An Agent-Based Simulation of the White-Selgin Model of Free Banking

By

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Nevertheless, Scotland never experienced a real monetary crisis […]; no depreciation of notes, no complaints and no inquiries into the sufficiency or insufficiency of the currency in circulation etc.

– Karl Marx, Grundrisse, 1939/1993

Abstract

The following work advances an agent-based simulation of the model of free banking put forth by Lawrence White (2008/1984) and George Selgin (1988). We define 'free banking' as a banking system where banks are allowed to issue banknotes competitively, and thus no single institution performs the functions of a central bank. We find this question particularly relevant in light of the recent technological developments in the area of cryptocurrencies which carry the implicit promise of a return to privately supplied money. First, we give a brief overview of the history of free banking, its main characteristics, and its apparent advantages. Next, we present an algorithmic, agent-based simulation of the White-Selgin model.[1] We find that the simulation's behaviour is roughly consistent with both the theoretical predictions and the historical experience, namely that (1) free banks do not expand the supply of liquid assets indefinitely, (2) the system evolves to a state of equilibrium with relatively few bank failures, and (3) the supply of liquid assets correlates positively with its demand. Finally, we draw conclusions with regard to the policy stance towards cryptocurrencies.

[1] For the Pyhon code of the simulation, please see the Jupyter Notebook file submitted along with to this document.

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Introduction

The recent rise of cryptocurrencies has rekindled the debate around privately issued money. The reaction to digital monies has been lukewarm, with both academics and policymakers voicing concerns over the ability of the private sector to supply the public with cash adequately (see, for example, Brainard, 2021; Bullard, 2021; Gordon & Zhank, 2021). However, some academics have emphasised the historical success of free banking, systems where the stock of money in the economy was privately supplied by note-issuing banks and without a central bank (see Selgin, 2013; Luther & White, 2014; Carter & Selgin, 2021).

Given the purported success of free banking systems at keeping the economy in monetary equilibrium by supplying money privately, we were surprised to see virtually no reference to this literature in the academic debate around cryptocurrencies. This apparent lack of awareness has spurred us to investigate the topic further. We have found ample evidence to suggest that free banking systems are no less effective at supplying the public with money and that they indeed might be superior in some aspects to ones where the state has a monopoly over the issue of banknotes. This would also imply that the recent advent of cryptocurrency technologies (and thus privately supplied money) is not nearly as dangerous to the financial system's stability as some commentators would suggest.

Free banking systems are marked by a single distinguishing feature: banknotes are supplied competitively by private banks. Consequently, the state does not have a monopoly on banknotes, nor does any institution perform the functions of a central bank. Under free banking, the supply of money and credit, and therefore interest rates, are mainly determined by market forces.

Contrary to a centrally banked economy, where the banking system operates with two distinct kinds of assets – banknotes, which also fulfil the function of reserves, and deposits – there are three such assets under free banking.

(1) Banknotes are issued by commercial banks (and not a central bank) and are redeemable for the issuing bank's assets.

(2) The money of ultimate redemption (also referred to as 'reserve currency', typically held in physical gold or silver), which constitutes the asset in which banknotes can be redeemed at the counter of the issuing bank;

(3) Deposits are less liquid than banknotes but typically bear a promise of paying interest.

While each bank's clients are entitled to redeem their banknotes for reserves, this rarely happens in a free banking system. More often than not, customers deposit the banknotes in their bank of choice. It is banks, then, who redeem the banknotes of their rivals in return for reserves. (Selgin, 1988, p. 156) The latter mechanism guarantees that no single bank can unilaterally expand its money supply beyond the public's demand for its banknotes and deposits (Selgin, 2017). If any bank tried to do so, it would quickly experience an outflow of reserves to rival banks, ultimately leading to insufficient liquidity and, possibly, to failure.

The surprising consequence of this mechanism is that under free banking, the supply of money is inversely related to the velocity of money (i.e. positively related with the nominal demand for money balances), which is in line with a monetary policy targeting a stable level of nominal spending (Selgin, 1988). This insight is opposed to the most common criticisms levelled at free banking, namely the idea that (1) free banks tend to expand the supply of banknotes indefinitely and (2) that free banking leads to frequent bank failures.

The following document aims to *illustrate* the workings of a free banking system through a relatively simple agent-based simulation. We want to emphasise that we do not intend this thesis to provide *proof* either for or against the success of free banking. We would merely like to demonstrate that even a relatively simple and intuitive model can display the main advantages that such an unregulated banking system can exhibit. We intend this to be a valuable source of inspiration for policymakers and researchers alike. In particular, we derive three working hypotheses from the literature that can be tested on the simulation: (1) that free banks do not expand the supply of liquid assets indefinitely, (2) that the system evolves to a state of equilibrium with relatively few bank failures, and (3) that the supply of liquid assets correlates positively with its demand (and thus negatively with the velocity of money).

Agent-based simulations have been gaining ground in social sciences in the recent past (see, for example, Farmer & Foley, 2009; Squazzoni, 2010; Dawid & Neugart, 2011; Cheng et al., 2012). The purpose of agent-based simulations is to simulate complex phenomena through relatively simple rules of interaction between heterogeneous agents. In our simulation, there are two classes of agents: banks and members of the population. Agents interact according to basic economic principles; banks set their interest rates and seek to increase their equity, and members of the population try to obtain loans at the lowest and deposits at the highest possible interest rates. The money demand of the population is set exogenously, and the primary endogenous output of the model is the money supply. In the simulation, banknotes are supplied competitively and issued by the banks; whenever banknotes are presented to the issuing bank's counter for redemption, the issuer redeems the notes for reserves. The banks' interest rate policy is determined by a function that sets the interest rate based on the properties of the bank's balance sheet. The parameters of this function are sorted randomly for each bank and are reset whenever a bank fails. This gives rise to evolutionary market selection between agents, where only the banks with a fit strategy manage to survive.

Our work shall proceed as follows. First, we draw the theoretical and historical contours of free banking systems by reviewing the academic literature. We go through the various descriptions of free banking and deduce our working definition for this project. Next, the history of free banking is briefly discussed, reviewing the most prominent examples of competitive note issuance. We then briefly describe the model that we believe best captures free banking theoretically and historically, thus discussing the workings of a free banking system. Next, a methodological overview of our work is given. We detail the importance of evolutionary models in our current framework, and we give a detailed, step by step explanation of the algorithm that we implemented.

In the third and most important section, we present the results of the simulation. We do this by running a large sample of simulations over various conditions – with constant exogenous variables, exogenous shocks, and, finally, fluctuating parameters. We illustrate our results by presenting graphs that visualise the most important outputs of the model, and we test our hypotheses by employing linear regression models.

Finally, we formulate policy recommendations based on the above work. We suggest that the best course of action for policymakers is to restrict themselves to keeping the cryptocurrency market functioning by imposing fundamental property rights, enforcing contracts, and protecting citizens from outright fraud, but without intervening overtly in its regulation.

1. Literature

1.1. Definition of free banking & historical overview

One distinctive feature sets apart all examples of free banking mentioned in the literature: banks in a free banking system have the right to issue redeemable claims on their assets. Hereafter, we shall refer to these claims as ’banknotes’, a terminology in line with the historical use of the term and the academic convention. The fact that banknotes are issued competitively in a free banking system also entails that the state does not have a monopoly on money and, consequently, that no institution performs the functions of a modern central bank.

While in most free banking systems specie (particularly gold or silver) has been the asset for which banknotes can be redeemed, a commodity standard in itself does not guarantee that the system operates under free banking (Selgin, 1988, p. 132), nor does a free banking system necessitate a commodity standard to work (Selgin & White, 1994, p. 1722). The reserve currency under free banking could either be specie or fiat money with a fixed supply (or regulated in a tight, non-discretionary fashion).[2] The vital criterion of free banking is for banks to be able to issue claims (i.e. banknotes and deposits) on the reserve currency and for the state to have next to no say in the supply of reserves.

The holders of banknotes can redeem their notes at the counter of the issuing bank at any given time.[3] The bank is then be obliged to pay out the specified amount of reserves in

[2] „By itself, […] free banking does not uniquely specify the base money regime. Base money could be gold or silver, as would be consistent with the evolution of a monetary system in which government had never intervened. Or it could be some fiat money, with the stock of fiat money permanently frozen (or otherwise determined by a strict rule) to eliminate any scope for discretionary monetary policy.” (Selgin & White, 1994, p. 1722)

[3] An exception to this is the so-called option clause, a provisional agreement between bank and customer that allows the bank in question to defer redemption to a pre-determined date. The option

exchange for its banknotes. Direct redemption, however, rarely happens in a mature free banking system. Instead, customers typically exchange unwanted banknotes with ones issued by their bank of choice or simply deposit them at the bank where they usually conduct business. The ultimate money of redemption *de facto* functions as an interbank clearing medium.[4] According to Selgin (1988, p. 31), specie currency in free banking is rarely used for day-to-day transactions. Instead, commodity money accumulates at banks and clearinghouses, which use it as a vehicle for settlements. „Only inside money is held in a mature free-banking system, and a large fraction of this money is deposit money" (p. 156).

The lack of a central bank or a monopoly on money does not mean that free banking systems were entirely free from regulations. Briones and Rockoff (2003, p. 279.) define free banking as "lightly regulated banking". The authors point out that "[t]here are numerous dimensions on which banking freedom can be measured" and that the term "applied […] to banking systems that in fact were regulated along many dimensions and were very far removed from true laissez-faire banking systems". They nevertheless concede that free banking systems "were subject to fewer regulations than the systems they replaced". They point out the freedom to issue banknotes as the characteristic feature of free banking. Table 1. is a summary of all the possible dimensions along which banking can be regulated.

clause was indeed one of the most important market institutions that prevented bank runs and general panics from happening. See, for example, Dowd (1988), and Selgin & White (1997).

[4] This function is facilitated by the presence of clearing houses.

Table 1. – *A checklist of banking restrictions.* Based on Briones & Rockoff (2003, p. 283).

I. Freedom to issue banknotes *A. Banks are allowed to issue both banknotes (paper money) and deposits (and not only deposits).* *B. Banks can freely contract the asset of redemption with their customers (i.e. they are not required to redeem notes and deposits in high-powered money).* *C. Banks have the ability, as per previous agreement with their customers, to delay redemption of banknotes or deposits (i.e. they are not required to redeem notes and deposits instantaneously)* *D. There are no restrictions on the denominations of banknotes (i.e.. banks can issue small-denomination notes as well).*
II. Freedom to lend *A. Banks are not required to back their notes with government bonds.* *B. Banks are not required to hold a minimum reserve of high-powered money.* *C. Banks can invest in long-term real assets such as real estate or corporate stocks. Banks are not limited to short-term nominal debts secured by real assets (the real bills doctrine).* *D. Bank lending is not subject to usury laws.* *E. Banks are not required to make their balance sheets public.*
III. Freedom of entry *A. Potential bankers can start a bank at the time or place of their choosing by following a standard procedure (i.e. bank charters do not require legislative action).* *B. Banks are allowed to open branches.* *C. Banks can be incorporated with limited liability.*
IV. Freedom from regulation by (or help from) a central bank *A. There is no government-owned or controlled central bank that regulates banks or acts as the lender of last resort.* *B. There is no privileged private bank that plays a similar role.*

The criterium of competitive note issue is also the one used in the comprehensive historical work edited by Kevin Dowd (1992a). In it, Schuler (1992a, Table 2.1) lists more than 60 modern examples of free banking, lasting „from a few years to over a century" (p. 9). The volume features prominent examples of free banking, including that of Australia 1817-1911, Canada 1817-1914, Colombia 1871-1886, China c. 1000-1935, France 1796-1803, Ireland 1693-1845, Scotland 1716-1845, Switzerland 1834-1907, and antebellum United States 1782-1863. Out of these, that of Canada and Scotland are by far the most pristine and well-researched examples of free banking (see Selgin, 2017, ch. 2.; Selgin & White 1994; Schuler, 1992b; White, 1992, 2008/1984)

1.2. The White-Selgin model of free banking

Sechrest (2008, pp. 6-10) notes two main competing theoretical models of free banking. The first, formulated by Hayek (1978), postulates a competitively issued, inconvertible paper currencies system. The second, first elaborated by White (2008/1984) and Selgin (1988, 2017), theorises competitively issued, *convertible* banknotes. In line with Sechrest, we shall refer to the latter as the White-Selgin model of free banking.[5]

The present work's theoretical foundations are to be found in the White-Selgin model of free banking. There are at least three compelling reasons to opt for this framework. First, as opposed to Hayek's formulation, the White-Selgin model is a realistic description of virtually every free banking system that is discussed in the literature. There have been no examples of free banking systems working according to Hayek's principles. Secondly, Selgin and White (1987) give a solid account of how their free banking system might evolve purely based on

[5] We nevertheless aknowledge that other authors have made significant contributions to the body of knowledge on free banking, including authors who agree with White and Selgin in their formulation.

mainstream economic praxeology. Third, some remarkable predictions of the White-Selgin model coincide with the behaviour of historical examples in free banking.

As we have stated above, free banks regularly redeem the banknotes of their competitors in exchange for reserves. This mechanism gives rise to the so-called *principle of adverse clearings*. In a free banking system, no individual bank can issue notes beyond the population's demand for its banknotes; if it did so, the notes would soon find their way back into the banking system, and the bank would experience a drain of reserves. Selgin (2017, p. 35) describes the mechanism the following way:

> Banks in a free-banking system may thus be likened to prisoners in a chain gang: escape is impossible for any single prisoner acting alone, and also for the group as a whole because of the difficulty its members will encounter in trying to coordinate their steps. The greater the size of the gang, the more difficult escape becomes.
>
> Because of [adverse clearings], the total volume of money and credit in a free-banking system cannot easily expand beyond limits consistent with a stable overall volume of payments. Once banks have expanded to the point at which their reserve cushions have fallen to some minimal, prudent level, they can expand further only if the demand to hold their notes or deposits increases […].

A free bank can only expand its money supply sustainably if it entices the public to hold more of its banknotes and deposits, either by offering competitive interest rates or better service than its competitors. Secondly, the money supply in a free banking system tends to equal the demand for cash balances.

Remarkably, a free banking system's behaviour is in line with a productivity-norm monetary policy that aims to keep nominal spending stable. Under free banking, whenever the demand for money balances rises (i.e. the velocity of money decreases), the banking system expands credit; whenever the demand for money balances decreases (i.e. the velocity of money increases), the banking system restricts credit. (Selgin, 1988, 2017; Sechrest, 2008)

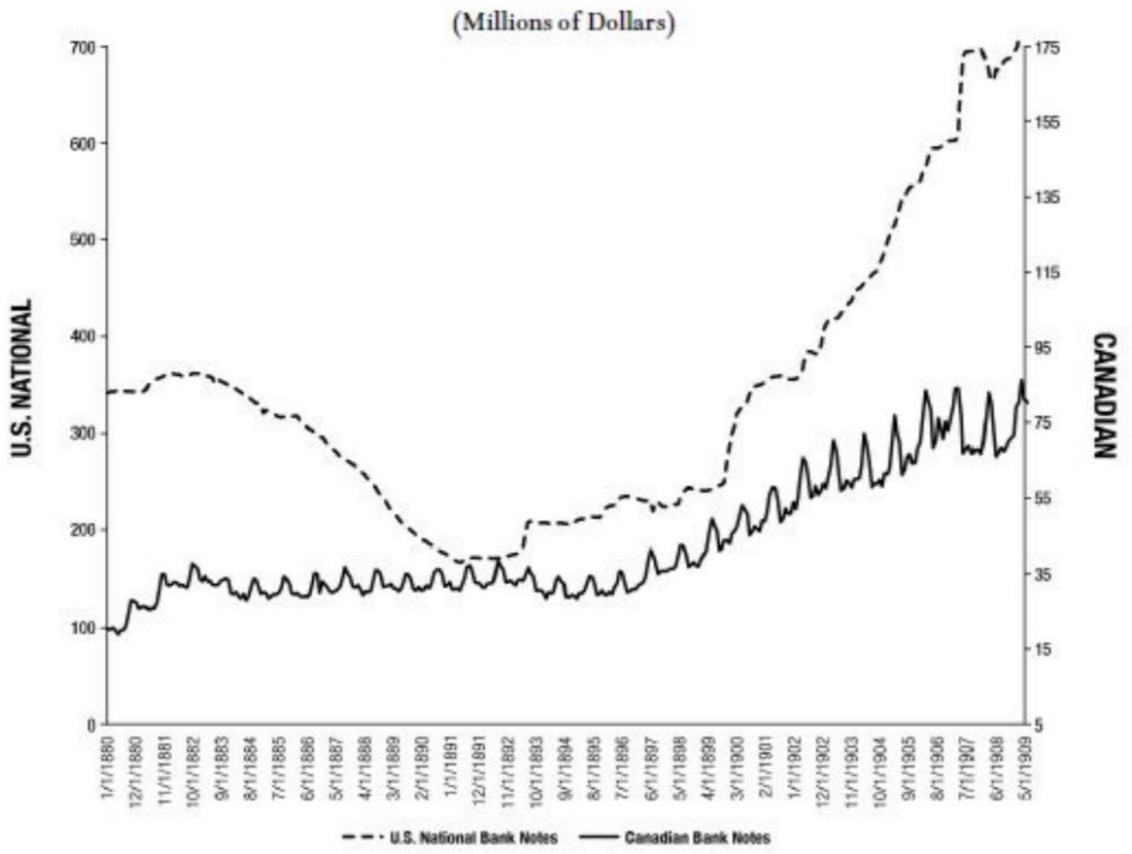

Figure 1. – *Banknotes in Circulation, 1880-1909, monthly, United States (left axis) and Canada (right axis).* Source: Selgin, 2017.

This is in line with the historical experience of free banking in Scotland and Canada who, unlike the centrally banked United States and England, were famously immune to financial crises. In Figure 1., we can see how the supply of banknotes in Canada followed both seasonal and secular trends in the velocity of money. For a survey of the arguments for and against free banking, see Briones and Rockoff (2005).

