A MODEL OF PRE-EMPTION OVER THE BUSINESS CYCLE

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Abstract

I present a model of pre-emptive collusion over the business cycle. By introducing competitive fringe into Markov-driven demand cycles, I characterize pre-emptive and non-pre-emptive equilibria. I show that under pro-cyclical, most-collusive prices, colluding firms can deter entry from less efficient firms in the boom periods. I also show that pre-emptive strategies cannot constitute equilibria when there are less than two incumbent firms. This further means that imperfectly colluding firms have better deterrence capabilities than oligopolies. The reason is that that pre-emption can only work through trigger strategies, a tool that is not credible for an oligopoly.

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1 INTRODUCTION

At the heart of collusion lies a trade-off, while a higher collusive price leads to higher potential profits, it also incentivizes cheating, weakening the incentive to collude. But if firms are patient enough, any level of collusion is possible as long as a one time gain from cheating is less than the punishment it would incur[1]. However, as we restrict the analysis to less than perfectly patient firms, perfect collusion becomes harder to maintain which results in profits lower than what the firms would earn if they formed an oligopoly[2]. This is especially true around the business cycle. Holding discount factor constant, introducing a cyclical demand can further reduce collusive profits.

Therefore, at first glance, it seems that staying in imperfect collusion and being unable to form an oligopoly only hurts the firms. However, in this paper, I show that in a business cycle setting, imperfect collusion allows the colluding firms to deter entry from outsider competition. This pre-emption is only possible through trigger strategies, a mechanism that is only credible under imperfect collusion.

This paper is tied to two related but distinct pieces of literature: pre-emption and collusion over the business cycle. The consensus in the literature on collusion over the business cycle is that the most collusive prices, depending on the parameters of the business cycle and some specific parameters, can be pro-cyclical or counter-cyclical (Knittel et al., 2010). But it is certain that the business cycle does necessarily affect collusive profits regardless of the direction of cyclicality (Bagwel and Staiger, 1997).

The literature on pre-emptive collusion mostly assumes that pre-emption is possible under some specific conditions. One instance is when the markets are not fully contestable; contestability is defined by the marginal cost advantage incumbent firms have over the entrants, high set-up costs, low scrap value, etc.. Klemperer (1987a,1987b,1989) shows that in low contestable markets if the switching costs incurred by the consumers are high enough, incumbent firms can deter entry. Bernheim (1984) illustrates that in-

[1]This concept is better known as the Incentive Compatibility (IC).

[2]In this paper with "Oligopoly" I am referring to a collusion of firms that behaves like a monopoly; an oligopoly maximizes its profits while ignoring collusive incentive compatibility constraints. In contrast, unlike an oligopoly, an imperfect collusion has to satisfy collusive incentive compatibility constraints.

cumbent firms can further decrease contestability by increasing sunk costs[3] of entry via advertisement. Another instance of pre-emption in low contestable markets is in a natural monopoly setting. When a first mover "installs durable capacity" to acquire the first-movers advantage, it can deter entry, given it captures all economies of scale (Spence 1977, 1979). Overall the literature of preemption[4], mostly assumes that preemption is only possible when the contestability of the market is low.

But as the degree of contestability rises (i.e as setup costs decrease, scrap value increase and marginal costs become less heterogeneous), since firms are assumed to be forward-looking, pre-emption becomes harder and impossible at fully contestable markets. Baumol et al. (1982) show that in fully contestable markets monopolistic behavior is impossible. The reason being that no strategy that leads to pre-emption is credible and can constitute a subgame-equilibrium. Therefore, pre-emptive strategies are only credible in markets with low contestability, at least in the *static* setting. More importantly, most of the literature search for pre-emptive strategies in static settings.

But modeling demand in a static setting is unrealistic since the real world has a business cycle. Moreover, introducing a cyclical demand may also increase colluding firms' pre-emptive ability. Specifically, I show that if a cyclical demand induces cyclicality in equilibrium prices, pre-emptive trigger strategies may arise with which the colluding firms can deter entry to high-cost firms.

To that end, I introduce a competitive fringe into an industry that has cyclical demand. I adopt a version of the business cycle model developed by Hamilton(1989), which is characterized by a Markov process that transitions from boom(recession) to recession(boom) with exogenous parameters. While in the real world, the business cycle is much more unpredictable, modeling the demand with a Markov process allows cyclicality and unpredictability while keeping the analysis simple.

The paper is organized as follows. Section 2 introduces the model, within which, the cyclical demand, colluding firms and the fringe firms are established. Section 3 identifies the non-pre emptive equilibrium, in which the colluding firms do not deter

[3]A sunk entry cost is an entry cost that cannot be recovered via scrap value in the exit.

[4]See Wilson (1992) for a more comprehensive literature review on preemption.

entry and fringe firms enter the market if they are profitable. Section 4 characterizes the pre-emptive trigger strategies that constitute an equilibrium in which the fringe firms that would be profitable under non-pre-emptive strategies find it impossible to enter the industry. I show that under the highest degree of pre-emption, fringe firms that would be profitable in booms but not in recessions can be deterred out of the market.

2 THE MODEL

2.1 Basics

I model an infinitely played Bertrand-pricing complete information game where $n \geq 2$ number of *colluding* firms and n_t number of fringe firms sell some homogeneous non-durable in each period $t \in \{0, 1, 2, \dots\}$. The colluding firms make pricing decisions while the fringe firms make entry/exit decisions. Fringe firms that are active in the market always sell one unit and the colluding firms satisfy the residual demand. The demand is driven by a business cycle and it can be in either a boom period or a recession period where the transition between periods is dictated by a Markov switch process. Both types of firms sell at the industry price which is determined by the lowest price set by the colluding firms.

2.2 Demand

Demand at time t is given by D_t. Market demand level can be either in a high demand period, called the boom, or a low demand period, called the recession period. The cyclical demand is governed with the following equation:

$$D_t = g_t D(p_t)$$

where $g_t = b$ in boom periods and $g_t = r$ in recessions. The transition between the two demand levels is determined by a Markov switching process according to following probabilities:

- $\rho = Prob(g_t = r | g_{t-1} = b) \in [0, 1]$

- $\alpha = Prob(g_t = b | g_{t-1} = r) \in [0, 1]$

where ρ (α) is the probability of switching from boom (recession) to recession (boom) and accordingly $1 - \rho$ ($1 - \alpha$) is the probability of staying in boom (recession) given previous period was a boom (recession) period. The Markov transition matrix for the demand level can also be easily visualized in table 1. Naturally, I assume that $b > r > 0$, boom demand level is higher than the recession demand level and demand is always positive.

TABLE 1. **Markov Transition Matrix for the Demand Level**

		Into State	
		Boom	Recession
From State	Boom	$1 - \rho$	ρ
	Recession	α	$1 - \alpha$

Notice that the Demand has no growth component, it merely changes between high demand and low demand periods. While it would be more realistic to model the business cycle with a growth component, for instance with $D_{t+1} = g_t D_t$, it would also require that the most collusive equilibrium prices grow over time. I discuss this further in the last section.

2.3 Firms

2.3.1 Colluding Firms

$n \geq 2$ *Colluding* firms indexed by $i = \{0, 1, 2, \dots\}$ with marginal cost normalized to 0, are playing an infinite Nash-Bertrand pricing game. The colluding firms are forward-looking with discount factor $\delta \in [0, 1]$. At the start of every period, firm i sets its price and if its price is amongst the lowest, when the price level is p^*, the number of firms that set the lowest price is n and there are n_t active fringe in the market, the firm's profit $\pi_{i,t}$ at time t is given by:

$$\pi_{i,t}(p^*, n_t) = \frac{(D_t(p^*) - n_t)p^*}{n}$$

that is, the colluding firms fully serve the residual demand and equally share the profits.

Firm $i's$ strategy for period t, $\sigma_{i,t}$, maps from the set of all possible past price levels, all past demand levels, all past possible number of active fringe firms and the current demand level into a set of possible price levels for the period t. Firm i chooses its strategy $\sigma_i = \{\sigma_{i,0}, \sigma_{i,1}, \sigma_{i,2}, \dots\}$ that maximizes its discounted lifetime profits according to given business cycle behavior, strategies of the colluding firms, number of colluding firms, and the fringe firm characteristics which is discussed in the following section.

