Can Persistent Real Rate Beliefs Explain Aggregate Consumption Dynamics?

A New-Keynesian DSGE Approach

by

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I, the undersigned, **Tibor Balazs**, candidate for the **Master of Arts in Economics** degree, declare herewith that the present thesis titled *"Can Persistent Real Rate Beliefs Explain Aggregate Consumption Dynamics? A New-Keynesian DSGE Approach"* is exclusively my own work, based on my research and only such external information as properly credited in notes and bibliography. I declare that no unidentified and illegitimate use was made of the work of others, and no part of the thesis infringes on any person's or institution's copyright.

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Vienna, 10 June 2025 **Tibor Balazs**

Abstract

The Euler equation represents a central piece in modern macroeconomic theory, underlying dynamic stochastic general equilibrium modeling conventionally used by central banks and governments worldwide. Despite its ubiquity, the canonical formulation of the Euler equation provides strikingly poor empirical performance in predicting consumption growth, putting in jeopardy both the empirical and theoretical results delivered by the microeconomics-founded macroeconomics research paradigm. The following thesis proposes a theoretical extension and evaluates the stochastic formulation's implications in a New-Keynesian DSGE framework. The results indicate that belief shocks explain approximately 25 percent of consumption growth dynamics, about 32 percent of inflation dynamics, and capture almost the total variation in policy rate dynamics.

Acknowledgments

I would like to thank my thesis supervisor, Dr. Tamas Istvan Konya, for guidance and support.

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List of Abbreviations

DSGE	Dynamic Stochastic General Equilibrium
CAPM	Capital Asset Pricing Model
MPC	Marginal Propensity to Consume
NKPC	New-Keynesian Phillips Curve
PIH	Permanent Income Hypothesis
RCT	Randomized Control Trial

Chapter 1

Introduction

The following thesis focuses on the research gap involving stochastic functional relationships connecting interest rates to consumption growth. Contrary to conventional approaches that attribute discrepancies to empirical shortcomings—such as omitted variables—or to theoretical deficiencies—such as misspecified micro-level consumption models—this thesis demonstrates that persistent interest rate beliefs play an important role in aggregate consumption dynamics. Historically, consumption research has revolved around the aggregate consumption's response to variations in the income process. In the following thesis, we shift the focus to the effects of stochastic extensions related to the interest rate and the effects of these interest rate processes on aggregate consumption growth and macroeconomic dynamics. We establish that interest rates and consumption growth are indeed related. Empirically, this relationship is best described by more complex functional forms potentially involving lags, expectations, learning, or other kinds of non-linearities or state dependencies, and most importantly some form of stochasticity, which will be the focus of the following thesis.

The relationship between consumption and interest rates has been one of the main focuses of macroeconomic theory since the dawn of macroeconomic science. The crucial nature of this relationship is twofold. First, the most self-evident reason is policy-related. As Romer (2012) argued, knowing how people's savings and consumption decisions depend on the prevailing interest rate has potentially important effects from a policy perspective:

”For example, many economists have argued that more favorable tax treatment of interest income would increase saving and thus increase growth. But if consumption is relatively unresponsive to the rate of return, such policies would have little effect. Understanding the impact of rates of return on consumption is thus important." (Romer, 2012, p. 380)

Moreover, standard growth theory tells us that finding the balance between optimal consumption and the savings rate is crucial from the standpoint of maximizing the present value of consumption, and consequently welfare.

There is a second, less self-evident reason why the relationship between the interest rate and consumption is of prime importance. The apparatus of modern macroeconomics (and many fundamental results in related fields such as the consumption capital asset pricing model in finance) hinges on the performance of the Euler equation. In particular, the canonical Euler equation states that consumers balance the marginal benefits of present and future consumption based on the prevailing interest rate. For our purposes, this merely implies the following: the consumer's intertemporal cost of consumption and intertemporal elasticity parameter, or their intertemporal preferences, determine the first-order condition that defines the Euler equation.

Historically, the empirical and theoretical analysis of this relationship has a long and illustrious tradition, ranging from *The General Theory of Employment, Interest, and Money* (Keynes, 1936), to *A Theory of the Consumption Function* (Friedman, 1957), *The "life cycle" hypothesis of saving* (Modigliani, 1963), and *Stochastic implications of the life cycle–permanent income hypothesis* (Hall, 1978). This literature culminated in the permanent income hypothesis, which has formed the basis for much of the consumption research literature in the second half of the 20th century.

Despite this illustrious research history, no clear and definitive answers have been arrived at regarding the ultimate nature of the relationship between aggregate consumption and interest rates. Many partial results have been derived placing the primary focus on stochasticity in the income processes following the footsteps of Keynes, but a general description of consumption behavior as constituted by its response to variations in income and the interest rate has remained elusive to this day. The broader problem—as the thesis will in part demonstrate—likely stems not simply from an empirical or theoretical misspecification in the equations describing consumer behavior; forecasting consumption growth is an inherently difficult multicausal problem hindered by all the technical issues related to time series analysis and the limitations posed by lack of RCT data in general. Nevertheless, as macroeconomic, and especially consumption research might still be considered to be in its relatively early days as far as research programs go, further analysis focusing on the research gap between consumption growth and stochastic extensions related to the interest rate process—as the thesis will demonstrate—might still hold valuable insights into the relationship governing macroeconomic phenomena and macroeconomic modeling in general.

The plan for the thesis is the following: first, we will introduce the most important milestones in the development of consumption theory in order to lay the groundwork for the theoretical framework and contextualize the thesis's contributions, and review the existing literature pertaining to the empirical and theoretical issues related to the connection between consumption and interest rates. Following the literature review, we will present the preliminary empirical analysis of the time series datasets used in the upcoming sections, and discuss their relevant statistical properties. After that we turn to the discussion of the proposed stochastic extension and delineate the theoretical framework that will serve as the basis for the empirical estimation and discussion in the following sections. Following the theoretical section, we will analyze the empirical performance of the proposed extension based on interest rate and consumption data from the United States in a New-Keynesian dynamic stochastic general equilibrium framework. Finally, we conclude the paper by discussing and summarizing its findings.

Chapter 2

Literature Review

2.1 The Context of Modern Consumption Theory

Modern consumption research began with John Maynard Keynes' groundbreaking work, *The General Theory of Employment, Interest, and Money*. Keynes focused on the behavior of macroeconomic aggregates and their corresponding comovements in his book. He postulated a theory in which the primary driver of the tendency to consume is aggregate income, in contrast to interest rates. Keynes emphasized his belief that there is only a weak correlation between interest rates and the inclination to consume (Keynes, 1936, p. 79).

In the theoretical framework proposed by Keynes, the propensity to consume depended on aggregate income and was postulated to be relatively stable across time. Keynes thought that the relationship between aggregate income and aggregate consumption should be fairly stable, that is, he thought that the relationship between aggregate income and consumption is time invariant, which we will empirically show is in fact not the case. He also postulated, well ahead of his time, that expectations and confidence play a crucial role in determining consumption decisions. He argued that, as opposed to rationality, it is mostly irrational considerations that are the main driving force behind short-run market fluctuations (Keynes, 1936, p. 70).

Keynes' theory would have predicted that consumption growth would decline as aggregate income grew due to the postulation of a falling average propensity of consumption as a function of income. The implications of Keynes' theoretical framework caused concern, as they would theoretically have led to so-called secular stagnation, or extended periods of sluggish growth. However, Keynes' theory seemed to perform well during the interwar period and in the years following the Second World War, consequently Keynes' influence and authority were not questioned. His policy prescriptions aligned with the zeitgeist of increasing state intervention; therefore, Keynes became an authority figure until his early passing in the late 1950s. Keynes' theoretical dismissal of the interest rate as a driving force behind intertemporal decision-making hindered theoretical progress in identifying important factors in consumption growth prediction—such as stochasticity in the interest rate process and decision rules that take into account these inherent stochasticities, an area where the following thesis seeks to contribute to the existing literature. Meticulous empirical work by Kuznets (1971) in the post-war period, however, appeared to refute the Keynesian model's prediction about extended periods of stagnation as growth did not seem to slow down as predicted by the Keynesian theoretical framework. This indicated fundamental issues in Keynesian consumption theory, which led to a surge in consumption research built on the foundation laid by Irving Fisher in his theory of interest rate.

Fisher (1930) alternatively argued that consumption does not primarily depend on current or disposable income. Specifically, the main driver of intertemporal consumption decisions is the intertemporal choice between present and future consumption, and the future income streams that the individual might reasonably expect, which is primarily determined by the interest rate. As we will demonstrate, Fisher's analysis provided the conceptual foundation for a subsequent analysis focusing on the relationship between savings and interest rate. Fisher argued that capital is simply discounted future income and as such, its main purpose is consumption, and is essentially unspent income, or in other words savings (Fisher, 1930, p. 15).

Opposing Keynes, Fisher believed that the interest rate is an important factor in intertemporal savings and consumption decisions, and thus a crucial factor in determining how much society allocates to consumption as opposed to savings. He argued that interest plays the role of a bridge between income and capital, in other words, savings (Fisher, 1930, p. 16). He emphasized that the interest rate and the concept of capital can be thought of as separate issues, capital conventionally related to the production side of the economy, while future income streams and time preferences conventionally related to the demand side of the economy. Fisher argued that "Capital wealth is merely the means to the end called income, while capital value (which is the sense in which the term capital is ordinarily used by interest theorists) is merely the capitalization of expected income" (Fisher, 1930, p. 40).

In a theoretically novel way, Fisher directly compares interest rates to prices in the sense of signifying the trade-off between consumption in present and future periods. Importantly, Fisher connects the concept of the interest rate directly to subjective preferences, which economically translates into people's time preferences. However, Fisher did not ignore the supply side of the economy. His aim was not to reduce the phenomenon of interest to only demand-side forces in the economy. He also put emphasis on the supply side forces that determine the interest rate, which are directly related to the available investment opportunities in the economy (Fisher, 1930, p. 40).

Moreover, he connects and details the concepts of time preference, or human impatience as he called it, and the rate of interest in the following way, and this served the bedrock of the later emerging optimality condition in intertemporal consumption choice and decisions:

"Time preference, or impatience, plays a central rôle in the theory of interest. It is essentially what Rae calls the "effective desire for accumulation," and what Böhm Bawerk calls the "perspective undervaluation of the future." It is the (percentage) excess of the present marginal want for one more unit of present goods over the present marginal want for one more unit of future goods. Thus, the rate of time preference, or degree of impatience, for present over future goods of like kind is readily derived from the marginal desirabilities of, or wants for, those present and future goods respectively." (Fisher, 1930, p. 40)

The novel conceptualization of Fisher of the interest rate as the price of intertemporal trade serves as a cornerstone for the log-linearized Euler equation approach and the subsequent stochastic extension proposed by this thesis.

A new generation of economists followed in his footsteps with an aim to mathematically formalize the theories verbally expounded by Fisher. Modigliani's (1963) and Milton Friedman's (1957) theories on consumption seemed to have spearheaded a new era in consumption research focusing on the individual decisions of households and consumers based on a heavier intertemporal mathematical optimization framework using Fisher's ideas to think about intertemporal consumption decisions instead of making large aggregate assumptions about the economy without micro foundations, such as Keynesian approaches previously proposed.

Both Friedman (1957) and Modigliani (1963) departed from the Keynesian approach in many ways. First, both Friedman and Modigliani approached the problem of consumption from a fundamentally microeconomic perspective in the sense that they both placed the focus on consumers' intertemporal optimizing behavior, breaking the tradition of thinking in large-scale macroeconomic aggregates catalyzed by Keynes. Second, and most importantly, they placed the focus on the issues and factors influencing and determining intertemporal optimizing behavior and consumption. Factors such as discounting, time preference, and expectations naturally enter into this framework, factors that have been expounded upon by Fisher in his monograph. Fisher's influence cannot be understated in the development of modern consumption theory. Although the approach laid out by Friedman and Modigliani led to a significant advance in consumption theory, it still left open the question as to the exact way how essentially forward-looking, microeconomics-based behavior should be incorporated into the mathematical framework describing intertemporal decision making, a research gap where the following thesis aims to contribute to this long line of research.

Along with Kuznets' (1971) empirical refutation of the simple Keynesian consumption function, building on Fisherian ideas about the relationships between interest rate, time preference, and intertemporal consumption and savings decisions, Modigliani (1963) and Friedman (1957) both conjectured a similar idea, and built their theories on a very similar supposition: they both believed that consumption is not only, or primarily, determined by current income, as opposed to Keynes' postulate, but has a fairly stable component, which Modigliani titled "wealth", and Friedman named "permanent income". Modigliani emphasized the "time shape" of consumption patterns, that is how young people would like to save, while old people would like to consume more. However, this pattern overpredicted old people's consumption growth. Friedman's theory, on the other hand, emphasized the difference between permanent and transitory income, and argued that average propensity to consume is hard to determine since it consists of the average of the permanent and temporary incomes, which are empirically hard to disentangle. Despite the fact that tax-policy effect studies supported Friedman's hypothesis empirically it still remained implausible that only these two factors would influence consumption.

Friedman's (1957) permanent income hypothesis was both a theoretical and empirical success. It explained phenomena that the simple Keynesian consumption function could not account for, thereby rendering Friedman's monograph a seminal contribution to consumption research. Durlauf and Blume (2008) argue that "Some consider Friedman's *A Theory of The Consumption Function* the best economics book since the 19th century, and perhaps earlier, because of its convincing and systematic applications of economic theory to important questions' (Durlauf and Blume, 2008, p. 942).

Despite the early empirical success of Friedman's (1957) model, some issues still persisted in the empirical analysis of consumption behavior. First, the empirical and theoretical apparatus to properly test or implement Friedman's hypothesis was not really available at the time, as consumption research was a relatively new area of economic research in the 20th century. Furthermore, many theoretical issues related to the possibility of structural models, such as the Lucas critique, were simply not yet uncovered in the early second half of the 20th century, rendering Friedman's analysis outdated in that aspect. Another issue related to the econometric techniques used by Friedman, such as the distributed lag methodology, has simply been supplanted as the state-of-the-art method related to macroeconomic modeling of consumption behavior as developments in modern macroeconomics shifted the focus on the analysis of the consumer's optimality condition as a way to recover some of the structural parameters of consumption models. As Durlauf and Blume (2008) describe, "Friedman's basic concept of permanent income and the details of his analysis of it (such as 'distributed lags') are less prevalent today, having been displaced by consumption Euler equations" (Durlauf and Blume, 2008, p. 943).

Mankiw and Campbell (1989) describe the process in which consumption Euler equations supplanted Friedman's approach. They argue that one of the fundamental reasons for this shift in approach occurred due to the Lucasian rational expectations revolution in macroeconomics. They note the impact Lucas' ideas played in the development of contemporary macroeconomics, and how intermediary theoretical and empirical results, such as Robert Hall's 1978 paper on the theoretical and empirical implementation of the Euler equation approach shifted perspectives in consumption research.

"(...) Robert Hall (1978) proposed a new approach to studying consumption that was firmly founded on the postulate of rational expectations and that was immune to the problems Lucas pointed out. Hall suggested that aggregate consumption should be modeled as obeying the first-order conditions for optimal choice of a single, fully rational, and forward-looking representative consumer. The new style of research based on this assumption-sometimes called the "Euler equation approach"-has dominated work on consumption during the past decade." (Mankiw and Campbell, 1989, p. 185).

