COLLECTIVE DYNAMICS IN EVOLUTIONARY PROCESSES AND STRATEGIC GROUP INTERACTIONS

By

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Author's Declaration

I, the undersigned, **Onkar Sadekar**, candidate for the PhD degree in Network Science declare herewith that the present thesis is exclusively my own work, based on my research and only such external information as properly credited in notes and bibliography. I declare that no unidentified and illegitimate use was made of the work of others, and no part of the thesis infringes on any person's or institution's copyright. I also declare that no part of the thesis has been submitted in this form to any other institution of higher education for an academic degree.

Onkar Sadekar
Vienna, 18th August 2025

Ideas, results, and figures appearing in this thesis are based on the publications listed below:

[I] A Civilini, **O Sadekar**, F Battiston, J Gómez-Gardeñes, V Latora, Explosive cooperation in social dilemmas on higher-order networks. *Physical Review Letters* 132 (16), 167401 (2024). [1]

[II] **O Sadekar**, A Civilini, J Gómez-Gardeñes, V Latora, F Battiston, Evolutionary game selection creates cooperative environments. *Physical Review E* 110 (1), 014306 (2024). [2]

[III] **O Sadekar**, S Chowdhary, MS Santhanam, F Battiston, Individual and team performance in cricket. *Royal Society Open Science* 11 (7), 240809 (2024). [3]

[IV] **O Sadekar**, A Civilini, V Latora, F Battiston, Drivers of cooperation in social dilemmas on higher-order networks. *Journal of the Royal Society Interface* 22 (227), 20250134 (2025). [4]

[V] F Battiston, V Capraro, F Karimi, S Lehmann, AB Migliano, **O Sadekar**, A Sánchez, M Perc, Higher-order social networks. (in press at *Nature Human Behaviour*)

[VI] **O Sadekar**, A Civilini, C Hilbe, V Latora, F Battiston, Collective dynamics in higher-order games. (submitted)

[VII] **O Sadekar**, V Latora, AB Migliano, L Vinicius, F Battiston, Higher-order interactions accelerate cultural innovations. (in preparation)

Other works not covered in this thesis include:

[VIII] C Gunasekaram, F Battiston, **O Sadekar**, *et al.*, Population connectivity shapes the distribution and complexity of chimpanzee cumulative culture. *Science* 386 (6724), 920-925 [5]

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"Space... The final frontier.
These are the voyages of the starship Enterprise.
Its continuing mission, to explore strange new worlds.
To seek out new life and new civilizations.
To boldly go where no one has gone before."
– CAPT. JEAN-LUC PICARD, STAR TREK: THE NEXT GENERATION

To all the trekkies and adventure-seekers of the world who dream and dare to venture into the unknown.

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Abstract

Complexity permeates every scale of our lives, and network science provides a versatile tool to reveal the hidden patterns and relationships that bind components of complex interconnected social systems. Yet traditional networks, limited to pairwise connections, cannot systematically capture the richness of non-dyadic, group-level interactions and their effects on social dynamics and system behavior. In response, a recent deluge of theoretical and empirical studies has highlighted how higher-order interactions enrich our understanding of real-world complexity. In this thesis, we investigate the collective dynamics arising as a consequence of group interactions through theoretical, computational, and empirical approaches. We begin by surveying digital data sources where higher-order methods have yielded fresh insights into social organization, group formation, and social phenomena such as contagion. Focusing on strategic behavior, we synthesize key ideas from evolutionary games in group structured populations and propose a new model that embeds group interactions directly into strategic dynamics. This model reveals the emergence of an explosive transition towards cooperation where multi-dimensional strategies and nested interaction patterns act as crucial drivers to further enhance pro-sociality in social dilemmas. To trace the evolutionary origins of these dilemmas, we propose a game selection mechanism in which more fit and cooperative games persist, showing that structured populations foster cooperative environments. Going beyond cooperation, we then investigate diffusion of innovations by coupling higher-order structure with dynamical recombination processes, finding that group interactions accelerate complex cultural recombinations while hindering simpler recombinations. Finally, to empirically investigate collective dynamics, we analyze group behavior in a particular team sport – cricket. We quantify various markers of individual and team success and provide evidence of emergent patterns such as the hot hand effect and the importance of specialists for team performance. By integrating group interactions into existing frameworks, this thesis hopes to advance our understanding of the mechanisms driving collective behavior in varied social and strategic contexts.

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– *Onkar* (sadekaronkar@gmail.com)
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CHAPTER 1
INTRODUCTION

At every scale of human life on Earth – from a group of innovators disrupting the tech industry to billions getting affected by a pandemic – social interactions shape our everyday life [6]. These interactions emerge from and are shaped by the interconnected components of the *complex system* that we are part of [7–9]. As Nobel laureate Murray Gell-Mann observes in *The Quark and the Jaguar*: "[complex systems] are the rich fabric of the world that we perceive directly and of which we are part." [10]. In his well-known essay *More is Different*, another laureate, Philip Warren Anderson, advocates an alternative approach to understanding complex systems, beyond reductionism [11]. Over the past few decades, foundational work in the field of complex systems has deepened our understanding of the world we inhabit [12–15]. Unsurprisingly, the physicist and science communicator Stephen Hawking predicted that the 21st century would be the century of complexity [16].

To understand such complex systems, the theory of network science has emerged as a central framework. It enables us to describe the plethora of connections, relationships, feedback, and influences that one part of a complex system exerts on another [17, 18]. Together they give rise to emergence – one of the most celebrated features of complexity – where the microscopic rules of the system fail to predict the rich variety of structural and dynamical behaviors at the macro scale [8, 9, 19]. The structure of these social relationships affects many aspects of human behavior, and perhaps more than any other paradigm lays bare the shortcomings of the 'economic man' perspective [20]. Humans do not act solely as self-interested optimizers; social ties compel us to weigh others' well-being. This line of thought leads to the perspective of the 'network man' who, driven by embeddedness in a network of social relations, exists and acts in a delicate balance between their well-being and the sympathy for the well-being of others. Ample evidence exists that maintaining this balance affects most of our actions, from who we vote for to what we eat and which partners we choose and why [21]. Apart from our behavior, the complex connectedness of modern human societies can be seen in the ease of global communication, and in the lightning speeds at which news and information as well as epidemics

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and financial crises spread [22].

The study of complex systems through the lens of network science has revealed diverse facets of social organization and dynamics. These include the emergence of scale-free structures in social networks [9, 23] and empirical validation of the “six degrees of separation” in real-world friendships [24, 25]. The rise of collective dynamics on such networked systems – ranging from vanishing epidemic thresholds [26] to the emergence of pro-sociality [27] – has drawn attention across many scientific domains. Indeed, these dynamical processes, inspired by a wide range of social phenomena, give rise to a rich landscape of collective behavior when studied on networks [28–30].

Yet despite these advances, most network models reduce interactions to pairwise ties, overlooking the true locus of complexity: groups [31]. The limitations of dyadic modeling approaches were already recognized in the early 70s by Atkin [32, 33] and Berge [34]. Only recently, however, has unprecedented access to high-resolution data enabled higher-order networks to emerge as a natural solution to capture and model the interconnected structure of groups that characterize many aspects of real-world social systems.

This thesis investigates collective dynamics in group interactions through empirical, theoretical, and computational lenses. We begin in **Chapter 2** by surveying the landscape of higher-order social networks. In particular, we trace the history of the idea of non-dyadic interactions and how the data deluge in recent years attracted the complexity scientists to investigate these group interactions. To further support the arguments, we investigate one example of digital data on high-frequency face-to-face contact networks and highlight some important empirical results that have been obtained from these temporal group dynamics. To highlight the relevance of higher-order interactions, we take one particular example of scientific collaboration patterns and analyze the different dimensions of this social network using the newly developed tools and methods from the higher-order theory literature. Beyond empirical observations, we discuss general modeling approaches for understanding the formation and evolution of social groups. Drawing inspiration from contagion processes on networks, we also highlight the emerging potential for modeling group-level contagion, moving past the traditional dichotomy of simple vs. complex contagion.

Up to this point, the focus remains largely on structural characteristics of group interactions and the impact of higher-order structures on basic dynamical processes. However, from a behavioral standpoint, these elementary processes do not fully capture the complexity and *strategic* reasoning involved in human decisions. Game theory has revolutionized the study of social behavior by formalizing strategic decision-making. Game theory introduces a formal mapping between choices (strategies) and their consequences (payoffs), enabling rational modeling. Subsequently, evolutionary models inspired by natural selection integrate adaptive changes in individual preferences. Lastly, incorporating structured interactions – such as small-world or scale-free topologies – into game-theoretic models has uncovered conditions that support cooperation in social dilemmas. Given this context, it is evident that group interactions

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play a vital role in strategic environments. Toward the end of **Chapter 2**, we describe relevant game-theoretic models involving group interactions and identify several forms of collective behavior that emerge.

Existing models of games with group interactions often rely on externally imposed group definitions. Even in those that employ hypergraphs, the models frequently oversimplify by assuming that payoffs scale linearly with group size. In **Chapter 3**, we detail our work on higher-order social dilemmas [1, 4]. We introduce a general modeling framework that links multiplayer games to hyperedges of corresponding order. The model incorporates realistic features, including tunable overlap among group memberships and the possibility for agents to hold multi-dimensional strategies. Through a combination of analytical insights and numerical simulations, we demonstrate a strong effect of higher-order interactions on the evolution of cooperation. Moreover, we identify structural and strategic factors that promote pro-social outcomes.

Social dilemmas are ubiquitous, yet their origins remain underexplored. A large fraction of the models in game theory literature start with the assumption that all players in the population are playing the same game. Even though this simplistic model has helped us uncover many interesting social phenomena as seen above, it is important to know how these games came into play in the first place. **Chapter 4** provides some evidence for the origins by proposing a new framework of evolutionary game selection [2]. Borrowing ideas from the concept of stochastic games, we model each player as being associated with both a strategy as well as a payoff matrix. Using co-evolutionary dynamics where both the strategies and games evolve based on the ability to replicate, we observe a synergy between cooperative strategies and cooperative games. Our analysis further revealed that clustering players with similar types of games can sustain pro-sociality in the long term. Our model also opens up avenues to incorporate this strategic selection of games in the presence of group interactions.

Even though our cooperative behavior is hypothesized to be one of the major drivers of success as a species, there are other ingredients that govern our daily life and social behavior. The dynamics of how culture emerges and spreads can help us understand such social behavior. Ranging from small-scale hunter-gatherer societies recombining different plants to create useful medicines to large teams in industry synthesizing a variety of techniques to produce useful gadgets, cultural recombination is central to the advancement of society. As such, the role of group interactions is vital to understanding how we recombine the knowledge from a diverse set of individuals to produce state-of-the-art technologies. The focus shifts in **Chapter 5** to incorporate higher-order interactions to the cultural recombination dynamics to produce newer items of higher value. We find that group interactions across the communities play an important role in generating novel items in modular partially-connected hypergraphs. However, this comes at a cost of only a few selected individuals having access to the new items. It would be interesting to understand this cost and benefit aspect in more detail in the future, possibly using game-theoretic approaches.

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So far, the discussion has addressed cooperative behavior, strategic dynamics, and innovation, all emerging from group interactions via simple mechanistic models. Yet real-world phenomena often involve subtleties and contextual factors that resist such modeling. Team sports offer a rich context for examining this complexity, particularly to investigate success and performance at individual and team level. Numerous factors such as intra-team coordination, strategic execution, counter-strategy adaptation, and personal reinvention across a player's career play an important part in shaping collective success. In **Chapter 6**, we turn to a case study of cricket – one of the world's most followed sports [35]. We analyze various indicators of individual and team performance, revealing patterns found in other creative domains (e.g., music, arts, and science), alongside insights into team composition and the division of labor. Finally, in **Chapter 7**, we synthesize the key findings of this thesis and outline future research directions emerging from this work.

In summary, this thesis explores collective dynamics in strategic group interactions across diverse social phenomena. It highlights the crucial role of how groups can affect the complexity and organization of collective behavior in real-world settings.

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CHAPTER 2
GROUP INTERACTIONS IN SOCIAL PHENOMENA AND EVOLUTIONARY PROCESSES

At a Glance

Humans outperform most other species in joint tasks involving coordination and cooperation. Collective action enabled us to survive against the odds from the onset of the early hunter-gatherers, and it continues to be a key pillar that facilitates our societal and technological progress. However, the models with which we describe and study collective human behavior fail to comprehensively capture group interactions, because the links of traditional social networks only account for pairwise interactions. Recent developments in network science have made it clear that a paradigm shift in the way we characterize human interactions is necessary, leading to the birth of higher-order social networks, where interactions are not limited to two people but can involve more than two people. Here we discuss recent experimental, data-driven and theoretical breakthroughs related to social networks beyond pairwise interactions that unveil the power of non-dyadic approaches to capture and better understand collective human behavior, from group formation and social contagion to norm adoption and the emergence of cooperation.

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2.1 Networks as the underlying framework of complex systems

Since the introduction of sociograms to describe social configurations by Moreno and Jennings [36], social network analysis has grown into a field of its own. New theories were proposed, starting with Granovetter's essays on the importance of weak ties for increasing the reach of marketing, politics, and information beyond the few that are accessible through strong connections [37], as well as pioneering experiments, such as Milgram's work on the small-world phenomenon [24]. Using nodes and links to describe individuals and their pairwise relationships, network science is nowadays a major paradigm in contemporary sociology and behavioral sciences, while at the same time being a vibrant research field in its own right [18,38–40].

Network science provides tools to characterize structural properties that critically shape dynamical processes unfolding on networks [41]. Key structural measures include local properties (e.g., node degree, clustering coefficient) and global features (e.g., average path length, network diameter) [39]. These structural parameters directly influence social processes modeled as network dynamics. For example, assigning oscillator frequencies based on node degree alters synchronization patterns [42]. Scale-free contact networks eliminate epidemic thresholds in large populations [26]. Heterogeneous network structures enhance cooperation in social dilemmas [27]. In short, network structure dictates social dynamics – a cornerstone of complex systems research.

Traditional social networks consist of agglomerates of dyads (or pairs), which together give rise to large interconnected webs of human relations. Yet, this theoretical framework is not well-suited to capture a crucial feature of human behavior, i.e. group interactions. In this chapter we discuss the limitations of the link as the single fruitful modeling paradigm for social interactions, and highlight the descriptive power of "higher-order interactions", where individuals can be bound in groups of two, but also three or hundreds, all at once.

2.2 Need for higher-order social networks

The limits of the classic network paradigm – and indeed the inherent irreducibility of higher-order interactions to pairwise interactions – become particularly evident when studying not only the structural organization of human relations, but also human behavior. Already in 1895, Gustave Le Bon pointed out that an individual immersed in a group for a long time loses their identity becoming subject to the 'magnetic influence' given out by the crowd [43]. A few years later Simmel further discussed the idea that group dynamics can not be reduced to the sum of dyadic relationships [44], and emphasized that groups of three can facilitate reconciliation and resolution of conflicts because of a third party (e.g. a mediating country facilitating communication to find a mutually acceptable solution to a conflict), but also create new conflicts (e.g. a beneficiary who chooses between two conflicting sides to change the power balance). Drawing from the ideas of 'gestalt psychology', Lewin postulated that when groups are formed, they indeed become a unified system which can not be understood by evaluating members individually [45]. In modern language, this translates into the ability to model new social phenomena and dynamics such as peer pressure, opinion formation and large-scale cooperation with the tools of higher-order social networks. On the other hand, in contrast to conventional social network analysis, which often infers group structures inductively from patterns of pairwise interactions, the hypergraph framework allows for a more direct, or deductive, representation of inherent group-level phenomena. This distinction is crucial for understanding social complexity that transcends dyadic relationships.

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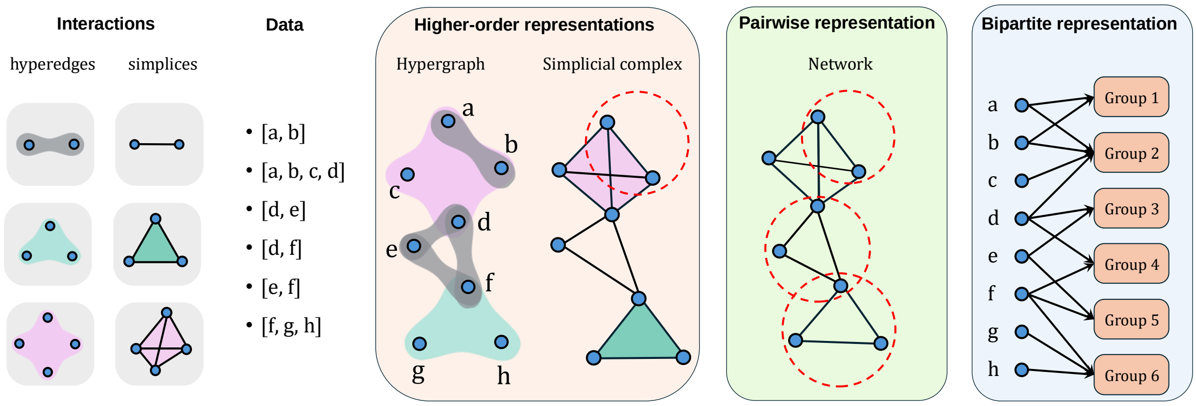

Figure 2.1: **Higher-order representations.** Given a dataset with non-dyadic interactions, they can be encoded via hyperedges or simplices. A k-hyperedge encodes an interaction among k individuals. A hypergraph, a collection of nodes and hyperedges, is the most flexible way to represent higher-order social networks. A simplicial complex, a collection of simplices, constrains the representation by enforcing the condition to have all possible subsets of the highest order interaction also included in the complex. This leads to inaccuracies, since all lower-order interactions are automatically considered, losing the ability to distinguish between overlapping and non-overlapping interactions (red dotted circle). A pairwise representation is obtained by projecting the group interactions into cliques of pairwise interactions, thus making it impossible to retrieve the size of the original groups (marked in red dotted circles). A bipartite projection allows to maintain the distinguishability between different groups, but here groups are represented in an indirect way, as a layer of nodes rather than edges / hyperedges.

In what follows, we first describe the vocabulary and key concepts behind higher-order interactions. We then delve into large-scale digital data as a trove of new opportunities for breakthrough explorations of human behavior, by taking a particular example of high-frequency contact social networks and collaboration and affiliation networks. We demonstrate the empirical value of this approach by applying higher-order metrics to academic collaboration networks. We continue our discussion about recent research where higher-order social networks have been employed to obtain new insights on social phenomena, allowing us to reveal new mechanisms for group formation as well as to improve the modeling of social contagion. We then shift the focus to give a detailed overview of collective dynamics arising out of strategic interactions in evolutionary processes. We highlight the fundamental breakthroughs in the field of games on graphs and list some of the major limitations of this approach. We then summarize some of the recent work where higher-order interactions have helped us to understand the emergence of cooperation better.

2.3 Higher-order interactions

Since its foundation, social network analysis has heavily relied on the mathematical framework of graph theory [46]. In its classical representation, a social network can be seen as a graph, that

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is a collection of actors, represented as nodes, and links, describing the pairwise interactions among them. Despite being widespread, this framework has clear limitations when describing real social systems, where social interactions often occur in larger groups. To better represent these *higher-order* interactions, we can make use of more complex mathematical objects, which naturally allows us to capture social relations beyond the dyadic level [31]. The natural candidates to describe higher-order social networks are hypergraphs. Formally, a hypergraph

$$\mathcal{H} = \{V, E\}$$

is a collection of nodes V, representing the agents in the system, and their interactions E, described as hyperedges, generalizations of links which can encode relations not only between pairs of nodes, but among an arbitrary K number of partners [34].

Despite the focus on simple graphs, social network analysis has already attempted to go beyond a simple characterization of relations among pairs. At the micro-scale, non-dyadic interactions have been investigated by looking at cliques, fully connected small subgraphs whose members are all socially linked to each other, or other small motifs [47], highlighting frequently observed patterns of social interactions. At the macro-scale, large attention has been devoted to the organization of individuals into social clusters, or communities [48, 49].

However, extracting information about the real higher-order structure of social networks from traditional graph representation might be misleading. We illustrate these limitations through an illustrative higher-order social network in Fig. (2.1). Hypergraphs [34] are the most flexible representation for higher-order social networks, allowing to encode interactions of arbitrary group sizes without any particular constraints. In the case of simplicial complexes, the system is encoded as a collection of simplices, which combinatorially not only describe an interaction among their members, but also among all possible subsets [50, 51]. For this reason, such a representation might not always be suitable, unless in those cases where the presence of a larger group interaction also implies the presence of all related interacting subgroups. The success of simplicial complexes is due to the fact that their particular structure makes them amenable to be analyzed following approaches based on topological data analysis, allowing to gain computational insights on the 'shape' of data. Yet, from a combinatorial perspective they appear as a particularly constrained type of hypergraphs, and hence are often unsuitable to properly describe the higher-order organization of real-world systems.

In a simple pairwise representation, groups are projected and represented as cliques of dyadic ties. This severely limits our understanding of the structure of interactions in the system, as the original groups can generally not be retrieved. For instance, transitivity may either indicate the presence of one higher-order interaction involving three partners, or arise from combining three distinct social interactions among three pairs of individuals. The two situations are both frequent in collaboration networks, where a triangle may be associated to a single paper co-authored by a team of three individuals, or to three distinct papers produced by pairwise

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collaborations. Differences become even more relevant when interactions are inferred from co-occurrence in social groups. If we take a group photo, a group meeting or an email chain and we draw dyadic links among all members of the group, this induces artificially high levels of transitivity in the system. Such distortions in network structure may lead to poor modeling choices when describing social dynamics which are strongly affected by group mechanisms.

It is also worth mentioning that past research has leveraged the language of pairwise networks in an attempt to explicitly describe higher-order interactions. This can be done by considering a particular type of bipartite graphs, where a first set of nodes describe individuals, and a second set of nodes accounts for groups, each individual being linked to the groups in which s/he participates [52, 53]. While such representation does not distort the data, direct higher-order representations are preferable as they recover and expand the original framework of social network analysis, where nodes are reserved for social actors, and (hyper)links are used to model interactions among them. Additionally hypergraphs allow for more intuitive grasp of many empirical features of real-world systems such as presence of nested and overlapping group structures, compared to bipartite projections. Moreover, such a representation does not reduce to a simple graph when only dyadic ties are present. Finally, we note that higher-order approaches are complementary to coarse-grained views of social networks based on communities [48] and hierarchical structures [54], since hyperlinks allow for the detailed model of groups of different sizes at the microscale.

Indeed, hypergraphs provide a natural representation of real social systems in their complexity, which smoothly reduce to traditional networks when only pairwise interactions are present. They allow researchers to inherit a generalized toolkit of consolidated measures of social network analysis, from degree to centrality measures [55, 56]. Recently, a substantial amount of research has been devoted to proposing more complex descriptors of higher-order connectivity. At the local scale, higher-order patterns may be measured through specific clustering coefficients [57], as well as extensions of motif analysis [58, 59]. At a larger scale, new algorithms allow us to capture hard [60, 61] and overlapping community [62, 63], or core-periphery structure [64] Finally, hypergraphs are particularly useful to effectively describe the temporal unfolding of several social phenomena, where higher-order social interactions change over time [65–68]. In parallel, efforts have been made to make such tools available to the research community, through libraries such as HGX [69], XGI [70], HyperNetX [71], and others.

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2.4 Digital data on high-frequency contact networks

Social structures, such as family, friends, etc, result in the same configuration of nodes recurring again and again over time [72]. We also know that the relationships during a meeting of a group of four people cannot be reduced to six pairwise relations [73]. These two observations suggest that it is meaningful to describe real-world contact networks using hypergraphs. Further, the hypergraph representation is particularly relevant when we include a temporal perspective of

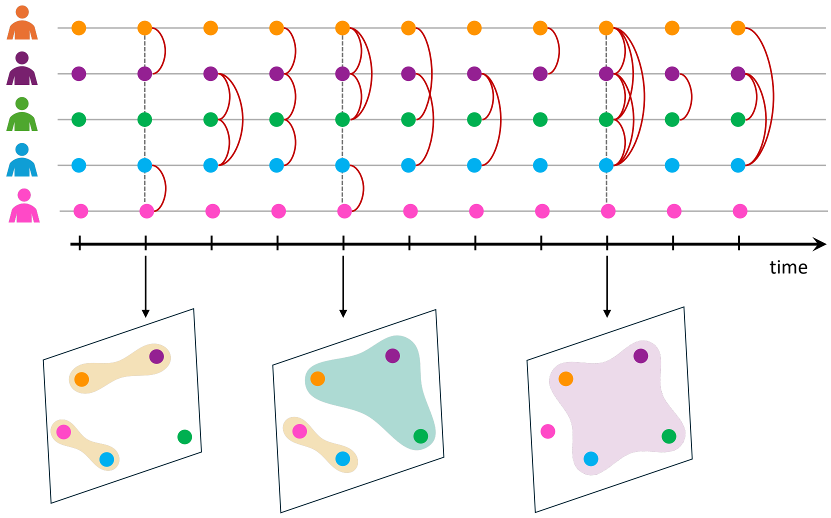

Figure 2.2: **Temporal higher-order networks.** Time-resolved contact data can be described by a temporal hypergraph, where hyperedges describing proximity or face-to-face interactions among individuals are extracted at each observation time.

how social interactions unfold. This intuition has been confirmed as technological progress has made it possible to collect datasets of large social systems with high time-resolution over extended periods of time. In perhaps the largest study of high-frequency contact networks the *Copenhagen Networks Study* [74], Sekara *et al.* [75] observed the interactions of about 1 000 freshman students in 5-minute time intervals over 36 months. In addition to physical proximity measured via Bluetooth, they also recorded virtual forms of social proximity, including phone calls, text messages, and social media interactions. They found the physical proximity network to be well described as temporal sequences of fully connected cliques or "gatherings" lasting up to 12 hours, with a gathering of size K corresponding to a meeting of K individuals. While the nomenclature is different, a gathering of size K is essentially a hypergraph of size K. The authors also identified repeated gatherings over time (denoted "cores"), corresponding to groups of individuals that would meet again and again across weeks and months. Analyzing their dataset in terms of gatherings and cores rather than simple dyads, allows one to define and predict the social trajectories of individuals [75]. This dataset is available to researchers [76].

The Copenhagen Networks Study is neither the first, nor the last study of high-frequency interaction data. Over the years, multiple field studies have used state-of-the-art technology to collect contact networks in diverse settings such as schools, universities, scientific conferences, hospitals, museums, and corporate offices. Below, we highlight some major datasets about the composition and evolution of groups. A key example is the pioneering work in reality mining [77] from MIT's MediaLab, where students in a MIT dormitory were equipped with sensing smartphones. Started in 2008, the Sociopattern project [78] collected longitudinal data of face-to-face interactions in a number of contexts such as workplace, scientific conference, and hospital [79]. Two datasets on contact networks at a scientific conference and a museum exhibition were collected and analyzed by Isella *et al.* [80], while Génois *et al.* [81] collected

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face-to-face data using wearable sensors in a corporate office. Similar data involving health-care workers and patients at a hospital was collected by Vanhems *et al.* [82]. Other examples of such data are the *StudentLife* dataset from Dartmouth University [83, 84], Marseilles high-school student dataset [85], and Lyon primary school student datasets [86]. In the recent years, the *DyLNet* project collected high resolution face-to-face data on preschool children over a period of 3 years [87]. Finally, high-frequency contact networks have also be been inferred from other sources, for example connection to WiFi-routers [88], or from co-location in GPS data [89].

More recently, higher-order representations have been directly leveraged to describe the evolution of social interactions in physical space with recurring groups, modeling them as a sequence of hyperedges of a hypergraph, as shown in Fig. (2.2). An analysis of face-to-face interactions across different contexts revealed that, no matter the size of social encounters, group interactions tend to be clustered closely in time, a phenomenon dubbed as burstiness [65]. In the recent past, Gallo *et al.* [67] proposed a systematic framework to measure correlations across time in higher-order networks. Using face-to-face data from multiple sources they analyzed the correlation of groups of different sizes across various time separations. Their analysis revealed that groups of similar sizes are significantly correlated even at a long time-scale, thus reinforcing signatures of past gatherings. Furthermore, using these temporal correlations among groups of different sizes, they highlighted the differences between group formation and group segregation depending on group size. While the previous model considered social interactions from a group-membership perspective, Iacopini *et al.* [68] studied temporal group dynamics from a node-centric perspective. In particular, they found that individuals often move from larger groups to smaller groups and that groups form and break over time in small incremental steps rather than any sudden changes, often forming large cores of central and tightly connected individuals [90].

Beyond humans, high-frequency proximity data have also allowed researchers to track the evolution of higher-order interactions in animal social networks [91]. An analysis of a vulturine guinea-fowl population has revealed that females and low-ranking group members take part preferentially in dyadic interactions, while males and more dominant group members are substantially more likely to engage in groups containing more than two individuals [92]. Beyond simple contacts, higher-order approaches have also been used to study non-dyadic communication patterns and vocal communication in birds, better revealing vocally coordinated group departures and informing models of cultural evolution of vocal communication [93].

2.5 Affiliation and collaboration networks

Affiliation networks, where individuals are associated to groups, are a primary example of social systems which can not be suitably described by simple graphs [46]. Indeed, affiliation to each group can be represented as a hyperedge of a social hypergraph as seen in Fig. (2.3). In

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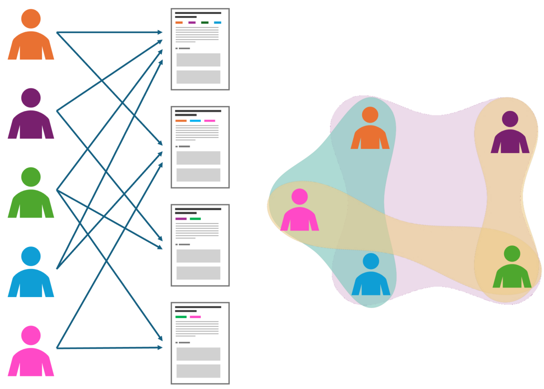

Figure 2.3: **Hypergraph of scientific collaborations**, where each hyperedge represents the set of co-authors of an article.

the early 1980s hypergraphs were first used to describe overlapping participation to voluntary organizations [94], ethnic groups [95] and religious celebrations [96]. This focus on group interactions served soon as a stimulus to develop new network tools, such as centrality measures explicitly taking into account higher-order social relationships [97–99]. In late 2000s, multipartite systems based on *folksonomy* (a system of users collaboratively tagging and annotating data) were used to develop a systematic framework to represent them as hypergraphs based on various projection protocols [100, 101]. Moreover, group memberships can be exploited to define similarity among individuals by introducing suitable association indices [102, 103].

Scientific collaboration networks are one of the most studied affiliation networks [104–106]. In many fields scientific advances are not achieved through the work of lone geniuses but through teamwork, with a tendency of pairwise collaborations to be less and less relevant compared to the outcome of larger groups [107, 108]. At the individual level, higher-order generalizations of local measures such as the node degree have been used to determine the relevance of scientists within scientific domains [109, 110]. At the team level, some collaboration patterns have been found to be prevalent [111], with a sizable number of groups of co-authors often working together exclusively as a single unit [58]. If a group of people represents a true social structure (family, friends, etc), we expect to see that same configuration of nodes recurring repeatedly over time [72]. This tendency of repeated instances of groups is typical of collaborations in science, holding true in workplaces where workers tend to form teams with similar sets of team members [112]. Starting from this observation, it has been possible to extract persistent collaborations by identifying overly abundant statistically significant higher-order interactions [113], finding that most of them are indeed non-dyadic, and geographically co-located with respect to non-persistent co-authorship [114].

Co-authorship networks have also been investigated through the eyes of topological data analysis, providing a characterization of the "shape" of collaborations [115]. Persistent homology, a recent computational technique to extract topological features of a simplicial complex at different spatial resolutions, has been applied to collaborations across different domains, to

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get insights on collaboration patterns across disciplines. Already in the 1970s Atkin pioneered works on the potential of higher-order interactions proposing a mathematical framework based on cohomology and q-analysis to encode higher-order interactions in affiliation data [32, 33]. Real collaboration hypergraphs were found to have peculiar structure, with more clustering and filled triangles than what is observed in randomized systems with the same number of nodes and connections [116]. An analysis of collaboration data from arXiv also showed that when three authors have collaborated as distinct pairs, there is a high chance that they also have published joint papers altogether [117]. The strength of such a "simplicial closure", a generalization of the well-known concept of structural holes for traditional networks [118], may differ according to the type of collaboration hypergraphs, and can also be used to differentiate social networks from biological systems [57]. Even if scientific fields were found to have quite different typical sizes for collaborations, the number of collaborative efforts in which each scientist takes part is generally comparable [117] (with the exception of large-scale experiments such as collaboration at CERN for physics), a finding which could be associated to a maximum capacity for attention.