2. Methodology

2.1. Setting up the simulation

The simulation is structured as follows. There are two kinds of agents; banks, and members of the population (or citizens), with ***I*** denoting the number of banks and ***J*** denoting the number of citizens (indexed as ***i*** and ***j***, respectively).

$$I, J \in \mathbb{N}^{+}$$

The maximum number of iterations is denoted by ***T***. Iterations are indexed by ***t.***

$$T \in \mathbb{N}^{+}$$

Each bank's balance sheet contains four elements: gold (a stand-in for reserves), claims against the population (or loans), banknotes, and equity (the variable that equalises the liability and asset sides of the equation). The interest rate portion of loans and deposit claims is not stored separately but instead added instantly to the balance sheet.

$$gold_{it} \in \mathbb{R}^{+}$$
$$loans_{ijt} \in \mathbb{R}^{+}$$
$$banknotes_{ijt} \in \mathbb{R}^{+}$$
$$deposits_{ijt} \in \mathbb{R}^{+}$$
$$equity_{it} = (gold_{it} + \sum_{j=0}^{J} loans_{ijt}) - (\sum_{j=0}^{J} deposits_{ijt} + \sum_{j=0}^{J} banknotes_{ijt})$$

In the above equations, $banknotes_{ijt}$ denotes the amount of notes issued by bank ***i*** and held by citizen ***j*** at time ***t,*** $loans_{ijt}$ is the amount of claims by bank ***i*** against citizen ***j*** at time ***t***, and $deposits_{ijt}$ is the amount of deposits held by citizen ***j*** at bank ***i*** at time ***t.*** The amount of gold owned by banks and each banks' equity is stored in a vector, while the amount of banknotes, deposits, and loans is stored in a relational matrix of dimensions ***I***×***J***×***3***.

Table 2. – *A schematic representation of a bank's balance sheet.*

Assets	***Liabilities***
Gold ***Loans***	***Equity*** ***Banknotes*** ***Deposits***

Each bank is randomly sorted six unique parameters (denoted by greek letters, where α_{it} is the first parameter of bank ***i*** in iteration ***t***) from a uniform distribution. The parameters describe each bank's strategy in setting deposit and loan interest rates relative to its capital and reserve ratios. Whenever a bank fails, new parameters are sorted from the same distribution (to simulate a new bank being set up in its place with a new strategy). Otherwise, parameters are inherited from the previous iteration. The parameters are stored in a matrix of dimensions ***6×I***.

$$\alpha_{it} \in [0,1)$$
$$\beta_{it} \in [0,1)$$
$$\gamma_{it} \in [0,2)$$
$$\delta_{it} \in [0,1)$$
$$\varepsilon_{it} \in [0,1)$$
$$\zeta_{it} \in [0,1)$$

The above parameters have been chosen based on the empirical observation of the optimal parameters of surviving banks. These parameters give ample space to failure; simulations with broader parameters reach the same conclusions but at the cost of greater computational complexity and longer runtimes.

The total amount of gold in the economy is constant and set exogenously. Throughout the simulations, we used 20 as a value for total gold. This value is arbitrary and does not vitally affect the simulation. Banks start the simulation with an equal share of the total amount of gold in the economy.

$$\sum_{i=0}^{I} gold_{it} = 20$$

$$gold_{i0} = \frac{20}{I}$$

Each citizen's demand for banknotes and deposits is set exogenously. We do this by defining a banknote multiplier and a deposit multiplier, which sets the ratio of total reserves to the total demand for liquid assets in the economy.

$$deposit\ multiplier \in \mathbb{R}^{+}$$

$$banknote\ multiplier \in \mathbb{R}^{+}$$

The reaction of the system to different values of deposit and banknote multipliers (i.e. levels of demand for cash balances) will be the primary subject of our investigation.

We then distribute the demand for assets randomly among citizens, according to:

$$deposit\ demand_j = \frac{a_j \sum_{i=0}^{I} gold_{it} * deposit\ multiplier}{\sum_{j=0}^{J} a_j}$$

$$banknote\ demand_j = \frac{a_j \sum_{i=0}^{I} gold_{it} * deposit\ multiplier}{\sum_{j=0}^{J} a_j}$$

Where

$$a_j \in [1,10]$$

We define the capital ratio and reserve ratio of banks as follows:

$$capital\ ratio_{it} = \frac{equity_{it}}{gold_{it} + \sum_{j=0}^{J} loans_{ijt}}$$

$$reserve\ ratio_{it} = \frac{gold_{it}}{gold_{it} + \sum_{j=0}^{J} loans_{ijt}}$$

or the ratio of capital and gold to total assets or liabilities.

Each bank sets its interest rates according to a sigmoid function that transforms its strategic parameters, capital ratio, and reserve ratio into two interest rates.

$$S(x) = \frac{1}{1+e^{x}}$$
$$di_{it} = S(capital\,ratio_{it} \,*\, \beta_{it} \,-\, reserve\,ratio_{it} \,*\, \alpha_{it} \,-\, \gamma_{it})$$
$$li_{it} = S(-\,\zeta_{it} \,-\, reserve\,ratio_{it} \,*\, \delta_{it} \,-capital\,ratio_{it} \,*\, \varepsilon_{it})$$

Where di_{it} denotes the interest rate set by bank i at time t on its deposits, and li_{it} denotes its loan interest rate. The total demand for credit is a function of the mean interest rate on loans, an exogenous parameter – the loan fraction –, and the total amount of gold in the economy.

$$loan\,fraction \in \mathbb{R}^{+}$$

$$credit\,demand_t = \frac{1}{\sum_{i=0}^{I} l_{it}} * \sum_{i=0}^{I} gold_{it} * loan\,fraction$$

All banks start with an equal amount of gold, zero deposits, zero loans, and zero notes.

2.2. Iterating the simulation

Each iteration of the algorithm has five phases.

I. BANKS EXTEND LOANS TO THE POPULATION

II. CITIZENS SPEND EXCESS BALANCES

III. CITIZENS MAKE DEPOSITS OR WITHDRAW NOTES FROM BANKS

IV. CITIZENS PAY BACK LOANS WITH EXCESS NOTES

V. INSOLVENT & ILLIQUID BANKS GO BANKRUPT

In the first part, randomly sorted citizens take out loans equal to a fixed increment of the demand for credit in the economy until the demand is entirely exhausted. Each citizen chooses the bank that offers the lowest interest rate on its loans, thus creating a bidding competition. Whenever a bank makes a loan, its interest rates are recalculated. Citizens receive the loan in banknotes from the issuing bank. They then owe the bank an amount equal to the loan plus interest. (See Figure 2.)

In part II., citizens compare their total holdings of liquid assets (total banknotes plus deposits held) to its desired size (defined by the *deposit demand* and *note demand* variables); if they have more than the desired amount and if they have available banknotes to spend, they transfer part of their banknotes to a randomly chosen citizen. If they have less liquid assets than needed, they do nothing. When spending, citizens prioritise the notes of banks at which they hold deposits and try to discard the ones where they have no accounts. (See Figure 3.)

In part III., citizens compare the desired amount of *deposits* to their current amount. If they have less than desired and have banknotes in their possession, they deposit the notes to the bank that offers the highest interest rate on its deposits until they reach the desired amount. The interest is instantly added to the amount they deposited, and they become entitled to withdraw it starting from the next iteration. If they have more than desired, they withdraw deposits (i.e. convert them to banknotes) from the bank that offers the lowest interest rate. (See Figure 4.)

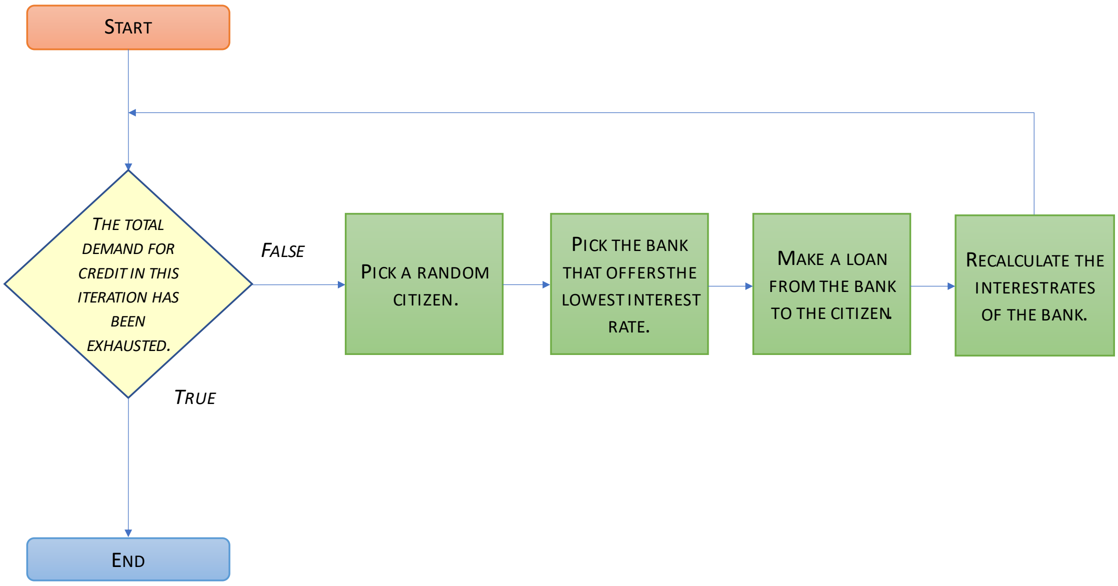

Figure 2. – *Part I. of the algorithm: banks make loans to citizens.*

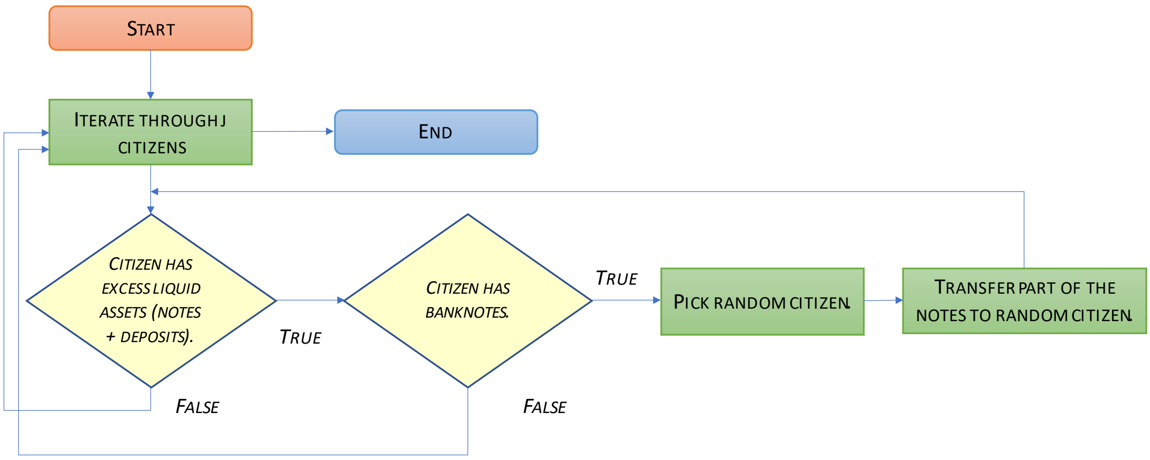

Figure 3. – *Part II. of the algorithm: citizens spend excess balances.*

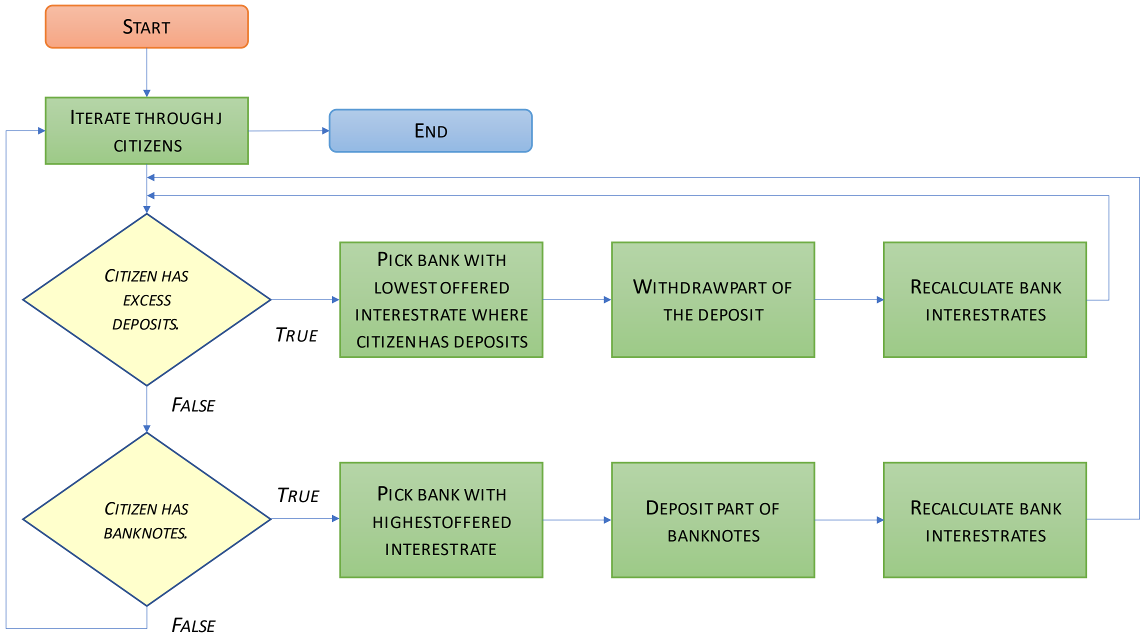

Figure 4. – *Part III. of the algorithm: citizens deposit or withdraw banknotes.*

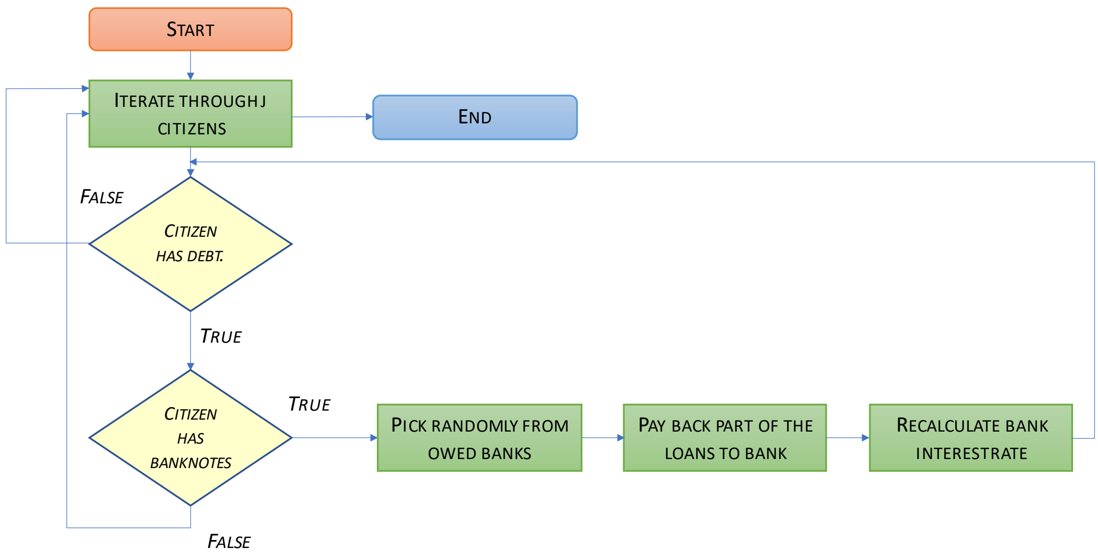

Figure 5. – *Part IV. of the algorithm: citizens pay back their loans.*

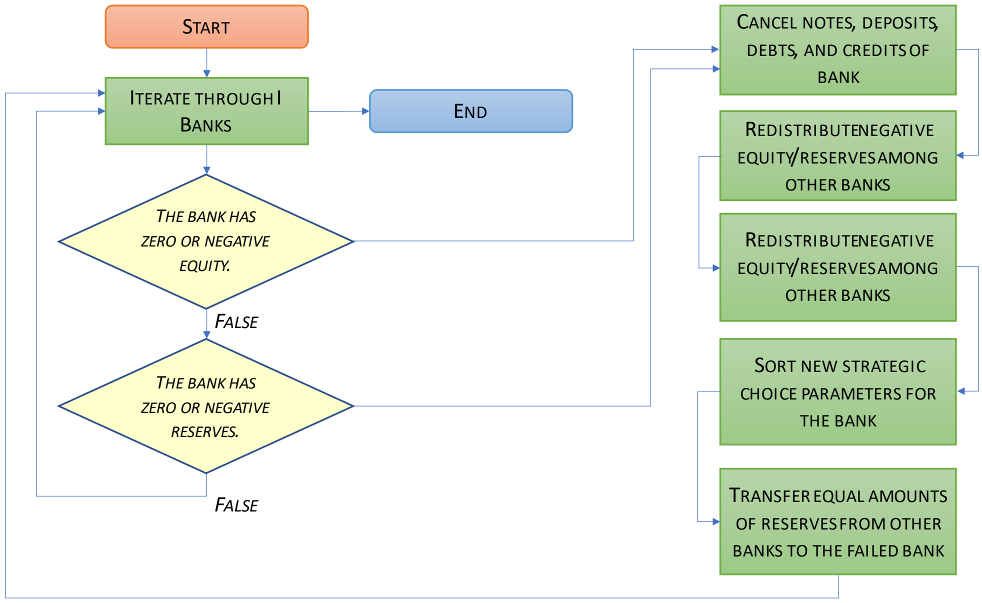

Figure 6. – *Part V. of the algorithm: setting up new banks in place of failed ones.*

In part IV, citizens pay back their loans. Whatever notes they may have will be used to pay back their debt. When paying back loans, they spend the notes of banks with whom they have little or no deposits. (See Figure 5.)

In part V., any bank with zero or negative capital (insolvent banks), or zero or negative reserves (illiquid banks) goes bankrupt. Its deposits, banknotes, and claims get cancelled. A new bank is set up in its place; a small part of other banks' reserves is set apart to finance the new bank. The bank gets sorted new choice variables. (See Figure 6.)

Figure 7. illustrates a schematic representation of the possible lifecycle of a banknote. In this example, Bank A extends a loan to Citizen I. in the form of banknotes. Citizen I. then spends the banknotes, which end up in the hands of Citizen II. and III. Citizen II. proceeds to deposit his newly acquired banknotes to Bank B. In return, Bank B confirms the deposited notes on his account. Bank B redeems the banknotes at Bank A's counter in exchange for reserves.

Citizen III. uses the banknotes to repay his debt with Bank C. Bank C, having received Bank A's banknotes, redeems the notes for reserves.

