Central European University

Credit Valuation Adjustment Wrong Way Risk:

Comparison of Hull-White's Wrong Way Risk Model with the Ruiz et al.'s Model

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Abstract

In this paper, the performance of two credit valuation adjustment (CVA) wrong way risk models were compared: Hull-White wrong way risk and Ruiz's et a1.'s model. In addition, performance of the models were compared to no wrong way risk credit valuation adjustment model. To capture high correlation between exposure and probability of default of the counterparty Russian Financial Crisis period (2014-2017) is considered in calibration of these two models. The effect of CVA wrong way risk was considered on cross currency swap with four up to ten year maturities. For Ruiz et a1.'s model exchange rate is taken as a market factor which drives positive correlation between hazard rate and exposure. On the stressed calibration of these two models Hull-White wrong way risk model always outperform Ruiz et al.'s model. Hull-White wrong way risk model shows higher CVA values as it captures indirectly positive correlation between exposure and probability of default through other market factors.

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1.Introduction

Counterparty credit risk (CCR) is the risk that counterparty will default before or at the time of final settlement date of its contractual obligations. As of today, by standard practice on the market over-the-counter derivatives dealers adjust the portfolio values by the risk of default of the counterparty. The adjustment is well known as Credit Valuation Adjustment (CVA). By the Basel Committee on Banking Supervision report (2011), on the time of subprime mortgage crisis 2007-2008 CVA losses were almost two-thirds of the credit crisis risk losses and other one-third were caused by defaults. After the crisis, Federal Reserve obligated certain over-the counter derivative contracts run through central clearing parties (CCPs).

Another valuation adjustment which is considered in the book value of a derivative is Debt Valuation Adjustment (DVA), which is beneficial for the dealer. DVA is an adjustment for the default risk of the dealer. Technically, from the perspective of the counterparty it is CVA adjustment to the dealer's default. So, the book value is calculated as no default value of the portfolio minus CVA plus DVA. The institution which controls the transactions between counterparty and a dealer is called International Swaps and Derivatives Association (ISDA). Through the Credit Support Annex (CSA) ISDA Master Agreement provide rules of posting the collateral. In my thesis, only CVA adjustment for the derivative was analyzed without any collateral from both sides.

The case when there is positive correlation between probability of default and exposure to the counterparty is known as wrong way risk (WWR). When the correlation is negative the risk referred as right way risk. For the performance of the trading desk, daily calculation of the wrong way risk CVA is computationally time consuming.

The purpose of this research is to see how simplified wrong way risk models for CVA can capture wrong way risk effectively for the stressed period comparing to CVA model with no wrong way risk effect. In this research, Hull-White wrong way risk model will be compared with Ruiz's wrong way risk model on the cross currency swap trade. The models will be assessed on effectiveness of capturing wrong way risk effect calibrating for stressed period on Russian financial crisis 2014-2017. In addition, models will be compared with no wrong way risk CVA model.

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There are several discussions on the Gregory's book (2009), and estimations provided by Canabarro and Duffie (2003) on the time needed to calculate wrong way risk CVA by simulating simultaneously correlated credit spreads and future exposures. As though, calculation of CVA usually assumes independence of credit spread of the counterparty and exposure to the counterparty. In fact, there is always correlation between these two risk factors. In this paper, models are considered which simplify the calculation of wrong way risk CVA. These types of risk affect four major areas of the banking system: pricing, which is considered previously as CVA adjustment to the default free market value of the portfolio; capital calculation, where Basel III rules incorporate this risk in the alpha multiplier, exposure management, by managing the credit risk on setting credit exposure limits; and initial margin, the similar way as exposure management incorporating risk through potential future exposures.

Wrong way risk and right way risk in most of the cases have impact on four asset classes: credit, equity, foreign exchange and commodities. For example, wrong way risk for credit is when dealer being on the long positions of CDS with the counterparty which has the common business or environment (i.e. the same country) with the reference entity. In case of distress of the environment (default of the country), the probability of default of the counterparty and the exposure will increase at the same time. Similarly, for equity when dealer is in a long position on a put option with counterparty and the reference entity of the option is the same counterparty. Just like on credit, wrong way risk on the foreign exchange is when the dealer is in a long position of cross currency swap with counterparty from a country with emerging economy, assuming that the dealer is a solid institution from a developed economy, for example, from G7. The dealer has positive cash flows in currency of developed economy and negative cash flows in currency of emerging economy. In case of credit distress, the currency of emerging economy will devaluate and the exposure will increase at the same time with credit spread of the counterparty. An example of right way risk in commodities is when the dealer taking long position with a commodity producer on commodity futures. In case of collapse of the price of the commodity the producer doesn't owe any cash, the credit spread of the producer may increase as well as the fair value of this position will decrease.

In this thesis, wrong way risk was analyzed for the foreign exchange swaps derivative which has impact on the CVA desk. Section II presents introduction of key concepts for the calculation of

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the CVA. Section III presents overall literature overview of the researches around the wrong way risk topic; section IV describes the data used for the analyses of wrong way risk; Section V describes the methodologies used to calculate wrong way risk; Section 6 describes results and findings. In addition, Appendix provides list of abbreviations.

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2.Key Concepts. Calculation of CVA

This section presents basic concepts in risk management which will be further used in this paper. First concept is hazard rate. Hazard rate is forward instantaneous probability of default for infinitely small time interval dt conditional on no prior default before or on time t.

The survival probability in the interval (t_1, t_2) is equal to:

$S(t_1, t_2) = \exp(-\int_{t_1}^{t_2} h(s)ds)$.

In this paper we consider only CVA adjustment, which will ignore dealer's own default risk adjustment DVA.

The general formula for calculation of CVA performed below:

$CVA = (1 - R) * E[I_{\{\tau \leq T\}} * V^+(\tau)]$ (1)

R is recovery rate, which is set up equal to 0.4. Recovery rate is the percentage of the value of security after default event comparing to before default time. Loss given default (LGD) is percentage of loss after default event, which is (1- recovery rate). CVA is an adjustment for counterparty credit risk that bears each firm on over-the-counter derivative trades.

$I_{\{\tau \leq T\}}$is the indicator of default and equal 1, if time of default is less than maturity, $V^+(\tau)$is the discounted to current time positive present value of the trade at time τ. The formula for $V^+(\tau)$ is $V^+(\tau) = \max(0, V(\tau))$, where $V(\tau)$ is present value of the trade at time τ. For the simplicity, the trade is not collateralized.

In the further calculations we take discretization of the formula with time steps of one year, where $t_0 = 0$, and $t_n = T$. The formula will be rewritten as

$$CVA = (1 - R) * \sum_{i=1}^{n} E[V^+(t_i) * q(t_i)]$$

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Where $q(t_i)$ is the probability of default of the counterparty within $(t_{i-1}, t_i]$. Further, formula for positive present value between time points t_{i-1} and t_i is approximated to $\bar{V}^+(t_i) = \frac{V^+(t_{i-1})+V^+(t_i)}{2}$. So, CVA will be rewritten as

$$CVA = (1 - R) * \sum_{i=1}^{n} E[\bar{V}^+(t_i) * q(t_i)]$$

Also, we consider regular CVA with zero correlation of probability of default and positive present value as Independent CVA. Independent CVA will be rewritten as

$$CVA_I = (1 - R) * \sum_{i=1}^{n} E[\bar{V}^+(t_i)] * E[q(t_i)]$$

Expectation operator is considered as taking average through simulated paths. In this paper, number of simulated paths is taken as 10 000.

Further, for understanding literature overview context several concepts of risk management is introduced. These concepts introduced by Banking Supervision Committee in response to the financial crisis of 2007-09 to control minimal capital requirements for credit risk. The concepts introduce in Basel III: international regulatory framework for banks. This framework consists set of rules how to measure exposure which defines Basel III rules. Wrong way risk is captured in calculation of minimal capital through alpha multiplier, which was analyzed in articles published by Federal Bank (see [8] Pykhtin, 2012). To understand general idea of the articles these concepts should be introduced first.

Credit exposure is the amount of loss in the event of counterparty's default.

Potential future exposure is one path of the simulated positive exposure.

Expected exposure (EE) is expectation of the positive exposure by taking mean of the simulated paths, it is $E[V^+(t_i)]$.

Effective expected positive exposure (EEPE) is the first year average non-decreasing expected exposure(NEE):

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$$EEPE = \sum_{i=1}^{\min(1\,year, maturity)} NEE(t_i) * w_i$$

Where $w_i = \frac{(t_i - t_{i-1})}{\min(1,T)}$, T is maturity in years and NEE is non-decreasing expected exposure.

$$NEE(t_i) = \max(NEE(t_{i-1}), E[V^+(t_i)].)$$

Expected positive exposure is expected exposure averaged through time:

$$EPE(T) = \frac{1}{T}\int_0^T E[V^+(t_i)]dt$$

For simplification, instead of integral we take summation through simulated time points.

Exposure at default (EAD) is defined in Basel III: international regulatory framework for banks as:

$$EAD = \propto * EEPE$$

By Basel III rules, alpha is set 1.4, which is the estimated ratio between economic capital from the jointly simulated market and credit risk factors, and economic capital when counterparty has positive exposure. For OTC derivatives, the Basel III capital is calculated as

$$Capital = EAD * CRW * SF$$

CRW will be credit risk weight that defined for each asset class and financial product. SF is scaling factor estimated as 1.06.

Initial Margin is the percentage of the value of the security that investor must have in the account for his own cash or marginable security on his account. The overall exposure to the counterparty will be diluted by initial margin. Cure period is the preliminary agreed period in the contract when the counterparty must close the difference between current posted collateral and exposed value when it is higher that collateral threshold (margin call, collateral call, etc.).