2.6 Higher-order analysis of real-world dataset

In this section we demonstrate the power of higher-order networks by analyzing collaboration patterns in different scientific domains, investigating arXiv co-authorship data (all papers uploaded between 2007 and 2022) in the fields of physics, computer science, statistics, and mathematics. For each domain, we construct a hypergraph $\mathcal{H}(\mathcal{V},\mathcal{E})$, where each hyperedge denotes the set of co-authors participating in a paper. In the following, we illustrate a variety of higher-order measures and approaches of increasing complexity, capturing different facets of the architecture of real-world collaboration systems.

2.7 Modeling group formation and evolution

There is a substantial literature on social mechanisms that describes the formation and evolution of ties in social networks [121–124]. Focusing, however, solely on dyadic interactions, this work does not incorporate the higher-order nature of many social interactions. Given the higher-order organization of real-world contact networks that we have summarized above, a stream of research has recently focused on proposing simple models able to reproduce the observed empirical patterns. Gallo *et al.* [67] introduced a model to generate a synthetic temporal hypergraph based on the memory of previous encounters. In particular, they showed that, considering a hyperedge update process based on the past occurrences of specific hyperedges of various sizes, reproduced real-life patterns of long term group correlations as well as dynamics of group aggregation and segregation. On the other hand, Iacopini *et al.* [68] proposed a model

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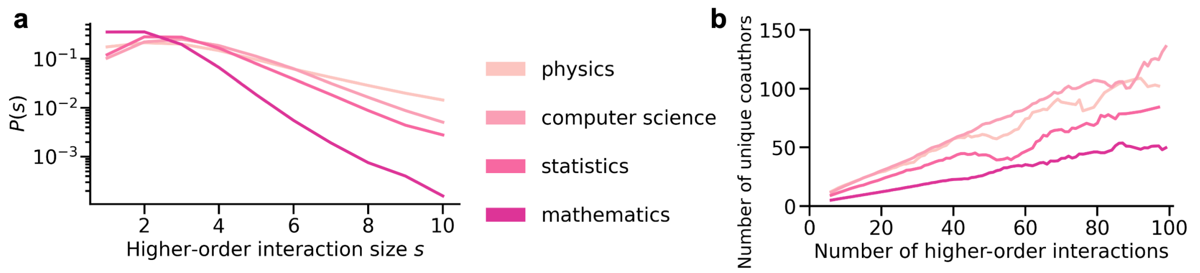

Figure 2.4: **Importance of higher-order approach. (a)** Probability distribution of interaction sizes for various disciplines. *Math* papers are typically written by the smallest teams. By contrast, *physics* papers are often produced by bigger collaborative efforts, as evidenced by the slower decay of $P(s)$. Switching from single papers to career trajectories of the authors, for each author **(b)** shows the number of unique coauthors as a function of their total number of papers. For a fixed number of interactions, we observe a hierarchy among fields, with mathematicians forming fewer unique connections in their career. The inversion between physicists and computer scientists indicates that physicists tend to work in larger teams and also have more persistent collaboration patterns.

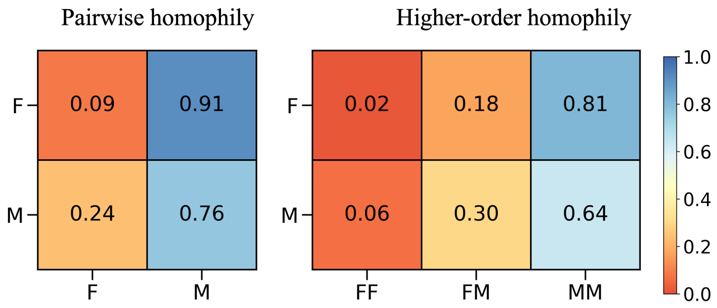

Figure 2.5: **Higher-order homophily.** Focusing on physics – the domain whose collaboration patterns display strongest higher-order character – the plot illustrates the higher-order dimension of homophily in social systems, evaluating gendered interactions separately for dyads and groups [119].

from a node-centric point of view. They considered that at each time step, an individual decides to either stay in the group or leave the old group and join a new group, based on the past history of time spent in the group as well as the trajectory of past encounters (often dubbed as *social memory*). Their model accurately reproduces the empirical patterns of group assembly and disassembly.

Another extension of this line of research concerns the introduction of signs, i.e., having positive and negative links (e.g., to represent friendships and enmities in a collective of people). One of the key mechanisms behind the dynamics of signed networks is social balance theory [125]: loosely speaking, the fact that the friend of my friend is my friend and the enemy of my enemy is also my friend [126]. This implies that some triangles are stable (three people who are all friends with each other, or two people who are friends and are enemies of a third one) and others are unstable (two enemies with a common friend or three enemies). Naturally,

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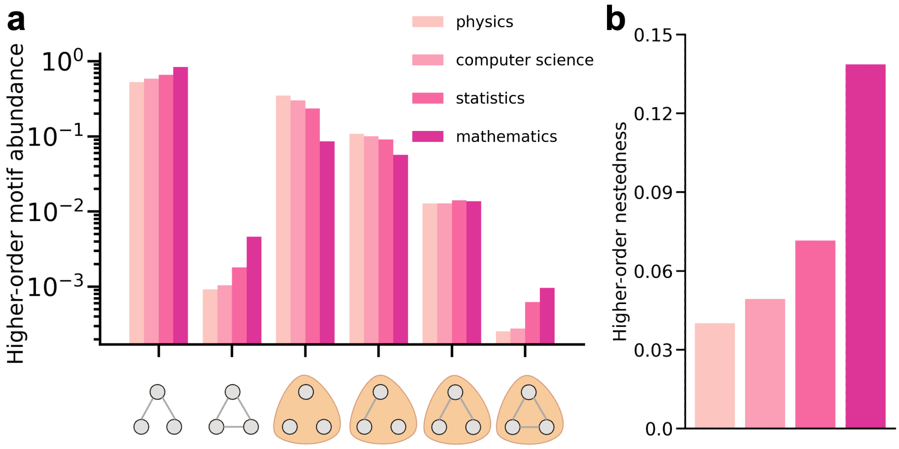

Figure 2.6: **Motif analysis and higher-order nestedness.** Analyzing patterns of interactions at the micro-scale, **(a)** displays the abundance of different higher-order motifs for subgraphs of three authors [58]. Motifs II and VI reveal that it is frequent for statisticians and mathematicians to work in pairs (II), and when a larger team is formed, its members typically also collaborate through pairwise interactions (VI), suggesting the presence of a mechanism known as simplicial closure [57]. By contrast, motif III shows that in physics and computer science groups do not require the presence of underlying dyadic ties. These findings are confirmed by looking at collaboration patterns at a larger scale by computing a measure of higher-order nestedness, which evaluates how much smaller groups are encapsulated in larger ones **(b)** [120] . Due to the cost of processing high dimensional data, it can be convenient to reduce a higher-order network by providing a simplified representation which still captures its essential higher-order structure.

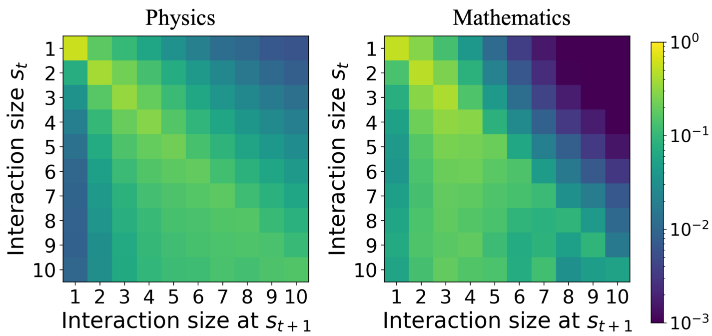

Figure 2.7: **Temporality in higher-order interactions.** By exploiting the temporal nature of the data, the heatmmap quantifies the transition probability $P(s_{t+1}|s_t)$ of switching team size in two consecutive papers [68]. For physics, authors who work in larger collaborations rarely revert back to smaller teams. By contrast, mathematicians more regularly alternate between groups of different sizes.

this calls for a study of triangle motifs in networks as drivers of relationship creation and destruction [127]. In addition, higher-order networks provide a natural formalism to include other motifs (squares, pentagons, cycles of any length) that have also been shown to be relevant in temporal signed networks [128].

A crucial feature neglected by network-based models is that in contact networks agents

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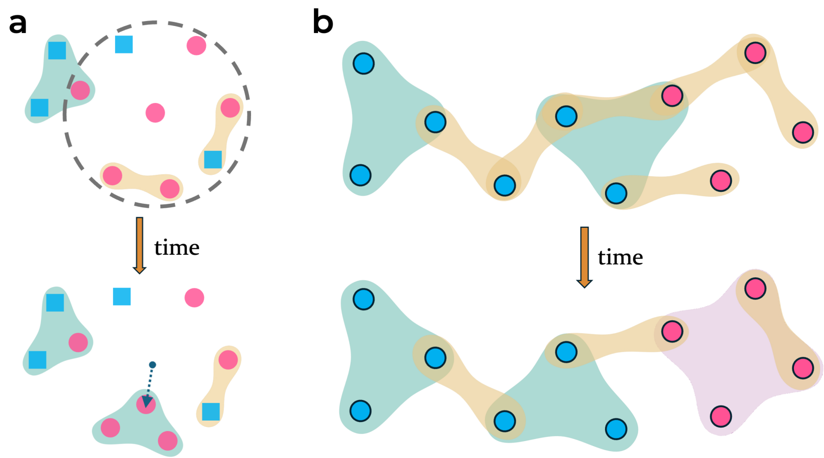

Figure 2.8: **Individual biases lead to formation of homophilic groups.** **(a)** Agent-based model describing the evolution of face-to-face interactions in physical space. An agent considers the groups lying within a spatial range (dotted circle on top), and decides to move and join one of them based on their attractiveness (dotted arrow at the bottom shows the movement). Group attractiveness depends on the properties of the group, such as its size and composition (here pink and blue can denote gender). **(b)** Initial snapshot (top) of the hypergraph where nodes with different inherent attributes are connected to each other through edges and hyperedges. (bottom) With time, the nodes rewire themselves dictated by group biases and preferences to form highly segregated hypergraphs with high homophily.

move in a physical environment. Indeed, simple frameworks based on mobile agents and individual attractiveness have been shown to successfully reproduce the temporal structure and bursty behavior of dyadic interactions [129]. Beyond dyads, the spatiotemporal features of groups in human face-to-face interactions can be captured by agent-based models where each group is characterized by an intrinsic degree of social appeal, the group attractiveness, based on which neighboring agents decide whether to join the group or walk away [119], as illustrated in Fig. (2.8) (a). The framework can reproduce many properties of groups in face-to-face interactions, including their distribution, the correlation in participation in both small and larger groups, and their persistence in time, which cannot be replicated by dyadic models.

The above models can be enriched to account for individual features such as gender, unveiling complex homophilic patterns in groups of different sizes [119] which are not included in standard pairwise measurement of homophily [130]. First, group-level interactions exacerbate homophily, the tendency of individuals to associate with similar others. In this way, homophily can exhibit multiplicative effects in the presence of a group, departing from traditional ways of measuring dyadic attractions [131, 132]. This can lead to social segregation and inequality as groups form around shared attributes as depicted in Fig. (2.8) (b). In a consolidated society where people associate with similar others in multiple shared features such as socioeconomic status, race, ethnicity etc, inequalities tend to become compounded [133] and higher-order interactions can amplify this compounding effect [134].

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2.8 Modeling social contagion

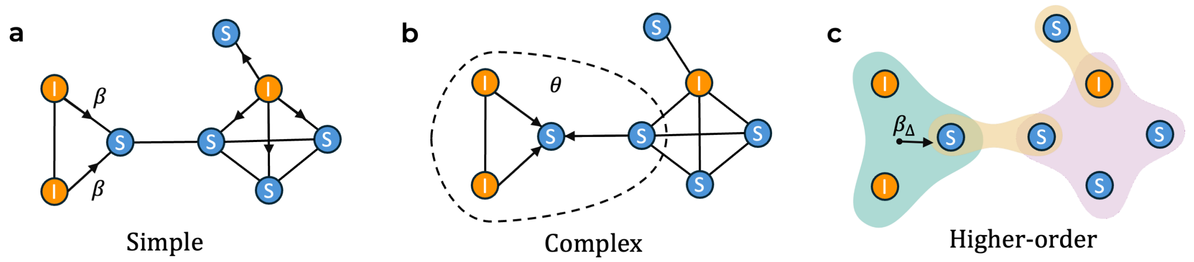

Figure 2.9: **Mechanisms of social contagion.** Social contagion models, where individuals can be either in a susceptible S or infected I state. **(a)** In simple contagion each link acts as an independent source of transmission, over which contagion occurs with probability β. **(b)** In complex contagion multiple exposures are required for transmission, and contagion happens if a sufficiently high fraction θ of contacts is infected. Nevertheless, the exact social structure is neglected, and all neighbors of a node are considered together regardless of whether they influenced a node as part of a group or not. **(c)** The microscopic structure of groups is considered in higher-order contagion models, where groups modeled as hyperedges can have different infection rate based on their size, allowing to model with probability β_Δ stronger (or weaker) transmission occurring in groups.

We now turn to the impact of group interactions on efforts to model the spreading of rumors, the adoption of norms and the diffusion of innovations. In biological contagion, such as epidemic spreading, the probability of infection between a pair of individuals i and j is proportional to the amount of time i and j spend together, in this sense the probability of infection is inherently dyadic [135]. Thus, when an individual is connected to multiple other agents, we can consider each link as an independent source of infection (Fig. (2.9) (a)). In the context of social contagion, the picture is less clear. Although initially considered similar and modeled in similar ways [136–138], we have now come to understand that 'complex' social spreading depends on the network configuration around a susceptible node [139–141]. Unlike the case of disease spreading, being exposed to a behavior for 10 hours by one person is different from being exposed to the same behavior for 1 hour by ten people. Multiple mechanisms for social contagion have been proposed, starting with the threshold model [142, 143], where multiple simultaneous exposures (and not just exposure to multiple sources) are needed for spreading. Opinion dynamics models [144], such as the voter model [145] or majority rule models [146] are other examples of complex interaction dynamics on networks. Theories of complex contagion, where exposures to multiple sources is required for contagion (Fig. (2.9) (b)), are supported by mounting experimental evidence that social spreading is different from disease spreading [147–153]. The detailed mechanisms behind 'complex contagion', however, are still not clear. In the computational domain, various epidemic models have been thoroughly explored but the 'toy models' studied in this domain (see [154] for an overview) are typically

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chosen for their analytical properties, rather than realistic properties.

Recently, however, the use of a framework based on higher-order interactions has shown great promise in allowing us to explicitly model group interactions at the microscopic scale. The crucial novelty is that groups of different sizes may be associated with unequal infection rates, reflecting different degrees of social influence and peer pressure (Fig. (2.9) (c)) [155]. The model [155] mimics a social reinforcement process where group pressure can have an additive effect with respect to traditional pairwise transmission. If collective social influence associated with higher-order interactions is low, the system behaves like a traditional SIS model. Indeed, a regime in which new ideas vanish soon is separated by a critical value of the pairwise transmissibility from an endemic phase, where they persist in the population. Such change is typically continuous, meaning that only a small fraction of the populations will be infected near the critical point. However, if groups impart a high social pressure, the epidemic threshold decreases and the transition separating the two regions becomes discontinuous, leading to large and abrupt changes in collective adoption.

This behavior can be explained analytically by describing the temporal evolution of infection using a mean-field approach, which shows the emergence of co-existence between endemic and non-endemic stable regimes. Importantly, the bistability has social consequences: depending on the number of initially infected individuals the propagation of a norm or behavior may either diffuse widely into the population, or die out. Differing from the traditional pairwise models of social contagion, this finding highlights the necessity of a critical mass in order to initiate social changes in society, as also observed for related dynamics of social conventions [156].

Originally obtained for homogeneous simplicial complexes, results have been generalized to heterogeneous simplicial complexes [157] and hypergraphs [158, 159], giving rise to a promising stream of new research aimed at characterizing contagion through better and more realistic models of social dynamics.

2.9 Evolutionary game theory as a playground for social behavior

Our focus so far has been on characterizing social processes through the lens of group-driven collective behavior. One important aspect in this regard is the strategic thinking that shapes the outcome of individual actions. Game theory was formulated to understand the problem of collective action in N-player interactions [160]. Indeed one of the first major contributions was to show the existence of so-called 'Nash equilibrium' states or stable actions in a N-player game [161]. In this section, we will give a brief overview of developments in game theory that have led to a deeper understanding of collective dynamics in strategic interactions.

Social dilemmas epitomize the tension between individual and collective rationality in game

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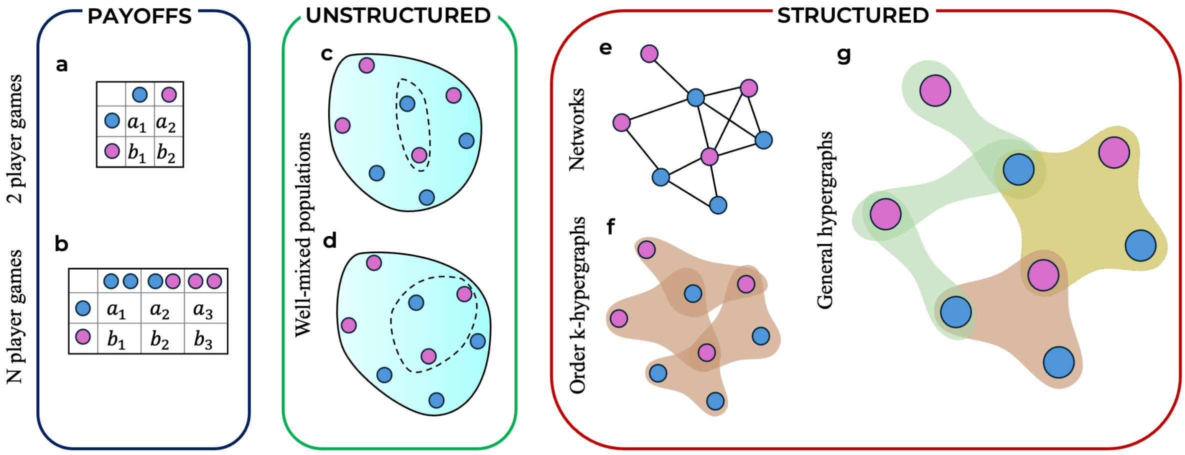

Figure 2.10: **Incorporating complex interactions into evolutionary game theory.** A game is characterized by two actions (denoted here by blue and pink circles) and corresponding payoffs ($\{a_i, b_i\}$). **(a)** The simplest case is for 2-player games, but in general the game can involve N-players. **(b)** For $N > 2$, we get a diverse set of situations including a mixed group of opponents for $N = 3$ with payoffs a_2 and b_2 respectively. Well-mixed populations offer a natural way to consider any game in unstructured interactions (panels **c** and **d**), where the players select their partners randomly at each time step denoted here by dashed curves. Structured populations offer a way to model the real-world organization of interaction patterns. **(e)** Networks are the most basic forms of structured interactions involving only dyadic ties which are best to model 2-player games. **(f)** Hypergraphs resolve this issue by allowing for non-dyadic ties of specific order k, such as $k = 3$, or any arbitrary mixture of orders as illustrated in panel **(g)** for order 2, 3, and 4-hyperedges. Higher-order interactions enable a systematic exploration of heterogeneous group interactions which shape the collective dynamics in evolutionary processes.

theory. The canonical social dilemma emerges when payoff rankings satisfy specific inequality conditions (Figure (2.10) (a)) [162, 163]. While two-player conditions are well-established, formal criteria for N-player social dilemmas were only rigorously defined in [164, 165]. In general, the condition for a N-player game to be called a social dilemma requires a defector earns more than a cooperator in a group of mixed players. Additionally, defectors earn more when they are fewer in number while cooperators earn more when they are more in number in a group. These conditions provide necessary and sufficient criteria to classify any multiplayer game as a social dilemma [166].

One of the major drawbacks of game theory is that the players engage in a single-shot game and are assumed to be perfectly rational. This is seldom true in the real-world organization of our world. People repeatedly interact with each other, while many entities in biology do not have intelligence to rationalize any action [167, 168]. The first limitation was overcome by extending the one-shot games to iterated games. In the famous tournament of Axelrod, computer programs with specific strategies competed against each other and it was found that 'tit-for-tat' was the most successful strategy [169]. These strategies require memory - a cognitive burden avoided in evolutionary approaches. Nowak and May pioneered an alternative

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framework removing both cognitive demands [170]. Inspired by the theory of natural selection, they proposed that there are only two types of players – those who always cooperate and those who always defect. Akin to the survival of the fittest, only those players who perform better than their neighbors survive and reproduce, thus eliminating any mechanism of memory and rationality. Surprisingly this simple system produced complex patterns on a spatial lattice showcasing the power of evolutionary dynamics.

The new era of evolutionary game theory has highlighted multiple pathways to the emergence, survival, and spread of cooperative behaviors in social dilemmas [171]. Over the years, game-theoretic models based on reciprocity [172, 173], image scoring [174–176], and reputation [177–179] have enhanced our understanding of pro-social behavior in collective action problems. In most works, players are assumed to interact in well-mixed, unstructured populations (Fig. (2.10) (b)), where interactions are random and unconstrained, leaving no room for memory or reputation dynamics. However, real-world social systems are far more complex. Networks underpin social relationships, forming the backbone of robust interactions [18,40]. A key mechanism in this context is network reciprocity, where structured interactions (Fig. (2.10) (c)) enable repeated engagements with the same neighbors. This allows cooperator clusters to thrive, surpassing the limitations of well-mixed models. The Nowak-May model exemplifies this, as lattices represent a highly regular network structure [170].

The exact mechanism of evolutionary updates is critical for understanding game dynamics on graphs [30]. Notably, the two most studied update rules—birth-death (BD) and death-birth (DB) – profoundly influence collective dynamics, with DB consistently promoting cooperation across diverse graph structures, from regular lattices to heterogeneous scale-free networks [180–182]. Similarly, shift updating on a circle has been shown to enhance cooperation [183, 184]. Another prominent update rule is imitation by comparison [185, 186]. Here, each player engages in pairwise games with all neighbors, accumulating payoffs from k interactions (where k is the node's degree). The player then randomly selects a neighbor and imitates their strategy with a probability proportional to the payoff difference, often modeled using a Fermi function to control noise in the imitation process. The most striking results emerge in scale-free networks, where cooperation thrives more robustly than in well-mixed or regular structures [27, 187]. This highlights how network heterogeneity can amplify pro-social behavior in social dilemmas.

2.10 Linear group games on structured populations

The public goods game (PGG) is a paradigmatic model of group social dilemmas [188]. Unlike the Prisoner's dilemma, PGG involves N players who contribute resources to a common pool; these resources are amplified by a synergy factor (r) and then shared equally among all group members. In well-mixed populations with random interactions, cooperation can be sustained only if r exceeds the group size. One of the earliest network-based PGG models was introduced

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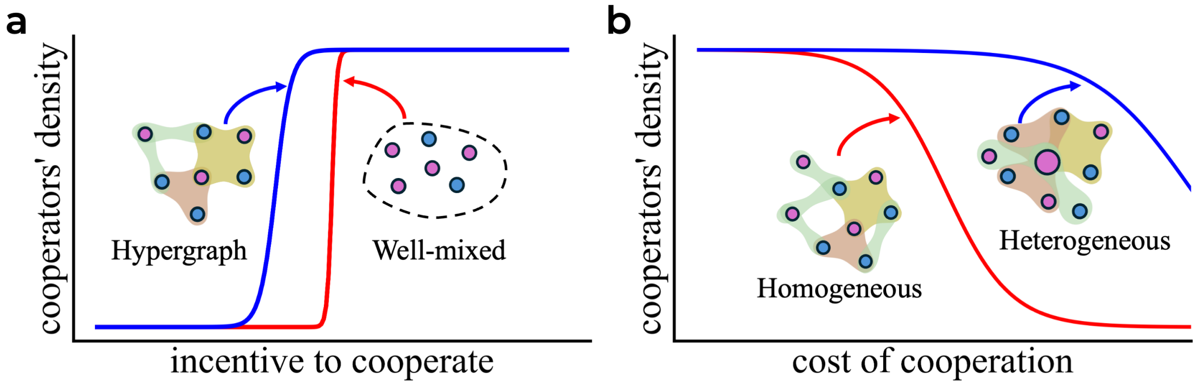

Figure 2.11: **Effect of structural organization on evolution of cooperation.** **(a)** Hypergraphs promote cooperation more than well-mixed populations. In particular, the critical synergy point in the public goods game dynamics is lowered when hypergraphs of various orders are considered. **(b)** Heterogeneous distribution of number of participating groups for a player in structured populations sustains cooperation beyond the observed sociality in homogeneously distributed populations owing to the propagation of cooperative behavior from the hubs.

by Santos *et al.* in their seminal work on scale-free networks [187]. Their key innovation was defining a group as the focal node and all its neighbors. Thus a player with degree k participated in k+1 games: one centered on themselves and k others centered on each neighbor. This setup creates a heterogeneous distribution of group sizes, which under specific conditions promotes cooperation more effectively than well-mixed populations. A critical factor was the initial resource allocation: assigning players a fixed endowment (rather than one proportional to their connections) boosted cooperation. Subsequent studies adopted this definition of group to explore PGG dynamics on networks [53, 189, 190]. Similar analyses were extended to other N-player dilemmas, including the snowdrift game [191, 192], stag hunt [193], and hawk-dove game [194], revealing how population structure shapes collective outcomes.

There are a few limitations to defining group games based on pairwise networks. First, the groups are represented artificially – they are superimposed on the underlying dyadic interaction pattern, creating dissonance between the structural and dynamical components of the system. Second, individuals who are not directly connected may still participate in the same game through common neighbors. This partially undermines the purpose of networks, where not only the presence of a link signifies a relationship but also its absence implies a lack of interaction. Third, such groups cannot represent the full spectrum of possible organizational structures in a population. For instance, local neighborhoods cannot model fully overlapping group interactions. Specifically, if node i is connected to nodes j,k,l, the only possible group is i,j,k,l. A subgroup like i,j,k cannot be formed in this representation. In other words, there is no one-to-one mapping from network-derived groups to all possible node-based groups.

Hypergraphs offer a natural framework to model group-structured games, overcoming the limitations of pairwise networks. Hyperedges encode interactions of arbitrary order, resolv-

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ing all three issues posed by dyadic representations. This framework was first leveraged by Unai-Alvarez et al. for linear public goods games on hypergraphs [195], using a payoff matrix analogous to Eq. (2.1). Their work established a direct correspondence between hyperedges (of any size) and public goods games (with any number of players), unifying the system's structural and dynamical components. Evolutionary dynamics on these hypergraphs accurately replicated the well-mixed replicator equation in mean-field settings. Notably, increasing the proportion of higher-order interactions (through larger hyperedges) enhanced cooperative behavior (Fig. (2.11) (a)). Similar patterns emerge when hypergraphs were constructed by promoting 3-node cliques to hyperedges [196].

$$\begin{array}{c} \\ C \\ \\ D \end{array} \begin{array}{c} \begin{array}{ccc} CC & CD & DD \end{array} \\ \begin{bmatrix} \frac{3rc}{3} - c & \frac{2rc}{3} - c & \frac{rc}{3} - c \\ \frac{2rc}{3} & \frac{rc}{3} & 0 \end{bmatrix} \end{array} \tag{2.1}$$

The heterogeneity of real-world interaction structures must be carefully considered. Unlike pairwise networks, hypergraphs offer multiple dimensions of heterogeneity that critically influence cooperation dynamics. This heterogeneity can emerge either from:

1. Variation in node participation (number of interactions per node),
2. Non-uniform distribution of interaction sizes.

For example, Unai-Alvarez et al. found that power-law hypergraphs with exponential cutoffs behaved similarly to uniform random hypergraphs, while scale-free hypergraphs unexpectedly suppressed cooperation [195]. Conversely, Burgio et al. observed enhanced cooperation with increased heterogeneity when using different hypergraph generation algorithms (Fig. (2.11) (b)) [196]. This apparent contradiction stems largely from hyperdegree-hyperdegree correlations – when present, they can restrict cooperator spread through topological trapping effects [195, 196].

The hypergraph framework has enabled numerous studies analyzing PGG through systematic modifications of model components. First, research demonstrated that heterogeneous investment – where agents contribute more to hyperlinks with cooperators – can enhance prosociality in spatial PGG [197, 198]. Interestingly, increased investment in cooperative hyperlinks mostly leads to increased cooperation [198]. Second, strategy space variations impact steady-state cooperation levels. While allowing different strategies across hyperlinks boosts cooperation (within limits) [199], combining reinforcement learning and reputation mechanisms yields more robust cooperative outcomes [200, 201]. Finally, zero-determinant (ZD) strategies were shown to emerge in mixed 2- and 3-player PGG scenarios [202], extending this celebrated concept to higher-order interactions.

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So far we have considered the simple case of linear public goods game where the payoffs of the players scale fraction of cooperators (ρ) linearly as follows:

$$\pi_C = r * \rho * c - c \quad (2.2)$$
$$\pi_D = r * \rho * c \quad (2.3)$$

While this model offers valuable mathematical and social insights into group dynamics, it fails to capture the true complexity arising from non-linear synergistic effects in groups. Linear payoffs reduce group interactions to simple sums of pairwise exchanges, overlooking scenarios where each additional cooperator multiplicatively affects group outcomes. For example, a cooperator's payoff $\pi_C = r \cdot f(\rho) \cdot c - c$ can capture non-linear benefits via $f(\rho)$, which may be quadratic, exponential, or take other forms – reflecting how contributions can snowball or diminish disproportionately with group compositions. This reveals a critical insight: the most consequential aspects of group interactions emerge from their inherent non-linearity. The following section explores how to model such non-linear dynamics and the rich complexity they introduce through higher-order interactions.

2.11 Non-linear higher-order social dilemmas on hypergraphs

Non-linearity is fundamental to complex systems, enabling diverse collective behaviors from spreading of ideas [155] to opinion synchronization [203] and consensus formation [204]. This section examines higher-order game models that systematically incorporate non-linear features. The simplest scenario exhibiting non-linearity involves 3-player interactions, which introduce two key effects:

1. *Structural effect*: Linearly increasing the number of 2-player and 3-player interactions changes the mixing ratio of interactions non-linearly. This is because each order m interaction contributes with weight m to the total count of interactions, thus skewing the proportions.

2. *Dynamical effect*: With strategies C (cooperate) and D (defect), 3-player games enable unique multi-body interactions. While opponent sets CC and DD can be decomposed into symmetric pairwise equivalents, the CD configuration represents a fundamentally irreducible 3-body interaction. This symmetry breaking is a hallmark of genuine higher-order dynamics.

To our knowledge, Civilini et al. were among the first to systematically exploit non-linearity in evolutionary dynamics at both structural and dynamical levels. Their foundational work considered a hypergraph mixing 2-player and 3-player games through 2- and 3-hyperedges

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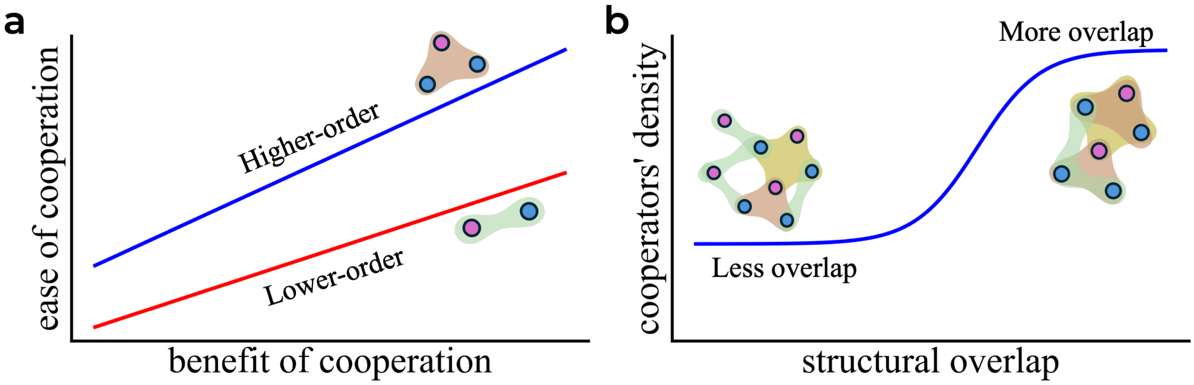

Figure 2.12: **Higher-order interactions enhance pro-social behavior.** **(a)** Groups promote cooperation more than pairwise interactions. For instance, we can analytically calculate the fixation probability of cooperators as a function of benefit associated with cooperative behavior. The qualitatively larger slope for higher-order interactions points to the synergistic benefits of groups. **(b)** Higher structural overlap between interactions of different orders can elevate cooperation in the system. The aggregation of payoffs across orders enables for the positive effects of groups to trickle down to lower-order interactions.

