models intend to capture and explain only specific part of the empirical data. The right combination of models creates a theory which can later be part of the normal science. In the 20th century many macroeconomic schools have emerged and went through the falsification testing. As Thomas Kuhn describes it (Kuhn, 1962) science develops in a cyclic way where model crisis is the natural part of selection. For example, until the 1970s macroeconomic modelling has been ruled by the Keynesianism and econometric forecasting but later this paradigm shifted.

As I see this, based on the nature of economic modelling, they only approximate the dynamics of real variables (inflation rate, GDP growth, investment, etc.) but never match the true process that drives them. These variables are potentially function of infinite number of other variables because of

- Large number of decision makers (households and firms)
- Interdependencies
- Stochasticity

In order to reduce the dimensions, models specify the significant variables and aggregates the decision makers by groups based on statistical data (e.g. distribution).

But the distribution of these aggregates can change drastically, and the system can jump from one equilibrium to another. It is inevitable to model the microeconomy with heterogenous agents as well.

There is a great similarity between the methodology that is used by economics and physics. Physics studies the motion of matter through time, and space aims to discover universal laws - just as economics does. Till the 19th century classical physics was concerned with topics like kinematics, dynamics, and only in the 20th century did it start researching the particles and energy on a very low scale (atomic physics). Similarly, economics should develop its frameworks on the small particles which build up the whole system (here, individuals). Hiroshi (2016) investigates this question and finds that the method of statistical physics can be usefully applied to the macroeconomy.

More and more focus is put on studying the peripheral regions of economics. Such branches are: behavioral economics, public and policy economics. These subfields help specifying the stability criteria of macroeconomic systems, for example: what are the effect of inequality on the stability of democracies or how do agents make their decision assuming that they are not totally rational. Heterogenous modelling is still in this periphery.

In this part of my thesis I will present the main papers of the heterogenous macroeconomic models with incomplete markets. In the first subsection the models are built on the Deaton (1991) model, where the heterogeneity comes from the individual idiosyncratic risk and liquidity constraint. These models investigate the precautionary motives of agents. In the second subsection, I review models that are centered around human capital accumulation and its effect on inequality.

## 2.1. Incomplete market models with precautionary savings

Deaton (1991) *Savings and Liquidity constraints*

This paper analyses the consumers saving motives when they are not allowed to borrow. The model is built on the intertemporal consumption maximizing framework with a representative agent. The Deaton model is the building block of the workhorse models in macroeconomics because it provides a general framework. It introduces an asymmetric restriction on assets (the inequality constraint) which leads to corner solution for the optimal consumption. Main finding of the model is that liquidity constraints generate precautionary motives which greatly changes the agent's decisions on savings. Figure 1

