Network Effects on Community Currency Systems

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A Dissertation Submitted in fulfilment of the requirements
for the Degree of Doctor of Philosophy in Network Science

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Network Effects on Community Currency Systems, Teodoro Criscione, 2025

Researcher declaration

I Teodoro Criscione certify that I am the author of the work Network Effects on Community Currency Systems. I certify that this is solely my own original work, other than where I have clearly indicated, in this declaration and in the thesis, the contributions of others. The thesis contains no materials accepted for any other degree in any other institution. The author declares that no unidentified and illegitimate use of the work of others was made and that no part of the thesis infringes on the copyright of any person or institution.

Abstract

The scope of this thesis is the study of community currencies for basic income pilots and cash transfer programmes. I empirically study and analyse the network structure and the dynamics of two community currency systems with the aim of assessing their socioeconomic impact by using network science tools. The first case study is the Sarafu token network, a community currency used in cash transfer programmes for humanitarian aid in Kenya during the COVID-19 emergency. The second case study is Circles UBI network, a pilot project for universal basic income in Berlin, which was also active during the COVID-19 emergency. In both cases, the use of a digital infrastructure allows for analysis of the topology of the economy underneath, study of user behaviour, and assess the state of the currency network. The contribution of this work is two-fold. First, I provide a comprehensive transaction network analysis of these two types of social innovation for humanitarian aid. Second, I provide a new approach for studying an economic system, in particular, related to the problems of recirculation and economic synergy. The tools introduced in the present study have a broad field of applications. They can help policymakers to assess similar projects, and in general those that include monetary interventions through a digital infrastructure.

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> “*The five colours make man’s eyes blind, the five notes make his ears deaf, the five tastes injure his palate.*” - **Lao Tzu**, **Tao Te Ching**

Teodoro Criscione
Vienna, January 25, 2025.

List of publications

In fulfilment of the requirements for the Doctorate degree in Network Science, the following publications were published.

Publication I. Mattsson, C. E. S., Criscione, T., & Ruddick, W. O. (2022). Sarafu Community Inclusion Currency 2020–2021. *Scientific Data*, *9*(1), 426. https://doi.org/10.1038/s41597-022-01539-4

Publication II. Criscione, T., Guterman, E., Avanzo, S., & Linares, J. (2022). Community currency systems: Basic income, credit clearing, and reserve-backed. models and design principles. https://doi.org/10.6094/FRIBIS/DiscussionPaper/8/04-2022

Publication III. Mattsson, C. E. S., Criscione, T., & Takes, F. W. (2023). Circulation of a digital community currency. *Scientific Reports*, *13*(1), 5864. https://doi.org/10.1038/s41598-023-33184-1

Publication IV. Avanzo, S., Criscione, T., Linares, J., & Schifanella, C. (2023). Universal basic income in a blockchain-based community currency. *Proceedings of the 2023 ACM Conference on Information Technology for Social Good*, 223–232. https://doi.org/10.1145/3582515.3609538

Publication V. Longo, A., Criscione, T., Linares, J., & Avanzo, S. (2024). Impact of a blockchain-based universal basic income pilot: The case of Circles UBI. *Care and Gender: Potentials and Risks of Universal Basic Income (UBI), Proceedings of the FRIBIS Annual Conference 2023*, *5*, 378. https://lit-verlag.de/isbn/978-3-643-91669-3/

Publication VI. Criscione, T. (2024). Topological components in a community currency network [submitted to Oxford Journal of Complex Networks]. https://arxiv.org/abs/2409.13674

Publication I (Mattsson, Criscione, and Ruddick, 2022) is a peer-reviewed data descriptor article, the declaration of co-authorship can be found in Figure 1. The description of Sarafu data with the relative detailed information provided in that paper is used to describe the context of the analysis (Chapter 3) and to interpret some of the results (Chapters 5 and 6).

Publication II (Criscione et al., 2022) is a discussion paper, the co-authorship declaration can be found in Figure 2. The paper is reported in the literature review (*Network Cycles in Economic Theory* paragraph).

Publication III (Mattsson et al., 2023) is a peer-reviewed scientific paper, the coauthorship declaration can be found in Figure 3. The paper is reported in the literature review (*Community Currency Networks* paragraph) as it partially inspired this follow-up work. Some of the findings are used to interpret the results of this work (Chapters 5 and 6).

Publication IV (Avanzo et al., 2023) is a peer-reviewed conference proceedings document, the declaration of co-authorship can be found in Figure 4. The description of the data from that paper is used in this work to describe the context of the analysis (Chapter 3). Moreover, similar to that work, the technique was used to analyse structural changes in a temporal network using the *causal fidelity* index (Chapter 3). Finally, information on the project history is used for the interpretation of the results (Chapter 5 and Chapter 6).

Publication V (Longo et al., 2024) is a peer-reviewed conference proceedings document, the declaration of co-authorship can be found in Figure 5. Information about the history of the project is used for the interpretation of the results (Chapter 5 and Chapter 6).

Publication VI (Criscione, 2024) is submitted for review to the Oxford Journal of Complex Networks. It is a single-author paper, so no co-authorship has to be declared. This paper is used for Chapter 4 (Sections 4.2 and 4.3). Besides **Publication VI** used as mentioned, the rest of this work is completely new and unpublished.

Declaration of Co-Authorship

Sarafu Community Inclusion Currency 2020–2021

The scientific article "*Mattsson, C.E.S., Criscione, T. & Ruddick, W.O. Sarafu Community Inclusion Currency 2020–2021. Sci Data 9, 426 (2022)*" is used in a thesis for a partial fulfillment of the requirements for the degree of Doctor of Philosophy in Network Science. Since the candidate is one of the Authors, the other Authors need to confirm their contribution. Please confirm the contributions reported in the following table.

Citation	***Mattsson, C.E.S., Criscione, T. & Ruddick, W.O. Sarafu Community Inclusion Currency 2020–2021. Sci Data 9, 426 (2022).***
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Technical Validation	Criscione, T.
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Figure 1: **Publication I.** Coauthorship declaration for Mattsson, C. E. S., Criscione, T., & Ruddick, W. O. (2022). Sarafu Community Inclusion Currency 2020–2021. *Scientific Data*, *9*(1), 426. https://doi.org/10.1038/s41597-022-01539-4

Declaration of Co-Authorship

Community Currency Systems: basic income, credit clearing, and reserve backed. Models and Design Principles.

The scientific article "*Teodoro Criscione, Eve Guterman, Sowuelu Avanzo, Julio Linares Community Currency Systems: basic income, credit clearing, and reserve backed. Models and Design Principles*" is used in a thesis for a partial fulfillment of the requirements for the degree of Doctor of Philosophy in Network Science. Since the candidate is one of the Authors, the other Authors need to confirm their contribution. Please confirm the contributions reported in the following table.

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Figure 2: **Publication II.** Coauthorship declaration for Criscione, T., Guterman, E., Avanzo, S., & Linares, J. (2022). Community currency systems: Basic income, credit clearing, and reserve-backed. models and design principles. https://doi.org/10.6094/FRIBIS/DiscussionPaper/8/04-2022

Declaration of Co-Authorship

Circulation of a digital community currency

The scientific article "*Mattsson, C.E.S., Criscione, T. & Takes, F.W. Circulation of a digital community currency. Sci Rep 13, 5864 (2023)*" is used in a thesis for a partial fulfillment of the requirements for the degree of Doctor of Philosophy in Network Science. Since the candidate is one of the Authors, the other Authors need to confirm their contribution. Please confirm the contributions reported in the following table.

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Declaration of Co-Authorship

Universal Basic Income in a Blockchain-Based Community Currency

The scientific article "*Sowelu Avanzo, Teodoro Criscione, Julio Linares, and Claudio Schifanella. 2023. Universal Basic Income in a Blockchain-Based Community Currency*" is used in a thesis for a partial fulfillment of the requirements for the degree of Doctor of Philosophy in Network Science. Since the candidate is one of the Authors, the other Authors need to confirm their contribution. Please confirm the contributions reported in the following table.

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Figure 4: **Publication IV.** Coauthorship declaration for Avanzo, S., Criscione, T., Linares, J., & Schifanella, C. (2023). Universal basic income in a blockchain-based community currency. *Proceedings of the 2023 ACM Conference on Information Technology for Social Good*, 223–232. https://doi.org/10.1145/3582515.3609538

Declaration of Co-Authorship

Impact of a blockchain-based Universal Basic Income pilot: The case of Circles UBI currency

The scientific article "*Alessandro Longo, Teodoro Criscione, Julio Linares, Sowelu Avanzo, 2024. Impact of a blockchain-based Universal Basic Income pilot: The case of Circles UBI currency"* is used in a thesis for a partial fulfillment of the requirements for the degree of Doctor of Philosophy in Network Science. Since the candidate is one of the Authors, the other Authors need to confirm their contribution. Please confirm the contributions reported in the following table.

Citation	***Alessandro Longo, Teodoro Criscione, Julio Linares, Sowelu Avanzo, 2024. Impact of a blockchain-based Universal Basic Income pilot: The case of Circles UBI currency. In Proceedings of the 2023 FRIBIS Conference. In publication.***
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4

Figure 5: **Publication V.** Coauthorship declaration for Longo, A., Criscione, T., Linares, J., & Avanzo, S. (2024). Impact of a blockchain-based universal basic income pilot: The case of Circles UBI. *Care and Gender: Potentials and Risks of Universal Basic Income (UBI), Proceedings of the FRIBIS Annual Conference 2023*, *5*, 378. https://lit-verlag.de/isbn/978-3-643-91669-3/

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Acronyms

CCS Community Currency System. 1, 2, 5, 81, 82, 85, 86

CCSs Community Currency Systems. 1, 4–8, 26, 81, 85, 87

GE Grassroots Economics. 28, 29

LWCC Largest Weakly Connected Component. 31, 49, 62, 77, 79

ROSCAs Rotating Savings and Credit Associations. 28

UBI Universal Basic Income. 2, 32, 85

1 Introduction

Community Currency Systems (CCSs) are payment systems that circulate in a limited geographic region in parallel with the official currency. A Community Currency System (CCS) is not enforced by the state but is based on agreements among members of the community (Blanc, 2011, 2012; Gomez, 2018; Gómez and Dini, 2016; Greco, 2001; Greco, 2013). CCSs have been explored as innovative methods for social and humanitarian interventions that could induce endogenous local development, empower local communities, and at the same time deliver humanitarian aid (Fare et al., 2015; Gomez, 2018; Lim and Nakazato, 2019; Martín Belmonte et al., 2021; Nakazato and Hiramoto, 2012; Nakazato and Lim, 2017; Ruddick et al., 2015; Ussher et al., 2021; Zeller, 2020). Furthermore, their countercyclic (or macrostabiliser) effect and the local multiplier effect may play an important role in improving the resilience of the local economy (Gelleri and Stodder, 2021; Groppa, 2013; Lucarelli and Gobbi, 2016; Martín Belmonte et al., 2021; Roca et al., 2024; Stodder, 2000; Stodder and Lietaer, 2016). In recent decades, several CCSs digitalised their payment infrastructure, opening new possibilities for research. As a payment system, the transaction network spans a directed, weighted, temporal network, where the nodes are the individuals or companies, and the time-stamped directed weighted edges correspond to transactions. In this work, the transactions are temporally aggregated into weighted directed edges; however, the analysis of some temporal dynamics is also included in the study of currency recirculation.

This work analyses two CCSs, the Sarafu token and the Circle UBI networks. Sarafu token network is a digital CCS used as a payment system in Kenya and organised by the non-profit organisation Grassroots Economics (Mattsson, Criscione, and Ruddick, 2022). In the period analysed, it was used as part of an emergency cash transfer programme during the COVID-19 emergency (Ruddick, 2021). The humanitarian aid campaign was co-designed with the Kenyan Red Cross and named *"Community Inclusion Currency"*. A cash transfer programme is used in emergency contexts to transfer money or vouchers to people in need which allow them to buy goods and services. A *"Community Inclusion Currency"* is a specific type of local voucher system implemented for those humanitarian cash transfer programmes, which can be used only in a predefined geographic region and/or within a local network of participants. It is argued that, once this local voucher is issued and its recirculation is bounded to a defined geographic area, it could also boost local development. Local development is triggered whenever an increase in the demand for goods and services satisfies the unused productive capacity of the region (Ussher et al., 2021). However, a very limited number of quantitative studies analysed the economic impact of local voucher systems in humanitarian cash transfer programmes.

The second project analysed is the Circles UBI Berlin pilot. Circles UBI was used as a digital CCS in Berlin and was organised by the Circles Coop cooperative. The Circles Coop was active in Berlin (Germany) from October 2021 until December 2023 (Avanzo et al., 2023; Longo et al., 2024; Papadimitropoulos and Perperidis, 2024). It was the first Universal Basic Income (UBI) project designed around a community currency network that adopted blockchain technology. The decentralised technology was designed to empower the self-organisation of the participants, who met regularly in democratic monthly assemblies and weekly markets (Avanzo et al., 2023; Longo et al., 2024; Papadimitropoulos and Perperidis, 2024). As summarised by Papadimitropoulos and Perperidis (2024) and Longo et al. (2024), a UBI must have four main characteristics: 1. universality, each individual is entitled to get it; 2. unconditionality, the distribution does not depend on certain requirements; 3. permanent throughout life and regularly disbursed; and 4. payment in cash which can be used to claim enough goods and services to cover 'basic' needs. During the COVID-19 emergency, the pilot project successfully helped to support some unemployed people and some businesses in the network, as reported by a recent qualitative study (Longo et al., 2024). It is worth mentioning that there are only two other similar projects of UBI adopting a local currency: Maricá Basic Income in Brazil (Balakrishnan et al., 2022) and REC Barcelona in Spain (Martín Belmonte et al., 2021; Roca et al., 2024). Quantitative studies on Maricá Basic Income are still ongoing, while studies on REC Barcelona are reported in the Literature Review.

The main contributions of this work can be summarised as follows. This work tries to cover a gap in the literature by providing quantitative analytical tools which can be used to assess the circulation of a digital community currency system. First, the role of cyclic structures is analysed by applying a novel topological categorisation for directed networks. Only a few recent studies focused on the analysis of the role of cyclic structures in transaction networks (Iosifidis et al., 2018; Mattsson et al., 2023), but limited to cycles of length 2, 3, 4, and 5. The author has already developed and tested this technique in a previous work (Criscione, 2024) which is based on the distinction between cyclic and acyclic components. Second, the role of recirculation is analysed between different components and time periods. In fact, the networks are divided into three periods each, according to the structural changes identified quantitatively using the *causal fidelity* index (Avanzo et al., 2023; Lentz et al., 2013) and qualitatively based on previous studies (Longo et al., 2024; Mattsson, Criscione, and Ruddick, 2022). Third, a novel measure of local circulation is introduced, called *circular network synergy*. As explained in the Literature Review, this metric is deeply connected to other existing economic phenomena, namely the *local multiplier effect* and the *synergy effect*. Finally, the work concludes with a comparison of the evolution of both networks, Sarafu and Circles, which was never done before.

The main research questions that are unfolded throughout this work are the following:

1. **RQ1**. *How did the network topology of the Sarafu and Circles networks change over time?* Structural changes are detected and used to split the temporal networks into three periods each. Each period is analysed separately by considering the aggregated network. After that, a topological categorisation uniquely assigns a label to nodes and edges. The categorisation is based on the detection of cyclic and acyclic components, and single-nodes.

2. **RQ2**. *How did the currency recirculation in these two networks change over time?* After having defined these topological components, the currency recirculation and its frequency are analysed in both networks.

3. **RQ3**. *How did the economic synergy in these two networks change over time?* A novel metric called *circular network synergy* is applied to measure the participation of users in cyclic components. This metric is also compared to evolving capacity metrics (*ascendency* and *systemic reserve*) and economic multiplier.

In Chapter 2, a comprehensive review of the literature is provided. The first part of the literature review introduces the concept of *local multiplier effect* and *synergy effect*, as described in the economic literature. These two concepts clearly describe a specific type of network effect, which could be partially caused by the presence of directed network cycles that allow for regular and consistent currency recirculation. The following part of the literature review presents the role of directed network cycles in payment systems and economic theory. The final part of the literature review is dedicated only to network science methods for community currency systems and the relative contribution of this work.

In Chapter 3, the data from the two transaction networks are presented. The most important information about the history of the projects is also reported. The networks are then divided into three periods, in line with structural changes that occur during the observed period. Some analytical information on degree, transactions, and volume distributions is also added.

In Chapter 4, the methods adopted in this work are presented. An analytical model using graph theory is used to show the limits of the economic multiplier as it is currently measured and estimated in economics. The topological categorisation and the study of recirculation are presented (from Criscione (2024)). Finally, a novel metric is introduced that aims to measure *economic synergy* in a transaction network, called *circular network synergy*.

In Chapter 5, the results of the analyses are shown for both projects, Sarafu token and Circles UBI networks. The sections reflect the methods used in this paper: topological analysis, recirculation, and *circular network synergy*. A comparison of the systems concludes the chapter. Finally, in Chapter 6 a discussion of the results concludes the work.

2 Literature Review

CCSs are payment systems that circulate in a limited geographic region in parallel with the official currency. A community currency is not enforced by the state but is based on agreements among members of the community (Blanc, 2011, 2012; Gomez, 2018; Gómez and Dini, 2016; Greco, 2001; Greco, 2013). Community currency systems have been explored as innovative methods for social and humanitarian interventions that could induce endogenous local development, empower local communities, and at the same time provide humanitarian aid (Fare et al., 2015; Gomez, 2018; Lim and Nakazato, 2019; Martín Belmonte et al., 2021; Nakazato and

Hiramoto, 2012; Nakazato and Lim, 2017; Ruddick et al., 2015; Ussher et al., 2021; Zeller, 2020). Furthermore, their countercyclic (or macrostabilizer) effect and the local multiplier effect may play an important role in improving the resilience of the local economy (Gelleri and Stodder, 2021; Groppa, 2013; Lucarelli and Gobbi, 2016; Martín Belmonte et al., 2021; Roca et al., 2024; Stodder, 2000; Stodder and Lietaer, 2016).

It is argued that CCSs can promote sustainable development, but empirical evidence is not yet sufficient (Michel and Hudon, 2015; Silva et al., 2024). The main systematic review of the topic reports about 3000 clusters of projects in 23 countries and 6 continents in 2015 (in Michel and Hudon (2015) from Seyfang and Longhurst, 2013). The authors (in Michel and Hudon (2015)) reviewed 48 major studies that attempt to assess the economic, social, and environmental impact of CCSs projects. One of the main conclusions is that most of the methodologies adopted in those studies are not sufficient to estimate the impact of a CCS project, due to the lack of control groups or similar techniques to test the statistical significance of empirical findings. In addition, a standardised and accepted impact assessment procedure is still lacking, although one was proposed and used in a few cases (Place and Bindewald, 2015), and another recent one was recently proposed but not yet applied (Diniz et al., 2024). With this premise, the authors did not find any study reporting a meaningful and significant economic and environmental impact. On the other hand, the impact on social sustainability seems to have some evidences: community building and empowerment, social inclusion, etc. Since this systematic review of the literature was published in 2015, a few progresses have been made in studying the countercyclic effect (or macrostabilizer) (Gelleri and Stodder, 2021; Stodder and Lietaer, 2016), estimating the *local multiplier effect* (Martín Belmonte et al., 2021; Roca et al., 2024), and analysing the socioeconomic network (Appleby et al., 2024; Criscione, 2024; Iosifidis et al., 2018; Lim and Nakazato, 2019; Mattsson, Luedtke, and Takes, 2022; Mattsson et al., 2023; Nakazato and Lim, 2017, 2024; Nakazato et al., 2022) of CCSs. However, statistical methods adopted in the field are still very few and this work aims to fill the gap by providing techniques for the impact assessment of CCS using network science tools.

Some studies on digital community currencies used network science techniques for their characterisation. One of the first studies focused on a convertible community currency, Tomamaecho, which was active in Hokkaido for only three months (Kichiji and Nishibe, 2008). Their findings confirmed that the transaction network was characterised by a power law decay of the degree distribution, dis-assortative behaviour, and a "small world" feature. The authors found that the ratio of the exponents of in-degree to out-degree distributions decreases with the velocity of the currency. They also measure the *network centralisation* (Freeman, 1978) of the transaction graph defined as the *"ratio of the sum of actual difference between the degree centrality of the most central actor and that of all the other actors in the network and the theoretical maximum possible sum of differences in actor degree centrality"* (from p.282, note 15 in Kichiji and Nishibe (2008)). One of their main findings is that network centralisation is positively correlated with transaction volume. Regarding the Sarafu data used in this paper, the decay of the power law and the disassortative behaviour were already confirmed in a previous work (Mattsson et al., 2023). In this work, the power law decay is confirmed and reported in both networks, Sarafu and Circles (see Chapter 3). This may also confirm the strong role of *preferential attachment* in CCSs, as suggested by recent agent-based simulations (Reyns, 2024). According to Reyns (2024), the formation of the network of CCSs is mainly driven by a value alignment between members, which redirects their consumption preferences towards the currency network. However, according to the author, the economic impact on members would be effective as long as a *club effect* is in place. In practice, a *club effect* means that members are capable of discounting goods and services to each other, and this is possible up to a certain size (of the network) and within certain financial limits. In this work, the *club effect* is explored from a network perspective through the concept of *circular network synergy* by noticing a strong connection with two other similar topics in the economic literature - i.e., the *local multiplier effect* and *synergy effect*.

Local Multiplier Effect In the economic literature, a *local multiplier effect* is defined as the long-term change in employment at the local level (usually, municipal level) due to an exogenous shock (Moretti, 2010) - e.g., flow of investments, subsidies, etc. According to Moretti (2010), due to the interplay between tradable and non-tradable sectors[1], the effect could turn

[1]The term *tradable sector* is used to indicate all those economic activities which produce goods and services for export. In contrast, the term *non-tradable sector* indicates economic activities mainly producing for local consumption.

positive, null, or even negative. After a shock expanding the economic network, if supply chains adapt to stay localised (or relocalize their activities), then there is a positive effect, which is also called *agglomeration externality (or spillover)* (Greenstone et al., 2010). The shift in expenditures towards local businesses (or not) is the key to understanding the *local multiplier effect* (Lafuente-Sampietro, 2021). However, most of the research on the topic is focused on estimating the multiplier effect using econometric techniques instead of directly measuring the changes happening in the economic network (Dijk, 2018; Moretti, 2010; van Dijk, 2017). Different approaches were recently adopted in the CCSs field by directly observing the transaction network, either through direct measurement or indirect estimation by using the *LM3* technique (Sacks et al., 2002).

In Martín Belmonte et al. (2021), the authors studied the *local multiplier effect*, the recirculation rate, and the circulation velocity of a local currency issued as a basic income pilot (called REC) in Barcelona, Spain. The REC was issued and used for a basic income pilot project that lasted 13 months between 2018 and 2019. During this period, 600 low-income families received 25% a guaranteed minimum income (called the Municipal Inclusion Subsidy) in local currency for a total amount of 789,592 REC. The total amount of volume generated in this period was equal to 901,004 REC and 643,532 REC were exchanged back to Euros by the partner businesses. The partner businesses (and other private users) were entitled to cash out, and they were committed by contract to use the currency in parity with Euros. Only direct beneficiaries of the subsidy in REC were not allowed to cash out in Euros. Since this basic income pilot and the disbursement in REC was over, the REC has been still circulating as a local currency even though it is no longer backed by Euros.

During the pilot, the monthly multiplier was calculated by taking the ratio of the transaction volume to the total of REC disbursements. A cumulative monthly multiplier at the end of the analysed period was equal to 2.11 (October 2019). The recirculation rate was calculated as the ratio of REC outgoing to REC ingoing per month, which in the observed period peaked at 33.4%. Finally, the velocity of circulation per month was calculated as the ratio between the transaction volume divided by the average amount of REC in circulation, multiplied by 12 to annualize the monthly value, and was equal to 5.80 in the observed period. In conclusion, the main quantitative indicators adopted indicated a positive effect. However, as stated by the authors, the main limitation of the study was the lack of control groups to assess the observed effects. To overcome this limitation, the same authors published a follow-up work in which control groups were used to estimate the *local multiplier effect* by using the *LM3* technique. The *LM3* technique (Sacks et al., 2002) is calculated by looking at all the ego networks (at depth three) of each receiver, namely three degrees of separation from the initial receiver of funds. In this way, researchers observe the percentage of expenditures kept in the local economy at each degree of separation from the initial receiver. In formula, the *LM3* is equal to

$$LM3 = \frac{I + E_1 + E_2}{I} \tag{1}$$

Where I is equal to the initial injection, E_1 is the amount of local expenditure of the first receiver, E_2 is the amount of local expenditure of those who gets funds from the first receiver. Using this technique, the authors could compare the *LM3* in REC with a control group of 4984 beneficiaries of a subsidy in Euros. The results show a slightly higher *LM3* in REC (2.09) than the *LM3* in Euros (1.94). In another study about two French CCSs (Lafuente-Sampietro, 2021), the formula used to calculate the *LM3* is based on the original definition of Keynesian multiplier in an open economy,

$$M = \frac{1}{(1 - c + m)} \tag{2}$$

Where c is the propensity to consume goods and services produced locally (within the network), while m is the propensity to consume goods and services produced outside the network. The authors proceed by calculating the average propensity to consume within the network (c) per each user as the ratio of expenditures in local products over the entire income. As a result, the two anonymous French CCSs have a yearly *local multiplier effect* of 2.05 and 2.34, respectively. A different approach is taken by Groppa (2013), where the multiplier is related to a completely different concept, the *monetary multiplier*. The *monetary multiplier* is the ratio between the amount of currency in circulation and in deposits over the total amount of currency created in the system. The main argument by Groppa (2013) is that a CCSs do not have any incentive to be hoarded (e.g. no banking system, no interest rates, etc.) and therefore by recirculating faster in the economy, they allow for a multiplier to be effective. However, the model is based on the assumption that what is not saved automatically recirculate, therefore it ignores any dynamic happening in the economic network, besides the financial one.

In summary, the literature around the *local multiplier effect* explored so far has two types of limitations. First, it is usually described as a network effect, but is not measured as that. It has been effectively estimated through econometric methods, but not directly measured. Second, direct measurement through the transaction network has been limited to a maximum of three degrees of separation from a sample of initial receivers, that is, LM3 technique (Sacks et al., 2002). This means that it measures the effect of recirculation occurring after only three steps in the network. From a network perspective, this is equivalent to considering only cycles and chains (or paths) of length 3 (see the Appendix Glossary for definitions 7.1; see also Chapter 4 for 'Limits of Economic Multiplier').

Synergy Effect The concept of *synergy* in economic literature has been used mainly in strategic management to indicate both the qualitative process and the result of an operational merging (or integration) of two or more systems (e.g. departments, industries, companies, working units, markets, etc.) and/or the integration of one element in an existing system (Ansoff and McDonnell, 1988; Geipele et al., 2018; Hernandez and Shaver, 2019). In general, a *synergy, synergic* or *synergistic effect* is measured as the resulting impact on the outcomes after the network is adaptively restructured into a new morphology. In institutional economics (Dopfer, 1991, the concept of *synergy* has been linked to Myrdal's concept of *circular cumulative causation* (Berger, 2009; Myrdal, 1957; Myrdal et al., 1944), which he explains by comparing the correlation among multipliers of different sectors after an exogenous shock occurs. The basic idea of *circular cumulative causation* is that firms from different sectors are circularly interconnected in their supply chain. This interconnectedness can have positive or negative effects, according to the exogenous shock that affects the network and the dynamics triggered in it. Both the *synergy effect* and the *local multiplier effect* have been recognised to be deeply connected to a specific type of network phenomenon. However, studies based on network theory are very few and limited. In Stan and Jivan (2012), *synergy* is described as a relational potential which is measured by the density of the ego-network[2] of a firm weighted by a 'quality' factor for each of its link; a 'quality' factor which needs to be estimated using qualitative methods.

A more advanced approach was to measure *synergy* in an economic network using information theory (Ivanova et al., 2019; Nijkamp and Reggiani, 1995). In these cases, several metrics derived from *entropy*, *relative entropy*, *conditional entropy*, and *mutual information* were used. Before exploring the literature on synergy, a brief explanation of these concepts is provided in the Appendix 7.2. In Ivanova et al. (2019), the Triple-Helix (TH) model of innovation is used to quantify the phenomenon of *economic synergy* caused by the mutual information occurring in three dimensions: governmental, technological and organisational. The basic idea behind this approach is that the redundancy of links among firms in multiple dimensions decreases the level of uncertainty in the system, and therefore innovation becomes less risky. The mutual information of a business network, which is happening at the intersection of these three dimensions is defined by the authors as

$$I_{G,O,T} = H_G + H_O + H_T - H_{G,O} - H_{G,T} - H_{O,T} + H_{G,O,T} \tag{3}$$

where the H is the Shannon entropy of each dimension. The firms in the business network are then grouped by geographic area (G), number of employees (organisational dimension, O) and technological class (T). The Shannon entropy of each dimension and cross-dimension is formulated as follows

$$H = -\sum_i p_i \log p_i \tag{4}$$

where i correspond to G, O, T, or any other combination of dimensions, as in Equation 3. The value of p_i is measured as the ratio of the number of firms in dimension i to the total number of firms in the whole system. For example, the value of $p_{GOT} = \frac{n_{GOT}}{N}$ is measured as the ratio of the number of firms n_{GOT} in the geographic area G, in the technological sector T, and in the organisational class O, to the total number of firms N in the entire system. It is therefore assumed

[2]The ego-network refers to the network of a single central agent, in this case a firm. An ego-network can include different levels of depth or degree of separation from the central agent. For example, if I include the suppliers and the customers, I will get an ego network of depth 1; but if I also include their first connections, then I will get an ego network of depth 2. The density of a network is the ratio of existing links over the theoretical maximum possible number of links a firm can have in its environment.

that there are links between firms which are similar across the three considered dimensions (geographical, organisational, and technological). According to the authors, this multilevel linkage creates redundancy which reduces the level of uncertainty and therefore makes it more profitable to invest in innovation. Following the previous definitions, the authors conclude that if *mutual information* (Equation 3) is negative, then this decrease in uncertainty can be called *economic synergy*. The authors estimate in this way the *synergy* in two different counties in Norway, and then they considered only national and only international firms. One of their main conclusions is that where the *synergy* of international firms is higher, the expected turnover of the county is also higher. A different approach was taken by Nijkamp and Reggiani (1995), where the authors use the concept of Shannon-Wiener formula to measure the "diversity of a system"[3]. The authors used this concept to characterise the *potential* (or productive capacity) of a network. The first level of analysis in Nijkamp and Reggiani (1995) is the minimisation of the costs per link. The authors first define the *potential* (productive capacity, space of possibilities) P_i of each link, which is a function of *network coverage* R_i (i.e., network benefits or externalities) and *network connectivity* C_i (i.e., synergy effect due to a particular topology of the network). In formula,

$$P_i = f_i(R_i, C_i) \tag{5}$$

In this view, the potential P_i defines the upper-bound and lower-bound of the performance (outcome, actual production) Y_i of each link i. Therefore, it is expected to have increasing scale returns in the range (P^{min}, P_*), decreasing scale returns in the range (P^*, P_{max}), and negative scale returns in the range (P^{max}, P_∞). A similar mathematical definition of *economic synergy* can be found in Yerznkyan et al. (2023).

The second level of analysis in Nijkamp and Reggiani (1995) is at the network level, one layer in a multilayer network. As defined by the authors, the potential of each link P_i depends on the "connectivity" of the entire network. Therefore, it is necessary to maximise this "connectivity effect" to increase the outcome at the individual and network level. To do this, the

[3] A definition of Shannon-Wiener entropy formula is provided in Appendix 7.2

authors suggest to measure it as diversity H of the network performance Y_i:

$$H = -\sum_i Y_i \ln Y_i \tag{6}$$

where $i = 1, ..., I$ is any of the links in the network. In this context, the authors interpret the ”diversity” of the system as the maximum combination of all link performance Y_i in the network. The network potential of a i-th link depends on the connectivity C_i. If the network topology changes by maximising Equation 6, a *synergy effect* is triggered. In simple words, according to the authors, the *synergy* can be triggered by creating new diversifying connections.

In summary, the mathematical formulation of *synergy* in economic theory defines it as a ”quality” of network connectivity, an effect which is due to the network topology. In the literature, this ”quality” has been measured by using the concepts of mutual information and the Shannon-Wiener formula. The maximisation of this network ”quality” can only happen at the network level due to changes in its topology. Nevertheless, one of the limitations of using such concepts is that they do not give precise information about the network topology, but only estimate a network externality. In this work, a measure of *synergy* and *local multiplier effect* is suggested, and it can be directly related to some precise topological characteristics of the economic network. In particular, the *circular network synergy* proposed in this work is measured by solving a *minimum-cost circulation problem* (Ahuja et al., 2014; Edmonds and Karp, 1972; Goldberg and Tarjan, 1989) in a transaction network and can be related to the presence of weighted directed network cycles (Fleischman and Dini, 2020; Simic and Milanovic, 1992).

Evolving Capacity in Ecological Networks As explained so far, *mutual information*, *entropy*, and *circular causation* have been linked to a *synergy effect* in economic networks. From a network topological perspective, the cycles in a flow graph could be used to describe the emergence of such phenomena. This approach was already partially suggested in the literature on ecological complexity. In Ulanowicz et al. (2009) and Zorach and Ulanowicz (2003), three metrics based on entropy, mutual information and conditional entropy are used to describe the so-called *evolving capacity* of a living complex system: *capacity* C, *ascendency* A, and *systemic reserve* ϕ. These metrics will also be used in this work to assess the *evolving capacity* of the currency network by comparing them with the *circular network synergy* and the economic multiplier.

In Ulanowicz et al. (2009) and Zorach and Ulanowicz (2003), the authors interpret the entropy of a probability mass function p_i as a measure of its *indeterminacy*, also measure of uncertainty around a random variable (see Appendix 7.2 for mathematical definitions). The *aggregate system indeterminacy* H (or the space of possibilities) is defined as

$$H = \sum_i h_i = -k \sum_i p_i \log p_i \tag{7}$$

where H is seen by the authors as the the total system capacity for change (or *evolving capacity*) for the ensemble of events i.

Considering the joint probability p_{ij} that event i and j occur, a measure of joint non-occurrence (*surprise*) is given by

$$s_{ij} = -k \log p_{ij} \tag{8}$$

If events i and j are completely independent of each other, then their joint probability can be defined by the product of their marginal distributions $p_{i.} = \sum_j p_{ij}$ and $p_{.j} = \sum_i p_{ij}$. The authors therefore assume that the surprise (Equation 8) is maximal when events i and j are totally independent.

However, if the two events are not totally independent, they can directly or indirectly influence each other. Similarly to the definition of mutual information (see Appendix 7.2, Equation 32), the difference between the maximal surprise s^*_{ij} and the actual surprise s_{ij} is interpreted as a measure of constraint of i over j (and vice versa).

$$\begin{aligned} x_{i|j} &= s^*_{ij} - s_{ij} \\ &= -k \log(p_{i.} p_{.j}) + k \log(p_{ij}) \\ &= -k \log \frac{p_{ij}}{p_{i.} p_{.j}} \end{aligned} \tag{9}$$

The average mutual information (or mutual constraint) X in the whole system is given by the weighted sum of all combinations $x_{i|j}$ of i and j by the respective joint probability p_{ij}

$$\begin{aligned} X &= \sum_{i,j} p_{ij} x_{i|j} \\ &= k \sum_{i,j} p_{ij} \log \frac{p_{ij}}{p_{i.} p_{.j}} \end{aligned} \tag{10}$$

Finally, due to the convexity of the logarithmic function, it can be stated that $H \geq X \geq 0$. If $H > X$, then the *conditional entropy* ψ is defined as follows

$$\begin{aligned} \psi &= H - X \\ &= -k \sum_{i,j} p_{ij} \log \frac{p_{ij}^2}{p_{i.} p_{.j}} \end{aligned} \tag{11}$$

In summary, the authors define *entropy* as capacity for evolution H, the indeterminacy of the system that allows for changes. This is equal to the sum $X + \psi$. In fact, X is the *mutual information* in the system, which the authors claim to be a measure of order, coherence and efficiency (i.e., used capacity, expressed potential, contraction of the space of possibilities). Finally, the authors interpret ψ as a measure of *conditional entropy*, which for them is also related to disorder, incoherence, and inefficiency (i.e., unused capacity, unexpressed potential, what is left outside such contraction of the space of possibilities). In practical terms, considering a weighted adjacency matrix T, the authors use these definitions above to derive network metrics by applying the following transformations:

$$\begin{aligned} p_{ij} &\sim \frac{T_{ij}}{T_{..}} \\ p_{i.} &\sim \frac{T_{i.}}{T_{..}} \\ p_{.j} &\sim \frac{T_{.j}}{T_{..}} \end{aligned} \tag{12}$$

where T_{ij} is the weighted link from i to j, $T_{i.} = \sum_j T_{ij}$ is the total outflow of node i, $T_{.j} = \sum_j T_{ji}$ is the total inflow of node i, and $T_{..} = \sum_{ij} T_{ij}$ is the total flow of the system. Therefore, the following network metrics are defined below,

$$C = T_{..}H = -\sum_{ij} T_{ij} \log \frac{T_{ij}}{T_{..}} \tag{13}$$

$$A = T_{..}X = -\sum_{ij} T_{ij} \log \frac{T_{ij}T_{..}}{T_{i.}T_{.j}} \tag{14}$$

$$\phi = T_{..}\psi = -\sum_{ij} T_{ij} log \frac{T_{ij}^2}{T_{i.}T_{.j}} \tag{15}$$

$$C = A + \phi \tag{16}$$

Note that the definitions of evolving capacity C, ascendency (or mutual constraint) A, and systemic reserve ϕ (or conditional entropy) are formulated according to condition 16. In fact, the authors speculate on the existence of an optimal balance between ascendency A and systemic reserve ϕ, by finding some empirical evidence in ecological networks which may prove a so-called "window of vitality" (Goerner et al., 2009). According to Ulanowicz (2009), "ascendency" (or mutual constraint) is associated with autocatalytic growth processes of living systems, a particular type of feedback loop in which there is selective pressure on each node. This means that the topology of the network is characterised by weighted directed network cycles, where the nodes in them are associated to *centripetality* (i.e., accumulation of resources by those in the cycle from the rest of the network) and *mutuality* (i.e., reciprocal transfer of resources among those within the cycle) (Ulanowicz, 2009). On the other hand, system reserve was associated with "redundancy", the existence of parallel directed simple paths (Kharrazi et al., 2020). It was suggested to extend this approach to the study of economic systems, especially to test their resilience as a dynamical equilibrium between ascendency and reserve, or in other words, exploitation of resources and creation of reserves (Fath, 2015; Goerner et al., 2009; Kharrazi et al., 2020).