Friedman's (1957) theory of the permanent income hypothesis, as Romer (2012) argues, however, did not prove to be the be-all-end-all of consumption theory:

"Based on Friedman's Theory, a temporary increase, or" windfall" income should have no effect on current consumption, as the consumer's consumption decision is based on its lifetime income. However, the consumer's current consumption and savings decisions are critical: saving is high when income is high relative to its average and when the current income is less when its lifetime average then saving is negative. According to both Friedman's and Modigliani's theory, the individual uses saving and borrowing to smooth the path of consumption. This is the key idea of the permanent income hypothesis." (Romer, 2012, p. 367)

Hall (1987) extended Friedman's permanent income hypothesis to accommodate for uncertainty in future income streams, and derived his famous results that the permanent income hypothesis—under uncertainty regarding future income streams—implies that consumption follows a random walk. An empirical issue with Hall's result was that consumption typically declines in a recession, or, in other words, with the decline of aggregate income, but typically recovers, implying its predictability, which runs counter to the argument that consumption is a random walk.

Friedman's permanent-income hypothesis along with Modiglianis life-cycle hypothesis spurred a large literature in consumption analyzing consumers intertemporal consumption and savings decisions. The main findings of these multi-decade long research are that the magnitude and frequency of the windfall payments play a crucial role in determining the percentage that people decide to save or consume out of it. Namely, people tend to react less to large and regular payments as opposed to small and irregular payments, most of which people tend to spend, in opposition to Friedman's permanent income hypothesis. Therefore, although the data has some relatively clear conclusions, it is not entirely certain whether Friedman's permanent income hypothesis or Halls random walk hypothesis stands unrefuted.

Building on Friedman's (1957) ideas about the permanent-income hypothesis, one of the first, and highly influential papers in this long line of consumption research was Bodkins'

(1959) Windfall Income and Consumption paper. Bodkins' paper put Friedman's permanent income hypothesis thesis to the test, according to which the marginal propensity to consume out of the windfall income would be relatively low. However, Bodkin found that the MPC out of this windfall income, which was relatively small and irregular, a one-time payment, was large, contradicting Friedman's permanent income hypothesis. Bodkin used data on families and WW2 insurance payments after the end of the Second World War. This result seems to suggest that even a temporary tax cut could stimulate the economy and be effective in increasing aggregate demand. Analyzing WW2 Israeli restitution payment and their effects on consumption patterns, Kreinin (1961) concluded that people saved most of the amount from the payment, although it is worth noting that the payment in Kreinin's case was large as compared to Bodkins' case. The early results on the effects of windfall payments on consumption seemed to have shown ambiguous results.

Subsequent research shifted the focus to expectations and the timing of the windfall incomes. Flavin (1981) found evidence against the permanent income hypothesis in the form of excess sensitivity of consumption to current income. Focusing similarly on the role of expectations, Wilcox (1989) analyzed the effects of pre-announced social security benefits on consumption. Wilcox again found seemingly substantial evidence against Friedman's permanent income hypothesis.

Interestingly, analyzing consumption and income seasonality in Thailand, Paxson (1993) finds evidence for consumption smoothing behavior, in line with the permanent income hypothesis. Paxson's article again draws attention to the importance of the regularity and expectations regarding the future payments: anticipated large fluctuations in income seem to have a smaller effect on current consumption than small unanticipated fluctuations. Souleles (1999), focusing on tax rebates instead of expectations regarding income, has found "significant evidence of excess sensitivity in the response of households' consumption to their income tax refunds" (Souleles, 1999, p. 956). Similarly, Parker (1999) focused on predictable changes in social security taxes in affecting consumption decisions using household-level consumption data and finds that: "Consumers do not perfectly smooth their demand for goods at quarterly frequencies across expected income changes (Parker, 1999, p. 969)".

In his seminal article, Shea (1995) using PSID data tests the permanent income hypothesis using union contract data. Shea finds that "(. . .) predictable wage movements are significantly correlated with consumption changes, contrary to the neoclassical consumption theory. We find that consumption responds more strongly to predictable income declines than to predictable income increases" (Shea, 1995, p. 186). Unlike Shea, Shapiro and Slemrod (2003) focus on tax rebates on consumption decisions and find that they had generally small effects on increasing aggregate consumption: "Only 21.8 percent of households report that the income tax rebate of 2001 led them mostly to increase spending" (Slemrod, 2003, p. 393).

Parker and his coauthors (Parker et al., 2001) focused similarly on the effects of tax rebates, using data from the Consumer Expenditure Survey and find evidence against the permanent income hypothesis. They find evidence against basic formulations of the rational expectations hypothesis. Collado and Browning (2001), using panel data from Spain, examined whether predictable large income changes within a year have a significant impact on consumption decisions, and they seemed to arrive at contradictory results to Parker and Souleles' previous analysis. Similarly to Collado and Browning, Hsieh (2003) examining the effects of Alaska's oil dividends finds evidence in support of consumption smoothing behavior, according to Friedman's permanent income hypothesis. Romer (2012) concludes some of the literature by arguing that arguably only aggregate data might be able to shed light on the above-described macroeconomic regularities, but thus far the literature seems to have produced inconsistent results (Romer, 2012, p. 420). However, the permanent income hypothesis still seemingly cannot be strongly rejected based on accumulated evidence. The historical progression described above reveals significant gaps in our knowledge regarding intertemporal consumption decisions and consumption growth predictions, areas which the following thesis, both theoretically and empirically, aims to address.

2.2 Empirical Connections Between Consumption and Interest Rates

Analytically, the relationship between the real interest rate and consumption can take on various forms, including but not limited to a random walk. This is a crucial intermediary result as it allows us to hypothesize certain functional relationships between aggregate consumption and interest rates and to empirically test them as they are substantiated by theory.

In this chapter we will parse and discuss the literature on how consumption growth and the interest rate are related empirically. Classical analyses of the empirical relationship between consumption growth and interest rate include Weber (1970), Gylfason (1981), Ferson (1983), Hansen and Singleton (1983), Hall (1988b), and Campbell and Mankiw (1989) and Attanasio and Weber (1993).

Weber (1970) crucially finds that interest rates do affect aggregate consumption and that income effects are most likely larger than substitution effects in an intertemporal consumption setting. Weber's analysis seemingly provides corroborative evidence in favor of an empirically significant relationship between consumption growth and the interest rate. Gylfason (1981) argues that based on his findings the interest rate is negatively related to aggregate consumption, which is intuitive, and positively related to inflation expectations. However, Ferson (1983) finds that there seems to be no empirical relationship between real returns on bonds and consumption growth rates.

Similarly to Ferson (1983), Hansen and Singleton (1983) approach the issue from the finance literature and consumption CAPM perspective. They build on previous asset pricing literature which focuses on the idea that consumers allocate consumption across time by trading financial assets. Along with Mehra and Prescott (1985), the authors find that the value of the coefficient of relative risk aversion is inconsistent with much of the empirical findings, which would imply a very high degree of relative risk aversion. Hall (1988) argues that Hansen and Singleton's analysis was not detailed enough, and notes that there is no strong evidence that the elasticity of intertemporal substitution is positive. In addition, he highlights the issue that is implied by the theoretical formulation of some consumption models where the elasticity of intertemporal substitution and relative risk aversion coefficients are, by definition, perhaps too closely related to each other, thereby distorting potential empirical estimates.

Mankiw and Campbell (1989) argue that a notable empirical regularity is the lack of a real empirical relationship between the real interest rate and consumption growth and propose a model in an attempt to explain the empirical facts about the relationship between the real interest rate and consumption growth. The authors criticize Hall's paper on theoretical grounds and suggest a behaviorally extended alternative model. But Mankiw and Campbell note that their model is an extension of previous models, as it allows a differentiation of consumers between consumers following rule-of-thumb behavior and consumers following perfect foresight behavior (but this is not our main focus in the paper, consequently, falls outside the scope of this thesis).

Based on the available empirical evidence, there seems to be a strong case for the observation that no strong empirical relationship between consumption growth, intertemporal substitution and interest rates, which is a problem, since the Euler equation relies upon this predicted relationship, putting in jeopardy much of the modeling apparatus ubiquitously used in both macroeconomics and finance. It is clear that the empirical relationship between consumption growth and interest rates is weak, and although potential sources of these weak correlations have been identified, no conclusive evidence has been presented as to the ultimate cause of this phenomenon. It is highly likely that this weak correlation is a multi-causal phenomenon of which in the following thesis we place the focus on potential stochasticities in the interest-rate process. Given the inconclusive nature of the previous empirical results, in the preliminary empirical analysis chapter we undertake a fresh empirical examination of US macroeconomic data in order to attempt to capture relatively recent empirical trends and relevant statistical properties of the analyzed macroeconomic aggregates.

Chapter 3

Empirical Analysis

In the following chapter we introduce the macroeconomic time series of the two chief variables of interest, namely, consumption and interest rates. After an in-depth discussion of the statistical and econometric properties of the two datasets we select and discuss the best-fitting autoregressive integrated moving average—ARIMA(p,d,q)—models in order to capture the most important empirical characteristics of the datasets.

3.1 Introduction to the Data

For the empirical analysis, we utilized quarterly interest rate and consumption data starting in 1982Q4 for the United States.[1] The data were provided by the International Monetary Fund (IMF) in the form of metadata by country from the International Financial Statistics (IFS) database. In terms of the interest rate variable, the dataset contains six key versions: the central bank policy rate, discount rate, money market rate, treasury bill rate, lending rate, and government bonds, out of which we decided to select the central bank policy rate as both a theoretical and practical monetary policy perspective this is the key policy variable the policy-makers can influence. Because the alternative short-term interest-rate series move in near-perfect lockstep throughout the sample, the choice of a specific rate is empirically inconsequential—the core results remain effectively identical whichever series is employed, as it will be demonstrated,

[1]The empirical conclusions remain qualitatively unchanged when the models are estimated over the entire data span (1954:Q3–2025:Q1). See Appendix A for full-sample results.

the correlations between the various versions of the interest rate variables are extremely strong, rendering each of them applicable to the present analysis. For the consumption variable we chose the household consumption expenditure including non-perishable goods, in a real, seasonally adjusted format, denominated in domestic currency.[2]

3.1.1 Interest Rates

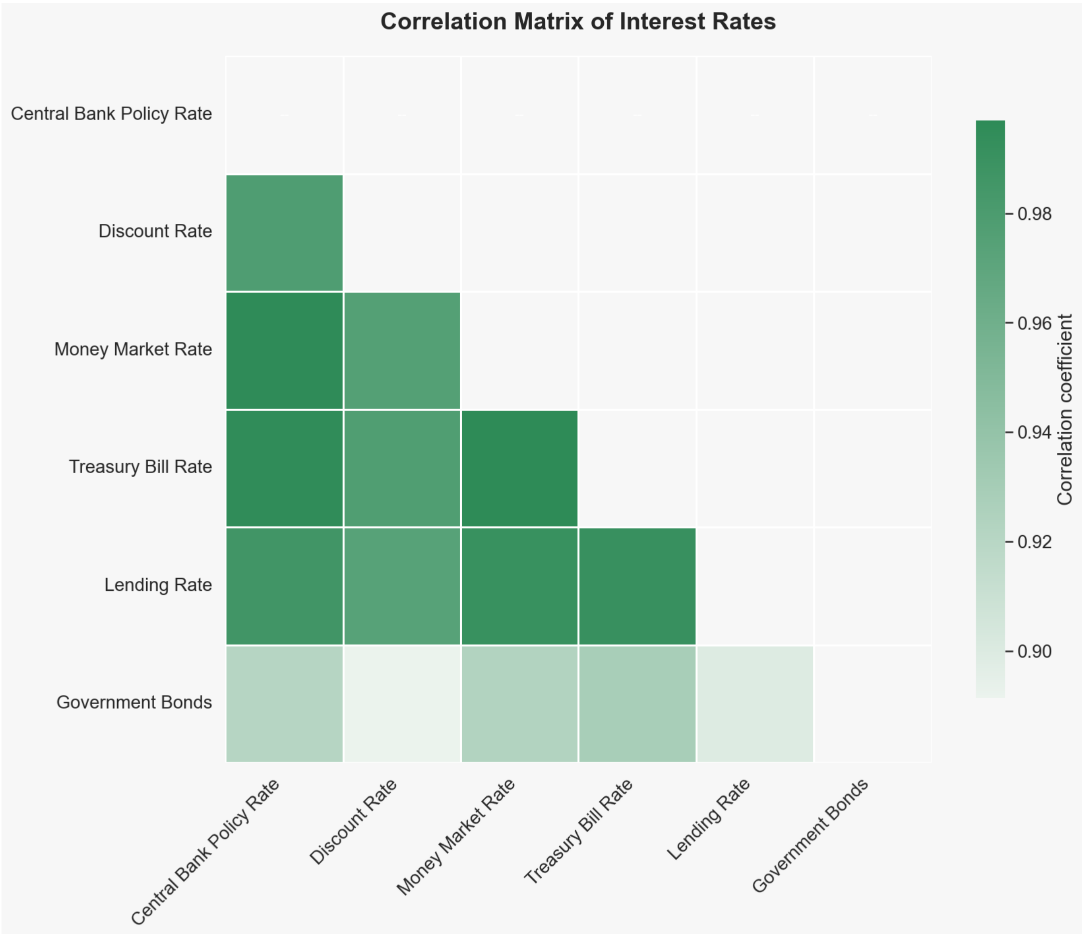

Figure 3.1: Correlation Heatmap Of Quarterly Interest Rates, 1982Q4–latest. The plot was created by the Python script, visualizing the raw data.

Figure (3.1) depicts the correlation heatmap of the selected interest rates in the given time period. Due to the very strong correlations among the various rates, it is apparent that monetary policy in general is closely related to other market instruments. We can see that the lending rate

[2]Consumption series: *Household Consumption Expenditure, incl. NPISHs, Real, Seasonally Adjusted, Domestic Currency* (IMF *International Financial Statistics*, code `NCP_R_SA_XDC`; units: millions).

is somewhat less correlated, which implies a lag in bank pricing behavior. Finally, government bond rate appears to show the weakest correlation, which is consistent with theory, as it is the most affected by non-monetary policy-related factors, such as inflation expectations, and other macroeconomic fundamentals. Based on the graph above, we can see that the interest rates are very strongly correlated, which implies extremely close co-movement in the data. This suggests that any of the above interest rate definitions would have been sufficient for the empirical analysis, but for the theoretical reasons described earlier, the policy rate has been selected as the main interest rate proxy variable.

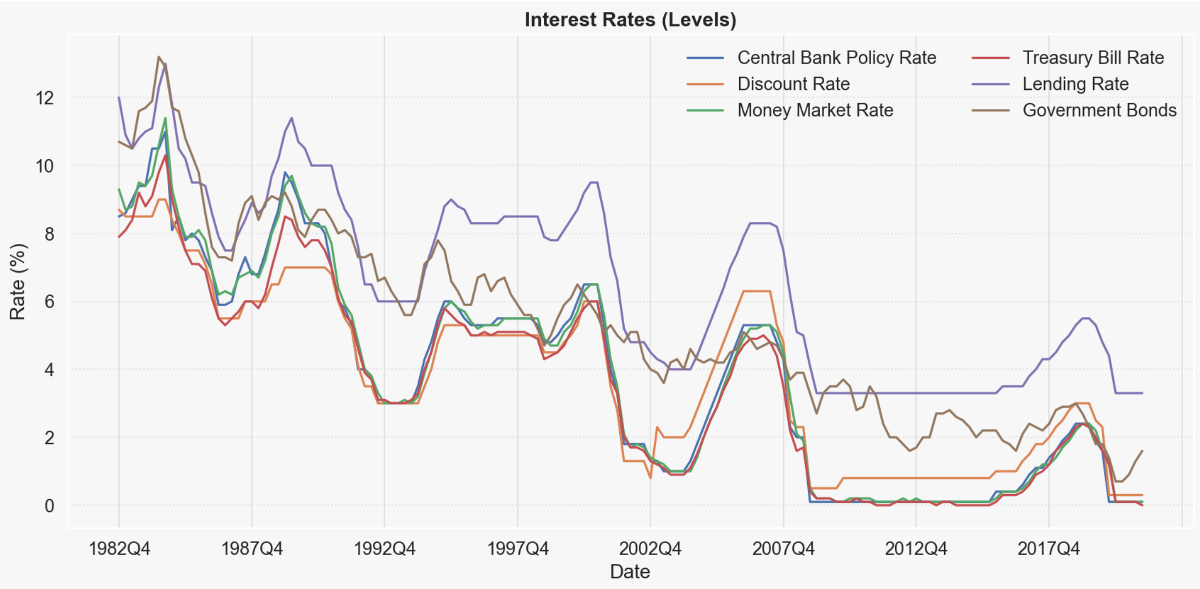

Figure 3.2: Quarterly Interest Rates in Levels, 1982Q4–latest. The above plots were created by the Python script, visualizing the raw data.