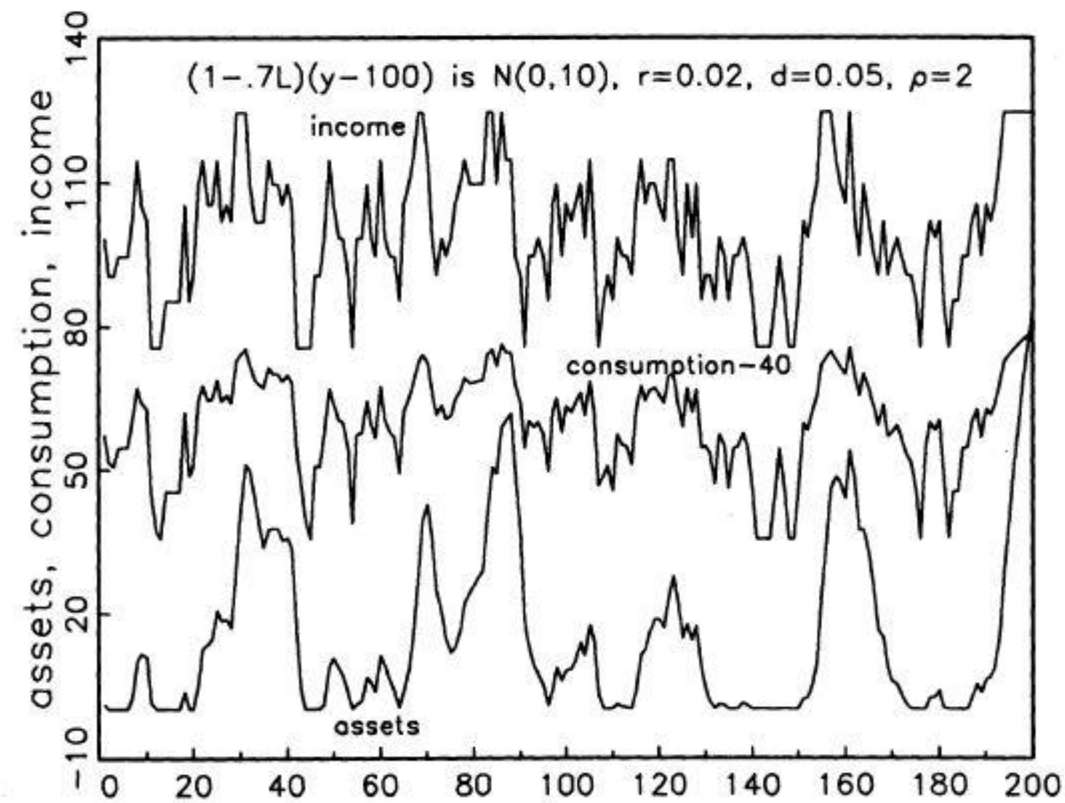


*Figure 1 Simulations of income, consumption and assets with positively autocorrelated income. Assets are used to smooth consumption. (Deaton, 1991)*

shows the buffer region of assets and that consumption is smoother when agents could accumulate enough assets beforehand. He also found that this motive is dependent on the stochastic labor income process that the agents receive. If it is a random walk, it is optimal for impatient liquidity constrained consumers simply to consume their income and the precautionary motive diminishes.

### Huggett (1993) *The Risk-free Rate in Heterogeneous Agent Incomplete Insurance Economies*

Huggett (1993) ask the following question in his paper: "Why has the average real risk-free interest rate been less than one percent?" He explains this with the failure of the calibrated representative-agent economies to explain the average return to equity and risk-free debt. His model follows the same idea as in Deaton (1991) that agents have liquidity constraint, but he develops a heterogenous agent-based framework. Agents interact through the asset market where the equilibrium interest rate is defined. "They are restricted in the level of their indebtedness. However, agents are not restricted from accumulating large credit balances." (Huggett, 1993). The interest rate should be low enough so in the general equilibrium the total assets can be equal to zero. He defines his concept of stationary equilibrium of wealth distribution which will be later used in my model in Section 3. Huggett's finding is that in his heterogenous framework the resulting risk-free rate is more than one percent below the rate in the comparable representative-agent economy.

## Aiyagari (1994) *Uninsured Idiosyncratic Risk and Aggregate*

Aiyagari (1994) embedded the Deaton (1991) model in a neoclassical production economy with idiosyncratic individual risk. By assuming a closed economy, so that aggregate investment equals aggregate saving, Aiyagari could pin down the capital stock. The equilibrium interest rate is defined by assuming that it is equal to the marginal productivity of capital[2]. Aiyagari produced a richer framework that could produce a closer match with the observed wealth distribution.

## Other incomplete market models

Krusell and Smith (1998) greatly contribute to the literature by developing a model where agents form rational expectation on the future interest rate in a heterogenous framework. "In a representative agent economy, equilibrium prices depend on a handful of aggregate state variables, and this problem is relatively simple to solve. However, once aggregation is abandoned, market-clearing prices become a function of the entire distribution of agents" (Heathcote, Storesletten, & Violante, 2009) Forecasting interest rate requires a law of motion for the distribution of agents. Heathcote et al. (2009) further report:

> *An important breakthrough came when Krusell and Smith (1998) proposed to approximate the numerical optimization problem by assuming a form of near-rational behavior: agents view prices as evolving only as functions of a finite set of moments of the distribution (e.g., its mean and variance), and they optimize given a forecasting rule that depends only on these moments. Krusell and Smith describe*



[2] Perfect market hypothesis

*an iterative procedure for computing the forecast- error-minimizing coefficients for this rule. This methodology greatly extends the range of questions that can be studied within this class of models and creates a rich environment to analyze the interaction between idiosyncratic and aggregate risk.*

In the same paper, Krusell and Smith extended their model with a stochastic intertemporal discount factor. Agents vary with respect to their $\beta$ so they are not homogenous anymore. Their model does a fairly good job of accounting for the Gini index[3] as their estimate is 0.82 and the empirical data is 0.79. The results can be interpreted as the source of inequality is the measure of discounting future utility less.

Castaneda et al. (2003) construct an incomplete market model which generates the distribution of US wealth. They model explicitly the quantitative properties of the U.S. Social Security system which allows agents to have less precautionary savings. They show that one way to replicate the high concentration of wealth observed in the US is to allow for a “rare event" in which individual income productivity becomes extremely high.