Regulatory credit sensitivity CS01 metrics are metrics that show credit exposure to a one basis point change in the credit spread of the counterparty. There are two types of regulatory credit sensitivities: partial by each tenor and parallel:

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$$partial\ regulatory\ CS01 = 0.0001 * t_i * \exp\left(-\frac{s_i * t_i}{LGD}\right) * \left(\frac{E[V^+(t_{i-1})] * D_{i-1} - E[V^+(t_{i+1})] * D_{i+1}}{2}\right)$$

$$parallel\ regulatory\ CS01 = 0.0001 * \sum_{i=1}^{T}\left(t_i * \exp\left(-\frac{s_i * t_i}{LGD}\right) - t_{i-1} * \exp\left(-\frac{s_{i-1} * t_{i-1}}{LGD}\right)\right) * \left(\frac{E[V^+(t_i)] * D_i + E[V^+(t_{i-1})] * D_{i-1}}{2}\right)$$

Where t_i is a tenor, D_i is a discounting factor, s_i is credit spread and T is the maturity.

Value at Risk (VaR) is maximum loss except the worse outcomes that has probability less the defined number q for certain period of time.

Expected loss (EL) is the estimated expected loss to the counterparty's default:

$$EL = PD * LGD * EAD$$

PD is probability of default.

Economic Capital (EC) is the capital amount needed for the company to remain financially stable given its risk profile.

Economic Capital is defined in the reviewed articles as $EC(L) = VaR_q(L) - EL$, where L is certain exposure measure, such as PnL of the trades of specific counterpart.

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3.Literature Overview

The literature overview introduced main frameworks that were used in industry to capture wrong way risk for various purposes.

3.1 Overview of previous researches

As the most cited article in the area of credit valuation adjustment wrong way risk was written by Hull and White (2012). Hull and White described simple model to capture WWR in CVA. Wrong way risk was presented as co-movement between hazard rate and portfolio value. Authors compared deterministic hazard rate versus stochastic by adding an idiosyncratic error, and found out negligible impact on the wrong way risk. The relationships of the hazard rate and portfolio value are performed in exponential and logarithmic functional dependence of hazard rate function from portfolio value; thus, the calculation does not affect the exposure simulation. As a result, hazard rate can be back-tested. For simplicity of the model the effect of wrong way risk is constructed as the positive correlation of the portfolio value to the counterparty and hazard rate. Calibration of this correlation parameter can be done by using expert judgement or historical data. In fact, calibration cannot capture information on the overall exposure of the counterparty to all dealers and in case when portfolio value did not rise for the dealer, but the overall exposure of the counterparty for all dealers rise, calibration parameter will not change. This sample bias could not be improved as the overall exposure of the counterparty is not public information. Calibration by historical data assumes that factors that influence to the hazard rate and exposure will be the same for future simulation and influence the same way. In the article, calibration is done by expert judgment.

For the research artificial 250 portfolios with 25 options were analyzed. Options have four different collateral configurations: uncollateralized, collateral threshold of 10 million USD with cure period 15 days, fully collateralized with cure period of 15 days, and initial margin of 5 million USD with cure period of 15 days. For all cases except when collateral threshold is 10 million USD the wrong way risk increases CVA when exposure increases. For uncollateralized case, the increase of CVA is on average between 0.3 to 0.4 times. In the cases of full collateralization and initial margin of 5 million USD the reason of increasing CVA is that in the cure period of 15 days the value of the portfolio can rise that causes the impact of wrong way

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risk. For the case of 10 million USD thresholds, the difference in WWR CVA and independent CVA is tend to increase and then decrease. For the portfolio value less than 10 million USD the wrong way risk impact is the same as for uncollateralized case, when the portfolio value exceeds 10 million, the gap between simulated higher value path and lower values path is much lower than in uncollateralized case. In addition, in the paper authors performed the CVA Greeks change on wrong way risk impact, but numbers and signs of the change were not explained due to the lack of intuition behind it. The model is simple, computationally fast and easy to perform backtesting results.

In addition, on the Hull and White proposed model has found deficiencies that were analyzed by Ghamami and Golberg (2014). In the paper, authors take the model from Hull and White and describe each aspect of the model in detail. They use the same scheme as in Hull and White for calibration of parameters in hazard rate function. They point out that wrong way risk makes an impact on credit quality for certain simulated paths, but wrong way risk does not change counterparty's credit quality from the market implied credit quality. Hull and White model implies that exposure and hazard rate are positively correlated, that means expectation of exposure multiplied by hazard rate is not lower than expected value of exposure multiplied by expected value of hazard rate. This does not imply that expected value of exposure multiplied by probability of default is not lower than expected value of exposure multiplied to expected value of probability of default. Authors provide artificial example when independent CVA is higher or equal than wrong way risk CVA from Hull and White model. The simulated results for in-the-money put options and forwards with specified configuration with low level of dependency of stochastic hazard rate on exposure shows that independent CVA exceeds wrong way risk CVA for each maturity for relatively small amount. If positive correlation of hazard rate on exposure is high then wrong way risk CVA is higher than independent CVA for each maturity.

Similar to Hull-White approach, the market driven effect of wrong way risk for different risk metrics were performed by Ruiz et al. (2013). Instead of simulating the hazard rate by the regression model which include the present value of the trade, authors considered to simulate hazard rate driven by market factors, such as equity prices, commodity indexes, foreign exchange, typical equity indexes of the countries, considering only one of these factors. They provide wide range of empirical studies finding relationship of the hazard rate with other market

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factors. Functional relationship was chosen from power, exponential, logarithmic, and linear. The criteria of fitness are R-squared and the smallest volatility of the error term. Power and exponential functional forms fit better than the other forms.

The effect of wrong way risk was captured in CVA, Initial Margin, Potential Future Exposure, effective expected positive exposure, parallel regulatory credit sensitivity CS01 metrics. Risk factors were simulated taking one factor geometric Brownian motion calibrated to the risk neutral measure as of start of the simulation date. The hazard rate calibrated using market credit default swap curves, the calibration of dependence coefficients of the hazard rate on market factors are done by using five years weekly historical data up to the start of the simulation period. For the analyses of wrong/right way risk they considered trades on four asset classes: equity, foreign exchange, commodity and credit. Authors also considered collateralized and uncollateralized cases. For the collateralized case, collateral is calculated from 10 day change of the forward price of the trade. The market factors will affect the value of the posted collateral.

For the collateralized case, wrong/right way risk make impact also by the change in collateral, changing from long position to short position of the trade could have the right way risk in both cases. As one of the trades, they considered "in-the-money" long FX forward trade, for the both collateralized and uncollateralized case, there is a wrong way risk. If the same trade is short and "out-of-the-money", then there is right way risk for uncollateralized and wrong way risk for collateralized case, because the distribution of the 10 day change is almost the same for short and long position. The same effect of symmetry of the distribution of 10 day change affects as right way risk for the commodity swaps. The right way risk is for short and long positions when the trade is collateralized. The effect of wrong/right way risk is substantial because of the strong correlation of hazard rate and commodity index. CVA Vega sensitivity affects substantially by wrong/right way risk for this trade, as drop in the commodity index price increased volatility of the index and increased credit spread.

Authors find interesting effect for the long position on collateralized put option. The effect of wrong way risk decreased and disappeared for put option with longer maturity. The high probability of default when equity price go down was waived out by the decreasing change in the 10-day forward price that decreased collateral amount posted. The symmetrical effect can be observed on short position of collateralized put option. The effect of the right way risk for short

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call option is strong for both collateralized and uncollateralized position. For collateralized case the effect of wrong way risk is strong because in case of decrease of the equity price, the 10 day option price change is very small, that will decrease the collateral value to post. Overall, Ruiz et al. (2013) provide framework to work with the effect of wrong way risk on risk metrics and sensitivities driven by the market data. In this research, Hull-White model, Ruiz one market factor model, Independent CVA models are replicated for cross-currency swap.

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3.2 Other researches around wrong way risk.

The most recent paper, which describes the efficient way of replicating results of full simulation by decomposing wrong way risk multiplier to CVA was written by Pang, Chen and Li (2015). In this paper, authors directly estimated correlation between exposure and probability of default. In the model performed, it is assumed that probability of default and exposures have finite first and second moments and the recovery rate is independent input for CVA calculation. By transforming the formula of wrong way risk CVA, it is shown that wrong way risk CVA is multiplication of independent CVA and a multiplier. This multiplier depends on robust correlation, mean values and volatilities of exposure and probability of default across time points. Robust correlation is weighted across volatilities and averaged across simulated time correlation. For computational efficiency it is assumed, that robust correlation is stable across time and they can independently simulate exposures and probability of defaults. For simulation purpose, the authors used CIR and Vasicek model for at the money interest rate swap with flat term structure and for default intensity Black-Karasinski model used. Underlying correlation is set as control variable, but the initial correlation could be found by Monte-Carlo simulation of independent and wrong way risk CVA. For relatively small changes in the underlying correlation, authors found piecewise exponential relationship of underlying instantaneous correlation and robust correlation to derive analytical wrong way risk CVA. This relationship performs stable results for the small changes in parameters of interest rate models. This method is not applicable in cases of drastic shock and full simulation is needed. The advantage of the model is computational efficiency when obtaining confidence interval for CVA and ratio of CVA wrong way risk to independent CVA. For the new levels of underlying correlation analytical formula with mapping to robust correlation used to obtain wrong way risk CVA without running full simulation. Sensitivity analysis of the models showed that for the given level of volatilities of exposure and probability of default, wrong way risk effect decreases when exposure and default probability increase, for the given level of exposures and default probabilities, wrong way risk effect increases with increase in volatilities of exposure and probability of default. As a particular example of interest rate swap described in the paper, increase the dependence of underlying processes increase wrong way risk. Authors get results of CVA ratio range from 1 to 2.8. When the underlying correlation is around 13-17 percent the

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multiplier will be 1.2. When the underlying correlation is around 25-35 percent multiplier is 1.4. One of the drawbacks of the model is that wrong way risk effect is caused by indirect dependence on the market data.