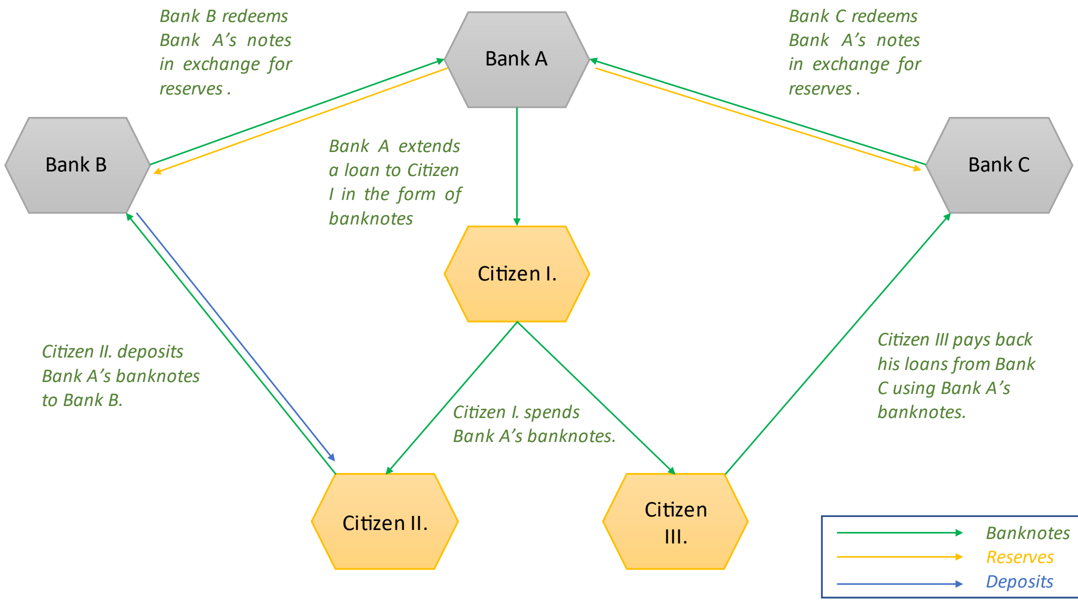

Figure 7. – *A schematic representation of the possible flow of banknotes*.

2.3. Limitations of the model

While the model described here is a good miniature representation of how a free banking system might work, it does have some limitations that deserve our attention. First, it fails to account for some critical economic phenomena that might affect the model's workings. The general price level, for example, is not factored in; the price level's responses to changes in the supply of money constitute a crucial feedback mechanism that would nudge the system towards clearing. Furthermore, the amount of reserves in the simulation is fixed.[6] In reality, the price of reserves (e.g. gold) is affected by the supply of money, and thus the supply (e.g. gold extraction) of reserves as well.

[6] This corresponds to a „global" simulation of free banking, as opposed to one in small, open economy.See, for example, Sechrest (2018)

Second, in the simulation, individuals are agnostic regarding the trustworthiness of banks; all notes circulate at par, and all notes are accepted at every bank's counter, and no bank has difficulty redeeming its notes. Both loans and deposits have infinity maturity, and citizens never default on their loans. Banks in the simulation do not have the option to borrow reserves when running out of liquidity. Finally, failed banks in the model are not taken over by competitors, a mechanism that often avoids total losses to the population in flesh-and-bones free banking.

Third, while the above examples can be regarded as simplifications or approximations of reality, there are some admittedly arbitrary elements to the simulation. For example, citizens prioritise deposits over banknotes, always keeping their deposit balances close to the desired level.[7] This may or may not be accurate in reality. Moreover, some variables, like the demand for liquid assets, are presented as exogenous, which may or may not be the case in reality.

With the above limitations in mind, we believe that the simulation is a reasonable approximation of a mature free banking system with a well-developed clearinghouse, good connections between banks, and an informed market for banknotes.

3. Results of the simulation

Let us restate the three working hypotheses that we have set out to test:

> *(1) Banks in a free banking system do not expand the supply of banknotes indefinitely;*

[7] As observed most clearly in Canada and Scotland, free banking systems tend to develop these instutions fairly quickly.

(2) Free banking systems tend toward equilibria relatively with few bank failures;

(3) The supply of banknotes in a free banking system is inversely associated with the velocity of circulation of banknotes, i.e. positively associated with the demand for cash balances.

The following subchapters will examine the simulation's behaviour under varying conditions and test the above-listed hypotheses.

3.1. Evolution of systems with exogenous variables held constant

First, we look at the algorithm's behaviour when exogenous variables are held constant over 10,000 iterations. We set the dimensions of the system to $I = 20$ and $J = 50$, the deposit multiplier to 1.5, and the note multiplier to 1.8. Figure 8. visualises the evolution of each bank's capital ratio and reserve ratio over time. We see that after an initial period of turbulence with frequent bankruptcies, the system evolves into equilibrium with virtually zero defaults and slight fluctuations, which is in line with our prediction.

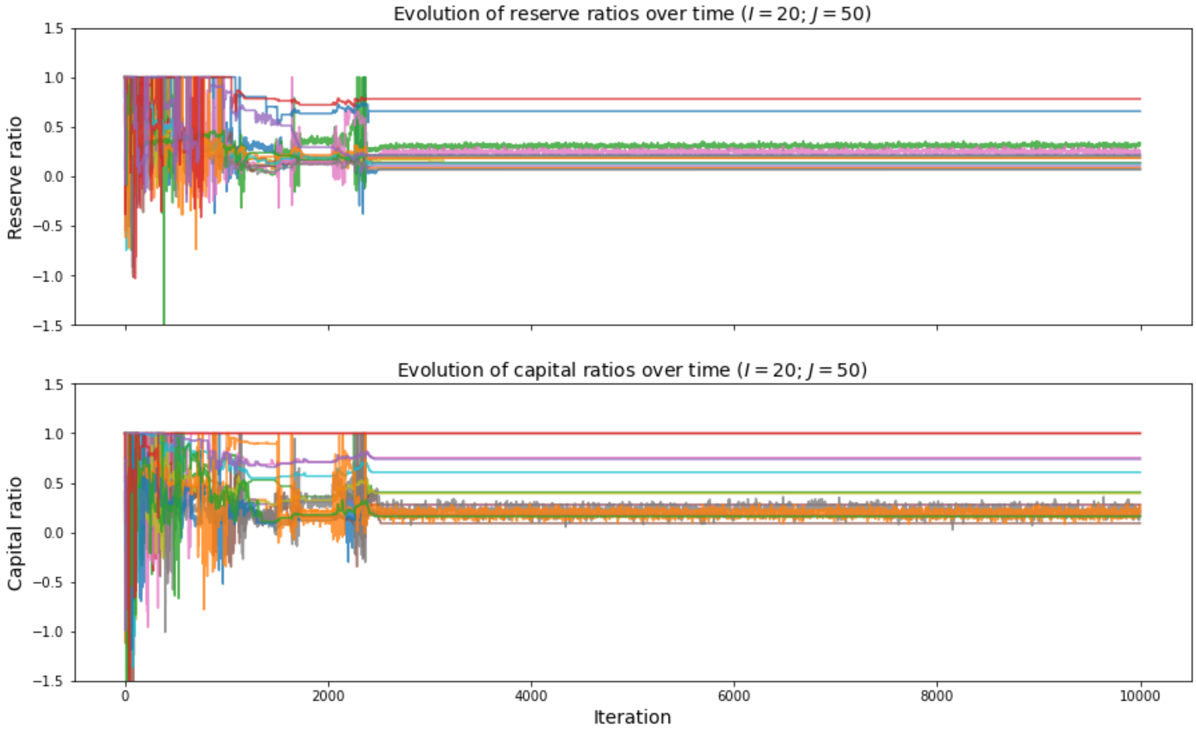

Figure 8. – *The evolution of reserve ratios (top) and capital ratios (bottom) over 10,000 iterations in a system of size I = 20 and J = 50, with each line representing a different bank.*

Figures 9. and 10. illustrate the evolution of the nominal demand and supply for liquid assets – the sum of banknotes and deposits – and its composition in the same system. While there is an oversupply of notes in the economy, banks do not expand their balance sheets indefinitely. As hypothesised, the supply of liquid assets in the system evolves to roughly equal the demand size. Finally, Figure 10. seems to confirm our hypothesis that the number of defaults approaches zero as the system grows more efficient.

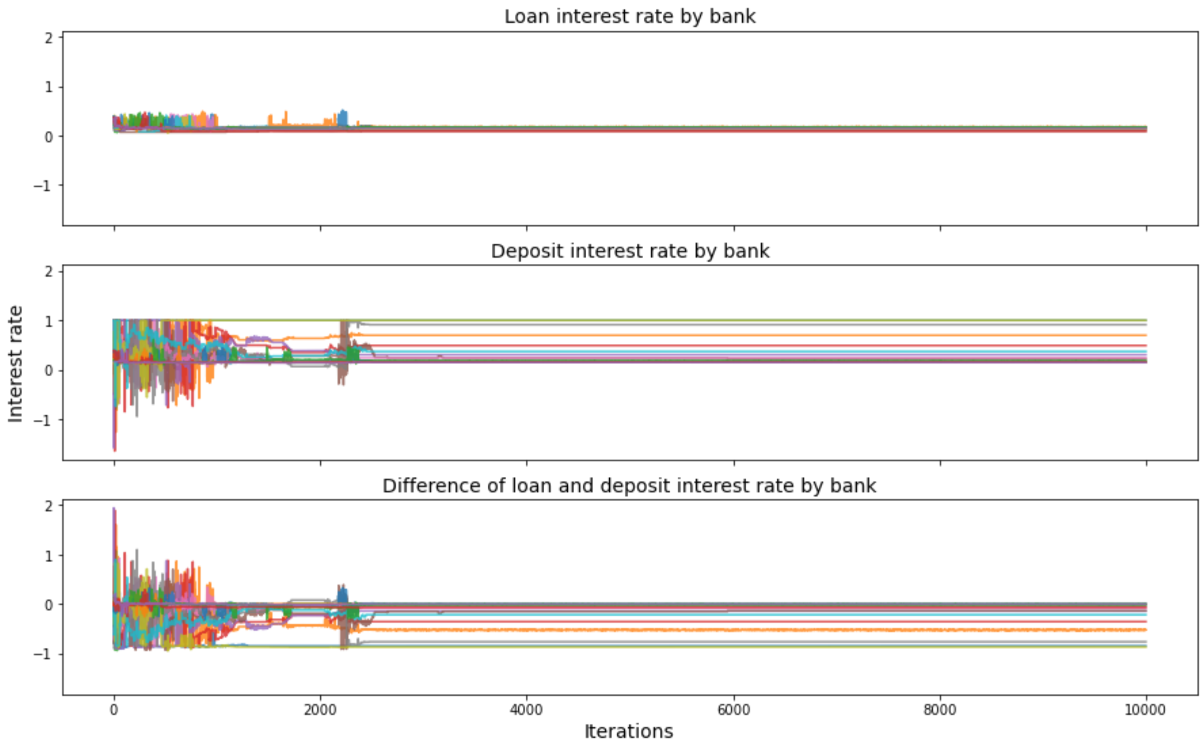

Figure 9. – *The deposit interest rates (top), loan interest rates (middle), and the difference of the two (bottom) over 10,000 iterations with each line representing a different bank.*

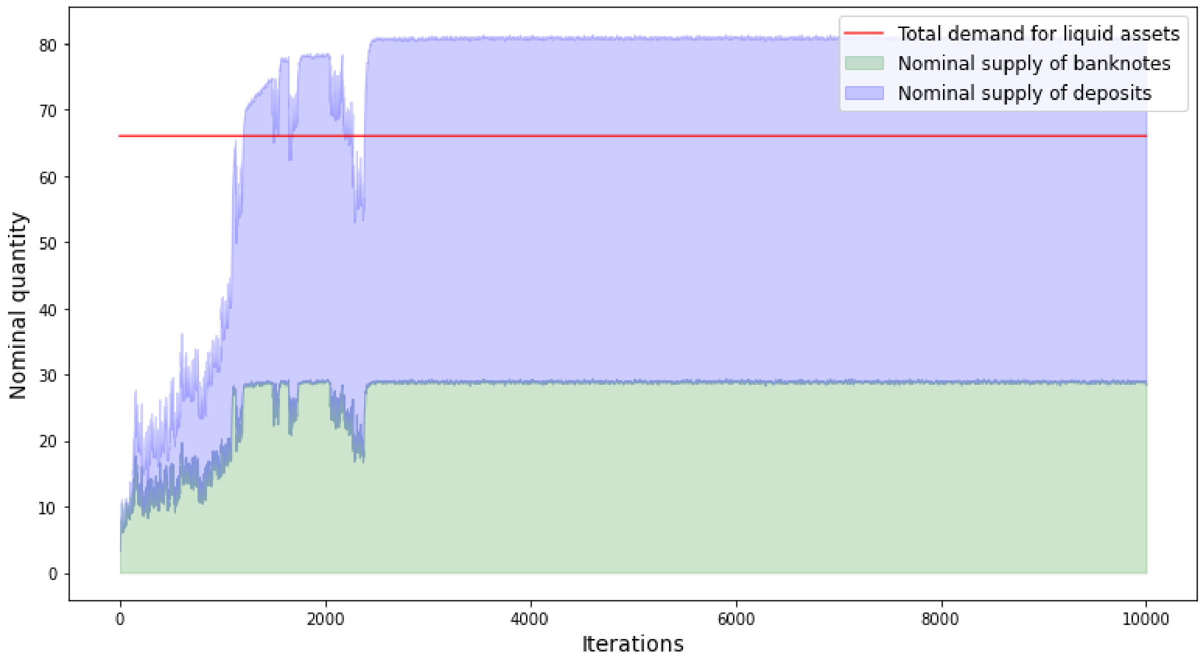

Figure 10. – *The evolution of the nominal supply of banknotes (green area), deposits (blue area), and the nominal demand for total liquid assets (banknotes plus deposits, red line).*

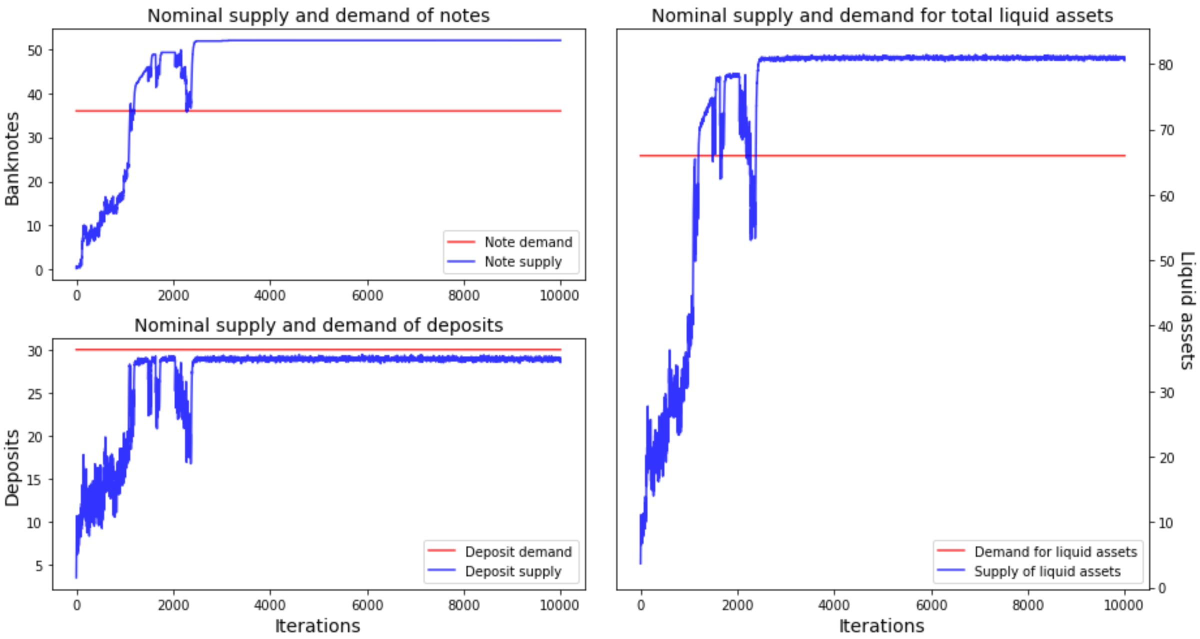

Figure 11. – *The evolution of the nominal supply and demand for banknotes (top left panel), deposits (bottom left panel), and total liquid assets (right panel).*

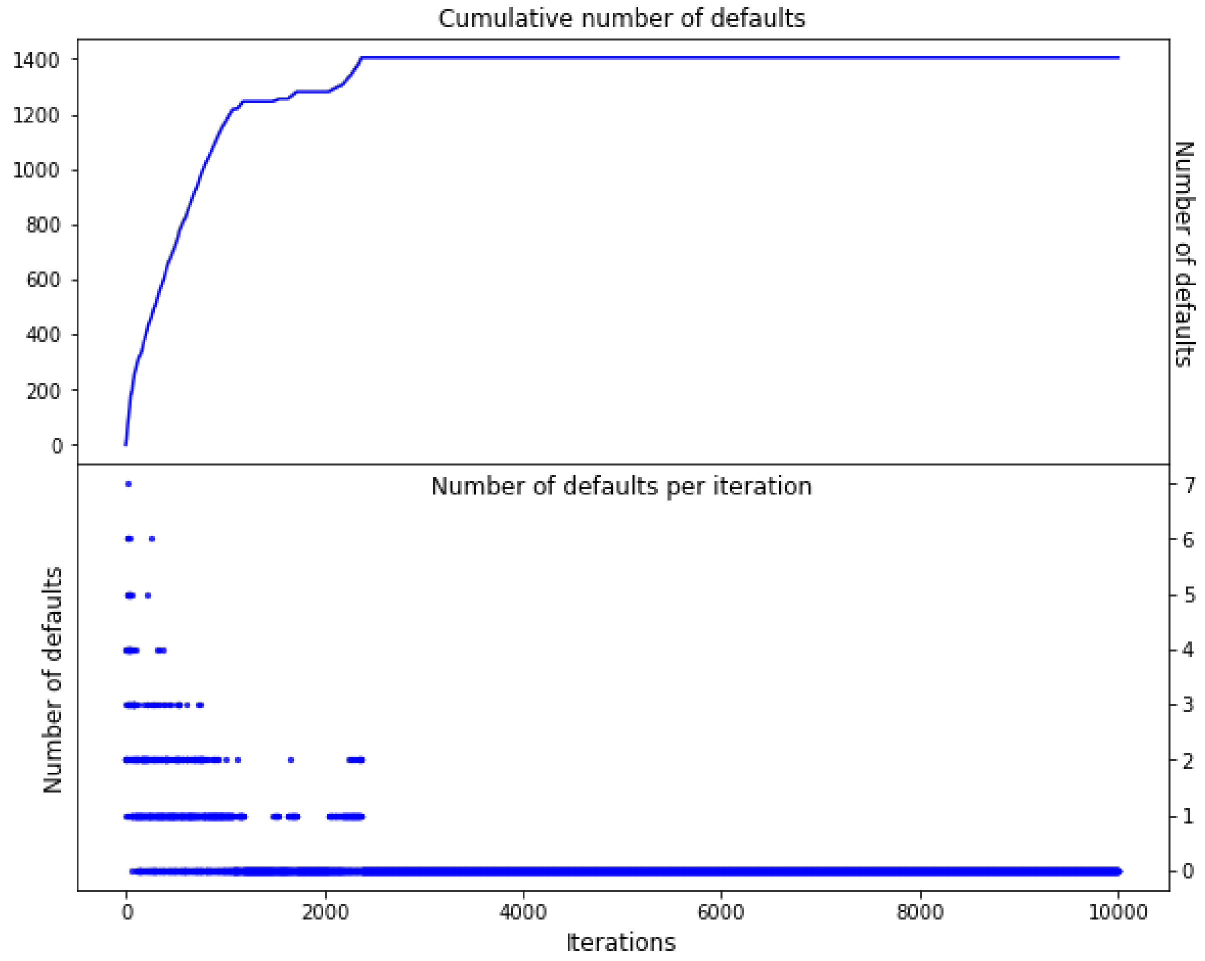

Figure 12. – *The evolution of the cumulative number of defaults (top panel) and the number of defaults per iteration (bottom panel), system of size I = 20 and J = 50.*

Next, we run 20 simulations of different system sizes and 10,000 iterations each. We then test our hypothesis by pooling the data of all 20 simulations and regressing the relative number of defaults over variable ***t***, the elapsed number of iterations. The results are summarised in Table 3. As expected, there is a significant negative association between the relative number of defaults per iterations over the ***t*** amount of iterations, with the P-value virtually equal to zero (due to the large sample size).