Figure (3.2) shows the time series of the interest rates in levels. We can see several noteworthy properties emerge from the plot above: first, we can see a general downward trend in the levels in the given time period for all of the interest rates. In addition, cycles are visible throughout the whole sample, which clearly correspond to periods of monetary policy interventions, such as the dot-com bubble in the early 2000s, the 2007-2008 global financial crisis, and the 2020 Covid-19 pandemic. The effects of these periods will become more apparent on the first-differenced time series. Another interesting phenomenon appears to be that the policy rate seems to be a leading indicator, which implies that monetary policy does influence the other interest rates across the economy. We can also see a relatively persistent spread between the bank lending rates and the other interest rates. Overall, government bond rates appear to be more stable post-2008, but notably, short-term rates are very close to zero, implying near liquidity trap conditions in the US economy. This implies that conventional tools of monetary policy might have been ineffective, compared to the periods with higher volatility in the pre-2000s period. As it will be empirically demonstrated, standard unit-root diagnostics (augmented Dickey-Fuller test, Akaike, Bayesian, and Hannan-Quinn information criteria) reject the theoretical presumption of stationarity: the policy rate exhibits a persistently non-stationary trajectory not only over our estimation window but also across the full 1954:Q3–2025:Q1 sample—a finding already documented in Appendix A's robustness footnote—and the inference is unaffected by the particular sub-period examined.

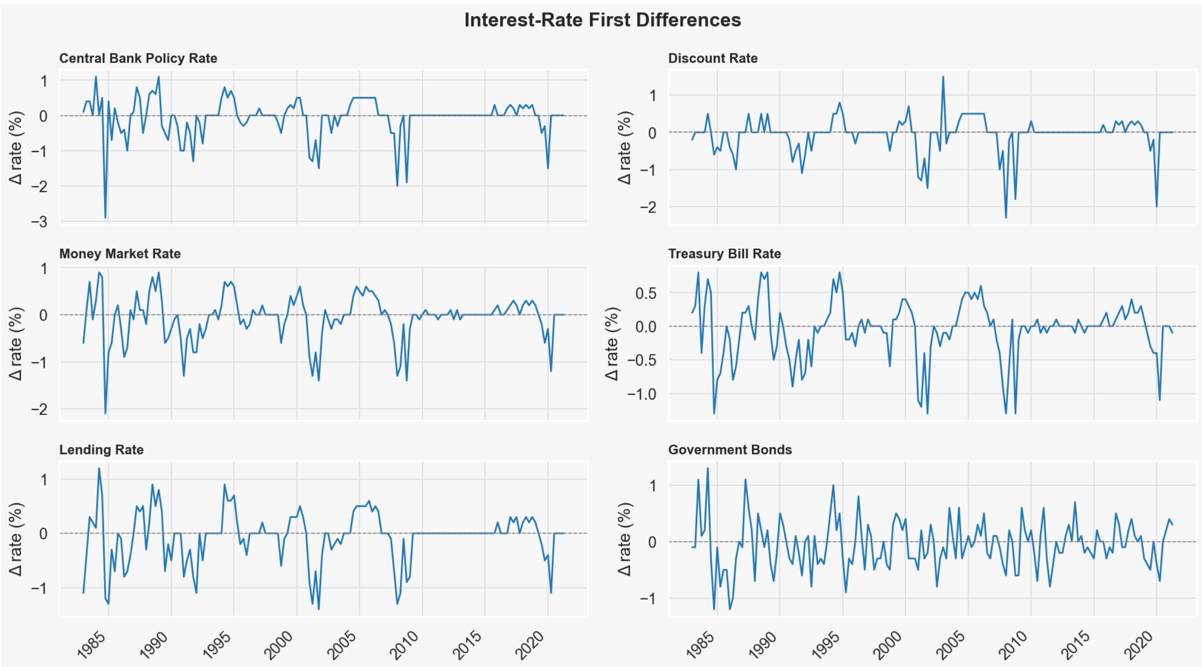

Figure 3.3: Quarterly Interest Rate in First-Differences, 1982Q4–latest. The above plots were created by the Python script, visualizing the raw data.

Figure (3.3) depicts the first-differenced time series of the interest rate variables. We can see high-frequency oscillations in all of the interest rates, albeit the range of the oscillations seems to be relatively small. As described earlier, the pre-2000s period appears to exhibit higher volatility in general. Nonetheless, at least three large spikes are apparent based on a visual inspection of the figure above: the first large positive spike occurred around 2003, presumably caused by the dot-com bubble, the second large negative spike occurred in 2008, at the time of the financial crisis, and the final negative large spike occurred around 2020, at the time of the Covid-19 pandemic. Interestingly, we can observe an asymmetricity property, namely, that large negative differences seem to be more frequent as compared to large positive ones. This might imply that in general it is easier for the central bank to cut rates as compared to increase interest rates after recessionary periods. It appears to be the case that the policy rate and the discount rate are strongly variable, which implies their use as primary policy tools for the central bank, while government bond rate and the lending rate appear to be less volatile. Finally, there appears to be a mean-reverting behavior in the interest rate process, since the differences appear to be oscillating around zero.

3.1.2 Consumption

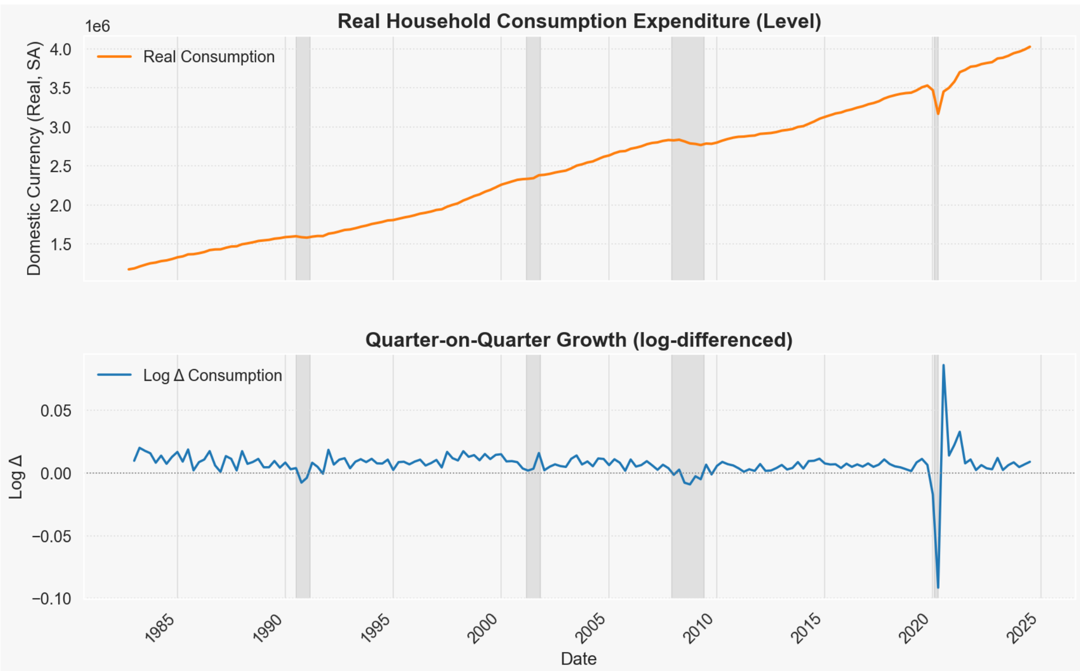

Figure 3.4: Real Household Consumption Expenditure in Levels, 1982Q4–latest. The above plot was created by the Python script, visualizing the raw data.

On figure (3.4) we can see the level and difference plots of the real household consumption variable in the given time period. We can see clear nonstationarity in the form of a positive time trend in the data, with relatively small downward spikes around the 2007-2008 financial crisis, and the 2020 Covid-19 pandemic. Overall, based on the graph above we can see strong evidence in favor of consumption smoothing behavior, which aligns with the theory of permanent-income hypothesis. These observations might imply that the effects of variations in the interest rate on consumption might be delayed, and more crucially, moderate.

The data shows relatively small variability, however the magnitude of the 2020 Covid-19 spike becomes apparent upon visual inspection. We can see that both the negative economic shock, and the subsequent relief policies caused an approximately more than three standard deviations large respective increase and decrease in consumption growth. Overall, we can see relatively stable secular growth with low volatility, with quarterly growth rates approximately between 0 and 2 percent. Similarly to the interest rate patterns, we can see a mean-reverting pattern around 0.5 and 1 percent per quarter, which would imply stationarity in consumption growth. The above findings appear consistent with both Hall's (1978) random walk hypothesis and Friedman's (1957) permanent income hypothesis.

3.1.3 Consumption and Interest Rates

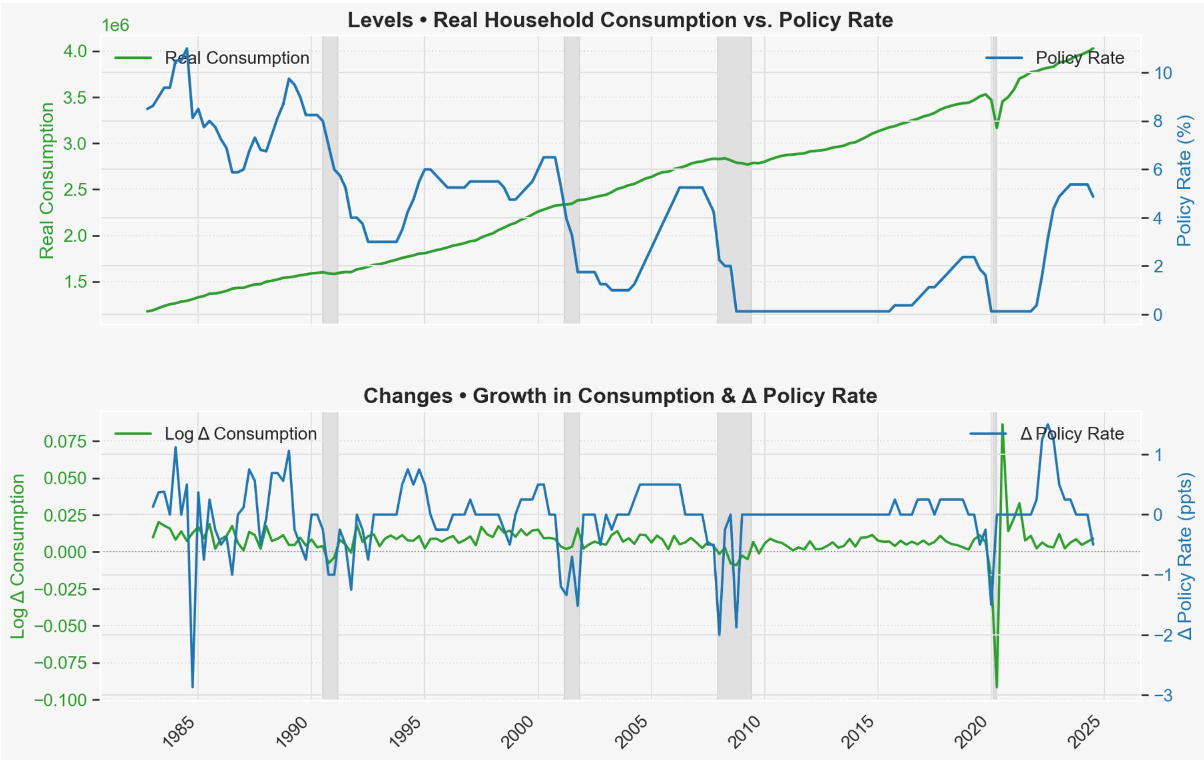

Figure 3.5: Quarterly Interest Rate (top) and Consumption (bottom), 1982Q4–latest. These plots were created by the Python script, visualizing the raw data.

On figure (3.5) we can see both the levels and first-differences of the real household consumption expenditure and central bank policy rate variables. We can see that the correlation, if any, will most likely be negative, but the variations in the two variables seem to appear largely independent upon visual inspection. Interestingly, we can see a clear divergence post-2008, around the periods when the policy rate seemed to hit the zero lower bound, but consumption continued to grow regardless. As there seems to be a very weak immediate correlation between the two variables, aligning with our previous observations, we can argue that in order to capture the effects of interest rates on consumption growth we need to introduce other channels, such as persistent beliefs.

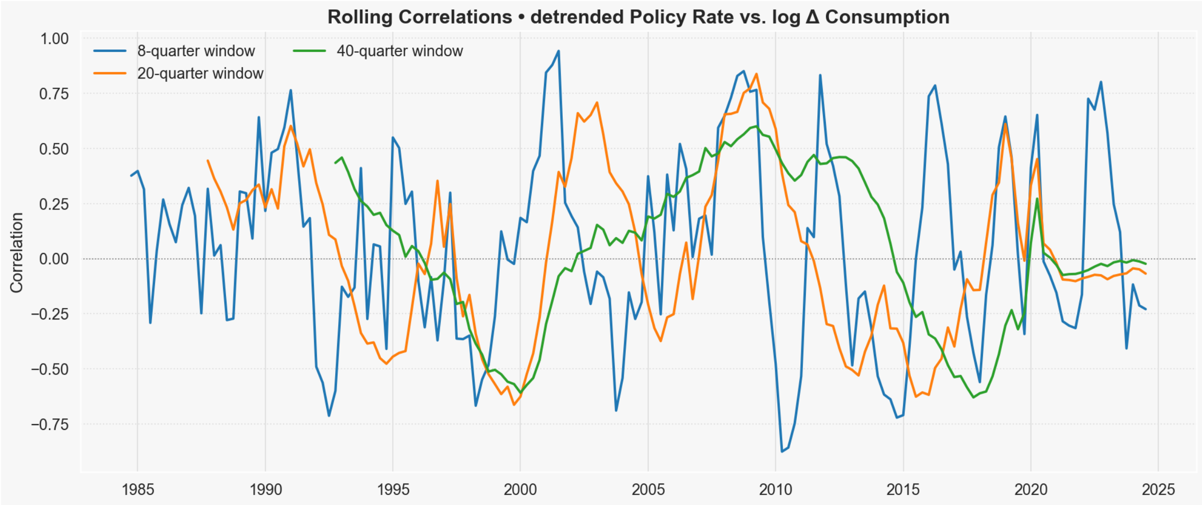

Figure 3.6: Rolling Correlations Between The Policy Rate and Consumption. These plots were created by the Python script, visualizing the raw data.

On figure (3.6) we can see the rolling correlation structure for different windows between the time-detrended policy rate and the log-differenced real household consumption expenditures variables. In contrast to the relatively weak contemporaneous correlation between the two variables, the rolling correlation plot reveals very strong cyclical correlations between the two variables. This result lends support for time-invariance arguments and the structural instability of the parameters designed to capture the connections among macroeconomic aggregates.