The broad analyses of wrong way risk on the banks portfolio were performed by Cespedes et al. (2010). In this article authors challenged the values of alpha multiplier for the Basel 2 internal based model. As per Final Rule, alpha is considered to capture three drawbacks of the regulatory based models, which are correlation between counterparty exposures and LGDs, correlation between counterparty exposures and defaults, and lack of granularity across. In the article, alpha is calculated to capture the effect of wrong way risk. The key advantages of the model are: counterparty exposure simulation methods used in this model have been widely accepted and are currently used in world's biggest traditional banks' risk management practices, computational efficiency, simplicity in implementation and transparency of the parameters used for estimation. Previous studies in this area range alpha from 1.1 to 2.5 for the concentrated exposures (BCBS (2005); Canabarro et al (2003); Wilde (2005); Fleck and Schmidt (2005)). Wilde(2005) considered credit default swaps and derived alpha in a range 1.25-1.27. Cespedes et al. (2010) considered to calculate alpha as economic capital of credit losses with simulated exposures divided by economic capital of credit losses with deterministic expected positive exposure. Authors for simplicity showed the calculations for the Gaussian copula single factor credit model with one year averaged exposures, where Gaussian copula single factor model is the mapping exposure and credit factor to standard normal distributed correlated processes and simulation of these processes. They also extended it for multiple credit risk factors and for multistep time horizons and implemented this way.

To create the wrong way risk exposure, matrix of simulated scenarios in rows and exposure of portfolios on counterparties in columns was used. To create a correlation of a single credit factor with scenarios total portfolio exposure was used as an aggregated market indicator. For the aggregated market indicator the weighted average counterparties exposures, or expected loss, or principal component of the matrix could be used. In addition, authors proposed to use matrix extending with all simulated market factors, and using "market index" for creating correlation with credit factor. Obvious drawback of this approach is to accurately define this "market index". Market indicator maps to the normalized measure which has standard normal distribution. For this normalized measure the thresholds are calibrated to identify the relevant scenario. Credit

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factor mapped to the normalized measure with standard normal distribution. To calculate economic capital they simulate joint normally distributed variables that are correspondent for credit factor and market indicator, and then determine exposure scenarios and defaulted counterparties to calculate portfolio losses and economic capital. For the calculation of alpha authors considered 1500 counterparties, 2000 market factors, 12 monthly time steps, with transactions of fixed income, foreign exchange, equity and credit derivatives. The portfolio concentrated in a small number of counterparties, by effective number of counterparties is 46 by Herfindahl Index. By the small number of effective counterparties gives impact of idiosyncratic contribution, though decreasing the impact of wrong way risk. As the first case, authors calculated alpha using systematic credit risk. As a second case, alpha calculated using both systematic credit risk and idiosyncratic risk. For the zero correlation, authors got alpha equal 1.02 for the first case and 1.03 for the second case. For all levels of correlation from -1 to 1, first case show alpha range from 0.89 to 1.27, second case alpha range from 0.72 to 1.3. The value 1.2 was for the correlation near 75%, which means that the Final Rule provides conservative approach. For the extension of the methodology part, other copulas can be used.

Pykhtin(2012) wrote a paper about similar approach on capturing general wrong way risk and the issues on overestimation of stress calibration of the wrong way risk exposure. As a general wrong way risk it is assumed wrong-way risk which observed when the trade position or book is affected by macroeconomic factors such as inflation, interest rates, etc. This paper divided into two part: first, describes method of incorporating general wrong way risk into Basel asymptotic single risk factor framework by simulating unconditional credit spread exposures, second, analyses of Basel III requirement on calibration for the stress period and provides reasonable arguments why this calibration does not capture the effect of general wrong way risk adequately. Method described in this paper put several key assumptions: individual counterparty's exposure is infinitesimally small comparing to total exposure of the portfolio and losses are driven by single systematic risk factor expressed in monotonic functional form. Other factors included as an idiosyncratic error. In the paper, author uses decreasing function of loss on risk factor. The Gordy's paper ([11] Gordy, 2003) describes rigorously that portfolio's credit Value at Risk at some confidence level does not depend on portfolio composition and can be expressed by expected loss conditional on systematic risk factor with the same percentile. Here, credit Value at Risk is expected loss to the counterparty default over given holding period for a given

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probability. For simplicity assumed that risk factor has standard normal distribution relating correlation by Gaussian copula to default time that is mapping to latent standard normal variable. Exposure at default includes wrong way risk as it is conditional on risk factor level. Considering single time point, expected exposure mapped to latent standard normal variable that is linked with Gaussian copula to the risk factor. In the methodology of the model, exposure distribution has been analyzed in two cases; first, when exposure is considered as normally distributed, second, when exposure distribution has no closed form. In the latter case, exposures sorted in increasing order and empirical constant piecewise cumulative and inverse cumulative function used. In addition, as an example, author take payer exposure of 5-year interest swap with value zero and semi-annual reset at inception, lognormally distributed forward rates, current interest rate is flat at 4 percent and volatility is 0.3. In both cases, expected exposure distribution conditional on risk level is shifted and narrowed. In addition, they get similar results at different level of correlation of risk factor and exposure. As a result, for correlation of 0.6, expected exposure is around 4.5-6.5 times higher of no general wrong way risk case, for correlation of 0.3, expected exposure is around 2.5-3 times. The calibration is introduced by clasterization of exposures with different correlation defining them in a range of no wrong way risk to extreme wrong way risk. In the paper, exposure at default is taken as expected positive exposure multiplied by alpha, which is dependent on systematic risk factor. Thus, putting the wrong way risk effect inside alpha, this is dependent on the level of risk factor. This simplification is one of the key disadvantages of the model, as WWR in the case of no correlation between probability of default time and risk factor will in theory produce the same results.

Second part of the paper shows that stress calibration does not adequately capture general wrong way risk. Two issues were pointed out why exposure at default calibration of stressed case does not apply properly. First, volatilities in a stressed period influence the exposure at default that follows the time of stress, not the period preceding it. Suppose the stressed period started at the first year. This means that when we calculate exposure at default for one year period as of today conditional on the first year stress case from now, the capital allocation will not be affected by the stressed volatilities, as stressed volatilities will impact exposure of default for the second year. Second, applying stress volatility to risk factors for unstressed period will lead to the different from the crisis values. Authors provided an example to support this point. For example, market risk factor was taken as log normally distributed stochastic process. Absolute volatility of

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risk factor is approximately equal to multiplication of value of risk factor at time zero, relative volatility, and square root of time. In unstressed case period before financial crisis 2007-2008, volatility of S&P 500 was relatively low and value was relatively high. In stressed case as of recent financial crisis 2007-2008, volatility of S&P 500 was relatively high and value was relatively low. So, by the logic applying stress calibration on normal case will give higher volatilities and high spot value of S&P 500, which obviously overestimate the exposures. Further, they compared expected exposure of interest rate swaps in three cases: normal calibration in unstressed period, normal calibration in a stressed period, and stressed calibration in unstressed period. This showed that much higher volatility applied to much lower swap rate in stressed period caused higher exposure than in unstressed period. The exposure applying stressed volatility to unstressed swap rate is around five times more than in other two case, showing inconsistency of this approach to capture general wrong way risk. This over evaluation could differ across asset classes.

In summation, the model has several key advantages to use, such as correlation of exposure and probability of default is controlled at the counterparty level, exposure at default expressed by closed form algorithm of unconditional exposure simulation, no joint simulation of exposures and risk factors is required. The key disadvantage of the model is that in the stressed case future exposure conditional on risk factor narrows the distribution, which is not in real case, as higher volatility of risk factors causes increases the volatility of distribution.

Another way of capturing wrong way risk in the stressed period was proposed by Sokol and Pykhtin (2012). For the diffusion-based models the article of Ehlers and Schönbucher (2006) showed that taking into account correlation of FX and default intensity couldn't capture the effect of jump of FX rate at time of default. As though, Sokol and Pykhtin (2012) analyzed the jump process in the time of default on systemically important counterparties. In their research, authors performed the models to capture wrong way risk for the credit exposures. Authors performed their analyses for systemically important counterparties (SIC), sovereigns and systemically important financial institution, such as Lehman Brothers. To see the effect of wrong way risk they considered expected exposures conditional on time of default. For the non-collateralized case trade exposure is calculated by simulating unconditionally risk factors and risk factor jumps, ignoring risk factor volatility effect. For the analyses, they compare the expected exposure of the 5 year forward contract modelling wrong way risk effect by changing

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correlation between risk factor and default intensity. For the collateralized case, authors take into account the margin period of risk, as two week period. Conditional on default risk factor value is calculated from unconditional values at look-back points applying jump and an increased volatility. Also, risk factor jump is adjusted by default predictability ratio as on the look back period of two weeks. In this research, foreign exchange rate was taken as a risk factor. The implied calibration of FX rate jump is possible in certain cases for sovereigns under certain assumptions (Ehlers and Schönbucher (2006)). These assumptions are: as hazard rate term structure is flat; zero correlations between interest rates, FX rate and default intensity; devaluation by a fixed proportion of local currency with respect to the domestic currency of a bank at time of default (as though, the same fixed proportion of decrease of CDS spreads with notional in USD to CDS spreads with notional in JPY).

For the analyses of the model with risk factor jump, authors took expected exposure 5-year cross-currency float-float swap, comparing cases with jump and no jump of risk factors. In addition, authors analyzed the effect of Lehman Brothers default to the market factors, such as indexes of stock prices of financial firms, stock prices of S&P 500, credit spreads of financial firms, credit spreads of CDX.NA.IG, Libor rates, T-Bill rates, equity volatilities, swaption volatilities for the period of the first appearance of the bad news, the defaulted time and 2 weeks after. The model introduced a jump for risk factor before valuation of the trade without margin and for the trade with margin. Jump is applied for a look back point of 2 weeks' time with increase in volatility. The model performed by Sokol and Pykhtin can describe the wrong way risk exposure to SIC and simple in the implementation into the structure of exposure simulation engines.