2.3.2 Fringe Firms

Fringe firms, indexed by $j = [0, Z]$ are a *continuum* of small firms that are heterogeneous in their marginal costs, c_j. At the beginning of the game, set of fringe firms is drawn from some function F. Function F determines the distribution of fringe firm marginal costs with support on $[0, Z]$, where Z is some big number, meaning there can exist fringe firms with 0 marginal costs. I assume nothing further about the fringe firm distribution. The set of fringe firms are drawn once and remain the same throughout the game and is observed by every firm at the start. The fringe firms are forward-looking with the common discount factor $\delta \in [0, 1]$.

The fringe firms that are out of the market are the *inactive* fringe, and those that are in the market are called the *active* fringe. If after observing all past prices, all past demand levels, the current market price and the current market demand level, an inactive fringe decides to enter the industry, it pays a setup cost of $\zeta > 0$ and then sells one unit at the industry price becoming active fringe. If an active fringe after observing the aforementioned variables decides to exit the industry, it receives a scrap value (or exit value) $\eta > 0$ and leaves the market becoming inactive. It is important to note at this point that if a fringe firm sells it can only exit the industry in the next period.

Fringe firms are price takers since they are capacity constrained. As they are able to sell only one unit, selling at a price higher than the market price would result in non-positive profits as the colluding firms do not have any capacity constraints and buyers would opt for the colluding firms' lower market price.

Since fringe firms are price takers, an inactive (active) fringe firm's strategy $\tau_{i,t}$, maps from the set of all possible past market demand levels, all past price levels set by the colluding firms, the current price levels of the colluding firms, the current market demand level[5] and colluding firms' strategies into a binary decision: {Enter(Stay), Out(Leave)}. As the price takers, fringe firm's objective is to choose its strategy $\tau_j = \{\tau_{j,0}, \tau_{j,1}, \tau_{j,2}, \dots\}$ that maximizes its lifetime discounted profits.

2.4 Timeline

For simplicity, I assume the game begins in a boom period and the timeline of the game is as follows:

1. Both fringe and colluding firms observe the current market demand level and whether the business cycle is in the boom or recession period.
2. Each colluding firm i sets its price $P_i \geq 0$.
3. Both the fringe and the colluding firms observe all P_i and the lowest P_i becomes the industry price.
4. Simultaneously, each outside fringe firm decides whether to enter or not and each active fringe firm decides whether to exit or not, inactive fringe firms that enter become active fringe and vice-versa.
5. Each active fringe sells one unit of the good at the industry price.
6. Firms whose price were among the lowest satisfy the residual demand, equally sharing the profits.

2.5 Equilibrium Concept

In this section and the following subsections, I define the equilibrium concept, the incentive constraints of the colluding firms and the firm dynamics on the equilibrium path for both firm types.

[5] Although the current market demand level does not affect per-period profits of the fringe, the reason for its inclusion will become clear when I characterize the pre-emptive equilibrium.

2.5.1 Basics and assumptions

Equilibrium is a set of strategies that constitute a subgame perfect Nash equilibrium. Consequently, in sections 3 and 4 I only consider the strategies that survive the one-shot deviation principle; The one-shot deviation principle dictates that, since there are two possible period types (boom and recession) if a firm under the given strategy profile does not find it profitable to deviate in the first boom and recession periods, then it will not find it profitable to deviate in any following periods. In other words, if a firm in any period $t > 2$ when the demand level is $g \in \{b, r\}$ finds it profitable to deviate then it should have deviated at the first period of when g starts. Following this logic, we only need to check for the first deviations in each period type[6]. Without loss of generality and for the sake of simplicity, I also restrict the equilibria to symmetric strategies that constitute collusive equilibria.

It is clear that due to the complete information nature of the game, on the equilibrium path deviation is impossible; Because on the equilibrium path incentive compatibility constraints must hold and a profitable deviation would be a violation of these constraints. Intuitively, since the colluding firms are identical, no firm would set a price so high that would make cheating profitable since they know doing so would be met with a deviation.

Proposition 1 *Off-equilibrium path, if the prices are such that starting from the next period perpetual price war will begin, no inactive fringe firm enters the market and all active fringe firms leave the market. (Proof: see Appendix A.1.)*

Intuitively, because the marginal prices of the colluding firms are normalized to zero, the Nash-Bertrand equilibrium price of the market is 0. Although the distribution of the fringe firms, F, allows for fringe firms with 0 marginal costs, entering (for inactive fringe) as well as not leaving (for the active fringe) the market produces a discounted profit of 0, whereas the scrap value and the setup costs are strictly positive $\eta, \zeta > 0$, meaning no inactive firm will enter and all active firms exit the market upon observing prices

[6] See Tirole and Fudenberg (1991) ch 4.2 for a better explanation of the principle, which they refer to as "one-stage deviation".

that signal an upcoming everlasting price war. In sections 3 and 4, I further assume, for simpler analysis, that the scrap value equals setup cost, $\zeta = \eta$.

Notice that in any demand level $g_t \in \{b, r\}$, in equilibrium, there can only be one most collusive price level, p_g^*. This means that, within a given demand level, the price level stays static for all periods until there is a switch in the demand level. And as the timeline of the game progresses, the only variable exogenously changing is the current market demand level between recession and boom levels. Although there may exist many equilibria in which the industry prices change within given periods, since I am after most collusive prices, in sections 3 and 4, I restrict the equilbria of interest to those with the static price level, $p_t \in \{p_b^*, p_r^*\}$:

$$p_t = \begin{cases} p_b^* & when\ g_t = b \\ p_r^* & when\ g_t = r \end{cases}$$

That is, on the equilibrium path, the equilibrium price at any period t equals either the static most collusive boom price or the static most collusive recession price. To further simplify the equilibria of interest, notice that since *on the equilibrium path* there are two possible price levels, $p_t \in \{p_b^*, p_r^*\}$, the number of active fringe can only take on two *static* values, $n_t \in \{n_t^b, n_t^r\}$, as the state of the demand switches between recession and boom. This simplification allows for easier analysis since in any equilibrium the colluding firms have two choice variables, the static boom price and the static recession price. Further, since the mass of active fringe firms in the market, on the equilibrium path, is a function of the current price level, at any point in time, in any non-pre-emptive equilibrium, the mass of active fringe firms, n_t is given by:

$$n_t = \begin{cases} n_t^b & when\ g_t = b \\ n_t^r & when\ g_t = r \end{cases}$$

This means that, for given equilibrium static price levels, $p_t \in \{p_b^*, p_r^*\}$, each fringe firm decides to be in the market or out of the market in respective boom and recession periods.

A very important component of the model that enables pre-emptive equilibria is the cyclicality of most collusive prices and hence, the cyclicality of the collusive ability. To recover the parameters that enable cyclicality, notice that the collusive ability in period t depends on colluding firms' expectations about the market demand level in the following periods. That is, an equilibrium with procyclical, most collusive prices requires that the expectations about future market demand levels are higher in booms relative to recessions. Therefore, to determine the variables necessary for such a specification one must compare expected market demand levels in booms with that of recessions[7]. I start by comparing expected next period demand level, given the system is in a boom period, with that of the recession period:

$$
\begin{aligned}
&E(g_{t+1}D_t(p^*)|g_t = b) - E(g_{t+1}D_t(p^*)|g_t = r) \\
=&[E(g_{t+1}|g_t = b) - E(g_{t+1}|g_t = r)]D_t(p^*) \\
=&(1 - \rho - \alpha)(b - r)D_t(p^*)
\end{aligned}
$$

Since $(b - r) > 0$, expected market demand level for subsequent periods in booms are higher than in recessions iff $(1 - \rho - \alpha) > 0$. Intuitively, if the probabilities were such that $(1-\rho-\alpha) = 0$, then the expected market demand level would be independent of the state firms are in, $E(g_{t+1}D_t(p^*)|g_t = b) - E(g_{t+1}D_t(p^*)|g_t = r) = 0$, which would imply no cyclicality in prices and collusive ability. But as the probability of switching from one demand level state to the other decreases to a small enough range ($i.e : \alpha, \rho << 0$), since it becomes more likely to stay in the given state, it becomes easier to collude in booms compared to recessions[8]. Hence, the assumption 1 ensures cyclicality in most collusive prices and collusive ability.