In summary, in the literature on ecological complexity, a relation between mutual information (also called "ascendency", systemic efficiency) and cycling in autocatalytic growth processes was theorised. This work also tries to cover this gap by empirically testing this hypothesis in economic networks. In Chapter 5, *ascendency* and *systemic reserve* are compared to *circular network synergy* and economic multiplier. The *circular network synergy* measures the percentage of volume flowing through weighted directed network cycles. This metric is based on a minimum-cost transformation of the real graph, which cancels out directed cycles based on their flow. The network metrics of evolving capacity (*ascendency* and *systemic reserve*) and *circular network synergy* could be used to explain the network causation behind economic phenomena such as *local multiplier effect* and *synergy effect*. In the next section, a brief review of the literature on cycles in payment systems is reported.

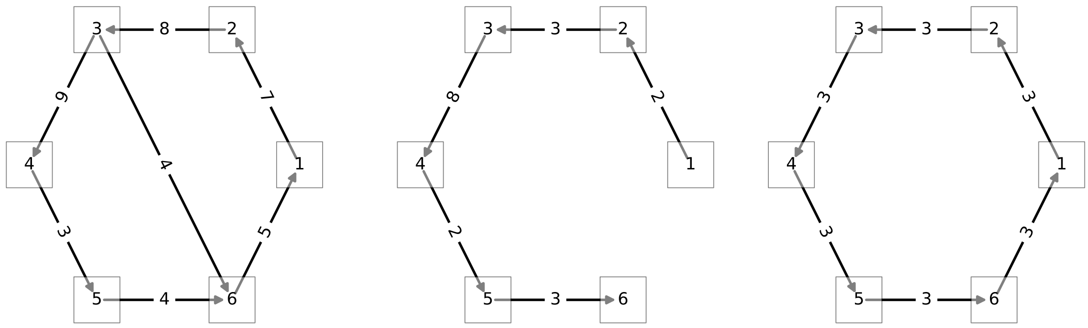
Network Cycles in Payment Systems In the previous paragraphs, the role of cycles in the *local multiplier effect* and *synergy effect* was explored. Both economic phenomena are caused by the recirculation of currency; therefore, most of the measurements adopted so far try to estimate directly or indirectly the potential embedded in currency flow cycling. As described in the previous paragraph, in this work, the *circular network synergy* index is used. The index is based on the ratio between the volume of transfers in the transformed graph (using the minimum-cost algorithm) and the volume of transfers in the original graph. As suggested by Fleischman et al. (2020) and Simic and Milanovic (1992), this operation is equivalent to clearing the network cycles of obligations and is one of the possible ”netting” techniques (or ”net settlement”).

The techniques for settling payments can be categorised into three main families (Roberds, 1999): gross settlement, net settlement, and gross settlement with queueing systems (or hybrid systems). Gross settlement techniques imply that each payment obligation is settled by a transfer of an offset amount within a limited predefined period after its issuance. Net settlement techniques (or netting) imply that the payment obligations are first collected by a central entity or ledger (e.g., clearing house, central bank) which is used to calculate net debt and net credit positions. This process reduces the number of transfers and the amount of liquidity necessary to settle all obligations. Finally, a gross settlement with a queuing system tries to integrate the benefits of having an immediate settlement (gross settlement) but partially keeping the benefit of a net settlement. In fact, using a queueing system, the net is calculated in real time considering the inflow and outflow of each participant. If there is enough credit in the account, the payment is immediately settled; otherwise, it is left in the queue.

The existence of netting techniques by merchants can be traced back to medieval and early modern European trade fairs (Börner and Hatfield, 2017). In a recent work (Börner and Hatfield, 2017), historians referred to it as *rescontre* procedure, and it was practised by merchants gathering about four times a year in several European capitals. On the first days, merchants would trade, while the last days were dedicated only to settlement of payments. In that work, two main procedures are described: cycle clearing and chain clearing.

In Figures 6 and 7, both procedures are described. In each figure, the net flow of each node reflects its debt position (negative net) or credit position (positive net). Cycle clearing is equivalent to the procedure described in Figure (6). The transaction graph (a) is the original graph. The minimum-cost transformation leads to graph (b). Figure (b) can be reached by subtracting the minimum due amount from all the obligations in the cycle. Note that in graph (b), the final flow satisfies the net flow of each node also in graph (a). In Figure 6(c), the obligation with the minimum amount (that is, 3) is used to discount all other obligations in the cycle, and the result is an empty graph. In fact, every node in the cycle owes the next node exactly 3 and expects 3 from a previous node. The net of each node is equal to zero. This means that if all nodes can coordinate, they can cancel all pending obligations.

In Figure (7), the chain clearing is represented. Assuming that every node owes the next node on the chain the same amount, every bold arrow is a debt relation (or obligation). For instance, A owes to B, B owes to C, etc. Consequently, A has a negative net flow, while E has a positive net flow. All other nodes have a net flow of zero. This means that A is expected to pay, while E is expected to receive money. The chain is cleared out by simply asking Node A to pay Node E, even if they do not know each other. Instead of solving the network in four transactions, the payment system can be resolved by simply allowing cash flow from A to E (dotted arrow). This operation is also called ”delegation” or ”novation”, because it requires the creation of a new link (or contract) between a debtor (A) and a creditor (E).



(a) (b) (c)

Figure 6: Application of the minimum-cost algorithm as cycle clearing.

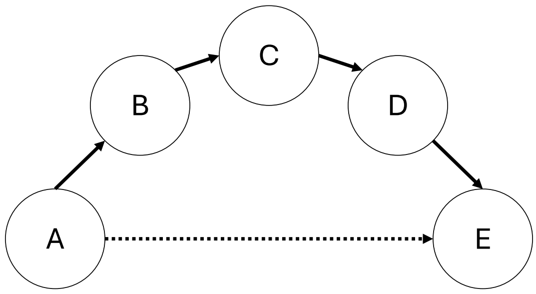


Figure 7: Chain clearing.

A first family of netting techniques includes *Deferred Net Settlement* (DNS) and *Batch Net Settlement* (BNS) techniques which are based on algorithms that can be launched at a fixed time of the day, after a certain amount of payments orders (or obligations) is reached, or prioritising older obligations by settling them in batches (Armour et al., 2016; Humphrey and Bank, 1995; Martin and McAndrews, 2008; Mikesell, 1948; Summers, 1994). Once the obligations are collected, the network is constructed and, finally, the netting procedure delivers a transformed graph. These netting techniques delivering a transformed graph can be based either on cycle detection and simplification (see Figure6) (Božić and Zrnc, 2023; Cui, 2021; Cui et al., 2017; Gazda, 2001; Shafransky and Doudkin, 2006), or based on the equivalent minimum-cost flow problem (Bottazzi et al., 2024; Buchman et al., 2024; Fleischman and Dini, 2020; Gavrila and Popa, 2021; Schara and Bric, 2018; Simic and Milanovic, 1992), or on heuristics methods based on the subset-sum problem (Amato et al., 2021; Guichon et al., 2023; Verhoeff, 2004) (see

Figure 7). A second family of netting techniques includes *Continuous Net Settlement* (CNS) techniques and *Liquidity Saving Mechanisms* (LMS) (or queuing systems) in *Real-Time Gross Settlement* (RTGS) environments (Armour et al., 2016; Humphrey and Bank, 1995; Martin and McAndrews, 2008; Mikesell, 1948; Summers, 1994). These net settlement techniques take advantage of the properties of temporal graph dynamics (Bech and Soramäki, 2001; Martin and McAndrews, 2008; Patcas, 2011; Patcas and Bartha, 2014) and temporal cycle detection (Jong, 2018). In addition to these two families, some other recent models for decentralised systems based on blockchain and zero-knowledge cryptography proposed different ways to deal with net settlement payment systems (Bottazzi et al., 2024; Buchman et al., 2024; Dandekar et al., 2012; Jong, 2018; Ramabaja, 2022).

In conclusion, the role of cycles in payment systems has a long history, but only in the last decades has been increasingly studied and applied. Cycles not only allow for the recirculation and autocatalytic growth of an economic network but can also be used to increase the efficiency of payment systems, reduce their liquidity costs, and risks. For this reason, in this work, a categorisation based on cyclic and acyclic components is implemented. Lastly, the implications for recirculation and *circular network synergy* are analysed. In the next paragraph, some theoretical findings on the role of network cycles in economic theory are reported.

Network Cycles in Economic Theory The relation between topology and dynamics in economic and financial networks has already been partially explored in the literature. In network game theory, degree centrality and sparseness of the network are proved to be the major causes of inequality, keeping all other conditions fixed (Cassese and Pin, 2024). In an agent-based model simulation, nodes with a high level of betweenness centrality on specific trading paths imposed a *mark-up* on their transactions, and therefore, affecting the formation of prices on the entire network (Cardoso et al., 2020). In a recent discussion paper (Criscione et al., 2022), the authors suggested that the presence of directed cycles in a payment system may theoretically be linked to a redistribution of economic power in it. In fact, economic power can be defined by the degree and the betweenness centrality[4]. Therefore, assuming equal weights on each edge,

[4] In a graph made of a unique directed cycle, every node has the same level of degree and betweenness centrality. In-degree and out-degree centrality can be related to monopolistic and monopsonistic power, while betweenness centrality is related to "brokerage" power.

a perfect market competition would imply that the nodes are involved in the same number of cycles, so that they all have the same economic power (Criscione et al., 2022).

In Jackson (2008)(pp. 384-388), a Nash stable directed cycle economy is reached only in a particular circumstance. A distance-based individual utility is a utility function in which agents get benefits from direct and indirect connections. When direct benefits are shared by both nodes (source and target), it is called the *two-way* flow utility. However, when benefits are only falling on the side of the sender/source, it is called the *one-way* flow utility. Considering a directed graph G with n players and a *one-way* distance-based utility function $u_i(G)$, where direct and indirect connections give the same benefit (i.e. absence of decay factor). In the formula, the utility (or payoff) of i is

$$u_i(G) = R_i(G) - cd_i(G) \tag{17}$$

where $R_i(G)$ is the number of players that can be reached by i through a directed path in G, $d_i(G)$ is the out-degree of i (i.e., the number of receivers from node i), and c is the marginal cost of each link. A directed Nash stable network is said to be *strictly Nash stable* when the removal or addition of one link by a node would strictly decrease its payoff. Therefore, the following cases are possible:

$$\begin{aligned} c < n-1 &: n\text{-cycle} \\ c > n-1 &: \text{null graph, only strictly Nash stable solution} \\ c < 1 &: n\text{-cycle is the only strictly Nash stable solution} \\ 1 < c < n-1 &: \text{both } n\text{-cycle and null graph are the only strictly Nash stable solutions} \end{aligned} \tag{18}$$

The n-cycle emerges when the cost of a link c is less than the number of remaining players $n-1$. It is also strictly Nash stable when $c < 1$. In simple words, in an economic network, when direct and indirect benefits are equivalent and the cost of each link is not too high ($c < n-1$), it is convenient for all the players to create a directed cycle.

The literature reported in this paragraph analysed the theoretical conditions and consequences of a network topology in an economic system. In particular, directed cycle formation was explored in very simple and limited scenarios. This emphasises the need to study the cycle components in this work and its impact on the economy. In the next paragraph, the literature review on network analysis of community currency systems is reported and relations to this work are briefly explained.

Community Currency Networks The very first work that examined the network structure of a community currency system was already mentioned at the beginning of this Literature Review (Kichiji and Nishibe, 2008). A second work focused on time banks (Collom, 2012). A time banking system is a specific type of community currency system in which time is used as a unit of account and organised as a mutual credit group. In that work (Collom, 2012), two sets of performance indicators were applied to the case of the Portland West Time Dollar Exchange in Portland, Maine. The first set reported the number of active members, new members joining each month, transaction volume, average transaction volume, and account balance per user. The second set reported the number of trading partners, the number of reciprocated links, the density of the ego network[5], and the diversity of services traded. In particular, the second set of indicators aims at measuring reciprocity and resilience in such a time-based currency. In this work, topological components are identified in the Sarafu and Circles networks. Each topological component is analysed by looking at the following network metrics: number of weakly connected components, number of nodes, number of directed links, number of transactions, and volume of exchange. In a third paper adopting such techniques, the network analysis mainly focused on detecting central players and identifying a rich-club of prominent users in the RozLEŤSe system active in Brno, Czech Republic (Franková et al., 2014). The identification of a rich-club was then used to study the resilience of the economic network by implementing a stress test using an experiment in which users have been removed from the system.

Some recent works focused on Sardex network, a business-to-business mutual credit system operating in Sardinia, Italy. In the most recent (Appleby et al., 2024), the connectivity of the network is analysed over time by looking at the average directed path length, average degree, diameter, clustering coefficient, and average degree centrality. The authors concluded that the

[5] See previous note on the definition of ”ego-network” and ”density”.

connectivity of the network increased with time. In another work on the Sardex network (Iosifidis et al., 2018), it was found that a statistically significant presence of directed cyclic motifs is beneficial for that payment system. Moreover, they define prominent nodes based on their participation in directed cycles. The findings suggest that the most prominent nodes in fact have a better performance over time. However, their analysis is focused only on static directed simple cycles of lengths 2, 3, 4, and 5. Following the same methods adopted in Criscione (2024), also in this work the role of cyclic and acyclic components is explored. In fact, a cyclic component can include many cycles of different lengths. In this work, the comparison between the cyclic and acyclic components is then carried out to show differences in the evolution of both networks and their relationship with *circular network synergy*, *ascendency*, *systemic reserve*, and economic multiplier.

A recent work analysed the transaction network of a mutual credit system (Hanbat LETS) in Korea as a multiplex network (Nakazato and Lim, 2024). The authors wanted to characterise the emergence of social capital through three main types of connections: bonding, bridging, and linking. In particular, they could distinguish by economic transactions, share of used goods, and provision of support. According to the authors, *bonding* can be related to dyadic reciprocity (that is, cycles of length 2) and transitive closure (that is, cycles of length 3). On the other hand, *bridging* can be related to formation of k-out-star and k-in-star. And finally, *linking* can be related to degree assortativity. In terms of the *bonding* process, the authors found that while transitive closure is significant in the transaction network, dyadic reciprocity is significant only from a multiplex perspective. In terms of *bridging* process, degree assortativity is significant at both the transactional and multiplex levels. The results suggest that different relational dimensions could complement each other and, therefore, a multiplex approach is advised in assessing the socioeconomic impact of similar projects. In Lim and Nakazato (2019) and Nakazato and Lim (2017), the authors try to assess the socioeconomic impact of a disaster response emergency community currency in Japan. To do so, they try to estimate how the social network grows according to the individual perception of community resilience. The authors detected a disassortative mixing based on the heterogeneous perception of community resilience, the absence of homophily, and a high level of clustering. In other words, people who were positive about the resilience of the community played a key role in the formation of social networks. In this work, only the transaction network is analysed, and only from its economic interpretations. However, it may not be completely excluded that studies on *recirculation* and *circular synergy* can also have implications for the literature on social capital and community resilience.

Similarly, a triadic census analysis in the Sarafu token network was recently performed comparing it with two other decentralised socioeconomic networks, the NFT (Non-Fungible Tokens) market and Steemit (Ba et al., 2023a). The authors analyse directed triads from a static and dynamic perspectives. As expected, dyadic reciprocity is higher in Sarafu (that is, cycles of length 2), which also has a larger strongly connected component and less chain-like structures than other online social networks. In particular, most of the open and closed triads that include a reciprocal dyad are significant. This is probably due to the fact that group accounts are included in the data, which are run by *Chamas* (i.e. rotating savings and credit groups). Finally, the authors report interesting statistics on the dynamics of the triadic closure process: on average there are 283 new links per day and a peak-day of 1370, on average there are also 540 closing triads per day with a peak-day of 7328, and finally, on average there is a triad/link ratio of 1.73 and a peak of 15. The triad/link ratio started to grow in July 2020 with a peak in January 2021, and decreased after that. Finally, 23% of the closures happen in less than a day and 89% in less than 3 months. These results are an important starting point for the analyses carried out in this work. First, group accounts are excluded in this work because the focus is on economic processes and not on financial processes. In fact, dyadic reciprocity between groups and individuals can be easily confused by borrower-lender transactions. Second, a static triadic census analysis is considered only for acyclic components to detect anomalies in the economic behaviour of agents excluded from trading cycles. Finally, the temporal pattern observed by Ba et al. (2023a) confirms the presence of structural changes that are reported in Chapter 3, and is used to split the network into three periods. As in Ba et al. (2023b) and Ba et al. (2022), each period corresponds to a particular internal policy framework, but also a different external policy change due to the COVID-19 emergency.

Another recent quantitative study on the Sarafu network focused on an inverse estimation of transfer velocity and effective balance (Mattsson, Luedtke, and Takes, 2022). In their work, the inverse estimation of the transfer velocity is defined as the average holding time of received funds and calculated on a ”first-in first-out” basis (Mattsson and Takes, 2021). Its findings suggest a high level of geographic and temporal heterogeneity in the usage of the currency. In particular, transfer velocity and effective balance generally had a sharp increase in the first half of 2020, but with some variations between urban and rural areas. Another study on the Sarafu network analysed some aspects of currency circulation (Mattsson et al., 2023). In particular, in that work the Sarafu network appears to be characterised by three main factors: geographic localisation, cycle motifs (of length 2, 3, 4, and 5), and structural correlations. Moreover, the authors detected key players using PageRank centrality: savings groups and faith leaders seem to play a key role in the circulation of Sarafu. In another work, the cooperative behaviour of the savings groups is analysed over time using Sankey diagrams (Ba et al., 2022, 2023b). In that work, the network was split into different periods according to the application of restrictions due to the emergency of COVID-19. The authors observed that the role of savings groups increased, especially when the strictest COVID-19 restrictions were implemented. The percentage of transactions from group accounts to users increased from 8% to 25%, the sectors of *food* and *shop* gained importance during the same period and finally, the geographical heterogeneity increased in terms of spending behaviour.

Unlike those previous works on the Sarafu token network, in this work only the transactions among users are considered (i.e. group accounts are excluded). Instead of considering the velocity of circulation (as in Mattsson, Luedtke, and Takes, 2022), *recirculation* operations are defined and analysed. Furthermore, like some of the other works mentioned above (Ba et al., 2022, 2023b), the circulation analysis is carried out in three different periods, where each period defines a temporally aggregated graph. In this work, topological components are identified, recirculation and *circular network synergy* are analysed. This is the main difference from previous works that focused on the activity of user and group accounts at the network level (Ba et al., 2022, 2023b; Mattsson et al., 2023). Since the main focus is on cyclic structures, the triadic census is only partially carried out on acyclic components (see Appendix 7.6). Finally, instead of focussing on cycle motifs (of lengths 2, 3, 4, and 5) as in a previous work on Sarafu data (Mattsson et al., 2023), in this work cyclic components are considered. As explained before, each cyclic component is defined here as a strongly connected component, where every node can be involved in one or more cycles of any length.

Some studies have been already carried out on Circles UBI network as well. The first study is a network analysis of the system (Avanzo et al., 2023), where the authors split the network into two periods by calculating the characteristic time through the causal fidelity index (Lentz et al., 2013). The causal fidelity index helped identifying structural changes in the network. In particular, from July 2021 until September 2023, a subsidy program for a small group of business partners was introduced, and the topology of the network slowly changed dramatically. The subsidy program would allow those businesses to cash-out monthly the 90% of accumulated Circles tokens to get Euros in exchange. The subsidized businesses became the most central actors in the system and their core number increased [6]. The network of subsidized businesses and relative partners increased in the first six months, and then stabilized in terms of volume and transactions. However, most of their Circles units were used to trade ”food” products and to cash-out. This means that they partially failed in creating a self-sustained business network. In two other qualitative studies, similar conclusions were reached. In Longo et al. (2024), a survey was carried out in November 2023, one month after the subsidy program stopped and one month before the Circles Coop was shut down. The study involved twenty-five individuals engaged in the system in different ways. There are three main findings that it is worth to report here. First, policy changes affected the engagement of members over time. For example, it is mentioned that the introduction of the subsidy program or a yearly *demurrage*[7] was a reason for engaging differently with the system. Second, the study reports the case of a person who lost her job during the COVID-19 emergency. She managed to engage in the system actively which helped her having access to products and services, otherwise unaffordable to her. As an example, access to locally produced food and holistic healthcare services. Third, the study also reports cases of economic *synergies*, where local businesses involved in Circles started to cooperate buying and selling each other products, or even starting new partnerships. Finally, it is worth to mention that the main motivation for these businesses to join was mostly political. As

[6] A *k*-core level identify a node with at least *k* in-degrees and *k*-out-degrees. In fact, it is a measure of connectivity which can be used to analyse the hierarchy of a network (Batagelj and Zaversnik, 2003).

[7] *Demurrage* is a negative interest rate on the deposit or holding tax. It is a ”forced” inflation rate which is used to avoiding currency hoarding.

already suggested by Reyns (2024), it was not an economic rationale to motivate businesses to join Circles, but their alignment in shared values. A similar conclusion was reached by another smaller qualitative study (Papadimitropoulos and Perperidis, 2024), where the perception of ”money as a commons” was found to be fundamental for people to join Circles. Based on thirteen interviewed people involved in Circles Coop, the authors pointed out some criticisms in the governance model as a major weakness. For example, it is reported that the project failed in actively creating *closed-loops* in the business network. On the other hand, there was strong alignment in shared values that allowed the system to attract a consistent amount of members and businesses.

Following the same techniques in Avanzo et al. (2023), in this work the Sarafu and Circles networks are split into three periods according to structural changes identified using the *causal fidelity index* (see Chapter 3). Each time period is then analysed separately by temporally aggregating the graph and studying its topology. The main goal is to observe and compare the topological changes, in accordance with some events that signed the history of the projects. In addition, a further goal is to observe how recirculation and *circular network synergy* changed over time. Qualitative studies about Circles network (Longo et al., 2024; Papadimitropoulos and Perperidis, 2024) and Sarafu network (Kiaka et al., 2024) are used to interpret and give context to the quantitative analysis presented in this paper.

Contribution At the beginning of this chapter, a study was reported that informed the reader about the lack of quantitative impact assessment techniques for CCSs (Michel and Hudon, 2015). More recent studies rely on the estimation of a *local multiplier effect*, but some limitations have been already pointed out (see also Chapter 4 on ’Limits of the Economic Multiplier’). Although these techniques are trying to measure a network effect, they fail in fully accounting, describing, and explaining the mechanisms behind. Similarly, the phenomenon of the *synergy effect* in economics was not adequately explored in a quantitative way. Some of the mathematical models exploring *synergy* relies on entropy and mutual information. In this work a *circular network synergy* index is suggested as a possible alternative. This metric is not only backed by economic theory, but is also explicitly implied in the study of payment systems. Indeed, the literature about ’cycles’ in payment systems and in economic theory was reported.

This chapter concluded with a brief review of the literature on other network methods used in the study of community currency networks. A special room was given to studies on Sarafu and Circles networks, which are the two case studies presented in this work.

In conclusion, the main contributions of this work can be summarised as follows. This work tries to cover a gap in the literature by providing quantitative analytical tools which can be used to assess the circulation of a digital community currency system. First, the role of cyclic structures is analysed by applying a novel topological categorisation for directed networks. Only a few recent studies focused on the analysis of the role of cyclic structures in transaction networks (Iosifidis et al., 2018; Mattsson et al., 2023), but limited to cycles of length 2, 3, 4, and 5. The author has already developed and tested this technique in a previous work (Criscione, 2024) which is based on the distinction between cyclic and acyclic components. Second, the role of recirculation is analysed between different components and time periods. In fact, the networks are divided into three periods each, according to the structural changes identified quantitatively using the *causal fidelity* index (Avanzo et al., 2023; Lentz et al., 2013) and qualitatively based on previous studies (see Chapter 3 for more details). Third, a novel measure of local circulation is introduced, called *circular network synergy*. As explained, this metric is deeply connected to other existing economic phenomena, namely the *local multiplier effect* and the *synergy effect*.

This work is focused on studying the circulation in two economic networks, the Sarafu and Circles community currency networks. For this reason, differently from previous works (Ba et al., 2022, 2023a, 2023b; Mattsson, Luedtke, and Takes, 2022; Mattsson et al., 2023), savings groups are excluded from the Sarafu network. This can help us identify the network effects behind the *local multiplier effect* and the economic *synergy effect* without the bias introduced by financial operations, i.e. borrowing and lending money. In fact, abstract monetary exchange for financial purposes does not reveal any information about the flow of real goods and services, and therefore about real and actual *economic synergies* happening in the system.

3 Data

3.1 Sarafu Network

The information in this section on the history of Sarafu and other technical details is taken from Mattsson, Criscione, and Ruddick (2022). The data used in this work are timestamped transactions from the Sarafu system in Kenya collected between 25 January 2020 and 15 June 2021 (Ruddick, 2021). The data also include some user data: geographical location, business sector, and gender. Grassroots Economics (GE) in collaboration with the Kenyan Red Cross (Mattsson, Criscione, and Ruddick, 2022) designed the currency as part of a COVID-19 disaster response intervention in the Mukuru kwa Njenga slum, Nairobi, and in Kisauni, Mombasa. In such humanitarian interventions, the Sarafu token was used as a cash transfer programme. In practice, new users could receive an initial payment after their registration: 400 Sarafu (from January until May 2020), 50 Sarafu (from May until June 2021) (Mattsson, Criscione, and Ruddick, 2022). In parity with the Kenyan Shillings, the Sarafu circulated locally among businesses, groups, and individuals.

It is also important to mention that since 2017 the Kinango area (Kwale county) has been targeted by Grassroots Economics for specific development interventions: donations have been collected to build community-owned assets with the purpose of enhancing community socioeconomic resilience (e.g., maize milling, refrigeration, water storage equipment, etc.)(Mattsson, Criscione, and Ruddick, 2022). In the data, Kinango, Mukuru kwa Njenga slum, and Kisauni are the most active geographical areas. About 86% of the users come from one of those treated areas, so any comparison with the untreated areas would be unbalanced.

In the Sarafu token network, a key role is played by savings groups, also called *Chamas* (Ba et al., 2022, 2023b; Mattsson, Criscione, and Ruddick, 2022; Mattsson et al., 2023). Local savings groups are very well known in Kenya for their old tradition (Anderson et al., 2009; Barinaga, 2020). *Chamas* are Rotating Savings and Credit Associations (ROSCAs), which are structured as informal cooperative groups where participants pool, invest, and lend their savings among each other. In the data, they are identifiable as *group accounts* (savings groups verified by GE) or *savings* business accounts (savings groups not verified by GE). From January until

July 2020, some donors backed the initial fund, so the cash-out in Kenyan shillings was limited to some users and vendors through savings groups, under certain constraints. In particular, savings groups were allowed to exchange back Kenyan shillings up to 30,000 Sarafu per month. However, between August and December 2020, savings groups could indicate a vendor from which to buy food. Subsequently, local vendors indicated by the *Chamas* could cash out their Sarafu to get Kenyan shillings back from GE. After December 2020, the currency exchange system stopped, and only in-kind donations kept going.

The main focus of this paper is to study the economic behaviour of Sarafu users. For this reason, the financial network of savings groups (or *Chamas*), admin operations, and vendors were removed from the dataset. The global network of standard operations has 40 767 nodes, 422 721 transactions, 296 991 019.65 in volume, 592 weakly connected components, and 619 strongly connected components. By excluding admin accounts, vendors, and savings group accounts, there is left a subnetwork with 39 355 nodes, 355 070 transactions (132 420 directed links), 175 704 135.68 of volume (in parity with Kenyan Shillings), 696 weakly connected components and 619 strongly connected components. In this subnetwork of individual users, 114 417 690.48 Sarafu were disbursed mainly to new members and to reward existing members. The users are grouped into 10 main geographic areas: Kilifi, Kinango (Kwale County), Kisauni (Mombasa County), Mombasa, Nairobi, Rural Counties, Mukuru (Nairobi County), Nyanza County, Turkana County, and other/unknown.

As described in a previous work (Mattsson et al., 2023), the degree distributions of this network are heavy-tailed. The degree distributions are built by aggregating all the transactions that happen between each pair of nodes and preserving their directionality. The in-degree and out-degree distributions can be well approximated by power laws, respectively, with exponents 1.53 and 1.47 (see Table 1). Similarly, the distribution of the number of transactions per link also behaves as a power law with an exponent of 1.44. This means that a very high number of links have one or few transactions happening on them, while a few links are responsible for a very high number of transactions. The distribution of volume per link (that is, total amount transferred) is also well approximated by a power-law distribution with exponent 1.85. The Pearson correlation between the distribution of the number of transactions and the total weight per edge is equal to 0.58 significant at 1% (p-value<0.01).

	Out-Degree		In-Degree		Transactions		Volume	
	LLR	p-value	LLR	p-value	LLR	p-value	LLR	p-value
lognormal	-1.33	0.18	-1.5	0.13	-0.25	0.8	-1.82	0.06
exponential	6.19	∼0.0	6.67	∼0.0	5.25	∼0.0	7.44	∼0.0
truncated power law	-1.29	0.26	-1.08	0.45	-1.29	0.26	-1.27	0.38
stretched exponential	1.2	0.22	2.34	0.018	0.39	0.69	2.31	0.02
lognormal positive	2.5	0.012	3.68	∼0.0	1.28	0.19	3.42	∼0.0

Table 1: Comparison of power-law distribution with other well-known distributions in Sarafu network. LLR is the log-likelihood ratio. The LLR is positive when it is more likely to be approximated by a power-law (first distribution in the ratio in this case), and negative otherwise. The p-value is the significance test of the result. Notice that when the LLR is negative, the p-value is too high, and therefore, the LLR is not significant. This means that the power-law distribution better represents the real data.

The temporal accessibility analysis is used to measure the ratio of shortest temporal paths over static ones, also called *causal fidelity* (Lentz et al., 2013). Following a similar procedure explained in Avanzo et al. (2023), the Sarafu network is split into three periods (see Figure 8). The first structural change is identified around 6 August 2020, right after the policy change for savings groups to cash-out was introduced. The second structural change is identified around 31 January 2021, right after the stop on the currency exchange system through vendors was introduced. These two dates identify three main periods for which the network can be split and temporally aggregated. The causal fidelity of the entire graph is only 14.3%. The identification of the two points in time is carried out following the instructions of previous works (Avanzo et al., 2023): after the first peak and dip, the first local maximum is considered (with causal fidelity 0.129); after the plateau, the last local maximum is considered before a permanent change in the trend (with causal fidelity 0.141). A network per each period is therefore identified:

- **Period 1.** 22 022 nodes, 171 223 transactions, 71 792 directed links, an exchanged volume of 83 583 846.76 Sarafu, and 16 067 969.07 Sarafu disbursed

- **Period 2.** 14 687 nodes, 122 031 transactions, 46 931 directed links, an exchanged volume of 82 030 707.46 Sarafu, and 7 060 433.27 Sarafu disbursed

- **Period 3.** 11 692 nodes, 61 816 transactions, 28 150 directed links, an exchanged volume of 10 089 581.45 Sarafu, and 1 653 613.64 Sarafu disbursed

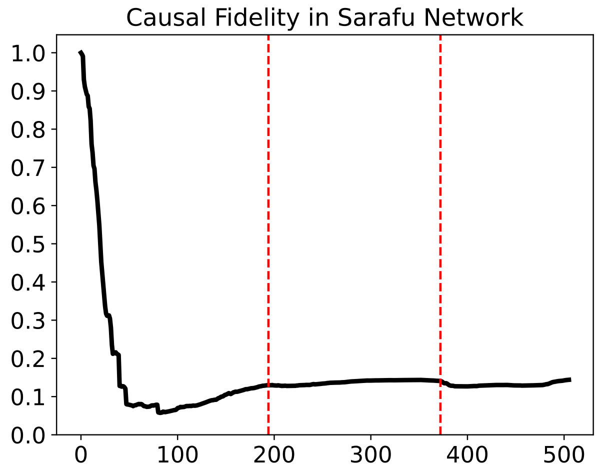


Figure 8: Causal fidelity in Sarafu network. Each time-step on the x-axis is equal to one day.

3.2 Circles UBI Network

The Circles UBI data used in this work are time-stamped transactions from 16 October 2020 to 14 December 2023. The blockchain technology used to store transaction data is a worldwide distributed database. However, the data used here refer only to the Berlin pilot. Only transaction data are used, but some additional information about 19 business partners in Berlin is available. In fact, only the Largest Weakly Connected Component (LWCC) is considered in the analysis, which includes all business partners in Berlin. The global network has 15 063 nodes, 80 824 transactions, 5 523 498.71 in volume, 1 454 weakly connected components, and 146 strongly connected components. The LWCC, which includes business partners in Berlin, has 8 598 nodes, 70 696 transactions, 4 686 294.87 of volume, and 146 strongly connected components. In LWCC the amount of Circles distributed as universal basic income was equal to 129 936 579.25.

A new user willing to be registered on the system must be endorsed by at least 3 other accounts (CirclesCoop, 2021). All endorsements in the system span a *Web of Trust*, which is a network of *trust* connections among members. The *Web of Trust* is excluded from the analysis of this work, but it is essential to understand the back-end functioning of the system. In fact, two users can exchange Circles only if they are connected to the *Web of Trust*. In the back-end, each account technically mints its own *personal* currency, which is automatically assigned after registration using Circles DApp [8]. In the front-end, the *personal* currencies are exchanged as Circles units (CRCs). The system is based on the Gnosis blockchain for the implementation of smart contracts [9].

Two types of accounts can be created on the Circles UBI network (Longo et al., 2024): *Individual accounts* and *Shared accounts*. *Individual accounts* are those that receive monthly UBI payments, which are designed to be matched to a single verified identity. *Shared accounts* do not receive the UBI, they are created by single individuals, but other can be connected to it. The *Shared accounts* are the most used by companies. However, this information is not present in the available data, except for the 19 business partners.

The number of business partners oscillated throughout the period (Longo et al., 2024). The business partners received a subsidy from July 2021 to September 2023 (Avanzo et al., 2023). The subsidy allowed them to cash out back in Euros on a monthly basis, under some constraints. The business partners belong to six main sectors (Avanzo et al., 2023): *Food and Beverage*, *Art*, *Care/Health*, *Consulting*, *Bikes*, *Books*.

From October 2020 until May 2022, the Circles system issued 8 CRCs per day per participant (Longo et al., 2024). After May 2022, the daily rate was increased to 24 CRC per day and a yearly *demurrage* (i.e., negative interest rate) policy was introduced. A 7% yearly rate of *demurrage* was meant to discourage hoarding and inactivity (CirclesCoop, 2022). In this period, also the cash out for subsidised business changed exchange rate from 1:1 to 1:10 (i.e., 10 CRCs for 1 Euro).

Also in the Circles network, the degree distributions are well-approximated by power-law distributions of exponents 1.62 (in-degree) and 1.55 (out-degree) (see Table 2). The distribution of the number of transactions per each link can also be fit by a power-law distribution with an exponent equal to 1.44. Similarly, the distribution of volume per link (that is, the total amount transferred) can be approximated by a power-law distribution with an exponent equal to 1.85.

[8] https://circles.garden/
[9] https://www.gnosis.io/

The Pearson correlation between the distributions of transactions and the volume of the links is equal to 0.4 (p-value<0.1).

	In-Degree		Out-Degree		Transactions		Volume	
	LLR	p-value	LLR	p-value	LLR	p-value	LLR	p-value
lognormal	-1.49	0.13	-1.04	0.29	-0.23	0.81	-1.75	0.078
exponential	7.36	∼0.0	6.57	∼0.0	4.72	∼0.0	6.35	∼0.0
truncated power law	-0.72	0.64	-0.97	0.39	-1.08	0.31	-0.96	0.54
stretched exponential	2.64	0.008	1.74	0.08	0.14	0.88	1.85	0.06
lognormal positive	3.69	∼0.0	2.86	0.004	0.78	0.43	2.77	0.005

Table 2: Comparison of power-law distribution with other well-known distributions in Circles network. LLR is the log-likelihood ratio. The LLR is positive when it is more likely to be approximated by a power-law (first distribution in the ratio in this case), and negative otherwise. The p-value is the significance test of the result. Notice that when the LLR is negative, the p-value is too high, and therefore, the LLR is not significant. This means that the power-law distribution better represents the real data.

Similarly to Sarafu, the temporal accessibility analysis is used here also to measure the ratio of the shortest temporal paths to static ones, also called *causal fidelity* (Lentz et al., 2013). Following a similar procedure explained in Avanzo et al. (2023), the Circles network is divided into three periods (see Figure 9). The first structural change is identified around 18 November 2021, four months after the beginning of the subsidy programme. The second structural change is identified around 29 June 2023, right after the announcement of the end of the subsidy programme (which took place effectively at the end of September 2023) (Longo et al., 2024). These two dates identify three main periods for which the network can be split and temporally aggregated. The causal fidelity of the entire graph is only 10.7%.

