1. Summary

This report details the processes and practices of an effective control infrastructure of Profit Attribute Analysis (PAA), which is a crucial component and an effective control infrastructure of Profit and loss (P&L) reporting regarding to trading desks. It forms a key to validate and explain revenues from trading desks and will be used in many control processes including trade activity review, residue analysis and valuation control to independently evaluate the fair value of books and records.

The purpose of Profit Attribute Analysis is to validate business-reported revenue against relevant risk sensitivities captured and monitored within each business. A better understanding of methodology behind and find an appropriate standard to control different risk sensitivities enables the Control Team to better manage their daily activities, and proper escalation of risk or booking errors to middle office, also help the market quantitative analysis team to build more accurate and efficient model capturing all the categories regarding different risks, challenge the front office discrepancies, most important, it will also help traders enhance their effectiveness of market hedges hence to justify their strategies.

2. PAA among the P&L Drivers

P&L validation is a crucial part of balance sheet reporting for every company. In investment banks, trading activity should be evaluating together with position-based and risk attribution commentaries, followed with verification of price testing and reconciliation between front office systems and accounting sub-ledgers.

PAA analysis helps to better understand P&L, and also an access to evaluate the effectiveness of market hedges, and the model performance from Quant analysis team. It is also used for back testing of value at risk for desks, as well we an important control method for challenging front office discrepancies, booking errors, and deficiencies.

As depict in the following table, Comprehensive of P&L mainly come from existing positions which mainly include revaluation due to changes in the specific risk factors and P&L from new positions which primarily driven by market gains or losses form transactions executed on the applicable day and commissions or fees also P&L from some other categories such as residual or unexplained P&L[1].

Comprehensive P&L	P&L from Existing Position	MTM Explained P&L	Credit Effect
			Credit Volatility
			Cross Terms & Other
			Equity Delta Effect
			Equity Forward Effect
			FX Effect
			Interest Rate Effect
			Theta
		Unexplained P&L	
	New Trade Activity P&L	Day 1 Trade	
		Trade Amendments	
		Fees & Commissions	
	Residual		

Table 1. Components of comprehensive daily profit and loss on trading desks.

The trade level reconciliations and PAA vs. P&L reviews from key processes around data integrity to ensure both completeness and accuracy of the data, daily PAA review forms an analysis-based procedure, focusing on drivers of P&L through risk positioning, market moves and trading activity.

3. Methodology on PAA

PAA allows the profit and loss to be attributed to different market factors by identifying the various risks, each risk attribution reflects the sensitivity to a unit change in the underlying factor and the daily change in the market factors.

A PAA methodology is a technical document that specifies the definitions and P&L calculation parameters housed in Front Office Risk System There are a combination of PAA methods applied and in use: risk-based attribution and revaluation-based attribution[2].

The P&L is explained by PAA in terms of risk measures produced by the model, and changed in the relevant market data, it is design to capture the P&L through the following risk categories:

- *Credit:* credit risk is the probable risk of loss resulting from a borrower /issuer's failure to pay or meet contractual obligations, termed as;
- *Interest Rate:* Interest rate risk is the risk from fluctuating interest rates, termed as greek 'i';
- *FX:* Foreign Exchange risk is a financial risk that exists when a financial transaction is denominated in a currency other than that of the base currency of the company, termed as greek 'f';

- *Time:* The chance that the cash flows from an investment won't be worth as much in the future because of changes in purchasing power, termed as 't', in the PAA methodology it indicates date of valuation of trades.
- *Other market data factors* such as recovery, possibly volatility for options, etc.

Greeks are the sensitivities of the PV to its market information typical risk sensitivities that calculated in a PAA include:

- Delta: Sensitivity respect to change in the underlying assets' price;
- Gamma: Sensitivity to rate of change in the delta with respect to changes in the underlying price;
- Vega: Sensitivity to change in the volatility of the underlying asset.
- Theta: Measures the sensitivity of the value of the derivative to the passage of time

Delta's - $\frac{\partial p}{\partial c}, \frac{\partial p}{\partial r}, \frac{\partial p}{\partial f}, \frac{\partial p}{\partial t}$ ......; Gamma's - $\frac{\partial^2 P}{\partial c^2}, \frac{\partial^2 P}{\partial r^2,}, \frac{\partial^2 P}{\partial f^2}, \frac{\partial^2 P}{\partial t^2}$ .....; Cross Gamma's - $\frac{\partial^2 P}{\partial c \partial r}, \frac{\partial^2 P}{\partial c \partial t}, \frac{\partial^2 P}{\partial r \partial f}, \frac{\partial^2 P}{\partial r \partial t}$.....