The Aiyagari (1994) type models have become workhorses of quantitative macroeconomics. Others have successfully used them in the field of fiscal policy or monetary policy (Guerrieri & Lorenzoni, 2017) since the crises showed: liquidity constraint is a general characteristic of households. As Heathcote (2009) argues: “They combine an explicit micro model of heterogeneous households' behavior with a full-blown equilibrium macro model, both micro data on individual allocations (e.g., earnings, wealth, consumption) and aggregate data from national accounts are generally used to discipline

[3] The benchmark model’s Gini index was 0.25

its parameterization." I believe it is justified that they should be used more commonly in macroeconomic modelling.

## 2.2. Source of heterogeneity – Human capital

The following two models find that the source of heterogeneity is the persistency of individual wealth through time: Individuals are destined to stay in their income deciles due to their initial human capital. Keane and Wolpin (1997) argued that factors predetermined at the time individuals enter the labor market account for 90% of lifetime earnings dispersion. If we accept his result this means that macroeconomic models should implement some degree of heterogeneity in initial conditions. More ambitiously, this heterogeneity should be endogenous and connected things like education choice or employment choice.

### Oded Galor (2004) *From Physical to Human Capital Accumulation: Inequality and the Process of Development*

It is a widely researched area, what is the relation between inequality and economic growth. The Classical approach - originated from Smith (1776) and developed by Kaldor (1957) and others - states that marginal propensity to save increases as people have more wealth. Inequality channels the capital to the ones whose saving rate is higher and increases the capital accumulation process. This has a positive effect on the process of development. Galor and Zeira (1993) showed that the credit market imperfections have contrary effect. Their main assumption is that "human capital accumulation and physical capital accumulation are fundamentally asymmetric". If the human capital is more evenly spread, it increases the productivity function.

Galor and Moav (2004) present a unifying theory of the effect of income distribution on the process of economic development. Since the prime engine of economic growth shifted from physical capital accumulation to human capital accumulation, the impact of inequality has significantly changed. This confirms the need for more complex, heterogenous models.

### Huggett et al. (2011) *Sources of Lifetime Inequality*

In the paper from (2011), Huggett et al. start with the following important question: "To what degree is lifetime inequality due to differences across people established early in life as opposed to differences in luck experienced over the working lifetime?" The question concerns not only sociologists, but it has a high priority for economists as well. Understanding the true answer would help improving the social policies that target reducing inequality and in general it would support analyzing the economic growth.

This model fits into the lifetime utility maximization framework extending it with a hump-shaped human capital accumulation process. The source of individual risk is the shocks to human capital because it defines the production function and the equilibrium wage. The model parameters and starting values are calibrated to empirical data. According to Huggett et. al the variation in the initial conditions account for more than 60% of the variation in lifetime earnings and among them the human capital is the most important. Learning ability and starting wealth have a weak effect on the expected lifetime wealth because they only have a rotational effect. The main component is still the initial human capital as in the model one standard deviation increase has a shifting effect of 47% on the expected lifetime earnings. The work of Huggett et al. underlines the general consideration that the investment in human capital is highly determinative for individuals.

## 3. A model with risky employment opportunity

In this section I develop a standard incomplete market model where agents can secure the next period income by endogenously choosing a low-wage but riskless employment. The model builds on the general set-up of macroeconomic models discussed before. Agents maximize lifetime discounted utility, accumulate savings meanwhile, they have an option to secure the next period endowment. This happens when their current period assets are too low and out of risk aversion. The model is a link between the complete and incomplete market models with precautionary savings and intends to analyze savings accumulation of individuals. Apart from the detailed settings which will be introduced latter, there exists two different job opportunities:

- Risky job offers high wage but with the chance of unemployment
- Riskless job gives low wage in the next period

Agents choose the optimal savings and the type of job for next period simultaneously based on the current period disposable income.

### 3.1. Motivation

The idea behind the model comes from the fact that previously summarized models[4] fail to replicate the empirical wealth distribution correctly. There are not enough incentives to accumulate high amount of capital while through the precautionary savings motive, there are too few fortuneless agents. In other words, the Gini index of wealth is too low. Giving



[4] Where agents' preferences are homogenous

a chance for choosing the riskless job would decrease the marginal propensity to save for the poor and increase the number of hand-to-mouth consumer in the economy.

In the model the agents' preferences are homogenous, their current wealth determines their wealth accumulation process. The motivation is that poor agents should endogenously choose to stay poor in the long run because they have low disposable income. When they decide on the type of employment, they select the one that gives higher expected discounted utility. Agents smooth their consumption and want to avoid the situation when they have no assets. Choosing the low-wage riskless employment would ensure that their consumption will not fall significantly in the next period. This way poor households have smaller marginal propensity to save.