(in proportion δ) - a model we will analyze in detail in Chapter 3. When studying specific social dilemmas like the Prisoner's dilemma or public goods game, we can constrain the payoff matrix's parameter space for tractability. Sheng et al. advanced this approach by formulating a specialized 3-player non-linear public goods game with payoff entries defined as [205]:

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{ccc} CC & CD & DD \end{array} \\ \begin{bmatrix} -c+\dfrac{(\delta_3^2+\delta_3+1)b}{3} & -c+\dfrac{(\delta_3+1)b}{3} & -c+\dfrac{b}{3} \\ \dfrac{(\delta_3+1)b}{3} & \dfrac{b}{3} & 0 \end{bmatrix} \end{array} \tag{2.4}$$

where $\delta_3 \neq 1$ accounts for various instances of non-linear effects in the public goods game. One of the major findings of the study was that allowing for non-linearity in 3-player games promotes cooperation in both order 3 hypergraphs as well as hypergraphs consisting of 2 and 3-hyperedges. Starting from the simple case of star networks, the work generalizes to clique networks and any arbitrary order hypergraphs to analytically calculate the critical benefit to cost ratio $\left(\frac{b}{c}\right)^*$ using the fixation probability theory. The study highlighted the critical role of higher-order interactions by demonstrating that it is always easier for cooperation to invade a population consisting of 3-player interactions (pure or mixture with 2-player interaction) compared to a purely dyadic interaction based populations (see Fig. (2.12) (a)).

While these models offer novel perspectives on multi-player interactions, they overlook several crucial real-world complexities. First, their strategies remain simplistic (uni-dimensional

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and memory-less), despite extensive evidence from both mathematical and social sciences emphasizing the need to consider diverse strategic landscapes [206–210]. Second, they fail to capture the rich structural organization of real interactions - particularly the multi-level, nested hierarchies of human social networks with their characteristic small-world properties, high clustering, and heterogeneous connectivity [8, 9, 15]. Understanding how these fundamental network features shape cooperation dynamics remains a critical open challenge.

Recent models have advanced our understanding of cooperative behavior by addressing these limitations. Building on the non-linear PGG framework from Ref. [205], Ma et al. introduced interaction-size-dependent strategies [211]. Their analytical derivation of the cost-to-benefit ratio's critical threshold revealed that specific ranges of non-linear factors selectively promote cooperation. Notably, increased overlap between different-sized interactions further enhanced cooperative outcomes (Fig. (2.12) (b)). This aligns with broader findings that strategically tuning the proportion of higher-order interactions consistently boosts cooperation [212–214]. Collectively, these studies demonstrate how hypergraph-based social dilemmas reveal richer, more nuanced cooperative landscapes than traditional pairwise models.

2.12 Other collective games

Non-cooperative game theory studies competitive situations wherein players participate in the game independently and try to maximize their gains or minimize their losses [160]. In this context, the Prisoners' dilemma and the public goods game provide a robust framework to model the tension between various elements of the social dilemma. However, there are other types of games which can be used to model different types of situations. For example, the stag hunt game models coordination between agents [215], while the snowdrift game models the emergence of anti-coordination [216–218]. Naturally, the higher-order equivalents of such games are equally important to explore the breadth of social interactions in both humans and non-human entities. In this section, we look at the impact of higher-order formalism in a variety of games. We also provide some alternative ways of utilizing the higher-order interactions beyond hypergraphs.

In certain decision-making scenarios, higher-order interactions can also be responsible for the emergence of irrational herd behaviors, such as those observed during financial bubbles. In Ref. [219], by studying an evolutionary model of group risk propensity on hypergraph with tunable hyperedge and hyperdegree distribution, the authors observed that depending on the level of heterogeneity of the hypergraph the model could dramatically deviate from the predicted Nash Equilibrium (NE) of the underlying group game. In particular, when the exponent γ of the power-law co-membership distribution (i.e., the distribution of the number of group co-members of each individual) exceeds the critical threshold $\gamma_c = 3$, the model undergoes a continuous phase transition deviating from the predicted NE to a new stationary state where the whole population becomes risk prone, imitating the strategy of the most connected nodes

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(the hubs). The observed emerging herd dynamics can be regarded as irrational from a game theoretic perspective, as it is followed by a collapse of the average income in the population.

The sender-receiver game serves as a foundational model for studying truthfulness evolution in social interactions. In its classical two-player formulation, the game involves sequential actions where the sender chooses to communicate truthfully or deceptively, while the receiver decides whether to believe the message. Recent work by Kumar et al. [220] has extended this framework to incorporate group dynamics through multiple interacting receivers organized in higher-order structures. Their analysis reveals that higher-order interactions can sustain honest communication even under deceptive incentives, particularly in regular structures like hyper-rings. However, this stabilizing effect exhibits nonlinear dependence on group size, gradually diminishing as the number of participants grows. These findings highlight the nuanced relationship between interaction topology and the evolutionary stability of truthfulness, where the benefits of higher-order organization must be carefully balanced against its associated costs. The naming game has similarly been demonstrated to facilitate norm emergence when incorporating group-level interactions, particularly in social contexts [156].

Uniform random hypergraphs assume independence between interactions of different orders, while simplicial complexes provide an alternative representation that requires all subsets of a k-order interaction to be present. This simplicial framework was first employed by Guo et al. [221] to model mixed games through strategy coherence in 2-simplices (3-body interactions). Their work demonstrated that tuning payoff structures and higher-order interaction frequencies diversifies individual strategies, ultimately enhancing pro-social behavior. Subsequent studies have expanded this approach, including: (a) Scale-free higher-order interactions promote cooperation in N-player snowdrift games [222], and (b) investigations of evolutionary dynamics on nested simplicial complexes [223] incorporating second-order reputation evaluation [224], adaptive structures [225], and interdependent complexes [214]. Collectively, these advances highlight how higher-order interactions fundamentally reshape the evolution of social behavior in groups.

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CHAPTER 3

MODELS OF HIGHER-ORDER GAMES

At a Glance

From climate change to public health, cooperation is key to solving major social challenges, but we have yet to solve the riddle of the emergence of these prosocial actions. In this work, we introduce a novel framework to examine the dependence of cooperation on various aspects of group interactions. By looking at strategies that build upon realistic conditions such as overlapping social interactions, we examine how groups of different sizes affect each other. This reveals that cooperation can be realizable in hierarchical group contexts more than the standard assumption of one-dimensional strategies. Our findings show that cooperation can emerge in group settings and may help guide strategies to promote collective action.

In the preceding chapter, we surveyed existing work on games in structured populations, establishing hypergraphs as the natural mathematical framework for modeling N-player interactions. Through our examination of public goods games, we demonstrated how increasing group size can enhance cooperative outcomes. However, current literature lacks a comprehensive theoretical framework for analyzing general N-player games on hypergraph structures. This chapter addresses this gap by developing a unified approach to study social dilemmas on hypergraphs, with particular emphasis on characterizing the structural properties and dynamical mechanisms that facilitate cooperation in group interactions.

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3.1 Modeling higher-order games on hypergraphs

A population of N players is represented as a hypergraph $\mathcal{H}(\mathcal{V},\mathcal{E})$, where the players are the nodes of the hypergraph $\mathcal{V}$, such that $|\mathcal{V}| = N$ and the set of hyperedges $\mathcal{E}$ represents the set of games played in the population. Hyperedges are a generalization of the network's edges to group interactions, a hyperedge representing a group of an arbitrary number of interacting nodes. In particular, we focus on the case where the hypergraph $\mathcal{H}$ consists of only

2-hyperedges and 3-hyperedges, respectively denoted by $\mathcal{E}^/$ and $\mathcal{E}^\Delta$. A 2-player game is associated with each 2-hyperedge, while a 3-player game to each 3-hyperedge. Henceforth, we will use superscripts $/$ and Δ to indicate pairwise and higher-order quantities respectively, while reserving the subscripts for node labels. Each player participates in all the hyperedges to which it belongs. Since every player is part of multiple hyperedges, they engage in a separate game for each one. Thus, the total number of games a player i takes part in equals their hyperdegree k_i. We define $k_i^/$ and k_i^Δ as the number of 2-player and 3-player games player i is involved in, respectively, satisfying $k_i^/ + k_i^\Delta = k_i$.

Each player i is associated with a strategy vector $s_i = [s_i^/, s_i^\Delta]$, which defines the strategy $s_i^/$ adopted by the player in pairwise games and s_i^Δ in 3-player games. We focus on social dilemma games, therefore each player can either choose to cooperate (strategy $s_i^{[\cdot]} = C$) or defect ($s_i^{[\cdot]} = D$) in each type of interaction independently, where $[\cdot] \in \{/, \Delta\}$.

Payoffs: The payoffs defining the symmetric 2-player game can be conveniently represented as a payoff matrix $M^/$:

$$M^/ = \begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \left[\begin{array}{cc} R^/, R^/ & S^/, T^/ \\ T^/, S^/ & P^/, P^/ \end{array} \right] \end{array}, \tag{3.1}$$

where the first entry in each cell denotes the payoff earned by the row player, while the second entry is the payoff for the column player. $R^/$ denotes the reward for mutual cooperation, while $P^/$ denotes the penalty for mutual defection. A cooperator earns a payoff $S^/$ (namely *sucker*'s payoff) against a defector, while the defector earns a payoff $T^/$ (*temptation* to defect) when facing a cooperator. The relative order of these payoffs determines the type of 2-player social dilemmas. In particular for our analysis, we will study Prisoner's dilemma games, defined by the ordering $T^/ > R^/ > P^/ > S^/$. To reduce the number of parameters in the game, we fix $R^/ = 1$ and $P^/ = 0$ to define the limit of the parameters as is typically done in the literature [1, 30, 212, 221, 226].

Analogously to the case of pairwise games, the payoffs in a 3-player game are determined by the combination of strategies of the players. However, with three players, the payoffs depend on the three strategies of the players considered simultaneously. Therefore, in the case of 3-player games the higher-order game payoff structure is represented as a $2 \times 2 \times 2$ payoff cube M^Δ, such that each axis denotes the possible strategies of each player [1]. For the sake of readability, we represent this payoff cube as two 2×2 payoff matrices $M^\Delta(C)$ and $M^\Delta(D)$ stacked on top of each other (Fig. (3.2)):

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$$
\begin{aligned}
M^{\Delta}(C) &= \begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \left[\begin{array}{cc} R^{\Delta}, R^{\Delta}, R^{\Delta} & G^{\Delta}, T^{\Delta}, G^{\Delta} \\ T^{\Delta}, G^{\Delta}, G^{\Delta} & W^{\Delta}, W^{\Delta}, S^{\Delta} \end{array} \right] \end{array}, \\
M^{\Delta}(D) &= \begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \left[\begin{array}{cc} G^{\Delta}, G^{\Delta}, T^{\Delta} & S^{\Delta}, W^{\Delta}, W^{\Delta} \\ W^{\Delta}, S^{\Delta}, W^{\Delta} & P^{\Delta}, P^{\Delta}, P^{\Delta} \end{array} \right] \end{array},
\end{aligned}
\tag{3.2}
$$

where $M^{\Delta}(C)$ and $M^{\Delta}(D)$ respectively denote the payoff matrices when the third player is cooperating or defecting. Each entry of the matrix is a 3-tuple with the payoffs of player 1 (row), player 2 (column), and player 3 respectively. As for pairwise games, R^{Δ} and P^{Δ} denote respectively the payoffs for mutual cooperation and mutual defection among 3 players. T^{Δ} is the higher-order temptation payoff for deviation from mutual cooperation by defecting and S^{Δ} is the higher-order sucker's payoff for deviation from mutual defection by cooperating. However, the 3-player game introduces some new payoffs which do not have any counterpart in the 2-player game. G^{Δ} denotes the payoff of a cooperator in a group with two defectors, i.e. for the strategy profile $\{C, C, D\}$ and all its permutations. On the other hand W^{Δ} denotes the payoff of a defector playing against two cooperators i.e. corresponding to $\{D, D, C\}$ and all its permutations [1].

Higher-order social dilemma: A social dilemma is a type of collective action problem where there is a tension between personal benefit and collective good. For instance, a 2-player Prisoner's dilemma represents a social dilemma since, given the payoff ordering $T^{/} > R^{/} > P^{/} > S^{/}$ defining the game, the rational choice (or Nash equilibrium) for a player is to choose defection, even though it would be more beneficial (i.e., it would bring a higher payoff) to cooperate for both players. However, generalizing the concept of social dilemma to multi-player games is not trivial. Over the years, different definitions [164, 227] of social dilemmas in multi-player game representation have been proposed. Here, we adopt the definition proposed by Ref. [166], according to which the payoffs of a 3-player game have to satisfy the following conditions to qualify as a social dilemma:

a. A focal player benefits when other members cooperate, regardless of its own strategic choice. This leads to the following payoff relationships in our framework:

$$R^{\Delta} \geq G^{\Delta} \geq S^{\Delta}, \tag{3.3}$$

$$T^{\Delta} \geq W^{\Delta} \geq P^{\Delta} \tag{3.4}$$

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b. Cooperating mutually yields a greater reward than mutual defection:

$$R^{\Delta} > P^{\Delta} \tag{3.5}$$

c. Within a group, defectors receive higher payoffs than cooperators:

$$T^{\Delta} > G^{\Delta}, \tag{3.6}$$
$$W^{\Delta} > S^{\Delta} \tag{3.7}$$

d. Switching from cooperation to defection results in a higher payoff. In a three-player setting, this translates to:

$$T^{\Delta} > R^{\Delta}, \tag{3.8}$$
$$W^{\Delta} > G^{\Delta}, \tag{3.9}$$
$$P^{\Delta} > S^{\Delta} \tag{3.10}$$

We characterize the higher-order Prisoner's Dilemma by maintaining the same relative ranking of payoffs as in the standard two-player case:

$$T^{\Delta} > R^{\Delta} > P^{\Delta} > S^{\Delta} \tag{3.11}$$

However, in the higher-order game, two additional payoffs, G^{Δ} and W^{Δ}, are present. If $G^{\Delta} < W^{\Delta}$, all the above conditions (a-d) hold, classifying the game as a *strong* social dilemma. Conversely, when $G^{\Delta} > W^{\Delta}$, condition d is not met, leading to what is known as a *relaxed* social dilemma. In general, the quantity $\alpha = W^{\Delta} - G^{\Delta}$ can be interpreted as a measure of the strength of the higher-order Prisoner's Dilemma (PD). Notably, the strong social dilemma has only full defection ($\{D,D,D\}$) as a Nash equilibrium, whereas in the relaxed social dilemma, besides full defection, the strategy profile $\{C,C,D\}$ also forms a Nash equilibrium [1,228,229]. We showcase all the possible fixed points arising out of this situation in Fig. (3.1).

Evolutionary dynamics: We evolve the system by using the Monte Carlo method for stochastic dynamics. At each time step, we select a random player f as the focal player and one of its neighbors m in either of the layers as the model player. They both play with all their neighbors in both orders of interaction and collect payoffs π_f and π_m respectively. Here, $\pi_f = \pi_f^{/} + \pi_f^{\Delta}$ and $\pi_m = \pi_m^{/} + \pi_m^{\Delta}$ denote the total payoff from both the orders [230] (see Fig. (3.2)). We define the transition probability based on the payoffs as the Fermi function: [186,231,232],

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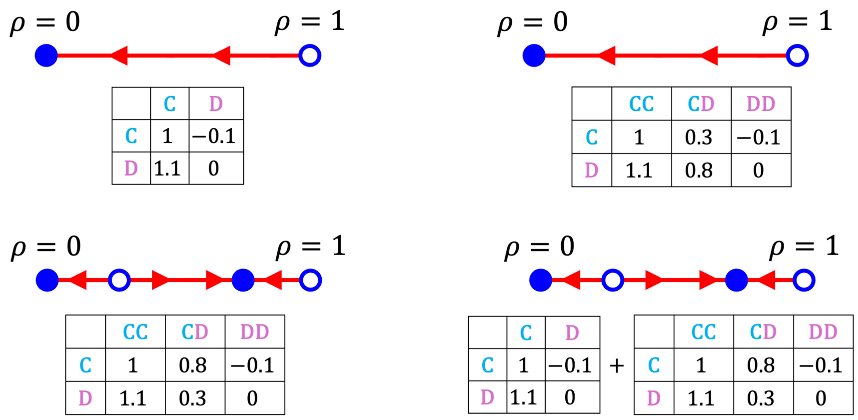

Figure 3.1: **Fixed points of higher-order games.** (top row) For classical two-player and three-player PD, full defection (filled blue circle) is the only stable equilibrium, while full cooperation (empty blue circle) remains unstable. (bottom row) In weak three-player PD ($\alpha < 0$) and mixed 2 and 3 player PD systems, the dynamics reveal an additional stable fixed point and corresponding unstable point (arrows indicate flow directions).

$$\Pi_t = \frac{1}{1+\exp[-w(\pi_m - \pi_f)]}, \tag{3.12}$$

where w quantifies the noise in the copying process. In the limit $w \to \infty$, Π_t approaches 1 if $\pi_m > \pi_f$ and 0 if $\pi_f > \pi_m$. In the other limit $w \to 0$, Π_t approaches 0.5, independent of the magnitudes of π_f and π_m, thus becoming a random process. We choose an intermediate value of noise, $w = 1/(k^{/} + k^{\Delta}) \approx 0.16$ [233–235].

To account for the dynamical coupling between interaction orders, we assume that the focal player can imitate the strategies of the model player associated with the two layers. To illustrate with an example, consider the update of the pairwise strategy of a focal player f denoted by $s_f^{/}$ (Fig. (3.2)). We now assume that the player can copy the pairwise strategy of the model player m denoted by $s_m^{/}$. However, here we propose that the dynamical coupling between the two layers also enables the focal player to copy the higher-order strategy of the model player denoted by s_m^{Δ}. We denote the probability of imitating the strategy of the model player from the other layer as p_{switch}, controlling the dynamical coupling between the two layers and allowing for inter-order imitation. The profile of all transition probabilities at a given time step can be summarized as follows,

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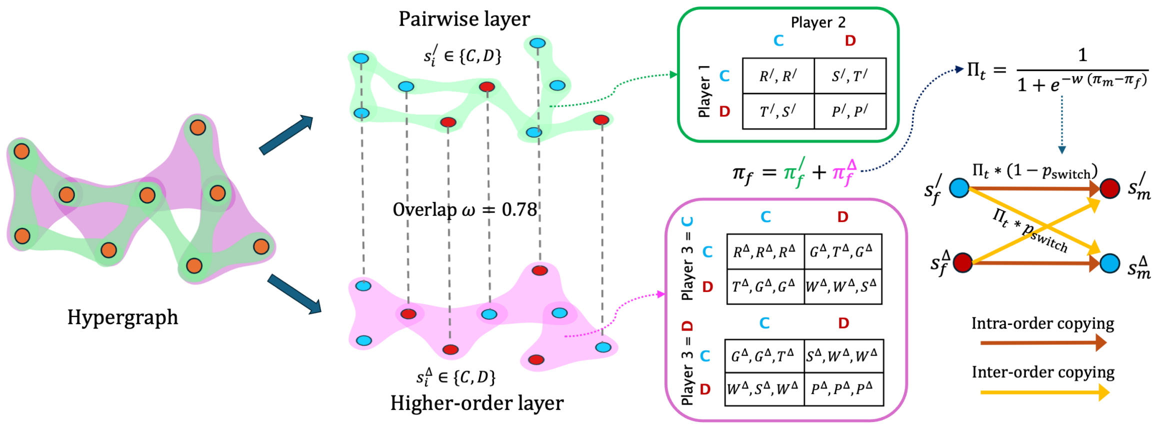

Figure 3.2: **Evolutionary dynamics of higher-order games.** Hypergraphs consisting of pairwise (green) and higher-order (pink) interactions connect individuals. A tunable fraction ω describes the structural overlap between the different orders. Each individual has order-specific strategies ($s_i^{/}$ and s_i^{Δ}) to either cooperate (C) or defect (D) with others and play the corresponding 2 or 3-player games with its neighbors. After accumulating the payoff across both orders, the player imitates the strategy in either order of a random neighbor based on the dynamical coupling p_{switch}.

$$s_f^{/} \to \begin{cases} s_m^{/} & \text{with } 0.5\cdot(1-p_{\text{switch}})\cdot\Pi_t \\ s_m^{\Delta} & \text{with } 0.5\cdot p_{\text{switch}}\cdot\Pi_t \end{cases} \tag{3.13}$$

$$s_f^{\Delta} \to \begin{cases} s_m^{\Delta} & \text{with } 0.5\cdot(1-p_{\text{switch}})\cdot\Pi_t \\ s_m^{/} & \text{with } 0.5\cdot p_{\text{switch}}\cdot\Pi_t \end{cases} \tag{3.14}$$

We obtain one full Monte-Carlo Step (MCS) by repeating the above procedure $2\cdot N$ times so that each player gets the opportunity to update both the pairwise and higher-order strategies once on average. The structural and dynamical components of our model are fully visualized in Fig. (3.2).

We characterize the pro-social behavior in our system using several descriptors. We introduce $\rho_0^{/}$ and ρ_0^{Δ} as the initial density of cooperators in the pairwise and higher-order layers respectively. Correspondingly, $\rho_0 = \frac{1}{2}[\rho_0^{/}+\rho_0^{\Delta}]$ denotes the initial mass of cooperators in the system. We always consider $\rho_0^{/} = \rho_0^{\Delta} = \rho_0 = 0.5$ unless stated otherwise. To quantify the stationary state properties of the system, *i.e.* a state where the observables of the system become time independent, we compute $\rho^{/}$ as the fraction of nodes cooperating in the pairwise interactions and ρ^{Δ} as the fraction of nodes cooperating in the higher-order interactions. We denote the overall cooperation level in the system as [230, 236],

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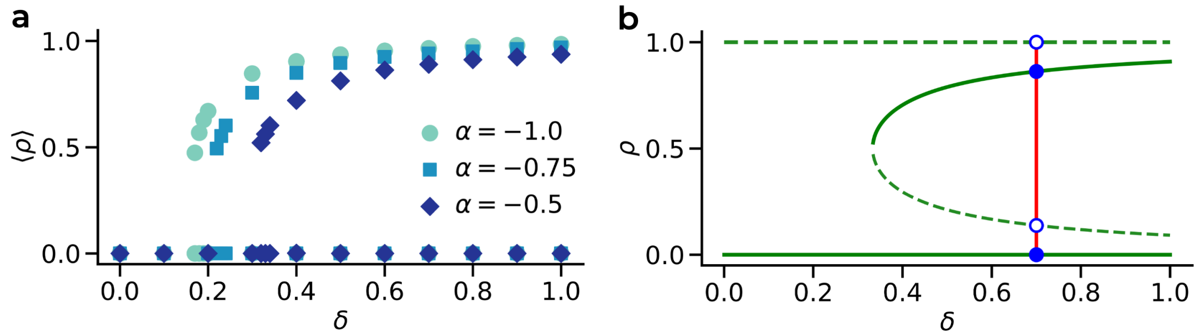

Figure 3.3: **Explosive transition in higher-order games.** **(a)** Stochastic simulations on hypergraphs of size $N = 1500$ showcase an explosive transition to a bistable state with one state showing high levels of cooperation beyond a critical fraction of higher-order interactions δ for various values of α. **(b)** The collective behavior of the system can be understood by looking at the evolutionary stable (thick lines, filled circles) and unstable (dashed lines, empty circles) states of the replicator equation in mean-field limit.

$$\rho = \frac{1}{2}\left[\rho^{/} + \rho^{\Delta}\right] \tag{3.15}$$

Furthermore, $\langle\rho\rangle$ denotes the ensemble average of ρ over M different independent runs.

3.2 Simplified case of higher-order games

We first look at a particular case of our model by considering the following simplifications:

- Each agent has a scalar strategy: $s_i^{/} = s_i^{\Delta} = s_i$, thus rendering p_{switch} meaningless.
- We consider the case of uniform random hypergraphs.
- To reduce the number of free parameters in the calculations, we consider that $R^{\Delta} = R^{/} = R$ and similarly for $S^{\Delta} = S^{/} = S$, $T^{\Delta} = T^{/} = T$, and $P^{\Delta} = P^{/} = P$.

We investigate the case of higher-order Prisoner's dilemma consisting of both 2-player and 3-player interactions. Figure (3.3) (a) reports the long-term behavior of the system obtained from stochastic evolutionary game dynamics on hypergraphs of size $N = 1500$. We observe that as we increase the fraction of higher-order interactions, δ, the population undergoes a phase transition to a bistable state, where one state is characterized by a high density of cooperators, while the other state is full defection. Note that we are considering the case of *weak* social dilemma, *i.e.* $\alpha < 0$. To gain deeper insight into the dynamics of the system, we write the mean-field replicator equation for the system as follows:

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$$\frac{d\rho}{dt} = \rho(1-\rho)\left[\pi_C - \pi_D\right], \tag{3.16}$$

where π_C and π_D, respectively denote the average payoff earned by a cooperator and defector. By substituting the expressions for average payoffs, we can write down the payoff difference as,

$$\pi_C - \pi_D = -\rho^2 c\delta + \rho(c\delta - b - 2S) + S, \tag{3.17}$$

where $a := -2\alpha$, $b := T - S - 1$ and $c := (a+b)$. By solving the quadratic equation for ρ, we find the non-trivial stationary solutions as

$$\rho^*_{\pm} = \frac{c\delta - b - 2S \pm \sqrt{(c\delta - b)^2 + 4S(b+S)}}{2c\delta}. \tag{3.18}$$

This leads to positive real-valued $\rho^*_{\pm}$ when:

$$\delta \geq \delta^{\text{th}}_1 = \frac{b + \sqrt{-4S(b+S)}}{c}. \tag{3.19}$$

A stability analysis of the solutions reveals that, while $\rho^*_D = 0$ and ρ^*_+ are stable, ρ^*_- and $\rho^*_C = 1$ are unstable stationary states as denoted in Fig. (3.3) (b) for $\delta = 0.7$. In particular, we find an excellent match between the data from stochastic simulations and solutions predicted by the mean-field replicator equation. The novel insight from this simplified scenario is that despite having $\{CCD\}$ as one of the Nash equilibria, the system still requires a minimum fraction of higher-order interactions to sustain cooperation, thus highlighting the important role of structure in collective cooperation [1]. In the next section, we dive deeper into the structural components of the model and examine how they influence the emergence of cooperation.

3.3 General case of higher-order games

The simplified model suffers from a variety of limitations. First, it considers that the agents choose the same strategy across groups of different sizes. Empirical research has shown that this is usually not the case [206–210]. Larger groups can exert peer pressure which does not necessarily grow linearly with group size [237–239]. Second, it considers that the groups of different sizes do not overlap. Even though these constraints ease the mathematical difficulty for analytical calculations as we saw before, multiple social interaction datasets consisting of higher-order relations have shown that real-world network structures might lay in between the two extreme cases of fully overlapping and fully non-overlapping [67,68,211,240,241]. Third, the model assumes that the payoffs obtained from group interactions are the same as pairwise interactions. From an economic utility perspective some interactions have an undue advantage

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since it is easier to coordinate actions in smaller groups. In the next two paragraphs, we discuss possible ways to overcome these limitations.

Comparability of pairwise and higher-order games: It is crucial that payoffs for hyperedges of different sizes remain comparable so that larger groups do not give an undue advantage (or disadvantage) simply because of their group interaction [237–239, 242]. For instance, consider a situation where a player cooperates with two cooperating players in two dyadic interactions earning a total payoff of $2R^{/}$. If instead the player cooperated with the same two cooperating players but in a 3-player interaction, it will earn a payoff of R^{Δ}. Since the evolutionary dynamics typically consists of aggregating payoffs to determine the fitness of strategies, if $2R^{/} \neq R^{\Delta}$, the total payoff earned by a player by interacting with the same players but with different types of interaction (pairwise or higher-order) will be different. Even though larger groups can have synergistic effects [237–239, 243] making cooperation naturally more or less advantageous, we want to focus on emergent collective behavior in absence of these phenomena. Inspired by this line of thinking, one can imagine that from a player's perspective, a single 3-player interaction is equivalent to two pairwise interactions. Here, we introduce a new payoff constraint for the pairwise and higher-order payoffs, $R^{\Delta} \sim R^{/} + R^{/} = 2R^{/}$ and similarly for $S^{\Delta} = 2S^{/}$, $T^{\Delta} = 2T^{/}$, and $P^{\Delta} = 2P^{/}$. We tune the values of G^{Δ} and W^{Δ} to explore the landscape of higher-order PD based on its strength mentioned above. Note that this payoff constraint is simply a choice and does not increase the number of parameters in the system.

Put together, we assign the following payoff entries,

- $T^{/}, R^{/}, P^{/}, S^{/} = 1.1, 1, 0, -0.1$
- $T^{\Delta}, R^{\Delta}, P^{\Delta}, S^{\Delta} = 2.2, 2, 0, -0.2$
- $W^{\Delta} = 0.7$ and denoting the social dilemma strength as $\alpha = W^{\Delta} - G^{\Delta} \in [-1.4, 0.3]$

Topological overlap in higher-order networks: We consider a random-regular hypergraph such that a player i has $k^{/}$ 2-hyperedges (or pairwise neighbors) and further participates in k^{Δ} 3-hyperedges (or triangles). We can represent any form of higher-order interactions as a multilayer hypergraph, where each layer consists of interactions of a specific size/order, as illustrated in Fig. (3.2). The advantage of such a representation is that we can adapt well-established measures from the theory of multiplex networks to characterize the structure of our system. We are interested in exploring how topological similarity between the pairwise and higher-order interactions impacts the emergence of cooperation in structured populations [240, 241, 244–248].

We denote $\mathcal{E}^{\Delta}_{\text{proj}}$ as the set of projection of 3-hyperedges unto their corresponding 2-hyperedges. In other words, if $e_{lmn} \in \mathcal{E}^{\Delta}$, then $e_{lm}, e_{mn}, e_{ln} \in \mathcal{E}^{\Delta}_{\text{proj}}$. We then define the topological overlap ω between $\mathcal{E}^{/}$ and $\mathcal{E}^{\Delta}$ as the size of the intersection between the sets of edges $\mathcal{E}^{/}$ and $\mathcal{E}^{\Delta}_{\text{proj}}$ normalized by the size of set of edges in the pairwise network [230, 244, 249, 250],

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$$\omega = \frac{|\mathcal{E}^{/} \cap \mathcal{E}^{\Delta}_{\text{proj}}|}{|\mathcal{E}^{/}|} \tag{3.20}$$

Thus, if all the pairs of players playing a higher-order game in $\mathcal{E}^{\Delta}$, also participate in 2-player games in $\mathcal{E}^{/}$, then $\omega = 1$. On the other hand, if none of the pair of players playing a higher-order game is participating together in a 2-player game $\omega = 0$. To fine-tune the topological overlap ω, we start by constructing a random-regular hypergraph with full overlap between pairwise and 3-player interactions/layers (i.e., with $\omega = 1$, corresponding to a 3-regular simplicial complex). We then decrease the level of overlap ω by performing a *criss-cross* rewiring of the pairwise interactions. To do that, we consider the pairwise layer in this hypergraph and find two edges $A-B$ and $C-D$, such that nodes A,B,C,D are all distinct and their corresponding subgraph is disjoint, i.e. there are no links from A or B to C or D. We then do a double swap rewiring of the old edges $A-B$ and $C-D$, such that $A-D$ and $B-C$ are now the new edges. The advantage of this rewiring method is that it preserves the degrees of all nodes, and it changes the overlap between the pairwise and higher-order layers of the hypergraph. We also ascertain that the graph remains connected following this edge swap even considering all links only and all 3-hyperedges only. We repeat this procedure multiple times to get the desired level of overlap in the system.

