SEEK THE NEWS:
INNOVATION DIFFUSION, SOCIAL LEARNING AND HETEROGENEOUS AGENTS

By
Hamed Ghorbani

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Supervisor: Professor Mats Köster

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ABSTRACT

The slow diffusion of innovation is a key puzzle in development economics and other disciplines as well. Various theories suggest factors like heterogeneity of agents, tendency to social learning, geographical distance, and network structure as influential. This study introduces a game-theoretic model where heterogeneous agents acquire information either directly or from peers, where distance impact both the cost and effectiveness of information acquisition. The model predicts that a unique chain equilibrium exists under certain conditions. In addition, the model can be used to justify the rationality of social learning and hierarchy formation and slow pace of innovation diffusion.

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1 Introduction

The reason behind slow innovation diffusion has been a key question for a long time in economics and other disciplines (Young, 2009). In development economics, specifically, poor rates of technology adoption is one of the central puzzles. While it is plausible to assume technology advancements in developed countries ease the process of technological progress in developing countries, the realized pace of progress is not as expected (Verhoogen, 2023). Nonetheless, the puzzling nature of slow innovation diffusion is not limited to developing economies and the same delays have been observed in agricultural communities in the United States in early twentieth century (Munshi, 2007; Ryan and Gross, 1943).

Numerous theories have been proposed by economists and researchers of other fields as explanations for the observed delays (Young, 2009). Despite the differences, there are some common parts in all the theories. Heterogeneity of individuals, social learning, geographical distance and network structure can be named as the popular themes that researchers have used to model the diffusion phenomena (Baptista, 2000; Bisztray et al., 2018; Munshi, 2007; Young, 2009).

I developed a simple and therefore tractable game-theoretic model to explain the innovation diffusion phenomena among heterogeneous agents. The model can be extended in various ways. But even in its simple form, the model can be employed to explain various phenomena, such as rationality of social learning and hierarchy formation and slow pace of innovation diffusion. One advantage of the model is that it allows for the strategic interaction of agents, which compared to widely-used contagion models, makes it more interesting from an economic perspective.

The model consist of heterogeneous agents which are able to acquire information from an information source (first-hand information) or learn from their peers (second-hand information). The source of heterogeneity is the agents' distance from the information source. I used distance as a proxy for all sources of heterogeneity, such as education level, TFP of firms, firms' size, physical distance, mental capacity of the individual and innovativeness of the individuals. In other words, I assumed all relevant characteristics of agents can be projected into the distance. Heterogeneity of agents, or distance, affects utility funtions through two channels: First, it defines the cost (disutility) that agents should bear to reach the information source; Second, the ability of agents to comprehend the information decreases as they get further away from the information source. That is, distance reduces the ability of agents to use the information.

To allow social learning, I assume agents can interact with other agents and acquire second-hand information from their peers. That is, if an agent's neighbor has acquired information or adopted a technology, the other agents can get the information from her. One advantage of getting second-hand information is the lower search cost of information acquisition. At the same time the transferable information depends on the distance between peers. That is, as agents get further away from each other, the efficiency of information sharing decrease.

I use Nash equilibrium solution concept to solve the game and prove that in the benchmark model a unique Nash equilibrium exist in which information flows through a chain of agents. Under different specifications, multiple Nash equilibria emerge. Whenever multiple Nash equilibria exist, I use risk-dominance refinement (Harsanyi and Selten, 1988) to select among the Nash equilibria.

In section 2 I review relevant literature on information diffusion. In section 3, I introduce the model itself in details. In section 4, I analyze a benchmark model in which the information flow through a unique Nash equilibrium. In section 5, I introduce a specification in which coordination problem arise. In section 6, I analyze some other specifications and compare the results with the benchmark model. In section 7, I assume that agents' distance comes from a random distribution and analyze its implications for the speed of innovation diffusion. Finally, in section 8, I review the implications of the model, caveats of the model, and possible direction of future extensions.

2 Literature Review

The diffusion of innovations has attracted the attention of researchers from various disciplines. Namely because of the long time it takes for an innovation to be accepted. One of the widely cited studies that considered the speed of innovation diffusion is Ryan and Gross (1943), who quantitatively studied the adoption of hybrid seed corn among Iowa farmers. They found that interpersonal relationships played an important role in adoption and documented an S-shaped curve for the rate of adoption.

Still, the puzzling nature of innovation diffusion is a central area of research. Young (2009) reviewed theoretical models of innovation diffusion and categorised them into three broad classes: contagion models, social influence models and social learning models.

One branch that has been widely used by researchers in all fields is contagion models. In contagion models, innovation diffuses through exposure to adopters. In other words, innovation spreads like a virus. Despite the random nature of these models, economists have studied them extensively because of their wide acceptance in other disciplines, comparing the predictions of these models with social learning models Young (2009).

Young (2009), after mentioning the caveats of contagion and social influence models, mention social learning models as "the most plausible from an economic point of view because it has a solid decision-theoretic foundation". Bikhchandani et al. (2021) has reviewed the progress of the social learning literature by economists.

To the best of my knowledge, the model developed in this thesis differs from famous innovation diffusion models in the literature because it allows for strategic interactions between agents, heterogeneity of agents, and allows agents to form a network[1]

Throughout the paper I use risk-dominance refinement of introduced by Harsanyi and Selten (1988). Although Harsanyi and Selten (1988), who first introduced payoff dominance and risk dominance, favoured the payoff dominance notation over the risk dominance notation, the risk dominance notation attracted the attention of game theorists in the early 1990s. For example, Myerson (1991) proposed a theoretical evolutionary justification for risk dominance and payoff dominance. An example of experimental research supporting the risk-dominance notation is the work of Straub (1995), in which he conducted several experiments to test whether the predicted strategies could be seen in the laboratory. Straub (1995)'s results shows that in most situations participants' strategies were consistent with the predictions.

3 Model

In this section, first I discuss the model and then discuss the possible interpretation of the model.

3.1 Setup

There is a information source and a set of heterogeneous agents $\mathcal{N}$ which play a static complete information game. Each agent $i \in \mathcal{N}$ has a $d_i \in (0, 1]$ distance from the information source. Agents can acquire first-hand information, by attaching to the information source, or they can get the second-hand information through other players.

Thus, the action set of agent agent i is $\{F, \{S_j\}_{j \in \mathcal{N}_{-\{i\}}}, Z\}$, where F refers to getting first-hand information, $\{S_j\}_{j \in \mathcal{N}_{-\{i\}}}$ refers to getting second-hand information from agent $j \in \{k | k \in \mathcal{N}, k \neq i\}$, and finally, Z refers to getting no information.

The utility function of getting first-hand information for agent i will be, which depends on d_i:

$$U_i^F = f_i = (1 - d_i) * I - d_i. \tag{1}$$

[1]However, the literature on social learning is vast and complex, and I admit that I haven't reviewed many such models.

For simplicity, I normalize I, the value of information to 1. Note that F in superscript of utility function in (1) refers to first-hand information, or direct linkage to the source.

Agent's i utility of getting second-hand information from agent j, which has acquired $I_j \in [0, 1)$ information, depends on $d_{i,j} = |d_i - d_j|$ which is the absolute distance between the agents:

$$U^S_{i,j} = s_{i,j} = (1 - d_{i,j}) * I_j - d_{i,j}, \tag{2}$$

Finally, the utility of getting no information is always zero for the agents:

$$U^Z_i = z_i = 0$$

I use the Nash equilibrium solution concept the risk-dominance refinement (Harsanyi and Selten, 1988) when there exist several Nash equilibria.

3.2 Model Interpretation

In this section, I discuss the interpretation of different parts of the model: Information source, distance, and the utility functions.

The information source in the model is a source which holds a positive value information, or innovation, for all the agents. It can be interpreted as a beneficial management practice that increases efficiency, a new technology that reduce the costs, or an information source which allow workers to update their beliefs about their outside options. Distance is a measure which suppose to proxy all possible search barriers and qualities of the agent, such as access to market, the socio-economic background of manager/worker, or the productivity of firms.

For the interpretation of the utility functions, I discuss two intuition that I had for developing this model[1].

3.2.1 Inability to Utilize

In the first approach, the discount factor of the information value can be interpreted as the inability of agents to utilize/comprehend the information. That is, the further the agent becomes from the information source, she becomes less confident of her ability to use the information. An example for this, might be an immigrant who does not search for the information because of her lack of institutional knowledge, language barriers or social capital that prevent her from comprehending or using the information effectively.

[1] In this thesis, and for the sake of simplicity, I use the interpretation in section 3.2.1. That is, I assume the utility functions are the true utilities that agents get.