Figure 8 in the appendix shows the distribution of income in USA in 2014. We can see that wages have a high variance, skewed to the left and there are a large amount of people earning the minimum wage. Incomplete market models like the Krusell-Smith (1998) operate with only one equilibrium wage but that can be one of the reasons why they fail to generate a correct wealth distribution[5]. A more complex model should implement this characteristic, as we have seen it from Castaneda (2003), they used four possible states of the endowment process. My model is far from capturing the whole income distribution, but it endogenizes the selection of endowments.

Rest of the section looks as the following: First, I characterize the individual decision problem and present the value functions and the policy functions for savings. Here the interest rate is exogenously given for the agents. In the second part, I simulate the



[5] In the original set-up

economy using the obtained decision rules and find the equilibrium interest rate where the aggregated savings matches an exogenous government bond value, following Huggett (1993). At last, I compare the simulation with the economy where there is no riskless employment and analyze the distribution of savings and consumption.

## 3.2. Individual decision problem

The model economy looks as the following. Consider a version of the stochastic growth model where in each period agents maximize the long-run discounted utility while they choose between two types of employment opportunity (risky or riskless). Agents live infinitely, can accumulate savings and receive endowment in each period based on their choice on employment in the previous period. The individual uncertainty comes from the risky job.

Agents' problem is to find the optimal consumption stream:

$$\max_{c_t;employment} \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{1}$$

Subject to

$$c_t + k_{t+1} = m_t \tag{2}$$

$$m_{t+1}^{risky} = (1 + r_t) * k_{t+1} + y_{t+1}^{risky} \tag{3}$$

$$m_{t+1}^{riskless} = (1 + r_t) * k_{t+1} + y_{t+1}^{riskless} \tag{4}$$

$$k_t \geq 0 \, , c_t \geq 0 \tag{5}$$

, where $U(c_t)$ is the utility function in the form of constant relative risk aversion (CRRA). $\beta$ is the discount factor; $c_t$, $k_t$ and $r_t$ denote consumption, savings[6] and interest rate and

[6] The asset can be interpreted as a credit balance with a central credit authority or as a one-period-ahead sure claim on consumption goods.

$m_t$ is the current period disposable income. Agents save the rest of their wealth and $m_{t+1}$ will either be equal to $m_{t+1}^{risky}$ or $m_{t+1}^{riskless}$ depending on the employment choice. Eq. (5) is the limit on savings and consumption. The size of endowments is exogenously given to make the model simpler. The expected wage for risky job in the next period is $E\left[y_{t+1}^{risky}\right] = P^{emp} * w^{risky} + (1 - P^{emp}) * 0$ [7], while $y_{t+1}^{riskless} = w^{riskless}$ . There is no correlation in the endowment process, which means that agents decision is state independent unlike in the Krusell-Smith model (1998)

The problem can be converted to the recursive Bellman form. Let $V(m_t)$ be the optimal value function (VF) for agents with total resources $m_t$; $V^{risky}(m_t)$ and $V^{riskless}(m_t)$ are the VFs when the risky and riskless employment were chosen, respectively. Clearly, these are different because the next period asset $m_{t+1}$ depends on our present choice. The agent picks the employment which leads to higher VF. These functions are the unique solutions of the following Bellman equations:

$$V(m_t) = \max_{1,2}\left(V^{risky}(m_t);\ V^{riskless}(m_t)\right) \tag{6}$$

$$V^{risky}(m_t) = \max_{k_{t+1}}\left\{U(m_t - k_{t+1}) + \beta E\left(V\left((1+r_t)k_{t+1} + y_{t+1}^{risky}\right)\right)\right\} \tag{7}$$

$$V^{riskless}(m_t) = \max_{k_{t+1}}\left\{U(m_t - k_{t+1}) + \beta\left(V\left((1+r_t)k_{t+1} + y_{t+1}^{riskless}\right)\right)\right\} \tag{8}$$

, subject to

$$m_t - k_{t+1} \geq 0 \ and \ k_{t+1} \geq 0 \tag{9}$$

, where I substituted $c_t = m_t - k_{t+1}$ , so that agents maximize utility with respect to $k_{t+1}$.