As before, the identification of the two points in time is carried out following the instructions of previous works (Avanzo et al., 2023). However, following the findings in Avanzo et al. (2023), the strategy slightly changed. After the first peak and dip, the causal fidelity slowly drops. This is probably due to attempt of Sybil attacks, namely fake accounts trying to hoard Circles in the first period. For this reason, the first local maximum is considered only later (with causal fidelity 0.129); after the plateau, the last local maximum is considered before a permanent change in trend (with causal fidelity 0.137)(see Figure 9). A network per each period is therefore identified:

- **Period 1.** 4074 nodes, 18 534 transactions, 8 943 directed links, an exchanged volume of 565 283.7 Circles, 4 415 449.7 Circles disbursed
- **Period 2.** 2 510 nodes, 15 884 transactions, 5 611 directed links, an exchanged volume of 2 444 959.09 Circles, 3 988 687.81 Circles disbursed
- **Period 3.** 3 622 nodes, 36 278 transactions, 17 590 directed links, an exchanged volume of 1 676 052.06 Circles, 2 302 230.16 Circles disbursed

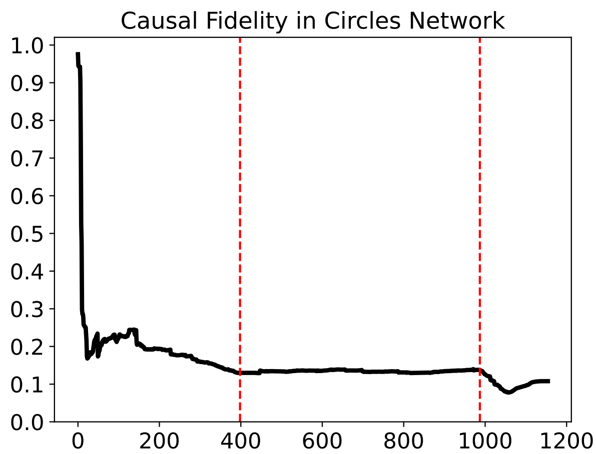


Figure 9: Causal fidelity in Circles network. Each time-step on the x-axis is equal to one day.

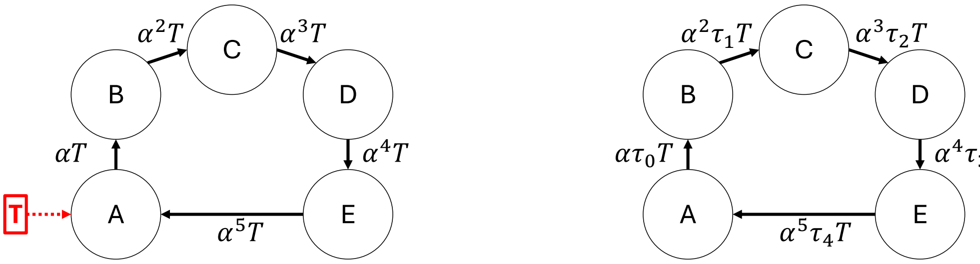
4 Methods

4.1 Limits of the Economic Multiplier

In this section, an analytical explanation of the limits of the *multiplier effect* is used to justify the main contributions of this work. Two economies, a directed cycle graph and a directed line graph, are presented as the simplest examples of cyclic and acyclic components. An injection of liquidity in both graphs is analytically represented. It is shown that in a directed cycle the volume of transactions depends on the number of nodes, their savings, and the number of times the currency recirculates throughout the graph. In the case of a directed line graph, this is not the case because it depends only on the number of nodes, while the rest of the liquidity is kept outside of the system and/or only by the last node in the chain. In simple words, in the short term, the two economies with the same number of nodes could show a similar volume, but in the long term the injection in a line graph economy fade out faster. This is crucial to understand the role of cycles in an economic network and to explain the meaning of autocatalytic growth. The usual way to measure the *multiplier effect* can be partially blinded to such network effects, and therefore it cannot tell much about the network 'quality' of economic growth induced by topological changes. This should convince the reader about the distinction of cyclic and acyclic components explained in the next section.

Consider first a directed graph $G = (N, E)$ which is structured as a directed cycle. It is assumed that every node moves sequentially, so that at each time step every node is spending α of its endowment. This implies that $1 - \alpha$ of its endowment is saved and the node spends it only if a predecessor sends some currency to it again. It is also assumed that $0 < \alpha < 1$. The sequential movement of actions is a useful simplification. Policymakers are interested in the recirculation that occurs after a specific amount of liquidity T is injected through monetary or fiscal interventions. In this simple model, it is assumed that there are only five nodes and only the first node A receives the initial injected liquidity T.

In the first round (a), the volume of each transaction sequentially decreases by a factor α (Figure 10(a)). In the second round, each node already has some savings from the previous iteration. Every node spends α of its endowment. In Figure 10(b), this is indicated by the



(a) First Round (b) Second Round

Figure 10: On the first round (a), the volume of each transaction sequentially decreases by a factor α. From the second round (b) on, this quantity is multiplied by τ factors which essentially depend on savings (from the previous node and the previous round).

factors τ. The analytical form of τ factors is the following.

$$\begin{aligned}
\tau_0 &= \alpha_5 + (1-\alpha) \\
\tau_1 &= \tau_0 + (1-\alpha) \\
\tau_2 &= \tau_1 + (1-\alpha) \\
\tau_3 &= \tau_2 + (1-\alpha) \\
\tau_4 &= \tau_3 + (1-\alpha)
\end{aligned} \tag{19}$$

Iterating this s rounds[10], the cumulative transaction volume $\hat{W}$ of a directed cycle graph with N nodes is given by the following formula.

$$\begin{aligned}
\hat{W} &= \sum_{i=1}^{s} W_i \\
&= \left(\alpha \frac{1-\alpha^{Ns}}{1-\alpha} + \sum_{t=2}^{s} (1-\alpha)^{(t-1)} \sum_{i=t}^{s} \alpha^{N(i-t)} \sum_{j=1}^{N} \alpha_j \prod_{k=2}^{t} \frac{N(i-t)+j+k-2}{k-1} \right) T
\end{aligned} \tag{20}$$

Where W_i is the transaction volume in round i, s is the number of rounds, T is the initial injection. Note that the last addend in Equation 20 depends on savings in an iterative way, and, in particular, the last multiplication factor follows Pascal's Triangle coefficients. The first addendum is the result of a partial sum of elements strictly less than 1, since it is assumed $\alpha < 1$. Note that this addend depends on the number of nodes N and rounds s.

[10] This means that the cycle is repeating s times. In other words, the currency flows s times through all nodes.

This result can be compared with a directed line graph with N nodes. There is no recirculation of currency, so savings are taken out of the system at each step, from every node. The result can be expressed by the following equation.

$$\begin{aligned}\hat{W} &= \sum_{i=1}^{s} W_i \\ &= \left[\alpha\left(1+\alpha^2+\alpha^3+\alpha^4+\ldots+\alpha^N\right)\right]T \quad = \left(\alpha\sum_{i=0}^{N-1}\alpha^i\right)T = \alpha\frac{1-\alpha^N}{1-\alpha}T\end{aligned} \tag{21}$$

where the last step is possible only because it is assumed that $\alpha < 1$, and therefore, the sum is converging. Note that in this case, the partial sum depends only on the number of nodes N.

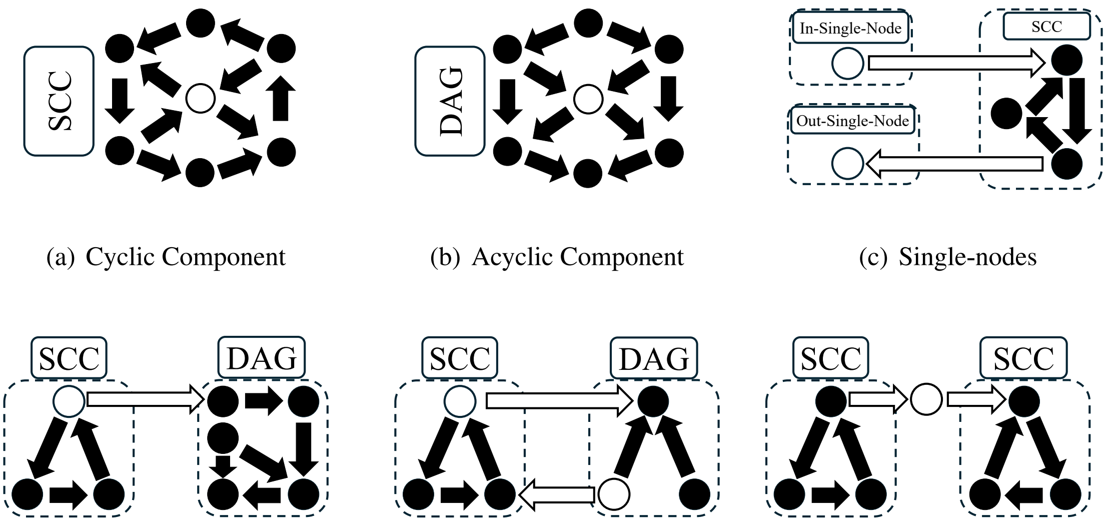
In conclusion, measuring the multiplier as the ratio of transaction volume to injected liquidity is a very limited way of assessing a policy intervention. See also Appendix 7.3, for a numerical example. In fact, the simple ratio does not take into account the topology of the graph that is growing underneath. This section shows that a cyclic component (i.e. a directed cycle graph in the example) and an acyclic component (i.e. a line graph in the example) can imply entirely different dynamics. This is especially important for long-term policy interventions. In fact, as shown before, in an acyclic graph the effect of the intervention slowly dies out with the number of nodes. However, in a cyclic graph, the role of the number of nodes, their savings, and the number of cyclic recirculations plays a key role in the long run. In the next section, the topological categorisation is presented. The topological categorisation is based on the assumption that the distinction between cyclic and acyclic components is extremely important for the study of economic networks, as has been shown here in this simple analytical model.

4.2 Topological Components

This section is taken entirely from Criscione (2024). In this work, the topological categorisation of cyclic components, acyclic components, and single-nodes is used for behavioural investigations on the economic network. Previous studies have already analysed the inner structure of directed networks (Broder et al., 2000; Donato et al., 2008; Dorogovtsev et al., 2001; Timár et al., 2017). According to the existing literature, directed networks are generally characterised by a *bow-tie* structure with a largest strongly connected component (*SCC*) as the core where nodes are mutually reachable from each other, a set of nodes which are only sending to the *SCC*, called *IN*-component, and a set of nodes which are only receiving from the *SCC*, called *OUT*-component. Attached to the *IN*- and *OUT*-components there are *tendrils* which can be either sets of nodes that a) can be reached only from the *IN*-component or b) sets of nodes which do not belong to the *SCC* but can reach nodes of the *OUT*-component. Other groups of nodes, called *tubes*, connect the *IN*-component with the *OUT*-component without passing through the *SCC*. Finally, isolated groups of nodes are just described as *disconnected* components. The *bow-tie* description was successfully applied to study the structure of directed graphs. In particular, it can be used to study some of their properties, such as their expected size, degree distributions, and resilience to random failures and targeted attacks (Dorogovtsev et al., 2001; Newman et al., 2001). A subsequent work on the web graph reviewed the *bow-tie* structure suggesting a *daisy* shape, where the *IN*- and *OUT*- components are highly fragmented into many *petals*, which are chains of nodes connected with the same component of origin (Donato et al., 2008). Finally, a more advanced version of the *bow-tie* (Timár et al., 2017) was recently introduced where *tendrils* and *tubes* are categorised based on their distance from the *IN*- and *OUT*-components. The results of these works also have important implications for the study of the resilience of directed networks.

In this work, the inner structure of a directed network is studied as well but from a completely different perspective. Contrary to the *bow-tie* description, the *core*, *IN*- and *OUT*- components, and relative connections are not considered in this work. Instead, the differentiation between cyclic and acyclic components is the key aspect considered here. A cyclic component is a portion of the network structured as a strongly connected component (SCC), whereas an acyclic component is a portion of the network structured as a directed acyclic graph (DAG). To understand the difference between cyclic and acyclic components consider the Figures 11(a) and 11(b). In Figure 11(b), the white node in the centre of a DAG can move and leave its position but cannot return to it following the direction of the arrows. On the other hand, in Figure 11(a), the white node in the centre of a SCC can move and leave its position and return to it by following at least 4 different directed *paths* which do not cross the same node twice. These directed paths identify directed simple *cycles*, because they start and end at the same white node position. For this reason, in this work, the cyclic component and the strongly connected component (SCC) are used as synonyms. Similarly, the acyclic component and directed acyclic graph (DAG) are also used as synonyms. Because only cyclic and acyclic components are identified, their connectivity with the rest of the network characterises the categorisation procedure adopted in this paper. In Table 3, the directed network is split into 14 different components: 11 including nodes and edges, 3 including only edges (see Table 3 for definitions). This is a comprehensive categorisation which uniquely assigns each node and edge into one and only one of those categories, and therefore, one and only one related network component.

The logic of this categorisation is explained in Figure 11 and Figure 12. In Figure 11, the objects and the relations among them are defined. The objects can be either a cyclic component (SCC, $\circlearrowright$)(Figure 11(a)), an acyclic component (DAG, $\langle\updownarrow\rangle$)(Figure 11(b)), or a single-node ($\bigcirc$)(Figure 11(c)). Every pair of objects can have three types of relation with each other. Please note that each relation considered here between each pair of objects cannot change the nature of the object itself. For instance, a cyclic component cannot become an acyclic component by adding any of the relations considered between them. The first type of relation considered here is a directed link ($\longrightarrow$) from one object to another (Figure 11(d)). A directed link corresponds to one or more transactions (flow of currency) from one object to another following the direction of that link. The second type of relation ($\leftrightarrows$) is a connection between two objects which includes links in the opposite direction, but without creating a cycle which involves nodes from both objects. This type of relation is illustrated in Figure 11(e), where we could imagine that the white nodes can move and jump into the opposite component following their white arrows, but we also notice that they eventually cannot return to their original position following the direction of the arrows. Obviously, this also implies that any two nodes from two different components cannot exchange back and forth; otherwise they would create a cycle of length 2. Finally, a third type of relation is a connection between two components that involves one node only ($\hookrightarrow$). In other words, one node is receiving from one component and sending to another component (see Figure 11(f)). For the last case, note the difference between Figure 11(c), Figure 11(d), and Figure 11(f). In Figure 11(d), one node connects a SCC to a DAG (that is, a chain of nodes is part of the DAG); in Figure 11(f) one node receives from one SCC and sends to another; finally, in Figure 11(c) a node is sending or receiving from a SCC. Obviously, a single-node sending to or receiving from another node in a DAG is part of that DAG itself. Similarly, a single-node sending or receiving from another single-node is a pair which constitutes a DAG. The categorisation procedure is explained in further detail in Figure 12.



(a) Cyclic Component (b) Acyclic Component (c) Single-nodes

(d) Directed link (⟶). (e) Double-link (⇆). (f) Link through one node (↬).

Figure 11: Objects and relations for the topological categorization. In Figures (a), (b), and (c), the considered objects are represented. In Figures (d), (e), and (f), the considered relations are represented. Figure from Criscione (2024).

In the previous paragraph, objects and relations for the topological categorisation were introduced. In this paragraph, each object is categorised based on its relationship to another object. In Figure 12, the logic of this categorisation is explained by showing 8 simple cases, where there are only two objects in relation to each other. First, four types of cyclic components are defined: *sccTin*, *sccTout*, *sccTmix*, and *scc0*. While *scc0* is simply a strongly connected component only connected to other SCCs or isolated, the other categories can emerge in one of the cases represented in Figures 12(a) and 12(b). A *sccTout* component emerges if a SCC sends to a DAG or an out-single-node (N.1 and N.2, Figure 12(a)). A *sccTin* component emerges if a SCC receives from a DAG or a in-single-node (N.3 and N.4, Figure 12(a)). A *sccTmix* component emerges if a SCC sends to and receives from a DAG, an in-single-node, and / or an out-single-node (N.5 and N.6, Figure 12(b)). Note that different types of SCCs can still be connected to each other without losing their identity. In fact, their identity is only defined by their connections with DAGs, in-single-nodes, and out-single-nodes.

Secondly, the typology for the acyclic components is also categorised as: *dagTin*, *dagTout*, *dagTmix*, and *dag0*. In this case, *dag0* is an isolated acyclic component, while the other categories can also emerge in one of the cases represented in Figure 12(a) and 12(b). A *dagTout* component emerges if a DAG receives from a SCC (N.1, Figure 12(a)). A *dagTin* component emerges if a DAG sends to a SCC (N.3, Figure 12(a)). A *dagTmix* component emerges if a DAG sends to and receives from a SCC (N.5, Figure 12(b)), obviously without creating a cycle with it. Unlike SCCs, when DAGs of different types connect to each other, they change their identity. For example, if a *dagTin* gets a connection to a *dagTout*, the resulting DAG would be a *dagTmix*.

The edges between SCC and DAG are considered as a separate category. From a cyclic component to an acyclic component (*edge_scc2dag* in N.1 and N.5 in Figure 12(a) and 12(b)) and from an acyclic component to a cyclic component (*edge_dag2scc* in in N.3 and N.5 in Figure 12(a) and 12(b)). Similarly, the edges between different SCCs (*edge_scc2scc*) are also considered separately (N.8 in Figure 12(c)).

The only special case left to be discussed is single-nodes, which are nodes that cannot really be associated to cyclic or acyclic components. Splitting a directed network only into cyclic and acyclic components (and link between them) is not sufficient to identify a comprehensive and unique categorisation for each node and edge in the network. Indeed, the inclusion of single-nodes complete its description. A single-node receiving from one SCC and sending to another SCC is called *bridge_scc* (N.7 in Figure 12 and in Figure 11(f)). A single-node only sending to SCCs is called *in-single-node* (N.4 in Figure 12 and Figure 11(c)). A single-node only receiving from SCCs is called *out-single-node* (N.2 in Figure 12 and Figure 11(c)). Any connection to a DAG will categorize the single-node as a part of that DAG itself. A single-node connected to another single-node will obviously create a DAG. In Table 3, a description of these categories is reported. Since each category corresponds to a uniquely identified network component, the sum of the volume of each component corresponds to the total volume of the network. For this reason, we conclude that this topological categorization completely and successfully describe the whole network under examination. Nevertheless, we do not exclude that future studies may find a better way to improve this technique and adapt it to different contexts and purpose.

Node	**Edge**	**Definition**
sccTmix	=	SCC sending to and receiving from a DAG, an in-single-node, and / or an out-single-node.
in-single-node	=	Single-node sending to a SCC.
dagTin	=	DAG sending to a SCC.
dag0	=	Isolated DAG.
out-single-node	=	Single-node receiving from a SCC.
scc0	=	SCC not connected neither with a DAG, an in-single-node, nor an out-single-node.
sccTin	=	SCC receiving from a DAG or an in-single-node.
dagTmix	=	DAG sending to and receiving from a SCC.
dagTout	=	DAG receiving from a SCC.
sccTout	=	SCC sending to a DAG or an out-single-node.
bridge_scc	=	Single-node connecting two or more SCCs.
–	edge_dag2scc	Link from a DAG to a SCC.
–	edge_scc2dag	Link from a SCC to a DAG.
–	edge_scc2scc	Link connecting two SCCs.

Table 3: Description of topological categories for edges and nodes. *SCC* is used as abbreviation for strongly connected component. *DAG* is used as abbreviation of directed acyclic component. Note that *edge_dag2scc*, *edge_scc2dag*, and *edge_scc2scc* do not have a corresponding node because their nodes are already assigned to different components. For example, one *edge_scc2dag* is made of one sender in a *SCC* and one receiver in a DAG. Similarly, *bridge_scc* is only one single-node receiving from one SCC and sending to another SCC. However, if there is a chain of nodes where the first node is receiving from a SCC and sending to another SCC through its last node, this is considered as *dagTmix*. Any connection between different types of SCCs does not change their nature. Any connection between different types of DAG does change their nature. Table from Criscione (2024).

4.3 Recirculation

This section is taken in part from Criscione (2024). In this work, the characterisation of the dynamics of the network is carried out using the recirculation time. The velocity of circulation was used in a recent work to describe the circulation of Sarafu using the same data (Mattsson, Luedtke, and Takes, 2022). In that work, the authors were interested in the time between one incoming operation and the first outgoing operation ("first-in, first-out" in Mattsson and Takes, 2021), and then averaging this quantity to define the "holding time" per each user. The authors

Symbol	SCC	DAG	Single-Node	Edge
N.1 ↻→⟨↕⟩	sccTout	dagTout		edge_scc2 dag
N.2 ↻→◯	sccTout		out-single-node	
N.3 ↻←⟨↕⟩	sccTin	dagTin		edge_dag2 scc
N.4 ↻←◯	sccTin		in-single-node	

(a)

Symbol	SCC	DAG	Single-Node	Edge
N.5 ↻⇆⟨↕⟩	sccTmix	dagTmix ***OR*** dagTout & dagTin ***OR*** dagTmix & any other DAG		edge_scc2 dag & edge_dag 2scc
N.6 ↻⇆◯	sccTmix		in-single-node & out-single-node	

(b)

Symbol	SCC	DAG	Single-Node	Edge
N.7 ↻⟜↻			bridge_scc	
N.8 ↻→↻				edge_scc2 scc

(c)

- ↻ Strongly Connected Component (SCC), Cyclic Component
- ⟨↕⟩ Directed Acyclic Graph (DAG), Acyclic Component
- ◯ Single-node
- → One directed link from one object to another
- ⇆ One directed link per each direction, but without creating a cycle involving nodes from both objects
- ⟜ Linked through one node

(d) Legend

Figure 12: Label per each topological component. The logic of this categorisation procedure is explained by showing 8 cases in Figures (a)-(c). In each case, there are only two objects in relation to each other. The objects can be either a cyclic component (SCC, ↻), an acyclic component (DAG, ⟨↕⟩), or a single-node (◯). Each pair of objects can have three types of relation exclusively. The first type of relation is a directed link (⟶) from one object to another. The second type of relation (⇆) is a connection which includes links in opposite direction between two objects (without creating a cycle). The third type of relation is a connection between two objects which involves one node only, which is receiving from one component and sending to another one (⟜). In reality, there is often a combination of these 8 cases represented above. For example, a strongly connected component can receive from a DAG and send to a single-node, and therefore, being identified as a *sccTmix*. Finally, consider also that a *bridge_scc* is a node receiving from a SCC and sending to another SCC, a behaviour which is described by the relation ⟜. As already mentioned, it is important to point out that a single-node sending to or receiving from another node in a DAG is part of that DAG itself, and one single-node sending or receiving from another single-node is a pair which constitutes a DAG. Figure from Criscione (2024).

then analyse the ”holding time” in relation to business sectors and geographic areas. As described in Figure 13, the recirculation time here is measured as the time difference between the first of all incoming operations and the last of all outgoing operations before another incoming operation arrives. Moreover, the recirculation time is not aggregated (or averaged) per user. In fact, each individual is assigned to one or more temporal categories, depending on the speed of their recirculation operations. Instead of focussing on circulation in business sectors and geographic areas, the recirculation time is then analysed in relation to predefined topological categories, as described in Section 4.2. The main intent is to observe how the recirculation speed changes in the network, revealing patterns between structure (or topology) and dynamics in the network. This temporal categorisation for recirculation operations can be used to analyse how recirculation itself changed over time (Research Question N.2, RQ2).

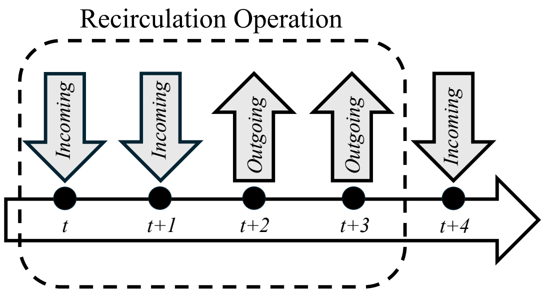


Figure 13: Illustration of a recirculation operation as defined in this work. One recirculation operation includes many incoming and outgoing operations. The node in the figure receives two incoming transactions at time *t* and *t+1*. After that, it sends currency twice at *t+2* and *t+3*. At *t+4*, it receives again some currency, so the recirculation operation closes.

The frequency (or speed) of recirculation is therefore defined as the time difference between the first incoming operation (at time *t* in Figure 13) and the last outgoing operation (at time *t+3* in Figure 13). In the example of Figure 13 the speed is equal to 3. This value is then used to categorise the recirculating operations from low to high frequency. In Table 4, the recirculation operations in Sarafu are categorised according to their speed in each period. Similarly, in Table 5, the categories of recirculation for Circles are reported. In the appendix, the distributions of the recirculation speed for Sarafu (Figure 52) and Circles (Figure 56) are reported. The distribution of recirculation speed per each operation is first ordered in ascending order, then its quartiles are detected. The acronym *HF* stands for *high frequency*. The acronym *LF* stands for *low frequency*. The suffices *Q1*, *Q2*, and *Q3* indicate the inter-quartile ranges considered in each category. Q1 for operations in the first quarter of the distribution (inter-quartile range 0-25%). Q2 for operations in the second quarter (interquartile range 25-50%). *High frequency* Q3 (HFQ3) for operations in the third quarter (interquartile range 50- 75%). And finally, *low frequency* Q3 (LFQ3) for operations in the last quarter of the distribution (interquartile range > 75%).

Period 1			
Freq.	**From**	**To**	**Inter-Quartile Range**
HFQ1	∼1 second	9 minutes, 15 second	0-25%
HFQ2	9 minutes, 15 seconds	3 hours, 21 minutes	25-50%
HFQ3	3 hours, 21 minutes	23 hours, 58 minutes	50-75%
LFQ3	23 hours, 58 minutes	23 weeks, 6 days	>75%
Period 2			
Freq.	**From**	**To**	**Inter-Quartile Range**
HFQ1	∼1 second	1 hour, 10 minutes	0-25%
HFQ2	1 hour, 10 minutes	15 hours, 48 minutes	25-50%
HFQ3	15 hours, 48 minutes	2 days, 6 hours	50-75%
LFQ3	2 days, 6 hours	24 weeks, 3 days	>75%
Period 3			
Freq.	**From**	**To**	**Inter-Quartile Range**
HFQ1	∼1 second	27 minutes, 3 seconds	0-25%
HFQ2	27 minutes, 3 seconds	19 hours, 41 minutes	25-50%
HFQ3	19 hours, 41 minutes	2 days, 23 hours	50-75%
LFQ3	2 days, 23 hours	17 weeks, 4 days	>75%

Table 4: Speed of recirculation in Sarafu.

Period 1			
Freq.	**From**	**To**	**Inter-Quartile Range**
HFQ1	∼1 second	3 hours, 31 minutes	0-25%
HFQ2	3 hours, 31 minutes	22 hours, 57 minutes	25-50%
HFQ3	22 hours, 57 minutes	5 days, 4 hours	50-75%
LFQ3	5 days, 4 hours	54 weeks, 2 days	>75%
Period 2			
Freq.	**From**	**To**	**Inter-Quartile Range**
HFQ1	∼1 second	6 hours, 13 minutes	0-25%
HFQ2	6 hours, 13 minutes	3 days, 12 hours	25-50%
HFQ3	3 days, 12 hours	2 weeks, 4 days	50-75%
LFQ3	2 weeks, 4 days	62 weeks, 6 days	>75%
Period 3			
Freq.	**From**	**To**	**Inter-Quartile Range**
HFQ1	∼1 second	55 minutes, 45 seconds	0-25%
HFQ2	55 minutes, 45 seconds	1 day, 7 hours	25-50%
HFQ3	1 day, 7 hours	1 week, 1 day	50-75%
LFQ3	1 week, 1 day	16.0 weeks, 5.0 days	>75%

Table 5: Speed of recirculation in Circles.

4.4 Circular Network Synergy

As mentioned in the Literature Review, it is possible to find a measure of the *synergy effect* and the *local multiplier effect* which can be directly related to the precise topological characteristics of the economic network that cause them. In this work, the *circular network synergy* is proposed, which is measured by solving a *minimum-cost circulation problem* in a transaction network (Ahuja et al., 2014; Edmonds and Karp, 1972; Goldberg and Tarjan, 1989; Simic and Milanovic, 1992). Following Fleischman and Dini (2020) and Simic and Milanovic (1992), the transaction network is considered as an obligation network, where all transactions are temporally aggregated, creating a directed weighted graph[11]. In an obligation network, every link represents a debt between two nodes and not an actual transfer of currency. As suggested by Fleischman and Dini (2020) and Simic and Milanovic (1992), the minimum-cost algorithm can be used to minimise the transfers and liquidity necessary to pay all debts. In particular, the algorithm works by taking advantage of the simplification of its cycles. In Figure 6, it is represented how the algorithm works. The cycle in Figure 6(a) is simplified, and the network is transformed

[11]The temporal aggregation implies that reciprocal links are solved. In fact, if A owes to B and B owes to A, then only the difference is considered.

into Figure 6(b).

The minimisation of transfers and liquidity necessary to clear out an obligation network is called in this work minimum-cost graph transformation. The maximum level of *circular network synergy* is reached in a topology where every node has the same value of inflow and outflow in the system, and therefore all cycles in the graph can be cancelled. In this optimal case, the minimum-cost graph transformation leads to a null graph. This optimal graph is represented in Figure 6(c). In economic terms, this is theoretically equivalent to an ideal perfect market competition, where a zero-profit condition is satisfied, i.e. revenues equal to costs, inflow equal to outflow (Criscione et al., 2022; Mas-Colell et al., 1995). From a network perspective, this situation implies that every node is involved in a *balanced* cyclic component (Fleischman and Dini, 2020), where all directed weighted cycles can be offset[12]. In short, it is assumed that the higher the number of cycles in the graph, the more similar the weights are, the higher its expressed *synergy*. Therefore, this optimal topology can be found using the minimum-cost transformation of the real graph.

In short, it is assumed that the higher the number of cycles in the graph, the similar weights they have, the higher its *synergy* interpreted as circular codependency. Therefore, the distance between the topology of the real graph and its optimal topology can be measured by using the minimum-cost transformation of the real graph. The ratio between the debt on the real graph W and the debt in the transformed graph W' is captured by the *circular network synergy*. This index is equal to 1 when the real network is close to its maximum *synergistic* potential, which means that it is made only of cyclic components that compensate for each other (Figure 6(c), as an example).

Consider a directed weighted graph $G = (V, E)$ with V as the set of nodes and E the set of edges, where $\forall i \in V$ and $\forall j \in V$, $(i, j) \in E$. The graph G is weakly connected, every node can reach every other node ignoring the direction of the arrows. Every link in G ($(i, j) \in E$) has a *capacity* u_{ij}, a *cost* c_{ij}, and an *actual final flow* x_{ij}. Every node has a net flow (or balance) b_i. A

[12]"Offsetting" a cycle means that all the obligations in it can be cleared out. In the literature, this operation is also called net settlement. More details are provided in the next paragraphs. As an example, if A owes to B, B owes to C, and C owes to A, then the three agents can agree on subtracting the minimum due amount among them. If they all owe the same amount, then the cycle is said to be "balanced" and no liquidity is necessary to pay out outstanding debts (Fleischman and Dini, 2020).

minimum-cost graph transformation problem can be formulated as follows (Kovács, 2015)

$$\begin{aligned} &\min \sum_{(i,j)\in E} c_{ij}x_{ij} \\ \text{subject to } &\sum_{ij\in E} x_{ij} - \sum_{ji\in E} x_{ji} = b_i, \forall i \in V, \\ &0 \le x_{ij} \le u_{ij}, \forall (i,j) \in E \end{aligned} \tag{22}$$

Here, the first constraint is called *flow conservation constraints* and the second *capacity constraints*. As described in Kovács (2015), to solve the problem it is assumed that the capacities are finite, the costs are non-negative, and there is an optimal solution with integer values. This also implies $\sum_{i\in V} b_i = 0$.

An obligation network G needs to be transformed using the minimum-cost algorithm to find a topology that minimises the number of transfers and the volume of currency required to pay all debts. The *capacity* of each link u_{ij} is equal to the amount of debt associated with that link w_{ij}, such that

$$W = \sum_{ij} w_{ij}, \forall (i,j) \in E \tag{23}$$

$$\sum_{ij\in E} w_{ij} - \sum_{ji\in E} w_{ji} = b_i, \forall i \in V \tag{24}$$

where W is the total amount of initial debt in the network. Note that Equation 24 is used to calculate the net balance of each node i. The value x_{ij} is the actual final transfer of currency that should take place after the graph transformation. The problem can be rewritten as follows.

$$\begin{aligned} &\min \sum_{(i,j)\in E} x_{ij} \\ \text{subject to } &\sum_{ij\in E} x_{ij} - \sum_{ji\in E} x_{ji} = b_i, \forall i \in V, \\ &0 \le x_{ij} \le w_{ij}, \forall (i,j) \in E \end{aligned} \tag{25}$$

The final transfer of currency must respect the net flow (or balance) of each node i, as described by Equation 24. In the second constraint, the edge (i, j) has a final actual flow x_{ij}, which can be any value between 0 and the initial amount of debt w_{ij}. Obviously, x_{ij} would be zero if it is part of one or more cycles ”balancing” each other (see Figure 6(c), as an example). The solution of this transformation of the graph is a transformed graph with a minimum-cost $G' = (V, E')$ for which $|E'| \leq |E|$ and $W' \leq W$, where $|E'|$ is the number of edges and W' is the total amount of debt in G'. The *circular network synergy* is therefore defined as

$$CNS = 1 - \frac{W'}{W} \tag{26}$$

W' is the volume of transactions in G'. G' is the minimum-cost transformation of the original graph G. In other words, W' is the amount of liquidity that the system requires to pay all outstanding debts. W is the volume of transactions in the original graph G.

The index *CNS* is equal to 1 when the amount of debt in G' is completely offset, and so $W' = 0$ (see Figure 6 (c)); while it is 0 when no cycles can be simplified and so $W' = W$ (that is, an acyclic graph). The *CNS* can also be expressed as the ratio $(W - W')/W$, which is the ratio between the amount of debt cleared using a minimum-cost algorithm $(W - W')$ and the total amount of debt in the system W.

In other words, it represents the relative amount of liquidity that can be saved by using a net settlement technique based on cycle clearing. It represents the percentage of volume passing through directed network cycles. It can also be interpreted as a measure of the distance of the real network from an optimal state where all debts can be compensated without using liquidity because every node is involved in offsetting cycles.

In this work, the *circular network synergy* is compared to other three metrics: the ”ascendency ratio”, the ”systemic reserve ratio”, and the economic multiplier. The ascendency ratio and the systemic reserve ratio are calculated following Ulanowicz et al. (2009). In particular, the ”ascendency” is calculated as a scaled mutual information of network flow, while the ”systemic reserve” is calculated as its conditional entropy. Both measures are normalised by the ”system capacity”, which is calculated as a scaled entropy of network flow. The ”synergy” (or *circular network synergy*) represents the percentage of volume that is going through weighted and directed cycles. And finally, the economic multiplier is the ratio of transaction volume to injected liquidity. The network metrics and the economic multiplier are calculated in LWCC only. The exclusion of components outside of LWCC allows one to focus on the main core of the economic network. Especially because of the significant presence of small *dag0* components which in both networks can be related to users who are simply trying the system. In Section 5, the comparison of these four metrics is reported with tables and graphs.

4.5 Significance

This section is taken in part from Criscione (2024). The statistical significance is computed by comparing the empirical results with null models. Three types of null models are built by adopting different edge-swap techniques. Each model is swapped 10 times the number of edges, and a creation of 100 models is created per each type (Milo et al., 2004). In the first type of null model, only the targets are swapped (”target-swap”), in the second only the sources (”source-swap”), and in the third either the source or target is swapped with a chance sampled from a uniform distribution (”both-swap”). When swapped, each directed link between two nodes carries with it the weights and timestamps of its transactions which took place between those two nodes.

The Z-Scores are calculated as the ratio between the difference of the empirical value (x) and the average of the null distribution (μ) over the standard deviation of the null distribution (σ):

$$Z = \frac{x - \mu}{\sigma} \tag{27}$$

The interpretation of the results for Z-Score is the following. When the Z-Score is close to zero, the empirical value is not statistically significant, so it is very likely drawn from the same distribution of the null models, which is close to random. When the Z-score has a very high positive value, it means that the network feature is over-represented with respect to the null models. Similarly, when the Z-score has a very high negative value, it means that the network feature is under-represented with respect to the null models. The Z-score can be read as the distance between the empirical value and the average of the null models, but expressed as the number of standard deviations.

5 Results

5.1 Sarafu Network

5.1.1 Topological Components

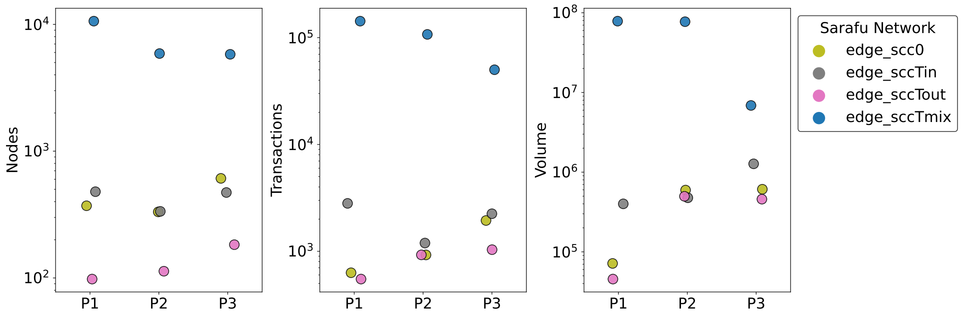
In this section, the topological analysis of the Sarafu network is presented. In Table 6, the characteristics of the topological categories presented in the previous paragraph are reported. The largest category in terms of volume is *sccTmix* with 162 627 643.89 Sarafu. In fact, the largest strongly connected component (LSCC) in all periods belongs to this category. Recall that *sccTmix* is a strongly connected component (or cyclic component) that sends to and receives from an acyclic component or a single-node. The size of LSCCs for each period is reported below.