As one of the recent articles where sovereign's default affects as a general wrong way risk on exposure to the counterparty was performed by Turlakov (2012). For the analyses, Turlakov used exposures in stressed market scenario conditional on the sovereign's default, which is assumed to be implemented in the bank's risk management system. In the paper, Turlakov described modelling of CVA wrong way risk in the case of tail events. He proposed the model taking Expected Positive Exposure as the sum of probability-weighted exposures in case of sovereign default of the counterparty and regular non-default case. To smoothen the wrong way risk exposure on stressed and unstressed cases author uses interpolating function that includes such parameter as the probability in a near default case. One key advantage of the model described is

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that it can be used with other wrong way risk models in the normal case. The main assumption of this model is that stress scenarios can be expressed in terms of stressed macroeconomic parameters and probabilities of these stress events. Here, probabilities of the stressed exposure happens is conditional probability on the counterparties default. To derive the conditional probability author use unconditional probability of default of the sovereign and multiply it with "coupling" parameter that captures dependence of the probability of default of counterparty and sovereign. The "coupling" parameter should be calibrated for each counterparty explicitly. The calibration takes into account historical data and expert judgment. As author proposed the calibration should be analogous to the credit rating, putting the numbers in the order if the counterparty has low, medium or high wrong way risk impact. As a result, the accurate calibration is the main deficiency of the model. The "coupling" parameter is the key element of the model to capture wrong way risk. The key advantage of the model is that this model could be used for the stress testing of the wrong way risk integrated models. Overall, accurate calibration is the key disadvantage of the models that were provided in the literature review. The next section will describe market data that was used to simulate market factors.

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4.Data description

For the data we take 4 year daily historical data from January 9, 2013 to December 29, 2016 that will be overall 984 data points. To match the data taken from different markets that has different holidays 246 business days per year is considered. The lack of data due to NYSE holidays average between two of the nearest data is used, and exclude time point with lack of data on the other markets. This approximation was used for 10 data point per year. Market data which is taken from Bloomberg was provided by Morgan Stanley. The market factor that we used were OIS USD daily for tenors 1-10 years, OIS RUB daily (RUONIA Overnight Interest Rate Swap) for tenors 1-10 years, RUB/USD fx rate, 5Y credit spread of SBER Bank of Russia, 5Y Treasury Rate.

Here, for the simplicity we assumed constant recovery rate at 40%. Wrong way risk effect arises when there is positive correlation of present value and probability of default. For Ruiz et al.'s model used regression model taking hazard rate as dependent variable from foreign exchange rate.

For the verification that the date fits the regression methods used in this paper, descriptive statistics and graphs are provided below. Here provided regression for daily change, whereas the estimate for weekly data showed very similar coefficient estimates. The Figure 1 shows relationship of log(CDS_5Y*10000) and Ruble/USD FX rate. Following statistics support the reasonableness of the choice.

Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)	Signif.
(Intercept)	6.52496	0.02543	256.6	<2e-16	***
FX rate	-35.0264	1.08498	-32.28	<2e-16	***
Adjusted R-squared:	0.5144	p-value:	<2.2e-16		

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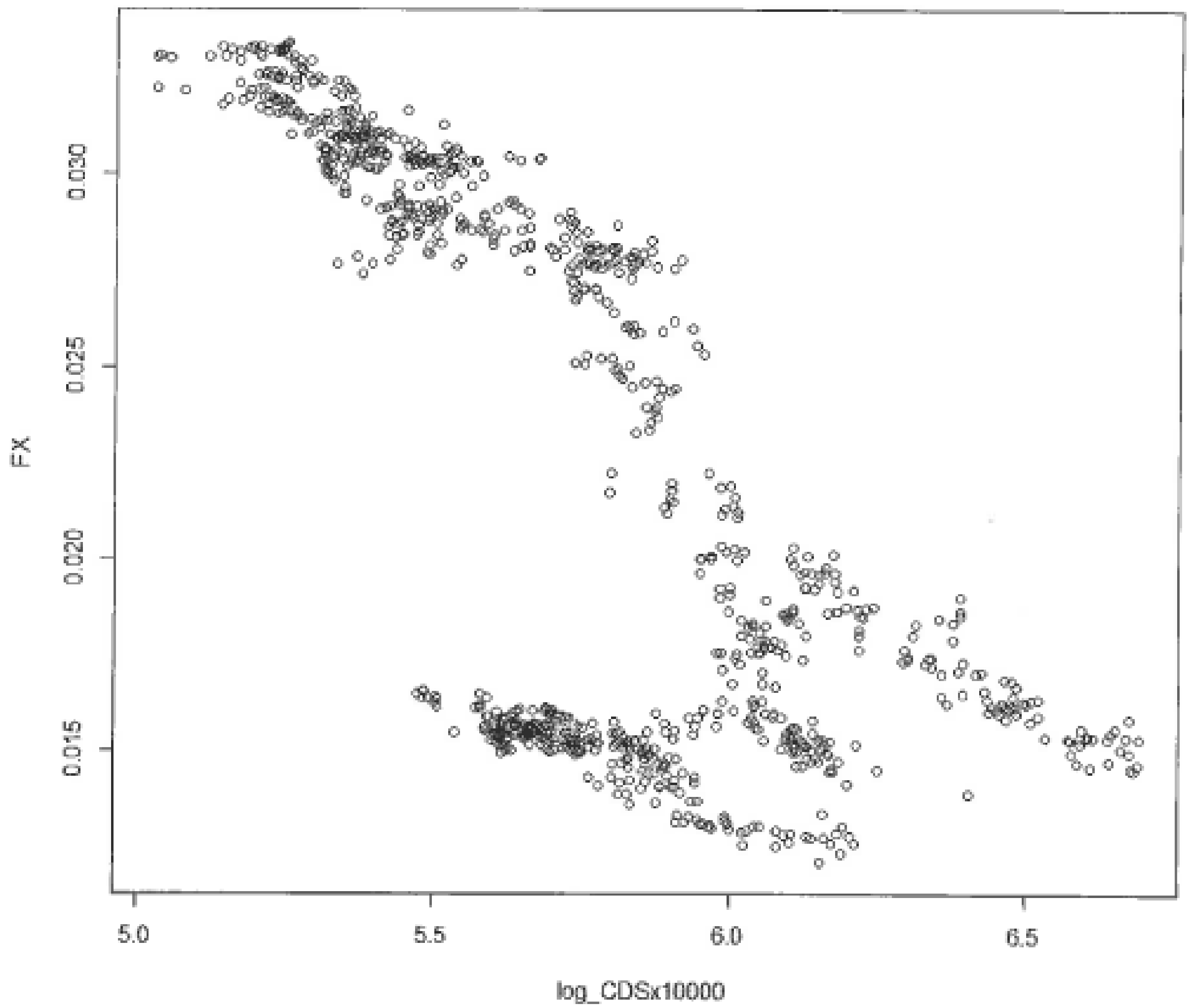

Figure 1: relationship of logarithm of CDS spread from FX rate.

For Hull-White wrong way risk model used regression model taking hazard rate as dependent variable from present value. The same analyses relevant to relationship of CDS and present value of the trade.

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Figure 2 shows relationship of log(CDS_5Y*10000) and present value of 4 year cross currency swap through the period from January 9, 2013 to December 29, 2016. Following statistics support the reasonableness of the choice.

Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)	Signif.
(Intercept)	5.058	1.78E-02	283.68	<2e-16	***
present value	1.43E-06	3.45E-08	41.42	<2e-16	***
Adjusted R-squared:	0.6357	p-value:	<2.2e-16		

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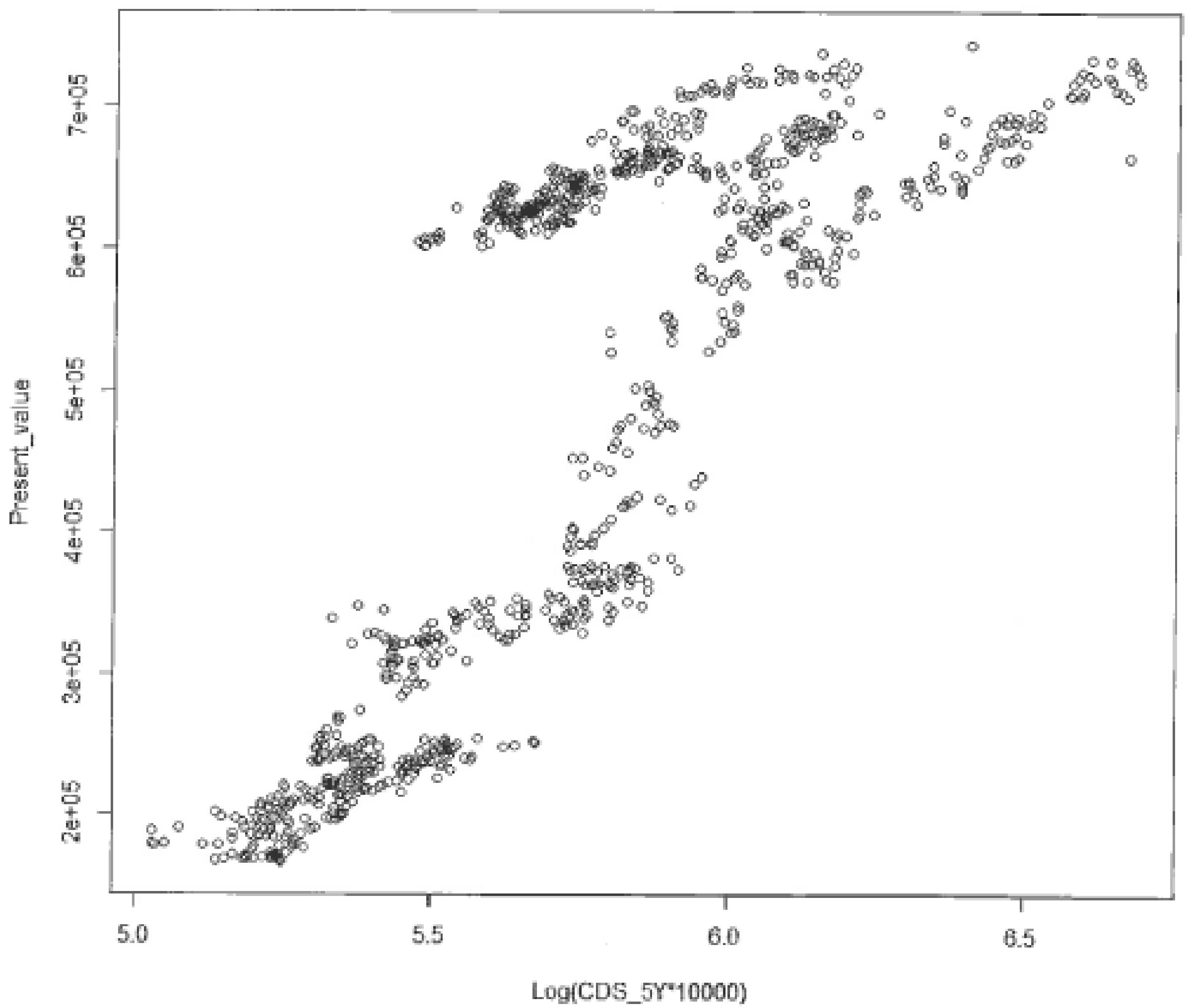