Table 3. – *Summary table of the OLS regression of the relative number of bankruptcies over the elapsed number of iterations, with different system sizes.*

OLS Regression Results						
Dep. Variable:	Relative number of bankruptcies			R-squared:		0.040
Model:	OLS			Adj. R-squared:		0.040
Method:	Least Squares			F-statistic:		8829.
No. Observations:	209958			AIC:		-6.904e+05
Df Residuals:	209956			BIC:		-6.903e+05
Df Model:	1					
	coef	***std err***	***t***	***P>|t|***	***[0.025***	***0.975]***
Constant	0.0750	0.000	367.318	0.000	0.075	0.075
Iterations	-3.321e-06	3.53e-08	-93.961	0.000	-3.39e-06	-3.25e-06
Omnibus:	27615.608	Durbin-Watson:	0.001			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	40254.288			
Skew:	1.071	Prob(JB):	0.00			
Kurtosis:	2.892	Cond. No.	1.15e+04			

Next, we test whether banks tend to expand the note supply indefinitely, or if the note supply is likely to adapt to the demand for notes. We test the hypothesis with systems of size ***I = 20, J = 50*** (a size shown to converge to equilibrium reliably), simulating over 25 different combinations for the exogenous values of the banknote multiplier and the deposit multiplier.[8]

[8] Please see the attached Jupyter notebook for a more detailed analysis.

Table 4. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with a system of size I = 20, J = 50.*

OLS Regression Results			
Dep. Variable:	Liquidity supply	R-squared:	0.072
Model:	OLS	Adj. R-squared:	0.072
Method:	Least Squares	F-statistic:	1.930e+04
No. Observations:	250000	AIC:	2.021e+06
Df Residuals:	249998	BIC:	2.021e+06
Df Model:	1		

	coef	***std err***	***t***	***P>|t|***	***[0.025***	***0.975]***
Constant	23.5065	0.277	84.931	0.000	22.964	24.049
Liquidity demand	0.7652	0.006	138.928	0.000	0.754	0.776
Omnibus:	81159.640	Durbin-Watson:	0.006			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	247929.618			
Skew:	-1.701	Prob(JB):	0.00			
Kurtosis:	6.497	Cond. No.	505.			

As expected, the association between the two variables is positive, with a coefficient of 0.72 (the ideal coefficient being one, which would mean a totally elastic supply of banknotes and deposits). The p-value of both coefficients is virtually zero, but this is likely due to the large sample size. The R-squared of the regression is 7.2%.

As we can see from Figure 13., the slope of the regression coefficient approaches one as we include more and more iterations in the model. Conversely, the regression constant tends toward zero. This is ideal; it shows that the supply of liquid assets tends to have a one-to-one relationship with its demand as time passes.

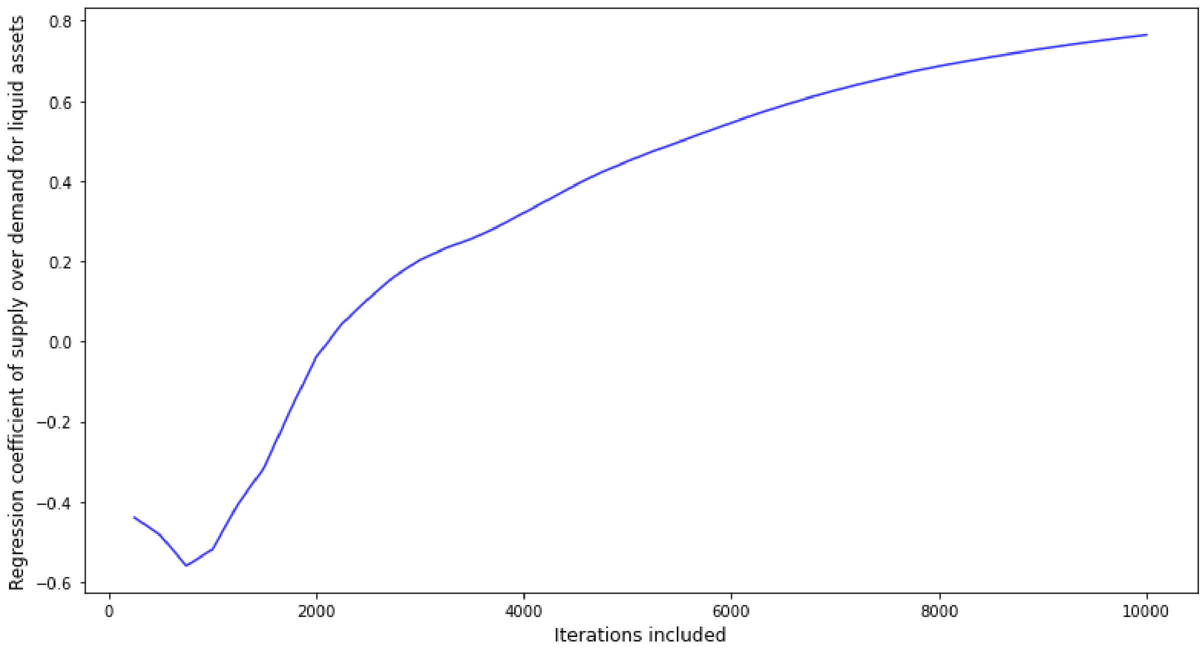

Figure 13. – *Evolution of the regression coefficient of the supply of liquid assets over its demand, as a function of iterations included in the OLS model.*

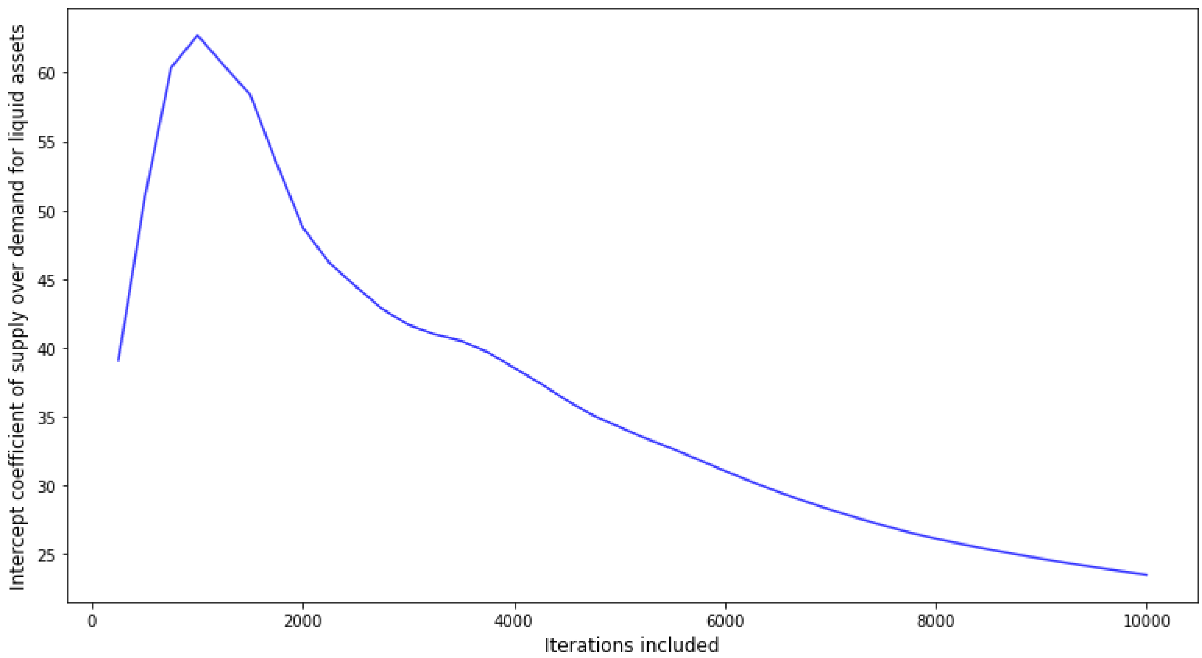

Figure 14. – *Evolution of the OLS constant of the supply of liquid assets over its demand, as a function of the iterations included in the model.*

3.2. Exogenous shocks to systems in equilibrium

When exogenous variables were held constant, the simulation tended to confirm our hypotheses – namely that bank failures approach zero over time and that the supply of liquid assets by the banking sector is determined by its demand. Next, we will test whether the same hypotheses hold when the system is subject to an exogenous shock in the demand for money.

To do so, we simulate systems of size ***I = 20***, ***J = 50*** over 10,000 iterations, each starting from a slightly different demand for liquid assets (i.e. combinations of the demand for banknotes and deposits). In the first instance, we increase the demand for deposits and banknotes by 25% at the 6000th iteration. This would correspond to a sudden increase in the demand for cash balances or a sudden drop in the circulation velocity of money. We then test our hypotheses using the same OLS model, the results of which can be found in Table 5.

Table 5. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with systems of size I = 20, J = 50, with a positive demand shock of 25% at iteration 6,000.*

OLS Regression Results			
Dep. Variable:	Liquidity supply	R-squared:	0.017
Model:	OLS	Adj. R-squared:	0.017
Method:	Least Squares	F-statistic:	5126.
No. Observations:	300000	AIC:	2.596e+06
Df Residuals:	299998	BIC:	2.596e+06
Df Model:	1		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Constant	44.9788	0.225	199.468	0.000	44.537	45.421
Liquidity demand	0.2839	0.004	71.599	0.000	0.276	0.292

Omnibus:	19359.161	Durbin-Watson:	0.006
Prob(Omnibus):	0.000	Jarque-Bera (JB):	22413.748
Skew:	-0.654	Prob(JB):	0.00
Kurtosis:	2.711	Cond. No.	384.

While the coefficient is still significant (likely due to the large sample size), its value is now equal to 0.28, thus much further from the desired level of 1. The R-squared of the model is a mere 1.7%, virtually zero. This would suggest that the supply of banknotes and deposits was insufficient to respond to the positive demand shock.

We can glean what happened from figures 15. and 16., which show the cumulative amount of bank failures, and the evolution of the nominal supply and demand of liquid assets, respectively. The simulated banking system had difficulty supplying adequate liquid assets after a sudden increase in demand. More often than not, the supply of money drops as bankruptcies sweep through the economy, for the supply of liquidity to creep back up to its desired level only later and more slowly. The nature of the simulation can largely explain this: given that banks are first selected in a stable environment with constant exogenous variables, an external shock needs to sort out the ones with adaptable strategies.

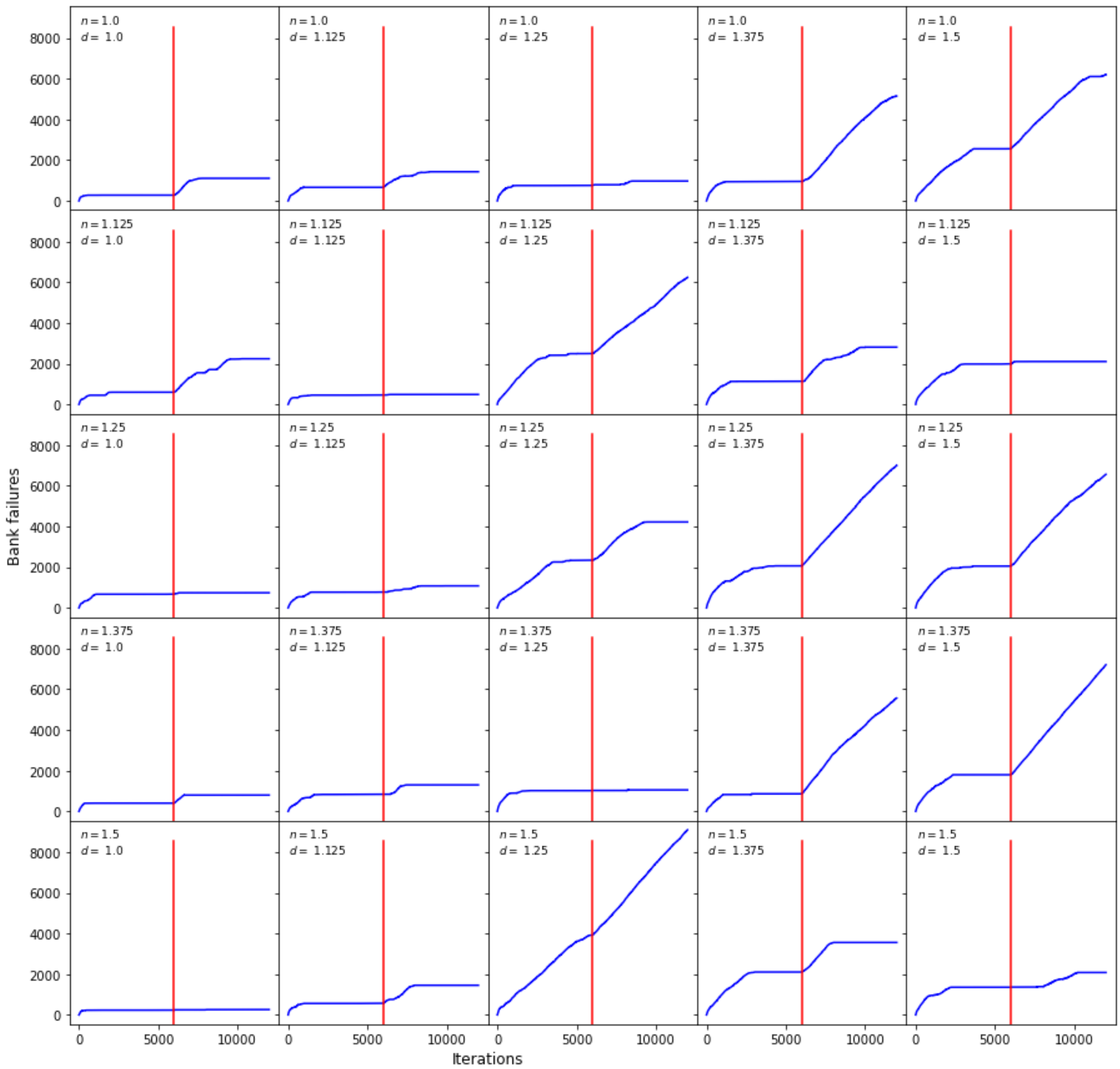

Figure 15. – *The evolution of the cumulative number of bank failures in a system of size I = 20 and J = 50, subject to a positive demand shock at t = 6,000 (red vertical line) and starting from different levels of money demand.*