For the first-hand information we have:

$$U_i^F = (1 - d_i) * I - d_i.$$

In which the first term, $(1 - d_i) * I$, is the true utility of getting information after taking into account the ability of agent to comprehend or utilize the information, and the second term, $-d_i$, is the cost (dis-utility) of reaching to the source or search cost.

The interpretation becomes much simpler this way: As agent get further away from the information source, her ability to comprehend/utilize the information decrease.

The same logic can be used to interpret the utility of second-hand information:

$$U_{i,j}^S = (1 - d_{i,j}) * I_j - d_{i,j}.$$

When $d_j < d_i$, if agent i believe agent j has acquired the information, the utility of getting second-hand information from agent j will be:

$$U_{i,j}^S = (1 - (d_i - d_j)) * (1 - d_j) - (d_i - d_j).$$

Using the same logic, the more distant the agents become from their peers their ability to comprehend or utilize the information decrease. That is, the agent utility of using her peer's information decrease because of two reasons: First, her peer have some distance from the information source, so he is not fully comprehended the information. Second, since the agent is far from her peer, she won't be able to utilize the information held by her peer.

3.2.2 Utility Misperception

In the second approach, the discount factor of the information value can be interpreted as the perceived value of the information.

Again, for the first-hand information we have:

$$U_i^F = (1 - d_i) * I - d_i.$$

That means, as agents get further away from the information source, in addition to the increase in search cost, their misperception of the value of information increase at the same time. That is, although the ultimate utility of information or technology is fixed for all agents, as agents get more distant from the information source, their misperceptions might prevent them from searching for the information.

The same logic can be used to interpret the utility of second-hand information.:

$$U^S_{i,j} = (1 - d_{i,j}) * I_j - d_{i,j}.$$

Same as before, when $d_j < d_i$, if agent i believe agent j has acquired the information, the utility of getting second-hand information from agent j will be:

$$U^S_{i,j} = (1 - (d_i - d_j)) * (1 - d_j) - (d_i - d_j).$$

Using the same logic, the more distant the agents become from their peers, their error about the value of their peer's information or technology increase. That is, an agent might believe information or technology that her peer is using is not valuable for two reasons: First, her peer have some distance from the information source, so the information held by him is not so valuable. In other words, the technology held by him might not be an advanced technology. Second, since the agent is far from her peer, she has misperception about the value of information held by her peer.

4 Benchmark Model

In the benchmark model, I assume there are $|\mathcal{N}| = N \geq 4$ agents which are located on the $(0, 1]$ interval such that the distance between any two neighbours is $\frac{1}{N}$. In other words, $\{d_1, d_2, ..., d_N\} = \{\frac{1}{N}, \frac{2}{N}, ..., 1\}$. I start with identifying the equilibrium and then discuss the results.

4.1 Equilibrium of the Benchmark

I prove a unique Nash equilibrium exist in which, except the first agent, who gets the first-hand information, all the other agents get second-hand information from their neighbour who has a lower distance to the information source. Thus, in equilibrium, a chain will form that leads to the information flow across all agents.

Proposition 1 (Chain equilibrium). *For any $4 \leq N$, there exist a unique Nash equilibrium in which player 1 gets first-hand information and any player $2 \leq i$ gets second-hand information from player $i - 1$.*

I first prove lemmas 1-3 and then prove proposition 1 using lemmas 1-3.

Lemma 1 (Acquisition Direction). *Consider player i. Suppose except i, strategy of the rest of players is fixed and for each agent, the fixed strategy maximize the utility of agents. Then, for agent i, getting second-hand information through agent $i - 1$ strictly dominates* *getting second-hand information any agent $i < j$.*

Proof. The utility of getting second-hand information for agent i from any other agent k is:

$$U_{i,k}^{S} = (1 - \frac{|k-i|}{N}) * I_k - \frac{|k-i|}{N}$$

For any $i < k$, $\max I_k < \max I_{i-1}$. That means, the maximum possible value for I_k is smaller than maximum possible value of I_{i-1}. Thus, for agent i, getting second-hand information from agent $i-1$ strictly dominates getting second-hand information from any agent $i < k$. $\square$

Lemma 2 (Length of Steps)**.** *Consider player i. Suppose that except agents i, any agent $j < i$ has acquired second-hand information through agent $j-1$ and agent 1 gets first-hand information from the information source. Then, for agent i, getting second-hand information through agent $i-1$ strictly dominates getting second-hand information from any agent $j < i-1$.*

Proof. Assuming each agent $j < i$ acquired information through $j-1$, agent's i utility of getting second hand information from any agent $i-m$, $m \in \{1, 2, i-1\}$, is:

$$U_{i,m}^{S} = (1 - \frac{m}{N}) * (1 - \frac{1}{N})^{i-m} - \frac{m}{N}.$$

The derivative of $U_{i,m}^{S}$ with respect to m is:

$$\begin{aligned}
\frac{\partial U_{i,m}^{S}}{\partial m} &= \frac{-1}{N} * (1 - \frac{1}{N})^{i-m} - (1 - \frac{m}{N}) * (1 - \frac{1}{N})^{i-m} * ln(1 - \frac{1}{N}) - \frac{1}{N} \\
&= (1 - \frac{1}{N})^{i-m} * [\frac{-1}{N} - (1 - \frac{m}{N}) ln(1 - \frac{1}{N})] - \frac{1}{N} \\
&= (1 - \frac{1}{N})^{i-m} * A - \frac{1}{N}.
\end{aligned}$$

Because $\frac{\partial A}{\partial m} < 0$ and $0 < \frac{\partial A}{\partial N}$,

$$\max_{m,N} A = \lim_{N \to \infty} A|_{m=1} < 0.$$

Thus, it can be argued that for any $N \in \mathcal{N}$ and $m \in \{1, 2, ..., i-1\}$, $U_{i,m}^{S}$ is decreasing in m and its maximum value occurs at $m = 1$. That said, if agent i knows that any agent $j < i$ will acquire information through agent $j-1$, its best response will be to get the second-hand information through agent $i-1$ and no other agent. $\square$

Lemma 3 (Domination of Outside Option). *Consider player i. Assuming that agent any agent $j < i$ has acquired the second-hand information from agent $j-1$. Then, for agent i, getting second-hand information through agent $i-1$ always dominates getting no information when $4 \leq N$.*

Proof. Under the mentioned setups, the utility of getting second-hand information from agent $i-1$ is:

$$U^S_{i,i-1} = (1 - \frac{1}{N})^i - \frac{1}{N},$$

and for any $4 \leq N$:

$$U^Z_i = z_i = 0 < U^S_{N,N-1} \leq U^S_{i,i-1}$$

□

Using lemmas 1-3, proof of proposition 1 will be straight forward:

Proof. Through lemma 1, agent 1 never gets the second-hand information from any agent $i < j$. In addition, when $4 \leq N$, getting no information is dominated by getting first-hand information from the source. So, it can be said that agent 1 always gets the first-hand information from the source.

Using lemma 1 and the fact that $U^F_2 < U^S_{2,1}$ (lemma 2), it can be argued that agent 2 strictly prefers getting second-hand information from agent 1, and from lemma 3 it is known that $S_{2,1}$ strictly dominates Z_2.

Since agent 1 get the first-hand information and agent 2 gets the second-hand information from agent 1, using lemmas 1-3 it can be easily proven that the best response of any agent $3 \leq j$ is to acquire information from agent $j-1$. That means, for $4 \leq N$, any agent $j \neq 1$ prefers to get second-hand information from agent $j-1$ rather than getting no information or getting information through any other agents. □

4.2 Discussion

In section 4.1 I examined the benchmark model and proved that the game has a unique Nash equilibrium in which the information diffuses through a chain. In the chain equilibrium, each agent acquire the information in the most efficient way and thus, the equilibrium is also Pareto efficient. In this section I discuss the implication of the benchmark model from the social welfare point of view.

4.2.1 Rationality of Social Learning

lemma 2 has one interesting implication which is rationality of social learning. As it has already proven, $\frac{\partial A}{\partial m} < 0$ in lemma 2 which implies agents will be better off by learning from their neighbours even if there was no search cost to bear. That means, learning from the neighbours increase the value of acquired information. In other words, under the assumptions of the model social learning and hierarchy formation are rationale.

4.2.2 Slow Information Diffusion

Although the game that I discussed is a static game and there exist no time, if one assume that the flow of information through the chain takes time, the model also explain the reason behind the slow process of innovation diffusion across communities. That means, since agents prefers to get the information through their neighbours instead of reaching to the information source themselves, the process of innovation diffusion gets slower[1]. I discuss this feature in section 7 from a different perspective.

