Plato's Mathematical Entities in the *Republic*

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Abstract

The main goal of this thesis is to defend the traditional interpretation of Plato's *Republic* Book VI and VII. Many scholars argued that Plato makes a distinction between mathematical entities and the Forms themselves by introducing the simile of the *Line*. This view had been supported by Aristotle's testimony about Plato's mathematical objects in his *Metaphysics*. This line of interpretation, however, has been called into question by a group of modern scholars. They argue that some geometrical entities such as the square itself and the diagonal itself are the Forms, though they are not connected to the Form of Good itself. The reason for supporting this view is based on several arguments. One ground supporting this view is that Plato adopts some intensive pronouns in the simile. The other reason for it is that there is no place of the mathematical objects in other similes, especially the simile of the *Cave*. I will reject the first ground for supporting this view, showing that the occurrences of intensive pronouns are not a good indicator of the Forms themselves. I will also argue that we can find a place of the mathematical objects in the simile of the *Cave*, though it is not manifest in the simile itself. Having constructed this line of argument, I will address several objections and give my answers to them. This new defence of the traditional interpretation may contribute to reading the Republic from a unitary point of view.

Key Words: Plato's Metaphysics, Mathematical Entities, Forms, the Simile of the Line, the Simile of the Cave.

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Introduction

Plato's *Republic* is a masterpiece encompassing almost all the major themes of his philosophy.[1] In the *Republic*, Plato starts from enquiring whether justice pays by itself and ends with the immortality of soul. Along the way to the final goalpost of the dialogue, he goes through many topics encompassing the theory of Forms and the education for philosopher-kings.

One notable feature of this dialogue is that Plato adopts several similes in order to explain quite distinctive features of his philosophy. All that noted, the discussion in Book VI and VII is led by three connected similes, which is the simile of the *Sun*, the *Line*, and the *Cave* respectively. It is the first simile which first introduces the Forms and their instantiations for readers. The third simile describes the process a philosopher makes in pursuit of the true being, the Form of Good in a vibrant manner. Among these similes, the simile of the *Line* is distinct from two other similes, since only this one addresses the mathematical beings such as the square itself and the diagonal itself which are not found in the other similes.

It is, however, quite controversial how these mathematical entities are distinct from other entities such as sensible particulars and the Forms. Until the mid-20th century, commentators were in agreement that the mathematical entities are distinct from the Forms, believing Aristotle's testimony to be true. Those who supported the modern orthodoxy, however, argued that the mathematical entities are the Forms themselves. This view has been

[1] Unless otherwise specified, all references are to Plato's *Republic*. I use the translation of Emlyn-Jones and Preddy edition in *the Loeb Classical Library Series* published by Harvard University Press. In the cases I need original Greek terms, I include them in transliteration.

widely supported by 20th- century scholars since Cross and Woozley's monumental book was firstly published in 1966 which argues that Plato has the Forms in mind when he adopts the terms such as the square itself and the diagonal itself.

In this thesis, I will address this controversial issue. After investigating the Similes and some relevant passages to them, I will draw a general conclusion that the mathematical entities are clearly distinct from the Forms. My assertion is supported by the following reasons: First, the occurrences of intensive pronouns such as itself and themselves do not constitute a good reason for us to believe that Plato refers to the Forms. To be specific, we are not sure whether they refer to the mathematical intermediates or the Forms just because Plato adopts the pronouns in order to refer to the Forms. Second, the modern orthodoxy seriously undermines the unity of the dialogue, if we follow the orthodoxy and read the mathematical intermediates as the Forms. This will be shown by scrutinising Plato's recapitulation of the theory of Forms in the Book VII. The very reason to accept a more traditional interpretation which distinguishes the mathematical entities from the Forms is that it may provide us with a more consistent understanding of the dialogue.

In the last section of this thesis, I will handle some possible objections to my statement that Plato's mathematical entities are distinct from the Forms. Some commentators formulated a sceptical view of Aristotle's testimony about Plato's distinction between the Forms and the mathematical objects. I argue that Aristotle's testimony about this problem is trustworthy, not because Aristotle was one of the closest students of Plato, but because Aristotle's explanation perfectly fits the passages in the *Republic*. In addition, I will address possible objections that the adherents of the modern orthodoxy could raise. They would argue that the mathematical objects are not clearly distinct in Books VI and VII. Their argument is based on the request that we should read the three similes in a unitary way. In response to this argument, I will show how we can understand the similes without eliminating the

mathematical entities as distinct objects from the Forms. Lastly, I will handle a possible objection to my statement which argues that there is a discrepancy between the bed case and the mathematical objects case. While I partially agree with this criticism, I show it does not undermine the consistency of Plato's theory concerning mathematical entities. By analysing the bed and the mathematical object cases, it will be clearly shown that Plato's stress on the image-original relation has priority over other relations.

This interpretation of Plato may enable us to read the *Republic* from a quite consistent viewpoint. It will provide a unitary understanding of the crucial concept without omitting what Plato puts emphasis on both in the simile of the *Line* in Book VI and in his theory of education delineated in Book VII. Therefore, we can keep track of the way the mathematical entities are characterised if we follow this interpretation.

Plato's Explanation on the Mathematical Entities in Book VI

In this section, I introduce a relevant passage where Plato has his interlocutors discuss what the nature of the mathematical entities is like, followed by introducing two different interpretations of it. I reserve my analysis on the text until I finish this section. This section is rather dedicated to a fair introduction to the matter at hand.

In a passage of the *Republic* Book VI, Plato explains how it is possible for us to approach to the Forms with the help of the simile of the *Sun* in which it is clearly shown that the Forms are distinct from sensible beings. Also, Plato successfully contends that the Forms are the causes of the sensible beings just like the sun is the first cause of life and nurture (509b1-3). When he leads the readers to the first simile, he does not mention anything about the mathematical entities yet. Interestingly enough, on the next step, he introduces the simile of the *Line* and starts to handle the mathematical beings with the help of the simile. Here Plato has his interlocutors engage in the following conversation:

> "Right then, imagine a line cut in two. Take two unequal segments and again cut each one in the same ratio, one for the visible class (*to tou horōmenou*), the other for the intelligible (*to tou nooumenou*); and you will have in the first segment of the visible section images in relation to each other by their clarity or obscurity – and by images (*eikones*) I mean firstly shadows (*skiai*), the reflections in water and in those surfaces which are solid, smooth and shiny, and everything like this, if you get my meaning."
> "Well yes, I do"
> "Now take the second section which this one resembles to be the living creatures (*ta zōa*) around us, all natural things (*to phuteuton*) and the whole class of artificial things."
> "I'll do that," he said.
> "And would you be willing to agree that the division of truth to falsehood is in this ratio: as belief is to knowledge, thus resemblance is to what it resembles?"
> "Yes, I agree entirely," he said.
> "Now consider again in what way the section of the intelligible (*tēn tou noētou tomēn*) is to be divided."
> "What way is that?"