Here we analyze the general model consisting of multi-dimensional strategies and tunable topological overlap along with flexible dynamical coupling between interactions of different orders. In particular, we also relax the constraints of payoff considered in the simplified case. For our simulations, we fix $k^{/} = 4$ and $k^{\Delta} = 2$ so that each player interacts with maximum 8 (=$k^{/} + 2k^{\Delta}$) other unique players.

3.4 Inter-order dynamical coupling mediates cooperation

We analyze the outcome of the evolutionary dynamics of our model by observing the system in the stationary state. We consider a population of $N = 1500$ individuals interconnected through edges and hyperedges. Each individual is connected to $k^{/} = 4$ individuals through pairwise interactions. Each individual is also connected to 4 neighbors through $k^{\Delta} = 2$ hyperedges of size 3. We evolve the system using the quasi-stationary method, often used in stochastic processes with absorbing states to find the stable point of the underlying dynamical process [251–254]. The quasi-stationary method dictates that if the system reaches an absorbing state, which in our case would be full cooperation or defection for either interaction order, the system is reverted (or teleported) to one of the previously visited states with a probability proportional to the time spent by the system in that particular state. We evolve the system for 10^5 MCS, starting from an equal number of cooperators and defectors for each type of interaction, and look at

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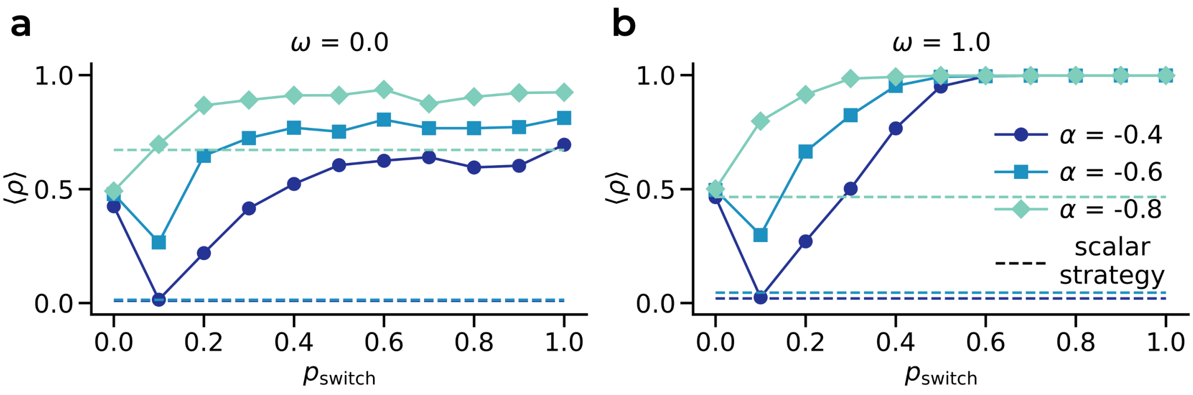

Figure 3.4: **Inter-order dynamical coupling mediates cooperation.** Total fraction of cooperative individuals $\langle \rho \rangle$ at the stationary state as a function of p_{switch} for various values of social dilemma strength α and two values of structural overlap: **(a)** $\omega = 0$ and **(b)** $\omega = 1$. In both panels, the dotted lines denote the reference levels of cooperation if the system consisted of a scalar-strategy profile instead of a 2-dimensional strategy vector for the individuals. Results are shown for the random-regular hypergraph with $N = 1500, k^{/} = 4, k^{\Delta} = 2$. The initial levels of cooperation in the system are $\rho_0^{/} = \rho_0^{\Delta} = 0.5$. All the results are plotted after averaging over $M = 400$ independent runs.

the average levels of cooperation in the last 10^4 MCS, defined as the stationary state across $M = 400$ independent runs.

We first investigate the effect of the dynamical coupling between the different orders of interactions in our model by tuning p_{switch}. Figure (3.4) depicts the stationary state cooperation levels $\langle \rho \rangle$ as a function of p_{switch} for various values of social dilemma strength α and structural overlap ω in multi-dimensional strategy systems. To highlight the advantages of our model, we also plot in colored dotted lines, the cooperation levels in a similar system except with a single/scalar strategy assigned to each node instead of a vector of strategies. In other words, we showcase the difference between the presence and absence of group-size-dependent strategies. Note that in the case where each node has only one associated strategy, the notion of p_{switch} is meaningless. First, we observe that depending on the value of α, the uni-dimensional scalar-strategy system shows a transition from full defection to high levels of cooperation [1]. Second, we notice that for both values of overlap $\omega = 0.0$ and $\omega = 1.0$, for $p_{\text{switch}} = 0$, the system displays 50% cooperative agents. We can understand this behavior by noticing that the optimum strategy for the pairwise PD game based on our chosen payoff values is pure defection, while for the higher-order game, since $\alpha < 0$, one of the Nash equilibria is to cooperate [1, 228].

The situation changes drastically when we increase the dynamical coupling in the system by changing p_{switch}. For very small values of $p_{\text{switch}} < 0.1$, the stationary state is non-trivially dependent on α as well as ω. We see a decrease in cooperation levels for all values of parameters except when the structural overlap is high. However, increasing the dynamical coupling even further ($0 < p_{\text{switch}} < 0.3$), the system shows signs of pro-social behavior as seen from an

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increased level of cooperation. Note that this increase is enhanced in the full structural overlap case ($\omega = 1.0$). It is crucial to observe that for all sets of parameters, the cooperation levels in the multi-dimensional strategy system are at least comparable to or greater than the same levels in scalar-strategy systems. Increasing the dynamical coupling even more ($p_{\text{switch}} > 0.3$), we notice that the cooperation levels saturate. The saturation value is dependent on α for $\omega = 0$, but for full structural overlap $\omega = 1$, the system displays full cooperation independent of α for a wide range of p_{switch}.

To summarize, hypergraph structure does not readily promote pro-social behavior in the absence of dynamical coupling ($p_{\text{switch}} = 0$). However, when we increase the dynamical coupling to a small non-zero value ($p_{\text{switch}} > 0.2$), the system prompts an increase in cooperative behavior. This increase is mediated by a nuanced balance between the dynamical coupling strength p_{switch} and structural overlap ω, where for higher values of overlap, cooperation is heavily preferred. Furthermore, we get higher levels of cooperation in our model compared to the scenario if the agents were described by a scalar strategy. All in all, our results display a rich interplay between the structural and dynamical components of the system to elevate levels of cooperation.

3.5 Inter-order structural overlap promotes pro-sociality

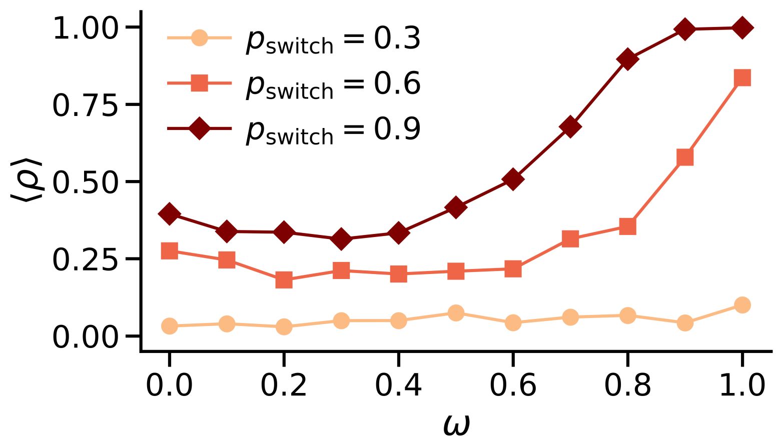

Figure 3.5: **Inter-order structural overlap promotes pro-sociality.** Density of cooperators $\langle\rho\rangle$ as a function of structural overlap ω for various values of p_{switch} and $\alpha = -0.1$.

Social behavior in groups is usually different from behavior in pairwise interactions [206, 237, 239]. One of the leading hypothesis for this difference is that a group is not merely a sum of its parts, but rather that emergent synergistic effects are manifested in the form of peer pressure which fundamentally change the interactions [255, 256]. In this context, it is interesting to understand scenarios where two individuals interact in social contexts with differing numbers of co-participants. Figure (3.5) shows how structural overlap between interactions of different

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orders controls the levels of cooperation in a system. This connection between group interactions and overlap arises because structural overlap determines how influence and behavioral reinforcement propagate across individuals. In systems with high overlap, the same individuals can participate in multiple groups, amplifying peer effects and stabilizing cooperative norms, whereas in low overlap limits such reinforcement, can reduce the persistence of cooperation.

Figure (3.5) shows the behavior of the system as a function of tunable topological overlap ω for various values of dynamical coupling p_{switch} and a fixed value of social dilemma strength $\alpha = -0.1$. We observe that for small values of $p_{\text{switch}} = 0.3$, there is no effect of the structural overlap on the stationary state cooperation levels. The cooperation levels are low and independent of ω. However, when we increase the dynamical coupling to $p_{\text{switch}} = 0.6$, the structural overlap between the different layers starts playing a crucial role. In particular, for high values of $\omega > 0.7$, the cooperative strategies are preferentially chosen more and we see elevated levels of cooperation in the system than before. When we tune the dynamical coupling to even higher values ($p_{\text{switch}} = 0.9$), we see a systematic increase in cooperation for almost all values of ω. Furthermore, the system shows full cooperation for very high values of $\omega > 0.9$.

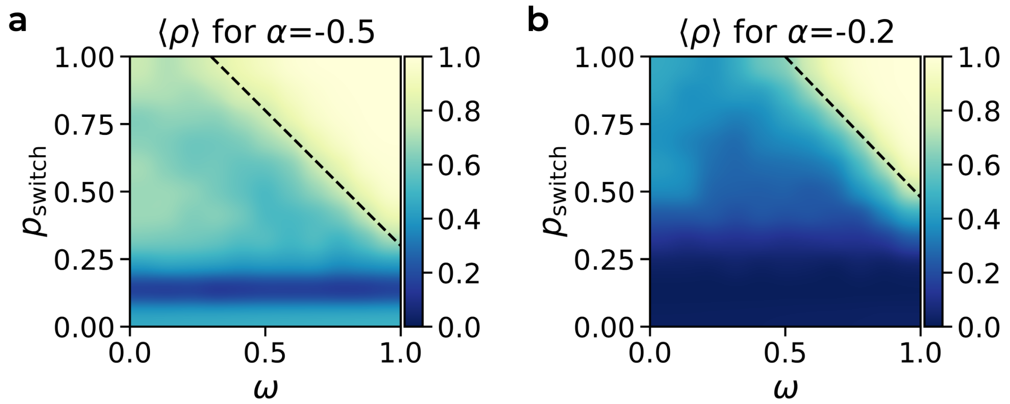

Figure 3.6: **Interplay between structural overlap and dynamical coupling.** Heatmap of the stationary state cooperation levels as a function of ω and p_{switch} for **(a)** $\alpha = -0.5$ and **(b)** $\alpha = -0.2$. Darker shades of blue denote lower levels of cooperation while lighter shades of yellow denote higher values of cooperation. The dotted line separates the area in which very high levels of cooperation ($\langle\rho\rangle > 0.8$) are observed.

To gain a deeper insight into the interplay between the dynamical coupling (p_{switch}) and structural coupling (ω), we plot a heatmap of stationary state cooperation for $\alpha = -0.5$ (Fig.(3.6) (a)) and $\alpha = -0.2$ (Fig.(3.6) (b)) as a function of both p_{switch} and ω. We use a black dotted line to separate areas of high cooperation levels ($\langle\rho\rangle > 0.8$) in the phase space. For a more *relaxed* social dilemma ($\alpha = -0.5$) we notice that for small values of dynamical coupling, *i.e.* $p_{\text{switch}} < 0.3$, topological overlap plays no part in the stationary state of the system. The cooperation levels fluctuate around the 0.5 for no dynamical coupling, while they drop down to 0.1 for small values of p_{switch}. However, when we increase the dynamical coupling ($p_{\text{switch}} > 0.5$), a high topological overlap promotes cooperation to a greater degree. The role of structural over-

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lap becomes more and more important as we increase the dynamical coupling. In particular, for the highest level of dynamical coupling $p_{\text{switch}} > 0.8$, we get full cooperation in the system even for relatively lower values of $\omega \sim 0.5$. When we increase the social dilemma strength to $\alpha = -0.2$ (Fig.(3.6) (b)), we get an overall decrease in cooperation levels. This is expected since increasing the social dilemma strength makes the temptation to defect even stronger. We notice that the region of very high cooperation levels separated by the black dotted line is now much smaller. Additionally, for lower values of dynamical coupling ($p_{\text{switch}} < 0.3$), the system displays high levels of defection independent of the level of structural overlap. Note that the white-yellow region persists even beyond $\alpha > 0$ (not plotted here), signifying that even for *strong* social dilemmas, high structural overlap and high dynamical coupling can promote cooperation.

All in all, our analysis reveals that edge-hyperedge overlap in hypergraphs is one of the major drivers of cooperation in social dilemma situations. The pro-social effects are most visible when the dynamical coupling is also high. These results hint at the importance of reinforcement and peer pressure present in real life to elevate cooperation levels.

3.6 Higher-order social dilemma strength modulates cooperative behavior

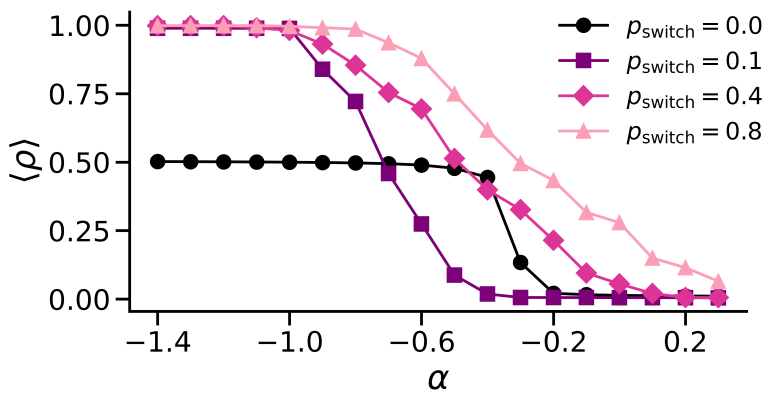

Figure 3.7: **Higher-order social dilemma strength modulates cooperative behavior**. Fraction of agents cooperating in the stationary state as a function of the higher-order social dilemma strength α for various values of dynamical coupling p_{switch} and $\omega = 0.5$.

We now turn our attention to exploring how changes in the strength of the social dilemma impact the cooperation levels observed in the population. In particular, we are interested in regimes close to $\alpha = 0$, where the higher-order Prisoner's dilemma transitions from being a *relaxed* social dilemma to a *strong* social dilemma.

Figure (3.7) showcases the behavior of the system as a function of α for various values of p_{switch} and structural overlap $\omega = 0.5$. First, we notice that in the absence of dynamical coupling

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$p_{\text{switch}} = 0$ (black line, circular symbols), the system shows half cooperation up to $\alpha = -0.4$ and then transitions to full defection for the rest of the values of α. The lack of any dynamical coupling pushes the system to a state characterized by full defection in pairwise games and full cooperation in higher-order games leading to 50% overall cooperation. However, for high social dilemma strength, we always get full defection which is not desirable. The trend is qualitatively different when we have non-zero values of dynamical coupling present in the system. Most importantly, for very low values of $\alpha < -1$, the system shows full cooperation for all values of $p_{\text{switch}} > 0$. However, as we start increasing the social dilemma strength, we notice a fall in the cooperation levels to $\langle \rho \rangle = 0$ for *strong* social dilemmas. This holds true for all values of p_{switch}. The important question to ask in this context is how the cooperation levels drop as we increase the social dilemma strength.

Figure (3.7) shows the trend of $\langle \rho \rangle$ for various values of p_{switch}. For $p_{\text{switch}} = 0.1$ (purple curve, square symbols), the cooperation drops relatively earlier and the cooperators lose majority ($\langle \rho \rangle < 0.5$) for $\alpha \sim -0.7$. The cooperation levels continue to plummet and we get full defection for values of α that still satisfy the conditions for a *relaxed* social dilemma. When we increase the dynamical coupling to $p_{\text{switch}} = 0.4$ (pink curve, diamond symbols), we see a relatively slower decay of cooperation levels. Furthermore, the cooperation is (almost) always higher than the case with no dynamical coupling. Additionally, we get non-zero values of cooperation even at high social dilemma strengths ($\alpha \sim -0.1$). Finally, when we tune the dynamical coupling to $p_{\text{switch}} = 0.8$ (light pink curve, triangular symbols), we get the slowest decay in cooperation levels. As a consequence, cooperation is always strictly higher than in the scenario with no dynamical coupling. Moreover, we get non-zero values of cooperation even in the *strong* social dilemma strength regimes ($\alpha \geq 0$).

To gain deeper insight into the mechanisms promoting pro-sociality, we plot the stationary state cooperation levels as a function of α and ω for $p_{\text{switch}} = 0.7$ in Fig. (3.8). We observe that for low values of social dilemma strengths ($\alpha < -0.9$), we get full cooperation independent of the value of structural overlap ω. This is expected since the temptation to defect is lower for relaxed social dilemmas. However, when we move closer to the limit of *relaxed* social dilemmas denoted by the white dotted line, i.e. $\alpha = 0$, we get some non-trivial patterns in cooperation levels. In particular for $-0.6 < \alpha < 0$, the cooperation levels are intermediate. However, higher structural overlap generally promotes cooperation more for a given value of α. For *strong* social dilemmas ($0 < \alpha < 0.3$), the system is mostly dominated by defectors with only 10% to 20% population cooperating in the long-time limit. However, even in this case, very high levels of structural overlap seem to sustain cooperation. In particular, for higher structural overlap, the system sustains cooperation for a long range of social dilemma strengths.

Put together, our results display rich emergent behavior as we tune the strength of the social dilemma. High social dilemma strength tends to inhibit cooperation in the system. However, high levels of structural overlap and large dynamical coupling can sustain pro-social behavior even in scenarios where cooperation is unfavourable due to a high temptation to defect.

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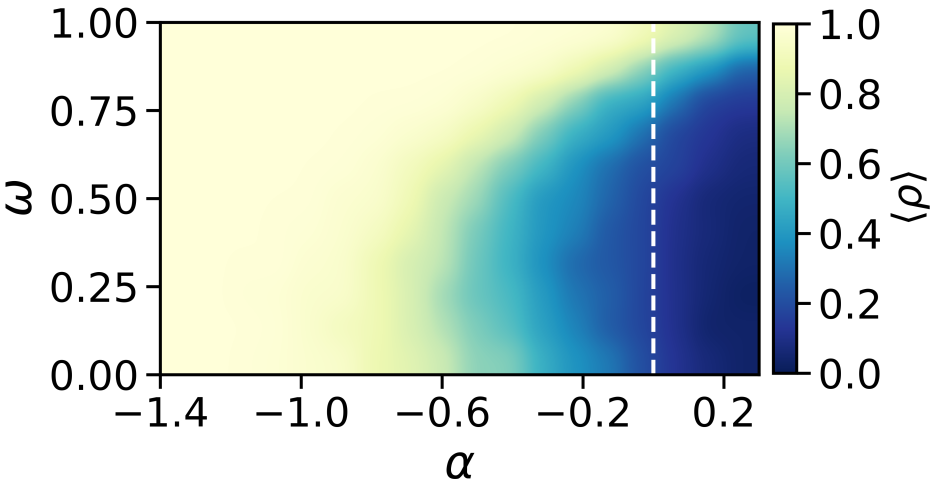

Figure 3.8: **Interdependence between social dilemma strength and structural overlap.** Heatmap showing the pattern of overall cooperation $\langle \rho \rangle$ in the system from low (blue) to high (yellow/white) as a function of topological overlap ω and strength of social dilemma α. The white dotted line denotes the transition from weak social dilemma ($\alpha < 0$) to strong social dilemma ($\alpha > 0$).

3.7 Discussion

Over the last few years, higher-order interactions have emerged as an important tool to effectively model networked populations [31, 257, 258]. In particular, research on dynamical processes such as epidemic spreading [155], synchronization [259], and more recently evolutionary games [1, 195, 219, 221] has showcased multiple instances where the presence of higher-order interactions fundamentally changes the stationary state properties of the system. While a much richer and more nuanced landscape of possibilities has opened up to examine the critical components of these high-dimensional processes, current works have not investigated in detail how the topology and dynamics of higher-order networks shape their evolutionary behavior.

Here, we proposed a new model for games on hypergraphs and showed how structural and topological features can lead a population towards cooperative behaviors. The key feature of the model is the representation of higher-order interaction structure as a multilayer hypergraph, where each layer represents the group interactions of a given size. The agents are endowed with different strategies depending on the order of the interaction. Such strategies are dynamically coupled through the introduction of the parameter p_{switch} which allows the agents to imitate the strategies of their neighbors across different orders. Finally, the hypergraph structure is controlled by tuning the overlap ω between the edges (size = 2) and hyperedges (size = 3) of the hypergraph.

Our model is characterized by a rich and nuanced interplay between the dynamical coupling p_{switch} and structural overlap ω across various strengths of social dilemma strength α. First,

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the multidimensional nature of strategic behavior allowed to promote cooperation. Second, increasing the dynamical coupling between the different orders of interactions increased the overall cooperation levels in the system. Third, higher structural overlaps further promoted cooperation in this multi-dimensional strategy system. Taken together, our results showed that each of the above three components can elevate cooperation, and the effect is further enhanced when these components are tuned simultaneously.

3.8 Future directions

In the preceding chapters, we examined recent advances in modeling N-player games within structured populations. The adoption of hypergraphs as a foundation for these multi-body strategic interactions has enhanced our comprehension of collective dynamics in evolutionary systems. Yet, we propose that this approach harbors even greater potential awaiting realization. Below, we enlist several promising avenues where this framework could yield novel insights.

Mixing different type of games

The representational richness of hypergraphs allows one to incorporate mixing of different types of games with varying numbers of players. This is a very unique and characteristic feature of higher-order interaction leading to many interesting questions in the context of social and biological situations. First, is there a way to represent the complex system consisting of various games with a simpler reduced set of games [260]? In other words, can one find conditions under which a combination of various N games can be instead represented by a smaller set of k $(< N)$ games? Second, beyond the dynamical reducibility, can we filter for structural redundancies informed by the evolution of the system based on the games that the players are participating in? Alternatively, can we highlight a subset of hyperedges which are most important to the dynamics of mixing various games? Third, which combination of games give rise to social dilemmas and is there a critical mixing ratio for the same? This could potentially point to a new mechanism for stabilizing cooperation where mixing a small fraction of higher-order ‘cooperative’ games to other lower-order strong social dilemmas could lead to creation of new cooperative stable points in the dynamics.

Higher-order game dynamics

So far, we have primarily focused on symmetric non-cooperative games with two strategies. Introducing additional complexity to this framework through asymmetry and multiple strategies has generated valuable theoretical contributions [217]. Extending these frameworks for higher-order interactions presents a promising direction for the future. Socially relevant concepts such as fairness and equality become more important in groups, since groups can exert peer-pressure

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beyond simple additive individual influence [130]. As such, including asymmetry can enable us to accurately model these systems mathematically and design better policies. On the other hand, going beyond two strategy frameworks with higher-order interactions is also much needed. It has been shown that apart from cooperation and defection, strategies such as punishment and abstinence can promote sociality [242]. A group interaction is a natural setting where a diverse set of players with non-binary strategies could influence the outcome in a non-trivial way.

Analytical approaches

While many novel features have been uncovered for 3-player games, the general case of N-player games remains elusive [1]. Two key challenges obstruct potential advancements: (1) payoff calculations grow computationally expensive in both time and space, and (2) analytical treatment becomes intractable as it requires solving higher-order polynomials. To provide a general solution for cooperation in hypergraphs, we may need to leverage inherent symmetries and implement strategic constraints in payoff matrices. Drawing inspiration from epidemic modeling, we could adapt compartment-based approaches and degree-based mean-field approximations [26] to evolutionary dynamics - particularly to quantify how group-structured interactions alter cooperation thresholds. Furthermore, we must rigorously characterize how network scale (small vs. large systems) fundamentally modulates the spread of cooperative behavior.

Higher-order structural features

Higher-order network reciprocity remains largely unexplored territory, with the topological influence of hypergraph structures on evolutionary dynamics still poorly characterized. Key structural features – including heterogeneity (in both group-size distributions and player participation in hyperedges) and modularity – represent critical avenues for investigation. The role of temporal interactions in cooperation dynamics similarly demands systematic study. Future research could leverage the well-developed theoretical tools of HOI and analyze these topics to form a general theory of higher-order network reciprocity. In this context, it would be important to investigate the major similarities and differences between pairwise and non-pairwise reciprocity and how they affect the collective dynamics through feedback. Another exciting direction would be to do a cost-benefit analysis of dyadic and non-dyadic interactions for optimal cooperation rate.

Applications to other disciplines

The evolutionary origins of social norms have become a focal point in game theory research in recent years [156, 261]. As norms emerge through the bottom-up convergence of diverse individual perspectives, game theory offers a powerful framework to systematically analyze

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their formation. However, given that norms are fundamentally group-level phenomena, traditional dyadic approaches fail to capture their full complexity—particularly regarding their emergence, stability, and potential extinction. Higher-order game theory provides a transformative approach to study norm dynamics in group-structured populations, revealing how network topology, payoff structures, and interaction heterogeneity influence these collective behaviors [262].

The potential applications extend into ecology, where game-theoretic models have historically inspired innovative solutions to biological challenges [263, 264]. Higher-order interactions may prove particularly transformative for understanding (1) how multi-species interdependencies (beyond pairwise relationships) affect ecosystem stability in mutualistic networks? and (2) The emergence of flocking, swarming, or herd immunity through multi-agent coordination. Recent work on higher-order ecological networks [257] and evolutionary dynamics [265] suggests these frameworks could deepen our understanding of biodiversity maintenance, species coexistence, and resilience to environmental shocks. This bidirectional flow – where ecological systems inspire game-theoretic models that in turn yield ecological insights – promises to open new frontiers in both fields.

All in all, higher-order interactions have presented a strong case to study collective dynamics in strategic situations. Future research should leverage these tools to avoid the pitfalls and navigate the rich landscape of evolutionary processes ubiquitously observed in real-life.

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CHAPTER 4

EVOLUTIONARY GAME SELECTION

At a Glance

Cooperation often emerges even in competitive settings, but most models of game theory assume that players participate in the same static game defined for the whole population and change their strategies based on the payoffs. Here, we 'reverse' the paradigm to propose a new framework where both strategies and the game itself evolves over time through evolutionary processes. We observe that this co-evolution gives rise to cooperative environments that would not emerge otherwise. When applied to structured populations, cooperation is strongly influenced by how individuals are linked to each other. We find that clustered groups of similar individuals and heterogeneous network structures amplify prosocial behavior. Our results reveal how the joint evolution of behavior and social context shapes cooperation in real-world systems, offering new insight into the origins of social dilemmas.

In the previous chapters, we looked at the influence of group interactions on emergence of cooperation in social dilemmas. One unusual yet important question to ask in this context is how are the specific payoff matrices of the social dilemmas chosen for the population? Furthermore, why does everyone in the population participate in the same type of social dilemma? In this chapter, we go one step deeper to understand the emergence of social dilemmas in the population. By using ideas from stochastic games, we associate payoff matrices to each individual. Allowing for co-evolution (and competition) between strategies and games, we try to explain the origins of ubiquitously observed social dilemmas in real-life.

4.1 Changing environments and game theory

Evolutionary Game Theory (EGT) combines game theory with Darwinian principles of natural selection and has made substantial contributions to diverse fields such as behavioral economics, social science, and biology [167,169,170,217,266]. Traditionally, EGT focuses on a population

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of players involved in a fixed game, evolving strategies over time, based on individual fitness [30, 267–269]. Fitness is generally considered a growing function of the player's payoff, and it depends on the game and strategies of interacting players. This approach has yielded valuable insights into social and biological interactions, particularly in the context of social dilemmas – a widely recognized framework for investigating cooperation. In a social dilemma, each member of a group faces a choice: to cooperate or defect. Cooperation benefits the group but comes at an individual cost, while defectors enjoy the collective benefits without incurring any personal sacrifice. Classic examples of social dilemmas include the Prisoner's Dilemma and the Snowdrift Game [169, 170].

When applied to understanding human behavior and puzzling aspects of social interactions, the simplified well-mixed approach of classical evolutionary game theory often leads to unrealistic predictions, and therefore, complex networks are used to describe patterns of interactions among players [30, 267–270]. This approach has proven instrumental in examining dynamical processes on real social networks, where players interact with their neighbors in a network structure [27, 185, 186, 271]. The concept of "centrality" of a player in a network, quantifying its importance or influence, has been explored to understand the spread of information, the diffusion of innovations, and the formation of social norms [272]. Moreover, traditional EGT models commonly rely on static, globally-defined payoff matrices [273, 274]. This simplification fails to account for the reality that individual players engage in multiple social contexts and scenarios—each contributing to their overall fitness and subject to change over time.

In the last few years, these ideas have begun to emerge in the literature of evolutionary game theory. For instance, models based on stochastic games have been proposed to describe situations where players are in heterogeneous or periodically switching game environments [275–283]. However, many of these studies consider only a limited number of available environments, lack a generalized approach to defining the different environments and deal with game changes as a random process only, thus neglecting mechanisms leading to the selection of these environments [281,284–294]. Thus, fundamental questions regarding how players' strategies and game environments mutually affect each other and co-evolve remain unanswered. In particular, the complex co-dependence between evolutionary strategy selection and evolutionary game selection dynamics itself, has not been studied previously.

In this work, we propose a new framework in which evolutionary dynamics acts both on strategies and game environments. This holistic perspective allows us to analyse how strategies and environments mutually co-evolve over time leading to the emergence of cooperative environments. Furthermore, we identify that the topological properties of social interaction networks play a crucial role in shaping individual behavior. Thus, our framework encompasses a comprehensive exploration of both intrinsic (behavioral) and extrinsic (contextual) factors that contribute to the emergence of pro-social behavior and social dilemmas.

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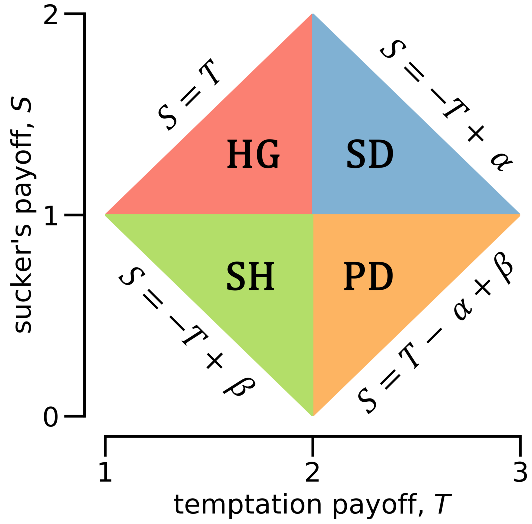

Figure 4.1: **Sampling games from a balanced phase space.** Each player is assigned one pair of values (T_i, S_i) from the game diamond. The equations for the boundaries of each game are derived from the inequalities between the payoffs. Here we set $\alpha = 4$, and $\beta = 2$.

4.2 Modeling evolutionary game selection and competition

Individual payoff matrices: Let us consider two-player games where each agent can choose between two strategies. Traditionally, symmetric games, in the context of social dilemmas have been described by the following payoff matrix,

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{bmatrix} R & S \\ T & P \end{bmatrix} \end{array} \tag{4.1}$$

where, C and D represent the two available strategies: cooperation and defection, respectively.

In particular, when facing another cooperator, a cooperator receives a payoff of R (namely, "reward"), while against a defector, the payoff is S (referred to as "sucker's payoff"). Conversely, a defector earns a payoff of T ("temptation") when facing a cooperator, and P ("penalty") when facing another defector.