4.2.3 Social Welfare

In this section, I discuss the implications of the model from social welfare perspective. For any $4 \leq N$, the average utility in such a network will be:

$$\begin{aligned}
\frac{\sum_{i=1}^{N} U_i}{N} &= \frac{\sum_{i=1}^{N} \frac{-1}{N} + (1-\frac{1}{N})^i}{N} \\
&= \frac{-1}{N} + \frac{\sum_{i=1}^{N}(1-\frac{1}{N})^i}{N} \\
&= \frac{-1}{N} + \frac{\frac{(1-\frac{1}{N})*[1-(1-\frac{1}{N})^{N-1}]}{1-(1-\frac{1}{N})}}{N} \\
&= \frac{-1}{N} + (1-\frac{1}{N}) * \left[1-(1-\frac{1}{N})^{N-1}\right] \\
&= \frac{-1}{N} + \left[1-\frac{1}{N}-(1-\frac{1}{N})^{N}\right] \\
&= 1-\frac{2}{N}-(1-\frac{1}{N})^{N},
\end{aligned}$$

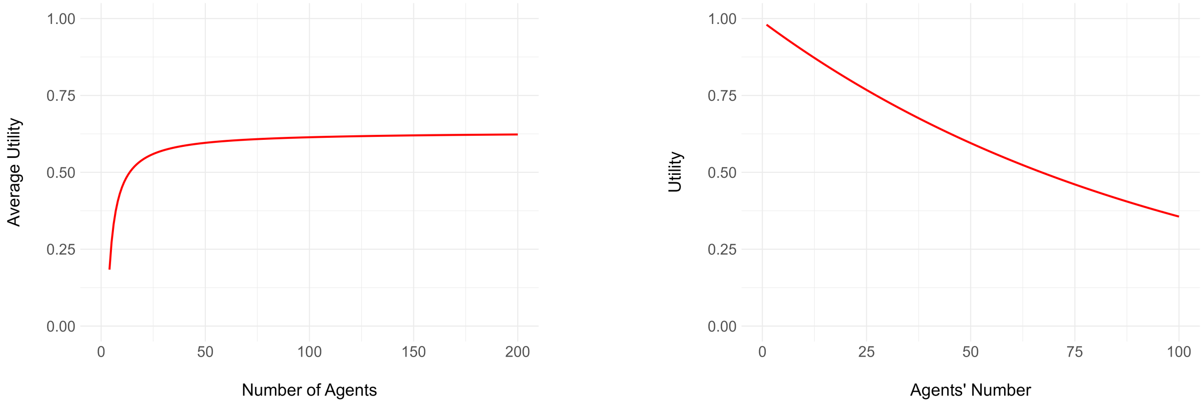
and

$$\lim_{N\to\infty} \frac{\sum_{i=1}^{N} U_i}{N} = 1-\frac{2}{N}-(1-\frac{1}{N})^{N} = 1-e^{-1} \approx 0.63$$

[1]If one assume agents have full present bias and the payoff of information is a one time lump-sum, the same equilibrium can be gained in a dynamic game

Considering the fact that $\frac{\sum_{i=1}^{N} U_i}{N}$ is increasing in N, I argue the maximum of average utility for the benchmark model is $1 - \frac{1}{e} \approx 0.63$.

In figure 1, the average utility of agents and individual utility of agents are illustrated. It can be seen from figure 1a that the convergence speed of average social welfare is high. The individual utility of agents, when $N = 100$ can be seen in the figure 1. It can be seen that even the utility of the farthest agent is considerable compared to her distance from the information source.



(a) Average Welfare as a Function of N (b) Individual Utilities when $N = 100$

Figure 1: Average utility and individuals utility

5 Free Riding and Coordination Problem

In this section, I analyze setups in which coordination problem can arise as a result of free riding opportunity. I use the risk-dominance refinement to choose between multiple equilibrium.

In section 5.1, I assume the agents are homogeneous and in second section 5.2, I assume agents are heterogeneous.

5.1 Homogeneous Agents

Assume there exist two agents with equal distances: $d_1 = d_2 = d$. Similar to section 4, each agent has three actions $\{F, S, Z\}$. Thus, the normal form game can be written in the following form:

	F	S	Z
F	f, f	f, s	$f, 0$
S	s, f	$0, 0$	$0, 0$
Z	$0, f$	$0, 0$	$0, 0$

In which $f = 1 - 2d$ and $s = 1 - d$. Clearly, $f < s$ and $f < 0$ for $d \in (\frac{1}{2}, 1]$. That means, if $d \in (\frac{1}{2}, 1]$, the Z strictly dominates F and the game can be reduced to:

	S	Z
S	$0,0$	$0,0$
Z	$0,0$	$0,0$

in which any action leads to a Nash equilibrium. Since this case is not interesting, I ignore this case for now and assume $d < 0.5$. When $d < 0.5$, $0 < f$ and thus F strictly dominates Z. Therefore, the game can be reduced to:

	F	S
F	f, f	f, s
S	s, f	$0,0$

The above game has two pure strategy Nash equilibria: $\{S, F\}$ and $\{F, S\}$. In the mixed strategy Nash equilibrium, probability of getting first-hand information (F) is:

$$p = \frac{f}{s} = \frac{1-2d}{1-d} = 1 - \frac{d}{1-d}$$

It can be easily proven that the mixed strategy Nash equilibrium risk dominates the pure strategy Nash equilibria using the algorithm introduced in Harsanyi and Selten (1988). According to Harsanyi and Selten (1988), if one define u_i and v_i as deviation losses of player i from the strong equilibrium points in a $2*2$ game, the mixed strategy Nash equilibrium will be the risk-dominant strategy if $u_1u_2 = v_1v_2$.

To prove whether the mixed strategy Nash equilibrium in the above mentioned game risk dominates other strategies, one should calculate u_i and v_i.

	U	V
U	a_{11}, b_{11}	a_{12}, b_{12}
V	a_{21}, b_{21}	a_{22}, b_{22}

Note that as opposed to the original example used in Harsanyi and Selten (1988), this game is an asymmetric coordination game and thus the strong equilibrium points of the game are $\{U, V\}$ and $\{V, U\}$. That means, the definitions of u_i and v_i change for our case:

$$\begin{cases} u_1 = a_{12} - a_{22} = f \\ u_2 = b_{12} - b_{11} = s - f \\ v_1 = a_{21} - a_{11} = s - f \\ v_2 = b_{21} - b_{22} = f \end{cases}$$

Using the above definitions, one can easily prove:

$$u_1 u_2 = v_1 v_2$$

Hence, the mixed strategy Nash equilibrium is the risk-dominant strategy in the new setting.

To conclude, in this section I proved that if one assume that $|\mathcal{N}| = 2$ and $d_1 = d_2 = d \leq 0.5$, in the risk dominant strategy each player get the first-hand information with probability $p = \frac{1-2d}{1-d}$. Considering the interesting properties of this equilibrium, from now on, for the homogeneous agents I only examine the mixed strategy Nash equilibrium which is the risk-dominant strategy of the game. In section 5.1.1, I study a case which $2 < N$

5.1.1 More than Two Homogeneous Agents

One question that might arise is the effect of number of homogeneous agents on the probability of getting first-hand information in the mix strategy Nash equilibrium. That means, assume there exist $2 < N$ agents with a fixed distance from information source d. The mix strategy equilibrium is a symmetric equilibrium in which the probability of getting first-hand information is the same for any agent, and agents choose their peers with a equal probability.