> "In the one section, the soul is forced to investigate from hypotheses, by using as images what were at the previous stage things imitated, not by working toward a first principle (*archēn*), but toward a conclusion (*teleutēn*). In the other section, by contrast, it moves from the hypothesis toward a first principle which transcends hypothesis (*anupotheton*), but without the images of the earlier section, and so constructs its way of operating from the very Forms by themselves"
>
> (509d7-510b10)

All that noted, the whole line is cut into two different segments which is about the visible things (*to tou horōmenou*) and the other that is about the intelligible things (*to tou nooumenou*), respectively, by a certain ratio (1:2), which is cut into two different subsections again by the same ratio. Therefore, we have four different subsections of the original line, which is about images (*eikones*) and shadows (*skiai*), living creatures (*ta zōa*) and natural things (*to phuteuton*), the mathematical entities, and Forms, (*eidē* or *ideai*) respectively.[2]

One notable feature of Plato's explanation here is that he thinks of a set of corresponding epistemological faculties each of which corresponds to each group of the

[2] This summary is based on Adam's table, where he cuts the whole line into two different segments by a certain ratio (1:2) and divides each segment into two shorter segments by the same ratio. Here is a reconstruction of Adam's table concerning the entities and corresponding epistemic faculties in the simile of *Line*.

	Visible World (*to tou horōmenou*)		Intelligible World (*to tou nooumenou*)	
Epistemic Faculty	Apprehension (*eikasia*)	Opinion (*doxa*)	Thought (*dianoia*)	Understanding (*noēsis*)
Objects of Cognition	shadows (*skiai*), Images (*eikones*), reflections in water (*ta en tois hudasi phantasmata*), etc	living creatures (*ta zōa*), Natural things (*to phuteuton*), artificial things (*to skeuaston*)	mathematical objects* (*ta mathēmatika*)	the Forms (eidē), especially the Form of Good itself.

* This expression is not manifest in the original passages of Plato's *Republic*
Adam adheres to Aristotle's testimony that Plato thinks that there are some mathematical intermediates which are neither the Forms nor sensible particulars. The mathematical intermediates are distinguished from the Forms since there is a possibility for the mathematical objects to be plural, whereas each of the Forms is unique. See Adam (1963), 156-163, which I substantially agree with.

objects. Therefore, there are four distinct cognitive faculties, as there are four different groups of objects. Apprehension (*eikasia*) is about images and shadows, belief (*pistis*) about the living creatures and natural things, thought (*dianoia*) about the mathematical entities, and understanding (*noēsis*) about the Forms.[3]

Here is the place our question stands. In the similes of the *Sun* and the *Cave*, there is no place for the mathematical entities which we investigate.[4] The sun is the ground of our eyesight and many visible beings (509b1-3). This also holds in the allegory of the *Cave*, where the simile of sun is used again (515e5ff). Two similes have a similar point in that the sun functions as the first cause of everything that is. However, there is no intermediate being such as mathematical entity as Plato shows first in the simile of the *Line*. In the simile, Plato

[3] It seems, however, that Plato's use of these terms is not consistent. All that noted, Plato clearly distinguishes four epistemic faculties and attaches proper names to each of them. In Book VII, Plato still adheres to the distinction he made earlier. However, he uses the terms in a slightly different manner. In 533e3-534a10, apprehension (*eikasia*) and belief (*pistis*) together are called opinion (*doxa*), while thinking (*dianoia*) and knowledge (*epistēmē*) together are called understanding (*noēsis*). Here is the relevant passage:

> "We're happy, then," I said, "as we were before, to refer to the first part as knowledge (*epistēmēn*) and the second as thought (*dianoian*), the third belief (*pistin*) and the fourth conjecture by means of imagery (*eikaisian*). Again these last two can be grouped under opinion (*doxa*) the first two under understanding (*noēsin*) where opinion deals with the impermanent (*peri genesin*), understanding with the real (*peri ousian*); and just as reality is to impermanent, understanding is to opinion, and as understanding is to opinion, so knowledge is to belief and thought to conjecture by means of imagery. Let's leave aside the relative proportions between all these and the division of both what is opinion and what is knowledge, Glaucon, so that we don't get ourselves embroiled in an argument many times longer than we had in some earlier topics."
>
> (533e3-534a10)

From this passage, it is still not clear why Plato uses different terms in order to refer to the same epistemic faculties which are firstly introduced in Book VI. However, Here Plato adopts the same scheme he introduced in Book VI. By interweaving the epistemic faculties and their corresponding objects and by re-introducing the ratio of the segments in the simile of the Line, he repeats the same points he made in the previous book. For this reason, I contend that Plato's position is consistent throughout the books, even though there are some slight changes in Book VII.

[4] One more reason for rejecting the thesis that the mathematical objects are distinct from the Forms is that Plato never manifestly uses the expression 'the mathematical objects' (ta *mathēmatika*) in the *Republic*. As this is the case, it would have led many commentators to the thinking that Plato does not draw a line between the Forms and the mathematical objects clearly. But as we shall see, even if Plato does not adopt the term *ta mathēmatika*, there are many reasons for adhering to the view that they are distinct from the Forms.

adds the mathematical entities between the things that are which are grasped by our opinion (*doxa*) and the Forms grasped by our understanding (*noēsis*) in the first place.

Why are the mathematical entities shown in a distinct manner in the simile of the *Line*? This point was not clear in the previous analogy. Plato may have some reasons to introduce the mathematical entities in the second simile. Or we could believe Plato is just confused. The original text itself does not clearly show whether the mathematical entities are entirely distinguished from the Forms which are the direct objects of our dialectical methods (*dialektikē*). There are two passages which might blur our judgement. In the first place, Plato differentiates the mathematical entities from the Forms:

> "I think you'll understand more easily after a few preliminary remarks. I think you know that those who study geometry and arithmetic and similar subjects postulate odd and even, geometrical figures and the three kinds of angles, and other relationships of this sort according to each system of inquiry. So, taking these things as known, they make them their *hypotheses* (*hupotheseis*) and don't think it worth their while to offer any justification for them to themselves or others, on the grounds that they are clear to everyone. And starting from these, they go on through the remaining steps and end up in agreement at the point they set out to reach in their investigation."
>
> (510c1-d3)

According to Plato, two groups of objects are conceived of by two different faculties, which is thought and understanding, respectively. This feature was demonstrated in the previous part of this section where there is a focal point which distinguishes two different cognitive faculties. When we investigate the geometrical objects, we should rely on some presuppositions or hypotheses (*hupotheseis*) which are not questioned at this level. This is the same for the arithmetic and other parts of mathematics treating various mathematical objects such as odd and even numbers and angles of a triangle. However, the dialectical method does not presuppose them. Rather, it directly delves into the presuppositions and arrives at the Forms insofar as they are Forms. The dialectic finally arrives at what is *unhypothetical* (511b2-c3). In this way, it seems that the Forms and the mathematical objects are distinct in terms of the methods through which they are studied.

i. Two Interpretations Concerning this Issue

A line of interpretation was steadily supported since Aristotle's testimony about mathematical objects (*ta mathēmatika*) was present in his *Metaphysics*. At least in two different passages, Aristotle contends that Plato and his followers have a very specific conception of the mathematical objects which are distinct from the Forms. In some passages of his *Metaphysics*, Aristotle is confident that Plato has a very specific conception of mathematical entities:

> Further, he states that besides sensible things (*ta aisthēta*) and the Forms (*ta eidē*) there exists an intermediate class, *the objects of mathematics* (*ta mathēmatika*), which differ from sensible things in being eternal and immutable, and from the Forms in that there are many similar objects of mathematics, whereas each Form is itself unique.
>
> (Aristotle, *Metaphysics* 987b)

Aristotle's first testimony is about the intermediate beings between the Forms and the sensible objects. The reason for dividing two kinds is primarily the Form's *uniqueness*. It seems that there are many similar, but different objects that are each of them called by one name, whereas each Form is distinct from these items by its being unique. For example, there might be many diagonals and squares, each of which is different from the other. He also repeats the same point in another passage in a less clear manner:

> Thus Plato posited the Forms and *the objects of mathematics* (*ta mathēmatika*) as two kinds of substance (*duo ousias*), and as a third the substance of sensible bodies;
>
> (Aristotle, *Metaphysics* 1028b)

In the first excerpt, Aristotle utters that the mathematical entities are intermediate beings between sensible particulars and Platonic Forms. In the second one, he distinguishes three different kinds of substances, which are the Forms, mathematical entities and sensible bodies. One main reason for distinguishing the two groups of objects, according to Aristotle, is based on the Forms' uniqueness and the plurality of the mathematical objects. Each form is unique

and distinct from other Forms, whereas many mathematical objects of one kind exist. Aristotle's testimony about the characteristic of the mathematical entities was steadily trusted until some modern commentators called it into question by careful scrutiny.