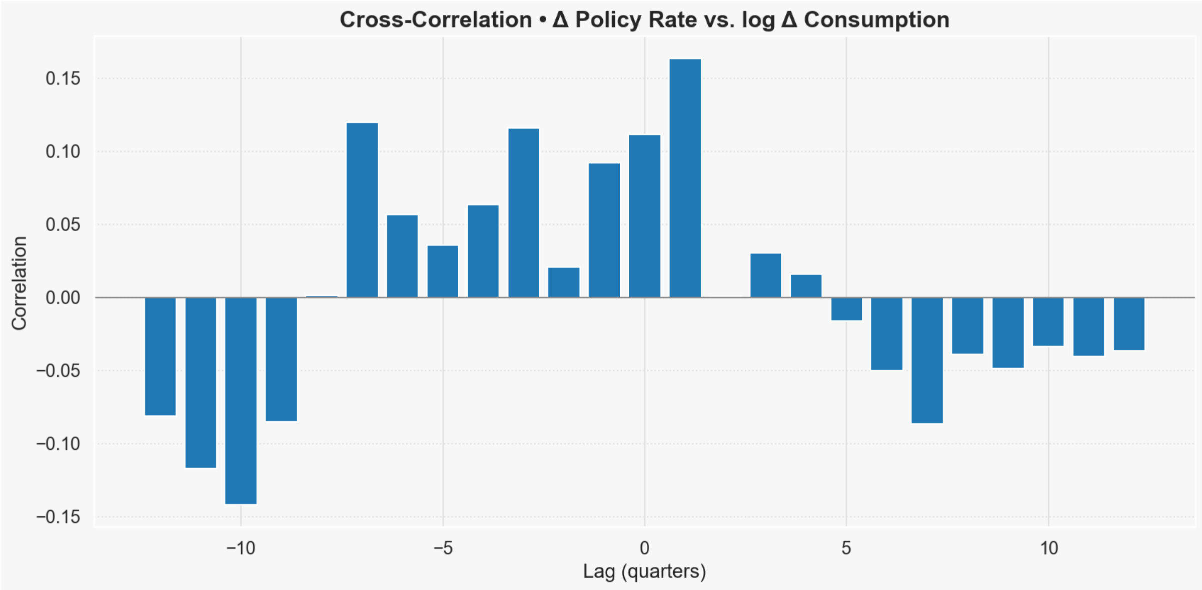

Figure 3.7: Cross-Correlations Between The Policy Rate and Consumption. These plots were created by the Python script, visualizing the raw data.

Figure (3.7) captures the cross-correlation structure of the two variables in the given time period. Based on visual inspection of the plot above we can observe two attributes of the relationship between consumption and interest rate: first, the relationship seems to be relatively weak over long time horizons, second, the relationship appears to be cyclical, which provides further support to exploring various stochastic extensions of the canonical models. In particular, the positive correlation at negative lags (-6 to -2) might suggest that increases in the policy rate are often preceded by higher consumption growth. This observation may imply some kind of reactive monetary policy rule, and crucially, the endogeneity of policy: policymakers increase the policy rate at the sign of overheating. The weak contemporaneous correlation seems to suggest that policy does not drive consumption, at least in the short run. Finally, we can observe a persistent negative correlation at positive lags (+4 to +10), which implies that higher interest rates entail lower long-run consumption growth. Thus, the effects of monetary policy are lagged by at least 4–8 quarters. Again, this supports the need for models that explicitly incorporate consumer beliefs.

3.2 Stationarity Analysis via ADF Tests

Table 3.1: Augmented Dickey–Fuller Test Results

Test Parameter	Interest Rate	Consumption
	ADF Stat = -1.6344	ADF Stat = 0.9124
p-value	0.4652	0.9932
# Lags Used	14	1
# Observations	151	164
Critical Value (1%)	-3.4744	-3.4709
Critical Value (5%)	-2.8809	-2.8793
Critical Value (10%)	-2.5771	-2.5763

Table (3.1) contains the results of the Augmented Dickey–Fuller (ADF) test on each series—in levels–to determine whether the time series are non-stationary. Both p-values exceed 0.05, therefore we cannot reject the presence of an unit root for either of the time-series, implying non-stationarity in levels. Both the augmented Dickey-Fuller test, and the automated recursive search algorithm of the SARIMAX(p,d,q) model, using the Akaike, Bayesian, and Hannan-Quinn information criteria, strongly corroborate the above observed non-stationarity result.[3]

[3]The non-stationarity results hold for the entire data span (1954:Q3–2025:Q1). See Appendix A for full-sample results.

3.3 ACF and PACF Plots

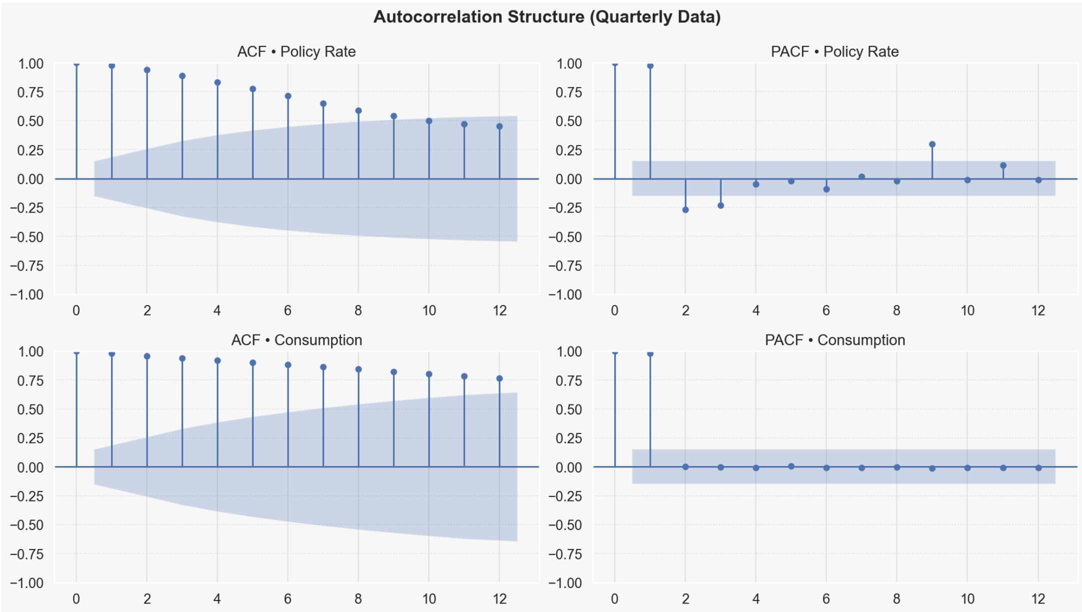

Figure 3.8: ACF and PACF of Interest Rate (top row) and Consumption (bottom row). Generated by the Python script.

Figure (3.8) depicts the plots for the autocorrelation and partial-autocorrelation functions for both the interest rate and consumption variables, up to 12 lags. Examining the ACF/PACF helps us understand the specific form of econometric model that might best capture the given time series. In this case, in alignment with the ADF-test results, we can see that the PACF cuts off abruptly after a single period, which strongly implies that both the level interest rate and consumption time series are integrated of order 1, in other words, first-differencing would render them stationary.

3.4 ARIMA Model Identification and Selection

To find the best-fitting ARIMA model, I employed a step-wise search (using `auto_arima`) to determine optimal (p, d, q) orders for the interest rate and consumption variables, guided by the Akaike Information Criterion (AIC).

3.4.1 Interest Rate: ARIMA(p, d, q)

The automated step-wise search-algorithm resulted in the following model selection:

$$\text{ARIMA}(3, 1, 1) \tag{3.1}$$

for interest rates. That is, the series is differenced once ($d = 1$) with three AR lags ($p = 3$) and one MA term ($q = 1$).

3.4.2 Consumption: ARIMA(p, d, q)

In the case of the consumption time series, the best fit was

$$\text{ARIMA}(0, 1, 0) \tag{3.2}$$

which, aligning with Hall (1978), proves to be a first-differenced random walk.

3.5 Final Model Estimations and Diagnostics

3.5.1 ARIMA(3,1,1) for Interest Rate

Table 3.2: SARIMAX Summary for Interest Rate (ARIMA(3,1,1))

Parameter	**Estimate**	**Comment**
ϕ_1 (AR.L1)	1.1768	Large positive AR(1) term
ϕ_2 (AR.L2)	0.0384	Very small, near zero
ϕ_3 (AR.L3)	-0.3033	Negative AR(3) coefficient
θ_1 (MA.L1)	-0.9283	Strong negative MA(1) term
σ^2	0.2390	Residual variance

Table (3.2) presents the parameter estimates for an ARIMA(3,1,1) model fitted to the interest rate time series.

3.5.2 ARIMA(0,1,0) for Consumption

Table 3.3: SARIMAX Summary for Consumption (ARIMA(0,1,0))

Parameter	Estimate	Comment
σ^2	1.603×10^9	Variance of differenced consumption

Table (3.3) displays the parameter estimates for an ARIMA(0,1,0) model for the consumption time series.

3.5.3 Residual Plots

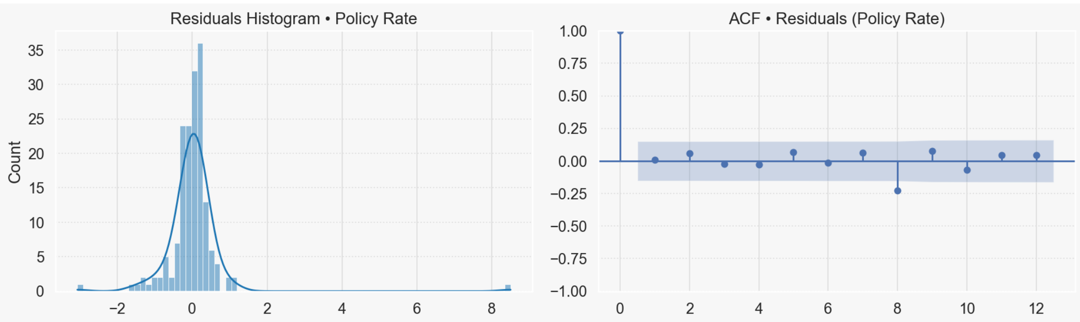

Figure 3.9: Histogram and ACF of Residuals (Interest Rate).

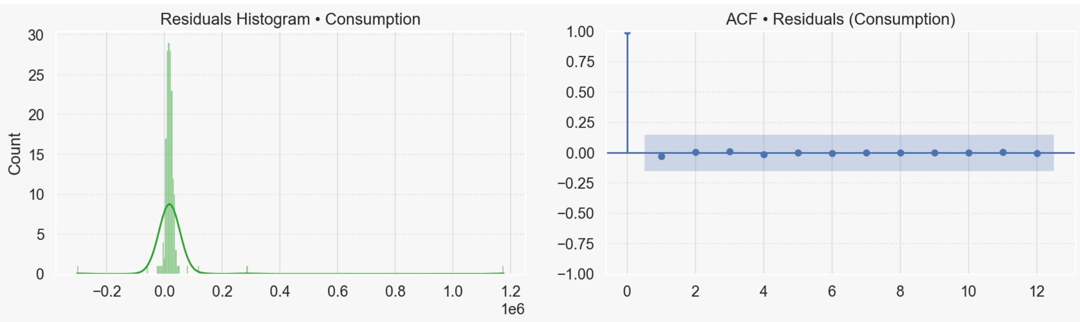

Figure 3.10: Histogram and ACF of Residuals (Consumption).

Figures (3.9) and (3.10) show histograms and autocorrelation plots of the model residuals for interest rate and consumption, respectively. Albeit the results of the Jarque-Bera tests (Jarque–Bera ≈ 544.70 (p-value $= 0.00$) and Jarque–Bera ≈ 19300.21 (p-value $= 0.00$)) seem to indicate signs of skewness and kurtosis in the residuals, based on visual inspection of the residuals the selected model specifications seem to capture interest rate and consumption dynamics reasonably well in a macroeconomic setting.

3.6 Final Functional Equations

Below are the final difference-equation forms of the two processes. Where $\nabla = 1 - B$ denotes the difference operator, and B the backshift operator.

3.6.1 Interest Rate (ARIMA(3,1,1))

$$\nabla r_t = 1.1768\,\nabla r_{t-1} + 0.0384\,\nabla r_{t-2} - 0.3033\,\nabla r_{t-3} + \varepsilon_t - 0.9283\,\varepsilon_{t-1} \tag{3.3}$$

Equation (3.3) shows that the differenced interest rate depends on three autoregressive lags plus an MA(1) shock, indicating both some level of persistence and non-stationarity, which economically implies that there are clear cycles in the interest rate process, most likely corresponding to periods of substantial quantitative easing, and monetary tightening on the part of the Federal Reserve.

3.6.2 Consumption (ARIMA(0,1,0))

$$\nabla c_t \;=\; c_t - c_{t-1} \;=\; \varepsilon_t \tag{3.4}$$

Equation (3.4) shows that the algorithmic search resulted in a model that could be best described as a random walk with no drift, which by-and-large coincides with both conventionally established theoretical and empirical results regarding consumption dynamics.

3.7 Summary

Based on the detailed empirical evaluation of the datasets, we can establish that both the interest rate and the consumption variables are non-stationary in levels. After differencing, interest rates show AR(3) and MA(1) dependence, while consumption is a random walk. These results partially align with the existing literature, insofar as they support the random-walk nature of consumption dynamics, but counter some existing observations insofar as the interest rate process, at least in the selected time interval, clearly contains cyclical periods rendering the variable non-stationary. The final model selections capture these characteristics, albeit diagnostics confirm some residual correlation for consumption; interest-rate residuals appear more white-noise-like. With ARIMA(3,1,1), future interest rates can be projected by iterating the difference equation. For consumption, we can simply treat changes as white noise.

Chapter 4

Theoretical Framework

The stochastically extended Euler equation provides a theoretical way to account for the empirical characteristics observed in the time series data, thereby connecting the preliminary empirical analysis and the theoretical framework implemented in the following empirical results chapter. In particular, the unit-root behavior in both time series lends further justification for the non-stationarity that the extended Euler equation is designed to absorb via the drifting-mean belief term. Furthermore, since the differenced dynamics fit two very different ARIMA profiles—ARIMA(3,1,1) for the interest rate and a random walk process for consumption—the introduction of belief-shock via the real rate channel allows the model to identify variations in consumption growth hitherto unaccounted for by pure consumption innovations. In addition, the lagged-comovement structure uncovered by the rolling and cross correlations show strong cyclical lead-lag patterns, which lend additional support for the explicit incorporation of persistent beliefs involving interest rates into the New-Keynesian DSGE framework. Empirically speaking, the persistence of belief shocks matches macroeconomic history in the sense that shifting long-run rate beliefs, as opposed to relatively transient policy surprises, appear to be the main driving force behind low-frequency movements. In the following chapter, we present the stochastic extension of the canonical Euler in which households update their beliefs about the mean real interest rate, which is followed by the delineation of the New-Keynesian dynamic stochastic general equilibrium framework in which the stochastically extended Euler equation will be embedded. This DSGE framework will computationally be implemented in the subsequent chapter in order to provide the structural estimates for the model's parameters.

4.1 Introduction to the Belief-Adjusted Euler Equation

The key contribution of the thesis is the incorporation of a belief-adjusted interest rate into the canonical Euler equation, which is then embedded into a New-Keynesian DSGE framework that allows for structural estimation of the model parameters. In the extended Euler equation the belief-updating process augments the canonical model by adding a slowly evolving perception about the interest rate process, which allows for a distinction between the transitory component of the policy rate $(i_t - \mathbb{E}_t\pi_{t+1})$, and the household's beliefs about the long-run real rate, μ_t. In our specification, changes in the effective real rate, $(i_t - \mathbb{E}_t\pi_{t+1} - \mu_t)$, depend not only on central bank actions but also on shifts in public expectations, which allows for an amplifying or dampening effect of interest rate changes on consumption growth.