[7] Unemployed agents earn 0 for that period

Obviously, equations (7) and (8) are contingent, because the expected next period VF involves the same decision as the current period decision on $\max_{1,2}\left(V^{risky}(m_t);\ V^{riskless}(m_t)\right)$ . That is: to calculate eq. (7) one needs eq. (8) as well. Moreover, eq. (7) does contain an expectation value because the uncertainty of $m_{t+1}^{risky}$ and eq. (8) does not since $y_{t+1}^{riskless}$ is certain. Partial equilibrium of the model is then - conditional on the model parameters - the pair of VFs: $V^{risky}(m_t)$ ; $V^{riskless}(m_t)$ which gives the policy function for $k_{t+1}^{risky}(m_t)$ and $k_{t+1}^{riskless}(m_t)$ and a decision rule on employment for eq. (6).

## 3.3. Computational strategy

There are many ways to solve Bellman equations. For this problem the value function iteration seems to be the best method, as individuals need to evaluate the VFs to compare them and make decision on which employment to select. One drawback is that this methodology is very slow because it does not utilize the Euler equation which gives the precise dynamics of consumption. In this sense my strategy is a brute force method but easier to present it.

Let's suppose that there exists a $m^*$ value where $V^{risky}(m^*) = V^{riskless}(m^*)$ . During my research I assumed that there can be only one such point which I explain with the shape of the utility function (continuously, monotonically increasing)[8]. Furthermore, when $m_t < m^*$ it implies that $V^{risky}(m_t) < V^{riskless}(m_t)$ , vice versa.

[8] Others have found complex settings where VFs can meet in two points (Chatterjee, Corbae, Nakajima, & Rios-Rull, 2007)

Define $p(m_t)$ as the decision rule for eq. (5):

$$p(m_t) = \begin{cases} 1, & m_t > m^* \\ 0, & m_t \le m^* \end{cases}$$

Having this switching point rewrites eq. (5) and appears in equations (6)-(7) as well:

$$V(m_t) = p(m_t) * \ V^{risky}(m_t) + \big(1 - p(m_t)\big) * V^{riskless}(m_t) \tag{10}$$

$$\begin{aligned} V^{risky}(m_t) = \ & \max_{k_{t+1}} \{U(m_t - k_{t+1}) + \ \beta E(p(m_t) * \ V^{risky}\big((1 + r_t)k_{t+1} + y_{t+1}^{risky}\big) \\ & + \big(1 - p(m_t)\big) * V^{riskless}\big((1 + r_t)k_{t+1} + y_{t+1}^{risky}\big))\} \end{aligned} \tag{11}$$

Assume that we have the $m^*$, the value function iteration is extended with a second step:

Step 1) In each iteration, using the old VFs and $m^*$, update our belief on the value functions parallelly.

Step 2) Compare the two new VFs and let the new $m^*$ be where they are equal. Repeat step 1) and 2) until there is no change in $m^*$ and the VFs show no significant change. (I used the 2-norm for calculating function distance)

The algorithm works correctly for the reason that: in step 1) VFs are still contraction mappings conditional on the contrary VF[9]. However, I could not produce a rigorous proof of this, more can be read from Stokey and Lucas ( (Stokey & Lucas, 1989), theorem 9.2). Finding the correct $m^*$ in step 2) is also criterion since in steady state the value function should not change.



[9] In the VFI, the contrary VF behaves as the first term, the utility function since the *method* can be *conditional on it*. Beta is still smaller than 1, the eq. has the form of v = Tv, Blackwell's theorem should hold.

## 3.4. Partial equilibrium

Crucial part of the analysis is the calibration of model parameters. For example, if the riskless wage is higher than the expected risky wage, agents have no incentive to take risks and the model collapses.

| Parameters | Values |
|---|---|
| IES | 1 |
| beta | 0.96 |
| $w^{risky}$ | 1 |
| Pr(employed) | 0.8 |
| $w^{riskless}$ | 0.75 |
| r | 2% |

*Table 1 Model parameters*

In Table 1 we can see the parameter values:

Intertemporal elasticity of substitution: 1 means logarithmic utility function

Beta: 0.96 is generally used in economic research papers

$w^{risky}$: it is normalized to 1 for simplicity

Probability of staying employed: I assumed that an average person stays employed at the same workplace for 4 periods. Using the formula[10] we get $p = 0.8$

$w^{riskless}$: for this example, I used the 0.75 to be a viable option

r: in the general equilibrium it will be endogenously selected, here I worked with a neutral value of 2%

[9] Probability of a k long series of employment, where in the next period the event should be the contrary, is $p^k * (1-p)$ the average is then: $(1-p)\sum_k kp^k = avg,$ here it equals to 4 when p = 0.8

For the grid of the state variable $m_t$ I chose the [0, 4] interval. The CRRA utility function takes the form of logarithmic function. The code produced a precise estimate for the desired functions.