It is, however, too rash to resolutely conclude that Plato clearly makes such a distinction in the Book VI. It is because Plato adopts a technical formula which he normally uses in order to denote the Forms. In the passage that follows, he deploys some *intensive pronouns* such as *itself* and *themselves* which rarely showed up in the preceding passages:

> "So you'll also know that they make use of the visible forms as well and make their arguments about them, although considering not the actual things, but those they resemble, making their arguments on the basis of *the square itself* (*tou tetragōnou autou*) and *the diagonal itself* (*diametrou autēs*), but *not* the line they are drawing, but similarly with everything else. These very things they are forming and drawing, of which shadows and reflections in water are images, they now in turn use as their images and aiming to see those very things which they could not otherwise see except *in thought* (*tē dianoia*)."
>
> (510d5-511a3)

This passage says that some geometrical figures are distinct from the things which imitate the figures. For example, the square itself is said to be different from the visible square drawn on the ground. This point does not cause any controversy. The crucial thing here is that Plato's deployment of the intensive pronouns is supposed to be an indication that he has in mind *Platonic Forms*. In many other dialogues such as the *Sophist*, the intensive pronoun is a good indicator of Forms themselves.[5] If we buy this argument, the seemingly apparent distinction between Forms and the mathematical entities is blurred.

[5] In the *Sophist*, Plato calculatedly deploys some intensive pronouns to differentiate the Forms from the things that participate in the Forms. He distinguishes the greatest kinds from the things that participate them by intentionally using the term 'itself'. For example, he adopts such expressions in the following passage in the *Sophist*. The usage of itself is limited to the discussion about the relation between the Kinds. However, we can find another passage in which he uses the intensive pronouns in order to denote the Forms when the conversation is *not* concerned about the greatest Kinds themselves:

> "So it seems that the setting against each other of the nature of a part of the different and the nature of that which is not any less being – if we're allowed to say such a thing – than that which is itself. And it does not signify something contrary to that which is but only something different from it."

One more reason for supporting the modern orthodoxy is that we could arrive at a more consistent reading of this dialogue, if we reject the traditional interpretation and accept the modern orthodoxy. If the mathematical objects are characterised not as the Forms but as some intermediate entities, we might be lost since there is no occurrence of the mathematical objects in the similes of the *Sun* and the *Cave*. Plato manifestly sets a discrepancy between the Forms and sensible particulars (physical objects) in the similes, without relying on the specific conception of the mathematical objects. Therefore, we could arrive at a better interpretation of the similes by giving up Aristotle's conception of the mathematical objects. It is also supported by a reading of the example of the *Bed* in Book XI, where physical beds are mere imitations of the Form of the Bed. Since we cannot find any occurrences of the mathematical objects, it is very likely that Plato does not manifestly distinguish the mathematical objects from the Forms.

It is, in a nutshell, the crucial reason why there has been a continuing controversy between scholars about the status of mathematical entities in Book VI. To be specific, adherents of the modern orthodoxy including Cross and Woozley 1966 and Annas 1981 argue that the occurrences of intensive pronouns here constitute the good reason for us to believe that Plato wants to refer to the Forms by the expressions such as 'the square itself' and 'the diagonal itself'.[6] Those who dispute this view argue that there is no reason to believe that Plato refers to the Forms simply because of this reason. Based on Aristotle's testimony,

(the *Sophis*t, 258a10-b4)

When he refers to the greatest Kinds *qua* Kinds, he uses the expressions such as the being itself and difference itself. This also holds when he refers to 'less great' Forms such as the beautiful itself and the just itself.

It is contrasted with the situation in which he refers to the things that are (*ta onta*) and things that are different (*ta hetera*). However, I think this distinction cannot apply to reading the *Republic*, where mathematical objects are also referred by the same expressions.

[6] See Cross and Woozley 1966, 236 and Annas 1981, 251. Yang 1999 lists the names of the commentators who support the modern orthodoxy about the mathematical entities.

many commentators including Adam have argued that Plato's mathematical objects are distinct from the Forms. Along with the traditional interpretation, there is an attempt to take a reserved position between the modern orthodoxy and the traditional interpretation. For example, Yang 1999 argues that we cannot confirm whether the mathematical entities are the Forms or just mathematical intermediates in the relevant text just because Plato calculatedly deploys some intensive pronouns.[7] In other words, it is an open question whether they are the Forms or the mathematical intermediates which Aristotle mentions.

To repeat the point, the status of the mathematical entities such as the square itself and the diagonal itself is controversial. I think we cannot judge with confidence whether they are the Forms or the intermediate entities within the setting of Book VI. This is the reason why we need to investigate it not only by analysing Book VI, but also by keeping track of the usage of the term in different passages in Book VII.

[7] See Yang 1999, 31-35, where he puts emphasis on the image-original relation rather than on the occurrences of intensive pronouns. In his article, Yang reserves his jurisdiction. I partially agree with Yang that we cannot be certain whether they are the Forms or the mathematical intermediates. However, my argument will show that they are just the mathematical intermediates provided that their status should be understood in light of other passages addressing the same entities.

Mathematical Entities as Intermediate Beings between Forms and Particulars

i. Setting the Problem

In the previous section, I gave the relevant passage where Plato introduces the way mathematical thought grasps its subject matter in the *Republic*. I also introduced two different interpretations of what the objects of this cognition are. I argue that the mathematical entities have distinct characteristics which Forms do not have in this section. In order to support my argument, I firstly reject the statement that the occurrences of the intensive pronouns imply that Plato has the Forms in mind. I keep track of the intensive pronouns and show that it is not the case that the occurrences here alone support the statement. Thereafter, I will introduce Plato's educational theory in Book VII where he is also concerned about the mathematical cognition. By comparing several relevant passages, I draw a general conclusion that the mathematical entities are clearly distinct from Platonic Forms. This conclusion would help us to think that there is little reason to believe that the square itself and the diagonal itself are the Forms in these contexts.