Assumption 1 $\alpha + \rho < 1$

I also assume that the probability of switching from boom to recession is higher than vice versa. This is a simplifying assumption, it allows me to ignore some of the IC constraints, which I discuss in the next section.

[7] To do so, I follow Bagwell and Staiger's (1995) formulation.

[8] See Bagwell and Staiger (1995) for a detailed discussion on how different probability levels may induce negative, positive and no cyclicality in most collusive prices.

Assumption 2 $\alpha > \rho$

I make three further simplifying assumptions for sections 3 and 4; I assume that the residual demand is no less than one for all periods and demand levels.

Assumption 3 $D(p_g) - n_t^g > 1, \ \forall t, g \in \{b, r\}$

This assumption overrules the case where the active fringe can grow large enough to leave nothing for the colluding firms, driving down the residual demand to 0. It also ensures that the mass of active fringe firms is less than demand, $D(p) < n_t$.

I also assume that the profits from residual demand are concave. This assumption ensures that the colluding firms solve for most collusive price levels. It is also needed for simpler first-order conditions, as without the concavity assumptions, IC constraints need not necessarily hold with equality.

Assumption 4 $\frac{\partial}{\partial p_g}[(D_t(p_g) - n_t^g)p_g] > 0, \frac{\partial^2}{\partial p_g \partial p_g}[(D_t(p_g) - n_t^g)p_g] < 0, \ \forall p \geq 0, g \in \{b, r\}$

I further assume that the scrap (exit) value to a fringe firm is zero if the industry is in a perpetual price war, where the market price equals zero. This assumption ensures that an inactive fringe would not enter the market if its entry would cause a perpetual price war. It will become further clear in the pre-emptive equilibrium section why this assumption is necessary.

Assumption 5 $\zeta = 0 \ \textit{if} \ p^* = 0$

Lastly, I assume that there are only two colluding firms and the entry costs equal exit scrap value. Having two colluding firms is a trivial assumption, it only serves to simplify the analysis. Setting entry costs to exit value also means that there are no sunk entry costs.

Assumption 6 n*=2,* $\zeta = \eta$

2.5.2 Colluding Firm Dynamics and Incentive Constraints

In any collusive equilibrium, the main equations of interest that one needs to keep track of are the incentive compatibility constraints, since if the collusive price is set too high then the incentive to undercut the opponents might out-weight the potential losses to the perpetual price war that would follow. Before describing the incentive constraints, I first characterize the value functions for the colluding firms. Let $V_{g,i}(p^*)$ denote firm $i's$ value function at the demand level-state $g \in \{b, r\}$ when the on-equilibrium path price level is given by $p^* = \{p^*_b, p^*_r\}$ where p^*_b and p^*_r corresponds to on-path boom and recession static price levels. Then boom and recession value functions for the colluding firms at the equilibrium price p^* can be shown as:

$$\begin{aligned} V_{b,i}(p^*) &= \pi(b, p^*, n_t^b) + \delta[(1-\rho)bV_{b,i}(p^*) + \rho r V_{r,i}(p^*)] \\ V_{r,i}(p^*) &= \pi(r, p^*, n_t^r) + \delta[(1-\alpha)rV_{r,i}(p^*) + \alpha b V_{b,i}(p^*)] \end{aligned} \tag{1}$$

where the n_t^g in the profit functions corresponds to the mass of active fringe in state $g \in \{b, r\}$. Equations (1) can also be shown as:

$$\begin{aligned} V_{b,i} &= \frac{\pi(b, p^*, n_t^b) + \delta\rho r V_{r,i}(p^*)}{1-\delta(1-\rho)b} \\ V_{r,i} &= \frac{\pi(r, p^*, n_t^r) + \delta\alpha b V_{b,i}(p^*)}{1-\delta(1-\alpha)r} \end{aligned}$$

substitute into each other, to express in profit levels, to get:

$$\begin{aligned} V_{b,i} &= \frac{(1-\delta(1-\alpha)r)\pi(b, p^*, n_t^b) + \delta\rho r\pi(r, p^*, n_t^r)}{A} \\ V_{r,i} &= \frac{(1-\delta(1-\rho)b)\pi(r, p^*, n_t^r) + \delta\alpha b\pi(b, p^*, n_t^b)}{A} \\ &\text{where } A = (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha\rho b r \end{aligned} \tag{2}$$

With the value functions expressed in profit levels, one can think about the incentive compatibility constraints. In any collusive equilibrium, we need *at least* 2 incentive compatibility (from now on *IC*) constraints satisfied; the firms should neither find it profitable to deviate in booms (IC 1) nor in (IC 2) recessions. To characterize IC 1 and IC 2, one must compare the in-period gains from deviation to discounted foregone profits. Recall that the profit function of a colluding firm at time t and demand level $g \in \{b, r\}$, under collusion, is defined by $\pi_{i,t}(p_g^*, n_t) = \frac{(D_t(p_g^*)-n_t)p^*}{n}$. Deviating means the cheating firm undercuts the other firms slightly and gets to serve the *whole* demand. Since deviation induces a perpetual price war, all the fringe firms, observing the failure to collude, leave the market, by proposition 1. Therefore, the cheating firm not only captures the share of other colluding firms but also shares of the fringe firms as well, hence it gets to serve the whole demand. So the one time in-period gain from deviation is given by $D_t(p_b^*)p_b^*$ in booms and $D_t(p_r^*)p_r^*$ in recessions. Deviating induces a perpetual price war for all subsequent periods which sets the price to Nash-Bertrand competition price. Since the marginal costs of the colluding firms are zero, at the Nash-Bertrand competition price, all firms earn 0 profit during the price war. This means, from the next period the deviating firm's expected net foregone profits are $V_b(p^*)$ in booms and $V_r(p^*)$ in recessions. Therefore, the two incentive compatibility constraints can be written as:

$$\begin{aligned} \text{IC 1}: \; D_t(p_b^*)p_b^* &\leq \delta[(1-\rho)bV_b(p^*) + \rho rV_r(p^*)] \\ \text{IC 2}: \; D_t(p_r^*)p_r^* &\leq \delta[(1-\alpha)rV_r(p^*) + \alpha bV_b(p^*)] \end{aligned} \tag{3}$$

2.5.3 Fringe Firm Dynamics

Not only are fringe firms price takers, but due to the nature of the game, they also play no role at the equilibrium. Since fringe firms make their decisions *after* the colluding firms set prices, and since inactive (active) fringe firms' binary decisions {In(Stay), Out(Leave)} are a function of the prices, fringe firm behavior depends entirely on the exogenous parameters and the colluding firm prices, which are exogenous to the fringe firms. That said, since the mass of fringe firms is a decision variable for the colluding firms[9]. I define the fringe firm Value functions in this section.