LSCC

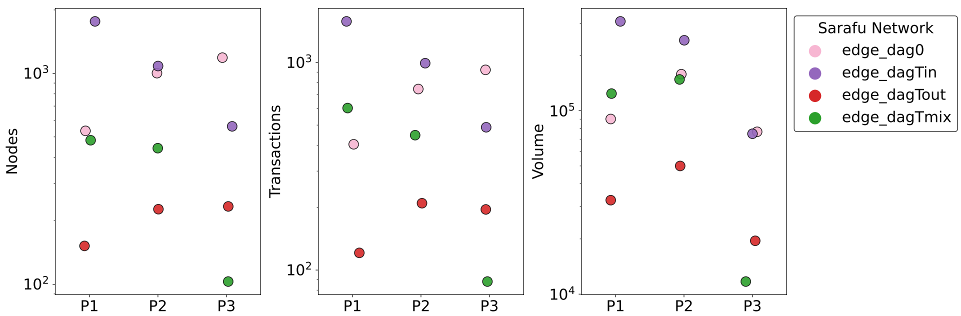
- **LSCC in Period 1** has 10 353 nodes, 142 115 transactions, 78 421 361.09 of exchanged volume, 9 911 926.52 Sarafu disbursed
- **LSCC in Period 2** has 5 432 nodes, 102 517 transactions, 75 693 128.54 of exchanged volume, 4 373 017.01 Sarafu disbursed
- **LSCC in Period 3** has 5 252 nodes, 45 249 transactions, and 5 827 070.75 of exchanged volume, 1 023 995 Sarafu disbursed

LWCC

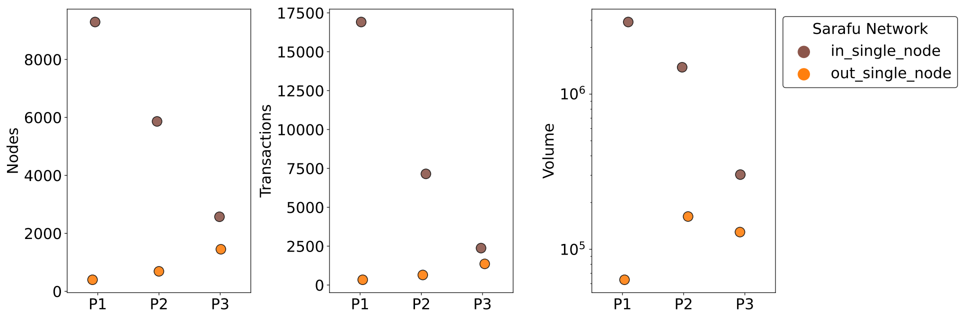
- **LWCC in Period 1** has 21 079 nodes, 170 074 transactions, 83 392 072.71 of exchanged volume, 15 624 709.96 Sarafu disbursed
- **LWCC in Period 2** has 12 618 nodes, 116 773 transactions, 80 713 105.97 of exchanged volume, 6 612 599.05 Sarafu disbursed
- **LWCC in Period 3** has 8 656 nodes, 52 647 transactions, and 75 36 134.35 of exchanged volume, 1 349 068.5 Sarafu disbursed



(a) Cyclic components



(b) Acyclic components



(c) Single nodes

Figure 14: Single-nodes, cyclic, and acyclic components in Sarafu network. The four largest categories (in terms of volume) are *sccTmix*, *dagTin*, *dag0*, and *in_single_nodes*. Notice the presence of *dagTmix* in the first two periods. Notice also the general drop in volume in the third period.

In Figure 14, the size of strongly connected components (or cyclic components, SCCs), directed acyclic graphs (or acyclic components, DAGs), and single-nodes is reported. It can be observed that, for all periods *sccTmix* among cyclic components, *dagTin* and *dag0* among acyclic components, and *in_single_node*s among single-nodes are the most relevant categories in terms of number of nodes, transactions, and volume exchanged. Among acyclic components, the main difference between periods is due to the presence of *dagTmix* which drops in the third period. Notice also how the volume moved by those four major categories was notably reduced in the third period as well. In the third period, there was a drastic policy change which was discussed in Chapter 3[13].

In Table 6, the second biggest category are *in_single_nodes* with a total exchanged volume of 4 709 640.97 Sarafu. These are users who sent their Sarafu to another user in a *sccTmix* or in *sccTin*. The third category is *sccTin* (2 148 894.71 Sarafu) followed by *scc0* (1 277 729.65 Sarafu), *edge_dag2scc* (1 042 953.26 Sarafu), and *sccTout* (1 001 581.49 Sarafu). This gives an impression of the importance of cyclic components in this economic network. Most of the exchanged volume is happening within strongly connected components: SCCs which are also receiving from acyclic component (*sccTin*)s and *in_single_nodes*, SCCs isolated from the rest of the network (*scc0*), SCCs which are also sending to acyclic components (*sccTin*) and *out_single_nodes*. The category *edge_dag2scc* indicates the volume transferred from acyclic components to cyclic components.

The above findings show that cyclic components play a fundamental role in this economic network. However, it is important to compare these findings with null models to understand the role played by randomness. First, in all the null models of different types, the strongly connected components generally disappear. Null models generally only have one large strongly connected component (LSCC) of *sccTmix* type. However, the null model LSCC usually has fewer transactions and fewer volume exchanged than the empirical one (see Figure 15(a)) . This first result tells us that the presence of *sccTmix* components in the empirical network is generally

[13]In Period 1, registered savings groups could cash-out the Sarafu of their members into Kenyan shillings. The monthly procedure was limited to a certain amount per group. In Period 2, savings groups could indicate one or more local businesses (vendors) which were entitled to cash out. Savings groups organised monthly collective purchases in Sarafu in those local businesses. At the end of the month, local businesses could cash out their Sarafu in Kenyan shillings. In Period 3, this currency exchange stopped.

	SCCs	WCCs	Nodes	Dir.Links	Transact.	Volume
edge_bridge_scc	0	27	100	75	98	74743.0
edge_dag0	0	1017	2725	1717	2073	325203.0
edge_dag2scc	0	551	2784	2422	3932	1042953.26
edge_dagTin	0	893	3410	2569	3063	624216.41
edge_dagTmix	0	144	1027	920	1139	283889.06
edge_dagTout	0	186	613	440	527	102052.47
edge_in_single_node	0	2167	17723	17651	26433	4709640.97
edge_out_single_node	0	786	2535	1763	2342	355133.03
edge_scc0	493	493	1313	1769	3494	1277729.65
edge_scc2dag	0	308	1028	778	1158	328492.14
edge_scc2scc	0	344	1425	1127	1716	801962.6
edge_sccTin	361	361	1286	2446	6248	2148894.71
edge_sccTmix	150	150	22294	112250	300338	162627643.89
edge_sccTout	113	113	394	946	2509	1001581.49

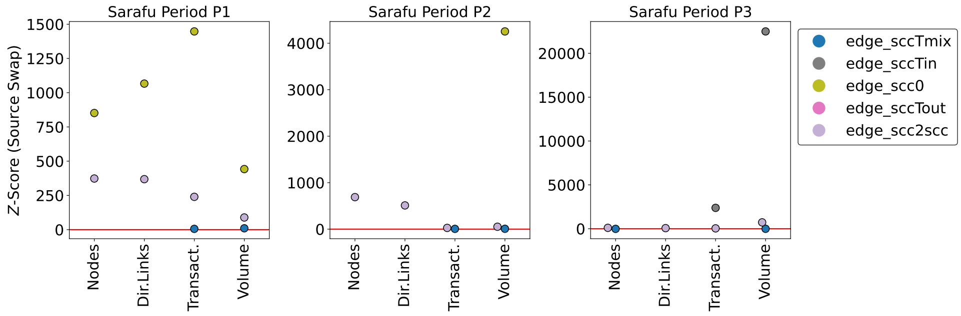
Table 6: Topological components in Sarafu network. For this table, the network was temporally aggregated across the whole observed period.

significant and over-represented (positive Z-score) (Figure 15(a)). The absence of some SCCs categories is not due to the lack of significance but to their complete absence in those null models. The result on acyclic component and single-nodes complements those observations. Generally, acyclic components and single-nodes are under-represented in empirical networks with the only exception of *dag0*. This means that the absence of these categories is not due to pure randomness. In summary, the role played by the cyclic components is statistically significant and over-represented, along with the *dag0* components.

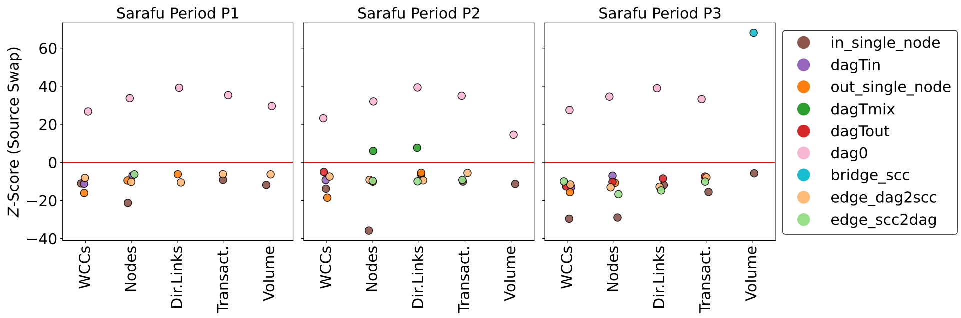
In conclusion, the presence of cyclic components in the Sarafu network is considerable and significant. This means that the cyclic structures in this currency network cannot simply be explained by randomness, but reflect a precise economic phenomenon. The only exception to this conclusion is the presence of *dag0*. In Criscione (2024), it was shown that this topological category is mostly associated with groups of users who are simply trying the system. In the next section, recirculation and one-time usage is analysed using these topological categories.

5.1.2 Recirculation

A recirculation operation is a collection made of incoming transactions followed by outgoing transactions before another incoming transaction follows. Recirculation operations are classified according to their speed, the time interval between the first incoming transaction and the

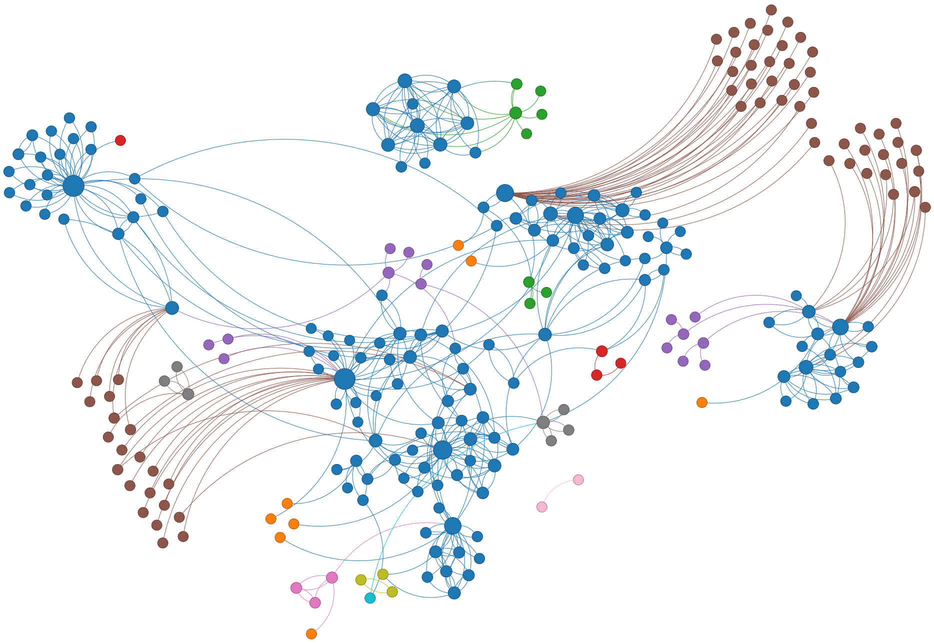


(a) Significance of cyclic components

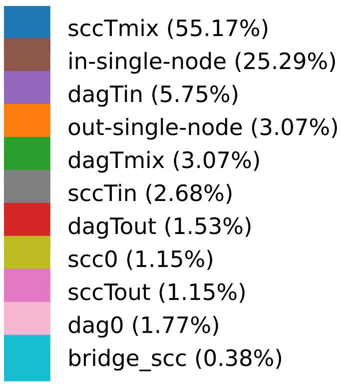


(b) Significance of acyclic components and single nodes

Figure 15: Significance of topological components in Sarafu over time. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. There is some little variations across all the other null models (see also Appendix, Figure 43). As explained before, this is due to the fact that this type of randomization generally eliminate strongly connected components. Therefore, in most of the cases, only one strongly connected component of *sccTmix* type is left. This implies that the absence of some cyclic component categories is due to their absence in the null models, and not to their lack of significance. Note also that the number of weakly and strongly connected components is not reported here, but in Appendix.



(a) Subgraph Sarafu Network



(b) Legend

Figure 16: Subgraph of 261 nodes and 524 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by aggregating the network on the whole observed period. Each link takes the color of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. In the legend, the proportion of users per each category in parenthesis.

last outgoing transaction per each account (see Chapter 4 for details). In Table 7, a summary of recirculation in Sarafu is reported. Note that a recirculation operation can involve more than two transactions (one incoming, one outgoing). Conversely, one transaction can be part of one or more recirculation operations, when the outgoing transaction in one operation becomes the incoming transaction of another. In those cases, one single transaction can be part of max two recirculation operations. In Table 7, in Period 1 the recirculating users were 22.7% of the total number of users[14] and exchanged between them 84.6% of the total volume[15]. In Period 2, recirculating users were 22.3% of the total number of users and exchanged among them 86% of the total volume. Finally, in Period 3, they were 33% of the total and exchanged 74% of the total volume. In summary, a small minority of users is responsible for most of the volume exchanged on the Sarafu network.

	Period 1			
	Operations	**Transactions**	**Volume**	**Users**
Tot.	203 350	150 837	76 203 610.65	18 458
Recirc. Only	–	115 968	70 761 076.1	5 003
	Period 2			
	Operations	**Transactions**	**Volume**	**Users**
Tot.	138 858	108 336	73 917 070.6	11 643
Recirc. Only	–	92 046	70 554 924.04	3 274
	Period 3			
	Operations	**Transactions**	**Volume**	**Users**
Tot.	69 197	50 424	8 415 057.45	8 203
Record. Only	–	42 832	7 554 553.09	3 906

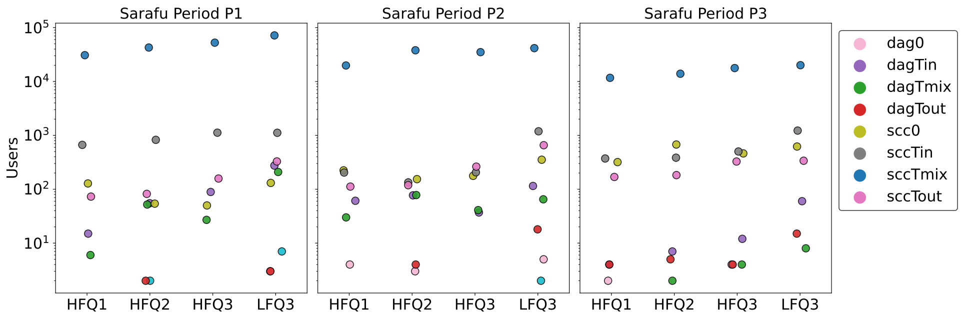
Table 7: Recirculation in Sarafu network. The *Recirc. Only* indicates operations happening only among recirculating users. In practice, the outgoing transaction of a recirculating user is also the incoming transaction for another recirculating user.

In Figure 17(a), the majority of recirculating users in all periods belong to cyclic components (or strongly connected components). In Figure 17(b), the number of transactions in recirculation operations across topological groups, periods, and temporal categories is generally dominated by cyclic components and *in_single_node*s. In the third period, also *out_single_node*s gained importance in recirculation. This means that some of those operations ended up in nodes that did

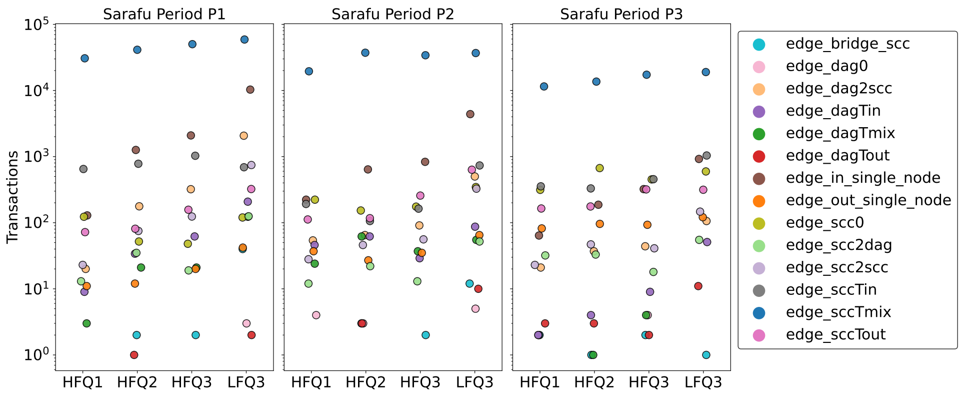
[14]This is the ratio of *Recirc. Only (users)* in Period 1 in Table 7 and total number of users in Period 1 (22 022, from Chapter 3)

[15]This is the ratio of Recirc. Only (Volume) in Period 1 in Table 7 and total volume in Period 1 (83 583 846.76, from Chapter 3)

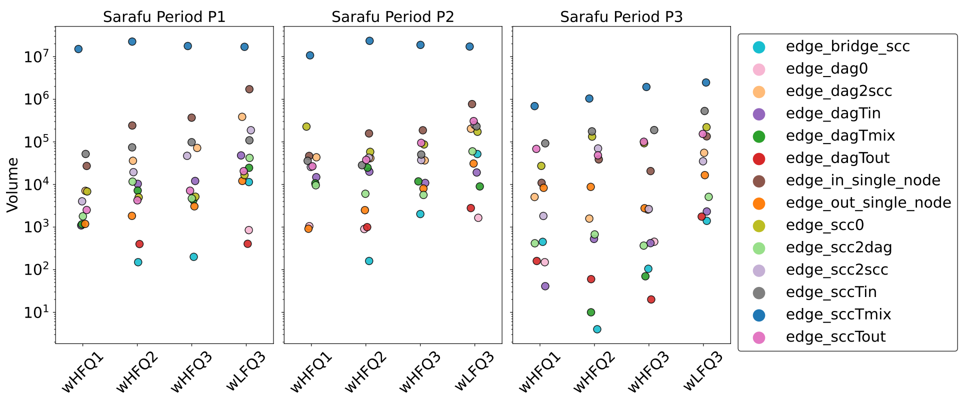
not recirculate afterwards. Similar conclusions can be reached by observing the recirculated volume in Figure 17(c). Looking at each topological group separately, it is possible to identify a very weak tendency to move more volume and have more transactions when the speed of recirculation slows down (from *HFQ1* to *LFQ3*). In summary, cyclic components played an important role in currency recirculation. Nonetheless, also *in_single_node*s and *out_single_node*s played a relevant role in initiating and finalising those recirculation operations.



(a) Users



(b) Transactions



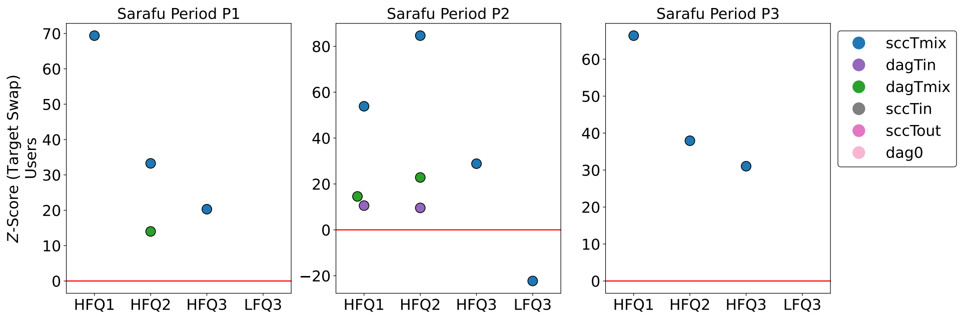
(c) Volume

Figure 17: Recirculation in Sarafu per topological group. When the same transaction is taking part into two recirculation operations its weight is split into two. This explains the meaning of the ”w-” as prefix of each temporal category in Figure (c).

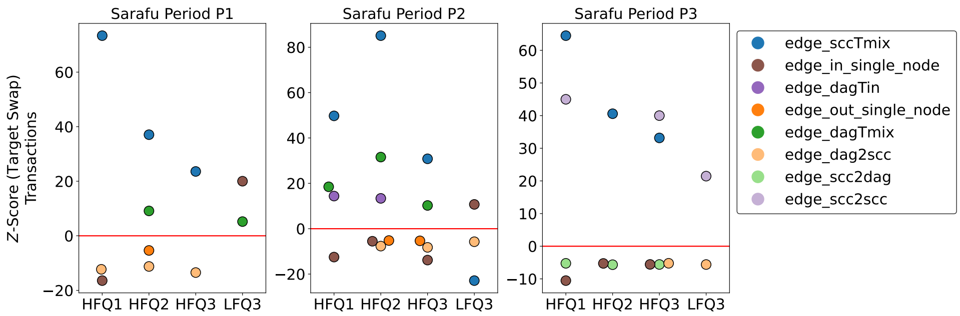
The role of cyclic components in recirculation could be due only to randomness, and for this reason the findings are compared to null models. Null models are created by randomising empirical networks. However, almost all null models show one type of strongly connected component, which is a unique large *sccTmix* component. This means that the existence of other types of cyclic component (and the recirculation in them) is already significant. In Figure 18(a), the number of recirculating users is significantly over-represented for temporal categories *HFQ1*, *HFQ2*, and *HFQ3*, across all periods. In other words, the presence of users in the *sccTmix* components that recirculating in less than ∼3 hours (*HFQ3*, Periods 1), ∼15 hours (*HFQ3*, Periods 2), ∼19 hours (*HFQ3*, Periods 3) is generally significant. A similar pattern can be observed for the number of transactions (Figure 18(b)) and the volume (Figure 18(c)).

However, there are four important observations to add. First, the number of users, transactions, and exchanged volume in *sccTmix* for the category of ”slow” recirculation operations (*LFQ3*) is under-represented in the first two periods and not significant in the last period. Second, in the ”faster” categories (*HFQ1*, *HFQ2*, *HFQ3*) also *dagTin* and *dagTmix* are playing a significant role in recirculation due to their over-representation. Third, *in_single_node*s are also playing a significant role in ”slow” recirculation operations (*LFQ3*, i.e. greater than or equal to ∼2-3 days) in Period 1. Finally, in terms of volume exchanged in recirculation, only transactions among cyclic components (*edge_scc2scc*) seem to play a significant role in the third period.

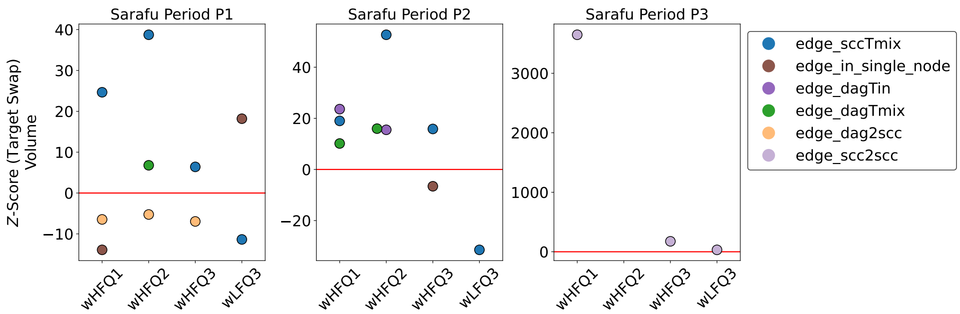
In summary, cyclic components play a statistically significant role in recirculation of the currency. Figure 17 shows that most of the recirculation occurs in these components. However, the existence of those components cannot generally be attributed to randomness because they are not present in null models. The only type of cyclic component that exists in all null models is *sccTmix*. For *sccTmix* components, the *Z*-Score shows an over-representation in number of users, transactions and volume. The only exception is the loss of significance for the exchanged volume in the third period.



(a) Users



(b) Transactions



(c) Volume

Figure 18: Significance of recirculation in Sarafu per topological group. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. Only null models with ”target-edge” swap are considered in this Figure. Other null models are compared in the Appendix (Section 7.7). Besides *sccTmix* components, consider that other types of strongly connected components are generally not present in null models due to the randomization of the empirical graph. Their absence in the plots is not due to lack of significance, but the opposite.

The last aspect to be considered in the analysis of currency recirculation is the presence of ’one-time’ users, namely users that participated in one transaction only before quitting the system (see also Tables 14 and 15 in the Appendix). This phenomenon can be explained by the presence of users creating fake accounts or simply a group of users trying out the system (see Criscione (2024) for more information). Figure 19 shows that this phenomenon is closely related to some specific topological groups. On the one hand, the majority of one-time users who made one outgoing transaction is categorised as *dagTin* and *in_single_node*. Both of those categories moved a considerable amount of value decreasing over time: *in_single_node* spent 1 145 684.21 (Period 1), 867 213 (Period 2), and 184 734 (Period 3); users in *dagTin* spent 156 395 (Period 1), 157 414 (Period 2), 67 127 (Period 3). On the other hand, the majority of one-time users who only received one incoming transaction is categorised as *dag0* and *out_single_node*. Both categories moved less volume than the previous ones: *out_single_node*s received 27 206.03 (Period 1), 43 394 (Period 2), and 65 599 (Period 3); in *dag0* the volume exchanged by one-time receivers was 32 186 (Period 1), 50 419 (Period 2), and 44 141 (Period 3). In summary, one-time users are associated with specific topological categories, mostly in acyclic components and single-nodes. In particular, one-time users as *in_single_node*s injected in the system more than 2 million Sarafu in the system in the first two periods. Also in this case, in the third period there is a considerable change in the internal dynamics, probably due to policy change discussed before.

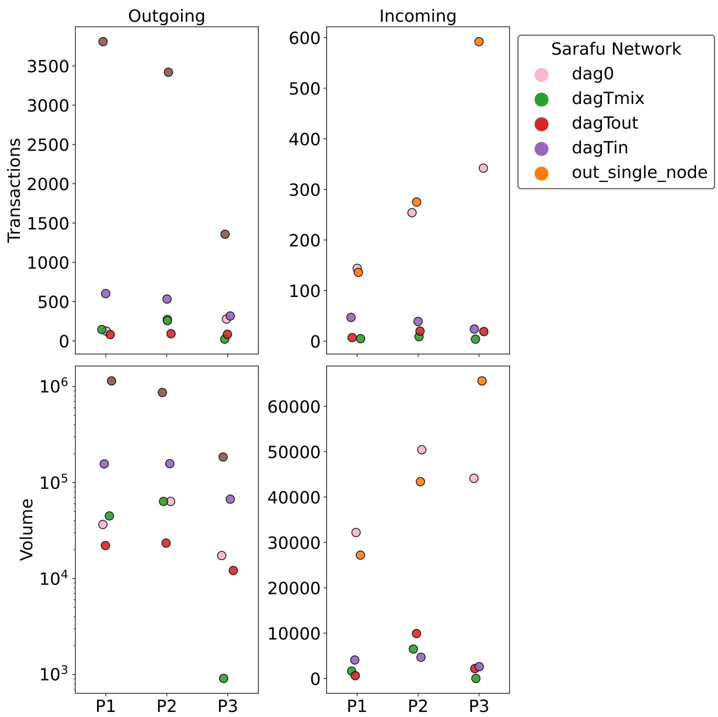


Figure 19: One-time users in Sarafu

In conclusion, recirculation in the Sarafu network occurs mostly in cyclic components, especially in the *sccTmix* components. This result is statistically significant for at least the first two periods. The role of other topological categories is generally marginal, except for *in_single_node*s which initiated recirculation of a considerable amount of Sarafu in the first two periods: 2 361 672.29 (Period 1), 1 162 150.42 (Period 2), and 205 043.1 (Period 3). In addition, users outside of the cyclic components can be partially associated with one-time usage. Especially, *in_single_node*s are associated with one-time outgoing transactions: 1 145 684.21 (Period 1), 867 213 (Period 2), and 184 734 (Period 3). This means that about 60% of the volume spent by *in_single_node*s was coming from one-time users. In the next section, the presence of cyclic components and recirculation is studied using some of the network performance metrics presented before in this work.

5.1.3 Circular Network Synergy

In this section, the state of the Sarafu network is analysed using the four metrics presented in the Methods section 4.4. That is, the ascendency ratio, the systemic reserve ratio, the *circular network synergy*, and the economic multiplier. The network metrics and the economic multiplier are calculated in LWCC only. The exclusion of components outside of LWCC allows one to focus on the main core of the economic network. Especially because of the significant presence of small *dag0* components which in both networks can be related to users who are simply trying the system.

In Figure 20, the *circular network synergy* is close to 0.6 in the first two periods. This means that about 60% of the flow was going through weighted directed cycles. In the first two periods, the *ascendency ratio* and the *reserve ratio* also agree towards 0.5. This means that in the first two periods, the system was converging toward a state of balance between used capacity (i.e. expressed potential) and unused capacity (i.e. unexpressed potential), as defined by Ulanowicz et al. (2009). Notice how the closer these two metrics are, the higher is the level of *circular network synergy*. In contrast, a larger difference between the *ascendency ratio* and the *reserve ratio* is associated with a lower level of *circular network synergy* and *multiplier*, like in the third period.

A weak monotonic relation of these network metrics to the economic multiplier can also be seen in Figure 20. From the previous section on recirculation, a considerable decrease in activity was observed in the third period across any topological category. This explains why both the multiplier and the *circular network synergy* decrease in Period 3. Another important topological change can explain the change from Period 1 to Period 2. As discussed in the previous sections on topological components and recirculation, the presence of *in_single_node*s and one-time users (in acyclic components and single-nodes) was high in the first two periods. This means that the multiplier was kept artificially high because it did not completely reflect the recirculation of currency. In addition, recirculation in *sccTmix* also loses importance in the third period, so these two factors together may have caused the multiplier to fall close to zero.

	Period 1	**Period 2**	**Period 3**
Ascendency Ratio	0.56	0.54	0.77
Reserve Ratio	0.44	0.46	0.23
Synergy	0.58	0.61	0.18
Multiplier	2.73	1.77	0.06

Table 8: Systemic metrics of Sarafu circulation over time.

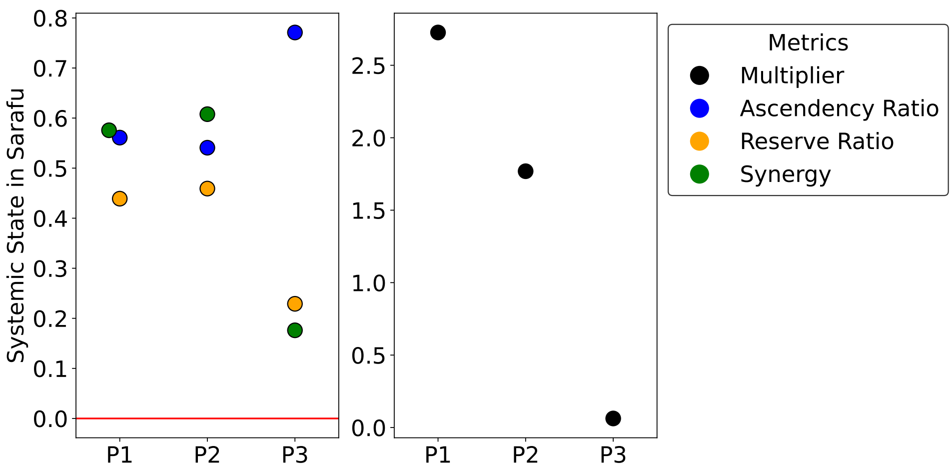


Figure 20: Systemic state of Sarafu over time. In the case of Sarafu, the injected liquidity is equal to the initial disbursement to new users and the rewards disbursed to registered users, minus the reclamation (i.e. cashing-out operations and demurrage). In Sarafu, in the first two periods an increasing level of ”synergy” is associated to a convergence of ”ascendency” and ”systemic reserve”. However, in the third period, a higher difference between the two is associated to a lower level of ”synergy”. The multiplier is decreasing over time, probably reflecting the decreasing level of exchanged volume.

In Figure 21, the network metrics are compared to three types of null models. A consistent result is the statistical significance and over-representation of *circular network synergy* across all null models and periods. On the other hand, the *ascendency ratio* and the *reserve ratio* have less significance with a Z-score oscillating between 5 and 10 in absolute value. The two metrics are complementary by design to each other, and this is also reflected in the graph. Finally, the *ascendency ratio* and the *reserve ratio* are not significant in Period 3 for source-edge swap null models and in Period 1 for target-edge swap null models. In this context, *circular network synergy* seems a more statistically significant network metric.

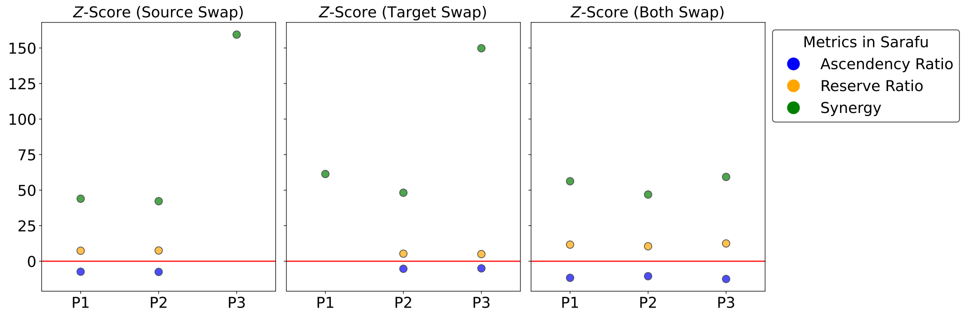


Figure 21: Significance of systemic state metrics of Sarafu over time. *Circular network synergy* is significantly over-represented in all three periods. In a complementary way, the *reserve ratio* is over-represented, while the *ascendency ratio* is under-represented. However, their Z-scores oscillates between 5 and 10 in absolute value.

5.2 Circles UBI Network

5.2.1 Topological Components

In this section, the topological analysis of the Circles network is reported. In Table 9, the largest topological group is *sccTmix* with a volume of 2 758 315.52 Circles units exchanged throughout the period. The second largest group is *in_single_node*s (629 161.09 Circles units), the third one is *dag0* (347 374.34 Circles units), and the fourth one is *edge_dag2scc* (312 393.91 Circles units). In each period, the largest strongly connected component was classified as *sccTmix* (in P1 and P2) and *sccTout* (in P3) and its size changed as follows.

LSCC

- **LSCC in Period 1** has 734 nodes, 10 029 transactions, 123 938.8 of exchanged volume, 1 145 999.19 Circles disbursed
- **LSCC in Period 2** has 391 nodes, 7 467 transactions, and 1 402 281.8 of exchanged volume, 912 023.8 Circles disbursed
- **LSCC in Period 3** has 1 398 nodes, 26 355 transactions, and 1 163 821.9 of exchanged volume, 1 327 801.51 Circles disbursed

LWCC

- **LWCC in Period 1** has 3490 nodes, 17 078 transactions, 522 612.39 Circles in exchanged volume, 4 109 380.76 Circles disbursed
- **LWCC in Period 2** has 1895 nodes, 14 874 transactions, 2 099 228.62 Circles in exchanged volume, 3 020 645.72 Circles disbursed
- **LWCC in Period 3** has 3 309 nodes, 35 535 transactions, 1 585 583.484 Circles in exchanged volume, 2 150 600.16 Circles disbursed

The first evident difference between the Sarafu and Circles networks is the lower presence of different types of cyclic components in the Circles network. In fact, *sccTin*, *sccTout*, and *scc0* components are generally smaller and less active in Circles than in Sarafu (see Figure 22(a)).

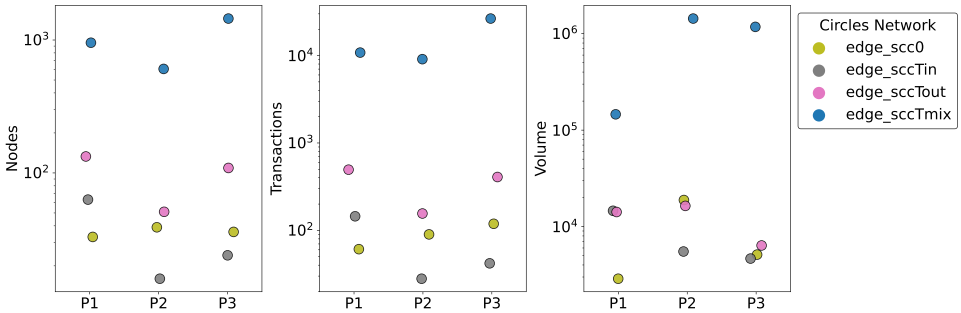
However, the number of nodes, transactions, and volumes moved by the acyclic components is very similar between the two networks (see Figure 22(b)). Finally, also single-nodes in Circles are generally less and with less volume than in Sarafu (see Figure 22(c)).

Figure 22 also shows the most relevant topological categories among cyclic and acyclic components, and single-nodes. Among cyclic components, Figure 22(a) shows that, after *sccTmix*, the second largest category is *sccTout* and the third one *scc0*. Cyclic components had a reduction in volume in the third period (negligible for *sccTmix* components). Figure 22(b) shows that the three largest categories of acyclic components are (in order of size, in terms of volume): *dag0*, *dagTmix*, and *dagTin*. The main difference between all periods is the negligible presence of *dag0* in the first and its importance in the second and third periods. Finally, Figure 22(c) shows that *out_single_node*s played a relevant role in the first and third period, but never moved a considerable amount of volume. On the other hand, even if in fewer numbers and with fewer transactions, *in_single_node*s moved the majority of volume across all periods.