The modified Euler equation incorporating the belief adjustment takes the form:

$$\nabla c_t = \frac{1}{\sigma}\big(i_t - \mathbb{E}_t\pi_{t+1} - \mu_t\big), \tag{4.1}$$

The slowly drifting component μ_t encapsulates the representative household's evolving perception of the "normal" long-run real interest rate. When a positive innovation $e_{\mu,t}$ lifts this benchmark, the contemporaneous real rate $\big(i_t - E_t\pi_{t+1}\big)$ is, by construction, left unchanged. The intertemporal wedge that governs optimal saving behaviour, namely $i_t - E_t\pi_{t+1} - \mu_t$ in the extended Euler equation (4.1), therefore contracts. A narrower wedge diminishes the perceived reward to waiting, so households re-optimise by reallocating expenditure toward the present. Accordingly, a positive belief shock manifests itself as an immediate acceleration of consumption growth.

4.2 Variables, Shocks, and Parameters

Table 4.1: Endogenous Variables, Observed Series, and Exogenous Shocks

Endogenous (state) variables	Interpretation
c_t	Consumption (log-linear)
π_t	Inflation rate (log-linear)
i_t	Nominal policy rate (annualised, log-linear)
μ_t	Subjective mean of the real rate (belief)
Observed series	
dc_t^{obs}	Detrended consumption growth
π_t^{obs}	CPI inflation
i_t^{obs}	Nominal interest rate
Exogenous shocks	
$e_c,\ e_\pi,\ e_a,\ e_\mu$	Consumption, Phillips, monetary, belief shocks

Structural parameters are β (discount factor), σ (intertemporal elasticity parameter), κ (slope of the NKPC), ϕ_π, ϕ_c (policy coefficients), and ρ_μ (belief persistence). Shock standard deviations are σ_j for $j \in \{x, \pi, a, \mu\}$.

4.3 New-Keynesian Log-Linear Equilibrium System

The New-Keynesian model implemented in the next chapter is summarized by the following four log–linear equations (Clarida, Gali Gertler 1999, pp. 1662-1665):

$$\text{Euler Equation:} \quad \nabla c_t = \frac{1}{\sigma}\Big(i_t - \mathbb{E}_t\pi_{t+1} - \mu_t\Big), \tag{4.2}$$

$$\text{NKPC:} \quad \pi_t = \beta\,\mathbb{E}_t\pi_{t+1} + \kappa\,c_t + e_{\pi,t}, \tag{4.3}$$

$$\text{Taylor rule:} \quad i_t = \phi_\pi\,\mathbb{E}_t\pi_{t+1} + \phi_c\,\mathbb{E}_t c_{t+1} + e_{a,t}, \tag{4.4}$$

$$\text{Belief updating:} \quad \mu_t = \rho_\mu\,\mu_{t-1} + e_{\mu,t}. \tag{4.5}$$

The equations above constitute the core of the theoretical framework implemented empirically in the next chapter. Equation (4.2) describes the extended Euler equation, while equation (4.3) captures the New-Keynesian Phillips curve, where π_t represents the quarter-on-quarter-demeaned inflation, while β captures the household discount factor. The parameter κ describes the slope of the New-Keynesian Phillips curve, while $e_{\pi,t}$ represents the cost-push shock. Equation (4.4) represents the Taylor rule with i_t capturing the nominal interest rate, and ϕ_π captures the policy response to expected inflation and ϕ_c represents the response to expected consumption growth, I include the expected log-deviation of consumption from its steady state in the Taylor rule so that the policy rate responds to a forward-looking, cleanly measured proxy for real demand. The $e_{\alpha,t}$ term represents the monetary-policy disturbance. Finally, in equation (4.5) μ_t introduces a stochastic trend in the real rate, with $e_{\mu,t}$ capturing a belief shock.

Measurement equations

$$dc_t^{\text{obs}} = c_t + e_{c,t}, \qquad \pi_t^{\text{obs}} = \pi_t, \qquad i_t^{\text{obs}} = i_t. \tag{4.6}$$

Together, equations (4.2)-(4.6) constitute a linear state–space model of the form $\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-1} + \mathbf{B}\boldsymbol{\varepsilon}_t$, $\mathbf{y}_t = \mathbf{C}\mathbf{s}_t + \boldsymbol{\eta}_t$, with $\mathbf{s}_t = (c_t, \pi_t, i_t, \mu_t)^\top$ and $\mathbf{y}_t = (dc_t^{\text{obs}}, \pi_t^{\text{obs}}, i_t^{\text{obs}})^\top$ (Kuttner 1994, p. 364), where $\mathbf{d}c_t^{\text{obs}}$ represents the detrended quarterly growth rate of real consumption (annualized and demeaned by the HP-filter trend). The measurement equation $dc_t^{\text{obs}} = c_t + e_{c,t}$ shows how it maps onto the model's log-linear consumption variable, while the error term captures the inherent statistical noise in the measurement of real consumption, as opposed to the policy, or inflation rate variables.

4.4 Calibration and Prior Distributions

The model is calibrated at the posterior mode of a previous run; these values also serve as prior means in the Bayesian estimation routine:

Table 4.2: Calibrated Parameter Values, Prior Distributions, and Literature Sources

Parameter	Value	Prior	Source(s)
β	0.99	*fixed*	Smets & Wouters (2007, Table 1)
σ	1.50	$\Gamma(1.50,\ 0.30)$	Smets & Wouters (2007, Table 1); Yogo (2004, pp.803–804)
κ	0.10	$\Gamma(0.10,\ 0.05)$	Gali & Gertler (1999, p.1665)
ϕ_π	2.00	$\mathcal{N}(2.00,\ 0.30^2)$	Taylor (1993, p.202)
ϕ_c	0.00	*fixed*	Woodford (2003, ch.3)
ρ_μ	0.70	Beta$(0.70,\ 0.10)$	Benati & Mumtaz (2007, Footnote 14)
σ_c	0.50	Inv-Gamma$(0.50,\ 2)$	Justiniano & Primiceri (2008, pp.40-41); Smets & Wouters (2007, Table 1)
σ_π	0.20	Inv-Gamma$(0.20,\ 2)$	*Ibid.*
σ_a	0.25	Inv-Gamma$(0.25,\ 2)$	*Ibid.*
σ_μ	0.05	Inv-Gamma$(0.05,\ 2)$	*Ibid.*

All priors are centered on the calibration and are sufficiently wide to allow the data to speak.

4.5 Steady State and Determinacy

Because the system is linear around a zero-inflation steady state, the `steady` command in Dynare simply sets $\bar{x} = \bar{\pi} = \bar{i} = \bar{\mu} = 0$. Running `check` confirms that exactly two eigenvalues lie outside the unit circle—equal to the number of non-predetermined forward-looking variables (x_t, π_t)—so the rational-expectations equilibrium is *determinate* (Blanchard & Kahn 1980).

4.6 State–Space Form for Bayesian Estimation

Stacking equations (4.2)-(4.6) we obtain

$$\mathbf{s}_t = \mathbf{A}\,\mathbb{E}_t\mathbf{s}_{t+1} + \mathbf{B}\,\boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t = (e_c, e_\pi, e_a, e_\mu)^\top. \tag{4.7}$$

Dynare's Kalman filter uses the measurement block (4.6) to compute the likelihood. With the `diffuse_filter` option and `varobs dc^{obs} pi^{obs} i^{obs}`, the initial-state covariance is left uninformative, which is appropriate for quarterly U.S. data from 1959 Q1 (Hamilton 1989, pp. 358-364).

Chapter 5

Estimation and Results

5.1 Overview

The following chapter presents the empirical estimation results for the New-Keynesian DSGE model with belief-adjusted real rates introduced in the previous chapter. First, we explain the estimation procedure, then describe the prior and posterior parameter summaries of the relevant structural parameters. Then, we turn to the delineation of the shock standard deviations and the impulse response analysis, which is followed by a detailed description of the historical decomposition figures. Finally, we discuss the robustness of the results.

5.2 Estimation Procedure

Using quarterly U.S. data (1982Q1–2023Q4, three observed series) Dynare's Metropolis–Hastings engine is run for 20 000 draws in two parallel chains. The estimation block in the `.mod` file reads

```
estimation(datafile='dataset', first_obs=2, diffuse_filter,
           mh_replic=20000, mh_nblocks=2, mh_drop=0.25,
           mode_compute=6, plot_priors=1);
```

The rank condition test (`check`) reports exactly two explosive roots for the two forward-looking variables (c_t, π_t), so the Blanchard–Kahn determinacy criterion is satisfied.

5.3 Prior and Posterior Summaries

5.3.1 Structural Parameters

Table 5.1: Posterior vs. Prior — Structural Parameters

	Prior Mean	Prior Std	Posterior Mode	Posterior Std
σ (IES^{-1})	1.50	0.30	1.433	0.294
κ (NKPC slope)	0.10	0.05	0.030	0.009
ϕ_π (Taylor)	2.00	0.30	1.832	0.148
ρ_μ (belief persistence)	0.70	0.10	0.981	0.006

Table (5.1) compares the prior *vs.* posterior structural estimates of the model parameters. The structural estimate for the intertemporal elasticity parameter remains essentially unchanged, which indicates that households' willingness to smooth intertemporal consumption was close to our initial specification. The slope of the Phillips curve decreases significantly, to approximately one-third of its prior value, which indicates that in our framework instead of demand-side, rather supply-side factors influence inflationary pressures the most. The Taylor-rule coefficient reveals that monetary policy responds less to inflationary pressures compared to the pre-specified parameter value, albeit the difference is not very large. Finally, the increase in the belief persistence parameter implies that in the model setting consumer beliefs are more persistent compared to the initially specified parameter, which translates to more gradual adjustment in the case of belief-shocks.

5.3.2 Shock Standard Deviations

Table 5.2: Posterior Modes — Shock Volatilities

Shock	Posterior Mode	Interpretation
σ_{e_c}	4.71	Large demand shocks (output gap)
σ_{e_π}	2.88	Moderate cost-push shocks
σ_{e_a}	0.08	Minor monetary shocks (rule error)
σ_{e_μ}	0.26	Belief-updating surprises

Table (5.2) presents the modes of posterior shock volatilities. The posterior mode for consumption is significantly larger compared to the other variances, which implies that demand-side disturbances drive much of the variation in consumption growth. The second largest variance belongs to inflation, which indicates that supply-side variations contribute the largest to inflationary pressures. The posterior shock volatility of the policy rule is very small, while the belief-updating variance is relatively larger. These, together with the posterior parameter estimates, translate to a relatively small, but very persistent belief-shock process, which translates to an increased cumulative effect.

5.4 Impulse-Response Analysis

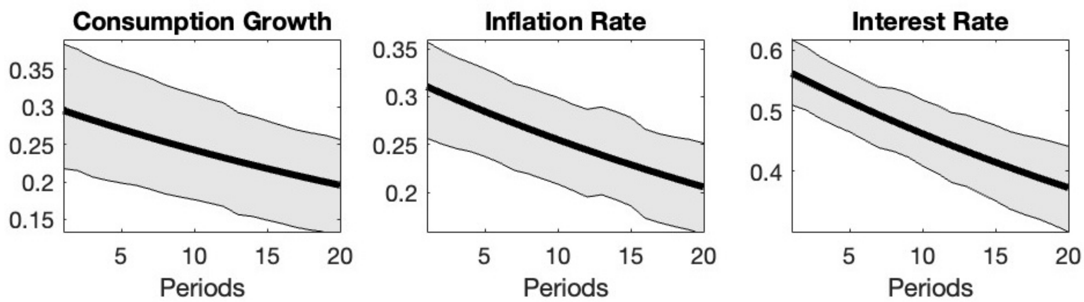

Figure 5.1: Impulse responses to a 1 s.d. belief shock e_μ.

Figure (5.1) shows that a positive belief shock (perceived higher real rate) leads to an approximately 0.3 pp jump in consumption growth, which fades only gradually as consumers frontload consumption to immediately benefit from the increased returns on savings. This is consistent with the relative smoothness observed in aggregate consumption data. Similarly, the inflation rate increases by about 0.3 pp, and decreases gradually following consumption growth. Economically, this implies that with a flat Phillips curve variations in aggregate demand drive inflation. Interestingly, the policy rate increases by about 0.55 pp, which is a larger jump observed in the previous variables, which is explained by the forward-looking Taylor rule that reacts to expected inflation (in order to accommodate the increase in aggregate consumption). Overall, we can argue that the effects of belief shocks in the present setting are similar to a persistent stimulus. A positive belief shock leads to a relatively persistent increase in both consumption growth and inflation, while the policy rate adjusts to accommodate for the inflationary pressure.

5.5 Historical Decompositions

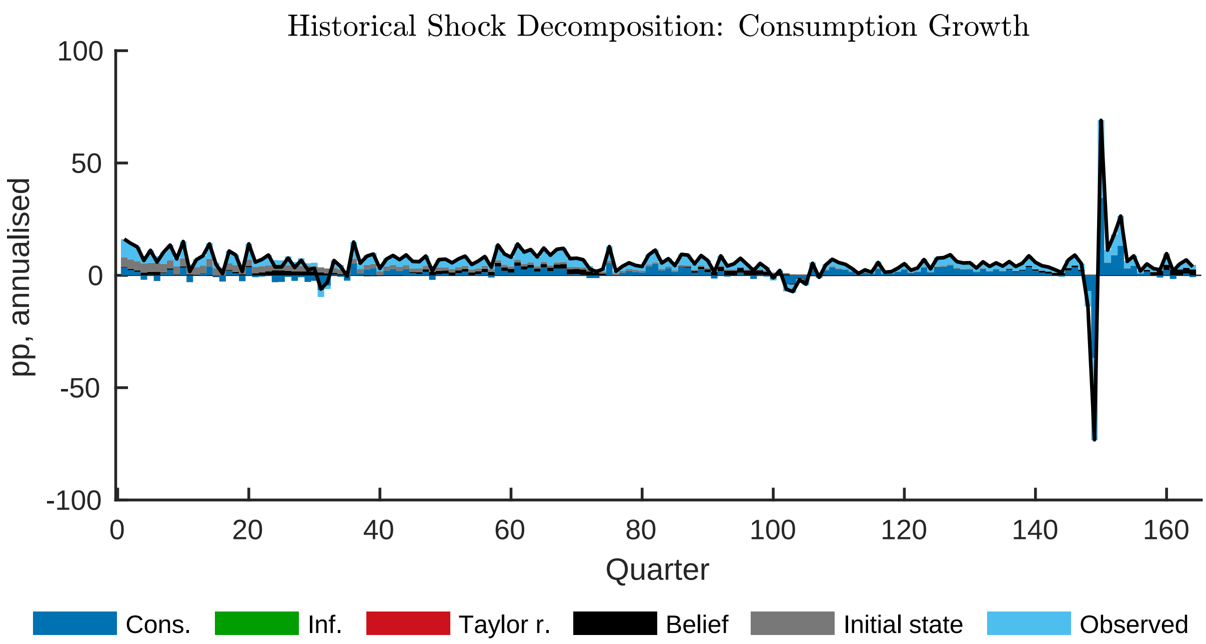

Figure 5.2: Historical Consumption Growth and Smoothed-Structural shocks Decomposition

Figure (5.2) depicts the historical decomposition of consumption growth. Observation reveals that exogenous shocks dominated the overall sample, which is consistent with demand-side explanations for variations in real economic activity in the sample. Variations in the inflation rate, policy rule do not seem to exert an observable effect on the historical variance of consumption growth in the presented framework, which aligns with the broader literature pinpointing the relatively small influence of monetary factors on real variables. Belief shocks, as the numerical percentage point breakdowns will demonstrate, steadily contribute to consumption growth dynamics. Finally, the initial values seem to influence the early periods, but after approximately 40-60 periods their effects seem to gradually decline.