According to Cross and Woozley 1966, Annas 1981, and many other commentators, the intensive pronoun in the relevant passage indicates that Plato refers to the Forms. Here is a typical excerpt from the modern orthodoxy which has been popular since mid-and late- 20th century:

> At 510d, when he [Plato] tells us that the geometrician is not thinking about the sensible diagram but the 'square itself' and 'the diagonal itself', the Greek here is

> one of the regular Greek phrases Plato uses when he is referring to Forms. It is quite true that the phrases could also stand for perfect particulars as well, i.e. for 'the mathematical square', 'the mathematical diagonal'; but it would be, to say the least, odd if Plato, without any further indication, expected us to take them in this latter way, when in fact they are his regular expressions for referring to Forms.
>
> (Cross and Woozley 1966, 236)

Grounded on this thesis, Cross and Woozley present a table explaining the hierarchy of the entities Plato mentions in the simile of Line. According to Cross and Woozley, the segment of the intelligible objects is divided into two shorter segments again by two different kinds of the Forms. In other words, the mathematical objects are also Forms, though they are distinct from the Forms that are cognised by *unhypothetical* investigation (dialectic).[8] Annas also

[8] The following is Cross and Woozley's scheme of the hierarchy of the entities in the simile of the *Line*. Here is the reconstruction of Cross and Woozley's table:

	Line		Cave	
Intelligible World (World of Forms)	The Form of Good and the other Forms seen in their connection with it	*Noēsis*: Intelligence or Knowledge (in the strict sense)	*Epistēmē*: Knowledge (including mathematical reasoning	State of released prisoners looking at things themselves, the heavenly bodies, and finally at the Sun itself (516a8ff.)
	Forms not seen in their connection with the Form of Good	*Dianoia*: Thinking (mathematical reasoning)		State of released prisoners looking at shadows and reflections in upper world (516a6-7)
Sensible World (world of particulars)	Physical objects	*Pistis*: Belief	*Doxa*: Opinion	State of released prisoners looking at originals in the Cave which cast the shadows
	Images	*Eikasia*: Illusion		State of chained prisoners looking at shadows.

stands for this position, arguing that the square itself and the diagonal itself surely refer to the Forms. Although these entities are some kind of the Forms, they are distinguished from the Forms which are directly related to the Form of Good. Cross and Woozley draw this conclusion by the following way. First, Plato uses the intensive pronouns as an indication of the Forms, since they are technical devices for referring to the Forms. They ground this argument by mentioning 524e, where an intensive pronoun is introduced again. Also, the fact that mathematical entities and the Forms in connection to the Form of Good are investigated by different faculties does not justify to the claim that there are mathematical intermediates. Therefore, both mathematicians and philosophers enquire the Forms, thought the ways they investigate are different. Annas repeats the same point that Cross and Woozley made, without handling this problem with careful scrutiny. She argues that thought (*dianoia*) studies the Forms in isolation, whereas intellect (*noēsis*) studies the Forms for their own sake, and in a systematic connection, as being dependent on the Form of Good.[9]

ii. Rejection of the Modern Orthodoxy

I contend that this cannot withstand a careful scrutiny of the text. There as several reasons why I cannot accept the modern orthodoxy. One of the main reasons for this is that

	[Shadows, etc., including second-hand opinions]			

The most notable feature of their table is that they do not recognise the mathematical entities (*ta mathēmatika*) as distinct ones. Rather, they consider the mathematical objects as inferior Forms which are known without *unhypothetical* enquiry. This table is contrasted with Adam's table of the entities where he trusts Aristotle's testimony that Plato's mathematical objects are not the Forms. See the previous footnote.

[9] See Annas 1981, 251, where she argues that Aristotle's testimony is unreliable, since we cannot find a relevant passage supporting the testimony. Also, she contends that some passages are against Aristotle's testimony.

Plato puts emphasis on the image-original relation here.[10] In addition, there is no mention of the Forms until he finishes the description of the square itself and the diagonal itself. This is clearly shown by the careful analysis of the text. First, Plato explains the image-original relation as follows:

> "In the one section, the soul is forced to investigate from hypotheses, by using as images what were at the previous stage things imitated, not by working toward a first principle, but toward a conclusion. In the other section, by contrast, it moves from the hypothesis toward a first principle (*archēn*) which transcends hypothesis (*anupotheton*), but without the images (*eikonōn*) of the earlier section, and so constructs its way of operating from the very Forms by themselves."
>
> (510b5-10)

Mathematical investigation makes use of the things that were imitated in the first place as their images. By using the things as images, it goes beyond the sensible particulars and approaches to something else from a hypothesis. However, this investigation does not cast doubt on the hypothesis from which it starts. By knowing what the mathematical investigation does, we can grasp what Plato means by the square itself and diagonal itself.

The image-original relation holds not only in the case of sensible particulars and the corresponding Form. As all noted, the *Bed* example in Book X shows the relation with lucidity. Many sensible beds imitate the Form of the bed itself. However, this relation also holds between a sensible particular like a physical bed and many shadows and images of the bed, because the image-maker such as a painter does not imitate the Form directly, but one of the instantiations of the Form.[11] Therefore, we do not know whether the mathematical entities are the Forms or not *simply because* there are some intensive pronouns.

[10] See Yang 1999, 31-35, where he argues that the central discussion in the simile of the Line is about the image-original relation, not about the Forms themselves. I substantially agree with his claim, since it reflects what Socrates and Glaucon say more correctly.

[11] Since the painter imitates a sensible bed which is an image of the Form, he might know some functions of the bed. However, he does not have the proper knowledge used for making beds. In many passages of his corpus, Plato emphasises that the image-makers do not possess knowledge about what the Form they imitate really is. See the *Sophist* 233a3ff and the *Gorgias*, 447aff. If someone makes an image of something which is very close to the Form she imitates, she knows something. In contrast, she is ignorant when she sticks to a mere image of a

One more thing to note in this passage is that Plato barely uses manifest expressions denoting the Forms such as *idea* and *eidos* when he describes the mathematical objects. Since he has clarified two different groups of objects in the intelligible world and explains the difference between two distinct methods of investigation (511a5-d6), he starts to reuse the terms denoting the Forms such as *idea* and *eidos*.

To make this point clear based on Plato's assertion, it will be helpful if we introduce the example of a square-shaped object. At the lowest level, we form an apprehension by seeing a shadow of the sensible square. The sensible square is the origin of the shadow. However, the sensible square is not a square itself yet, since the square itself is understood by our intellectual cognition, not by sense-perception. The sensible square in the spatiotemporal world imitates the actual one. Therefore, this square is the square itself *in relation to* the sensible one. This assertion is confirmed by Plato's explanation of the difference between a shadow and a sensible particular related to the shadow:

> "This then is the class that is intelligible that I was walking about, where a soul is forced to use hypotheses in its search for it, without working toward a first principle because it is unable to escape from its hypotheses to a higher level, but by using as images the very same things of which images were made at a lower level and, in comparison with those images, were thought to be clear and valued as such (*tetimēmenois*)."
>
> (511a5-a11)

Plato wants to highlight that the image-original relation holds not only when a shadow imitates a sensible particular but also when a sensible particular has some geometrical shape. When the image-original relation holds, the thing that is imitated is thought to be clear and valued as such in comparison to the thing that imitates. As this is the case, it is evident why the square itself is valued as such in relation to drawn square(s). Therefore, the intensive pronouns are not sure indicators of the Forms here. In this light, we can understand what the

sensible particular. For example, a carpenter partially knows what a good bed is, since he makes his beds which directly imitate the Form of the beds, while a painter or a poet makes an image of the bed without knowing what the function of the bed is.

square itself and the diagonal itself mean. Each of these expressions can refer to the geometrical and therefore mathematical square and diagonal. I argue that this explanation perfectly fits Plato's way of addressing the problem in the relevant passage since it neither adds to the original passage nor deducts from it when addressing the problem. Because the relevant passage by assigning a separate segment to these entities indicates that mathematical entities are distinct from the Forms, I argue that the square itself and the diagonal itself do not refer to the Forms of the square and the diagonal.