I assume each agent get the first-hand information with probability p, and connect to one of her peers to get the second-hand information with probability $\frac{1-p}{N-1}$. Each agent, when deciding about her mix strategy would solve the below equality by choosing a proper p:

$$\frac{f}{s} = Q(1, N, p) = p + \frac{(N-2)(1-p)}{N-1} Q(2, N, p)$$

In which $Q(1, N, p)$ is the probability that other agents might have the information. In other words, if player 1 decide to get second-hand information, the probability that other agents have the information is $Q(1, N, p)$. When $N = 2$, $Q(1, N, p) = p$[1] because except agent 2, there exist no other agent who agent 1 can his information through it. To give an example, assume $N = 3$:

$$\begin{aligned} \frac{f}{s} = Q(1, 3, p) &= p + \frac{(1)(1-p)}{2} Q(2, 3, p) \\ &= p + \frac{1-p}{2} p \end{aligned}$$

[1] $Q(m, N, p)$ can be defined for $m \in \{k | k \in \mathbf{N}, k < N\}$

If one fix $\frac{f}{s}$ while increasing N, it can be easily argued that the difference between $Q(1, N, p)$ and $Q(2, N, p)$ should decrease. However, $Q(2, N, p)$ is always smaller than $Q(1, N, p)$. In other words, assuming $\xi(1, 2, N, p) = Q(1, N, p) - Q(2, N, p)$ it can be argued that[1]:

$$\xi(1, 2, N, p) > 0$$

$$\frac{\partial \xi(1, 2, N, p)}{\partial N} < 0$$

$$\frac{\partial^2 \xi(1, 2, N, p)}{\partial N^2} > 0$$

Using the above conditions, one can re-write the equality:

$$\frac{f}{s} = Q(1, N, p) = p + \frac{(N-2)(1-p)}{N-1} \big[Q(1, N, p) - \xi(1, 2, N, p) \big]$$

The above equality can be written in another form. Using the fact that $Q(N-1, N, p) = p$:

$$\frac{f}{s} = Q(1, N, p) = p + \frac{(N-2)(1-p)}{N-1} \sum_{1}^{N-2} \frac{(N-3)!(1-p)^{i-1}p}{(N-2-i)!(N-1)^{i-1}} \tag{3}$$

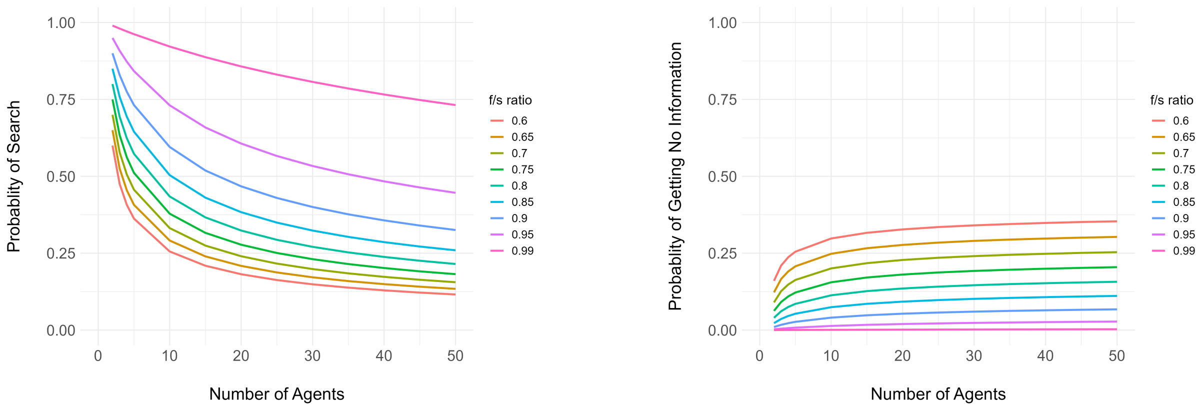
Although I couldn't analytically solve the above inequality for a p, some interesting results can be derived from it. Specifically, one can argue:

$$\frac{f}{(N-1)(s-f)+f} < p < \frac{f}{s}$$

That is, lower band and upper band of p have analytical solutions. The exact solutions of 3 and failure to collect information[2] can be seen in figure 2. In the next sections, whenever it is needed, I used numerical methods to find the solution for p.

[1] $\frac{\partial^2 \xi(1,2,N,p)}{\partial N^2} > 0$ doesn't affect the proof

[2] The probability of failure to collect information is $1 - (p + (1-p) * Q(1, N, p))$



(a) Probability of search (b) Probability of failure

Figure 2: Probability of search and failure to collect information

5.2 Heterogeneous Agents

In this section, I assume $d_1 < d_2 = \frac{1}{2}$. Similar to section 4, each agent has three actions $\{F, S, Z\}$. Thus, the normal form game can be written in the following form:

	F	S	Z
F	f_1, f_2	$f_1, s_{2,1}$	$f_1, 0$
S	$s_{1,2}, f_2$	$0, 0$	$0, 0$
Z	$0, f_2$	$0, 0$	$0, 0$

In which:

$$f_1 = 1 - 2d_1 \quad , \quad s_{1,2} = (1 - d_{1,2})(1 - d_2) - d_{1,2}$$
$$f_2 = 1 - 2d_2 \quad , \quad s_{2,1} = (1 - d_{2,1})(1 - d_1) - d_{2,1}$$

If $s_{1,2} < f_1$ in the above game, there exist no coordination problem and NE of the game is $\{F, S\}$. However, if $f_1 < s_{1,2}$ there exist two pure strategy Nash equilibria and one mixed strategy Nash equilibrium. Nonetheless, it can be proved that whenever $d_1 < d_2 = \frac{1}{2}$, the risk-dominant strategy is $\{F, S\}$[1].

That means, if an agent has advantage over the other agent, despite the existence of coordination problem, the risk-dominant strategy is a pure strategy Nash equilibrium.

6 Coordination Problem and Benchmark Model

In this section, I assume that $|\mathcal{N}| = 2N$, $N \to \infty$, and at each $d_i = \frac{[\frac{i+1}{2}]}{N}$, $i \in \{1, ..., 2N\}$, there exist two agents. After finding the equilibrium of the game, I compare

[1] I didn't proved it analytically, but using numerical methods it can be verified that $u_1 u_2 \neq v_1 v_2$ and strategy $\{F, S\}$ risk-dominates other strategies.

the average social utility with results of benchmark model.

Let's consider the action set of agents at $d = \frac{1}{N}$. Player 1, located at $d_1 = \frac{1}{N}$, has $2N+1$ actions: $\{F, \{S_k\}_{k\in\mathcal{N}_{-\{1\}}}, Z\}$ and Player 2, located at $d_2 = \frac{1}{N}$, has the same number of actions. Action set and utility payoffs for player 1 and 2 can be seen in the table:

	F	S_1	S_3	$\cdots$	S_{2N}	Z
F	f_1, f_2	$f_1, s_{2,1}$	$f_1, \hat{s}_{2,3}$	$\cdots$	$f_1, \hat{s}_{2,2N}$	$f_1, 0$
S_2	$s_{1,2}, f_2$	$0, 0$	$\hat{s}_{1,2}, \hat{s}_{2,3}$	$\cdots$	$\hat{s}_{1,2}, \hat{s}_{2,2N}$	$0, 0$
S_3	$\hat{s}_{1,3}, f_2$	$\hat{s}_{1,3}, \hat{s}_{2,1}$	$\hat{s}_{1,3}, \hat{s}_{2,3}$	$\cdots$	$\hat{s}_{1,3}, \hat{s}_{2,2N}$	$\hat{s}_{1,3}, 0$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
S_{2N}	$\hat{s}_{1,2N}, f_2$	$\hat{s}_{1,2N}, \hat{s}_{2,1}$	$\hat{s}_{1,2N}, \hat{s}_{2,3}$	$\cdots$	$\hat{s}_{1,2N}, \hat{s}_{2,2N}$	$\hat{s}_{1,2N}, 0$
Z	$0, f_2$	$0, 0$	$0, \hat{s}_{2,3}$	$\cdots$	$0, \hat{s}_{2,2N}$	$0, 0$

In the above table $\hat{s}_{i,j}$ indicates that the payoff of player i depends on the information held by agent j. That means:

$$\hat{s}_{i,j} = (1 - d_{i,j})I_j - d_{i,j}$$

Since $I_j < 1$, one can argue that for any $j \in \{j | j \in \mathcal{N}, j \notin \{1, 2\}\}$, $\hat{s}_{1,j} < 1 - 2d_{1,j} \leq 1 - \frac{2}{N} = f_1$. On the other hand, it is obvious that $0 < f_1$. That means, for player 1 and 2, pure strategy F strictly dominates strategy Z and any strategy S_j if $3 \leq j$. Thus, the game can be written in the following form:

	F	S_1
F	f_1, f_2	$f_1, s_{2,1}$
S_2	$s_{1,2}, f_2$	$0, 0$

And because both player have distance $d_1 = \frac{1}{N}$ from the information source, the game can be simplified even further:

	F	S
F	f, f	f, s
S	s, f	$0, 0$

The above game is the one that has been examined in section **??**, and its risk-dominant equilibrium is:

$$p(F) = \frac{N-2}{N-1} \quad , \quad p(S) = \frac{1}{N-1}$$

Using the exact reasoning for removing dominated strategies, one can argue that for agents 3 and 4 the game will be:

	F	S_1	S_2	S_3
F	f_3, f_4	$f_3, s_{4,1}$	$f_3, s_{4,2}$	$f_3, s_{4,3}$
S_1	$\hat{s}_{3,1}, f_4$	$\hat{s}_{3,1}, \hat{s}_{4,1}$	$\hat{s}_{3,1}, \hat{s}_{4,1}$	$\hat{s}_{3,1}, \hat{s}_{4,1}$
S_2	$\hat{s}_{3,2}, f_4$	$\hat{s}_{3,2}, \hat{s}_{4,1}$	$\hat{s}_{3,2}, \hat{s}_{4,1}$	$\hat{s}_{3,2}, \hat{s}_{4,1}$
S_4	$s_{3,4}, f_4$	$\hat{s}_{3,4}, \hat{s}_{4,1}$	$\hat{s}_{3,4}, \hat{s}_{4,1}$	$0, 0$

As opposed to the game that I discussed in section 4, coordination problem makes players 3 and 4 uncertain whether player 1 and 2 aquired the information or not. The probability that agents 1 and 2 fail to acquire information is $p(S_1)^2 = \frac{1}{(N-1)^2}$. One assumption that I'll make to proceed is the risk-neutrality of the agents[1]. Using the risk-neutrality assumption, one can argue:

$$\begin{aligned}
\hat{s}_{3,1} = \hat{s}_{3,2} = \hat{s}_{4,1} = \hat{s}_{4,2} &= (1 - d_{3,1})\mathbf{E}[I_1] - d_{3,1} \\
&= (1 - \frac{1}{(N-1)^2})(1 - d_{3,1})I_1 - d_{3,1} \\
&= (1 - \frac{1}{(N-1)^2})(1 - \frac{1}{N})^2 - \frac{1}{N} \\
&= \frac{(N-1)^2 - 1}{(N-1)^2}(\frac{N-1}{N})^2 - \frac{1}{N} \\
&= \frac{N^2 - 2N}{N^2} - \frac{1}{N} \\
&= \frac{N-3}{N}
\end{aligned}$$

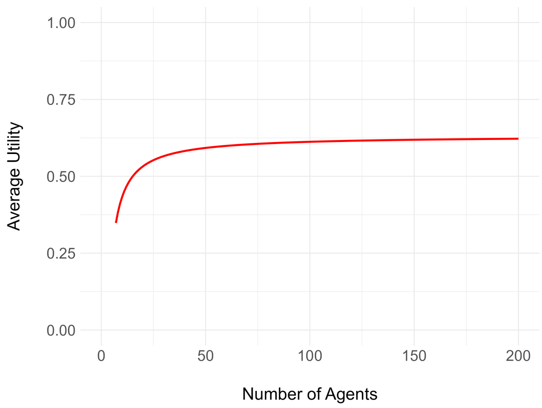
Since

$$\begin{aligned}
f_3 = f_4 &= (1 - \frac{2}{N}) - \frac{2}{N} \\
&= 1 - \frac{4}{N} < \hat{s}_{3,1} = \frac{N-3}{N},
\end{aligned}$$

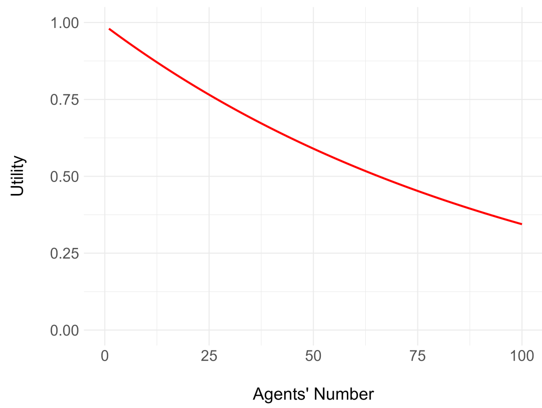
it can be argued that F is strictly dominated by S_1 and S_2. Although I couldn't complete the proof analytically, numerical results support existence of unique chain equilibrium for the new specification. However, in the new specification the chain equilibrium exist when $7 \leq N^2$. That means, lemma 1-3 and proposition 1 hold for the new specification whenever $7 \leq N$.

Because the free riding problem is not severe in this specification, the average expected social welfare and expected utility of individuals won't be affected by much as it can be seen through 3

[1]One should note that risk aversion and risk dominance are two separated notions (Straub, 1995).
[2]I tested the uniqueness of equilibrium for $7 \leq N \leq 2000$



(a) Average Welfare as a Function of N

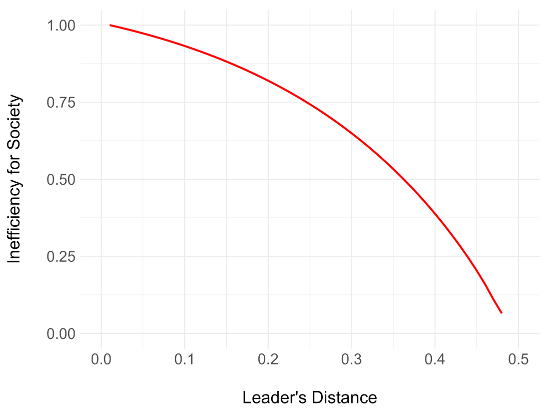


(b) Individual Utilities when $N = 100$

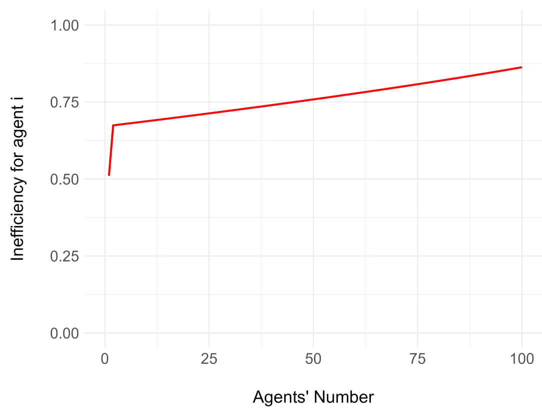
Figure 3: Average utility and individuals utility (N pairs of homogeneous agents)

6.1 Severity of Free Riding Problem

In this section, I vary the distance of first pair of agents from the information and analyze its effect from a social welfare perspective. That means, I assume the distance of first pair of agents, which I call them leaders, is c, and the difference between each two neighbours is $\frac{1-c}{N}$[1]. Note that as c gets larger, the severity of free riding problem increases for the first pair of agents, and it affect the average social welfare. In figure 4 the average social welfare of the specification in this section is compared with the benchmark model.



(a) Aggregate Loss as a Function of c



(b) Individual Loss ($c = 0.25$)

Figure 4: Welfare Effects of Free Riding ($N = 100$)

As it can be seen through figure 4, as leaders get more distant from the information source, the loss in aggregate welfare gets larger.

[1]Numerical results support the existence of unique chain equilibrium for large N. For example, for $N = 100$ and $c < 0.47$, the unique chain equilibrium exist

7 Random Heterogeneity

In this section, I assume that agent's distances comes from a random distribution. I assume the underlying distribution of agents' distances is a normal distribution[1].

I first generate 1000 random variables from a standard normal distribution and after re-scaling the random numbers to fit into $(0, 1]$ interval, I round them to the second decimal. The distribution of agents on $(0, 1]$ interval can be seen through figure 5