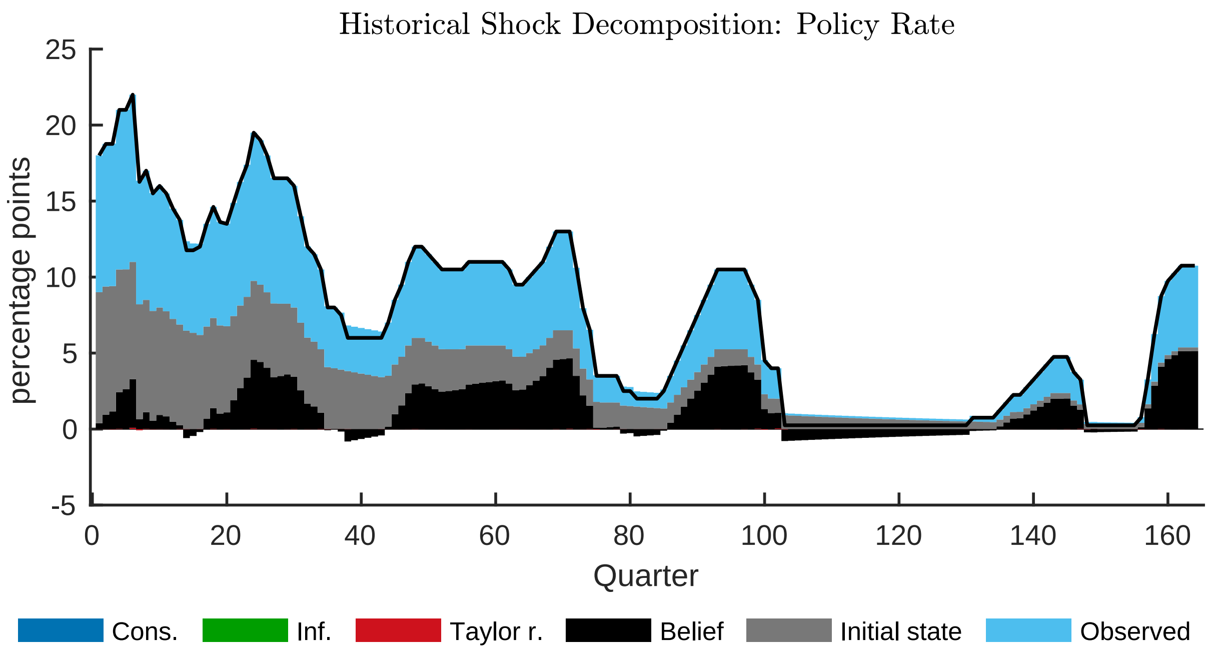

Figure 5.3: Historical Interest Rate and Smoothed Structural-Shocks Decomposition

Figure (5.3) shows the historical decomposition of interest rate. The initial periods can be explained macroeconomically as the legacy of the Volcker-era. However, following the initial periods, belief shocks appear to consistently contribute to the variations in the interest rate, which implies that evolving beliefs about the real rate provide crucial signals for monetary policy in the policy rate determination process. Variations in the inflation rate, and policy rule shocks, along with demand-side shocks seem to have exerted empirically insignificant effects on the variations in the interest rate in the examined period in our theoretical DSGE framework.

Finally, the effects of the initial values appear strong in the first half of the sample, but its effect gradually declines in subsequent periods.

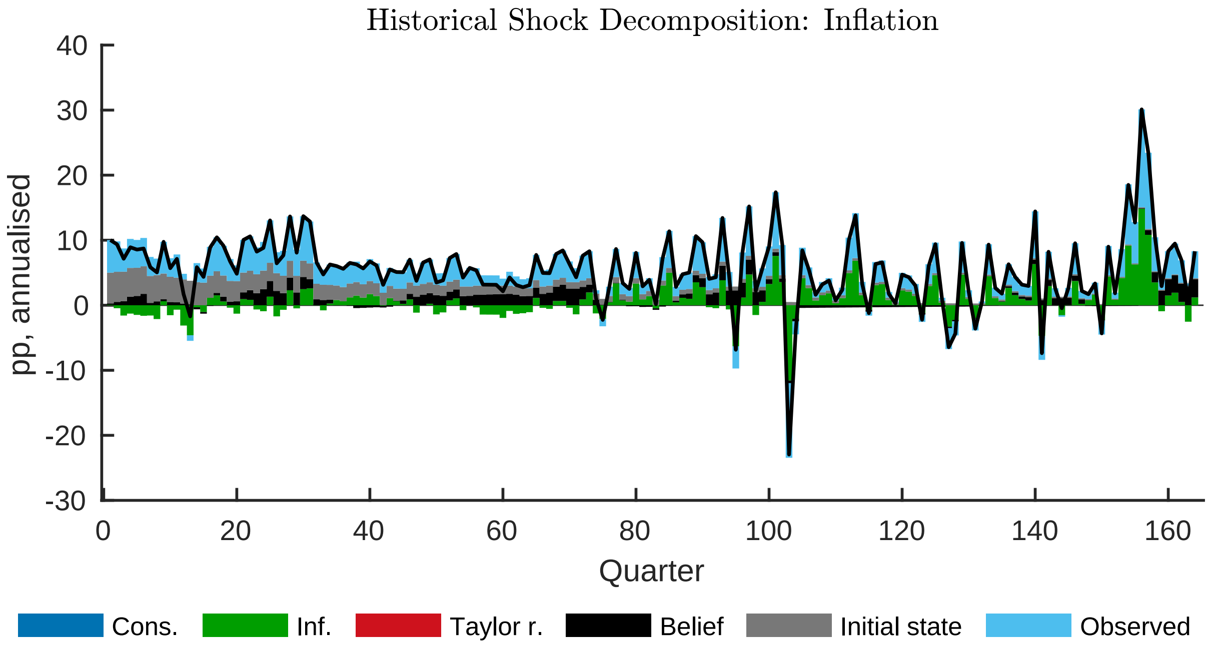

Figure 5.4: Historical Inflation Rate and Smoothed Structural-Shocks Decomposition

Figure (5.4) highlights the historical decomposition of the inflation rate. Based on the figure we can say that inflation appears to be driven by supply-side forces and belief shocks appear to be modest in the examined period. Variations in demand-side forces, or in the policy rule seem to have exerted statistically insignificant effects in the variations on the inflation process as depicted by the historical decomposition above. Interestingly, along with the natural effects of inflation itself, belief shocks seem to have exerted a modest, albeit apparent effects on inflationary dynamics in the examined period in our theoretical DSGE framework.

5.6 Time-Varying Share of Belief Shocks

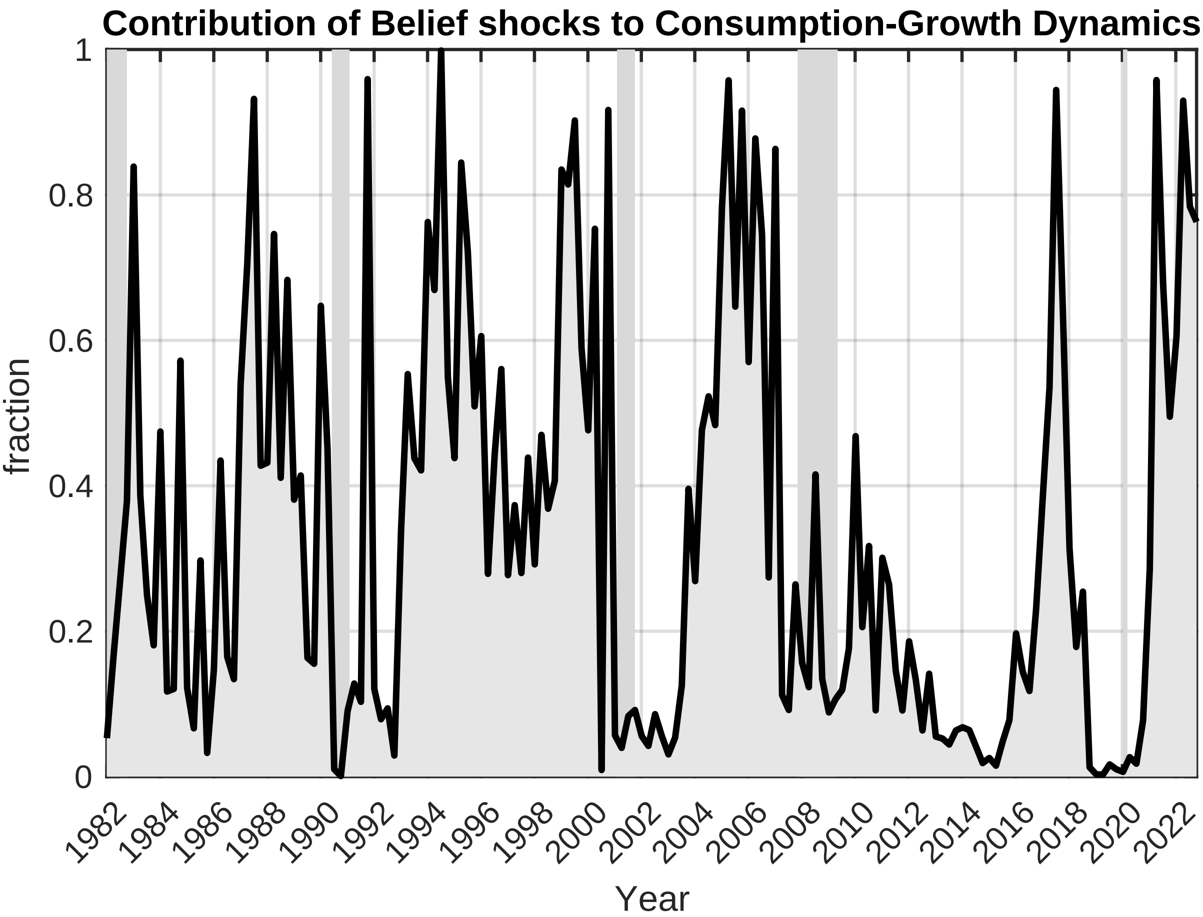

Figure 5.5: Contribution of Belief Shocks to Consumption Growth Dynamics

Figure (5.5) depicts the time-varying contribution of belief shocks to consumption growth dynamics. The contributions appear to be strongly episodic and often dominant. The highest contributions seem to be around the late 1980s boom, the dot-com recession of the early 2000s, the relatively short tightening period of the mid 2000s, and finally the post-pandemic period in the early 2020s. On some occasions, the contribution collapses to zero, such as in the downturn of the early 90s, and post Great Recession period, which implies that other shocks dominated the US macroeconomy in these years. Overall, belief shocks appear to be a relevant driver of consumption growth dynamics, with relatively large, but periodic real effects.

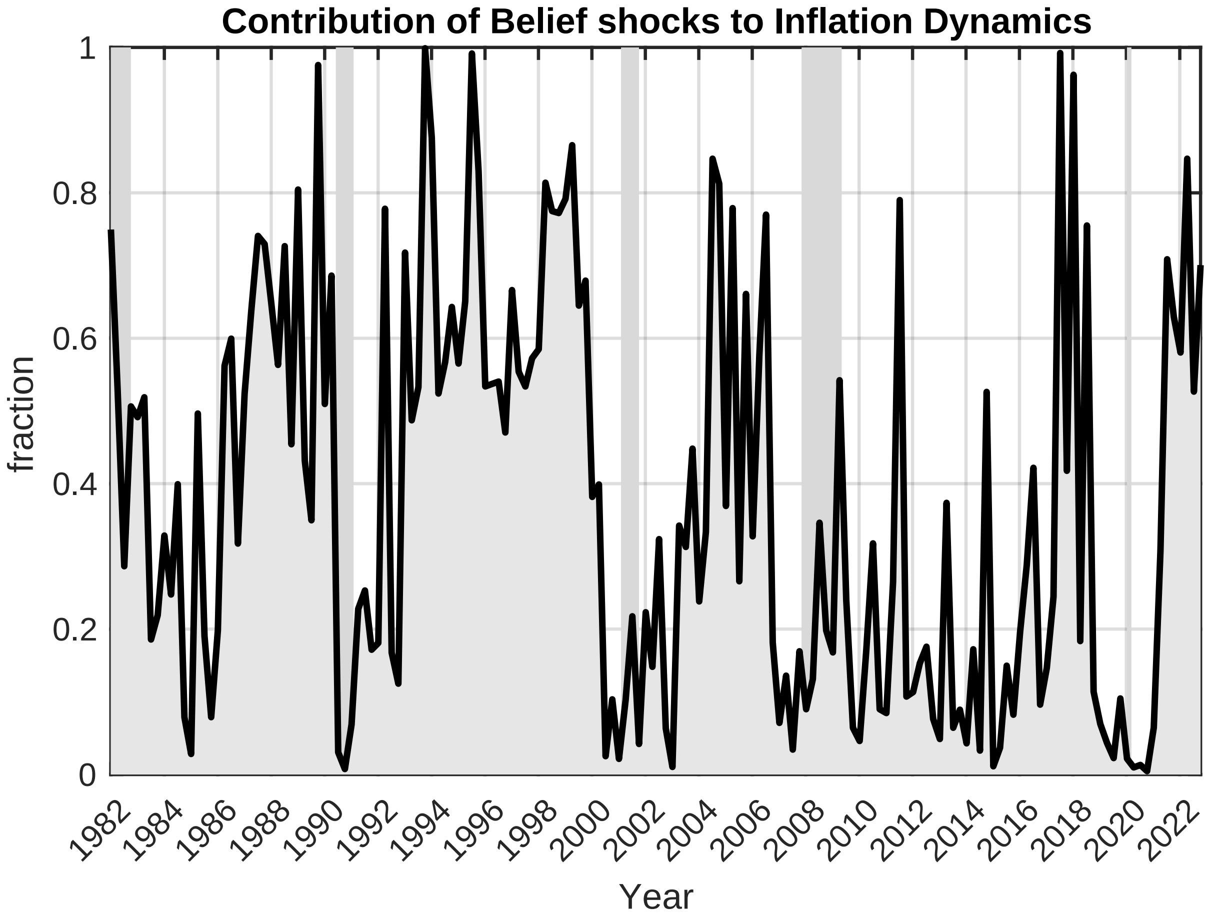

Figure 5.6: Contribution of Belief Shocks to Inflation Dynamics

Figure (5.6) shows the time-varying contribution of belief shocks to inflation dynamics. Based on the figure above, we can argue that belief shocks contribute importantly, albeit not dominantly to inflation dynamics, which is consistent with the cost-push shock narrative laid out earlier. We can see that the contribution of belief shocks to inflation dynamics generally flares up preceding recessionary periods, such as the early 1990s, early 2000s dot-com bubble, and finally before the Financial Crisis of 2007-2008. The contribution seems to be highly volatile, and periodic, ranging from a fraction of less than ten percent to almost one hundred percent, which implies that the contribution of belief shocks to inflation dynamics plays an important role in capturing monetary trends in the macroeconomy.