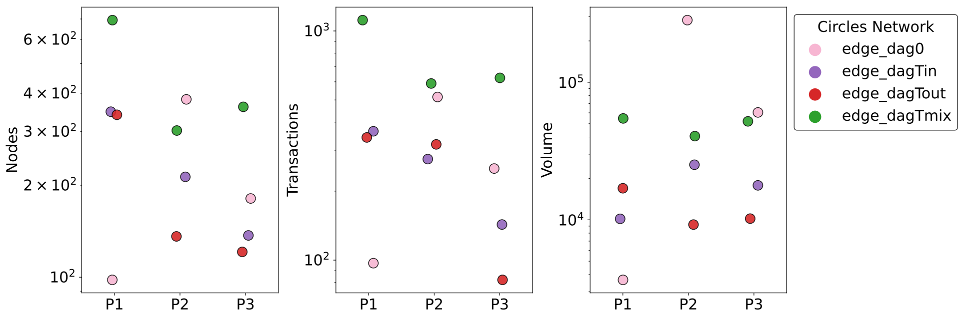
These findings can be summarised and explained as follows. First, the presence of less diverse cyclic components than in Sarafu could be a hint of less economic synergy. Second, a new user who wants to join and test the system needs to be validated by at least three other members of the *Web of Trust* (see Chapter 3). This means that existing members often were onboarding new users by sending a request in the *Web of Trust* and sending them Circles units to let them try the system. If the new member would not further engage with the system, then this could be visible as *out_single_node*[16]. In fact, in the first and third periods the number of *out_single_node*s was high, but their volume was low. This probably means that in both periods there was a great deal of effort to welcome new members. The third finding is about the presence of *dag0* and *dagTmix*, especially in the second and third period. The increasing number of *dag0* in the second and third periods may be related to the group of users trying the system out or using it marginally (e.g., as a secondary currency).

The findings should be compared with null models to understand the role of randomness in them. In Figure 23, the empirical network is compared with null models created using a "target-edge" swap technique (other null models can be found in Appendix 7.5). In Circles,

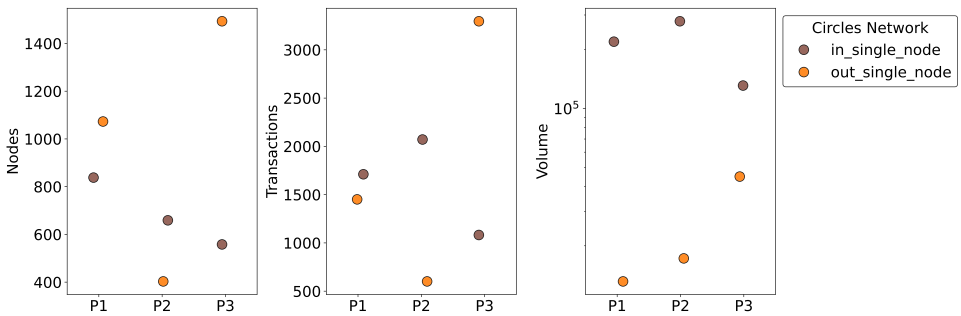
[16] As a reminder, a *out_single_node* is a node which is only receiving from another node in a cyclic component of type either *sccTmix* or *sccTout*



(a) Cyclic components



(b) Acyclic components



(c) Single nodes

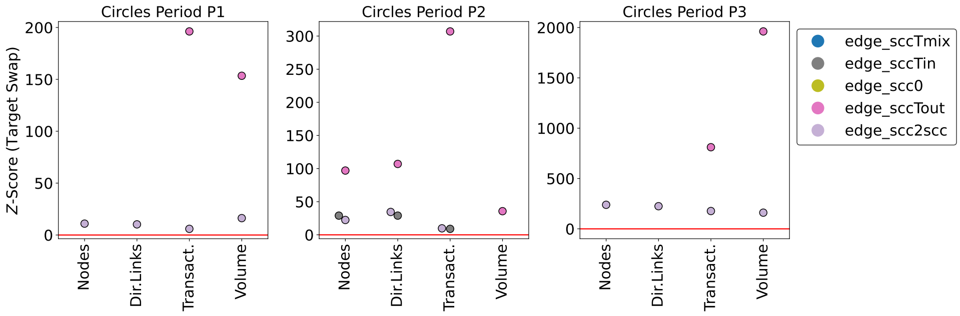
Figure 22: Single-nodes, cyclic, and acyclic components in Circles network. The four largest categories (in terms of volume) are: *sccTmix*, *in_single_node*s, *dag0*, and *edge_dag2scc*. Notice the presence of *dag0* in the last two periods. Notice the drop in volume in the third period.

	SCCs	WCCs	Nodes	Dir.Links	Transact.	Volume
edge_bridge_scc	0	14	65	53	106	10831.73
edge_dag0	0	207	661	497	863	347374.34
edge_dag2scc	0	161	1277	1868	3686	312393.91
edge_dagTin	0	206	698	518	784	53111.81
edge_dagTmix	0	123	1357	1433	2329	147258.56
edge_dagTout	0	172	597	451	745	36392.48
edge_in_single_node	0	209	2055	2371	4864	629161.09
edge_out_single_node	0	375	2969	3424	5348	75478.18
edge_scc0	52	52	108	115	270	26847.66
edge_scc2dag	0	158	1359	1764	3195	152728.97
edge_scc2scc	0	77	430	447	825	74720.8
edge_sccTin	44	44	103	118	215	24757.6
edge_sccTmix	94	94	3010	18422	46407	2758315.52
edge_sccTout	78	78	293	663	1059	36922.16

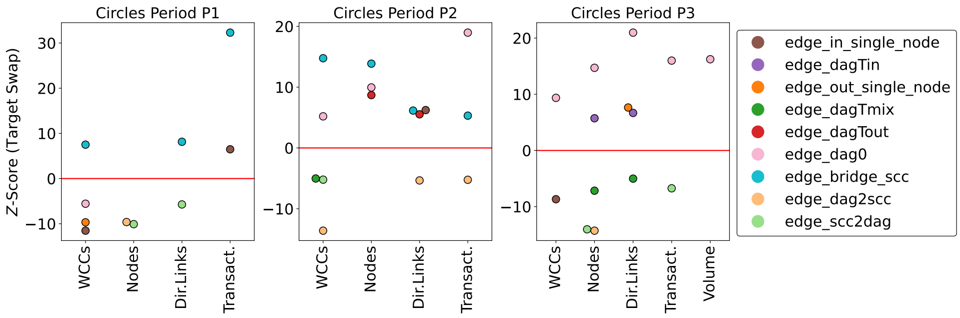
Table 9: Topological components in Circles network. Temporal aggregation includes the whole observed period.

as in Sarafu, randomisation often destroys most of the cyclic components, leaving only one large strongly connected component of type *sccTmix*. This implies also that the calculation of *Z*-Score may fail because the standard deviation is equal to zero and not because of lack of significance. Figure 23(a) shows that *sccTout* and *edge_scc2scc* play a significant role in this network across all periods. Figure 23(b) shows also *bridge_scc* is over-represented in the empirical network. The significant presence of *edge_scc2scc* and *bridge_scc* can be explained by the general absence of many different strongly connected components in null models. In Figure 23(b), the role played by *dag0*, *dagTin*, and *out_single_node* changed over time: from being under-represented in Period 1 to being over-represented in Period 3. This could confirm the explanation provided before about the role played by *dag0* and *out_single_node*. Although it is marginal, the presence of *dagTin* could be associated with a systemic inability to close 'loops'.

In summary, cyclic components in Circles have less diversity than in Sarafu, but their presence is still statistically significant. A particular role in this network is played by *in_single_node*s, *out_single_node*s, *dagTin*, and *dag0*. *In_single_node*s are users who spent their Circles units without accepting them back. *Out_single_node*s could be explained by the onboarding process of new members. *DagTin* could be related to missing "linkages" in the supply chain or economic network. While *dag0* could be connected to small groups of users simply trying out



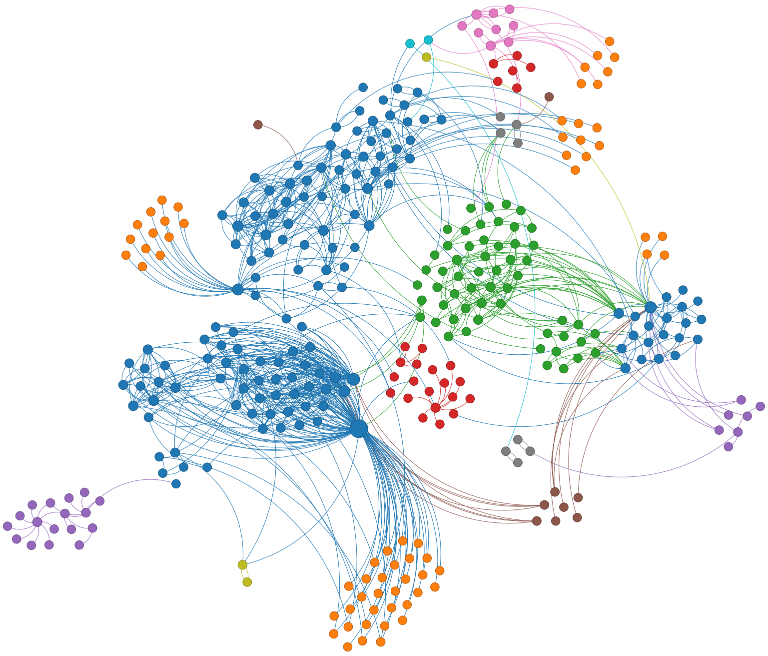
(a) Significance of cyclic components



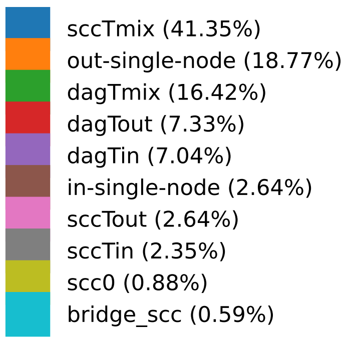
(b) Significance of acyclic components and single nodes

Figure 23: Significance of topological components in Circles over time. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. There is some little variations across all the other null models (see also Appendix, Figure 49). As explained before, this is due to the fact that this type of randomization generally eliminate strongly connected components. Therefore, in most of the cases, only one strongly connected component of *sccTmix* type is left. This implies that the absence of some cyclic component categories is due to their absence in the null models, and not to their lack of significance. Note also that the number of weakly and strongly connected components is not reported here, but in Appendix.

the system or marginally using it. In the next section, the recirculation in Circles network is explored with a particular regard to its topological structure.



(a) Subgraph Circles UBI Network



(b) Legend

Figure 24: Subgraph of 341 nodes and 825 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by aggregating the network on the whole observed period. Each link takes the color of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. In the legend, the proportion of users per each category in parenthesis.

5.2.2 Recirculation

In this section, the recirculation in Circles is analysed with respect to its topological structure. A recirculation operation is made by an incoming transaction followed by an outgoing transaction.

Recirculation operations are classified according to their speed, the time interval between the first incoming transaction and the last outgoing transaction (see Section 4 for details). In Table 10, a description of the recirculation operation in Circles over time is reported. Notice that in Period 2 the highest volume of recirculated currency occurred. However, in this period, fewer users were involved in recirculation. This means that the currency was circulating a lot among fewer people. In the third period, the opposite was true: many users participated in the recirculation of currency, even though the total volume was lower. It should also be considered that Period 2 is the longest one (∼16 months), while Period 3 is the shortest (∼6 months). This means that recirculation performed overall better in Period 3 than in Period 2.

In Table 10, in Period 1 the recirculating users were 20.5% of the total[17] and exchanged between them 21.4% of the total volume[18]. In Period 2, recirculating users were 18.3% of the total number of users and exchanged 52. 6% of the total volume. Finally, in Period 3, they were 31.1% of the total and exchanged among them 57.7% of the total volume. In summary, a small minority of users were responsible for half of the volume exchanged on the Circles network in the last two periods, while in the first period the amount of recirculation was negligible.

	Period 1			
	Operations	**Transactions**	**Volume**	**Users**
Tot.	14 570	11 668	305 339.46	1 994
Recirc. Only	–	9 137	121 012.09	837
	Period 2			
	Operations	**Transactions**	**Volume**	**Users**
Tot.	13 457	11 674	1 839 116.48	1 380
Recirc. Only	–	7 510	1 286 352.73	461
	Period 3			
	Operations	**Transactions**	**Volume**	**Users**
Tot.	26 313	23 096	1 314 987.48	2 433
Recirc. Only	–	18 138	967 279.39	1 129

Table 10: Recirculation in Circles network. The *Recirc. Only* indicates operations happening only among recirculating users. In practice, the outgoing transaction of a recirculating user is also the incoming transaction for another recirculating user.

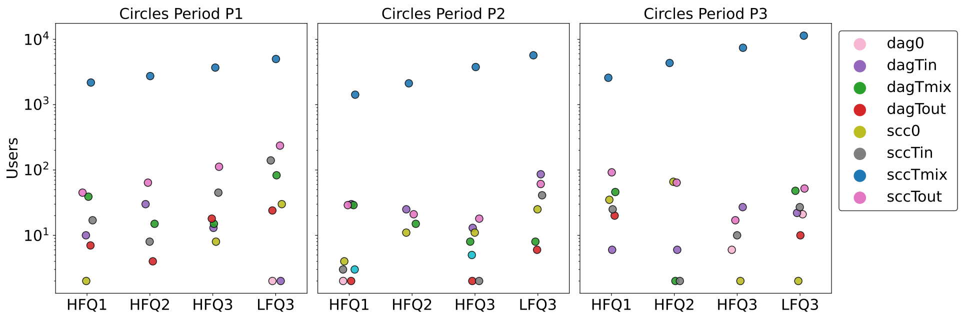
In Figure 25, almost all topological categories increase in terms of users, transactions, and

[17]This is the ratio Recirc. Only (Users) in Period 1 in Table 10 and total number of users in Period 1 (4 074, from Chapter 3)

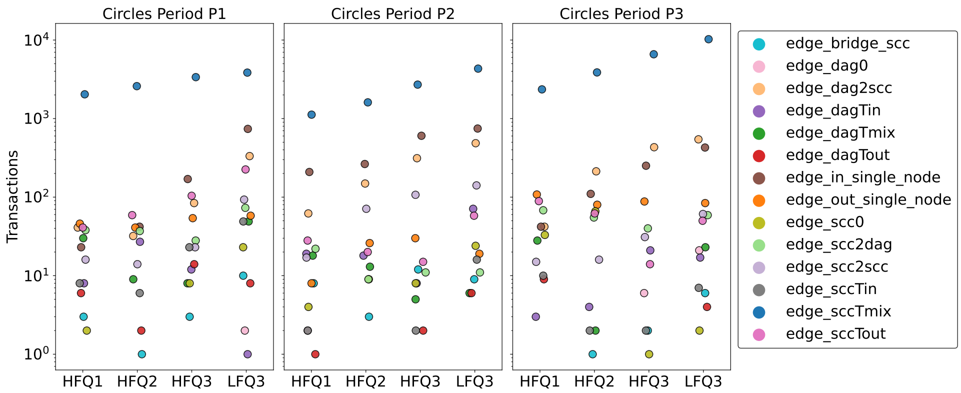
[18]This is the ratio of Recirc. Only (Volume) in Period 1 in Table 10 and total volume in Period 1 (565 283.7, from Chapter 3)

volume with respect to their speed of recirculation. For instance, ”fast” operations (*HFQ1*) have generally less users, transactions and volume than ”slow” operations (*LFQ3*). This is generally what we expect in a normal economic network: most users recirculate currency on a weekly basis, and large transaction volume moves slower and less frequently. This was not the case in Sarafu, where we generally observed the same activity across temporal categories. One possible explanation is that users were rewarded and / or penalised weekly (and then monthly) for their usage (see also Criscione (2024) and Mattsson, Criscione, and Ruddick (2022)).

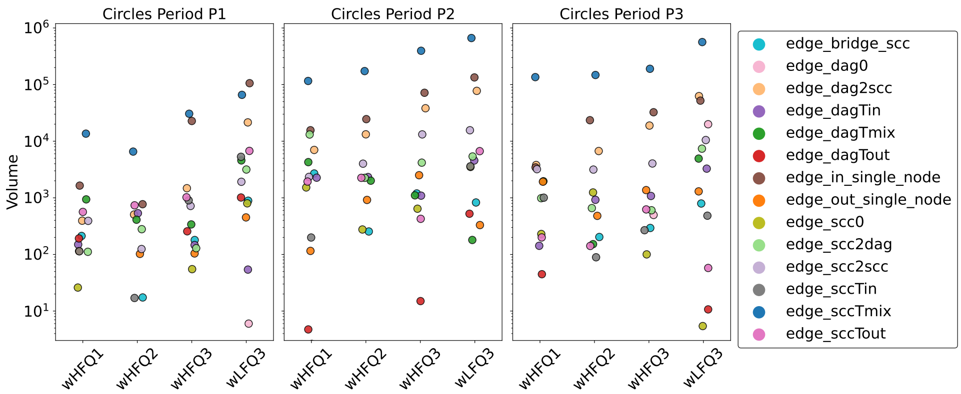
Figure 25(a) shows that the most important categories in terms of number of users are *sccTmix*, *sccTout*, *dagTmix*, *dagTin*, and marginally *sccTin*, with very little variation in all periods and temporal categories. Figure 25(b) shows the most important categories in terms of number of transactions are *sccTmix*, *in_single_node*s, *edge_dag2scc*, and *out_single_node*s. There are very little variations between periods. For instance, in the *edge_scc2scc* components in Periods 1 and 2 there were more transactions happening than in Period 3. Similarly, in Period 1, *sccTout* had a more important role in terms of number of transactions. These observations are also confirmed in Figure 25(c), the categories with more number of transactions also moved more volume, with the only exception of *out_single_node*s. In particular, in the *sccTmix* components there was very little recirculation in Period 1 (10 months long; 116 597.67 Circles units in volume), the highest amount in Period 2 (19 months long; 1 349 120.72 Circles units in volume) and a very high amount in a short time in Period 3 (6 months long; 1 037 278.57 Circles units in volume). In Circles, as in Sarafu, the second largest category that played a key role in recirculation is *in_single_node*. Also *in_single_node*s provided the highest amount of liquidity used for recirculation operations especially in the second period (245 694.91 Circles units in volume), while in the first and third periods the volume sent to cyclic components was similar (131 050.86 Circles units in volume, Period 1; 111 469.08 Circles units in volume, Period 2).



(a) Users



(b) Transactions

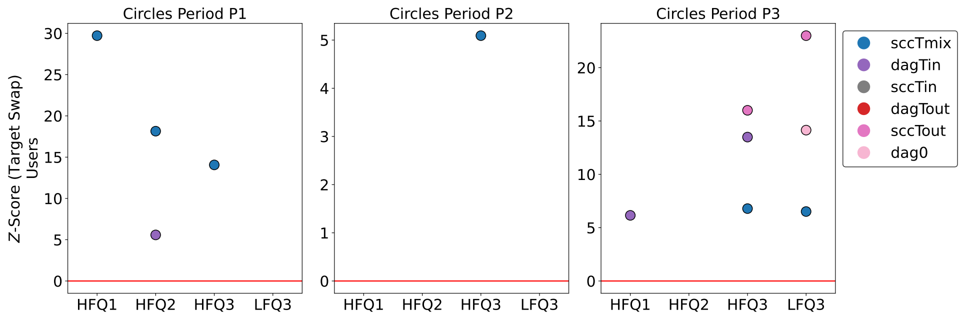


(c) Volume

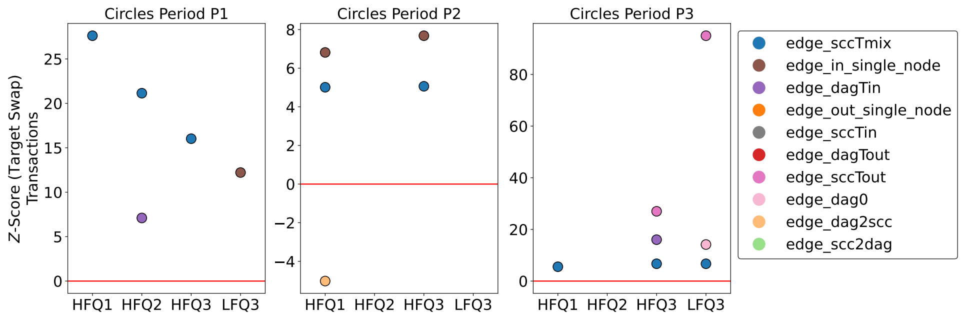
Figure 25: Recirculation in Circles per topological group. When the same transaction is taking part into two recirculation operations its weight is split into two. This explains the meaning of the "w-" as prefix of each temporal category in Figure (c).

The statistical significance of these findings can be studied by comparing them with null models. In the first period, the number of users (Figure 26(a)) and the number of transactions (Figure 26(b)) is over-represented for the *sccTmix* category in the "fastest" recirculation operations (less than HFQ3; that is, less than ~ 5 days in Period 1). In the second period, the number of transactions (Figure 26(b)) is over-represented for the categories of *in_single_node*s and *sccT-*

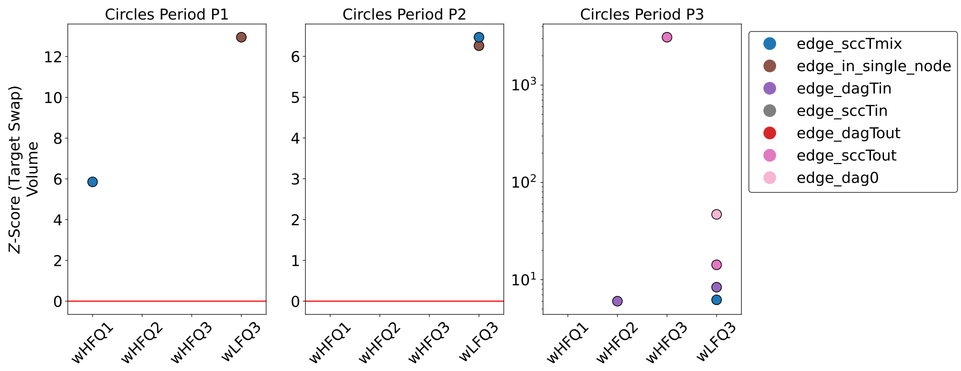
mix (less than HFQ3; that is, less than $\sim$ 2 weeks in Period 2). In the third period, the presence of recirculation operations becomes significant for *sccTout*, *dagTin*, *dag0*, and *sccTmix*. In this period, most of the significant volume is moved by "slow" operations for more than $\sim$ 1 week (more than *LFQ3* in Period 3). In the last period, *sccTout*, *dagTin* and *dag0* made a significant contribution to the number of transactions and volume exchanged. In this period, the LSCC is of *sccTout* type.



(a) Users



(b) Transactions



(c) Volume

Figure 26: Significance of recirculation in Circles per topological group. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. Only null models with ”target-edge” swap are considered in this Figure. Other null models are compared in the Appendix (Section 7.7). Besides *sccTmix* components, consider that other types of strongly connected components are generally not present in null models due to the randomization of the empirical graph. Their absence in the plots is not due to lack of significance, but the opposite.

Recirculation findings need to be complemented with data on one-time usage. Figure 27 shows that most of the one-time users are associated to *in_single_node*s, *out_single_node*s, and *dag0* components. However, *out_single_node*s moved a limited amount of volume[19], and this could confirm the explanation that this is a practice associated simply with onboarding new

[19] 2 355.29 Circles units in Period 1; 2 870.23 Circles units in Period 2; 4 351.25 Circles units in Period 3

members. The high presence of one-time users in *dag0* could be associated with a group of members who tested the system and their activity increased during the last two periods[20]. Finally, the major role played by *in_single_node*s is left to be explained. An explanation could be the creation of fake accounts to exploit the system, as in Sarafu (Criscione (2024)). However, peer validation in Circles should theoretically avoid these kind of practices (i.e. Sybil attacks) (see Chapter 3). The question is left open, but the data show the presence of accounts that were used one time to send Circles units to users in strongly connected components. In general, *in_single_nodes* moved the largest share of volume among one-time users: 29 906.96 in Period 1, 27 903.65 in Period 2, and 33 832.5 in Period 3 (see also Tables 16 and 17 in Appendix).

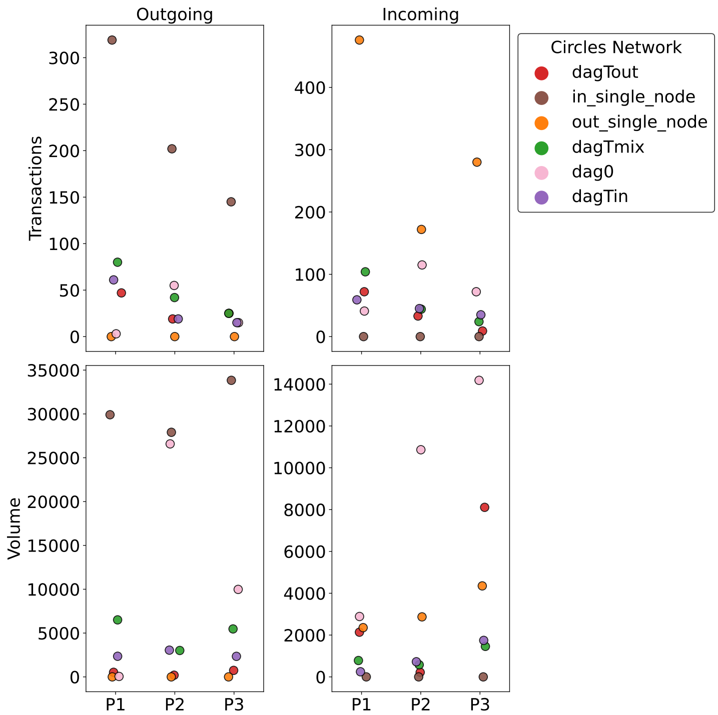


Figure 27: One-time users in Circles

In conclusion, in this section the recirculation in Circles network was analysed. The main findings can be summarised as follows. First, recirculation is happening at a slower pace than in Sarafu. Second, a large part of the recirculated volume is moved by *in_single_node*s (similar to Sarafu) and *dag0*, and this is statistically significant mostly in the third period. Third, connections across strongly connected components moved a significant amount of volume mostly in the second period (*bridge_scc*, *edge_scc2scc*). Fourth, the presence of one-time users in

[20] 2 939.1 Circles units in Period 1; 37 443.63 Circles units in Period 2; 24 162.51 Circles units in Period 3

*in_single_node*s, *out_single_node*s, and *dag0* components is not negligible. In the next section, the systemic state of Circles over time is also analysed in relation to these findings.

5.2.3 Circular Network Synergy

In this section, the state of the Circles network is analysed using the four metrics presented in the Methods section 4.4. That is, the ascendency ratio, the systemic reserve ratio, the *circular network synergy*, and the economic multiplier. The network metrics and the economic multiplier are calculated in LWCC only. The exclusion of components outside of LWCC allows one to focus on the main core of the economic network. Especially because of the significant presence of small *dag0* components which in both networks can be related to users who are simply trying the system.

In Figure 28, the values of the network metrics are reported for each period. In Period 1, the *circular network synergy* is almost zero. This means that in the first period, the number of directed weighted cycles is negligible. Only in Period 2, the *circular network synergy* grows to 1%, and in Period 3 it reaches 7%. This means that only in the last two periods has some economic synergy been created. This is also reflected by the slow convergence of the "ascendency ratio" and the "systemic reserve ratio", as in Sarafu. In fact, according to Ulanowicz et al., 2009 these two values should balance each other; the first can be interpreted as used capacity (i.e. expressed potential) and the second as unused capacity (i.e. unexpressed potential). Consequently, the multiplier was also growing with time. The network metrics adopted in this section fail to capture the spike of the multiplier in Period 2, but flipped their position showing a spike in "systemic reserve ratio". One possible explanation comes from a previous work (Avanzo et al., 2023). In that work, the authors observed that in that period most of the subsidised business partners were mostly accepting Circles as a means of payment but regularly cashing out at the end of the month. The authors also observed a core-periphery structure in that period, where the core is made up of the subsidised businesses. Thus, in the second period, the increase in the multiplier is probably not related to any relevant synergistic effect.

	Period 1	Period 2	Period 3
Ascendency Ratio	0.58	0.43	0.51
Reserve Ratio	0.42	0.57	0.49
Synergy	0.00	0.01	0.07
Multiplier	0.12	0.26	0.14

Table 11: Systemic metrics of Circles circulation over time.

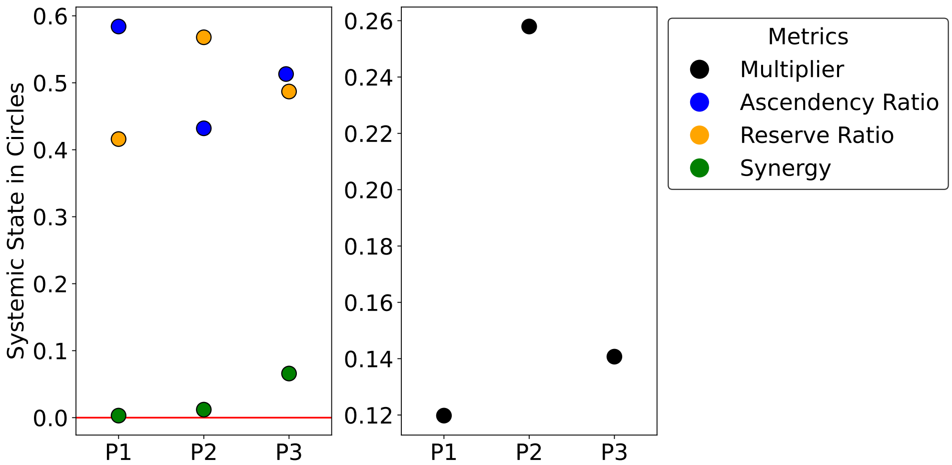


Figure 28: Systemic state of Circles over time. In the case of Circles, the injected liquidity is equal to the universal basic income provided in Circles units. In Circles, an increasing level of ”synergy” is associated to a convergence of ”ascendency” and ”systemic reserve” as well. This means that probably in the last period the system was going towards a stabilization phase. This explains also the increasing level of multiplier over time. Nonetheless, the level of ”synergy” is still below 0.1 and the multiplier below 0.26. In conclusion, even though the system was going towards a maturity phase, the evidence of a ”synergy effect” and/or ”multiplier effect” was still too low.

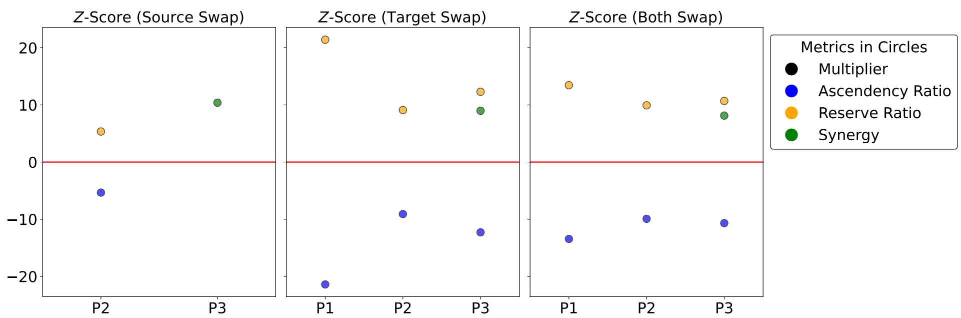


Figure 29: Significance of systemic state metrics of Circles over time. *Circular network synergy* is significant only in the third period. In a complementary way, the *reserve ratio* is over-represented, while the *ascendency ratio* is under-represented. This can be interpreted as if the system had a lot of unused capacity, especially in the first two periods.

Finally, the significance of the network metrics is compared with three types of null models. In Period 1, the network metrics are not significant for ”source-edge” swap null models. In the second period, *circular network synergy* is not statistically significant, while in the third period it is. This confirms the suspect that the subsidy programme in Period 2 probably failed to create economic synergy in the network. In other words, the presence of weighted directed cycles was too low to be statistically significant. In Period 3, only close to the end of the subsidy programme, the synergetic effect started to be significant. This is partially confirmed by a recent qualitative study (Longo et al., 2024), in which some economic synergy among businesses was reported in that last period.

5.3 Comparison

In this section, a brief comparison of the results for the Sarafu and Circles networks is reported. In the Sarafu network, the empirical findings can be summarised as follows. Across all periods, cyclic components are the largest, and their presence is statistically significant and over-represented. Other topological categories are generally under-represented, with the only exception of *dag0*, namely isolated acyclic components. This is also reflected in the recirculation analysis, with more than 20% of the users recirculating around 85% of volume in the first two periods. However, in Period 3 the share of recirculated volume decreased to 74%, and generally recirculation had less statistical significance. A particular role in recirculation was played by *in_single_node*s which sent to cyclic components about 2 million of Sarafu in the first two periods and then almost stopped in Period 3. About 60% of *in_single_node*s were accounts used only once. These findings are also reflected in the systemic state analysis. First, the highest multiplier was in Period 1, when the injection by *in_single_node*s was the highest. Second, the highest *circular network synergy* level is in Period 2, when the recirculated volume was also the highest. Finally, both multiplier and *circular network synergy* decreased drastically in Period 3, when recirculated volume reduced, and volume in LWCC shrunk by 1 order of magnitude. For a detailed recap, Table 12 summarises the main findings and also provides some context. For all the reasons mentioned above, it is possible to conclude that Sarafu was indeed used as a primary currency, especially in the first two periods. A primary currency is a currency used as means of payment to finalise daily (ordinary and extraordinary) trades.

In Circles network, the empirical findings can be summarised as follows. Period 1 is a period of not significant activity. In Period 2, a core-periphery structure is observed. Unlike in Sarafu, in Periods 2 and 3, the presence of acyclic components is as significant as that of cyclic components. In these two last periods, 20-30% of the users recirculated about 50% of volume. Most of one-time users are located in *dag0*, *in-* and *out- single-nodes*, with volume peak in Period 2. The level of *circular network synergy* (CNS) is very low in Circles, but slowly increases in the last two periods, and statistically significant only in Period 3. In Period 2, the multiplier and the *systemic reserve ratio* show a peak, not registered by the CNS. This is probably due to one-time usage and circulation initiated by *in_single_nodes* and happening through acyclic components. In this period, the network was also shaped as a core-periphery structure (subsidised businesses in the core; Avanzo et al., 2023), which is also reflected in the low number of recirculating users. The CNS is significant only in Period 3, when the number of recirculating users and the volume of recirculation increased. For a detailed recap, Table 13 summarises the main findings and also provides some context. For all the reasons mentioned above, it is possible to conclude that Circles was indeed used as a secondary currency, especially in Periods 2 and 3. A secondary currency is a currency used to complement the means of payment to facilitate otherwise unaffordable trades. This is also confirmed by a recent qualitative study (Longo et al., 2024), in which the authors found that some users were using Circles to buy products that were otherwise unaffordable to them.

There are a few key considerations that can be made in the comparison of the two systems. In the first period, both systems were launched adopting different strategies. Sarafu token was launched during the first wave of COVID-19 emergency as part of a community disaster response strategy in collaboration with the Kenyan Red Cross. In this period, the recirculation was apparently very efficient, but also caused by two factors. First, because of a system of rewards and penalties (demurrage), we cannot measure the exact economic impact of such stimulated and fast recirculation. Second, one-time usage from single-nodes extensively contributed to such recirculation, too. On the other hand, the Circles network had a very slow start. The most mature period for Circles was the second period when the subsidy programme was fully implemented. Only in this period does recirculation reach the maximum absolute amount in terms of volume, but with the minimum number of recirculating users. In this second period, single-nodes were also contributing to recirculation, but in a comparable lesser amount than in Sarafu. The speed of recirculation was also much slower than in Sarafu. In both systems, the last period seems to be a period of consolidation. Although volume was decreasing in both cases, the number of recirculating users was increasing.

The adoption of an open-access digital currency comes with some drawbacks. In fact, in both networks, there is a large amount of one-time users and isolated components[21] which can also be a source of bias for the calculation of the economic multiplier. Furthermore, in similar projects, the presence of single-nodes and acyclic components needs to be accurately considered in the evaluation process. Single-nodes are very numerous in both networks, they represent users only interested in receiving or sending, therefore lacking further reciprocity with the network[22]. The acyclic components are groups of nodes among which the currency flows only in one direction. On the one hand, monitoring such situations can be useful in understanding where and how to intervene to close loops in the supply chains. On the other hand, acyclic components may also indicate some 'fire sale' when users are just trying to get rid of the currency as soon as possible (increasing also the multiplier), but no one is willing to accept it back. Last but not least, especially when associated to rewards or regular income, a digital currency is exposed to Sybil attacks and identity theft. This means that the creation of malicious software can also be used to take advantage of those systems. Unfortunately, such drawbacks for any currency system cannot be fully solved as long as the medium is perceived as a financial asset in itself, instead of a personal commitment towards a real economic community.

In conclusion, the use of CCSs for humanitarian aid can induce endogenous local development. A digital CCS allows for a quantitative assessment of the impact in the local economy. In this work, a novel measure was introduced for this scope, that is, the *circular network synergy*. This measure can help identify the main network process behind the *synergy effect* and the *local multiplier effect*, which is the creation of circular flow (or weighted and directed network cycles). For this reason, a monetary intervention must be complemented with suitable network

[21]For example, in both networks there are many isolated acyclic components *dag0*, which are probably linked to group of users simply trying out the system.

[22]In the context of an humanitarian project, this is an expected outcome. However, it is important to monitor such single-nodes to avoid overwhelming local businesses. In such a scenario, the synergy between local businesses needs to be sustained to avoid gridlocks and deadlocks in the payment system.

monitoring tools and with side projects aiming to stimulate *economic synergies* by reinforcing circular linkages in the local supply chain.

In the case of the Sarafu network, in Periods 1 and 2, the reward and cash-out programmes in place seem to have created some synergies, but partially depending also on liquidity injected by *in_single_node*s which sustained recirculation operations. In Period 3, the Red Cross Sarafu purchase programme stopped and the synergy decreased to 18%. In the following periods not treated in this work, the Sarafu network evolved into a decentralised producer voucher system (Ruddick, 2023a, 2023b). In the case of the Circles UBI network, in the first two periods there is no significant synergetic process going on. This means that the subsidy program[23] generally failed to trigger local recirculation, and therefore a *synergistic effect*. However, in Period 3, a low and significant level of synergy was observed.

In summary, the presence alone of a CCS is not sufficient for its recirculation, but there need to be some side projects to trigger local *economic synergy* among local businesses. In both cases, the cash-out policy was the necessary but not sufficient condition for their success. Local businesses may prefer to keep the currency as long as a synergistic effect is in place. This means that they can reliably spend and accept the currency with ordinary business partners and, therefore, they can satisfy their demand locally. This ultimately requires an analysis of the supply chain network, a preventive supply chain design, and a constant effort in supply chain management. Finally, the findings confirm that a reward program[24] in this context is highly discouraged, according also to previous studies (Criscione, 2024, Kiaka et al., 2024, Barinaga, 2020).