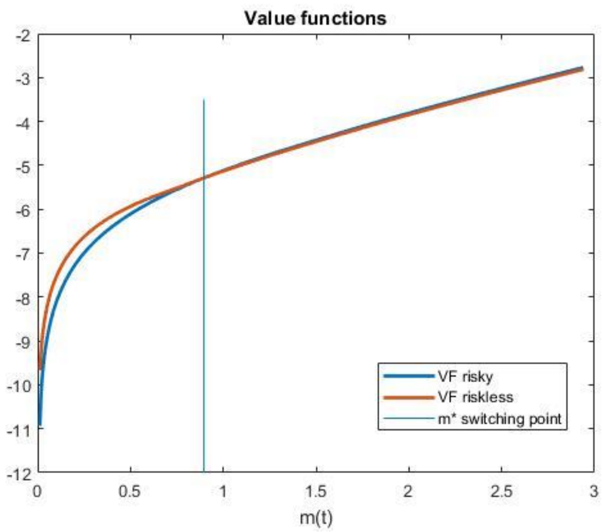


*Figure 3 Risky and riskless value functions*

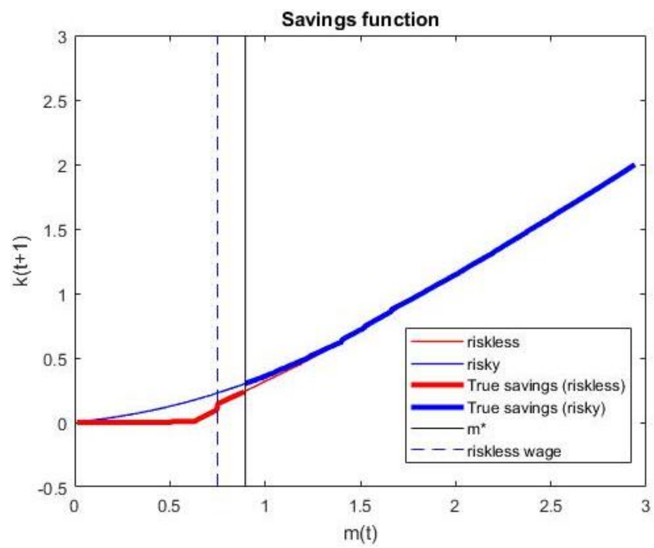


*Figure 2 Representative agent's savings function*

Figure 1 shows the VFs and their intersection at $m^* = 0.93$. Until that point the $V^{riskless}(m_t)$ is higher. In Figure 2 we can see the saving function (thick lines). Agents consume all their disposable income[11] when they are poor until a point where they start saving. The motivation is to accumulate savings through multiple periods, depending on the model setup. After reaching $m^*$ they choose the risky job and their marginal propensity to save increases. Still, they can become immediately unemployed and fall back into the described cycle. This will be presented in Section 3.5. as well.

## 3.5. General equilibrium and comparison with benchmark economy

In the general setup there are continuum of agent of total mass equal to one. Agent interact through the savings market where the interest rate is defined by the total demand

[11] When the curve is flat

equal to the supply. The model employs a predefined government budget (1.5 unit per individual) which leads to that the aggregated savings are positive.

The general equilibrium concept relies on Huggett's assumption (1993). A stationary equilibrium is defined by him as: "where the probability measure $\varphi$ and the price of credit q remain unchanged". Translated to this model it means that: find the interest rate where the savings market is unchanged meanwhile the probability distribution of individuals should be stationary as well. Obviously, agents should still behave according to their decision rules from Section 3.4.

For the model solution I used 5000 simulations with 4000 individuals. I iterated through the possible values of interest rate r and found the one where the model is in stationary equilibrium. We can see the results below.