The ordering of the entries of the payoff matrix defines four different types of games, namely:

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$$\begin{aligned}
&\text{Prisoner's Dilemma (PD): } T > R > P > S \\
&\text{Snowdrift (SD): } T > R > S > P \\
&\text{Stag-hunt (SH): } R > T > P > S \\
&\text{Harmony Game (HG): } R > T > S > P
\end{aligned} \tag{4.2}$$

In particular, we notice that for each game, either $T > R$ or $T < R$ and $S > P$ or $S < P$, leading to four distinct quadrants in the T-S phase space. In our framework, differently from the classical approach where just one game (i.e. a single payoff matrix) is taken into account, we consider a distribution of games. To do so we associate to each player i a randomly generated game/payoff matrix. We note that the games can alternatively be represented in terms of 'dilemma strengths' $D_g(= T - R)$ and $D_r(= P - S)$ instead of T and S [295–297]. The two representations are equivalent under translational and rotational symmetry in the phase space. If we define different games by generating the entries of the payoff matrix completely at random, the total payoff (i.e. the sum of the payoff matrix entries) for each game will, in general, be different. This introduces a bias, where the players associated with the games with a larger total payoff are favored. Since in our model not only the strategies but also the games evolve, this bias will lead the evolutionary dynamics towards a trivial outcome where the whole population plays the game with the largest total payoff (as we verified numerically). In our model, we remove this bias by considering games described by payoff matrices where the matrix entries (i.e. the payoffs) are drawn from symmetric distributions, and their sum (total payoff) is fixed. This is achieved by using the following payoff matrix to define the game associated with player i:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} \alpha - T_i & S_i \\ T_i & \beta - S_i \end{bmatrix} \end{array} \tag{4.3}$$

where $T_i \in [\beta/2, \alpha - \beta/2]$ and $S_i \in [\beta - \alpha/2, \alpha/2]$ are continuously distributed as shown in Fig. (4.1).

In this way, the sum of the elements in each column of the payoff matrix (Eq. (4.3)) is equal to $R + T = \alpha$ for the first column, and to $S + P = \beta$ for the second column. As a consequence, for fixed values of α and β the total payoff (sum of all matrix entries) is a constant equal to $\alpha + \beta$. By varying the values of T_i and S_i we can define all four types of games. In particular, we can verify this by substituting the expressions $R = \alpha - T$ and $P = \beta - S$ into the payoff inequalities given by Eqs. (4.2). This allows us to derive the conditions for the different social

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dilemmas, while adhering to the constraint of a constant total payoff:

1. Prisoner's Dilemma (PD): $T > \alpha/2$, $S < \beta/2$, and $S > T + \beta - \alpha$

2. Snowdrift (SD): $T > \alpha/2$, $S > \beta/2$, and $S < \alpha - T$

3. Stag-hunt (SH): $T < \alpha/2$, $S < \beta/2$, and $S > \beta - T$

4. Harmony Game (HG): $T < \alpha/2$, $S > \beta/2$, and $S < T$

We notice that $\alpha > \beta$ is a necessary condition for the existence of T and S, solutions of the system of inequalities defining each game. By representing these conditions in the T-S space we obtain a "games diamond" as shown in Fig. (4.1).

Co-evolutionary game dynamics: We represent the state of player i as $\Omega_i = (g_i, s_i)$. Each player i within the population is distinctly identified by the game g_i linked to it and its chosen strategy s_i, which can be either cooperation C or defection D. As we saw in the previous section, given fixed values of α and β, the game g_i is completely determined by the payoffs T_i and S_i and the payoff matrix Eq. (4.3). Henceforth, we will denote both the game and its corresponding payoff matrix by g_i.

The system evolves by agents replicating strategies s_i and games g_i from one another through an asynchronous update process [298]. At each time step we randomly select a focal player f and a model player m. We studied the evolutionary dynamics both for well-mixed populations, where each player can interact with all the other players, and for structured populations, where the players are represented as the nodes of a network and they interact if an edge connects them. In particular, while in a well-mixed scenario the model player is randomly chosen from the entire population, in a structured population the model player is randomly drawn from the neighbors of the focal one.

The focal (respectively, model) player collects total payoff π_f (respectively, π_m) by playing with each of its opponents. In structured populations, the focal player and the model player interact with their k_f and k_m neighbors, respectively. In well-mixed populations, they engage with k randomly selected agents. Each player participates in two games against every opponent j: one using their own payoff matrix g_f (g_m for the model), and another using the opponent's payoff matrix g_j. The focal player updates its strategy through a two-phase stochastic pairwise comparison process, through which he/she can adopt the strategy and *the game* of the model player.

The state update consists of two steps. First, we determine whether the game, the strategy, or both will be updated: the game is selected for an update with probability p_g, and the strategy is selected with an independent probability of p_s. Note that the case $p_g = 0$, $p_s = 1$ recovers the usual evolutionary game dynamics where only the strategies evolve. Given that environments usually change at a slower rate than the individual behaviors [289, 299, 300], we restrict

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ourselves to scenarios where $p_g \leq p_s$. Then, the adoption of the model game and/or strategy occurs with a probability that increases with $\pi_m - \pi_f$, the payoff difference between the model and focal players. In particular, the probability of adoption is described by the so-called Fermi function [27, 231, 301, 302]:

$$\Pi = \frac{1}{1+\exp\left(-w(\pi_m - \pi_f)\right)}\,, \tag{4.4}$$

where w governs the strength of selection. Note that the Fermi update rule is not deterministic and (especially for low values of w) the focal player can adopt games and strategies from the model player even when $\pi_m < \pi_f$. Thus, at each step of the evolutionary dynamics, there are 4 possible transitions from the original state of the focal player $\Omega_f(g_f, s_f)$, to a new state where both the strategy and the game can be those of the model player, each occurring with the following probabilities:

$$\Omega_f(g_f, s_f) \rightarrow \begin{cases} \Omega_f(g_m, s_m) & \text{with } p_s \cdot p_g \cdot \Pi^2 \\ \Omega_f(g_m, s_f) & \text{with } p_g \cdot \Pi \cdot (1 - \Pi \cdot p_s) \\ \Omega_f(g_f, s_m) & \text{with } p_s \cdot \Pi \cdot (1 - \Pi \cdot p_g) \\ \Omega_f(g_f, s_f) & \text{with } (1 - \Pi \cdot p_s) \cdot (1 - \Pi \cdot p_g) \end{cases} \tag{4.5}$$

We repeat the simulation step until the system reaches a quasi-stationary state in which we compute the relevant macroscopic order parameters. In stochastic processes featuring absorbing states, quasi-stationary methods are employed to identify the system's stable states [251, 252, 303]. If the system transitions into an absorbing state, the quasi-stationary methodology reverts it to a prior state, with a probability proportional to the time spent in that state. This approach yields a distribution (namely, quasi-stationary distribution) where the probability of each state is proportional to the time spent by the system in that state. Previous research has shown that the local maxima of the quasi-stationary distribution asymptotically approaches the stable fixed point inherent to the system's stochastic dynamics with growing system size [219, 253, 254].

4.3 Co-evolutionary dynamics in well-mixed populations

We consider well-mixed populations where we assign a game chosen uniformly at random from the game's diamond in Fig. (4.1) to each player. We start our analysis of the model dynamics with the case $p_g = 0$ and $p_s = 1$. In the rest of the paper we always choose $p_s = 1$ unless stated otherwise. In Fig. (4.2) (a) we observe that for $p_g = 0$, independently of the initial fraction of cooperators, the trajectories converge to a quasi-stationary state where the population cooperates roughly half of the time. Thus, independently from the initial number of cooperators in the system, when $p_g = 0$ we always find players cooperating roughly half of the

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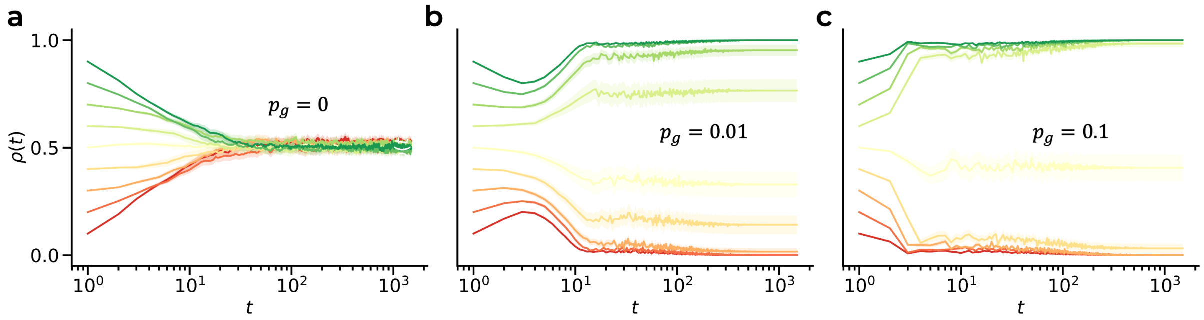

Figure 4.2: **Temporal co-evolution of cooperation.** Fraction of cooperators $\rho(t)$ over time in a well-mixed population for **(a)** $p_g = 0$, **(b)** $p_g = 0.01$, and **(c)** $p_g = 0.1$. The solid lines denote the averages while the shaded region is the standard error over 64 independent runs of the co-evolutionary game dynamics. Here we set $p_s = 1$, $N = 2500$, and $k = 4$.

time. The error bars of the plot denote the standard error over 64 runs for each ρ_0.

In contrast to the scenario where $p_g = 0$, a slight increase of p_g breaks down the symmetry of the evolutionary outcome under changes in the initial configuration. For instance, even for a very small value of $p_g = 0.01$ in Fig. (4.2) (b), we observe a variety of quasi-stationary states in which the final density of cooperators strongly depends on the initial levels of pro-social behavior (i.e., the initial fraction of cooperators). When p_g is further increased the former effect becomes more pronounced and (see Fig. (4.2) (c) for $p_g = 0.1$) trajectories seem to bifurcate in two sets based on the number of initial cooperators. In particular, when the initial fraction of cooperators $\rho_0 = 0.5$, we observe that the cooperation level remains the same. However, if cooperators are in the minority, the system tends to eliminate cooperators while, when we start with a majority of cooperators, cooperation prevails in the long run.

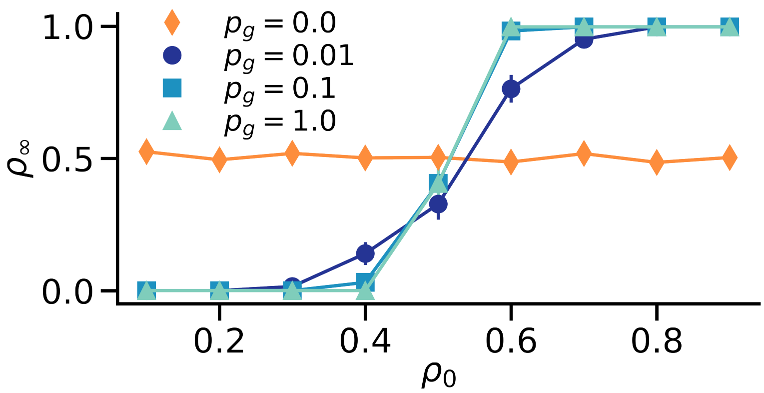

Figure 4.3: **Emergence of cooperation in presence of game selection.** Quasi-stationary state fraction of cooperators ρ_∞ as a function of the initial cooperators' density ρ_0 for various values of p_g, the probability of selection of game update, in a well-mixed population. The parameters are $p_s = 1$, $N = 2500$, and $k = 4$ and averaged over 64 runs.

To have a better understanding of this effect we define $\rho_\infty = \lim_{t\to\infty} \langle \rho(t) \rangle$ as the time-averaged cooperators' density in the long time limit and explore the dependence of ρ_∞ as a function of

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ρ_0 for various p_g values. In Fig. (4.3) we illustrate the symmetry-breaking phenomenon that occurs when $p_g > 0$. In particular, as soon as $p_g > 0$ we observe that a critical initial fraction of cooperators of $\rho_0 \approx 0.5$ is needed to have a majority of cooperators in the steady state of the system. Moreover, as p_g increases the curves ρ_∞ depend more and more non-linearly on ρ_0, resembling a step function when $p_g \to 1$. In this limit of strong game selection, the form of the curves ρ_∞ pinpoint a reinforcement effect in which a slight bias towards cooperation (defection) in the initial configuration leads to a dynamical reinforcement of cooperators (defectors).

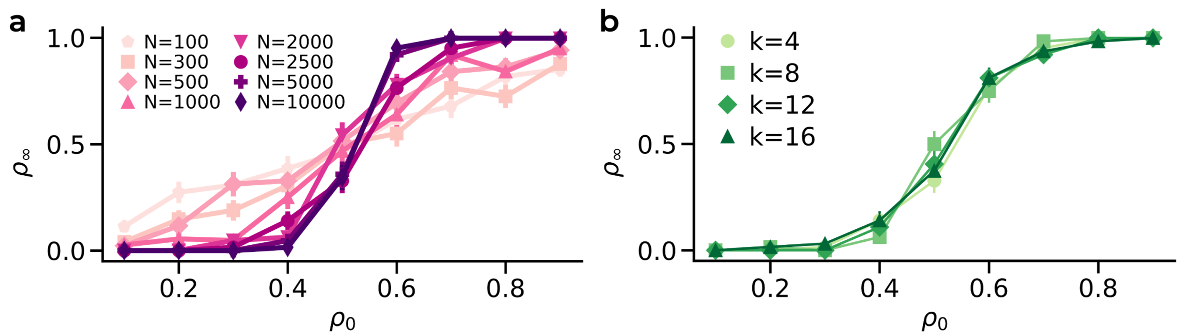

Figure 4.4: **Finite size effect and degree effect on co-evolutionary dynamics.** Effect of **(a)** system size and **(b)** average degree on the quasi-stationary fraction of cooperators ρ_∞ for $p_g = 0.01$. The parameters are $p_s = 1$, $N = 2500$, and $k = 4$. All the results are for a well-mixed population and averaged over 64 runs.

Fig. (4.4) (a) shows the dependence of system size N on the cooperators' density in the quasi-stationary state, ρ_∞ for $p_g = 0.01$. We notice that the transition becomes sharper as the system approaches the thermodynamic limit $N \to \infty$. The finite size effects become less relevant beyond $N \sim 1000$. Subsequently, we fix $N = 2500$ for the rest of the manuscript. Fig. (4.4) (b) shows the effect that the average number of interactions has on the degree of cooperation for $N = 2500$ and $p_g = 0.01$. We see that there is no effect on the quasi-stationary state density for different values of the average degree. Consequently, we fix $\langle k \rangle = 4$ for the rest of the manuscript unless stated otherwise.

Now we explore the effects of the co-evolutionary dynamics in game selection when $p_g > 0$. In particular, in Fig. (4.5) (a), we show the distribution in the T-S plane of the surviving games, where the colors denote the number of initial cooperators, ρ_0, of the corresponding realization. First, we observe that the quasi-stationary state consists primarily of players playing the SH, and also PD or HG depending on ρ_0. Moreover, it is interesting to notice that the surviving games are densely distributed near the corners of the phase space. In particular, we observe a transition of surviving games from PD-SH to HG-SH as a function of ρ_0. Fewer initial cooperators lead to games closer to PD, which has full defection as the Nash equilibrium. On the other hand, an initial majority of cooperators brings the system closer to HG, which has full cooperation as the Nash equilibrium.

Finally, we analyse in depth the dependence of the selected games as a function of p_g

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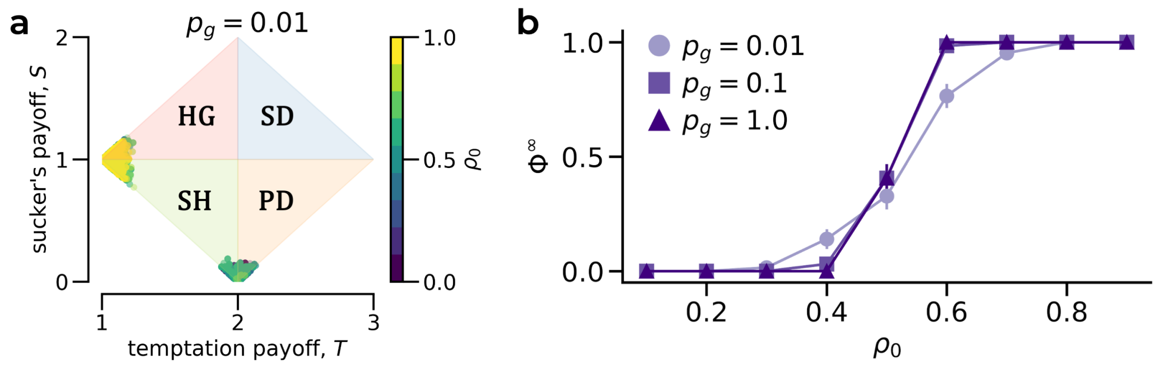

Figure 4.5: **Game selection in well-mixed populations.** **(a)** Surviving games in a well-mixed population for various values of initial cooperators. Blue points denote initial conditions with fewer cooperators than defectors, while yellow dots denote more initial cooperators. **(b)** Surviving cooperative games fraction Φ^∞ as a function of ρ_0 for the same values of p_g. The parameters are $p_s = 1$, $N = 2500$, and $k = 4$.

and ρ_0. The game g_i played by each individual i is defined by the values of T_i and S_i and can be classified as cooperative or non-cooperative. To this aim, we classify a game g_i as cooperative if the associated Nash equilibrium (mixed or pure) has a majority of cooperators, otherwise g_i is regarded as non-cooperative. In particular, a cooperative game holds when $S > T - 1$. Figure (4.5) (b) shows the fraction of cooperative games (environments) Φ^∞ among the surviving games as a function of ρ_0 for different values of $p_g > 0$. It is interesting to see that the curves are quantitatively very similar to the trend of ρ_∞ shown in Fig. (4.3), implying a strong correlation between the selected games and strategies for all values of p_g.

In conclusion, the presence of co-evolutionary dynamics in well-mixed populations provides a way out for the survival of cooperation. However, this effect requires that $\rho_0 \approx 0.5$, which is an unrealistic critical mass of initial cooperators to achieve pro-social behavior. In the next section, we show how the presence of structured populations can enhance pro-social behavior at a lower cost (i.e. initial cost of cooperation).

4.4 Strategy and game co-evolution on structured populations

Well-mixed populations are not the best representation of real-world systems, since individuals do not interact at random, but according to well-defined structural patterns. Well-mixed populations prevent repeated interactions between players since model players are chosen at random by each focal agent at each time step. Moreover, the opponents of the focal and model players are also drawn randomly from the population at each time step. Networks constitute a natural framework for analyzing these interaction paradigms. When networks are used to represent the population structure, the neighbors of each agent are fixed. This implies that the set of

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possible model players for each focal player, and the set of possible opponents of each node, do not change in time. We will show that this can greatly impact the update of both strategies and games. In the context of evolutionary game dynamics it is widely known that networked interactions provide a structural way to sustain cooperation through various mechanisms such as punishing those who defect, clustering the cooperators together, reinforcement of pro-social behavior [30, 185, 269, 271, 285, 304–306]. This effect, also known as 'network reciprocity', can lead to pro-social behavior even in situations where, without network structure, cooperation can not be sustained.

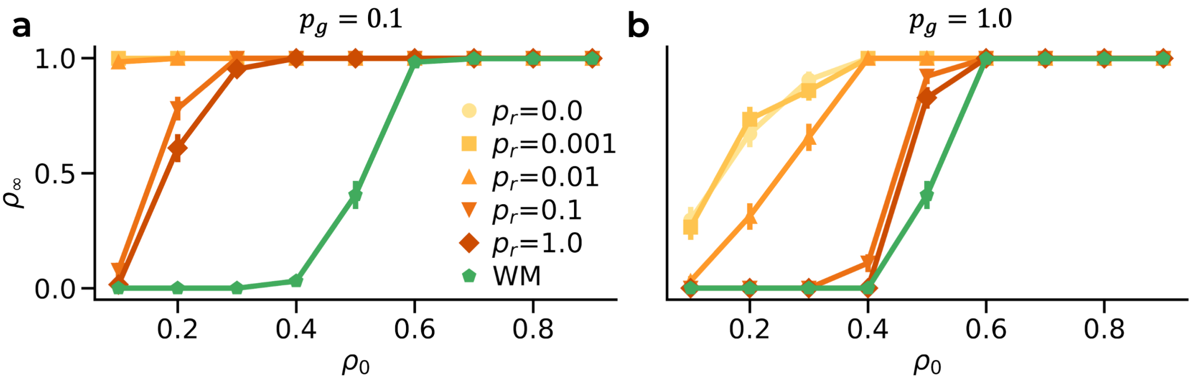

Figure 4.6: **Effect of small-worldness on game selection.** Fraction of cooperators in the quasi-stationary state ρ_∞ as a function of ρ_0 for **(a)** $p_g = 0.1$ and **(b)** $p_g = 1$ for different rewiring values p_r in a lattice along with well-mixed populations for comparison. Simulations are for populations of $N = 2500$ individuals and $p_s = 1$ averaged over 64 runs.

Inspired by this phenomenon, we look at how a structured population can result in an enhancement of cooperative behavior in the context of evolutionary game selection. Specifically, we are interested in how interacting with fixed neighbors and the structure of different network topologies change the quasi-stationary state compared to a well-mixed population. In the next part, we will explore how clusters of similar types of players (in our case having identical strategies and games) can emerge and how the targeted placement of cooperators on heterogeneous networks can amplify pro-social behavior.

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4.5 The effect of small-world networks

We start by examining co-evolutionary dynamics on 2D lattices, a topology that was first investigated in the context of evolutionary games by Nowak and May [170], as these structures can be easily represented as players on top of a surface, allowing us to easily visualize and investigate how spatial correlations affect the emergence of collective behavior [288, 307–310]. Initially proposed by Watts and Strogatz [8], the link-rewiring mechanism systematically alters network structure going from a well-ordered periodic structure (lattice) to a disordered random structure displaying the so-called 'small-world' phenomenon, where all agents in the system are at most few steps far apart from one another. By using a variation of the Watts-Strogatz model where we tune the probability of rewiring the edges p_r in such a way that the degree of each node remains fixed, we change the local (clustering) as well as the global (shortest path length) structural properties, thus breaking the locally homogeneous patterns inherent to simple lattices

As we have shown previously, cooperators' density in the quasi-stationary state is highly correlated to the surviving game. Hence, from now on we only show the fraction of cooperators in the quasi-stationary state, ρ_0, since the trend for the fraction of surviving cooperative games is practically the same. Figure (4.6) shows the fraction of cooperators in the steady state ρ_∞ for various values of p_r and two values of p_g. We also report the results for well-mixed populations for comparison.

For the smallest value of p_g (Fig. (4.6) (a)), well-mixed populations showcase a sigmoidal curve with the onset of cooperation around $\rho_0 \approx 0.4$. However, for rewired lattices, ρ_∞ is consistently higher than that of well-mixed populations when we start from a minority of initial cooperators. In particular, we notice that for very small values of p_r we always get a full cooperative state independent of the initial density of cooperators. Interestingly for intermediate values of p_r, we can get a majority of cooperators with around 20% or fewer players starting as cooperators. However, the difference between random regular networks ($p_r = 1$) and well-mixed populations is quite large, pointing to other possible mechanisms for the enhancement of pro-social behavior beyond lattice rewiring.

The former picture changes when game selection is fully activated, $p_g = 1$, as shown in Fig. (4.6) (b). Differently from the previous case, when $p_g = 1$ random regular networks ($p_r = 1$) exhibit a very similar trend to well-mixed populations. On the other hand, rewired lattices (intermediate values of p_r) even though sustain cooperation better than random regular networks in this case, are far from their performance as boosters of cooperation as for $p_g = 0.1$.

In summary, the former results indicate that networks boost pro-social behavior, elevating the cooperation level in the population. Interestingly, smaller but non-zero values of p_g increase the cooperative behavior to a greater extent in locally ordered homogeneous structures (lattices) than in the case of large p_g values. These results point to a balance between the propensity of changing game and the heterogeneity of interaction patterns.

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4.6 Clustering games on a 2D lattice

Figure (4.7) depicts the initial snapshots of a 2D lattice when, initially, individual games are (a) randomly placed, or (c) clustered, such that the number of neighbors having the same game type as of a given node is higher. Since we consider square lattices with periodic boundary conditions, all players are structurally equivalent and there are no boundary effects. In both cases, 20% of the players are initially cooperators, located randomly on the lattice. We quantify

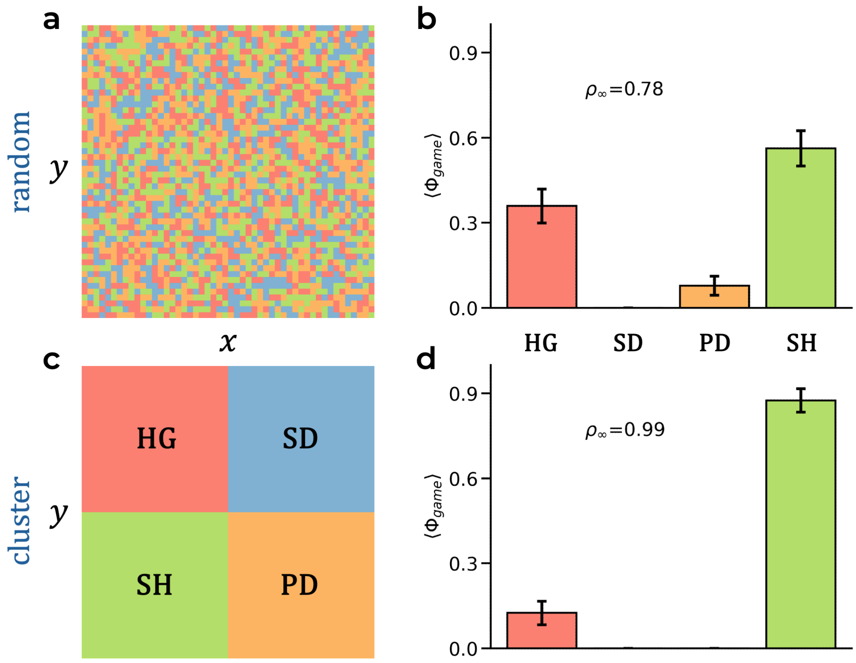

Figure 4.7: **Clustering games promotes emergence of cooperative environments.** Surviving games on 2D square lattices for initial conditions with clustered or random games and $\rho_0 = 0.2$. Initial snapshot of **(a)** randomly allocated games and **(c)** clustered games on a lattice of size 50×50. **(b), (d)** The fraction of players adopting a particular game type in long-time limit averaged over 64 runs in lattices. The text denotes the density of cooperators. Here, $p_g = p_s = 1$.

the quasi-stationary state distribution of games by defining $\langle \Phi_{game} \rangle$ as the average fraction of sites occupied by a given *game*. We calculate the average over 64 independent runs and the error bars denote the standard error.

From panels Fig. (4.7) (b) and (d) it is clear that SH games prevail the majority of times. When games are randomly placed, the Stag-hunt and the Harmony games dominate the final configuration of the lattice. In this particular scenario, Prisoner's Dilemma (PD) games also survive. When the games are initially clustered on the lattice, the small fraction of PD games disappear, while the proportion of the Stag-hunt games increases. The density of cooperators is further increased when games are clustered. Notably, the Snowdrift game always goes extinct, regardless of the initial configuration.

In conclusion, clustering games on a lattice seems to promote the prevalence of cooperative environments. Moreover, while cooperators' density is generally higher on a lattice compared to structures without spatial correlations, game clustering further amplifies the selection of cooperation-friendly environments. This behavior is important since it further corroborates the effect of homophily in real-world systems [311, 312].

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4.7 Targeted placement of cooperators

Multiple empirical investigations have revealed the presence of heterogeneous and power-law degree distributions in social networks [9, 41]. A heterogeneous degree distribution implies the presence of *hubs*, nodes with a particularly high degree with respect to the average degree of the network. Such nodes with high degrees play a central role in the dynamics taking place on the network by effectively disseminating information, opinions, and behaviors throughout the network [27, 271]. Here, we explore the effect of the strategic initial placement of a small fraction of cooperators (less than or equal to 0.005 times the size of the population) on the evolution of pro-social behavior. In particular, we consider configurations with cooperators initially placed on the hubs of the network. In addition, we investigate how changing the level of degree-heterogeneity of the network affects the co-evolutionary dynamics of the system for the placement strategy mentioned above.

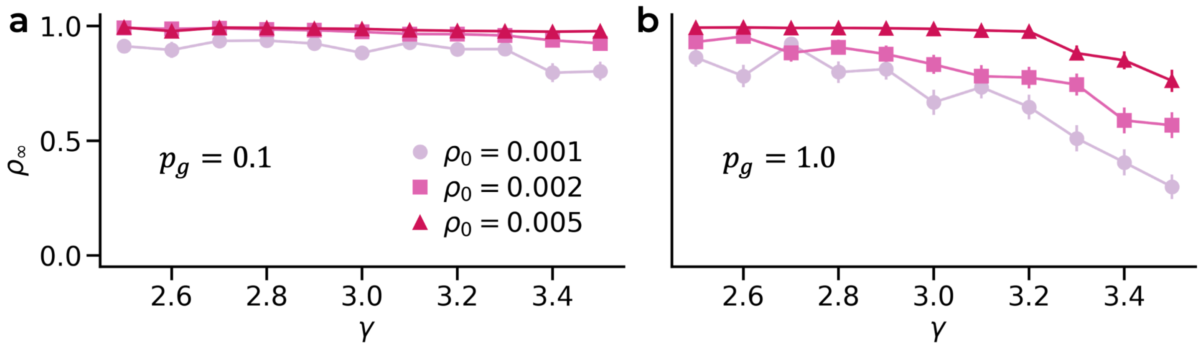

Figure 4.8: **Targeted placement of cooperators on scale-free networks.** Cooperation levels in scale-free networks for strategic placements of cooperators. A small fraction of cooperators (0.001 to 0.005 times the size of the population $N = 2500$) is initially placed on the hubs for **(a)** $p_g = 0.1$ and **(b)** $p_g = 1.0$.

Figure (4.8) (a) $p_g = 0.1$ and (b) $p_g = 1$ shows the effect of strategically placing cooperators in power-law degree distributed networks as a function of their heterogeneity, captured by the value of the exponent γ of the power-law. For a small probability of game updating, $p_g = 0.1$, placing cooperators on the hubs ensures a very high level of cooperation across all values of γ (Fig. (4.8) (a)). For example, starting with just three (0.1%) cooperators among 2500 players leads to a final count of approximately 2000 cooperators ($\approx$ 80%). When game updating is always active, $p_g = 1$, and cooperators are placed on the hubs, cooperation levels for a similar initial cooperative mass decrease as the heterogeneity of the graph reduces (increasing γ), as illustrated in Fig. (4.8) (b).

To gain more insight into the mechanisms for the pro-social behavior, we plot the degree distribution $P(k)$ for various values of γ in Fig. (4.9) (a). We see that $P(k) \sim k^{-\gamma}$, since the degree distribution follows a straight line in a log-log plot. Note that the slope of the distribution

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quantifies the heterogeneity and it decreases with increasing values of γ [41]. On the other hand, Fig. (4.9) (b) illustrates the average degree of the initial cooperators, $\langle k(\rho_0)\rangle$, as a function of γ. We observe that $\langle k(\rho_0 = 0.001)\rangle \sim 150$ for $\gamma = 2.5$, but it drops to ~ 20 for $\gamma = 3.5$.

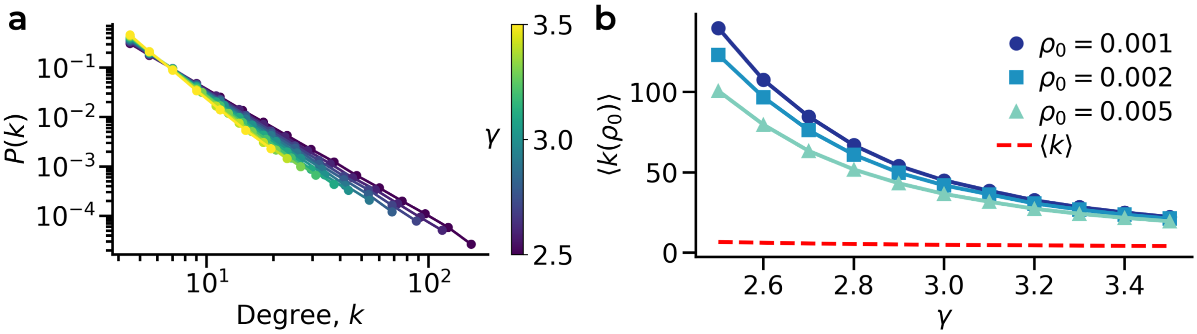

Figure 4.9: **Ingredients influencing the emergence of cooperation on scale-free networks. (a)** Degree distribution of scale free graphs for various values of γ in a log-log scale. **(b)** The average degree of initial cooperators $\langle k(\rho_0)\rangle$ as a function of γ for various values of ρ_0. The red dashed line shows the average degree $\langle k\rangle$ of the networks.