[23] It is called ”subsidy programme”, but in fact, it is a cash-out policy for a limited number of local businesses.

[24] The reward scheme was applied mostly in three cases: 1. bringing new users, 2. usage in volume, 3. recirculation, measured as triadic closure

	Period 1 (P1)	**Period 2 (P2)**	**Period 3 (P3)**	**All Periods**
Dates	25/01/2020 - 06/08/2020 (6 months)	06/08/2020 - 31/01/2021 (6 months)	31/01/2021 - 15/06/2021 (5 months)	
Nodes	22022	14687	11692	
Transactions	171223	122031	61816	
Directed Links	71792	46931	28150	
Volume	83 583 846.76	82 030 707.46	10 089 581.45	
Fidelity	$0.999 \leq X < 0.129$	$0.129 \leq X < 0.141$	$0.141 \leq X < 0.143$	
Policy	Until 05/2020: 400 SRF at registration. After 05/2020: 50 SRF at registration. Until 07/2020: cash-out through savings groups. Reward schemes in place.	After 07/2020 until 12/2020: cash-out through vendors. After 12/2020: cash-out programme stopped, only in-kind donations. Reward schemes in place.	Only in-kind donations.	
Topology	(+)in_single_nodes. (+)dag0(*).	(-)in_single_nodes. (+)out_single_nodes. (+)dag0(*).	(–)Volume. (-)in_single_nodes. (+)out_single_nodes. (+)dag0(*).	(+)sccTmix, dagTmix, dag0, in_single_nodes(*). High diversity and big size of cyclic components.
Recirculation	Users:22.7%. Volume:84.6.%. (+)in_single_nodes, mixed(*).	Users:22.3%. Volume:86%. (–)Volume, (–)Users. (+)in_single_nodes, mixed(*).	Users:33%. Volume:74%. (–)Volume, (++)Users.	(+)sccTmix,fast(*). One-time users:in_single_nodes, dagTin, dag0, out_single_nodes. 60% of the volume spent by in_single_nodes was coming from one-time users.

Table 12: Summary of main findings in Sarafu network. ”(+)” indicates an increased importance for the component considered. ”(++)” stands for ”more than the previous period”. ”(*)” indicates a statistical significance in any of the topological features considered (nodes, links, transactions, volume). SRF stands for Sarafu tokens. In Recirculation, ”fast”, ”slow”, ”mixed” indicate where the presence was significant in fast operation (HFQ1, HFQ2, HFQ3), slow operations (LFQ3), or in mixed categories.

	Period 1 (P1)	**Period 2 (P2)**	**Period 3 (P3)**	**All Periods**
Dates	16/10/2020 - 18/11/2021 (11 months)	18/11/2021 - 29/06/2023 (16 months)	29/06/2023 - 14/12/2023 (6 months)	
Nodes	4074	2510	3622	
Transactions	18534	15884	36278	
Directed Links	8943	5611	17590	
Volume	565 283.7	2 444 959.09	1 676 052.06	
Fidelity	$0.975 \leq X < 0.129$	$0.129 \leq X < 0.137$	$0.137 \leq X < 0.107$	
Policy	Until 05/2022: 8 CRC/day basic income. Business subsidy programme starts in 07/2021 (cash-out at 1:1).	After 05/2022: 24 CRC/day basic income; cash-out subsidy at 1:10. Full implementation of business subsidy programme.	Announcement end of subsidy programme from 09/2023.	
Topology	(+)acyclic(*).	(+)cyclic(*). (-)acyclic(*). (+)dag0(*). (+)out_single_nodes.	(+)dag0(*). (+)out_single_nodes(*).	(+)sccTmix(*). Less diverse and less relevance of cyclic components than in Sarafu.
Recirculation	Users:20.5%. Volume:21.4%. (+)in_single_nodes, slow (*). Only fast operations in sccTmix(*).	Users:18.3%. Volume:52.6%. (++)Volume;(–)Users. (+)in_single_nodes, mixed(*).	Users:31.1%. Volume:57.7%. (++)Users;(–)Volume. (+)acyclic, slow(*). (+)dag0, slow(*).	(+)sccTmix (sccTout in P3)(*). Slow recirculation dominant across all periods. One-time users: in_single_nodes, out_single_nodes, and dag0 components.

Table 13: Summary of main findings in Circles network. This table shows how the structure and dynamics of the network changed accordingly to changes in policy. ”(+)” indicates an increased importance for the component considered. ”(++)” stands for ”more than the previous period”. ”(*)” indicates a statistical significance in any of the topological features considered (nodes, links, transactions, volume). CRC stands for Circles units. In Recirculation, ”fast”, ”slow”, ”mixed” indicate where the presence was significant in fast operation (HFQ1, HFQ2, HFQ3), slow operations (LFQ3), or in mixed categories.

6 Conclusion

The contribution of this work covers several aspects related to the study of economic networks, payment systems, and especially CCSs. In Chapter 2, it was mentioned that the literature on CCSs generally lacks quantitative methods to test the socioeconomic impact of those projects. In addition, there is a lack of methods to test the statistical significance of quantitative findings. In this work, an assessment method based on network science was proposed and empirically tested on two community currency networks, the Sarafu token in Kenya and Circles UBI in Berlin, Germany.

Sarafu token network is a digital CCS used as a payment system in Kenya and organised by the non-profit organisation Grassroots Economics (Mattsson, Criscione, and Ruddick, 2022). In the period analysed, from 25 January 2020 to 15 June 2021, it was used as part of an emergency cash transfer programme during the COVID-19 emergency (Ruddick, 2021). Circles UBI was used as a digital CCS in Berlin to distribute universal basic income. It was organised by the Circles Coop cooperative. The Circles Coop was active in Berlin (Germany) from 16 October 2020 to 14 December 2023 (Avanzo et al., 2023; Longo et al., 2024; Papadimitropoulos and Perperidis, 2024).

In both networks, three main periods are identified (Data section 3) which also corresponds to periods where different policies were applied. This can facilitate the interpretation of the results that are mainly based on static metrics (that is, aggregating the temporal networks). After that, each network was divided into topological components that can reflect different types of involvement within the system (Methods section 4.2). The temporal behaviour of recirculation was also analysed (Methods section 4.3). This technique can be used to understand where and how fast recirculation is occurring in the network. Finally, network metrics and economic multiplier were measured in the networks. In particular, a novel measure called *circular network synergy* was introduced (Methods section 4.4).

In the economic literature, the *local multiplier effect* and the *synergy effect* have been described as economic network phenomena, but never directly measured as such. The *circular network synergy* (CNS) is suggested here to cover this gap (Section 4.4). CNS measures the percentage of volume that flows through weighted directed cycles. The advantage of using *circular network synergy* to assess the socio-economic impact of CCS projects is twofold. First, it can be related to an autocatalytic growth process[25], as mentioned in the Literature Review. In fact, it overcomes the limitations of the economic multiplier (Section 4.1). Second, its statistical significance can be tested using appropriate null models (Section 4.5).

The *circular network synergy* (CNS) was also tested in relation to the topology of the network and the recirculation of the currency, in response to **RQ1** and **RQ2**. To do this, the topology of the network was analysed by distinguishing it into cyclic and acyclic components, and single-nodes (Section 4.2). These techniques were developed in a previous work by the author and were successfully tested to identify anomalies in the Sarafu network (Criscione, 2024). Empirical evidence shows that the CNS is related to the structure of cyclic components and the recirculation that occurs in them. This means that a change in the number of nodes, edges, and volume in cyclic components can affect the *economic synergy* of the network, and it can be measured by the CNS.

The relation between *circular network synergy* (CNS), the evolving capacity network metrics, and the economic multiplier have also been measured in the LWCC of both networks (in response to **RQ3**). The evolving capacity of the networks has been measured using two existing metrics, the *ascendency ratio* and the *systemic reserve ratio* (Ulanowicz et al., 2009). The first can be interpreted as a measure of the evolving capacity used, while the second can be interpreted as the unused evolving capacity. In this work, empirical evidence also shows that a convergence between the two appears to be associated with an increase in *circular network synergy*. More research is needed to explain this phenomenon. However, the relationship between the CNS and the multiplier worked as expected. The CNS and the multiplier are generally monotonically related; however, there are some important differences to point out. The CNS measures autocatalytic growth (that is, the volume of currency going through network cycles), while the multiplier is only a general measure of growth. In other words, the multiplier grows when the increased exchanged volume flows through any simple path in the network, not only network cycles. The CNS grows when the volume exchanged through network cycles increases.

[25] A growth in volume sustained by cyclic structures, and therefore, currency recirculation.

As explained in the Methods section 4.1 and in the Appendix section 7.3, the flow of currency through network cycles allows recirculation over time (autocatalytic growth), whereas the flow of currency through acyclic paths does not.

The policy recommendations resulting from this work are mainly three. First, the estimation and measurement of the economic multiplier as a policy assessment technique should be complemented with network-based metrics, especially for the study of CCSs. Second, *circular network synergy* can be used not only to measure the socioeconomic impact of monetary and fiscal interventions, but also to assess the state of an entire economic network and eventually design specific network interventions. *Circular network synergy* measures the amount of liquidity that could be saved by simply coordinating the local business network using a multilateral compensation framework. In fact, the assessment of *circular network synergy* can be used to build and monitor the local business network, eventually boosting local development by directly stimulating economic network effects discussed in this work [26]. Similar projects reduce the dependency of local businesses on money, credit, the banking sector, and financial markets, ultimately protecting local economies from global crises (Lucarelli and Gobbi, 2016).

For the reasons mentioned above, a third policy recommendation is to introduce a local payment system that adopts net settlement techniques for local business networks. Similar projects have recently sprouted in Europe also with the support of local governments, such as in Bosnia and Herzegovina (Božić and Zrnc, 2023), in Slovenia (Fleischman et al., 2020), in Romania (Gavrila and Popa, 2021), but also as private initiatives such as Cycles (Buchman et al., 2024), Local Loop Merseyside[27], and LedgerLoop (Jong, 2018). In fact, the introduction of credit and debt clearing houses for local business networks can be beneficial to the economy in many aspects. First, it reduces the need for liquidity to settle firm obligations, thus reducing liquidity costs and risks (Božić and Zrnc, 2023). Second, they reduce internal debt in the economy[28], especially improving the financial condition of small and medium firms (Božić and Zrnc, 2023; Fleischman et al., 2020). Third, by helping firms save more liquidity, it gives them incentives to increase their level of investment (Božić and Zrnc, 2023). Finally, it gives local

[26] *local multiplier effect*, *synergy effect*, *agglomeration externality*, *circular cumulative causation*, etc.

[27] https://localloop-merseyside.co.uk/

[28] The 'netting' itself is justified by reducing the number of transactions and the amount of liquidity necessary to settle all the payments.

businesses the incentive to search for local partners, potentially increasing the level of *circular network synergy* and therefore triggering a long-term and sustainable local multiplier effect (as shown in the Results section 5). In conclusion, this work can also have policy implications for policies related to local economic development and financial stability, both in developing and developed countries (Gaffeo et al., 2022; Lucarelli and Gobbi, 2016).

More research is needed to empirically test *circular network synergy* and its relationship to the *local multiplier effect* and *synergy effect* in economic networks. In addition, the algorithm used in this work requires additional improvements to be used in larger networks. This algorithm works only on static networks, while other solutions have been proposed that could be more appropriate for temporal networks (Jong, 2018; Patcas and Bartha, 2014). In fact, future research is also needed to provide a systematic review of those algorithms and their applications. Finally, the CNS should also be analysed in relation to other network and economic metrics (e.g., the Gini index). As an example, the relationship between the CNS, the *ascendency ratio*, and the *systemic reserve ratio* observed in this work could be further explored using numerical and agent-based simulations.

7 Appendix

7.1 Glossary

In the following section, a comprehensive glossary is reported to explain the context of the methods adopted in this work. This section is partially taken from Criscione (2024).

- **Graph, directed graph, network** (Newman, 2018). A graph G of size V is defined by a pair $G = (V, E)$ where V is a set with elements $v_i \in V$ with $i = 1, 2, ..., N$ (*vertices*) and the set E consists of pairs of vertices v_i, v_j (*edges*). The graph is *undirected*, if the pair is unordered and it is *directed*, if the pair is ordered. The following synonyms are used: ”network” for graph, ”link” for edge, and ”node” for vertex. Graphs are represented by points (vertices) connected by lines (edges); for directed graphs arrows are drawn instead of lines.

- **Subgraph**. If $G = (V, E)$ is a graph, $G' = (V', E')$ is its subgraph if $G' \subseteq G$ and $E' \subset E$ such that if $e_{ij} \in E'$ then $\{i, j\} \in G'$.

- **Strongly Connected Component (*SCC*)** (Newman, 2018). In a directed graph, two nodes A and B are *path equivalent* if there is a path from A to B and from B to A. A *SCC* is the set of mutually path equivalent nodes. The detection algorithm for *SCC* by Nuutila and Soisalon-Soininen (1994) and Tarjan (1972) is implemented in NetworkxHagberg et al., 2008. *SCC*s have implications for the study of human behaviour when studying currency flow. Every node in a *SCC* is involved in at least one of its directed simple cycles, the length of which can vary between 2 to the size of the *SCC* itself. Consequently, every node in a *SCC* is both ”sending to” and ”receiving from” at least another node in the same *SCC*. In the context of a transaction network, single isolated nodes are not present. There are four types of SCCs identified in this paper: *sccTin*, *sccTout*, *sccTmix*, and *scc0*. If it is a strongly connected component (SCC) receiving from a DAG (or a *in-single-nodes*), it gets the suffix *-in*. If the SCC is sending to a DAG (or a *out-single-nodes*), it gets the suffix *-out*. If the SCC is both sending to and receiving from one or more DAGs (or single-nodes), it gets the suffix *-mix*. Otherwise, if it is not connected to DAGs or a single-node, the SCC component gets the suffix *-0*. In this work, *strongly connected component* is used a synonym of *cyclic component*.

- **Path** (Newman, 2018). A path on a directed graph is a sequence of adjacent edges following the same direction, where neither nodes nor edges are repeated.

- **Cycle** (Newman, 2018). A cycle on a directed graph is a sequence of adjacent edges following the same direction, where the last vertex is the same as the first one. A *simple cycle* is a cycle where no other vertex is repeated except the first (=last) one. A *simple cycle* is therefore a path where only the first (=last) node is repeated.

- **Directed Acyclic Graph (DAG)** (Newman, 2018). A DAG is a directed network without cycles. For algorithms, see NetworkxHagberg et al., 2008, Karrer and Newman (2009) and Newman (2018). In this work, a DAG is a group of nodes which are connected within themselves in acyclic way. If connected to a strongly connected component, a DAG does not get involved in any of its cycles. There are four types of DAGs identified in this paper: *dagTin*, *dagTout*, *dagTmix*, and *dag0*. If it is a DAG receiving from one or more SCCs, it gets the suffix *-out* because it is a flow going 'out' from a SCC. If the DAG is sending to one or more SCCs, it gets the suffix *-in* because it is a flow going into a SCC. If the DAG is both receiving and sending to one or more SCCs of different type, it gets the suffix *-mix*. Otherwise, when isolated, the DAG component gets the suffix *-0*. In this work, *directed acyclic graph* is used a synonym of *acyclic component*.

- **Single-node** (Newman, 2018). A single-node is a node connected to one strongly connected component without getting involved in any of its cycles. There are three types of *single-nodes* identified in this paper. If the single-node is sending to a strongly connected component, it is called *in-single-nodes* because it is a flow going into a SCC. If the single-node is receiving from a strongly connected component, it is called *out-single-nodes* because it is a flow going out from a SCC. If the single-node is receiving from a SCC and sending to another SCC (without closing a cycle), then it is called *bridge_scc*.

7.2 Entropy, Mutual Information, and Conditional Entropy

In this section a brief explanation of the four concepts used in the literature review on the *synergy effect* is provided following Cover and Thomas (2005). *Entropy* is defined as a measure of uncertainty around a random variable X. Considering a discrete random variable X in the alphabet χ and with a probability mass function $p(x) = Pr(X = x), x \in \chi$, then the entropy of X can be defined as

$$H(X) = -\sum_{x \in \chi} p(x) \log p(x) \tag{28}$$

where the log is often on base 2, so that entropy is expressed in bits. Considering two events, the definition can be extended to a pair of random variables (X,Y) which is considered a random variable with a single vector value[29]. In this way, the *joint entropy* becomes

$$\begin{aligned} H(X,Y) &= -\sum_{x \in \chi} \sum_{y \in \mathscr{Y}} p(x,y) \log p(x,y) \\ H(X,Y) &= H(X) + H(Y|X) \\ H(X,Y) &= H(Y) + H(X|Y) \end{aligned} \tag{29}$$

If one of the two events (Y) is conditioned by the other event (X), then a *conditional entropy* is defined as

$$\begin{aligned} H(Y|X) &= -\sum_{x \in \chi} \sum_{y \in \mathscr{Y}} p(x,y) \log p(y|x) \\ H(Y|X) &= H(X,Y) - H(X) \end{aligned} \tag{30}$$

The measure of the distance between two probability mass functions, $p(x)$ and $q(x)$, is called *relative entropy*, and it defined as

$$D(p||q) = -\sum_{x \in \chi} p(x) \log \frac{p(x)}{q(x)} \tag{31}$$

It can also be considered as a measure of inefficiency of assuming that the distribution is q when the true distribution is actually p. Therefore, this measure is always nonnegative and is zero if and only if $p = q$. Furthermore, the *mutual information* is the relative entropy of a joint

[29] It maps from a probability space to a vector of numbers.

distribution (X,Y) and the distribution of the product between X and Y, such that

$$\begin{aligned}
I(X,Y) &= \sum_{x\in\chi}\sum_{y\in\mathscr{Y}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)} \\
I(X,Y) &= H(X)-H(X|Y) \\
I(X,Y) &= H(Y)-H(Y|X) \\
I(X,Y) &= H(X)+H(Y)-H(X,Y) \\
I(X,Y) &= I(Y,X)
\end{aligned} \tag{32}$$

The *mutual information* measures the reduction in the uncertainty of X due to the knowledge of Y. As shown in Equation 32, the opposite is also true, since $I(X,Y) = I(Y,X)$. This means that it also measures the reduction in the uncertainty of Y due to the knowledge of X.

Finally, the Shannon-Wiener formula is derived by the definition of entropy, but with some differences to apply for the study of socio-ecological systems (Spellerberg and Fedor, 2003):

$$H(X) = -\sum_{i=1}^{S} p(i)\ln p(i) \tag{33}$$

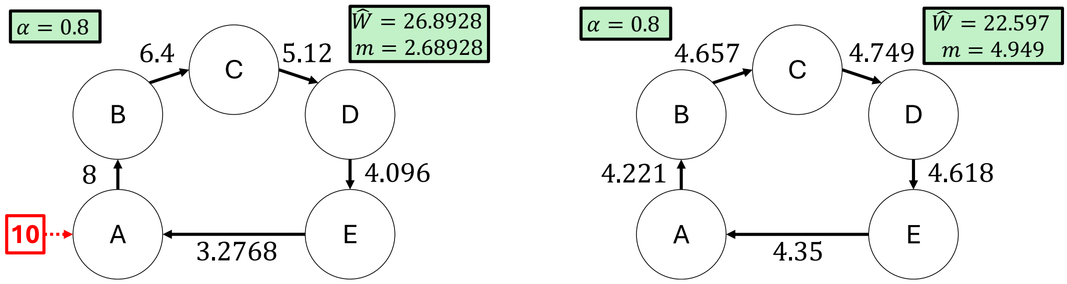
where S is the total number of species or categories and p_i is the proportion of individuals belonging to the i-th specie or category. Note that the natural logarithm is used in this formulation and that $p(i)$ is interpreted as a fraction of the i-th individuals on the total population.

7.3 Limits of the Economic Multiplier

In this section, a numerical example of the limits of the economic multiplier is provided (see the Methods section 4.1 for further details). In Figure 30, an economy structured as a cycle graph is represented. In Figure 31, an economy structured as a line graph is represented. A fixed propensity for consumption α is assumed, which is equal to 0.8. The initial injection in both economies is equal to 10. It is assumed that liquidity is injected only into the first node A. It is also assumed that nodes spend sequentially only if they receive some currency from another node. Given these assumptions, the cycle economy is the only one that allows for the recirculation of currency. In fact, the recirculation of currency can potentially go to infinite (if any rational number can be considered as an acceptable amount for a transaction). For practical reasons, only the first two periods of the cycle economy are considered.

In the first period, the cycle economy and the line economy have the same transaction volume and, therefore, the same multiplier (2.68928). In the same period, the *circular network synergy* is 0.61 in the cycle economy, while obviously it is zero in the line economy. In the second period, the *circular network synergy* in the cycle economy increases to 0.93 and the multiplier to 4.949. In the line economy, there is no second period. In such an economy, recirculation cannot happen in this network topology, and the currency that is not spent goes out of the economy.

This simple example shows that *circular network synergy* (CNS) captures a network structure and a dynamic otherwise ignored by the multiplier. In fact, in the first period the multiplier in both economies is equal, while the CNS is not. The CNS can actually measure the percentage of volume potentially recirculated. This static CNS is actually agnostic of the sequential order of payments (e.g., first A, second B, etc.). Therefore, it considers all the obligations simultaneously. However, this index does show its efficacy in measuring the volume of circular exchanges induced by the initial injection.



(a) First Period (b) Second Period

Figure 30: Example of a cycle graph economy. The topology of the network as a cycle graph allows for recirculation over time. At each round the economic multiplier increases.

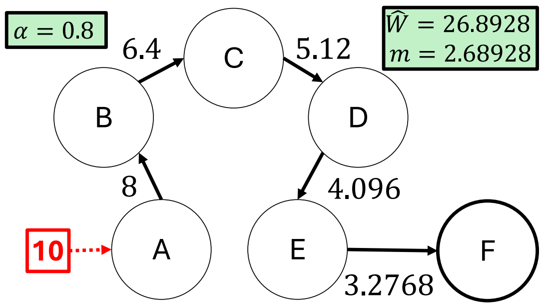
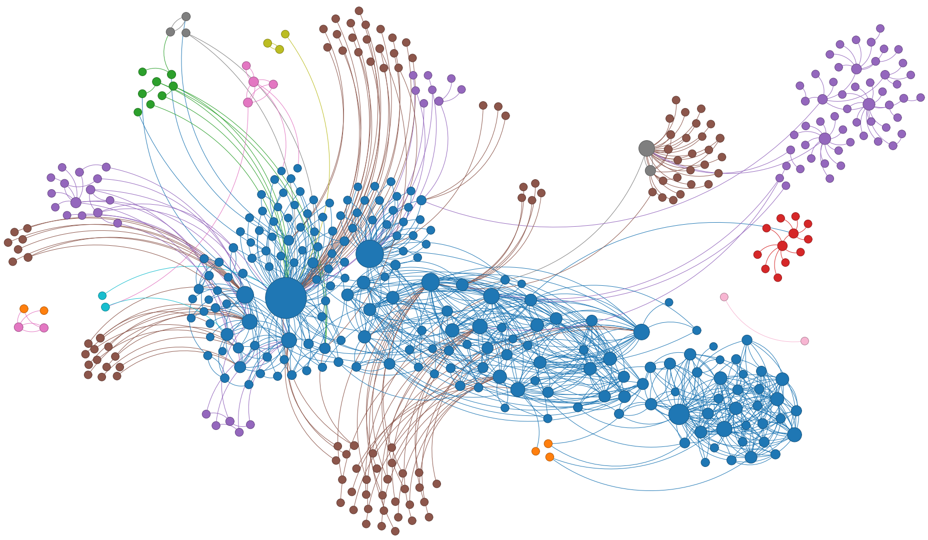


Figure 31: Example of a line graph economy. The topology of the network as a line graph does not allow for recirculation over time. The node F accumulates currency which is not put again into circulation (e.g., import, hoarding, emigration, financial investment, death). Note that the multiplier induced on the first round in the cycle graph is equal to the multiplier in the line graph.

7.4 Topological Changes per Period

7.4.1 Sarafu Network

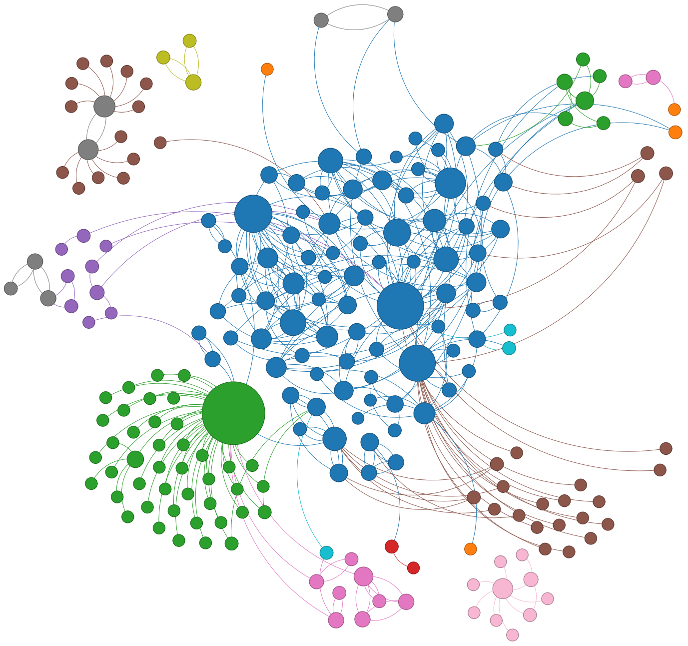
32 33 34



(a) Subgraph Sarafu Network (Period 1)

(b) Legend

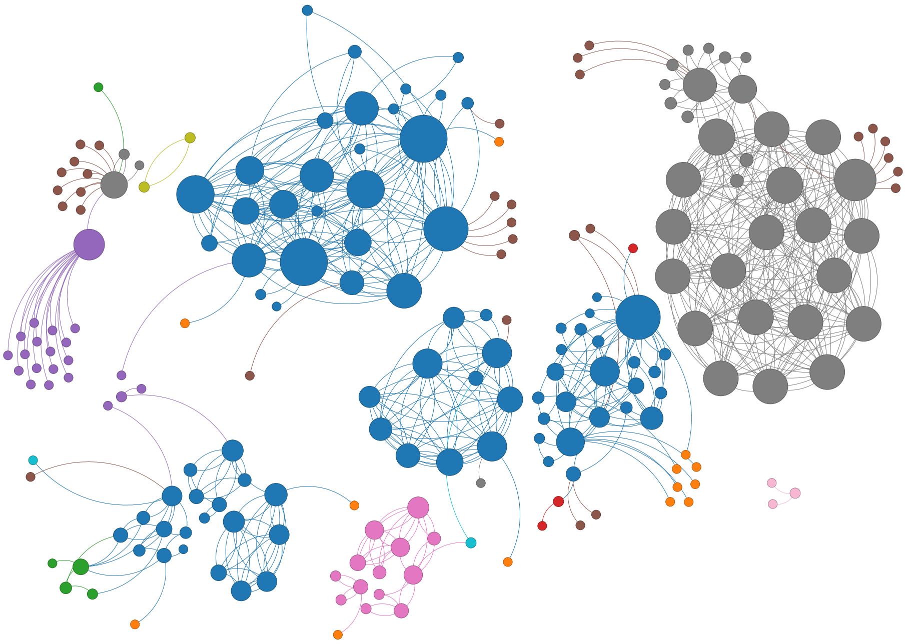
Figure 32: Subgraph of 431 nodes and 937 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by sampling nodes from the network in Period 1. Each link takes the colour of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. **Period:** 25/01/2020 - 06/08/2020 (6 months). Cash-out through savings groups. Reward schemes in place. Full lockdown (Ba et al., 2023b). **Topology:** Large Cyclic Component, LSCC (sccTmix). Significance: sccTmix*, scc0*, dag0*. **Recirculation:** 22.7% of the users recirculated 84.6% of volume. Significance: sccTmix* (<LFQ3), dagTmix* (<LFQ3), and in-single-nodes* (>LFQ3) (LFQ3: 23 hours, 58 minutes). **Synergy*:** 0.58. **Multiplier**: 2.73. (*): Positive Z-score.



(a) Subgraph Sarafu Network (Period 2)

(b) Legend

Figure 33: Subgraph of 217 nodes and 380 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by sampling nodes from the network in Period 2. Each link takes the colour of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. **Period:** 06/08/2020 - 31/01/2021 (6 months). Cash-out through vendors. Reward schemes in place. Partial lockdown (Ba et al., 2023b). **Topology:** Acyclic Uprise, LSCC (sccTmix). Significance: sccTmix*, scc0*, dag0*, dagTmix*. **Recirculation:** 22.3% of the users recirculated 86% of volume. **Significance:** sccTmix* (<LFQ3), dagTmix* (<LFQ3), dagTin* (<LFQ3), in-single-nodes* (LFQ3) (LFQ3: 2 days, 6 hours). **Synergy***: 0.61 **Multiplier**: 1.77. (*) Positive Z-score.



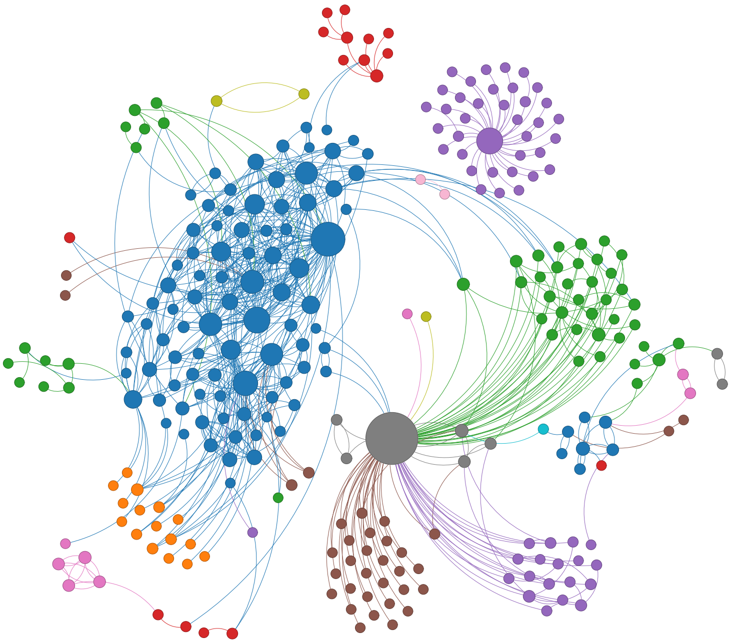
(a) Subgraph Sarafu Network (Period 3)

(b) Legend

Figure 34: Subgraph of 211 nodes and 682 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by sampling nodes from the network in Period 3. Each link takes the colour of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. **Period:** 31/01/2021 - 15/06/2021 (5 months). Only in-kind donations. **Topology:** Cyclic Fragmentation, LSCC (sccTmix). Significance: sccTmix*, sccTin*, sccTout*, scc0*, dag0*. More components of sccTmix(*). **Recirculation:** 33% of the users recirculated 74% of volume. Significance: sccTmix* (<LFQ3) (LFQ3: 2 days, 23 hours). **Synergy***: 0.18 **Multiplier**: 0.06. (*) Positive Z-score.

7.4.2 Circles Network

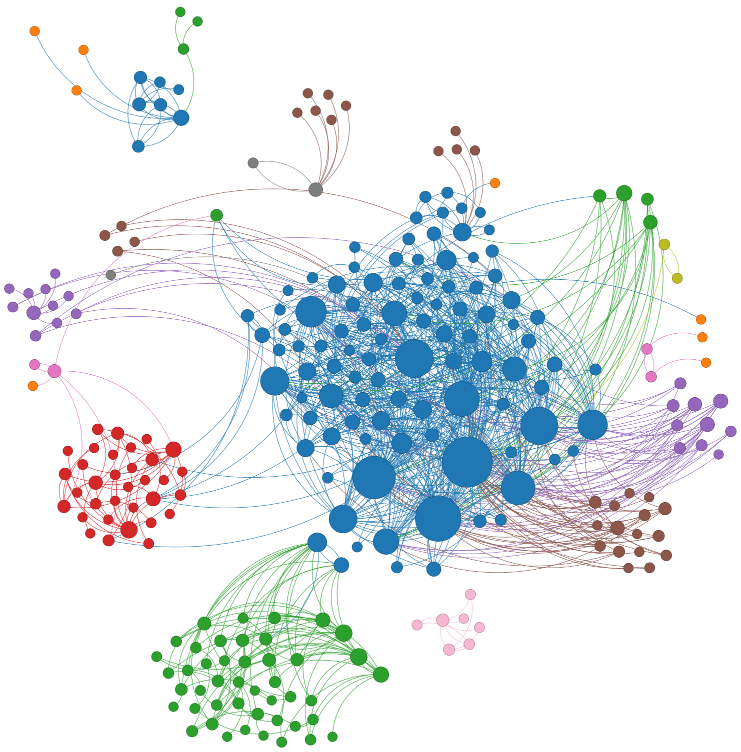
35 36 37



(a) Subgraph Circles Network (Period 1)

(b) Legend

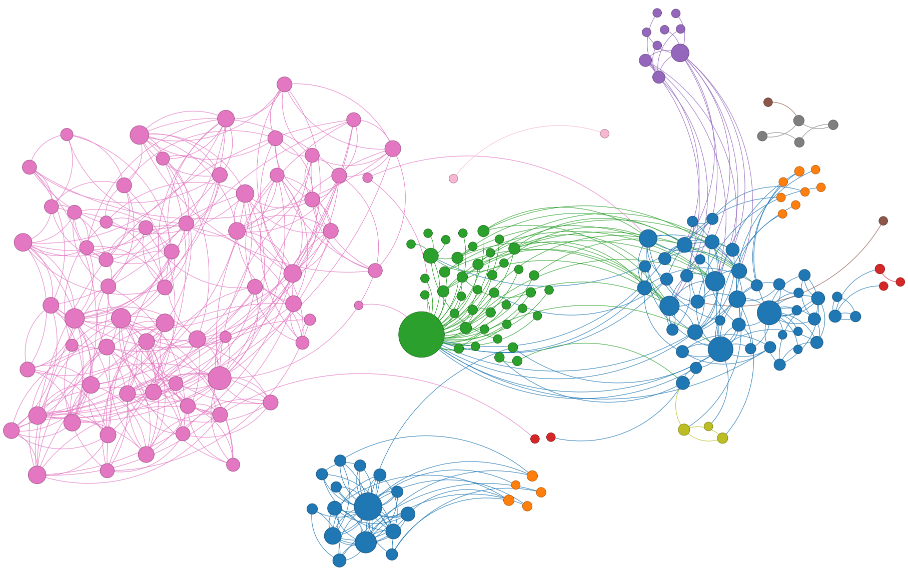
Figure 35: Subgraph of 288 nodes and 651 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by sampling nodes from the network in Period 1. Each link takes the colour of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. **Period:** 16/10/2020 - 18/11/2021 (11 months). 8 CRC/day basic income. The business subsidy programme started on 07/2021 (cash-out at 1:1)(Longo et al., 2024). **Topology:** Low-key state, LSCC (sccTmix). Significance: sccTout*, in-single-nodes*. **Recirculation:** 20% of the users recirculated 21.4% of volume. Significance: sccTmix* (<LFQ3) and dagTin* (<LFQ3), and in-single-node* (LFQ3) (LFQ3: 5 days, 4 hours). **Synergy:** ∼0.00. **Multiplier:** 0.12. (*) Positive Z-score.



(a) Subgraph Circles Network (Period 2)

(b) Legend

Figure 36: Subgraph of 267 nodes and 898 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by sampling nodes from the network in Period 2. Each link takes the colour of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. **Period:** 18/11/2021 - 29/06/2023 (16 months). After 05/2022: 24 CRC/day basic income. Cash-out subsidy programme at 1:10. **Topology:** Core-periphery (Avanzo et al., 2023), LSCC (sccTmix). Significance: dag0*, dagTout*, dagTin*, in-single-nodes*, sccTin*, sccTout*. **Recirculation:** 18.3% of the users recirculated 52. 6% of volume. Significance: sccTmix* and in-single-nodes* (any speed) (LFQ3: 2 weeks, 4 days). **Synergy:** 0.01. **Multiplier:** 0.26. (*) Positive Z-score.



(a) Subgraph Circles Network (Period 3)

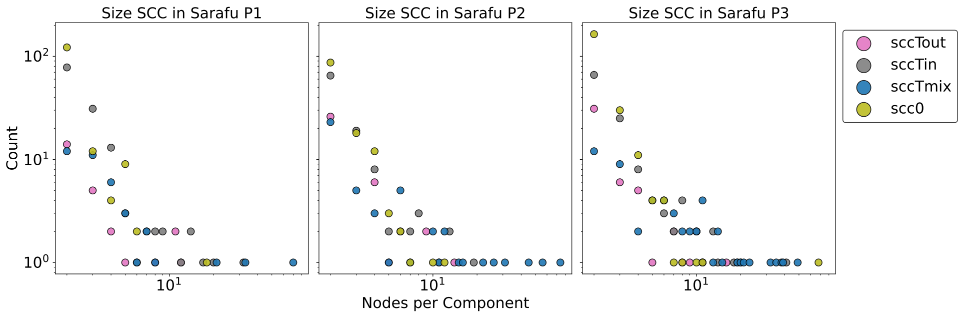
(b) Legend

Figure 37: Subgraph of 202 nodes and 535 directed links. The subgraph is created by merging ego graphs at depth 2 and 3 of 11 nodes, one random node per each topological category. The subgraph is made by sampling nodes from the network in Period 3. Each link takes the colour of the source (i.e. node sending). This plot is made out of a sample of ego graphs, hence, it is very likely that some connections are missing within the same components. **Period:** 29/06/2023 - 14/12/2023 (6 months). Announcement end of the subsidy programme from 09/2023 (Longo et al., 2024). **Topology:** Fragmentation, LSCC (sccTout). Significance: sccTout*, dag0*, dagTin*, out-single-nodes*. **Recirculation:** 31. 1% of the users recirculated 57. 7% of volume. Significance: sccTmix* (<LFQ3), sccTout* (<LFQ3), dagTin* (<LFQ3), and dag0* (LFQ3) (LFQ3: 1 week, 1 day). **Synergy:** 0.07. **Multiplier:** 0.14. (*) Positive Z-score.