Figure 2: relationship of logarithm of CDS spread from present value.

The following time series in Figures 3, 4, 5 show correlated spikes along the historical time periods.

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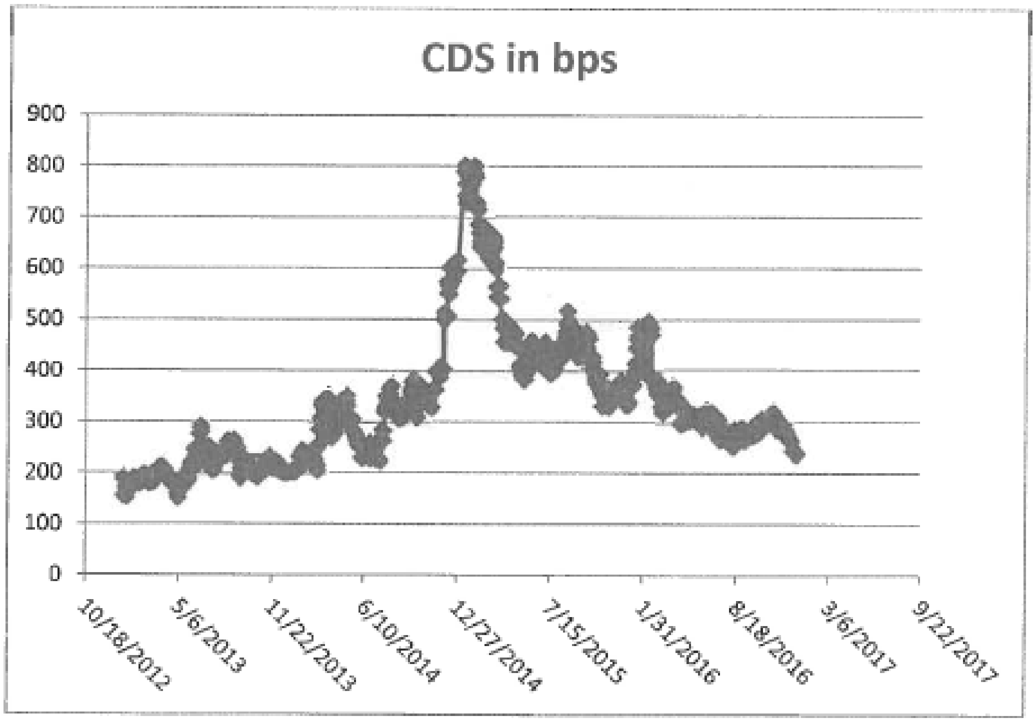

Figure 3: Time series of CDS spread of SBER Bank from January 9, 2013 to December 29, 2016

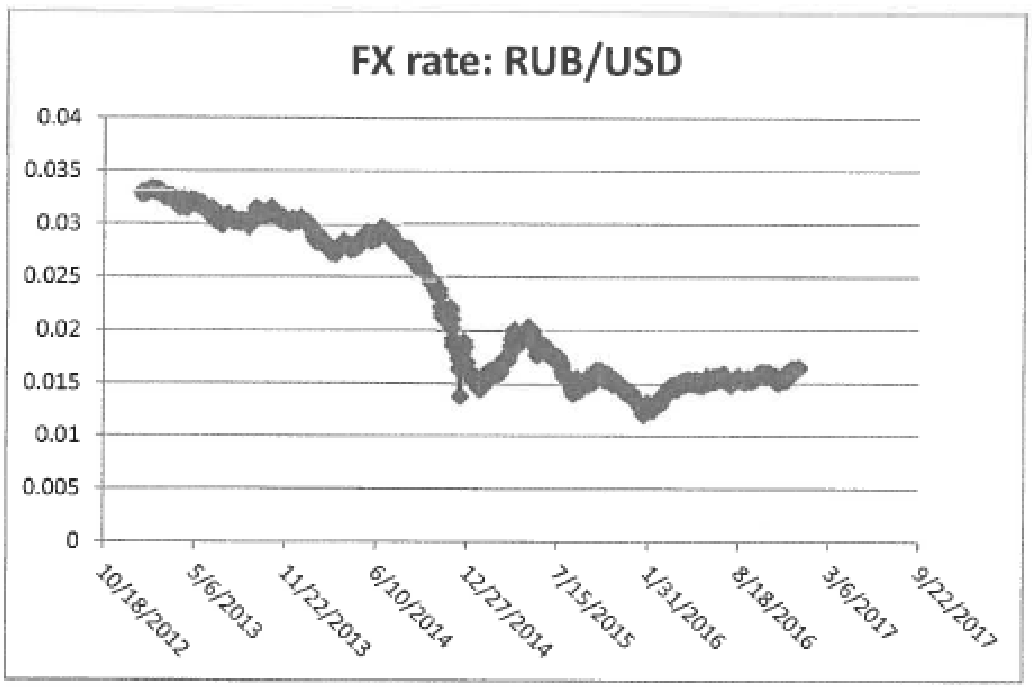

Figure 4: Time series of exchange rate RUB/USD from January 9, 2013 to December 29, 2016

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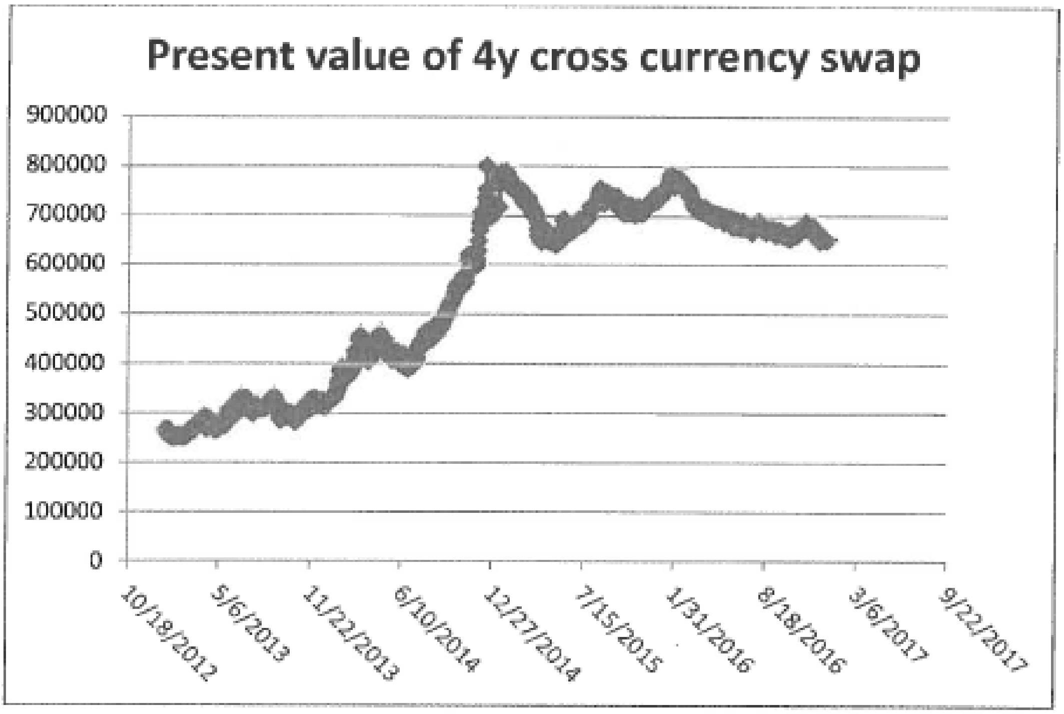

Figure 5: Time series of present value of 4-year cross currency swap from January 9, 2013 to December 29, 2016.