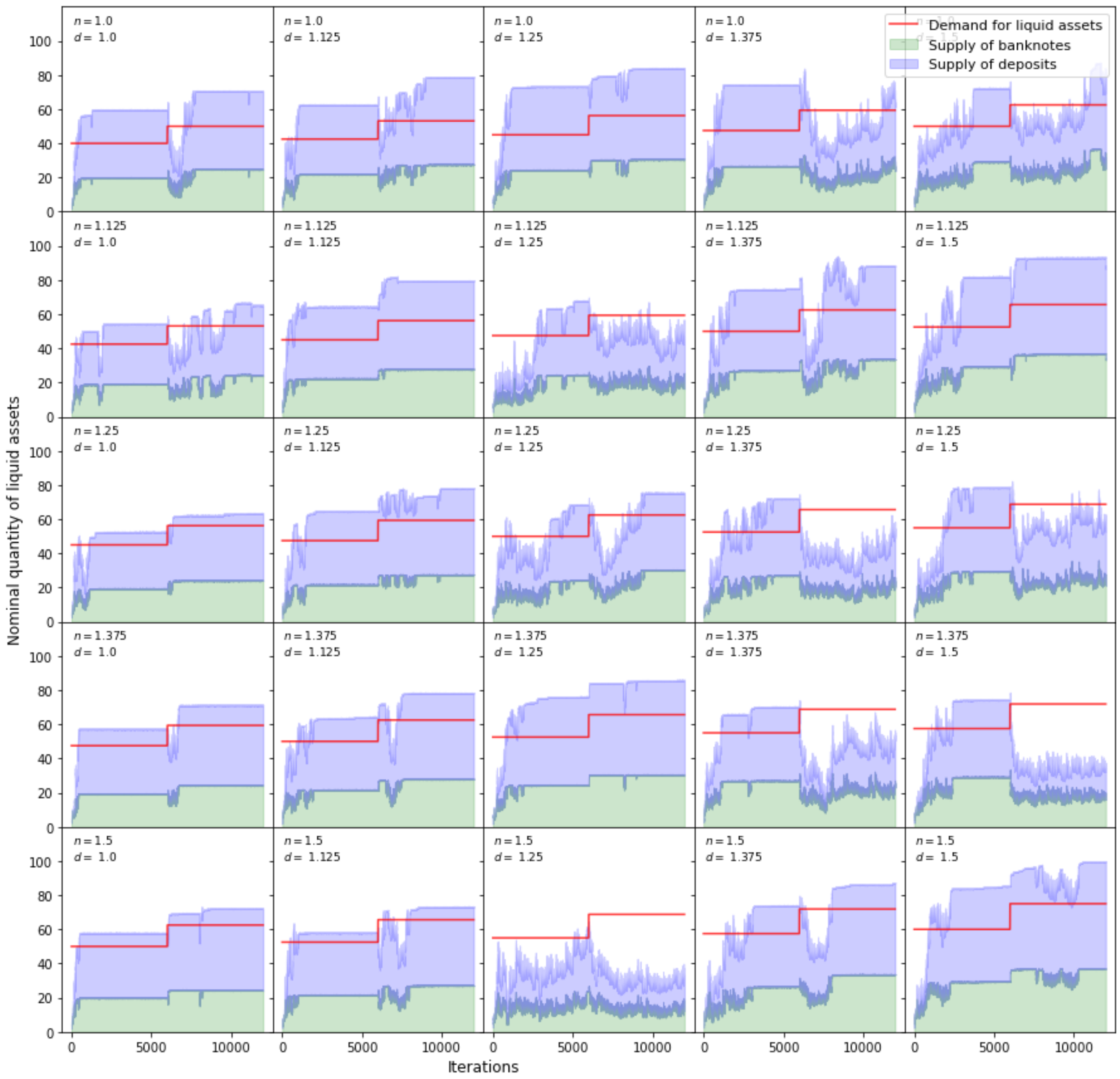

Figure 16. – *The evolution of the nominal supply of banknotes (green area) and deposits (blue area), and the nominal demand for total liquid assets deposits (red line) in a system of size I = 20 and J = 50, subject to a positive demand shock at t = 6,000, and starting from different levels of money demand.*

3.3. Evolution of systems subject to exogenous fluctuations

Finally, we simulate and test systems where the exogenous values of the demand for banknotes and deposits fluctuate according to sine waves of varying frequencies. Figure 17. illustrates how the simulation adapted to the different combinations of frequencies. Table 6.

summarises the OLS regression of the demand for notes and deposits over the supply of liquid assets. Both the visualisation and the regression model confirm that the system did react to changes in the demand for money, but it did so in a relatively inelastic way -

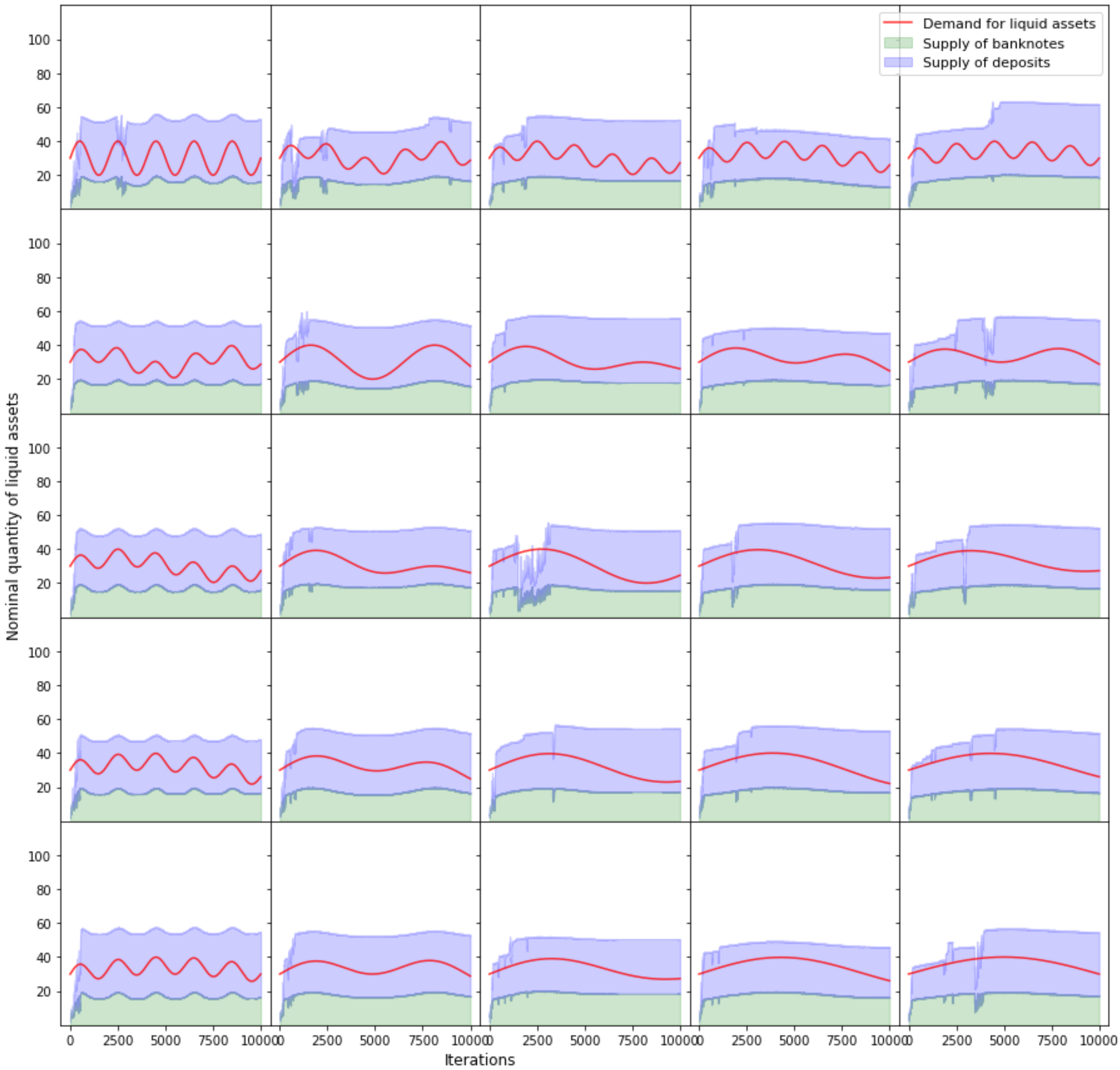

Figure 17. – *The evolution of the nominal supply of banknotes (green area) and deposits (blue area), and the nominal demand for total liquid assets (red line) in a system of size I = 20 and J = 50, subject to fluctuating demand.*

Table 6. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with systems of size I = 20, J = 50, with fluctuating demand for liquid assets according to sine waves of varying frequency.*

OLS Regression Results			
Dep. Variable:	Liquidity supply	R-squared:	0.001
Model:	OLS	Adj. R-squared:	0.001
Method:	Least Squares	F-statistic:	306.6
No. Observations:	250000	AIC:	1.715e+06
Df Residuals:	249998	BIC:	1.715e+06
Df Model:	1		

	coef	std err	t	P>|t|	[0.025	0.975]
Constant	48.4297	0.097	499.815	0.000	48.240	48.620
Liquidity demand	0.0515	0.003	17.509	0.000	0.046	0.057

Omnibus:	154209.719	Durbin-Watson:	0.008
Prob(Omnibus):	0.000	Jarque-Bera (JB):	1801908.472
Skew:	-2.838	Prob(JB):	0.00
Kurtosis:	14.864	Cond. No.	214.

Conclusion & Policy Recommendations

The theory and the practical experience of free banking suggest that unregulated banking with competitive note issuance is not inferior to central banking. Our simulations partially confirmed this insight. The agent-based model evolved into a state with very few bank failures and a money supply strongly correlated with money demand. This happened even though the banks' strategies were randomly selected and the only force to nudge the system towards equilibrium was market selection. When subject to a positive demand shock, the simulated banking system could expand the supply of money relatively quickly. On the other hand, banks had a much harder time adapting to negative shocks; the system typically failed to contract the money supply. This failure, however, is likely due to the idiosyncrasies of the model, which are largely arbitrary.

Our simulation worked with some strong modelling assumptions. The general price level, for example, is not factored in; the amount of reserves is fixed, and there is no open market for reserves (as opposed to, for example, gold). Individuals are agnostic regarding the trustworthiness of banks; all notes circulate at par, and all notes are accepted at every bank's counter. Both loans and deposits have infinity maturity, and citizens never default on their loans. Banks in the simulation do not have the option to borrow reserves when running out of liquidity. Finally, failed banks in the model are not taken over by competitors, a mechanism that often avoids total losses to the population in flesh-and-bones free banking. We view our simulation as complementary to the empirical, historical, and theoretical evidence presented above in light of these limitations.

This experience would suggest that a laisser-faire approach to the cryptocurrency market might be a viable path to take for policymakers. Cryptocurrencies are, after all, explicit attempts to create privately issued money, not at all unlike in the era of free banking. If we wish to recreate the financial stability that characterised free banking systems such as that of Scotland or Canada, governments must keep themselves to enforce fundamental market institutions in the realm of cryptocurrencies. This includes protecting property rights, protecting customers from fraud, intellectual property, the enforcement of contracts, and taxation. Further regulating cryptocurrencies might forever deprive policymakers of a historic opportunity to bring back, at least partly, a functioning private market for money.

Appendices

Appendix A. – Negative demand shock

We repeat the previous experiment, but this time with a *negative* demand shock of 25%. The results of the simulations are summarised in Table 7. We can see that the system does a much worse job adapting to the negative shocks; there is a negative association between the supply and demand for liquid assets, the coefficient's value being 1.41. This result suggests that the supply of banknotes and deposits in the simulation is highly inelastic even in the face of a sudden increase in the public's willingness to hold money. This picture, however, is only partially confirmed by Figure 18. In it, we see that systems with lower multipliers had an easier time adapting to the shock.

Table 7. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with systems of size I = 20, J = 50, with a negative demand shock of 25% at iteration 6,000.*

OLS Regression Results			
Dep. Variable:	Liquidity supply	R-squared:	0.063
Model:	OLS	Adj. R-squared:	0.063
Method:	Least Squares	F-statistic:	2.000e+04
No. Observations:	300000	AIC:	2.388e+06
Df Residuals:	299998	BIC:	2.388e+06

	coef	std err	t	P>|t|	[0.025	0.975]
Constant	78.9694	0.137	575.876	0.000	78.701	79.238
Liquidity demand	-0.4367	0.003	-141.433	0.000	-0.443	-0.431

Omnibus:	43899.773	Durbin-Watson:	0.007
Prob(Omnibus):	0.000	Jarque-Bera (JB):	80456.762
Skew:	-0.942	Prob(JB):	0.00
Kurtosis:	4.699	Cond. No.	258.

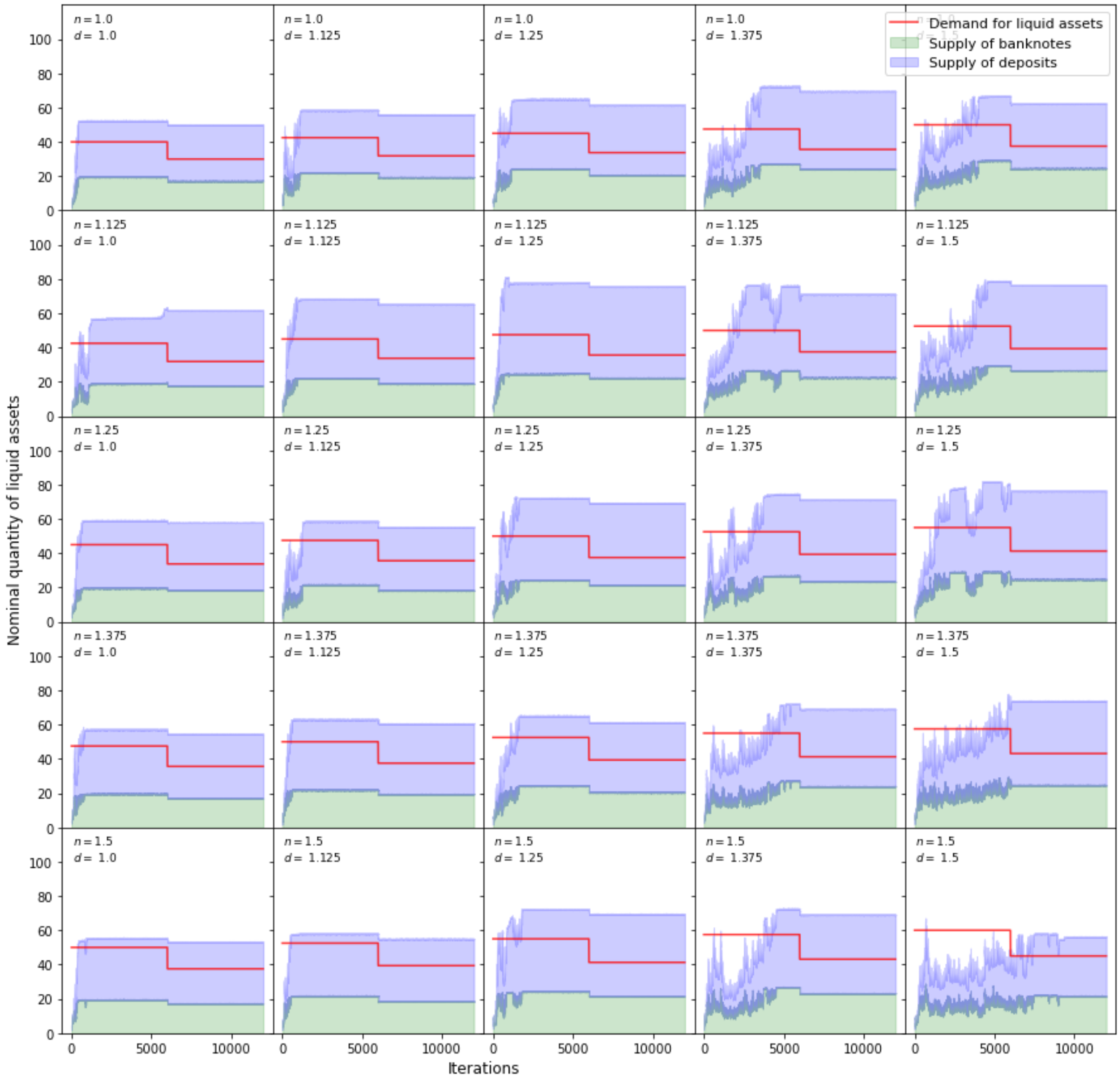

Figure 18. – *The evolution of the nominal supply of banknotes (green area) and deposits (blue area), and the nominal demand for total liquid assets deposits (red line) in a system of size I = 20 and J = 50, subject to a negative demand shock at t = 6,000, and starting from different levels of money demand.*

Appendix B. – List of Tables

Table 1. – *A checklist of banking restrictions.* Based on Briones & Rockoff (2003, p. 283).

I. Freedom to issue banknotes
A. Banks are allowed to issue both banknotes (paper money) and deposits (and not only deposits).
B. Banks can freely contract the asset of redemption with their customers (i.e. they are not required to redeem notes and deposits in high-powered money).
C. Banks have the ability, as per previous agreement with their customers, to delay redemption of banknotes or deposits (i.e. they are not required to redeem notes and deposits instantaneously)
D. There are no restrictions on the denominations of banknotes (i.e.. banks can issue small-denomination notes as well).
II. Freedom to lend
A. Banks are not required to back their notes with government bonds.
B. Banks are not required to hold a minimum reserve of high-powered money.
C. Banks can invest in long-term real assets such as real estate or corporate stocks. Banks are not limited to short-term nominal debts secured by real assets (the real bills doctrine).
D. Bank lending is not subject to usury laws.
E. Banks are not required to make their balance sheets public.
III. Freedom of entry
A. Potential bankers can start a bank at the time or place of their choosing by following a standard procedure (i.e. bank charters do not require legislative action).
B. Banks are allowed to open branches.
C. Banks can be incorporated with limited liability.
IV. Freedom from regulation by (or help from) a central bank
A. There is no government-owned or controlled central bank that regulates the banks or acts as the lender of last resort.
B. There is no privileged private bank that plays a similar role.

Table 2. – *A schematic representation of a bank's balance sheet.*

Assets	***Liabilities***
Gold ***Loans***	***Equity*** ***Banknotes*** ***Deposits***

Table 3. – *Summary table of the OLS regression of the relative number of bankruptcies over the elapsed number of iterations, with different system sizes.*

OLS Regression Results			
Dep. Variable:	Relative number of bankruptcies	R-squared:	0.040
Model:	OLS	Adj. R-squared:	0.040
Method:	Least Squares	F-statistic:	8829.
No. Observations:	209958	AIC:	-6.904e+05
Df Residuals:	209956	BIC:	-6.903e+05
Df Model:	1		

	coef	***std err***	***t***	***P>|t|***	***[0.025***	***0.975]***
Constant	0.0750	0.000	367.318	0.000	0.075	0.075
Iterations	-3.321e-06	3.53e-08	-93.961	0.000	-3.39e-06	-3.25e-06
Omnibus:	27615.608	Durbin-Watson:	0.001			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	40254.288			
Skew:	1.071	Prob(JB):	0.00			
Kurtosis:	2.892	Cond. No.	1.15e+04			

Table 4. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with a system of size I = 20, J = 50.*

OLS Regression Results			
Dep. Variable:	Liquidity supply	R-squared:	0.072
Model:	OLS	Adj. R-squared:	0.072
Method:	Least Squares	F-statistic:	1.930e+04
No. Observations:	250000	AIC:	2.021e+06
Df Residuals:	249998	BIC:	2.021e+06
Df Model:	1		

	coef	***std err***	***t***	***P>|t|***	***[0.025***	***0.975]***
Constant	23.5065	0.277	84.931	0.000	22.964	24.049
Liquidity demand	0.7652	0.006	138.928	0.000	0.754	0.776

Omnibus:	81159.640	Durbin-Watson:	0.006
Prob(Omnibus):	0.000	Jarque-Bera (JB):	247929.618
Skew:	-1.701	Prob(JB):	0.00
Kurtosis:	6.497	Cond. No.	505.