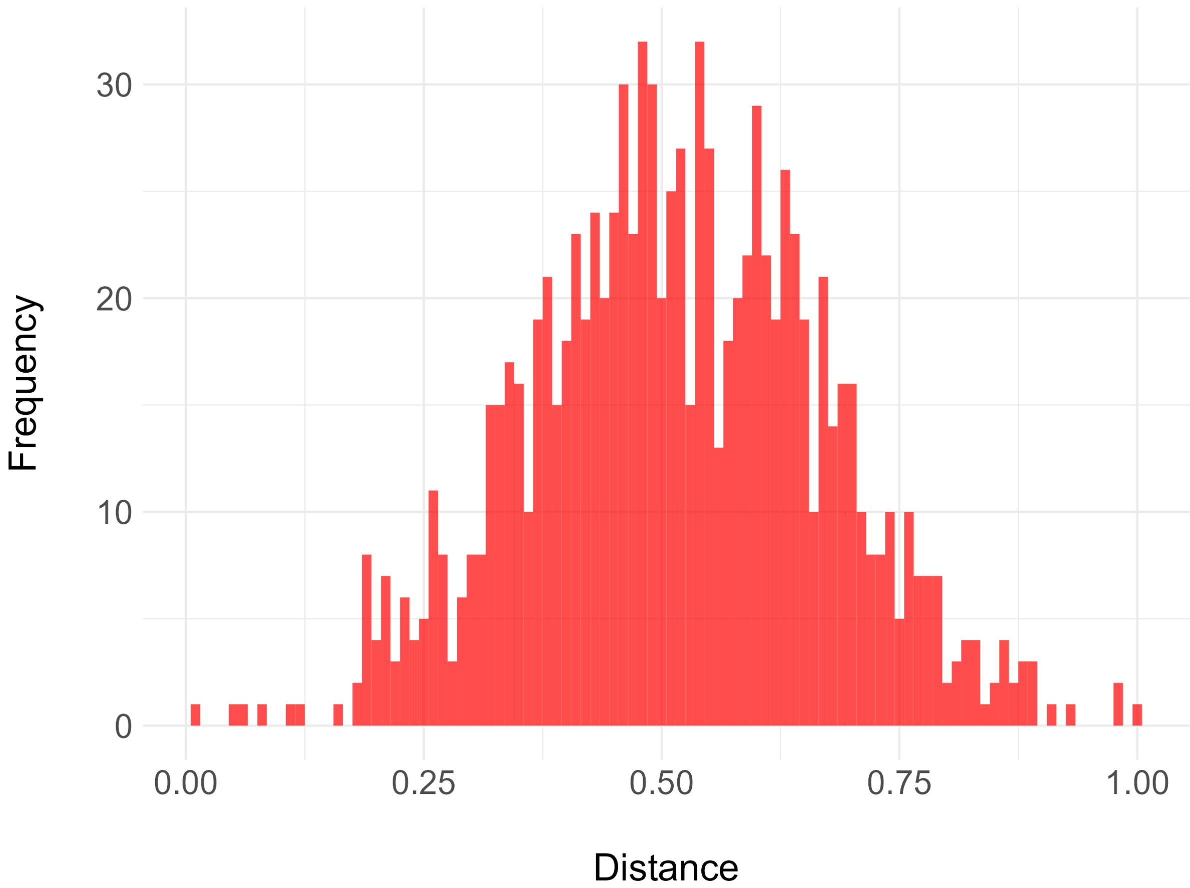


Figure 5: Distribution of Agents' distances on $(0, 1]$ interval (Normal distribution)

As it can be seen through figure 5, there may exist numerous homogeneous agents at each point on $(0, 1]$ interval. For the homogeneous agents I numerically solve the below equation to find the proper p[2]:

$$Q(1, N, p) = p + \frac{(N-2)(1-p)}{N-1} \sum_{1}^{N-2} \frac{(N-3)!(1-p)^{i-1}p}{(N-2-i)!(N-1)^{i-1}}$$

[1]One major caveat to this section is that I didn't do any analytical or numerical prove that the equilibrium will be a chain equilibrium. I only assume that a chain equilibrium exist

[2]The equation has been discussed in section 5.1.1

The individuals' expected utility and average social welfare is presented in figure 6. It can be seen that the average social welfare has decreased[1], but the loss is not so high[2].

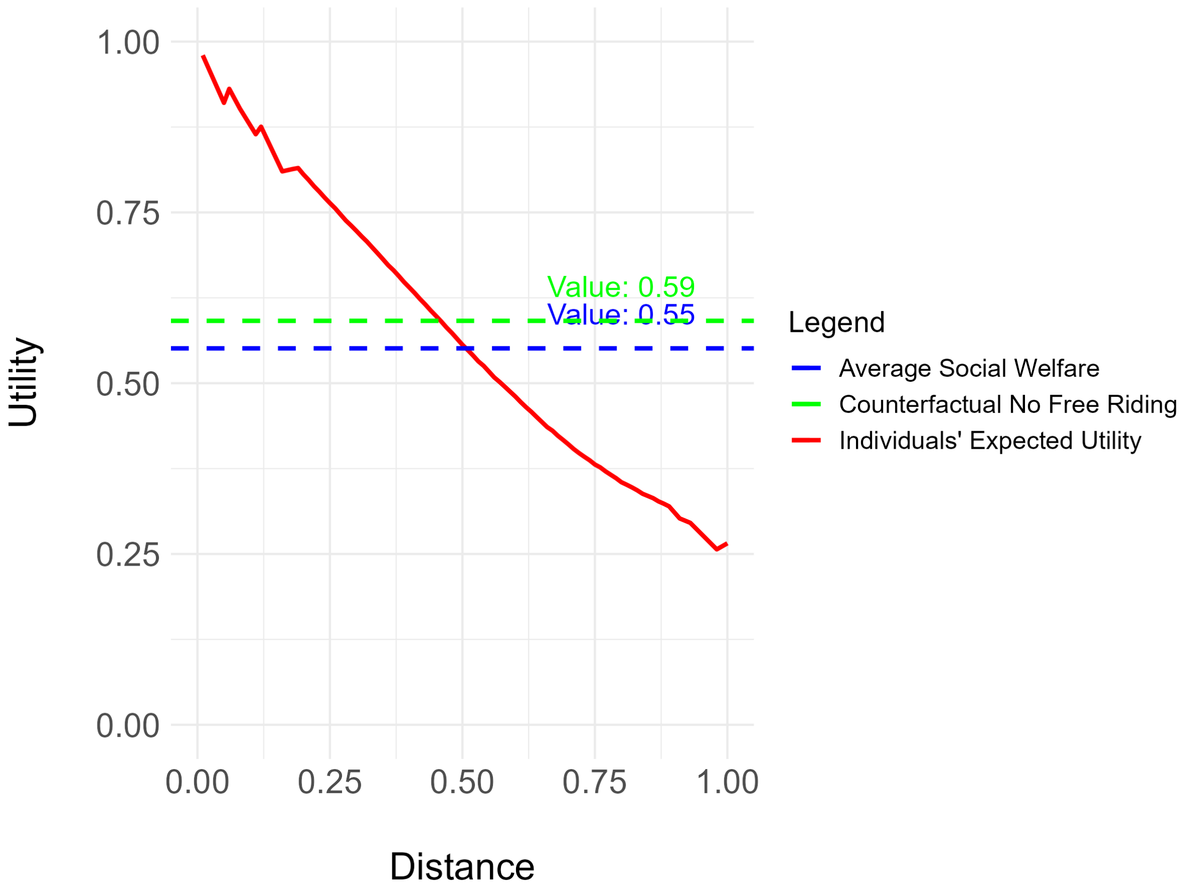


Figure 6: Expected Utility of Agents as a Function of Distance (Normal distribution)

7.1 Slow Information Diffusion

If one assume that information sharing between homogeneous agents occurs instantly and information diffusion through any other link take 1 period, the diffusion curve will be as 7. As it can be seen, the shape will be S-shaped curve which is what is expected to see for the pace of innovation diffusion. However, the result is not satisfying because of two reasons. First, I haven't prove that the equilibrium will be a chain equilibrium. Second, the original game is a static game which time has not been incorporated in agents' utilities. That means, if agents knew that they will learn about the information by a delay, they should've incorporate this into their utility function.

[1]In the benchmark model the average social welfare was 0.63

[2]Note that the loss depends on the generated random values. To calculate the expected utilities, one should do Monte Carlo simulation.