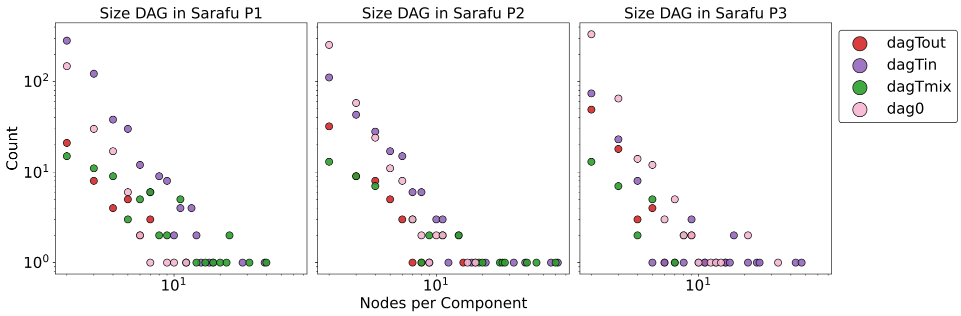
7.5 Topological Components

7.5.1 Sarafu Network

38 39 40 41 42 43



(a) Cyclic components



(b) Acyclic components

Figure 38: Size of single-nodes, cyclic, and acyclic components in Sarafu network. In Figure (a), it is possible to notice that across all periods *scc0* and *sccTin* generally have less nodes, but are more numerous than other categories; while *sccTmix* generally have more nodes, but they are much fewer than other categories. For this plot, the largest strongly connected components are excluded. In Figure (b), it is possible to notice that across all periods *dag0* and *dagTin* generally have less nodes, but are more numerous than other categories; while *dagTmix* generally have more nodes, but they are much fewer than other categories.

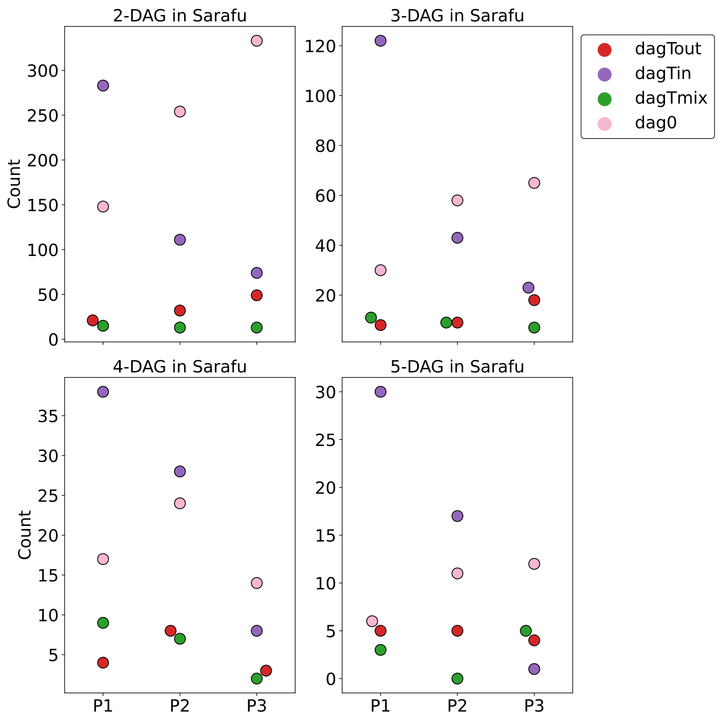


Figure 39: DAGs over time for size 2,3,4 and 5 in Sarafu Network. The *dagTin* and *dag0* dominate across all three periods. However, the number of *dag0* increases, while the *dagTin* decreases over time.

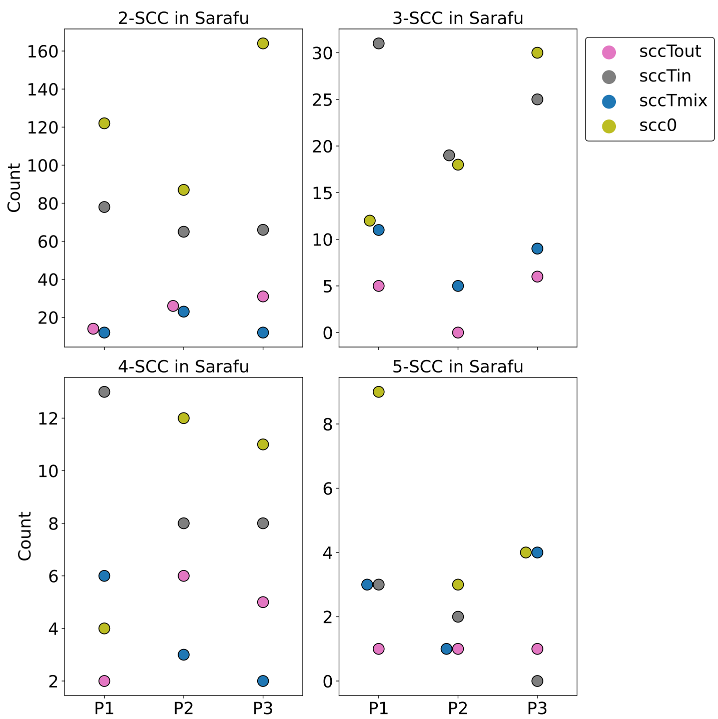
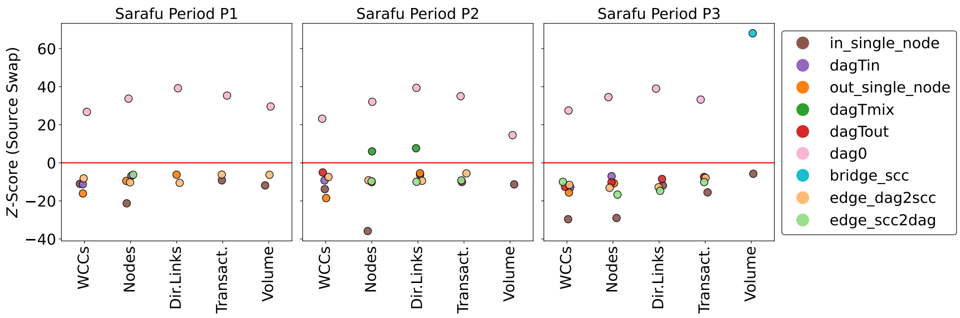
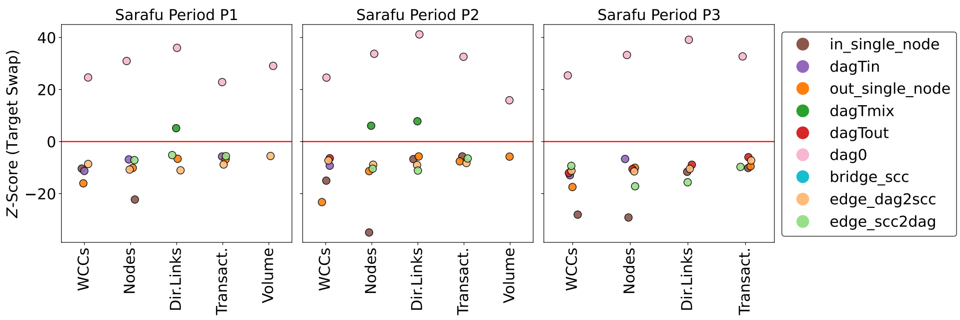


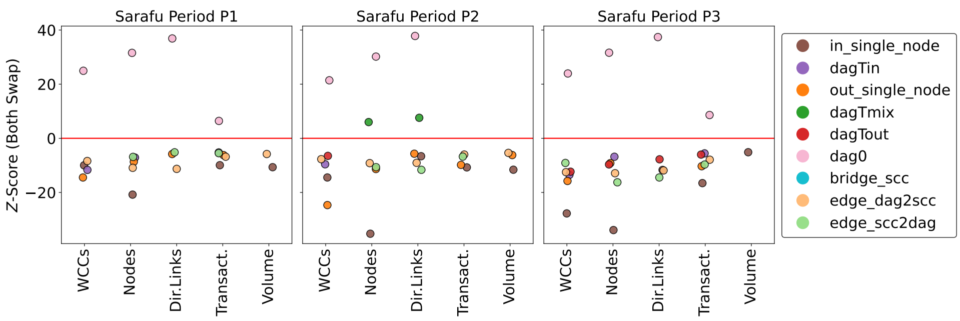
Figure 40: SCCs over time for size 2,3,4 and 5 in Sarafu Network. Remember that the largest strongly connected component (*sccTmix* type) is excluded from this plot. The number of *scc0* and *sccTin* dominate all other categories across all periods.



(a) "Source"-Edge Swap

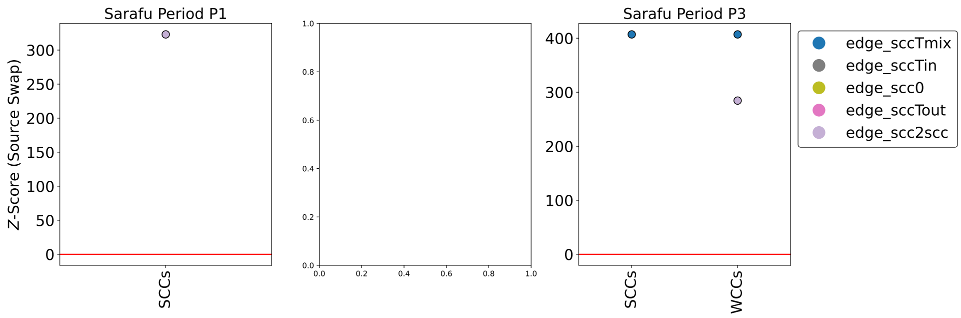


(b) "Target"-Edge Swap

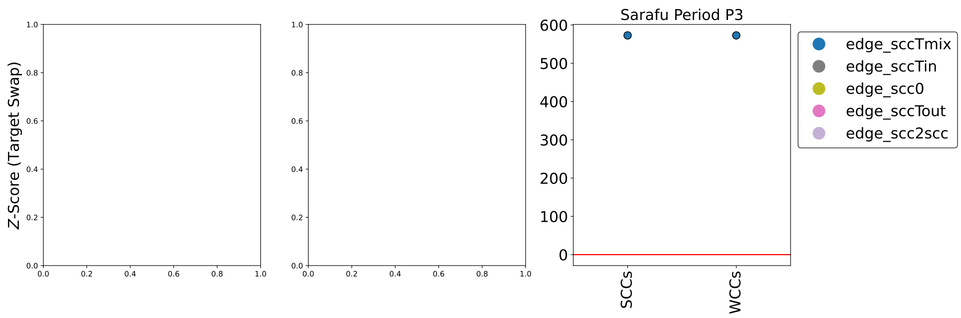


(c) "Both"-Edge Swap

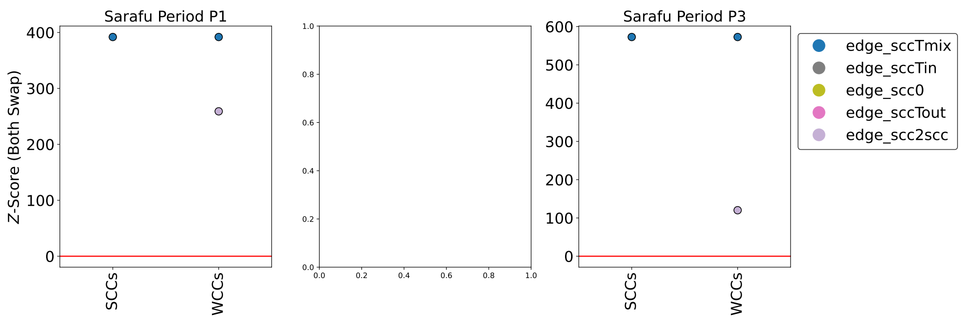
Figure 41: Significance of topological components in Sarafu over time (SCCs excluded). Comparison of different types of null models. Consider that, in the null models the randomisation often creates only one unique strongly connected component of *sccTmix* type. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. The *dag0* components are largely over-represented with respect to all null models and across all periods. Some other categories are slightly under-represented in the empirical network: *edge_dag2scc*, *edge_scc2dag*, *in_single_node*s, *out_single_node*s, *dagTin*, and *dagTout* (only second and third periods). In the second period only, *dagTmix* are slightly over-represented in the empirical network with respect to all the null models.



(a) ”Source”-Edge Swap

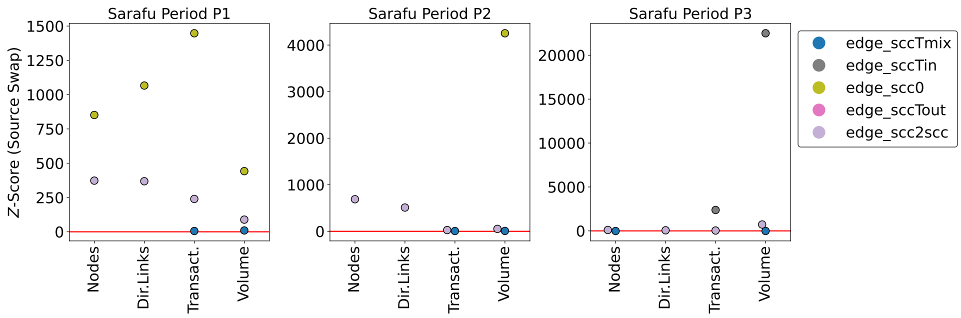


(b) ”Target”-Edge Swap

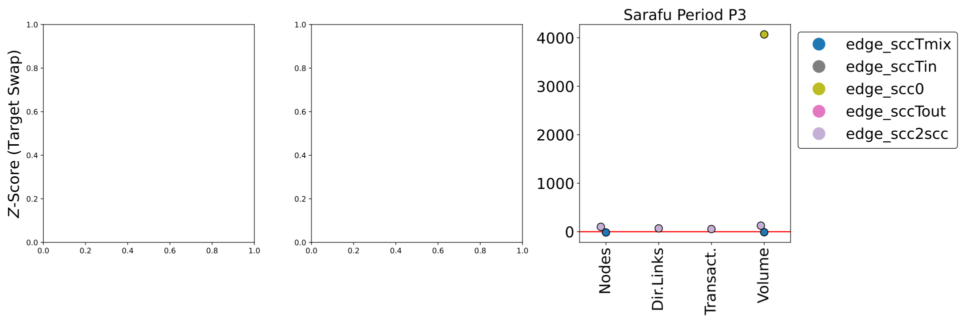


(c) ”Both”-Edge Swap

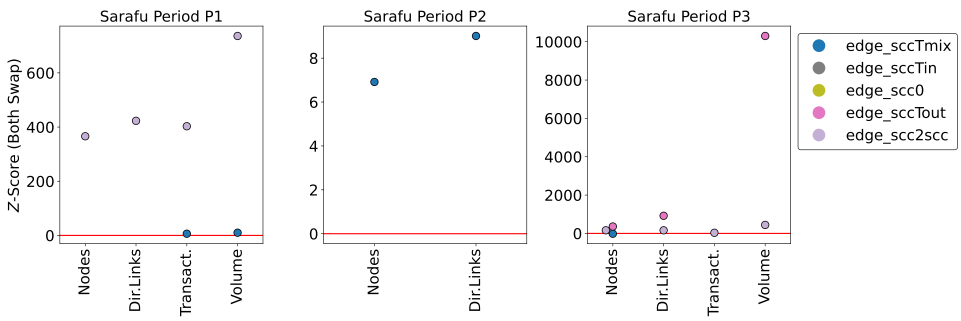
Figure 42: Significance of topological components in Sarafu over time (SCCs only, part 2). Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot. Consider that, in the null models the randomisation often creates only one unique strongly connected component of *sccTmix* type. In terms of number of components only, the *sccTmix* and *edge_scc2scc* are over-represented in the empirical network, especially in Periods 1 and 2.



(a) ”Source”-Edge Swap



(b) ”Target”-Edge Swap

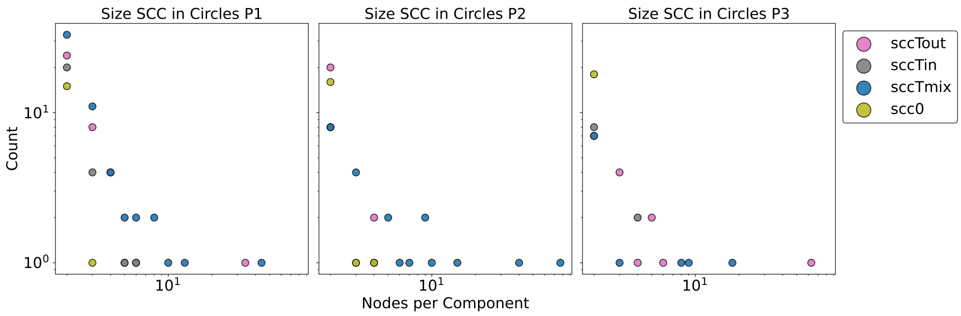


(c) ”Both”-Edge Swap

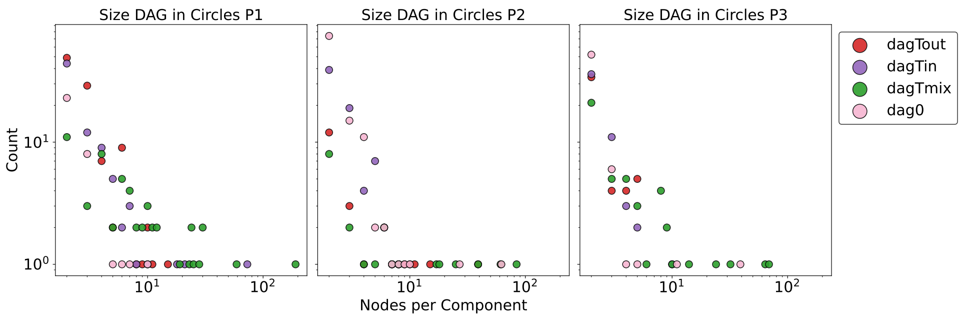
Figure 43: Significance of topological components in Sarafu over time (SCCs only, part 3). Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot. Consider that, in the null models the randomisation often creates only one unique strongly connected component of *sccTmix* type. In the first period, *scc0*, *edge_scc2scc*, and *sccTmix* are generally over-represented with respect to all the null model except one (*target-edge swap null model type*). In the second and third period, these topological categories partially lose significance, but with very little variation across all null models. As explained before, this is due to the fact that this type of randomisation generally eliminate strongly connected components. Therefore, in most the cases, only one strongly connected component of *sccTmix* type is left.

7.5.2 Circles Network

44 45 46 47 48 49



(a) Cyclic components



(b) Acyclic components

Figure 44: Size of single-nodes, cyclic, and acyclic components in Circles network.

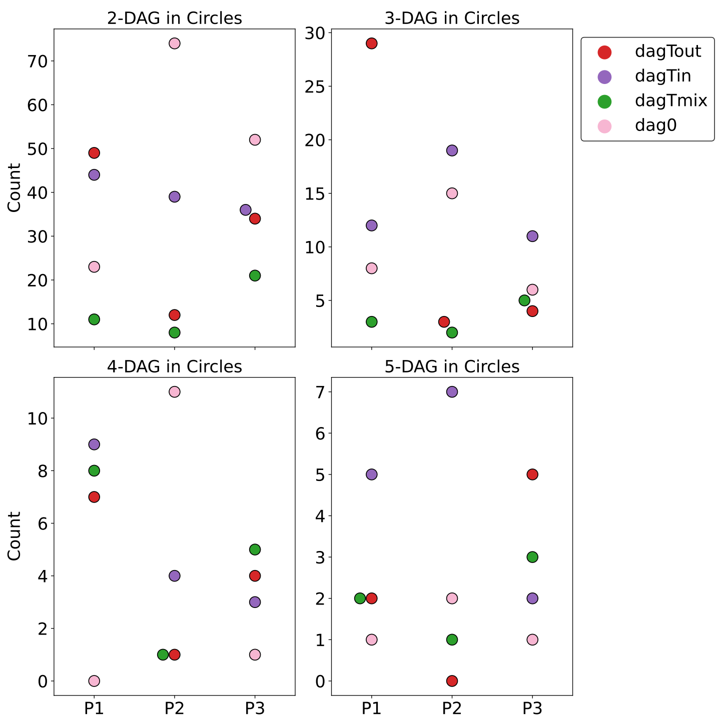


Figure 45: DAGs over time for size 2,3,4 and 5 in Circles Network. In the first period, the number of *dagTout* and *dagTin* components is generally larger than others across all sizes. In the second period, *dag0* and *dagTin* are generally larger than others across all sizes. In the third period, the number of DAGs generally shrinks and the difference across DAG-types is reduced. Although, the number of *dagTout* of size 5 increases.

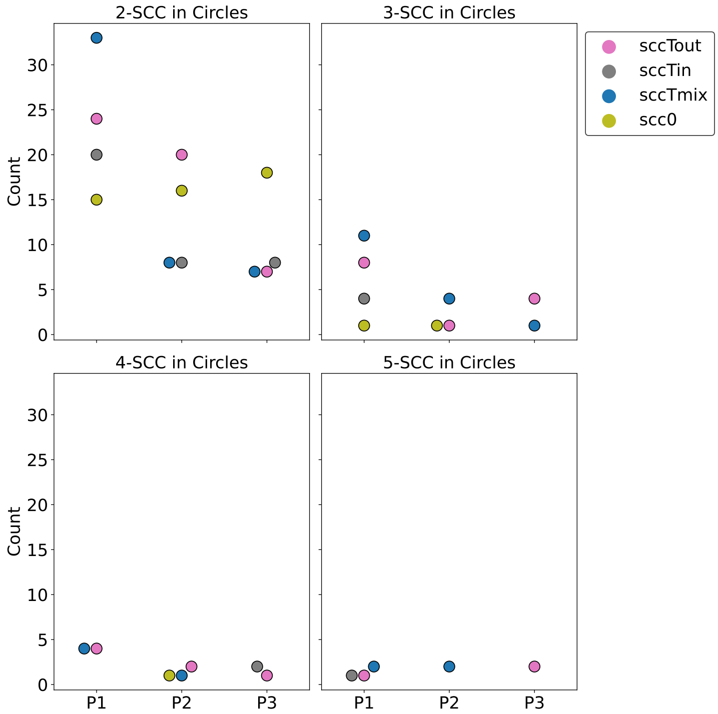
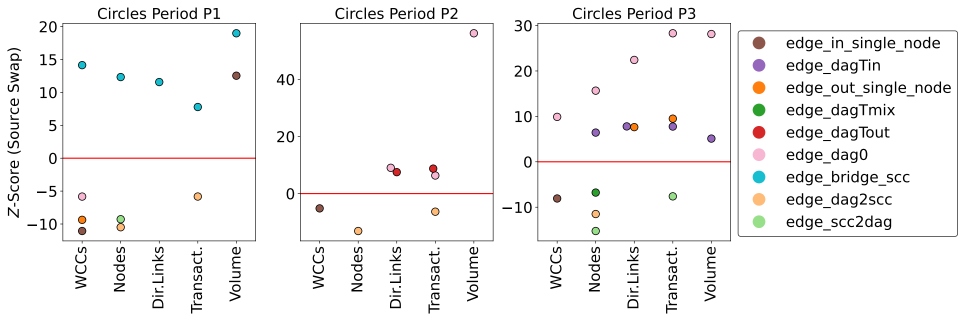
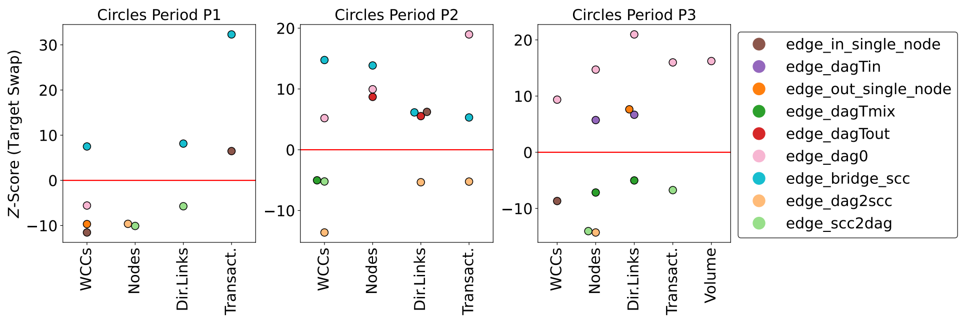


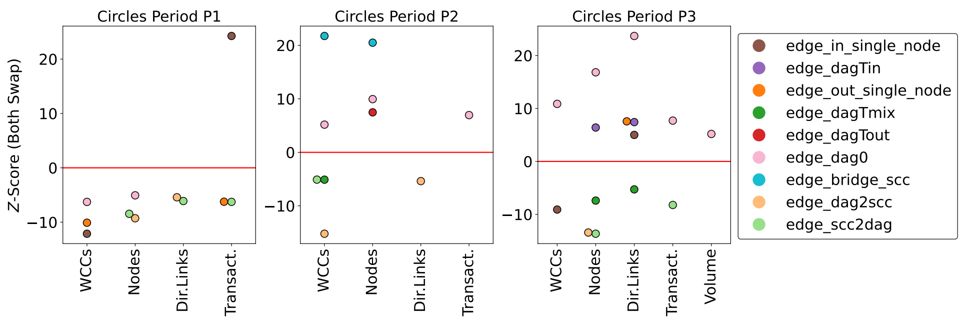
Figure 46: SCCs over time for size 2,3,4 and 5 in Circles Network. The majority of SCCs have size 2 for which *scc0* components gain more importance over time.



(a) ”Source”-Edge Swap

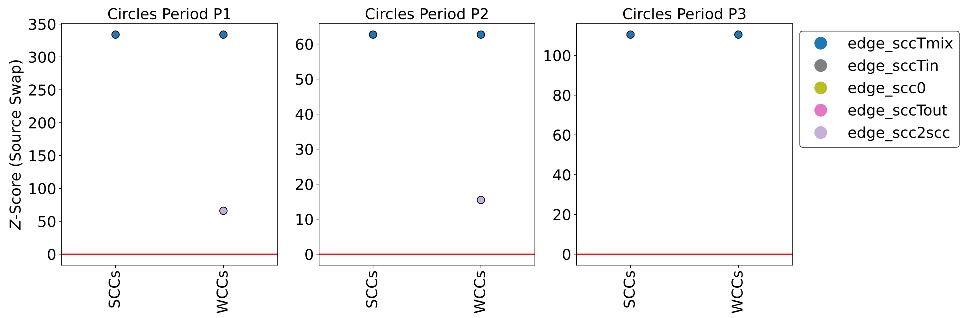


(b) ”Target”-Edge Swap

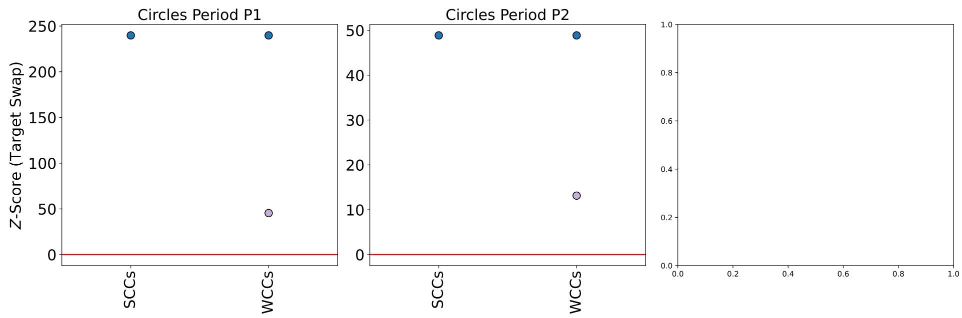


(c) ”Both”-Edge Swap

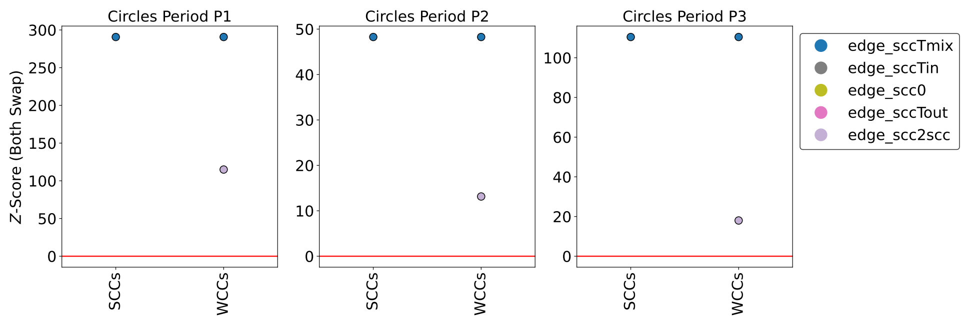
Figure 47: Significance of topological components in Circles over time (SCCs excluded, part 1). Comparison of different types of null models. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. There is some difference across model types. This means that the results of the empirical network are very sensitive to different kind of randomisation techniques. In the first period, generally the presence of nodes connecting different SCCs (*bridge_scc*) seems to play a relevant role (over-representation). The other categories are generally under-represented or not significant (i.e., absent in the plot). For instance, *edge_dag2scc* and *edge_scc2dag* are generally under-represented. The only exception is the number of transactions in *in_single_node*s which is over-represented across all null model types. In the second period, besides *bridge_scc*s, *dagTmix*, and *dagTout* components are generally over-represented in the empirical network. Also here, the connection from DAGs to SCCs (*edge_dag2scc*) are generally under-represented. Finally, in the third period, *dag0*, *dagTin* and *out_single_node*s are generally over-represented. While, *dagTmix* and *edge_scc2dag* are generally under-represented.



(a) "Source"-Edge Swap

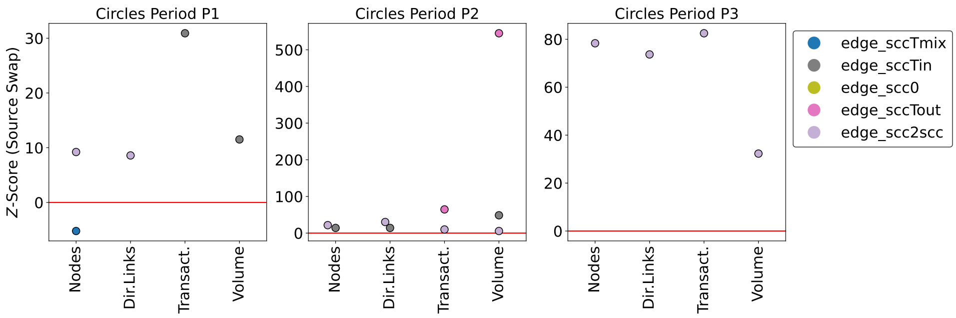


(b) "Target"-Edge Swap

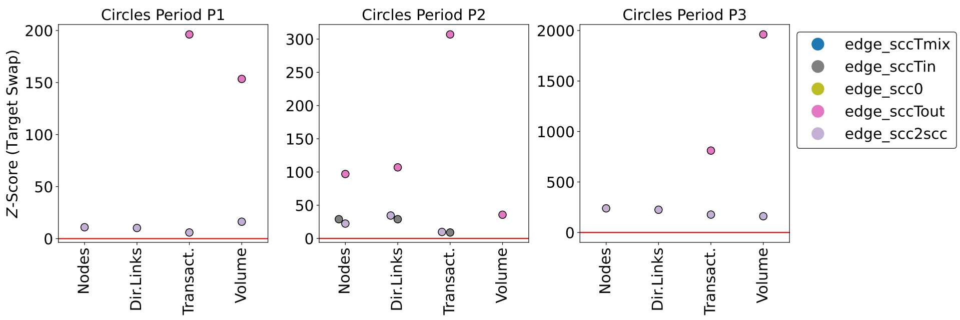


(c) "Both"-Edge Swap

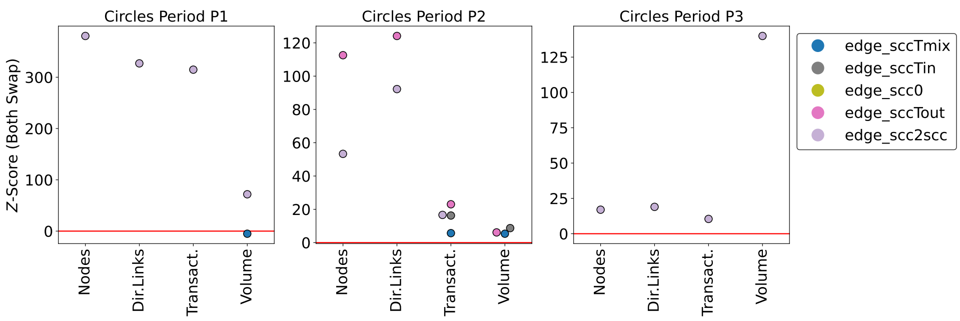
Figure 48: Significance of topological components in Circles over time (SCCs only, part 2). Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot. Consider that, in the null models the randomisation often creates only one unique strongly connected component of *sccTmix* type. The number of strongly connected components of type *sccTmix* is over-represented across all periods with respect to all the null models. The only exception is at Period 3 for "target-edge" swap configuration models.



(a) ”Source”-Edge Swap



(b) ”Target”-Edge Swap



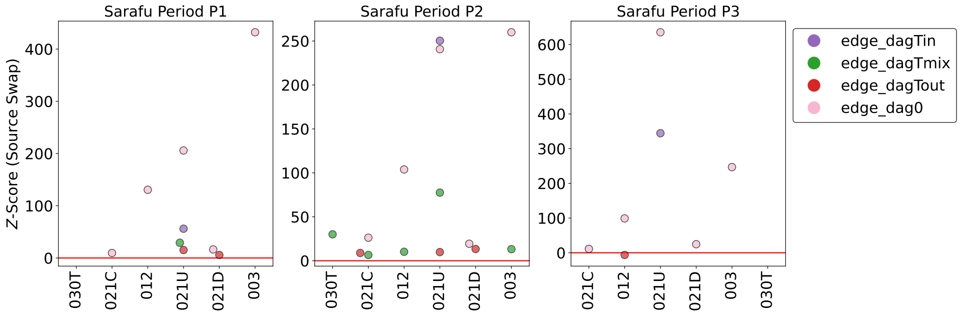
(c) ”Both”-Edge Swap

Figure 49: Significance of topological components in Circles over time (SCCs only, part 3). Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot. Consider that, in the null models the randomisation often creates only one unique strongly connected component of *sccTmix* type. The presence of *sccTout*, *sccTin*, and *edge_scc2scc* seem to be generally over-represented, especially in Periods 1 and 2.

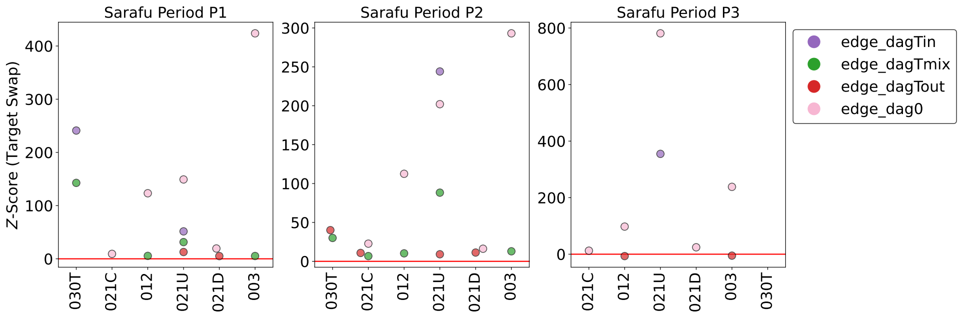
7.6 Triadic Census in DAGs

The dyadic triad 012 is a simple dyad (from A to B). The triad 021C is a “brokerage” interaction (from A to C through B - where B is the ”broker”). The triad 021U represents one central user collecting the currency of the other two (from A to B, from C to B, where B is the ”collector”). The triad 021D represents one central user sending to two other users (from B to A, from B to C - where B is the ”distributor”). The triad 003 is the ”empty” triad, a measure of potential triads over existing ones. The triads 030T is the only closed triad, where one node is sending to a dyad (source and target of a dyad are both receiving from one third node).

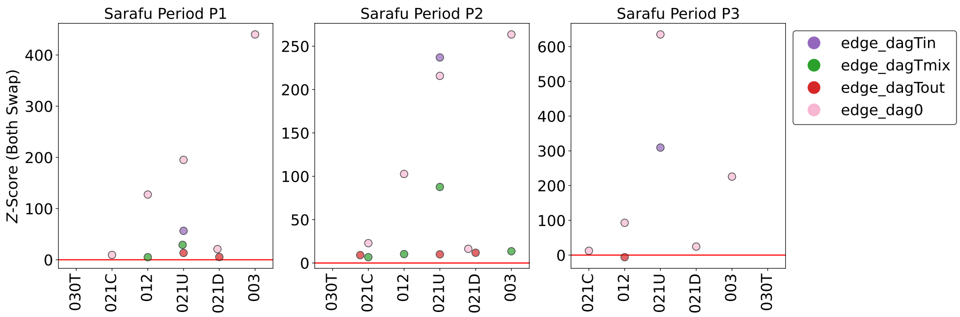
7.6.1 Sarafu Network



(a) "Source"-Edge Swap



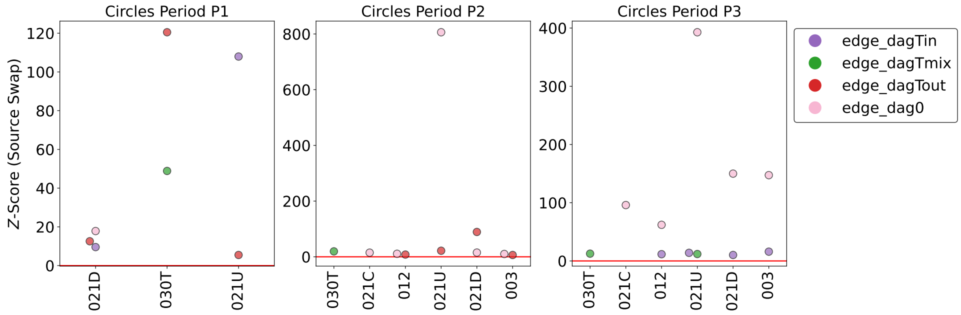
(b) "Target"-Edge Swap



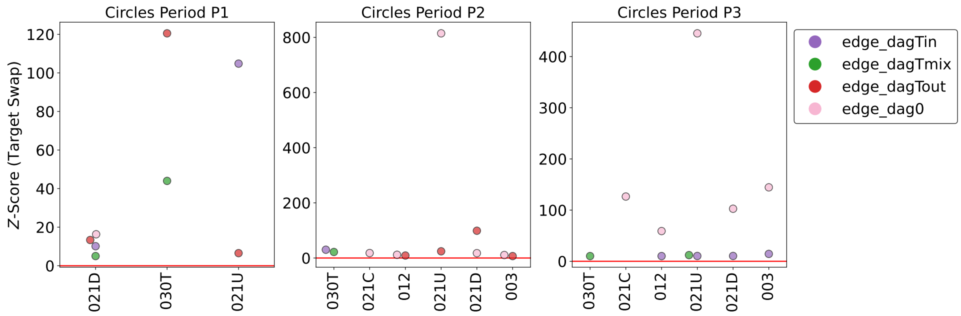
(c) "Both"-Edge Swap

Figure 50: Significance Triads in DAGs in Sarafu Over Time. Comparison of different types of null models. Value of the *Z*-Score less than 5 in absolute value are excluded from the plot. The main consistent finding across all the null models is the significant over-representation of triads in *dag0* and *dagTin*. Besides the "empty" triad (003), the most frequent triad in those component is the "collector" triad (021U).