Table 5. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with systems of size I = 20, J = 50, with a positive demand shock of 25% at iteration 6,000.*

OLS Regression Results			
Dep. Variable:	Liquidity supply	R-squared:	0.063
Model:	OLS	Adj. R-squared:	0.063
Method:	Least Squares	F-statistic:	2.000e+04
No. Observations:	300000	AIC:	2.388e+06
Df Residuals:	299998	BIC:	2.388e+06

	coef	std err	t	P>|t|	[0.025	0.975]
Constant	78.9694	0.137	575.876	0.000	78.701	79.238
Liquidity demand	-0.4367	0.003	-141.433	0.000	-0.443	-0.431

Omnibus:	43899.773	Durbin-Watson:	0.007
Prob(Omnibus):	0.000	Jarque-Bera (JB):	80456.762
Skew:	-0.942	Prob(JB):	0.00
Kurtosis:	4.699	Cond. No.	258.

Table 6. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with systems of size I = 20, J = 50, with fluctuating demand for liquid assets according to sine waves of varying frequency.*

OLS Regression Results						
Dep. Variable:	Liquidity supply	R-squared:	0.001			
Model:	OLS	Adj. R-squared:	0.001			
Method:	Least Squares	F-statistic:	306.6			
No. Observations:	250000	AIC:	1.715e+06			
Df Residuals:	249998	BIC:	1.715e+06			
Df Model:	1					
	coef	std err	t	P>|t|	[0.025	0.975]
Constant	48.4297	0.097	499.815	0.000	48.240	48.620
Liquidity demand	0.0515	0.003	17.509	0.000	0.046	0.057
Omnibus:	154209.719	Durbin-Watson:	0.008			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	1801908.472			
Skew:	-2.838	Prob(JB):	0.00			
Kurtosis:	14.864	Cond. No.	214.			

Table 7. – *Summary table of the OLS regression of the nominal liquidity supply over the liquidity demand with systems of size I = 20, J = 50, with a negative demand shock of 25% at iteration 6,000.*

OLS Regression Results						
Dep. Variable:	Liquidity supply	R-squared:	0.063			
Model:	OLS	Adj. R-squared:	0.063			
Method:	Least Squares	F-statistic:	2.000e+04			
No. Observations:	300000	AIC:	2.388e+06			
Df Residuals:	299998	BIC:	2.388e+06			
	coef	std err	t	P>|t|	[0.025	0.975]
Constant	78.9694	0.137	575.876	0.000	78.701	79.238
Liquidity demand	-0.4367	0.003	-141.433	0.000	-0.443	-0.431
Omnibus:	43899.773	Durbin-Watson:	0.007			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	80456.762			
Skew:	-0.942	Prob(JB):	0.00			
Kurtosis:	4.699	Cond. No.	258.			

Appendix C. – List of Figures

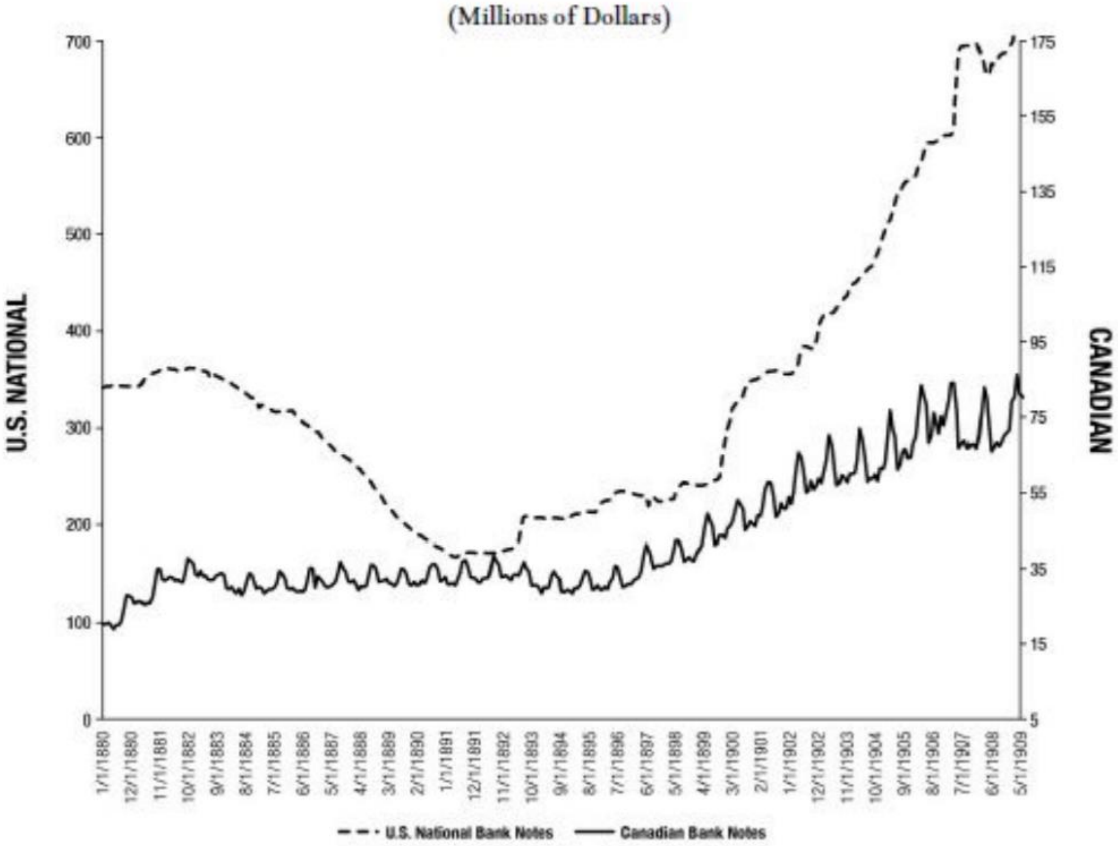

Figure 1. – *Banknotes in Circulation, 1880-1909, monthly, Canada (right axis) and United States (left axis).* Source: Selgin, 2017.

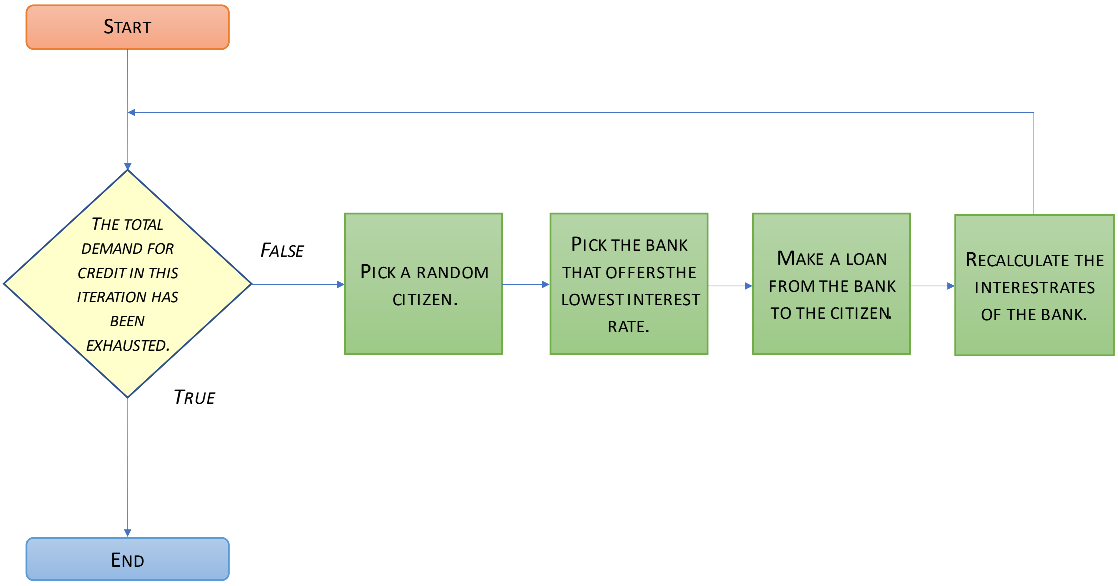

Figure 2. – *Part I. of the algorithm: banks make loans to citizens.*

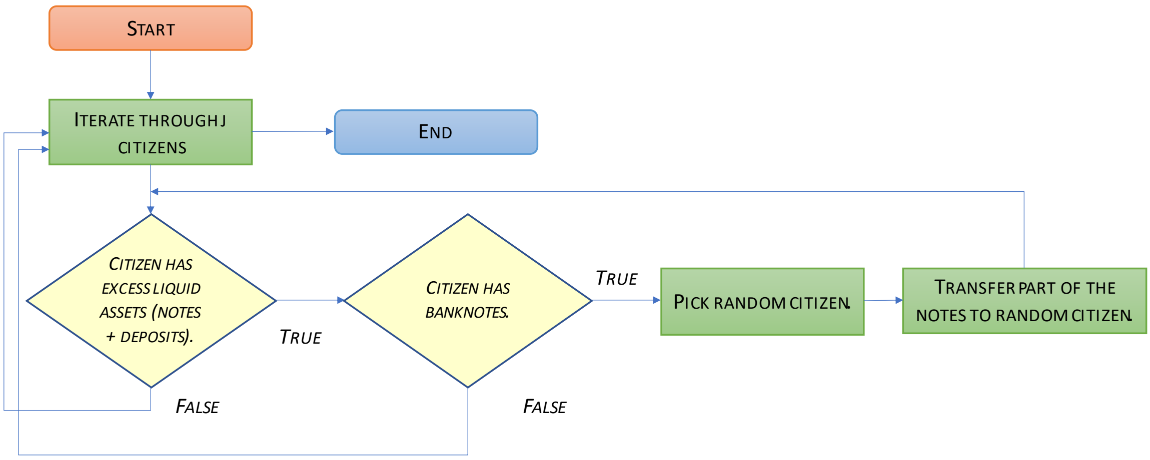

Figure 3. – *Part II. of the algorithm: citizens spend excess balances.*

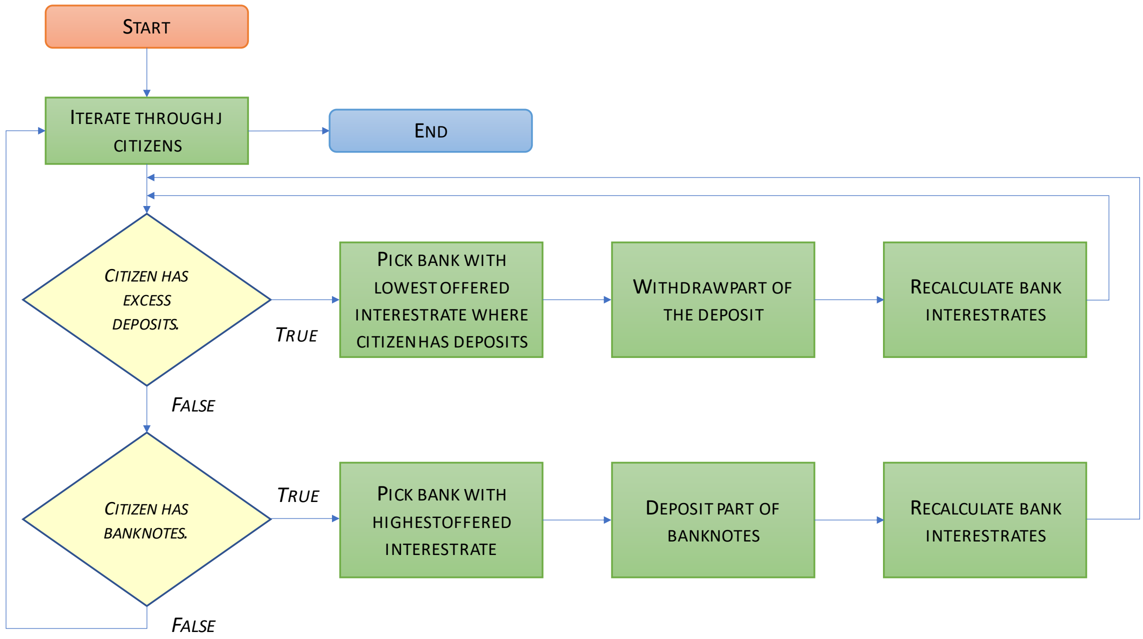

Figure 4. – *Part III. of the algorithm: citizens deposit or withdraw banknotes.*

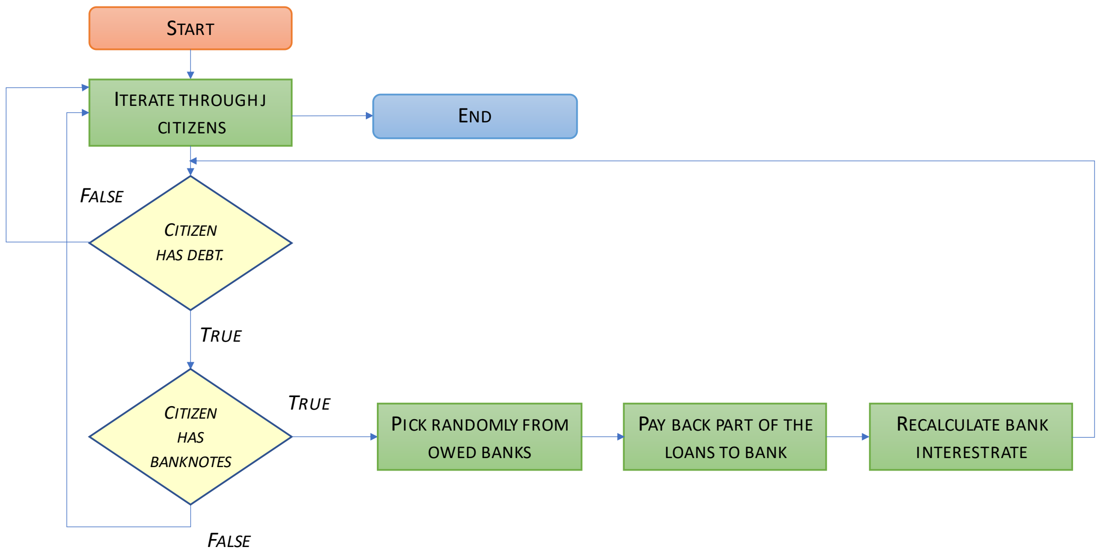

Figure 5. – *Part IV. of the algorithm: citizens pay back their loans.*

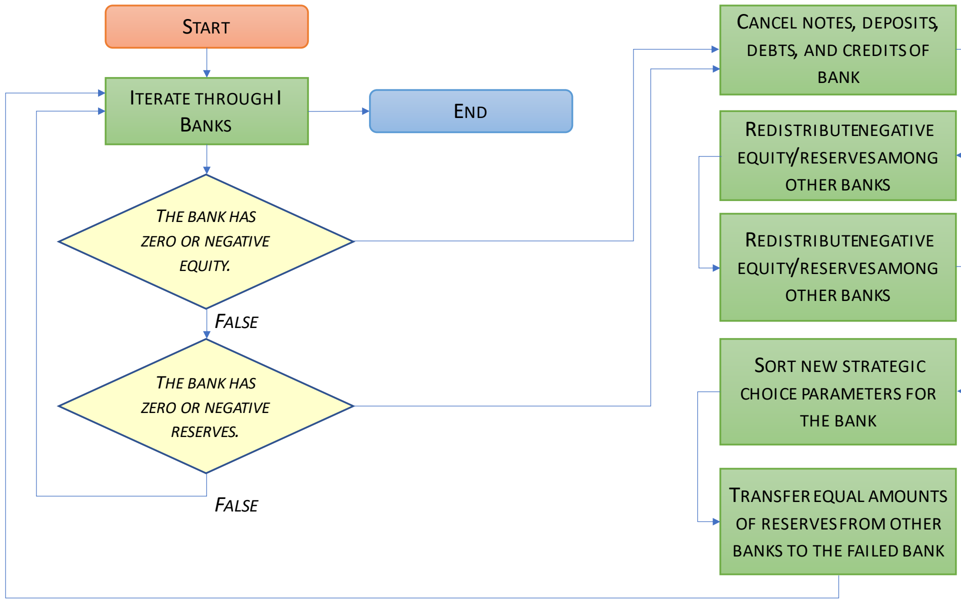

Figure 6. – *Part V. of the algorithm: setting up new banks in place of failed ones.*