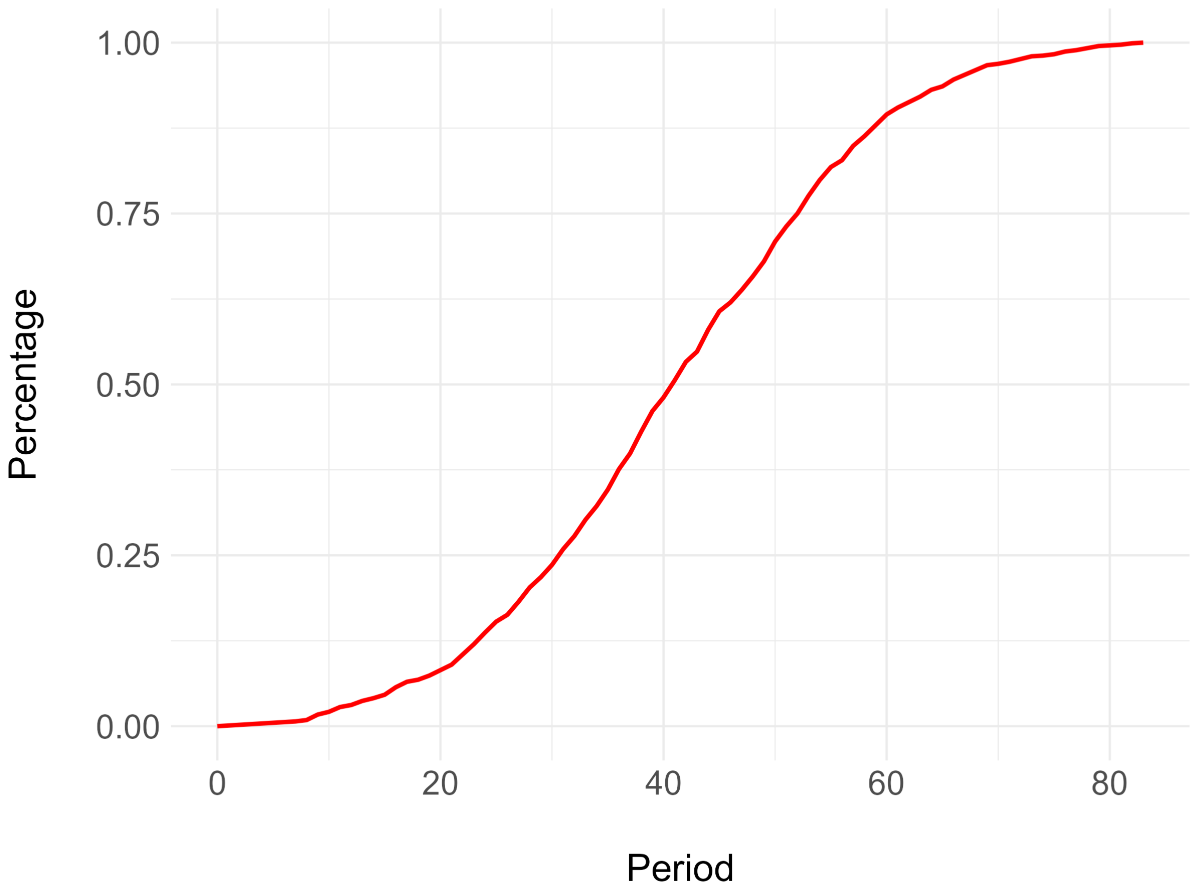


Figure 7: Percentage of agents that learned about the innovation at each time interval (Normal distribution)

7.2 Skewed Distribution

In this section, I assume that agent's distances comes from a χ^2_4 random distribution. I assume the underlying distribution of agents' distances is a χ^2_4 [1].

I first generate 1000 random variables from a χ^2_4 distribution and after re-scaling the random numbers to fit into $(0, 1]$ interval, I round them to the second decimal. The distribution of agents on $(0, 1]$ interval can be seen through figure 8

[1]Similar to section 7, one major caveat to this section is that I didn't do any analytical or numerical prove that the equilibrium will be a chain equilibrium. I only assume that a chain equilibrium exist

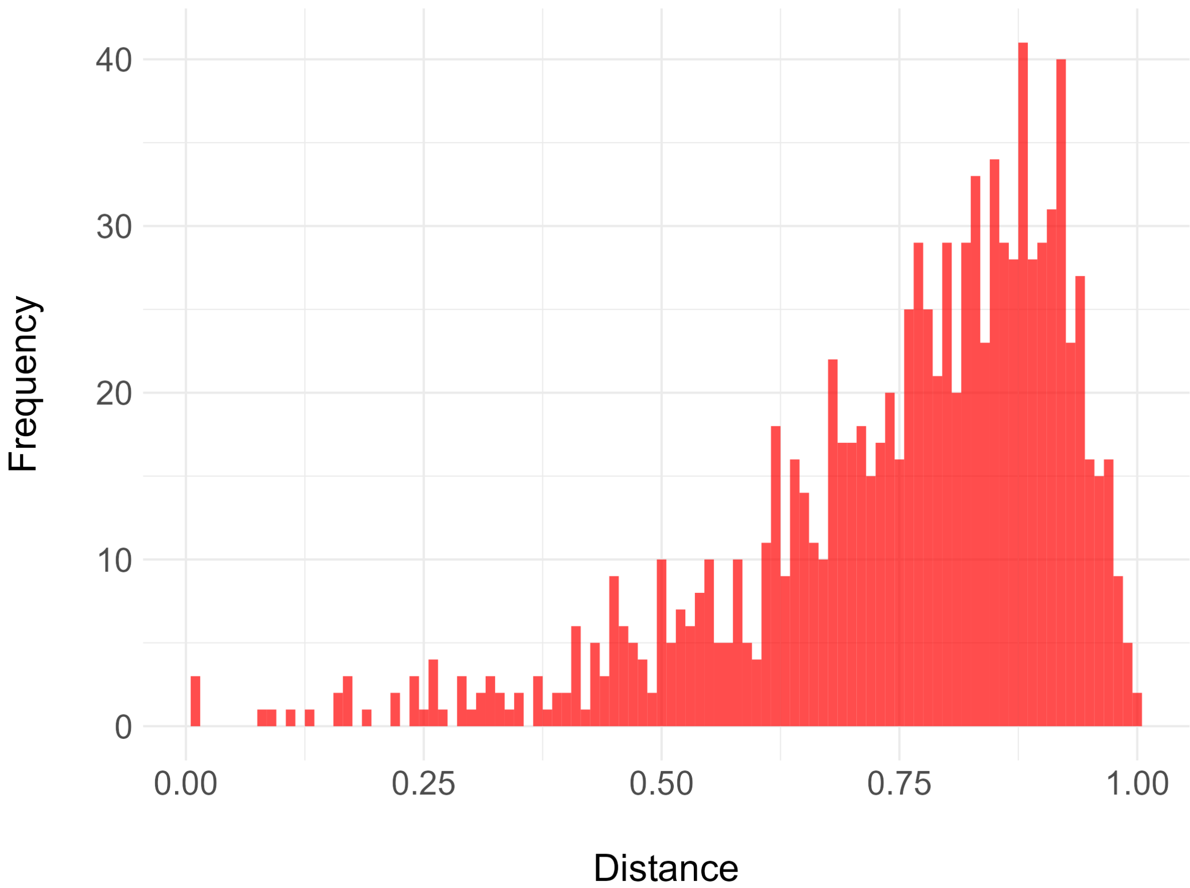


Figure 8: Distribution of Agents' distances on $(0, 1]$ interval (Chi Squared distribution)

As it can be seen through figure 8, there may exist numerous homogeneous agents at each point on $(0, 1]$ interval. For the homogeneous agents I numerically solve the below equation to find the proper p[1]:

$$Q(1, N, p) = p + \frac{(N-2)(1-p)}{N-1} \sum_{1}^{N-2} \frac{(N-3)!(1-p)^{i-1} p}{(N-2-i)!(N-1)^{i-1}}$$

The individuals' expected utility and average social welfare is presented in figure 9. It can be seen that the average social welfare has decreased[2], but the loss is not so high[3].

[1]The equation has been discussed in section 5.1.1

[2]In the benchmark model the average social welfare was 0.63

[3]Note that the loss depends on the generated random values. To calculate the expected utilities, one should do Monte Carlo simulation.