The P&L is defined as $\Delta P = P(c', r', f', t') - P(c, r, f, t)$

Risk Based PAA Methodology: Taylor series are used to compute ΔP:

$$\Delta P = \frac{\partial p}{\partial c}\Delta c + \frac{\partial p}{\partial r}\Delta r + \frac{\partial p}{\partial f}\Delta f + \frac{\partial p}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 P}{\partial C^2}(\Delta c)^2 + \frac{1}{2}\frac{\partial^2 P}{\partial r^2,}(\Delta r)^2 + \frac{1}{2}\frac{\partial^2 P}{\partial f^2}(\Delta f)^2 + \frac{1}{2}\frac{\partial^2 P}{\partial t^2}(\Delta t)^2$$
$$+ \frac{\partial^2 P}{\partial c \partial r}\Delta c \Delta r + \frac{\partial^2 P}{\partial c \partial f}\Delta c \Delta f + \frac{\partial^2 P}{\partial c \partial t}\Delta c \Delta t + \frac{\partial^2 P}{\partial r \partial f}\Delta r \Delta f + \frac{\partial^2 P}{\partial r \partial t}\Delta r \Delta t + \frac{\partial^2 P}{\partial f \partial t}\Delta f \Delta t$$
$$+ \cdots$$

Taylor series is infinite, but in practice for PAA computations and in RB PAA it is restricted to at most second order terms:

$$\Delta P \approx Dleta's + Gamma's + CrossGamma's$$

Brute Force PAA Methodology: The BF PAA is an approach designed to capture the P&L by mechanically shifting the market data items. By definition the P&L can be decompose to:

Credit P&L = $P(c', r, f, t) - P(c, r, f, t)$

Interest Rate P&L = $P(c, r', f, t) - P(c, r, f, t)$

FX P&L = $P(c, r, f', t) - P(c, r, f, t)$

Theta P&L = $P(c, r, f, t') - P(c, r, f, t)$

Credit-IR P&L = $P(c', r', f, t) - P(c, r, f, t) -$ Credit P&L$-$ Interest Rate P&L

The P&L under the BF PAA will be:

$$\Delta P = Credit\ P\&L + Interest\ Rate\ P\&L + FX\ P\&L + Theta\ P\&L + CrossTerms\ P\&L$$

4. Comparison of PAA Methodologies

To be more directly see the discrepancy of two mythologies, consider a trade:

- Pays a single risk-free amount N at time T.

- Suppose the interest rate is constant at Rt=r for all t.

The PV of the trade is $P(r, t) = Ne^{-r\theta(T-t)}$ where $\theta(T-t) = \frac{T-t}{365.25}$, consider trade with N=10M and T= 23-Oct-19, The PV over two business days will be

t1=23-Apr-19 and r1=5%	t2=24-Apr-19 and r2=10%
P(r1,t1)=9,752,766	P(r2,t2)=9,514,246

The P&L is $\Delta P = $ -238,676

Based on Risk-based PAA:

$\Delta P = \frac{\partial p}{\partial t}\Delta t + \frac{\partial p}{\partial r}\Delta r + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}(\Delta r)^2 + \frac{\partial^2 P}{\partial r \partial t}\Delta r\Delta t$, using the Greeks with Δr=5% and Δt=1

RB PAA P&L = -238,619, with unexplained of -58;

BF PAA :

$\Delta P = Credit\ P\&L + Interest\ Rate\ P\&L + FX\ P\&L + Theta\ P\&L + CrossTerms\ P\&L$

= -238,676 which gives a ZERO unexplained.

Thus, based on the projection, BF methodologies seems more accurately describes the profit attribution categories.

In the meantime, there are some constrains when applying both methodologies in reality, that is, the constrain of time on calculating, BF PAA is more accurate and need extremely more time to compute compare to RB PAA. Considering the condition, another applicable way of P&L reporting might be using RB PAA and keep a threshold for adjusting according to the unexplained residuals.

PAA analysis is suitable for market driven attributions, in addition to these effects, there is additional PAA analysis performed on residuals and trading activity. Both of those form part of the exception review for trades breaching thresholds.

5. Other Market Factor Attributions for PAA

- Bond-CDS basis

For cash instruments which trade on price, the Bond-CDS basis, a spread applied to the credit or the interest rate curve in order to match the quoted price, may incorporate in PAA process to improve the accuracy of the model.