Let $V_{j,g}^F(p^*)$ denote the Value function of the fringe firm j in demand level g when the equilibrium price level is $p^* = \{p_b^*, p_r^*\}$. Then value to a fringe firm of both types is simply the profit from selling one unit at the emerging industry price plus the discounted continuation value and the exit value:

[9] As the per-period profit is a function of mass of active fringe, $\pi_{i,t}(p_g^*, n_t) = \frac{(D_t(p_g^*)-n_t)p^*}{n}$.

$$\begin{aligned}
V_{j,b}^F(p^*) &= p_b^* - c_j + \delta[(1-\rho)V_{j,b}^F(p^*) + \rho V_{j,r}^F(p^*)] + \eta \\
&= \frac{p_b^* - c_i + \delta\rho V_{j,r}^F(p^*) + \eta}{1-\delta(1-\rho)} \\
V_{j,r}^F(p^*) &= p_r^* - c_j + \delta[(1-\alpha)V_{j,r}^F(p^*) + \alpha V_{j,b}^F(p^*)] + \eta \\
&= \frac{p_r^* - c_i + \delta\alpha V_{j,b}^F(p^*) + \eta}{1-\delta(1-\alpha)}
\end{aligned}$$

substitute into each other, to express in exogenous terms, to get:

$$\begin{aligned}
V_{j,b}^F(p^*) &= \frac{[1-\delta(1-\alpha)](p_b^* - c_i + \eta) + \delta\rho(p_r^* - c_i + \eta)}{B} \\
V_{j,r}^F(p^*) &= \frac{[1-\delta(1-\rho)](p_r^* - c_i + \eta) + \delta\alpha(p_b^* - c_i + \eta)}{B} \qquad (4) \\
&\text{where } B = (1-\delta(1-\alpha))(1-\delta(1-\rho)) - \delta^2\alpha\rho
\end{aligned}$$

3 NON-PRE-EMPTIVE EQUILIBRIUM

Definition 3.1 *Non-Pre-Emptive Equilibrium: An equilibrium is non-pre-emptive if the strategies that constitute the equilibrium are such that, an entry decision of a fringe firm at time t does not affect the price level in subsequent periods.*

In other words, the strategies are such that they create a price level $p^* = \{p_b^*, p_r^*\}$ and price level in period $t+1$ does not depend on the number of entries in period t, but rather only on the current market demand level g_t. This also means the number of fringe firm entries does not affect the profitability of fringe firms in the following periods.

Let $p^* = \{p_b^*, p_r^*\}$ be the most collusive price level. Since the game starts in a boom period, firm i solves[10]:

$$\begin{aligned}
&\max_{p^*=\{p_b^*,p_r^*\}} \quad V_b(p^*) \\
&\text{s.t.} \quad n_t^g = F(V_g^F(p^*)) \\
&\text{IC } 1: \; D_t(p_b^*)p_b^* \leq \delta[(1-\rho)bV_b(p^*) + \rho r V_r(p^*)] \\
&\text{IC } 2: \; D_t(p_r^*)p_r^* \leq \delta[(1-\alpha)rV_r(p^*) + \alpha b V_b(p^*)]
\end{aligned}$$

[10] I omit the index subscript for the Value functions from now on

Proposition 2 *Under Assumption 1, in any most collusive equilibrium, IC 1 holds with equality:*

$$\text{IC 1}: \; D_t(p_b^*)p_b^* = \delta[(1-\rho)bV_b(p^*) + \rho r V_r(p^*)]$$

(Proof: see Appendix A.2.).

Intuitively, since increasing prices lead to higher profits, colluding firms increase the prices up to the point where they become indifferent between collusion and cheating.

Proposition 3 *Under assumption 1 and assumption 2, IC 1 implies IC 2 (Proof: see Appendix A.3.)*

With IC 1 implying IC 2, since I assume the system starts in a boom period, if a firm would find it profitable to deviate in the recession period, it would deviate in the first period. Therefore we only need the first incentive compatibility constraint to hold and can safely ignore the second constraint; if in a boom period a firm finds it not profitable to deviate, it is even less profitable to deviate in recessions. The Lagrangian and its first derivatives can be expressed as:

$$\mathcal{L} = V_b(p^*) + \mu[(1-\rho)bV_b(p^*) + \rho r V_r(p^*) - \frac{1}{\delta}D_t(p_b^*)p_b^*)]$$

First-Order Conditions:

$$\frac{\partial \mathcal{L}}{\partial p_b^*}: \; \frac{\partial V_b'(p^*)}{\partial p_b^*} + \mu[(1-\rho)b\frac{\partial V_b'(p^*)}{\partial p_b^*} + \rho r\frac{\partial V_r'(p^*)}{\partial p_b^*} - \frac{1}{\delta}D_t(p_b^*) - \frac{1}{\delta}p_b^* D_t'(p_b^*)] = 0$$
$$\frac{\partial \mathcal{L}}{\partial p_r^*}: \; \frac{\partial V_b'(p^*)}{\partial p_r^*} + \mu[(1-\rho)b\frac{\partial V_b'(p^*)}{\partial p_r^*} + \rho r\frac{\partial V_r'(p^*)}{\partial p_r^*}] = 0$$

Plugging in the partial derivatives of the Value functions, which are derived in Appendix B, I find the most collusive equilibrium boom price (p_b^*) is given by:

$$p_b^* = \frac{1}{C}\Bigg[(1-\mu(1-\rho)b)\Bigg([1-\delta(1-\rho)r](D_t(p_b^*) - n_t^b(p^*)\Bigg)+$$
$$+\mu\rho r\Bigg((1-\delta(1-\alpha)b)(-\frac{\partial n_t^r(p^*)}{\partial p_b})\Bigg) + \delta\rho r[D_t(p_b^*) - n_t(p_b^*)] - 2AD_t(p_b^*)\Bigg]$$

where

$$\begin{aligned}
C &= (1-\mu(1-\rho)b)\Bigg[(1-\delta(1-\rho)r)[D_t'(p_b^*) - \frac{\partial D_t'(p_b^*)}{\partial p_b^*}] - \delta\alpha r\frac{\partial n_t^r(p^*)}{\partial p_b^*}\Bigg]+ \\
&+ \delta\mu\rho^2 r^2(D_t'(p_b^*) - \frac{\partial n_t^b(p^*)}{\partial p_b^*} - 2AD_t'(p_b^*) \\
A &= (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2 \\
\frac{\partial n_t^b(p^*)}{\partial p_b} &= \frac{1-\delta(1-\alpha)}{B}F'(V_{b,j}^F(p^*)) \\
\frac{\partial n_t^r(p^*)}{\partial p_b} &= \frac{\delta\alpha}{B}F'(V_{r,j}^F(p^*))
\end{aligned}$$

and the most collusive equilibrium price of the recession period, p_r^*, is given by:

$$p_r^* = \frac{1}{E}\Bigg[(1+\mu(1-\rho)b)\Bigg((1-\delta(1-\rho)r(-\frac{\partial n_t^b(p^*)}{\partial p_r^*}p_b^*) + \delta\alpha r[D_t(p_r^*) - n_t^r(p^*)]\Bigg]$$
$$+\mu\rho r\Bigg[\delta\rho r(-\frac{\partial n_t^b(p^*)}{\partial(p_r^*)}p_b^*) + (1-\delta(1-\alpha)b)[D_t(p_r^*) - n_t^r(p^*)]\Bigg]$$

where

$$\begin{aligned}
E =& (1+\mu(1-\rho)b)\Bigg[\delta\alpha r D_t'(p_r^*) - \frac{\partial n_t^r(p^*)}{\partial p_r^*}] + \mu\rho r[(1-\delta(1-\alpha)b)D_t'(p_r^*) - \frac{\partial n_t^r(p^*)}{\partial p_r^*}]\Bigg] \\
\frac{\partial n_t^b(p^*)}{\partial p_r} &= \frac{\delta\rho}{B}F'(V_{b,j}^F(p^*)) \\
\frac{\partial n_t^r(p^*)}{\partial p_r} &= \frac{1-\delta(1-\rho)}{B}F'(V_{r,j}^F(p^*))
\end{aligned}$$

4 PRE-EMPTIVE EQUILIBRIUM

Definition 4.1 *Pre-Emptive Equilibrium: An equilibrium is pre-emptive if the strategies that constitute the equilibrium are such that, an entry decision of a fringe firm at time t may change the price levels in subsequent periods.*

In non-pre-emptive equilibrium the dynamic price level $p^* = \{p_b^*, p_r^*\}$ alternates between boom and recession *optimal* most-collusive price levels and this switch is only governed by the current market demand level, g_t. This means fringe firms are free to enter and exit and they do so depending on profitability. However, in a pre-emptive equilibrium, the colluding firms can design trigger strategies so that some higher-cost fringe firms, even though they find it profitable to enter at the emerging price levels, cannot enter the market due to trigger strategies of the colluding firms.