To summarize, the collective behavior of surviving games and strategies depends heavily on complex network features such as small-world behavior, scale-free nature of the degree distributions, and the presence of strategy-degree correlations. In addition, increasing p_g reinforces the effect of the degree heterogeneity in determining if cooperation is enhanced or diminished with respect to the strategic placement of initial cooperators on the hubs of the network.

4.8 Discussion

In the last decades, evolutionary game theory has provided valuable insights into understanding why agents choose cooperation despite personal incentives to defect. However, most existing studies focus on the evolution of strategies for specific, globally defined and static payoff matrices, disregarding changing environments and game conditions. Although some recent works have considered game heterogeneity through stochastic formulations, a comprehensive framework explaining the origin and emergence of these games and the dynamic relationship between games and strategies has remained elusive.

Our work contributes to bridging this gap by introducing a co-evolutionary framework where both strategies and games co-evolve and undergo evolutionary selection. We propose a simple model for game competition with various types of games ensuring an unbiased payoff distribution in all games. By adjusting the propensity to change the game (p_g) while maintaining a fixed probability of changing strategies (p_s), we discover fundamental changes in the system's evolution in well-mixed populations. In particular, even a small probability of switching games leads to a bifurcation of the quasi-stationary state, depending on the critical initial

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mass of cooperators. Additionally, we observe that the system tends to select more cooperative environments when there are enough initial cooperators, and the games and strategies influence each other, leading to a strong correlation between surviving games and strategies.

Beyond well-mixed scenarios, we find that structured populations enhance cooperation levels for small values of p_g. In particular, locally homogeneous graphs such as lattices lead to a state with a majority of cooperators even with a low initial mass of cooperators. However, this effect diminishes with the increase in the game selection propensity p_g and the disruption of the regular lattice structure through a rewiring process, which creates shortcuts. In the specific case of 2D lattices, we also observed that clustering similar games promotes cooperation and leads to the survival of cooperative games.

In the case of scale-free networks we have found that the enhancement of cooperation decreases as the value of p_g increases. The drastic difference in connection patterns of high-degree nodes compared to the rest of the nodes reinforces pro-social behavior.

In summary, our findings shed light on the complex mechanisms shaping evolutionary processes and the interplay between strategic decision-making and mutating environments that define choices. Our work contributes to exploring the origins of social dilemmas prevalent in social settings. For the future, considering additional features such as community structure [313], time-varying [314], and higher-order interactions [1] may offer further insights into co-evolutionary processes of strategies and games in real-world systems [263]. We hope that our work inspires more research on co-evolutionary dynamics as an avenue to tackle the puzzle of cooperation.

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CHAPTER 5

CULTURAL RECOMBINATION AND HIGHER-ORDER INTERACTIONS

At a Glance

From ancient spears to cutting-edge hypercars, human history has been shaped by technological innovation. While individual ingenuity plays a role, innovators are typically embedded in complex networks that shape and transmit culture, making it essential to understand the social and structural drivers of these cultural processes. Prior research has shown that networks facilitate the diffusion of knowledge and the cumulative build-up of cultural traits, eventually giving rise to complex innovations. However, these models often overlook group interactions mechanisms, where rare but simultaneous encounters among individuals with diverse knowledge can spark high-level cultural breakthroughs. We address this gap by introducing a model that systematically incorporates group interactions into cultural recombination, showing that higher-order structures are essential for sustained cultural accumulation.

In previous chapters, we have primarily examined cooperation and pro-social behavior. We now investigate how group interactions influence a distinct dynamical process – cultural innovation. Transmission of knowledge and technological advancements has played a pivotal role in shaping human cultural history. This raises a fundamental question: what structural factors govern the evolutionary emergence of innovation among interconnected individuals? To address this, we develop a minimal model of cultural recombination that highlights the critical importance of higher-order interactions in cultural dynamics.

5.1 Introduction

Understanding how complex innovations emerge in human societies remains an open challenge in cultural evolution research [315, 316]. Unlike biological evolution, cultural advancement operates through *recombination processes*, where existing knowledge components combine

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into novel configurations [317–319]. This theoretical framework posits that cumulative culture arises not from *de-novo* inventions but through systematic integration of existing cultural traits – a process accelerated by social connectivity and knowledge exchange [320–322].

Mathematical models have uncovered deeper structural principles underlying cultural transmission and innovation [323, 324]. Research on collaborative learning has shown that performance scales non-linearly with group connectivity, where learning efficiency depends critically on the network's architecture [325]. Concurrently, a complementary line of inquiry modeled the innovation space as a network of adjacent possibles [326, 327], revealing that innovation dynamics can reproduce Zipf's and Heap's exponents through mechanisms like Polya's Urn models [328]. Experimental work further demonstrated the "Goldilocks principle" in team science, showing that intermediate group sizes maximize recombinant innovation, while both small and large groups lead to diminished returns [329].

Seminal experimental work by Derex and Boyd provided empirical validation of this theory [330]. Their cultural recombination paradigm demonstrated how isolated subpopulations developing distinct technological lineages could only achieve breakthrough innovations through cross-group interactions. This work established that social connections serve as critical conduits for cultural synthesis, with transmission pathways directly impacting innovation rates. Subsequent agent-based simulations further quantified how different network structures can either constrain or facilitate these recombinant processes [331].

Despite these advances, a critical modeling gap persists. Most agent-based and theoretical models still focus on *pairwise* interactions [332–334], even though empirical evidence shows that higher-order collaborations play a decisive role in driving breakthrough innovations in science, technology, and traditional societies [335, 336]. This raises the question: How do group-level interactions beyond dyads accelerate cultural recombination? Existing network paradigms capture dyadic knowledge transfer but fail to represent the synergistic dynamics of multi-agent collaboration, where $n > 2$ individuals simultaneously contribute complementary expertise. These higher-order interactions may be especially important for "crossover innovations" that integrate knowledge from previously disconnected domains – phenomena observed in Derex and Boyd's experiments but never systematically modeled using group dynamics. This limitation calls for a new formalism capable of capturing these collective mechanisms.

In this study, we bridge this gap by developing a hypergraph-based cultural recombination model that incorporates three key advances:

1. Extending Derex-Boyd paradigms to k-lineage recombination systems.
2. Formalizing group interaction effects through higher-order structures.
3. Quantifying tradeoffs between innovation speed and knowledge diffusion.

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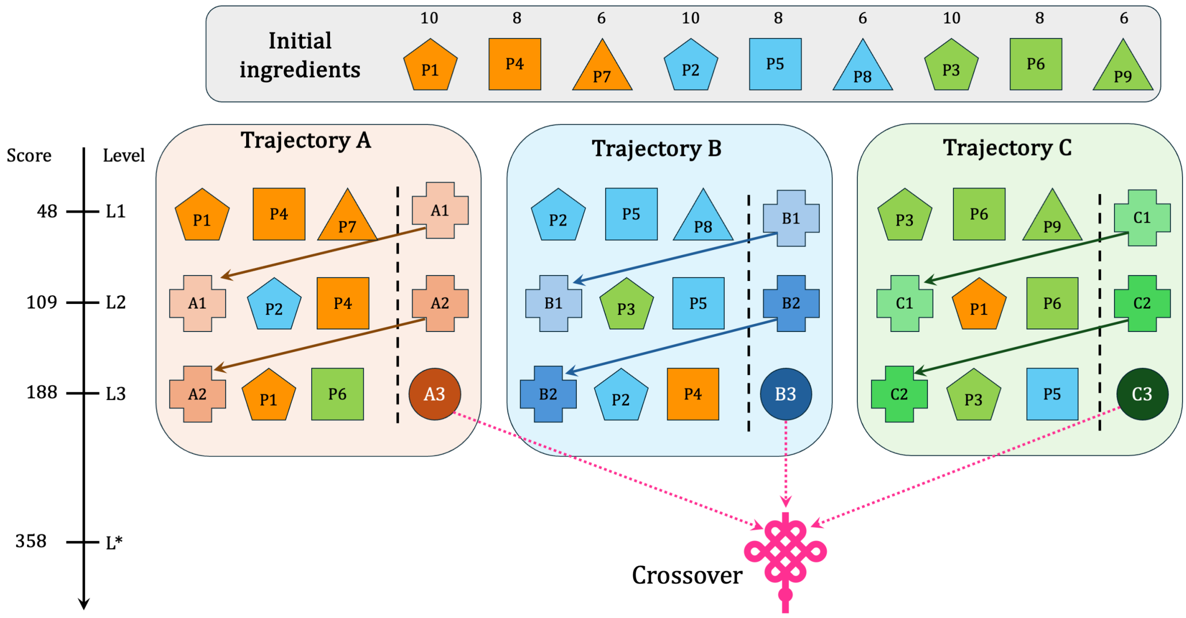

Figure 5.1: **Higher-order cultural recombination:** Various pathways for recombination starting from an initial set of ingredients $\{P1, P2, \cdots, P9\}$ to get the $L3$ (level 3) innovations – $A3$, $B3$, $C3$ by stepwise recombinations. In order to get the higher-order crossover innovation $L*$, one needs all three highest recombinations from the three trajectories.

5.2 Modeling higher-order cultural recombination

Structural organization of the system

A population of N individuals is modeled as a hypergraph $\mathcal{H}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ represents the set of vertices such that $|\mathcal{V}| = N$, and $\mathcal{E}$ denotes the set of hyperedges. Unlike traditional networks, hypergraphs generalize interactions by allowing hyperedges to connect not just pairs but any arbitrary number of individuals. For simplicity, we restrict our analysis to hypergraphs composed of two types of hyperedges: pairwise interactions ($\mathcal{E}^{/}$) and three-way interactions ($\mathcal{E}^{\Delta}$).

In this work, we use the hunter–gatherer society as a prototypical example to illustrate the spread of innovation, but in theory this model could be applied in diverse settings. To reflect the camp-based structure typical of hunter-gatherer societies, we partition the population into M camps, each containing $\frac{N}{M}$ individuals. The fraction of higher-order interactions, δ, is defined as [1]:

$$\delta = \frac{3 \cdot |\mathcal{E}^{\Delta}|}{3 \cdot |\mathcal{E}^{\Delta}| + 2 \cdot |\mathcal{E}^{/}|}, \tag{5.1}$$

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where $|\mathcal{A}|$ denotes the cardinality of the set $\mathcal{A}$. To introduce modularity akin to real-world camp structures, we impose a constraint such that a fraction α of interactions occurs within camps, while the remaining $1-\alpha$ occur between camps. When $\alpha = 1$, camps are entirely isolated, whereas $\alpha = 0.5$ corresponds to a non-modular, random interaction pattern. In our simulations, we select $\alpha = 0.9$ to emulate the empirically observed partial connectivity in social systems [337].

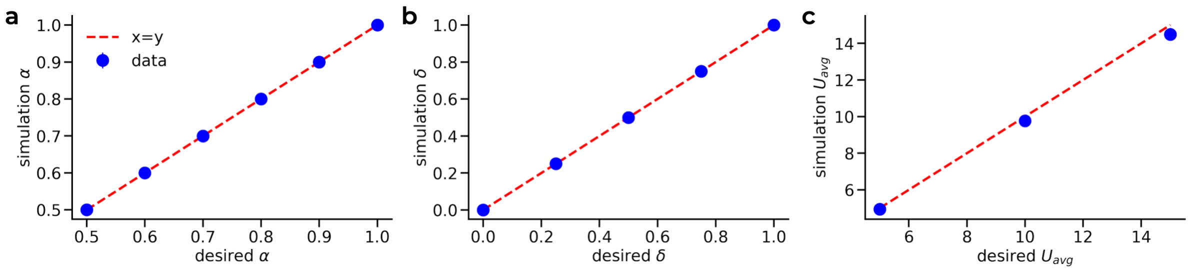

Figure 5.2: **Validation of hypergraph generation algorithm:** Match between the expected and simulated structural properties of the hypergraph generated by the algorithm described in the main text. We observe that the match between the desired and calculated quantities is excellent for all three structural parameters – **(a)** α **(b)** δ, and **(c)** U_{avg}.

Traditionally, hypergraph connectivity is quantified by the hyperdegree k. However, the relationship between δ and k is non-trivial due to the weighted contributions of different interaction orders. Specifically, for large δ, individuals tend to have more social connections for the same k and α, which could confound our analysis of cultural diffusion dynamics. To address this, we introduce an alternative measure, U, representing the number of *unique* connections per individual. The distinction between these measures is formalized as:

$$\begin{aligned} k &= \frac{3\cdot|\mathcal{E}^{\Delta}|+2\cdot|\mathcal{E}^{/}|}{N} \\ U &= \frac{6\cdot|\mathcal{E}^{\Delta}|+2\cdot|\mathcal{E}^{/}|}{N}, \end{aligned} \tag{5.2}$$

where the factor of 6 accounts for the fact that each three-agent interaction contributes 6 unique connections (2 per individual). This approximation holds for sparse hypergraphs, where overlaps between interactions are negligible – a condition typically satisfied in social networks.

The hypergraph generation algorithm proceeds as follows:

1. Distribute N nodes (agents) into M camps.
2. Solve for $|\mathcal{E}^{/}|$ and $|\mathcal{E}^{\Delta}|$ using U and δ:
   - $|\mathcal{E}^{/}_{expected}| = \dfrac{N\cdot U\cdot(1-\delta)}{2\cdot(1+\delta)}$

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- $|\mathcal{E}^{\Delta}_{expected}| = \dfrac{N \cdot U \cdot \delta}{3 \cdot (1+\delta)}$

3. Randomly create hyperedges, selecting individuals within the same camp (probability α) or across camps (probability $1-\alpha$) until the expected number of hyperedges is attained.

As illustrated in Fig. (5.2), our algorithm reliably constructs hypergraphs with the desired properties. Unless otherwise specified, we adopt the parameters $N = 1000$, $M = 25$, $U = 10$, and $\alpha = 0.9$ throughout this study.

Dynamical evolution of the system

All individuals are initially endowed with nine preliminary ingredients $\{P1, P2, \ldots, P9\}$, each possessing specific fitness values as illustrated in Fig. (5.1). The interaction dynamics among agents are governed by the hypergraph structure described previously. The system evolves through the following micro-level process:

1. At each micro timestep, a focal agent i is selected uniformly at random from the population.

2. One hyperedge containing the focal agent is chosen randomly from all hyperedges incident to i.

3.
   - For a 2-hyperedge ($\mathcal{E}^{/}$), the interaction occurs between agents i and j.
   - For a 3-hyperedge ($\mathcal{E}^{\Delta}$), the interaction involves agents i, j, and k.

4.
   - In 2-agent interactions:
     - With probability 0.5, i selects 2 ingredients and j selects 1 ingredient
     - With probability 0.5, i selects 1 ingredient and j selects 2 ingredients
   - In 3-agent interactions, each agent (i, j, and k) selects exactly 1 ingredient.
   - All the items are selected with probability proportional to the fitness values of the ingredients.

5. If the selected ingredients form a valid new triad (as defined in Fig. (5.1)), all participating agents add this triad to their repertoire.

6. Each agent shares newly discovered triads with all camp-mates through existing connections.

This sequence constitutes one micro timestep. A macro timestep is completed when every agent has had one opportunity to serve as the focal agent. The simulation continues until the target crossover triad emerges in the population. To ensure statistical robustness, we perform 500 independent simulation runs and report ensemble averages.

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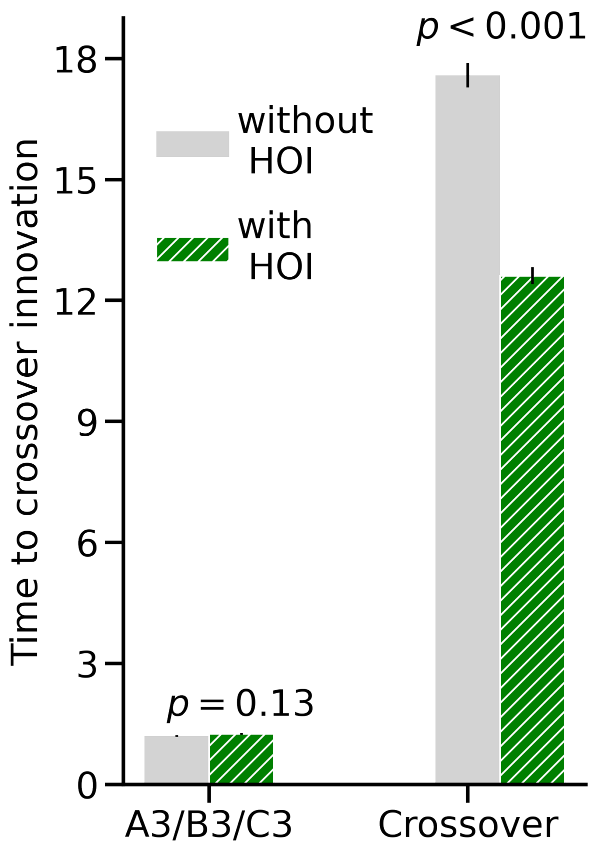

Figure 5.3: **Acceleration of innovation with higher-order interactions:** Bar plots showing the time per agent to reach the level 3 and crossover innovations with ($\delta = 0.5$) and without ($\delta = 0$) higher-order interactions. Crossover innovations are greatly accelerated in the presence of group interactions. The text above the bars shows the p-values obtained from Welch's t-test.

Cultural recombination paths

The cultural recombination model, first introduced by Derex and Boyd in their foundational work [330], provides a powerful framework for analyzing how connectivity patterns influence the diffusion and evolution of cultural innovations. Their original formulation featured two distinct cultural lineages where agents, initially endowed with basic elements, could combine these elements to produce increasingly valuable cultural artifacts. The model's key innovation lay in its requirement for a crossover event between the highest-value artifacts from both lineages, thereby capturing the critical role of population connectivity in cultural synthesis.

Our work extends this paradigm in two ways that reflect realistic features of cultural transmission:

- **Multi-lineage Dynamics**: We generalize the model to three independent cultural lineages ($A3, B3, C3$), enabling more complex evolutionary pathways that better mirror real-world cultural systems where innovations often emerge from the intersection of multiple traditions.

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- **Higher-Order Interactions**: The three-lineage structure naturally incorporates higher-order dependencies in cultural transmission, where breakthrough innovations frequently require the simultaneous recombination of elements from multiple distinct knowledge domains.

5.3 Acceleration of cultural recombination

We investigate how higher-order interactions (HOI) affect innovation times in the population. Figure (5.3) shows the time (in macro timesteps) for successful recombinations to emerge, comparing scenarios without HOI ($\delta = 0$) and with HOI ($\delta = 0.5$). For reaching the highest recombination in individual lineages ($A3$, $B3$, or $C3$), the average time across 500 runs is 1.20 for $\delta = 0$ versus 1.25 for $\delta = 0.5$, with no significant difference (Welch's t-test: t=-1.52, p-value=0.13). This suggests HOI provide no advantage for lineage-specific innovations. However, HOI majorly accelerate crossover recombinations, reducing the average time from 17.6 to 12.6 timesteps (a 30% improvement), with a statistical significance (Welch's t-test: t=13.26, p-value$< 10^{-3}$). Thus, while HOI don't speed up incremental lineage progress, they expedite complex breakthrough innovations.

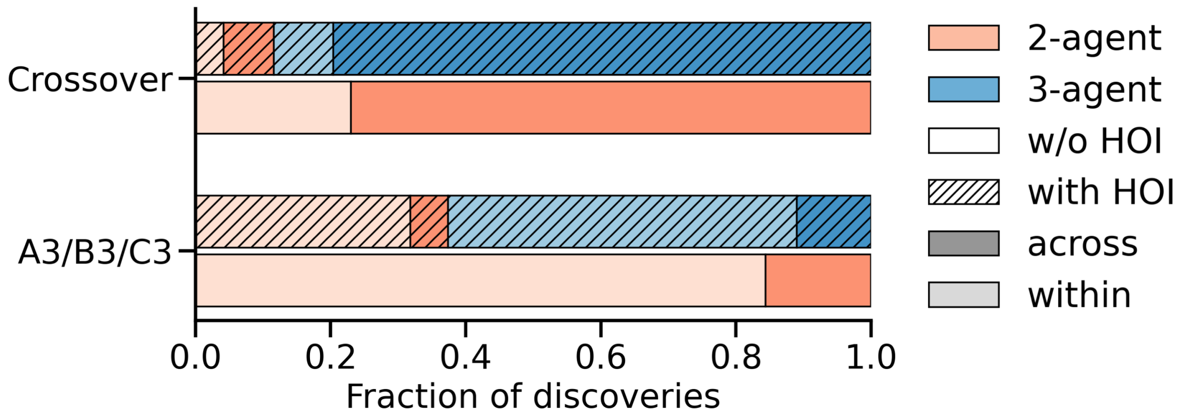

Figure 5.4: **Pathways to innovation:** Fraction of successful recombinations segregated by every possible interaction types – (i) 2-agent within camp: light orange; (ii) 3-agent within camp: light blue; (iii) 2-agent between camps: dark orange; (iv) 3-agent between camps: dark blue. Striped bars represent $\delta = 0.5$, solid bars $\delta = 0$. 3-agent across camps interactions in the presence of group interactions contribute the most for complex crossovers.

To understand how HOI accelerate innovations, we analyze the interaction pathways leading to successful recombinations. For each innovation (e.g., $A3$ from $A2$, $P1$, and $P6$), we examine whether it emerged from 2-agent or 3-agent interactions, and whether participants were from the same or different camps. Figure (5.4) reveals three key patterns. First, lineage recombinations ($A3/B3/C3$) occur primarily within camps (lighter colors), while crossovers emerge mainly through between-camp interactions (darker colors), regardless of HOI presence. Sec-

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ond, HOI increase crossover innovations, with 3-agent interactions (blue) accounting for over 50% of successes for both lineage and crossover recombinations. Third, 3-agent between-camp interactions (dark blue) drive nearly 80% of crossover events, particularly notable given our $\alpha = 0.9$ parameter means most interactions (90%) occur within camps, highlighting the disproportionate importance of rare between-camp HOI for breakthrough innovations.

5.4 Trade-off between speed and spread

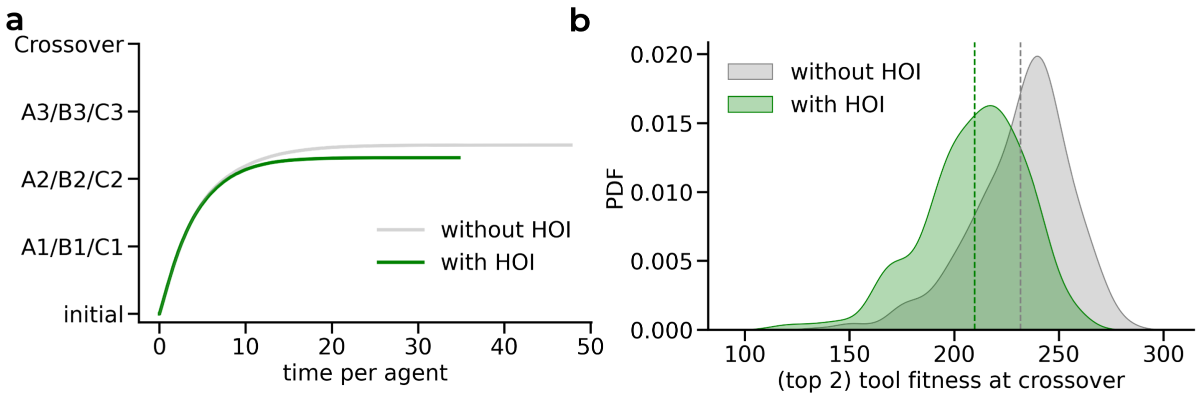

Figure 5.5: **Population state at the time of crossover: (a)** Average level of tool-set in the population as a function of time until the crossover recombination is achieved. Even though higher-order interactions accelerate the complex crossover, most of the agents are equipped with lower fitness tools. **(b)** This is evidenced by the rightward shifted distribution of top 2 tools in the repository of the agents at the time of crossover without higher-order interactions.

Building on our finding that higher-order interactions (HOI) accelerate crossover recombinations, we now examine the population state when these innovations first emerge. Figure (5.5) (a) presents the temporal evolution of system observables, revealing key dynamical patterns. Panel a shows the population's average innovation level as a function of time, averaged across runs until the maximum recombination time. Both interaction structures (with and without HOI) exhibit similar fitness trajectories initially. However, the earlier crossover occurrence in HOI systems prevents subsequent diffusion of other high-fitness ingredients, ultimately lowering overall innovation level compared to systems without HOI. The stationary state analysis in Figure (5.5) (b) further elucidates these findings. Panel b demonstrates that agents in non-HOI systems possess higher average fitness across their entire repertoire when crossovers first occur as seen by examining the top two highest-value ingredients (Figure (5.5) (b)). These results point at a potential tradeoff: while HOI accelerate breakthrough innovations, they may simultaneously limit the spread of other high-fitness lineage tools. Consequently, populations without HOI achieve crossovers with greater overall cultural complexity, as measured by the prevalence of high-fitness tools.

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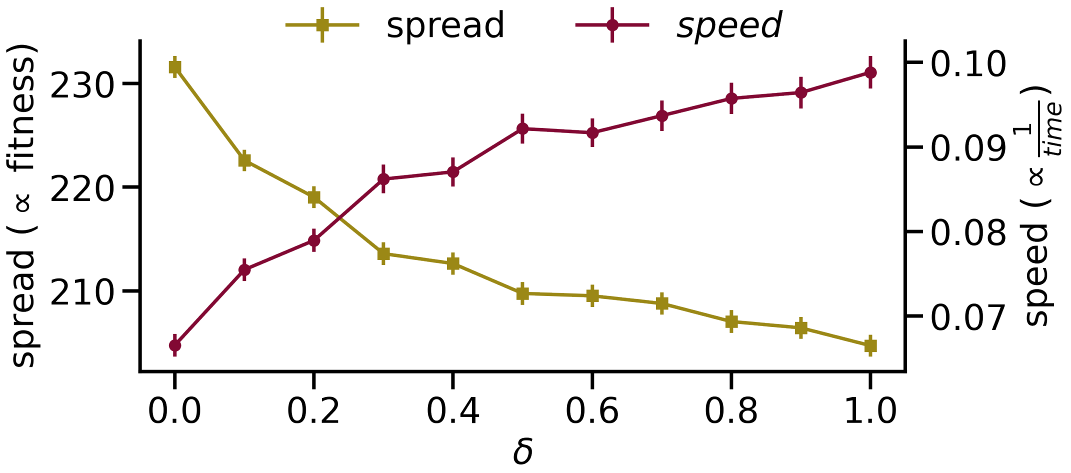

Figure 5.6: **Tradeoff between spread and speed of recombinations:** The average fitness of agents (as a proxy for spread of culture) shows a declining trend as a function of fraction of higher-order interactions δ. On the other hand, the speed of cultural innovations, taken as inversely proportional to time shows a monotonic increase with more group interactions highlighting an intrinsic tension between innovation and distribution.

Figure (5.6) quantifies this tradeoff across the full spectrum of interaction structures. The crossover speed – inversely proportional to the time – increases monotonically with δ, confirming the role of HOI in accelerating complex recombinations. Conversely, average fitness decreases with δ, demonstrating how higher-order interactions systematically reduce cultural complexity at breakthrough moments. This intrinsic tradeoff between innovation speed and cultural accumulation suggests the existence of an optimal δ value that balances these competing objectives – a promising direction for future cost-benefit analyses of cultural transmission networks.

5.5 Robustness of the model

Different items for recombination

In Fig. (5.1), we observed the recombination pathways leading to crossover innovations, such as $\{A1, P2, P4\}$ producing $A2$. While our results demonstrate that HOI accelerate cultural recombinations, a potential concern is whether this acceleration depends on our specific choice of recombination rules. We address this through a comprehensive robustness analysis.

The space of possible alternative lineages is combinatorially vast. With 9 initial ingredients, there are $\binom{9}{3} = 84$ possible ways to select ingredients for a single $L1$ recombination. Considering all lineages $(A*, B*, C*)$ and all levels $(*1, *2, *3)$, there are 9 simple recombinations that are necessary for the final crossover complex recombination. Restricting recombinations to original ingredients yields $84^9 \approx 10^{17}$ possible configurations. If we include newly formed

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ingredients in the recombination pool, the possibilities become truly intractable.

To make this analysis feasible, we implement two constraints: (1) preserving the $L1$ recombination structure (since ingredient relabeling produces equivalent systems), and (2) requiring $L1$ innovations to participate in $L2$ recombinations and $L2$ in $L3$ recombinations. This reduces the problem to filling 12 positions (4 variable ingredients per lineage across 3 lineages) with 9 items, allowing repetition. For instance, for the A lineage, the replaceable items would be $P2$ and $P4$ in making $A2$ and $P1$ and $P6$ for making $A3$. The correct combinatorial count is $\binom{n+r-1}{r} = \binom{20}{12} = 125,970$ possible configurations, where $n = 9$ items and $r = 12$ positions. While still substantial, we sample 100 configurations uniformly for computational tractability.

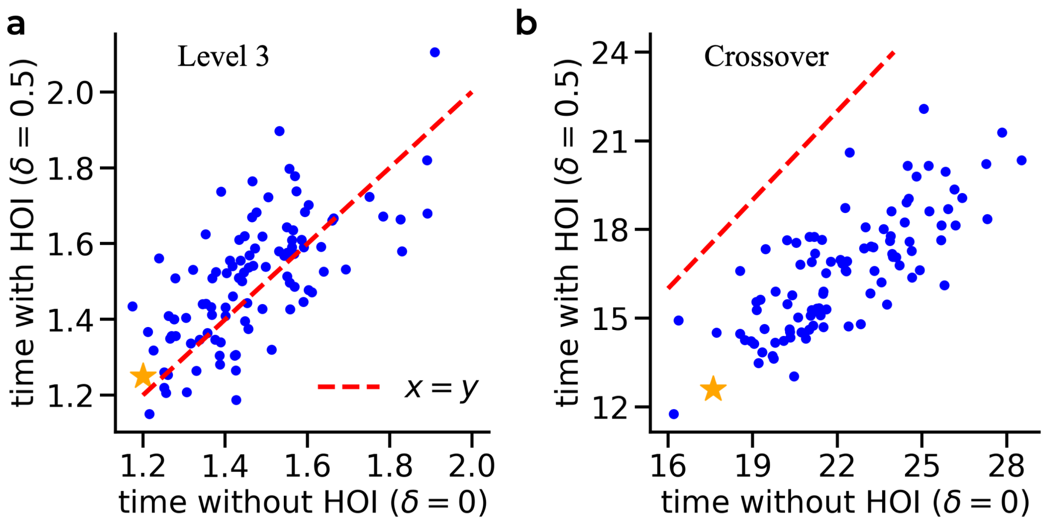

Figure 5.7: **Robustness with respect to item choices: (a)** Time to reach level 3 recombinations with and without higher-order interactions for 100 random trajectories of cultural recombinations. We see that the points are evenly spread on both sides of the $x = y$ line. **(b)** The same plot as before but for time to reach complex crossover recombinations. Notice that all the random choices point to speed-up of cultural recombination in presence of higher-order interactions. The stars denote the choice used in rest of the simulations.

Figure (5.7) reveals two key findings about innovation times across these configurations. First, for $L3$ innovations (highest lineage level), points cluster near the $x = y$ line, confirming HOI generally don't affect lineage-specific progression. Second, and crucially, all crossover (L^*) innovations occur faster with HOI - with no exceptions in our sample. This provides robust evidence that the observed acceleration of cultural recombination is a genuine effect of higher-order interactions, not an artifact of specific ingredient choices.

2-lineage model

The cultural recombination model was originally proposed by Derex and Boyd with two cultural lineages [330], later extended by Migliano *et al.* to incorporate structural constraints

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through multi-level networks [331]. Our work generalizes this framework in two key aspects: (1) introducing a third cultural lineage, and (2) incorporating higher-order interaction patterns through hypergraphs. Having demonstrated the crucial role of HOI by comparison with dyadic networks, we now investigate whether structural (hypergraph) or dynamical (third lineage) components contribute more to accelerated cultural diffusion.