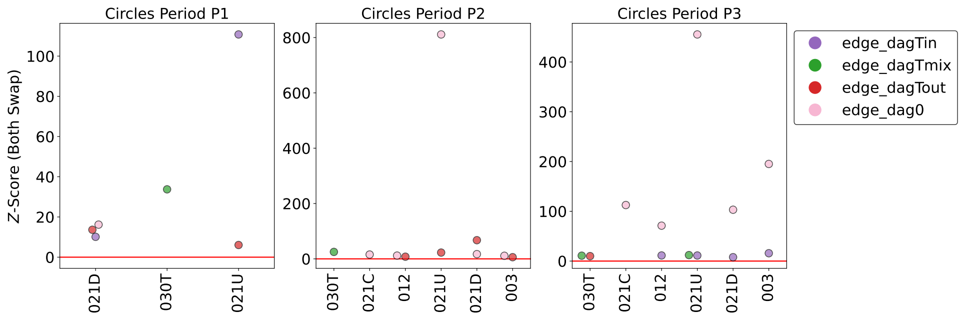
7.6.2 Circles Network



(a) "Source"-Edge Swap



(b) "Target"-Edge Swap



(c) "Both"-Edge Swap

Figure 51: Significance Triads in DAGs in Circles Over Time. Value of the Z-Score less than 5 in absolute value are excluded from the plot. In the first period, the most significant triad is the 030T in *dagTout* and *dagTmix*, while the "collector" triad is over-represented in *dagTin* components. In the second period, the "collector" triad is the most frequent triad among all types, and mostly appearing in *dag0* components. The second most frequent triad is the "distributor" triad (021D) appearing in *dagTin*. Finally, in the third period, the *dag0* is definitively the component with more triads than others, among which the most frequent are (in order of importance): "collector" (021U), "empty" (003), "broker" (021C), and "distributor" (021D) triads.

7.7 Recirculation

7.7.1 Sarafu Network

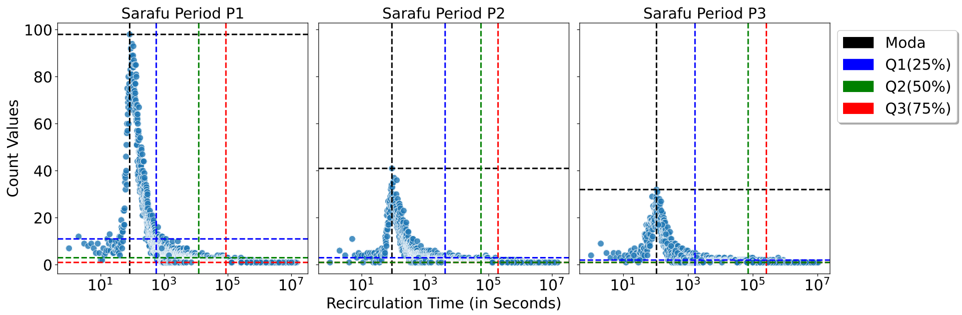
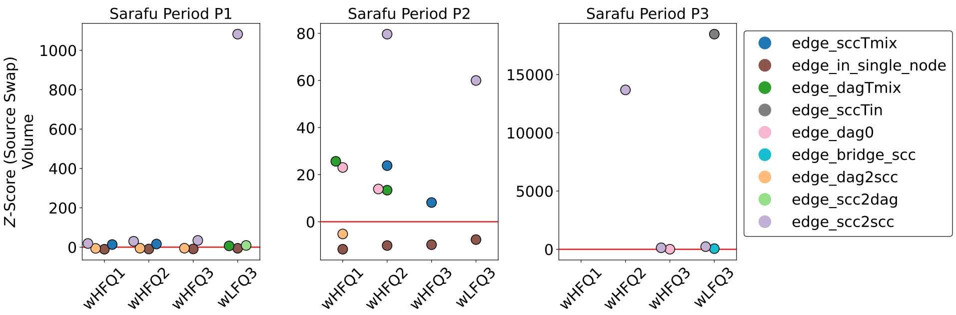
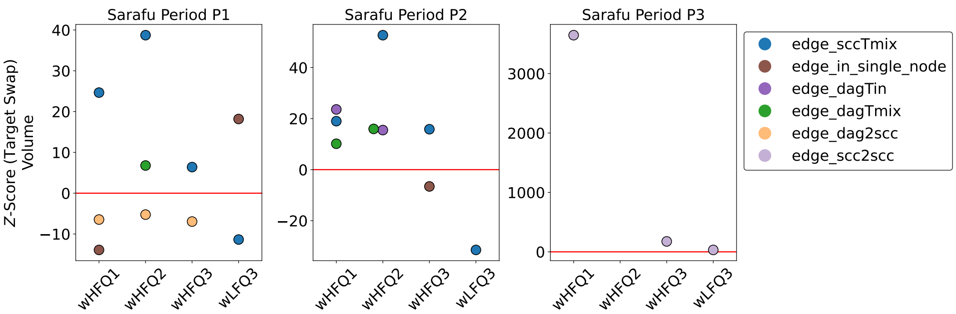


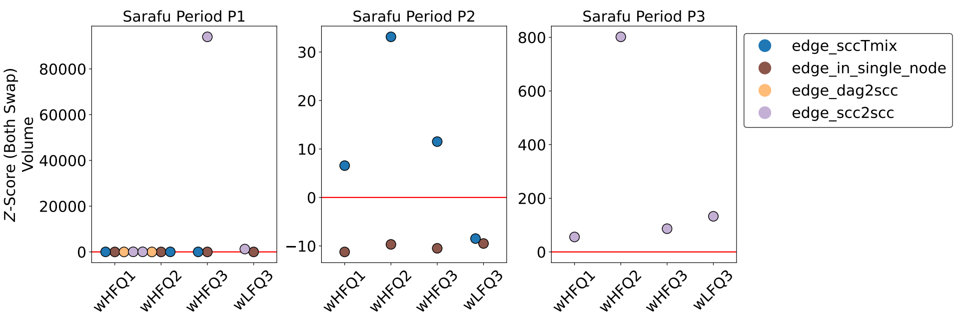
Figure 52: Frequency of recirculation in Sarafu. The mode in Period 1 is at 81 seconds (1 minute, 21 seconds) which happened 98 times. 75 operations took place in less than 10 seconds. The mode in Period 2 is at 89 seconds (1 minute, 29 seconds) which happened 41 times. 42 operations took place in less than 10 seconds. The mode in Period 3 is at 105 seconds (1 minute, 45 seconds) which happened 32 times. 42 operations took place in less than 10 seconds. In each period, the mode is visible as a peak in the plot. In each period, all the 'fastest' users and the users active at the mode of the distribution belong to cyclic components. As explained in Criscione (2024) and Kiaka et al. (2024), these peaks could be related to a few groups of users who were regularly meeting to simulate transactions and unlock rewards. A practice that was qualitatively described in Kiaka et al. (2024). The reward system was adopted in Period 1, reduced in Period 2, and abandoned in Period 3. This is why probably we notice a decrease in the number of fast operations. The presence of operations happening in less than 10 seconds hints either to users who used a software to simulate transactions and unlock rewards, or a recording error. In fact, at the beginning some of the data had to be moved manually in batches from a SQL database to a blockchain (Mattsson, Criscione, and Ruddick, 2022).



(a) "Source"-Edge Swap

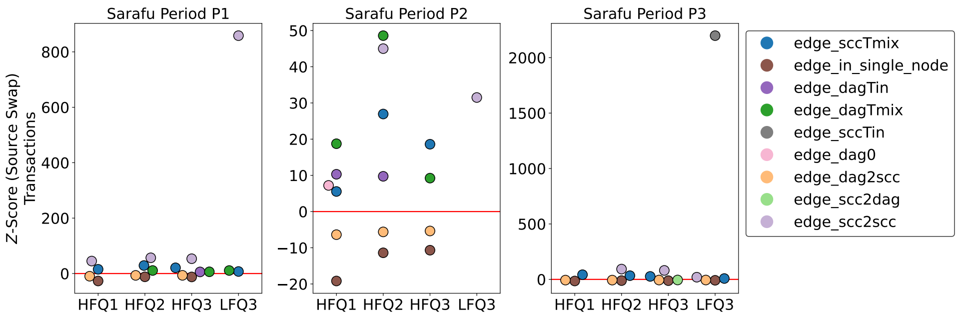


(b) "Target"-Edge Swap

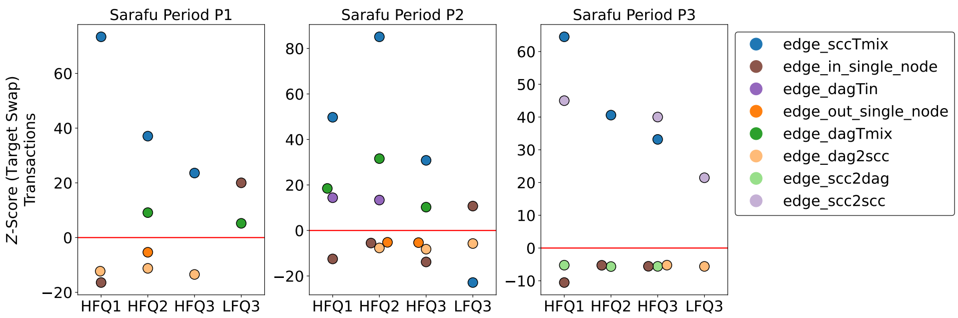


(c) "Both"-Edge Swap

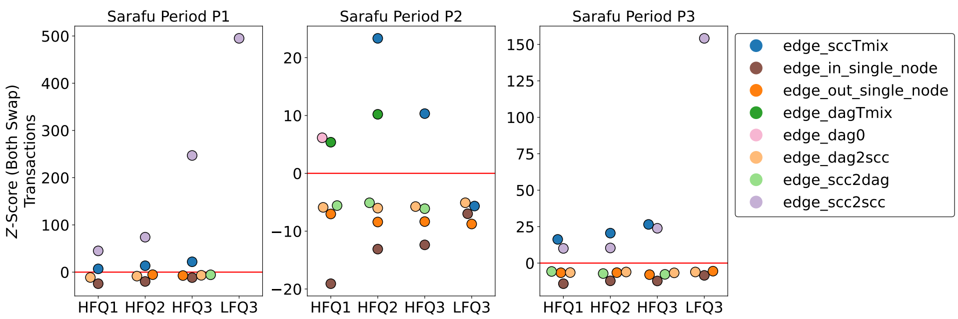
Figure 53: Significance of recirculating volume in Sarafu per topological group. Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot.



(a) ”Source”-Edge Swap

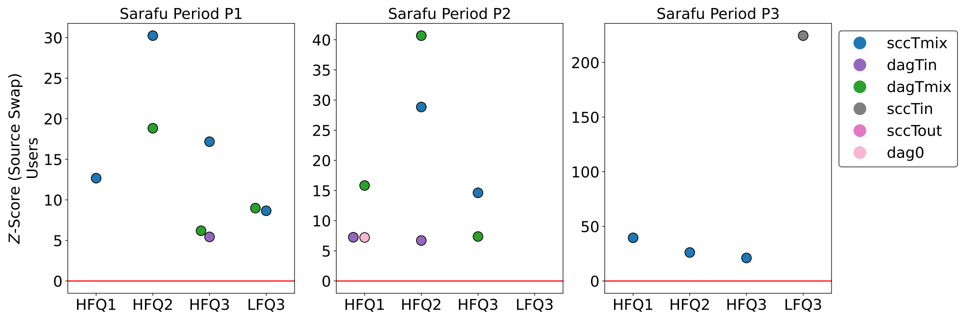


(b) ”Target”-Edge Swap

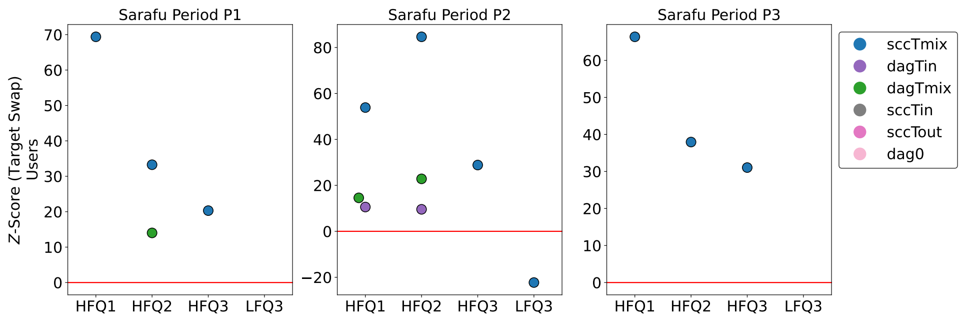


(c) ”Both”-Edge Swap

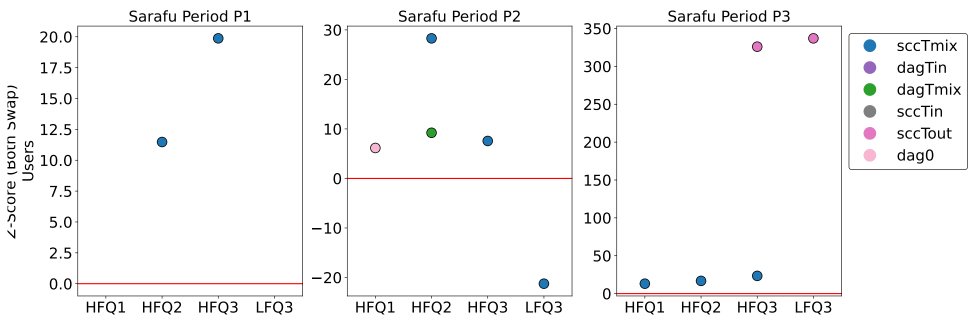
Figure 54: Significance of recirculating transactions in Sarafu per topological group. Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot.



(a) "Source"-Edge Swap



(b) "Target"-Edge Swap



(c) "Both"-Edge Swap

Figure 55: Significance of recirculating users in Sarafu per topological group. Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot.

7.7.2 Circles Network

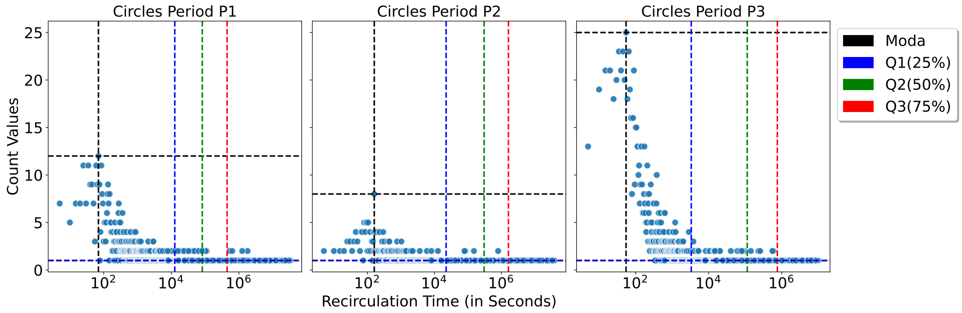
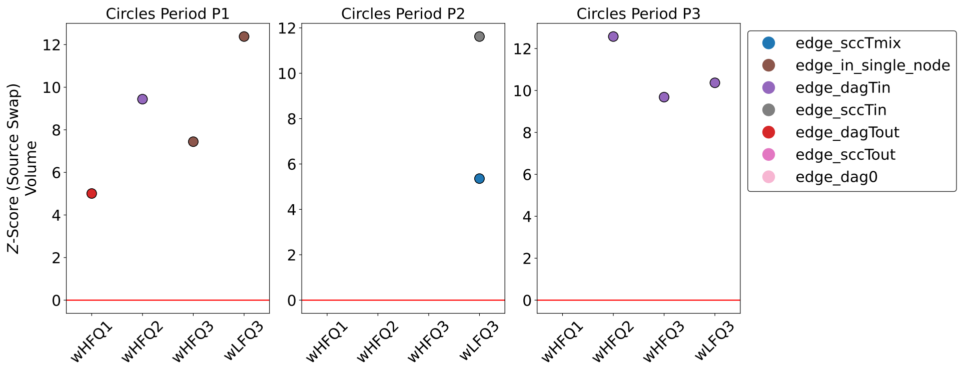
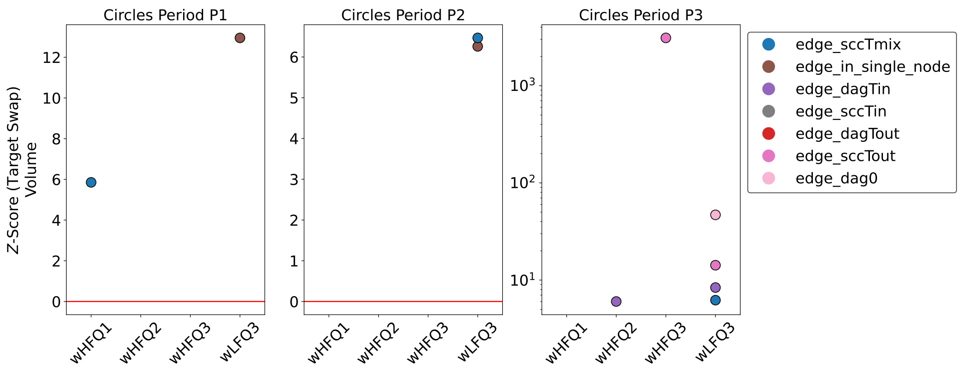


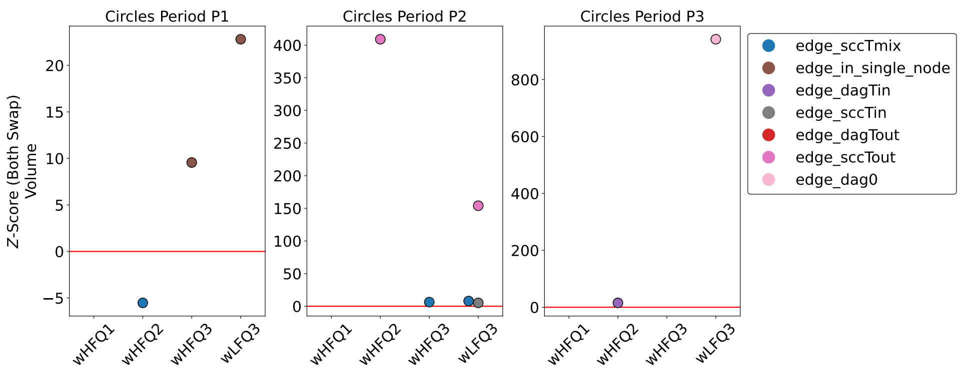
Figure 56: Frequency of recirculation in Circles. The mode in Period 1 is at 70 seconds (1 minute, 10 seconds) which happened 12 times. 12 operations took place in less than 10 seconds. The mode in Period 2 is at 160 seconds (2 minutes, 40 seconds) which happened 8 times. 5 operations took place in less than 10 seconds. The mode in Period 3 is at 55 seconds, which happened 25 times. 30 operations took place in less than 10 seconds. In each period, the mode is visible as a peak in the plot. Generally, 'fastest' operations and the users active at the mode of the distribution belong to cyclic components, except for Period 2 when there are also users from *dag0* and *dagTmix*. As observed in Chapter 5, in Period 3 the network was generally very active, most of the recirculation happened in *sccTmix* components. *In-* and *out-single-node*s are generally more present in recirculation happening in Period 2 and Period 3, respectively. In Period 3, there could have been a general attempt to revive the network by onboarding new members and re-involving existing ones. High-frequency recirculation happening in less 10 seconds could also be associated to software trying to simulate transactions. Although, the *web-of-trust* technology embedded in the back-end of the platform should limit Sybil attacks.



(a) "Source"-Edge Swap

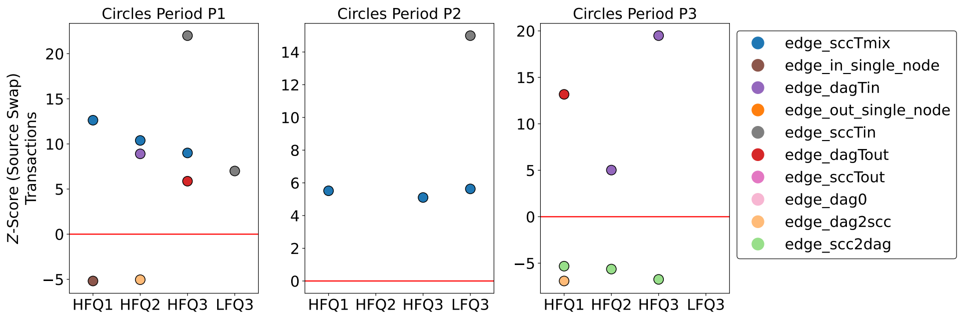


(b) "Target"-Edge Swap

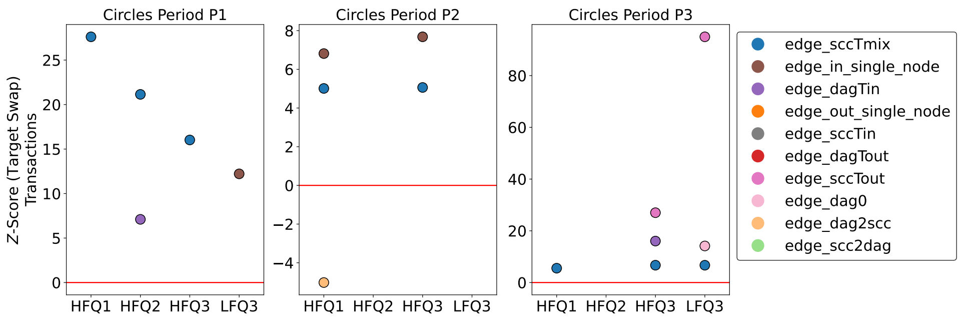


(c) "Both"-Edge Swap

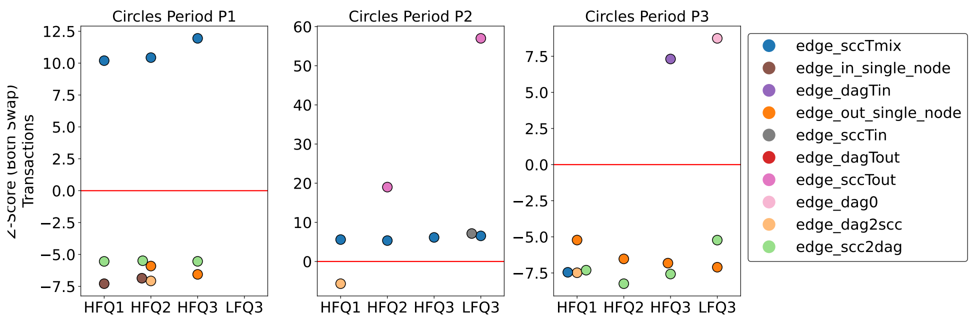
Figure 57: Significance of recirculating volume in Circles per topological group. Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot.



(a) ”Source”-Edge Swap

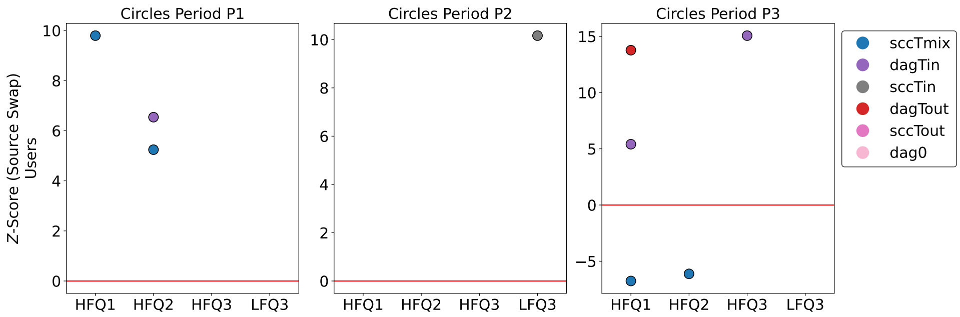


(b) ”Target”-Edge Swap

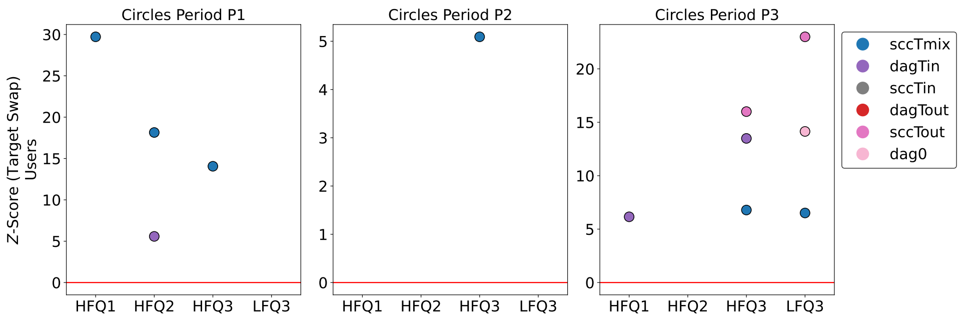


(c) ”Both”-Edge Swap

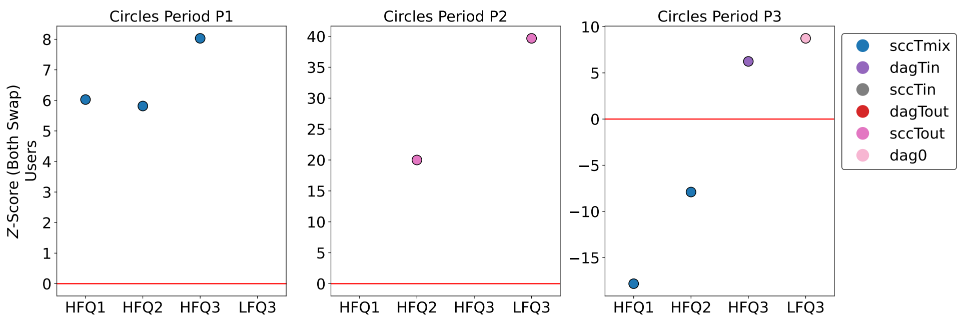
Figure 58: Significance of recirculating transactions in Circles per topological group. Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot.



(a) ”Source”-Edge Swap



(b) ”Target”-Edge Swap



(c) ”Both”-Edge Swap

Figure 59: Significance of recirculating users in Circles per topological group. Comparison of different types of null models. Value of the Z-Score less than 5 in absolute value are excluded from the plot.

7.8 One-time usage

7.8.1 Sarafu Network

14 15

Period	Category	(1)OT Users	(2)Tot. OT Users	(3)Tot. Users (Category)	(4)Tot. Users (Period)	((1)/(2))%	((1)/(3))%	((1)/(4))%
P1	**dag0**	400	5229	534	22022	7.65	74.91	1.82
P1	**out_single_node**	136	5229	182	22022	2.6	74.73	0.62
P1	**dagTin**	649	5229	1765	22022	12.41	36.77	2.95
P1	**dagTmix**	149	5229	482	22022	2.85	30.91	0.68
P1	**dagTout**	87	5229	152	22022	1.66	57.24	0.4
P1	**in_single_node**	3808	5229	7344	22022	72.82	51.85	17.29
P2	**dag0**	753	5395	1003	14687	13.96	75.07	5.13
P2	**out_single_node**	275	5395	370	14687	5.1	74.32	1.87
P2	**dagTin**	571	5395	1084	14687	10.58	52.68	3.89
P2	**dagTmix**	266	5395	442	14687	4.93	60.18	1.81
P2	**dagTout**	111	5395	227	14687	2.06	48.9	0.76
P2	**in_single_node**	3419	5395	4884	14687	63.37	70.0	23.28
P3	**dag0**	913	3332	1188	11692	27.4	76.85	7.81
P3	**out_single_node**	592	3332	803	11692	17.77	73.72	5.06
P3	**dagTin**	341	3332	561	11692	10.23	60.78	2.92
P3	**dagTmix**	26	3332	103	11692	0.78	25.24	0.22
P3	**dagTout**	103	3332	234	11692	3.09	44.02	0.88
P3	**in_single_node**	1357	3332	1726	11692	40.73	78.62	11.61

Table 14: One-Time (OT) Users in Sarafu. One-Time Users in Period 1 are 23.74% of the total. One-Time Users in Period 2 are 36.73% of the total. One-Time Users in Period 3 are 28.5% of the total.

Period	Category	(1)OT Volume	(2)Tot. OT Volume	(3)Tot. Volume (Category)	(4)Tot. Volume (Period)	((1)/(2))%	((1)/(3))%	((1)/(4))%
P1	**edge_dag0**	68751.0	1471326.24	90146.0	83583846.77	4.67	76.27	0.08
P1	**edge_out_tendril**	27206.03	1471326.24	63563.03	83583846.77	1.85	42.8	0.03
P1	**edge_dagTin**	160476.0	1471326.24	307070.41	83583846.77	10.91	52.26	0.19
P1	**edge_dagTmix**	46530.0	1471326.24	124045.96	83583846.77	3.16	37.51	0.06
P1	**edge_dagTout**	22679.0	1471326.24	32532.0	83583846.77	1.54	69.71	0.03
P1	**edge_in_tendril**	1145684.21	1471326.24	2916768.35	83583846.77	77.87	39.28	1.37
P2	**edge_dag0**	114004.0	1290051.47	158278.0	82030707.468	8.84	72.03	0.14
P2	**edge_out_tendril**	43394.0	1290051.47	162545.0	82030707.468	3.36	26.7	0.05
P2	**edge_dagTin**	162097.0	1290051.47	242246.0	82030707.468	12.57	66.91	0.2
P2	**edge_dagTmix**	70044.0	1290051.47	148134.1	82030707.468	5.43	47.28	0.09
P2	**edge_dagTout**	33299.47	1290051.47	49987.47	82030707.468	2.58	66.62	0.04
P2	**edge_in_tendril**	867213.0	1290051.47	1489981.524	82030707.468	67.22	58.2	1.06
P3	**edge_dag0**	61496.0	396812.0	76779.0	10089581.45	15.5	80.09	0.61
P3	**edge_out_tendril**	65599.0	396812.0	129025.0	10089581.45	16.53	50.84	0.65
P3	**edge_dagTin**	69742.0	396812.0	74900.0	10089581.45	17.58	93.11	0.69
P3	**edge_dagTmix**	934.0	396812.0	11709.0	10089581.45	0.24	7.98	0.01
P3	**edge_dagTout**	14307.0	396812.0	19533.0	10089581.45	3.61	73.25	0.14
P3	**edge_in_tendril**	184734.0	396812.0	302891.1	10089581.45	46.55	60.99	1.83

Table 15: One-Time (OT) Volume in Sarafu. One-Time Volume in Period 1 is 1.76% of the total. One-Time Volume in Period 2 is 1.57% of the total. One-Time Volume in Period 3 is 3.93%.

7.8.2 Circles Network

16 17

Period	Category	(1)OT Users	(2)Tot. OT Users	(3)Tot. Users (Category)	(4)Tot. Users (Period)	((1)/(2))%	((1)/(3))%	((1)/(4))%
P1	**dag0**	63	1281	98	4074	4.92	64.29	1.55
P1	**out_tendril**	476	1281	735	4074	37.16	64.76	11.68
P1	**dagTin**	120	1281	348	4074	9.37	34.48	2.95
P1	**dagTmix**	184	1281	694	4074	14.36	26.51	4.52
P1	**dagTout**	119	1281	340	4074	9.29	35.0	2.92
P1	**in_tendril**	319	1281	665	4074	24.9	47.97	7.83
P2	**dag0**	237	813	382	2510	29.15	62.04	9.44
P2	**out_tendril**	172	813	256	2510	21.16	67.19	6.85
P2	**dagTin**	64	813	213	2510	7.87	30.05	2.55
P2	**dagTmix**	86	813	302	2510	10.58	28.48	3.43
P2	**dagTout**	52	813	136	2510	6.4	38.24	2.07
P2	**in_tendril**	202	813	502	2510	24.85	40.24	8.05
P3	**dag0**	135	693	181	3622	19.48	74.59	3.73
P3	**out_tendril**	280	693	846	3622	40.4	33.1	7.73
P3	**dagTin**	50	693	137	3622	7.22	36.5	1.38
P3	**dagTmix**	49	693	361	3622	7.07	13.57	1.35
P3	**dagTout**	34	693	121	3622	4.91	28.1	0.94
P3	**in_tendril**	145	693	353	3622	20.92	41.08	4.0

Table 16: One-Time Users in Circles. One-Time Users in Period 1 are 31.44% of the total. One-Time Users in Period 2 are 32.39% of the total. One-Time Users in Period 3 are 19.13% of the total.

Period	Category	OT Volume	Tot. OT Volume	Tot. Volume (Category)	Tot. Volume (Period)	((1)/(2))%	((1)/(3))%	((1)/(4))%
P1	**edge_dag0**	2939.11	47748.505	3656.694	565283.709	6.16	80.38	0.52
P1	**edge_out_tendril**	2355.293	47748.505	13167.642	565283.709	4.93	17.89	0.42
P1	**edge_dagTin**	2597.85	47748.505	10162.03	565283.709	5.44	25.56	0.46
P1	**edge_dagTmix**	7288.66	47748.505	54628.622	565283.709	15.26	13.34	1.29
P1	**edge_dagTout**	2660.63	47748.505	16967.883	565283.709	5.57	15.68	0.47
P1	**edge_in_tendril**	29906.961	47748.505	219358.016	565283.709	62.63	13.63	5.29
P2	**edge_dag0**	37443.635	75997.101	283320.569	2444959.098	49.27	13.22	1.53
P2	**edge_out_tendril**	2870.233	75997.101	17252.894	2444959.098	3.78	16.64	0.12
P2	**edge_dagTin**	3774.392	75997.101	25138.608	2444959.098	4.97	15.01	0.15
P2	**edge_dagTmix**	3592.441	75997.101	40610.512	2444959.098	4.73	8.85	0.15
P2	**edge_dagTout**	412.747	75997.101	9228.998	2444959.098	0.54	4.47	0.02
P2	**edge_in_tendril**	27903.651	75997.101	278721.63	2444959.098	36.72	10.01	1.14
P3	**edge_dag0**	24162.519	82214.163	60397.084	1676052.066	29.39	40.01	1.44
P3	**edge_out_tendril**	4351.253	82214.163	45057.652	1676052.066	5.29	9.66	0.26
P3	**edge_dagTin**	4089.429	82214.163	17811.174	1676052.066	4.97	22.96	0.24
P3	**edge_dagTmix**	6929.11	82214.163	52019.427	1676052.066	8.43	13.32	0.41
P3	**edge_dagTout**	8849.353	82214.163	10195.606	1676052.066	10.76	86.8	0.53
P3	**edge_in_tendril**	33832.499	82214.163	131081.45	1676052.066	41.15	25.81	2.02

Table 17: One-Time (OT) Volume in Circles. One-Time Volume in Period 1 is 8.45% of total volume. One-Time Volume in Period 2 is 3.11% of total volume. One-Time Volume in Period 3 is 4.91% of total volume.

Additional information

The author declares no competing interests.

Data availability

The Sarafu data 2020-2021 (Ruddick, 2021) are available for download on UK Data Service (UKDS) under End User Licence (https://reshare.ukdataservice.ac.uk/855142/) after registration. A data description paper is also available for download (Mattsson, Criscione, and Ruddick, 2022). The Circles data are publicly available and can be downloaded following the instruction in one of the official repositories (https://github.com/CirclesUBI/circles-analysis).

Software availability

All software used in this study is available under an open source licence:

- `networkx v.3.1.` (Hagberg et al., 2008)
- `scipy v.1.9.1.` (Virtanen et al., 2020)
- `numpy v.1.23.0` (Harris et al., 2020)
- `powerlaw v.1.5` (Alstott et al., 2014)
- `seaborn v.0.11.2` (Waskom, 2021)
- `matplotlib v.3.5.2` (Hunter, 2007)
- `pandas v.1.4.4.` (Reback et al., 2022)
- `pycirclize v.1.4.0` (Wang, 2022)
- `gephi v.0.10` (Bastian et al., 2009)

Supplementary material

The code used for this work is available in this public GitHub repository: https://github.com/TeodoroCriscione/github_PhD_thesis_Teodoro_Criscione.git. Copyright Notice: CC BY-NC-

ND 4.0. The code is licenced under a Creative Commons "Attribution- NonCommercial-NoDerivatives 4.0 International" licence (https://creativecommons.org/licences/by-nc-nd/4.0/deed.en). The following Python packages are required: networkx v.3.1, numpy v.1.23.0, pandas v.1.4.4, collections, datetime, os, random.

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