Under assumption 1 and 2, boom price levels are higher than recession price levels, that means in non-pre-emptive equilibrium, in booms, there are more active fringe firms than in recessions since n_t^g is a function of price levels in $g_t \in \{b, r\}$. In recessions the mass of active fringe is given by n_t^r, which in booms is n_t^b, obviously $n_t^b > n_t^r$. Consider the fringe firms that are unprofitable in recession price levels, the mass of fringe firms in $n_t^b - n_t^r$ belong to this category. These firms can only be profitable under boom prices. Consequently, if the colluding firms were to play p_r^* in boom periods, then fringe firms belonging to this category $(n_t^b - n_t^r)$ would never enter the market, and the only fringe in the market would be those that belong to n_t^r. So one pre-emptive trigger strategy for the colluding firms is to play the most collusive boom and recession prices, $p^* = \{p_b^*, p_r^*\}$, if the only active fringe firms in the market are those that belong to n_t^r, or those are profitable in both recessions and booms, and play the recession price, p_r^*, in booms, if $n_t > n_t^r$; In other words, play the recession price in booms if the mass of fringe firms exceed n_t^r - the mass of fringe firms that are profitable in both period types.

Before designing the trigger strategies, let T_t denote the set of periods in which the current market demand level has been active. Since the game starts in a boom period, if we are at period $t = 5$ and market demand level has been in the boom cycle since the start, then $T_5 = \{0, 1, 2, 3, 4, 5\}$, that is, in period $t = 5$, T_5 holds the set of consecutive same-type demand levels. Then a trigger strategy that pre-empts the mass of fringe firms belonging to $n_t^b - n_t^*$ can be captured in a two price equilibrium in the following way. Let p^*, p_{pre}^* denote price levels, respectively, under on-equilibrium path where the fringe firms belonging to $n_t^b - n_t^*$ are successfully deterred out of the market and under the triggered pre-emption period, where mass of fringe firms exceed n_t^* in a boom period:

$$p_t = \begin{cases} p^* & if\ n_t \leq n_t^*,\ \forall t \in T_t \\ p_{pre}^* & if\ \exists t \in T_t \text{ s.t } n_t > n_t^* \end{cases}$$

where $p^* = \{p_b^*, p_r^*\}$ for $g_t = \{b, r\}$ and $p_{pre}^* = \{p_r^*, p_r^*\}$ for $g_t = \{b, r\}$. This can also be shown as:

$$p_t = \begin{cases} p_r^* & if\ g_t = r \\ p_b^* & if\ g_t = b \text{ and } n_t \leq n_t^*,\ \forall t \in T_t \\ p_{pre}^* & if\ g_t = b \text{ and } n_t > n_t^r,\ \forall t \in T_t \end{cases}$$

That is, the strategy of the colluding firms is to play p^* if the mass of active fringe does not exceed n_t^* and to play p_{pre}^* otherwise. Clearly, under the highest degree of pre-emption, colluding firms would set $n_t^* = n_t^r$ meaning only the fringe firms who are profitable in recessions are not deterred out of the market. Furthermore, if setting $n_t^* = n_t^r$ constitutes an equilibrium, it is the strongest pre-emptive equilibrium and anything in the range of $n_t^* \in [n_t^r, n_t^b)$ can constitute a pre-emptive equilibrium. In other words, colluding firms can design trigger strategies such that they deter any upper portion of the $[n_t^r, n_t^b)$ range.

Since equilibrium most-collusive price levels depend on the mass of active fringe and since under pre-emptive equilibrium mass of active fringe may be less, the equilibrium prices may be different in pre-emptive equilibrium than in non-pre-emptive equilibrium. But before solving for pre-emptive equilibrium prices, notice that, for trigger strategies to work we need to make sure they are credible. Since the trigger strategy sets the boom price to the recession price in booms for the duration of the boom, one must make sure that the colluding firms have no incentive to cheat in this period. That is why we need another IC. I call it IC_c for Incentive Compatibility constraint for credibility since if IC_c does not hold, fringe firms know that p^*_{pre} will not be played, hence the threat is not credible.

$$IC_c: \; D_t(p^*_b)p^*_b \leq \delta[(1-\rho)bV_b(p^*_{pre}) + \rho rV_r(p^*)]$$

If the variables are such that IC_c does not hold then the pre-emption is not credible since fringe firms know trigger strategy price level, p^*_{pre}, will never be played.

With all the IC constraints defined, one can move onto solving for pre-emptive equilibrium most collusive prices. Since p^*_{pre} can be anything between $p^*_{pre} = \{p^*_r, p^*_r\}$ and $p^*_{pre} = \{p^*_r, p^*_b\}$, with the former being the most pre-emptive equilibrium and the latter being equivalent to the non-pre-emptive equilibrium, I solve for the most-pre-emptive equilibrium prices.

Since the game starts in the boom period and the cutoff point for mass of active fringe is endagenously defined by the mass of fringe firms that would be profitable under $p^*_{pre} = \{p^*_r, p^*_r\}$, under highest degree of pre-emption firm i solves:

$$\begin{aligned}
&\max_{\{p^*_b, p^*_r\}} \quad V_b(p^*) \\
&\text{s.t.} \quad n^*_t = F(V^F_b(p^*_{pre})) \\
\text{IC 1}: \; & D_t(p^*_b)p^*_b \leq \delta[(1-\rho)bV_b(p^*) + \rho rV_r(p^*)] \\
\text{IC 2}: \; & D_t(p^*_r)p^*_r \leq \delta[(1-\alpha)rV_r(p^*) + \alpha bV_b(p^*)] \\
IC_c: \; & D_t(p^*_b)p^*_b \leq \delta[(1-\rho)bV_b(p^*_{pre}) + \rho rV_r(p^*)]
\end{aligned}$$

Proposition 4 *IC_c implies IC 1 (Proof: see Appendix A.1.).*

Since IC 1 implies IC 2 and IC_c implies IC 1, we can safely ignore IC 1 and IC 2. Then the Lagrangian can be written as:

$$\mathcal{L} = V_b(p^*) + \mu[(1-\rho)bV_b(p^*_{pre}) + \rho rV_r(p^*) - \frac{1}{\delta}D_t(p^*_b)p^*_b)]$$

First-Order Conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial p_b^*} &: \ \frac{\partial V_b'(p^*)}{\partial p_b^*} + \mu[(1-\rho)b\frac{\partial V_b'(p_{pre}^*)}{\partial p_b^*} + \rho r\frac{\partial V_r'(p^*)}{\partial p_b^*} - \frac{1}{\delta}D_t(p_b^*) - \frac{1}{\delta}p_b^* D_t'(p_b^*)] = 0 \\
\frac{\partial \mathcal{L}}{\partial p_r^*} &: \ \frac{\partial V_b'(p^*)}{\partial p_r^*} + \mu[(1-\rho)b\frac{\partial V_b'(p_{pre}^*)}{\partial p_r^*} + \rho r\frac{\partial V_r'(p^*)}{\partial p_r^*}] = 0
\end{aligned}$$

Plugging in the derivatives of the value functions Defined in Appendix C, the FOC for p_b^* can be written as:

$$\begin{aligned}
p_b^* :& \frac{\partial V_b(p^*)}{\partial p_b^*} + \mu(1-\rho)b\frac{\partial V_b(p_{pre}^*)}{\partial p_b^*} + \mu\rho r\frac{\partial V_r(p^*)}{\partial p_b^*} - \frac{\mu}{\delta}D_t'(p_b^*)p_b^* + \frac{\mu}{\delta}D_t(p_b^*) = 0 \\
& \frac{1}{A}\left[\frac{\partial \pi(p_b, n_t^*)}{\partial p_b^*}(1-\delta(1-\alpha))r + \delta\rho r\frac{\partial \pi(p_r, n_t^*)}{\partial p_b^*}\right] + \\
& + \frac{\mu(1-\rho)b}{1-\delta(1-\rho)b}\left[\frac{\partial \pi(p_r, n_t^*)}{\partial_b^*} + \frac{\delta\rho r(1-\delta(1-\rho)b}{A}\frac{\partial \pi(p_r, n_t^*)}{\partial_b^*} + \frac{\delta^2\alpha\rho br}{A}\frac{\partial \pi(p_b, n_t^*)}{\partial_b^*}\right] + \\
& + \frac{\mu\rho r}{A}\left[(1-\delta(1-\rho)b)\frac{\partial \pi(p_r, n_t^*)}{\partial_b^*} + \delta\alpha b\frac{\partial \pi(p_b, n_t^*)}{\partial_b^*}\right] = \frac{\mu}{\delta}[D_t'(p_b^*)p_b^* + D_t(p_b^*)]
\end{aligned}$$