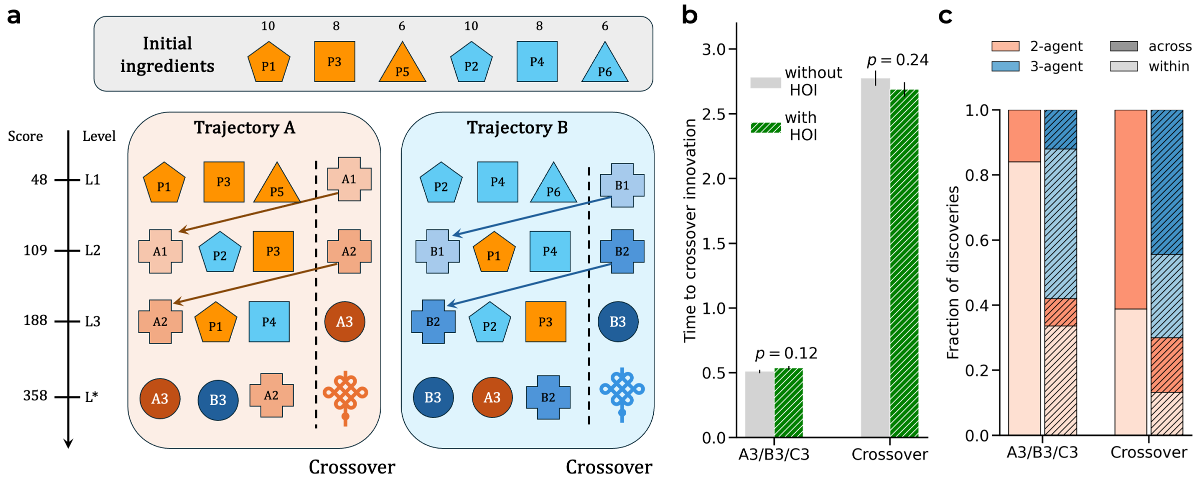

Figure 5.8: **Comparison to two-trajectory model:** **(a)** The two trajectory model with two possible crossovers as originally proposed in Ref. [330]. **(b)** The time to reach crossover recombinations does not get affected by the presence of higher-order interactions. **(c)** Furthermore the 3-agent across camp interactions contribute lesser than before for complex recombinations.

To isolate these effects, we implement the original Migliano *et al.* recombination model [331] on both pairwise graphs and hypergraphs. Figure (5.8) reveals several important findings. First, while hypergraphs show slight acceleration of crossover events (L^*), the difference is not statistically significant (p-value = 0.24). Second, the fraction of recombinations arising from 3-agent interactions remains substantially lower in this two-lineage model compared to our three-lineage system. These results demonstrate that the Migliano *et al.* framework shows no effect of HOI on cultural diffusion, highlighting how both structural (hypergraph) and dynamical (third lineage) components are essential for the accelerated innovation patterns we observed.

This analysis suggests that higher-order organization must be present in both the interaction structure and cultural dynamics to impact innovation rates. Future work should systematically explore how different mixtures of higher-order interactions in both the hypergraph structure and cultural recombination rules affect diffusion dynamics, potentially revealing optimal configurations for cultural evolution.

Diffusion of culture on real-world hypergraphs

To validate our model's applicability to real-world scenarios, we implement it on two empirical interaction datasets with distinct social structures. The first dataset captures proximity

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interactions among high school students in France, recorded at 20-second intervals over five school days [338]. This dataset exhibits natural community structure, with students primarily interacting within their 9 distinct classes, while breaks and recess periods facilitate cross-class interactions. This structure provides an ideal test case for our model's ability to capture both strong within-group ties and weaker between-group connections that characterize many real social systems. The second dataset documents face-to-face interactions within a hunter-gatherer community in the Philippines, recorded at 2-minute intervals [337]. This dataset includes additional kinship information between individuals, allowing us to examine how family relationships mediate interaction patterns. The hunter-gatherer social structure, with its characteristic blend of frequent within-camp interactions and occasional between-camp connections, offers a valuable contrast to the school environment and tests our model's generalizability across different social contexts.

Table 5.1: **Weighted hypergraph structural properties for the two datasets.**

	High school - 2013	**Hunter Gatherer - Didikeg**
N	329	49
M	9	6
$\langle k_{\text{interact}} \rangle$	52.49	4.06
$\langle k_{\text{transmit}} \rangle$	24.48	0.49
U	35.24	1.10
$\delta_{\text{unweighted}}$	0.36	0.44
δ_{weighted}	0.02	0.24
α_{PW}	0.71	0.41
α_{HO}	0.80	0.24

We construct weighted hypergraphs from both datasets by aggregating the durations of each interaction, filtering to retain only 2- and 3-hyperedges. The structural properties of the resulting hypergraphs are summarized in Table (5.1). For the cultural evolution dynamics, we proceed similarly to our previous analysis with two important modifications specific to each population. For high school students, we enforce that new recombinations are only shared with classmates, while hunter-gatherers share innovations exclusively with immediate family members. This leads to distinct transmission degrees ($\langle k_{\text{transmit}} \rangle$) that differ from interaction degrees ($\langle k_{\text{interact}} \rangle$). The selection of interaction follows a weighted scheme where choices are proportional to the aggregated duration of each contact. The table presents both weighted (δ_{weighted}) and unweighted ($\delta_{\text{unweighted}}$) fractions of higher-order interactions. Consistent with empirical observations of social interactions, we find $\delta_{\text{weighted}} < \delta_{\text{unweighted}}$, reflecting that three-agent

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interactions typically have shorter durations than dyadic contacts [67, 68]. This duration difference leads to lower probability weighting for higher-order interactions in the weighted case.

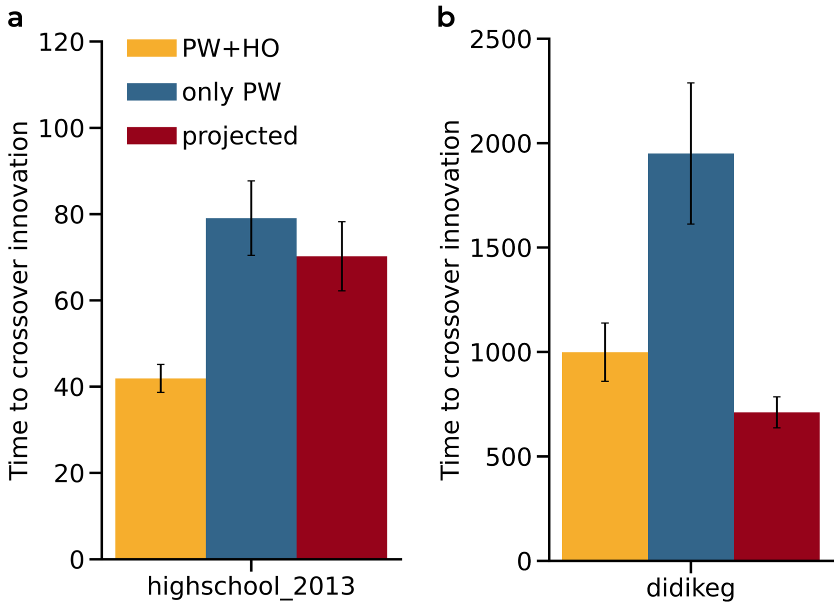

Figure 5.9: **Validation on empirical hypergraphs:** Results for time to reach crossover for **(a)** High-school dataset and **(b)** Hunter-gatherer dataset with three possible structural choices – pairwise + higher-order, only pairwise, and pairwise + projected higher-order interactions. We notice that the high school dataset with clear camp-like structure accelerates innovations. On the other hand, the fluid society of hunter-gatherers does not affect the time to reach crossover.

We implement our simulations using the empirically-derived structural patterns and present the results in Fig. (5.9), examining three distinct network configurations: (a) the complete hypergraph containing both pairwise and higher-order interactions, (b) a restricted network with only pairwise interactions, and (c) a projected pairwise network obtained by flattening all higher-order interactions into dyads.

The high school dataset, characterized by strong modular structure with class-based interaction patterns, demonstrates particularly interesting dynamics. Here we observe acceleration in crossover recombination for the hypergraph case compared to the other configurations. This acceleration correlates with the dataset's structural properties, particularly the substantial fraction of between-class interactions ($\alpha_{PW} = 0.71$ for pairwise and $\alpha_{HO} = 0.8$ for higher-order interactions). The preservation of these between-group connections in the hypergraph representation appears crucial for facilitating the rapid spread of innovative recombinations across different class communities. The hunter-gatherer dataset reveals more complex patterns. While the full hypergraph shows reduced time to crossover compared to the pairwise-only case, the projected network exhibits even faster innovation rates. This counterintuitive result stems from the dataset's exceptionally low between-family interaction fractions ($\alpha_{PW} = 0.41$ and $\alpha_{HO} = 0.24$), which effectively eliminates any meaningful modular structure. In such weakly structured populations, the projection process appears to create artificial bridging connections that facilitate

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faster diffusion.

These contrasting results highlight the importance of carefully considering both the presence of higher-order interactions and the underlying community structure when modeling cultural transmission. The high school case demonstrates how hypergraphs can better capture real-world interaction patterns that accelerate innovation in modular populations, while the hunter-gatherer results caution against simplistic network projections that may introduce artificial dynamics. Further analysis of diverse real-world datasets will be essential to fully validate our model's effectiveness across different social contexts and interaction regimes.

5.6 Discussion

Understanding the diffusion of innovations, cultural traits, and knowledge represents an open challenge in social behavior research. Unlike epidemic spreading processes where single exposures can trigger transmission, cultural adoption typically requires more complex mechanisms involving repeated exposures and cognitive integration. The cultural recombination model introduced by Derex and Boyd [330], and later extended to network structures by Migliano *et al.* [331], successfully captured key aspects of how complex innovations emerge through pairwise interactions. However, these frameworks overlooked the potentially crucial role of higher-order group dynamics in cultural transmission, despite empirical evidence suggesting their importance in real-world social systems [339, 340]. Our generalized model addresses this gap by incorporating both higher-order interactions and cumulative recombination processes. The results demonstrate that including these group interactions accelerates the emergence of complex cultural innovations compared to purely dyadic models. This acceleration, however, comes with an important tradeoff: while higher-order interactions facilitate rapid breakthrough innovations, they simultaneously reduce the population-wide diffusion of lower-value recombinations. This reveals an intrinsic tension between the speed of cultural evolution and the breadth of cultural dissemination that warrants careful consideration in future studies of social learning dynamics.

While our model demonstrates robustness across various structural and dynamical configurations, several important limitations warrant discussion. First, the current framework employs simplified interaction patterns that assume homogeneous structural properties: all camps maintain identical sizes, degree distributions follow uniform patterns [9], and interactions of different orders remain mostly non-overlapping [241]. These idealizations, while computationally convenient, may not fully capture the heterogeneous and interdependent nature of real-world social systems. Future extensions incorporating realistic structural variability – such as skewed camp sizes, heterogeneous degree distributions, and overlapping interaction orders – could yield valuable insights into how organizational complexity affects cultural transmission. Second, the model currently neglects the temporal dimension of social interactions, treating all contacts as static and concurrent. Although this static approximation reduces computational

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overhead, it overlooks the dynamic, time-dependent nature of real social engagements [68,156]. This omission is particularly relevant given our focus on hierarchical recombination processes, where the timing and sequencing of interactions likely influence both the pace and pathways of cultural transmission. Incorporating temporal interaction patterns would not only enhance realism but also provide crucial insights into how interaction rhythms modulate innovation diffusion rates. Figure (5.10) showcases a preliminary result with the *Sociopatterns* temporal hypergraph dataset. Note that the time to reach complex crossover is significantly higher for temporal hypergraphs compared to a static hypergraph. However, to isolate and understand the specific mechanisms through which temporality influences cultural diffusion, further analysis on synthetic temporal hypergraphs is required. Such controlled settings would allow us to disentangle the respective roles of burstiness, temporal reachability, and the evolution of group structure in shaping diffusion dynamics.

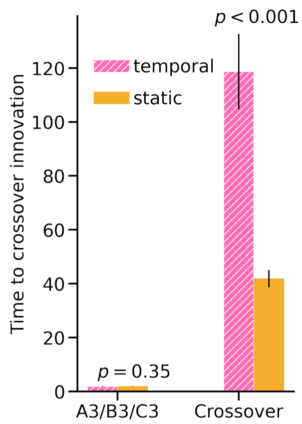

Figure 5.10: **Acceleration of innovation with static interactions:** Bar plots showing the time per agent to reach the level 3 and crossover innovations with static and temporal hypergraphs. Crossover innovations are greatly accelerated in the presence of static interactions. The text above the bars shows the p-values obtained from Welch's t-test.

In summary, our model demonstrates the potential of higher-order interactions to accelerate cultural evolution. While we have specifically examined interactions up to third order (triadic), the results suggest a broader theoretical principle: k-order interactions among k distinct cultural lineages may be generally required to observe substantial acceleration in crossover recombination. This hypothesis aligns with our finding that triadic interactions among three lineages yield faster innovation rates compared to dyadic models. Future research should prioritize experimental validation of these computational insights, particularly through controlled cultural transmission studies that systematically vary interaction orders and lineage diversity. Such empirical work would not only test the generalizability of our findings but also provide crucial ground truth for understanding how complex cultural traits emerge and propagate in real-world social systems.

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CHAPTER 6

COLLECTIVE DYNAMICS IN SPORTS – THE CASE OF CRICKET

At a Glance

Team sports provide a natural laboratory for studying collective dynamics, where individual success is deeply tied to group performance. Using detailed data from One-Day International (ODI) cricket, we examine the influence of personal trajectories and team compositions on success. We uncover "hot streaks" of high performance clusters in time, and use early performance indicators to forecast long-term careers showing that sidelined players often return stronger. Our analysis shows that balanced teams – those where contributions are widely shared – tend to perform better, and that leadership enhances individual performance selectively. These findings reveal how cooperation, role specialization, and group cohesion drive collective outcomes, illustrating core principles of social dynamics in action.

In the preceding chapters, we examined various models of strategic and group interactions that give rise to cooperation and innovation. The overarching theme of this thesis is collective dynamics, and many of the insights discussed earlier are fundamental to the survival and advancement of our species. One of the distinguishing characteristics of humans is the desire to test the limits of our performance and to push them further. Since the era of ancient Greece, humans have engaged in diverse forms of sport for both competition and entertainment.

In this chapter, we turn to cricket, a sport widely popular across several regions, to explore collective dynamics as they emerge from both individual and team-level contributions. Cricket, like many complex systems, involves numerous interdependent variables – ranging from player abilities to contextual factors such as ground conditions – which influence outcomes but are difficult to capture fully in a model. While simulating a full match or a player's career is practically infeasible, we can still uncover meaningful patterns by averaging over individual players and career trajectories. Moreover, simple null models can be applied to specific aspects of the dynamics. Although these models do not encapsulate the full complexity of the sport,

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they allow us to detect the presence or absence of relevant signals in the empirical data.

By explicitly linking these empirical analyses with modeling insights, this chapter illustrates how theoretical frameworks of collective behavior can inform, and be informed by, real-world data. This integrative perspective highlights the broader relevance of collective dynamics in understanding human performance and team-level phenomena.

6.1 Sports and human behavior

The inception of sports, notably the Olympic Games in ancient Greece, played a pivotal role in cultural and societal bonding [341, 342]. As societies evolved, sports mirrored changes in social structures, becoming more organized and diverse [343, 344]. Recent digital technology advancements and enhanced data acquisition capabilities have ushered a new era of sports analytics, providing valuable insights into athlete and team performance [345–349]. In baseball, the 'Moneyball' revolution popularized the strategic use of data analytics, profoundly altering team management and player evaluation [350–352]. Premier leagues in other sports such as basketball (NBA) and American football (NFL) have similarly embraced analytics to optimize player performance, team strategies, and in-game decisions, leading to stylistic shifts in play [353–355]. Intellectual games like chess have also advanced with the introduction of sophisticated chess engines [348, 356, 357].

One-Day International (ODI) Cricket, the world's second most-followed sport after soccer [35, 358], enjoys widespread popularity primarily in Commonwealth countries, including India, Australia, New Zealand, the United Kingdom, South Africa, the West Indies, Sri Lanka, and Pakistan. The availability of match data, driven by amateur and professional enthusiasts, has fostered various analyses. One of the major lines of research has been predicting the match outcomes of ODI matches using a variety of techniques such as machine learning [359], logistic regression using pre-match covariates [360, 361], and logistic regression using *in-game* dynamic variables [362]. Other studies have tried to uncover specific patterns based on performance such as the *hot-hand effect* [363] and ranking of the players [364] or model the dynamics of the game [365]. A few studies have tried to investigate in detail the batting [366, 367] and bowling [368] aspects of the game. With the advent of shorter and faster formats of the game such as T20, some attention has been devoted to investigating the effect of premier leagues on social media activity [369] as well as on other formats of the game [370]. Going beyond the specificities of ODI, researchers have tried to examine the role of injuries in cricket [371–373]. Despite this, there remains a substantial gap in the understanding of individual performances and their contributions to team success in One-day international cricket. In this work, we track players' careers, unveiling universal patterns of performance and identifying correlates of team-level success.

The progression of a player's skill level enhances their likelihood of surpassing previous performance peaks [363]. Conversely, as players age, their athletic prowess may diminish,

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potentially impeding their ability to exceed past achievements [371]. This contrast of skill development against physical decline poses a critical question: at which point in their careers do players deliver their best performance? We address this question in Sec.(6.3) by making use of tools and methodologies from data science and extreme value theory [374, 375] already deployed in diverse fields like science of science [376–378], arts [379–381], and sports [348, 382, 383].

Early identification of talent and skills can provide key advantages in many competitive settings, from firm growths [384], information spreading [385] to sports, where nurturing talent in young players can lead to higher returns [386–388]. In Sec. (6.4) we extend this inquiry to investigate the relationship between a player's initial performance and their overall career trajectory to capture whether it is possible to see hints of future performance based on early-career display.

Fluctuations in team composition frequently arise due to injuries and variations in player performance [389]. While injuries often occur unpredictably, a decline in performance typically manifests more gradually and may not be immediately apparent. A fitting inquiry in this context is whether it is possible to discern patterns in a player's performance preceding their expulsion from the team. We study this aspect in Sec. (6.5).

The composition of an effective team encompasses skilled players under proficient leadership. Case studies across various domains, including sports, science, and business, have illustrated scenarios, where moderately weak teams achieve success under adverse conditions, guided by strategic leadership [112, 390–392]. Here, we ask the reverse question, is a player's performance affected by the burden of leadership in Sec. (6.6). However, from a collective perspective, the strategic composition of a team is crucial for its effective functioning. The concept of utilizing specialists – individuals who perform highly specific roles – extends beyond sports into various domains of human activity, including business organizations, scientific research, software development, and even hunter-gatherer societies [393–395]. We analyze the role of three specialists – openers, all-rounders, and wicket-keepers in Sec. (6.7).

So far we have predominantly focused on individual contributions to team success in ODI cricket. We now broaden our perspective to analyse team success, setting aside specific individualistic factors that contribute to victory. Given that each cricket match culminates in a definitive outcome, our interest lies in discerning patterns of wins and losses for teams. We apply established metrics from the literature to quantify patterns in team success in Sec. (6.8). Collectively, our work presents one of the first comprehensive analyses of individual and team performance in ODI cricket.

6.2 Data collection and pre-processing

Data collection: We extracted data on 4418 One Day International (ODI) matches in men's cricket, involving 2863 unique players by web scrapping howstat.com (Ref. [396]), an open-

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access repository for cricket statistics, using *BeautifulSoup* and *urllib*, two python libraries. This dataset encompasses records of all ODI matches played from their inception in 1971 until March 2024. For each match, we extracted information including date, teams, runs scored, wickets taken, and overs played by each team, along with player names and their contributions in terms of batting and bowling. This included details like batting position, number of balls faced, number of fours and sixes hit, number of overs bowled, number of runs conceded, number of maiden overs, and number of wickets taken. Furthermore, data on the captain and wicket-keeper for each match were also collected.

Although the primary aggregation of data is at the team level for each match, the dataset is also suitable for an in-depth exploration of individual player trajectories and careers. This allows for a multifaceted analysis of performance trends, the impact of various factors on player and team success, and the evolution of the sport over more than five decades.

Classification of players: We give a brief description of our methodology to classify players. For curious readers, we direct to Ref. [397] for the exact role of each type of player. Note that a player can be simultaneously classified into multiple categories. For e.g., a batsman can be an all-rounder, a captain, an opener, and a wicket-keeper.

Batsman: We consider players as batsmen who have played at least 25 matches and batted at position 7 or above in at least 50% of their matches. We have 580 batsmen in our dataset after applying this criteria.

Bowlers: We classify players as bowlers who have played at least 25 matches and bowled in at least 50% of the matches they played. We have 551 bowlers in our dataset after applying this criterion.

All-rounders: We consider all-rounders as players who have played at least 25 matches and are classified as both batsmen and bowlers using the above definitions. We have 117 all-rounders in our dataset.

Captains: In our analysis, we consider players as captains if they have captained the team in at least 15 matches. The information about captaincy is available as metadata on the website.

Openers and Non-openers: We characterize openers as players who have batted at first or second position at least 10 times in their career, while non-openers as players who have batted at positions three to six at least 10 times in their career.

Wicket-keepers: In our analysis, we consider players as wicket-keepers based on the metadata available for each match on the website

Fielders: All players on the bowling side who are not bowling or wicket-keeping are classified as *fielder* in our analysis.

Data normalization: Team scores per match have generally increased over the decades (see Fig. (6.1)). This observed variation is potentially due to a confluence of factors, including modifications in-game regulations (fielding rules for example) [398], advancements or changes

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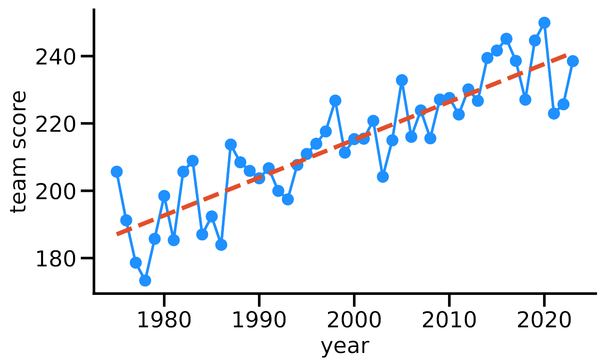

Figure 6.1: **Inflation of performance across decades.** We observe a 40% increase in the total team score from the 1980s to the 2020s. To ensure a fair comparison for players from different eras we normalize all the scores such that the average team score over the years is constant.

in the equipment used (notably the cricket bat) [399], and the reduction in the dimensions of the playing field 65-75 meters compared to 80-85 meters in stadiums earlier as spectators are interested in high scoring matches [400]. To facilitate a robust and equitable comparison of player and team performances across distinct time periods, we account for the inflation in run-scoring by implementing a normalization procedure on all batting statistics. Specifically, we multiplied runs scored by the batsmen and the runs given by a bowler in a given year by a normalization factor $nf = \frac{\langle Team\ runs\rangle_{all}}{\langle Team\ runs\rangle_{year}}$ such that the average team score is constant over the examined time period. Here, $\langle Team\ runs\rangle_{all}$ is the average team score across all years, while $\langle Team\ runs\rangle_{year}$ is the average team score in the given year. This procedure was originally introduced to correctly assess the impact of scientific papers published in different years [401]. Our approach ensures that the comparative analysis of players' performance from different eras is conducted in an unbiased manner that controls for background temporal trends.

6.3 Temporal patterns of top performances

We designate N^*, N^{**}, and N^{***} as the match numbers corresponding to a player's best, second-best, and third-best performances, respectively. For batsmen, this is marked as their highest run score. For bowlers, it corresponds to the highest number of wickets taken. In instances of identical runs or wickets, the performance involving respectively fewer balls played or fewer runs conceded is considered. Additionally, to account for variations in the career lengths of the individual players, we normalize the timing of peak performances (N^*, N^{**}, and N^{***}) with overall career lengths (N).

We are interested in two questions, (1) Does the best performance (N^*) occur at a specific time within a player's career? and (2) Are the best performances closely related in time? To answer the first question, we calculate the probability distribution function of N^* as well as N^*/N and compare it with the randomized dataset. In order to establish a baseline for the

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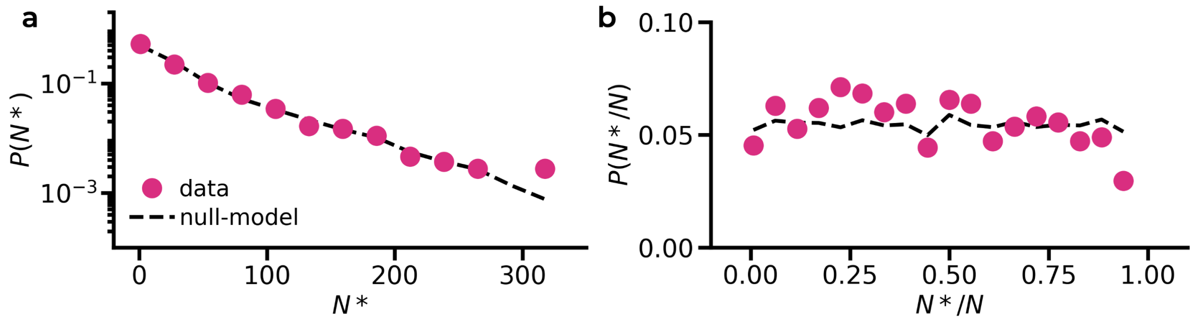

Figure 6.2: **Random impact rule.** **(a)** Distribution of the match number (N^*) of the top performance for a player's career. **(b)** Distribution of the time of the top performance in a career N^* normalized by the career length N. Real data (pink circles) is indistinguishable with a randomized null model which removes temporal correlations (black dotted line) suggesting that the best performance can occur at any time within the career.

measures, we shuffle the timestamps of individual performances for each player 100 times keeping the actual values of performance the same. In this way, we are breaking the temporal correlation between different performances. For each shuffling, we find the time stamp for the best (N^*), second best (N^{**}), and third best (N^{***}) performances of a player. We compare this randomly shuffled data to the original dataset to establish the similarities and differences. This analysis was originally introduced in Ref. [377] to test the presence of *hot-streaks* in time series data. We use the K-S test to check if the distributions obtained from the original and randomized datasets differ.

The probability distribution function $P(N^*)$ for all players is shown in Fig. (6.2) (a). We observe that $P(N^*)$ is a monotonically decreasing function, suggesting a much higher likelihood of peak performance occurring earlier rather than later in a career. However, this analysis does not account for variations in career lengths. When normalizing the timing of peak performance relative to overall career length, we observe a uniform distribution, as shown in Fig. (6.2) (b). This pattern, previously dubbed as the 'random impact rule' for scientific careers [376], suggests that the timing of peak performance is generally unpredictable and can occur at any point in a career. The *Kolmogorov-Smirnov test* (K-S test for short) is a non-parametric test used to determine if two *unpaired* sampled distributions come from the same underlying distribution [402]. The null hypothesis is that the two distributions are the same and the p-value gives the probability that the samples in question are taken from the same distribution. The K-S test comparing the data to the null-model gives p-value > 0.05 in both cases, thus giving an indication that the null hypothesis of the two distributions being statistically the same can not be rejected [403].

On the other hand, to investigate the second question, we divide the differences between the top performances ΔN, i.e. $|N^* - N^{**}|$ and $|N^* - N^{***}|$, normalized by the career length (N) into 5 bins. Within each bin, we compute the ratios of the number of players in the data ($n_{\Delta N}^{data}$)

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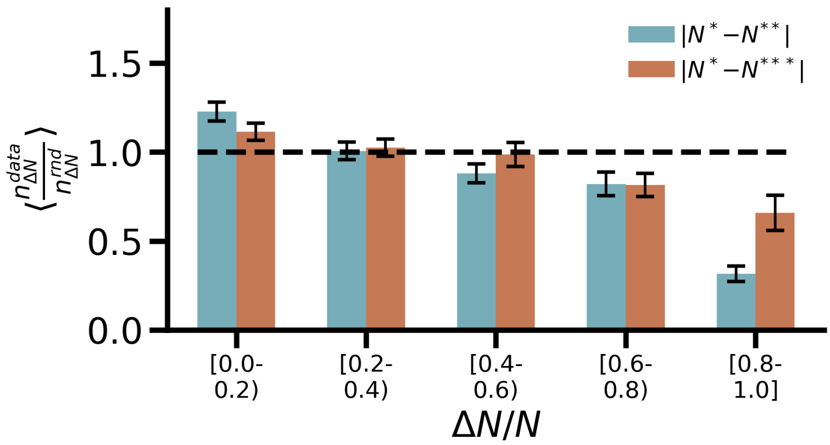

Figure 6.3: **Evidence for individual hot streaks.** Ratio of number of players for the original data ($n_{\Delta N}^{data}$) and randomized data ($n_{\Delta N}^{rnd}$) having normalized differences ($\Delta N/N$) between best performances ($|N^* - N^{**}|$ in blue and $|N^* - N^{***}|$ in brown) in each particular bin. Values higher than 1 for small $\Delta N/N$ indicate that the best performances are clustered, highlighting the presence of *hot streaks* in cricket careers.

and the number of players in the randomized data ($n_{\Delta N}^{rnd}$). We run the Wilcoxon signed rank test to determine if the values of the ratios differing from 1 are significant. In Fig. (6.3) we plot the ratios of the number of players in the bins of $\Delta N/N$ for the original and the randomized dataset. We notice that, for $\Delta N/N \in [0.0, 0.2)$ (indicating top performances closely related in time) we observe a ratio > 1, signifying that the number of players having small gaps in their best performances is higher than expected by chance on a randomized data. In other words, they exhibit *hot-streaks* – brief periods of time accentuated by best performances of comparable magnitude. In contrast, the ratio of the number of players for $\Delta N/N \in [0.6, 1.0]$ (indicating that best performances are far away in time) is always less than one. This implies that fewer players have long gaps between best performances compared to random expectations.The *Wilcoxon signed rank test* is a non-parametric test used to determine if the location of means of two *paired* distributions is the same [404]. Assuming the null hypothesis that the means are the same, it returns the probability that the null hypothesis is true. The Wilcoxon signed rank test indicates a significant deviation of ratios from 1 (p-value < 0.001) for $\Delta N/N \in [0.0, 0.2)$ and $\Delta N/N \in [0.6, 1.0]$ for both $|N^* - N^{**}|$ and $|N^* - N^{***}|$. Our results are robust to the number of bins.

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6.4 Early career observations predicts long-term performance

We are interested in determining the relationship between a player's early career trajectory and their overall career performance. In Fig. (6.4) (a) and (b) we correlate early and overall career statistics. Our analysis considers players who have participated in a minimum of 50 matches. We focus on their mean performance during the initial 25 matches. For batsmen, we look at the

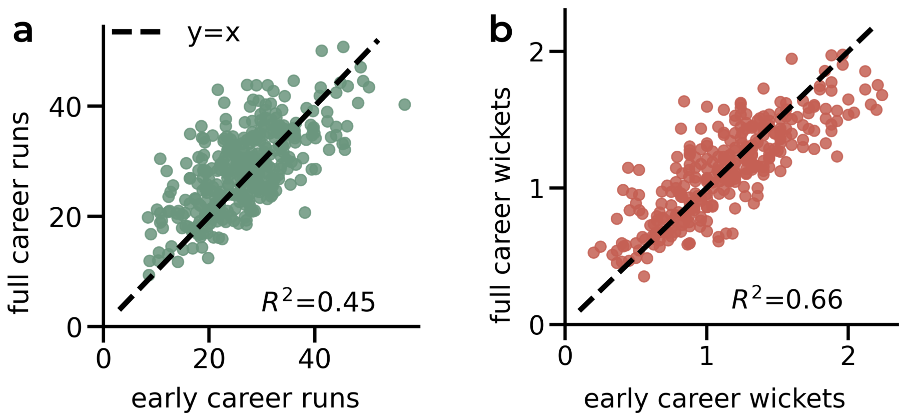

Figure 6.4: **Impact of early career performance.** Scatter plots showing the correlation between early career and full career performance for **(a)** batsmen and **(b)** bowlers respectively. The R^2 values indicate that performances in different stages of the career are correlated.

average runs scored per match in this early phase, whereas for bowlers, we look at the average number of wickets taken per match within the same period. Concurrently, we examine the full-career performance of these players.