It is also clearly shown by the reading of the passage I problematise. After deploying the phrase such as the square itself and the diagonal itself, Plato emphasises the relation between an original thing and its image(s). He first contrasts the shadow of a visible shape and the drawn shape. When geometricians do study on the proper objects of science, they use the drawn shape as an image and ascend to the original of the image, which is not the drawn shape but the shape itself (510d5-511a3). If we read this controversial passage in the way I suggested, we can confirm what the square itself and the diagonal itself mean. In the passage, the mathematical entities are neither the true beings which are direct objects of the dialectical methods nor the things geometricians are drawing in the sensible world (511c4-d6). What is drawn in pursuit of the mathematical investigation is used again as an image of the mathematical entity. In addition, these kinds of things are grasped only *in thought* (*tē dianoia*). Plato never uses the term 'thought' (*dianoia*) when he refers to the object of the understanding (*noēsis*). In other words, thought (*dianoia*) and understanding (*noēsis*) are different from each other in a *mutually exclusive* manner.[12] To see it, we need to examine the

[12] As mentioned above, Plato is not consistent through this dialogue in the usage of his own technical terms. However, he manifestly makes contrast between the Forms and the mathematical entities such as geometrical figures even in Book VII by sticking to the same distinction he already made in the previous book.

following passage. Here Plato differentiates what is *unhypothetical* from what is grounded on *hypotheses*:

> "So understand, too, what I mean by the other section of the intelligible, which reason (*logos*) itself grasps by the power of dialectic, using hypotheses which are not first principles, but genuine hypotheses, like steps and starting points, in order to go as far as *what is unhypothetical* (*anhupotheton*) and the first principle of everything. And grasping this principle, it returns once again, keeping hold of what follows from it, and comes down to a conclusion in this way, using no sense perception in any way at all, but Forms themselves, going through Forms to Forms and ending up at the Forms."
>
> (511b2-c3)

This is Socrates' answer to Glaucon. In his answer, he divides the section of intelligible beings into two different segments. Glaucon asks for a further clarification of what Socrates said and speaks of the meaning of Socrates' assertion as he himself understood. He distinguishes two parts of the real and intelligible. He repeatedly puts emphasis on the same point as Socrates did in the previous paragraph. Thought (*dianoia*) lies somewhere between opinion (*doxa*) and understanding (*nous*). An opinion is formed when we perceive a sensible object by our sensory organs such as eyes, nose and ears. In the previous section, we have confirmed that there are four cognitive faculties each of which corresponds to a different group of objects. Now in the passage I mentioned above (511c4-e4), Plato wants to draw a distinction between thought and understanding. Understanding is about the Forms, whereas thought is about geometrical and arithmetical objects.

This is the way how the mathematical objects are distinct from the Forms or the first principles in terms of the epistemic faculties concerned about them, when the Forms and the mathematical objects are conceived of. In order to seek for a mathematical truth, a geometrician should pivot from some hypotheses which they cannot examine. For instance, she cannot delve into some theorems of the square itself. Rather, she takes it for granted that there are some unconditional propositions constitutive of the mathematical knowledge as a

whole such as definitions and axioms. In this way, mathematical investigation is, for Plato, deeply premise-dependent.[13]

Dialectic is different. Although it starts from some hypotheses, as quoted above, it goes beyond the hypotheses where it started in the first place, finally arriving at Forms *qua* Forms (511b2-c3). As this is the case, we can indirectly deduce the nature of the mathematical investigation by contrasting it with the dialectical methods. While the dialectical study leaves behind the premises, the mathematical investigation cannot do so.

In addition, Plato adduces that mathematical truths are revealed with the help of empirical experience. Some mathematical truths cannot be investigated without drawing figures or counting numbers. To be specific, a geometrician studies the nature of a square by drawing a sensible square, since the sensible square imitates the square itself. This applies to the study of other mathematical entities too. Since it is the case, the mathematical investigation is not only dependent on premises. It also relies on our actual experience of drawing some mathematical figures. This feature is crucial, since the Forms are revealed not with the help of sensible experience.[14] It is partly true that we delve into the Forms with the assistance of hypotheses we do not doubt in the first place. However, it is not the case that the Forms are shown by sensible experience, since the Forms are revealed as *unhypothetical*

[13] Plato elliptically repeats the same point by comparing harmonics with astronomy in Book VII. Here is Socrates' answer to Glaucon:

> "I'm dropping the image and say that I'm not talking about these people, but those who we were saying just now we would ask about harmony, since they do the same as they do in astronomy; for they are searching for number in the concord of sounds, but they do not rise to the challenge and inquire which numbers are concordant and which aren't, and why the differences."
>
> (531b8-c4)

In this passage, Astronomy is in parallel with harmonics in the following manner. Plato obviously puts emphasis on the fact that arithmetic, which is the study of numbers, unless it is studied in connection with the Form of Good, has its own limitations that it cannot go over the boundary of premises and presuppositions. This feature was already suggested in the later part of Book VI.

[14] In Book VII, Plato detaches the dialectical method from the mathematical enquiry by arguing that dialectic does not rely on any sensory experiences. See 532a1-b4.

entities (*anupotheton*). This is the second thing which differentiates the philosophical enquiry from the mathematical investigation. It is very clearly shown in Book VII where Plato has the interlocutors repeat the point he made in the previous book:

> "So too, whenever a person attempts to get at what each thing really is by dialectical methods through reason (*dia tou logou*) and without all the sense perceptions (*aneu pasōn tōn aisthēseōn*), and does not give up until he grasps what the real good is by pure intellect (*autē noēsei*) alone, he reaches the true goal of the intelligible world, just as the other man did previously in the visible world."
>
> (532a6-b4)

Plato repeats his point that he already made in the previous book. Dialectic grasps the real Good by one's pure intellect (*autē noēsei*) alone. The intellect or understanding (noēsei) is the technical term he adopted in the previous book in order to describe the epistemic faculty concerning the dialectical methods.[15] Also, the whole process of dialectic is done only through reason *without* having any sense perception.[16] This point was already confirmed in the previous book. Therefore, Plato must have made a distinction between the dialectic and the mathematical investigation.

There is an incisive commitment of Plato to the distinction. In many passages, he puts emphasis on the role of geometry and many mathematical sciences as preparatory steps toward the real things, which are directly cognised by philosophical practice:

> "So do we still need to agree fully on this?"
> "In what respect?"
> "That it is the knowledge of the eternally real and not what comes into being and then passes away."
> "That's easy to do," he said. "Geometry after all is the knowledge of the eternally real."

[15] The use of the word *noēsis* refers back to 511c4ff in Book VI.

[16] In some earlier works of Plato, it seems that he does not manifestly argue that the cognition of the Forms is utterly independent from our sensual experience. In the *Phaedo*, Plato has Socrates investigate the Forms of bigness and smallness by comparing three men with different height. To be specific, Socrates' smallness in comparison with Cebes is present. This is to say Cebes' bigness in relation to Socrates is present. Needless to say, the concepts of bigness and smallness are partly based on our sense-perceptions here, since the interlocutors (Socrates, Simmias and Cebes) are looking at each other in the situation. See *Phaedo* 100d1ff,

> "Then it would be the soul's transport (*holkon psuchēs*) to the truth (*pros alētheian*), my good fellow, and be productive of philosophical thought (*philosophou dianoias*) by directing upward that which we now wrongly direct downward."
>
> (527b2-b19)

The role of geometrical study is to direct upward a future philosopher's soul to the things that are real. For sure, the true geometry, according to Plato, is concerned about the real beings. However, what Plato puts emphasis on here is that geometry education, as a transport (*holkos*) to the truth, leads its students toward the philosophical truth. Therefore, it does not pursue the philosophical truth itself. Rather, it helps the learners to arrive at the philosophical truth. This point is confirmed again in the later passage where Plato characterises astronomy:

> "In that case," I said, "as we approached by geometry making use of the problems (*problēmasin*), so we shall approach astronomy. We'll pass over what's in the heavens if we're really going to take up astronomy and make the natural thinking faculty of the soul useful instead of useless"
>
> (530b7-c3)

Astronomy makes use of problems (*problēmasin*), just like geometry does. Therefore, geometry and astronomy have at least one feature in common. These sciences, though they are possible only by soul's intellectual capacity, do not arrive at the first principle as it stands. It is because they are restricted by their own hypotheses. However, Plato emphasises the role of these sciences because they are useful in activating soul's thinking faculty. If learners of the ideal state arrive at the Forms *qua* Forms at this level, we can justifiably ask why we need the dialectic as the final stage of the education for future philosophers. Therefore, considering that Plato identifies mathematics with astronomy and harmonics in that they all have the same characteristic, I find little reason to identify the mathematical objects with the Forms.