simplifying:

$$\begin{aligned}
p_b^* :& \left[\frac{1-\delta(1-\alpha)r}{A} + \frac{\mu(1-\rho)b^2\delta^2\alpha\rho r}{A[1-\delta(1-\rho)b]} + \frac{\mu\alpha\rho br}{A}\right]\frac{1}{2}\left[D_t'(p_b^*)\underline{p_b^*} + D_t(p_b^*) - \frac{\partial n_t(p^*)}{\partial p_b^*}\underline{p_b^*} - n_t^*(p^*)\right] + \\
+ & \left[\frac{\delta\rho r}{A} + \frac{\mu(1-\rho)b}{1-\delta(1-\rho)\rho} + \frac{\mu(1-\rho)b\delta\rho r}{A} + \frac{\mu\rho r[1-\delta(1-\rho)b]}{A}\right]\frac{1}{2}(-\frac{\partial n_t(p^*)}{\partial p_b^*}p_r^*) = \frac{\mu}{\delta}[D_t'(p_b^*)\underline{p_b^*} + D_t(p_b^*)]
\end{aligned}$$

and solving for p_b^*:

$$p_b^* = \frac{-A_1 D_t(p_b^*) + A_1 n_t^*(p^*) + A_2 \frac{\partial n_t(p^*)}{\partial p_b^*} p_r^* - \frac{\mu}{\delta} D_t(p_b^*)}{A_1 D_t'(p_b^*) - A_1 \frac{\partial n_t(p^*)}{\partial p_b^*} - \frac{\mu}{\delta} D_t'(p_b^*)}$$

$$\begin{aligned}
&where: \\
A_1 &= \frac{1}{2}\left[\frac{1-\delta(1-\alpha)r}{A} + \frac{\mu(1-\rho)b^2\delta^2\alpha\rho r}{A[1-\delta(1-\rho)b]} + \frac{\mu\alpha\rho b r}{A}\right] \\
A_2 &= \frac{1}{2}\left[\frac{\delta\rho r}{A} + \frac{\mu(1-\rho)b}{1-\delta(1-\rho)\rho} + \frac{\mu(1-\rho)b\delta\rho r}{A} + \frac{\mu\rho r[1-\delta(1-\rho)b]}{A}\right] \\
A &= (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2 \\
\frac{\partial n_t^*(p^*)}{\partial p_b} &= F'(V_{r,j}^F(p^*))\frac{\partial V_{r,j}^F(p^*)}{\partial p_b^*} \\
&= \frac{\delta\alpha}{B} F'(V_{r,j}^F(p^*))
\end{aligned}$$

Similarly, the FOC for p_r^* can be written as:

$$\begin{aligned}
p_r^* :& \frac{\partial V_b(p^*)}{\partial p_r^*} + \mu(1-\rho)b\frac{\partial V_b(p_{pre}^*)}{\partial p_r^*} + \mu\rho r\frac{\partial V_r(p^*)}{\partial p_r^*} = 0 \\
& \frac{1}{A}\left[\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_r^*}(1-\delta(1-\alpha))r + \delta\rho r\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*}\right] + \\
& + \frac{\mu\rho r}{1-\delta(1-\rho)b}\left[\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*} + \frac{\delta\rho r}{A}\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*} + \frac{\delta^2\alpha\rho b r}{A}\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_r^*}\right] = 0
\end{aligned}$$

simplifying:

$$\begin{aligned}
p_r^* :& \left[\frac{1-\delta(1-\alpha)r}{A} + \frac{\mu(1-\rho)b^2\delta\alpha}{A} + \frac{\mu\alpha\rho^2\delta^2 r^2 b}{A[1-\delta(1-\rho)b]}\right]\left[-\frac{1}{2}\frac{\partial n_t^*(p^*)}{\partial p_r^*}p_b^*\right] + \\
& + \left[\frac{\delta\rho r}{A} + \frac{\mu(1-\rho)b(1-\delta(1-\rho)b}{A} + \frac{\mu\rho r}{1-\delta(1-\rho)b} + \frac{\delta\mu^2 r^2}{A}\right]\frac{1}{2} * \\
& * \left[D_t'(p_r^*)\underline{p_r^*} + D_t(p_r^*) - \frac{\partial n_t(p^*)}{\partial p_r^*}\underline{p_r^*} - n_t^*(p^*)\right] = 0
\end{aligned}$$

and solving for p_r^*:

$$p_r^* = \frac{A_3 \frac{\partial n_t(p^*)}{\partial p_r^*} p_b^* - A_4[D_t(p_r^*) - n_t(p^*)]}{A_4[D_t'(p_r^*) - \frac{\partial n_t(p^*)}{\partial p_r^*}]}$$

$$where:$$

$$A_3 = \left[\frac{1-\delta(1-\alpha)r}{A} + \frac{\mu(1-\rho)b^2\delta\alpha}{A} + \frac{\mu\alpha\rho^2\delta^2r^2b}{A[1-\delta(1-\rho)b]}\right]$$

$$A_4 = \left[\frac{\delta\rho r}{A} + \frac{\mu(1-\rho)b(1-\delta(1-\rho)b}{A} + \frac{\mu\rho r}{1-\delta(1-\rho)b} + \frac{\delta\mu^2r^2}{A}\right]$$

$$A = (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2$$

$$\frac{\partial n_t^*(p^*)}{\partial p_r} = F'(V_{r,j}^F(p^*))\frac{\partial V_{r,j}^F(p^*)}{\partial p_r^*} = \frac{1-\delta(1-\rho)}{B}F'(V_{r,j}^F(p^*))$$

5 CONCLUSION

I develop a model of pre-emption over the business cycle. By characterizing the pre-emptive firms as imperfectly colluding, zero marginal cost firms and the deterred firms as infinitesimally small fringe firms with heterogeneous marginal cost, I set up a contestable industry with a cyclical demand. The demand is governed by a business cycle with exogenous parameters. I show that around the business cycle, under pro-cyclical most-collusive prices and other assumptions, colluding firms can deter entry to a mass of fringe firms in boom periods that are on the higher cost side of the distribution.

There are two important components of pre-emption, cyclical demand and imperfect collusion. Cyclical demand's role is somewhat clear, it divides the mass of fringe firms into those that are only profitable in boom periods, and those that are profitable in all periods. Under the highest degree of pre-emption, colluding firms deter the former from entry by threatening a reversion to recession price levels in boom periods. It is worthwhile to mention that, if prices were counter-cyclical the pre-emptive equilibrium would still be feasible, but pre-emption would take place in recessions, instead of booms. The second important aspect of pre-emption is that it is only possible in the setting of imperfectly colluding firms. As the trigger strategy that threatens a reversal to lower price levels would not be credible for a monopoly or an oligopoly.

The biggest limitation of the model is the exogenous business cycle. While modeling demand as a Markov-driven switch process with exogenous parameters greatly simplifies the analysis, a more interesting direction for future research would be to explore pre-emption over the business cycle with dynamic cyclical demand. Also, although there is cyclicality in demand, it is only changing between high demand (boom) and low demand (recession) periods. Though having a growth component, ensures that the emerging equilibrium prices are static within period demand level types and hence the equilibrium is easier to solve, further research could introduce a growth level to the demand cycles where dynamic price levels would emerge within the high growth and low growth period levels.

A Proof of Propositions

A.1 Proposition 1

Let $V^F_{j,\text{PW}}$ denote the value function of an active fringe firms under price. Since in the price war $p_t = 0,\ \forall t$:

$$V^F_{j,\text{PW}} = 0 - c_j + \delta V^F_{j,\text{PW}}$$
$$= -\frac{c_j}{1-\delta}$$

and

$$\zeta > -\frac{c_j}{1-\delta},\ \forall c_j \ \in \ F_{[0,C]}$$

Since the scrap value is strictly greater than the value of the fringe firms under price war, all active fringe firms leave. Also since entry cost is strictly greater than the value to the inactive fringe firms, no inactive fringe enters the market.