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5.Modelling wrong way risk

5.1 Simulation of the market factors

In this paper, I considered four up to ten year cross-currency swaps with SBER Bank of Russia. In the maturity date exchange of notional is included. Counterparty pays in USD 1Year OIS rate + 50 basis points each year and receives fixed 2% rate in RUB. For the simulation of both rates we used Vasicek model as mean reversion process for interest rates, which is shown in the equation (1), (2). To avoid arbitrage, FX rate evolution is derived as difference between foreign and domestic risk free interest rates. For calculation of correlation of these market factors, calibration of the parameters of the Vasicek model Ordinary Least Square Method (OLS) is used by historical data, which will be provided in section 8.3. As Vasicek model parameters have known issue on accurate calibration on current market condition, for simulation Vasicek model parameters were calibrated historically by the last quarter of 2016. This assumes that market condition didn't change Vasicek parameters for the last quarter staying relatively stable. The start of the simulation is December 29, 2016.

$$dr_{1t} = \lambda_1 * (\mu_1 - r_{1t}) * dt + \sigma_1 * dW^{USD}{}_{1t} \qquad (1)$$

$$dr_{2t} = \lambda_2 * (\mu_2 - r_{2t}) * dt + \sigma_2 * dW^{RUB}{}_{2t} \qquad (2)$$

$$dlnFX^{RUB/USD}{}_t = \left(r_{1t} - r_{2t} - \frac{1}{2}\sigma_3^2\right) * dt + \sigma_3 * dW^{USD}{}_{3t} \qquad (3)$$

Time interval delta is considered as one business day. Here Brownian motion W^{RUB} is under foreign risk-neutral measure P^{RUB} and W^{USD} is under domestic risk-neutral measure P^{USD}. Using Girsanov's Theorem to change risk-neutral measure we get:

$$\frac{dP^{USD}}{dP^{RUB}} = \frac{FX^{USD/RUB}{}_t}{FX^{USD/RUB}{}_0} \, e^{(r_2 - r_1)t} = e^{-\frac{1}{2}\sigma_3^2 * t + \sigma_3 * W^{RUB}{}_{3t}}$$

This implies the processes are defined as:

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$$dW^{RUB}{}_{2t} = dW^{USD}{}_{2t} + corr23 * \sigma_3 * dt$$

Equation (2) can be rewritten as:

$$dr_{2t} = \lambda_2 * (\mu_2 - r_{2t}) * dt + corr23 * \sigma_3 * \sigma_2 * dt + \sigma_2 * dW^{RUB}{}_{2t} \qquad (2')$$

$$dr_{2t} = \lambda_2 * \left(\mu'_2 - r_{2t}\right) * dt + \sigma_2 * dW^{RUB}{}_{2t} \qquad (2'')$$

In this way all the market factors simulated in one probability space. Further on, the upper subscripts are lifted. After the calibration I used calibrated parameters to retrieve series of dW_{1t}, dW_{2t}, dW_{3t}.

r_{1t} is OIS USD daily, and r_{2t} is OIS RUB daily. $dW_{it} = \sqrt{dt} * \epsilon_i$, where ϵ_i is correlated standard normal distribution. $dr_{it}, dlnFX_t -$ are taken as daily changes on these market factors. As there is intersection of holidays in business days of stock market in Russia and US, we took 246 business days in one year; so, $dt = 1/246$

Reverting formula (1), (2), (3):

$$dW_{1t} = (dr_{1t} - \lambda_1 * (\mu_1 - r_{1t}) * dt)/\sigma_1$$

$$dW_{2t} = (dr_{2t} - \lambda_2 * \left(\mu'_2 - r_{2t}\right) * dt)/\sigma_2$$

$$dW_{3t} = (dlnFX_t - \left(r_{1t} - r_{2t} - \frac{1}{2}\sigma^2{}_3\right) * dt)/\sigma_3$$

We get time series $dW_{1t}, dW_{2t}, dW_{3t}$ from which we can get estimated unbiased correlation. Here $\mu_1, \lambda_1, \sigma_1, \mu_2, \lambda_2 \sigma_2$ we get from OLS calibration; σ_3 is taken from unbiased volatility estimate of $dlnFX_t$; $dr_{1t}, r_{1t}, dr_{2t}, r_{2t}, dlnFX_t$ are linearly transformed market data. Correlation matrix looks like this

correlation	dW1	dW2	dW3
dW1	1	-0.0381	-0.0125
dW2	-0.0381	1	-0.1003
dW3	-0.0125	-0.1003	1

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Simulation of correlated normally distributed variables is described in Section 8.1. We assume that historical correlation will persist at the future time and will be constant.

Vasicek model has closed form solution and for each short rate we get:

$r(t+\Delta) = r(t) * e^{-\lambda*\Delta} + \mu * \left(1 - e^{-\lambda*\Delta}\right) + \sigma * \sqrt{\frac{1-e^{-2\lambda*\Delta}}{2\lambda}} * \varepsilon_t$, where ε_t is random variable with standard normal distribution.

Derivation of closed form solution to Vasicek model is performed in Section 8.2. Calibration of Vasicek parameters is described in Section 8.3

To get discounting factor in the arbitrage free short term model, we will introduce the price of zero coupon bond

$$P(t,r,T) = A(t,T) * e^{-B(t,T)*r}$$

$$A(t,T) = \exp\{\left(\mu - \frac{\sigma^2}{2*\lambda^2}\right) * [B(t,T) - T + t] - \frac{\sigma^2}{4*\lambda} * B(t,T)^2\}$$

$$B(t,T) = \frac{1}{\lambda}[1 - \exp(-\lambda * (T-t))]$$

The derivation of discounting factor is described in section 8.4.

Foreign exchange rate using backward Ito-Doeblin formula for (3) we can derive the form:

$$FX_t = FX_0 * \exp(\left(r_{1t} - r_{2t} - \frac{{\sigma_3}^2}{2}\right) * t + \sigma_3 * W_t) \qquad (3')$$

For all models considered in this paper we assume hazard rate with flat term structure.

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5.2 Present value of the trade

Cross currency swap is the sum of discounted cash flows of two legs. Here, formula shown for 4 year cross currency swap, although generalization for n years is straightforward::

$$\begin{aligned}\mathrm{PV(t)} = &\sum_{s=\max(t,1)}^{4} Notional * [P_1(t,s) * (I_{|s-t|<1} * (r_1(t,s) + 0.005) * (s-t) + I_{|s-t|\geq 1} \\ &* (r_1(t,s) + 0.005))] + Notional * P_1(t,4) \\ &- \sum_{s=\max(t,1)}^{4} [Notional/FX(0)] * [P_2(t,s) * FX(t) * (I_{|s-t|<1} * (0.02) * (s-t) \\ &+ I_{|s-t|\geq 1} * (0.02))] - [Notional/FX(0)] * P_2(t,4) * FX(t)\end{aligned}$$

Notional is 1 000 000.

Where $r_1(t,s) = \frac{P_1(t,s-1)}{P_1(t,s)} - 1$

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5.3 Independent CVA model

For calculation of CVA we assume constant term structure of hazard rates. Hazard rates in this case are derived by approximation: $h(t,T) = CDS_{5y}/(1 - Recovery\_rate)$. Derivation of simplified form of hazard rate is described in Section 8.5. Here shown CVA for 4 year cross currency swap, although generalization for n years is straightforward:

$$CVA = (1 - R) * \frac{1}{10\,000} \sum_{j=1}^{10\,000} \sum_{i=1}^{4} PD(t_i, t_{i-1}) \frac{P_1(0, t_{i-1}) * PV_j^{+}(t_{i-1}) + P_1(0, t_i) * PV_j^{+}(t_i)}{2}$$

Probability of default is calculated as

$$PD(t_2, t_1) = \max(0, \exp(-h * t_1) - \exp(-h * t_2))$$

The formula for $PV^{+}(t)$ is $PV^{+}(t) = \max(0, PV(t))$

Here hazard rate is not simulated and there is no case through the time horizon of simulation when probability of default is 1. In that case, we have option not to put condition in CVA formula that future present value will be zero as in previous time point counterparty defaulted their obligations.

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5.4 Wrong Way Risk CVA: Hull-White model

Hull-White model calculates CVA for 4 year cross currency swap this way:

$$CVA = (1 - R) * \frac{1}{10\,000} \sum_{j=1}^{10\,000} \sum_{i=1}^{4} PD(t_i, t_{i-1}) \frac{P_1(0, t_{i-1}) * PV_j^{+}(t_{i-1}) + P_1(0, t_i) * PV_j^{+}(t_i)}{2} * I_{\{t_{i-1} \geq \tau\}}$$

$I_{\{t_{i-1} \geq \tau\}}$is the indicator that default didn't happened previous time point. If default happened previous time, then future present values of the trade should be equalized to zero.

As one of the advantages of this model, it will not influence simulated present values of the trade. Probability of default is calculated as

$$PD(t_2, t_1) = \max(0, \exp(-h * t_1) - \exp(-h * t_2))$$

For the Hull-White model hazard rate has exponential relationship:

$$h_j(t) = \exp(a(t) + b * PV'_j(t))$$

Where j is simulated path, PV is the present value of the trade at time t, b is the coefficient which we get running OLS, taking historical value of present value for the 4 year cross currency swap, and credit spread at time frame mentioned in section 4. In this research calibration implies correlation between hazard rate and present value from the crisis period.

$a(t)$ is the calibration parameter which adjusts the probability of default by implied market probability of default. This parameter should satisfy equation:

$$\frac{1}{m} * \sum_j^m \left[\exp\left(-\sum_i^k h_j \Delta t\right)\right] = \exp\left(-\frac{s * t_k}{1-R}\right) \qquad \text{for } 1 \leq k \leq n$$

Where s is 5 year CDS spread.

Here, n is the maturity, and m is the number of simulations.

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In this research we consider ordinary case when the CDS market is not liquid for any tenor of the counterparty and we take hazard rate flat through the tenor structure, taking spread of 5 year teno r to derive it. As a support, we provide correlation of 5 year tenor and 6 month tenor correlation. Correlation year by year doesn't change showing 0.9098, 0.9729, 0.9338, 0.9080 for the 4 year st ress period. This shows that the co-movement of hazard rates should provide similar impact on w rong way risk, as we could calibrate for each hazard rate by tenor.

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5.5 Historical calibration of parameters for Hull-White wrong way risk model.

For other models performed in this paper, historical calibration can be done by using historical market data and OLS method. For Hull-White model we also need historical data of present values of 4-year cross currency swap, which is not available in the market as it is over-the-counter fixed income derivative.