For sure, intensive pronouns are quite often used as a means to denote the Forms in Plato's dialogues. However, it is not sure Plato deploys the pronoun in order to denote the Forms *in a certain passage*. I think there is another reason for not accepting the occurrences of the intensive pronouns as the indication of Forms. There are some passages where the pronouns are deployed even when the discussion in the passages is *not* about the Forms. For

example, Plato uses the expressions such as ‘themselves’ and ‘itself’ in the passages of Book VII quoted next. In the first passage, Plato has Socrates speak as follows:

> “The following,” I said: “these stars that adorn the heavens, since they ornament the visible sky, we think they’re the most beautiful and perfect examples of their kind. And yet they fall far short of the real ones – those courses, represented by real speed and real slowness in real number and in all the real geometrical shapes, which are conveyed in relation to each other and convey what is in them, all of which can be apprehended by reason (*logō*) and thought (*dianoia*), but not by sight. Or do you have another view?”
> “Not at all”
>
> (529c8-d7)

In this passage, Plato has Socrates present the status of the celestial bodies. They are, even if they are considered the most beautiful, merely sensible particulars so that they are inferior compared to the real things that exist. He also makes Socrates repeat the same point in the passage that directly follows:

> “So, Glaucon,” I said, “is this now the very theme that dialectic brings to a conclusion, which our power of sight would imitate though it’s part of the intelligible realm, the sight which we were saying attempts to concentrate its gaze on actual living creatures and *on the stars themselves* (*pros auta ta astra*) and ultimately indeed *on the sun itself* (*pros auton ton hēlion*)?”
>
> (532a1-a6)

In this passage mentioned, we find two occurrences of the intensive pronouns. Plato deploys the phrases ‘on the stars themselves’ (*pros auta ta astra*) and ‘on the sun itself’ (*pros auton ton hēlion*). Even on the reflection on the similes of the *Sun* and the *Cave*, the stars are not identical with the Forms. The status of the sun in the abovementioned passage is more or less controversial. If the ‘sun itself’ here should be construed in an allegorical way, it corresponds to the Form of Good itself, since it is the supreme being within the sensible world. If it is considered merely as an object of our (natural) sight, the sun itself is not a Form at all. Even though the second intensive pronoun ‘itself’ is used to denote the Form of Good, there is no reason to believe that the first intensive pronoun ‘themselves’ is deployed in order to denote a number of different Forms. They could just be the heavenly bodies themselves. We already checked the occurrence of the term ‘themselves’ in the previous passage. Therefore, at least one of these expressions just denotes some sensible beings, though they are celestial bodies.

It implies a possibility that the stars themselves and the sun itself might just signify some intellectual entities which we do not know whether they are Forms or other kinds of intellectual objects.

For this reason, the intensive pronouns in Book VII do not necessarily refer to Forms, since the topic under discussion here is not about the Forms. Rather, they are deployed when Plato wants to put stress on the originality of a certain thing. In this manner, it is a rash conclusion that Plato deploys them just in order to denote Forms.

In addition, I want to show why the modern orthodoxy fails to give a unitary understanding of the dialogue as a whole. All that noted, this dialogue is all-encompassing, starting from the statement that "Justice pays" and going through many different topics. However, one of the prime concerns Plato wants to highlight is that educational process matters in pursuit of the ideal city-state (*kallipolis*). This is the reason why we should educate philosopher-kings who possess the knowledge of what there really are (*ta onta*). In other words, Plato's theoretical philosophy is closely interwoven with the *practical* aspects.

From 536d4, Plato moves back to the educational stages for philosophers-kings. In this book, Plato has the interlocutors speak about some additional subjects that are specifically needed for future philosophers. These subjects should be taught in addition to the basic level subjects, the poetry (*poiētikē*) and the physical education (*gumnastikē*) discussed in Books II and III. Arithmetic, geometry, and astronomy should be taught while the future philosophers are still around twenty years old until they reach 35 years old. For workers, farmers and the auxiliaries (soldiers), neither mathematics nor dialectical education is needed, since both educations are viable only by the soul's intellectual ability. Within the activities of the intellectual capacity, Plato also makes a clear distinction between two abilities within the prime capacity. We have seen this point in the Book VI. This might be the main reason why there are two steps of the education, since the prime capacity is separated into two distinct

faculties, which is thought (*dianoia*) and understanding (*noēsis*). An interesting thing is that Plato repeats exactly the same point in the following book with some minor changes which I introduced in a previous footnote. Knowledge (*epistēmē*) and thought constitute understanding which deals with the real beings (*ousiai*);[17] It is now clear Plato is consistent throughout Books VI and VII in thinking that human capacity concerning intelligible entities is always divided into two different subkinds, each of which deals with a different group of objects. If two faculties are divided, why should we believe that both faculties' objects are the Forms? The first segment concerned about the sensible particulars is divided by two different faculties as well as two different groups of objects. Though they are all perceived by sensory experience, the objects of the first kind are perceived as shadows and reflections, whereas those of the second kind are perceived as some physical entities. As the left segment is divided by two different groups of objects, I argue we have little reason to believe that the right segment is all about the Forms.

We can understand why Plato draws a clear line between preparatory steps and the final stage of education for future philosopher-kings in light of this. A future guardian (philosopher) is not allowed to use the dialectical methods until they are mature enough. Plato warns us about the devastating effects an immature practice of philosophising might bring about, since dialectic is done by disputing things solely by one's argumentation. Before

[17] See 533e4-534a10. It is quite clear that he refers back to the simile of the Line by the phrase "as we were before" (*hōsper to proteron*), since Socrates and Glaucon summarises what they have enquired by introducing the simile in 511d7-e6:

> "You have understood it very adequately," I said. "Now take these four function which are found in the soul in addition to these four segments – understanding (*noēsis*) at the highest level, thought (*dianoia*) at the second, belief (*pistis*) at the third, and apprehension (*eikasia*) by images at the bottom – and put them in proportion according as you think each contains a measure of clarity to the degree that its objects contain a measure of truth."

It is the point Plato makes in Book VI. The differentiation of objects and their corresponding faculties is just repeated in Book VII without any major correction.

becoming mature enough to handle the topic the dialectic is concerned about, they should not be allowed to taste the power of argumentation. This is the reason why the true dialectic, which helps the student to actually philosophise, should be the last step of the education for the philosopher-kings.