The scatter plots in Fig. (6.4) reveal a correlation between early and full career average performance metrics. We calculate the linear regression coefficients $R^2 = 0.45$ for (a) batsmen and $R^2 = 0.66$ for (b) bowlers. This finding suggests that a player's initial performance may serve as an indicator of their subsequent career performance. However, notable differences emerge when comparing batsmen and bowlers. In general, approximately 55% of batsmen are observed to improve upon their early career averages, whereas this number drops to about 45% for bowlers.

6.5 Effect of drop and re-entry

Persistence (or lack thereof) of team composition can have a substantial effect on individual as well as team performance. Figures (6.5) (a) and (b) show the average performance trajectories of batsmen and bowlers, respectively, both prior to their removal from the team and subsequent to their rejoining. We consider all players who experienced a temporary exclusion from the team for a minimum of three matches before making a return.

We observe a consistent downtrend in performance during the pre-removal phase, with average runs and wickets demonstrating a monotonic decline, amounting to an approximate 19% reduction for both batsmen and bowlers compared to 5 matches before the exclusion. Notably, the lowest performance levels are recorded in the match just before the player's exclusion. By contrast, players tend to exhibit a strong comeback performance after reinstatement to the team. In particular, the initial performance post-return exhibits a substantial elevation, with batsmen registering an approximate 36% improvement and bowlers showing a 30% enhance-

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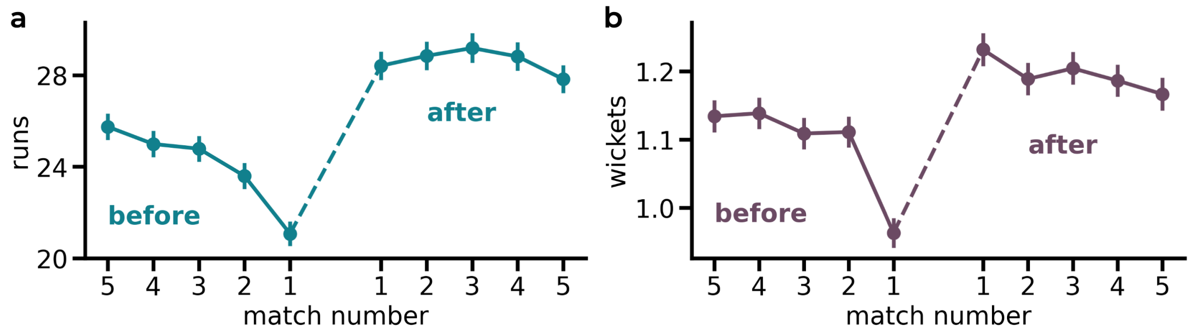

Figure 6.5: **Effect of drop and re-entry.** **(a)** Average runs scored and **(b)** average number of wickets taken by individual players (who were dropped for at least 3 matches) before they were dropped and after they were reinstated in the team. We observe a dip in the performance prior to the drop and a strong comeback in performance after re-entering the team.

ment compared to their final pre-removal performance. This enhanced performance appears to be stable in subsequent matches, consistently displaying higher performance compared to the pre-removal, suggesting that short temporal exclusion may improve performance in the longer run.

6.6 Role of leadership

Proficient leadership can enhance performance in teams of skilled individuals. We consider captains who have led their team in a minimum of 15 matches. Within our dataset, we identify 172 captains, comprising 71% batsmen and 29% bowlers, indicating a higher propensity for batsmen to be chosen as captains – more than twice as often as bowlers. Despite these differences, captain-bowlers remain in this position for longer than the batsmen on average (211 vs 232 matches). We first compare the career performance of captains with other players for both batsmen and bowlers. The *Mann-Whitney U test* is a non-parametric test of the null hypothesis that the *unpaired* distributions underlying the two samples are the same [405]. This test checks the null hypothesis if one of the distributions is stochastically larger or smaller than the other. We use the Mann-Whitney *U* test to determine if the distribution of captain performances is the same as that of the players' performance distribution. To investigate the influence of leadership, we divide the career trajectories of captains into 3 phases, before, during, and after the captainship. We are interested if the performance qualitatively changes during the three phases.

Figure (6.6) (a) presents a comparison of the career average performances between captains and non-captains (referred to as players). Quantitatively, the average runs scored by captain-batsmen (31) exceed those of players (26) by approximately 16%. Conversely, captain-bowlers secure, on average, 18% fewer wickets than non-captain-bowlers (0.96 versus 1.13). We have considered only the matches where players get a chance to perform. For both batsmen

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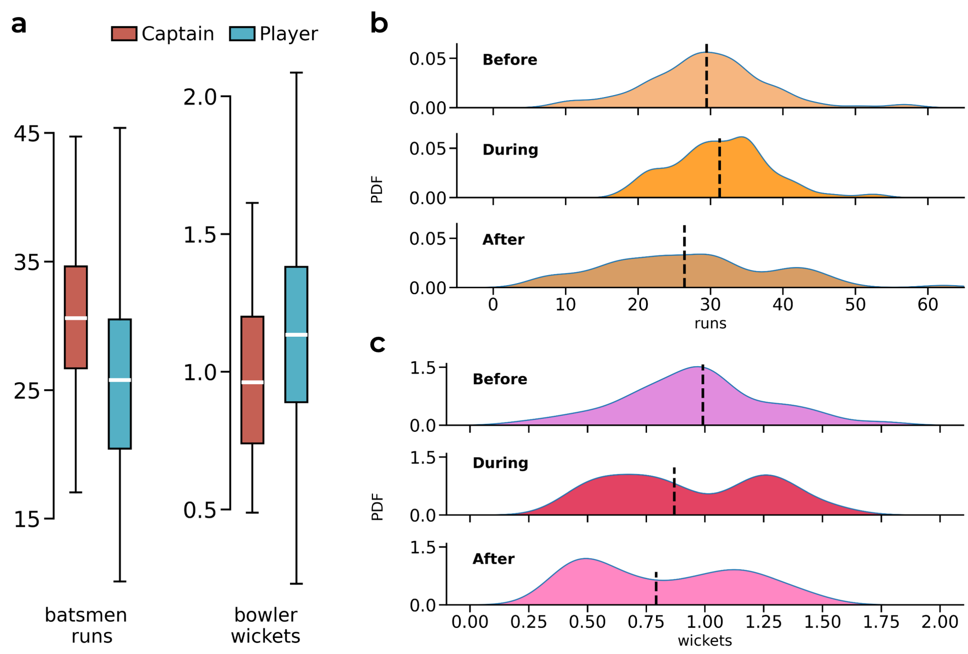

Figure 6.6: **Leadership affects player performance.** **(a)** Boxplot showing the average runs scored and average wickets taken by captain and non-captain players. The average trend for batsmen and bowlers is the opposite with captain-batsmen performing better than their non-captain counterparts. PDFs of performance for captains who are **(b)** batsman and **(c)** bowlers during various phases of their career. The dotted black lines denote the median of the distribution. The performance of captain-batsmen improves during captaincy, while it declines for captain-bowlers.

and bowler, the Mann-Whitney U test yields p-value < 0.001. This disparity suggests differing pathways to captaincy, where batsmen may need to consistently outperform peers, while bowlers' captaincy seems less dependent on individual performance.

For both batsmen (Fig. (6.6) (b)) and bowlers (Fig. (6.6) (c)). We observe that batsmen typically enhance their performance upon assuming captaincy, whereas bowlers exhibit a decline. A closer examination of Fig. (6.6) (c) reveals a bimodal distribution during the captaincy phase for bowlers, indicating a dichotomy in skill sets among captain-bowlers. Post-captaincy, the trajectories diverge for batsmen and bowlers. For batsmen, average performance markedly decreases post-captaincy, even falling below their pre-captaincy levels. The decrease is even more substantial for bowlers.

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6.7 Contribution of specialists

Specialists, providing focused contributions to group endeavors are often key for team success. Here we consider two types of specialists – openers and all-rounders. Batsmen are categorized as openers if they occupy the first or second position in the batting lineup, while positions 3 to 6 are considered non-openers. We focus on the pace at which they accumulate these

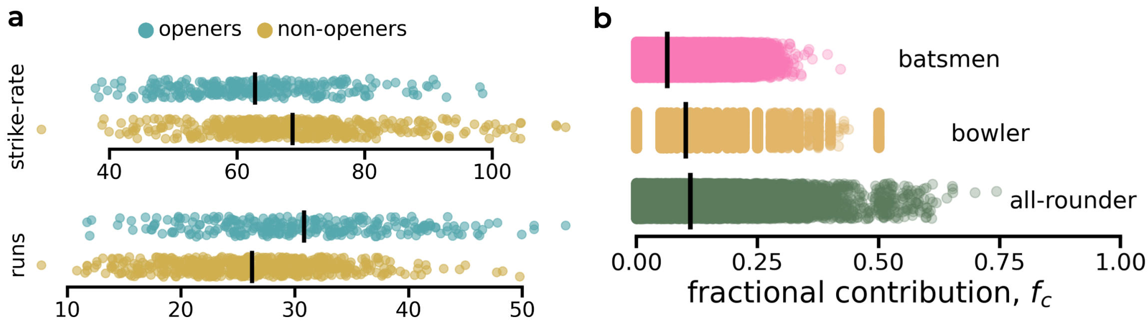

Figure 6.7: **Contribution of specialists.** **(a)** Runs and strike rates of opener and non-opener batsmen. The solid black lines denote the mean of the distributions. Openers score more runs at a slower pace, while non-openers play aggressively at the cost of lower contribution to team scores. **(b)** Fractional contribution f_c of players to the team scores for batsman, bowler, and all-rounder. All-rounders contribute more to the team.

runs. The run-scoring rate is quantified as the number of runs per 100 balls faced, commonly referred to as the *strike rate* in cricket terminology. This metric is frequently used to assess a player's defensive or aggressive playing style. We use the Mann-Whitney U test to compare the distributions of runs (and strike rate) scored by openers and non-openers.

Figure (6.7) (a) elucidates the variations in batting patterns between opening batsmen and their non-opening counterparts. Our analysis reveals that openers average a strike rate of around 63 runs per 100 balls, whereas non-openers exhibit a higher average of 69 runs per 100 balls. This suggests that non-openers score approximately 9% more rapidly than their opening counterparts. However, the average score for openers stands at approximately 31 runs, in contrast to 26 runs for non-openers, indicating a reduction of about 17% for batsmen coming later in the batting order. The Mann-Whitney U test yields statistical significance level given by p-value < 0.001.

All-rounders are players who make important contributions in both aspects of the game – batting and bowling. In contrast, specialized batsmen and bowlers are characterized by their higher contribution in only one of these areas. We quantify the fractional contribution for each type of player as follows. We introduce the fractional contribution f_c as,

$$f_c = \frac{1}{2} \times \left(\frac{\text{runs scored}}{\text{total team runs}} + \frac{\text{wickets taken}}{\text{total team wickets}} \right), \tag{6.1}$$

so that a player's maximum possible contribution (f_c) is normalized within the range of 0 to 1. It is important to note that, given this framework, specialized batsmen and bowlers have an upper limit of $f_c = 0.5$ in a best-case scenario. We use the K-S test to analyse the differences between the distributions of fractional contributions of batsmen and bowlers against all-rounders.

Figure (6.7) (b) presents the distribution of the contributions of all players across all matches.

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Among these, batsmen demonstrate the lowest mean contribution ($f_c \approx 0.06$), with bowlers exhibiting a slightly higher average contribution of $f_c \approx 0.1$. Notably, all-rounders surpass both groups with an average contribution of $f_c \approx 0.11$. Furthermore, there are instances where all-rounders singularly account for more than 50% of the team's performance, evidenced by instances of $f_c > 0.5$. The K-S test reveals that the contributions from batsmen and bowlers are significantly different (p-value < 0.001) from those of all-rounders. We also note that non-specialists contribute less than the specialists in both departments.

6.8 Patterns of team success

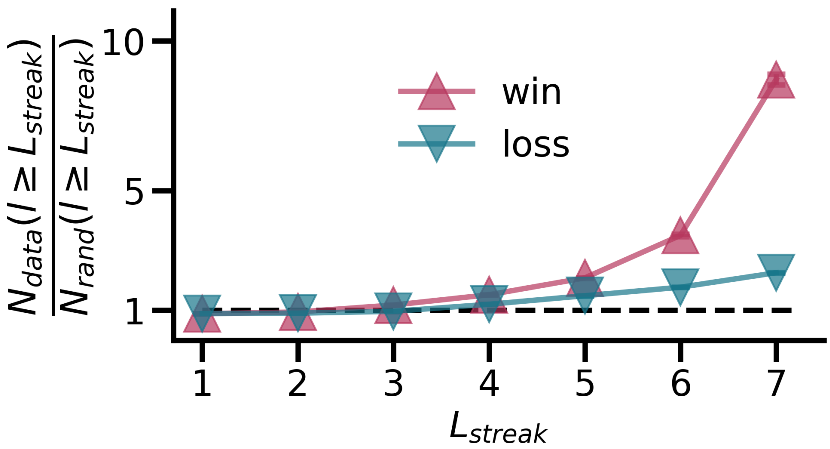

Figure 6.8: **Hot and cold streaks for team success.** Occurrences of hot streaks (wins denoted by red) or cold streaks (losses denoted by blue) of a given length L_{streak} compared to a null model. $N(l \geq L_{streak}$ denotes the number of times when the team had the same result for l or more matches, averaged over teams. We observe that teams demonstrate both hot and cold streaks. The fluctuations in the numbers calculated as the standard error are smaller than the symbols.

Similar to the bursts of exceptional performance observed in individual players (as discussed in Sec.(6.3)), we assess the propensity of teams to consecutively win (or lose) a series of matches. We quantify this tendency by defining the ratio $\frac{N_{data}}{N_{rand}}$, where N_{data} represents the frequency of a team winning (or losing) l or more consecutive matches. In contrast, N_{rand} is the analogous frequency within a randomized null model defined as follows. We consider all the teams that have played at least 200 matches. By keeping the number of wins constant, we shuffle the timestamps of the match results 10^4 times for each team. For each reshuffling, we compute the number of consecutive wins (hot) and losses (cold) denoted by L_{streak}. By taking the average across 10^4 reshuffles, we compute N_{rand} to highlight the significance of N_{data} – the number of streaks of length l or higher in the actual dataset. Thus, the ratio $\frac{N_{data}}{N_{rand}}$ indicates the likelihood of a team experiencing a streak of at least l consecutive wins or losses compared to a random outcome.

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Figure (6.8) reveals that teams are more prone to winning or losing matches in sequence than would be expected by chance. Notably, the probability of winning 7 or more consecutive matches is nine times higher than chance, while the likelihood of losing 7 or more consecutive matches is about three times the random expectation. This observation suggests that teams exhibit *hot streaks* in their winning performances. Intriguingly, we also observe *cold streaks*, where teams undergo multiple successive losses.

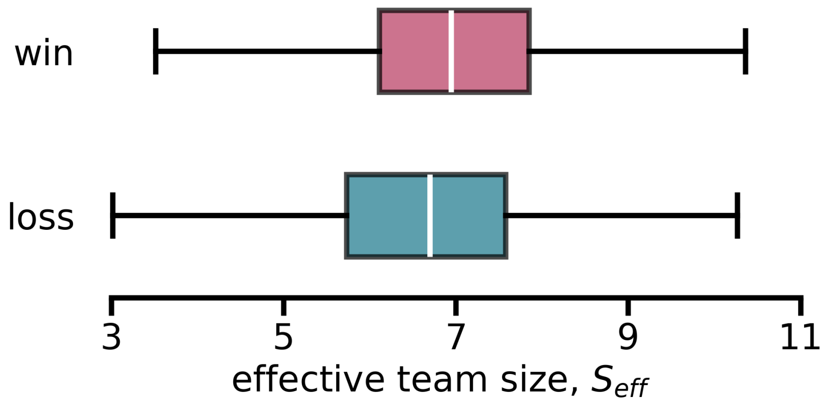

Figure 6.9: **Effective team size.** Differences between *effective team size* S_{eff} for winning and losing teams. Winning teams have a larger S_{eff} on average indicating that a larger number of players contribute significantly in winning teams than in losing teams.

To gain more insight into the team composition, we employ the concept of effective size S_{eff}. *Effective team size* is a well-established metric in the team analysis literature [406, 407] used to characterize the heterogeneous contributions of individuals to the team. *Effective team size* quantifies the essential number of players in a team, effectively measuring the redundancy in team composition. Mathematically, we denote the contribution of the player to the team score as f_c, such that $\sum_c f_c = 1$, as per Eq. (6.1). The *effective team size* S_{eff} is defined in such a way that it equals the actual team size if contributions are evenly distributed among all players and reduces to 1 if a single player is solely responsible for the entire performance. A common approach to calculate S_{eff} is to use the formula $S_{eff} = 2^H$, where $H = -\sum_c f_c \log_2 f_c$. Statistically, S_{eff} represents the entropy of the distribution of contributions f_i from team players. To compute the statistical differences between the S_{eff} of winning and losing teams, we use the Mann-Whitney U test. Additionally, we also compute the effect size $r = \frac{U}{n_1 n_2}$, where U is the test statistic and n_1 and n_2 are the sample sizes respectively.

As illustrated in Fig. (6.9) via a boxplot, the distributions of S_{eff} for winning and losing teams differ notably. The median S_{eff} for winning teams (≈ 6.9) is approximately 4% greater than that for losing teams (≈ 6.6). This indicates that successful teams require a broader array of individual contributions. The Mann-Whitney U test indicates that the differences are significant (p-value < 0.001) and the effect size $r = 0.56$ indicates a probability that the winning side is

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56% more likely to have higher S_{eff} than the losing side.

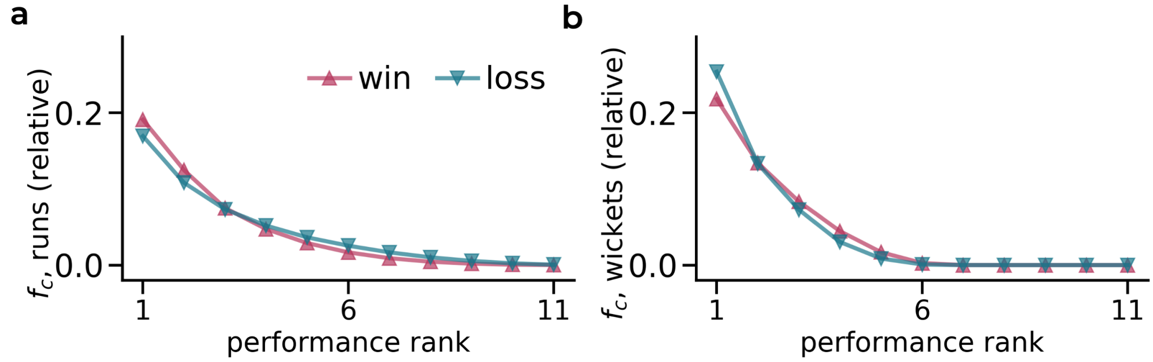

Figure 6.10: **Top performer's contribution to team.** Individual fractional contributions (f_c) to the winning and losing teams ranked by the contribution. **(a)** The top batsman from the winning team contributes higher to his team than their losing counterparts. **(b)** The top bowler from the losing team on the other hand contributes more to his team than that of the top bowler from the other team.

As a final analysis, we look at the relative contribution of the individuals to the team ranked by their performance in Fig. (6.10). We use the fractional contribution f_c for each batsman and bowler and order them from highest to lowest for both the winning as well as the losing team. We see some notable differences. Top performing bowlers from both the teams contribute more to their teams, while the batsmen tend to share the burden more homogeneously as evidenced by the long tail of the distribution. Furthermore, the top batsmen from the winning team slightly outperforms his counterpart from the losing team. However, the roles are reversed for bowlers, where the star bowler with most wickets from the losing side usually contributes more to his team than his winning side counterpart, possibly hinting at one of the reasons for the loss.

6.9 Discussion

In the last few years, the advent of detailed sports analytics has revolutionized our understanding of human behavior in sports [350–352]. It offers insights into physical performance, cognitive processes, and team dynamics, moving beyond traditional training methods [353, 354]. This shift towards a data-driven approach in sports reflects a broader societal trend in optimizing human potential, combining historical practices with modern scientific methodologies.

In this study, we quantified the individual and team performance in ODI cricket, examining various aspects of the game. Due to the absence of quantifiable performance metrics such as the Elo rating system in chess, we validated the use of runs scored and wickets taken as indicators of batting and bowling performance, respectively. Our analysis revealed a significant increase in average runs scored over time. To negate this inflation in run-scoring and facilitate fair comparisons, we implemented a normalization of all performances involving runs.

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This rescaled data corroborated the "random impact rule" observed in other domains, indicating that a player's peak performance can occur at any point during their career [376, 377]. However, this best performance often co-occurs with other comparable exemplary performances, pointing to the existence of *hot streaks* in individual cricket players. Across a range of domains such as science [377, 378], arts [380, 381], and sports [348, 363], individual careers display *hot streaks*, where exceptional performances tend to occur in bursts which are clustered in time. Our analysis showed that, while the best performance of a cricket player's career may manifest at any point within their career, peak performances tend to occur in rapid sequence over a short period of time.

Our exploration into the predictors of individual performance unveiled a strong correlation between early career achievements and overall career trajectory. However noteworthy differences were observed between batsmen and bowlers with batsmen improving their performance from early careers more than the bowlers. Such disparities may be attributed to the distinct elements inherent in the roles of batsmen and bowlers. Batsmen, for instance, may leverage accumulated experience and refined skills to augment run-scoring, while bowlers may depend more on innovation and strategic creativity to increase their wicket tally. This distinction potentially accounts for the observed trend of a higher proportion of batsmen surpassing their initial career performance compared to bowlers. Alternatively, our observations can also be explained if some players are simply better than the rest. Thus, their performance in various stages of their careers will correlate with their full career performance. This effect is also known as the *Q-model* in the context of scientific careers [376]. Additionally, we confirmed that players are often excluded from teams due to declining performances, yet they typically exhibit a sustained enhancement in performance following their comeback.

We investigated the relationship between player performance and captaincy. Our data suggested that batsmen who ascend to captaincy roles demonstrate higher performance levels, both overall and during their tenure as captain, in contrast to non-captain-batsmen. This pattern was not mirrored among bowlers, highlighting the differential impact of captaincy on distinct player types. These observations suggest a close correlation between individual performance and captaincy tenure for batsmen, where they often lead through exemplary performance. Conversely, bowlers may experience captaincy as a burden, reflected in their individual statistics. This could account for the observed differences in performance between captains and non-captains among batsmen and bowlers. An alternative explanation might be that captains tend to bowl at the wrong time to relieve the team pressure. However, to test this hypothesis one would need fine-grained temporal information about the progression of a match, which is not present in our data.

Focusing on the role of specialists, our findings indicated that opening batsmen typically score more runs but at a slower rate, whereas subsequent batsmen tend to adopt more aggressive strategies. Collectively, these findings illustrate a strategic balance between defensive and attacking approaches contingent on a player's batting position. Openers tend to adopt a more

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conservative approach, possibly due to the initial uncertainty of the pitch conditions, while subsequent batsmen often adopt a more aggressive strategy, building upon the foundational efforts of the openers.

All-rounders enhance the team's score by applying their skills across both facets of the game. They were observed to contribute more to team performance compared to batsmen and bowlers. While bowlers generally contribute more towards the team's collective efforts than batsmen, all-rounders emerge as pivotal players, often driving the team's success with contributions that exceed 50% of the total team scores. In the realm of fielding, wicket-keepers emerged as pivotal in effecting dismissals, thereby bolstering the impact of bowlers on team success. In summary, team success depends on the successful coordination of different types of – often specialized – contributions. Although most players are predominantly recognized for their batting or bowling abilities, fielding is an integral aspect of a team's overall success. Indeed, a commonly reiterated phrase in cricket states, 'catches win matches', emphasizing the significance of player dismissals through effective fielding. Wicket-keepers, by contributing to a substantial number of total dismissals, enhance the bowlers' efforts and, consequently, the overall team performance. These findings underscore the vital role of specialists in ODI cricket. Future research could delve into a more detailed examination of their role and their strategic impact on the dynamics of the game.

Additionally, our analysis reveals, similarly to individual performances, also teams tend to win or lose matches in clusters, thus identifying both *hot* and *cold* streaks during seasons. We observed that winning teams were characterized by more evenly distributed player performances, as indicated by higher *effective team sizes*. A lack of such collective effort often results in teams losing matches. Moreover the data reveals a redundancy of 3 to 4 players in most teams.

Our results might be subject to certain limitations. Cricket is a multi-faceted game, and while our analysis focuses on a wide range of indicators, it still does not consider all aspects of the game such as home advantage and differences among spinners and fast bowlers. Furthermore, the volume of One Day International (ODI) matches, and the resulting dataset size is not fully extensive. Future research could benefit from incorporating data from diverse cricket formats such as Test matches, T20 games, and franchise leagues like the IPL, which might yield deeper insights into varied player performance patterns. Supplementing our analysis with data from various professional levels may reveal further patterns in players' mobility and skill levels. Additionally, our dataset focuses on end-match statistics, omitting the nuanced temporal dynamics within individual games. While the final scorecard offers substantial information, some facets of performance may only become apparent with a more granular data analysis.

All in all, our work reveals intriguing patterns of individual and team performance in One Day International cricket. We believe that our methodologies developed here could be readily applied without substantial modifications to other formats of the game such as T20 and Test cricket, as well as other team sports. Particularly suitable are teams with high specialization

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such as baseball, American football, and volleyball. A comparative analysis of our results with a broader literature on sports can improve our understanding of human behavior. We hope that our analysis motivates further research into synergistic individual efforts, in sports and more broadly in team dynamics.

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CHAPTER 7

CONCLUSIONS

Social interactions are the fundamental building blocks of complex systems that govern everything from human societies and economies to ecosystems and technological networks. In this thesis, we looked at the emergence of collective dynamics in the presence of group interactions in strategic and evolutionary processes. In this final chapter, we tie all the works together highlighting the commonality as well as important differences between them. We also suggest some interesting future directions worth pursuing.

Network science has established itself as a key methodology for analyzing complex systems, offering both conceptual elegance and a robust mathematical foundation that has provided transformative insights across disciplines. However, the conventional network paradigm is fundamentally limited in its capacity to represent group-level dynamics – a critical shortcoming addressed by the formalization of higher-order interactions (HOI) as an extension of network theory. In **Chapter 2**, we systematically investigated the potential of HOI frameworks, demonstrating their capacity to modulate social processes through group interactions and enable novel modeling approaches particularly in the case of cooperation dynamics. Building on this foundation, in **Chapter 3**, we developed a higher-order evolutionary game model that revealed emergent cooperative equilibria in social dilemma scenarios previously unattainable in dyadic frameworks. Exploring the origins of such social dilemmas, we displayed the versatility of evolutionary processes by considering a game selection model in **Chapter 4**. We examined the emergence of social dilemmas when they are competing against each other for survival and found that structured populations enable more cooperative environments. In **Chapter 5** we extended the investigation of group interactions to evolutionary cultural recombination process. Employing hypergraph-based modeling frameworks, we demonstrated how higher-order connectivity structures facilitates accelerated knowledge diffusion dynamics compared to traditional pairwise networks. Our quantitative analysis revealed the critical role of non-dyadic social structures in shaping cultural transmission patterns. Going beyond the modeling processes, in **Chapter 6**, we looked at a specific case of collective dynamics – in sports by using

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cricket as a prototypical example. We uncovered various patterns of individual and team performance highlighting the delicate balance between individuals' roles in the team.

Building on the above work, we highlight four key directions for future analysis using the models and concepts developed in this thesis.

Higher-order games and social experiments

It would be worthwhile to combine different types of social dynamics beyond social dilemmas in future. In particular, integrating the mechanism of hypergraph formation with game theoretic cost-and-benefit analysis would provide fresh directions for the structural organization of real-world higher-order systems. Even though the theoretical models and simulations enrich our understanding of evolutionary processes, robust experimental verification of these theories is an equally important aspect of the field. Some major open questions in this area include whether group interactions facilitate or hinder cooperation, and how repeated group interactions differ from repeated pairwise interactions in structured populations. When designing experiments to address these questions, it is important to consider the fundamental differences in the experimental setup of higher-order games [408, 409]. First, the display of information should be carefully designed. Given the increasing amount of information with increasing size of interactions, it is necessary to not overload the participants with extra data but still provide enough information to make a decision. Second, the structural aspects of hypergraphs should be carefully chosen and integrated into the experimental settings as the previous experiments with games on networks did not show strong signatures of evolution of pro-sociality. Third, appropriate control scenarios should be constructed to establish the significance of the results (if any). All of these considerations would hopefully help one design a good experimental setup to test out strategic behavior in group settings.

Team dynamics

Many scientific and societal breakthrough can not be obtained by single individuals, but need collective efforts of larger teams [107, 112, 410]. Despite the growing interest in teams, from science to management studies most research has so far considered teams mostly as static entities, neglecting their dynamics and temporal evolution. For instance, in science of science most analysis consider the set of co-authors of a scientific article as an entirely different unit, and link their compositional properties to success regardless of their previous collaboration history. In organization theory, some studies have performed multi-period observations of team activities through surveys of team members, but this approach is clearly limited to collect fine-grained data about team activities over time [411]. New temporal data from science to open-source software developments [392], escape rooms [412] have already opened the way to study temporal individual trajectories of networked individuals involved in group interactions. Beyond this, modeling frameworks such as temporal hypergraphs applied to management and innovation

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systems, and even sports, will allow to characterize the dynamics and evolution of entire teams over time, as well as their interplay and interactions, allowing to better understand the collective nature and emergence of embeddedness [413], social capital [414] and Matthew effects [415] in social networks.

Cumulative culture evolution

Human culture is uniquely complex due to its cumulative nature, shaped by contributions from many individuals and requiring recombination of information [336]. Higher-order interaction frameworks offer new insights into cumulative cultural evolution, including how knowledge is shared and innovated within hunter-gatherer societies [331]. Hunter-gatherer groups are key to understanding cumulative culture, as human cognitive and cooperative skills evolved within the foraging niche over thousands of years. [337, 416, 417]. For example, through generations of collective problem-solving, Congo hunter-gatherers developed extensive medicinal plants knowledge, even though no single individual holds all of this information [418]. Like most Western cultural traits, hunter-gatherer culture was built collectively over generations. Higher-order network models have potential to clarify the group dynamics that drive cultural accumulation beyond dyadic exchanges. They could trace how information flows within hunting groups, storytelling events, rituals, or collaborative tool-making. These models may also identify the optimal group sizes, compositions, or interactions that enhance knowledge transfer and foster innovation. [330, 419, 420]. Temporal hypergraphs, for example, can monitor how changes in group compositions influence cultural resilience and innovation, highlighting the role of intergenerational or interpopulation transfers [322]. These processes may impact the rates of innovation and recombination, leading to cultural complexification. Future research should explore how group-level homophily or heterophily affects access to cultural information. Computational models incorporating higher-order effects could simulate how group structures influence cultural evolution. By focusing on these dynamics, researchers can develop more comprehensive theories, addressing key questions in human evolution, such as why cumulative culture emerged in the hunter-gatherer niche and remains rare in other species.

Collective dynamics in team sports

Team sports offer a compelling natural laboratory for studying cooperation and synergy in competitive environments—a dynamic that mirrors many real-world collaborative systems. The wealth of available sports data provides unprecedented opportunities to analyze collective behavior under precisely recorded competitive pressures. A particularly underexplored dimension is how team members coordinate through group-centered strategies rather than individual excellence alone. While our cricket analysis revealed important insights, comparative studies across different sports could uncover universal principles versus sport-specific dynamics, particularly in quantifying team synergy and the interplay between specialists and generalists –

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findings that could inform other domains of collective behavior. This raises a fundamental question: how essential is the "team" in team sports? Does success stem primarily from skilled individuals performing discrete roles (as often seen in cricket), or does it require continuous, interdependent coordination (characteristic of football)? Answering this could reshape our understanding of collective performance. Moreover, moving beyond aggregate match statistics to examine micro-scale temporal patterns could reveal how teams navigate adversity and how dominance shifts emerge during play. Such fine-grained analysis could uncover the hidden rhythms of team coordination and crisis response that traditional metrics miss.

Science is a collective endeavor and I hope that this thesis – which is a result of numerous fruitful collaborations – is a little step towards uncovering the depths of complex systems and the collective dynamics emerging out of them.

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