For that purpose we calculate present value at historical time points for cross currency swap with constant 4 year tenor (as it will not decrease its value because of paid cash flows).

Generalization for n- year swap can be easily derived.

$$PV'(\mathrm{t}) = \sum_{s=1}^{4} Notional * P'_1(t,s) * (r'_1(t,s) + 0.005) + Notional * P'_1(t,4)$$

$$- \sum_{s=1}^{4} \left[\frac{Notional}{FX(0)}\right] * FX(t) * P'_2(t,s) * (0.02) - [Notional/FX(0)] * P'_2(t,4)$$

$$* FX(t)$$

Where FX is historical rate,

$P'_i(t,s) = (1 + r_i(t,s))^{\wedge -s}$, $r_i(t,s)$ is historical rate at time t with tenor s.

$r'_i(t,s) = \frac{P'_i(t,s-1)}{P'_i(t,s)} - 1$ retrieved historical 1 year forward rates.

For discounting of the fixed leg will be done taking MosPrime Rate (Moscow Prime Offered Rate is the National Foreign Exchange Association (NFEA) fixing of reference rate based on the offer rates of Russian Ruble deposits as quoted by Contributor Banks — the leading participants of the Russian money market to the first class financial institutions.) For the floating leg discounting is taken as OIS rate.

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5.6 Wrong way risk CVA: Ruiz et al.'s model.

The modeling structure is the same as Hull-White wrong way risk model. Here instead of present value, correlation with market factor foreign exchange rate influence the probability of default.

Ruiz's model calculates CVA 4-year cross currency swap this way:

$$CVA = (1 - R) * \frac{1}{10\,000} \sum_{j=1}^{10\,000} \sum_{i=1}^{4} PD(t_i, t_{i-1}) \frac{P_1(0, t_{i-1}) * PV_j^{+}(t_{i-1}) + P_1(0, t_i) * PV_j^{+}(t_i)}{2} * I_{\{t_{i-1} \geq \tau\}}$$

$I_{\{t_{i-1} \geq \tau\}}$ is the indicator that default didn't happened previous time point. If default happened previous time, then future present values of the trade should be equalized to zero.

As one of the advantages of this model, it will not influence simulated present values of the trade. Probability of default is calculated as

$$PD(t_2, t_1) = \max(0, \exp(-h * t_1) - \exp(-h * t_2))$$

For the Hull-White model hazard rate has exponential relationship:

$$h_j(t) = \exp(a(t) + b * FX_j(t))$$

Where j is simulated path, FX is foreign exchange rate at time t, b is the coefficient which we get running OLS, $a(t)$ is the calibration parameter which adjusts the probability of default by implied market probability of default. In this research calibration implies correlation between hazard rate and present value from the crisis period. This parameter should satisfy equation:

$$\frac{1}{m} * \sum_j^m \left[\exp\left(-\sum_i^k h_j \Delta t\right)\right] = \exp\left(-\frac{s * t_k}{1-R}\right) \qquad \text{for } 1 \leq k \leq n$$

Where s is 5 year CDS spread.

Here, n is the maturity, and m is the number of simulations.

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6.Results and Conclusion

In this paper, we compared performance of Hull-White wrong way risk and Ruiz et al.'s model calibrated by stress period for one product, which is cross currency swap. s with different maturities were considered.

Underlying correlation of interest rates, exchange rate and model parameters of hazard rate were historically calibrated taking 4 year data from January 9, 2013 to December 29, 2016. Correlation for FX and hazard rate is -0.7175 (for Ruiz model) and 0.7975 is correlation n between present value and hazard rate (for Hull White model).

In this paper, to see the effect of wrong way risk by Hull White model and Ruiz model we take ratio of wrong way risk CVA to independent CVA. The table 1 shows the ratios for different maturities.

Maturity	4	5	6	7	8	9	10
Ruiz et al.'s model ratio	1.1088	1.1407	1.1894	1.2275	1.2972	1.3518	1.4293
Hull White model ratio	1.2058	1.2709	1.439	1.6924	1.9142	1.9895	2.1883

Table 1: Ratio of Wrong way risk CVA (Hull-White model and Ruiz et al.'s model) to Independent CVA

From the table 1 it is clearly seen than wrong way risk effect increases with increase in maturity of the trade. This increase can be explained in increase of volatility of forward FX rate and present value of the trade. Increase in volatility of present value and FX rate are regular cases when increasing simulation time period. From formula $(3')$ can be seen increase of FX volatility by increase in time t. The results are in the range of what was performed in previous researches (see Section 3).

Further on, we change the correlation of FX rate and hazard rate for Ruiz et al.'s model and present value and hazard rate for Hull White model wrong way risk. For example, for Hull-White wrong way risk hazard rate is described by the formula:

$$h_j(t) = \exp(a(t) + b * PV'_j(t))$$

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, where $b = \frac{cov(h_j(t),PV'_j(t))}{var(PV'_j(t))} = \frac{corr\left(h_j(t),PV'_j(t)\right)*\sigma\left(h_j(t)\right)*\sigma\left(PV'_j(t)\right)}{var(PV'_j(t))}$. In this case, we can divide b by current correlation and multiply by correlation we want to see and hazard rate adjust by updated parameter $a(t)$ to match implied market probability of default. The same way correlation is changed for Ruiz et al.'s model.

Under 10 000 simulation the correlation has been changed from 0 to 1 by 0.05 step.

Hull White model results are shown in table 2. It can be seen that by gradual increase in maturity of the trade and correlation the effect of wrong way risk increases. Increase in maturity of the swap also increases the exposure. Also, here it can be seen that in low correlation level the wrong way risk has negative effect that was described in Ghamami and Goldberg. As from the Hull-White model of positive correlation of hazard rate and present value we can derive this:

$$E[PV_t * h_t] \geq E[PV_t] * E[h_t]$$

As a result, the model doesn't imply this

$$E\left[PV_t * h_t * e^{-\int_0^t h_t * dt}\right] \geq E[PV_t] * E[h_t * e^{-\int_0^t h_t * dt}]$$

Knowing that it is purely modelling issue we improve the performance of the model slightly by flooring the ratio by 1.

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Marturity / Correlation	4	5	6	7	8	9	10
0	1.027	0.987	1.002	0.998	0.992	0.990	1.003
0.05	1.011	0.969	0.992	0.992	0.991	0.991	1.008
0.1	1.009	0.962	0.979	0.988	1.013	1.026	1.060
0.15	0.997	0.962	0.991	1.010	1.072	1.084	1.122
0.2	0.992	0.969	1.018	1.060	1.121	1.146	1.206
0.25	0.989	0.977	1.043	1.104	1.179	1.212	1.282
0.3	0.986	0.976	1.087	1.143	1.251	1.276	1.386
0.35	1.001	1.029	1.123	1.184	1.322	1.363	1.445
0.4	1.021	1.049	1.149	1.261	1.392	1.441	1.522
0.45	1.027	1.080	1.200	1.309	1.464	1.500	1.592
0.5	1.056	1.097	1.250	1.372	1.538	1.567	1.675
0.55	1.061	1.144	1.245	1.398	1.575	1.652	1.775
0.6	1.098	1.175	1.290	1.462	1.650	1.733	1.859
0.65	1.127	1.219	1.318	1.487	1.709	1.799	1.967
0.7	1.142	1.224	1.384	1.556	1.715	1.832	1.998
0.75	1.157	1.251	1.425	1.606	1.866	1.948	2.101
0.8	1.163	1.267	1.441	1.644	1.893	1.972	2.059
0.85	1.199	1.319	1.469	1.715	1.962	2.068	2.220
0.9	1.242	1.379	1.528	1.764	2.012	2.047	2.305
0.95	1.289	1.335	1.619	1.812	2.104	2.214	2.363
1	1.309	1.428	1.640	1.860	2.140	2.196	2.387

Table 2: Dependence ratio of Hull-White CVA wrong way risk to Independent CVA on maturity of the cross currency swap and correlation between hazard rate and present value of the trade.

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Improved Hull-White CVA can be seen in table 3.

Marturity / Correlation	4	5	6	7	8	9	10
0	1.027	1.000	1.002	1.000	1.000	1.000	1.003
0.05	1.011	1.000	1.000	1.000	1.000	1.000	1.008
0.1	1.009	1.000	1.000	1.000	1.013	1.026	1.060
0.15	1.000	1.000	1.000	1.010	1.072	1.084	1.122
0.2	1.000	1.000	1.018	1.060	1.121	1.146	1.206
0.25	1.000	1.000	1.043	1.104	1.179	1.212	1.282
0.3	1.000	1.000	1.087	1.143	1.251	1.276	1.386
0.35	1.001	1.029	1.123	1.184	1.322	1.363	1.445
0.4	1.021	1.049	1.149	1.261	1.392	1.441	1.522
0.45	1.027	1.080	1.200	1.309	1.464	1.500	1.592
0.5	1.056	1.097	1.250	1.372	1.538	1.567	1.675
0.55	1.061	1.144	1.245	1.398	1.575	1.652	1.775
0.6	1.098	1.175	1.290	1.462	1.650	1.733	1.859
0.65	1.127	1.219	1.318	1.487	1.709	1.799	1.967
0.7	1.142	1.224	1.384	1.556	1.715	1.832	1.998
0.75	1.157	1.251	1.425	1.606	1.866	1.948	2.101
0.8	1.163	1.267	1.441	1.644	1.893	1.972	2.059
0.85	1.199	1.319	1.469	1.715	1.962	2.068	2.220
0.9	1.242	1.379	1.528	1.764	2.012	2.047	2.305
0.95	1.289	1.335	1.619	1.812	2.104	2.214	2.363
1	1.309	1.428	1.640	1.860	2.140	2.196	2.387

Table 3: Floored by one ratio of Hull-White CVA wrong way risk to Independent CVA dependence on maturity of the cross currency swap and correlation between hazard rate and present value of the trade.