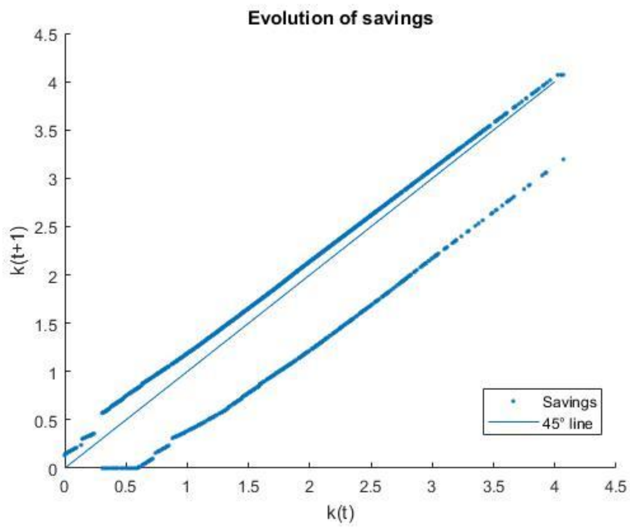


*Figure 4 Optimal decision rule for savings*

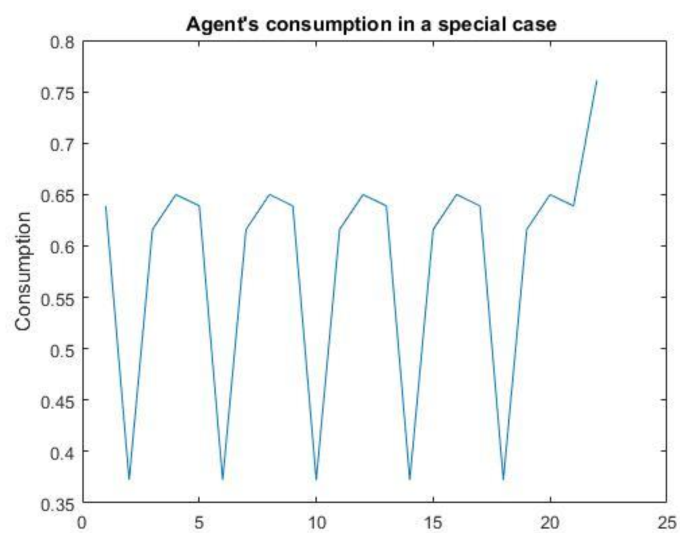


*Figure 5 Negative cycle of low consumption*

Figure 5 shows how savings evolve from period to period. It presents that savings are naturally bounded. The two lines are for employed and unemployed agents. In Figure 6 we can see how a poor agent's consumption behaves after unemployment. At the start, the agent experiences unemployment and due to his low savings, the consumption falls as well. After 3 periods he accumulates enough savings to choose the risky employment but unfortunately, he instantly becomes unemployed and cycle starts again. At the fifth try he finally receives the risky wage and starts accumulating wealth.

I compared the stationary distribution of savings with a benchmark economy where there are only risky employments. The introduction of secured employment drastically changes the wealth distribution. There are more agents in the left and right tails, the number of poor agents doubled. The Gini index increases from 0.18 to 0.22. The number of agents with low savings significantly increases due to the insuring effect of the riskless employment.

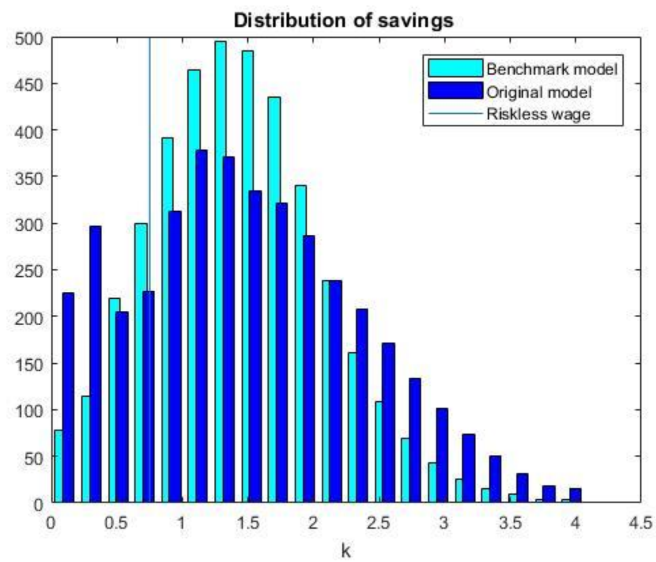


*Figure 6 Stationary distribution of savings in the two economies*

Table 2 in the appendix shows the detailed effects of the secured employment. The equilibrium interest rate increases from 2.6% to 3.7% because individuals are less motivated to save. The distribution of consumption shows no significant change apart from that there are less unfortunate individuals whose consumption is very low; agents still smooth their consumption.

## 3.6. Application and further possible extensions

These findings can be translated as the effect of minimum wage. Agents have less incentive to follow the precautionary motive. As I see, assuming that minimum wage earners are always employed, due to high demand for low skilled labor, the model can be used to present their behavior. The model shows that they do not accumulate enough savings thanks to the insuring effects and can stay poor for periods.