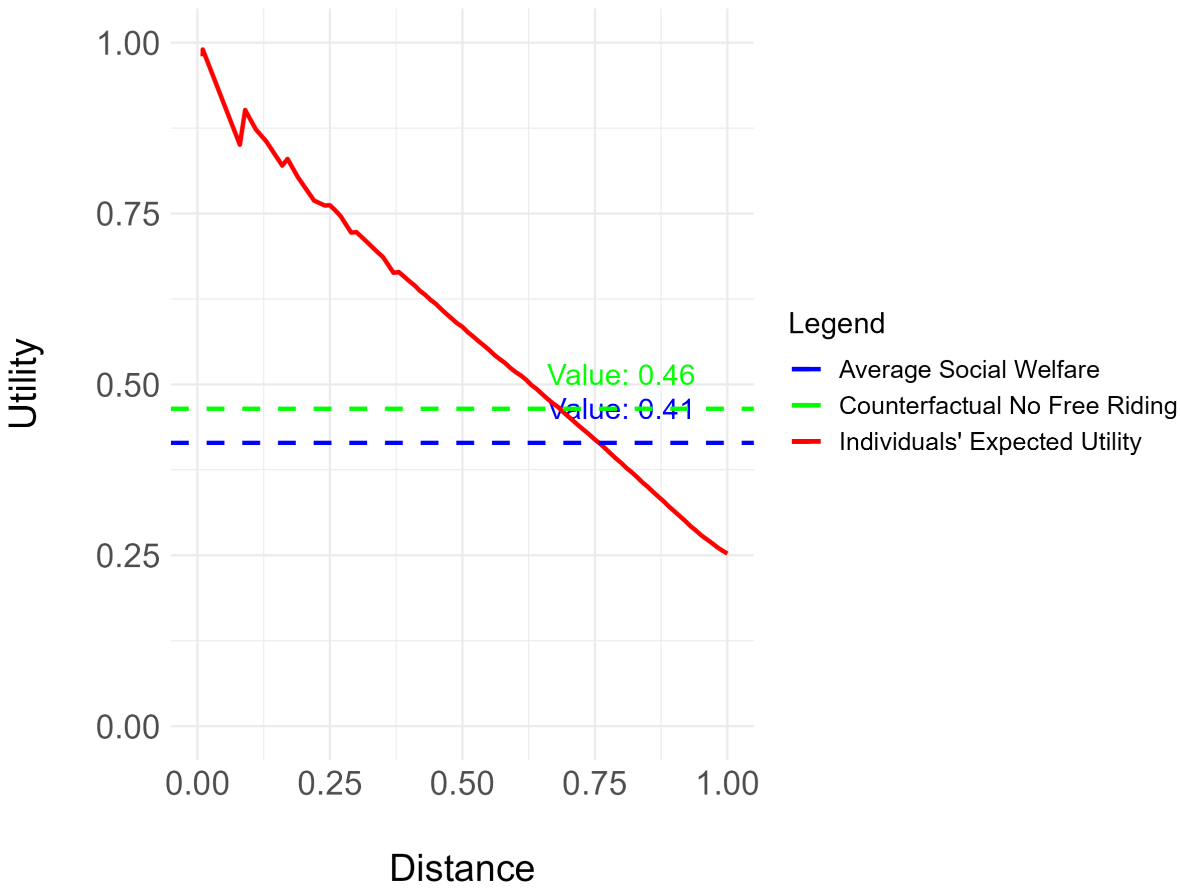


Figure 9: Expected Utility of Agents as a Function of Distance (Chi Squared distribution)

Using the same assumptions as in 7.1 the diffusion curve will be as 10. As it can be seen, the pace of innovation diffusion is slower compared to the normal distribution in figure 7

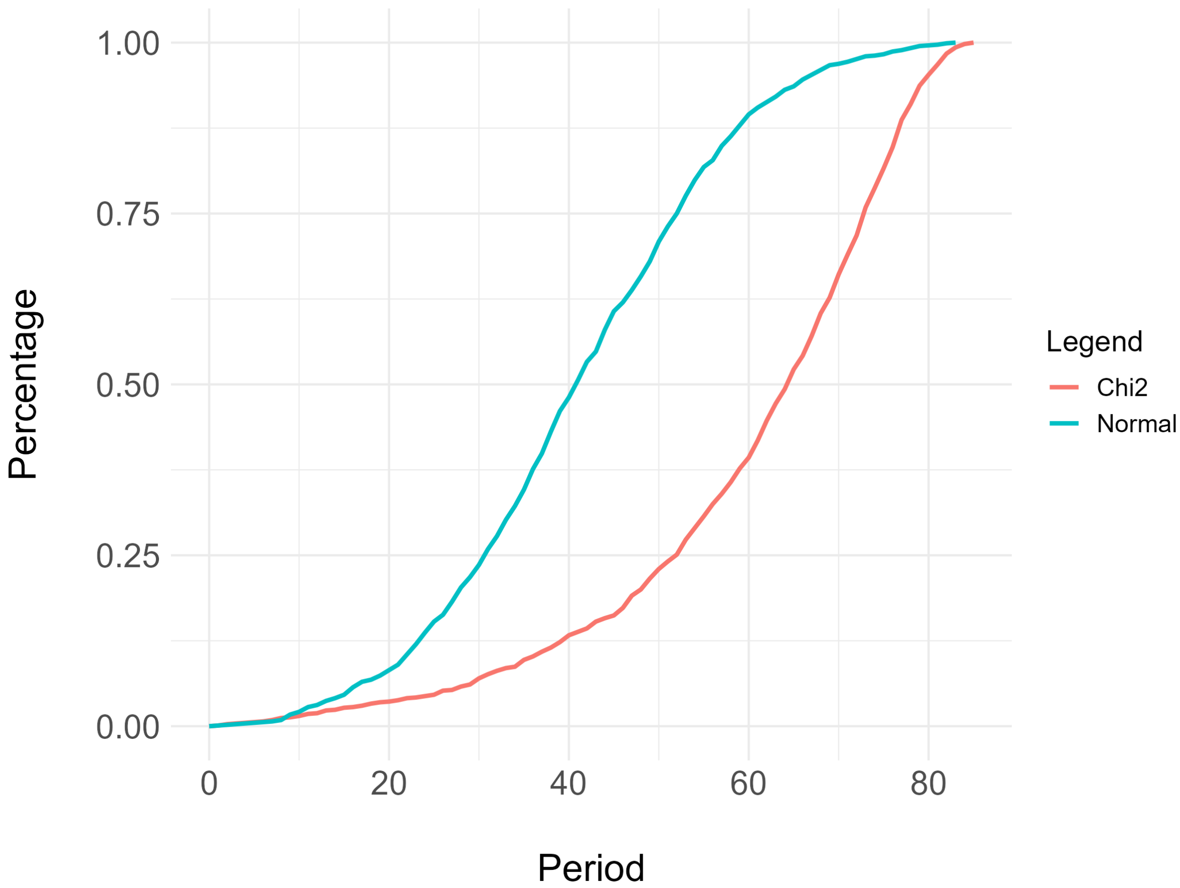


Figure 10: Comparing innovation diffusion pace for Normal and Chi-squared distribution (y-axis is percentage of agents which learned about the information)

8 Conclusions

To study the innovation diffusion process, I developed a static game-theoretic model with heterogeneous agents. The model justifies the rationality of social learning and hierarchy formation. That means, the model predicts the most efficient way for innovation diffusion is through learning from neighbours and a chain equilibrium. Although the model does not incorporate time in utility functions of agents, under some assumptions the model can be used to justify the slow process of innovation diffusion.

The model predicts existence of free riding and coordination problem in the innovation diffusion process. Although the effect of coordination failure might be negligible or even positive for society, free riding might cause a significant negative effect on welfare of society under some specifications. For example, if one assume the underlying distribution of agents' heterogeneity is a skewed distribution, the negative effect of free riding becomes significant.

Although I could analytically prove the existence of unique chain equilibrium for basic model in section 4, the results presented in sections 6 through 7 should be approached with some skepticism[1].

Simplicity of the model allow for extension in several ways. One possible extension to the model can be incorporating time into utility functions of agents to study the effects on the speed of diffusion. Another possible extension might be adding a fixed loss of information through acquisition of information through peers.

[1]Specifically, for section 7 I only assumed that the chain equilibrium exist and it is the unique equilibrium of the game

References

Baptista, Rui (2000) "Do innovations diffuse faster within geographical clusters?" *International Journal of Industrial Organization*, 18 (3), 515–535, 10.1016/S0167-7187(99)00045-4.

Bikhchandani, Sushil, David Hirshleifer, Omer Tamuz, and Ivo Welch (2021) "Information Cascades and Social Learning," Working Paper 28887, National Bureau of Economic Research, 10.3386/w28887.

Bisztray, Márta, Miklós Koren, and Adam Szeidl (2018) "Learning to import from your peers," *Journal of International Economics*, 115, 242–258, 10.1016/j.jinteco.2018.09.010.

Harsanyi, John C and Reinhard Selten (1988) *A general theory of equilibrium selection in games*, 1: The MIT Press.

Munshi, Kaivan (2007) "Chapter 48 Information Networks in Dynamic Agrarian Economies," in *Handbook of Development Economics*, 4, 3085–3113: Elsevier, 10.1016/S1573-4471(07)04048-X.

Myerson, Roger B. (1991) *Game Theory: Analysis of Conflict*: Harvard University Press, `http://www.jstor.org/stable/j.ctvjsf522`.

Ryan, Bryce and Neal C Gross (1943) "The diffusion of hybrid seed corn in two Iowa communities.," *Rural sociology*, 8 (1), 15.

Straub, Paul G. (1995) "Risk dominance and coordination failures in static games," *The Quarterly Review of Economics and Finance*, 35 (4), 339–363, `https://doi.org/10.1016/1062-9769(95)90048-9`.

Verhoogen, Eric (2023) "Firm-Level Upgrading in Developing Countries," *Journal of Economic Literature*, 61 (4), 1410–1464, 10.1257/jel.20221633.

Young, H. Peyton (2009) "Innovation Diffusion in Heterogeneous Populations: Contagion, Social Influence, and Social Learning," *American Economic Review*, 99 (5), 1899–1924, 10.1257/aer.99.5.1899.