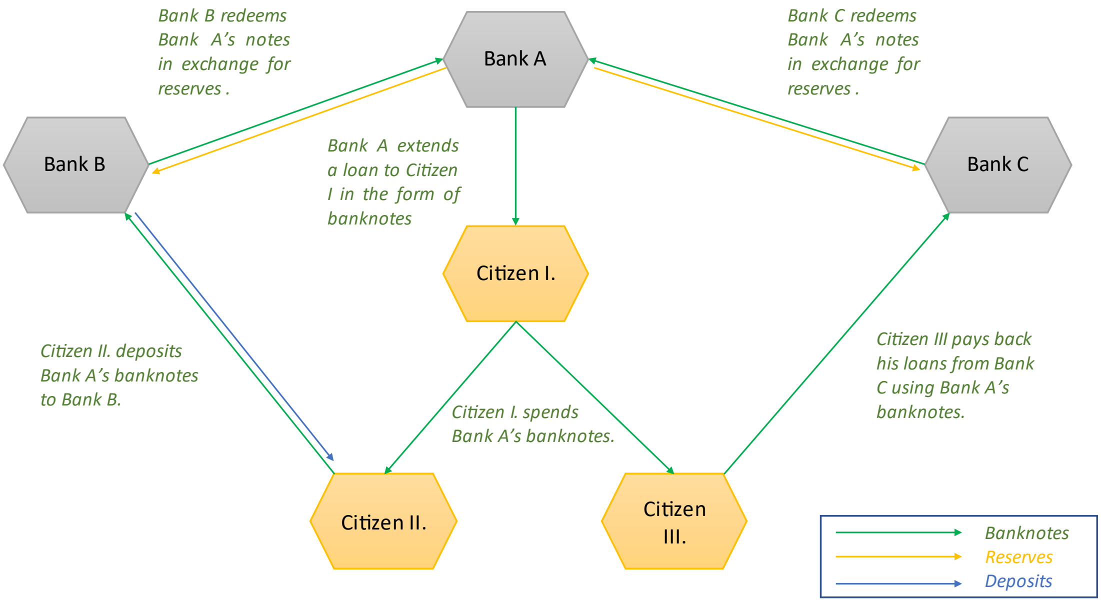

Figure 7. – *A schematic representation of the possible flow of banknotes.*

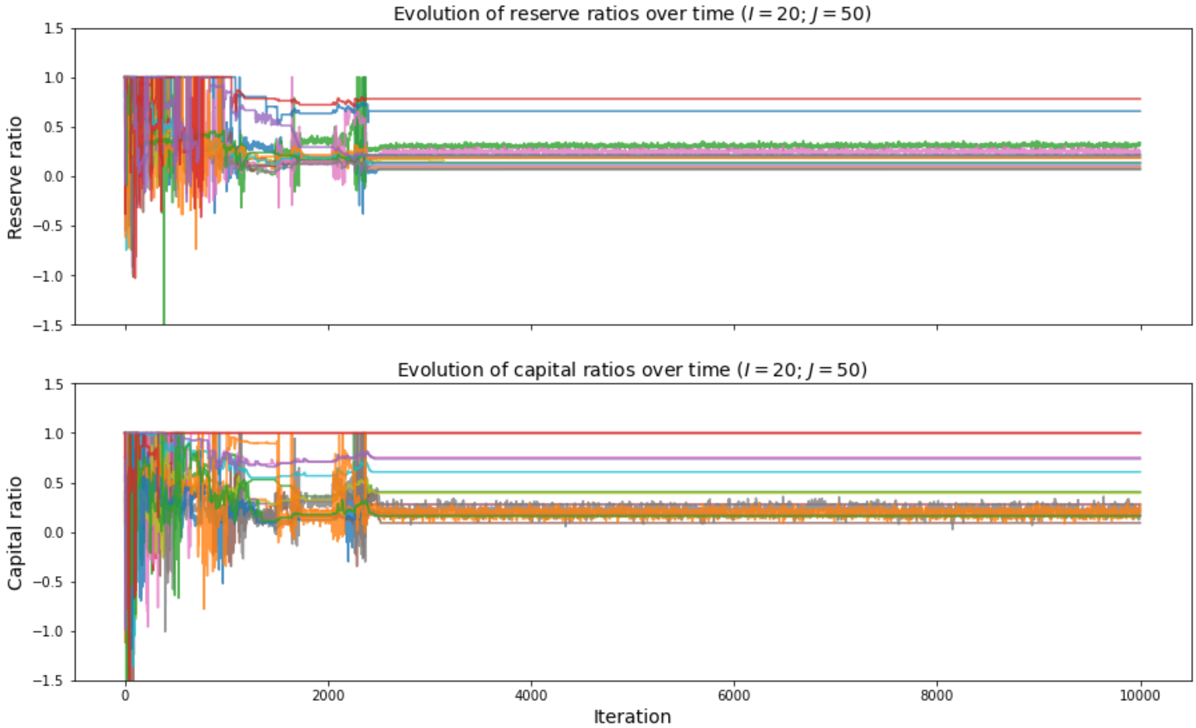

Figure 8. – *The evolution of reserve ratios (top) and capital ratios (bottom) over 10,000 iterations in a system of size I = 20 and J = 50, with each line representing a different bank.*

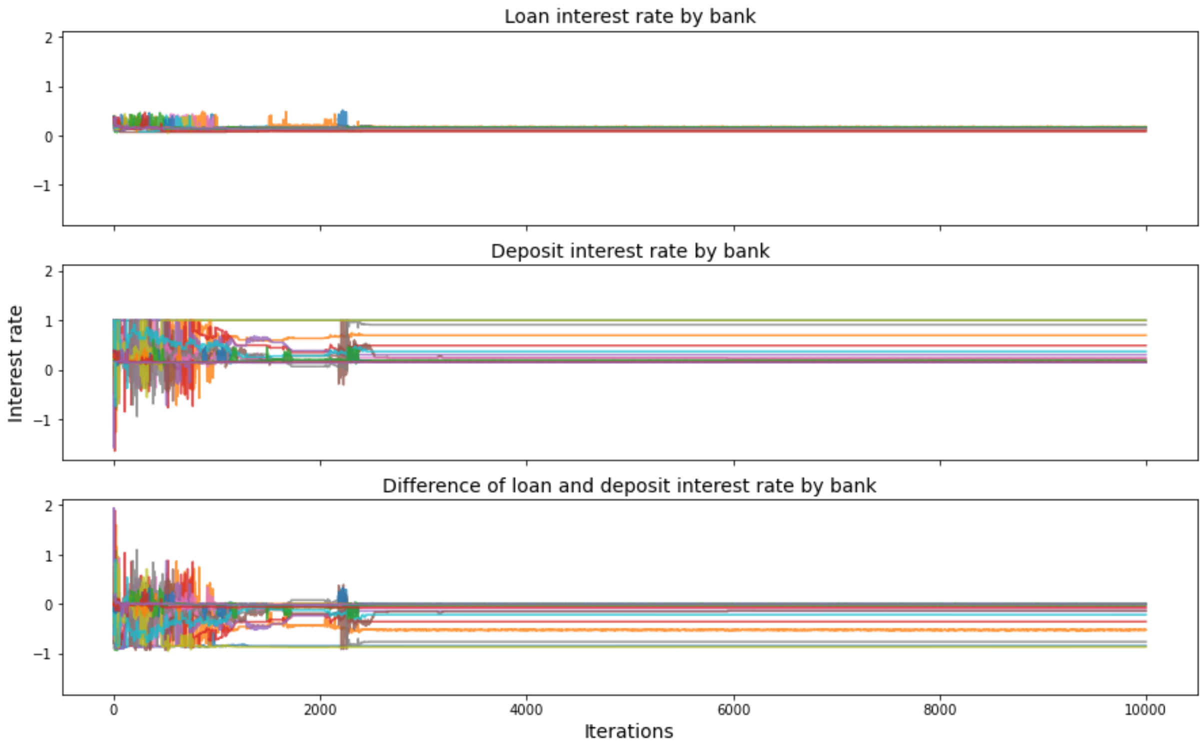

Figure 9. – *The deposit interest rates (top), loan interest rates (middle), and the difference of the two (bottom) over 10,000 iterations in a system of size I = 20 and J = 50, with each line representing a different bank.*

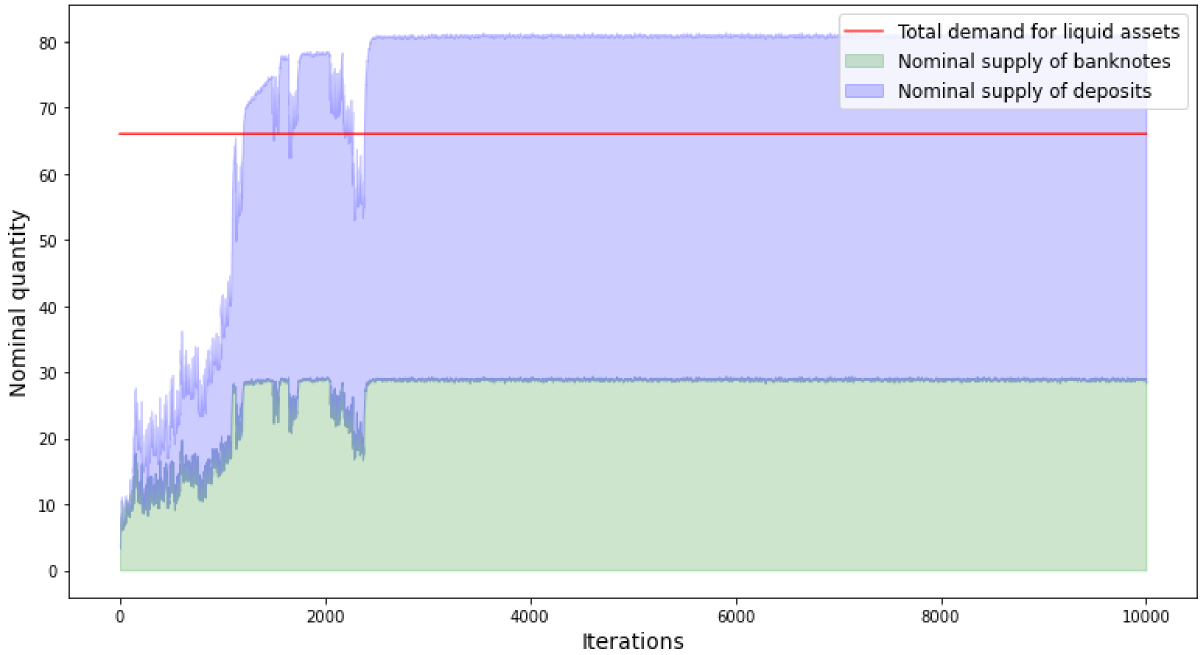

Figure 10. – *The evolution of the nominal supply of banknotes (green area), deposits (blue area), and the nominal demand for total liquid assets (banknotes plus deposits, red line), system of size I = 20 and J = 50.*

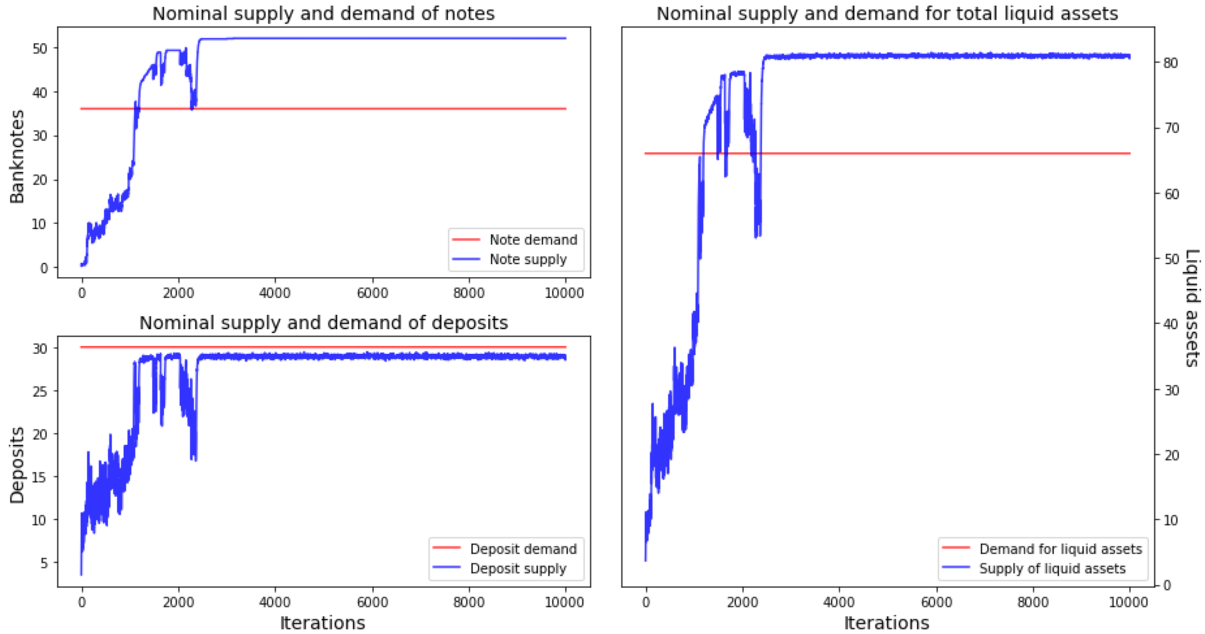

Figure 11. – *The evolution of the nominal supply and demand for banknotes (top left panel), deposits (bottom left panel), and total liquid assets (right panel).*

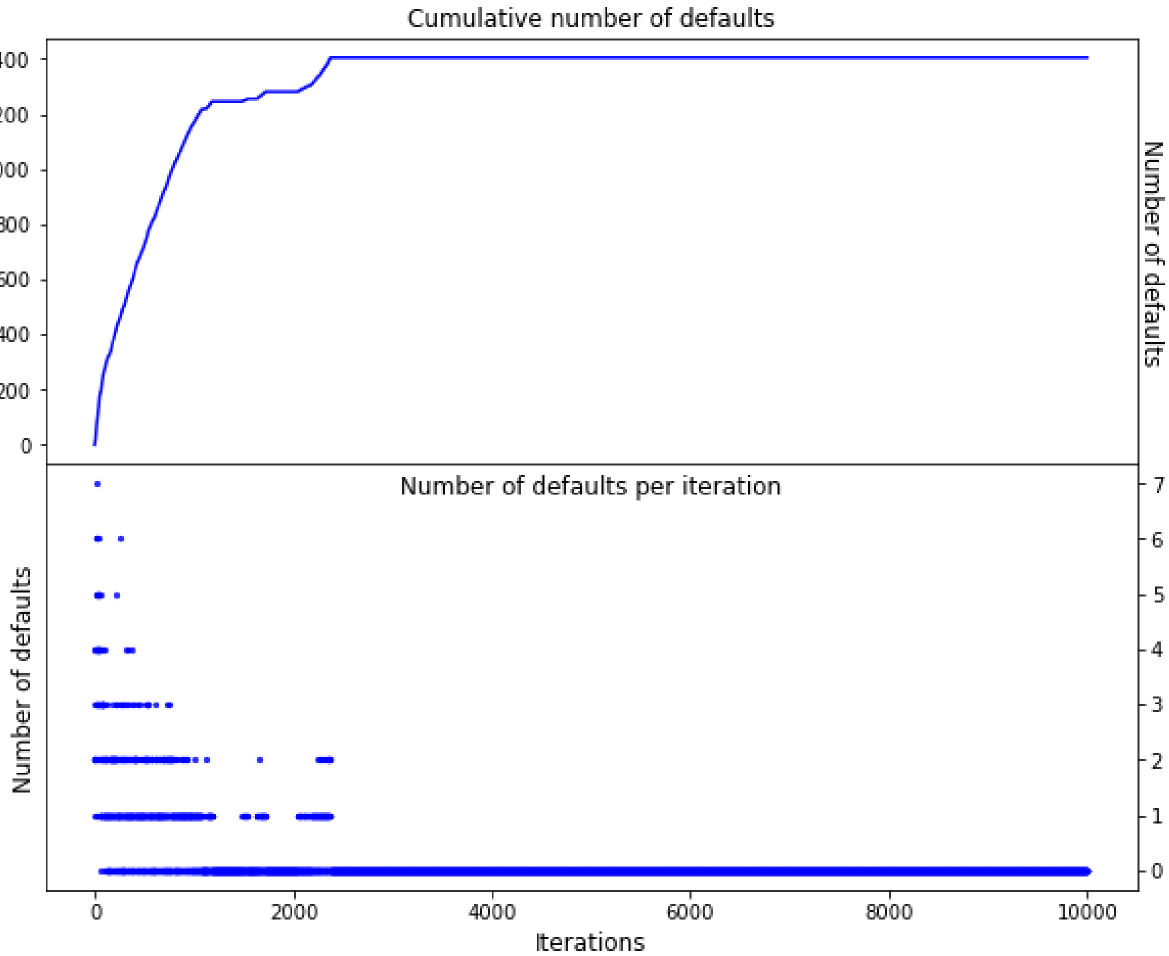

Figure 12. – *The evolution of the cumulative number of defaults (top panel) and the number of defaults per iteration (bottom panel), system of size I = 20 and J = 50.*

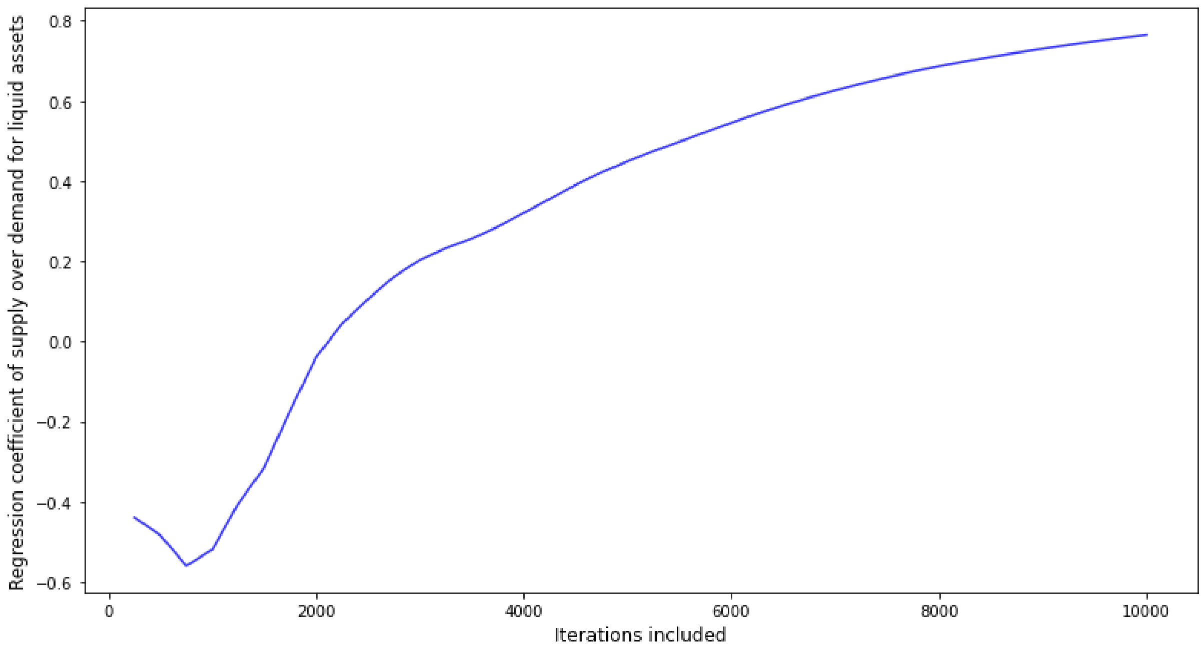

Figure 13. – *Evolution of the regression coefficient of the supply of liquid assets over its demand, as a function of the iterations included in the OLS model.*