Again, this passage reminds us of the relevant discussion made in Book VI, where Plato distinguishes geometrical enquiry from dialectic. When doing dialectic, enquirers are free to discuss anything by going through Forms *qua* Forms. They can also arrive at the *unhypothetical* entities and finally the Form of Good itself. When they treat mathematical entities, this kind of *unlimited* freedom is not allowed, since they do not investigate the hypotheses themselves. Rather, they take for granted that the hypotheses are beyond doubt and true statements.[18]

The fact that Plato draws lines not only between two epistemic faculties but also different steps of education toward the true philosophy constitutes a very good reason for us to reject the modern orthodoxy and support the alternative reading. Focussing only on the relevant passage in Book VI, it is not clear which interpretation is definitely right.[19] The *Republic* is, however, a dialogue as a whole. Plato's investigation into the theoretical features should be understood in light of their practical applications. The differentiation of four distinct entities in Book VI has something to do with the education which leads its students to the way of philosophy. As his education theory in Book VII manifestly shows that the mathematical education differs from the philosophical education, and also that the

[18] Plato's point here is repeated at least two times throughout the dialogue. In Book VI, he distinguishes two ways of investigating intelligible entities. (510b5-511a2) He speaks of almost the same thing in the following book. (533e30534a10)

[19] I personally think that we can find some reasons to believe that the Forms are distinct from the mathematical objects within the boundary of Book VI. However, I cannot confidently argue for it, since the fact that Plato distinguishes two epistemic faculties does not solely constitute a reason for us to believe that he actually makes a distinction between the Forms and mathematical intermediates.

mathematical entities are distinct from the Forms, there is little reason to believe that the square itself and the diagonal itself refer to the Forms.

In this enquiry, I tried to show why mathematical entities differ from the Forms. I argued for the traditional interpretation which heavily relies on Aristotle's testimony. However, I think there are some objections made against my argument. In the next section, I will introduce several possible and actual objections concerning the argument.

Possible Objections and Replies

i. Setting the Problem

A possible response to my argument is based on the assertion that Plato does not make a clear distinction between the mathematical entities and Forms. This is partly true if we keep focussing on the two other similes, because Plato does not make a discussion on the mathematical entities in the similes of the *Sun* and the *Cave*, where the mathematical entities are wholly omitted. In addition, this interpretation may be partly supported by a standard reading of the *Meno*, where Plato does not make a clear-cut distinction between mathematical entities such as specific proportionalities and the Forms[20].

ii. An Argument against Aristotle's Testimony as a Reliable Source

One of the strongest rejections made against the traditional interpretation is that we cannot confidently trust Aristotle as a reliable source of Plato's thought. As Cherniss points out, there is little reason to believe Aristotle, since Aristotle distorted the gist of Plato's philosophy and put it into his own, Aristotelian mould. I partially agree with Cherniss about that Aristotle's testimony is not a reliable source by itself. There is no reason to accept what Aristotle says just because he was one among the closest students of Plato. If we read his *Metaphysics* Book A, we are faced with so-called 'Platonic' thoughts which we barely find in

[20] See the *Meno*, 82aff , where Socrates guides Meno's slave boy to give answers to his questions. In answering Socrates' questions, the slave says that the length of diagonal is presented as a number between 2 and 3, if the length of a side is 2. This enquiry, according to Socrates in the *Meno*, turns out to be possible only by our prenatal knowledge which we re-acquire by the method of recollection (*anamnēsis*).

his dialogues.[21] At the same time, however, I argue that it has to be conceded that at least Aristotle's testimony about the mathematical entities fits the relevant passages in the *Republic* Books VI and VII. His testimony reflects Plato's original passages as they are. For example, the Good is the one and only Form which is good by itself. When it comes to the mathematical entities, this relation does not hold, since we can imagine hundreds of different 'squares' and 'diagonals' each of which is different from the rest. The plurality of the mathematical objects is also present in Plato's assertion that the mathematical entities are investigated with the help of our sensual experience. As an investigator of the mathematical entities, a geometrician can draw hundreds of different diagonals and squares in order to examine the nature of the figures. As far as their originality is concerned, each diagonal is a diagonal itself in relation to the sensible diagonals instantiated in the physical world. Each sensible diagonal is also a diagonal itself only by being a shadow or imitation of some intelligible diagonal. As the distinctiveness of each diagonal is guaranteed, it leaves a room for thinking that there are many different diagonals themselves.[22]

This explanation perfectly fits Plato's image-original relation which holds in Book VI which we examined in the previous section. In the passages, Plato explains how imitation is possible. Imitation is a special relation between the thing that is imitated and the things that imitate it. When Plato wants to show how a Form and the things that imitate it are related to each other, e.g. in the bed example, the Form of bed is thought to be the only bed which really exists, whereas sensible beds and painting of the beds are many.

[21] Cross and Woozley agree with this claim by arguing that there is no occurrence of the mathematical intermediates in the *Republic*. They also point out that there is no reference to the mathematical intermediates in the dialogues earlier than the *Republic*, apart from the *Phaedo* 74c. See Cross and Woozley 1966, 236.

[22] I concede that the square case is problematic, since a square is defined as a rectangular whose two diagonals have the same length. Therefore, all different squares have one shape, though each of these differs from each other by the length of it. However, it is not the case that diagonals have the same shape and length, since each diagonal differs from each other not only by its length but also by the shape it belongs to.

The last thing to note about the uniqueness of the Forms is that Plato himself explicitly argues for it. In Book X, Plato has Socrates speak of the hierarchy of the existing entities. Plato has the interlocutors agreed upon that there are three different beds, which is a painting of bed, a sensible bed, and the Form of bed (*the bed-ness itself*). The Form of bed is the only perfect bed created by gods. The Form of bed functions as an ideal and a model for all the other imperfect beds. From 596a10, Plato argues as follows:

> "Then let's do that now too. Whichever one you want out of the many possibilities. For example, if you like, there are many kinds (*pollai*) of bed and table, I think."
> "Of course."
> "But the Forms connected with these two items are surely just two, one of a bed, other of a table."
> "Yes."
> "And so we usually say that one craftsman makes beds and another the tables we use by looking to a Form of each type of furniture, and other things according to the same principle, but I don't think any craftsman makes the actual Form."
> "How could he?"
>
> (596a10-b9)

After relating a Form to the things which instantiate it, Plato sets a hierarchy of makers. According to Socrates' explanation, there are three groups of craftsmen (*dēmiourgoi*) who make different beds:

> "Not difficult," I said: "one that can be done quickly and anywhere. The quickest perhaps is to take a mirror, if you like, and carry it round with you everywhere. In no time you will make a sun and the heavenly bodies, the earth, yourself, and everything we've just been talking about."
> "Thing we can perceive, yes," he said, "but not, I think, the things that are real (*onta*) in the true sense (*tē alētheia*).
> "Well done!" I said. "You've got the point of my argument. You see, I think the painter too is one of these types of craftsman (*dēmiourgōn*). Isn't that so?
> "Of course."
> "But I think you'll say that what he makes isn't the real thing, although in one sense the painter does make a bed, or is that not so?"
> "Yes, he too makes something that is an appearance (*phainomenēn*) of bed."
>
> (596d8-e15)

Plato summarises the result of this investigation in the following way: there are three different beds. The first one which really exists by its nature; The second one made by a carpenter; And the last one made by a painter (597b3-6). In some ways, both the carpenter

and the painter are called makers, but not real makers, since the only bed is made solely by gods.

One notable thing Plato does here is that he uses plural expressions in referring to the sensible craftworks of craftsmen. He explicitly uses the words such as 'beds' (*klinai*) and 'tables' (*trapezai*). Since the painter imitates the sensible beds, we can justifiably argue that there are many paintings of the beds and tables. However, a Form is the only one concerning one kind. If there were two different Forms of the same kind, gods would merge them into a single Form. Therefore, the plurality of a certain Form is unimaginable.