Results of Ruiz et al.'s model can be seen in table 4. The results have the same pattern on increasing by maturity and correlation. The wrong way risk effect is lower due to the reason that Ruiz model captures influence of global market factor, whereas Hull-White model also captures idiosyncratic risk of particular counterparty.

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Marturity / Correlation	4	5	6	7	8	9	10
0	1.010	0.989	0.985	0.995	0.996	1.008	1.010
-0.05	1.012	0.987	0.999	0.996	0.997	0.995	1.005
-0.1	1.015	0.982	1.000	1.005	1.004	1.006	1.022
-0.15	1.023	0.992	1.006	1.014	1.017	1.027	1.041
-0.2	1.024	0.993	1.019	1.031	1.030	1.050	1.077
-0.25	1.027	0.997	1.021	1.041	1.046	1.073	1.103
-0.3	1.032	1.012	1.040	1.054	1.076	1.105	1.137
-0.35	1.040	1.032	1.055	1.080	1.098	1.127	1.166
-0.4	1.055	1.033	1.078	1.104	1.127	1.170	1.207
-0.45	1.056	1.050	1.097	1.115	1.157	1.207	1.242
-0.5	1.067	1.048	1.101	1.146	1.186	1.235	1.272
-0.55	1.085	1.056	1.112	1.183	1.215	1.265	1.312
-0.6	1.084	1.092	1.155	1.182	1.234	1.292	1.362
-0.65	1.096	1.092	1.176	1.216	1.243	1.319	1.369
-0.7	1.108	1.111	1.181	1.217	1.283	1.355	1.403
-0.75	1.121	1.133	1.207	1.252	1.297	1.375	1.454
-0.8	1.130	1.156	1.212	1.289	1.331	1.400	1.485
-0.85	1.150	1.152	1.267	1.321	1.378	1.438	1.474
-0.9	1.156	1.169	1.222	1.346	1.392	1.474	1.561
-0.95	1.157	1.204	1.258	1.327	1.402	1.499	1.619
-1	1.177	1.194	1.273	1.331	1.426	1.521	1.600

Table 4: Dependence ratio of Ruiz et al.'s CVA wrong way risk to Independent CVA on maturity of the cross currency swap and correlation between hazard rate and present value of the trade.

We can see the non-FX wrong way risk effect, we take the difference of the Hull-White and Ruiz et al.'s ratios (see table 5). Increase in maturity of the swap increases non-FX wrong way risk CVA impact of this trade. As present value of cross currency swap is dependent on interest rates these increases through maturity can be explained by influence of interest rates and idiosyncratic risk of the trade to hazard rate. Non-FX wrong way risk CVA impact increase on maturity will

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be influenced by increase in volatility of simulated forward rate on a longer time horizon.

Correlation of hazard rate and RUB risk free interest rate is 0.866576.

Correlation of hazard rate and USD risk free interest rate is 0.033727.

Maturity	4	5	6	7	8	9	10
Difference of Hull-White to Ruiz's model	0.097	0.1302	0.2496	0.4649	0.617	0.6377	0.759

Table 5: Difference between the ratios of Hull-White CVA wrong way risk and Ruiz et al.'s model's CVA.

Overall, in this paper we showed the performance Hull-White wrong way risk CVA and Ruiz's et al.'s models calibrated on the Russian Financial Crisis period. The effect of CVA wrong way risk was considered on cross currency swap with four up to ten year maturities. On the stressed calibration of these two models Hull-White wrong way risk model always outperform Ruiz et al.'s model. Ruiz et al.'s model captures positive correlation of probability of default and exposure to the counterparty through one global market factor, which is foreign exchange rate. Hull-White wrong way risk model captures positive correlation of probability of default and exposure directly through present value, which imply capturing positive correlation through interest rates (other market factors) and idiosyncratic risk of the trade.

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8. Appendix

8.1 Simulation of correlated normally distributed variables.

For every N×N real symmetric matrix, the eigenvalues are real and the eigenvectors can be chosen such that they are orthogonal to each other. Thus a real symmetric matrix A can be decomposed as $A = Q\Lambda Q^T$, where Q is an orthogonal matrix whose columns are the eigenvectors of A, and Λ is a diagonal matrix whose entries are the eigenvalues of A.

For simulation of correlated Normal distributed variables, correlation matrix is used.

Correlated random variables calculated this way: $dW = Q\sqrt{\Lambda} * N$, where N is 3x[number_of_time_points] standard normally distributed uncorrelated random numbers.

8.2 Closed form solution to Vasicek model

Derivation of closed form solution to Vasicek model is shown below.

$$dr_t = \lambda * (\mu - r_t) * dt + \sigma * dW_t$$

Let's multiply both sides by $e^{\lambda * t}$:

$$e^{\lambda * t} * dr_t + \lambda * r_t * e^{\lambda * t} * dt = \lambda * \mu * e^{\lambda * t} * dt + \sigma * e^{\lambda * t} * dW_t$$

$$d(e^{\lambda * t} r_t) = \lambda * \mu * e^{\lambda * t} * dt + \sigma * e^{\lambda * t} * dW_t$$

$$e^{\lambda * t} r_t = r(0) + \lambda * \mu * \int_0^t e^{\lambda * s} * ds + \sigma * \int_0^t e^{\lambda * s} * dW_s$$

$$r_t = r(0) * e^{-\lambda * t} + \lambda * \mu * \int_0^t e^{-\lambda * (t-s)} * ds + \sigma * \int_0^t e^{-\lambda * (t-s)} * dW_s$$

$$E[r(t)] = r(0) * e^{-\lambda * t} + \mu * (1 - e^{-\lambda * t})$$

$$Variance[r(t)] = \sigma^2 * e^{-2\lambda * t} * \int_0^t e^{2\lambda * s} * dW_s = \frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda * t})$$

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8.3 Calibration of Vasicek parameters.

For OLS above could be performed as $Y = B * X + A + \varepsilon_t$,

Where $A = \mu * (1 - e^{-\lambda*\Delta})$, $B = e^{-\lambda*\Delta}$, $\varepsilon_t = \sigma * \sqrt{\frac{1-e^{-2\lambda*\Delta}}{2\lambda}} * \varepsilon_t$, $Y = r(t + \Delta)$, $X = r(t)$

Further for simplicity we can take intermediary variables a, b, sd

a <- $(n * \sum xy - \sum x * \sum y) / (n * \sum xx - (\sum x)^2)$

b <- $(\sum y - a * \sum x) / n$

sd <- sqrt$((n * \sum yy - (\sum y)^2 - a * (n * \sum xy - \sum x * \sum y)) / (n * (n - 2)))$

To get λ, μ, σ

λ <- -log(a)/ dt

μ <- b/(1-a)

σ <- sd * sqrt(-2*log(a)/ dt /(1-a^2))

In this calibration we imply that today's instantaneous rate for tomorrow is the same as tomorrow's instantaneous rate.

8.4 Derivation of discounting factor.

Let's introduce discount process $D(t) = e^{-\int_0^t r(s)ds}$

Then $dD(t) = -r(t)D(t)dt$. For no arbitrage condition martingale property should hold for discounted value of the bond. Using first Ito product rule, then Ito-Doeblin formula, we get the following:

$$d(D(t) * P(t, r(t), T)) = P(t, r(t), T) * dD(t) + D(t) * dP(t, r(t), T)$$

$$= D(t)\left[-r * P * dt + P_t * dt + P_r * dr + \frac{1}{2} * P_{rr} * dr * dr\right]$$

$$= D(t)\left[-r * P + P_t + \frac{1}{2} * P_{rr} * \sigma^2 + P_r * \lambda * (\mu - r)\right] * dt + D(t)\sigma dW$$

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For martingale property holds term under dt should be equal to 0.

$$r * P = P_t + \frac{1}{2} * P_{rr} * \sigma^2 + P_r * \lambda * (\mu - r)$$

With final time condition: $P(T, r(T), T) = 1$. This partial differential equation can be derived by simply using discounted Feynman-Kac theorem.

The solution for this partial differential equation should be the form:

$$P(t, r, T) = A(t, T) * e^{-B(t,T)*r}$$

With final values $(T, T) = 1$, $B(T, T) = 0$.

$$r * A * e^{-B(t,T)*r}$$

$$= A_t * e^{-B(t,T)*r} - A * B_t * r * e^{-B(t,T)*r} - A * B * \lambda * (\mu - r) * e^{-B(t,T)*r}$$

$$+ \frac{1}{2} * A * B^2 * \sigma^2 * e^{-B(t,T)*r}$$

Term under r should be equal to zero, which gives two equations:

$$B * \lambda - B_t - 1 = 0$$

$$A_t - A * B * \lambda * \mu + \frac{1}{2} * A * B^2 * \sigma^2 = 0$$

From these two equations we derive solutions to $A(t, T)$, $B(t, T)$.

$$A(t, T) = \exp\{\left(\mu - \frac{\sigma^2}{2*\lambda^2}\right) * [B(t, T) - T + t] - \frac{\sigma^2}{4*\lambda} * B(t, T)^2\}$$

$$B(t, T) = \frac{1}{\lambda}[1 - \exp(-\lambda * (T - t))]$$

8.5 Hazard rate simplified form

To derive this formula, we should remind that present value of CDS at the initiation is 0, which is:

$$PV_{premium}(t, T) + PV_{default}(t, T) = 0$$

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$$PV_{premium}(t,T) \cong CDS_{spread} * \int_t^T B(t,u)S(t,u)du$$

$$PV_{default}(t,T) = -(1 - recovery_{rate}) * h * \int_t^T B(t,u)S(t,u)du$$

Where $B(t,u)$, $S(t,u)$ are price of the bond and survival probability of the counterparty, respectively. Solving this assuming hazard rate has flat term structure, we get: $h(t,T) = CDS_{5y}/(1 - Recovery\_rate)$.

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