To illustrate this, consider the same example, trade with N=100M, maturity T=23-Oct - 19, and r1=5%. If the cash-flow is risky, as an example we use a spread as c=200bps.

The PV is: $P(c, r, t) = Ne^{-(r+c)\theta(T-t)}$

If the trade is priced in the market at 95, then the basis b will be the shift applied to the credit curve such that the PV matches the quoted price. So we have c+b=95, to match the price, we require b=324 bps.

Once the basis is found, we treat it also as any other market data value. Thus, the PV of the trade will depend on an additional component P(c,r,f,t,b). The corresponding PAA will include a basis attribution component P&L:

RB PAA : $\frac{\partial p}{\partial b}$ and $\frac{\partial^2 P}{\partial b^2}$

BF PAA : P(c,r,f,t,b')-P(c,r,f,t,b)

Also, there will be cross terms regarding bond CDS basis. in the meantime, there can be daily changes to the basis without underlying changes in the bond price, such as changes in the credit spreads, interest rates or pull to par affects.

- Theta

Theta is a risk term which calculates the changes in PV of the trade, by moving forward one business day. At time t, for market data (c,r,f) the theta is calculated as

Theta = P(c,r,f, t+1) − P(c,r,f,t) + Cash (t+1)

Conceptually, in the even the market data remains constant, theta will give the P&L.

For cash instruments, pull to par affects will be used to work out if Theta will be positive or negative. If a bond trades at a premium, then the Theta will be negative and vice versa for a bond trading at a discount.

For CDS instruments, if coupon payments were received, then the Thera will be positive and vice versa for a short position.

There are other market data factors: recovery rate, volatility rate for options, basis for bonds etc. All these variables may cause an unexplained in the PAA model and will be incorporated to the P&L buckets.

- Term Structure

In general, market components such as the credit curve (Ct) and interest rate curve (Rt) have term structures. The term structures of these curves will be factored into the PV of the trade, such that we have P(Ct, Rt,f,t), the curve term structure will also be incorporated inito the Talyor Series P&L explanation. Most of the risk and P&L will be concentrated to tenors and buckets closest to the maturity of the trade.

Most single names CDS have roll dates at Mar 20th and Sep 20th in calendar year will have an implicit change to the credit curve term structure. On these dates the five-year maturity date on CDS contracts are rolled forward by six months, this leads to an implied change in the

underlying credit curve tenor points. For Theta and credit risk calculations a business day prior to the single name rolled dates the impending change in the credit curve will be taken into account.

6. Residual Review for PAA

Interpretation of the PAA residual differs depending upon the attribution methodology being used. Where a Brute Force methodology is used the PAA residual represents the component of reported P&L not explained and attributed as a result of revaluation stresses. The primary cause of residuals in a Brute Force PAA is due to not calculating all possible cross effects between market parameters. Where a Risk Based methodology is used the PAA residual represents the difference between the predicted P&L derived from market risk sensitivities, and the actual P&L reported. Residual on a Risk Based methodology suggest an inconsistency between market risk parameters and reported P&L. This can be due to omission of risk parameters (or not using parameters to sufficiently high order) or model performance issues resulting in inaccuracies in reported risks or valuations.

Due to the non-linear nature of derivatives and the calculation methodology for PAA, there is always a small remaining residue. Residuals are reviewed at both an aggregate portfolio level for the business, and on an exception basis for individual trades breaching the prescribed level thresholds.

Each trade causing an exception should be investigate, analyzed and commented upon. There are some common cause of residues due to known issues, such as PAA deficiencies, as there is continuous development of the PAA, and a number of known attributes are being added; or data quality issues or modelling issues within front office risk systems may causing timing issues, or trades misallocations. Exception of cases may include default of a company casing credit volatility change drastically, or derivative contracts moving to its maturity date, etc. All the exception cases should be escalated and investigated to quant analysis team, then allocate manually by controllers to correct PAA attribution.

The result of daily PAA can be further consolidated and to assist to identify common or recurring issues. Trending over time can indicate further insight into recurring issues on certain products or models, help to set appropriate threshold for controllers, and contribute to VaR back testing and business performance evaluation for the whole company.

References:

[1] Market Quantitative Analysis: Profit Attribution Analysis for Credit Flow and EMCT Trades. Citibank, March 30. 2017

[2] EMCT PAA Training Material, Citibank. April 23th, 2019