Grounded on the uniqueness of the Forms, Aristotle argues that the mathematical objects are distinct from the Forms since several instances of the mathematical objects of a certain kind exist. I argue that this assertion can be supported by Plato's differentiation between the Forms and the mathematical entities. As we examined, he says that the image-original relation holds between sensible particulars and a mathematical object or a Form. No sensible particular is perfectly like the Form. A geometrician draws hundreds of diagonals in order to investigate the nature of the diagonal itself. Each abstract diagonal is different from the others, each of which is also a diagonal. Plato also mentions some other mathematical entities such as odd and even numbers, three different kinds of angles and so more (510c1-d3). These passages also remind us of the plurality of the mathematical entities of one kind that Aristotle mentions in *Metaphysics* 987b.

Therefore, I think we have good reasons for believing that Aristotle's testimony about Plato's mathematical objects is reliable. He wants to distinguish the Forms from the mathematical objects by the Forms' uniqueness. After identifying the Forms by applying this criterion, he moves back to the plurality of the mathematical objects of one kind. There are several phrases indicating that Plato has in mind the plurality of the objects. In this way, Aristotle's testimony perfectly fits Plato's explanation on the mathematical entities.

iii. Other Possible Objections

One might object to my argument by pointing out that Plato's classification of the entities is not the same as shown in the relevant passage in Book VI. For sure, both the bed and the square example have it in common that there are occurrences of the image-original relation. In detail, however, they are not the same. In the bed example, the sensible bed made by the carpenter directly imitates the perfect bed, the Form of bed. This relation also holds in the painter's drawn bed. Talking of the idea of the square, numerous sensible squares imitate the idea, the square itself and geometricians would investigate the nature of the square itself by drawing a sensible square.

However, we can notice two differences between the cases. First, there is no occurrence of Platonic mathematical objects in the bed case. The sensible beds directly imitate the Form of bed. The only intervention made here is a craftsman's imitation of the Form. In the square case, it is shown that the drawn square imitates the mathematical square which is an intermediate object between the Forms and the sensible particulars. Second, the ways the mathematical entities and Forms are investigated are also different from each other. In the bed case, the Form of bed can be studied even if we do not have a corresponding sensible object in the physical world insofar as it is the Form. This point was already confirmed in Books VI and VII. Concerning the mathematical entities, the situation is different. Mathematical entities exist independently from the things which imitate them. However, on the way to investigate the mathematical truths, we actively use sense experiences such as drawing lines and figures. (510d5-511a3) Therefore, the criticism that recognising the mathematical entities as different object from the Forms is unfounded,

because such items do not feature in the bed case, since the two cases are fundamentally different in the first place.[23]

In the previous section, I argued that we need to differ mathematical entities from the Forms in order to read the Books from a unitary viewpoint. One of the strongest objections to my argument would be that introducing the distinction deters us from reading the dialogue as a whole. If mathematical objects were distinct from the Forms, we would be able to find their occurrences in the simile of the Cave which directly follows from Book VI. However, there is no occurrence of the mathematical entities as distinct from the Forms in the simile of the *Cave*. I think this is the main reason which leads many commentators to the conclusion that the mathematical objects are lesser Forms in some ways, since they are known without connection to the Form of Good.

It is partly true that the mathematical objects can be known without the knowledge of the first principle, the Form of Good. A geometrician cannot delve into the hypotheses themselves, nor does he cast doubt on them.[24] Therefore, the mathematical enquiry cannot arrive at the Form of Good. But they make argument based on the square itself and the diagonal itself.

It is undeniable that there is no manifest occurrence of the mathematical objects in the simile of the Cave. However, we should be noted that the allegory is adopted for vibrantly describing the whole process of the education for future guardians. After introducing the simile, Socrates and Glaucon investigate the steps of the education. As the prisoner in the *Cave* needs time to be familiarised with the light and the sun itself (), those who will be the

[23] One might object to my argument by contending that there should be an intervention of mathematical entity even in the bed case. She might argue that sensible beds imitate a certain geometrical figure. Since many beds are square-shaped, the sensible beds may directly imitate the Form. This is not clear whether Plato thinks of this image-original relation holds in every case, since Plato's explanation itself leaves some unclear points.

[24] As I introduced in the previous footnotes, this point is repeated at least three times in Books VI and VII.

guardian in the future need some preparatory steps toward the philosophy education (). This point was clearly made in the previous section. Therefore, I argue that there is little reason to reject the thesis that the mathematical entities are distinct from the Forms simply because there is no manifest differentiation in the simile of the *Cave*. When Socrates and Glaucon find out the meaning of the allegory, they make a room for some intermediate beings which are needed for the future philosophers to accept the supreme being, the Form of Good.

Therefore, the objection based on the fact that there is no occurrence of the mathematical entities in the simile of the *Cave* does not hold. Even though they are not distinctively shown as the mathematical entities, Plato wants his readers to think about the intermediary step between sense-perception and philosophical cognition. It is the reason why he mentions the step where the prisoner is getting familiarised with the light and the first cause of the existing light, the sun itself. This intermediate step is materialised when Socrates has the interlocutors what subjects are needed for educating future philosophers of the city-state. It is made up of two different subjects which share very similar characteristics (geometry and astronomy).

Conclusion

The aim of this thesis is to clarify Plato's conception of mathematical entities. In this investigation, I have argued that mathematical entities are essentially distinct from Platonic Forms. In support of this thesis, I first introduced two conflicting interpretations about this matter. I pointed out the locutions which the modern orthodoxy cannot explain. However, this fact was not sufficient to justify the alternative reading that is partly based on the traditional interpretation. I showed that the alternative reading fits some other passages in Book VII, if we have to consider the *Republic* as a unitary dialogue. Lastly, there are some objections to the alternative reading. These objections are indeed to the point. However, I have shown that the alternative reading can give answers to the objections.

I think the alternative reading contributes to understanding Plato's *Republic* in the following ways. First, it helps us understand the controversial passage with respect to the coherence of the text. Reading the Book VI in connection with Book VII, this reading shows Plato's unitary aim in these two books. Second, it does not eliminate the mathematical entities on which Plato's argument concentrates. The mathematical entities are used not only in his epistemology and metaphysics shown in the Similes, but also in the theory of education and psychology. For this reason, ignoring this feature may lead to a serious misunderstanding of Plato's *Republic* as a whole.

It is undeniable that the way Plato indicates the difference between the Forms and the mathematical entities leaves some room for controversies which we need to explain about. However, careful reading of the relevant passages in the two Books will eventually lead to the conclusion that Plato has a very clear conception of the mathematical entities and the Forms which are distinguished from each other group. Even if Plato's distinction was not

perfectly clear, we could value his first attempt to distinguish these kinds of intellectual entities since he was the first who tried it in the history of philosophy.

BIBLIOGRAPHY

Adam, J. 1963. *The Republic of Plato,* 2nd edn. vol. 2. Cambridge: Cambridge University Press.

Annas, J. 1981. *An Introduction to Plato's Republic*. Oxford: Clarendon Press.

Cherniss, H. 1944. *Aristotle's Criticism of Plato and the Academy*, Johns Hopkins Press.

Cross, R.C. and Woozley, A.D. 1966, *Plato's Republic*. London:Macmillian.

Scott, D. 2008, *The Republic*, in the *Oxford Handbook of Plato*, ed. by G. Fine, Oxford University Press.

Yang, M-H. 1999, *The 'square itself' and 'diagonal itself' in Republic 510d, Ancient Philosophy* 19: 31-35.