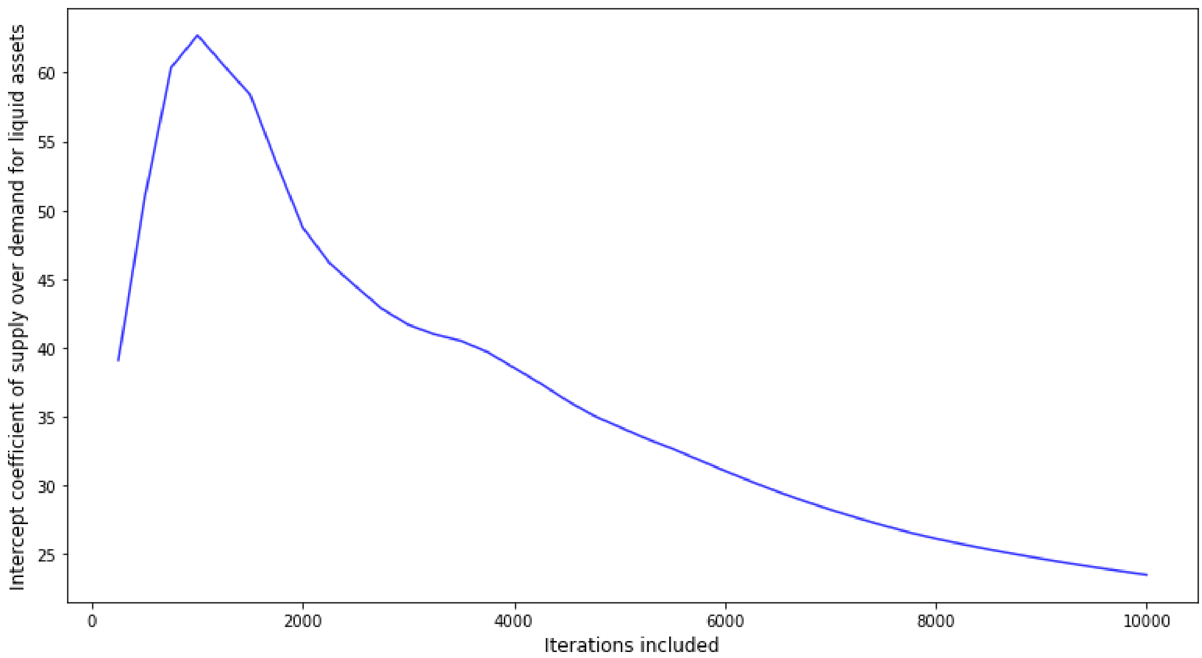

Figure 14. – *Evolution of the OLS constant of the supply of liquid assets over its demand, as a function of the iterations included in the model.*

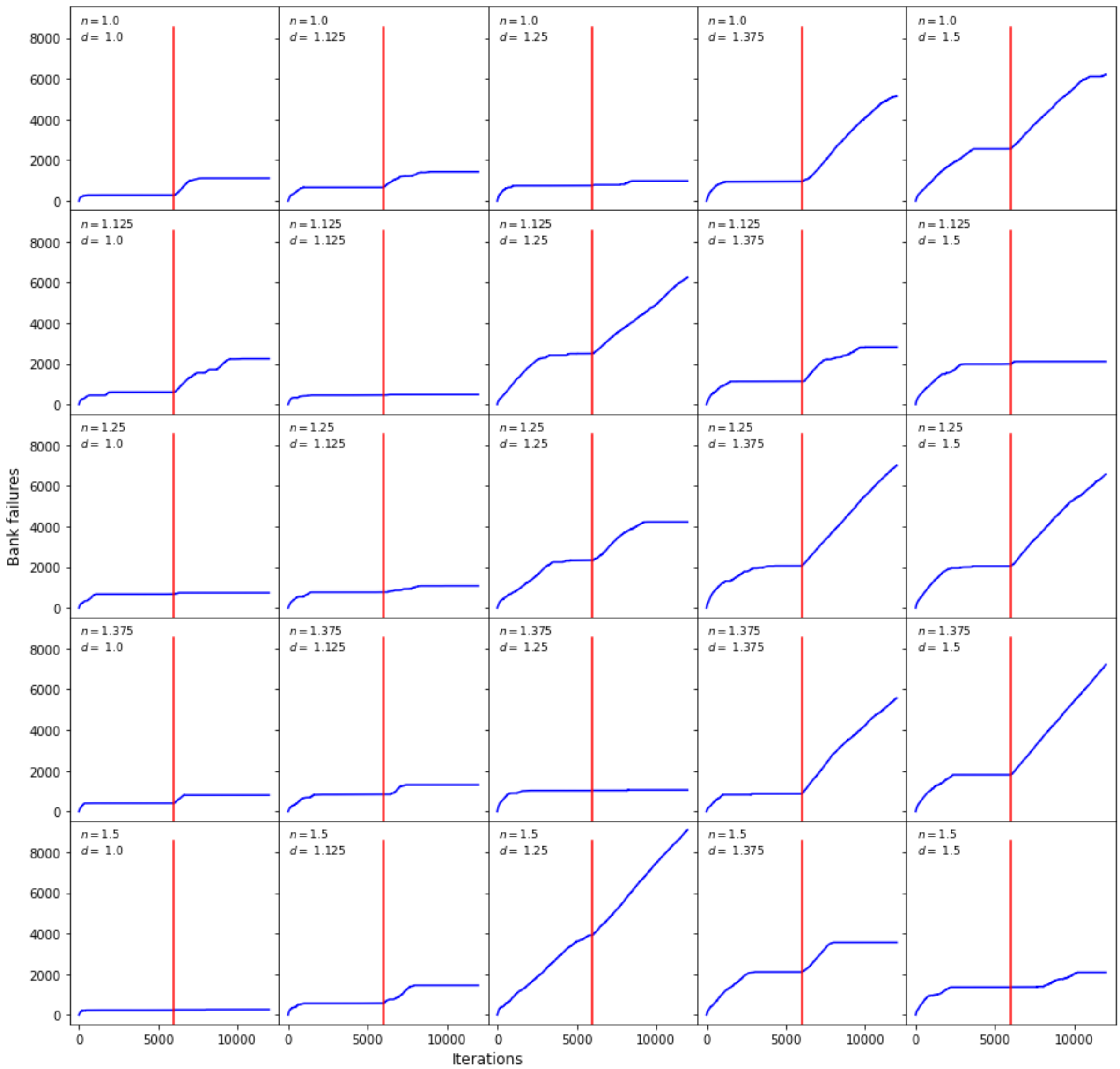

Figure 15. – *The evolution of the cumulative number of bank failures in a system of size I = 20 and J = 50, subject to a positive demand shock at t = 6,000 (red vertical line) and starting from different levels of money demand.*

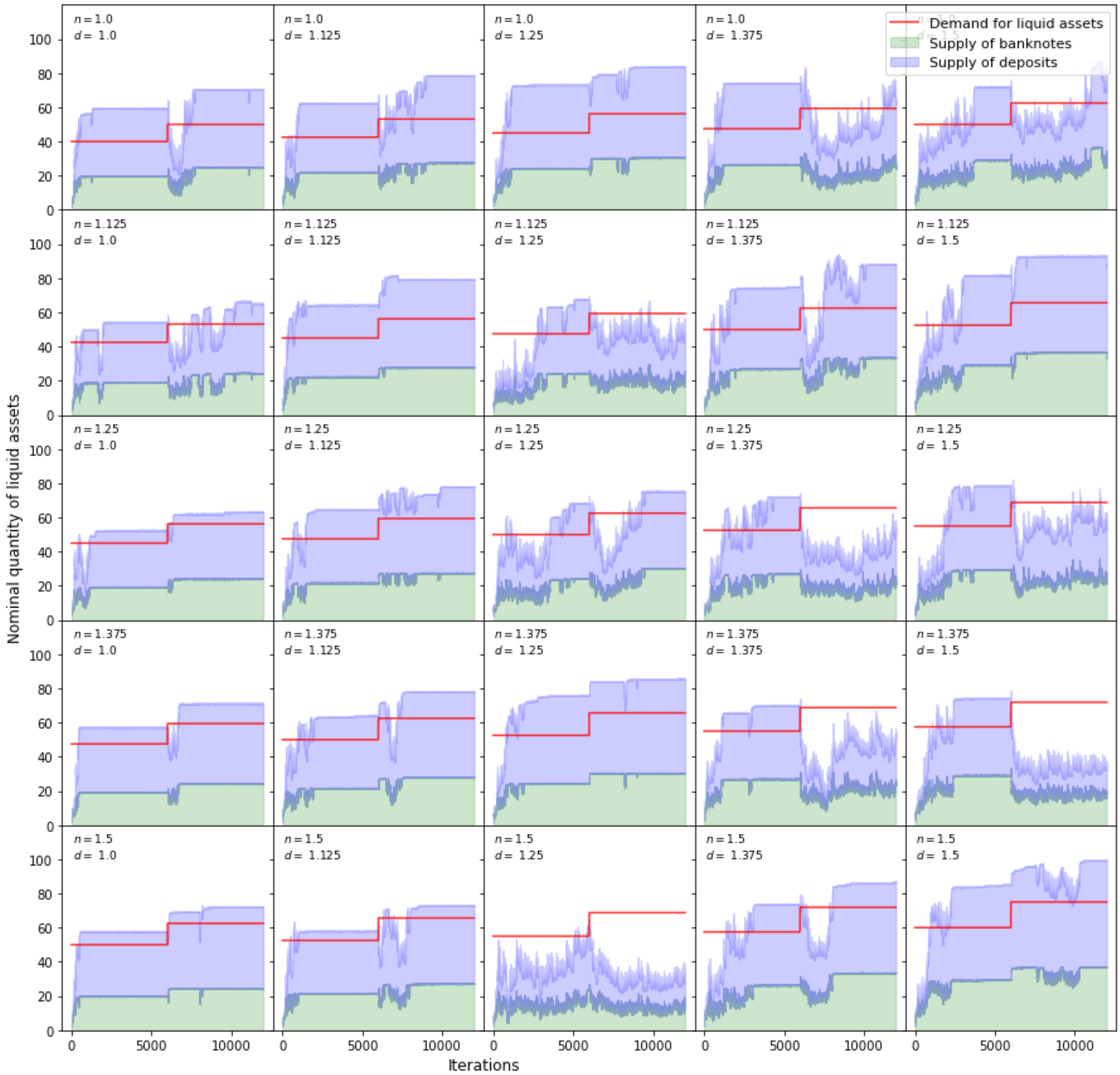

Figure 16. – *The evolution of the nominal supply of banknotes (green area) and deposits (blue area), and the nominal demand for total liquid assets deposits (red line) in a system of size I = 20 and J = 50, subject to a positive demand shock at t = 6,000, and starting from different levels of money demand.*

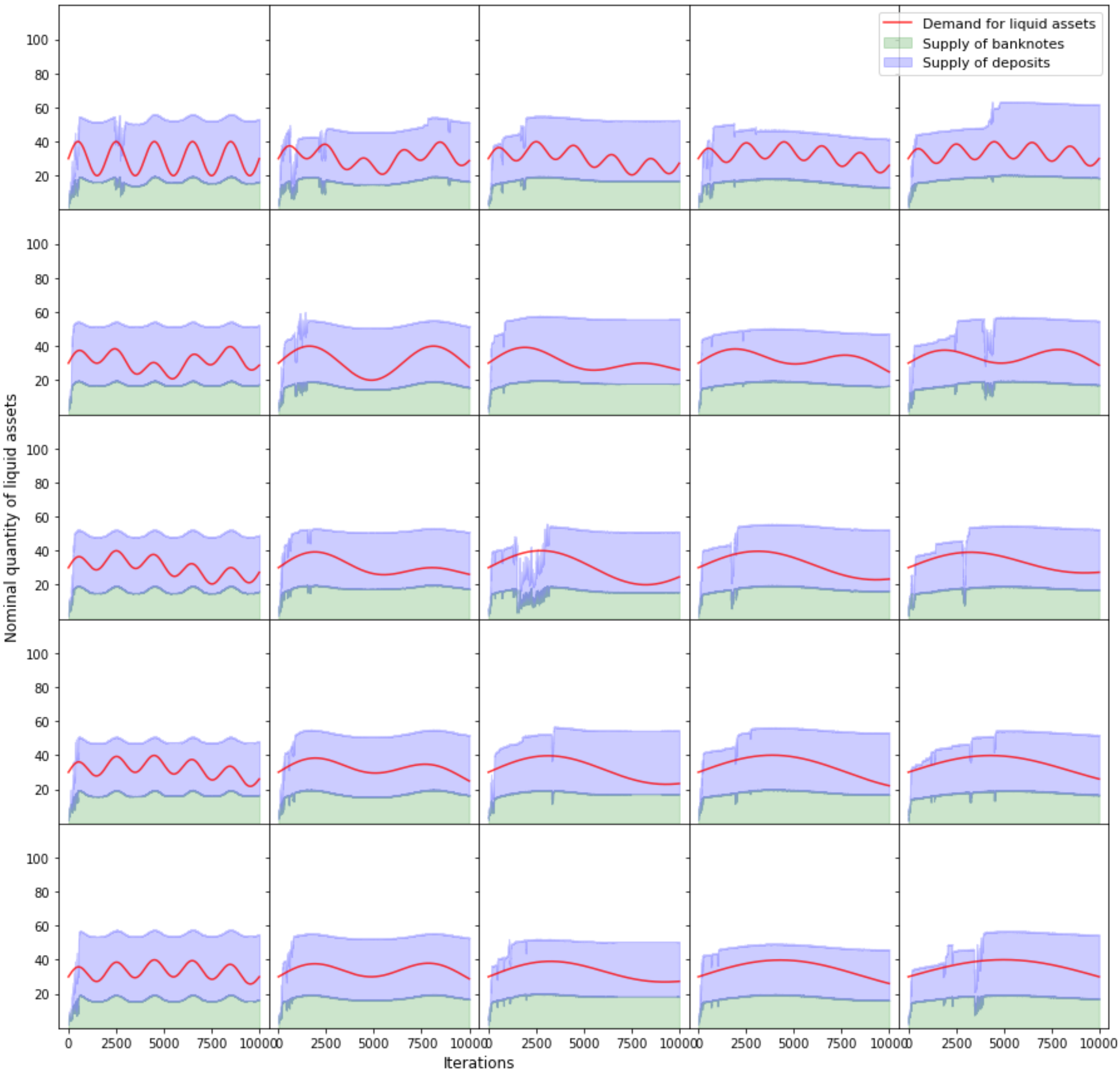

Figure 17. – *The evolution of the nominal supply of banknotes (green area) and deposits (blue area), and the nominal demand for total liquid assets (red line) in a system of size I = 20 and J = 50, subject to fluctuating demand.*

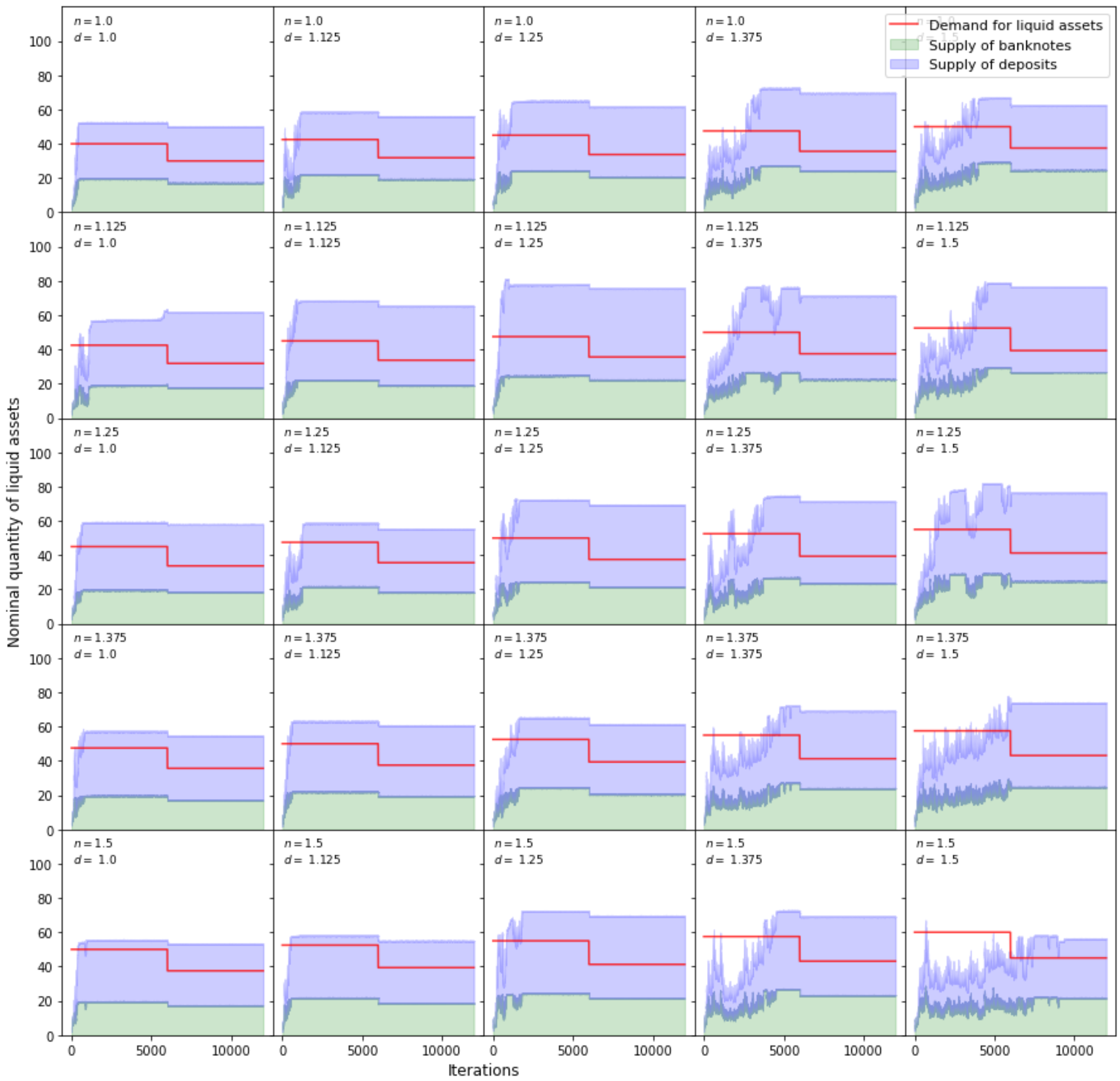

Figure 18. – *The evolution of the nominal supply of banknotes (green area) and deposits (blue area), and the nominal demand for total liquid assets deposits (red line) in a system of size I = 20 and J = 50, subject to a negative demand shock at t = 6,000, and starting from different levels of money demand.*

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