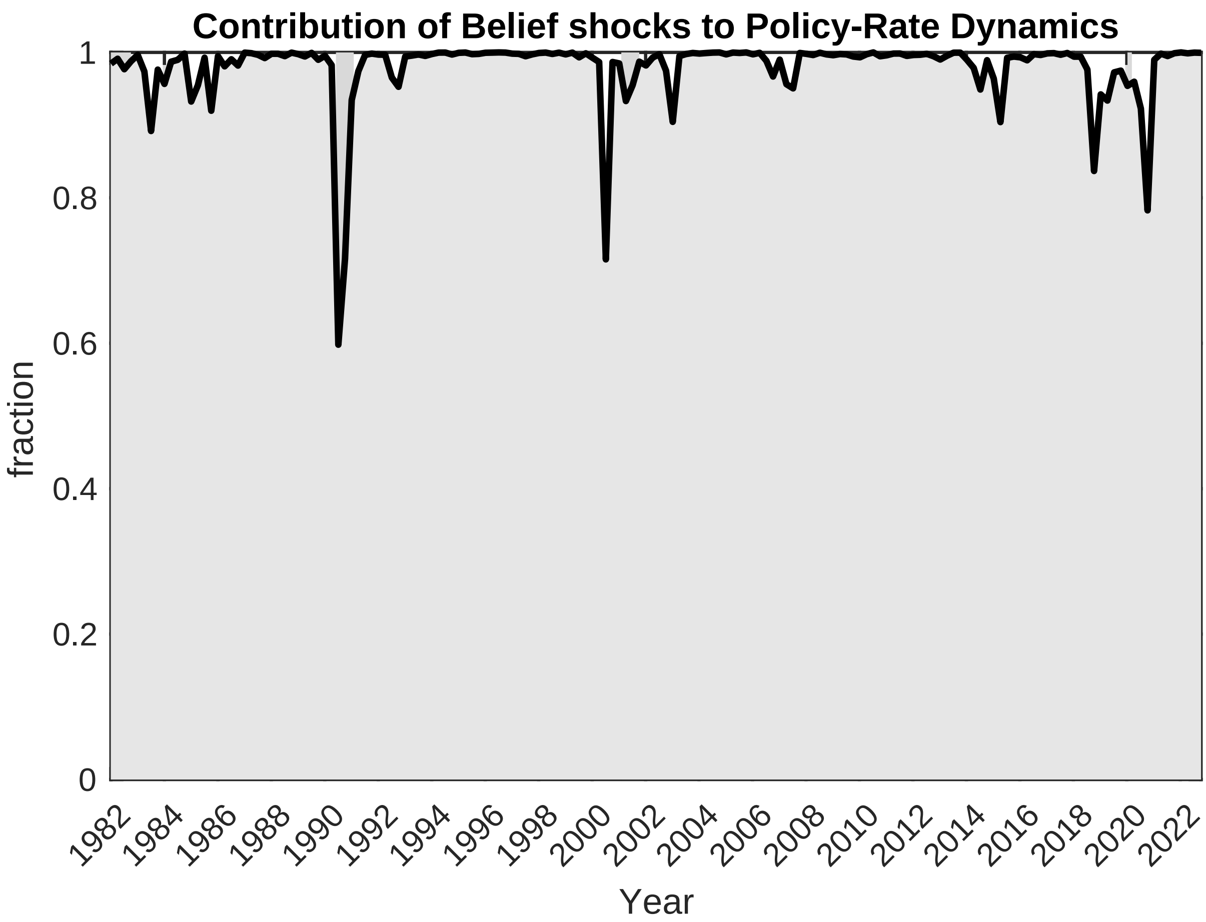

Figure 5.7: Contribution of Belief Shocks to Policy-Rate Dynamics

Figure (5.7) shows the time-varying contribution of belief shocks to policy-rate dynamics. The policy-rate appears to be dominated by belief shocks throughout the entire sample period, which lends further support for the persistence of monetary policy and non-stationarity of the policy rate process. Throughout the entire sample, the policy-rate appears to be dominated by the persistent belief process, which provides strong support for the empirical observation that interest rates are generally persistent, and especially the monetary policy-rate is best captured by a near unit-root time series process of which consumers can rationally form a persistent belief about.

5.7 Mean, Maximum, and Absolute Belief Shock Contributions

Table 5.3: Average, Maximum and Relative Shock Contributions

	Mean abs. (pp)			Max abs. (pp)			Share abs. var. (%)		
Shock	dc	π	i	dc	π	i	dc	π	i
e_c	2.49	0.00	0.00	36.64	0.00	0.00	74.86	0.00	0.00
e_π	0.00	1.95	0.00	0.00	14.83	0.00	0.00	68.41	0.00
e_a	0.01	0.00	0.01	0.06	0.00	0.09	0.19	0.01	0.58
e_μ	0.83	0.90	1.64	2.59	2.82	5.13	24.95	31.58	99.42
Initial	1.19	1.29	2.35	4.36	4.73	8.62	0.00	0.00	0.00

Table (5.3) contains the magnitude of each variable's shock contribution in percentage point format. Based on the table above we can reinforce our previous conclusion regarding the importance of demand shocks, as on average demand shocks add about 2.5 percentage points, with a peak contribution of 37 percentage points during the Covid-19 demand collapse. In contrast to consumption growth, cost-push shocks seemingly dominate inflation volatility, as they explain almost 2 percentage points of the mean absolute inflation movement. The disturbances in the Taylor-rule appear to contribute relatively little in the case of each variable. Belief shocks, in contrast, appear to be a relatively strong low-frequency force as they add approximately 0.8 percentage points to both consumption growth and inflation, but most importantly, 1.6 percentage points to the policy rate in mean terms. The maximum 5 percentage point swing (during the Volcker transition) confirms the importance of long-run beliefs in terms of the interest rate environment. Overall, consumption growth, the policy-rate, and the inflation rate dynamics appear to be significantly, albeit with time-varying strength, affected by shocks to the persistence belief process: belief shocks explain approximately 25 percent of consumption growth dynamics, about 32 percent of inflation dynamics, and capture almost the total variation in policy rate dynamics.

5.8 Robustness

As part of testing for robustness, re-estimating with alternative priors, (twice-as-wide priors) for (σ, ϕ_π) and a flatter Beta prior for ρ_μ nudges posterior modes by less than 0.02 and leaves the marginal data density unchanged at the fourth decimal. Furthermore, as part of the Karhunen-Loéve tail trimming method, discarding the top 0.5 of draws for σ_{e_c} does not alter the IRF shapes.

Chapter 6

Conclusion

The theoretical framework presented in this thesis provided a stochastic extension to the canonical Euler equation in a New-Keynesian dynamic stochastic general equilibrium setting. The proposed extension lets households learn a drifting mean of the real interest-rate, accounting for the non-stationarity observed in interest-rate data throughout modern economic history. The empirical estimation of the structural parameters implied that the belief-persistence parameter becomes essentially a unit root, which means that households alter their beliefs extremely gradually. In contrast, the slope of the consumption variable of the Phillips curve appears to be small, which implies that variations in demand-side forces might not capture the whole story behind inflationary trends. These two observations together might imply that household beliefs, especially in the medium to long run, might hold a larger role in inflationary dynamics than previously believed. The key results indicate that belief shocks explain approximately 25 percent of consumption growth dynamics, about 32 percent of inflation dynamics, and capture almost the total variation in policy rate dynamics. Theoretically speaking, the expectations-based Taylor rule implemented in our DSGE model yielded a unique equilibrium, which strengthens the argument for forward-looking, instead of reactive, policy rules. Overall, we can argue that a near-unit-root belief process, along with an expectation-driven Taylor rule and the Euler equation implies that central bank policy becomes effective when it focuses on shaping beliefs, as opposed to implementing large contemporaneous moves in the policy rate.

Appendix

Appendix A

Interest Rate and Consumption Time Series

Table A.1: SARIMA(1,1,1)(0,0,0)$_4$ results — Federal Funds Effective Rate (DFF)

Parameter	**Coef.**	**Std. Err.**	z	p**-value**	**95% CI**
ϕ_1 (AR L1)	-0.311	0.070	-4.410	<0.001	$[-0.449,\ -0.173]$
θ_1 (MA L1)	0.670	0.049	13.724	<0.001	$[0.574,\ 0.765]$
σ^2	0.620	0.019	33.412	<0.001	$[0.584,\ 0.656]$
Model fit and diagnostics					
Observations (T)	283				
Log-likelihood	-332.875				
AIC	671.749				
BIC	682.675				
HQIC	676.131				
Ljung–Box Q(L1)	0.01 ($p = 0.94$)				
Jarque–Bera	3919.82 ($p < 0.001$)				
Heteroskedasticity H	0.29 ($p < 0.001$)				
Data source: FRED, series `DFF`	**Sample:** 1954:Q3–2025:Q1				

Table A.2: SARIMA(0,2,2)(0,0,1)$_4$ results — Real Personal Consumption Expenditures (PCECC96)

Parameter	**Coef.**	**Std. Err.**	z	p**-value**	**95% CI**
θ_1 (MA L1)	-1.1354	0.013	-87.739	<0.001	$[-1.161,\ -1.110]$
θ_2 (MA L2)	0.1674	0.021	7.926	<0.001	$[0.126,\ 0.209]$
Θ_1 (MA S. L4)	-0.1315	0.028	-4.668	<0.001	$[-0.187,\ -0.076]$
σ^2	1.242×10^4	258.153	48.103	<0.001	$[1.19 \times 10^4,\ 1.29 \times 10^4]$
Model fit and diagnostics					
Observations (T)	283				
Log-likelihood	-1724.881				
AIC	3457.762				
BIC	3472.316				
HQIC	3463.599				
Ljung–Box Q(L1)	0.01 ($p = 0.91$)				
Jarque–Bera	89214.60 ($p < 0.001$)				
Heteroskedasticity H	48.97 ($p < 0.001$)				
Data source: FRED, series `PCECC96`	**Sample:** 1954:Q3–2025:Q1				

Appendix B

Bayesian Estimation Details

B.1 Prior Distributions

Table B.1: Priors Used in `Dynare` Estimation

Parameter	Distribution	Mean	Std/Shape
σ	Gamma	1.50	0.30
κ	Gamma	0.10	0.05
ϕ_π	Normal	2.00	0.30
ρ_μ	Beta	0.70	0.10
σ_{e_c}	Inv-Gamma	0.50	2
σ_{e_π}	Inv-Gamma	0.20	2
σ_{e_a}	Inv-Gamma	0.25	2
σ_{e_μ}	Inv-Gamma	0.05	2

B.2 MCMC Settings

- **Number of chains:** 2
- **Draws per chain:** 20 000 (25 % burn-in, 15 000 kept)
- **Proposal scale (`jscale`):** 0.67 (adaptive)
- **Acceptance ratios:** 33.4 % (chain 1), 32.9 % (chain 2)
- **Effective sample sizes:** ≥ 500 for all parameters

B.3 Convergence Diagnostics (Geweke p-values)

Table B.2: Selected Geweke Test Results (chain 1)

Parameter	No taper	4 % taper	8 % taper
σ	0.23	0.83	0.83
κ	0.00	0.47	0.44
ϕ_π	0.00	0.52	0.53
ρ_μ	0.00	0.44	0.43
σ_{e_c}	0.00	0.22	0.23

All p-values lie above 0.05 once a 4 % spectral taper is applied, indicating no residual non-convergence.