A.2 Proposition 2

Suppose in equilibrium IC 1 holds with strict inequality: $D(p_b^*)p_b^* < \delta[(1-\rho)bV_b(p^*) + \rho r V_r(p^*)]$. Then let, for some small $\epsilon > 0$, $p_\epsilon^* = p_b^* + \epsilon$ be the new boom price. It is clear that for small enough ϵ, IC 1 still holds with inequality $D(p_b^*)p_b^* \leq \delta[(1-\rho)bV_b(p^*) + \rho rV_r(p^*)]$, while by assumption 4, profits are higher under p_ϵ^* than under p_b^*. Therefore p_b^* cannot be the most collusive boom price.

A.3 Proposition 3

Recall from 3 that:

$$\text{IC 1}: \; D_t(p_b^*)p_b^* \leq \delta[(1-\rho)bV_b(p^*) + \rho rV_r(p^*)]$$

$$\text{IC 2}: \; D_t(p_r^*)p_r^* \leq \delta[(1-\alpha)rV_r(p^*) + \alpha bV_b(p^*)]$$

Notice that, by convexity of demand and pro-cyclicality of most collusive prices, $D(p_b^*)p_b^* \geq D(p_r^*)p_r^*$. Also, by Proposition 2, $D(p_b^*)p_b^* = \delta[(1-\rho)bV_b(p^*) + \rho rV_r(p^*)]$, then proving

$$(1-\rho)bV_b(p^*) + \rho rV_r(p^*) \geq (1-\alpha)rV_r(p^*) + \alpha bV_b(p^*)$$

equates to proving that IC_1 implies IC_2:

$$(1-\rho)bV_b(p^*) + \rho rV_r(p^*) \geq (1-\alpha)rV_r(p^*) + \alpha bV_b(p^*)$$

$$\rightarrow rV_r(p^*)(1-\alpha-\rho) \leq bV_b(p^*)(1-\rho-\alpha)$$

since by assumption 1, $(1-\alpha-\rho) > 0$,

$$\rightarrow rV_r(p^*) \leq bV_b(p^*)$$

$$\rightarrow V_r(p^*) \leq V_b(p^*)$$

with the Value functions from Appendix B:

$$\rightarrow [1-\delta(1-\alpha)b]\pi(r,p^*,n_t^r) + \delta\rho b\pi(b,p^*,n_t^b) \leq [1-\delta(1-\rho)r]\pi(b,p^*,n_t^b) + \delta\alpha r\pi(r,p^*,n_t^r)$$

$$\rightarrow [1-\delta-\delta\alpha b-\delta\alpha r]\pi(r,p^*,n_t^r) \leq [1-\delta-\delta\rho r-\delta\rho b]\pi(b,p^*,n_t^b)$$

$$\rightarrow \rho[b+r]\pi(b,p^*,n_t^b) \leq \alpha[b+r]\pi(r,p^*,n_t^r)$$

since $\pi(b,p^*,n_t^b) \leq \pi(r,p^*,n_t^r)$ and $b+r>0$:

$$\rightarrow \rho \leq \alpha$$

Since the last inequality holds by Assumption 2, all inequalities hold, ending the proof for Proposition 3.

A.4 Proposition 4

Assume IC_c holds with equality, $D(p_b^*)p_b^* = \delta[(1-\rho)bV_b(p_{pre}^*) + \rho rV_r(p^*)]$ then from IC 1:

$$
\begin{aligned}
(1-\rho)bV_b(p_{pre}^*) + \rho rV_r(p^*) &\leq (1-\rho)bV_b(p^*) + \rho rV_r(p^*) \\
\rightarrow V_b(p_{pre}^*) &\leq V_b(p^*)
\end{aligned}
$$

which holds true since $p_{pre}^* \leq p^*$, profits under p^* are no less than under p_{pre}^*.

B Non-Pre-Emptive Equilibrium Value Functions and Their Partial Derivatives

Recall the value functions from (2), assuming only two colluding firms:

$$
\begin{aligned}
V_{b,i} &= \frac{(1-\delta(1-\rho)r)\pi(b,p^*,n_t^b) + \delta\alpha r\pi(r,p^*,n_t^r)}{A} \\
&= \frac{(1-\delta(1-\rho)r)[D_t(p_b^*) - n_t^b]\frac{p_b^*}{2} + \delta\alpha r[D_t(p_r^*) - n_t^r]\frac{p_r^*}{2}}{A} \\
V_{r,i} &= \frac{(1-\delta(1-\alpha)b)\pi(r,p^*,n_t^r) + \delta\rho b\pi(b,p^*,n_t^b)}{A} \\
&= \frac{(1-\delta(1-\alpha)b)[D_t(p_r^*) - n_t^r]\frac{p_r^*}{2} + \delta\rho b[D_t(p_b^*) - n_t^b]\frac{p_b^*}{2}}{A} \\
\text{where } A &= (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2 \\
\text{and } n_t^g &= F(V_g^F(p^*))
\end{aligned}
$$

since the first derivative of n_t^g is always a constant, I keep it as it is, then the first-order derivatives become:

$$\frac{\partial V_{b,i}(p^*)}{\partial p_b^*} = \frac{1}{A}[(1-\delta(1-\rho)r)\frac{\partial \pi_i(b,p^*,n_t^b)}{\partial p_b^*} + \delta\alpha r\frac{\partial \pi_i(r,p^*,n_t^r)}{\partial p_b^*}]$$
$$\frac{\partial V_{b,i}(p^*)}{\partial p_r^*} = \frac{1}{A}[(1-\delta(1-\rho)r)\frac{\partial \pi_i(b,p^*,n_t^b)}{\partial p_r^*} + \delta\alpha r\frac{\partial \pi_i(r,p^*,n_t^r)}{\partial p_r^*}]$$
$$\frac{\partial V_{r,i}(p^*)}{\partial p_b^*} = \frac{1}{A}[(1-\delta(1-\alpha)b)\frac{\partial \pi_i(r,p^*,n_t^r)}{\partial p_b^*} + \delta\rho r\frac{\partial \pi_i(b,p^*,n_t^b)}{\partial p_b^*}]$$
$$\frac{\partial V_{r,i}(p^*)}{\partial p_r^*} = \frac{1}{A}[(1-\delta(1-\alpha)b)\frac{\partial \pi_i(r,p^*,n_t^r)}{\partial p_r^*} + \delta\rho r\frac{\partial \pi_i(b,p^*,n_t^b)}{\partial p_r^*}]$$
where $A = (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2$

and $\pi_i(g,p^*,n_t^g) = \frac{1}{2}(D_t(p_g^*) - n_t^g)p_g^*$

simplifying while keeping the first derivatives of the number of fringe firms as a constant:

$$\frac{\partial V_{b,i}(p^*)}{\partial p_b^*} = \frac{1}{2A}[(1-\delta(1-\rho)r)[D_t(p_b^*) + p_b^*D_t'(p_b^*) - \frac{\partial n_t^b(p^*)}{\partial p_b^*}p_b^* - n_t(p_b^*)] - \delta\alpha r\frac{\partial n_t^r(p^*)}{\partial p_b^*}p_r^*]$$
$$\frac{\partial V_{b,i}(p^*)}{\partial p_r^*} = \frac{1}{2A}[(1-\delta(1-\rho)r)[-\frac{\partial n_t^b(p^*)}{\partial p_r^*}p_b^*] + \delta\alpha r[D_t'(p_r^*)p_r^* + D_t(p_r^*) - \frac{\partial n_t^r(p^*)}{\partial p_r^*}p_r^* - n_t^r(p^*)]]$$
$$\frac{\partial V_{r,i}(p^*)}{\partial p_b^*} = \frac{1}{2A}[\delta\rho r[D_t(p_b^*) + p_b^*D_t'(p_b^*) - \frac{\partial n_t^b(p^*)}{\partial p_b^*}p_b^* - n_t(p_b^*)] - (1-\delta(1-\alpha)b)\frac{\partial n_t^r(p^*)}{\partial p_b^*}p_r^*]$$
$$\frac{\partial V_{r,i}(p^*)}{\partial p_r^*} = \frac{1}{2A}[\delta\rho r)[-\frac{\partial n_t^b(p^*)}{\partial p_r^*}p_b^*] + (1-\delta(1-\alpha)b)[D_t'(p_r^*)p_r^* + D_t(p_r^*) - \frac{\partial n_t^r(p^*)}{\partial p_r^*}p_r^* - n_t^r(p^*)]]$$
where $A = (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2$

and $\frac{\partial n_t^g}{\partial p_g^*} = \frac{F(V_g^F(p^*))}{p_g^*}$ is always a constant