The model can be improved in many ways. The idiosyncratic shocks could be modelled as in the Krusell-Smith (1998) model. The state independent individual shocks can be transformed to a Markov process where currently employed agents have higher chance to stay employed. It would greatly change the endogenous choice of individual while the model would be more realistic. Moreover, the model can be extended with a firm side which allows choosing the interest rate and wages endogenously according to marginal productivities[12]. In the future it will be interesting to investigate how the average number of agents in riskless employment changes depending on the wage ratio and interest rate. Furthermore, the model can be extended with multiple job markets, with different employment probabilities or altered in such a way that agents are permitted to borrow.



[12] Assuming perfect markets

At last, the solving methodology can be revisited as there are better, more complex solving methods to deal with Bellman equations. One way is shown by Carroll (2006) where he avoids the root-finding operations of VFI and greatly reduces the running time.

# 4. Conclusion

The main goal of the thesis was to develop a model where agents select their employment and analyse its effect on the precautionary savings. The model is a link between the complete and incomplete market models as the riskless employment behaves as an employment insurance. The setup produces a larger inequality in the distribution of wealth; the first decile of agents has half the savings, compared to a benchmark model while the interest rate increases. The Gini index increased from 0.18 to 0.22. Individuals have less incentives to follow a precautionary motive because they expect to choose the secured employment if the idiosyncratic shocks are too negative. Furthermore, agents' saving rate is smaller while they are employed in the riskless position which leads to a lower marginal propensity to save. That is also a key element of the empirical data.

At last, I discussed further possible improvements on the model such as: the individual shocks should follow a Markov process. Also, model parameters can be calibrated more precisely. Can a modified model answer the persistence of poverty? It would be interesting to investigate how the poor agents behave if the riskless wage changes.

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# Appendix

*Figure 7 Illustration of approximation. Polynomial approximation of sinus function can be quite misleading further from the point.*

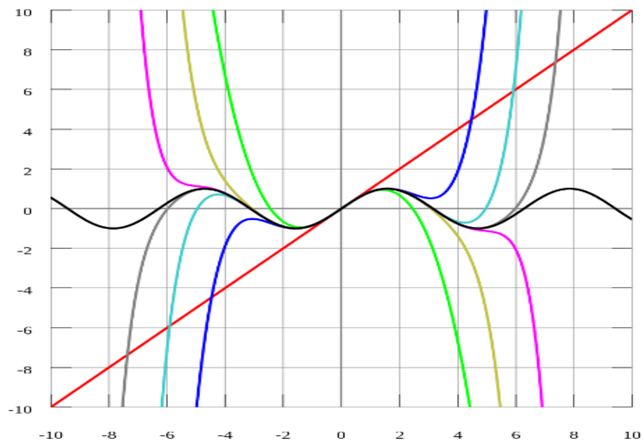


*Figure 8 Distribution of household income in the US 2014 (Distribution of Household Income, 2015)*

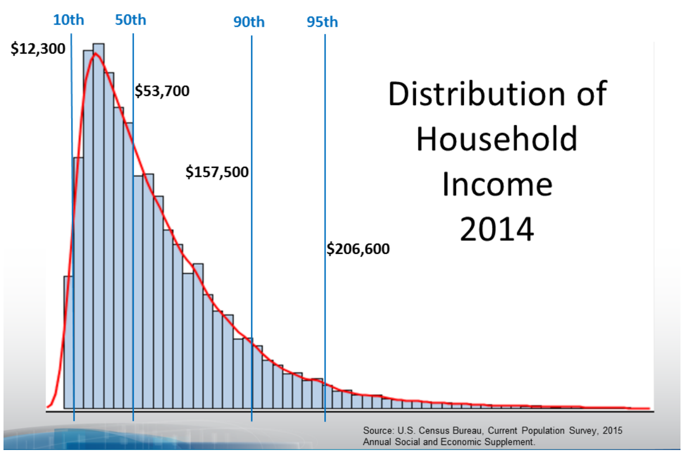

*Table 2 Comparison of outcome in the general equilibrium á la Huggett*

| | Mean/Value | Decile 1 | Decile 4 | Decile 9 | Gini index |
|---|---|---|---|---|---|
| | Model with riskless employment | | | | |
| r | 3.7% | - | - | - | - |
| savings | 1.5 | 0.341 | 1.205 | 2.6767 | 0.222 |
| consumption | 0.8509 | 0.6405 | 0.8531 | 0.9819 | 0.047 |
| | Benchmark model | | | | |
| r | 2.6% | - | - | - | - |
| savings | 1.5 | 0.6012 | 1.2339 | 2.2801 | 0.1812 |
| consumption | 0.8375 | 0.6605 | 0.842 | 0.9652 | 0.043 |