Appendix C

Supplementary Figures

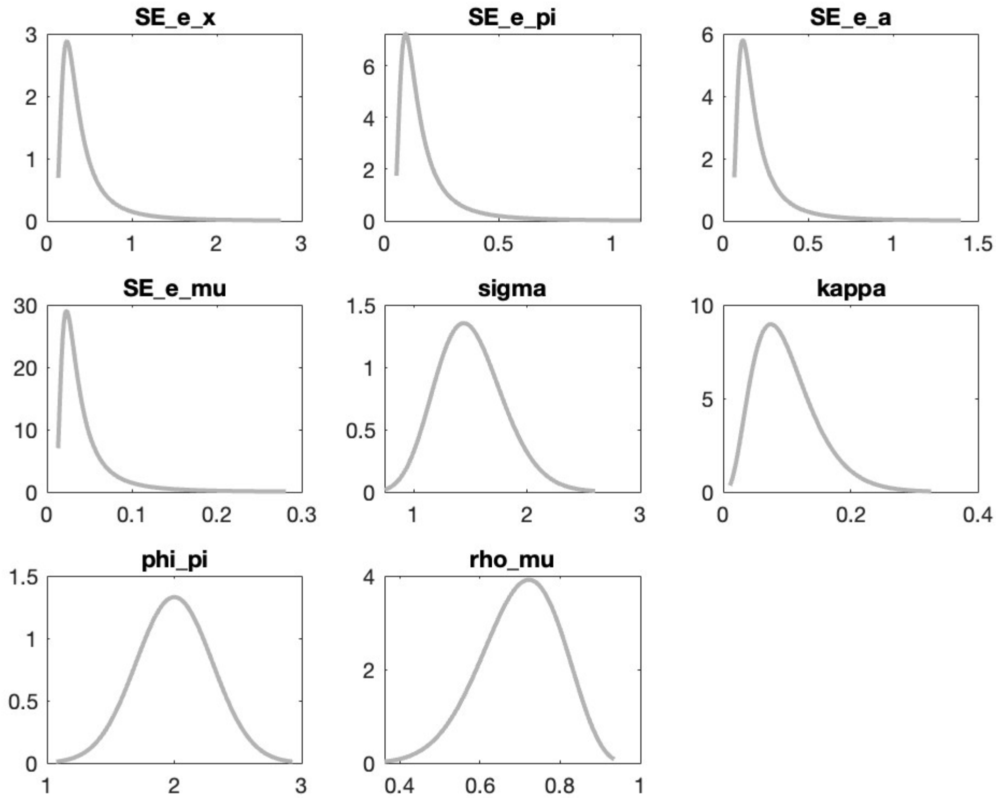

Figure C.1: Priors

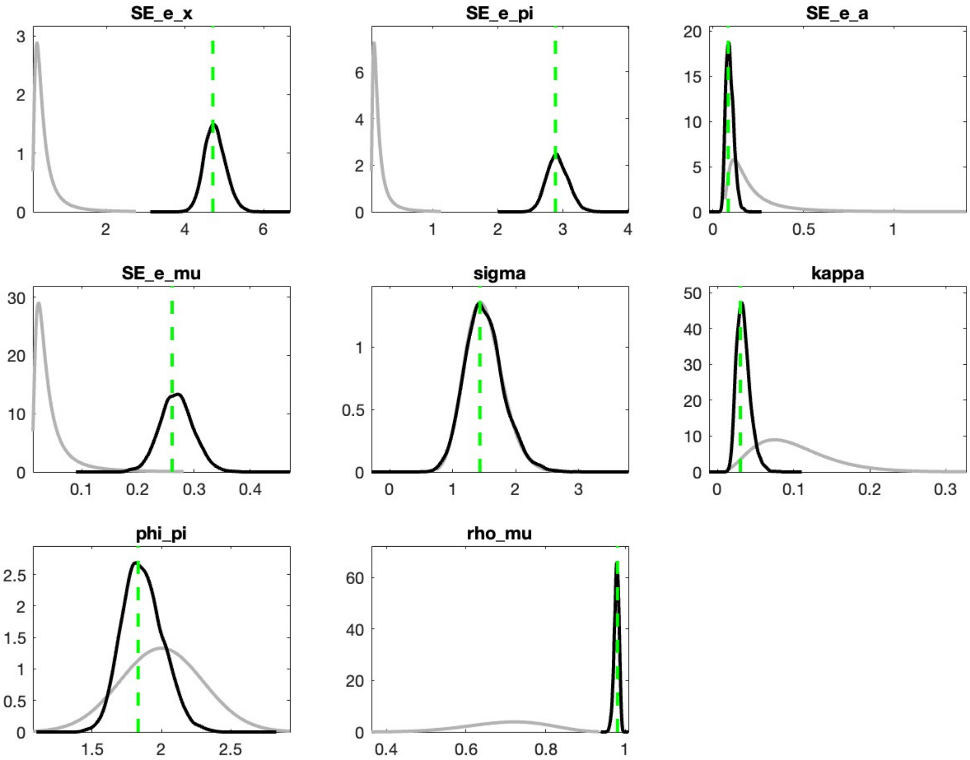

Figure C.2: Priors and Posteriors

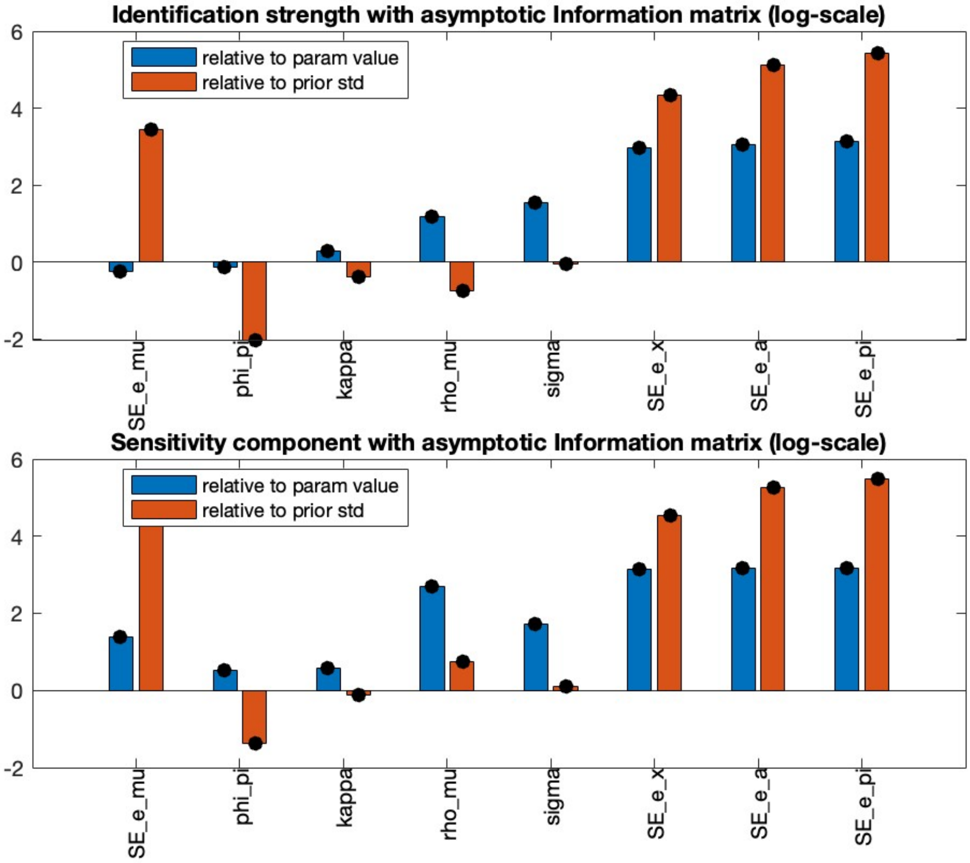

Figure C.3: Prior Mean - Identification Using Info From Observables

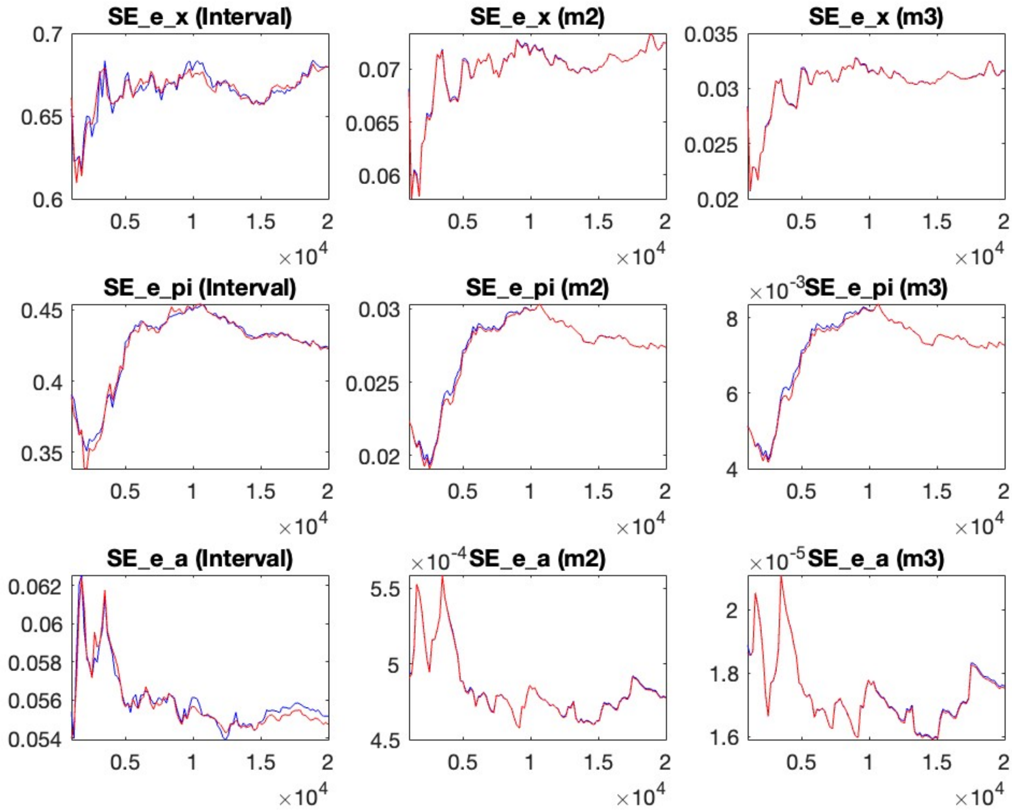

Figure C.4: Univariate Convergence Diagnostic (Brooks and Gelman, 1998)

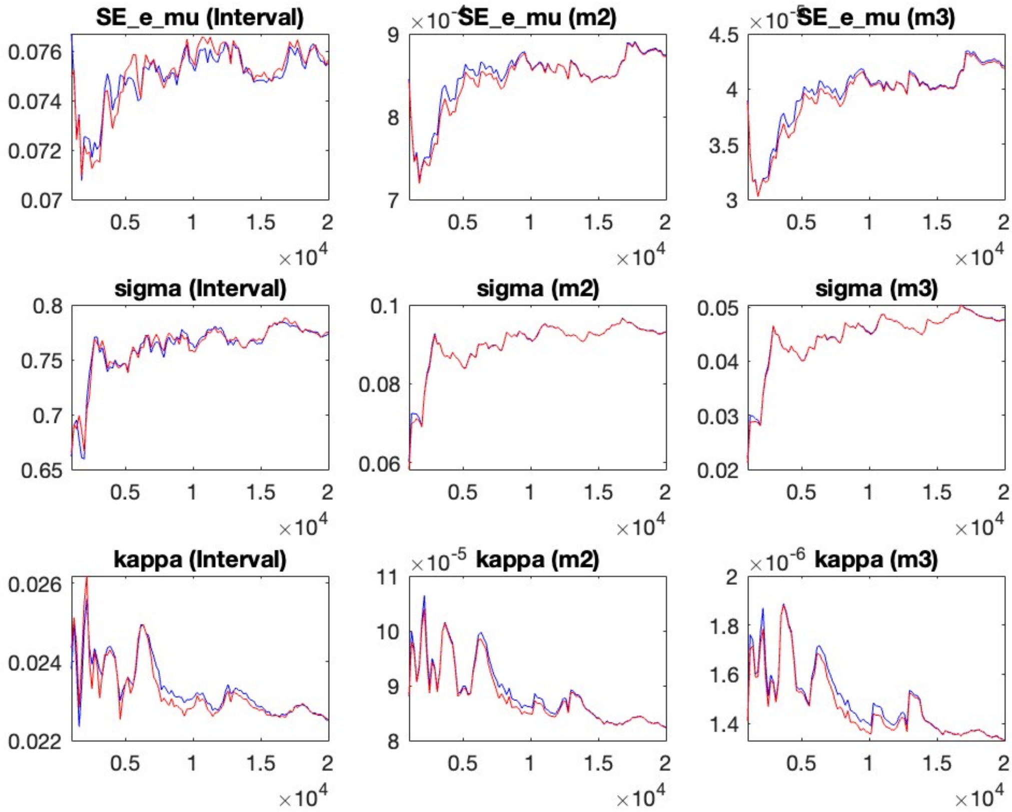

Figure C.5: MCMC Univariate Convergence Diagnostic 2 (Brooks and Gelman, 1998)

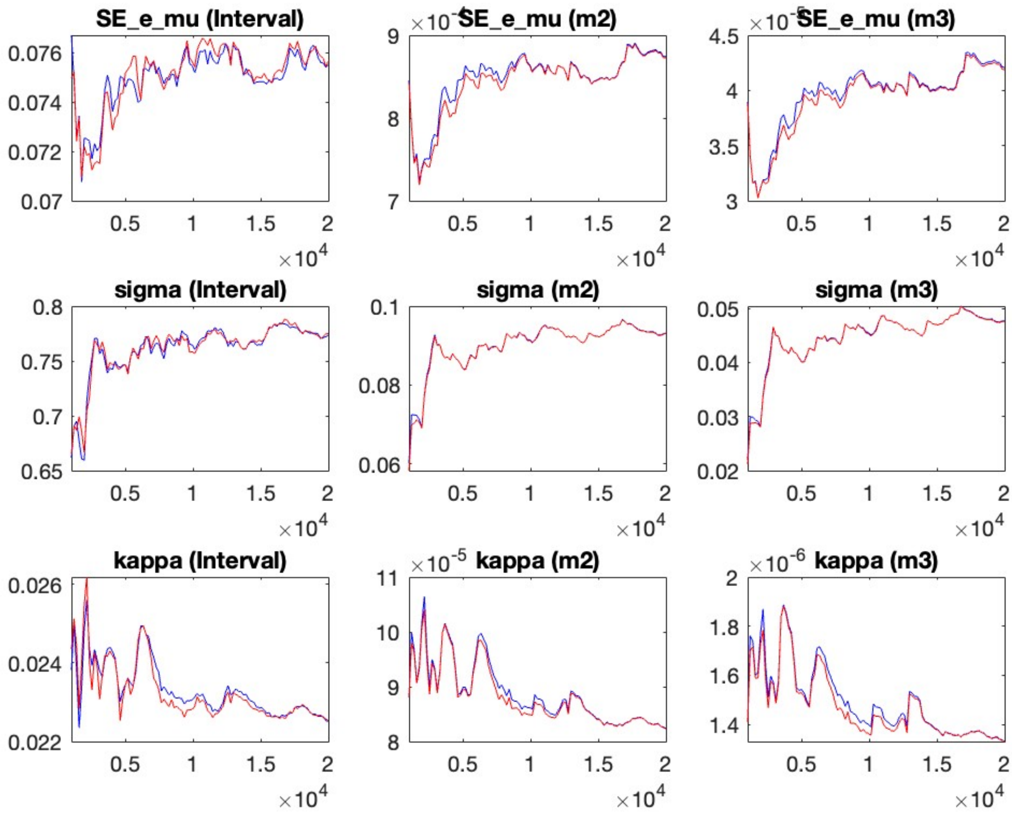

Figure C.6: MCMC Univariate Convergence Diagnostic 3 (Brooks and Gelman, 1998)

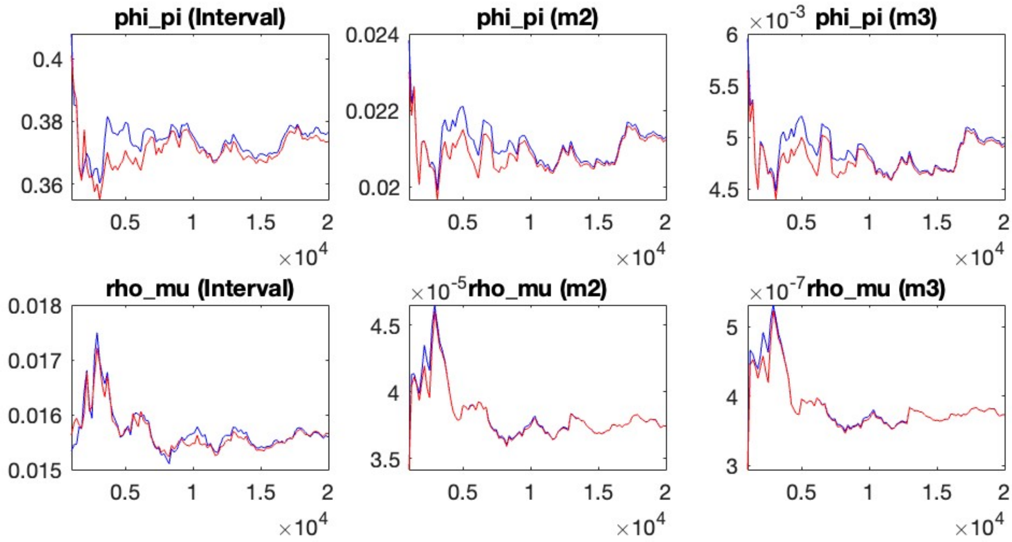

Figure C.7: MCMC Univariate Convergence Diagnostic 4 (Brooks and Gelman, 1998)

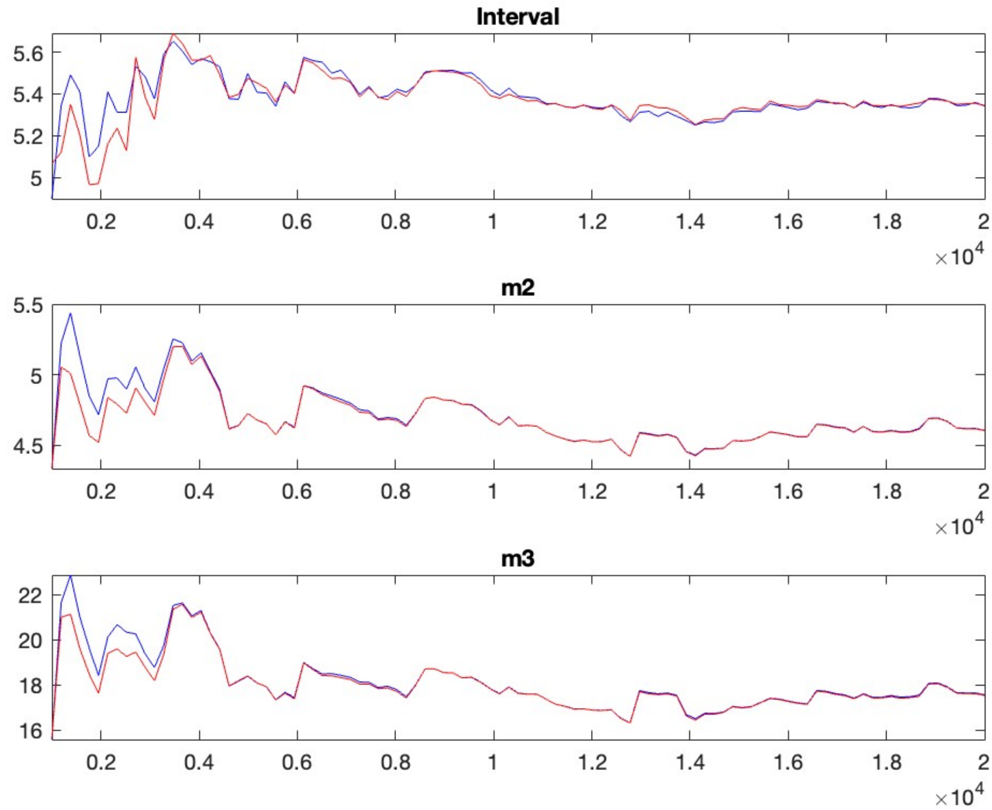

Figure C.8: Multivariate Convergence Diagnostic

Appendix D

Data Sources and Transformations

D.1 Series Definitions

Table D.1: Key Variables Used in Estimation

Variable	Source	Transformation
Policy rate	`IMF / ISF`	annualised level, demeaned HP gap
Inflation	`IMF / ISF`	$\Delta \log P_t \times 400$
Real consumption	`IMF / ISF`	$\Delta \log C_t$

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