C Pre-Emptive Equilibrium Value Functions and Profit functions and their Partial derivatives

Colluding firm Value functions under $p_t = \{p^*, p_{pre}^*\}$ where $p_{pre}^* = \{p_r^*, p_r^*\}$ and $p^* = \{p_r^*, p_b^*\}$:

$$V_b(p^*) = \pi(p_b^*, n_t^*) + \delta[(1-\rho)bV_b(p^*) + \rho rV_r(p^*)]$$
$$V_r(p^*) = \pi(p_r^*, n_t^*) + \delta[(1-\alpha)rV_r(p^*) + \alpha bV_b(p^*)]$$
$$V_b(p_{pre}^*) = \pi(p_r^*, n_t^*) + \delta[(1-\rho)bV_b(p_{pre}^*) + \rho rV_r(p^*)]$$

Simplifying Value functions:

$$V_b(p^*) = \frac{\pi(p_b^*, n_t^*) + \delta\rho r V_r(p^*)}{1 - \delta(1-\rho)b}$$
$$V_r(p^*) = \frac{\pi(p_r^*, n_t^*) + \delta\alpha b V_b(p^*)}{1 - \delta(1-\alpha)r}$$
$$V_b(p_{pre}^*) = \frac{\pi(p_r^*, n_t^*) + \delta\rho r V_r(p^*)}{1 - \delta(1-\rho)b}$$

Substituting into each other to express in profit terms:

$$V_b(p^*)[1-\delta(1-\rho)b] = \pi(p_b^*, n_t^*) + \delta\rho r \frac{\pi(p_r^*, n_t^*) + \delta\alpha b V_b(p^*)}{1-\delta(1-\alpha)r}$$

$$V_b(p^*)[1-\delta(1-\rho)b][1-\delta(1-\alpha)r] = [1-\delta(1-\alpha)r]+$$
$$+\,\pi(p_b^*, n_t^*) + \delta\rho r\pi(p_r^*, n_t^*) + \delta^2\rho\alpha r b V_b(p^*)$$

$$\underline{V_b(p^*)} = \frac{1}{A}\left([1-\delta(1-\alpha)r]\pi(p_b^*, n_t^*) + \delta\rho r\pi(p_r^*, n_t^*)\right)$$

$similarly:$

$$\underline{V_r(p^*)} = \frac{1}{A}\left([1-\delta(1-\rho)b]\pi(p_r^*, n_t^*) + \delta\alpha b\pi(p_b^*, n_t^*)\right)$$

$and:$

$$\underline{V_b(p_{pre}^*)} = \frac{1}{1-\delta(1-\rho)b}\left(\pi(p_r^*, n_t^*) + \frac{\delta\rho r}{A}[(1-\delta(1-\rho)b)\pi(p_r^*, n_t^*) + \delta\alpha b\pi(p_b^*, n_t^*)]\right)$$

$where:$

$$A = (1-\delta(1-\alpha)b)(1-\delta(1-\rho)r) - \delta^2\alpha^2\rho^2$$

Partial derivatives of the value functions:

$$
\begin{aligned}
\frac{\partial V_b(p^*)}{\partial p_b^*} &= \frac{1}{A}\left[[1-\delta(1-\alpha)r]\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_b^*} + \delta\rho r\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_b^*}\right] \\
\frac{\partial V_b(p^*)}{\partial p_r^*} &= \frac{1}{A}\left[[1-\delta(1-\alpha)r]\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_b^*} + \delta\rho r\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*}\right] \\
\frac{\partial V_r(p^*)}{\partial p_b^*} &= \frac{1}{A}\left[[1-\delta(1-\rho)b]\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_b^*} + \delta\alpha b\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_b^*}\right] \\
\frac{\partial V_r(p^*)}{\partial p_r^*} &= \frac{1}{A}\left[[1-\delta(1-\rho)b]\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*} + \delta\alpha b\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_r^*}\right] \\
\frac{\partial V_r(p_{pre}^*)}{\partial p_b^*} &= \frac{1}{1-\delta(1-\rho)b}\left[\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_b^*} + \frac{(\delta\rho r)[1-\delta(1-\rho)b]}{A}\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_b^*} + \frac{\delta^2\alpha\rho br}{A}\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_b^*}\right] \\
\frac{\partial V_r(p_{pre}^*)}{\partial p_r^*} &= \frac{1}{1-\delta(1-\rho)b}\left[\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*} + \frac{(\delta\rho r)[1-\delta(1-\rho)b]}{A}\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*} + \frac{\delta^2\alpha\rho br}{A}\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_r^*}\right]
\end{aligned}
$$

Where since $\pi(p_g^*, n_t^*) = \frac{1}{2}(D_t(p_g^*) - n_t^*(p^*)p^*$ for $g \in \{b, r\}$ Partial derivatives of the profit functions are:

$$
\begin{aligned}
\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_b^*} &= \frac{1}{2}(D_t'(p_b^*)p_b^* + D_t(p_b^*) - \frac{\partial n_t(p^*)}{\partial p_b^*}p_b^* - n_t(p^*)) \\
\frac{\partial \pi(p_b^*, n_t^*)}{\partial p_r^*} &= -\frac{1}{2}\frac{\partial n_t(p^*)}{\partial p_r^*}p_b^* \\
\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_b^*} &= -\frac{1}{2}\frac{\partial n_t(p^*)}{\partial p_b^*}p_r^* \\
\frac{\partial \pi(p_r^*, n_t^*)}{\partial p_r^*} &= \frac{1}{2}(D_t'(p_r^*)p_r^* + D_t(p_r^*) - \frac{\partial n_t(p^*)}{\partial p_r^*}p_r^* - n_t(p^*))
\end{aligned}
$$

D Partial Derivatives of Mass of Active Fringe Firms

Partial Derivatives of the (4), Fringe firm value functions:

$$
\begin{aligned}
\frac{\partial V_{b,j}^F(p^*)}{\partial p_b^*} &= \frac{1-\delta(1-\alpha)}{B} \\
\frac{\partial V_{b,j}^F(p^*)}{\partial p_r^*} &= \frac{\delta\rho}{B} \\
\frac{\partial V_{r,j}^F(p^*)}{\partial p_b^*} &= \frac{\delta\alpha}{B} \\
\frac{\partial V_{r,j}^F(p^*)}{\partial p_r^*} &= \frac{1-\delta(1-\rho)}{B}
\end{aligned}
$$

Partial Derivatives of the number of active firms:

$$
\begin{aligned}
\frac{\partial n_t^b(p^*)}{\partial p_b} &= F'(V_{b,j}^F(p^*))\frac{\partial V_{b,j}^F(p^*)}{\partial p_b^*} \\
&= \frac{1-\delta(1-\alpha)}{B}F'(V_{b,j}^F(p^*)) \\
\frac{\partial n_t^b(p^*)}{\partial p_r} &= F'(V_{b,j}^F(p^*))\frac{\partial V_{b,j}^F(p^*)}{\partial p_r^*} \\
&= \frac{\delta\rho}{B}F'(V_{b,j}^F(p^*)) \\
\frac{\partial n_t^r(p^*)}{\partial p_b} &= F'(V_{r,j}^F(p^*))\frac{\partial V_{r,j}^F(p^*)}{\partial p_b^*} \\
&= \frac{\delta\alpha}{B}F'(V_{r,j}^F(p^*)) \\
\frac{\partial n_t^r(p^*)}{\partial p_r} &= F'(V_{r,j}^F(p^*))\frac{\partial V_{r,j}^F(p^*)}{\partial p_r^*} \\
&= \frac{1-\delta(1-\rho)}{B}F'(V_{r,j}^F(p^*))
\end{aligned}
$$

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